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Pantograph Adjustment

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Technical Bulletin No. 7 presents a method for systematically adjusting a pantograph which does not accurately reproduce dimensional figures.

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Pantograph Adjustment

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A METHOD is presented for systematically adjusting a pantograph which does not accurately reproduce dimensional figures. The accuracy of the pantograph depends on (1) whether or not the three scales are accurately indexed at their respective bearings and (2) whether or not all six bearings are maintained in perfect vertical alignment at all times.

If a pantograph produces errors, they can usually be reduced significantly by the application of numerical corrections to the three scale settings as explained herein. It may be obvious that it is not possible to adjust completely with three settings such an instrument as this which has nine sources of error. However, the vertical misalignment of the bearings is difficult to analyze and difficult to correct mechanically.

It was found during the study that some of the rectangular arms or bars of the pantograph had a definite sag or bow downward which naturally would cause the bearings at the ends to be deflected from the vertical.

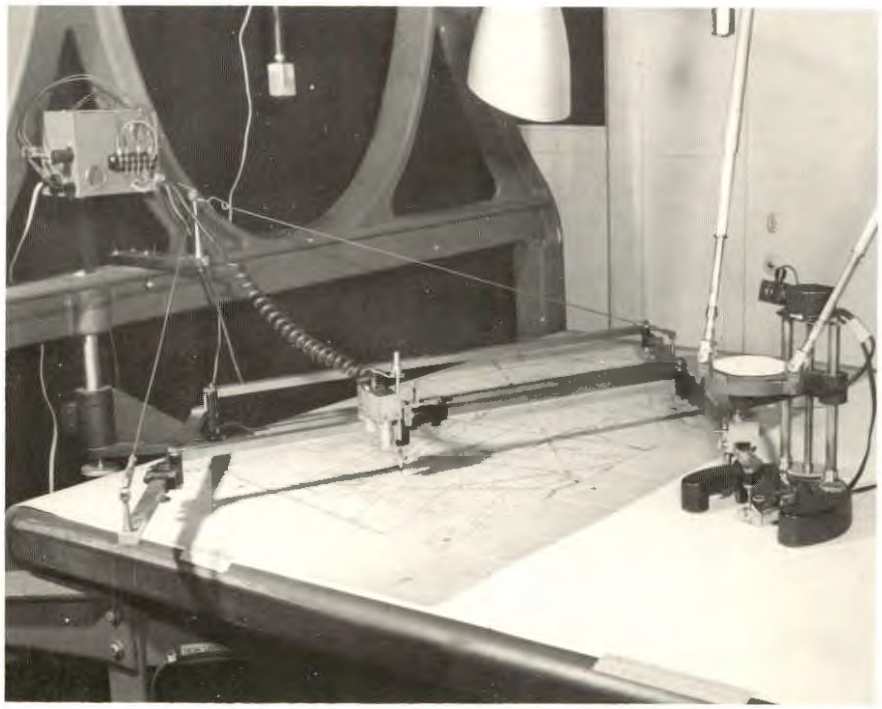
Although this study was conducted with regard to the pantograph as used with the Kelsh Plotter, it also applies to any other pantograph. Moreover, the general method has a very broad application to many different kinds of devices and processes and is not limited to the three unknown parameters of this particular instrument.

CONCLUSIONS

In a typical example, a standard residual error of $\pm$ 0.25 mm. was reduced to $\pm$ 0.14 mm. in which the maximum errors were normally about three times as great. The time required for the study of one instrument is about one work day. The study also indicated the minimum residual error which might be hoped for after adjustment, and which might serve as a criterion for rebuilding, discarding, or replacing the instrument.

THEORY AND COMPUTATION

As one can measure or observe the errors of a pantograph in several places in its working area, and as one can change each of the three variable linear settings a unit amount one at a time and observe its effect on the errors, it seems reasonable that one should be able to determine the corrections that are needed to reduce a set of initial



Type of pantograph tested.

errors to a minimum. Therefore, the following equation may be expressed:

$$(a - d)\,A + (b - d)\,B + (c - d)\,C = -d.$$

Here A, B, and C are the unknown number of units by which the three pantograph scales should be corrected to reduce the initially observed error d to zero. The symbol a is used to represent the error at any given place which is observed after the A setting is changed 1 mm.; b and c indicate similarly the errors observed by altering the B and C scales one at a time. The symbol d represents the error observed when all three scales are in their normal operating setting. Thus the value $(a - d)$ represents the change of error caused at that point by a unit change in the A scale.

As the equation contains three unknowns, the errors must be observed at three different places in order to provide at least as many equations as there are unknowns. In this test, however, the errors were observed at the six significant places of the rectangular working area. Moreover, the errors were observed in two directions, x and y. (See the diagram.) Thus in all there were 12 equations in three unknowns. Two of the points were used as index points for the purpose of observing the changes, hence three equations for the index points were not used in the solution. As a result, the remaining nine equations were used and these were reduced to three normal equations by standard methods. The solution of the normal equations gives the values of the three numerical adjustments which most nearly fit all nine conditions according to the principal of least squares.

The table of observed discrepancies are the actual readings of the errors as observed in the four separate conditions. The units are in 0.001 inch and are measured with a lineoscope. The observation equations were formed as indicated by the basic equation stated above from the quantities given in the table of observed discrepancies. The solution procedure of the normal equations is not indicated. Upon substitution of the computed adjustments in the nine condition equations, residual discrepancies were obtained as indicated. To test this further the pantograph was actually adjusted and the discrepancies were observed as before. The observed residual discrepancies are tabulated along with the computed values and indicate a general similarity. It is noted that point 5 has the largest error not only in the table of observed discrepancies, but also in the observation equations, and in the residual discrepancies. This may be partly explained as the pantograph is in its poorest operating position geometrically at point 5. In other words at this point all six bearings of the pantograph are approaching a straight line which does not give a rigid figure.

It is realized that the adjustment is valid only at the single ratio setting of the pantograph at which the study is made. However, it seems reasonable to assume that the adjustment is valid over a small range of settings in the immediate vicinity of the ratio of a test.

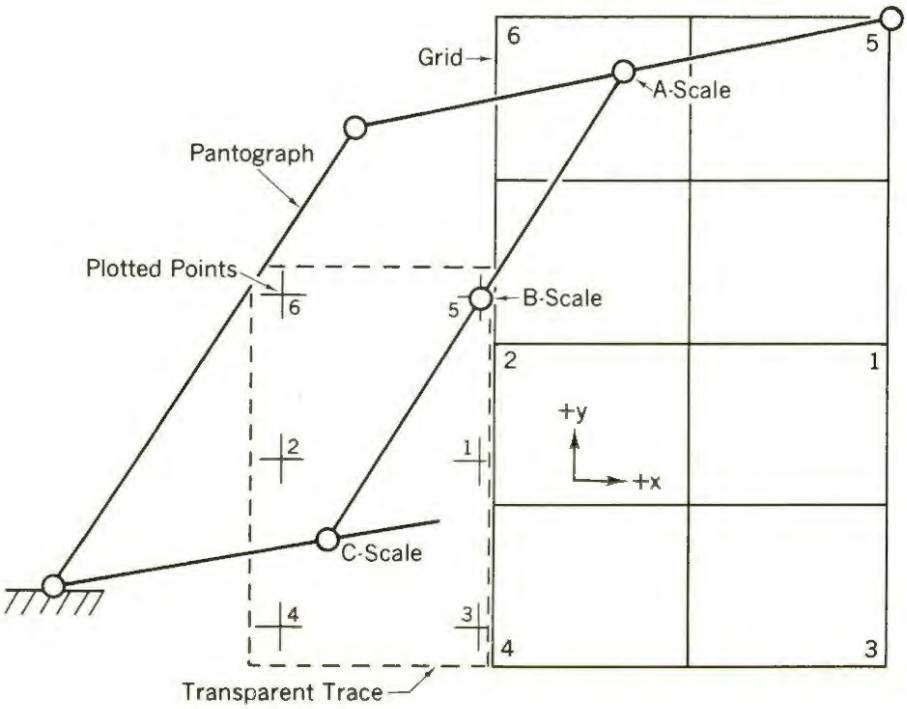


Diagram of test plot.

PROCEDURE

A pantograph was set at one-half scale. A rectangular map grid plotted on stable material comprised the basic grid. The grid intersections 1, 2 . . . . 6 were selected having an even number of grid squares between the extreme points. The intersections were plotted at the reduced scale with the pantograph on a smaller sheet of transparent vinylite.

Actually four plottings were recorded with different settings of the *A*, *B*, and *C* verniers, as explained above. For convenience all the plottings were made on the same sheet of material. By moving the sheet slightly out of register, the different sets of points were identified by means of different symbols and colors.

The sheet was then laid over the master grid as a comparison and for measuring the discrepancies. In this study, point 2 was placed in coincidence with a grid intersection, and point number 1 was placed on line. Then the x- and y-errors were measured with a lineoscope for all the other points to yield the data for the computation as tabulated in the Observed Discrepancies.

After the computation, the three scales were changed as indicated. The points were then replotted to check the efficacy of the computation and to demonstrate the residual error remaining in the instrument.

Successive computations and corrections would not be expected to reduce the residual discrepancies significantly.

Observed Discrepancies

Discrepancies (0.001 inch)

Point		*a*	*b*	*c*	*d*
1	x	-28	+ 2	- 5	-19
	y	0	0	0	0
2	x	0	0	0	0
	y	0	0	0	0
3	x	-21	+18	-15	-12
	y	- 9	-10	- 6	- 3
4	x	0	-24	-24	- 3
	y	-20	-16	-11	- 9
5	x	-38	-18	- 5	-31
	y	-13	0	+23	- 1
6	x	-21	-31	+11	-11
	y	+ 6	+ 9	+18	+ 7

Observation Equations

$a - d$	$b - d$	$c - d$		$-d$
$-9A$	$+21B$	$+14C$	=	+19
- 9	+ 6	- 3		+12
- 6	- 7	- 3		+ 3
+ 3	-21	-21		+ 3
-11	- 7	- 2		+ 9
- 7	+13	+26		+31
-12	+ 1	+24		+ 1
-10	-20	+22		+11
- 1	+ 2	+11		- 7

Arm Settings (mm.)

A	*361*	360	360	360
B	360	*361*	360	360
C	360	360	*361*	360

*Normal Equations**

$$+622A - 92B - 823C = -719$$

$$-92A + 1590B + 696C = +584$$

$$-823A + 696B + 2516C = +1135$$

Solution of the Normal Equations

$A = -1.1$ mm., $B = +0.3$ mm., $C = 0.0$ mm.

Residual Discrepancies

Observed, before adjustment (0.001 inch)

-19, 0, 0, 0, -12, -3, -3, -9, -31, -1, -11, +7

Observed, after adjustment

-3, 0, 0, 0, 0, -2, -6, +1, -19, +12, -6, +9

Computed, after adjustment

-3, 0, 0, 0, -2, -1, +1, 0, -18, 0, -5, +5

*Method described in Manual of Geodetic Astronomy, by A. J Hoskinson and J. A. Duerksen, C&GS Special Pub. No. 237, 1947, page 138.

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