NOAA Reprint of

Basic Geodesy

Rockville, Md.
September 1977
NOAA Reprint of
Basic Geodesy

Rockville, Md.
September 1977

DEPARTMENT OF DEFENSE
Defense Mapping School
Fort Belvoir, Virginia

4MF1-F-010-010
April 1973
STUDENT PAMPHLET

Reprinted September 1977
U.S. Department of Commerce
National Oceanic and Atmospheric Administration
Although geodesy is the oldest among earth sciences, dating back to the Sumerians of 5000 years ago, its subject and even its name have been widely unknown. Man's present ventures into space, his commercial explorations on the continental shelves and his increasing knowledge of science and its relationship to the world around him have made him aware of geodesy's increasing contribution.

This pamphlet is intended to present a concise overview of the various interrelated topics dealt with in modern geodesy. It explains the significance and impact of geodetic concepts using general theory approach. It is hoped that this initiation into the Mysteries of Geodetic Concepts will be welcomed by newcomers to the field before they delve into technical textbooks and also by those who do not intend to become geodesists but wish to develop a general understanding of what geodesists are talking about. The reader is referred to the complementary pamphlet "Geodesy for the Layman," published by ACIC, which extends the concepts of "Basic Geodesy."

The Defense Mapping School of the Defense Mapping Agency expresses its gratitude to Mrs. Irene K. Fisher for preparation of the original text and to the Defense Mapping Agency Topographic Center for technical assistance.

TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREFACE</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER I The Shape and Size of the Earth</td>
<td>2</td>
</tr>
<tr>
<td>CHAPTER II The Gravity Field of the Earth</td>
<td>14</td>
</tr>
<tr>
<td>CHAPTER III Point Positioning</td>
<td>23</td>
</tr>
</tbody>
</table>
I N T R O O U C T I O N

Geodesy is one of the Earth Sciences. Other familiar Earth Sciences or GEO-sciences are: GEOgraphy, which describes what you find where on the earth; land and water, climate, vegetation, and man's habitat in relation to these. GEOlogy, which studies the rock formations and their history. GEOmorphology, which describes the landforms and their evolution. GEOphYSics, which studies the physical forces that shape the earth. GEOdesy is less well known, although it is the oldest of the geosciences. What does it do?

The following is an "Initiation into the Mysteries of Geodetic Concepts". It is centered on three major topics:

1. The shape and size of the earth.
2. The gravity field of the earth.
3. Point positioning.

Each of these will be discussed in separate chapters which follow.

PREFACE

Although geodesy is the oldest among earth sciences, dating back to the Sumerians of 5000 years ago, its subject and even its name have been widely unknown. Man's present ventures into space, his commercial explorations on the continental shelves and his increasing knowledge of science and its relationship to the world around him have made him aware of geodesy's increasing contribution.

This pamphlet is intended to present a concise overview of the various interrelated topics dealt with in modern geodesy. It explains the significance and impact of geodetic concepts using general theory approach. It is hoped that this "Initiation into the Mysteries of Geodetic Concepts" will be welcomed by newcomers to the field before they delve into technical textbooks and also by those who do not intend to become geodesists but wish to develop a general understanding of what geodesists are talking about. The reader is referred to the complementary pamphlet "Geodesy for the Layman," published by AC1C, which extends the concepts of "Basic Geodesy."

The Defense Mapping School of the Defense Mapping Agency expresses its gratitude to Mrs. Irene K. Fisher for preparation of the original text and to the Defense Mapping Agency Topographic Center for technical assistance.
Geodesy is one of the Earth Sciences. Other familiar Earth Sciences or GEO-sciences are:

GEOgraphy, which describes what you find where on the earth: land and water, climate, vegetation, and man's habitat in relation to these.

GEOlogy, which studies the rock formations and their history.

GEOmorphology, which describes the landforms and their evolution.

GEOphysics, which studies the physical forces that shape the earth.

GEOdesy is less well known, although it is the oldest of the geosciences. What does it do?

The following is an "Initiation into the Mysteries of Geodetic Concepts". It is centered on three major topics:

1. The shape and size of the earth.
2. The gravity field of the earth.
3. Point positioning.

Each of these will be discussed in separate chapters which follow.
CHAPTER I - THE SHAPE AND SIZE OF THE EARTH

Geodesy developed from practical needs: It was always necessary to establish property lines, mainly for tax purposes. Roads and buildings need advance planning. The location of a nation's resources must be determined and recorded. To get from one place to another, we want to know which way and how far to go. We can outline these needs as follows:

CADASTRAL - property lines, taxes
CIVIL ENGINEERING - roads, buildings
RESOURCES - What? Where? How much?
NAVIGATION - Which way? How far?

For such needs we may assume that the earth is flat - as long as we stay within our immediate vicinity. But for long distances and large areas, this simple assumption does not work. It contradicts the following observations:
3D OBSERVATION. Traveling north by day, a man's shadow at noon becomes longer. Realizing that the earth is curved, people jumped to the conclusion that it is curved equally all over, like a sphere. Our observations suggest that the earth is like a sphere. The next question then arises: how big is that sphere? Eratosthenes, who lived in Alexandria, Egypt, in the third century B.C., was a genius who computed the size of the earth from watching a sun dial cast its shadow at noon on a certain day, the summer solstice in June. Of course, he also knew a few other things:

1. First he knew that there was a place called Syene (now Aswan) south of Alexandria, where the sun's rays reached the bottom of a deep well at noon of that same day.
2. He knew how many days it took a camel caravan to travel the distance between Syene and Alexandria.
3. He also knew some geometry. For those who remember high school geometry, it will be easy to follow Eratosthenes' reasoning from Figure 4. The others will just have to believe this famous story about the "father of geodesy".

TRAVELING NORTH AT NIGHT

2D OBSERVATION. Traveling north at night, the north pole star appears to get higher in the sky, as measured from the horizon up. In the figure above, Angle 2 is larger than Angle 1.
Eratosthenes
3rd Century B.C.

5000 stadia = 1/10 nautical mile

Circumference = 15,000 nautical miles

16° to 20° big

Circumference = 21,600 nautical miles = 40,000 km.

When watching the shadow of the pointer on the spherical bowl of the sun dial, Eratosthenes estimated that the length of the shadow, a circular arc subtended by the angle α of the sun's rays, was the fiftieth part of a full circle. Similarly, the terrestrial arc traveled by the camels (5000 stadia, subtended by an equal angle α at the center of the earth) must be the fiftieth part of the earth's circumference. Thus, the latter must be 250,000 stadia.

A stadium was a length unit in antiquity. There were several different stadia in use, and the experts do not know which one Eratosthenes used. If it was about 1/10 of a nautical mile, then Eratosthenes' result was about 16% too big. But the actual number is really beside the point. The important and amazing achievement is the method of combining astronomical angles with measured distances. This basic method was used for centuries with the only improvements being made in the accuracy of the angle and distance measurements.

In the 17th century, it was possible to measure distances and gravity precisely enough to notice that the curvature of the earth was not the same at different places. This meant that the earth was not a sphere after all! What else could it be?

30 Observation. Traveling north by day, a man's shadow at noon becomes longer.

Realizing that the earth is curved, people jumped to the conclusion that it is curved equally all over, like a sphere.

Our observations suggest that the earth is like a sphere. The next question then arises: how big is that sphere?

Eratosthenes, who lived in Alexandria, Egypt, in the third century B.C., was a genius who computed the size of the earth from watching a sun dial cast its shadow at noon on a certain day, the summer solstice in June. Of course, he also knew a few other things:

1. First he knew that there was a place called Syene (now Aswan) south of Alexandria, where the sun's rays reached the bottom of a deep well at noon of that same day.

2. He knew how many days it took a camel caravan to travel the distance between Syene and Alexandria.

3. He also knew some geometry.

For those who remember high school geometry, it will be easy to follow Eratosthenes' reasoning from Figure 4. The others will just have to believe this famous story about the "father of geodesy".
When watching the shadow of the pointer on the spherical bowl of the sun dial, Eratosthenes estimated that the length of the shadow, a circular arc subtended by the angle $\alpha$ of the sun's rays, was the fiftieth part of a full circle. Similarly, the terrestrial arc traveled by the camels (5000 stadia, subtended by an equal angle $\alpha$ at the center of the earth) must be the fiftieth part of the earth's circumference. Thus, the latter must be 250,000 stadia.

A stadium was a length unit in antiquity. There were several different stadia in use, and the experts do not know which one Eratosthenes used. If it was about 1/10 of a nautical mile, then Eratosthenes' result was about 16% too big. But the actual number is really beside the point. The important and amazing achievement is the method of combining astronomic angles with measured distances. This basic method was used for centuries with the only improvements being made in the accuracy of the angle and distance measurements.

In the 17th century, it was possible to measure distances and gravity precisely enough to notice that the curvature of the earth was not the same at different places. This meant that the earth was not a sphere after all! What else could it be?
IF NOT A SPHERE, WHAT ELSE?

FIGURE 5

The French had measured distances in France, and they were convinced that the earth was pointed towards the poles, like an egg. The British had Newton's new theory of gravitation and they were just as convinced that the earth was flattened at the poles, like a grapefruit. An international dispute arose when the so-called "earth elongators" were pitted against the "earth flatteners". The French Academy of Sciences decided in a magnificently simple way to solve the argument by sending one expedition to Lapland in the north, and another to Peru near the equator, to measure and compare arcs. If a 1° arc in the north were shorter than a 1° arc near the equator, the French would be right. If the arc in the north were longer, the British would be right. As it turned out, the British were right - the earth was flatter at the poles. A flattened earth model, or oblate ellipsoid, has been used since.

These two famous expeditions attracted the attention of the educated world. The French dramatist, poet and reformer Voltaire, well known for his witty and skeptical quips, commented about the protagonists; Newton for the British versus the Cassinis, a family of geodesists, for the French. When the leader of the Lapland expedition came home, Voltaire praised him as the man "who flattened the earth and the Cassinis". Much later when the leader of the more strenuous Peru expedition returned, Voltaire commented: "You have found by prolonged toil, what Newton had found without even leaving his home."
In the early 1950's AMS (The Army Map Service) added exciting new information about the southern hemisphere. A field party in the Sudan completed a missing link in the triangulation along the 30th meridian through Africa and IAGS (The Inter-American Geodetic Survey) completed a long triangulation arc through Central and South America. These two long arcs from way in the north to way down south, more than 1000 long, were analyzed at AMS and a new Figure of the Earth was derived which was somewhat smaller than had been internationally thought before.

Another exciting piece of information was added by AMS in 1958 at the time when the first satellites went up. It was found that the earth was a little less flattened than had been thought before.

If an ellipse is rotated about its minor axis, it forms an ellipsoid of revolution. The customary ellipsoidal earth model has its minor axis parallel to the rotational axis of the earth. The size of such an ellipsoid is usually given by the length of the two semi-axes or by the semi-major axis and the flattening.
In the early 1950's AMS (The Army Map Service) added exciting new information about the southern hemisphere. A field party in the Sudan completed a missing link in the triangulation along the 30th meridian through Africa and IAGS (The Inter-American Geodetic Survey) completed a long triangulation arc through Central and South America. These two long arcs from way in the north to way down south, more than 100° long, were analyzed at AMS and a new Figure of the Earth was derived which was somewhat smaller than had been internationally thought before.

Another exciting piece of information was added by AMS in 1958 at the time when the first satellites went up. It was found that the earth was a little less flattened than had been thought before.
THREE SURFACES

Actually, things are quite complicated. When we talk about a pear-shape or an ellipsoid, we obviously do not mean the shape produced by the mountains and valleys, the topography. Since we can measure the elevations of places above sea level (this is what is recorded on topographic maps), we can discount them and inquire into the shape of what is left: that is, the sea-level surface itself, as if it were extended from the sea shore into the land areas without those elevations above it. This sea-level surface is also called the GEOID. The shape of the geoid is what we mean by the Figure of the Earth.

We have found from many measurements that the shape of this geoid is very irregular as compared with an ellipsoid, and we describe these irregularities by the distances from the much smoother ellipsoid. These distances are called GEOIDAL HEIGHTS. Thus we distinguish three surfaces: the topography, the geoid, and the ellipsoid. Topographic maps give the elevations above sea level (the geoid). Geoidal maps give the geoidal heights in relation to the ellipsoid. Both together give the total height of the topography above the ellipsoid at any point.

When the satellite data were analyzed further, it turned out that the nearest point in a satellite's orbit, the perigee, was always nearer to the earth when the satellite was over the northern hemisphere than when it was over the southern hemisphere. This indicates an asymmetry in the earth's shape. It is a little narrower in the north than in the south.

Once, one had thought that the earth was a sphere, and then it seemed to be rather like a grapefruit. Now we found that it was slightly different from a grapefruit, rather like a pear.
Actually, things are quite complicated. When we talk about a pear-shape or an ellipsoid, we obviously do not mean the shape produced by the mountains and valleys, the topography. Since we can measure the elevations of places above sea level (this is what is recorded on topographic maps), we can discount them and inquire into the shape of what is left: that is, the sea-level surface itself, as if it were extended from the sea shore into the land areas without those elevations above it. This sea-level surface is also called the GEOID. The shape of the geoid is what we mean by the Figure of the Earth.

We have found from many measurements that the shape of this geoid is very irregular as compared with an ellipsoid, and we describe these irregularities by the distances from the much smoother ellipsoid. These distances are called GEOIDAL HEIGHTS.

Thus we distinguish three surfaces: the topography, the geoid, and the ellipsoid. Topographic maps give the elevations above sea level (the geoid). Geoidal maps give the geoidal heights in relation to the ellipsoid. Both together give the total height of the topography above the ellipsoid at any point.
The geoid with its irregular ups and downs makes one think rather of an irregular potato than a pear. To describe its shape, we use an ellipsoid as an approximation, but we have to pick one of the right size and shape. In the figure above an ellipsoid that fits very well, for instance, in America, does not necessarily fit in Europe. Some of these ellipsoids and areas where in use are as follows:

Clarke 1866 (North and Central America, Greenland)
International 1924 (Hayford 1909) (Europe, individual states in South America)
Modified Clarke 1880 (Africa)
Everest 1830 (India, Southeast Asia, Indonesia)
Bessel 1841 (China, Korea, Japan)
Krasovskiy 1942 (U.S.S.R. and adjacent countries)
Australian National Spheroid 1965 (Australia)
South American Ellipsoid 1969 (South America)
THE EARTH'S ATTRACTION

CHAPTER II THE GRAVITY FIELD OF THE EARTH

The earth's attraction--called gravity--causes things to fall. Remember the story about Newton sitting under an apple tree? When an apple fell and hit him, it started him thinking of a new theory of gravity.

A heavy plumb bob, suspended by a string, is attracted by the earth and therefore pulls that string into a straight downward (vertical) direction.

Gravity pulls on each molecule of water in a glass, and the water arranges itself to form a level (horizontal) surface. These two directions, the vertical and the horizontal, are given by nature. They are very useful in engineering and surveying. We use either the plumb line or the level to determine them, for instance, to secure straight walls, horizontal floors, or for grading roads, controlling river beds, and so on.

When the artificial satellites were launched into orbit in the late 1950's, an earth model fitting the world as a whole was needed for the manned space flights Mercury, Gemini, and Apollo. AMS's Mercury Datum (Fischer Ellipsoid 1960) was picked by NASA and DoD for this program.

Recently, AMS (now called TOPOCOM) computed an updated version, a 1968 modification of the Mercury Datum (Fischer Ellipsoid 1960). Its semi-major axis is 6,378,150 meters and its flattening is 1/298.3. The geoidal heights are shown on the figure above in meters. They range from about 80 meters below the ellipsoid, to about 60 meters above.
In surveying, the level determines the relative height of two places. A measuring rod is set up at the seashore and at the next point inland. Markings are determined on the two rods at the same level. The difference in the rod readings gives the height of the second ground point as compared with the first. By repeating this leveling procedure step by step from the ocean shore across the land we can determine the elevation of any point above sea level.

You have now learned the story of our developing knowledge of the size and shape of the earth. The shape of the earth is partly attributed to the force of gravity. The way we study the gravity field is the subject of the next chapter.
The mountains and valleys, water and land, and the different kinds of rock complicate geodetic theory because they affect gravity and gravity affects the shape of the geoid.

A mountain mass near the plumb line (or near a level) will pull it out of direction. A very dense mass buried underneath the surface will do the same. At the seashore the difference between the denser land mass and the less dense water will have the same effect.

We only see the plumb line (or the level) as it is affected by the pull of all the irregular, known and unknown masses around it, but we do not see offhand by how much it has been deflected by these masses. This DEFLECTION OF THE VERTICAL is the basic problem that affects our computations unless we find ways to correct for it.

As we have seen, gravity is very useful for finding positions and elevations, but it is also very misleading to us by falsifying positions and directions. The science of geodesy is necessary to explain and evaluate the influence of gravity and to determine corrections to apply to our measurements.

The earth's attraction--called gravity--causes things to fall. Remember the story about Newton sitting under an apple tree? When an apple fell and hit him, it started him thinking of a new theory of gravity.

A heavy plumb bob, suspended by a string, is attracted by the earth and therefore pulls that string into a straight downward (vertical) direction.

Gravity pulls on each molecule of water in a glass, and the water arranges itself to form a level (horizontal) surface.

These two directions, the vertical and the horizontal, are given by nature. They are very useful in engineering and surveying. We use either the plumb line or the level to determine them, for instance, to secure straight walls, horizontal floors, or for grading roads, controlling river beds, and so on.
A ball thrown into the air will eventually fall down due to the attraction of the earth. Its path depends on the power and direction of the throw, which is gradually overcome by the force of gravity. A misjudgment of either one will make the ball miss its goal.

Likewise, the path of a missile can be computed from the power and direction of the thrust, which is gradually overcome by the force of gravity. If the gravity is different from what we think it is, our computations may give incorrect results. At launch, the missile may also be deflected into another direction because the vertical is not where we thought it was. During flight, the missile is subject to the pull of known and unknown masses and may be pulled out of its pre-computed trajectory. That makes it miss the target.

Satellites are also subject to these irregular gravitational pulls which have a tendency to change their predicted orbits. Unless we have a thorough understanding of the nature of these gravitational irregularities which cause orbital perturbations, we cannot successfully predict future orbits. This makes such procedures as spacecraft rendezvous and docking difficult and affects our ability to use satellites for geodetic purposes.

In surveying, the level determines the relative height of two places. A measuring rod is set up at the seashore and at the next point inland. Markings are determined on the two rods at the same level. The difference in the rod readings gives the height of the second ground point as compared with the first. By repeating this leveling procedure step by step from the ocean shore across the land we can determine the elevation of any point above sea level.
Gravity holds the water to the earth and our feet to the ground so we don't fall off into space. About five-sevenths of the earth's surface is covered by oceans. The level surface which coincides with mean sea level is called the GEOID. Other similar level surfaces can be imagined at any elevation, for example, the water surface of mountain lakes.

The higher a level surface is above the geoid, the further removed it is from the irregularities in the earth's structure; thus the warping will be less pronounced.

Satellites sample the earth's external gravity field along their specific orbits at great heights above the geoid. From these samples a gravity model is derived which helps us predict the gravity effect on other orbits.

In geodesy, the level and the plumb line help us determine where we are on the earth. The angle between the vertical and the equator is approximately our latitude, as seen in the geometry of the figure above.

The opposite of the DOWN direction is the UP direction. The point directly overhead is the ZENITH. We look for the north polestar and determine its angular distance in the sky from our zenith or from our horizon. The height of the north polestar above the horizon, i.e., the angle between its direction and the level, is equal to our latitude on the globe.

This is only true, however, for a perfectly homogeneous earth. In reality the earth is not that perfect, and nature has a way of fooling us, as you will see.
FIGURE 16

The mountains and valleys, water and land, and the different kinds of rock complicate geodetic theory because they affect gravity and gravity affects the shape of the geoid.

A mountain mass near the plumb line (or near a level) will pull it out of direction. A very dense mass buried underneath the surface will do the same. At the seashore the difference between the denser land mass and the less dense water will have the same effect.

We only see the plumb line (or the level) as it is affected by the pull of all the irregular, known and unknown masses around it, but we do not see offhand by how much it has been deflected by these masses. This DEFLECTION OF THE VERTICAL is the basic problem that affects our computations unless we find ways to correct for it.

As we have seen, gravity is very useful for finding positions and elevations, but it is also very misleading to us by falsifying positions and directions. The science of geodesy is necessary to explain and evaluate the influence of gravity and to determine corrections to apply to our measurements.
Astrogeodetic methods determine deflections of the vertical directly by comparing positions obtained from studying stars with those of triangulation surveys. From these deflections a detailed shape of the geoid can be derived. This method is limited to the land areas of the earth.

The figure above shows deflections along the Andes Mountains in South America. Their magnitudes and their directions reflect the steep slope of the topography from the high mountain ranges to the ocean shore and down to the deep ocean trenches. In the flat parts of Argentina the deflections are small and random.

![HIT OR MISS](image)

**FIGURE 17**

A ball thrown into the air will eventually fall down due to the attraction of the earth. Its path depends on the power and direction of the throw, which is gradually overcome by the force of gravity. A misjudgment of either one will make the ball miss its goal.

Likewise, the path of a missile can be computed from the power and direction of the thrust, which is gradually overcome by the force of gravity. If the gravity is different from what we think it is, our computations may give incorrect results. At launch, the missile may also be deflected into another direction because the vertical is not where we thought it was. During flight, the missile is subject to the pull of known and unknown masses and may be pulled out of its pre-computed trajectory. That makes it miss the target.

Satellites are also subject to these irregular gravitational pulls which have a tendency to change their predicted orbits.

Unless we have a thorough understanding of the nature of these gravitational irregularities which cause orbital perturbations, we cannot successfully predict future orbits. This makes such procedures as spacecraft rendezvous and docking difficult and affects our ability to use satellites for geodetic purposes.
In summary, the three methods of studying the effects of the earth's gravity field are:

- **Satellite Gravity**, from orbits
- **Gravimetry**, from surface gravity
- **Astrogeodetic Deflections**, from triangulation

All of these three methods have their advantages and disadvantages, their special applications and their limitations. Depending on the purpose, the required detail and accuracy, one or another or a combination of methods will be used. With this information we should be able to accurately locate any point on the earth.

**CHAPTER III - POINT POSITIONING**

When driving along or hiking in unknown territory, your location can be of vital importance to you. An answer such as "You are right next to a huge anthill" will not be very helpful to find your way home, even if it is correct and useful from another aspect. You need an answer in relation to some known reference such as the nearest town or highway. You will want to know how far you are from that town; and in what direction you should turn off from your present path - or from the north direction.

Geodesy studies the effect of these irregular masses in the earth so that corrections can be made for them. For that purpose we start with a smooth and homogeneous earth model, such as a sphere or, more customarily, an ellipsoid. The real earth is studied in its deviations from such a model. If the direction of the force of gravity is different from that for the model, we call it a deflection of the vertical, measured in seconds of arc. If the intensity of the force of gravity is different from that on the model, we call it a gravity anomaly, measured in milligals.

The deflection of the vertical is the angle between the observed vertical direction of the plumb line and the normal to the ellipsoidal model. Considering the UP direction, it is the angle between the astronomic and geodetic zenith. Since the plumb line and the level surface form right angles with each other, a deflection of the vertical causes a corresponding warping of the level surface. The angle of the deflection equals the angle of the warping. Thus, the shape of that warped surface can be traced out, step by step, if the deflections are known.
Gravity holds the water to the earth and our feet to the ground so we don't fall off into space. About five-sevenths of the earth's surface is covered by oceans. The level surface which coincides with mean sea level is called the GEOID. Other similar level surfaces can be imagined at any elevation, for example, the water surface of mountain lakes.

The higher a level surface is above the geoid, the further removed it is from the irregularities in the earth's structure; thus the warping will be less pronounced.

Satellites sample the earth's external gravity field along their specific orbits at great heights above the geoid. From these samples a gravity model is derived which helps us predict the gravity effect on other orbits.
When property lines are established by land surveying which starts at different points in the country, it may happen that when the two systems meet, they won’t fit together. The two survey nets are based on the different datum points.

When America entered World War II, the maps we had of France and Germany did not fit together at the boundary, because they were based on different geodetic systems with different datum points.

After the war, AMS assisted the European countries achieve an overall adjustment for a uniform continental datum, the European Datum of 1950. There are several other areas where we do not have good maps, or where those we have do not fit together reliably, because they are based on different geodetic systems. That’s another reason why Geodesy is necessary.

Gravimetry, as distinct from satellite gravity, deals with surface gravity. It measures directly the intensity of gravity at surface points, and thus gives detailed information of its variation from place to place. This is particularly useful for prospecting and for near surface effects.

Deflections of the vertical and the geoidal shape can be computed from gravity anomalies, provided there is a worldwide coverage of gravity measurements. Surface gravity data is only available, however, for the areas shown in red in the figure above. It is easy to see how incomplete our knowledge really is. Theories about extrapolation, or projection, into unobserved areas must take the place of actual observations at this time.
Astrogeodetic methods determine deflections of the vertical directly by comparing positions obtained from studying stars with those of triangulation surveys. From these deflections a detailed shape of the geoid can be derived. This method is limited to the land areas of the earth.

The figure above shows deflections along the Andes Mountains in South America. Their magnitudes and their directions reflect the steep slope of the topography from the high mountain ranges to the ocean shore and down to the deep ocean trenches. In the flat parts of Argentina the deflections are small and random.
In summary, the three methods of studying the effects of the earth's gravity field are:

- SATELLITE GRAVITY, from orbits
- GRAVIMETRY, from surface gravity
- ASTROGEODETC DEFLECTIONS, from triangulation

All of these three methods have their advantages and disadvantages, their special applications and their limitations. Depending on the purpose, the required detail and accuracy, one or another or a combination of methods will be used. With this information we should be able to accurately locate any point on the earth.

CHAPTER III - POINT POSITIONING

When driving along or hiking in unknown territory, your location can be of vital importance to you. An answer such as "You are right next to a huge anthill" will not be very helpful to find your way home, even if it is correct and useful from another aspect. You need an answer in relation to some known reference such as the nearest town or highway. You will want to know how far you are from that town; and in what direction you should turn off from your present path - or from the north direction.
Instead of describing the position of a point P in terms of latitude, longitude, and height as we do in surveying and practical applications in the field, we can also use a Cartesian coordinate system in $x, y, z$. The latter is often used within the process of computations, especially in satellite computations.

One coordinate system can be converted into another by mathematical conversion formulas. But if one is incorrect, for instance, through the neglect of a large geoidal height, then the other will be incorrect also. Even if the positional inaccuracy of a specific point may seem tolerable in itself, it may snowball into significant errors in certain applications.

In 1920 the Zero Milestone Monument was established on the Ellipse in Washington, D. C. It is the starting point for the measurement of distances over all the highways radiating from Washington. Driving towards or from Washington the milestones can be seen along the way, giving distances from the Ellipse.

Figure 23, a map of "The Nation's Capital, Washington, D. C." printed by AMS in 1965 shows the location of the Zero Milestone at the edge of the Ellipse, and distance-circles around it at one-mile intervals. At the bottom of this map is a simple compass card, such as the one on the top of the Zero Milestone giving the north-south direction, a perpendicular to it representing the east-west direction, and further subdivisions for the northeast - southwest and northwest - southeast directions.

Notice on the map, for instance, that Massachusetts Avenue leads northwest from the Zero Milestone, and that half-way between the 2-mile and 3-mile circle on Massachusetts Avenue there is the U. S. Naval Observatory. The position of the Observatory is therefore 2 1/2 miles northwest of the Zero Milestone Monument on the Ellipse. In technical language, we might call the Zero Milestone a datum point.
By observing the stars, the astronomic position of a point P is obtained by relating the vertical direction at P, the astronomic zenith (Z_a), to the celestial sphere. The vertical direction is defined by a plumb line which passes through point P or by an imaginary line drawn perpendicular to the axis of a horizontal bubble level.

By performing triangulation, trilateration and/or traverse in a geodetic survey, the geodetic position of a point P may be obtained. The position is based on a computation using a specific reference ellipsoid. It is defined by the normal to the ellipsoid at P, the geodetic zenith (Z_g).

These two zenith directions at a point P are usually not identical. The angle between them is the astrogeodetic deflection of the vertical; it equals the angle of intersection between the geoidal and ellipsoidal surfaces at point P.
Since the angle between the astronomic and the geodetic zenith (the astrogeodetic deflection) equals the angle of intersection between the geoidal and ellipsoidal surfaces, it is possible to compute the separation between these two surfaces step by step from the deflection values. Thus a consistent system of geodetic positions can be derived.

In surveying, we need more accuracy in determining the distance and direction from one point to another. If the two points are far away from each other, a set of intermediate points is established. Some of the various procedures used are:

1. Triangulation which establishes a chain of triangles. The procedure starts from the given point A with a carefully measured base line and its azimuth (its direction from north). Then all the other angles in the chain of triangles are measured, and from them the final distance and direction from A to B may be computed.

2. Trilateration which involves measuring the sides of a chain of triangles or other polygons. From them the distance and direction AB may be computed.

3. Traverse which involves measuring distances, and angles between them, without triangles for the purpose of computing the distance and direction from A to B.

For greater accuracy in a triangulation chain, more than one base line may be established. Trilateration is strengthened by some astronomic or solar azimuth measurements or a connecting traverse measurement.
The astrogeodetic method can connect all points of a land mass into a consistent geodetic system of positions, but it cannot span the oceans, unless the land masses are close together. This can be done by satellite geodesy.

There are different types of satellite techniques; some primarily measure directions, others measure distances.

When property lines are established by land surveying which starts at different points in the country, it may happen that when the two systems meet, they won't fit together. The two survey nets are based on the different datum points $O_I$ and $O_{II}$.

When America entered World War II, the maps we had of France and Germany did not fit together at the boundary, because they were based on different geodetic systems with different datum points.

After the war, AMS assisted the European countries achieve an overall adjustment for a uniform continental datum, the European Datum of 1950. There are several other areas where we do not have good maps, or where those we have do not fit together reliably, because they are based on different geodetic systems.

That's another reason why Geodesy is necessary.
DIRECTIONS BY SATELLITE

FIGURE 32

In geometric-optical satellite systems (BC-4 and Baker-Nunn Cameras) the moving satellite is photographed against a star background from two ground stations A and B simultaneously, which fixes the pair of directions AS and BS for many satellite positions.

These many pairs of directions determine as many planes through the same as yet unknown straight line AB, so that the direction from a known station A to an unknown station B can be computed.

By repeating this determination of directions from known to unknown stations a world net of stations can be formed, comparable to a huge triangulation net.

THE HORIZONTAL POSITION

FIGURE 26

For geodetic systems large enough to be affected by the curvature of the earth, we use an ellipsoidal earth model and designate on it the position of any point, in terms of latitude, longitude, and height, with zero starting references for each.

LATITUDE refers to a set of circles parallel to the equator, called parallels, the way you slice a tomato. The numbering starts at the equator and goes to 90° north and 90° south.

LONGITUDE refers to a set of ellipses (or circles if the earth model is taken as a sphere) called meridians, the way you divide an orange. Their numbering starts customarily with the meridian through Greenwich in England and goes either to 360° eastward or to 180° east and 180° west.

The HORIZONTAL POSITION of a point is at the intersection of a parallel and a meridian and is therefore expressed in terms of latitude and longitude.

The complete position must include a third value, the vertical position. This is the height of the point above or below the ellipsoid. It must also include the specifications of the ellipsoid itself. The latter, together with the complete position of a particular point, the datum point, is called a GEODETIC DATUM.
FIGURE 27

The figure above is a meridional section through the earth. The height $H$ of the point $P$ above the ellipsoid is measured along the normal to the ellipsoid. The point $P'$ on the ellipsoid itself has a zero height. $P$ and $P'$ have the same horizontal position (same latitude and longitude), but different vertical positions. The total height $H$ of a point $P$ above the ellipsoid is not directly observed. Leveling determines one part, the elevation $h$ above mean sea level, which is also called the GEOID. The other part, the GEOIDAL HEIGHT $N$, that is the separation between the geoid and the ellipsoid, must be computed separately.

Failure to compute the geoidal height will make the height coordinate of the point $P$ incorrect. Theoretically, geoidal heights may vary between $+100$ meters and $-100$ meters, but in practice some geodetic systems may have several hundred meters of geoidal heights in some areas. For example, the old South American Datum had geoidal heights of about 300 meters in Chile. Therefore, South America accepted a new continental datum in 1969, including a new reference ellipsoid, where geoidal heights are less than 50 meters. In Southeast Asia, the Indian Datum is still used with more than 300 m of geoidal heights, although better fitting datums have been computed.

Geodesy can correct these problems.
In a dynamic (moving) satellite system the orbit of the satellite is determined first from equations of motion which relate the satellite's position to the center of the earth and include the orbit's perturbations due to an estimate of the earth's irregular gravity field. One type of dynamic satellite system is named Doppler. The Doppler satellite is tracked from a ground station P, and its nearest distance is deduced from the Doppler Effect of its approach and departure. This effect, named after the Austrian physicist Doppler, is the same principle by which the siren of an ambulance seems to sound higher to us as it approaches getting lower as it recedes.

By tracking several satellites, the position of the ground station P is linked to their orbits and in turn to the center of the earth, yielding geocentric coordinates. If two ground points are thus linked to the same geocentric coordinate system, their relative position to each other can be computed.

Instead of describing the position of a point P in terms of latitude, longitude, and height as we do in surveying and practical applications in the field, we can also use a Cartesian coordinate system in x, y, z. The latter is often used within the process of computations, especially in satellite computations.

One coordinate system can be converted into another by mathematical conversion formulas. But if one is incorrect, for instance, through the neglect of a large geoidal height, then the other will be incorrect also. Even if the positional inaccuracy of a specific point may seem tolerable in itself, it may snowball into significant errors in certain applications.
By observing the stars, the astronomical position of a point P is obtained by relating the vertical direction at P, the astronomical zenith (Z\textsubscript{a}), to the celestial sphere. The vertical direction is defined by a plumb line which passes through point P or by an imaginary line drawn perpendicular to the axis of a horizontal bubble level.

By performing triangulation, trilateration and/or traverse in a geodetic survey, the geodetic position of a point P may be obtained. The position is based on a computation using a specific reference ellipsoid. It is defined by the normal to the ellipsoid at P, the geodetic zenith (Z\textsubscript{g}).

These two zenith directions at a point P are usually not identical. The angle between them is the astrogodetic deflection of the vertical; it equals the angle of intersection between the geoidal and ellipsoidal surfaces at point P.
Various navigation methods are used to determine a ship's position at sea. Most ancient is celestial navigation, that is, deriving one's position from the sun and the stars. The longitude determination posed the toughest problem throughout the centuries until a reliable portable chronometer was constructed in the 18th Century. A chronometer keeps track of the passing of celestial (star time) and is used for determining the astro­nomic and solar position of a ship at sea. Dead reckoning is a way of keeping track of the distance and direction traveled since leaving port.

Today Doppler satellites are used in connection with a system of tracking stations (TRANET). Most common among the various electronic techniques presently used to establish the position of ships and planes with reference to shore stations are hyperbolic systems (LORAN, DECCA, OMEGA). Simultaneous signals from a master station and at least two slave stations are received aboard ship and interpreted as differences in distance from the master station and each slave station. Thus the ship finds itself on numbered lanes of at least two sets of confocal hyperbolas with the shore stations as foci. The intersection of the appropriate lanes gives the ship's position.

Since the angle between the astronomic and the geodetic zenith (the astrogeodetic deflection) equals the angle of intersection between the geoidal and ellipsoidal surfaces, it is possible to compute the separation between these two surfaces step by step from the deflection values. Thus a consistent system of geodetic positions can be derived.
The astrogeodetic method can connect all points of a land mass into a consistent geodetic system of positions, but it cannot span the oceans, unless the land masses are close together. This can be done by satellite geodesy.

There are different types of satellite techniques; some primarily measure directions, others measure distances.
In summary, there are many different topics that need to be studied for accurate POINT POSITIONING. For a specific problem, one must choose among several methods and give careful thought to questions such as these: Do the various methods really do what their proponents say they do? Are there ways of checking the results? Checking against what? Which method for which purpose? Could there be systematic errors? Does it matter or does it not? What exactly do we mean by a position?


These and similar questions must be investigated by Geodesy.

In geometric - optical satellite systems (BC-4 and Baker-Nunn Cameras) the moving satellite is photographed against a star background from two ground stations A and B simultaneously, which fixes the pair of directions AS and BS for many satellite positions. These many pairs of directions determine as many planes through the same as yet unknown straight line AB, so that the direction from a known station A to an unknown station B can be computed.

By repeating this determination of directions from known to unknown stations a world net of stations can be formed, comparable to a huge triangulation net.