Gravity Disturbances at Altitude and at the Surface

Theresa M. Damiani*
NOAA-National Geodetic Survey, 1315 East-West Hwy, SSMC3, Silver Spring, MD 20910; *Contact: theresa.damiani@noaa.gov

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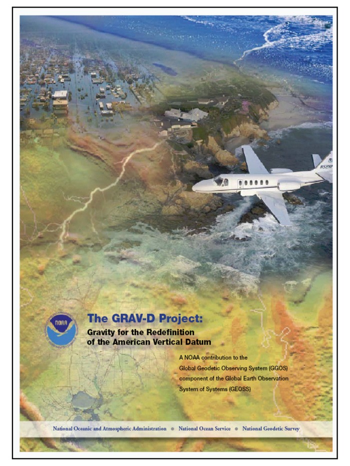
1. Background

The GRAV-D (Gravity for the Redefinition of the American Vertical Datum) Project of the U.S. National Geodetic Survey plans to collect airborne gravity data across the entire U.S. and its holdings over the next decade. The goal of the project is to create a gravimetric geoid model to use as the national vertical datum by 2022. The project plan and more details are available: http://www.ngs.noaa.gov/GRAV-D



GRAV-D (as of August 2013) has publicly released full-field gravity products from these high-altitude flights for >15% of the country. The full-field gravity (FFG) at altitude product is versatile because it allows the user to calculate any disturbance or anomaly that is appropriate for their application- based on any datum and height above the datum desired.

IMPORTANCE OF THIS STUDY:
Accurate combination (or comparisons) of airborne gravity data with marine or terrestrial data require accounting for normal gravity at the measurement points and differences in the data spectral content.

This poster compares available methods of calculating gravity disturbances and assesses them for their errors across all latitudes and from zero to 11 km altitude. In the conclusions, we recommend the most accurate method for calculating disturbances.

Also, before comparing two sets of gravity disturbances collected at different heights, an additional step must be taken to filter or continue the data sets until they have matching signal content.

2. Normal Gravity and the Free-Air Correction

Normal gravity is gravity due to an ellipsoidal representation of the Earth, at or above the surface of the ellipsoid. A common ellipsoid is WGS-84. Gravity is negative here, by the convention that positive is directed upward.

A scalar **gravity disturbance** (δg) is the difference between scalar measured full-field gravity (g) and scalar normal gravity (γ) at the exact measurement point (P) :

$$\delta g_P = g_P - \gamma_P \qquad (1)$$

g_P is provided online as the official GRAV-D airborne gravity product.

Normal gravity has often been calculated on the surface of the ellipsoid (γ_0) and then a height (or "free-air") correction applied. The correction can be represented as a series (Heiskanen and Moritz, 1967). Thus, **the first-order free-air correction** approximates the vertical gradient in normal gravity over the distance (h) between the measurement and surface of the ellipsoid, such that:

$\delta g_{FA} \approx \gamma' h_P$ (2), where $\gamma' = \delta\gamma/\delta h$ and h_P is ellipsoidal height of point (P)

Plugging into Equation (1):

$$\delta g_P \approx g_P - \gamma_0 + \delta g_{FA} \qquad (3)$$

The second order and third order forms of the free-air correction (in brackets) are such that:

$$\delta g_P \approx g_P - \gamma_0 + [\,(\gamma' h_P) + (1/2\,\gamma'' h_P^2)\,] \qquad (4)$$

$$\delta g_P \approx g_P - \gamma_0 + [\,(\gamma' h_P) + (1/2\,\gamma'' h_P^2) + (1/3\,\gamma''' h_P^3)\,] \qquad (5)$$

The accuracy of the gravity disturbance is dependent on our ability to either accurately:
1. calculate normal gravity at the measurement point; or
2. calculate normal gravity at the surface and calculate the derivates of noraml gravity with respect to height

3. Available Methods

Given:

The value of γ_0, normal gravity at the surface of the ellipsoid, is given to a precision of 1 microGal by the Somigliana-Pizetti formula. In the literature, it is written in two ways, which are equivalent: Heiskanen and Moritz (1967) Eqn 2-78; Hackney and Featherstone (2003) Eqn 5.

An ellipsoid must be chosen as the reference. WGS-84 is used here.

A. Linear Free-Air Correction

The least precise correction, but one derived from experimental data, is a linear (first-order) approximation of the vertical gravity gradient near the surface of the Earth:
γ' = 0.3086 mGal/meter

$$\delta g_{FA} \approx 0.3086\, h_P \qquad (6)$$

B. Heiskanen and Moritz (H&M) 1967 Explicit 2nd order Free-Air Correction

They give a 2nd-order free-air correction, for "small" heights above the ellipsoid (H&M Equation 2-124):

$$\delta g_{FA} \approx -2\gamma_a/a\ \ [\,1 + f + m + (-3f + 5/2\ m)\sin^2\Phi\,]\ h + 3\gamma_a h^2/a^2 \qquad (7)$$

where
γ_a is normal gravity at the equator at the surface of the ellipsoid,
a, f, and m are properties of the ellipsoid
Φ is latitude
h is ellipsoidal height of the measurement

Cons: This equation approximates γ_0 and its derivatives in terms of γ_a and parameters of the ellipsoid. Also, the second derivative of normal gravity is derived for a spheroid, not an ellipsoid.

C. NGS-Improved H&M Free-Air Correction

Subtracting γ_0 from H&M Eqn 2-132 for γ_P, we can solve for second order and third order free-air corrections without approximating γ_0:

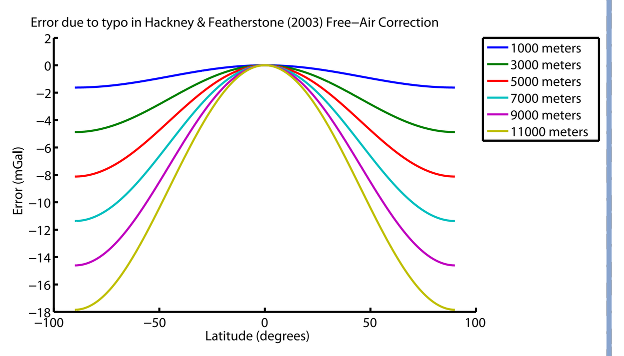
$$\delta g_{FA} \approx -2\gamma_0/a\ \ [\,1 + f + m - 2f\sin^2\Phi\,]\ h + 3\gamma_0 h^2/a^2 \qquad (8)$$

$$\delta g_{FA} \approx -2\gamma_0/a\ \ [\,1 + f + m - 2f\sin^2\Phi\,]\ h + 3\gamma_0 h^2/a^2 + 4\gamma_0 h^3/a^3 \qquad (9)$$

These equations still approximate the normal gravity derivatives, but can be solved using the precise Somigliana-Pizetti γ_0 value. The second and third derivatives of normal gravity are derived for a spheroid, not an ellipsoid. Adding a 4th term changes the correction by < 0.06 microGals (for maximim altitude of 11km).

Important Note: My Equation 8 is the same as presented in Featherstone and Dentith (1997) and Featherstone (1995). However, Hackney and Featherstone's (2003) Equation 6, which should be the same, has a typo where γ_a should be γ_0.

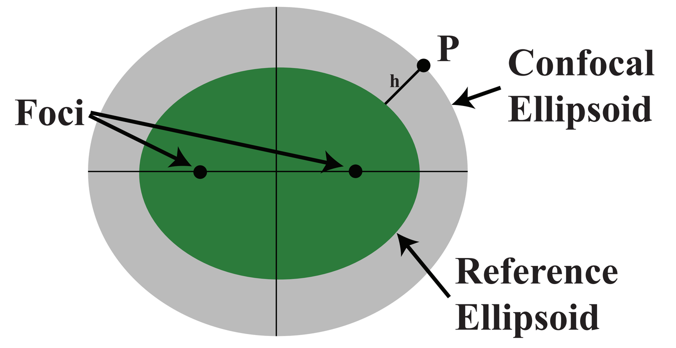
The result of this typo is that there is an error in the free-air correction that grows with increasing latitude when using the 2003 version as printed (see right).



D. Confocal Ellipsoid for γ_P

For any point (P) in space, an ellipsoid can be drawn through that point that has the same foci as the reference ellipsoid. This "confocal" ellipsoid has an associated Somigliana-Pizetti equation that will precisely calculate normal gravity on confocal ellipsoid's surface.

Inside the National Geospatial Intelligence Agency's (NGA's) Fortran routines of the hsynth_WGS84.f program (NGA, 2010; Pavlis, et al., 2012), is a subroutine called "radgrav" that calculates: 1. a confocal ellipsoid that passes through any point (P) and 2. γ_P, normal gravity at P.



An advantage of this method is that the gravity disturbance can be calculated without applying a free-air correction, using equation (1). But, for comparisons, the free-air correction from this method would be:

$$\delta g_{FA} = \gamma_0 - \gamma_P \qquad (10)$$

E. Harmonic Synthesis of Reference Ellipsoid's Zonal Coefficients

Spherical harmonics divide a sphere into a series of compartments, as in the figure below (Heiskanen and Moritz, 1967). The ellipsoid, being a simple surface that only varies with respect to latitude, can be represented well with the first 10 even zonal harmonics (2, 4, 6, ..., 20). To be zonal, these harmonics all have orders equal to zero.

WGS-84 Spherical Harmonic Coefficients

degree (n)	order (m)	normalized C_{nm}	normalized N_{nm}
2	0	-0.48416678176D-03	0
4	0	0.79030375916D-06	0
6	0	-0.16872497152D-08	0
8	0	0.34605251221D-11	0
10	0	-0.26500241424D-14	0
12	0	-0.41079006120D-16	0
14	0	0.44717732450D-18	0
16	0	-0.34636255309D-20	0
18	0	0.24114560041D-22	0
20	0	-0.16024329348D-24	0

In NGA's hsynth_WGS84.f program, a subroutine called "grs" can calculate the harmonic coefficients for an ellipsoid. Once the coefficients are obtained (shown in table above), the hsynth_WGS84.f program can be run with those coefficients as the input to obtain a very accurate value of normal gravity at any point on or above the ellipsoid, γ_P.

This method also offers the advantages of Method D, using no free-air correction to calculate the disturbance. Again, for comparions only, the free-air correction for this method would be Equation 10, as well.

F. H&M 1967 Equation for γ_P

They give a explicit 2nd order equation for γ_P (H&M Equation 2-123):

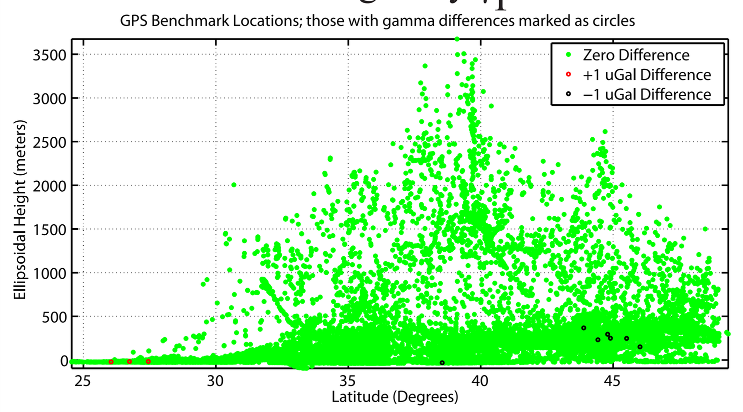
$$\gamma_P \approx \gamma_0\ [\ 1 - 2/a\ (\ 1 + f + m + -2f\sin^2\Phi\ )\ h + 3\ h^2/a^2\ ] \qquad (11)$$

This equation is most accurate when solved with the γ_0 from the Somigliana-Pizetti formula. It has the same advantages of Methods D&E, however contains approximations that make it less accurate.

4. Results

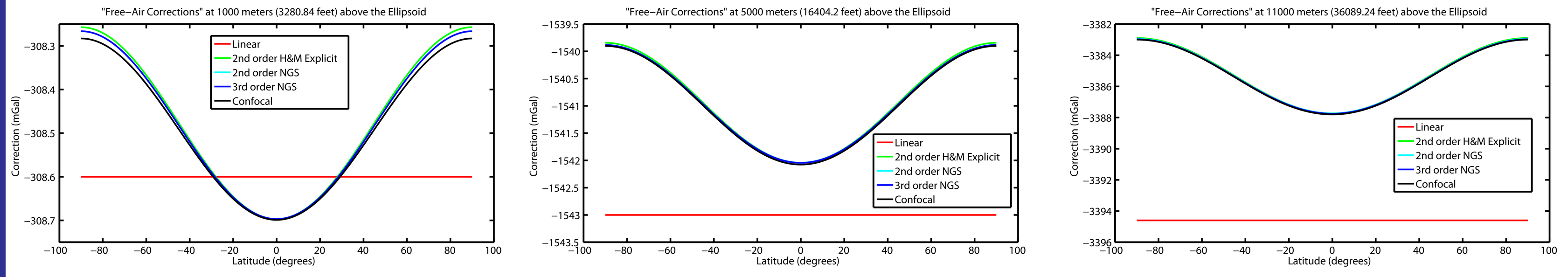
Comparison of Two Most Accurate Methods (D & E)

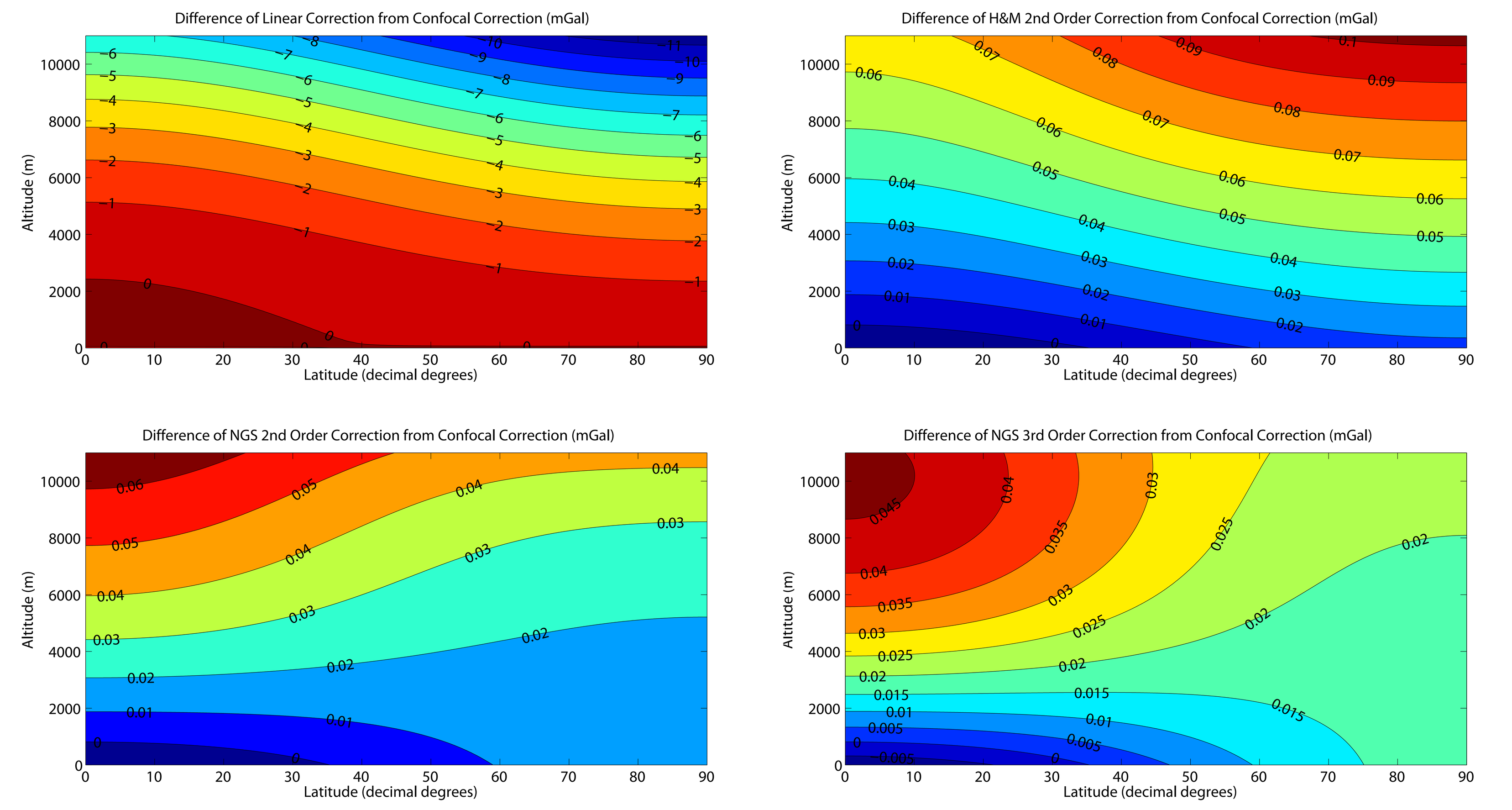
Since Methods D & E both provide a disturbance without needing a free-air correction, these are the two most accurate methods available. For comparison, a test data set with the locations of 2789 of NGS' most reliable GPS Benchmarks in the continental United States was used to calculate normal gravity γ_P with both Methods D and E, and the results compared.



The results of the comparison showed that Methods D and E returned identical results in mGal (to three decimal places) for all but 10 of the GPS Benchmarks. The remaining 10 have a +1 or -1 microGal difference due to rounding error. **Thus, the two methods both yeild results precise to 1 microGal.**

Comparison of Methods A through C with Method D





5. Conclusions

- The 2nd order NGS formula (also, Featherstone & Dentith, 1997) is more accurate at nearly all latitudes and altitudes compared to the Heiskanen and Moritz 2nd order formula. The exception is that the two 2nd order corrections are nearly equivalent at very low latitudes (< +/- 10 degrees).
- The addition of the 3rd term to the NGS formula further benefits higher latitudes (> +/- 50 deg.).
- **Recommended correction: Confocal or Zonal Coefficient Methods**
If not available, then use the NGS 3rd order formula (error <0.05 mGal)

Disturbances at different heights must still be continued to equal height before comparison.

6. References and Acknowledgements

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