Practical Aspects of Strapdown Inertial Gravimetry

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Overview

- **Why Strapdown?**
- Kalman-Filter design
- A few practical ideas…
- System analysis
  - Observability
  - Error propagation simulations
- Thermal calibration
  - Motivation
  - A simple approach
  - Parametric approaches (BSC, BSSC)
  - Sample-based approach
- Test data sets
- Results

Why Strapdown?

- small
- "low" cost
- lightweight
- low power consumption
- autonomous operation
- little maintenance required
- vector gravimetry

- robust against turbulence
- allows gravimetry during altitude changes

Why not?

- **poor long-term stability**
  - do a bias adjustment per line?
  - do a drift adjustment per line?
  - linear interpolation based on errors of closure
  - sensor calibration

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Aconcagua

Kalman-Filter design *("P-approach")*

System State:

$$\{\delta\vec{p}^{\,n}, \delta\vec{v}^{\,n}, \delta\vec{\psi}^{\,n}, \delta\vec{b}_a^{\,b}, \delta\vec{b}_\omega^{\,b}, \delta\Delta\vec{g}^{\,n}\}^T$$

RC+RW

RC
RC+RW
1st order GM

3rd order GM

- 3D position
- 3D velocity
- 3D attitude
- 3D accelerometer bias
- 3D gyro bias
- 3D gravity disturbance
- 3D leverarm *(opt.)*

State transition model:

$$\mathbf{dx}_{k+1} = (\mathbf{I} + \mathbf{F}_k dt)\mathbf{dx}_k + \mathbf{w}_k$$

$$\mathbf{F}_{9x9} = \left[\begin{array}{c|c|c} \mathbf{F}_{11} & \mathbf{I} & \mathbf{0} \\ \mathbf{F}_{21} & \mathbf{F}_{22} & -[\mathbf{C}_b^n \mathbf{f}_{ib}^b \times] \\ \mathbf{F}_{31} & \mathbf{F}_{32} & -\mathbf{\Omega}_{in}^n \end{array}\right]$$

$$\mathbf{F}_{24x24} = \left[\begin{array}{c|cc|ccc} & \mathbf{0} & \mathbf{0} & \mathbf{0} & & \\ \mathbf{F}_{9x9} & -\mathbf{C}_b^n & \mathbf{0} & \mathbf{I} & \mathbf{0}_{9x3} & \mathbf{0}_{9x3} \\ & \mathbf{0} & -\mathbf{C}_b^n & \mathbf{0} & & \\ \hline \mathbf{0}_{6x9} & -\beta_a\mathbf{I} & \mathbf{0} & \mathbf{0}_{6x3} & \mathbf{0}_{6x3} & \mathbf{0}_{6x3} \\ & \mathbf{0} & -\beta_g\mathbf{I} & & & \\ \hline \mathbf{0}_{3x9} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0}_{3x9} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0}_{3x9} & \mathbf{0} & \mathbf{0} & -\beta_{dg}^3\mathbf{I} & -3\beta_{dg}^2\mathbf{I} & -3\beta_{dg}\mathbf{I} \end{array}\right]$$

Observation model: $\vec{z} = H \cdot \vec{x} + \vec{v}$

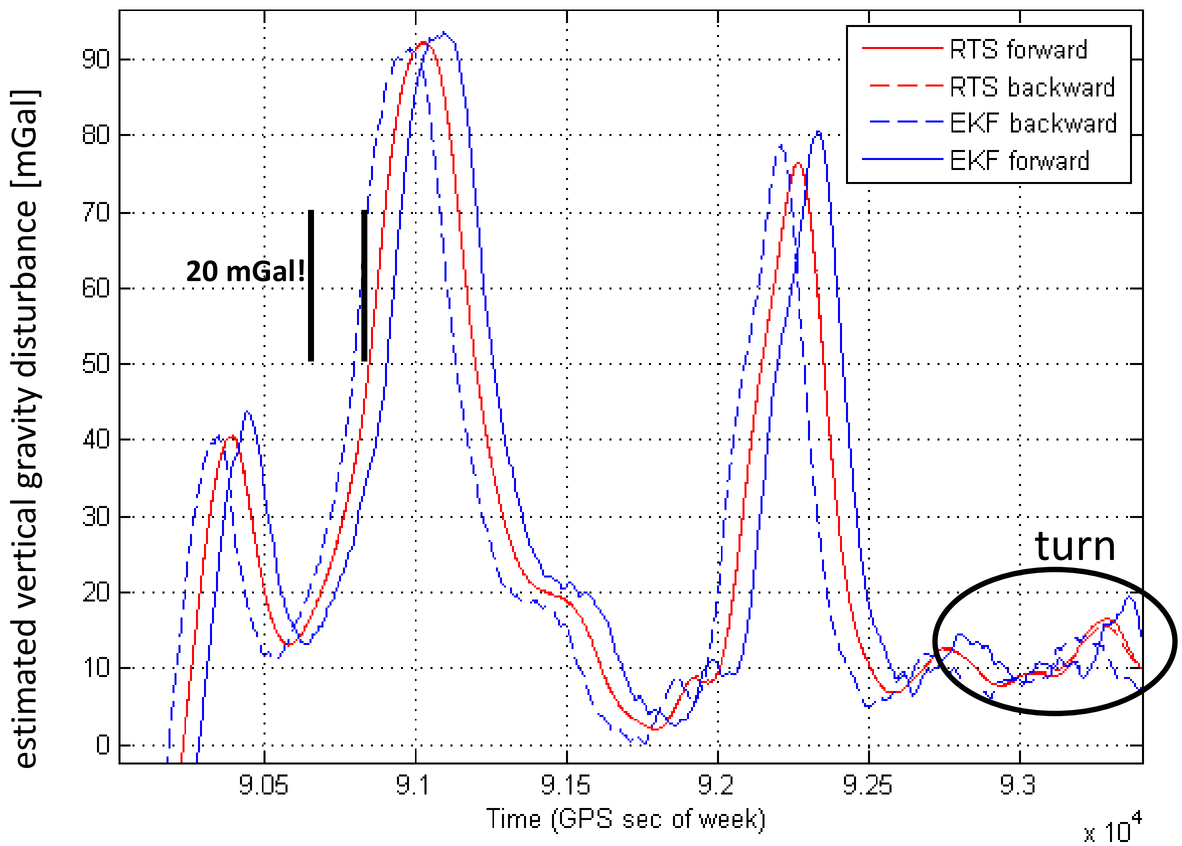
$$H_{\text{coordinate}} = \begin{bmatrix} I & 0 & [\vec{l}^{\,n}\times] & 0 & 0 & 0 \end{bmatrix}$$

$$H_{\text{velocity}} = \begin{bmatrix} 0 & I & [-C_b^n \Omega_{ib}^b \vec{l}^{\,b}\times] & 0 & C_b^n \cdot [\vec{l}^{\,b}\times] & 0 \end{bmatrix}$$?

Kalman Smoother

Why a Smoother?

Malaysia 2014 (DoY 237)



A few practical ideas…

gravity in initial and final aircraft position is usually known – it may be introduced as an observation here!

to enable the removal of a linear drift, the IMU must be running from start to end

Thermal drifts of accelerometers and FOG ➔ avoid temperature changes during the flight!

Let the device warm for > 5 hours before the flight, if possible.

If lever arm is <u>not</u> perfectly known, do not introduce it as such!

Internal IMU clocks are drifting! Precise time stamps are crucial (few msec). Clock drifts can also be temperature dependent!

ZUPT? Aircraft is shaking in parking positions, so $v \neq 0$!

GNSS antenna position is not <u>really</u> "static" in parking position!

rate of gravity change is related to <u>position change</u>, not time…

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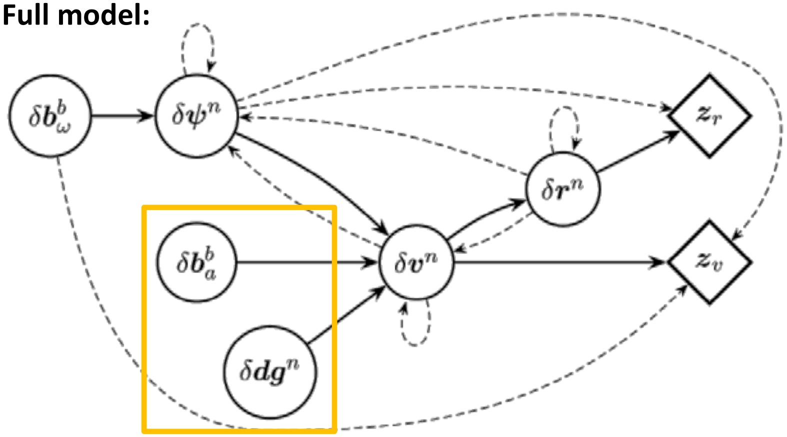
Aconcagua

System analysis (1/5)

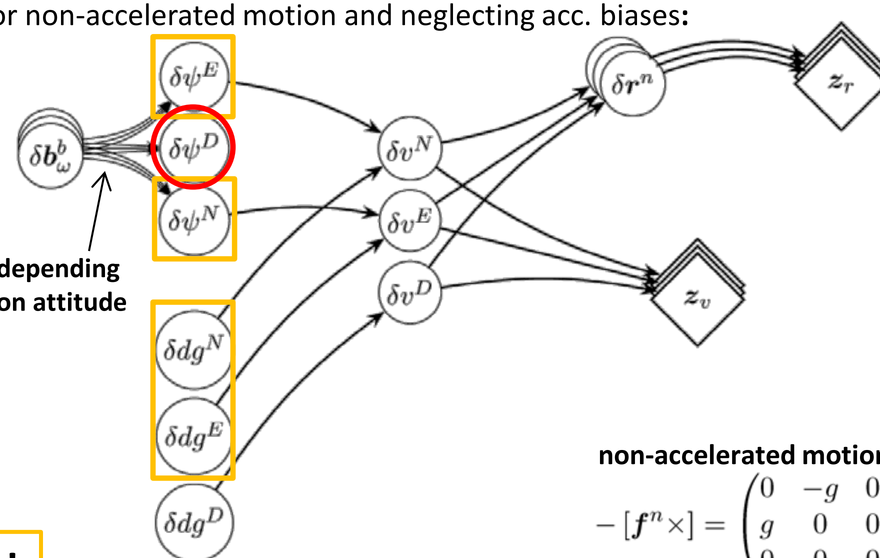
- Observability
  - Is the system state (fully) observable?
  - Or only a subspace?
  - Structure graph observability conditions:
    1. each state must have a path to an output node (= an observation)
    2. for any set of nodes A and its targets T(A): **|T(A)| ≥ |A|** (otherwise: *contraction*)

Full model:



contraction!

Simplified model,
for non-accelerated motion and neglecting acc. biases**:**



non-accelerated motion:

$$-[f^n \times] = \begin{pmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

System analysis (2/5)

- Observability (2): algebraic definition

$$\Xi = \begin{pmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_{k+1}\boldsymbol{\Phi}_k \\ \boldsymbol{H}_{k+2}\boldsymbol{\Phi}_{k+1}\boldsymbol{\Phi}_k \\ \vdots \\ \boldsymbol{H}_{k+N-1}\boldsymbol{\Phi}_{k+N-2}\boldsymbol{\Phi}_{k+N-3}\dots\boldsymbol{\Phi}_k \end{pmatrix}$$

$$r = n - \mathrm{rank}(\Xi^*)$$

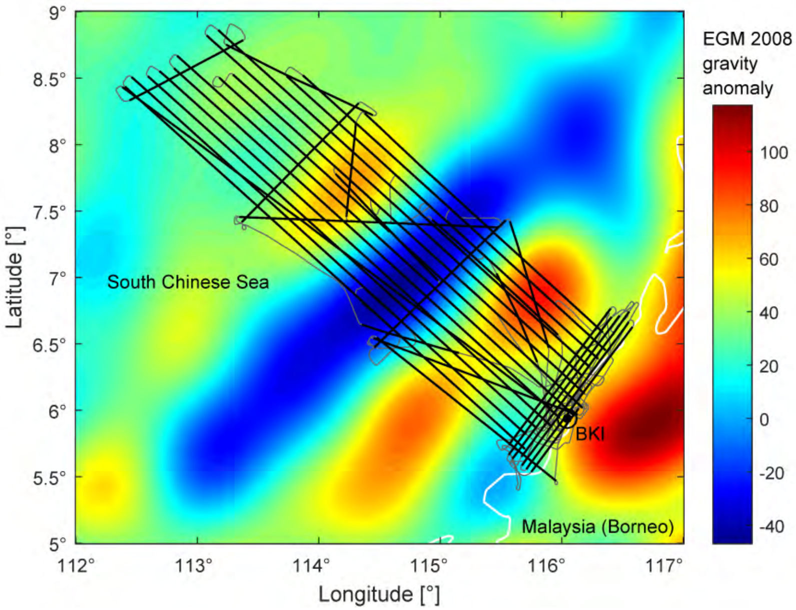
- coordinate and velocity observations
- evaluated for different sets of active states
- 4 basic scenarios for an aerogravity flight:

scenario	w/ attitude changes	w/ horizontal accelerations
S1	✗	✗
S2	✓	✗
S3	✗	✓
S4	✓	✓

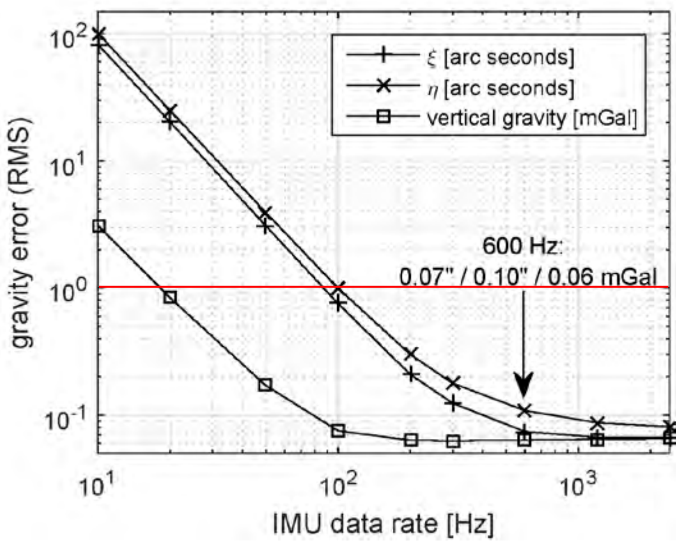
	active states				rank deficiency r for the simplified / full model			
	$\delta\boldsymbol{\psi}^n$ N/E/D	$\delta\boldsymbol{b}_a^b$ x/y/z	$\delta\boldsymbol{b}_\omega^b$ x/y/z	$\delta\boldsymbol{dg}^n$ N/E/D	S1	S2	S3	S4
0	● ● ○	○ ○ ○	○ ○ ○	○ ○ ○	0 / 0	0 / 0	0 / 0	0 / 0
1	● ● ●	○ ○ ○	○ ○ ○	○ ○ ○	1 / 0	1 / 0	0 / 0	0 / 0
2	● ● ●	● ● ○	○ ○ ○	○ ○ ○	3 / 1	1 / 0	1 / 0	0 / 0
3	● ● ●	○ ○ ●	○ ○ ○	○ ○ ○	1 / 0	1 / 0	0 / 0	0 / 0
4	● ● ●	● ● ●	○ ○ ○	○ ○ ○	3 / 1	1 / 0	1 / 0	0 / 0
5	● ● ●	○ ○ ○	● ● ●	○ ○ ○	2 / 2	1 / 0	1 / 1	0 / 0
6	● ● ●	● ● ○	● ● ●	○ ○ ○	4 / 4	1 / 0	2 / 2	0 / 0
7	● ● ●	○ ○ ●	● ● ●	○ ○ ○	2 / 2	1 / 0	1 / 1	0 / 0
8	● ● ●	● ● ●	● ● ●	○ ○ ○	4 / 4	1 / 0	2 / 2	0 / 0
9	● ● ●	○ ○ ○	○ ○ ○	● ● ○	3 / 1	3 / 1	1 / 0	1 / 0
10	● ● ●	● ● ○	○ ○ ○	● ● ○	5 / 3	3 / 1	3 / 2	1 / 0
11	● ● ●	○ ○ ●	○ ○ ○	● ● ○	3 / 1	3 / 1	1 / 0	1 / 0
12	● ● ●	● ● ●	○ ○ ○	● ● ○	5 / 3	3 / 1	3 / 2	1 / 0
13	● ● ●	○ ○ ○	● ● ●	● ● ○	4 / 4	3 / 2	2 / 2	1 / 1
14	● ● ●	● ● ○	● ● ●	● ● ○	6 / 6	3 / 2	4 / 4	1 / 1
15	● ● ●	○ ○ ●	● ● ●	● ● ○	4 / 4	3 / 2	2 / 2	1 / 1
16	● ● ●	● ● ●	● ● ●	● ● ○	6 / 6	3 / 2	4 / 4	1 / 1
17	● ● ●	○ ○ ○	○ ○ ○	○ ○ ●	1 / 0	1 / 0	0 / 0	0 / 0
18	● ● ●	● ● ○	○ ○ ○	○ ○ ●	3 / 1	1 / 0	1 / 0	0 / 0
19	● ● ●	○ ○ ●	○ ○ ○	○ ○ ●	2 / 1	1 / 0	1 / 1	0 / 0
20	● ● ●	● ● ●	○ ○ ○	○ ○ ●	4 / 2	1 / 0	2 / 1	0 / 0
21	● ● ●	○ ○ ○	● ● ●	○ ○ ●	2 / 2	1 / 0	1 / 1	0 / 0
22	● ● ●	● ● ○	● ● ●	○ ○ ●	4 / 4	1 / 0	2 / 2	0 / 0
23	● ● ●	○ ○ ●	● ● ●	○ ○ ●	3 / 3	1 / 0	2 / 2	0 / 0
24	● ● ●	● ● ●	● ● ●	○ ○ ●	5 / 5	1 / 0	3 / 3	0 / 0
25	● ● ●	○ ○ ○	○ ○ ○	● ● ●	3 / 1	3 / 1	1 / 0	1 / 0
26	● ● ●	● ● ○	○ ○ ○	● ● ●	5 / 3	3 / 1	3 / 2	1 / 0
27	● ● ●	○ ○ ●	○ ○ ○	● ● ●	4 / 2	3 / 1	2 / 1	1 / 0
28	● ● ●	● ● ●	○ ○ ○	● ● ●	6 / 4	3 / 1	4 / 3	1 / 0
29	● ● ●	○ ○ ○	● ● ●	● ● ●	4 / 4	3 / 2	2 / 2	1 / 1
30	● ● ●	● ● ○	● ● ●	● ● ●	6 / 6	3 / 2	4 / 4	1 / 1
31	● ● ●	○ ○ ●	● ● ●	● ● ●	5 / 5	3 / 2	3 / 3	1 / 1
32	● ● ●	● ● ●	● ● ●	● ● ●	7 / 7	3 / 2	5 / 5	1 / 1

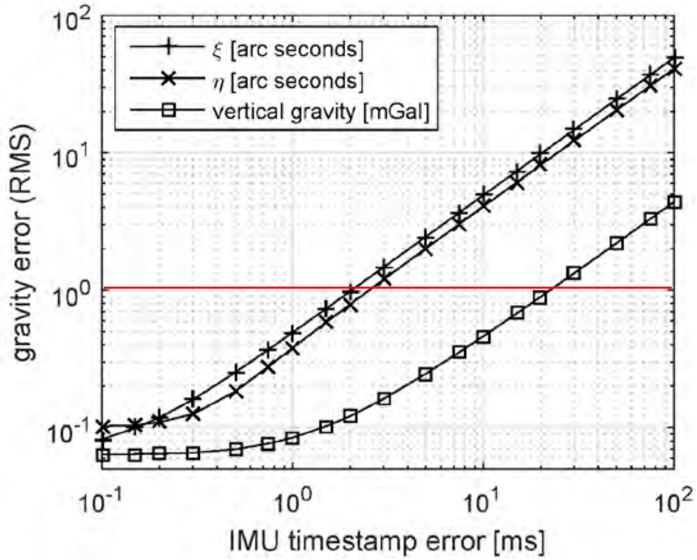
System analysis (3/5)

Simulation-based error propagation analysis

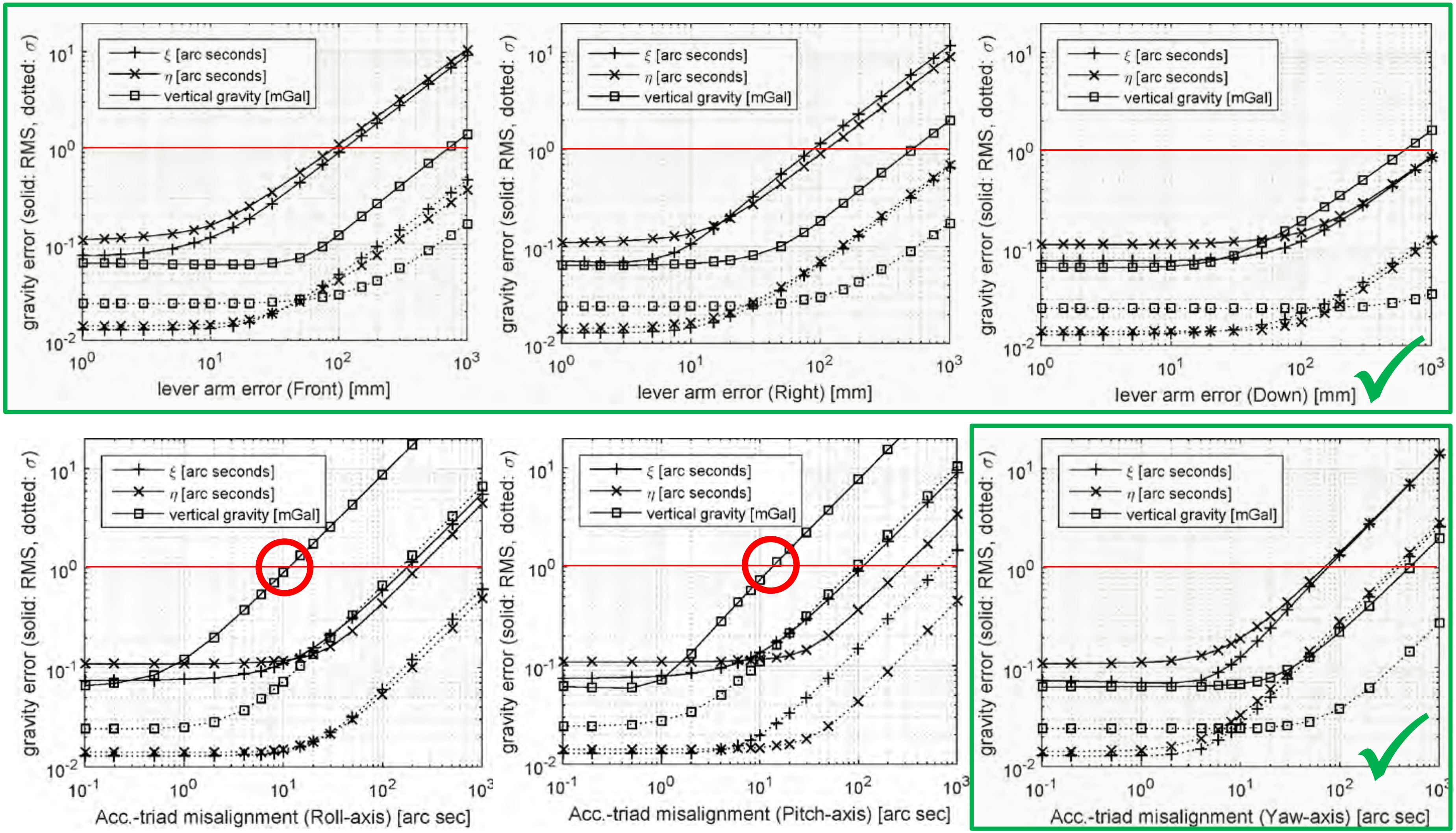


- 12 flights
- 13,000 km (9,600 km on lines)
- 41 h airborne (31 h on lines)
- average speed: 87 m/s
- used as basis for ground-truth simulation
  - spline-"imitation" of attitude and velocity
  - spline-"imitation" of EGM gravity disturbance
  - exact determination of sensor measurements
    - ➔ realistic flight characteristics





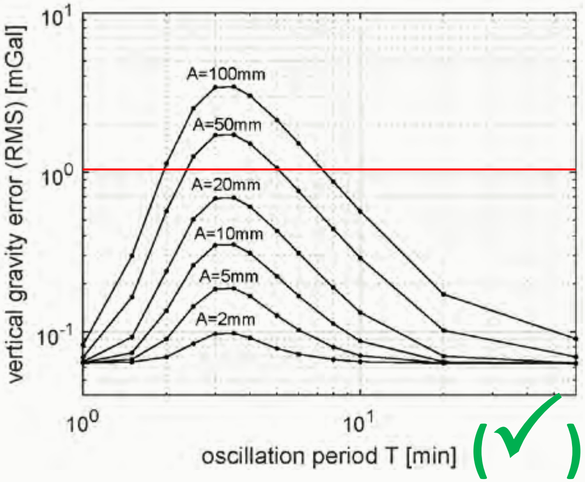
System analysis (4/5)



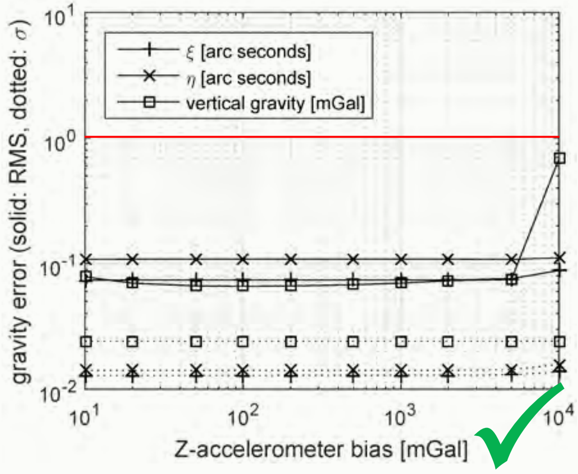
System analysis (5/5)

Vertical lever arm oscillation:

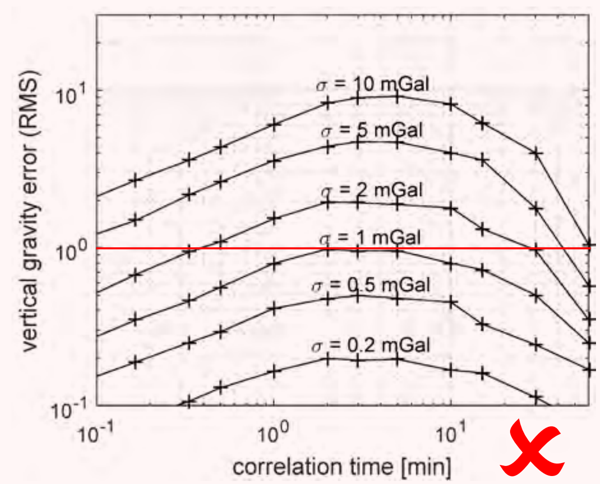
$$\Delta l^{\mathrm{Down}}(t) = A\sin(2\pi t/T)$$



Constant Z-accelerometer bias:



Correlated Z-accelerometer Gaussian noise:



- Accelerometer bias *changes* on the lines are (almost) undetectable
  - more precisely: Gravity *changes* and accelerometer bias *changes* are almost inseparable
- A reduction of in-flight accelerometer bias *changes* is desirable
- **Calibrations** may help to reduce such changes

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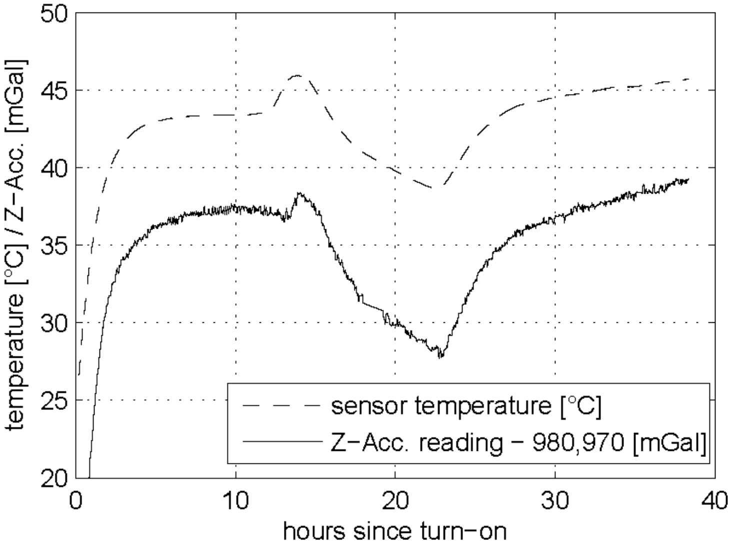
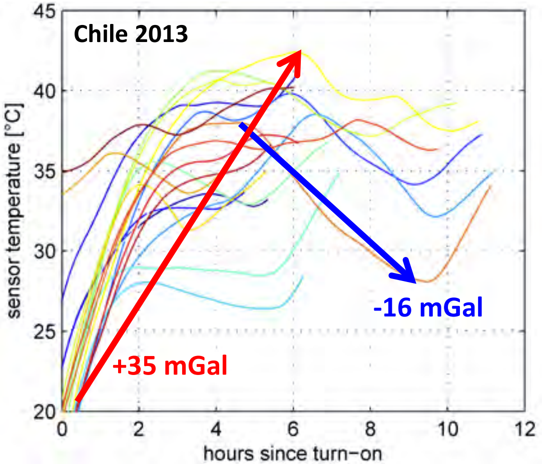
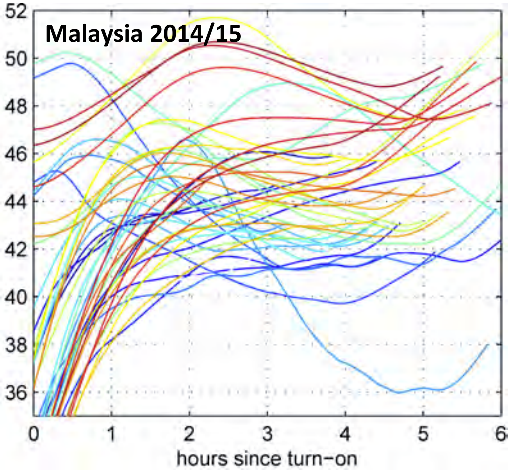
Thermal calibration - Motivation

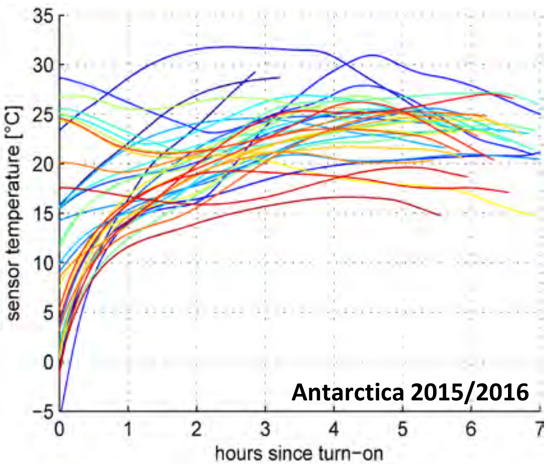


strong correlation between sensor temperature and QA-2000 accelerometer bias!

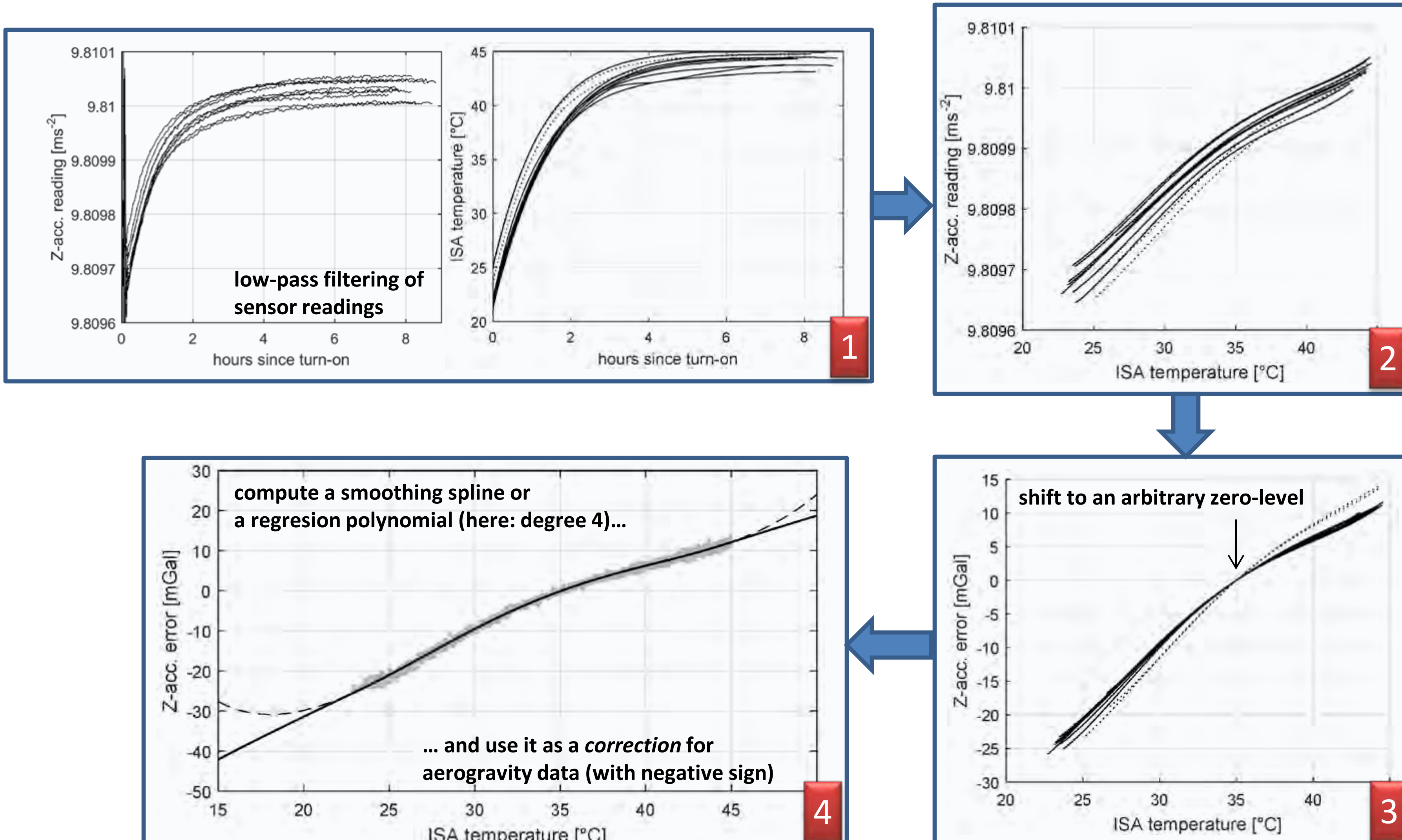
➔ **Try to calibrate this effect!**







Thermal Calibration: A Simple Approach



BSC-calibration

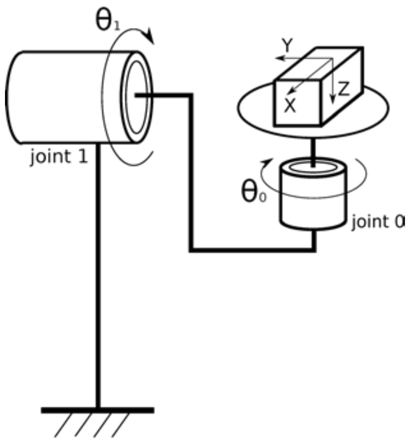
- Calibration of **b**ias, **s**cale factor and **c**ross-coupling [1)]
  - least squares estimate with 3x3 parameters
  - based only on **scalar** observations of gravity
- requires…
  - IMU at rest in different poses (e.g. face/edge/corner down ➔ **26 poses**)
  - ground-truth **scalar** gravity
- Adv

QA-2000 (in field)	bias X / Y / Z [mGal]	scale fact. X / Y / Z [ppm]	cross coupling θ_{YZ} / θ_{ZX} / θ_{ZY} [arcsec]
run #1	-1.6 / -2.4 / -6.5	22.5 / 19.5 / 18.8	0.1 / -3.4 / -0.4
run #2	-2.3 / 0.0 / -6.9	20.3 / 19.3 / 22.9	0.9 / -3.5 / -0.1
mean (σ)	**-2.0 / -1.2 / -6.7 (1.0)**	**21.4 / 19.4 / 20.8 (1.9)**	**0.5 / -3.4 / -0.3 (0.37)**

1) Shin, Eun-Hwan, and Naser El-Sheimy. "A new calibration method for strapdown inertial navigation systems." *Zeitschrift für Vermessungswesen.–2002.–Zfv* 127.1 (2002): 41-50.

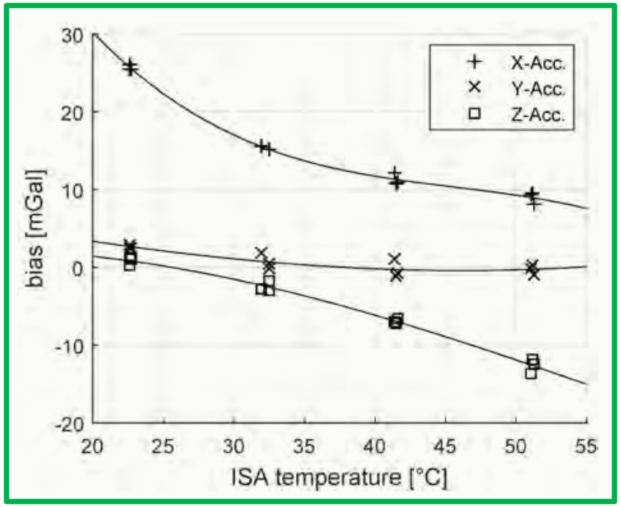
Thermal BSC-calibration (1/2)

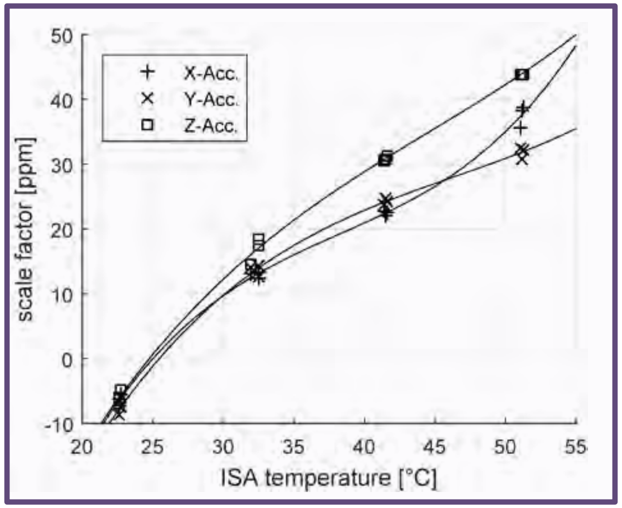
- **Repeat this calibration at different sensor temperatures…**

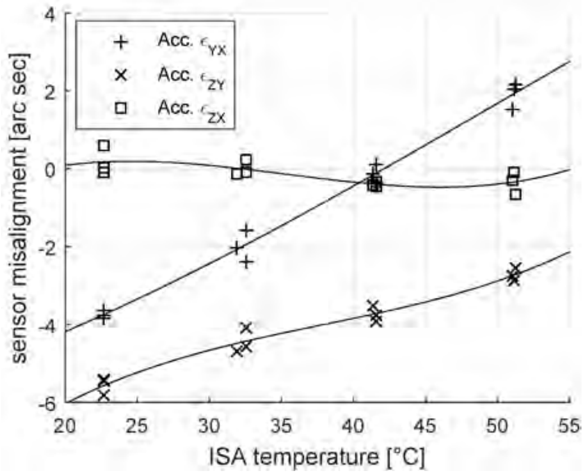


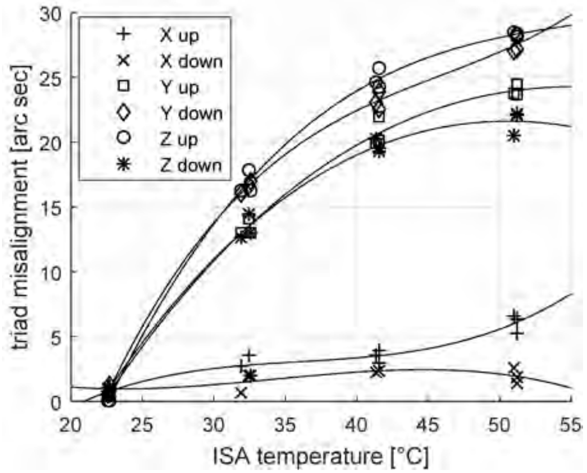
- **26 poses**
- **4 oven temperatures:**
  10°C, 20°C, 30°C, 40°C
- **3 repetitions** for each temperature
  (to check the repeatability)
- **1 minute** per pose
- sensor temperature has to be **constant** for each set
  - add 3 hours waiting time after temperature change
- **total: ~ 19 hours** (incl. table motion times)

Thermal BSC-calibration (2/2)









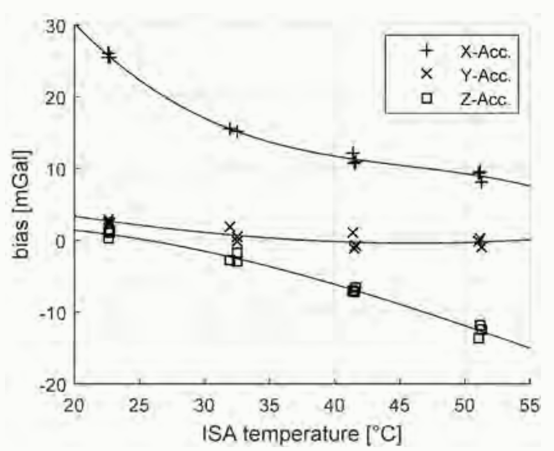
repeatability	σ
bias	**0.7 mGal**
scale factor	**0.8 ppm**
cross coupling	**0.25"**

residuals of observations (RMS)	**78 mGal**

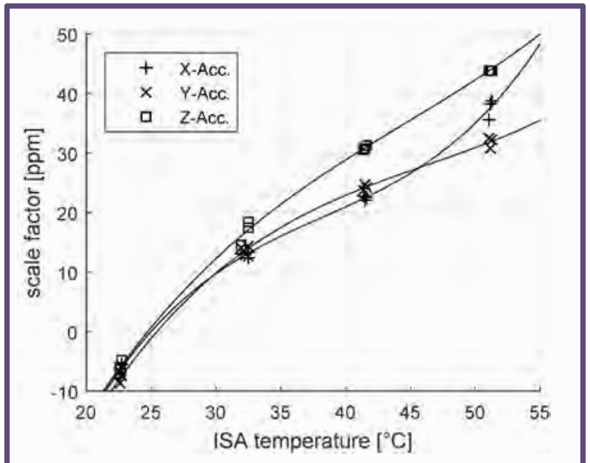
Thermal BSSC-calibration (1/2)

Idea: Extend the error model by an **additional scale factor** (for negative sensor readings)

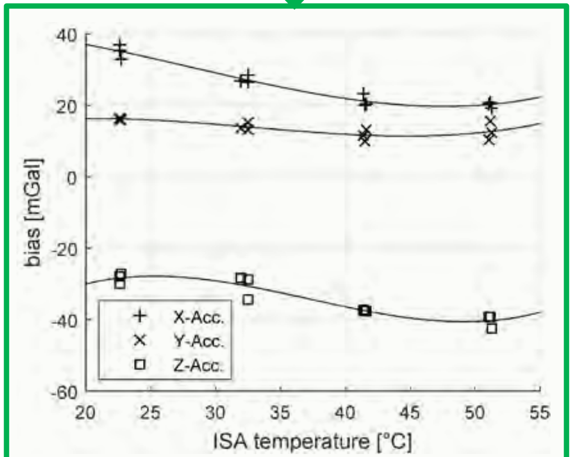
BSC



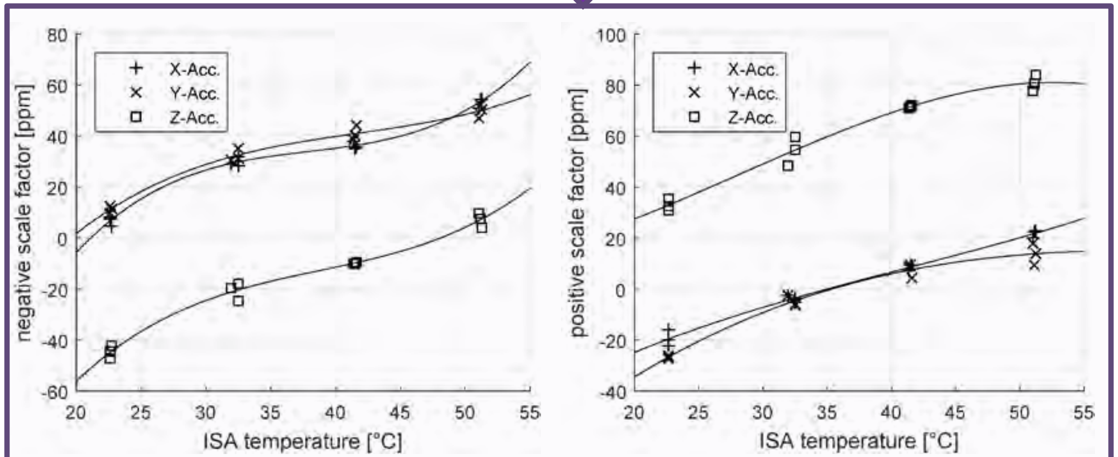
BSC



BSSC



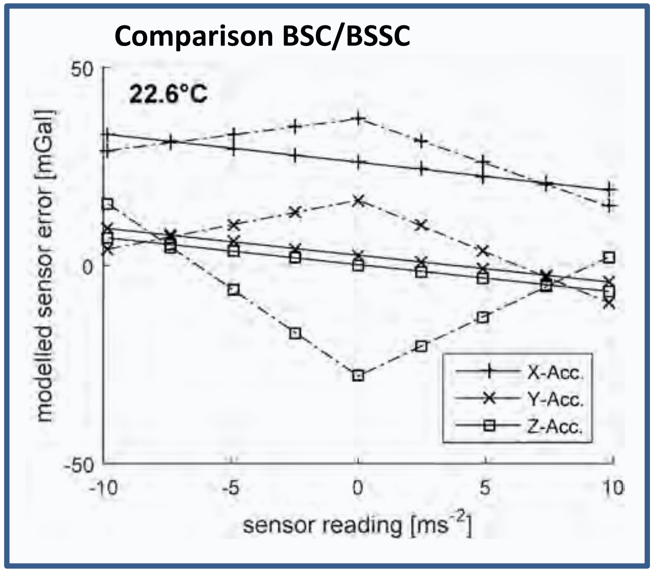
BSSC



Thermal BSSC-calibration (2/2)

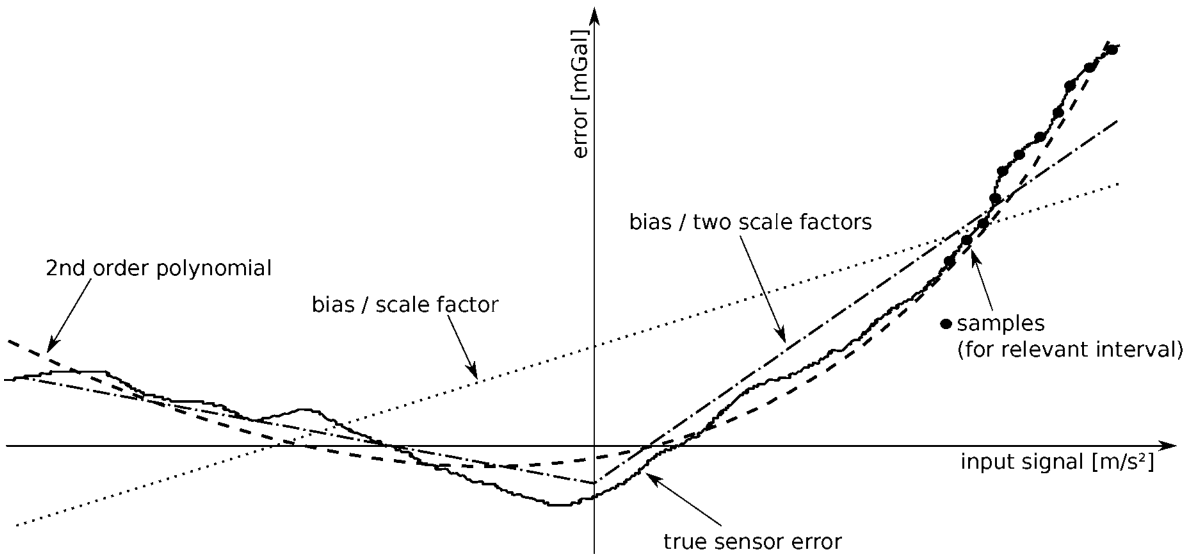
repeatability	σ
bias	**1.4 mGal**
pos. scale factor	**2.4 ppm**
neg. scale factor	**2.2 ppm**
cross coupling	**0.25"** ✓

residuals of observations (RMS)	**35 mGal** (BSC: 78 mGal) ✗



Discussion

- bias + scale factor **or** bias + 2 x scale factor are **both unsatisfactory** error models!
  - cross-coupling might change among different poses
  - the good repeatability suggests that we can do better…
- so? add *even more* parameters to the model?



Sample-based calibration (1/5)

- Idea: build a **look-up table** of sensor errors

1. select a **sample space**
2. define a **set of samples** to be included in the table
3. for each sample: **measure the error** (measured quantity – true quantity)
   - masks *all* influences in a single value (per sensor)
   - error is 3-D for a sensor triad
4. if required, **smooth** the sample data
5. use **interpolation** to get correction (=neg. error) for *any* coordinate in sample space

Sample-based calibration (2/5)

1. select a **sample space**
   - reasonable dimensions are
     - temperature **(T)** (strong dependencies for accelerometers, should *always* be included!)
     - sensor reading **(S)**
     - roll angle **(R)**
     - pitch angle **(P)**

T (1-D)
→ used for "simple approach"

TS (2-D)
…still no (explicit) attitude modeling…

TRP (3-D)
vehicle accelerations unmodelled

TPS (3-D)
…if error is independent of roll angle

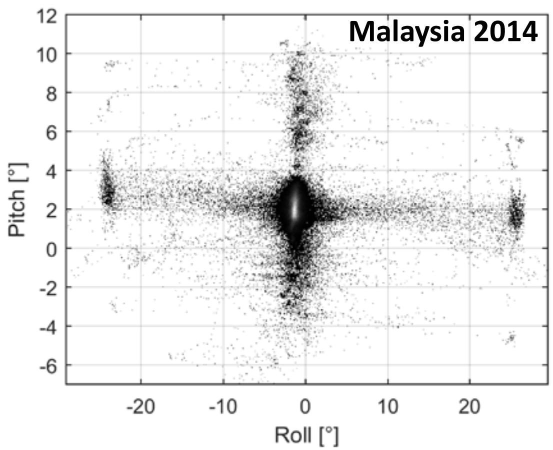
TRS (3-D)
…if error is independent of pitch angle

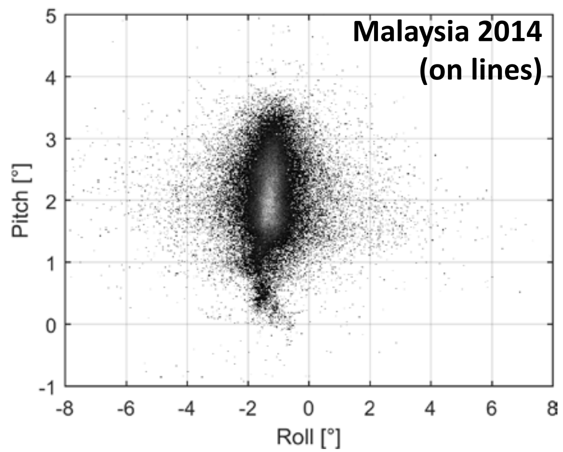
TRPS (4-D)
difficult/impossible to calibrate

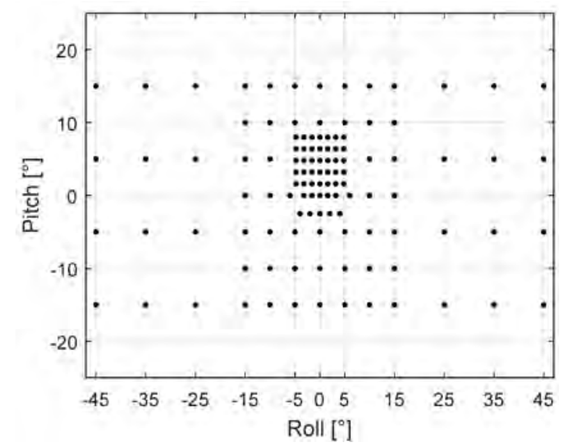
Sample-based calibration (3/5)

TRP (3-D)

2. define a **set of samples**
   - choose a set of **relevant** samples (reduce calibration time)
   - fixed-wing airborne gravimetry:
     - ambient temperatures: **-15°C…40°C**
       (equiv. sensor temperature: **-1°C…52°C**)
     - roll: max. **-45°...45°** (on the lines: **-5°…5°**)
     - pitch: max: **-15°…15°** (on the lines: **0°…5°**)





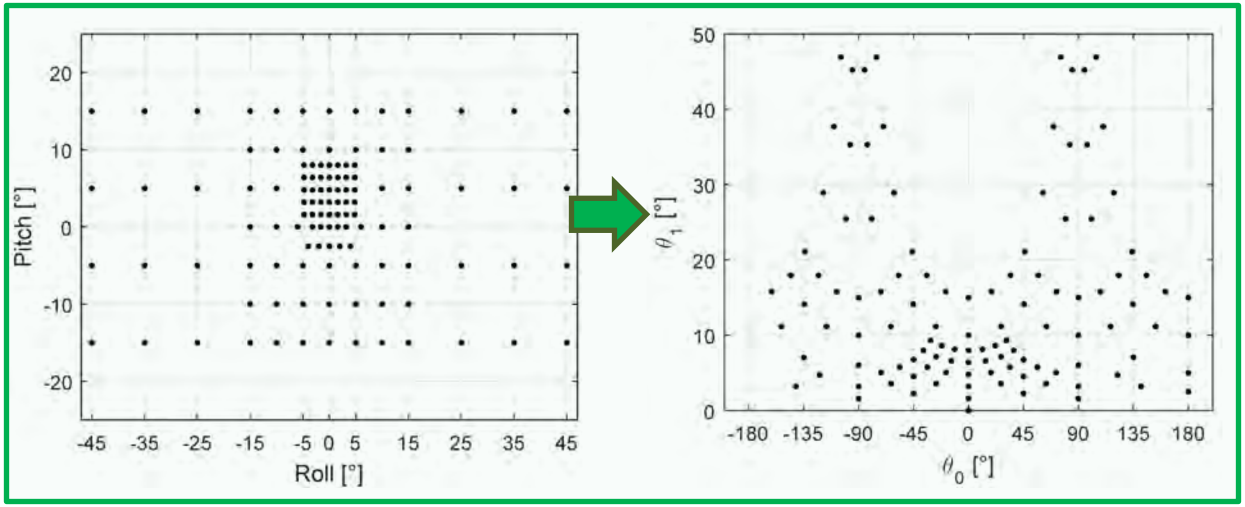


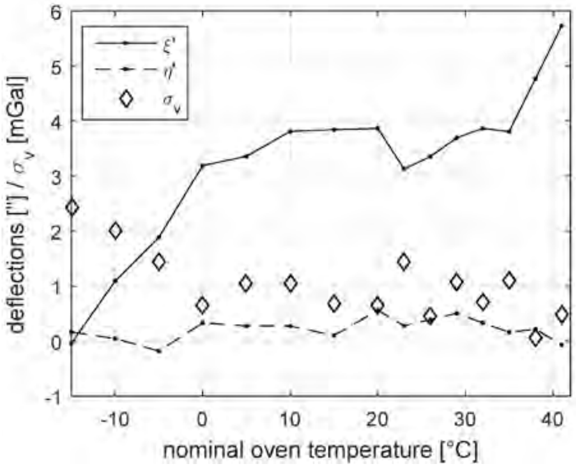
➔ 15 temperatures, 114 poses

Sample-based calibration (4/5)

TRP (3-D)

3. **measure the errors**
   - overall calibration duration: 64 hours
   - with a 2-DOF table, *heading* changes can not be avoided to set up roll/pitch angles
   - deflections of the vertical may distort the calibration
   - estimate deflections
     - additional horizontal poses
     - adjustment with deflections and biases as parameters





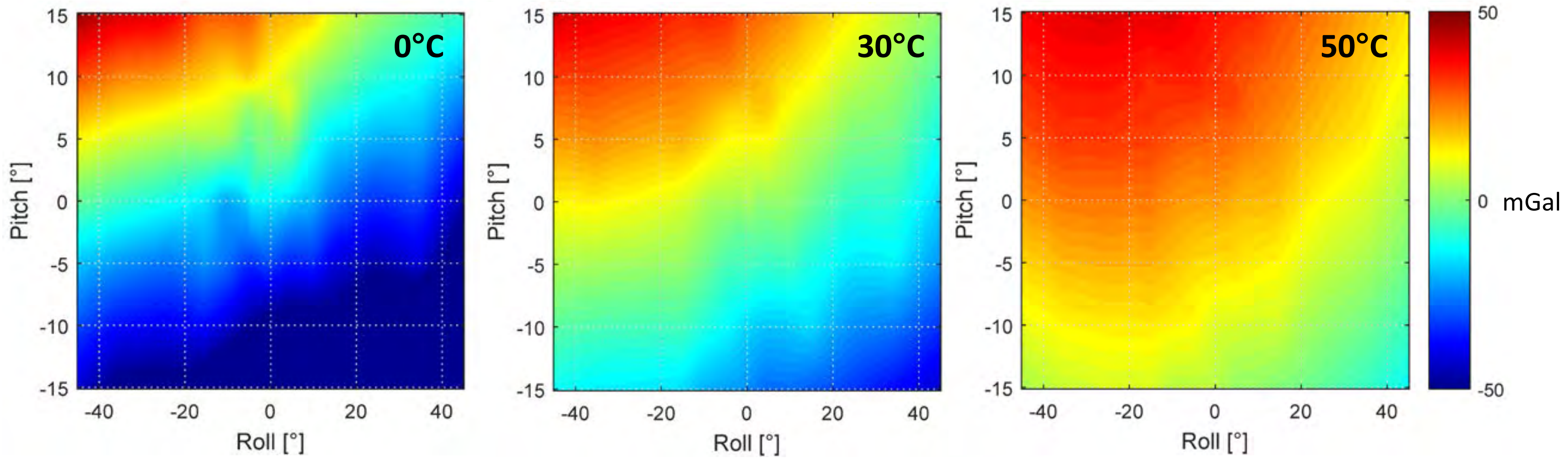
Sample-based calibration (5/5)

TRP (3-D)

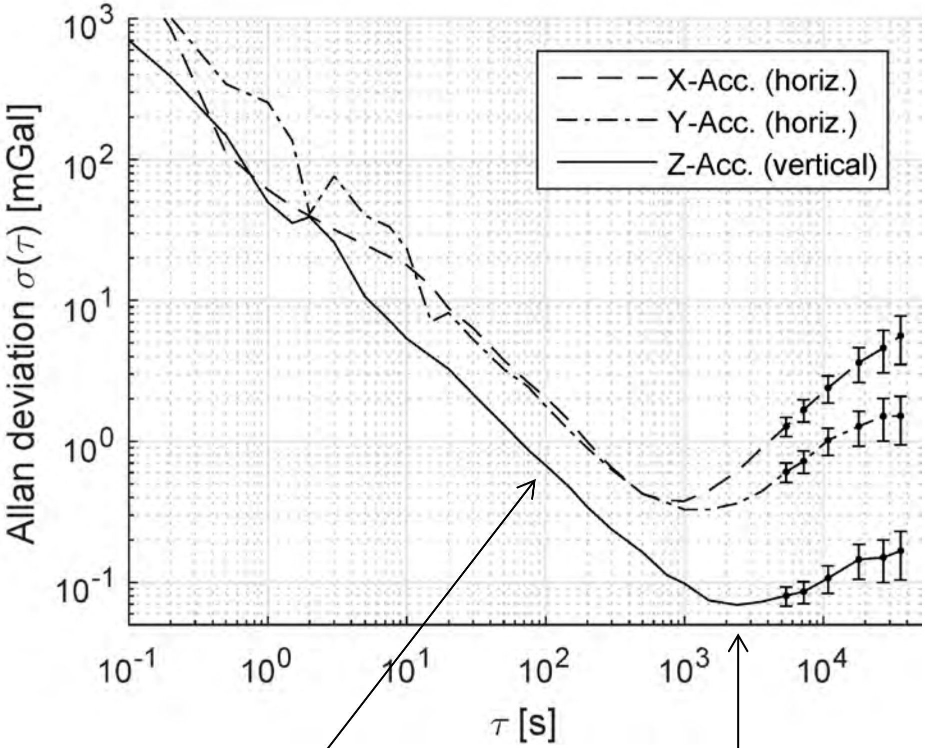
4. / 5. smooth / interpolate

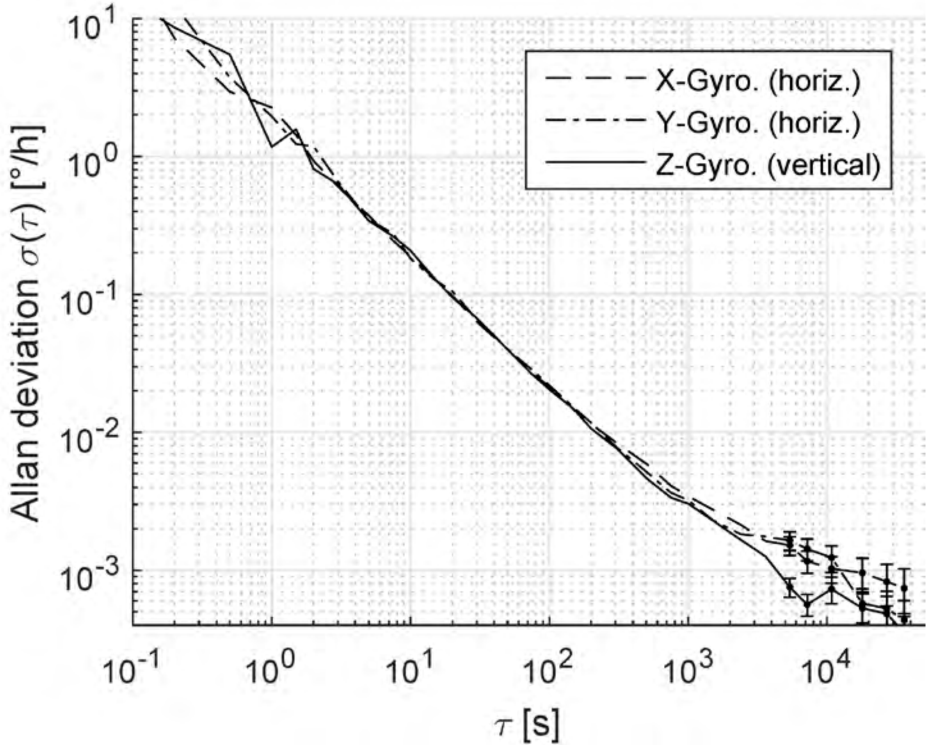
- interpolate to regular grid in **TRP**-space
- apply e.g. a 3-D Gaussian low-pass filter
  - sensor noise may depend on temperature!
  - horizontally aligned accelerometers are sensitive to small random bends (1" → 4 mGal)

Example: Z-accelerometer errors [mGal] (no smoothing)**:**



iMAR RQH: Allan standard deviation





0.7 mGal @100 s

68 µGal @2500 s

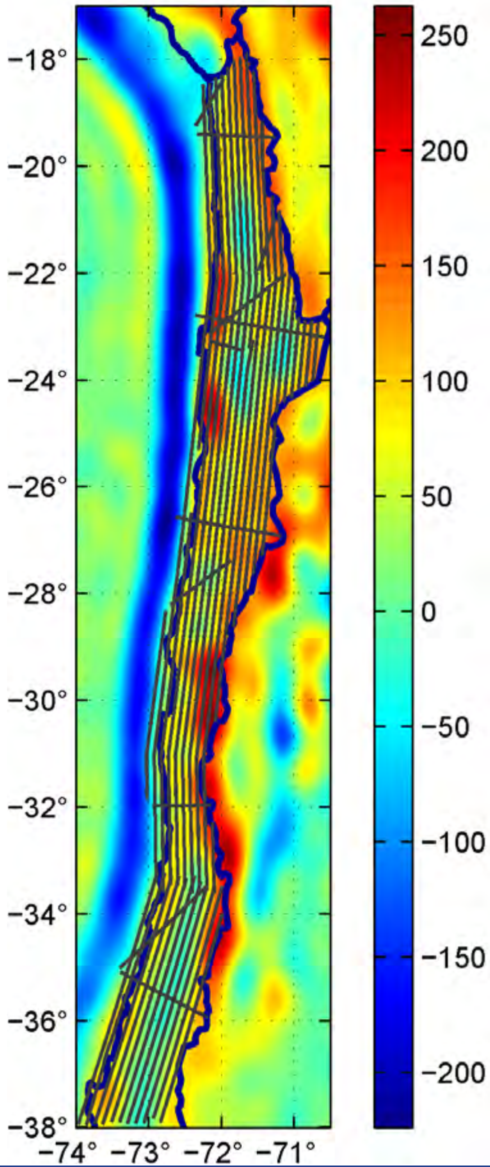
Overview

- Why Strapdown?
- Kalman-Filter design
- A few practical ideas…
- System analysis
  - Observability
  - Error propagation simulations
- Thermal calibration
  - Motivation
  - A simple approach
  - Parametric approaches (BSC, BSSC)
  - Sample-based approach
- **Test data sets**
- **Results**

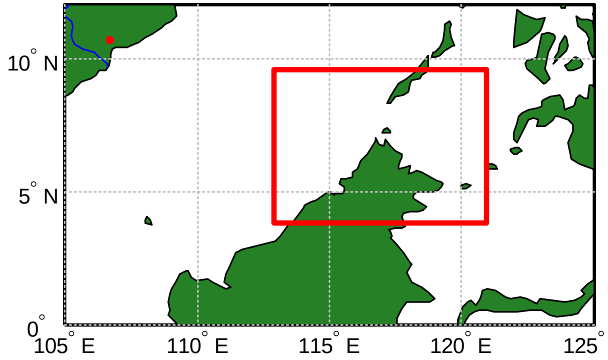
Aconcagua

Test data sets (1/4): Chile 2013

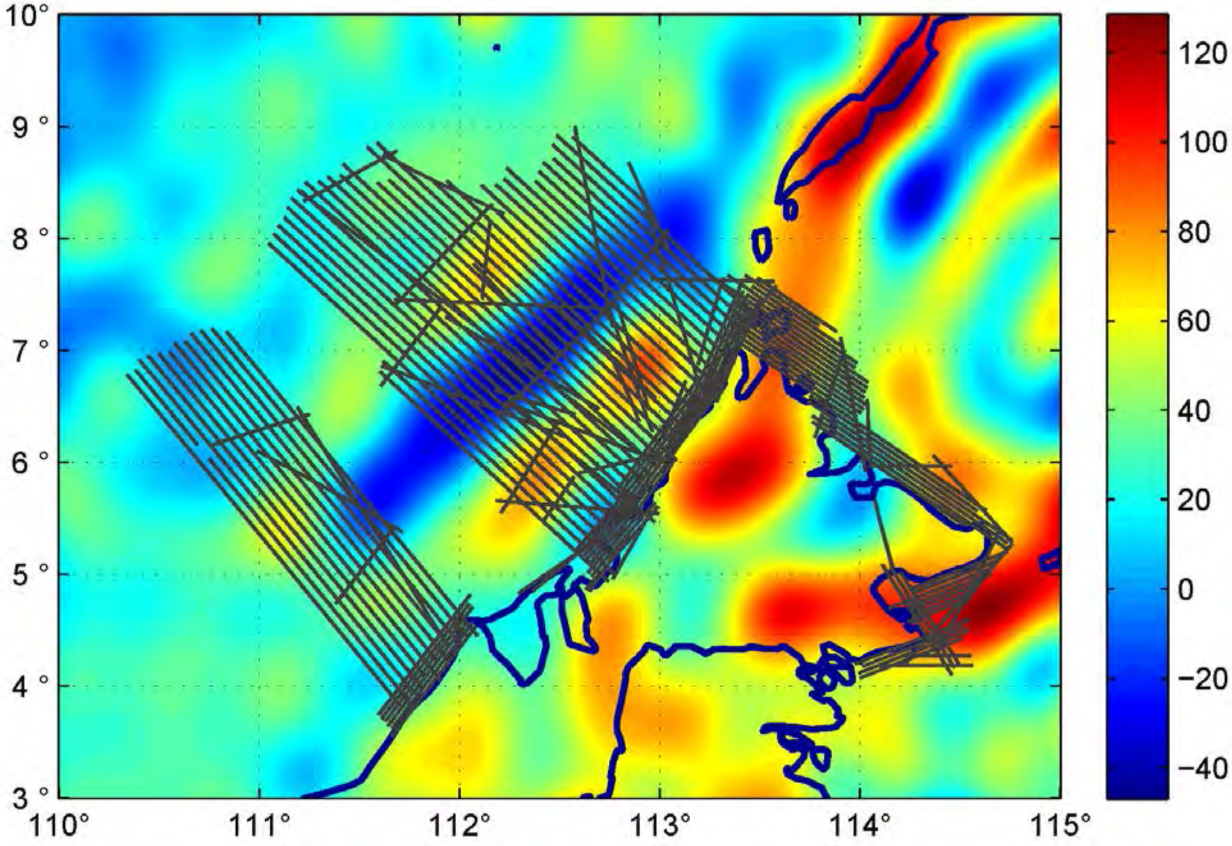
- 24 flights
- 88 flightlines
- 30,000 km / 80 h
- ~200 kts
- 3,300 – 16,000 ft
- 20 km line spacing



Test data sets (2/4): Malaysia 2014/2015

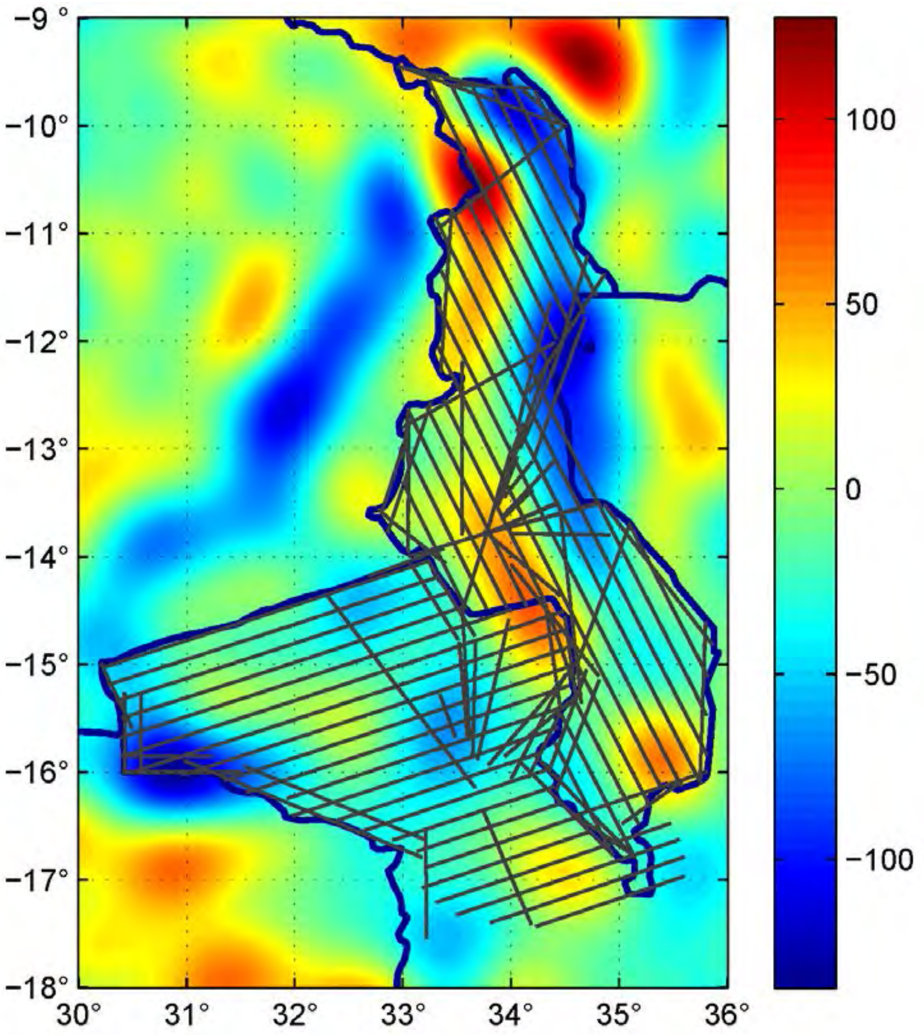


- 46 flights
- 181 flightlines
- 34,800 km, 110 h
- 170 kts
- 3,000-8,000 ft (mostly 6,000 ft)
- 5 / 10 km line spacing



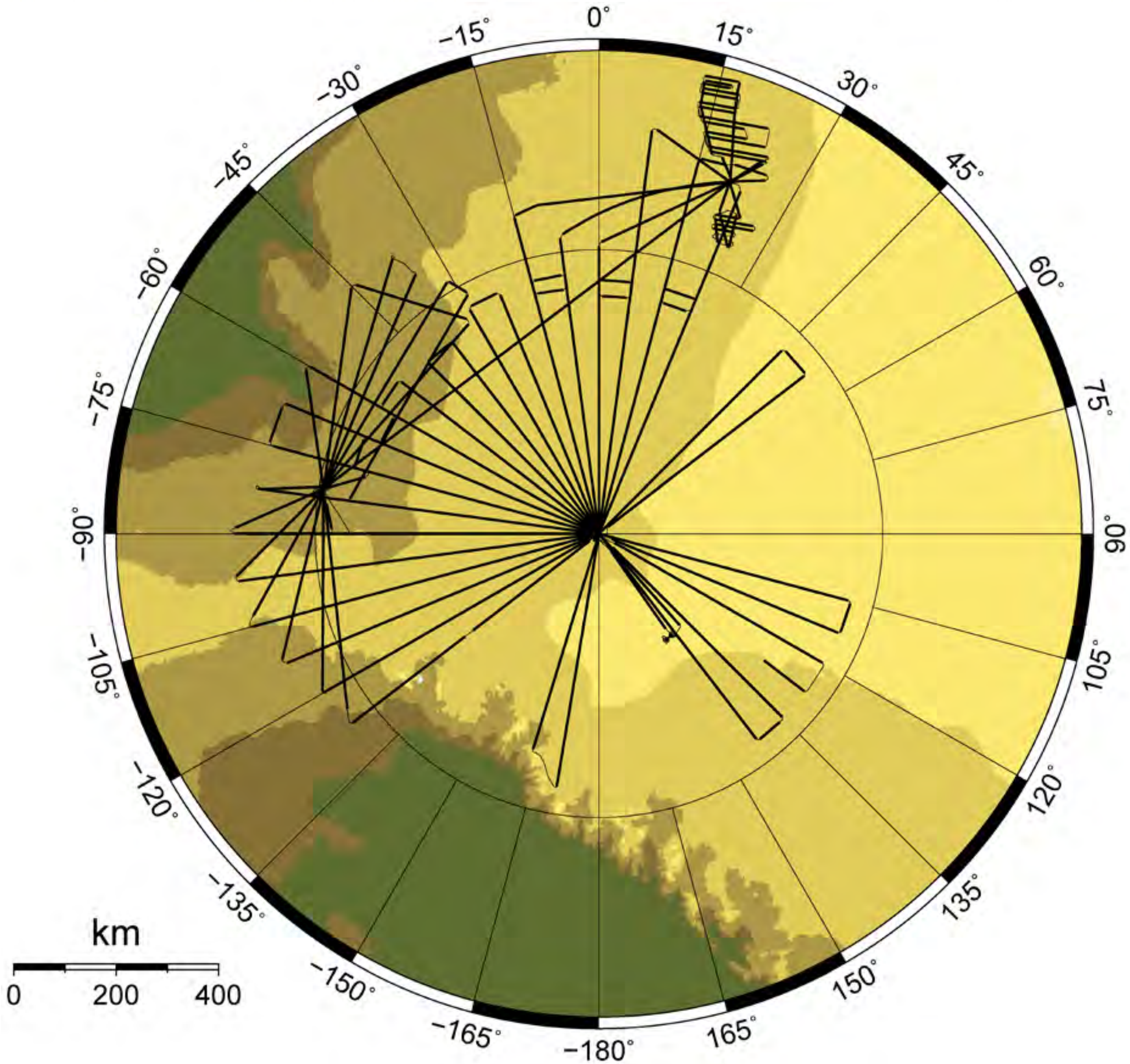
Test data sets (3/4): Mozambique/Malawi 2015

- 21 flights
- 130 flightlines
- 21,000 km, 78 h
- 150 kts
- 4,700 – 15,300 ft
- 20 km line spacing



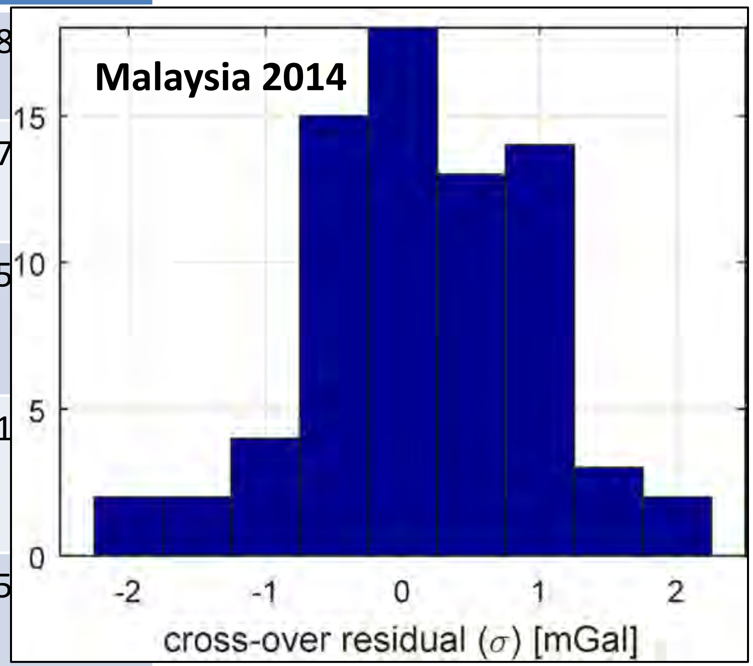
Test data sets (4/4): Antarctica Polar Gap

- 32 flights
- 121 flightlines
- 32,400 km, 78 h
- 130 kts
- 1.5 – 4.3 km



Results: Thermal Calibrations (1/2)

X-over RMS (σ) [mGal] Δh < 100 m	without calibration
Chile 2013 **(*)**	6.8
Malaysia 2014	3.7
Malaysia 2015 (33 out of 34 flights)	3.5
Mozambique/Malawi 2015 **(*)** (18 out of 21 flights)	4.1
Antarctica PolarGap 2015/2016	4.5



(*) 15" topographic reductions applied in the EKF (30' RTM)

Results: Thermal Calibrations (2/2)

Chile 2013

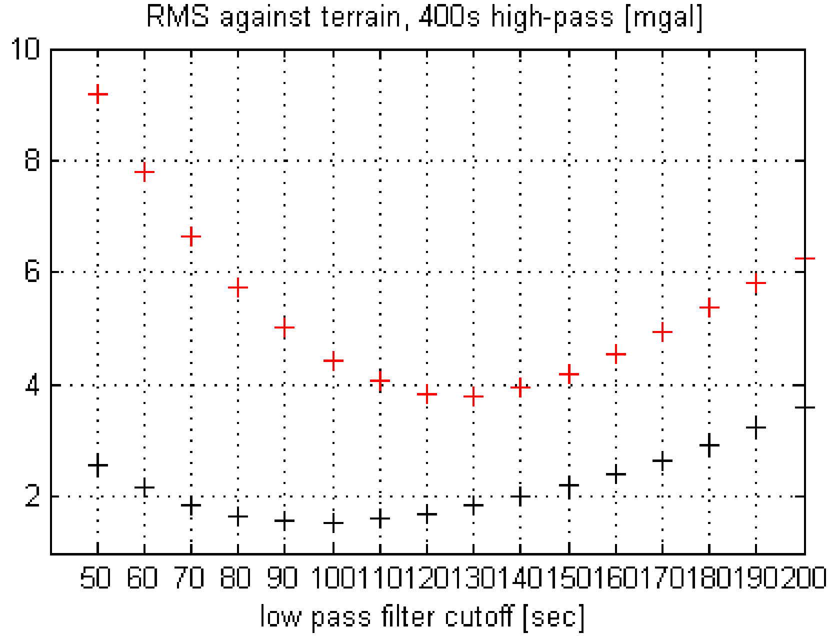
Errors of Closure
with/without STC
of Z-accelerometer

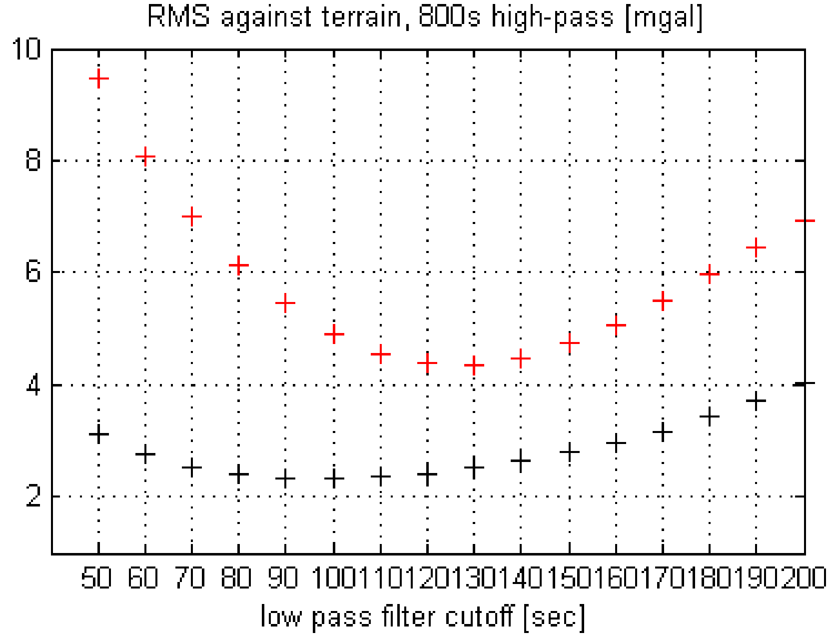
flight number	day of year	sensor temperature min. [°C]	sensor temperature max. [°C]	error of closure without thermal corr. [mGal]	error of closure with thermal corr. [mGal]	flight duration [h:mm]	drift without thermal corr. [mGal/h]	drift with thermal corr. [mGal/h]
1	285	22.7	33.6	10.8	0.5	4:15	2.55	0.12
2*	287	28.2	39.0	-	-	5:41	-	-
3	288	26.9	41.1	14.5	0.3	5:01	2.88	0.06
4	289	22.7	33.7	6.3	-1.9	3:17	1.91	-0.56
5*	290	15.3	39.4	-	-	10:53	-	-
6	291	14.8	38.5	18.3	1.3	9:46	1.88	0.14
7	292	16.4	28.4	10.3	2.0	5:13	1.97	0.39
8	294	20.0	38.9	19.8	1.6	4:31	4.38	0.35
9	295	17.8	34.8	20.6	3.1	6:09	3.35	0.50
10	296	19.2	36.8	26.2	1.2	5:54	4.44	0.20
11	297	18.0	41.2	35.8	1.6	4:45	7.55	0.33
12	298	20.7	40.7	23.5	-1.9	9:16	2.53	-0.20
13	299	16.6	42.4	22.6	-3.2	9:42	2.33	-0.33
14	301	19.6	35.0	24.8	2.1	4:33	5.44	0.47
15**	301	33.5	36.1	-0.3	1.5	2:41	-0.10	0.58
16	302	19.4	38.0	15.3	-1.0	10:18	1.49	-0.09
17	303	20.1	38.2	28.3	-1.4	9:04	3.11	-0.16
18	304	15.6	37.3	31.4	-1.6	5:22	5.83	-0.30
19	305	16.3	40.2	30.9	-0.2	9:24	3.29	-0.02
Mean		20.2	37.5	20.0	0.24	6:37	3.22	0.09
Min.		14.8	28.4	-0.3	-3.17	2:41	-0.10	-0.56
Max.		33.5	42.4	35.8	3.10	10:53	7.55	0.58
σ				9.3	1.73		1.79	0.32
RMS				22.0	1.74		3.69	0.33

Results: Resolution RQH vs. LCR

Chile 2013 (DoY 288)

LCR, IMU





Summary

- For an off-the-shelf navigation-grade strapdown IMU, …
  - the **1 mGal level** appears to be possible (without adjustment)
    - calibrations are required to reach this level, unless the IMU is thermally stabilized
    - Standard parametric error models (bias/scale factor) can be insufficient!
  - Some IMU errors are relevant for airborne gravimetry…
    - in-flight drifts,
    - sensor triad misalignments / sensor cross-couplings,
    - timestamping errors,
    - leverarm instability (?), …
  - … while others are (usually) not
    - constant accelerometer biases,
    - constant lever arm errors (up to 10 cm),
    - discretization errors (coning/sculling).

Outlook

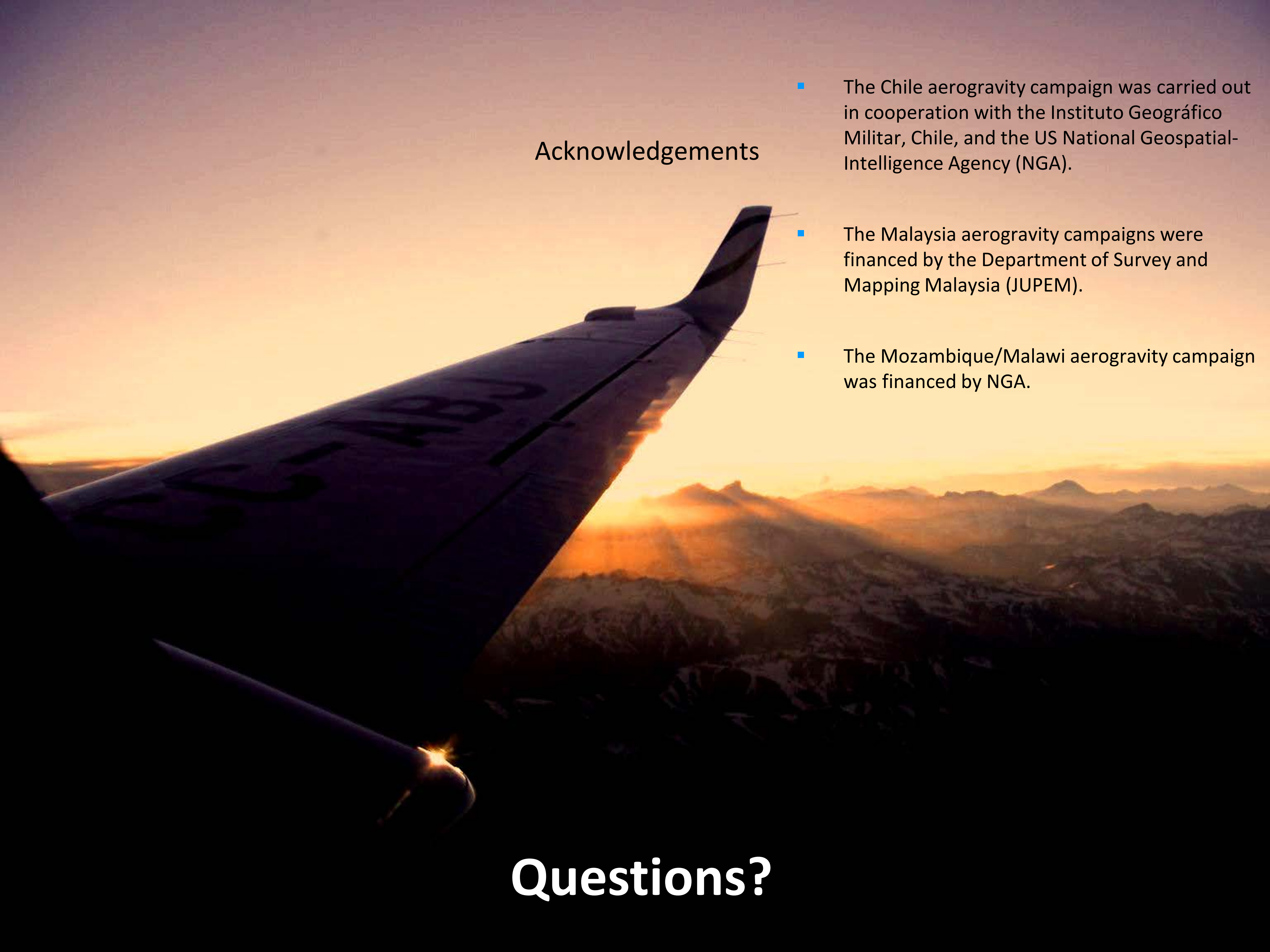
A **good prospect** for strapdown gravimetry!

➔Strapdown gravimetry on miniature drones? (new RQH housing: 7.5 kg)

➔ Getting 1 mGal from a combination of tactical-grade strapdown IMU (2-3 kg) and GGM?

➔ Thermal stabilisation for fixed-wing aerogravity. Higher power/space/price.

➔ Enabling a reliable 1 mGal-accuracy



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- The Malaysia aerogravity campaigns were financed by the Department of Survey and Mapping Malaysia (JUPEM).
- The Mozambique/Malawi aerogravity campaign was financed by NGA.

Questions?