# Heuristic Weighting and Data Conditioning in the National Geodetic Survey Rapid Static GPS Software

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**Abstract:** NOAA's National Geodetic Survey (NGS) has developed the Rapid Static GPS software for use as the major processing engine in the OPUS-RS utility (online positioning user service—rapid static) (http://www.ngs.noaa.gov/OPUS/OPUS-RS.html). The software was written specifically to support the computation of static positions from GPS tracking sessions as short as 15 min, while using reference station data from the NGS archive of continuously operating reference stations (CORS). When the reference stations are close (50 km) to the user's station, it is relatively easy to obtain an accurate solution. However, the CORS stations in the NGS archive are separated by 200 km or more in many areas of the country. In this situation, much care must be taken in conditioning the data sets and in selecting appropriate weights for the observations and constraints. This paper describes methods and weights that have been found to work well for most (but not quite all) data sets, and, therefore, can be used in an automated procedure such as OPUS-RS.

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## Introduction

NOAA's National Geodetic Survey (NGS) has used the Program for Adjustment of GPS Ephemerides (PAGES) software (National Geodetic Survey 1999) for the computation of both orbits and positions from GPS tracking data for many years. This program is also the major processing engine for the NGS on-line positioning user service (OPUS) utility (http://www.ngs.noaa.gov/OPUS/OPUS.html). The OPUS utility is designed to handle baselines of several hundred km in length, but requires long (at least 2 h) tracking sessions to get accurate results (Soler et al. 2006).

At a series of continuously operating reference station (CORS) user forums, many OPUS users had asked for the capability to handle shorter data sets (as short as 15 min). It was known that accurate differential positioning could be done with very short data sets over very short baselines (this is the basis for many RTK programs). The challenge was to compute accurate positions (within a few cm) from short data sets using reference stations from the NGS CORS archive (Snay and Soler 2008). This network of reference stations provides baseline lengths of 100–200 km in many areas, but in areas where the CORS network is sparse, the baseline lengths are much longer.

Research conducted by the satellite positioning and inertial navigation (SPIN) group at the Ohio State University (Wielgosz et al. 2004; Kashani et al. 2005; Grejner-Brzezinska et al. 2005, 2007) indicated that the challenge could be met, at least for areas in which the reference station data is well behaved. The new NGS rapid static GPS software RSGPS is based on the ideas and methodology developed by the SPIN group and implemented in its multipurpose GPS (MPGPS) software. RSGPS became the major processing engine for the OPUS-RS web utility (http://www.ngs.noaa.gov/OPUS/OPUS-RS.html), which became operational in Jan. 2007.

During the development of RSGPS and OPUS-RS, it was found that the reference station GPS tracking data obtained from the CORS sites are not always as good as the data from the Ohio CORS sites that had been selected by the SPIN group for their analysis. Additional features were added to condition the data by detecting cycle slips and by filtering the error-prone range observations. Furthermore, a series of experiments was carried out to determine a weighting scheme that would work with almost every data set.

## RSGPS Software

RSGPS performs a network adjustment of GPS tracking data contained in RINEX files (Gertner 2001; Strang and Borre 1997, p. 585). Special features are:

1. RSGPS uses P-code range observations as well as phase observations on both L1 and L2 frequencies.
2. After the network adjustment is available (called the float solution for all parameters), RSGPS uses the well-known LAMBDA algorithm (de Jong and Tiberius 1996; Chang et al. 2005) to find the integer values of the ambiguities. The W-ratio described in Wang et al. (1998) is used to validate the integer ambiguities selected by the LAMBDA process.

RSGPS has two processing modes: network and rover. The network mode is intended for adjustment of the observations from a set of reference stations. After the integer values of the ambiguities are computed by LAMBDA, the float solution and its cofactor matrix are updated with the constraints that the ambiguities must take on these integer values. RSGPS then uses the geometry free equations [Shaer 1999, p. 26; Strang and Borre, 1997, Eq. (15.12); Leick 1995, Eq. (9.55)] to find the double difference ionospheric delays.

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In the network mode, four types of information are saved for possible later use in the rover mode. These are:

1. Reference station coordinates and their covariance matrix.
2. Tropospheric refraction parameters and their covariance matrix at the reference stations.
3. Integer valued double difference ambiguities on baselines between reference stations.
4. Double difference ionospheric delays and their covariance matrix at each epoch.

In the rover mode, RSGPS uses the parameters determined in the previous network mode run and forms constraints to the solution containing one or more new rover stations. The user may specify that all, some, or none of the information saved from the network mode adjustment are to be used. In the OPUS-RS application, constraints are formed from the tropospheric refraction, ambiguity, and double difference ionospheric delay information (but not from the reference station coordinates).

RSGPS can spatially interpolate the tropospheric refraction from the network solution to one or more new stations (rovers). When this program option is selected, a plane is fit to the values at the reference stations, and values at the rover stations are computed from this plane. This requires that at least three reference stations be used in the network solution. Other models for predicting tropospheric refraction are possible, but were not used in RSGPS.

RSGPS will also spatially interpolate the double difference ionospheric delay from the reference stations to one or more rover stations. As with the tropospheric delay, a plane is fit to the delays from the network solution. The double difference delay at the reference station is identically zero. In the OPUS-RS application, the predicted tropospheric and ionospheric refraction delays are always computed and used to form constraints.

The reference satellite at a particular epoch chosen by the rover solution may differ from the one chosen by the network solution. In this case, double difference ionospheric delays are computed from $I_{ij}^{kl}=I_{ij}^{km}-I_{ij}^{lm}$, where $m$=reference satellite in the network solution; and $l$=reference satellite in the rover solution. If one or both of the double difference delays $I_{ij}^{km}$ and $I_{ij}^{lm}$ are not available from the network solution, no predicted delay at the rover can be computed and no constraint is applied.

## Methodology

RSGPS uses the double difference (DD) observation equations in the form described by Wielgosz et al. (2004), Eq. (1), based on the undifferenced mathematical model given by Leick [1995, Eq. (10.1)]. Four DD observations [range on both carriers ($P_{1,ij}^{kl}$ and $P_{2,ij}^{kl}$) and phase on both carriers ($\varphi_{1,ij}^{kl}$ and $\varphi_{2,ij}^{kl}$)] must be available at each epoch. Their observation equations are written

$$\begin{aligned}
&\lambda_1\varphi_{1,ij}^{kl} && -\rho_{ij}^{kl}-T_{ij}^{kl}+I_{ij}^{kl} && -\lambda_1 N_{1,ij}^{kl} && =0\\
&\lambda_2\varphi_{2,ij}^{kl} && -\rho_{ij}^{kl}-T_{ij}^{kl}+I_{ij}^{kl}(f_1^2/f_2^2) && -\lambda_2 N_{2,ij}^{kl} && =0\\
&P_{1,ij}^{kl} && -\rho_{ij}^{kl}-T_{ij}^{kl}-I_{ij}^{kl} && && =0\\
&P_{2,ij}^{kl} && -\rho_{ij}^{kl}-T_{ij}^{kl}-I_{ij}^{kl}(f_1^2/f_2^2) && && =0
\end{aligned} \tag{1}$$

where $f_1$ and $f_2$=L1 and L2 carrier frequencies; and $\lambda_1$ and $\lambda_2$=corresponding wavelengths.

The unknown parameters that appear in these observation equations fall into four groups:

1. Corrections to a priori station coordinates (earth centered, earth fixed coordinate system) contained in the geometric range $\rho$.
2. Corrections to station specific tropospheric refraction parameters $T_i^w$, contained in the double difference tropospheric refraction $T_{ij}^{kl}$ through Eqs. (2) and (3) below.
3. Double difference ambiguities $N$ (in cycles) on L1 and L2 for each double difference combination and each continuous span of data.
4. Double difference ionospheric delays $I$ for each double difference combination at each epoch. It is assumed that the stations are spaced sufficiently far apart and the observations are separated by enough time that the double difference ionospheric delays are uncorrelated. The OPUS-RS application selects observations spaced 30 sec apart, irrespective of the observation interval found in the input rinex files.

All unknown parameters are subject to a priori constraints applied as weighted constraint equations. The user may control the numerical values of the weights.

The least-squares solution is carried out by the method of matrix partitioning for sparse matrices as described by Schwarz (1985). In the notation used there, the group of global unknowns $\dot{\mathbf{X}}$ comprises the station position, tropospheric refraction, and ambiguity groups. The double difference ionospheric delay unknowns make up the local unknowns $\ddot{\mathbf{X}}$. The observations are processed epoch by epoch, with each epoch contributing the four double difference observation equations for each baseline and satellite combination, according to Eq. (1). Since the observation equations for a single epoch involve only the double difference ionospheric delay for that epoch, the submatrix $\ddot{\mathbf{N}}$ is block diagonal, and the computing method detailed in Schwarz (1985) can be applied. In particular, at each epoch:

1. The observation equations for just that epoch are formed.
2. The corresponding partial normal equations are formed.
3. The partial normal equations are reduced by the elimination of the double difference ionospheric delay unknowns.
4. The partial reduced normal equations are added to those already accumulated, so that at the end, the total set of reduced normal equations is available.

After the observations from each epoch have been processed, RSGPS may compute a solution using all the observations processed so far. The float solution obtained by solving the normal equations accumulated so far is passed to the LAMBDA algorithm. This sequential process allows the analyst to watch the evolution of the solutions (both float and fixed integer ambiguities) and the evolution of the LAMBDA validation statistics. If all goes well, the solutions will converge to stable values and the validation statistics will indicate that the LAMBDA selection of the best set of integer ambiguities is valid.

The method of matrix partitioning used by RSGPS is algebraically equivalent to the method of generalized least squares described in the publications of the OSU SPIN group. However, it takes advantage of knowledge of the structure of the observation and normal equations, and is, thus, more appropriate for application in a production environment.

## Reference Station

All double differences are formed with respect to a single reference, or hub, station, which may be specified by the program user. In the network mode, the default is that the first named reference station is the hub. In the rover mode, the default is that the first named rover station is the hub. The selection of the hub stations affects which double differences can be formed; choosing a hub

in the middle of the network usually produces the largest number of double difference observations.

## Reference Satellites

At each epoch, the satellite that has the highest elevation angle as seen at the hub station is selected as the reference satellite. This means that the reference satellite may change several times over the course of a tracking session. The DD ionospheric unknowns at each epoch refer to the reference satellite at that epoch. DD ambiguity unknowns all refer to the reference satellite at the first epoch. If the reference satellite changes and the old reference satellite is still visible at the first epoch after the change, RSGPS will enforce continuity across the change. If the old reference satellite is not visible, RSGPS will insert a cycle slip for all ambiguities.

## Tropospheric Refraction

Tropospheric refraction is modeled according to the international earth rotation service recommendation (McCarthy and Petit 2004, Sec. 9.2). The double difference delay is

$$T_{ij}^{kl} = T_i^k - T_i^l - T_j^k + T_j^l \tag{2}$$

and the one-way delay is

$$T_i^k = m_h(e_i^k)T_i^h + m_w(e_i^k)T_i^w \tag{3}$$

Here $T_i^h$=hydrostatic delay at the zenith; and $T_i^w$=zenith wet delay at station $i$; while $m_h$ and $m_w$=hydrostatic and wet mapping functions, respectively (functions of the elevation angle $e_i^k$ from station $i$ to satellite $k$). The zenith hydrostatic delay accounts for about 90% of the total tropospheric delay, but it can be accurately computed from surface pressure and temperature (Leick 1995, p. 308; McCarthy and Petit 2004, p. 100). A priori values of both the hydrostatic and wet components can be computed from seasonal values of pressure and temperature. It is difficult to separate the corrections to both the hydrostatic and wet components, since the mapping functions are very similar at moderate elevation angles. In RSGPS, the hydrostatic zenith delay is fixed at its a priori value $T_i^{h0}$, and the wet zenith delay is modeled as an a priori value plus an unknown correction $T_i^w = T_i^{w0} + \delta T_i^w$. Any errors in the computed hydrostatic zenith delay will be largely absorbed by the wet zenith delay $\delta T_i^w$.

## Rover Constraints

In the rover mode, seven types of constraints are applied:

1. Constraints on the input coordinates of the rover station(s). These must be specified by the user, since they depend on the accuracy of the input coordinates.

   The remaining constraints are those derived from the results of the network solution:
2. Constraints on tropospheric refraction values at reference stations.
3. Constraints on tropospheric refraction values predicted for rover stations.
4. Constraints on DD ionospheric delays involving only reference stations.
5. Constraints on DD ionospheric delays predicted for baselines involving the rover station.
6. Constraints on DD ambiguities involving only reference stations. In the rover mode solution, these are constrained to the integer values determined in the network solution.
7. Constraints on DD ambiguities for baselines involving the rover station.

It may appear at first that these constraints are redundant, since they are being applied in an adjustment in which the reference station data are also being reprocessed. However, these constraints are largely statistically independent of the reference station data, because:

1. The constraints are obtained from the network solution with integer fixed ambiguities, not the float solution.
2. The data being reprocessed do not necessarily match the time span of the data used in the network solution. The network solution typically uses 1–2 h of data from the reference stations. The rover solution uses only the reference station data that matches the time span of the rover data set, typically 15 min.
3. The constraints on predicted values involve a geographic interpolation based on the best fitting plane, and the prediction error generally dominates the other error sources.

In the rover adjustment, the tropospheric refraction parameter at each reference station is constrained to its value from the network adjustment (after fixing ambiguities at integer values). These values are also used to predict the value at the rover station. The prediction is performed by reducing the values obtained at the CORS sites in the network solution to sea level, fitting a plane to those values, using the plane to interpolate to the rover horizontal position, and raising the interpolated value to the input elevation of the rover.

Similarly, the double difference ionospheric delays determined in the network mode adjustment (after fixing the ambiguities to integer values) are used to constrain the double difference ionospheric delays in the rover mode adjustment. For the delays at the rover station (which did not take part in the network mode adjustment), the delay is predicted by fitting a plane to the values from the network solution.

Last, the integer ambiguities determined in the network adjustment are used to form constraints for the rover adjustment. The rover solution may have a different hub station (say B) and reference satellite (say r) than those used in the network solution. Therefore, RSGPS computes

$$N_{1,AB}^{kr} = N_{1,AZ}^{km} - N_{1,AZ}^{rm} - N_{1,BZ}^{km} + N_{1,BZ}^{rm} \tag{4}$$

$$N_{2,AB}^{kr} = N_{2,AZ}^{km} - N_{2,AZ}^{rm} - N_{2,BZ}^{km} + N_{2,BZ}^{rm} \tag{5}$$

where $m$=reference satellite; and $Z$=hub station used in the network solution. Here, the ambiguities $N_{1,AB}^{kr}$ and $N_{2,AB}^{kr}$ are known exactly, since they are integer values.

The rover adjustment typically has one more station (the rover) than the network adjustment. Therefore, there will be some ambiguities in the rover adjustment for which constraints are not applied; all others are constrained.

## Weights

The weighting schemes and values described below are used by OPUS-RS when processing GPS dual-frequency phase and range observations.

A. Phase observations: in RSGPS, each one-way phase observation is considered to be statistically independent of every other observation. Observations on L1 are considered to be independent of those on L2. RSGPS assigns greater variance (less weight) to observations with lower elevation angles, allowing for the greater effect of unmodeled refraction and multipath on these observations. Thus, the variances of the one-way phase observations on L1 and L2 are given by

$$\mathrm{var}(\varphi_{1,i}^k) = (\sigma_{\mathrm{L1}}/\sin(e_i^k))^2 \tag{6}$$

$$\mathrm{var}(\varphi_{2,i}^k) = (\sigma_{\mathrm{L2}}/\sin(e_i^k))^2 \tag{7}$$

Here $\sigma_{\mathrm{L1}}$ and $\sigma_{\mathrm{L2}}$=configurable parameters whose default value is 0.01 cycle; and $e_i^j$=elevation angle of satellite $k$ as seen from station $i$. The weight of each observation is the inverse of its variance.

B. Range observations: the same scheme is used for the range observations

$$\mathrm{var}(P_{1,i}^k) = (\sigma_{\mathrm{P1}}/\sin(e_i^k))^2 \tag{8}$$

$$\mathrm{var}(P_{2,i}^k) = (\sigma_{\mathrm{P2}}/\sin(e_i^k))^2 \tag{9}$$

where $\sigma_{\mathrm{P1}}$ and $\sigma_{\mathrm{P2}}$=also configurable parameters with default values $\sigma_{\mathrm{P1}}$=0.2 m and $\sigma_{\mathrm{P2}}$=0.256 m. However, the weights of the range observations may be modified by the adaptive weighting scheme described below.

C. Double difference observations: the covariance matrix of the double difference observations is computed by linear error propagation [see Leick (1995), Sec. 10.2.1]. This covariance matrix is full, reflecting the correlation of the double differences. However, there is no covariance between a double difference phase observation on L1 and one on L2 (or P1 or P2). Thus, there are four covariance matrices at each epoch, and the dimension of each is the number of double differences at that epoch. These are inverted to produce four independent weight matrices.

D. Weighting of a priori values: a priori constraints are applied to all unknown parameters in both network and rover modes. The standard deviations of these constraints are all configurable. The following values are used by OPUS-RS:

1. The default standard deviation of a reference station a priori XYZ coordinate is 0.02 m in each coordinate, which is the estimated error in the coordinates of the national CORS network.
2. The a priori value of the zenith wet delay at each station is typically in the range 0.15 to 0.35 m. The default standard deviation is 0.025 m.
3. The a priori value of each double difference ionospheric delay is 0.0. A default standard deviation of 0.4 m is used. This value seems to be satisfactory even during periods of high ionospheric activity (because the large delays cancel out in the double differences). A larger value may be selected if the stations are spaced far apart.
4. The a priori value of an ambiguity is computed by comparing the first phase measurement to the distance computed from a priori values of the other parameters. Its accuracy depends on the accuracy of the other parameters through the equations

$$\sigma_{N1}(\mathrm{ref}) = 0.5 * \mathrm{sqrt}[\sigma_x^2(\mathrm{ref}) + \sigma_I^2 + \sigma_T^2 + (2.0 * \lambda_1)^2]/\lambda_1$$

$$\sigma_{N2}(\mathrm{ref}) = 0.5 * \mathrm{sqrt}[\sigma_x^2(\mathrm{ref}) + \sigma_I^2 + \sigma_T^2 + (2.0 * \lambda_2)^2]/\lambda_2 + 1.0$$

$$\sigma_{N1}(\mathrm{rov}) = 0.5 * \mathrm{sqrt}[\sigma_x^2(\mathrm{rov}) + \sigma_I^2 + \sigma_T^2 + (2.0 * \lambda_1)^2]/\lambda_1$$

$$\sigma_{N2}(\mathrm{rov}) = 0.5 * \mathrm{sqrt}[\sigma_x^2(\mathrm{rov}) + \sigma_I^2 + \sigma_T^2 + (2.0 * \lambda_2)^2]/\lambda_2 \tag{10}$$

E. Weights for rover mode constraints:

1. The coordinates of the rover station(s) are constrained according to the standard deviations specified by the user.
2. The vector of a priori values of the tropospheric refraction parameters at the reference stations and its covariance matrix is extracted from the vector of values saved from the network solution. The covariance matrix is inverted to form the weight matrix for these constraints. The a priori constraints described in the previous section are not used.
3. For predicted values of the tropospheric refraction parameter (those involving the rover station), the default standard deviation is 0.01 m.
4. The vector of ionospheric refraction delays, and its covariance matrix, is taken from the network solution. The covariance matrix is inverted to form the weight matrix for these constraints. If an a priori ionospheric delay cannot be found in the data saved from network solution, the ionospheric delay constraints described in the previous section are used.
5. For predicted values of the ionospheric refraction delays (those involving the rover station), the assigned standard deviation is the greater of (a) 50% of the double difference ionospheric delay on the nearest baseline, or (b) 0.05 m. This heuristic scheme is based on the reasoning that the standard deviation of the prediction should be greater when the ionosphere is more variable.
6. The vector of ambiguities is taken from the network solution and used to apply constraints on the ambiguities among the reference stations appearing in the rover adjustment. Being integers, these ambiguities are treated as errorless.
7. Ambiguities on baselines involving the rover station are subject to the reasonableness ambiguity constraint described in the previous section.

## Conditioning the Input Data

A. Adaptive weighting for ranges: range observations (P1 and P2) may contain both large isolated errors (in the tens of meters) and significant multipath effects. The factor $1/\sin(e_i^k)$ used in the initial weighting is intended to give less weight to observations at low elevation angles, since these are the ones most likely to contain multipath errors. However, even this measure may not be enough to protect against blunders and large multipath effects.

The ranges are examined before the main adjustment begins. At each epoch, and for each station, the ranges are corrected for effects such as nominal tropospheric refraction, station antenna offsets, antenna calibration, offset of the satellite antenna from the center of mass, and satellite clock error. The ionosphere free (IF) range combination [Leick (1995), Eq. (9.46)] is written for each station and satellite

$$P_{\mathrm{IF},i}^k = \frac{f_1^2}{f_1^2 - f_2^2} P_{1,i}^k - \frac{f_2^2}{f_1^2 - f_2^2} P_{2,i}^k = \rho_i^k - cdt_i - cdt^k + T_i^k \tag{11}$$

where $f_1$ and $f_2$=carrier frequencies on L1 and L2; $P_{1,j}^k$ and $P_{2,j}^k$=pseudorange measurements on L1 and L2; $\rho_i^k$=distance from station $i$ to satellite $k$; $dt_i$=clock correction at station $i$;

$dt^k$=satellite clock correction for satellite $k$; $T_i^k$=tropospheric refraction delay; and $c$, as usual, = speed of light.

The largest part of the tropospheric refraction in this combination can be computed from a priori values in such a way that the remaining refraction is 10 cm or less. Furthermore, we assume that the errors in the station coordinates are small (less than 1 m). This is certainly justified for reference stations taken from the CORS, and is also justified for the rover station if we start the adjustment with a good approximate position. Thus, the only unknown left is the station clock correction $dt_i$. The observations at this station (and this epoch) are used to solve for the station clock correction. The residuals to the ion-free range observations are computed, and from these, the residuals to the P1 and P2 observations are found. If the absolute value of a residual is larger than the variance already assigned to the observation, the residual is substituted for the variance. This means that the weight of this observation in the main adjustment is the inverse of its residual. This has the effect of deweighting observations with large residuals, whether caused by isolated blunders or large multipath effects.

B. Cycle slip detection: RSGPS examines both the one-way phase observations and the double difference phase observations for cycle slips.

The one-way cycle slip detector computes the time rate of change of the ion-free linear combination of the phases. If the rate of change between two epochs changes by more than one cycle at the L1 frequency, a cycle slip is detected.

The double difference cycle slip detector uses the double difference phase observations. At each epoch, both the ion-free and the geometry-free (GF) combinations are computed for each satellite (other than the reference satellite) and baseline. These are monitored from one epoch to the next. A cycle slip at epoch $t_{k-1}$ is detected if

$$|d\varphi_{ij,\mathrm{IF}}^{kl}(t_k) - d\varphi_{ij,\mathrm{IF}}^{kl}(t_{k-1})| > \sigma_{\mathrm{DDIF}} \tag{12}$$

or

$$|d\varphi_{ij,\mathrm{GF}}^{kl}(t_k) - d\varphi_{ij,\mathrm{GF}}^{kl}(t_{k-1})| > \sigma_{\mathrm{DDGF}} \tag{13}$$

where $\sigma_{\mathrm{DDIF}}$ and $\sigma_{\mathrm{DDGF}}$=configurable parameters (default values are 0.05 and 0.5 cycle, respectively). Here

$$d\varphi_{ij,\mathrm{IF}}^{kl} = d\varphi_{i,\mathrm{IF}}^{k} - d\varphi_{i,\mathrm{IF}}^{l} - d\varphi_{j,\mathrm{IF}}^{k} + d\varphi_{j,\mathrm{IF}}^{l} \tag{14}$$

$$d\varphi_{i,\mathrm{IF}}^{k} = \frac{cf_1}{f_1^2 - f_2^2} d\varphi_{1,i}^{k} - \frac{cf_2}{f_1^2 - f_2^2} d\varphi_{2,i}^{k} \tag{15}$$

and

$$d\varphi_{1,i}^{k} = \varphi_{1,i}^{k} - d_i^{k0} \tag{16}$$

where $d_i^{k0}$=distance from station $i$ to satellite $k$ computed from a priori values. A cycle slip is also detected if a cycle slip in any of the four one-way phases that go into these double differences was present. When a cycle slip occurs, a new ambiguity is introduced, increasing the number of unknown parameters.

C. Short data spans: short data spans can occur if a satellite sets soon after the beginning of the time span to be processed by the program; if a satellite rises near the end of the time span; if a cycle slip occurs near the beginning or end; or if two cycle slips occur close together in time. Each new data span creates two new unknowns (ambiguities on L1 and L2).

**Fig. 1.** Configuration of reference stations for the COLB test (day 8-31-2007)

Since it is often difficult to solve for the ambiguity unknowns on short data spans, RSGPS deletes all data and unknowns for these particular data spans. A short data span is defined as one less than one-third of the total number of epochs.

D. Trimming the data set and forming double differences: at each epoch, the available observations are examined. An observation from a station to a satellite at an epoch is used only if all four observation types L1, L2, P1, and P2 are present (except that C1 may be used if P1 is not present).

The set of satellites seen by each station is formed, and the intersection of these sets is computed. Only satellites in this intersection are used. Thus, a satellite is used at a particular epoch only if it is seen by all the stations at that epoch. The resulting set of observations contains $4(n_{\mathrm{sta}}-1)(n_{\mathrm{sat}}-1)$ observations, where $n_{\mathrm{sat}}$=number of satellites seen by all the stations, and $n_{\mathrm{sta}}$=number of stations (and $n_{\mathrm{sta}}-1$=number of independent baselines).

## Performance

The ability of the software, with the weights and heuristics described above, was tested at two rover sites, COLB (Columbus, Ohio) and GNVL (Gainesville, Fla.). Both of these are National CORS sites, so their coordinates are well known (within 2 cm horizontal and 4 cm vertical). A set of reference stations was selected for each rover site (Figs. 1 and 2).

For each rover site, a full day's data was retrieved from the CORS archive and broken into 96 data sets of 15 min each. These represented the rover data sets. For each data set, 1 h of data, centered at the midpoint of the rover data set, was retrieved from the publicly available CORS archive for each reference station.

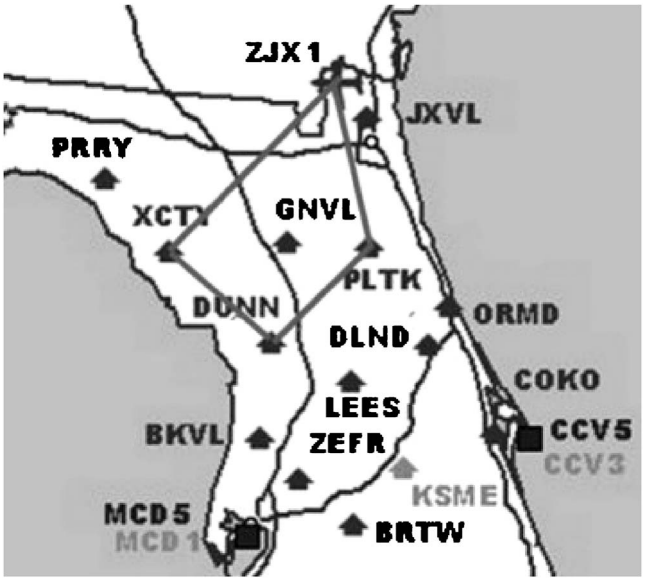


**Fig. 2.** Configuration of reference stations for the GNVL test (day 8-13-2005)

The reference station data were adjusted in the network mode as described above. The rover data were then adjusted, together with the reference station observation data sets, in the rover mode. Initially, the known coordinates (adopted CORS coordinates) of the rover station were purposefully assigned a 2.0 m bias in each coordinate. If the correction to any coordinate was greater than 3 cm, the solution was repeated, using the corrected coordinates from the previous solution as the new a priori coordinates. Two rover mode solutions were sufficient to reach convergence in almost all cases. The resulting coordinates were compared to the unbiased known coordinates, with the results shown in the second and third columns of Table 1.

These tests show that RSGPS is capable of solving for the coordinates of the unknown station with centimeter accuracy. For the COLB rover, there are no errors greater than a few centimeters, and the rms errors are quite small.

The situation is not quite as good for the GNVL rover. Here, five of the 96 solutions appear to be outliers (defined as solutions for which a horizontal coordinate is in error by more than 5 cm or the height is in error by more than 10 cm).

The reasons for these occasional outliers are not completely understood. Some possibilities being investigated include:

1. Multipath effects beyond those accounted for by the adaptive weighting scheme.
2. Unmodeled tropospheric refraction.

Many of the outlier solutions were investigated. In almost all cases, it was possible to obtain an accurate solution by deleting one or two satellites or by changing the weights on the a priori parameter values or on the constraints. Unfortunately, the software has no way of detecting when a solution is in error. While errors can be detected for those tracking stations for which we have external means of determining the coordinates, we have not found a means to detect outliers reliably for unknown rover stations.

The accuracy of a solution depends largely on whether the correct integer ambiguities have been determined. Although the integer ambiguities are treated as errorless, this is not really the case. In principle, the vector of integer ambiguities determined by the LAMBDA method has the greatest probability of being correct, but this probability is not 100%; other vectors of integer ambiguities also have some probability of being correct (Verhagen 2005).

The *W*-ratio of integer ambiguity validation was designed as a measure of the probability that the ambiguities determined by LAMBDA are correct. It was at first thought that the *W*-ratio is distributed as Student's *t*. This has since been shown to be incorrect, and the determination of the probability of the integer ambiguities being correct remains an open problem (Verhagen 2004).

In these tests, the outlying solutions were often associated with low values (less than 3.0) of the *W*-ratio. However, the association is not perfect; there were both good solutions with low *W*-ratios and outlying solutions with acceptable *W*-ratios. All that can be said at the moment is that solutions with low values of the *W*-ratio should be treated with caution, and the search for reliable measures of integer ambiguity validation should be continued.

There are a number of ways to reduce the likelihood of outliers. One is to use more reference stations (although in these experiments, the series of solutions at GNVL, using four reference stations, contained five outliers, while the series at COLB, using only three reference stations, contained none).

Another approach is to use a longer data span. The fourth column of Table 1 shows the results when 30 min rover data sets are used. Here, the number of outliers is reduced from five to one. The other statistics are also improved.

**Table 1.** Summary of Two RSGPS Tests; All Time Units Are Given in Minutes; All Tabulated Statistics Are Given in m

| Unknown (rover) station | COLB | GNVL | |
|---|---|---|---|
| CORS reference stations | PKTN, SIDN, WOOS | XCTY, ZJX1, DUNN, PLTK | |
| Date | 8/31/2007 | 8/13/2005 | |
| Average distance from rover to reference stations | 112 km | 81 km | |
| Time span of rover data | 15 | 15 | 30 |
| Time span of network data | 60 | 60 | 80 |
| Number of solutions | 96 | 96 | 48 |
| Maximum latitude error | 0.012 | 0.077 | 0.035 |
| Maximum longitude error | 0.011 | 0.096 | 0.072 |
| Maximum height error | 0.068 | 0.169 | 0.152 |
| Average latitude error | −0.005 | −0.007 | −0.006 |
| Average longitude error | −0.005 | 0.006 | 0.003 |
| Average height error | −0.023 | 0.003 | 0.008 |
| RMS latitude error | 0.004 | 0.014 | 0.009 |
| RMS longitude error | 0.002 | 0.017 | 0.014 |
| RMS height error | 0.021 | 0.051 | 0.048 |
| Number of outliers | 0 | 5 | 1 |

## Conclusions

The rapid static GPS method can be used to find the position of unknown rover stations with an accuracy of a few centimeters using as little as 15 min of tracking data and reference stations separated by 200 km. However, the rover data set must be carefully conditioned to ensure that it is free of cycle slips, short data spans, and excessive multipath effects. Furthermore, it is necessary to constrain the a priori values of all parameters. With careful selection of the weights associated with these constraints, accurate solutions can be achieved for almost all input data sets.

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