NOAA Technical Memorandum NOS NGS - 54

A PROGRADE GEOSAT EXACT REPEAT MISSION?

Carl Wagner

Rockville, MD
August 1990

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A PROGRADE GEOSAT EXACT REPEAT MISSION?

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ABSTRACT. Prograde/retrograde orbit pairs that yield close ground tracks at low latitudes are cited for Exact Repeat Missions (ERM). In particular, the prograde orbit which most closely matches the Geosat ERM (108° inclination) is found to have an inclination of 65.1°. This orbit can stay within ±10 km of the Geosat ERM ground track to ±28° latitude. The objective is to duplicate this well-covered retrograde track as much as possible, thereby maximizing its oceanographic use in the event another retrograde mission will not be possible. A prograde Geosat ERM can support a heavier satellite and allow the P_1 tide to be determined from collinear altimetry, as compared to the current mission which tracks to somewhat higher latitudes and on which the P_1 tide is stationary.

INTRODUCTION

Since the highly successful launch of the altimeter bearing Geodetic Satellite A (Born et al. 1987) by the U.S. Navy in 1985 into a retrograde orbit ($I = 108°$), there has been discussion of a follow-on mission. Originally, Geosat B or Spinsat was to be in the same orbit principally to satisfy the same security concerns for Geosat A, that its Exact Repeat Mission (ERM) must duplicate the 17-day repeat track of the earlier unrestricted Seasat in the summer of 1978. In addition, of course, ground track continuity enhances the oceanographic applications of these missions. However, Geosat B was found to be too heavy for launch into a retrograde orbit. To save the mission, it was proposed that a prograde Geosat ERM would be an attractive alternative for civilian oceanographers (Bruce Douglas, NGS, personal communication, 1990).

Unfortunately, the same circumstance which restricts the weight of retrograde launches, namely the rotation of the Earth, also restricts the possibilities of close track-matching for its prograde orbit-pair to lower latitudes only. The estimation of the inclination of these closely matching prograde orbits and their resulting track deviations (departures from the reference retrograde track) follows below.

ORBIT BIAS FOR EQUATORIALLY MATCHED TRACKS

As figure 1 shows, to achieve an exact match of geographic tracks at the Equator, it is necessary to bias the corresponding prograde orbit inclination by $\alpha_R + \alpha_P$ less than the ideal everywhere-matched value for a "frozen" (nonrotating) Earth, of $180° - I_R$. From figure 1, I find that the two components of this deflection bias are given by

$$\cos \alpha_{R,P} = \frac{[1-(r_{R,P}\,\dot{\theta}_{R,P}/v_{R,P}) \cos I_{R,P}]}{\{\sin^2 I_{R,P} + [(r_{R,P}\,\dot{\theta}_{R,P}/v_{R,P}) - \cos I_{R,P}]^2\}^{1/2}} \tag{1}$$

where I is the orbit inclination and α the inclination bias.

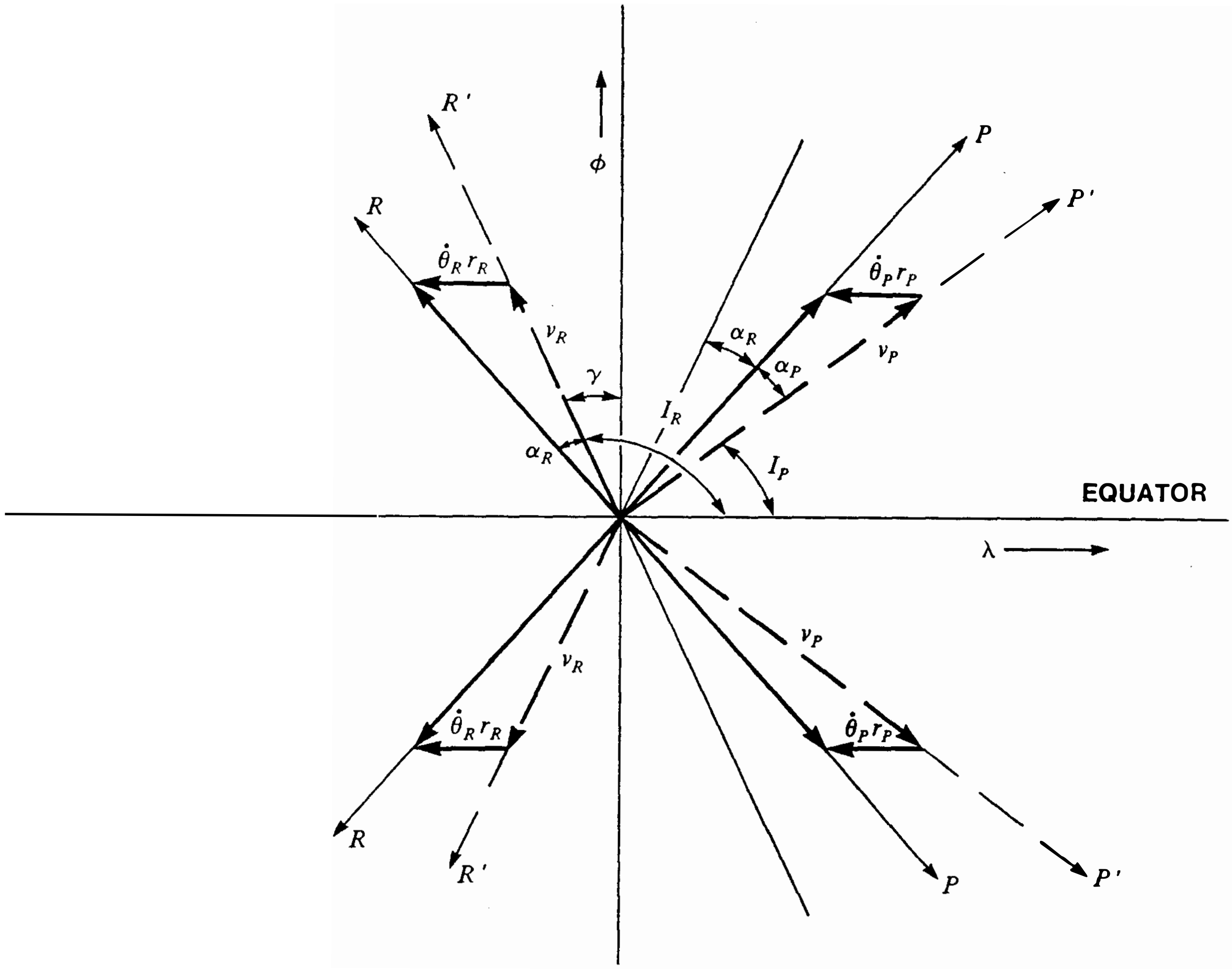


Figure 1. Retrograde/prograde orbit pairs with close ground tracks. Shown are horizontal velocity vectors of satellites at the Equator with respect to the Earth's surface (solid) and inertial space (dashed). The retrograde satellites are labeled *R*, the prograde *P*. The velocities of the satellites with respect to inertial space are v_P and v_R; their radii from the center of the Earth are r_P and r_R, and the effective Earth rotation rates for their orbits are $\dot{\theta}_P$ and $\dot{\theta}_R$.

The corresponding equatorially matching prograde orbit inclination I_p is then given as:

$$I_p = 180^\circ - I_R - \alpha_R - \alpha_P \ . \tag{2}$$

In eq. (1) $r_{R,P}$ is the radius to the satellite (retrograde and prograde), and v_R and v_P are their circumferential velocities at the Equator. The effective rotation rates of the Earth for these orbits (accounting for the rotation of their orbit planes) are

$$\dot{\theta}_{R,P} = \dot{\theta}_e - \dot{\Omega}_{R,P} \tag{3}$$

where $\dot{\Omega}$ is the precession rate of the orbit's node and $\dot{\theta}_e$ is the actual rotation rate of the Earth.

These rates and satellite velocities (for a near circular orbit) are given by (Wagner 1987, p. 8138):

$$\dot{\Omega} = \frac{3}{2} n\, C_{20} (r_e/a)^2 \cos I \tag{4}$$

where C_{20} is the principal geopotential term in the Earth's oblateness, r_e is the Earth's mean equatorial radius, and a is the mean semimajor axis of the (near circular) orbit,

$$n = \mu^{1/2} a^{-3/2} \{1 - \frac{3}{2} C_{20} (r_e/a)^2 \, (1 - \frac{3}{2} \sin^2 I)\} \tag{5}$$

where n is the mean motion of the satellite and μ is the Gaussian gravitational constant for the Earth,

$$v = a(n + \dot{\omega}) \tag{6}$$

where $\dot{\omega}$ is the mean motion of the satellite's perigee,

$$\dot{\omega} = -\frac{3}{4} n\, C_{20} (r_e/a)^2 \, (5 \cos^2 I - 1) \tag{7}$$

$$C_{20} = -1082.63 x 10^{-6} \tag{8}$$

$$\mu = 398600.5 \ km^3/s^2 \tag{9}$$

$$r_e = 6378.138 \ km \tag{10}$$

$$\dot{\theta}_e = 0.7292115 \ x \ 10^{-4} \ rad/s \tag{11}$$

To determine a for an Exact Repeat Mission of R (integer) revolutions in D (integer) synodic (or nodal) days, where one nodal day is

$$T_N = \frac{2\pi}{\dot{\theta}_e - \dot{\Omega}} , \tag{12}$$

the algebraic equation

$$n + \dot{\omega} = 2\pi \frac{R}{DT_N} = [R(\dot{\theta}_e - \dot{\Omega})/D] , \quad or$$
$$(R/D) = \frac{[n + \dot{\omega}]}{[\dot{\theta}_e - \dot{\Omega}]} \tag{13}$$

must be solved for a, given the inclination I of the orbit and the mean motions, eqs. (4), (5), and (7), which are accurate to first order in the oblateness, or 10^{-6} (see e.g., Cutting *et al.* 1978 or Stewart 1985).

In practice, the initial spherical Earth approximation:

$$(R/D) = (n/\dot{\theta}_e) = [\mu/a^3]^{1/2}/\dot{\theta}_e$$

yielding

$$a_0 = \mu^{1/3}[(R/D)\dot{\theta}_e]^{-2/3} \tag{14}$$

may be used to start an iterative solution of eq. (13), namely

$$a_{i+1} = \mu^{1/3}[R\dot{\theta}_e/D]^{-2/3}\left[1 - \frac{3}{2}C_{20}(r_e/a_i)^2\left(1 - \frac{3}{2}\sin^2 I\right)\right]^{2/3} \cdot$$
$$\cdot\left[1 + C_{20}(r_e/a_i)^2\left\{\frac{3}{2}(R/D)\cos I - \frac{3}{4}(5\cos^2 I - 1)\right\}\right]^{2/3} . \tag{15}$$

However, ignoring terms in C_{20}^2 and higher powers permits an estimate of the mean orbit a to order 100 m:

$$a \sim a_0\left[1 - C_{20}(r_e/a_0)^2\{4\cos^2 I - (R/D)\cos I - 1\}\right] . \tag{16}$$

In any case, formal convergence of a to 1 m from eqs. (14) and (15) is obtained by the third iteration over all inclinations and ERM specifications *R/D*.

In particular, for the Geosat ERM with $I = 108°$, $R = 244$ and $D = 17$, the mean semimajor axis a is found from eq. (15) to be 7162.578 km (784.440 km altitude). The ERM cycle time, DT_N, from eq. (12) is found to be 17.0505 days.

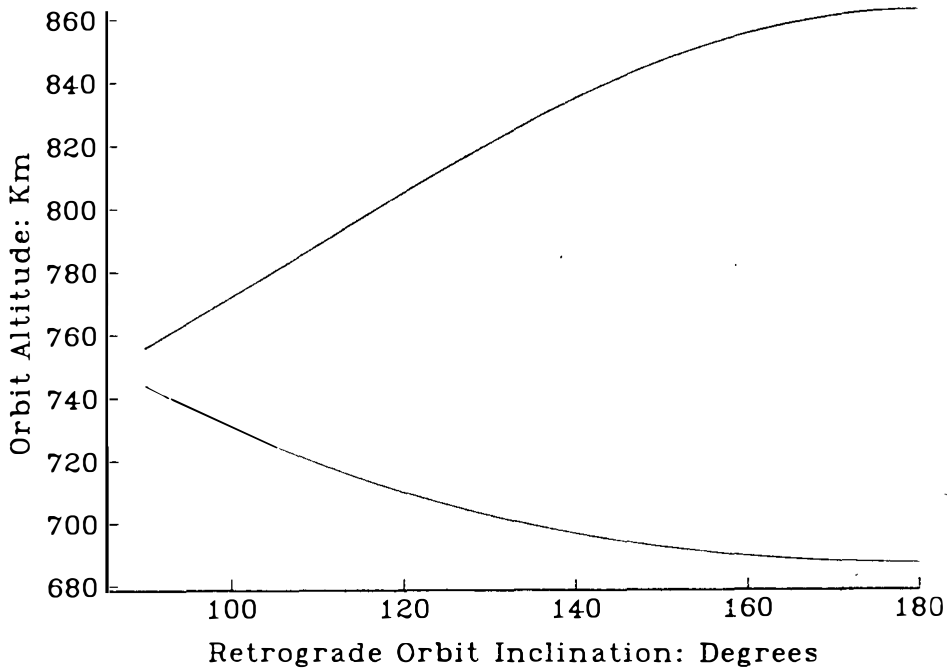


Figure 2a. Orbit altitude for prograde/retrograde ERM pairs. The Exact Repeat Mission is 244 revolutions in 17 days. The upper curve is for the retrograde orbit, the lower curve for the equatorially matching prograde orbit.

But because of the effects of oblateness on the mean motions [eqs. (4), (5), and (7)], the mean semimajor axis of the frozen Earth prograde Geosat ERM orbit (at 180° - 108° = 72°) is [from eqs. (14) and (15)] only 7107.716 km (729.578 km altitude). Thus the prograde Geosat not only has the launch advantage over the retrograde ERM of gaining a measure of the Earth's rotation velocity, but the required altitude to achieve the 244 revolutions in a 17-day orbit is also significantly less than for the retrograde mission. (See fig. 2a.)

Continuing with the estimation of the equatorially matching prograde ERM orbit (both its mean semimajor axis and its inclination), a great simplification of the nonlinear system determining these parameters [namely eqs. (1), (2), and (13)] is effected by assuming initially a spherical Earth as well as a near circular orbit, implying that:

$$\dot{\theta}_P a_P / v_p = \dot{\theta}_R a_R / v_R = \dot{\theta}_e a_R / v_R = \dot{\theta}_e a_R^{3/2} \mu^{-1/2} = p_R \ . \tag{17}$$

In addition, for a close Earth satellite $p_R << 1$ and $\alpha_{p,R} << 1$. (Note that for a 24-hour orbit, $p = 1$ and α can be large). With these assumptions, the solution of eq. (1) is simply

$$\alpha_R = \alpha_P = p_R \sin I_R \tag{18}$$

and the prograde inclination from eq. (2) is

$$I_P = 180° - I_R - \frac{360 p_R \sin I_R}{\pi} \quad . \tag{19}$$

With these simplifications and using a_R = 7163 km (which would be available in any case from the well-tracked Geosat orbit), the initial estimate of the prograde Geosat ERM's inclination, from eq. (19) is 64.4°. (See figs. 2b, c.) A much more precise value, based on eq. (15) or (16) is 64.58°, significantly less than the frozen Earth estimate of 72°.

Unfortunately the bias from the frozen Earth value of inclination, due to the Earth's rotation, means that the prograde track will depart significantly from the retrograde at high latitudes. Thus for the Geosat-pair, the difference is more than 7° at maximum latitude. But the equatorially matched tracks are still tangent to one another (geographically) at the Equator and stay close for a considerable distance before the departure becomes excessive.

In the following discussion, I use an arbitrary reference figure of 10 km to define acceptable closeness to the retrograde track. It is about the maximum deviation of the 17-day Seasat orbit from the Geosat ERM. Thus a track within these limits would satisfy military security considerations since all Seasat and Geosat ERM altimetry has been released publicly. Closeness requirements from a scientific standpoint are more difficult to define since they depend on cross track sea surface gradient errors that are highly variable. Certainly with such a mission it will be necessary when comparing sea surface heights over "close" tracks to remove a detailed "permanent" surface (predominantly the geoid) from the height in both tracks before assessing oceanographic variability.

Nevertheless, since the prograde orbit always has a smaller maximum latitude than the retrograde, it is always possible to "fine tune" its bias by raising its inclination somewhat and still maintain acceptable closeness to the retrograde track to greater latitudes than the simple equatorially matching track.

ORBIT BIAS ESTIMATION FOR ACCEPTABLE DEPARTURES

Figure 3 shows the evolution of equatorially matching track-pairs to high latitude for the Geosat ERM, as generated by numerical integration from a typical node point. On this and subsequent figures, I show a departure zone surrounding the retrograde track. Evidently the prograde track stays "close" to the retrograde reference for more than ±20° in latitude. While it is difficult to determine analytically the departure (or geographic separation) of the tracks as a function of the latitude, it is fairly straightforward to do this by an iterative procedure outlined in figure 4.

Let t_1 be the time forward in the prograde track from the node N (or N') to reach latitude ϕ. In this time, for a circular orbit, the argument of latitude will be $f_P = t_1 (n_P + \dot{\omega}_P)$, where the two mean motions are computed from eqs. (5) and (7) for inclination I_P and mean semimajor axis a_P [determined for the ERM by eq. (15) or (16)].

The inclination I_P will be close to the equatorially matching value found previously. Here we seek to find an optimum change from this equatorially matching track which maximizes the time spent within a specified acceptable departure from the reference retrograde track.

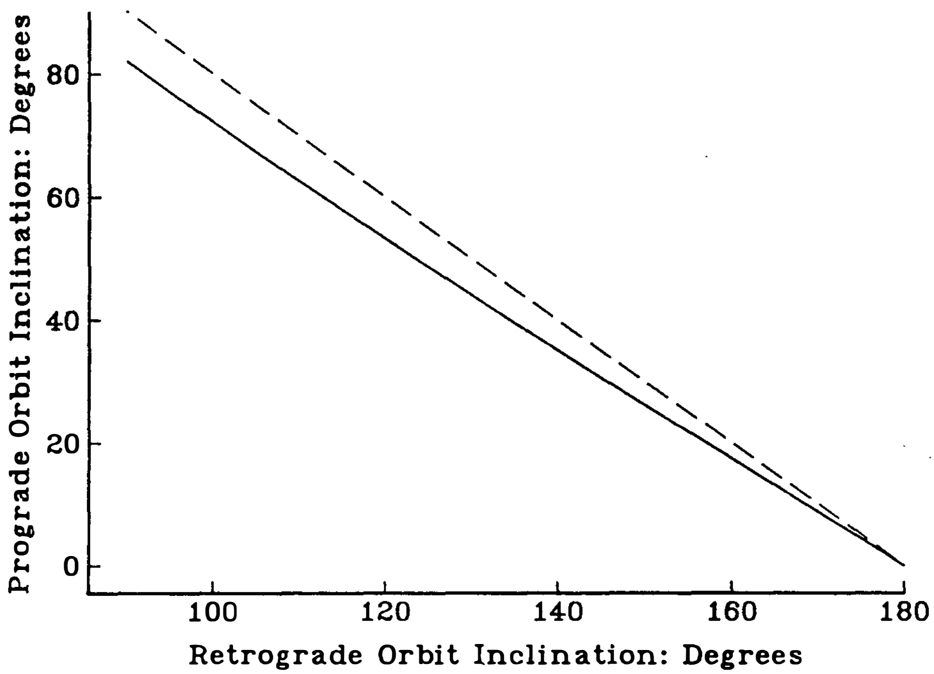


Figure 2b. Inclination of matching ERM orbit pairs. The ERM completes 244 revolutions in 17 days. The solid curve is for the rotating Earth, the dashed (line) for a "frozen" Earth.

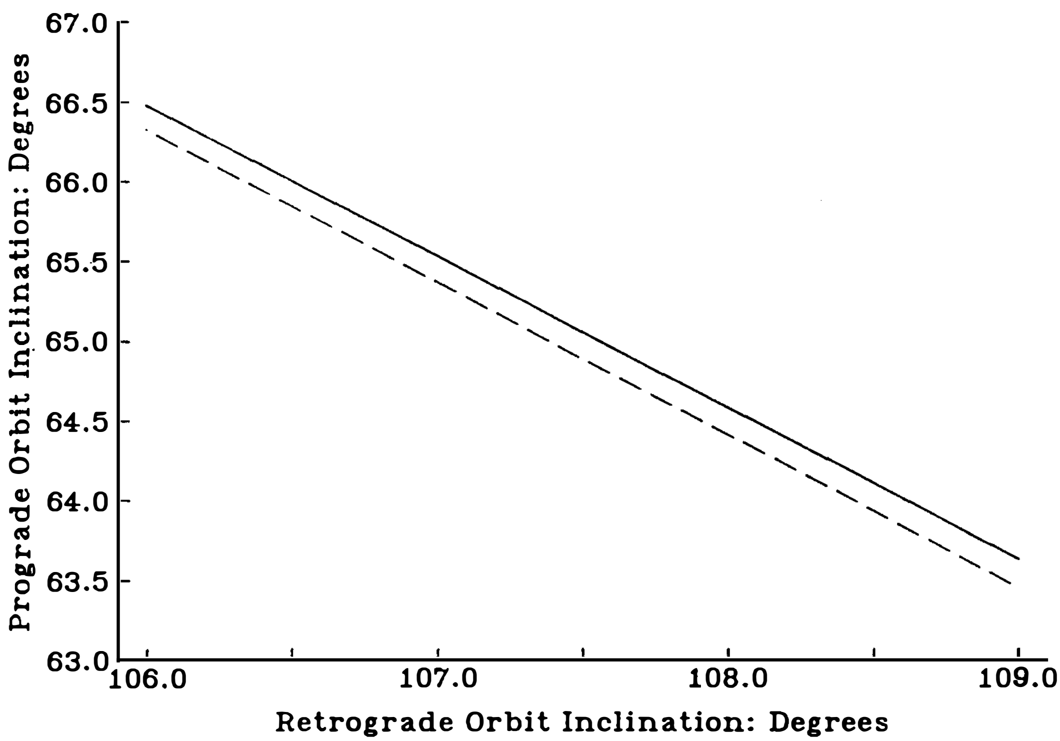


Figure 2c. Inclination of matching ERM orbit pairs. The solid curve refers to the estimate based on a rotating, oblate Earth. The dashed curve refers to the estimate based on a rotating spherical Earth.

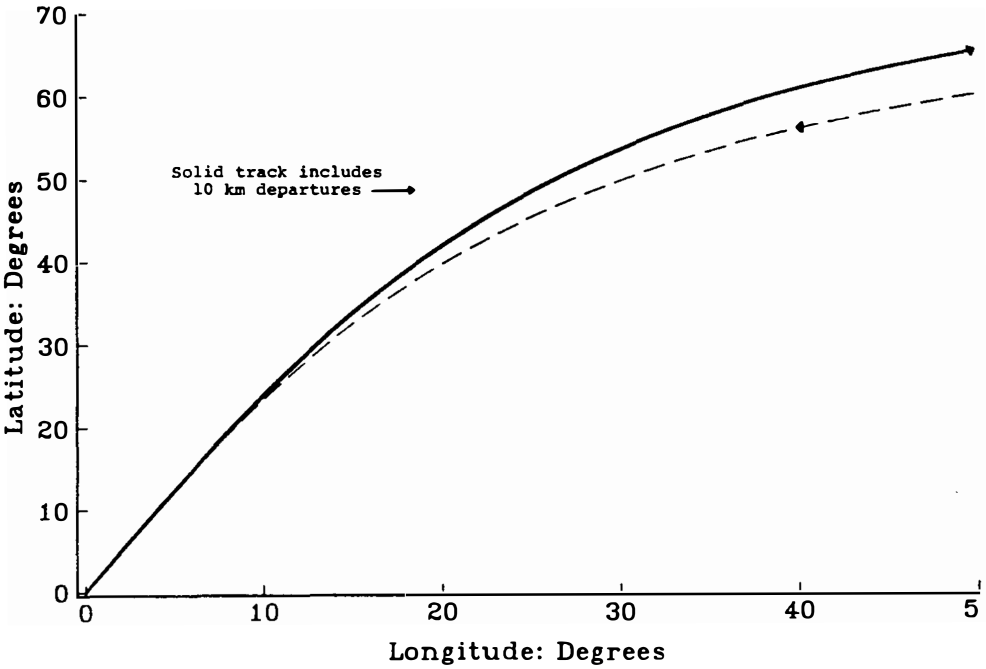


Figure 3. Geosat ground tracks for prograde/retrograde orbit pairs. The solid curve shows the track near a node for the retrograde ERM orbit of 244 revolutions in 17 days at 108° inclination as determined by numerical integration. Also shown are ±10 km parallel tracks. The dashed track arises from a 64.6° inclined orbit of 244 revolutions in 17 days (ERM), also determined from numerical integration.

Including the small rotation of the orbit plane in this short time, the geographic longitude in the prograde track (with respect to N') at t_1 is

$$\lambda'_P = \Delta\lambda_P - \dot{\theta}_P t_1 = \cos^{-1}[(1 - \sin^2\phi/\sin^2 I_P)^{1/2}/\cos\phi] - [\dot{\theta}_P \sin^{-1}(\sin\phi/\sin I_P)]/(n_P + \dot{\omega}_P). \tag{20}$$

Similarly, the geographic longitude back along the retrograde track (with respect to N') at t_2 <u>before</u> reaching N' (at ϕ) is

$$\lambda'_R = \Delta\lambda_R + \dot{\theta}_R t_2 = \cos^{-1}[(1 - \sin^2\phi/\sin^2 I_R)^{1/2}/\cos\phi] + [\dot{\theta}_R \sin^{-1}(\sin\phi/\sin I_p)]/(n_R + \dot{\omega}_R). \tag{21}$$

Finally, the geographic distance, d, between these points of the track (at ϕ) is

$$d = \cos^{-1}[\sin^2\phi + \cos^2\phi\cos(\lambda'_R - \lambda'_P)]. \tag{22}$$

Evidently, from the curvatures of the tracks, if $\lambda_R' > \lambda_P'$ the crossover point has not been reached and the minimum distance (or departure) of the prograde track at ϕ, λ'_P can be found by <u>decreasing</u> f_R until that minimum distance is reached. Alternately, if $\lambda_R' < \lambda_P'$ the crossover has been passed and the departure can be found by <u>increasing</u> f_R. In either case, for

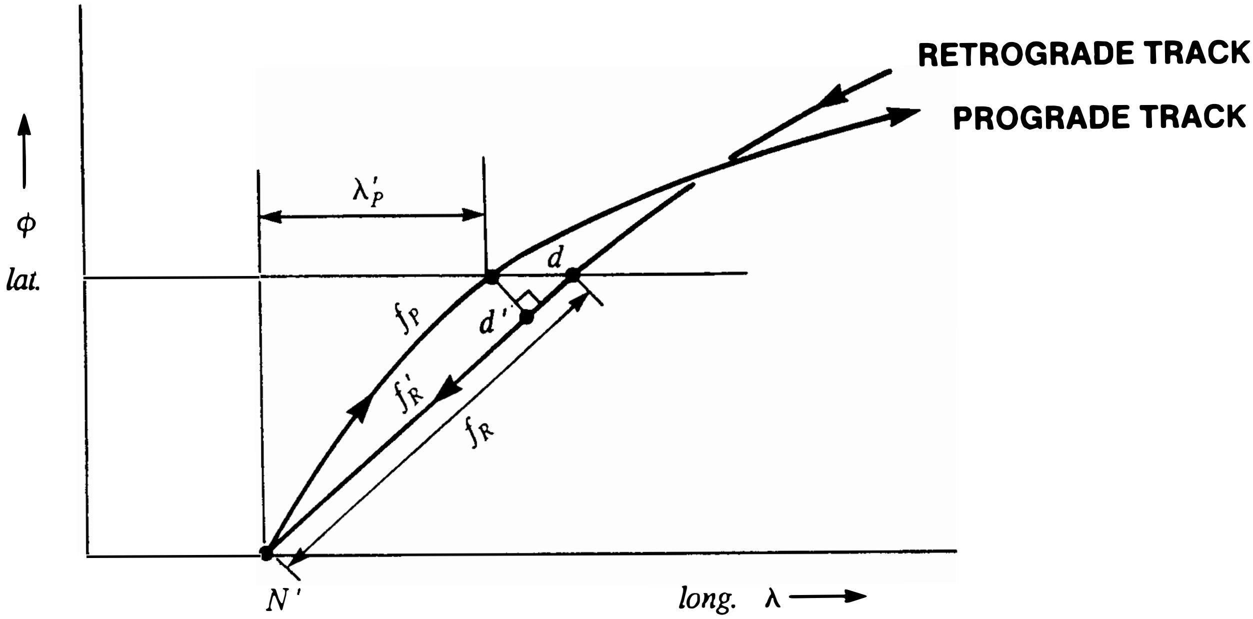


Figure 4a. Geographic tracks of a prograde/retrograde orbit pair.

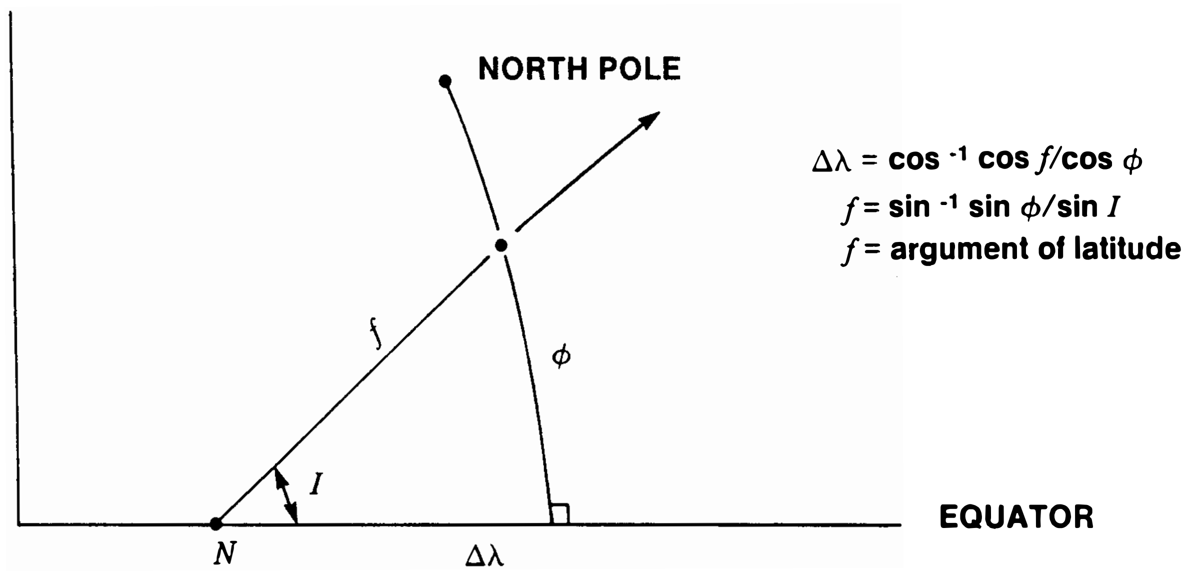


Figure 4b. Inertial track of an orbit.

each trial f_R' beyond f_R, the latitude in the retrograde track is determined from

$$\sin\phi_R = \sin f_R' \sin I_R . \tag{23}$$

Then the new retrograde longitude is found from eq. (21) with ϕ_R replacing ϕ, and the new distance between the track points from

$$d' = \cos^{-1}[\sin\phi \sin\phi_R + \cos\phi\cos\phi_R\cos(\lambda_R' - \lambda_P')] . \tag{24}$$

Figure 5 shows the results of these departure calculations for fine-tuned Geosat ERM pairs. (See also figs. 6a, b, c.) For example, the departure can be kept to within ±10 km as far as latitude ±27.6° if the biased prograde orbit has an inclination of 65.07°. The mean altitude of such an orbit (from fig. 2a) is 722 km. The ERM cycle time for 244 revolutions is also reduced from that of the retrograde Geosat, from 17.0505 days to only 16.8193 days.

ALIASED TIDAL PERIODS

Since the oblateness of the Earth is of order 10^{-3}, the rotation of the orbit plane of a close Earth satellite is of order 10^{-3} times its mean motion, or about 5° a day. Polar orbiting satellites experience no plane rotation and those that are slightly retrograde have positive rotation rates near 1° per day, which can make them nearly synchronous with tides that depend on the Sun.

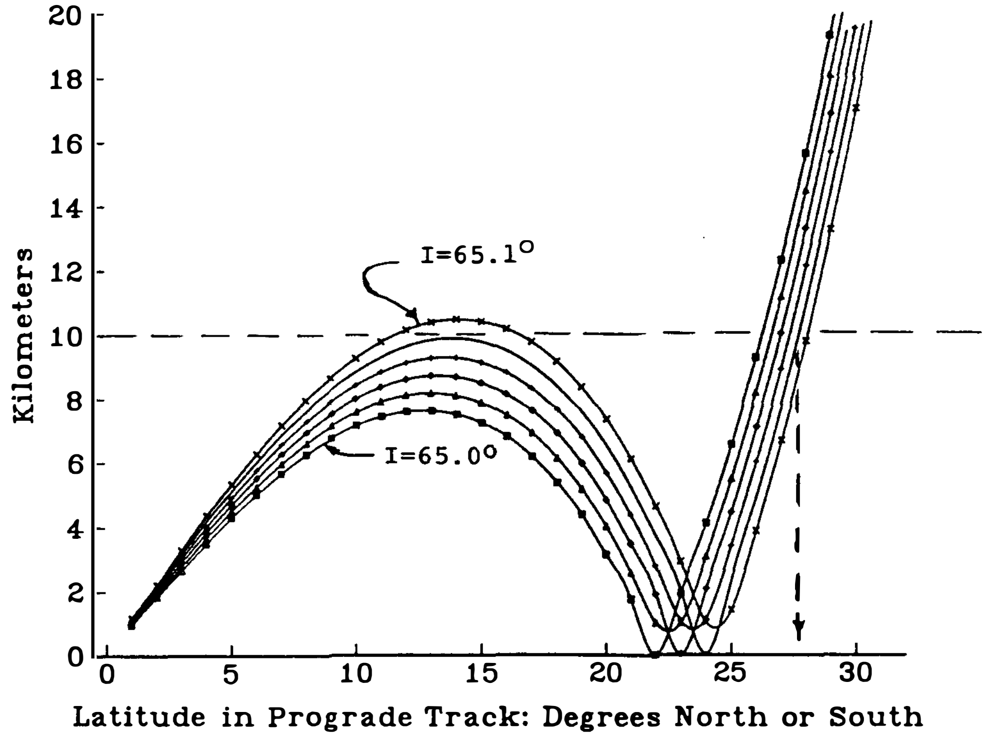
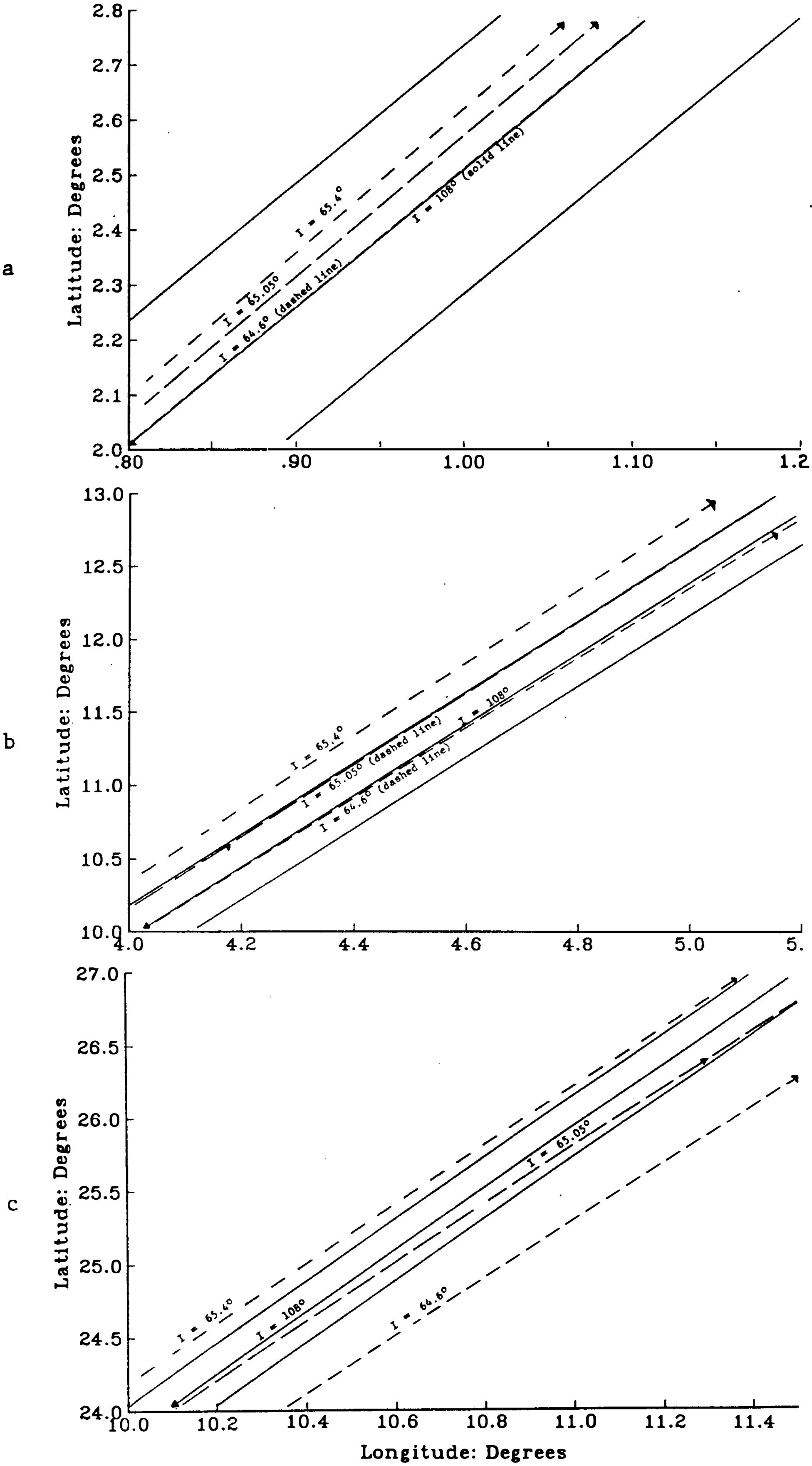


Figure 5. Departure of prograde track from the Geosat ERM retrograde reference. Shown are the track separations for nonmatched prograde orbits away from the Equator as determined by iteration from eqs. (20) through (24). The matching prograde orbit (at the Equator) has an inclination of 64.6°. Notice these overbiased tracks cross over the retrograde track between ±20° - 25° latitude. The matched track has a uniformly increasing departure to high latitude.



Figures 6a, b, c. Geosat ground tracks for retrograde/prograde orbit pairs. The ERM for the pairs are 244 revolutions in 17 days, but an oblate Earth is assumed and the tracks are determined by numerical integration. The black solid curves are tracks for the retrograde orbit (I = 108°) and ±10 km departures.

For example, table 1 shows the aliased periods (in ERM cycles) for the 11 principal lunisolar tides on the retrograde Geosat A orbit. These are the periods as seen by the altimeter returning every 17.0505 days (for the retrograde mission) and 16.8193 days (for the prograde mission) to the same geographic point over the Earth. Thus, in the retrograde mission a single ERM cycle of 17.0505 days corresponds to 32.9462 M_2 cycles. This ERM samples an M_2 wave moving at -0.0538 M_2 cycles every ERM cycle. The sampling is complete only after 1/0.0538 = 18.6 ERM cycles. (See table 1.)

Table 1.--Aliased tide periods for Geosat ERM pairs [repeat cycle: 244 revolutions/17 days. cycle times: I = 108°, 17.0505^d; I = 65.1°, 16.8193^d]

Tide name	Equilibrium amplitude (cm)	Period (hr)	ERM aliased period ERM cycles I = 108°	I = 65.1°
M_2	24.2	12.42060	18.60	2.00
K_1	14.2	23.93447	10.29	7.42
S_2	11.3	12.00000	9.90	2.77
0_1	10.1	25.81934	6.62	2.73
P_1	4.7	24.06589	261.96	4.41
N_2	4.6	12.65835	3.05	9.02
M_f	4.2	327.8406	4.03	4.33
K_2	3.1	11.96724	5.15	3.71
M_m	2.2	661.301	1.6	2.57
SS_a	1.9	4383.00	10.7	10.9
Q_1	1.9	26.86816	4.3	42.04

On the other hand, for the prograde mission (1 ERM cycle = 16.8193 days) the aliased M_2 period as seen by the altimeter is only 2 ERM cycles. But note that for the principal solar (diurnal) tide P_1 the aliased period in the retrograde mission is 262 ERM cycles. It is essentially stationary with respect to repetitive passes (of the same kind, ascending or descending) over the same geographic point. Determining P_1 from overlap differences of altimetry yields no information. If differences of crossover altimetry are used, however, only the same phase difference is repeatedly sampled and the two harmonic constants of P_1 cannot be distinguished.

On the other hand, in the prograde mission a single P_1 wave is completely sampled by one kind of pass in only 4.41 ERM cycles (about 74 days). Notice that both principal solar tides (P_1 and S_2) have significantly smaller aliased periods in the prograde mission which should improve their determination, compared to the retrograde Geosat. In addition, the variety of periods provided by data from both missions should greatly benefit the separation of all tidal components from altimeter measurements.

SUMMARY

The feasibility of a prograde mission with ground track closely matching the current retrograde Geosat ERM has been demonstrated. The prograde mission permits greater payload weight both because the launch is in the direction of the Earth's rotation and the Earth's oblateness permits the required mean motions to be achieved at significantly lower altitude. An additional benefit of the prograde Geosat is its ability to determine all solar tide components. However, the rotation of the Earth also reduces the zone where the prograde track can be kept acceptably close to the retrograde. For the Geosat ERM this zone is between ±28° latitude for ±10 km track departure. In addition, the prograde mission does not sample data to as high a latitude as the current retrograde Geosat.

REFERENCES

Born, G. H., Mitchell, J. M., and Heyler, G., 1987: The Geosat-ERM mission design. *J. Astronaut. Sci., 35(2)*, 119-134.

Cutting, E., Born, G. H., and Frautnick, J. C., 1978: Orbit analysis for Seasat A. *J. Astronaut. Sci., 26(4)*, 315-342.

Stewart, R. H., 1985: *Methods of Satellite Oceanography.* Univ. of California Press, Los Angeles, CA, 301.

Wagner, C. A., 1987: Geopotential orbit variations: Applications to error analysis. *J. Geophys. Res., 92(B8)*, 8138.