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Aerotriangulation Adjustment of Instrument Data by Computational Methods

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Technical Bulletin Series

This series of Technical Bulletins is inaugurated to present to the personnel of the Coast and Geodetic Survey and to others the results of research and development in the various fields of the Bureau's activities. It is designed for quick publication and dissemination and supersedes The Journal of the Coast and Geodetic Survey, the last issue of which was published in October 1957. Since many of the bulletins will deal with new practices and new techniques, the views expressed will be those of the author and will not necessarily represent final Bureau policy.

Technical Bulletin No. 1 is a result of the cooperative effort of several units and individuals of the Photogrammetry Division in seeking a computational method of aerotriangulation adjustment, which could be coded for automatic computation, to replace the burdensome graphical-numerical method.

Aerotriangulation Adjustment of Instrument Data by Computational Methods

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A NONLINEAR propagation of residual errors in the process of aerotriangulation with stereoscopic plotting instruments, results in relatively large errors in the center of a spanned distance, when a linear transformation is used to adjust the instrument coordinates to geodetic control at the ends of the strip. Through analysis of the effects of small residual errors in relative orientation, it can be shown that this nonlinear propagation of error follows second-degree curves in the x - y and x - z planes of a spacial rectangular coordinate system, and a third-degree curve for scale correction along the axis of flight. In addition to these three primary error curves, each of the three error components contains linear secondary effects from the other two primary curves.

The resultant three-dimensional errors of instrument positions could probably be determined directly by a complex spacial analysis, but the reasons for separating the horizontal and vertical adjustments are:

1. The horizontal and vertical control stations are normally geographically separated.
2. The previously mentioned secondary effects from the primary vertical error curve are negligible, because the width of the strip is from 10 to 100 times the differences in terrain height.
3. Vertical bridging is seldom considered feasible, because the vertical accuracy requirements are usually higher than the horizontal accuracy requirements, while the geometry of aerotriangulation makes the vertical accuracy lower.
4. The initial cost of developing and programming a simultaneous adjustment for the electronic computer would be much greater.

The Coast and Geodetic Survey has been using a semigraphic method of adjustment which is a modification of the method developed by the Army Map Service.[1]

A numerical method of adjustment has now been developed to replace and improve upon the semigraphic method, in which the nonlinear errors of aerotriangulation were determined by bending a spline to simulate the errors at three or more horizontal control points. The numerical method replaces the curved spline by its equation, recognizing the "scale" curve to be third degree and the "azimuth" curve to be second degree.

The numerical method has been tested and will soon be programmed for the IBM-650 electronic computer. The computer input data will consist of the geodetic plane coordinate positions of control points and the plotting instrument (x) and (y) coordinates of control and model pass points. When necessary, the plotting instrument coordinates will be adjusted in advance by desk calculator for the effects of *BZ-falloff* on elevated points. The computer output will be the final adjusted geodetic plane coordinate positions of all model pass points and residual errors of position at all control points, based upon a least-squares adjustment of their magnitudes. Actually, the residual errors at control points will be displayed by the computer when they are determined, so that the operator can eliminate a bad control station and redetermine the equations of the correction curves before the final positions of the model pass points are computed.

In the numerical method, it is first necessary to transform the plotting instrument coordinates and the geodetic plane coordinates of the ground control stations into the axis-of-flight system from which the coefficients of correction for the photogrammetric points are computed and finally their geodetic plane coordinates determined.

In devising the method described, the Geodesy Division cooperated with the Photogrammetry Division in solving sets of test simultaneous equations and in programming and coding the complete solution.

[1] Aerotriangulation Adjustment by G. C. Tewinkel in *Photogrammetric Engineering*, April 1957 and *The Journal*, Coast and Geodetic Survey, October 1957.

TRANSFORMATION OF INSTRUMENT COORDINATES

The plotting instrument coordinate axes system is only approximately parallel to the axis of flight, because it is aligned with the line-of-centers in the first model of the strip. The origin of the instrument coordinate system is so placed that the entire strip will lie in the first quadrant, and all coordinates will have positive values. On the other hand, the errors are propagated along the axis of flight and are symmetrical with respect to it.

A universal transformation was chosen to permit full freedom in labeling the instrument coordinate axes regardless of the relationship between the direction of flight and the geodetic plane coordinate system. At the same time the axis-of-flight will always be the x-axis, a necessary condition for complete automation of the curve-fitting procedure. The computation program is also simplified by placing the origin of the axis-of-flight system at the center of the flight line.

Let (x, y) be the original instrument coordinates of a point and (x', y') its coordinates in the new axis-of-flight system. The axis-of-flight is established so that the (y') coordinates of the centers of the terminal photographs are both zero and the (x') coordinates are equal in value and opposite in sign. That is

$$x_1' = -\frac{\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]^{\frac{1}{2}}}{2} \tag{1}$$

$$x_2' = -x_1' \tag{2}$$

$$y_1' = y_2' = 0 \tag{3}$$

(Note: Subscript (1) designates the initial photograph center, and (2) the terminal photograph center.)

Using auxiliaries for the frequently used (x) and (y) differences,

$$\Delta x' = x_1' - x_2' \tag{4}$$

$$\Delta y' = y_1' - y_2' \tag{5}$$

$$\Delta x = x_1 - x_2 \tag{6}$$

$$\Delta y = y_1 - y_2 \tag{7}$$

then

$$x_1' = -\frac{(\Delta x^2 + \Delta y^2)^{\frac{1}{2}}}{2} \tag{8}$$

The coordinates of a point (a) in the axis-of-flight system are

$$x_a' = ax_a - by_a + c \tag{9}$$

$$y_a' = bx_a + ay_a + d \tag{10}$$

where,

$$a = \frac{\Delta x' \Delta x + \Delta y' \Delta y}{\Delta x^2 + \Delta y^2} \tag{11}$$

$$b = \frac{\Delta x \Delta y' - \Delta y \Delta x'}{\Delta x^2 + \Delta y^2} \tag{12}$$

$$c = x_1' - ax_1 + by_1 \tag{13}$$

$$d = y_1' - bx_1 - ay_1 \tag{14}$$

TRANSFORMATION OF GEODETIC PLANE COORDINATES

The geodetic plane coordinates of the ground control stations are transformed into the axis-of-flight system in order that the effects of the nonlinear errors can be measured. This transformation requires a scale change as well as a rotation and translation of axes. The terminal control points are used for the transformation with the result that the errors at the ends of the strip become zero and the error is greatest at the center of the strip.[2]

The data for the transformation are the coordinates of the two terminal control points in both systems.

Let (X_1, Y_1) and (X_n, Y_n) be the geodetic plane coordinates of the initial and terminal control stations, and (x_1'', y_1'') and (x_n'', y_n'') be the instrument coordinates in the axis-of-flight system. The auxiliaries are then calculated from the following equations:

$$\Delta x'' = x_1'' - x_n'' \tag{15}$$

$$\Delta y'' = y_1'' - y_n'' \tag{16}$$

$$\Delta X = X_1 - X_n \tag{17}$$

$$\Delta Y = Y_1 - Y_n \tag{18}$$

The coordinates of a point (a) in the geodetic plane coordinate system are

$$X_a = ax_a'' - by_a'' + c \tag{19}$$

$$Y_a = bx_a'' + ay_a'' + d \tag{20}$$

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[2] For equations of this transformation and its inverse, see footnote 1, above.

where,

$$a = \frac{\Delta X \Delta x'' + \Delta Y \Delta y''}{(\Delta x'')^2 + (\Delta y'')^2} \quad (21)$$

$$b = \frac{\Delta x'' \Delta Y - \Delta y'' \Delta X}{(\Delta x'')^2 + (\Delta y'')^2} \quad (22)$$

$$c = X_1 - a x_1'' + b y_1'' \quad (23)$$

$$d = Y_1 - b x_1'' - a y_1'' \quad (24)$$

The coordinates of a point (a) in the axis-of-flight system are

$$x_a'' = \frac{a}{a^2 + b^2} X_a + \frac{b}{a^2 + b^2} Y_a - \frac{ac + bd}{a^2 + b^2} \quad (25)$$

$$y_a'' = -\frac{b}{a^2 + b^2} X_a + \frac{a}{a^2 + b^2} Y_a - \frac{ad - bc}{a^2 + b^2} \quad (26)$$

Transformation of the geodetic plane coordinates of the initial and terminal control points (X_1, Y_1 and X_n, Y_n) into the axis-of-flight system gives coordinate values identical with the plotting instrument coordinates of those points because the transformation is so defined. The disparities between the instrument coordinates and the transformed geodetic coordinates at other control stations are due to the nonlinearity in the plotting instrument solution and can be considered as corrections to be applied to the plotting instrument coordinates.

Let c_x represent the correction to be applied to the x' coordinate and c_y the correction for the y' coordinate, then

$$c_x = x'' - x' \quad (27)$$

and

$$c_y = y'' - y' \quad (28)$$

DETERMINATION OF COEFFICIENTS FOR CORRECTION CURVE EQUATIONS

A curve fitting procedure is used for developing the equations of the correction curves. The "x-scale" curve is recognized to be of third degree in x and the "azimuth" curve to be of second degree in x. The general forms of the two primary equations are

$$c_x' = Ax^3 + Bx^2 + Cx + F \quad (29)$$

$$c_y' = Dx^2 + Ex + G \quad (30)$$

In addition to these primary corrections, there are secondary corrections resulting from the product of the y coordinates of the individual points and the rate of change or derivative of the opposite primary curve. The secondary corrections are

$$c_x'' = -d(c_y')y = -(2Dx + E)y \quad (31)$$

$$c_y'' = d(c_x')y = (3Ax^2 + 2Bx + C)y \quad (32)$$

The total corrections are the sum of the primary and secondary corrections and may be written

$$c_x = c_x' + C_x'' = Ax^3 + Bx^2 + Cx + D(-2xy) + E(-y) + F \quad (33)$$

$$c_y = c_y' + c_y'' = A(3x^2y) + B(2xy) + Cy + Dx^2 + Ex + G \quad (34)$$

If the coefficients A through G are regarded as unknowns, two equations can be written for each control point for which the corrections c_x and c_y are known. A minimum of 4 control points and 8 equations are required for the determination of the 7 coefficients A through G. The solution of these equations is an established IBM-650 subroutine which makes a least-squares minimization of the errors in the absolute terms of the individual equations which, in this case, are the errors of c_x and c_y. If there are more than the necessary 4 control points in the strip, there will simply be more condition equations in the least-squares adjustment. Inasmuch as the residuals are computed and displayed for each equation, an error in the identification of 1 of the control points will be discovered and that point will be eliminated from the solution.

A minimum of 3 control points can be used for a less precise solution. If only 3 points are used, only 6 condition equations can be written and a maximum of 6 unknowns can be permitted. This restriction of the adjustment makes it necessary to consider the scale curve to be of second rather than of third degree. The same computer program applies in this case but the column with the coefficient A is removed from the condition equations. The coefficient A is associated with the x^3 term in the primary curve and its derivative in the secondary curve. The error introduced by the use of only 3 control points is not precisely known but it is probably negligible for strips of less than 10 models. When the IBM-650 program is completed, it will be feasible to determine the reliability of the 3-point solution for several different lengths of strips using data from an overcontrolled strip.

CALCULATIONS OF FINAL GEODETIC PLANE COORDINATES OF PHOTOGRAMMETRIC POINTS

With the values of the coefficients A through G now known, the correction equations (33) (34) can be used to calculate the corrections, c_x and c_y for all photogrammetric points in the strip. The omission of the corrections in the input data will result in their computation automatically.

The corrections are algebraically added to the instrument coordinates to obtain the adjusted instrument coordinates

$$x'' = x' + c_x \qquad (35)$$

$$y'' = y' + c_y \qquad (36)$$

Following the adjustment of the instrument coordinates of all photogrammetric points in the strip, an inverse transformation is made to obtain the desired geodetic plane coordinates. The equations for this transformation are the same as those used for the forward transformation of the geodetic coordinates of the control points (see equations (19) and (20).

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