THE $M_2$ TIDE FROM GEOSAT ALTIMETRY

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Rockville, MD 20852
June 1990
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NOAA Technical Memorandum NOS NGS-53

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ABSTRACT. More than 2,000,000 global Geosat altimeter observations taken in 1986-88 have been analyzed for the $M_2$ tide at overlapping points in its Exact Repeat Mission. The results are compatible with Schwiderski's 1980 tide-gage-plus-theory model at a level of 6 cm (rms discrepancy). The gross features of that model (amphidromes and highs) are unchanged. The altimeter solutions are for about 1,000 independent time series, each covering separate 5° by 5° areas of the deep oceans. They have been calibrated through two roughly equal and independent subset analyses employing different arcs of the ERM configuration. Examination of the longest full series (108 records) shows that with further densification in time, significant correction to many other ocean tidal components will be possible.

INTRODUCTION

The tides of the world’s oceans have been a continuing concern and source of scientific analysis since Newton’s day. They have intrinsic interest as force probes to which the Earth reacts and reveals how it is constructed. These reactions must be properly accounted for in precision geodynamics investigations. But just as important, the large lunisolar (so-called deterministic) tides are major “noise” sources for more subtle sea level inquiries, such as “El Nino” and other “ocean weather” effects including secular climate changes.

The last thorough review of lunisolar tidal theory and results was given by Schwiderski (1980). Although it is now more than 30 years since the first artificial Earth satellite, by 1980 there had been no global confirmation of these results from satellite measurements. [Long term analysis of tracking data had confirmed, by the mid-1970's, long wavelength tide parameters important to the slow degradation of the Earth’s spin (e.g., Goad and Douglas 1978).] But with the advent of Seasat (1978) and Geosat (1985), the first operational and precise altimetric measurements of the sea’s surface could be made from space, promising a revolution in tidal research. Until 1980, the detailed tidal results had been either entirely theoretical (with ideal boundary conditions) or partly theoretical with boundary conditions given by measured tide gage data at scattered ocean locations and empirical “friction” coefficients to rationalize the fit between the theory and the measurements.

Although there have been many regional studies (e.g., Diamante and Nee 1981; Brown 1983; Sanchez and Cartwright 1988), only two serious attempts have been made to construct a global tide model using the new satellite altimeter data (Mazzega 1985; Cartwright and Ray 1990). Mazzega’s solution, however, used only limited Seasat data in the summer of 1978 and only partially resolved a complex and significant orbit signal. Its resolution, therefore, was limited to the broadest global scales. With the Geosat Exact Repeat Mission (ERM) beginning late in 1986, the simple orbit error signal as seen from difference-altimetry could be easily eliminated and ocean variability directly studied in detail for the first time over many seasons and years (e.g., Cheney et al. 1989). The Cartwright and Ray solution from Geosat ERM altimetry in its first year is of this type and similar to the work here.
Previous satellite studies have relied primarily on the altimeter-derived sea heights either directly along-track (e.g., Mazzega 1985) or indirectly from their differences at track crossovers and overlaps (e.g., Brown 1983). (See Discussion.) In this paper, the global $M_2$ ocean tide is evaluated from long term sampling of the altimetry only at overlapped (collinear) points on the ocean-track in the ERM. The approach is simple: Following Brown (1983), the difference of altimetrically determined sea heights over the same track point at different times (with proper corrections for a multitude of path and other errors) provides open ocean time series fully equivalent to a conventional in-situ gage for tidal analysis.

The aim in this (ERM) overlap tidal analysis was to arrive at a significant number of such time series of differences averaged over discrete ocean areas as small as practical.

Currently, more than 2,000,000 differences have been processed over a 1.6 year period (1986-88) and averaged into about 1000 5° by 5° ocean areas to achieve adequate resolution of the $M_2$ tide at that scale. Fortunately, the tide in most of the open ocean is a very broad scale phenomenon and a 5 degree resolution is not that restrictive generally. Nevertheless, the results achieved here should be considered just an indication of what can be accomplished when the full ERM data set is analyzed.

METHOD AND DATA REDUCTION

The analysis began with the 1-second (6.5 km) averaged altimetric sea heights (with respect to the Earth ellipsoid) on the Geophysical Data Records (GDR) (Cheney et al. 1987). Using all the media and tide corrections and subtracting a detailed geoid height available on the GDR, these measurements (order of 50 m) were reduced to a Sea Topographic Height (STH) (order of a few meters). In particular, sea level corrections were made for both dry and wet tropospheric and ionospheric retardation of the altimeter signal (order of 200, 10, and 10 cm respectively). Corrections of 1 percent of the significant wave height were also made to account for electromagnetic bias (Cheney et al. 1989). Most important, an inverted barometer correction at 1 cm/in/bar (order of 10 cm) was also applied (from GDR pressure data) and found to significantly reduce the sea level variances in most of the arcs after removal of the orbit signal. The 11 component ocean tide model of Schwiderski (1980) (order of 50 cm) and a solid tide model derived from the equilibrium tide of Cartwright and Taylor (1971) (order of 20 cm) were also removed from the sea height measurements.

In this process the data were also screened to include only deep ocean areas (greater than 2,000 m depth), and to exclude STH greater than 10 m and slopes greater than 0.1 m/km (both absolute). This screening also eliminated heights near trenches, seamounts, and fracture zones which could not be reliably interpolated (before differencing) from one overlap (ERM) cycle to the next.

The STH data were then accumulated in arcs of from 1 to 2 days, identified as day-segments of the 17-day ERM cycle. Eight such day-segments were used: days 1,4,6,10,14 and days 7 and 8, 12 and 13, and 16 and 17. Figure 1 summarizes the results of this initial processing. The large STH values here are due both to near 1 cycle per revolution (cpr) orbit error from the models employed to calculate the radial ephemeris of Geosat and errors in the detailed geoid. Note the significant improvement in the STH when Gem-T1 (Marsh et al. 1988; Haines et al. 1990) was used with the same limited tracking data as for the original GDR ephemeris (Cheney et al. 1987). Further improvement of Geosat orbits should be possible (to perhaps 50 cm radially) with more global Doppler tracking and correction of their station coordinates.

Using the day-segments of the first cycle as base (November 8-25, 1986), successive cycle segments were registered onto the base arcs. For each 1-second STH of the successive arcs the point of its closest approach to the base arc (always less than 1 km away) was found and the STH difference (successor minus base) was taken, interpolating the base height to these closest points.
Figure 1. Global sea level topography from Geosat altimetry. Shown are sea topographic height results (rms) in 1- and 2-day arcs (of the Exact Repeat Mission). The measured sea height has been corrected for tides, ionosphere, troposphere, wave height, and inverse barometer effects. A reference geoid is also subtracted, from a 180 by 180 model containing gravimetric and Geos-3 altimetric information. The upper symbols are for 1-day arcs using the original Geosat GDR ephemeris (Gem 10 gravity field). The lower symbols are for 1- and 2-day arcs using a radial ephemeris with the Gem T1 gravity field. The dominating signal in these data arcs, even with the Gem T1 ephemeris, is 1 cpr orbit error.

These STH differences in each arc pair were again dominated by two orbit frequencies close to 1 cpr. But before proceeding the differences were pass-averaged in 2° by 2° ocean areas to provide both a fine resolution and sufficient smoothing for erratic mesoscale current activity (eddies, rings, and meanders). Then the 2-degree data were relieved of these orbit signatures by a least-squares fitting process with known frequencies. Figure 2 shows the results of this step. Note the initial rise and fall of these average residuals occur in the first 18-21 ERM cycles or around 1 year. Since the dominant M₂ tide has a period of 12.4206 hours, it is readily seen that the "aliased" period for this tide as repeatedly sampled in the mission is 317 days, or at cycle No. 19.6. Of course both the M₂ and possible yearly ocean weather signals here are residuals to the Schwiderski (1980) 11 (luni-solar) tide model in the first instance and the crude instantaneous "inverted barometer" model in the second.

It should be mentioned that the detailed time series solutions themselves for each area are formally capable of distinguishing the aliased M₂ from the unaliased yearly "weather" tide. The series generally includes both ascending and descending passes from a variety of day-segments of the ERM. They accomplish this resolution by sensing the M₂ differences with respect to many initial phases while the yearly weather signal over the initial ERM cycle is essentially at the same phase. This favorable feature of time series analysis over finite areas was first pointed out to me by Laury Miller. (See also Cartwright and Ray 1990: pp. 3071-3073.)
Figure 2. Global sea level differences from Geosat altimetry after orbit correction. Shown are the
sea topographic height differences with cycle 1 (global rms of 2° by 2° averages) for the following
day-segment arcs of the exact repeat mission cycle: Boxes, day 1: Pluses, day 4: Diamonds, day 6:
Triangles, day 10: Octagons, day 14: Crosses, days 7 and 8: Asterisks, days 12 and 13: Slashed
Boxes, days 16 and 17. Note the initial signature (to cycle 22) showing a period close to that for
$M_2$ (aliased to 18.6 ERM cycles).

Nevertheless, as a matter of interest, assuming that all of the initial rise of these difference-
residuals (in fig. 2 to a point about halfway into the aliased $M_2$ period) is due to the wanted $M_2$
correction we seek, the average power of that correction can be easily estimated. Using 11.5 and
17.5 cm as the minimum and maximum values, the power (rms) of this correction should be $(17.5^2
- 11.5^2)^{1/2}/2$, or 6.6 cm. This is compatible with Schwiderski's (1980) claim of 5 cm accuracy for this
dominant tide but needs further verification from the actual solutions and their error assessment.

Finally to densify the area series the orbit-reduced 2-degree data were pass-averaged into
$5^\circ$ by $5^\circ$ bins yielding 43,654 records (average time series: 35 records) of difference pairs. Each
set of time series records was then analyzed independently (by least squares fitting) for the
two (correction) components of an $M_2$ harmonic. For all grid point solutions a mild a priori constraint
of zero (with 30 cm error, the power of the full $M_2$ tide) was also applied, which significantly
constrained only those few grid solutions with less than about 10 records.

Calibration

In addition to the full series solutions, the data at each grid point were broken into two roughly
equal portions (interleaved in time), each with independent cycle segments and each analyzed
separately for the same $M_2$ tide (as a correction to the Schwiderski 1980 GDR values). If the time
series were affected by white noise only one would expect that the diagonals of the least squares inverse for the harmonics (as scaled by the residuals, their formal error) would be a reliable estimate of the true variances of these parameters. In this case the distribution of the differences of these independent solutions divided by the root-sum of (scaled) parameter “variances” should be unit-normal (z). Probably most of the error sources for such a simple M₂ evaluation from these time series are far from white noise (e.g., other tides are influential including failure of the inverted barometer assumption). Therefore, I used the actual average value of z (rms) in each such grid comparison to get an initial estimate of the true calibration of the formal error of the full grid solution. With additional conservative factors these "measured" calibrations (of formal errors) were applied to the results of the full series solutions.

**M₂ SOLUTIONS**

Before presenting the full series solutions, I give a typical example of a calibration analysis with the two independent subset time series consisting of A: the 1-day arcs of the cycle-day-segments 1,4,6,10 and the 2-day arcs of the cycle-day-segment 7 and 8, and B: the 1-day arcs of the cycle-day-segment 14, and the 2-day arcs of the cycle-day-segments 12 and 13, and 16 and 17. The total A set consisted of 22,567 records (average time series: 19 records). The total B set consisted of 21,087 records (average: 18).

Figure 3 shows the way the orbit reduced residual tide data (5 degree averaged) scatter about the two independent M₂ solutions for a South Pacific station, plotted with respect to the fraction of an

![Graph](image-url)

**Figure 3.** Residual M₂ time-difference series and tide in the South Pacific; latitude: - 42.5°, longitude: 202.5°. The asterisk data are from set A cycle-day-segments and refer to the solid curve M₂ solution as an ordinary time series. The boxes are from set B and refer, likewise, to the dashed curve solution. For the purposes of this plot, the cycle-1 errors (of each difference pair) are assumed to be zero.
M₁ cycle starting at the beginning of the Geosat ERM. Clearly, even though set B lacks full M₁ coverage it is compatible with set A, and the two solutions are likewise compatible. In fact the error-calibration factor for these independent solutions is only 0.8 (rms of the factors for the cosine and sine parameters). In estimating the (error) calibration factors for the full solutions I arbitrarily took 0.75 of this measured factor, reasoning that the full set should be closer to a "white noise" condition than any partial set. In fact, most of the measured factors were greater than 1: their average was 1.7. But to avoid fortuitous circumstances (such as this case) where the factor was less than 1 (apparently better than "white noise") and to maintain a conservative result the minimum full set factor was set at 1.5. And finally, to avoid the small-record circumstance where the residuals might be unreasonably low, the minimum calibrated residuals (actual residuals, rms, times the error calibration factor) were set at 7.5 cm in scaling the formal errors of the parameters in the full series solutions.

There were 1,253 full set 5° by 5° time-difference series, which yielded independent components of M₁ at these points and calibrated error estimates as just described. Figure 4 shows how the power of these solutions (rms correction) varies with the number of observations in each time series. Clearly, the grid points with less than about 10 records have unreliable solutions (a conclusion borne out in what few significant calibrations could be made with such sparse data).

But even for solutions with greater numbers there is also a noticeable trend towards smaller power. Again though, this trend is considerably reduced when only the most reliable of the solutions (as judged by the calibration) are selected. A good example of this was given in figure 3, where the full set results have calibrated errors of less than 1.5 cm in each coefficient. Clearly, the power of the full set solution has not changed significantly from the subset ones. I find this is
Figure 5. Power of $M_2$ corrections from best solutions (5° by 5°): Box widths show power linearly with maximum = 20 cm (rms). Global rms of corrections = 6 cm. In most regions the largest corrections correspond to the largest amplitudes of the reference tide (Schwiderski, 1980). Overall result shows the gross features of the reference tide are unchanged.

generally true for 846 of the full set time series (of at least 10 observations each) whose calibrated errors (rms) are all less than 5 cm. The global distribution of the power of these “best” corrections is displayed in figure 5. The global rms power of these corrections is only 6 cm. Thus, in a general sense the power expectation for these solutions from the orbit-reduced data has been confirmed.

It should also be noted, however, that while the overall 6 cm power of the $M_2$ correction is certainly significant, the average reduction in the residual differences at a given grid point is small, from 14 cm to only 12 cm (rms). On the other hand, I have also examined a number of the (current) longest time series for a combination of other tidal components both lunisolar and "weather."

I find that reasonable combination solutions for all of them can be obtained (with virtually unchanged $M_2$ solutions) which yield difference residuals in the vicinity of only 6 cm (rms). This last figure, or $(6/\sqrt{2}) = 4$ cm for "one-way" error, seems to represent the irreducible minimum "noise" for oceanographic signals in these data. It is certainly compatible with a host of uncertainties in media modeling, principally ionosphere and wet troposphere, as well as electronic bias from asymmetrical reflection off finite ocean waves (e.g., Tapley et al. 1982).

**ADDITIONAL DISCUSSION**

What is the advantage of overlapping altimetry (in an ERM) for tidal analysis as opposed to using difference-altimetry at track crossovers or straight altimetry (without differences)? If straight altimetry is used, then the geoid error (currently of the order of 50 cm (Rapp, 1986)] and the full orbit error (at about the same level of accuracy (Marsh et al. 1988)) become problematic burdens to analysis. Both are of complex wave structure through the former is more "white" while the latter
is mainly of long wavelength (less than 2 cpr). Crossover analysis of altimetric height differences solves the geoid error problem, but retains more than half of the orbit error as a "red noise" source, eliminating only the geographically correlated part of this error at all frequencies (e.g., Tapley and Rosborough, 1985). But with overlapping altimetry, not only is the geographically correlated part eliminated but all gravitational frequencies in the orbit error are cancelled since they repeat exactly each ERM cycle. The only orbit frequency which does not repeat exactly is the fundamental 1 cpr term which is easily removed empirically as we have done here.

However, our simple uncoupling of the orbit reduction from the tidal evaluation may have a damping and distorting effect on the subsequent analysis. The ocean tide is also a generally long wave phenomenon but its dominant power, as seen by the satellite's altimeter along-track, is about 2 cpr. Still there is also some power at 1 cpr and even longer wavelengths which, we have found, can be absorbed by the orbit correction necessary (here) to eliminate the major source of error introduced by the imperfect orbit determination process.

In many simulations of this "absorption" process using a complete $M_2$ model as "signal," it was found that the power of this signal for the 1-day arc reductions is indeed reduced by about 3 percent but less for the 2-day arcs. But the distortion of the resulting data proved to be more serious. Recovering a simulated (full) $M_2$ tide from the same day-segment (and pass direction) of ERM overlaps over a full (aliased) $M_2$ period showed errors of order 10 percent from this orbit-absorption process. However, with 2-day arcs and multi-aspect and segment data, both the distortion and the power loss are less. Most importantly, the correction signal, presumably a small fraction of the full signal is what we correct in the Schwiderski (1980) model, reducing still further this distorting effect (to the level of only a few centimeters).

I have implied that the $M_2$ corrections here apply to the Schwiderski (1980) ocean tide model. But the altimeter actually senses a "geocentric tide" or the sum of ocean and body tides. Further, the body (or ocean bottom) response reflects both the direct solid Earth tide and the indirect ocean loading effect. But while the solid tide on the Geosat GDRs does not include the ocean loading effect (Ray and Sanchez 1989), since it does not exceed 4 cm in the open oceans (rms of about 1.5 cm), it is only a minor constituent (if any) of the $M_2$ corrections here.

It should also be said that I am neglecting here a small (~5 percent) 18-year modulation of the $M_2$ tide from the movement of the moon's orbital node (e.g., Schwiderski 1983, pp. 253) which affects my data at a level of ~1 cm (rms) during 1986-87.

In the future we would like to match the 1° by 1° resolution of the tidal theorists. When a finer resolution is achieved meaningful comparisons can be made with the scattered deep sea tide gage data at isolated islands and ocean bottom locations. Averaging over at least 1° is necessary to eliminate the random fluctuations (up to 50 cm) of eddies and rings. Unfortunately, all the data have not yet been processed in the 17.0505 day ERM cycles over a sufficiently long time to achieve this ideal resolution. Roughly, every 1° by 1° area in the open oceans (outside the polar regions) is covered by an ascending and a descending arc in each Geosat ERM cycle. Complete data over the $M_2$ aliased period of 317 days would provide about 37 time series records for each 1° by 1° area, which should be adequate for an accurate resolution of the $M_2$ tide there. Using the "1 second" Geophysical Data Records of the Geosat project (Cheney et al. 1987) this resolution would require processing some 11,000,000 such records.

REFERENCES


