Absolute Ionosphere Slant Delays From Ambiguous Carrier Phase Data

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Topics of Discussion

• Motivation for NGS ionosphere

• Model/Equations

• Comparisons/Analysis

• Conclusions
Geodetic need for ionosphere delays

- Dominant Frequency-dependent signals in GPS:
  - Ambiguities
  - Ionosphere

- Difficult to separate quickly

- NGS decision: model the ionosphere to get ambiguities faster

- Data wasn’t an issue: CORS
Nearly every part of the ionosphere above CONUS is viewed by CORS 12+ times daily (some >100 times a day)

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Tools and Terms

• Terms:
  – Track = Sequential L1&L2 data for one CORS/SV combo without extended loss of lock
  – TECS = Total Electron Content along satellite/receiver vector

• CORS yields about 20-30k tracks every day
Smith, D.A., Absolute Ionosphere, ION NTM 2005
Primary Objective

• Model absolute TECS data and maintain high resolution details of TECS for every track over CONUS

• Focus on fast, accurate ionosphere delays; not on modeling 4-D electron distribution
Getting TECS from carrier data

• For 1 track, between any two epochs (i, j):
  – $i_j \Delta \text{TECS} = k(\text{“}40.3\text{”}, f_1, f_2) \times (i_j \Delta L_1 - i_j \Delta L_2)$

• Thus, every track has:
  – Very accurately known shape of $\partial \text{TECS}/\partial \text{time}$ (from carrier phase data)
  – One unknown TECS bias

• As per the Primary Objective:
  – Solve 1 TECS bias per track
• 5 (of \(\infty\)) possible TECS curves for a particular track
• Same shape, unknown bias
Solving for biases

- Consider: Two receiver-satellite vectors of two different tracks “sufficiently close” to each other in time & space.
  - Call this a crossover

- Assumption at a crossover:
  - $\text{TECS}(t, \text{track } a) = f[\text{TECS}(t \pm dt, \text{track } b)]$
  - “sufficiently close” must be defined
  - Find an acceptable mapping function “$f$”
Mapping Functions

• Any mapping function can be used
  – Linear or non-linear
  – But, how good is your mapping function?

• NGS currently using the “vertical column equality” assumption
  – Crossovers defined by nearness of the two vectors at their 300 km altitude points
  – “Sufficiently close” generally at 0.1° x 0.1° x 60 sec

Smith, D.A., Absolute Ionosphere, ION NTM 2005
\[ \text{TECS}_1 \times \cos(z_1') = \text{TECR}_1 = \text{TECR}_2 = \text{TECS}_2 \times \cos(z_2') \]
Using Crossovers

• By itself, one crossover has:
  – 1 condition ( \( \text{TECS}_1 = f [\text{TECS}_2] \) )
  – 2 unknowns (TECS biases for 2 tracks)
  – Thus, unsolvable as is

• Need conditions \( \geq \) unknowns

• Closed polygons is the solution
-3 Tracks
-Crossovers A, B, C occur in sequential order
-Not as rare as it looks
-Forms a "closed polygon" of tracks
-*Uniquely* solvable in *absolute* TECS space
Polygon Crossover Equations

\[
\begin{bmatrix}
A \Delta \text{TECS} \cos A_1 z' - 3A \Delta \text{TECS} \cos A_3 z' \\
B_1 \Delta \text{TECS} \cos B_1 z' - 2B \Delta \text{TECS} \cos B_2 z' \\
C_2 \Delta \text{TECS} \cos C_2 z' - 3C \Delta \text{TECS} \cos C_3 z'
\end{bmatrix}
\begin{bmatrix}
-\cos A_1 z' \\
-\cos B_1 z' \\
0
\end{bmatrix}
+ \begin{bmatrix}
0 \\
+ \cos B_2 z' \\
0
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix}

Smith, D.A., Absolute Ionosphere, ION NTM 2005
-4 Tracks (unknowns)
-5 Crossovers (conditions)
-Redundancy = Least Squares Adjustment in *absolute* TECS space
Initial Tests (NGS)

- Small “tracknets” of 10-12 tracks formed
- Proof-of-concept
- Absolute delays converted to double difference delays
- DD delays good to $0.1 \pm 0.01$ TECUs against “truth” (Ambiguity resolving software)

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Purple = Iono implied after knowing ambiguities

Yellow = Iono from this method

Match to 0.01 - 0.1 cyc
Initial Tests (OSU)

- The Ohio State University compared various Ionosphere estimates at Ohio CORS stations
- Double-difference mode
- Crossovers restricted to 40 degrees above the horizon
  - Avoids erroneous biases from low-elevation crossovers
  - Reduces number of tracks immediately solvable from tracknets (unsolved tracks need interpolation from nearby solved tracks)
$\text{TECS}_1 \times \cos(z_1') = \frac{\text{TECR}_1}{\text{TECR}_2} = \text{TECS}_2 \times \cos(z_2')$

"Large" $z'$ makes the mapping of $\text{TECS}_1$ into $\text{TECS}_2$ questionable.
"Small" $z'$ makes the mapping of TECS$_1$ into TECS$_2$ more reliable.

$\text{TECS}_1 \times \cos(z'_1) = \text{TECR}_1 = \text{TECR}_2 = \text{TECS}_2 \times \cos(z'_2)$
Report for NOAA/NGS

On:

Accuracy analysis of various NGS ionosphere estimation models

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Smith, D.A., Absolute Ionosphere, ION NTM 2005
This model

NOAA’s "Magic" model

Fig. 22 NGSa DD iono (day-time)

Fig. 23 NGSa DD iono differences from the “truth” (day-time)

Fig. 24 NGSb DD iono (day-time)

Fig. 25 NGSb DD iono differences from the “truth” (day-time)
Initial Tests (Results)

- In double-differenced mode, this method yields ~0.3 TECU agreement with independent estimates of the ionosphere

- Caveats:
  - One outlying bias can skew results of many tracks
  - Cycle slip detection/correction may be too strict
  - This method behaved worse in A.R. than MAGIC
Absolute TECS Sensitivity Analysis

• While mathematically consistent, this method is sensitive to choices:

  – What is a crossover?
    • “Sufficiently close” definition

  – How are the mapping functions applied?
    • Which one is used and where is it applied?
Absolute TECS Sensitivity Analysis
(Crossover definition)

- Sensitivity to definitions of “sufficiently close”
  - Tested 5 different definitions for day 298 of 2004

<table>
<thead>
<tr>
<th>Definition</th>
<th>Solvable Tracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.20^\circ \times 0.20^\circ \times 300\text{ s}$</td>
<td>14,657 tracks solvable</td>
</tr>
<tr>
<td>$0.15^\circ \times 0.15^\circ \times 150\text{ s}$</td>
<td>13,941 tracks solvable</td>
</tr>
<tr>
<td>$0.10^\circ \times 0.10^\circ \times 60\text{ s}$</td>
<td>12,698 tracks solvable</td>
</tr>
<tr>
<td>$0.05^\circ \times 0.05^\circ \times 30\text{ s}$</td>
<td>9,129 tracks solvable</td>
</tr>
<tr>
<td>$0.01^\circ \times 0.01^\circ \times 10\text{ s}$</td>
<td>0 tracks solvable</td>
</tr>
</tbody>
</table>

Sensitivity of TECS values: ±1.98 TECU

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Absolute TECS Sensitivity Analysis (Mapping Function Location)

- Sensitivity to location of mapping function
  - Tested 5 different locations for day 298 of 2004

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Number of Tracks Solvable</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>12,041</td>
</tr>
<tr>
<td>300</td>
<td>12,698</td>
</tr>
<tr>
<td>350</td>
<td>12,680</td>
</tr>
<tr>
<td>400</td>
<td>12,905</td>
</tr>
<tr>
<td>450</td>
<td>13,044</td>
</tr>
</tbody>
</table>

Sensitivity of TECS values: ±1.26 TECU

Smith, D.A., Absolute Ionosphere, ION NTM 2005
• After internal testing, a prototype production was established at NGS (Nov 1, 2004) to encourage independent validations
• Daily solutions (~15k crossovers, ~30k tracks)
  – Sparse matrix solution = 2 minutes
  – Reading data/uncompressing/gridding/making pretty pictures = 3 hours
• “ICON” (Ionosphere over CONus)
• www.ngs.noaa.gov/ionosphere
Absolute Comparison with IGTEC

- ~ 1 month of data (Dec 2004)
- ICON – IGTEC
- Daily bias between models ~ -3 to -4 TECU
- Daily $\sigma$ around bias ~ ±2 to 3 TECU
- Possible causes:
  - Resolution differences
  - Model errors
7,261,965 Differences between TECS(ICON) and TECS(IGTEC) for 2004 Nov 29

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Absolute Comparison with MAGIC(NGS)

- Data generally unavailable currently
- ICON – MAGIC
- Daily bias between models \(\sim +1\) TECU
- Daily \(\sigma\) around bias \(\sim \pm 2\) to 3 TECU
- Possible causes:
  - Resolution differences
  - Model errors
2,716,181 Differences between TECS(ICON) and TECS(MAGIC) for 2004 Nov 29
• As a secondary product, ICON produces radial TEC (TECR) on a grid in IONEX and GIF formats

• Mostly for analysis: Grid to slant delays introduce another error source
  – Useful for seeing outliers, storms and small ionosphere features
Smith, D.A., Absolute Ionosphere, ION NTM 2005
Smith, D.A., Absolute Ionosphere, ION NTM 2005
Summary and Conclusions

• Absolute TECS is mathematically determinable from ambiguous carrier phase data under 4 assumptions:
  – Network of Ground Stations
  – Dual Frequency
  – $i,j \Delta \text{TECS} = k(“40.3”, f_1, f_2) \times (i,j \Delta L_1 - i,j \Delta L_2)$
  – $\text{TECS}_1 = f[\text{TECS}_2]$ when “sufficiently close”
Summary and Conclusions

• Relying on simple cosine mapping functions, a model for the ionosphere can be computed as an entire network
  – to ~4 TECU RMS (absolute)
  – to ~0.3 TECU RMS (5 cm on L1) agreement with Double Difference estimates, subject to cycle-slip fixing and outlier biasing

• Interpolation can yield ± 5 cm (L1) biases from nearby tracks
Future Work

• Removal of outliers and general improvement in regional correlation

• Usefulness of method in A.R. must be improved

• Move from daily solutions to progressive epoch by epoch solutions
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- Dru.Smith@noaa.gov
- www.ngs.noaa.gov/ionosphere

Questions?

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Conclusions

- Average a-posteriori $\sigma_{\text{bias}}$ of $\pm 1.1$ TECU reasonable, but larger than hoped for
- Sub-TECU crossover residuals show tight “locking” or consistency of tracknet
- Overall noise in grids needs improvement
- General conclusion:
  - “Promising” but not by any means “done”
  - Initial analysis indicates near-horizon crossovers are the primary error source (TECS=TECR/$\cos z'$ unreliable)

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Summary and Conclusions (cont)

• Further sensitivity studies:
  – Removing near-horizon crossovers (nearly done)
  – Shell height
  – CORS thinning

• Independent tests forthcoming:
  – Against other ionosphere models
  – In ambiguity resolving software

• Production:
  – Daily solutions expected to begin in Fall 2004
<table>
<thead>
<tr>
<th>Time</th>
<th>OBSERVATION DATA</th>
<th>RINEX VERSION / TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000Jul12</td>
<td>G (GPS)</td>
<td></td>
</tr>
<tr>
<td>MEGH</td>
<td>2004O128</td>
<td>05:10:03 UTC/CH / RUN BY / DATE</td>
</tr>
<tr>
<td>XE</td>
<td>MARKER NAME</td>
<td></td>
</tr>
<tr>
<td>XE</td>
<td>MARQUE NUMBER</td>
<td></td>
</tr>
<tr>
<td>XE</td>
<td>CORS/NGS/NROA</td>
<td></td>
</tr>
<tr>
<td>3753120424</td>
<td>OAR/FSL</td>
<td></td>
</tr>
<tr>
<td>3382145803</td>
<td>TRIMBLE 4000881 7.19</td>
<td></td>
</tr>
<tr>
<td>-39454.742 -227170.335 -355679.03513</td>
<td>APPROX POSITION XYZ</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>ALT / WAVELENGTH FACT L1/L2</td>
<td></td>
</tr>
<tr>
<td>1 7 0 0 0.000000</td>
<td>INTERVAL</td>
<td></td>
</tr>
<tr>
<td>10.0000</td>
<td>GMT</td>
<td></td>
</tr>
<tr>
<td>FORCED MODULO DECLENDATION TO 3 SECONDS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is an RINEX file, not strictly RINEX. The difference is that an L1 variable, representing the computed ionosphere delay on L1, in cycles of L1, has been introduced. This value should generally always be positive. It was computed by David Smith, NOAA/NOSA with the following parameters (see D. Smith for details):

- Year = 2004 Day of Year = 007
- Time (Hm) = 05:10:03
- Track Cleaning Criteria Index = 001
- Crossover Spacing Criteria Index = 001
- Track Formation Criteria Index = 009
- LSA Weighting Scheme Index = 004

Flag for post-LSA interpolation: 000

END OF HEADER
CORS Network

- Currently 400+ 24/7 receivers
  - Dual frequency, carrier-phase
  - Multi-agency
  - Administered by NGS
  - All 50 states, Central America, others
  - *Ideally suited to serve as an ionosphere monitoring network for geodetic applications in the USA*

Smith, D.A., Absolute Ionosphere, ION NTM 2005
\[ i R_k = b_k + i r + c(i \delta t) + i T + i I_k (+i m_k) = \lambda_k i \Phi_{k}^{\text{RINEX}} \] (biased range, m, epoch "i", freq "k")

\[ I_k = -\frac{40.3}{f_k^2} \text{TECS} \quad \text{(m)} \]

\[ \therefore \lambda_1 i \Phi_1^{\text{RINEX}} - \lambda_2 i \Phi_2^{\text{RINEX}} = (b_1 - b_2) + (i I_1 - i I_2) \]

\[ \therefore i \text{TECS} = \left( \frac{1}{40.3} \right) \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right)^{-1} \left[ \lambda_1 i \Phi_1^{\text{RINEX}} - \lambda_2 i \Phi_2^{\text{RINEX}} \right] \]

\[ -\left( \frac{1}{40.3} \right) \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right)^{-1} (b_1 - b_2) \]

\[ \therefore i.j \Delta \text{TECS} = i \text{TECS} - i \text{TECS} \]

\[ = \left( \frac{1}{40.3} \right) \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right)^{-1} \left( \lambda_1 i.j \Delta \Phi_1^{\text{RINEX}} - \lambda_2 i.j \Delta \Phi_2^{\text{RINEX}} \right) \]
Closed Polygons

• Altimetry or Leveling ($\Delta H$ & $H$-equality):
  – # conditions = # vertices – 1

• Ionosphere ($\Delta\text{TECS}$ & $\text{TECR}$-equality)
  – # conditions = # vertices

• Any time that a closed polygon is formed we have:
  – # Conditions = # Unknowns
Polygon Crossover Equations

\[
\begin{bmatrix}
\Delta TECS_A \cos A_1 z' - \Delta TECS_A \cos A_3 z' \\
\Delta TECS_B \cos B_1 z' - \Delta TECS_B \cos B_2 z' \\
\Delta TECS_C \cos C_2 z' - \Delta TECS_C \cos C_3 z'
\end{bmatrix}

= \begin{bmatrix}
- \cos A_1 z' & 0 & + \cos A_3 z' \\
- \cos B_1 z' & + \cos B_2 z' & 0 \\
0 & - \cos C_2 z' & + \cos C_3 z'
\end{bmatrix} \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix}
\]

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Polygon Crossover Equations

- The existence of the \( \cos z' \) values on the RHS allows for matrix inversion
  - (as opposed to \(+1, 0\) and \(-1\) for altimetry)
- Solvability
- Can we have redundancy?
  - YES
A good fit between P-R and carrier phase
A poor fit between P-R and carrier phase
Initial Tests

- Parameters:
  - Crossover height = 300 km
  - Crossover definition: $0.1^\circ \times 0.1^\circ \times 1$ min
  - Cut-off angle: $10^\circ$ (for data and crossovers)
Initial Tests
(all contain the 4 base tracks)

- **Solution 1** (smallest tracknet possible containing the 4 base tracks)
  - 8 tracks, No polygons, PR-fit 6 of 8 tracks
- **Solution 2**
  - 10 tracks, 2 polygons, PR-fit 7 of 10 tracks
- **Solution 3**
  - 10 tracks, 2 polygons, no PR-fitting
- **Solution 4**
  - 10 tracks, 2 polygons, PR-fit 1 of 10 tracks
Formal $\sigma_{bias}$ estimates for first tracknet tests (in TECU)

<table>
<thead>
<tr>
<th>Track #</th>
<th>Soln 1 (PR fit to 6 of 8; no polygons)</th>
<th>Soln 2 (PR fit to 7 of 10; 2 polygons)</th>
<th>Soln 3 (No PR fit; 2 polygons)</th>
<th>Soln 4 (PR fit to 1 of 10; 2 polygons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4300 (base)</td>
<td>3.5</td>
<td>2.9</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>4303 (base)</td>
<td>8.8</td>
<td>4.7</td>
<td>0.2</td>
<td>2.1</td>
</tr>
<tr>
<td>9484 (base)</td>
<td>9.3</td>
<td>4.6</td>
<td>0.2</td>
<td>2.0</td>
</tr>
<tr>
<td>9487 (base)</td>
<td>9.4</td>
<td>3.1</td>
<td>0.1</td>
<td>1.3</td>
</tr>
<tr>
<td>2253</td>
<td>13.6</td>
<td>5.9</td>
<td>0.3</td>
<td>2.5</td>
</tr>
<tr>
<td>10146</td>
<td>9.7</td>
<td>3.3</td>
<td>0.1</td>
<td>1.4</td>
</tr>
<tr>
<td>11416</td>
<td>6.5</td>
<td>4.9</td>
<td>0.2</td>
<td>2.0</td>
</tr>
<tr>
<td>12565</td>
<td>6.1</td>
<td>3.9</td>
<td>0.2</td>
<td>1.6</td>
</tr>
<tr>
<td>2224</td>
<td>-</td>
<td>4.3</td>
<td>0.2</td>
<td>1.7</td>
</tr>
<tr>
<td>11580</td>
<td>-</td>
<td>3.0</td>
<td>0.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Initial Tests (cont)

- Individual ionosphere delays for each SV/CORS combo were estimated:
  - $I_{4300}(SV1/GODE)$, $I_{4303}(SV2/GODE)$, $I_{9484}(SV1/RED1)$, $I_{9487}(SV2/RED1)$ all estimated individually (as well as for all other tracks in the tracknet)

- Double Difference delays were then computed:
  - $I_{DD}=(I_{4300}-I_{9484})-(I_{4303}-I_{9487})$ computed and compared to independent estimates from NGS ambiguity resolving software
First tracknet tests

- Pseudo-range fitting tends to bias the tracknet
- Better fit to Double Difference estimated ionosphere by using just polygons and no P-R fitting
Full day solution (cont)

• Interpolation from tracks to grids and/or other tracks:
  – Track-to-grid-to-Track
    • Useful for grid-distributed Ionosphere model and animations
    • 0.00 ± 0.38 TECU (±6 cm on L1)
  – Track-to-Track
    • Useful for RINEX-distributed Ionosphere model
    • 0.00 ± 0.25 TECU (±5 cm on L1)

• Full day solution was gridded and animated

Smith, D.A., Absolute Ionosphere, ION NTM 2005
Example 2. 17:00-18:00 UT (day-time)

The “truth” DD ionospheric delays are presented in figure 14 with the corresponding satellite elevation map in figure 15. Figures 16–25 represents the derived DD ionosphere form each method and the difference from the “truth” (in pairs). The mean and standard deviation of the ionospheric residuals from the “truth” are shown in Table 2.

Fig. 14 “Truth” DD iono (day-time)

Fig. 15 Satellite elevations

“Truth” (Iono after ambiguity fixing)
Smoothed Pseudorange Estimates

Fig. 16 P4 DD iono (day-time)

Fig. 17 P4 DD iono differences from the “truth” (day-time)

OSU’s MPGPS method

Fig. 18 GIM DD iono (day-time)

Fig. 19 GIM DD iono differences from the “truth” (day-time)