A New High Resolution Gravimetric Geoid Model for the United States Using the EGM96 Potential Coefficient Model

by

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Presentation

Undulations from Potential Coefficients
Height Anomaly vs Geoid Undulation
Mathematical consistency

EGM96 vs. Beta Models in the U.S.

The G96SSS and GEOID96 Geoid Models

Conclusions
Undulations from Potential Coefficients

Simplified Form:

\[
\zeta(r, \theta, \lambda) = \frac{GM}{r_\gamma} \sum_{n=0}^{\infty} \left( \frac{a}{r} \right)^n \sum_{m=-n}^{n} C_{nm} Y_{nm}(\theta, \lambda)
\]

Traditionally:

\[
N(\theta, \lambda) = \zeta(r_{\text{geoid}}, \theta, \lambda)
\]

\[
\Delta g(\theta, \lambda) = \Delta g(r_{\text{geoid}}, \theta, \lambda)
\]

Advantage: Mathematical compatibility of N and \(\Delta g\)
Disadvantage: Inaccurate undulation estimate

Recently:

\[
\zeta(\theta, \lambda) = \zeta(r = r_{\text{surface}}, \theta, \lambda)
\]

\[
N(\theta, \lambda) = \zeta(\theta, \lambda) + \frac{\Delta g_B}{\gamma} h
\]

Advantage: More theoretically correct
Compute undulations from coefficients alone
- Corrected from surface height anomaly to geoid undulation
- Compare against 2497 GPS/level benchmarks

EGM96 vs. Models X01-X05

<table>
<thead>
<tr>
<th>Model</th>
<th>Tilt</th>
<th>Azimuth</th>
<th>RMS about plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>X01</td>
<td>0.40 ppm</td>
<td>338</td>
<td>26.5 cm</td>
</tr>
<tr>
<td>X02</td>
<td>0.32 ppm</td>
<td>336</td>
<td>29.8 cm</td>
</tr>
<tr>
<td>X03</td>
<td>0.35 ppm</td>
<td>334</td>
<td>26.2 cm</td>
</tr>
<tr>
<td>X04</td>
<td>0.35 ppm</td>
<td>335</td>
<td>26.0 cm</td>
</tr>
<tr>
<td>X05</td>
<td>0.35 ppm</td>
<td>335</td>
<td>26.1 cm</td>
</tr>
<tr>
<td>EGM96</td>
<td>0.41 ppm</td>
<td>343</td>
<td>27.0 cm</td>
</tr>
</tbody>
</table>

!! 90% of this tilt is removed during the remove-compute-restore procedure !!
Undulation Differences, EGM96 - EGM-X05
Tests with Surface Data

- EGM96 vs. X05 difference shows features at $n=40$
- Peak-to-Peak magnitudes of 1 meter

Oklahoma Investigation ($34.5^\circ-38^\circ ; 259.5^\circ-262^\circ$):

1) Residual geoid undulations ($2' \times 2'$ grid, from 1-D spherical FFT):

<table>
<thead>
<tr>
<th></th>
<th>Ave</th>
<th>RMS about Ave</th>
</tr>
</thead>
<tbody>
<tr>
<td>X05 ($n=360$)</td>
<td>13.1 cm</td>
<td>12.9 cm</td>
</tr>
<tr>
<td>EGM96 ($n=360$)</td>
<td>18.5 cm</td>
<td>16.6 cm</td>
</tr>
</tbody>
</table>

2) Residuals wrt 16 GPS/BMs (Corrected for 43.4 cm NAVD88 bias):

<table>
<thead>
<tr>
<th></th>
<th>Ave</th>
<th>RMS about Ave</th>
</tr>
</thead>
<tbody>
<tr>
<td>X05 ($n=360$)</td>
<td>-9.1 cm</td>
<td>13.3 cm</td>
</tr>
<tr>
<td>EGM96 ($n=360$)</td>
<td>-17.5 cm</td>
<td>14.9 cm</td>
</tr>
</tbody>
</table>

✓ Surface data can provide checks on geopotential models
High Resolution Geoid Models

G96SSS:
- 1.8 Million gravity measurements (marine, land, altimetry)
- 30" DTED updated with Canadian Rockies data
- EGM96
- 1-D Spherical Stokes’ FFT for "remove-compute-restore"
- 2’ x 2’ spacing (2’ x 4’ in Alaska)
- ITRF94 (1996.0)

GEOID96:
- Begin with G96SSS
- 2951 GPS/Level Benchmarks
- Converts NAD83 (86) into NAVD88
- Relative to non-geocentric GRS-80 ellipsoid
(centered) G96SSS residuals wrt ITRF94/NAVD88 GPS/Level Benchmarks
Results / Conclusions

- Data detects differences between EGM96 and EGM-X05
  ➔ All but longest wavelength removable by FFT
- 2951 GPS/BMs & G96SSS show 15.5 cm RMS
  ➔ 45 cm NAVD88 bias
- GPS/BMs with G96SSS detect biases in GPS network
  ➔ Correlated with statewide GPS surveys north of 42°
- GEOID96 computed from G96SSS with collocation of 2951 GPS/BMs
- GEOID96 shows 5.5 cm Gaussian noise (GPS) with 2.5 cm correlated error (GPS/geoid) randomizing at 40 km