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Vertical Adjustment of Instrument Aerotriangulation by Computational Methods

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Technical Bulletin No. 10 develops the vertical adjustment of aerotriangulation by computational methods in much the same manner as the horizontal adjustment was developed in Technical Bulletin No. 1.

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Vertical Adjustment of Instrument Aerotriangulation by Computational Methods

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A NONLINEAR propagation of errors in the process of aerotriangulation with stereoscopic plotting instruments, results in relatively large errors at the center of a spanned distance, if a linear transformation is used to adjust the instrument coordinates to geodetic control at the ends of the strip. Through analysis of the effects of small residual errors in relative orientation, it can be shown that this nonlinear propagation of error follows second-degree curves in the *xy* and *xz* planes of a spacial rectangular coordinate system, and a third-degree curve for scale correction along the axis of flight.[1] In addition to these three primary error curves which define the error at points along the axis of flight or *x*-axis, each of the three error components contains linear secondary effects from the other two primary curves which contribute to the error at points not on the *x*-axis.

The Coast and Geodetic Survey used a modification of the Army Map Service semigraphic method[2] for the adjustment of these nonlinear errors until 1958 when a completely mathematical method of adjustment was developed[3] for the errors in the *xy* or horizontal plane. The horizontal adjustment was programmed for solution with the IBM-650 electronic computer to reduce the adjustment time for a strip from days to minutes.

The vertical adjustment was not programmed at that time because previous experience with the semigraphic method indicated that the C-factor (flight altitude divided by smallest contour interval which meets national map accuracy standards) would usually be too low for our mapping requirements. While it is still considered necessary to provide a minimum of three vertical control points in each model when C-factors greater than 1,200 are required, a plotting instrument technique has been adopted which has increased the accuracy of vertical bridging enough to make it useful for medium- and small-scale mapping in rough terrain. The new instrument technique compensates for earth curvature, thereby reducing the magnitude of the vertical error curve by 90 percent. For example, the middle ordinate of the error curve was reduced from 2,000 feet to 200 feet for a 33 model strip of 1:40,000 scale photography. It now seems that a C-factor of 600 will be attainable when pairs of vertical control points are placed on opposite sides of the strip in every fifth model. The accuracy will increase as the density of the control is increased so that a C-factor of 900 is expected when pairs of vertical control points are placed in every third model.

INSTRUMENTAL EARTH CURVATURE COMPENSATION

In the ordinary method of *free* aerotriangulation, each added diapositive is oriented relative to the preceding one so that the successive centers of projection will form an arc in instrument space just as the path of the airplane followed an arc around the earth. Inasmuch as the plotting instrument coordinate measuring system is rectangular (not curved like the earth), the aerotriangulation can and should be made more nearly compatable by making it similar to a cylindrical or transverse Mercator projection. This is accomplished by tipping the fixed projector of each successive model to compensate for earth curvature before the next diapositive is oriented. The required *phi* motion is very small (0.02^g per 10,000 feet of flying height), but sufficient to keep the nadiral axis of each succeeding diapositive parallel with the vertical coordinate axis of the plotting instrument. It should be noted that this technique does not restrict the freedom of the aerotriangulation in any way. It does, however,

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Mr. Harris is Assistant Chief, Research Branch, Photogrammetry Division. The formulation was developed under the direction of Capt. L. W. Swanson, Chief, Photogrammetry Division. The author acknowledges the technical assistance of Morton Keller, Cartographer. In devising the method described, the Geodesy Division cooperated with the Photogrammetry Division in solving sets of simultaneous equations and in programming the complete solution.

[1]Some theoretical and practical problems in photogrammetric bridging by Dr. J. M. Zarzycki, *Photogrammetric Engineering*, December 1955.

[2]Aerotriangulation adjustment by G. C. Tewinkel, *Photogrammetric Engineering*, April 1957 and *The Journal*, Coast and Geodetic Survey, October 1957.

[3]Aerotriangulation adjustment of instrument data by computational methods by W. D. Harris, *Technical Bulletin No. 1*, Coast and Geodetic Survey, January 1958.

plumb the model so that differences in elevation are measured on the vertical coordinate axis. Production tests show that it reduces our vertical error curves by approximately 90 percent without otherwise altering their shape. Incidentally, this flattening of the curve has made the preliminary adjustment of ***horizontal*** instrument coordinates for ***BZ-falloff*** unnecessary even in mountainous terrain.

VERTICAL ADJUSTMENT

Theoretically the vertical error curve should be quadratic like the azimuth curve, but experience with the semigraphic method of adjustment has shown that the additional freedom permitted by the use of a third-degree adjustment curve is necessary for a satisfactory solution.

The similarities between the vertical and horizontal adjustments made several of the established computer horizontal subroutines adaptable also to the vertical adjustment program. In fact, an alteration in the computer instructions for the horizontal program now causes the previously computed ***axis-of-flight*** coordinates of all points to be punched out as a byproduct of the horizontal adjustment. These coordinates are punched on cards which become the input cards for the vertical adjustment program.

TRANSFORMATION OF GROUND CONTROL COORDINATES

Inasmuch as the vertical error curve is to be considered independent of, and uncorrelated with, the azimuth and scale curves, the transformation of the ground control coordinates into the axis-of-flight system is simply a matter of applying a scale factor g to bring the ground elevation to the same scale and unit of measurement as that of the instrument system and adding the instrument coordinate system index k, the instrument coordinate assigned to the datum elevation.

Let Z be the ground elevation of the vertical control station and z'' be the correct coordinate in the axis-of-flight system then,

$$z'' = Z / g + k \tag{1}$$

where g, the scale of the aerotriangulation, is determined from the auxiliaries $\Delta x''$, $\Delta y''$, ΔX, and ΔY (equations 15 through 18 of reference 3) as the ratio

$$g = \left(\frac{\Delta X^2 + \Delta Y^2}{\Delta X''^2 + \Delta Y''^2} \right)^{\frac{1}{2}} \tag{2}$$

DETERMINATION OF CORRECTIONS FOR VERTICAL CONTROL POINTS

The disparities between the transformed vertical ground coordinates and the plotting instrument coordinates are, of course, the errors of the aerotriangulation which are to be minimized by the least-squares curve fitting adjustment. These errors can he considered corrections be applied to the plotting instrument coordinate The correction c_z is expressed as

$$c_z = z'' - z' \tag{3}$$

where z' is the plotting instrument coordinate.

DETERMINATION OF COEFFICIENTS FOR CORRECTION CURVE EQUATIONS

A least-squares curve fitting procedure is used for developing the equation of the curved correction surface. The trace of this surface on the xz-plane is assumed to be a curve of third degree about which an accelerating or second degree twist surface is generated by linear or straight line elements perpendicular to the x-axis. The equation for this surface is

$$c_z = Ax^3 + Bx^2 + Cx + Dx^2y + Exy + Fy + G \tag{4}$$

If the coefficients A through G are regarded as unknowns, a minimum of seven equations are required for their determination. One equation can be written for each vertical control station so a minimum of seven control points are required. If more than seven control points are available, a correction surface will be generated which will fit the control points in such a way that the algebraic sum of the disparities between the control points and the surface will be zero and the sum of their squares will be a minimum.

The electronic computer program for the formation and summation of the normalized observation equations as well as the solution of these normal equations for the coefficients A through G was duplicated from the horizontal adjustment.[4]

As in the horizontal adjustment program, by reducing the degree of the correction surface, the adjustment can be made with a minimum of five control points. This is accomplished by removing the columns Ax^3 and Dx^2y from the condition equations. This economy on control is not recommended on strips more than eight models long.

CALCULATION OF CORRECTIONS FOR PHOTOGRAMMETRIC POINTS

With the values of the coefficients A through G now known, the correction equation (4) can be used to calculate the instrument coordinate correction c_z for all photogrammetric points in the strip.

[4] See footnote 3 above.

The computed corrections are then added to the instrument coordinates.

$$z'' = z' + c_z \tag{5}$$

ıere z'' is the adjusted elevation in the instrument coordinate system.

The adjusted instrument coordinates are finally transformed into the geodetic system through the use of equation (1)

$$Z = g\,(z'' - k) \tag{6}$$

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