

# Methods and other considerations to correct for higher-order ionospheric delay terms in GNSS

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# Outline

- Introduction
- Is it necessary to routinely correct the I2+ terms in precise GNSS modeling?
- How should the I2+ correction be applied?
- How should the I2+ terms be computed?
- Conclusions

# Introduction

- The first order ionospheric term ( $I_1$ ) is the main contribution of the GNSS ionospheric refraction (99.9%, [2,1,7]).
- $I_1$  can be removed when considering the carrier phase or the code ionospheric free combinations of dual frequency measurements ( $L_c$  and  $P_c$ ).
- However, because of the increasing accuracy demand in precise GPS positioning, the study of the impact of the higher ionospheric terms –up to few cm in range- has become relevant.
- We are going to focus our attention in the **higher order ionospheric terms ( $I_2+$ )**, in particular in the **second-order ionospheric term ( $I_2$ )**, which is the most important one (typically more than 90%, and which uncertainty can reach up to more than 40% depending on the computation strategy), basically depending on the **STEC** and the **magnetic field projection** over the propagation direction at the ionospheric pierce point.
- We will discuss the **motivations** of correcting higher order ionospheric terms, **different approaches** to do it, and **potential recommendations to IGS Analysis Centers** when possible.

# Basic equations

$$P_i = \rho + \frac{q}{f_i^2} + \frac{s}{f_i^3} + \frac{r}{f_i^4}$$

$$L_i = \rho + n_i \lambda_i - \frac{q}{f_i^2} - \frac{1}{2} \frac{s}{f_i^3} - \frac{1}{3} \frac{r}{f_i^4}$$

$$q = 40.3 \int N dL$$

$$s = 7527 \cdot c \int N |\mathbf{B}_0| \cos \theta_B dL$$

$$P_c = \frac{f_1^2}{f_1^2 - f_2^2} P_1 - \frac{f_2^2}{f_1^2 - f_2^2} P_2$$

$$= \rho - \frac{s}{f_1 f_2 (f_1 + f_2)} - \frac{r}{f_1^2 f_2^2}$$

$$L_c = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2$$

$$= \rho + \frac{s}{2 f_1 f_2 (f_1 + f_2)} + \frac{r}{3 f_1^2 f_2^2} + n_c \lambda_c$$

$$r = 2437 \int N^2 dL + 4.74 \cdot 10^{22} \int N B_0^2 (1 + \cos^2 \theta_B) dL$$

$$I_2 = - \frac{s}{2 f_1 f_2 (f_1 + f_2)} \approx - \frac{7527 c B_0 \cos \theta \cdot STEC}{2 f_1 f_2 (f_1 + f_2)} = \alpha B_0 \cos \theta \cdot STEC$$

$$I_3 = - \frac{r}{3 f_1^2 f_2^2} \approx - \frac{2437 N_{\max} \eta \cdot STEC}{3 f_1^2 f_2^2}$$

- Both I2 and I3 terms (I2+=I2+I3) are defined here as the corrections to be applied (added) to Lc(L3) measurements, to remove the higher order iono effects.
- In the I3 approximation ([4]):
  - The magnetic field term can be neglected at sub-mm error level.
  - The shape term  $\eta$  is around 0.66 and Nmax can be estimated from VTEC, through the slab thickness ( $H=VTEC/N_{\max}$ ).
  - It can be easily adapted to include the ionospheric bending correction (typically up to few mm at low elevation, [6]).
- $P_i, L_i, P_c, L_c$  are the pseudoranges and carrier phases for frequencies  $i$  and I1-free combination,  $B_0$  is the magnetic field at the ionospheric pierce point,  $\theta$  is the angle formed between  $B_0$  and the propagation direction,  $N$  is the electron density,  $f_i$  the corresponding frequencies, STEC is the Slant Total Electron Content ( $N$  integrated along the ray path), etc...

# Questions

- 1) Is it necessary to routinely correct the  $I_2+$  terms in precise GNSS modeling?
- 2) How should the  $I_2+$  correction be applied?
- 3) How should the  $I_2+$  terms be computed?

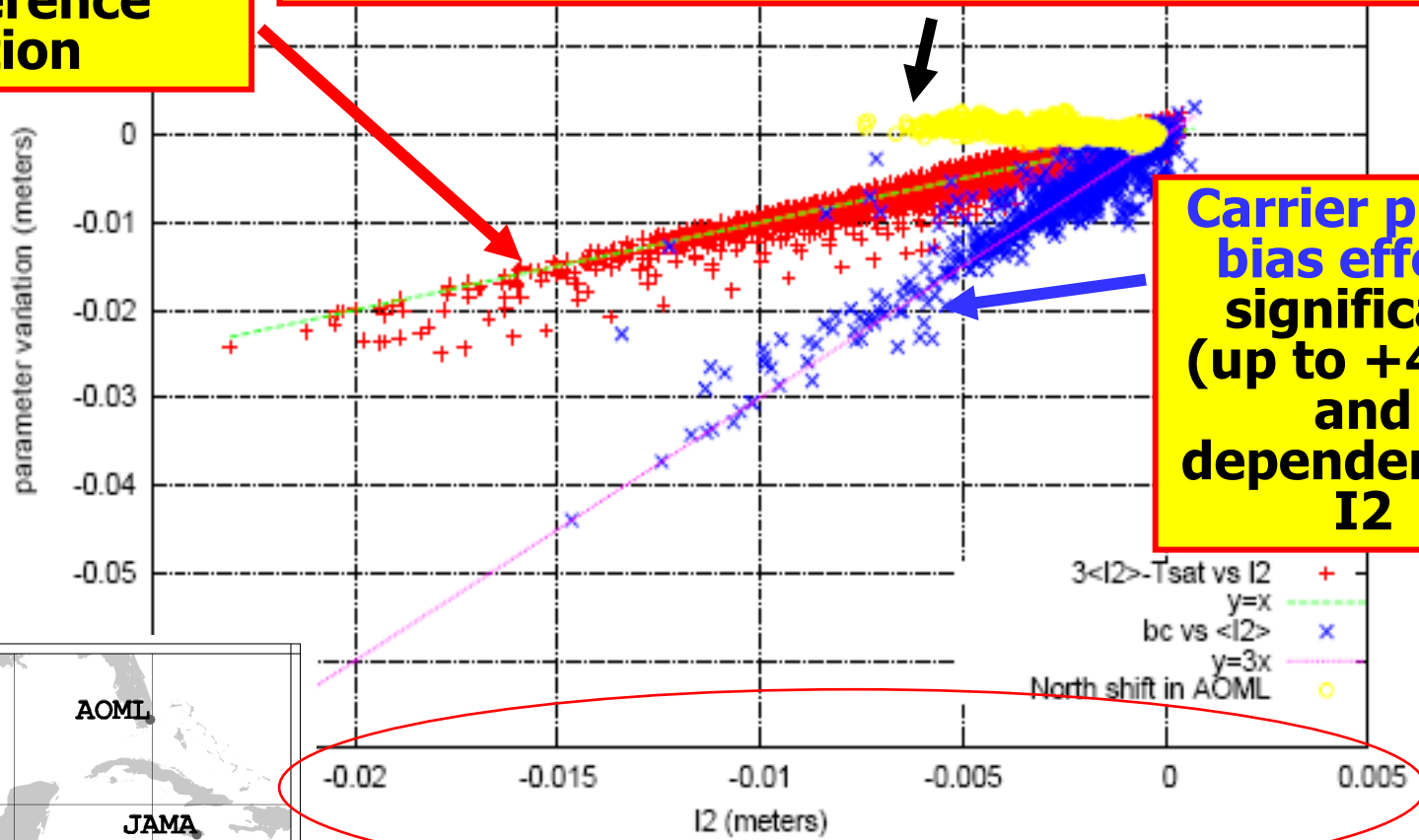
1) Is it necessary to routinely correct the I2+ terms in precise GNSS modeling?

# I2 effects on subdaily differential estimation

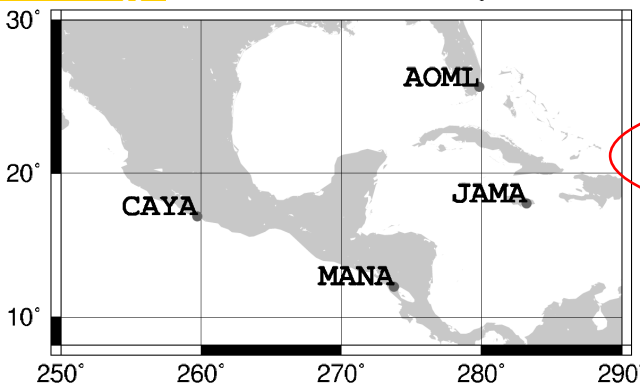
**Satellite clock effect:** significant (up to +2cm) and dependent on I2 at reference station

**Coordinates** (north shift of AOML): Small effect (up to ~1mm) and **NO** significant dependence on I2 at reference stat. The small observed effect depend on the relative I2 value, regarding to the reference station (I2-I2ref).

**Carrier phase bias effect:** significant (up to +4cm) and dependent on I2

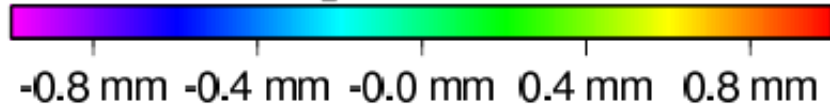


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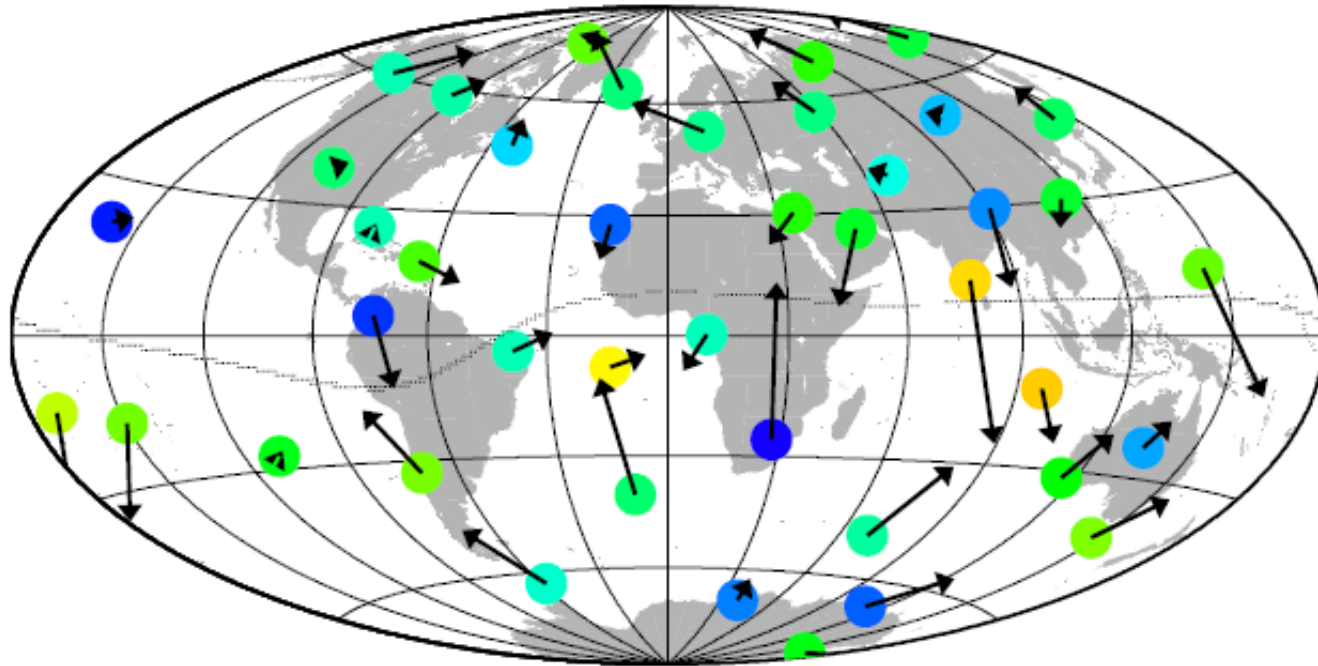


# Mean I2 effect on receiver positions (21 months, 2002 -03)

radial\_component

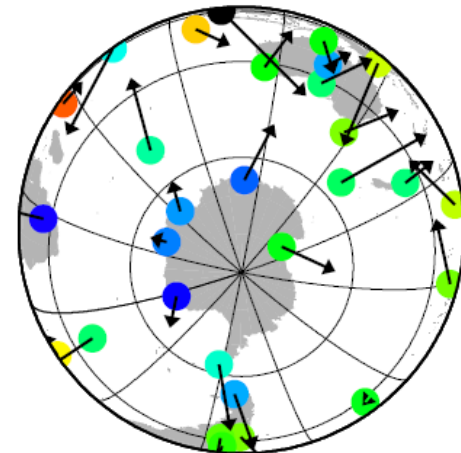
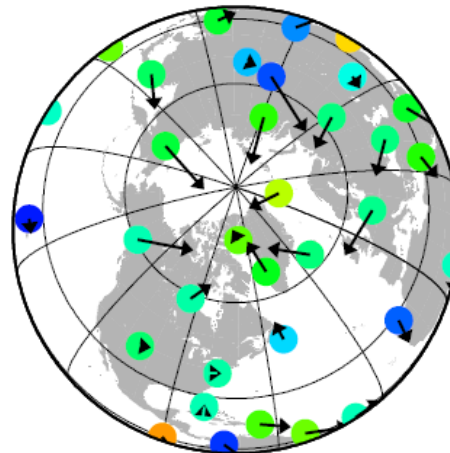


1mm →



Among I2 processing complete for all the geodetic parameters, realistic magnetic field model (see below) and more homogeneous distribution of receivers are some of the hints of these computations.

**Receiver position effect: Confirming previous results with differential scenario, the dependence on the difference of I2 values wrt neighbour receivers, producing long term effects at mm level and few tenths of mm for daily repeatability effect, in terms of intranet apparent deformation.**





# Subdaily residual I2 effect on satellite orbits and clocks (averaged on year 2003)

Latitude / degrees



Local Time / hours

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**NORTH**

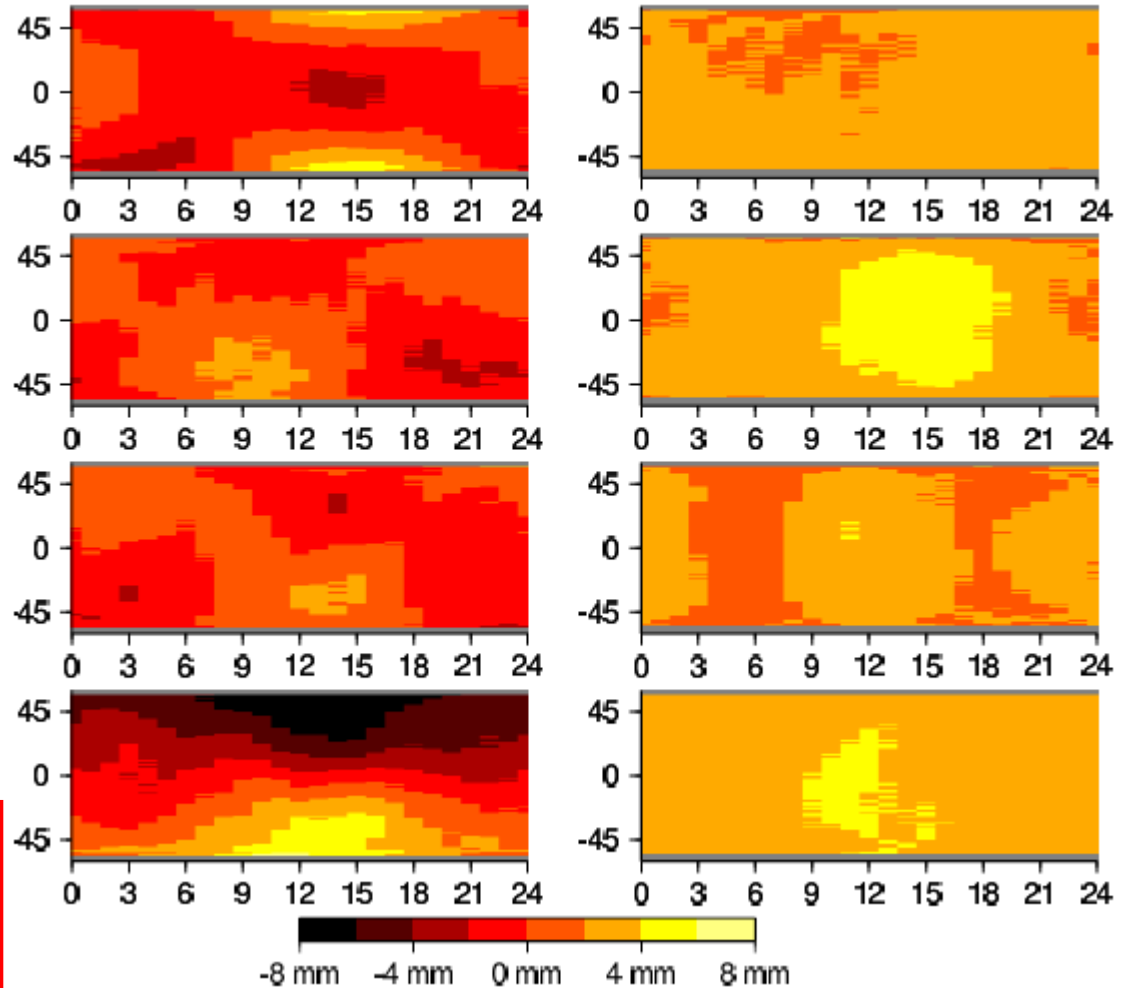
**EAST**

**RADIAL**

**CLOCKS**

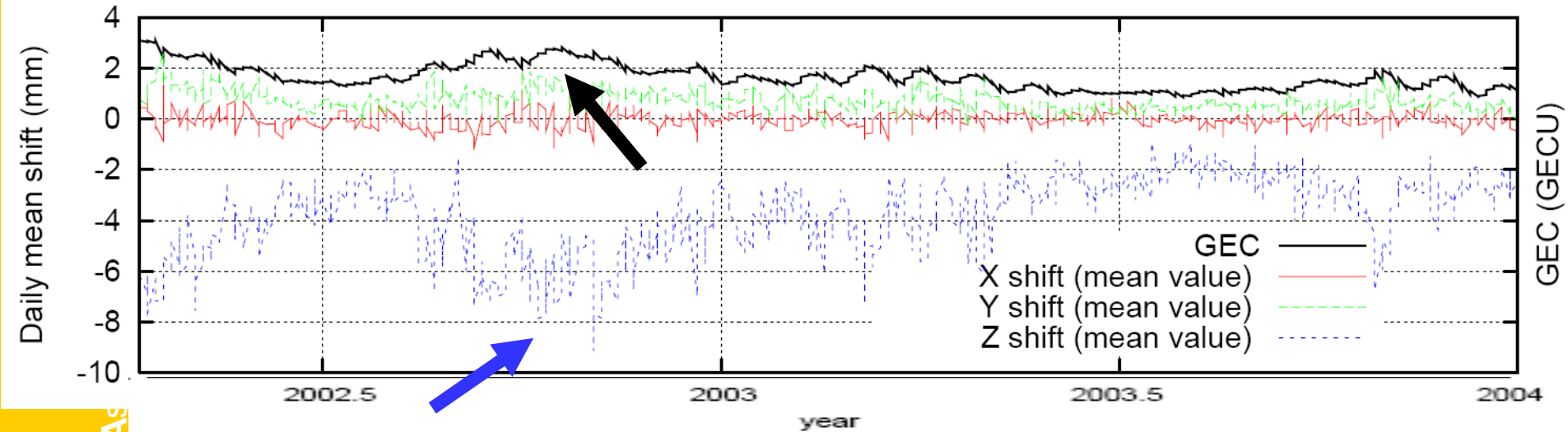
**BIAS**

**STD.DEV.**



**Confirming importance of I2 effect on satellite clocks and orbits (up to 1 cm and several mm, latitudinal signatures)**

# I2 effect on Satellite Orbit estimation



The overall I2 effect (orbit displacement + dynamical integration) produces a **general averaged Z-displacement of the orbits** of several millimeters. It is correlated with the **Global Electron Content** (GEC, VTEC integrated along the overall Ionosphere, computations from 2002.3 to 2004).

# 1) Is it necessary to routinely correct the I2+ terms in precise GNSS modeling?

- Yes, I2+ should be systematically corrected, in order to avoid biases of up to several centimeters in range, which produced errors in the estimations up to centimeter level in satellite dependent parameters (orbits, clocks, associated geocenter) and errors up to millimeter level in station coordinates. Also I2+ estimations could eventually help in ambiguity fixing.
- Such I2+ induced errors in precise GNSS modelling appear at different scales, associated with the corresponding TEC variation: subdaily, quasi-monthly (Solar rotation), seasonal, Solar Cycle (in particular the subdaily and seasonal variations can be important, not only on Solar Maximum conditions). In particular an apparent intranet deformation at millimeter level is produced linked to the spatial variability of I2+ effect.
- In general, when processing a global network of stations using double difference GPS positioning, the majority of the computation time is taken up by performing the least squares analysis rather than modelling the observations: it makes sense to spend a little more time on the modelling to ensure the investment of CPU time in the least squares adjustment provides the best result.

## 2) How should the I2+ correction be applied?

- The I2+ correction should be applied **in a consistent manner**, by using GNSS products (such as satellite orbits and clocks) computed after applying these corrections on GNSS measurements.
- There are two possible strategies (among others with specific coefficients at different regions as e.g. Europe, see [6] and [6']), applicable at global scale :

✓(1) **Forming a new linear combination of observables  $L_c^*(L3^*), P_c^*(P3^*)$ , with coefficients dependent on magnetic field** (and on STEC for I3) to cancel out I1+I2+ effects. **This is a compact approach** but with a **new carrier phase bias which is variable**, depending on the magnetic field, ray geometry and STEC (or ionospheric phase combination ambiguity).

$$\tilde{L}_c = L_c - \alpha B_0 \cos\theta (L_I - B_I) = \hat{L}_c + \alpha B_0 \cos\theta B_I = \rho^* + B_c$$

$$\hat{L}_c = \rho^* + B_c - \alpha B_0 \cos\theta B_I$$

✓ The method of Brunner and Gu does not require calculation of the TEC and is time dependent. It is done by introducing a new 'dynamic' LC combination, which includes the second order term, though it introduces an error into the bias term. This is the default method implemented in GIPSY, though the TEC option is available as well.

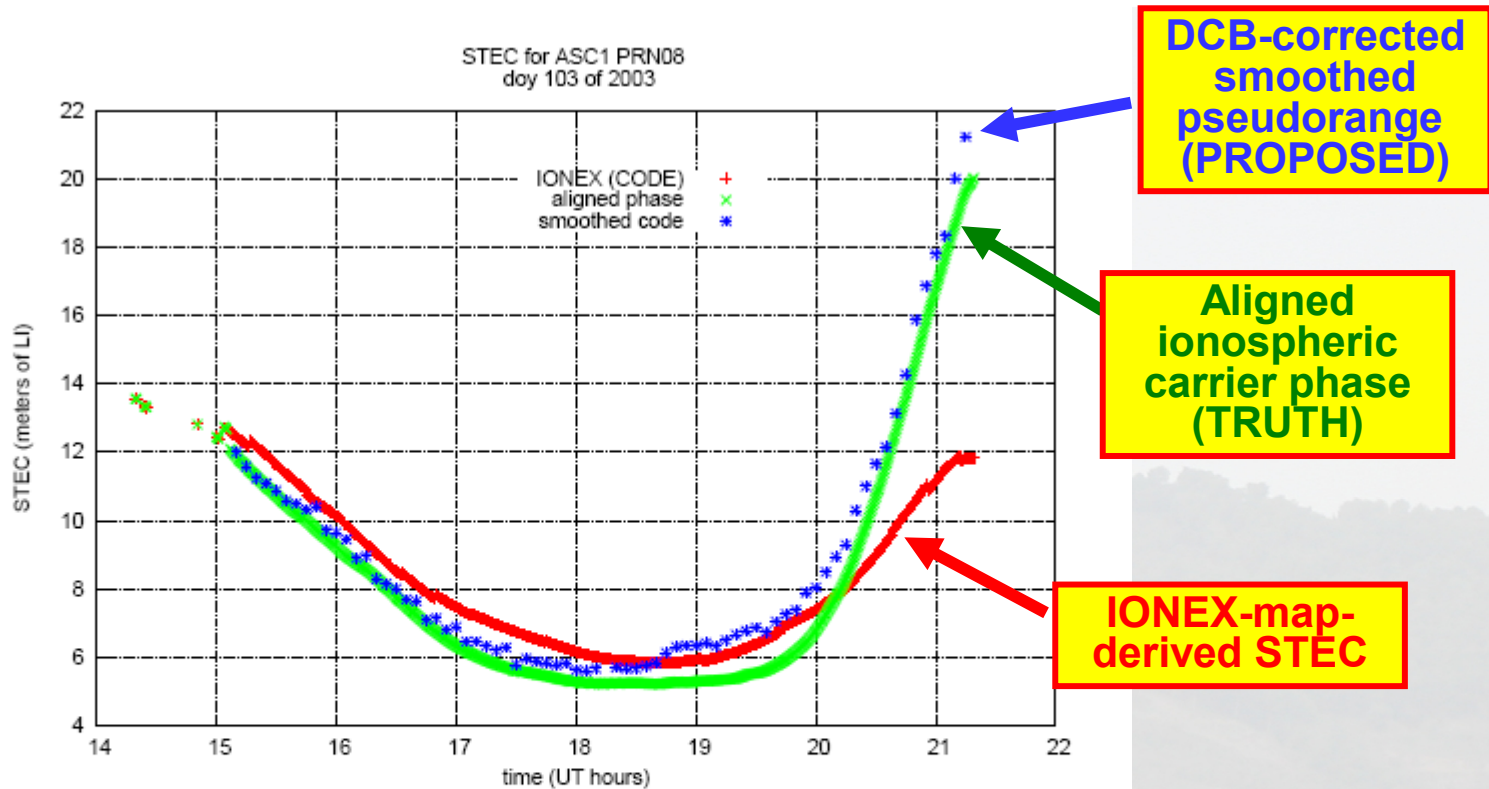
✓(2) **Correcting each measurement separately (again using magnetic field and electron content), before to perform the I1-ionospheric free combinations  $L_c(L3)$ .**

$$L_c = L_c - \alpha B_0 \cos\theta \cdot STEC$$

• As the magnitude and sign of the correction term depend on the signal's direction (intersection angle with the magnetic field) and the actual TEC, **it would be preferable to use a model that allows to correct each individual measurement in order to maintain the carrier phase ambiguities as constant values, but you can need an external source for STEC** (see next section).

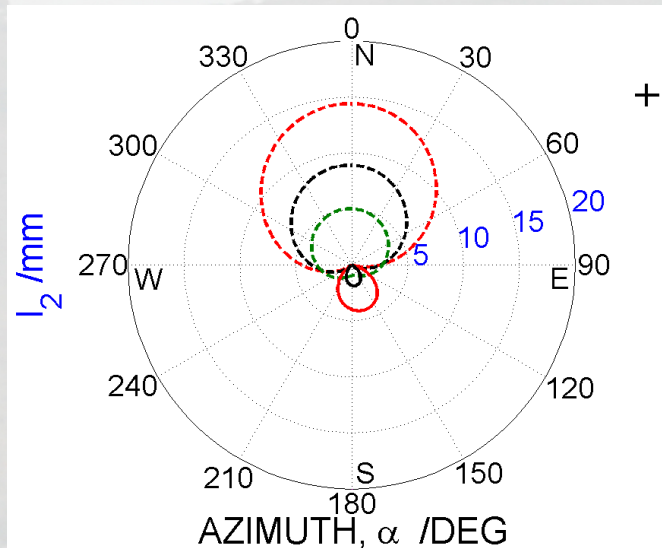
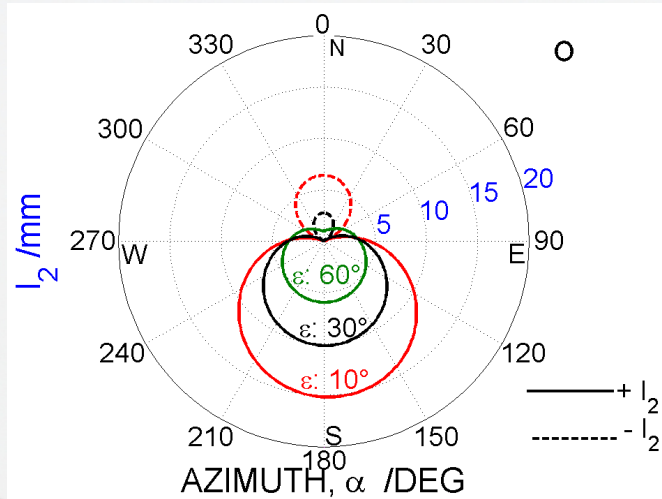
3) How should the  $I_2+$  terms be computed?

# A simple and accurate approach to compute and apply the I2 correction: STEC term

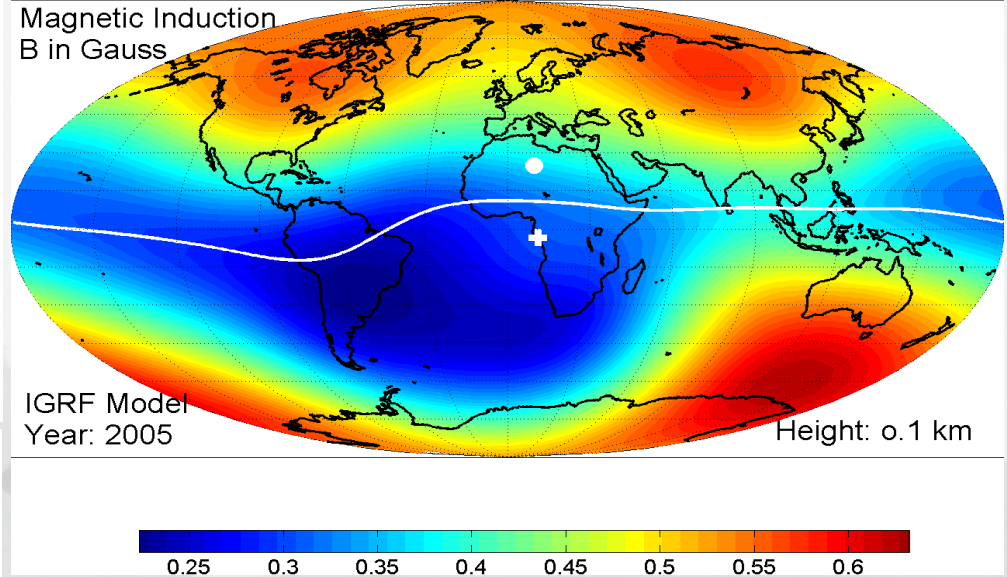


The Slant Total Electron Content, STEC, can be computed in a simple and accurate-enough way, from carrier-smoothed geometry-free combination of pseudoranges (PI≡P4), after removing previously estimated interfrequency bias values for transmitter and receiver ( $D, D'$ , quite stable on time). This approach is not affected by the single-layer accuracy limitations in VTEC IONEX format, and can be understood as a new I1+I2 free carrier phase combination:  $Lc' = \alpha \cdot L1 + \beta \cdot L2 + \gamma(B) \cdot P1 + \delta(B) \cdot P2 + \varepsilon(B) \cdot (D + D')$

# $I_2$ at geomagnetic conjugate points (receiver positions)



TEC<sub>v</sub>: 100 TECU,  $\epsilon$  : elevation



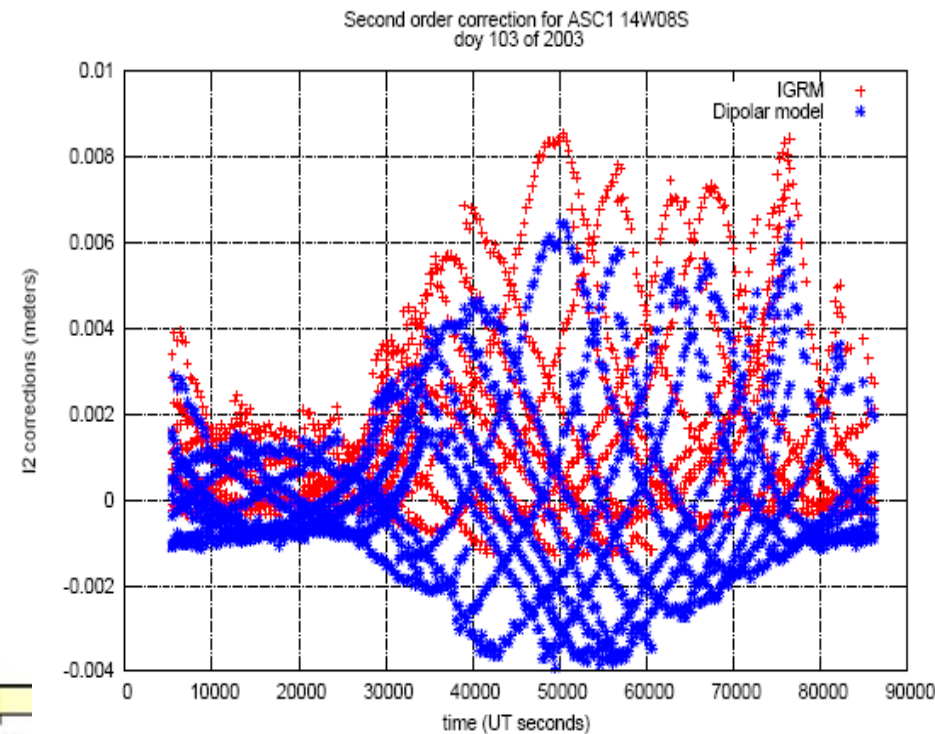
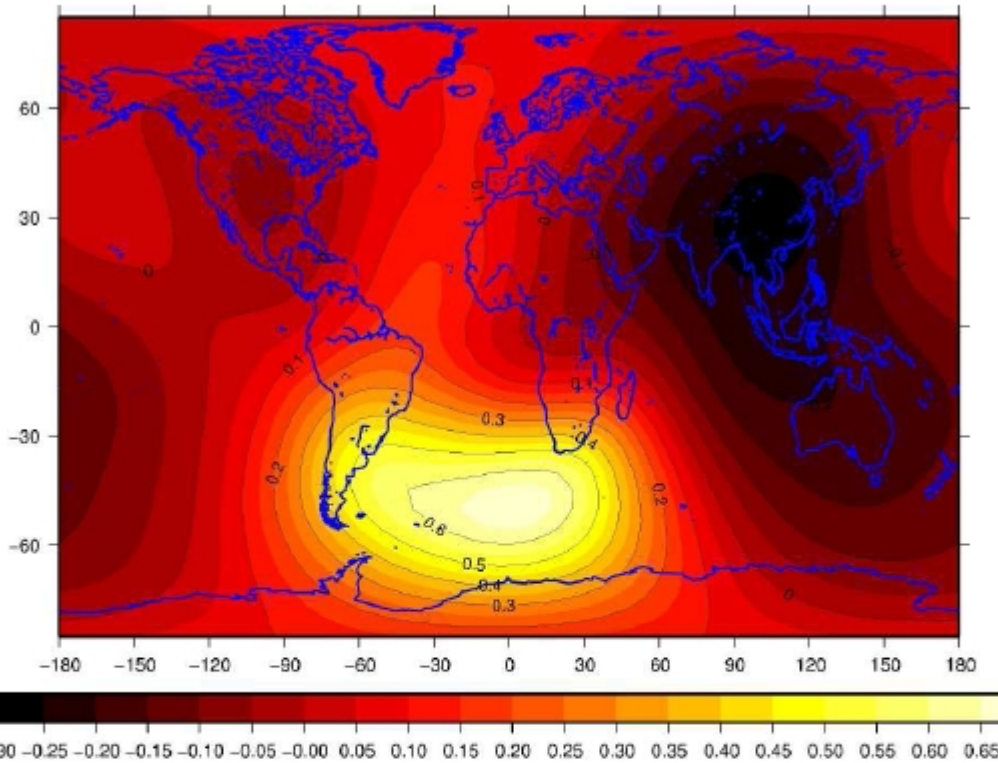
**Position:**

latitude: 25° N  
longitude: 10° E

**Conjugate Position:**

latitude: 3.4° S  
longitude: 11° E

# A simple and accurate approach to compute and apply the I2 correction: Magnetic field term



**The Magnetic Field,  $B$ , is the second dependence of I2, which can be computed by using the more accurate International Geomagnetic Reference model (IGRM), reducing the error up to 60% regarding the previously used dipolar model (this is specially evident at the Atlantic South Anomaly -see relative error of dipolar model at left hand plot, and comparison of I2 corrections in Ascension Island, ASC1, at right hand plot-).**



### 3) How should the I2+ terms be computed?

- The correction for both Lc/L3 and Pc/P3 observations can be computed in different ways.
- Once the iono single-layer simplification is taken, there are two sources to deal with: the **electron content source to compute the STEC**, and the **magnetic field source** to compute the magnetic field projection along the observation line-of-sight, at the iono pierce point.
- Different possibilities appear: to use **VTEC maps (IGS GIMS or regional maps provided by other services) to provide STEC vs. using Pi(P4) DCB-corrected observations**, for computing STEC; and by using a **dipolar geomagnetic model versus a more realistic one (such as the IGRM)** to compute the magnetic term.
- In particular the usage of DCB-corrected Pi(P4) observations is good enough to provide an STEC estimate (even better at low latitudes and elevations, compared with the usage of actual IONEX VTEC maps, and it can be easily applied in real-time), combined with the use of a **more realistic geomagnetic model such as IGRM** (I2 improvement greater than 50% in certain regions such as South Atlantic).

# Summarizing approaches to compute I2+

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	STEC source	Don't require an external source (suitable for real-time)	Can be assumed a constant iono-free carrier phase ambiguity?	Scope	I2+ correction error (assuming a nominal I2+ value of 2cm and a realistic geomagnetic field model –such as IGRM-)
Correcting measurements from I2+, including STEC	<i>Klobuchar model</i>	✓OK	✓OK	<i>Global</i>	<b>30-50% (~6 to 10 mm)</b>
(Kedar et al. 2003 [7], Fritsche et al. 2005 [4])	IGS VTEC in IONEX format	<b>NO</b>	✓OK	Global	10-20% (~2 to 4 mm)
(Hernandez-Pajares et al. 2007 [5])	DCB corrected PI(P4)	✓OK	✓OK	Global	~0.1 to 1 mm (depending whether smoothed Pi is used or not)
New “dynamic” Lc combination (Brunner & Gu 1991 [2])	-	✓OK	<b>NO</b>	Global	0.1% of Bi(B4) (from 1 to 10 mm for first LI measurement aligned to BI up to <b>unbounded values</b> otherwise)
(Hoque & Jakowski 2007 [6,6'])	-	✓OK	✓OK	Regional	~1 to 3 mm (depending on user positions)

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# Is I2+ related with N/S deformation in IGS combined SINEX frame?

Could be I2+ a possible explanation for the larger variation in the weekly network-averaged N/S component of the IGS combined SINEX frame (after removing Helmert differences)? There is a large common-mode annual signal in the N/S but not in E/W and much smaller in the U/D. The dispersion among ACs also seems to be largest in the N/S component.

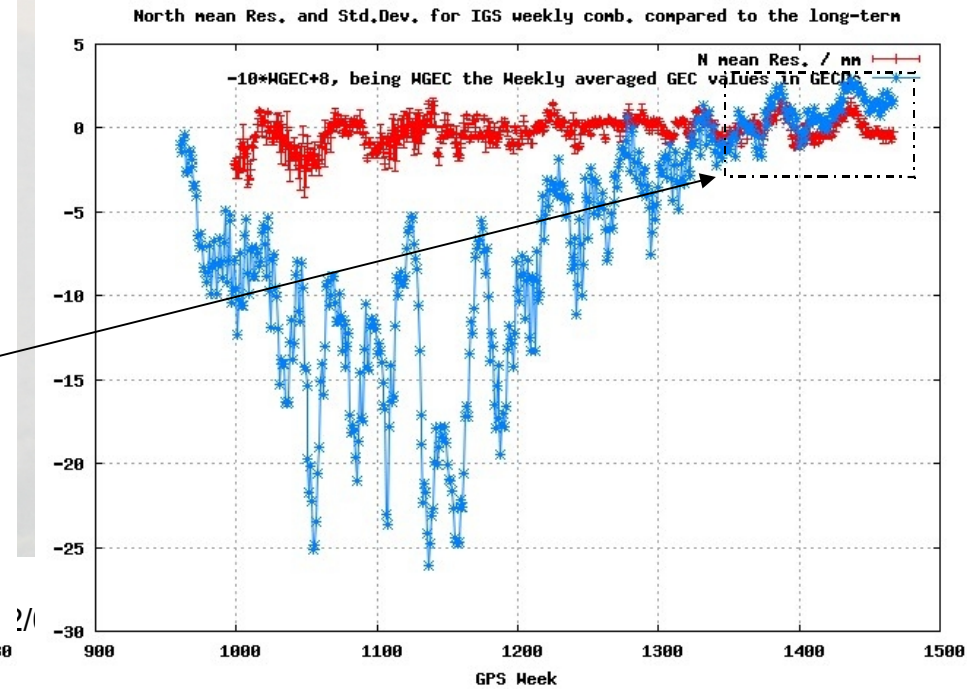
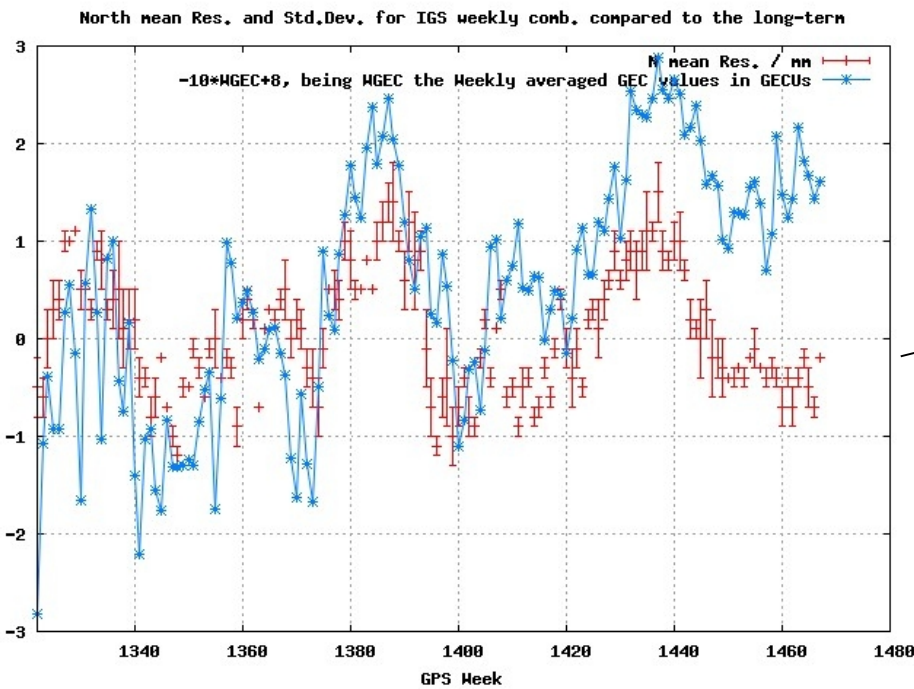
At “short term” (left-hand plot, since GPSweek 1350): **Apparent correlation with -GEC, qualitatively compatible with I2 origin** in a similar way that has been found between geocenter/satellite and GEC ([4], [8], [5]).

The apparent correlation seems quite good for some receivers.

Moreover in the residuals for receivers around the geomagnetic equator the signature is much more clear, than considering all of them. This is again qualitatively compatible with the potential I2 origin ([5]).

Within this period a geomagnetic-latitudinal effect compatible with [5] is observed, being significant in the periods with higher GEC (such as GPSweek 1440) and practically flat for low GEC periods (GPSweeks 1432 and 1450).

At “long term” (right-hand plot, since GPSweek 1000), **such apparent correlation is not evident & too large effects.**



# Conclusions

- It is necessary to incorporate the higher order ionospheric correction (I2+) as a standard term in precise GNSS models because: (1) the contribution of the I2+ effect is not negligible (several centimeters in range), (2) the algorithms are easy to implement. And (3) the I2+ effect, is significant to eliminate associated spurious trends, improving the GNSS accuracy, in particular on satellite clocks and orbits.
- The I2+ correction should be applied consistently, with GNSS products (such as satellite orbits and clocks) computed after applying these corrections on GNSS measurements. It would be preferable to use a model that allows to correct each individual measurement.
- The I2+ correction should be computed in a simple and accurate way, at least from IGS AC & ACCs perspectives. As it depends on STEC and magnetic field:
  - Regarding to the magnetic field, it should be computed from a more realistic model (such as the IGRM) than the dipolar one.
  - Regarding to the STEC, among other sources, it can be computed from IGS VTEC maps in IONEX formats, or by correcting the geometry free combination of pseudoranges  $P_i(P4)$  by the corresponding interfrequency biases. The use of  $P_i(P4)$  is suitable for real-time application and is in general accurate enough for computing I2+ (and it can be still better at low latitudes and elevations).
- Potential relationship of I2+ effect with weekly network-averaged N/S component of the IGS combined SINEX frame (after removing Helmert differences): only at short term, last 2 years or so, **but not for long term**. More studies are needed: the reprocessing by correction the I2+ term in long time AC series can clarify this potential origin.

# References

- [1] Bassiri, S., and G. Hajj (1993), High-order ionospheric effects on the global positioning system observables and means of modeling them, *Manuscr. Geod.*, 18, 280– 289.
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- [6'] Hoque M. M. and N. Jakowski, Mitigation of higher order ionospheric effects on GNSS users in Europe, *GPS Solutions*, 12 (2), DOI 10.1007/s10291-007-0069-5, 2007.
- [6''] Jakowski N., F. Porsch, and G. Mayer, Ionosphere - Induced -Ray-Path Bending Effects in Precision Satellite Positioning Systems, *SPN 1/94*, 6-13, 1994
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# Backup slides

# Recommendations on higher ionospheric order terms for IGS ACs

- The higher order ionospheric correction (I2+) should be incorporated as an standard model term.
- The I2+ correction should be applied consistently.
  - With GNSS products, such as satellite orbits and clocks, computed after applying these corrections on GNSS measurements.
- The I2+ correction should be computed in a simple and accurate way:
  - The magnetic field should be computed from a more realistic model (such as the IGRM) than the dipolar one.
  - The slant ionospheric delay (STEC) can be computed from VTEC maps (such as those computed by IGS in IONEX format).
  - It can be preferable, in particular for low elevation, low latitude usage or when no external GIM or TEC source is available (such in real-time), to compute STEC from the carrier smoothed geometry free combination of pseudoranges  $P_i(P4)$ , corrected by the corresponding interfrequency biases.

# I<sub>2</sub> estimation using STEC and a regional (European) correction model

$$I_2 = \frac{7527c \cdot \overline{B \cos \theta} \cdot STEC}{2f_1 f_2 (f_1 + f_2)}$$

$\overline{B \cos \theta}$  is the average longitudinal component of  $B$  along ray path

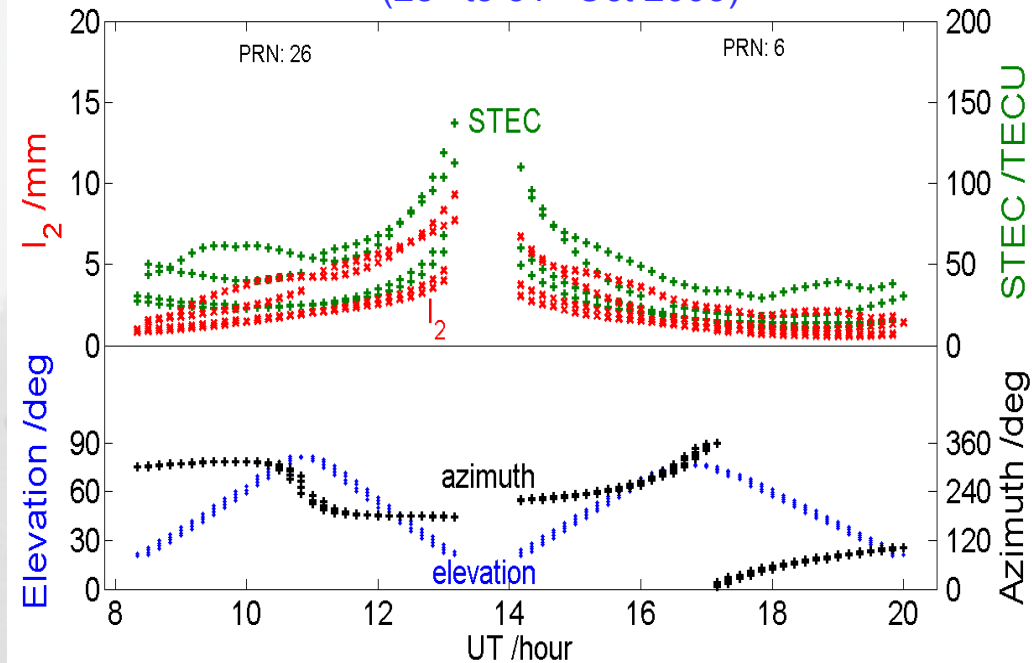
Regional correction model:

$$\begin{aligned} \overline{B \cos \theta} &= \overline{B \cos \theta}(\varepsilon, \alpha, \phi, \lambda) \\ &= -y_1 \cdot \cos \alpha + \left| \sqrt{r_1^2 - y_1^2 \cdot \sin^2 \alpha} \right| - 2r_2 \cdot \cos \alpha' \end{aligned}$$

where  $\varepsilon$ ,  $\alpha$ ,  $\alpha'$ ,  $\phi$  and  $\lambda$  are user-to-satellite elevation, azimuth, modified azimuth and user latitude and longitude. The coefficients for parameters  $r_1$ ,  $r_2$ ,  $y_1$  can be derived for specific regions as e.g. Europe. Specific ionospheric – geomagnetic relationships, typical for a region are reproduced in the coefficients (Hoque and Jakowski 2007)

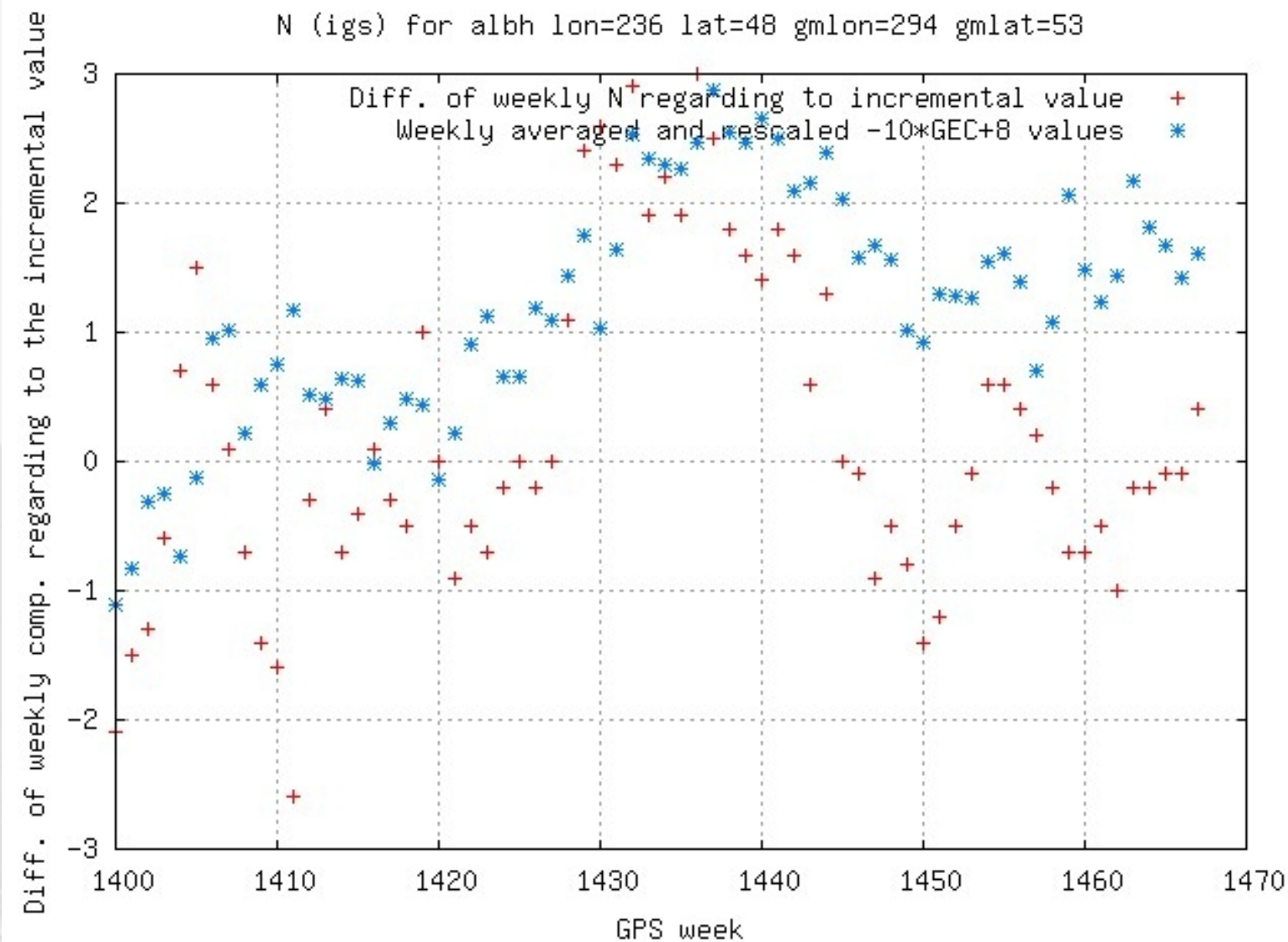
The regional correction **needs only slant TEC** (derivable from direct link related TEC estimations at user level or from external VTEC maps via mapping function) as input.

I<sub>2</sub> values at Matera (Italy) during Halloween storm  
(28<sup>th</sup> to 31<sup>st</sup> Oct 2003)

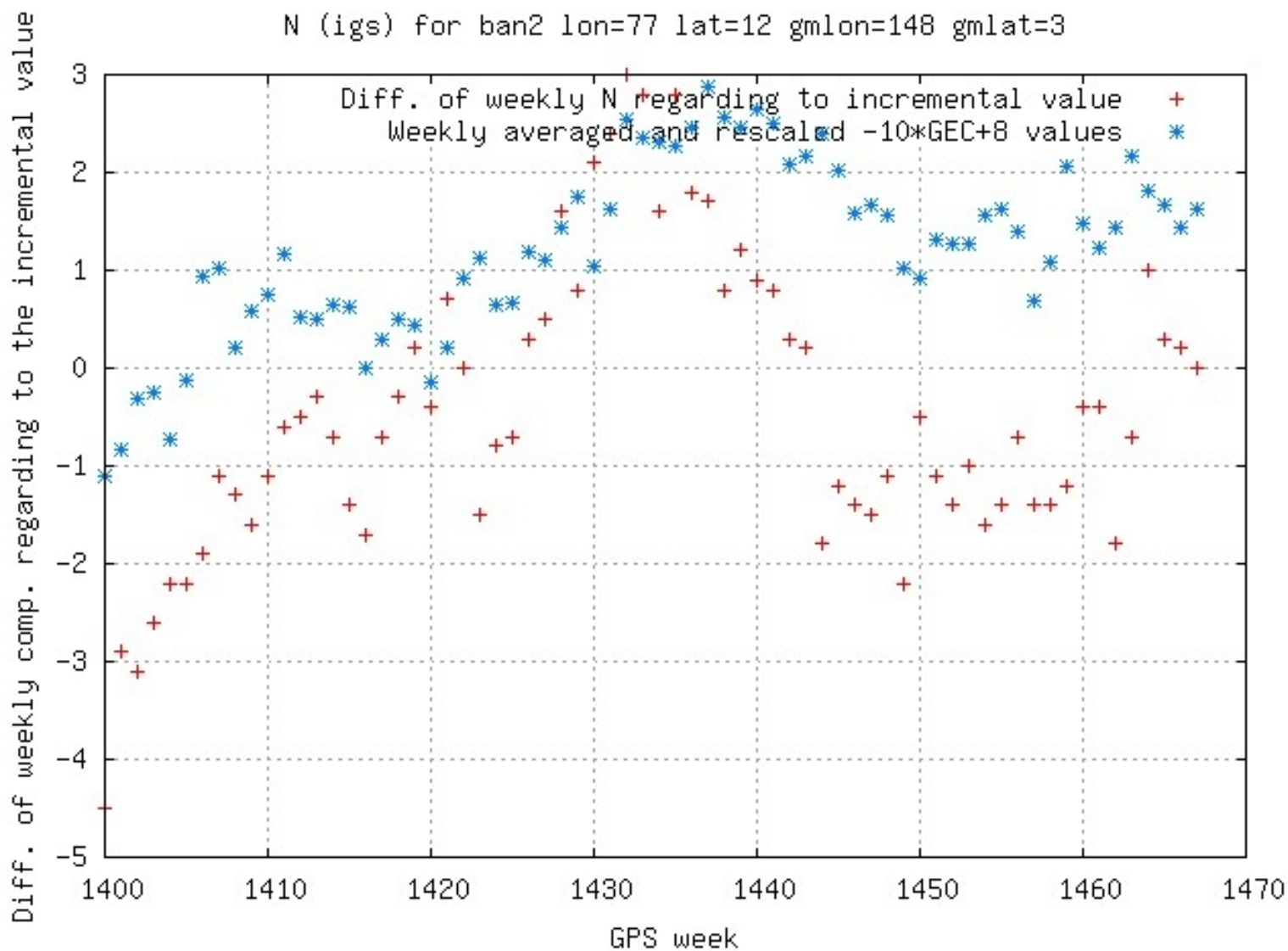




# Backup slide 1: potential I2+ origin...



# Backup slide 2: potential I2+ origin...



# Backup slide 3: potential I2+ origin...

