# ELEVATIONS FROM ZENITH DISTANCES

(MACHINE COMPUTATION)

WITH

# 6 - PLACE NATURAL TANGENT TABLES

## 0° – 45°


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G - 56


U. S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
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# Computation and Adjustment of Elevations from Zenith Distance Observations

Zenith distances are abstracted on form 29 and checked in the field. The abstracts, therefore, contain the starting data for office computation. It is sometimes necessary to compute in the office the values on form 29 for the column headed "Reduction to line joining stations". This column is used only when the observations are reciprocal. Each value in the column is an angle which in seconds equals $-\frac{t - o}{s \sin 1''}$, t being the height of the telescope above the station mark, o the height of the object observed upon above its station mark, and s the geodetic distance between the stations concerned. The computation of the value for $-\frac{t - o}{s \sin 1''}$ can be simplified by first dividing the reciprocal of sin 1" by s (in meters), which reduces to $\frac{206265}{s}$. This value should be written on computation form 29C just under station 2. In order to obtain the correction to the observed zenith distance, the value of $\frac{206265}{s}$ is multiplied by $-(t-o)$. The correction has the opposite algebraic sign to $t-o$.

The value for s, which is the distance between station 1 and station 2 in meters, should be written on form 29C. $\zeta_1$ is the zenith distance of station 2 as observed from station 1, and $\zeta_2$ is the zenith distance of station 1 as observed from station 2. After $\zeta_1$ and $\zeta_2$ are written on form 29C, $\zeta_2 - \zeta_1$ is obtained and then $1/2\ (\zeta_2 - \zeta_1)$. The algebraic sign should be carried with this value as this determines the sign of the difference in elevation between station 1 and station 2. The natural tangent of 1/2 ( $\zeta_2$ - $\zeta_1$) is multiplied by s on the calculating machine and the result placed in the keyboard. This is the approximate value of $h_2 - h_1$. Factor A is a multiplier, the value for which is determined by the elevation of station 1 ($h_1$). Factor B is a multiplier whose value is determined by the difference in elevation of stations 1 and 2. C is a multiplier determined by s. The values of A, B, and C are found in attached tables. The approximate value of $h_2 - h_1$ is then multiplied by the factors A, B and C. In order to simplify this operation, $h_2 - h_1$ may be multiplied by $A + B + C - 2$ when $h_2 - h_1$ is positive or $A - B + C$ when $h_2 - h_1$ is negative. (This approximation is satisfactory in all cases.) The resulting value is the difference in elevation between station 1 and station 2 or ($h_2 - h_1$).

A table for finding the correction to $h_2 - h_1$ directly will be found included in this manual with the A, B and C tables. This table can be used with differences of $h_2 - h_1$ up to 1500 meters and distances up to about 35,000 meters.

The value of $h_2$ is the algebraic sum of $h_1$ and ($h_2 - h_1$).

The weight of a computed difference in elevation is proportional to $\frac{1}{s^2}$. In order to get a value convenient to use, s should first be divided by a multiple of ten. The same divisor must be used in determining the weight for all differences in elevation used in any one adjustment of elevations. A convenient value for $\frac{1}{s^2}$ for distances of 50,000 meters or less will be obtained by dividing s (in meters) by 10,000 before finding the reciprocal. The result should then be multiplied by 10 and carried to two decimal places. The same result can be obtained by log p = 9 - 2 log s.

In the determination of (0.5-m), $\alpha$ and mean $\phi$ are the azimuth from station 1 to station 2 and the mean latitude of the stations, respectively. The value of $\rho \sin 1''$ will be found in attached tables, and $\frac{\rho \sin 1''}{2}$ is 1/2 this value.

The azimuths used in the tables for $\rho \sin 1''$ are from $0^\circ$ to $90^\circ$. For azimuths between $90^\circ$ to $180^\circ$, subtract from $180^\circ$; between $180^\circ$ and $270^\circ$, subtract $180^\circ$ from azimuth; and between $270^\circ$ and $360^\circ$, subtract azimuth from $360^\circ$. The refraction value (0.5-m) is the product of $\zeta_1 + \zeta_2 - 180^\circ$ in seconds times $\frac{\rho \sin 1''}{2}$ divided by s (in meters). The weight (p) of (0.5-m) is proportional to $s^2$ and a convenient value may be obtained by dividing s by 10,000 and squaring the result. This weight should be carried to two decimal places.

When two or more determinations of the elevation of a station are made, a weighted mean elevation of the station should be computed and used in subsequent computations of elevations of other stations from that station. The weighted mean elevation of a station is found by taking the sums of the products of the elevations by their respective weights and dividing by the sum of the weights. For example, if the elevations of a station are determined to be 987.62 meters, 985.23 meters and 988.16 meters and the respective weights of elevation computations are 3.16, 0.57, and 5.73 the weighted mean elevation of the station would be

$$\frac{987.62 \times 3.16 + 985.23 \times 0.57 + 988.16 \times 5.73}{3.16 + 0.57 + 5.73}$$

or

987.80 meters.

Any individual elevation determination should not vary from the weighted mean value by an amount much greater than 0.1 meter per mile of the length of the observed line, for class 2 vertical angle elevations.

An example of a nonreciprocal computation is included with the vertical angle elevation determinations.

$\zeta_1$ is the observed zenith distance of station 2 as observed from station 1. $\alpha$ and mean $\phi$ are the azimuth of station 1 to station 2 and mean latitude of the stations respectively. (0.5-m) is the weighted mean refraction for station 1 obtained from the reciprocal observation forms for station 1. s is the geodetic distance of station 1 to station 2. $\rho \sin 1''$ is obtained from attached tables using $\alpha$ and mean $\phi$ as argument. k in seconds is $\frac{(0.5-m)s}{\rho \sin 1''}$. The computation of the remainder of the form is self explanatory and similar to the reciprocal computation except the term (t-o) which is obtained from form 29 (field abstract).

When nonreciprocal observations are used in an adjustment with reciprocal observations the weight of the nonreciprocal observations should be divided by 3.

After all differences of elevations have been computed, both from the reciprocal and nonreciprocal observations, and their weights determined, the next step is the adjustment of these differences of elevation by the method of least squares. An example of such an adjustment from the formation of the equations to the determination of the final elevations is given in detail on the following pages for the differences of elevation represented by the figure given.

The adjustment of vertical observations is made by means of observation equations. Elevations approximating the final values are first assumed for the different stations. To these assumed values are added x's to be determined by the adjustment. These observation equations are formed by comparing the differences of the assumed elevations with the differences determined by computation.

In the first table shown with the adjustment sketch are given the stations already fixed in elevation and their elevations. In the second table are given the names of the stations whose elevations are to be fixed by the adjustment and in the second column of this table their assumed elevations and correction symbols. The last two columns of this table are filled in after the adjustment is completed.

If any of the stations of the scheme have been determined in elevation directly from first order leveling their elevations should be held fixed in the adjustment. A careful check should be made before an adjustment is started to ascertain whether any of the stations have been fixed by leveling.

On the formation of equation sheet, columns 2 and 3 contain the names of the stations between which observations were made. Column 4 contains the symbols; station 1 symbols are minus and station 2 are plus. Fixed stations have no symbol. Column 5 is obtained from the table of fixed and assumed elevations. Columns 6 and 8 are found on the elevation computation forms. Column 7 is column 5 minus column 6 and columns 9 and 10 are filled in after adjustment.

The table for formation of normal equations corresponds to correlates (for convenience is referred to hereafter as correlates) and is set up from the preceding table. (Columns 2 and 3 correspond to 7 and 8). Columns 4 to 8 are from the symbol column in preceding table, pN is the product of p and N and $\Sigma$ is the horizontal sum across a line of N and any term under the x columns. p $\Sigma$ is simply the product of p and $\Sigma$ .

The normals are formed in the following manner from the formation of normals table. The value +82.61 in the first column is obtained by squaring each coefficient in the first x column, multiplying by its weight and taking the sum, in this case (6.32+7.17+50.66+6.73+3.65+8.08)= +82.61. The values +29.90, +79.01, +88.41, and +59.52 are obtained in the same manner and are called diagonal terms.

The remainder of the values in columns 2 to 5 of the normal equations are product terms, resulting from multiplying each correlate equation by each of the other correlate equations and by the corresponding weights. For example, equation ($x_1$) times ($x_3$) times the weight of the coefficients equals $(-1)x(+1)x(3.65)= -3.65$.

The $\eta$ term is the sum of the products of each term in an equation times pN and $\Sigma$ for each equation is the sum of the products of each term in the equation times p $\Sigma$ .

The solution of the normals is the same as in condition equations. The x's thus obtained (which are equivalent to the C's in condition equation solutions) are the corrections to be applied to the various assumed elevations.

A check on the solution may be obtained by taking the sum of the products, of the upper times lower terms, in the solution, of the $\eta$ column. The sum of these products ( $\eta$ x $\eta$ ) should equal the sum of the products of each $\eta$ in the normals times its x.

A further check may be applied which is

$$(pN^2) = (pv^2) + (\eta x \eta) \quad \text{(in forward solution)}$$

After the assumed elevations have been corrected, the v's for each observation should be computed to four decimal places and listed on the sheet with the formation of equations. Each v may be easily obtained in the following manner. Place the elevation of station 1 in the machine as a negative number, and elevation of station 2 as a positive number, and subtract observed difference of elevation algebraically. The remainder is the v for that observation.

Each pv should be computed for each observation and listed (product of v and p). The sum of all the pv's at each station should be approximately zero. If the sum of pv's around the point at any station differs from zero by more than 50 in the fourth decimal place, the supervisor should be consulted to see if the check is adequate.

If the probable error of the observations is needed, the sum of the ($pv^2$'s) may be obtained by accumulating the individual products, v(pv) in the machine.

Class 1 elevations (determined by spirit leveling) should be listed to hundredths of meters and tenths of feet.

Class 2 elevations (determined by reciprocal vertical angles) should be listed to tenths of a meter and nearest foot.

Class 3 elevations (determined by non-reciprocal vertical angles) should be listed to nearest meter and nearest foot.

Elevations should be listed on the geographic position sheets in column headed "seconds in meters" when that column has not been used, otherwise they should be listed after the name and date of triangulation station.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 C

# COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

(By calculating machine)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Lewes | Marian | Alcan | Lewes | Alcan | Rocky |
| | | | | | | |
| Station 2, obs. | Rocky | Rocky | Rocky | Grassy | Grassy | Grassy |
| | | | | | | |
| $\zeta_1$ | 89 03 00.6 | 91 59 51.3 | 86 10 55.5 | 88 48 24.3 | 88 18 26.5 | 89 06 40.0 |
| $\zeta_2$ | 91 02 38.9 | 88 05 38.9 | 93 50 14.4 | 91 20 15.2 | 91 48 45.9 | 90 58 54.4 |
| $\zeta_2-\zeta_1$ | +1 59 38.3 | −3 54 12.4 | +7 39 18.9 | +2 31 50.9 | +3 30 19.4 | +1 52 14.4 |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | +0 59 49.2 | −1 57 06.2 | +3 49 39.5 | +1 15 55.5 | +1 45 09.7 | +0 56 07.2 |
| $\tan \frac{1}{2}(\zeta_2-\zeta_1)$ | 0.017403 | 0.034077 | 0.066904 | 0.022089 | 0.030600 | 0.016326 |
| $s$ | 12575.0 | 11813.2 | 4442.7 | 18913.7 | 16228.8 | 12191.9 |
| $A$ | | | | | | |
| $B$ | | | | | | |
| $C$ | | | | | | |
| | | | | | | |
| $h_2-h_1$ | +218.87 | −402.64 | +297.27 | +417.85 | +496.68 | +199.08 |
| $h_1$ | 866.45 | 1487.82 | 788.10 | 866.45 | 788.10 | 1085.34 |
| $h_2$ | 1085.32 | 1085.18 | 1085.37 | 1284.30 | 1284.78 | 1284.42 |
| $\frac{1'}{s^2}=$ p of $(h_2-h_1)$ | 6.32 | 7.17 | 50.66 | 2.80 | 3.80 | 6.73 |
| $\alpha$ and mean $\phi$ | 72° 61° | 59° 61° | 57° 61° | 69° 61° | 35° 61° | 28° 61° |
| $\zeta_1+\zeta_2-180°$ | 5′ 39.″5 | 5′ 30.″2 | 1′ 09.″9 | 8′ 39.″5 | 7′ 12.″4 | 5′ 34.″4 |
| $\zeta_1+\zeta_2-180°$ in sec. | 339.5 | 330.2 | 69.9 | 519.5 | 432.4 | 334.4 |
| $\rho \frac{\sin 1''}{2}$ | 15.499 | 15.495 | 15.494 | 15.498 | 15.485 | 15.482 |
| $s$ | 12575.0 | 11813.2 | 4442.7 | 18913.7 | 16228.8 | 12191.9 |
| $(0.5-m)$ | 0.4184 | 0.4331 | 0.2438 | 0.4257 | 0.4126 | 0.4246 |
| p of $(0.5-m)$ * | 1.58 | 1.40 | 0.20 | 3.58 | 2.63 | 1.49 |

← 1085.34 w.m. → ← 1284.50 w.m. →

$$h_2-h_1 = s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)\ A\ B\ C$$

$$(0.5-m) = \frac{\zeta_1+\zeta_2-180° \text{ in sec. } \rho \frac{\sin 1''}{2}}{s}$$

* Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 C

## COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

(By calculating machine)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Grassy | Rocky | Easy | Grassy | Rocky | |
| | | | | | | |
| Station 2, obs. | Easy | Easy | Lake | Lake | Lake | |
| | | | | | | |
| $\zeta_1$ | 92 50 51.3 | 91 01 47.8 | 88 39 47.2 | 91 12 24.5 | 90 21 28.2 | |
| $\zeta_2$ | 87 13 01.2 | 89 05 25.4 | 91 23 51.0 | 88 52 45.4 | 89 43 27.8 | |
| $\zeta_2-\zeta_1$ | -5 37 50.1 | -1 56 22.4 | +2 44 03.8 | -2 19 39.1 | -0 38 00.4 | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | -2 48 55.0 | -0 58 11.2 | +1 22 01.9 | -1 09 49.6 | -0 19 00.2 | |
| $\tan\frac{1}{2}(\zeta_2-\zeta_1)$ | 0.049176 | 0.016927 | 0.023866 | 0.020315 | 0.005528 | |
| $s$ | 9724.7 | 16546.8 | 9127.7 | 12910.6 | 11127.9 | |
| $A$ | | | | | | |
| $B$ | | | | | | |
| $C$ | | | | | | |
| | | | | | | |
| $h_2-h_1$ | -478.30 | -280.13 | +217.87 | -262.33 | -61.52 | |
| $h_1$ | 1284.50 | 1085.34 | 805.95 | 1284.50 | 1085.34 | |
| $h_2$ | 806.20 | 805.21 | 1023.82 | 1022.17 | 1023.82 | |
| $\frac{1}{s^2}=p$ of $(h_2-h_1)$ | 10.57 | 3.65 | 12.00 | 6.00 | 8.08 | |
| $\alpha$ and mean $\phi$ | 69° 61° | 63° 61° | 24° 61° | 25° 61° | 85° 61° | |
| $\zeta_1+\zeta_2-180°$ | 3' 52".5 | 7' 13".2 | 3' 38".2 | 5' 09".9 | 4' 56".0 | |
| $\zeta_1+\zeta_2-180°$ in sec. | 232.5 | 433.2 | 218.2 | 309.9 | 296.0 | |
| $\rho\,\frac{\sin 1''}{2}$ | 15.498 | 15.496 | 15.481 | 15.481 | 15.501 | |
| $s$ | 9724.7 | 16546.8 | 9127.7 | 12910.6 | 11127.9 | |
| $(0.5-m)$ | 0.3705 | 0.4057 | 0.3701 | 0.3716 | 0.4123 | |
| $p$ of $(0.5-m)$ * | 0.95 | 2.74 | 0.83 | 1.67 | 1.24 | |

805.95 w.m. 1023.44 w.m.

$$h_2-h_1 = s \tan\tfrac{1}{2}(\zeta_2-\zeta_1)\ A\ B\ C$$

$$(0.5-m) = \frac{\zeta_1+\zeta_2-180° \text{ in sec. } \rho\,\frac{\sin 1''}{2}}{s}$$

* Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 C

## COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

(By calculating machine)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Easy | Lake | Easy | Lake | Takhini E.B. | |
| | | | | | | |
| Station 2, obs. | Takhini E.B. | Takhini E.B. | Takhini W.B. | Takhini W.B. | Takhini W.B. | |
| | | | | | | |
| $\zeta_1$ | 91 22 25.3 | 94 18 20.5 | 90 52 39.0 | 92 10 59.7 | 89 52 12.6 | |
| $\zeta_2$ | 88 39 30.6 | 85 43 54.0 | 89 11 11.0 | 87 53 04.9 | 90 16 07.6 | |
| $\zeta_2-\zeta_1$ | -2 42 54.7 | -8 34 26.5 | -1 41 28.0 | -4 17 54.8 | +0 23 55.0 | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | -1 21 27.4 | -4 17 13.2 | -0 50 44.0 | -2 08 57.4 | +0 11 57.5 | |
| $\tan \frac{1}{2}(\zeta_2-\zeta_1)$ | 0.023699 | 0.074962 | 0.014758 | 0.037529 | 0.003479 | |
| $s$ | 5766.3 | 4732.1 | 7951.1 | 8930.4 | 4727.3 | |
| $A$ | | | | | | |
| $B$ | | | | | | |
| $C$ | | | | | | |
| | | | | | R | |
| $h_2-h_1$ | -136.67 | -354.78 | -117.35 | -335.19 | +16.45 | |
| $h_1$ | 805.95 | 1023.44 | 805.95 | 1023.44 | 668.91 | |
| $h_2$ | 669.28 | 668.66 | 688.60 | 688.25 | 685.36 | |
| $\frac{1}{s^2}=$ $p$ of $(h_2-h_1)$ | 30.08 | 44.66 | 15.82 | 12.54 | 44.76 | |
| $\alpha$ and mean $\phi$ | 2° 61° | 57° 61° | 38° 61° | 76° 61° | | |
| $\zeta_1+\zeta_2-180°$ | 1' 55.9" | 2' 14.5" | 3' 50.0" | 4' 04.6" | | |
| $\zeta_1+\zeta_2-180°$ in sec. | 115.9 | 134.5 | 230.0 | 244.6 | | |
| $\rho \frac{\sin 1''}{2}$ | 15.476 | 15.494 | 15.486 | 15.500 | | |
| $s$ | 5766.3 | 4732.1 | 7951.1 | 8930.4 | | |
| $(0.5-m)$ | 0.3111 | 0.4404 | 0.4480 | 0.4245 | | |
| $p$ of $(0.5-m)$ * | 0.33 | 0.22 | 0.63 | 0.80 | | |

668.91 W.m. 688.45 W.m.

$$h_2-h_1 = s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)\ A\ B\ C$$

$$(0.5-m) = \frac{\zeta_1+\zeta_2-180° \text{ in sec. } \rho \frac{\sin 1''}{2}}{s}$$

* Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 C

# COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

(By calculating machine)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Easy | Takhini E.B. | Takhini W.B. | Lake | | |
| | | | | | | |
| Station 2, obs. | Ibex | Ibex | Ibex | Ibex | | |
| | | | | | | |
| $\zeta_1$ | 88 55 52.4 | 87 53 41.2 | 86 08 26.6 | 89 59 13.6 | | |
| $\zeta_2$ | 91 09 23.0 | 92 10 47.1 | 93 53 17.5 | 90 06 38.0 | | |
| $\zeta_2-\zeta_1$ | +2 13 30.6 | +4 17 05.9 | +7 44 50.9 | +0 07 24.4 | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | +1 06 45.3 | +2 08 33.0 | +3 52 25.5 | +0 03 42.2 | | |
| $\tan\frac{1}{2}(\zeta_2-\zeta_1)$ | 0.019421 | 0.037411 | 0.067713 | 0.001078 | | |
| $s$ | 12044.3 | 9903.4 | 5177.8 | 13962.5 | | |
| $A$ | | | | | | |
| $B$ | | | | | | |
| $C$ | | | | | | |
| | | | | | | |
| $h_2-h_1$ | +233.94 | +370.55 | +350.65 | + 15.05 | | |
| $h_1$ | 805.95 | 668.91 | 688.45 | 1023.44 | | |
| $h_2$ | 1039.89 | 1039.46 | 1039.10 | 1038.49 | | |
| $\frac{1}{s^2}=$ p of $(h_2-h_1)$ | 6.89 | 10.20 | 37.30 | 5.13 | | |
| $\alpha$ and mean $\phi$ | 57° 61° | 85° 61° | 86° 61° | 83° 61° | | |
| $\zeta_1+\zeta_2-180°$ | 5' 15."4 | 4' 28."3 | 1' 44."1 | 5' 51."6 | | |
| $\zeta_1+\zeta_2-180°$ in sec. | 315.4 | 268.3 | 104.1 | 351.6 | | |
| $\rho\frac{\sin 1''}{2}$ | 15.494 | 15.501 | 15.501 | 15.501 | | |
| $s$ | 12044.3 | 9903.4 | 5177.8 | 13962.5 | | |
| $(0.5-m)$ | 0.4057 | 0.4199 | 0.3116 | 0.3903 | | |
| p of $(0.5-m)$ * | 1.45 | 0.98 | 0.27 | 1.95 | | |

1039.20 w.m.

$$h_2-h_1 = s\tan\tfrac{1}{2}(\zeta_2-\zeta_1)\ A\ B\ C$$

$$(0.5-m) = \frac{\zeta_1+\zeta_2-180° \text{ in sec. } \rho\frac{\sin 1''}{2}}{s}$$

* Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 D

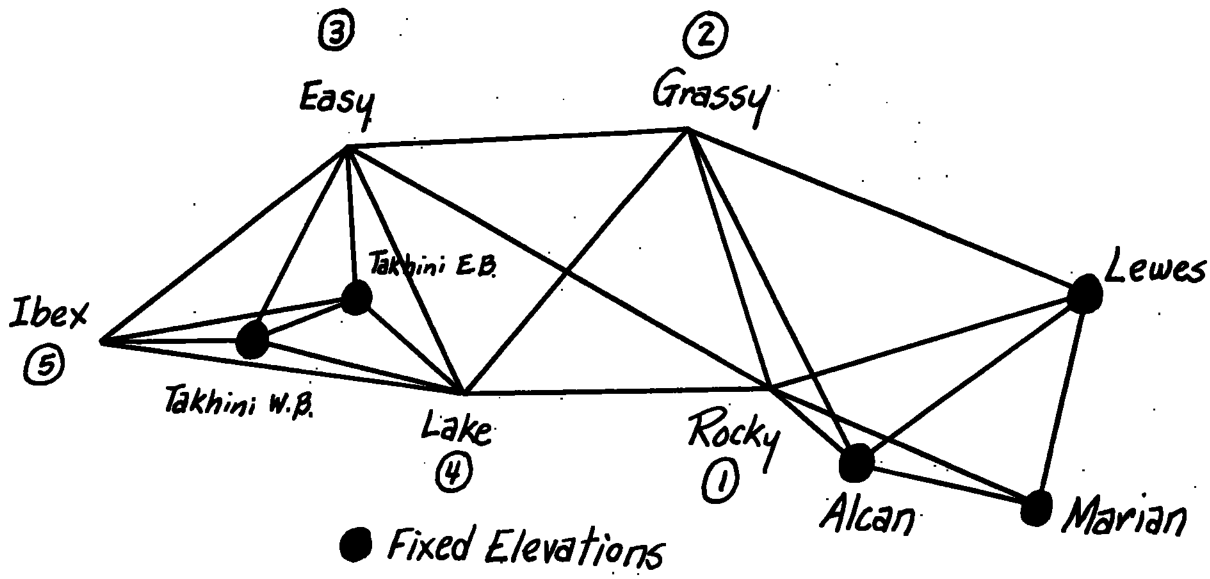
## COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

(By calculating machine)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | Tanacross Astro | Tana W.B. | Dodo | | | |
| | | | | | | |
| Station 2, obs. | Peak E | Peak E | Peak E | | | |
| | | | | | | |
| Object sighted | ground | ground | ground | | | |
| | | | | | | |
| $\zeta_1$ | 88 15 51 | 88 30 43 | 88 52 26 | | | |
| $\alpha$ and mean $\phi$ | 47° 63° | 45° 63° | 15° 63° | | | |
| | | | | | | |
| $(0.5-m)$ | 0.4257 | 0.4077 | 0.4107 | | | |
| $s$ | 19618.6 | 22141.3 | 15953.4 | | | |
| $\rho \sin 1''$ | 30.986 | 30.984 | 30.965 | | | |
| $k$ in secs. | 269.5 | 291.3 | 211.6 | | | |
| $(90°-\zeta_1+k)$ | +1 48 38 | +1 34 08 | +1 11 06 | | | |
| $\tan (90°-\zeta_1+k)$ | 0.031611 | 0.027389 | 0.020685 | | | |
| $A$ | | | | | | |
| $B$ | | | | | | |
| $C$ | | | | | | |
| $h_2-h_1$ | +620.24 | +606.51 | +330.04 | | | |
| $h_1$ | 470.75 | 475.50 | 761.18 | | | |
| $t-o$ | + 1.48 | +9.44 | +1.45 | | | |
| Corrected elevation | 1092.47 | 1091.45 | 1092.67 | | | |
| | | | | | | |
| $\frac{1}{s^2}=p$ of $(h_2-h_1)$ | 2.60 | 2.04 | 3.93 | | | |
| Weighted mean elevation of sta. obs. | | 1092.3 | | | | |

$$k \text{ in secs.} = \frac{(0.5-m)\, s}{\rho \sin 1''}$$

$$h_2-h_1 = s \tan (90°-\zeta_1+k)\, A\, B\, C$$




## Fixed Elevations

| Station | Elevation meters |
|---|---|
| Alcan | 788.10 |
| Marian | 1487.82 |
| Lewes | 866.45 |
| Takhini E.B. | 667.56 |
| Takhini W.B. | 687.41 |

## Assumed and Adjusted Elevations

| Station | Assumed Elev. meters | Adjusted Elev. meters | Adopted Elev. meters |
|---|---|---|---|
| 1 Rocky | $1085+x_1$ | 1085.1514 | 1085.2 |
| 2 Grassy | $1284+x_2$ | 1283.9652 | 1284.0 |
| 3 Easy | $806+x_3$ | 804.6190 | 804.6 |
| 4 Lake | $1023+x_4$ | 1022.5075 | 1022.5 |
| 5 Ibex | $1039+x_5$ | 1038.0830 | 1038.1 |

# Formation of Equations

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | From station 1 | To station 2 | Symbol | Assumed $h_2 - h_1$ | Observed $h_2 - h_1$ | N assumed minus obser. | Weight $P$ | Adjusted minus obser. $V$ | $pV$ |
| 1 | Lewes | Rocky | $+x_1$ | +218.55 | +218.87 | −0.32 | 6.32 | −0.1686 | −1.0656 |
| 2 | Marian | Rocky | $+x_1$ | −402.82 | −402.64 | −0.18 | 7.17 | −0.0286 | −0.2051 |
| 3 | Alcan | Rocky | $+x_1$ | +296.90 | +297.27 | −0.37 | 50.66 | −0.2186 | −11.0743 |
| 4 | Lewes | Grassy | $+x_2$ | +417.55 | +417.85 | −0.30 | 2.80 | −0.3348 | −0.9374 |
| 5 | Alcan | Grassy | $+x_2$ | +495.90 | +496.68 | −0.78 | 3.80 | −0.8148 | −3.0962 |
| 6 | Rocky | Grassy | $-x_1+x_2$ | +199.00 | +199.08 | −0.08 | 6.73 | −0.2662 | −1.7915 |
| 7 | Grassy | Easy | $-x_2+x_3$ | −478.00 | −478.30 | +0.30 | 10.57 | −1.0462 | −11.0583 |
| 8 | Rocky | Easy | $-x_1+x_3$ | −279.00 | −280.13 | +1.13 | 3.65 | −0.4024 | −1.4688 |
| 9 | Easy | Lake | $-x_3+x_4$ | +217.00 | +217.87 | −0.87 | 12.00 | +0.0185 | +0.2220 |
| 10 | Grassy | Lake | $-x_2+x_4$ | −261.00 | −262.33 | +1.33 | 6.00 | +0.8723 | +5.2338 |
| 11 | Rocky | Lake | $-x_1+x_4$ | −62.00 | −61.52 | −0.48 | 8.08 | −1.1239 | −9.0811 |
| 12 | Easy | Takhini E.B. | $-x_3$ | −138.44 | −136.67 | −1.77 | 30.08 | −0.3890 | −11.7011 |
| 13 | Lake | Takhini E.B. | $-x_4$ | −355.44 | −354.78 | −0.66 | 44.66 | −0.1675 | −7.4806 |
| 14 | Easy | Takhini W.B. | $-x_3$ | −118.59 | −117.35 | −1.24 | 15.82 | +0.1410 | +2.2306 |
| 15 | Lake | Takhini W.B. | $-x_4$ | −335.59 | −335.19 | −0.40 | 12.54 | +0.0925 | +1.1600 |
| 16 | Easy | Ibex | $-x_3+x_5$ | +233.00 | +233.94 | −0.94 | 6.89 | −0.4760 | −3.2796 |
| 17 | Takhini E.B. | Ibex | $+x_5$ | +371.44 | +370.55 | +0.89 | 10.20 | −0.0270 | −0.2754 |
| 18 | Takhini W.B. | Ibex | $+x_5$ | +351.59 | +350.65 | +0.94 | 37.30 | +0.0230 | +0.8579 |
| 19 | Lake | Ibex | $-x_4+x_5$ | +16.00 | +15.05 | +0.95 | 5.13 | +0.5255 | +2.6958 |

# Table for formation of Normal Equations

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| | $N$ | $p$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $pN$ | $\Sigma$ | $p\Sigma$ |
| 1 | -0.32 | 6.32 | +1 | | | | | -2.0224 | +0.68 | +4.2976 |
| 2 | -0.18 | 7.17 | +1 | | | | | -1.2906 | +0.82 | +5.8794 |
| 3 | -0.37 | 50.66 | +1 | | | | | -18.7442 | +0.63 | +31.9158 |
| 4 | -0.30 | 2.80 | | +1 | | | | -0.8400 | +0.70 | +1.9600 |
| 5 | -0.78 | 3.80 | | +1 | | | | -2.9640 | +0.22 | +0.8360 |
| 6 | -0.08 | 6.73 | -1 | +1 | | | | -0.5384 | -0.08 | -0.5384 |
| 7 | +0.30 | 10.57 | | -1 | +1 | | | +3.1710 | +0.30 | +3.1710 |
| 8 | +1.13 | 3.65 | -1 | | +1 | | | +4.1245 | +1.13 | +4.1245 |
| 9 | -0.87 | 12.00 | | | -1 | +1 | | -10.4400 | -0.87 | -10.4400 |
| 10 | +1.33 | 6.00 | | -1 | | +1 | | +7.9800 | +1.33 | +7.9800 |
| 11 | -0.48 | 8.08 | -1 | | | +1 | | -3.8784 | -0.48 | -3.8784 |
| 12 | -1.77 | 30.08 | | | -1 | | | -53.2416 | -2.77 | -83.3216 |
| 13 | -0.66 | 44.66 | | | | -1 | | -29.4756 | -1.66 | -74.1356 |
| 14 | -1.24 | 15.82 | | | -1 | | | -19.6168 | -2.24 | -35.4368 |
| 15 | -0.40 | 12.54 | | | | -1 | | -5.0160 | -1.40 | -17.5560 |
| 16 | -0.94 | 6.89 | | | -1 | | +1 | -6.4766 | -0.94 | -6.4766 |
| 17 | +0.89 | 10.20 | | | | | +1 | +9.0780 | +1.89 | +19.2780 |
| 18 | +0.94 | 37.30 | | | | | +1 | +35.0620 | +1.94 | +72.3620 |
| 19 | +0.95 | 5.13 | | | | -1 | +1 | +4.8735 | +0.95 | +4.8735 |

## Normals

| | 1 | 2 | 3 | 4 | 5 | $n$ | $\Sigma$ |
|---|---|---|---|---|---|---|---|
| 1 | +82.61 | -6.73 | -3.65 | -8.08 | | -21.7649 | +42.3851 |
| 2 | | +29.90 | -10.57 | -6.00 | | -15.4934 | -8.8934 |
| 3 | | | +79.01 | -12.00 | -6.89 | +97.0705 | +142.9705 |
| 4 | | | | +88.41 | -5.13 | +23.2797 | +80.4797 |
| 5 | | | | | +59.52 | +42.5369 | +90.0369 |

## Solution

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| +0.1514 | -0.0348 | -1.3810 | -0.4925 | -0.9170 | | | |
| +0.15145 | -0.03477 | -1.38100 | -0.49249 | -0.91698 | | | |
| 1 | 2 | 3 | 4 | 5 | $n$ | $\Sigma$ | |
| +82.61 | -6.73 | -3.65 | -8.08 | | -21.7649 | +42.3851 | |
| $x_1$ | +0.08147 | +0.04418 | +0.09781 | | +0.26347 | -0.51307 | |
| | +29.90 | -10.57 | -6.00 | | -15.4934 | -8.8934 | |
| | +29.3517 | -10.8674 | -6.6583 | | -17.2666 | -5.44036 | |
| | $x_2$ | +0.37025 | +0.22685 | | +0.58827 | +0.18537 | |
| | | +79.01 | -12.00 | -6.89 | +97.0705 | +142.9705 | |
| | | +74.8251 | -14.8222 | -6.89 | +89.7160 | +142.8289 | |
| | | $x_3$ | +0.19809 | +0.09208 | -1.19901 | -1.90884 | |
| | | | +88.41 | -5.13 | +23.2797 | +80.4797 | |
| | | | +83.1731 | -6.4948 | +35.0058 | +111.6841 | |
| | | | $x_4$ | +0.07809 | -0.42088 | -1.34279 | |
| | | | | +59.52 | +42.5369 | +90.0369 | |
| | | | | +58.3784 | +53.5316 | +111.9100 | |
| | | | | $x_5$ | -0.91698 | -1.91698 | |

Factors for use in computing elevations by vertical angles.

(Machine method)

| Elevation of $h_1$ | A | $s \tan\frac{1}{2}(\zeta_2 - \zeta_1)$ | B | s | C |
|---|---|---|---|---|---|
| meters | | meters | | meters | |
| 0 | 1.000000 | 0 | 1.000000 | 0 | 1.000000 |
| 100 | 16 | 100 | 8 | 75,000 | 12 |
| 200 | 31 | 200 | 16 | 130,000 | 35 |
| 300 | 47 | 300 | 24 | 168,000 | 58 |
| 400 | 63 | 400 | 31 | 198,000 | 81 |
| 500 | 78 | 500 | 39 | 225,000 | 104 |
| 600 | 94 | 600 | 47 | 250,000 | 127 |
| 700 | 110 | 700 | 55 | 270,000 | 150 |
| 800 | 125 | 800 | 63 | 290,000 | 173 |
| 900 | 141 | 900 | 71 | | |
| 1000 | 1.000157 | 1000 | 1.000079 | | |
| 1100 | 172 | 1100 | 86 | | |
| 1200 | 188 | 1200 | 94 | | |
| 1300 | 204 | 1300 | 102 | | |
| 1400 | 219 | 1400 | 110 | | |
| 1500 | 235 | 1500 | 118 | | |
| 1600 | 251 | 1600 | 126 | | |
| 1700 | 267 | 1700 | 133 | | |
| 1800 | 282 | 1800 | 141 | | |
| 1900 | 298 | 1900 | 149 | | |
| 2000 | 1.000314 | 2000 | 1.000157 | | |
| 2100 | 329 | 2100 | 165 | | |
| 2200 | 345 | 2200 | 173 | | |
| 2300 | 361 | 2300 | 181 | | |
| 2400 | 376 | 2400 | 188 | | |
| 2500 | 392 | 2500 | 196 | | |
| 2600 | 408 | 2600 | 204 | | |
| 2700 | 423 | 2700 | 212 | | |
| 2800 | 439 | 2800 | 220 | | |
| 2900 | 455 | 2900 | 228 | | |
| 3000 | 1.000470 | 3000 | 1.000236 | | |
| 3100 | 486 | 3100 | 243 | | |
| 3200 | 502 | 3200 | 251 | | |
| 3300 | 517 | 3300 | 259 | | |
| 3400 | 533 | 3400 | 267 | | |
| 3500 | 549 | 3500 | 275 | | |
| 3600 | 564 | 3600 | 283 | | |
| 3700 | 580 | 3700 | 290 | | |
| 3800 | 596 | 3800 | 298 | | |
| 3900 | 611 | 3900 | 306 | | |
| 4000 | 1.000627 | 4000 | 1.000314 | | |

CORRECTION TO $h_2-h_1$
in centimeters

(correction always added numerically to $h_2-h_1$)

$h_1$ in meters

| $h_2-h_1$ in meters | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | $h_2-h_1$ in meters |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1500 | 18 | 20 | 22 | 25 | 27 | 29 | 32 | 34 | 36 | 39 | 41 | 44 | 46 | 48 | 51 | 53 | +1500 |
| +1400 | 15 | 18 | 20 | 22 | 24 | 26 | 29 | 31 | 33 | 35 | 37 | 40 | 42 | 44 | 46 | 48 | +1400 |
| +1300 | 13 | 15 | 17 | 19 | 21 | 23 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | +1300 |
| +1200 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | +1200 |
| +1100 | 9 | 11 | 13 | 15 | 16 | 18 | 20 | 22 | 23 | 25 | 27 | 28 | 30 | 32 | 34 | 35 | +1100 |
| +1000 | 8 | 9 | 11 | 13 | 14 | 16 | 17 | 19 | 20 | 22 | 24 | 25 | 27 | 28 | 30 | 31 | +1000 |
| + 900 | 6 | 8 | 9 | 11 | 12 | 13 | 15 | 16 | 18 | 19 | 20 | 22 | 23 | 25 | 26 | 28 | + 900 |
| + 800 | 5 | 6 | 8 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | 18 | 19 | 20 | 21 | 23 | 24 | + 800 |
| + 700 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | + 700 |
| + 600 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | + 600 |
| + 500 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | + 500 |
| + 400 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | 9 | 9 | 10 | 11 | + 400 |
| + 300 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | + 300 |
| + 200 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | + 200 |
| + 100 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | + 100 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| - 100 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | - 100 |
| - 200 | | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | - 200 |
| - 300 | | | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | - 300 |
| - 400 | | | | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | - 400 |
| - 500 | | | | | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | - 500 |
| - 600 | | | | | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | - 600 |
| - 700 | | | | | | | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 13 | - 700 |
| - 800 | | | | | | | | 4 | 5 | 6 | 8 | 9 | 10 | 11 | 13 | 14 | - 800 |
| - 900 | | | | | | | | | 5 | 6 | 8 | 9 | 11 | 12 | 13 | 15 | - 900 |
| -1000 | | | | | | | | | | 6 | 8 | 9 | 11 | 13 | 14 | 16 | -1000 |
| -1100 | | | | | | | | | | | 8 | 9 | 11 | 13 | 15 | 16 | -1100 |
| -1200 | | | | | | | | | | | | 9 | 11 | 13 | 15 | 17 | -1200 |
| -1300 | | | | | | | | | | | | | 11 | 13 | 15 | 17 | -1300 |
| -1400 | | | | | | | | | | | | | | 13 | 15 | 18 | -1400 |
| -1500 | | | | | | | | | | | | | | | 15 | 18 | -1500 |
| | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | |

$h_1$ in meters

| $h_2-h_1$ in meters | 1500 | 1600 | 1700 | 1800 | 1900 | 2000 | 2100 | 2200 | 2300 | 2400 | 2500 | 2600 | 2700 | 2800 | 2900 | 3000 | $h_2-h_1$ in meters |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1500 | 53 | 55 | 58 | 60 | 62 | 65 | 67 | 69 | 72 | 74 | 77 | 79 | 81 | 84 | 86 | 88 | +1500 |
| +1400 | 48 | 51 | 53 | 55 | 57 | 59 | 62 | 64 | 66 | 68 | 70 | 73 | 75 | 77 | 79 | 81 | +1400 |
| +1300 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | +1300 |
| +1200 | 40 | 41 | 43 | 45 | 47 | 49 | 51 | 53 | 55 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | +1200 |
| +1100 | 35 | 37 | 39 | 41 | 42 | 44 | 46 | 47 | 49 | 51 | 53 | 54 | 56 | 58 | 60 | 61 | +1100 |
| +1000 | 31 | 33 | 35 | 36 | 38 | 39 | 41 | 42 | 44 | 46 | 47 | 49 | 50 | 52 | 53 | 55 | +1000 |
| + 900 | 28 | 29 | 30 | 32 | 33 | 35 | 36 | 37 | 39 | 40 | 42 | 43 | 44 | 46 | 47 | 49 | + 900 |
| + 800 | 24 | 25 | 26 | 28 | 29 | 30 | 31 | 33 | 34 | 35 | 36 | 38 | 39 | 40 | 41 | 43 | + 800 |
| + 700 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 34 | 35 | 36 | 37 | + 700 |
| + 600 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | + 600 |
| + 500 | 14 | 15 | 15 | 16 | 17 | 18 | 18 | 19 | 20 | 21 | 22 | 22 | 23 | 24 | 25 | 26 | + 500 |
| + 400 | 11 | 11 | 12 | 13 | 13 | 14 | 14 | 15 | 16 | 16 | 17 | 18 | 18 | 19 | 19 | 20 | + 400 |
| + 300 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 13 | 13 | 14 | 14 | 15 | + 300 |
| + 200 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 8 | 9 | 9 | 10 | + 200 |
| + 100 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | + 100 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| - 100 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | - 100 |
| - 200 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | - 200 |
| - 300 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 13 | 13 | - 300 |
| - 400 | 8 | 9 | 9 | 10 | 12 | 11 | 12 | 13 | 13 | 14 | 14 | 15 | 16 | 16 | 17 | 18 | - 400 |
| - 500 | 10 | 11 | 11 | 12 | 13 | 14 | 15 | 15 | 16 | 17 | 18 | 18 | 19 | 20 | 21 | 22 | - 500 |
| - 600 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 24 | 25 | - 600 |
| - 700 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | - 700 |
| - 800 | 14 | 15 | 16 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 31 | 33 | - 800 |
| - 900 | 15 | 16 | 18 | 19 | 20 | 22 | 23 | 25 | 26 | 28 | 29 | 30 | 32 | 33 | 35 | 36 | - 900 |
| -1000 | 16 | 17 | 19 | 20 | 22 | 24 | 25 | 27 | 28 | 30 | 31 | 33 | 35 | 36 | 38 | 39 | -1000 |
| -1100 | 16 | 18 | 20 | 22 | 23 | 25 | 27 | 28 | 30 | 32 | 34 | 35 | 37 | 39 | 41 | 42 | -1100 |
| -1200 | 17 | 19 | 21 | 23 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 41 | 43 | 45 | -1200 |
| -1300 | 17 | 19 | 21 | 23 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | -1300 |
| -1400 | 18 | 20 | 22 | 24 | 26 | 29 | 31 | 33 | 35 | 37 | 40 | 42 | 44 | 46 | 48 | 51 | -1400 |
| -1500 | 18 | 20 | 22 | 25 | 27 | 29 | 32 | 34 | 36 | 39 | 41 | 44 | 46 | 48 | 51 | 53 | -1500 |
| | 1500 | 1600 | 1700 | 1800 | 1900 | 2000 | 2100 | 2200 | 2300 | 2400 | 2500 | 2600 | 2700 | 2800 | 2900 | 3000 | |

$h_1$ in meters

# Tables for Refraction Computations

## $\rho \sin 1''$

### Latitude

| Azimuth (degrees) | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 30.801 | 30.806 | 30.811 | 30.816 | 30.821 | 30.826 | 30.831 | 30.837 | 30.842 |
| 5 | .801 | .806 | .811 | .817 | .822 | .827 | .833 | .838 | .843 |
| 10 | .805 | .810 | .815 | .820 | .825 | .830 | .835 | .840 | .846 |
| 15 | .811 | .816 | .820 | .825 | .830 | .835 | .840 | .845 | .850 |
| 20 | .818 | .823 | .828 | .832 | .837 | .842 | .847 | .852 | .857 |
| 25 | .828 | .832 | .836 | .840 | .845 | .850 | .855 | .860 | .864 |
| 30 | .838 | .843 | .847 | .851 | .855 | .860 | .864 | .868 | .873 |
| 35 | .850 | .854 | .858 | .862 | .866 | .870 | .874 | .878 | .882 |
| 40 | .862 | .867 | .870 | .874 | .877 | .881 | .885 | .889 | .893 |
| 45 | .876 | .879 | .882 | .886 | .889 | .893 | .897 | .900 | .904 |
| 50 | .889 | .892 | .895 | .898 | .902 | .904 | .908 | .911 | .914 |
| 55 | .902 | .904 | .907 | .910 | .913 | .916 | .919 | .921 | .925 |
| 60 | .914 | .916 | .919 | .921 | .924 | .926 | .929 | .932 | .934 |
| 65 | .924 | .926 | .930 | .931 | .934 | .936 | .939 | .941 | .944 |
| 70 | .934 | .936 | .938 | .940 | .942 | .944 | .946 | .949 | .951 |
| 75 | .941 | .944 | .946 | .947 | .949 | .951 | .953 | .955 | .957 |
| 80 | .947 | .949 | .951 | .952 | .954 | .956 | .958 | .960 | .961 |
| 85 | .951 | .952 | .954 | .956 | .957 | .959 | .961 | .963 | .964 |
| 90 | .951 | .954 | .955 | .956 | .959 | .960 | .962 | .963 | .966 |

| Azimuth (degrees) | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 30.848 | 30.853 | 30.858 | 30.864 | 30.870 | 30.875 | 30.880 | 30.886 | 30.891 |
| 5 | .848 | .854 | .860 | .865 | .870 | .875 | .881 | .887 | .892 |
| 10 | .851 | .857 | .862 | .867 | .872 | .878 | .883 | .889 | .894 |
| 15 | .855 | .861 | .866 | .871 | .877 | .882 | .887 | .892 | .897 |
| 20 | .862 | .867 | .872 | .877 | .882 | .887 | .892 | .897 | .902 |
| 25 | .869 | .874 | .878 | .883 | .888 | .893 | .898 | .902 | .907 |
| 30 | .877 | .882 | .887 | .891 | .896 | .900 | .905 | .909 | .914 |
| 35 | .887 | .891 | .895 | .899 | .904 | .908 | .912 | .916 | .921 |
| 40 | .897 | .901 | .905 | .909 | .913 | .916 | .921 | .925 | .929 |
| 45 | .907 | .911 | .914 | .918 | .922 | .926 | .929 | .933 | .936 |
| 50 | .918 | .921 | .924 | .928 | .931 | .934 | .938 | .941 | .944 |
| 55 | .928 | .931 | .934 | .936 | .940 | .943 | .946 | .949 | .952 |
| 60 | .937 | .940 | .943 | .945 | .948 | .951 | .954 | .956 | .959 |
| 65 | .946 | .949 | .952 | .954 | .956 | .958 | .961 | .963 | .967 |
| 70 | .954 | .956 | .958 | .960 | .962 | .964 | .966 | .969 | .971 |
| 75 | .959 | .961 | .963 | .966 | .968 | .969 | .971 | .973 | .976 |
| 80 | .963 | .966 | .968 | .969 | .971 | .973 | .975 | .977 | .979 |
| 85 | .966 | .968 | .970 | .972 | .973 | .976 | .978 | .979 | .981 |
| 90 | .967 | .969 | .971 | .973 | .975 | .976 | .978 | .980 | .982 |

## Tables for Refraction Computations

### $\rho$ sin 1"

**Latitude**

| Azimuth (degrees) | 50° | 51° | 52° | 53° | 54° | 55° | 56° | 57° | 58° |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 30.897 | 30.902 | 30.907 | 30.913 | 30.918 | 30.923 | 30.929 | 30.934 | 30.939 |
| 5 | .897 | .903 | .908 | .914 | .919 | .924 | .929 | .934 | .939 |
| 10 | .899 | .904 | .910 | .915 | .920 | .925 | .930 | .935 | .940 |
| 15 | .902 | .908 | .913 | .918 | .923 | .928 | .933 | .938 | .942 |
| 20 | .907 | .911 | .916 | .921 | .926 | .931 | .936 | .941 | .945 |
| 25 | .912 | .917 | .921 | .926 | .931 | .936 | .940 | .944 | .949 |
| 30 | .919 | .923 | .927 | .932 | .936 | .941 | .945 | .949 | .954 |
| 35 | .925 | .929 | .934 | .938 | .942 | .946 | .950 | .954 | .958 |
| 40 | .933 | .936 | .941 | .944 | .948 | .952 | .956 | .959 | .963 |
| 45 | .940 | .944 | .947 | .951 | .954 | .958 | .961 | .965 | .968 |
| 50 | .948 | .951 | .954 | .958 | .961 | .963 | .967 | .970 | .973 |
| 55 | .955 | .958 | .961 | .963 | .967 | .970 | .973 | .976 | .978 |
| 60 | .962 | .965 | .967 | .970 | .973 | .975 | .978 | .981 | .983 |
| 65 | .968 | .971 | .973 | .976 | .978 | .980 | .983 | .985 | .987 |
| 70 | .973 | .976 | .978 | .980 | .982 | .984 | .986 | .988 | .991 |
| 75 | .978 | .980 | .982 | .984 | .986 | .988 | .990 | .992 | .993 |
| 80 | .981 | .983 | .985 | .987 | .988 | .991 | .992 | .994 | .996 |
| 85 | .983 | .985 | .986 | .988 | .991 | .992 | .993 | .996 | .997 |
| 90 | .983 | .986 | .987 | .989 | .991 | .993 | .994 | .996 | .998 |

| Azimuth (degrees) | 59° | 60° | 61° | 62° | 63° | 64° | 65° | 66° |
|---|---|---|---|---|---|---|---|---|
| 0 | 30.944 | 30.948 | 30.953 | 30.958 | 30.962 | 30.966 | 30.971 | 30.975 |
| 5 | .944 | .949 | .954 | .958 | .962 | .966 | .971 | .975 |
| 10 | .945 | .950 | .954 | .959 | .964 | .968 | .972 | .976 |
| 15 | .947 | .951 | .956 | .961 | .965 | .969 | .974 | .977 |
| 20 | .950 | .954 | .959 | .963 | .967 | .971 | .975 | .979 |
| 25 | .955 | .958 | .962 | .966 | .970 | .974 | .978 | .981 |
| 30 | .957 | .961 | .965 | .969 | .973 | .976 | .980 | .984 |
| 35 | .961 | .966 | .969 | .973 | .976 | .980 | .984 | .986 |
| 40 | .966 | .970 | .973 | .976 | .980 | .984 | .986 | .989 |
| 45 | 30.971 | 30.974 | 30.978 | 30.981 | 30.984 | 30.986 | 30.990 | 30.992 |
| 50 | .976 | .979 | .982 | .985 | .988 | .990 | .993 | .996 |
| 55 | .981 | .983 | .986 | .988 | .991 | .993 | .996 | 30.998 |
| 60 | .985 | .988 | .990 | .992 | .995 | 30.997 | 30.999 | 31.001 |
| 65 | .989 | .991 | .993 | .996 | 30.998 | 31.000 | 31.002 | .003 |
| 70 | .993 | .995 | .996 | 30.998 | 31.001 | .002 | .004 | .006 |
| 75 | .996 | .997 | 30.999 | 31.001 | .003 | .004 | .006 | .008 |
| 80 | .998 | .999 | 31.001 | .003 | .004 | .006 | .007 | .008 |
| 85 | .999 | 31,001 | .002 | .003 | .005 | .006 | .008 | .009 |
| 90 | 30.999 | 31.001 | 31.003 | 31.004 | 31.006 | 31.007 | 31.008 | 31.010 |

## NATURAL TANGENTS

| | 0° | Δ per sec. | 1° | Δ per sec. | 2° | Δ per sec. | 3° | Δ per sec. | 4° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .000 000 | 4.85 | .017 455 | 4.85 | .034 921 | 4.85 | .052 408 | 4.85 | .069 927 | 4.87 |
| 1 | 000 291 | 4.85 | 017 746 | 4.85 | 035 212 | 4.85 | 052 699 | 4.87 | 070 219 | 4.87 |
| 2 | 000 582 | 4.85 | 018 037 | 4.85 | 035 503 | 4.87 | 052 991 | 4.87 | 070 511 | 4.88 |
| 3 | 000 873 | 4.85 | 018 328 | 4.85 | 035 795 | 4.85 | 053 283 | 4.87 | 070 804 | 4.87 |
| 4 | 001 164 | 4.83 | 018 619 | 4.85 | 036 086 | 4.85 | 053 575 | 4.85 | 071 096 | 4.88 |
| 5 | 001 454 | 4.85 | 018 910 | 4.85 | 036 377 | 4.85 | 053 866 | 4.87 | 071 389 | 4.87 |
| 6 | 001 745 | 4.85 | 019 201 | 4.85 | 036 668 | 4.87 | 054 158 | 4.87 | 071 681 | 4.87 |
| 7 | 002 036 | 4.85 | 019 492 | 4.85 | 036 960 | 4.85 | 054 450 | 4.87 | 071 973 | 4.88 |
| 8 | 002 327 | 4.85 | 019 783 | 4.85 | 037 251 | 4.85 | 054 742 | 4.85 | 072 266 | 4.87 |
| 9 | 002 618 | 4.85 | 020 074 | 4.85 | 037 542 | 4.87 | 055 033 | 4.87 | 072 558 | 4.88 |
| 10 | 002 909 | 4.85 | 020 365 | 4.85 | 037 834 | 4.85 | 055 325 | 4.87 | 072 851 | 4.87 |
| 11 | 003 200 | 4.85 | 020 656 | 4.85 | 038 125 | 4.85 | 055 617 | 4.87 | 073 143 | 4.87 |
| 12 | 003 491 | 4.85 | 020 947 | 4.85 | 038 416 | 4.85 | 055 909 | 4.85 | 073 435 | 4.88 |
| 13 | 003 782 | 4.83 | 021 238 | 4.85 | 038 707 | 4.87 | 056 200 | 4.87 | 073 728 | 4.87 |
| 14 | 004 072 | 4.85 | 021 529 | 4.85 | 038 999 | 4.85 | 056 492 | 4.87 | 074 020 | 4.88 |
| 15 | 004 363 | 4.85 | 021 820 | 4.85 | 039 290 | 4.85 | 056 784 | 4.87 | 074 313 | 4.87 |
| 16 | 004 654 | 4.85 | 022 111 | 4.85 | 039 581 | 4.87 | 057 076 | 4.87 | 074 605 | 4.88 |
| 17 | 004 945 | 4.85 | 022 402 | 4.85 | 039 873 | 4.85 | 057 368 | 4.87 | 074 898 | 4.87 |
| 18 | 005 236 | 4.85 | 022 693 | 4.85 | 040 164 | 4.87 | 057 660 | 4.87 | 075 190 | 4.88 |
| 19 | 005 527 | 4.85 | 022 984 | 4.85 | 040 456 | 4.85 | 057 952 | 4.85 | 075 483 | 4.87 |
| 20 | 005 818 | 4.85 | 023 275 | 4.85 | 040 747 | 4.85 | 058 243 | 4.87 | 075 775 | 4.88 |
| 21 | 006 109 | 4.85 | 023 566 | 4.85 | 041 038 | 4.87 | 058 535 | 4.87 | 076 068 | 4.88 |
| 22 | 006 400 | 4.85 | 023 857 | 4.85 | 041 330 | 4.85 | 058 827 | 4.87 | 076 361 | 4.87 |
| 23 | 006 691 | 4.83 | 024 148 | 4.85 | 041 621 | 4.85 | 059 119 | 4.87 | 076 653 | 4.88 |
| 24 | 006 981 | 4.85 | 024 439 | 4.87 | 041 912 | 4.87 | 059 411 | 4.87 | 076 946 | 4.87 |
| 25 | 007 272 | 4.85 | 024 731 | 4.85 | 042 204 | 4.85 | 059 703 | 4.87 | 077 238 | 4.88 |
| 26 | 007 563 | 4.85 | 025 022 | 4.85 | 042 495 | 4.87 | 059 995 | 4.87 | 077 531 | 4.88 |
| 27 | 007 854 | 4.85 | 025 313 | 4.85 | 042 787 | 4.85 | 060 287 | 4.87 | 077 824 | 4.87 |
| 28 | 008 145 | 4.85 | 025 604 | 4.85 | 043 078 | 4.87 | 060 579 | 4.87 | 078 116 | 4.88 |
| 29 | 008 436 | 4.85 | 025 895 | 4.85 | 043 370 | 4.85 | 060 871 | 4.87 | 078 409 | 4.88 |
| 30 | 008 727 | 4.85 | 026 186 | 4.85 | 043 661 | 4.85 | 061 163 | 4.87 | 078 702 | 4.87 |
| 31 | 009 018 | 4.85 | 026 477 | 4.85 | 043 952 | 4.87 | 061 455 | 4.87 | 078 994 | 4.88 |
| 32 | 009 309 | 4.85 | 026 768 | 4.85 | 044 244 | 4.85 | 061 747 | 4.87 | 079 287 | 4.88 |
| 33 | 009 600 | 4.85 | 027 059 | 4.85 | 044 535 | 4.87 | 062 039 | 4.87 | 079 580 | 4.88 |
| 34 | 009 891 | 4.83 | 027 350 | 4.85 | 044 827 | 4.85 | 062 331 | 4.87 | 079 873 | 4.87 |
| 35 | 010 181 | 4.85 | 027 641 | 4.87 | 045 118 | 4.87 | 062 623 | 4.87 | 080 165 | 4.88 |
| 36 | 010 472 | 4.85 | 027 933 | 4.85 | 045 410 | 4.85 | 062 915 | 4.87 | 080 458 | 4.88 |
| 37 | 010 763 | 4.85 | 028 224 | 4.85 | 045 701 | 4.87 | 063 207 | 4.87 | 080 751 | 4.88 |
| 38 | 011 054 | 4.85 | 028 515 | 4.85 | 045 993 | 4.85 | 063 499 | 4.87 | 081 044 | 4.87 |
| 39 | 011 345 | 4.85 | 028 806 | 4.85 | 046 284 | 4.87 | 063 791 | 4.87 | 081 336 | 4.88 |
| 40 | 011 636 | 4.85 | 029 097 | 4.85 | 046 576 | 4.85 | 064 083 | 4.87 | 081 629 | 4.88 |
| 41 | 011 927 | 4.85 | 029 388 | 4.85 | 046 867 | 4.87 | 064 375 | 4.87 | 081 922 | 4.88 |
| 42 | 012 218 | 4.85 | 029 679 | 4.85 | 047 159 | 4.85 | 064 667 | 4.87 | 082 215 | 4.88 |
| 43 | 012 509 | 4.85 | 029 970 | 4.87 | 047 450 | 4.87 | 064 959 | 4.87 | 082 508 | 4.88 |
| 44 | 012 800 | 4.85 | 030 262 | 4.85 | 047 742 | 4.85 | 065 251 | 4.87 | 082 801 | 4.88 |
| 45 | 013 091 | 4.85 | 030 553 | 4.85 | 048 033 | 4.87 | 065 543 | 4.88 | 083 094 | 4.87 |
| 46 | 013 382 | 4.85 | 030 844 | 4.85 | 048 325 | 4.87 | 065 836 | 4.87 | 083 386 | 4.88 |
| 47 | 013 673 | 4.85 | 031 135 | 4.85 | 048 617 | 4.85 | 066 128 | 4.87 | 083 679 | 4.88 |
| 48 | 013 964 | 4.83 | 031 426 | 4.85 | 048 908 | 4.87 | 066 420 | 4.87 | 083 972 | 4.88 |
| 49 | 014 254 | 4.85 | 031 717 | 4.87 | 049 200 | 4.85 | 066 712 | 4.87 | 084 265 | 4.88 |
| 50 | 014 545 | 4.85 | 032 009 | 4.85 | 049 491 | 4.87 | 067 004 | 4.87 | 084 558 | 4.88 |
| 51 | 014 836 | 4.85 | 032 300 | 4.85 | 049 783 | 4.87 | 067 296 | 4.88 | 084 851 | 4.88 |
| 52 | 015 127 | 4.85 | 032 591 | 4.85 | 050 075 | 4.85 | 067 589 | 4.87 | 085 144 | 4.88 |
| 53 | 015 418 | 4.85 | 032 882 | 4.85 | 050 366 | 4.87 | 067 881 | 4.87 | 085 437 | 4.88 |
| 54 | 015 709 | 4.85 | 033 173 | 4.87 | 050 658 | 4.85 | 068 173 | 4.87 | 085 730 | 4.88 |
| 55 | 016 000 | 4.85 | 033 465 | 4.85 | 050 949 | 4.87 | 068 465 | 4.88 | 086 023 | 4.88 |
| 56 | 016 291 | 4.85 | 033 756 | 4.85 | 051 241 | 4.87 | 068 758 | 4.87 | 086 316 | 4.88 |
| 57 | 016 582 | 4.85 | 034 047 | 4.85 | 051 533 | 4.85 | 069 050 | 4.87 | 086 609 | 4.88 |
| 58 | 016 873 | 4.85 | 034 338 | 4.87 | 051 824 | 4.87 | 069 342 | 4.88 | 086 902 | 4.90 |
| 59 | 017 164 | 4.85 | 034 630 | 4.85 | 052 116 | 4.87 | 069 635 | 4.87 | 087 196 | 4.88 |
| 60 | 017 455 | | 034 921 | | 052 408 | | 069 927 | | 087 489 | |

**NATURAL TANGENTS**

| | 5° | Δ per sec. | 6° | Δ per sec. | 7° | Δ per sec. | 8° | Δ per sec. | 9° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .087 489 | 4.88 | .105 104 | 4.90 | .122 785 | 4.92 | .140 541 | 4.93 | .158 384 | 4.98 |
| 1 | 087 782 | 4.88 | .105 398 | 4.90 | 123 080 | 4.92 | 140 837 | 4.95 | 158 683 | 4.97 |
| 2 | 088 075 | 4.88 | 105 692 | 4.92 | 123 375 | 4.92 | 141 134 | 4.95 | 158 981 | 4.97 |
| 3 | 088 368 | 4.88 | 105 987 | 4.90 | 123 670 | 4.93 | 141 431 | 4.95 | 159 279 | 4.97 |
| 4 | 088 661 | 4.88 | 106 281 | 4.90 | 123 966 | 4.92 | 141 728 | 4.93 | 159 577 | 4.98 |
| 5 | 088 954 | 4.90 | 106 575 | 4.90 | 124 261 | 4.93 | 142 024 | 4.95 | 159 876 | 4.97 |
| 6 | 089 248 | 4.88 | 106 869 | 4.90 | 124 557 | 4.92 | 142 321 | 4.95 | 160 174 | 4.97 |
| 7 | 089 541 | 4.88 | 107 163 | 4.92 | 124 852 | 4.92 | 142 618 | 4.95 | 160 472 | 4.98 |
| 8 | 089 834 | 4.88 | 107 458 | 4.90 | 125 147 | 4.93 | 142 915 | 4.95 | 160 771 | 4.97 |
| 9 | 090 127 | 4.90 | 107 752 | 4.90 | 125 443 | 4.92 | 143 212 | 4.93 | 161 069 | 4.98 |
| 10 | 090 421 | 4.88 | 108 046 | 4.90 | 125 738 | 4.93 | 143 508 | 4.95 | 161 368 | 4.97 |
| 11 | 090 714 | 4.88 | 108 340 | 4.92 | 126 034 | 4.92 | 143 805 | 4.95 | 161 666 | 4.98 |
| 12 | 091 007 | 4.88 | 108 635 | 4.90 | 126 329 | 4.93 | 144 102 | 4.95 | 161 965 | 4.97 |
| 13 | 091 300 | 4.90 | 108 929 | 4.90 | 126 625 | 4.92 | 144 399 | 4.95 | 162 263 | 4.98 |
| 14 | 091 594 | 4.88 | 109 223 | 4.92 | 126 920 | 4.93 | 144 696 | 4.95 | 162 562 | 4.97 |
| 15 | 091 887 | 4.88 | 109 518 | 4.90 | 127 216 | 4.93 | 144 993 | 4.95 | 162 860 | 4.98 |
| 16 | 092 180 | 4.90 | 109 812 | 4.92 | 127 512 | 4.92 | 145 290 | 4.95 | 163 159 | 4.98 |
| 17 | 092 474 | 4.88 | 110 107 | 4.90 | 127 807 | 4.93 | 145 587 | 4.95 | 163 458 | 4.97 |
| 18 | 092 767 | 4.90 | 110 401 | 4.90 | 128 103 | 4.93 | 145 884 | 4.95 | 163 756 | 4.98 |
| 19 | 093 061 | 4.88 | 110 695 | 4.92 | 128 399 | 4.92 | 146 181 | 4.95 | 164 055 | 4.98 |
| 20 | 093 354 | 4.88 | 110 990 | 4.90 | 128 694 | 4.93 | 146 478 | 4.97 | 164 354 | 4.97 |
| 21 | 093 647 | 4.90 | 111 284 | 4.92 | 128 990 | 4.93 | 146 776 | 4.95 | 164 652 | 4.98 |
| 22 | 093 941 | 4.88 | 111 579 | 4.90 | 129 286 | 4.93 | 147 073 | 4.95 | 164 951 | 4.98 |
| 23 | 094 234 | 4.90 | 111 873 | 4.92 | 129 582 | 4.92 | 147 370 | 4.95 | 165 250 | 4.98 |
| 24 | 094 528 | 4.88 | 112 168 | 4.92 | 129 877 | 4.93 | 147 667 | 4.95 | 165 549 | 4.98 |
| 25 | 094 821 | 4.90 | 112 463 | 4.90 | 130 173 | 4.93 | 147 964 | 4.97 | 165 848 | 4.98 |
| 26 | 095 115 | 4.88 | 112 757 | 4.92 | 130 469 | 4.93 | 148 262 | 4.95 | 166 147 | 4.98 |
| 27 | 095 408 | 4.90 | 113 052 | 4.90 | 130 765 | 4.93 | 148 559 | 4.95 | 166 446 | 4.98 |
| 28 | 095 702 | 4.88 | 113 346 | 4.92 | 131 061 | 4.93 | 148 856 | 4.97 | 166 745 | 4.98 |
| 29 | 095 995 | 4.90 | 113 641 | 4.92 | 131 357 | 4.92 | 149 154 | 4.95 | 167 044 | 4.98 |
| 30 | 096 289 | 4.90 | 113 936 | 4.90 | 131 652 | 4.93 | 149 451 | 4.95 | 167 343 | 4.98 |
| 31 | 096 583 | 4.88 | 114 230 | 4.92 | 131 948 | 4.93 | 149 748 | 4.97 | 167 642 | 4.98 |
| 32 | 096 876 | 4.90 | 114 525 | 4.92 | 132 244 | 4.93 | 150 046 | 4.95 | 167 941 | 4.98 |
| 33 | 097 170 | 4.90 | 114 820 | 4.90 | 132 540 | 4.93 | 150 343 | 4.97 | 168 240 | 4.98 |
| 34 | 097 464 | 4.88 | 115 114 | 4.92 | 132 836 | 4.93 | 150 641 | 4.95 | 168 539 | 4.98 |
| 35 | 097 757 | 4.90 | 115 409 | 4.92 | 133 132 | 4.93 | 150 938 | 4.97 | 168 838 | 4.98 |
| 36 | 098 051 | 4.90 | 115 704 | 4.92 | 133 428 | 4.95 | 151 236 | 4.95 | 169 137 | 5.00 |
| 37 | 098 345 | 4.88 | 115 999 | 4.92 | 133 725 | 4.93 | 151 533 | 4.97 | 169 437 | 4.98 |
| 38 | 098 638 | 4.90 | 116 294 | 4.90 | 134 021 | 4.93 | 151 831 | 4.97 | 169 736 | 4.98 |
| 39 | 098 932 | 4.90 | 116 588 | 4.92 | 134 317 | 4.93 | 152 129 | 4.95 | 170 035 | 4.98 |
| 40 | 099 226 | 4.88 | 116 883 | 4.92 | 134 613 | 4.93 | 152 426 | 4.97 | 170 334 | 5.00 |
| 41 | 099 519 | 4.90 | 117 178 | 4.92 | 134 909 | 4.93 | 152 724 | 4.97 | 170 634 | 4.98 |
| 42 | 099 813 | 4.90 | 117 473 | 4.92 | 135 205 | 4.95 | 153 022 | 4.95 | 170 933 | 5.00 |
| 43 | 100 107 | 4.90 | 117 768 | 4.92 | 135 502 | 4.93 | 153 319 | 4.97 | 171 233 | 4.98 |
| 44 | 100 401 | 4.90 | 118 063 | 4.92 | 135 798 | 4.93 | 153 617 | 4.97 | 171 532 | 4.98 |
| 45 | 100 695 | 4.90 | 118 358 | 4.92 | 136 094 | 4.93 | 153 915 | 4.97 | 171 831 | 5.00 |
| 46 | 100 989 | 4.88 | 118 653 | 4.92 | 136 390 | 4.95 | 154 213 | 4.95 | 172 131 | 4.98 |
| 47 | 101 282 | 4.90 | 118 948 | 4.92 | 136 687 | 4.93 | 154 510 | 4.97 | 172 430 | 5.00 |
| 48 | 101 576 | 4.90 | 119 243 | 4.92 | 136 983 | 4.93 | 154 808 | 4.97 | 172 730 | 5.00 |
| 49 | 101 870 | 4.90 | 119 538 | 4.92 | 137 279 | 4.95 | 155 106 | 4.97 | 173 030 | 4.98 |
| 50 | 102 164 | 4.90 | 119 833 | 4.92 | 137 576 | 4.93 | 155 404 | 4.97 | 173 329 | 5.00 |
| 51 | 102 458 | 4.90 | 120 128 | 4.92 | 137 872 | 4.95 | 155 702 | 4.97 | 173 629 | 4.98 |
| 52 | 102 752 | 4.90 | 120 423 | 4.92 | 138 169 | 4.93 | 156 000 | 4.97 | 173 928 | 5.00 |
| 53 | 103 046 | 4.90 | 120 718 | 4.92 | 138 465 | 4.93 | 156 298 | 4.97 | 174 228 | 5.00 |
| 54 | 103 340 | 4.90 | 121 013 | 4.92 | 138 761 | 4.95 | 156 596 | 4.97 | 174 528 | 5.00 |
| 55 | 103 634 | 4.90 | 121 308 | 4.93 | 139 058 | 4.93 | 156 894 | 4.97 | 174 828 | 5.00 |
| 56 | 103 928 | 4.90 | 121 604 | 4.92 | 139 354 | 4.95 | 157 192 | 4.97 | 175 128 | 4.98 |
| 57 | 104 222 | 4.90 | 121 899 | 4.92 | 139 651 | 4.95 | 157 490 | 4.97 | 175 427 | 5.00 |
| 58 | 104 516 | 4.90 | 122 194 | 4.92 | 139 948 | 4.93 | 157 788 | 4.97 | 175 727 | 5.00 |
| 59 | 104 810 | 4.90 | 122 489 | 4.93 | 140 244 | 4.95 | 158 086 | 4.97 | 176 027 | 5.00 |
| 60 | 105 104 | | 122 785 | | 140 541 | | 158 384 | | 176 327 | |

**NATURAL TANGENTS**

| | 10° | Δ per sec. | 11° | Δ per sec. | 12° | Δ per sec. | 13° | Δ per sec. | 14° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .176 327 | 5.00 | .194 380 | 5.03 | .212 557 | 5.07 | .230 868 | 5.12 | .249 328 | 5.15 |
| 1 | 176 627 | 5.00 | 194 682 | 5.03 | 212 861 | 5.07 | 231 175 | 5.10 | 249 637 | 5.15 |
| 2 | 176 927 | 5.00 | 194 984 | 5.03 | 213 165 | 5.07 | 231 481 | 5.12 | 249 946 | 5.15 |
| 3 | 177 227 | 5.00 | 195 286 | 5.03 | 213 469 | 5.07 | 231 788 | 5.10 | 250 255 | 5.15 |
| 4 | 177 527 | 5.00 | 195 588 | 5.03 | 213 773 | 5.07 | 232 094 | 5.12 | 250 564 | 5.15 |
| 5 | 177 827 | 5.00 | 195 890 | 5.03 | 214 077 | 5.07 | 232 401 | 5.10 | 250 873 | 5.17 |
| 6 | 178 127 | 5.00 | 196 192 | 5.03 | 214 381 | 5.08 | 232 707 | 5.12 | 251 183 | 5.15 |
| 7 | 178 427 | 5.00 | 196 494 | 5.03 | 214 686 | 5.07 | 233 014 | 5.12 | 251 492 | 5.15 |
| 8 | 178 727 | 5.02 | 196 796 | 5.05 | 214 990 | 5.07 | 233 321 | 5.10 | 251 801 | 5.17 |
| 9 | 179 028 | 5.00 | 197 099 | 5.03 | 215 294 | 5.08 | 233 627 | 5.12 | 252 111 | 5.15 |
| 10 | 179 328 | 5.00 | 197 401 | 5.03 | 215 599 | 5.07 | 233 934 | 5.12 | 252 420 | 5.15 |
| 11 | 179 628 | 5.00 | 197 703 | 5.03 | 215 903 | 5.08 | 234 241 | 5.12 | 252 729 | 5.17 |
| 12 | 179 928 | 5.02 | 198 005 | 5.05 | 216 208 | 5.07 | 234 548 | 5.12 | 253 039 | 5.15 |
| 13 | 180 229 | 5.00 | 198 308 | 5.03 | 216 512 | 5.08 | 234 855 | 5.12 | 253 348 | 5.17 |
| 14 | 180 529 | 5.02 | 198 610 | 5.03 | 216 817 | 5.07 | 235 162 | 5.12 | 253 658 | 5.17 |
| 15 | 180 829 | 5.02 | 198 912 | 5.05 | 217 121 | 5.08 | 235 469 | 5.12 | 253 968 | 5.15 |
| 16 | 181 130 | 5.00 | 199 215 | 5.03 | 217 426 | 5.08 | 235 776 | 5.12 | 254 277 | 5.17 |
| 17 | 181 430 | 5.02 | 199 517 | 5.05 | 217 731 | 5.07 | 236 083 | 5.12 | 254 587 | 5.17 |
| 18 | 181 731 | 5.00 | 199 820 | 5.03 | 218 035 | 5.08 | 236 390 | 5.12 | 254 897 | 5.17 |
| 19 | 182 031 | 5.02 | 200 122 | 5.05 | 218 340 | 5.08 | 236 697 | 5.12 | 255 207 | 5.15 |
| 20 | 182 332 | 5.00 | 200 425 | 5.03 | 218 645 | 5.08 | 237 004 | 5.13 | 255 516 | 5.17 |
| 21 | 182 632 | 5.02 | 200 727 | 5.05 | 218 950 | 5.07 | 237 312 | 5.12 | 255 826 | 5.17 |
| 22 | 182 933 | 5.02 | 201 030 | 5.05 | 219 254 | 5.08 | 237 619 | 5.12 | 256 136 | 5.17 |
| 23 | 183 234 | 5.00 | 201 333 | 5.03 | 219 559 | 5.08 | 237 926 | 5.13 | 256 446 | 5.17 |
| 24 | 183 534 | 5.02 | 201 635 | 5.05 | 219 864 | 5.08 | 238 234 | 5.12 | 256 756 | 5.17 |
| 25 | 183 835 | 5.02 | 201 938 | 5.05 | 220 169 | 5.08 | 238 541 | 5.12 | 257 066 | 5.18 |
| 26 | 184 136 | 5.02 | 202 241 | 5.05 | 220 474 | 5.08 | 238 848 | 5.13 | 257 377 | 5.17 |
| 27 | 184 437 | 5.00 | 202 544 | 5.05 | 220 779 | 5.08 | 239 156 | 5.13 | 257 687 | 5.17 |
| 28 | 184 737 | 5.02 | 202 847 | 5.03 | 221 084 | 5.10 | 239 464 | 5.12 | 257 997 | 5.17 |
| 29 | 185 038 | 5.02 | 203 149 | 5.05 | 221 390 | 5.08 | 239 771 | 5.13 | 258 307 | 5.18 |
| 30 | 185 339 | 5.02 | 203 452 | 5.05 | 221 695 | 5.08 | 240 079 | 5.12 | 258 618 | 5.17 |
| 31 | 185 640 | 5.02 | 203 755 | 5.05 | 222 000 | 5.08 | 240 386 | 5.13 | 258 928 | 5.17 |
| 32 | 185 941 | 5.02 | 204 058 | 5.05 | 222 305 | 5.08 | 240 694 | 5.13 | 259 238 | 5.18 |
| 33 | 186 242 | 5.02 | 204 361 | 5.05 | 222 610 | 5.10 | 241 002 | 5.13 | 259 549 | 5.17 |
| 34 | 186 543 | 5.02 | 204 664 | 5.05 | 222 916 | 5.08 | 241 310 | 5.13 | 259 859 | 5.18 |
| 35 | 186 844 | 5.02 | 204 967 | 5.07 | 223 221 | 5.08 | 241 618 | 5.12 | 260 170 | 5.17 |
| 36 | 187 145 | 5.02 | 205 271 | 5.05 | 223 526 | 5.10 | 241 925 | 5.13 | 260 480 | 5.18 |
| 37 | 187 446 | 5.02 | 205 574 | 5.05 | 223 832 | 5.08 | 242 233 | 5.13 | 260 791 | 5.18 |
| 38 | 187 747 | 5.02 | 205 877 | 5.05 | 224 137 | 5.10 | 242 541 | 5.13 | 261 102 | 5.18 |
| 39 | 188 048 | 5.02 | 206 180 | 5.05 | 224 443 | 5.08 | 242 849 | 5.15 | 261 413 | 5.17 |
| 40 | 188 349 | 5.03 | 206 483 | 5.07 | 224 748 | 5.10 | 243 158 | 5.13 | 261 723 | 5.18 |
| 41 | 188 651 | 5.02 | 206 787 | 5.05 | 225 054 | 5.10 | 243 466 | 5.13 | 262 034 | 5.18 |
| 42 | 188 952 | 5.02 | 207 090 | 5.05 | 225 360 | 5.08 | 243 774 | 5.13 | 262 345 | 5.18 |
| 43 | 189 253 | 5.03 | 207 393 | 5.07 | 225 665 | 5.10 | 244 082 | 5.13 | 262 656 | 5.18 |
| 44 | 189 555 | 5.02 | 207 697 | 5.05 | 225 971 | 5.10 | 244 390 | 5.13 | 262 967 | 5.18 |
| 45 | 189 856 | 5.02 | 208 000 | 5.07 | 226 277 | 5.10 | 244 698 | 5.15 | 263 278 | 5.18 |
| 46 | 190 157 | 5.03 | 208 304 | 5.05 | 226 583 | 5.10 | 245 007 | 5.13 | 263 589 | 5.18 |
| 47 | 190 459 | 5.02 | 208 607 | 5.07 | 226 889 | 5.08 | 245 315 | 5.15 | 263 900 | 5.18 |
| 48 | 190 760 | 5.03 | 208 911 | 5.05 | 227 194 | 5.10 | 245 624 | 5.13 | 264 211 | 5.20 |
| 49 | 191 062 | 5.02 | 209 214 | 5.07 | 227 500 | 5.10 | 245 932 | 5.15 | 264 523 | 5.18 |
| 50 | 191 363 | 5.03 | 209 518 | 5.07 | 227 806 | 5.10 | 246 241 | 5.13 | 264 834 | 5.18 |
| 51 | 191 665 | 5.02 | 209 822 | 5.07 | 228 112 | 5.10 | 246 549 | 5.15 | 265 145 | 5.20 |
| 52 | 191 966 | 5.03 | 210 126 | 5.05 | 228 418 | 5.10 | 246 858 | 5.13 | 265 457 | 5.18 |
| 53 | 192 268 | 5.03 | 210 429 | 5.07 | 228 724 | 5.12 | 247 166 | 5.15 | 265 768 | 5.18 |
| 54 | 192 570 | 5.02 | 210 733 | 5.07 | 229 031 | 5.10 | 247 475 | 5.15 | 266 079 | 5.20 |
| 55 | 192 871 | 5.03 | 211 037 | 5.07 | 229 337 | 5.10 | 247 784 | 5.13 | 266 391 | 5.18 |
| 56 | 193 173 | 5.03 | 211 341 | 5.07 | 229 643 | 5.10 | 248 092 | 5.15 | 266 702 | 5.20 |
| 57 | 193 475 | 5.03 | 211 645 | 5.07 | 229 949 | 5.10 | 248 401 | 5.15 | 267 014 | 5.20 |
| 58 | 193 777 | 5.02 | 211 949 | 5.07 | 230 255 | 5.12 | 248 710 | 5.15 | 267 326 | 5.18 |
| 59 | 194 078 | 5.03 | 212 253 | 5.07 | 230 562 | 5.10 | 249 019 | 5.15 | 267 637 | 5.20 |
| 60 | 194 380 | | 212 557 | | 230 868 | | 249 328 | | 267 949 | |

## NATURAL TANGENTS

| | 15° | Δ per sec. | 16° | Δ per sec. | 17° | Δ per sec. | 18° | Δ per sec. | 19° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .267 949 | 5.20 | .286 745 | 5.25 | .305 731 | 5.30 | .324 920 | 5.35 | .344 328 | 5.42 |
| 1 | 268 261 | 5.20 | 287 060 | 5.25 | 306 049 | 5.30 | 325 241 | 5.37 | 344 653 | 5.42 |
| 2 | 268 573 | 5.20 | 287 375 | 5.25 | 306 367 | 5.30 | 325 563 | 5.37 | 344 978 | 5.43 |
| 3 | 268 885 | 5.20 | 287 690 | 5.25 | 306 685 | 5.30 | 325 885 | 5.37 | 345 304 | 5.43 |
| 4 | 269 197 | 5.20 | 288 005 | 5.25 | 307 003 | 5.32 | 326 207 | 5.35 | 345 630 | 5.42 |
| 5 | 269 509 | 5.20 | 288 320 | 5.25 | 307 322 | 5.30 | 326 528 | 5.37 | 345 955 | 5.43 |
| 6 | 269 821 | 5.20 | 288 635 | 5.25 | 307 640 | 5.32 | 326 850 | 5.37 | 346 281 | 5.43 |
| 7 | 270 133 | 5.20 | 288 950 | 5.27 | 307 959 | 5.30 | 327 172 | 5.37 | 346 607 | 5.43 |
| 8 | 270 445 | 5.20 | 289 266 | 5.25 | 308 277 | 5.32 | 327 494 | 5.38 | 346 933 | 5.43 |
| 9 | 270 757 | 5.20 | 289 581 | 5.25 | 308 596 | 5.30 | 327 817 | 5.37 | 347 259 | 5.43 |
| 10 | 271 069 | 5.22 | 289 896 | 5.25 | 308 914 | 5.32 | 328 139 | 5.37 | 347 585 | 5.43 |
| 11 | 271 382 | 5.20 | 290 211 | 5.27 | 309 233 | 5.32 | 328 461 | 5.37 | 347 911 | 5.43 |
| 12 | 271 694 | 5.20 | 290 527 | 5.25 | 309 552 | 5.30 | 328 783 | 5.38 | 348 237 | 5.43 |
| 13 | 272 006 | 5.22 | 290 842 | 5.27 | 309 870 | 5.32 | 329 106 | 5.37 | 348 563 | 5.43 |
| 14 | 272 319 | 5.20 | 291 158 | 5.25 | 310 189 | 5.32 | 329 428 | 5.38 | 348 889 | 5.45 |
| 15 | 272 631 | 5.22 | 291 473 | 5.27 | 310 508 | 5.32 | 329 751 | 5.37 | 349 216 | 5.43 |
| 16 | 272 944 | 5.20 | 291 789 | 5.27 | 310 827 | 5.32 | 330 073 | 5.38 | 349 542 | 5.43 |
| 17 | 273 256 | 5.22 | 292 105 | 5.25 | 311 146 | 5.32 | 330 396 | 5.37 | 349 868 | 5.45 |
| 18 | 273 569 | 5.22 | 292 420 | 5.27 | 311 465 | 5.32 | 330 718 | 5.38 | 350 195 | 5.45 |
| 19 | 273 882 | 5.20 | 292 736 | 5.27 | 311 784 | 5.33 | 331 041 | 5.38 | 350 522 | 5.43 |
| 20 | 274 194 | 5.22 | 293 052 | 5.27 | 312 104 | 5.32 | 331 364 | 5.38 | 350 848 | 5.45 |
| 21 | 274 507 | 5.22 | 293 368 | 5.27 | 312 423 | 5.32 | 331 687 | 5.38 | 351 175 | 5.45 |
| 22 | 274 820 | 5.22 | 293 684 | 5.27 | 312 742 | 5.33 | 332 010 | 5.38 | 351 502 | 5.45 |
| 23 | 275 133 | 5.22 | 294 000 | 5.27 | 313 062 | 5.32 | 332 333 | 5.38 | 351 829 | 5.45 |
| 24 | 275 446 | 5.22 | 294 316 | 5.27 | 313 381 | 5.32 | 332 656 | 5.38 | 352 156 | 5.45 |
| 25 | 275 759 | 5.22 | 294 632 | 5.27 | 313 700 | 5.33 | 332 979 | 5.38 | 352 483 | 5.45 |
| 26 | 276 072 | 5.22 | 294 948 | 5.28 | 314 020 | 5.33 | 333 302 | 5.38 | 352 810 | 5.45 |
| 27 | 276 385 | 5.22 | 295 265 | 5.27 | 314 340 | 5.32 | 333 625 | 5.40 | 353 137 | 5.45 |
| 28 | 276 698 | 5.22 | 295 581 | 5.27 | 314 659 | 5.33 | 333 949 | 5.38 | 353 464 | 5.45 |
| 29 | 277 011 | 5.23 | 295 897 | 5.27 | 314 979 | 5.33 | 334 272 | 5.38 | 353 791 | 5.47 |
| 30 | 277 325 | 5.22 | 296 213 | 5.28 | 315 299 | 5.33 | 334 595 | 5.40 | 354 119 | 5.45 |
| 31 | 277 638 | 5.22 | 296 530 | 5.27 | 315 619 | 5.33 | 334 919 | 5.38 | 354 446 | 5.45 |
| 32 | 277 951 | 5.23 | 296 846 | 5.28 | 315 939 | 5.32 | 335 242 | 5.40 | 354 773 | 5.47 |
| 33 | 278 265 | 5.22 | 297 163 | 5.28 | 316 258 | 5.33 | 335 566 | 5.40 | 355 101 | 5.47 |
| 34 | 278 578 | 5.22 | 297 480 | 5.27 | 316 578 | 5.35 | 335 890 | 5.38 | 355 429 | 5.45 |
| 35 | 278 891 | 5.23 | 297 796 | 5.28 | 316 899 | 5.33 | 336 213 | 5.40 | 355 756 | 5.47 |
| 36 | 279 205 | 5.23 | 298 113 | 5.28 | 317 219 | 5.33 | 336 537 | 5.40 | 356 084 | 5.47 |
| 37 | 279 519 | 5.22 | 298 430 | 5.28 | 317 539 | 5.33 | 336 861 | 5.40 | 356 412 | 5.47 |
| 38 | 279 832 | 5.23 | 298 747 | 5.27 | 317 859 | 5.33 | 337 185 | 5.40 | 356 740 | 5.47 |
| 39 | 280 146 | 5.23 | 299 063 | 5.28 | 318 179 | 5.35 | 337 509 | 5.40 | 357 068 | 5.47 |
| 40 | 280 460 | 5.22 | 299 380 | 5.28 | 318 500 | 5.33 | 337 833 | 5.40 | 357 396 | 5.47 |
| 41 | 280 773 | 5.23 | 299 697 | 5.28 | 318 820 | 5.35 | 338 157 | 5.40 | 357 724 | 5.47 |
| 42 | 281 087 | 5.23 | 300 014 | 5.28 | 319 141 | 5.33 | 338 481 | 5.42 | 358 052 | 5.47 |
| 43 | 281 401 | 5.23 | 300 331 | 5.30 | 319 461 | 5.35 | 338 806 | 5.40 | 358 380 | 5.47 |
| 44 | 281 715 | 5.23 | 300 649 | 5.28 | 319 782 | 5.35 | 339 130 | 5.40 | 358 708 | 5.48 |
| 45 | 282 029 | 5.23 | 300 966 | 5.28 | 320 103 | 5.33 | 339 454 | 5.42 | 359 037 | 5.47 |
| 46 | 282 343 | 5.23 | 301 283 | 5.28 | 320 423 | 5.35 | 339 779 | 5.40 | 359 365 | 5.48 |
| 47 | 282 657 | 5.23 | 301 600 | 5.30 | 320 744 | 5.35 | 340 103 | 5.42 | 359 694 | 5.47 |
| 48 | 282 971 | 5.25 | 301 918 | 5.28 | 321 065 | 5.35 | 340 428 | 5.40 | 360 022 | 5.48 |
| 49 | 283 286 | 5.23 | 302 235 | 5.30 | 321 386 | 5.35 | 340 752 | 5.42 | 360 351 | 5.47 |
| 50 | 283 600 | 5.23 | 302 553 | 5.28 | 321 707 | 5.35 | 341 077 | 5.42 | 360 679 | 5.48 |
| 51 | 283 914 | 5.25 | 302 870 | 5.30 | 322 028 | 5.35 | 341 402 | 5.42 | 361 008 | 5.48 |
| 52 | 284 229 | 5.23 | 303 188 | 5.30 | 322 349 | 5.35 | 341 727 | 5.42 | 361 337 | 5.48 |
| 53 | 284 543 | 5.23 | 303 506 | 5.28 | 322 670 | 5.35 | 342 052 | 5.42 | 361 666 | 5.48 |
| 54 | 284 857 | 5.25 | 303 823 | 5.30 | 322 991 | 5.35 | 342 377 | 5.42 | 361 995 | 5.48 |
| 55 | 285 172 | 5.25 | 304 141 | 5.30 | 323 312 | 5.37 | 342 702 | 5.42 | 362 324 | 5.48 |
| 56 | 285 487 | 5.23 | 304 459 | 5.30 | 323 634 | 5.35 | 343 027 | 5.42 | 362 653 | 5.48 |
| 57 | 285 801 | 5.25 | 304 777 | 5.30 | 323 955 | 5.37 | 343 352 | 5.42 | 362 982 | 5.50 |
| 58 | 286 116 | 5.25 | 305 095 | 5.30 | 324 277 | 5.35 | 343 677 | 5.42 | 363 312 | 5.48 |
| 59 | 286 431 | 5.23 | 305 413 | 5.30 | 324 598 | 5.37 | 344 002 | 5.43 | 363 641 | 5.48 |
| 60 | 286 745 | | 305 731 | | 324 920 | | 344 328 | | 363 970 | |

## NATURAL TANGENTS

| | 20° | Δ per sec. | 21° | Δ per sec. | 22° | Δ per sec. | 23° | Δ per sec. | 24° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .363 970 | 5.50 | .383 864 | 5.57 | .404 026 | 5.65 | .424 475 | 5.72 | .445 229 | 5.80 |
| 1 | 364 300 | 5.48 | 384 198 | 5.57 | 404 365 | 5.63 | 424 818 | 5.73 | 445 577 | 5.82 |
| 2 | 364 629 | 5.50 | 384 532 | 5.57 | 404 703 | 5.65 | 425 162 | 5.72 | 445 926 | 5.82 |
| 3 | 364 959 | 5.48 | 384 866 | 5.57 | 405 042 | 5.63 | 425 505 | 5.73 | 446 275 | 5.82 |
| 4 | 365 288 | 5.50 | 385 200 | 5.57 | 405 380 | 5.65 | 425 849 | 5.72 | 446 624 | 5.82 |
| 5 | 365 618 | 5.50 | 385 534 | 5.57 | 405 719 | 5.65 | 426 192 | 5.73 | 446 973 | 5.82 |
| 6 | 365 948 | 5.50 | 385 868 | 5.57 | 406 058 | 5.65 | 426 536 | 5.73 | 447 322 | 5.82 |
| 7 | 366 278 | 5.50 | 386 202 | 5.57 | 406 397 | 5.65 | 426 880 | 5.73 | 447 671 | 5.82 |
| 8 | 366 608 | 5.50 | 386 536 | 5.58 | 406 736 | 5.65 | 427 224 | 5.73 | 448 020 | 5.82 |
| 9 | 366 938 | 5.50 | 386 871 | 5.57 | 407 075 | 5.65 | 427 568 | 5.73 | 448 369 | 5.83 |
| 10 | 367 268 | 5.50 | 387 205 | 5.58 | 407 414 | 5.65 | 427 912 | 5.73 | 448 719 | 5.82 |
| 11 | 367 598 | 5.50 | 387 540 | 5.57 | 407 753 | 5.65 | 428 256 | 5.75 | 449 068 | 5.83 |
| 12 | 367 928 | 5.52 | 387 874 | 5.58 | 408 092 | 5.67 | 428 601 | 5.73 | 449 418 | 5.83 |
| 13 | 368 259 | 5.50 | 388 209 | 5.58 | 408 432 | 5.65 | 428 945 | 5.73 | 449 768 | 5.82 |
| 14 | 368 589 | 5.50 | 388 544 | 5.58 | 408 771 | 5.67 | 429 289 | 5.75 | 450 117 | 5.83 |
| 15 | 368 919 | 5.52 | 388 879 | 5.58 | 409 111 | 5.65 | 429 634 | 5.75 | 450 467 | 5.83 |
| 16 | 369 250 | 5.52 | 389 214 | 5.58 | 409 450 | 5.67 | 429 979 | 5.73 | 450 817 | 5.83 |
| 17 | 369 581 | 5.50 | 389 549 | 5.58 | 409 790 | 5.67 | 430 323 | 5.75 | 451 167 | 5.83 |
| 18 | 369 911 | 5.52 | 389 884 | 5.58 | 410 130 | 5.67 | 430 668 | 5.75 | 451 517 | 5.85 |
| 19 | 370 242 | 5.52 | 390 219 | 5.58 | 410 470 | 5.67 | 431 013 | 5.75 | 451 868 | 5.83 |
| 20 | 370 573 | 5.52 | 390 554 | 5.58 | 410 810 | 5.67 | 431 358 | 5.75 | 452 218 | 5.83 |
| 21 | 370 904 | 5.52 | 390 889 | 5.60 | 411 150 | 5.67 | 431 703 | 5.75 | 452 568 | 5.85 |
| 22 | 371 235 | 5.52 | 391 225 | 5.58 | 411 490 | 5.67 | 432 048 | 5.75 | 452 919 | 5.83 |
| 23 | 371 566 | 5.52 | 391 560 | 5.60 | 411 830 | 5.67 | 432 393 | 5.77 | 453 269 | 5.85 |
| 24 | 371 897 | 5.52 | 391 896 | 5.58 | 412 170 | 5.68 | 432 739 | 5.75 | 453 620 | 5.85 |
| 25 | 372 228 | 5.52 | 392 231 | 5.60 | 412 511 | 5.67 | 433 084 | 5.77 | 453 971 | 5.85 |
| 26 | 372 559 | 5.52 | 392 567 | 5.60 | 412 851 | 5.68 | 433 430 | 5.75 | 454 322 | 5.85 |
| 27 | 372 890 | 5.53 | 392 903 | 5.60 | 413 192 | 5.67 | 433 775 | 5.77 | 454 673 | 5.85 |
| 28 | 373 222 | 5.52 | 393 239 | 5.58 | 413 532 | 5.68 | 434 121 | 5.77 | 455 024 | 5.85 |
| 29 | 373 553 | 5.53 | 393 574 | 5.60 | 413 873 | 5.68 | 434 467 | 5.75 | 455 375 | 5.85 |
| 30 | 373 885 | 5.52 | 393 910 | 5.62 | 414 214 | 5.67 | 434 812 | 5.77 | 455 726 | 5.87 |
| 31 | 374 216 | 5.53 | 394 247 | 5.60 | 414 554 | 5.68 | 435 158 | 5.77 | 456 078 | 5.85 |
| 32 | 374 548 | 5.53 | 394 583 | 5.60 | 414 895 | 5.68 | 435 504 | 5.77 | 456 429 | 5.87 |
| 33 | 374 880 | 5.52 | 394 919 | 5.60 | 415 236 | 5.68 | 435 850 | 5.78 | 456 781 | 5.85 |
| 34 | 375 211 | 5.53 | 395 255 | 5.62 | 415 577 | 5.70 | 436 197 | 5.77 | 457 132 | 5.87 |
| 35 | 375 543 | 5.53 | 395 592 | 5.60 | 415 919 | 5.68 | 436 543 | 5.77 | 457 484 | 5.87 |
| 36 | 375 875 | 5.53 | 395 928 | 5.62 | 416 260 | 5.68 | 436 889 | 5.78 | 457 836 | 5.87 |
| 37 | 376 207 | 5.53 | 396 265 | 5.60 | 416 601 | 5.70 | 437 236 | 5.77 | 458 188 | 5.87 |
| 38 | 376 539 | 5.55 | 396 601 | 5.62 | 416 943 | 5.68 | 437 582 | 5.78 | 458 540 | 5.87 |
| 39 | 376 872 | 5.53 | 396 938 | 5.62 | 417 284 | 5.70 | 437 929 | 5.78 | 458 892 | 5.87 |
| 40 | 377 204 | 5.53 | 397 275 | 5.60 | 417 626 | 5.68 | 438 276 | 5.77 | 459 244 | 5.87 |
| 41 | 377 536 | 5.55 | 397 611 | 5.62 | 417 967 | 5.70 | 438 622 | 5.78 | 459 596 | 5.88 |
| 42 | 377 869 | 5.53 | 397 948 | 5.62 | 418 309 | 5.70 | 438 969 | 5.78 | 459 949 | 5.87 |
| 43 | 378 201 | 5.55 | 398 285 | 5.62 | 418 651 | 5.70 | 439 316 | 5.78 | 460 301 | 5.88 |
| 44 | 378 534 | 5.53 | 398 622 | 5.63 | 418 993 | 5.70 | 439 663 | 5.80 | 460 654 | 5.87 |
| 45 | 378 866 | 5.55 | 398 960 | 5.62 | 419 335 | 5.70 | 440 011 | 5.78 | 461 006 | 5.88 |
| 46 | 379 199 | 5.55 | 399 297 | 5.62 | 419 677 | 5.70 | 440 358 | 5.78 | 461 359 | 5.88 |
| 47 | 379 532 | 5.53 | 399 634 | 5.62 | 420 019 | 5.70 | 440 705 | 5.80 | 461 712 | 5.88 |
| 48 | 379 864 | 5.55 | 399 971 | 5.63 | 420 361 | 5.72 | 441 053 | 5.78 | 462 065 | 5.88 |
| 49 | 380 197 | 5.55 | 400 309 | 5.62 | 420 704 | 5.70 | 441 400 | 5.80 | 462 418 | 5.88 |
| 50 | 380 530 | 5.55 | 400 646 | 5.63 | 421 046 | 5.72 | 441 748 | 5.78 | 462 771 | 5.88 |
| 51 | 380 863 | 5.55 | 400 984 | 5.63 | 421 389 | 5.70 | 442 095 | 5.80 | 463 124 | 5.90 |
| 52 | 381 196 | 5.57 | 401 322 | 5.63 | 421 731 | 5.72 | 442 443 | 5.80 | 463 478 | 5.88 |
| 53 | 381 530 | 5.55 | 401 660 | 5.62 | 422 074 | 5.72 | 442 791 | 5.80 | 463 831 | 5.90 |
| 54 | 381 863 | 5.55 | 401 997 | 5.63 | 422 417 | 5.70 | 443 139 | 5.80 | 464 185 | 5.88 |
| 55 | 382 196 | 5.57 | 402 335 | 5.63 | 422 759 | 5.72 | 443 487 | 5.80 | 464 538 | 5.90 |
| 56 | 382 530 | 5.55 | 402 673 | 5.63 | 423 102 | 5.72 | 443 835 | 5.80 | 464 892 | 5.90 |
| 57 | 382 863 | 5.57 | 403 011 | 5.65 | 423 445 | 5.72 | 444 183 | 5.82 | 465 246 | 5.90 |
| 58 | 383 197 | 5.55 | 403 350 | 5.63 | 423 788 | 5.73 | 444 532 | 5.80 | 465 600 | 5.90 |
| 59 | 383 530 | 5.57 | 403 688 | 5.63 | 424 132 | 5.72 | 444 880 | 5.82 | 465 954 | 5.90 |
| 60 | 383 864 | | 404 026 | | 424 475 | | 445 229 | | 466 308 | |

## NATURAL TANGENTS

| | 25° | Δ per sec. | 26° | Δ per sec. | 27° | Δ per sec. | 28° | Δ per sec. | 29° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .466 308 | 5.92 | .487 733 | 6.00 | .509 525 | 6.12 | .531 709 | 6.23 | .554 309 | 6.33 |
| 1 | 466 662 | 5.90 | 488 093 | 6.00 | 509 892 | 6.10 | 532 083 | 6.22 | 554 689 | 6.35 |
| 2 | 467 016 | 5.92 | 488 453 | 6.00 | 510 258 | 6.12 | 532 456 | 6.22 | 555 070 | 6.33 |
| 3 | 467 371 | 5.90 | 488 813 | 6.02 | 510 625 | 6.12 | 532 829 | 6.23 | 555 450 | 6.35 |
| 4 | 467 725 | 5.92 | 489 174 | 6.00 | 510 992 | 6.12 | 533 203 | 6.23 | 555 831 | 6.35 |
| 5 | 468 080 | 5.90 | 489 534 | 6.02 | 511 359 | 6.12 | 533 577 | 6.22 | 556 212 | 6.35 |
| 6 | 468 434 | 5.92 | 489 895 | 6.02 | 511 726 | 6.12 | 533 950 | 6.23 | 556 593 | 6.35 |
| 7 | 468 789 | 5.92 | 490 256 | 6.02 | 512 093 | 6.12 | 534 324 | 6.23 | 556 974 | 6.35 |
| 8 | 469 144 | 5.92 | 490 617 | 6.02 | 512 460 | 6.13 | 534 698 | 6.23 | 557 355 | 6.35 |
| 9 | 469 499 | 5.92 | 490 978 | 6.02 | 512 828 | 6.12 | 535 072 | 6.23 | 557 736 | 6.37 |
| 10 | 469 854 | 5.92 | 491 339 | 6.02 | 513 195 | 6.13 | 535 446 | 6.25 | 558 118 | 6.35 |
| 11 | 470 209 | 5.92 | 491 700 | 6.02 | 513 563 | 6.12 | 535 821 | 6.23 | 558 499 | 6.37 |
| 12 | 470 564 | 5.93 | 492 061 | 6.02 | 513 930 | 6.13 | 536 195 | 6.25 | 558 881 | 6.37 |
| 13 | 470 920 | 5.92 | 492 422 | 6.03 | 514 298 | 6.13 | 536 570 | 6.25 | 559 263 | 6.37 |
| 14 | 471 275 | 5.93 | 492 784 | 6.02 | 514 666 | 6.13 | 536 945 | 6.23 | 559 645 | 6.37 |
| 15 | 471 631 | 5.92 | 493 145 | 6.03 | 515 034 | 6.13 | 537 319 | 6.25 | 560 027 | 6.37 |
| 16 | 471 986 | 5.93 | 493 507 | 6.03 | 515 402 | 6.13 | 537 694 | 6.25 | 560 409 | 6.37 |
| 17 | 472 342 | 5.93 | 493 869 | 6.03 | 515 770 | 6.13 | 538 069 | 6.27 | 560 791 | 6.38 |
| 18 | 472 698 | 5.93 | 494 231 | 6.03 | 516 138 | 6.15 | 538 445 | 6.25 | 561 174 | 6.37 |
| 19 | 473 054 | 5.93 | 494 593 | 6.03 | 516 507 | 6.13 | 538 820 | 6.25 | 561 556 | 6.38 |
| 20 | 473 410 | 5.93 | 494 955 | 6.03 | 516 875 | 6.15 | 539 195 | 6.27 | 561 939 | 6.38 |
| 21 | 473 766 | 5.93 | 495 317 | 6.03 | 517 244 | 6.15 | 539 571 | 6.25 | 562 322 | 6.38 |
| 22 | 474 122 | 5.93 | 495 679 | 6.05 | 517 613 | 6.15 | 539 946 | 6.27 | 562 705 | 6.38 |
| 23 | 474 478 | 5.95 | 496 042 | 6.03 | 517 982 | 6.15 | 540 322 | 6.27 | 563 088 | 6.38 |
| 24 | 474 835 | 5.93 | 496 404 | 6.05 | 518 351 | 6.15 | 540 698 | 6.27 | 563 471 | 6.38 |
| 25 | 475 191 | 5.95 | 496 767 | 6.05 | 518 720 | 6.15 | 541 074 | 6.27 | 563 854 | 6.40 |
| 26 | 475 548 | 5.95 | 497 130 | 6.03 | 519 089 | 6.15 | 541 450 | 6.27 | 564 238 | 6.38 |
| 27 | 475 905 | 5.95 | 497 492 | 6.05 | 519 458 | 6.17 | 541 826 | 6.28 | 564 621 | 6.40 |
| 28 | 476 262 | 5.95 | 497 855 | 6.05 | 519 828 | 6.15 | 542 203 | 6.27 | 565 005 | 6.40 |
| 29 | 476 619 | 5.95 | 498 218 | 6.07 | 520 197 | 6.17 | 542 579 | 6.28 | 565 389 | 6.40 |
| 30 | 476 976 | 5.95 | 498 582 | 6.05 | 520 567 | 6.17 | 542 956 | 6.27 | 565 773 | 6.40 |
| 31 | 477 333 | 5.95 | 498 945 | 6.05 | 520 937 | 6.17 | 543 332 | 6.28 | 566 157 | 6.40 |
| 32 | 477 690 | 5.95 | 499 308 | 6.07 | 521 307 | 6.17 | 543 709 | 6.28 | 566 541 | 6.40 |
| 33 | 478 047 | 5.97 | 499 672 | 6.05 | 521 677 | 6.17 | 544 086 | 6.28 | 566 925 | 6.42 |
| 34 | 478 405 | 5.95 | 500 035 | 6.07 | 522 047 | 6.17 | 544 463 | 6.28 | 567 310 | 6.42 |
| 35 | 478 762 | 5.97 | 500 399 | 6.07 | 522 417 | 6.17 | 544 840 | 6.30 | 567 694 | 6.42 |
| 36 | 479 120 | 5.95 | 500 763 | 6.07 | 522 787 | 6.18 | 545 218 | 6.28 | 568 079 | 6.42 |
| 37 | 479 477 | 5.97 | 501 127 | 6.07 | 523 158 | 6.17 | 545 595 | 6.30 | 568 464 | 6.42 |
| 38 | 479 835 | 5.97 | 501 491 | 6.07 | 523 528 | 6.18 | 545 973 | 6.28 | 568 849 | 6.42 |
| 39 | 480 193 | 5.97 | 501 855 | 6.07 | 523 899 | 6.18 | 546 350 | 6.30 | 569 234 | 6.42 |
| 40 | 480 551 | 5.97 | 502 219 | 6.07 | 524 270 | 6.18 | 546 728 | 6.30 | 569 618 | 6.42 |
| 41 | 480 909 | 5.98 | 502 583 | 6.08 | 524 641 | 6.18 | 547 106 | 6.30 | 570 004 | 6.43 |
| 42 | 481 268 | 5.97 | 502 948 | 6.07 | 525 012 | 6.18 | 547 484 | 6.30 | 570 390 | 6.43 |
| 43 | 481 626 | 5.97 | 503 312 | 6.08 | 525 383 | 6.18 | 547 862 | 6.30 | 570 776 | 6.42 |
| 44 | 481 984 | 5.98 | 503 677 | 6.07 | 525 754 | 6.18 | 548 240 | 6.32 | 571 161 | 6.43 |
| 45 | 482 343 | 5.97 | 504 041 | 6.08 | 526 125 | 6.20 | 548 619 | 6.30 | 571 547 | 6.43 |
| 46 | 482 701 | 5.98 | 504 406 | 6.08 | 526 497 | 6.18 | 548 997 | 6.32 | 571 933 | 6.43 |
| 47 | 483 060 | 5.98 | 504 771 | 6.08 | 526 868 | 6.20 | 549 376 | 6.32 | 572 319 | 6.43 |
| 48 | 483 419 | 5.98 | 505 136 | 6.10 | 527 240 | 6.20 | 549 755 | 6.32 | 572 705 | 6.45 |
| 49 | 483 778 | 5.98 | 505 502 | 6.08 | 527 612 | 6.20 | 550 134 | 6.32 | 573 092 | 6.45 |
| 50 | 484 137 | 5.98 | 505 867 | 6.08 | 527 984 | 6.20 | 550 513 | 6.32 | 573 478 | 6.45 |
| 51 | 484 496 | 5.98 | 506 232 | 6.10 | 528 356 | 6.20 | 550 892 | 6.32 | 573 865 | 6.45 |
| 52 | 484 855 | 5.98 | 506 598 | 6.08 | 528 728 | 6.20 | 551 271 | 6.32 | 574 252 | 6.45 |
| 53 | 485 214 | 6.00 | 506 963 | 6.10 | 529 100 | 6.22 | 551 650 | 6.33 | 574 639 | 6.45 |
| 54 | 485 574 | 5.98 | 507 329 | 6.10 | 529 473 | 6.20 | 552 030 | 6.32 | 575 026 | 6.45 |
| 55 | 485 933 | 6.00 | 507 695 | 6.10 | 529 845 | 6.22 | 552 409 | 6.33 | 575 413 | 6.45 |
| 56 | 486 293 | 6.00 | 508 061 | 6.10 | 530 218 | 6.22 | 552 789 | 6.33 | 575 800 | 6.45 |
| 57 | 486 653 | 6.00 | 508 427 | 6.10 | 530 591 | 6.20 | 553 169 | 6.33 | 576 187 | 6.47 |
| 58 | 487 013 | 6.00 | 508 793 | 6.10 | 530 963 | 6.22 | 553 549 | 6.33 | 576 575 | 6.45 |
| 59 | 487 373 | 6.00 | 509 159 | 6.10 | 531 336 | 6.22 | 553 929 | 6.33 | 576 962 | 6.47 |
| 60 | 487 733 | | 509 525 | | 531 709 | | 554 309 | | 577 350 | |

## NATURAL TANGENTS

| | 30° | Δ per sec. | 31° | Δ per sec. | 32° | Δ per sec. | 33° | Δ per sec. | 34° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .577 350 | 6.47 | .600 861 | 6.60 | .624 869 | 6.75 | .649 408 | 6.88 | .674 509 | 7.05 |
| 1 | 577 738 | 6.47 | 601 257 | 6.60 | 625 274 | 6.75 | 649 821 | 6.90 | 674 932 | 7.05 |
| 2 | 578 126 | 6.47 | 601 653 | 6.60 | 625 679 | 6.73 | 650 235 | 6.90 | 675 355 | 7.07 |
| 3 | 578 514 | 6.48 | 602 049 | 6.60 | 626 083 | 6.75 | 650 649 | 6.90 | 675 779 | 7.07 |
| 4 | 578 903 | 6.47 | 602 445 | 6.62 | 626 488 | 6.77 | 651 063 | 6.90 | 676 203 | 7.07 |
| 5 | 579 291 | 6.48 | 602 842 | 6.62 | 626 894 | 6.75 | 651 477 | 6.92 | 676 627 | 7.07 |
| 6 | 579 680 | 6.47 | 603 239 | 6.60 | 627 299 | 6.75 | 651 892 | 6.90 | 677 051 | 7.07 |
| 7 | 580 068 | 6.48 | 603 635 | 6.62 | 627 704 | 6.77 | 652 306 | 6.92 | 677 475 | 7.08 |
| 8 | 580 457 | 6.48 | 604 032 | 6.62 | 628 110 | 6.77 | 652 721 | 6.92 | 677 900 | 7.07 |
| 9 | 580 846 | 6.48 | 604 429 | 6.63 | 628 516 | 6.75 | 653 136 | 6.92 | 678 324 | 7.08 |
| 10 | 581 235 | 6.50 | 604 827 | 6.62 | 628 921 | 6.77 | 653 551 | 6.92 | 678 749 | 7.08 |
| 11 | 581 625 | 6.48 | 605 224 | 6.63 | 629 327 | 6.78 | 653 966 | 6.93 | 679 174 | 7.08 |
| 12 | 582 014 | 6.48 | 605 622 | 6.62 | 629 734 | 6.77 | 654 382 | 6.92 | 679 599 | 7.10 |
| 13 | 582 403 | 6.50 | 606 019 | 6.63 | 630 140 | 6.77 | 654 797 | 6.93 | 680 025 | 7.08 |
| 14 | 582 793 | 6.50 | 606 417 | 6.63 | 630 546 | 6.78 | 655 213 | 6.93 | 680 450 | 7.10 |
| 15 | 583 183 | 6.50 | 606 815 | 6.63 | 630 953 | 6.78 | 655 629 | 6.93 | 680 876 | 7.10 |
| 16 | 583 573 | 6.50 | 607 213 | 6.63 | 631 360 | 6.78 | 656 045 | 6.93 | 681 302 | 7.10 |
| 17 | 583 963 | 6.50 | 607 611 | 6.65 | 631 767 | 6.78 | 656 461 | 6.93 | 681 728 | 7.10 |
| 18 | 584 353 | 6.50 | 608 010 | 6.63 | 632 174 | 6.78 | 656 877 | 6.95 | 682 154 | 7.10 |
| 19 | 584 743 | 6.52 | 608 408 | 6.65 | 632 581 | 6.78 | 657 294 | 6.93 | 682 580 | 7.12 |
| 20 | 585 134 | 6.50 | 608 807 | 6.63 | 632 988 | 6.80 | 657 710 | 6.95 | 683 007 | 7.10 |
| 21 | 585 524 | 6.52 | 609 205 | 6.65 | 633 396 | 6.80 | 658 127 | 6.95 | 683 433 | 7.12 |
| 22 | 585 915 | 6.52 | 609 604 | 6.65 | 633 804 | 6.78 | 658 544 | 6.95 | 683 860 | 7.12 |
| 23 | 586 306 | 6.52 | 610 003 | 6.67 | 634 211 | 6.80 | 658 961 | 6.97 | 684 287 | 7.12 |
| 24 | 586 697 | 6.52 | 610 403 | 6.65 | 634 619 | 6.80 | 659 379 | 6.95 | 684 714 | 7.13 |
| 25 | 587 088 | 6.52 | 610 802 | 6.65 | 635 027 | 6.82 | 659 796 | 6.97 | 685 142 | 7.12 |
| 26 | 587 479 | 6.52 | 611 201 | 6.67 | 635 436 | 6.80 | 660 214 | 6.95 | 685 569 | 7.13 |
| 27 | 587 870 | 6.53 | 611 601 | 6.67 | 635 844 | 6.82 | 660 631 | 6.97 | 685 997 | 7.13 |
| 28 | 588 262 | 6.52 | 612 001 | 6.67 | 636 253 | 6.80 | 661 049 | 6.97 | 686 425 | 7.13 |
| 29 | 588 653 | 6.53 | 612 401 | 6.67 | 636 661 | 6.82 | 661 467 | 6.98 | 686 853 | 7.13 |
| 30 | 589 045 | 6.53 | 612 801 | 6.67 | 637 070 | 6.82 | 661 886 | 6.97 | 687 281 | 7.13 |
| 31 | 589 437 | 6.53 | 613 201 | 6.67 | 637 479 | 6.82 | 662 304 | 6.98 | 687 709 | 7.15 |
| 32 | 589 829 | 6.53 | 613 601 | 6.68 | 637 888 | 6.83 | 662 723 | 6.97 | 688 138 | 7.15 |
| 33 | 590 221 | 6.53 | 614 002 | 6.67 | 638 298 | 6.82 | 663 141 | 6.98 | 688 567 | 7.13 |
| 34 | 590 613 | 6.55 | 614 402 | 6.68 | 638 707 | 6.83 | 663 560 | 6.98 | 688 995 | 7.17 |
| 35 | 591 006 | 6.53 | 614 803 | 6.68 | 639 117 | 6.83 | 663 979 | 6.98 | 689 425 | 7.15 |
| 36 | 591 398 | 6.55 | 615 204 | 6.68 | 639 527 | 6.83 | 664 398 | 7.00 | 689 854 | 7.15 |
| 37 | 591 791 | 6.55 | 615 605 | 6.68 | 639 937 | 6.83 | 664 818 | 6.98 | 690 283 | 7.17 |
| 38 | 592 184 | 6.55 | 616 006 | 6.70 | 640 347 | 6.83 | 665 237 | 7.00 | 690 713 | 7.17 |
| 39 | 592 577 | 6.55 | 616 408 | 6.68 | 640 757 | 6.83 | 665 657 | 7.00 | 691 143 | 7.15 |
| 40 | 592 970 | 6.55 | 616 809 | 6.70 | 641 167 | 6.85 | 666 077 | 7.00 | 691 572 | 7.18 |
| 41 | 593 363 | 6.57 | 617 211 | 6.70 | 641 578 | 6.85 | 666 497 | 7.00 | 692 003 | 7.17 |
| 42 | 593 757 | 6.55 | 617 613 | 6.70 | 641 989 | 6.83 | 666 917 | 7.00 | 692 433 | 7.17 |
| 43 | 594 150 | 6.57 | 618 015 | 6.70 | 642 399 | 6.85 | 667 337 | 7.02 | 692 863 | 7.18 |
| 44 | 594 544 | 6.55 | 618 417 | 6.70 | 642 810 | 6.87 | 667 758 | 7.02 | 693 294 | 7.18 |
| 45 | 594 937 | 6.57 | 618 819 | 6.70 | 643 222 | 6.85 | 668 179 | 7.00 | 693 725 | 7.18 |
| 46 | 595 331 | 6.57 | 619 221 | 6.72 | 643 633 | 6.85 | 668 599 | 7.02 | 694 156 | 7.18 |
| 47 | 595 725 | 6.58 | 619 624 | 6.70 | 644 044 | 6.87 | 669 020 | 7.03 | 694 587 | 7.18 |
| 48 | 596 120 | 6.57 | 620 026 | 6.72 | 644 456 | 6.87 | 669 442 | 7.02 | 695 018 | 7.20 |
| 49 | 596 514 | 6.57 | 620 429 | 6.72 | 644 868 | 6.87 | 669 863 | 7.02 | 695 450 | 7.18 |
| 50 | 596 908 | 6.58 | 620 832 | 6.72 | 645 280 | 6.87 | 670 284 | 7.03 | 695 881 | 7.20 |
| 51 | 597 303 | 6.58 | 621 235 | 6.72 | 645 692 | 6.87 | 670 706 | 7.03 | 696 313 | 7.20 |
| 52 | 597 698 | 6.58 | 621 638 | 6.73 | 646 104 | 6.87 | 671 128 | 7.03 | 696 745 | 7.20 |
| 53 | 598 093 | 6.58 | 622 042 | 6.72 | 646 516 | 6.88 | 671 550 | 7.03 | 697 177 | 7.22 |
| 54 | 598 488 | 6.58 | 622 445 | 6.73 | 646 929 | 6.88 | 671 972 | 7.03 | 697 610 | 7.20 |
| 55 | 598 883 | 6.58 | 622 849 | 6.73 | 647 342 | 6.88 | 672 394 | 7.05 | 698 042 | 7.22 |
| 56 | 599 278 | 6.60 | 623 253 | 6.73 | 647 755 | 6.88 | 672 817 | 7.05 | 698 475 | 7.22 |
| 57 | 599 674 | 6.58 | 623 657 | 6.73 | 648 168 | 6.88 | 673 240 | 7.03 | 698 908 | 7.22 |
| 58 | 600 069 | 6.60 | 624 061 | 6.73 | 648 581 | 6.88 | 673 662 | 7.05 | 699 341 | 7.22 |
| 59 | 600 465 | 6.60 | 624 465 | 6.73 | 648 994 | 6.90 | 674 085 | 7.07 | 699 774 | 7.23 |
| 60 | 600 861 | | 624 869 | | 649 408 | | 674 509 | | 700 208 | |

## NATURAL TANGENTS

| ′ | 35° | Δ per sec. | 36° | Δ per sec. | 37° | Δ per sec. | 38° | Δ per sec. | 39° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0′ | .700 208 | 7.22 | .726 543 | 7.40 | .753 554 | 7.60 | .781 286 | 7.80 | .809 784 | 8.03 |
| 1 | 700 641 | 7.23 | 726 987 | 7.42 | 754 010 | 7.62 | 781 754 | 7.82 | 810 266 | 8.03 |
| 2 | 701 075 | 7.23 | 727 432 | 7.42 | 754 467 | 7.60 | 782 223 | 7.82 | 810 748 | 8.03 |
| 3 | 701 509 | 7.23 | 727 877 | 7.42 | 754 923 | 7.62 | 782 692 | 7.82 | 811 230 | 8.03 |
| 4 | 701 943 | 7.23 | 728 322 | 7.42 | 755 380 | 7.62 | 783 161 | 7.83 | 811 712 | 8.05 |
| 5 | 702 377 | 7.25 | 728 767 | 7.43 | 755 837 | 7.62 | 783 631 | 7.82 | 812 195 | 8.05 |
| 6 | 702 812 | 7.23 | 729 213 | 7.42 | 756 294 | 7.62 | 784 100 | 7.83 | 812 678 | 8.05 |
| 7 | 703 246 | 7.25 | 729 658 | 7.43 | 756 751 | 7.63 | 784 570 | 7.83 | 813 161 | 8.05 |
| 8 | 703 681 | 7.25 | 730 104 | 7.43 | 757 209 | 7.63 | 785 040 | 7.83 | 813 644 | 8.07 |
| 9 | 704 116 | 7.25 | 730 550 | 7.43 | 757 667 | 7.63 | 785 510 | 7.85 | 814 128 | 8.07 |
| 10 | 704 551 | 7.27 | 730 996 | 7.45 | 758 125 | 7.63 | 785 981 | 7.83 | 814 612 | 8.07 |
| 11 | 704 987 | 7.25 | 731 443 | 7.43 | 758 583 | 7.63 | 786 451 | 7.85 | 815 096 | 8.07 |
| 12 | 705 422 | 7.27 | 731 889 | 7.45 | 759 041 | 7.65 | 786 922 | 7.87 | 815 580 | 8.08 |
| 13 | 705 858 | 7.27 | 732 336 | 7.45 | 759 500 | 7.65 | 787 394 | 7.85 | 816 065 | 8.07 |
| 14 | 706 294 | 7.27 | 732 783 | 7.45 | 759 959 | 7.65 | 787 865 | 7.85 | 816 549 | 8.08 |
| 15 | 706 730 | 7.27 | 733 230 | 7.47 | 760 418 | 7.65 | 788 336 | 7.87 | 817 034 | 8.08 |
| 16 | 707 166 | 7.28 | 733 678 | 7.45 | 760 877 | 7.65 | 788 808 | 7.87 | 817 519 | 8.10 |
| 17 | 707 603 | 7.27 | 734 125 | 7.47 | 761 336 | 7.67 | 789 280 | 7.87 | 818 005 | 8.10 |
| 18 | 708 039 | 7.28 | 734 573 | 7.47 | 761 796 | 7.67 | 789 752 | 7.88 | 818 491 | 8.08 |
| 19 | 708 476 | 7.28 | 735 021 | 7.47 | 762 256 | 7.67 | 790 225 | 7.87 | 818 976 | 8.10 |
| 20 | 708 913 | 7.28 | 735 469 | 7.47 | 762 716 | 7.67 | 790 697 | 7.88 | 819 462 | 8.12 |
| 21 | 709 350 | 7.30 | 735 917 | 7.48 | 763 176 | 7.67 | 791 170 | 7.88 | 819 949 | 8.10 |
| 22 | 709 788 | 7.28 | 736 366 | 7.48 | 763 636 | 7.68 | 791 643 | 7.90 | 820 435 | 8.12 |
| 23 | 710 225 | 7.30 | 736 815 | 7.48 | 764 097 | 7.68 | 792 117 | 7.88 | 820 922 | 8.12 |
| 24 | 710 663 | 7.30 | 737 264 | 7.48 | 764 558 | 7.68 | 792 590 | 7.90 | 821 409 | 8.13 |
| 25 | 711 101 | 7.30 | 737 713 | 7.48 | 765 019 | 7.68 | 793 064 | 7.90 | 821 897 | 8.12 |
| 26 | 711 539 | 7.30 | 738 162 | 7.48 | 765 480 | 7.68 | 793 538 | 7.90 | 822 384 | 8.13 |
| 27 | 711 977 | 7.32 | 738 611 | 7.50 | 765 941 | 7.70 | 794 012 | 7.90 | 822 872 | 8.13 |
| 28 | 712 416 | 7.30 | 739 061 | 7.50 | 766 403 | 7.70 | 794 486 | 7.92 | 823 360 | 8.13 |
| 29 | 712 854 | 7.32 | 739 511 | 7.50 | 766 865 | 7.70 | 794 961 | 7.92 | 823 848 | 8.13 |
| 30 | 713 293 | 7.32 | 739 961 | 7.50 | 767 327 | 7.70 | 795 436 | 7.92 | 824 336 | 8.15 |
| 31 | 713 732 | 7.32 | 740 411 | 7.52 | 767 789 | 7.72 | 795 911 | 7.92 | 824 825 | 8.15 |
| 32 | 714 171 | 7.33 | 740 862 | 7.50 | 768 252 | 7.70 | 796 386 | 7.93 | 825 314 | 8.15 |
| 33 | 714 611 | 7.32 | 741 312 | 7.52 | 768 714 | 7.72 | 796 862 | 7.92 | 825 803 | 8.15 |
| 34 | 715 050 | 7.33 | 741 763 | 7.52 | 769 177 | 7.72 | 797 337 | 7.93 | 826 292 | 8.17 |
| 35 | 715 490 | 7.33 | 742 214 | 7.53 | 769 640 | 7.73 | 797 813 | 7.95 | 826 782 | 8.17 |
| 36 | 715 930 | 7.33 | 742 666 | 7.52 | 770 104 | 7.72 | 798 290 | 7.93 | 827 272 | 8.17 |
| 37 | 716 370 | 7.33 | 743 117 | 7.53 | 770 567 | 7.73 | 798 766 | 7.93 | 827 762 | 8.17 |
| 38 | 716 810 | 7.33 | 743 569 | 7.52 | 771 031 | 7.73 | 799 242 | 7.95 | 828 252 | 8.18 |
| 39 | 717 250 | 7.35 | 744 020 | 7.53 | 771 495 | 7.73 | 799 719 | 7.95 | 828 743 | 8.18 |
| 40 | 717 691 | 7.35 | 744 472 | 7.55 | 771 959 | 7.73 | 800 196 | 7.97 | 829 234 | 8.18 |
| 41 | 718 132 | 7.35 | 744 925 | 7.53 | 772 423 | 7.75 | 800 674 | 7.95 | 829 725 | 8.18 |
| 42 | 718 573 | 7.35 | 745 377 | 7.55 | 772 888 | 7.75 | 801 151 | 7.97 | 830 216 | 8.20 |
| 43 | 719 014 | 7.35 | 745 830 | 7.53 | 773 353 | 7.75 | 801 629 | 7.97 | 830 708 | 8.18 |
| 44 | 719 455 | 7.37 | 746 282 | 7.55 | 773 818 | 7.75 | 802 107 | 7.97 | 831 199 | 8.20 |
| 45 | 719 897 | 7.37 | 746 735 | 7.57 | 774 283 | 7.75 | 802 585 | 7.97 | 831 691 | 8.20 |
| 46 | 720 339 | 7.37 | 747 189 | 7.55 | 774 748 | 7.77 | 803 063 | 7.98 | 832 183 | 8.22 |
| 47 | 720 781 | 7.37 | 747 642 | 7.57 | 775 214 | 7.77 | 803 542 | 7.98 | 832 676 | 8.22 |
| 48 | 721 223 | 7.37 | 748 096 | 7.55 | 775 680 | 7.77 | 804 021 | 7.98 | 833 169 | 8.22 |
| 49 | 721 665 | 7.38 | 748 549 | 7.57 | 776 146 | 7.77 | 804 500 | 7.98 | 833 662 | 8.22 |
| 50 | 722 108 | 7.37 | 749 003 | 7.58 | 776 612 | 7.77 | 804 979 | 7.98 | 834 155 | 8.22 |
| 51 | 722 550 | 7.38 | 749 458 | 7.57 | 777 078 | 7.78 | 805 458 | 8.00 | 834 648 | 8.23 |
| 52 | 722 993 | 7.38 | 749 912 | 7.57 | 777 545 | 7.78 | 805 938 | 8.00 | 835 142 | 8.23 |
| 53 | 723 436 | 7.38 | 750 366 | 7.58 | 778 012 | 7.78 | 806 418 | 8.00 | 835 636 | 8.23 |
| 54 | 723 879 | 7.40 | 750 821 | 7.58 | 778 479 | 7.78 | 806 898 | 8.02 | 836 130 | 8.23 |
| 55 | 724 323 | 7.38 | 751 276 | 7.58 | 778 946 | 7.80 | 807 379 | 8.00 | 836 624 | 8.25 |
| 56 | 724 766 | 7.40 | 751 731 | 7.60 | 779 414 | 7.78 | 807 859 | 8.02 | 837 119 | 8.25 |
| 57 | 725 210 | 7.40 | 752 187 | 7.58 | 779 881 | 7.80 | 808 340 | 8.02 | 837 614 | 8.25 |
| 58 | 725 654 | 7.40 | 752 642 | 7.60 | 780 349 | 7.80 | 808 821 | 8.03 | 838 109 | 8.25 |
| 59 | 726 098 | 7.42 | 753 098 | 7.60 | 780 817 | 7.82 | 809 303 | 8.02 | 838 604 | 8.27 |
| 60 | 726 543 | | 753 554 | | 781 286 | | 809 784 | | 839 100 | |

## NATURAL TANGENTS

| | 40° | Δ per sec. | 41° | Δ per sec. | 42° | Δ per sec. | 43° | Δ per sec. | 44° | Δ per sec. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .839 100 | 8.25 | .869 287 | 8.52 | .900 404 | 8.78 | .932 515 | 9.07 | .965 689 | 9.37 |
| 1 | 839 595 | 8.28 | 869 798 | 8.52 | 900 931 | 8.78 | 933 059 | 9.07 | 966 251 | 9.38 |
| 2 | 840 092 | 8.27 | 870 309 | 8.52 | 901 458 | 8.78 | 933 603 | 9.08 | 966 814 | 9.38 |
| 3 | 840 588 | 8.27 | 870 820 | 8.53 | 901 985 | 8.80 | 934 148 | 9.08 | 967 377 | 9.38 |
| 4 | 841 084 | 8.28 | 871 332 | 8.52 | 902 513 | 8.80 | 934 693 | 9.08 | 967 940 | 9.40 |
| 5 | 841 581 | 8.28 | 871 843 | 8.55 | 903 041 | 8.80 | 935 238 | 9.08 | 968 504 | 9.38 |
| 6 | 842 078 | 8.28 | 872 356 | 8.53 | 903 569 | 8.82 | 935 783 | 9.10 | 969 067 | 9.42 |
| 7 | 842 575 | 8.30 | 872 868 | 8.55 | 904 098 | 8.82 | 936 329 | 9.10 | 969 632 | 9.40 |
| 8 | 843 073 | 8.30 | 873 381 | 8.55 | 904 627 | 8.82 | 936 875 | 9.12 | 970 196 | 9.42 |
| 9 | 843 571 | 8.30 | 873 894 | 8.55 | 905 156 | 8.82 | 937 422 | 9.10 | 970 761 | 9.42 |
| 10 | 844 069 | 8.30 | 874 407 | 8.55 | 905 685 | 8.83 | 937 968 | 9.12 | 971 326 | 9.43 |
| 11 | 844 567 | 8.32 | 874 920 | 8.57 | 906 215 | 8.83 | 938 515 | 9.12 | 971 892 | 9.43 |
| 12 | 845 066 | 8.30 | 875 434 | 8.57 | 906 745 | 8.83 | 939 062 | 9.13 | 972 458 | 9.43 |
| 13 | 845 564 | 8.32 | 875 948 | 8.57 | 907 275 | 8.83 | 939 610 | 9.13 | 973 024 | 9.43 |
| 14 | 846 063 | 8.32 | 876 462 | 8.57 | 907 805 | 8.85 | 940 158 | 9.13 | 973 590 | 9.45 |
| 15 | 846 562 | 8.33 | 876 976 | 8.58 | 908 336 | 8.85 | 940 706 | 9.15 | 974 157 | 9.45 |
| 16 | 847 062 | 8.33 | 877 491 | 8.58 | 908 867 | 8.85 | 941 255 | 9.13 | 974 724 | 9.45 |
| 17 | 847 562 | 8.33 | 878 006 | 8.58 | 909 398 | 8.87 | 941 803 | 9.15 | 975 291 | 9.47 |
| 18 | 848 062 | 8.33 | 878 521 | 8.60 | 909 930 | 8.87 | 942 352 | 9.17 | 975 859 | 9.47 |
| 19 | 848 562 | 8.33 | 879 037 | 8.60 | 910 462 | 8.87 | 942 902 | 9.15 | 976 427 | 9.48 |
| 20 | 849 062 | 8.35 | 879 553 | 8.60 | 910 994 | 8.87 | 943 451 | 9.17 | 976 996 | 9.47 |
| 21 | 849 563 | 8.35 | 880 069 | 8.60 | 911 526 | 8.88 | 944 001 | 9.18 | 977 564 | 9.48 |
| 22 | 850 064 | 8.35 | 880 585 | 8.62 | 912 059 | 8.88 | 944 552 | 9.17 | 978 133 | 9.50 |
| 23 | 850 565 | 8.37 | 881 102 | 8.62 | 912 592 | 8.88 | 945 102 | 9.18 | 978 703 | 9.48 |
| 24 | 851 067 | 8.35 | 881 619 | 8.62 | 913 125 | 8.90 | 945 653 | 9.18 | 979 272 | 9.50 |
| 25 | 851 568 | 8.37 | 882 136 | 8.62 | 913 659 | 8.90 | 946 204 | 9.20 | 979 842 | 9.52 |
| 26 | 852 070 | 8.38 | 882 653 | 8.63 | 914 193 | 8.90 | 946 756 | 9.18 | 980 413 | 9.50 |
| 27 | 852 573 | 8.37 | 883 171 | 8.63 | 914 727 | 8.90 | 947 307 | 9.20 | 980 983 | 9.52 |
| 28 | 853 075 | 8.38 | 883 689 | 8.63 | 915 261 | 8.92 | 947 859 | 9.22 | 981 554 | 9.53 |
| 29 | 853 578 | 8.38 | 884 207 | 8.63 | 915 796 | 8.92 | 948 412 | 9.22 | 982 126 | 9.52 |
| 30 | 854 081 | 8.38 | 884 725 | 8.65 | 916 331 | 8.92 | 948 965 | 9.22 | 982 697 | 9.53 |
| 31 | 854 584 | 8.38 | 885 244 | 8.65 | 916 866 | 8.93 | 949 518 | 9.22 | 983 269 | 9.55 |
| 32 | 855 087 | 8.40 | 885 763 | 8.65 | 917 402 | 8.93 | 950 071 | 9.22 | 983 842 | 9.53 |
| 33 | 855 591 | 8.40 | 886 282 | 8.67 | 917 938 | 8.93 | 950 624 | 9.23 | 984 414 | 9.55 |
| 34 | 856 095 | 8.40 | 886 802 | 8.67 | 918 474 | 8.93 | 951 178 | 9.25 | 984 987 | 9.55 |
| 35 | 856 599 | 8.42 | 887 322 | 8.67 | 919 010 | 8.95 | 951 733 | 9.23 | 985 560 | 9.57 |
| 36 | 857 104 | 8.40 | 887 842 | 8.67 | 919 547 | 8.95 | 952 287 | 9.25 | 986 134 | 9.57 |
| 37 | 857 608 | 8.42 | 888 362 | 8.67 | 920 084 | 8.95 | 952 842 | 9.25 | 986 708 | 9.57 |
| 38 | 858 113 | 8.43 | 888 882 | 8.68 | 920 621 | 8.97 | 953 397 | 9.27 | 987 282 | 9.58 |
| 39 | 858 619 | 8.42 | 889 403 | 8.68 | 921 159 | 8.97 | 953 953 | 9.25 | 987 857 | 9.58 |
| 40 | 859 124 | 8.43 | 889 924 | 8.70 | 921 697 | 8.97 | 954 508 | 9.27 | 988 432 | 9.58 |
| 41 | 859 630 | 8.43 | 890 446 | 8.68 | 922 235 | 8.97 | 955 064 | 9.28 | 989 007 | 9.58 |
| 42 | 860 136 | 8.43 | 890 967 | 8.70 | 922 773 | 8.98 | 955 621 | 9.27 | 989 582 | 9.60 |
| 43 | 860 642 | 8.43 | 891 489 | 8.72 | 923 312 | 8.98 | 956 177 | 9.28 | 990 158 | 9.62 |
| 44 | 861 148 | 8.45 | 892 012 | 8.70 | 923 851 | 8.98 | 956 734 | 9.30 | 990 735 | 9.60 |
| 45 | 861 655 | 8.45 | 892 534 | 8.72 | 924 390 | 9.00 | 957 292 | 9.28 | 991 311 | 9.62 |
| 46 | 862 162 | 8.45 | 893 057 | 8.72 | 924 930 | 9.00 | 957 849 | 9.30 | 991 888 | 9.62 |
| 47 | 862 669 | 8.47 | 893 580 | 8.72 | 925 470 | 9.00 | 958 407 | 9.32 | 992 465 | 9.63 |
| 48 | 863 177 | 8.47 | 894 103 | 8.73 | 926 010 | 9.02 | 958 966 | 9.30 | 993 043 | 9.63 |
| 49 | 863 685 | 8.47 | 894 627 | 8.73 | 926 551 | 9.00 | 959 524 | 9.32 | 993 621 | 9.63 |
| 50 | 864 193 | 8.47 | 895 151 | 8.73 | 927 091 | 9.02 | 960 083 | 9.32 | 994 199 | 9.65 |
| 51 | 864 701 | 8.47 | 895 675 | 8.73 | 927 632 | 9.03 | 960 642 | 9.33 | 994 778 | 9.65 |
| 52 | 865 209 | 8.48 | 896 199 | 8.75 | 928 174 | 9.02 | 961 202 | 9.32 | 995 357 | 9.65 |
| 53 | 865 718 | 8.48 | 896 724 | 8.75 | 928 715 | 9.03 | 961 761 | 9.35 | 995 936 | 9.65 |
| 54 | 866 227 | 8.48 | 897 249 | 8.75 | 929 257 | 9.05 | 962 322 | 9.33 | 996 515 | 9.67 |
| 55 | 866 736 | 8.50 | 897 774 | 8.75 | 929 800 | 9.03 | 962 882 | 9.35 | 997 095 | 9.68 |
| 56 | 867 246 | 8.50 | 898 299 | 8.77 | 930 342 | 9.05 | 963 443 | 9.35 | 997 676 | 9.67 |
| 57 | 867 756 | 8.50 | 898 825 | 8.77 | 930 885 | 9.05 | 964 004 | 9.35 | 998 256 | 9.68 |
| 58 | 868 266 | 8.50 | 899 351 | 8.77 | 931 428 | 9.05 | 964 565 | 9.37 | 998 837 | 9.68 |
| 59 | 868 776 | 8.52 | 899 877 | 8.78 | 931 971 | 9.07 | 965 127 | 9.37 | 999 418 | 9.70 |
| 60 | 869 287 | | 900 404 | | 932 515 | | 965 689 | | 1.000 000 | |