U.S. DEPARTMENT OF COMMERCE
Frederick H. Mueller, Secretary

COAST AND GEODETIC SURVEY
H. Arnold Karo, Director

A Singular Geodetic Survey

LANSING G. SIMMONS

TECHNICAL BULLETIN NO. 13

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Technical Bulletin Series

This series of Technical Bulletins was inaugurated to present primarily to the personnel of the Coast and Geodetic Survey and incidentally to others technical information related to the Bureau's scientific and technical activities. Since many of the bulletins deal with new practices and new techniques, the views expressed are those of the authors and do not necessarily represent final Bureau policy.

Technical Bulletin No. 13 describes in some detail the planning, operations, and results of a remarkably accurate horizontal control survey which covered an area of approximately four thousand square miles. While the requirements called for an accuracy of 1:400,000, the final results were better than 1:1,000,000. This was partly due to the numerous lines measured with the Geodimeter. These measurements were made, for the first time, from the tops of tall steel surveying towers.

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A Singular Geodetic Survey

LANSING G. SIMMONS, Mathematician

U.S. Coast and Geodetic Survey

THE PROBLEM

ON NOVEMBER 27, 1959, the Director of the U. S. Coast and Geodetic Survey received a letter from Capt. F. R. Gossett, his liaison officer at Cape Canaveral, Fla., requesting a statement of accuracies of certain triangulation stations in Florida, as related to points on the Cape. The Air Force was planning a test of its *Azusa Mod II*, an electronic missile-tracking device, and wanted a method of checking its operation. To attain this check, it was planned that the missile would be positioned in flight by photographing it against the star background simultaneously with nine ballistic cameras, situated near triangulation stations at from about 40 to 50 miles from the Azusa installation at the Cape.

It was stated that accuracies of 1:100,000 were required. An investigation of the triangulation tying in these camera sites indicated that accuracies better than 1:50,000 could not be assured. A plan to strengthen the existing work was set ˆ rth. It consisted mainly in measuring certain the longer lines by Tellurometer and in some minor revision of the triangulation.

By the middle of December 1959 the accuracy requirements had risen to 1:200,000 and the estimated cost to obtain this was nearly five times that required for 1:100,000. This was not the end! Another letter dated February 1, 1960, from the Director of Range Development, Patrick Air Force Base, Fla., requested an accuracy of 1:400,000 between the Cape and nine outlying ballistic camera sites. This immediately created a situation piled high with problems to be solved. It meant "giving it the works," and the estimated cost went up another 50 percent.

Now, accuracies of 1:400,000 over a scheme of triangulation some 90 miles long and 45 miles wide probably have never been achieved before. To assure such results, in an absolute sense, would require the measurement of many long base lines to control the triangulation lengths. Those sides of the triangles which form sort of a traverse from the Cape directly to the camera sites would have to be measured. This, in the area under consideration, would be highly impracticable if not impossible. The clearing of trees and brush and the staking of so many long base lines across country present virtually unsurmountable problems. The next best possibility considered was the Geodimeter. In this connection two questions arose: (1) could it be operated successfully from the top of a Bilby steel tower? and (2) is the speed of light, a basic constant for Geodimeter measurements, known accurately enough?

To help answer the first question, a Geodimeter party operating in the Middle West was instructed to experiment with Bilby towers. The Chief of Party, Ens. James S. Midgely, reported on February 27 that such measurements could be made from the regular tripod of the tower, although hauling the gear to the top and operating it in the small space on the platform presented some problems. As to the second question, the accuracy of the constant, the speed of light, it is believed that this is known probably to 1:500,000, possibly better. But the important thing is that this triangulation was to be the basis for checking out electronic equipment which itself employs the speed of electromagnetic waves. In this respect it is probably better to control the scale of the triangulation by Geodimeter rather than by precise base tapes.

After careful consideration of what was required and how to meet these requirements, it was decided that the area be covered completely by a network of not single triangles but of checked figures preferably quadrilaterals with two diagonals. Also, the triangulation observations must be made on at least each of two nights. One part in 400,000, translated into angle measurement, is 0".5, an accuracy that cannot be assured by one night's observation, primarily because of the uncertainties of refraction. Moreover, many sides of the triangulation must be measured by Geodimeter, preferably those sides which form a sort of traverse from the Cape to each of the three groups of outlying camera sites and between the camera groups themselves. All plans having been made and details decided upon, a series of Instructions were written for the triangulation reconnaissance, triangulation observations, and Geodimeter measurements.

In order that the nine ballistic cameras be placed on a Cartesian Coordinate System with the instrumentation at the Cape, it became necessary

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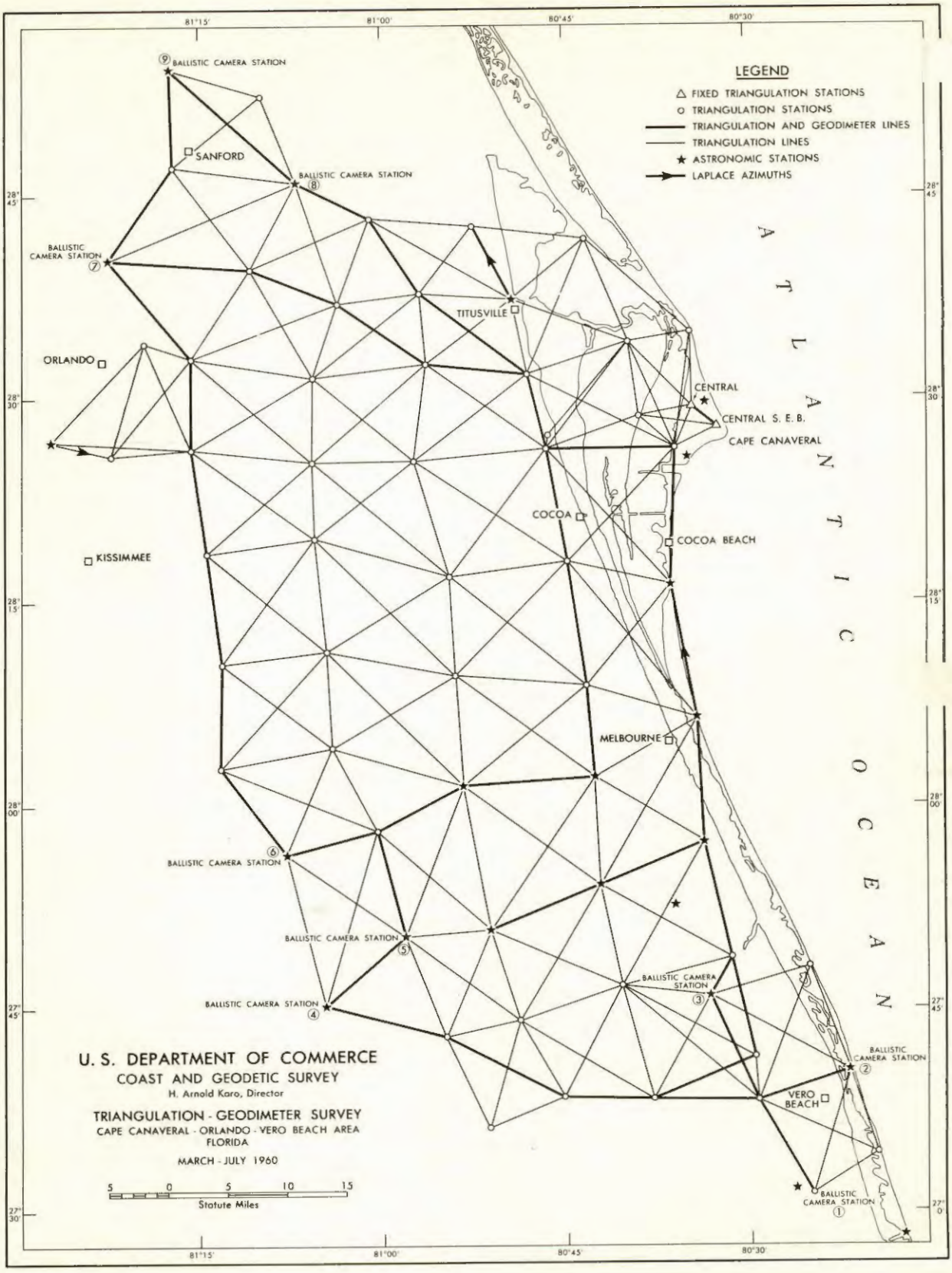
Mr. Simmons is Chief Mathematician, Geodesy Division. The writer acknowledges particularly the assistance of Comdr. Glenn W. Moore and Ens. James S. Midgley for material related to field operations; and of Joseph L. Stearn, mathematician in direct charge of the triangulation adjustment, for certain statistical data the accuracy attained.



TRIANGULATION DIAGRAM OF AREA

to determine not only the topographic elevations the cameras, but also the relative geoid heights between the Cape and the camera sites. This necessitated spirit leveling to determine topographic elevations and about 15 astronomic latitudes and longitudes to develop, by astro-geodetic leveling, the relation between the geoid and the spheroid at the camera locations to within about two or three feet. Thus, leveling and astronomic Instructions were also prepared. Those parts of the field instructions which are of technical interest are reproduced in the next section.

THE INSTRUCTIONS

Reconnaissance

10 February 1960

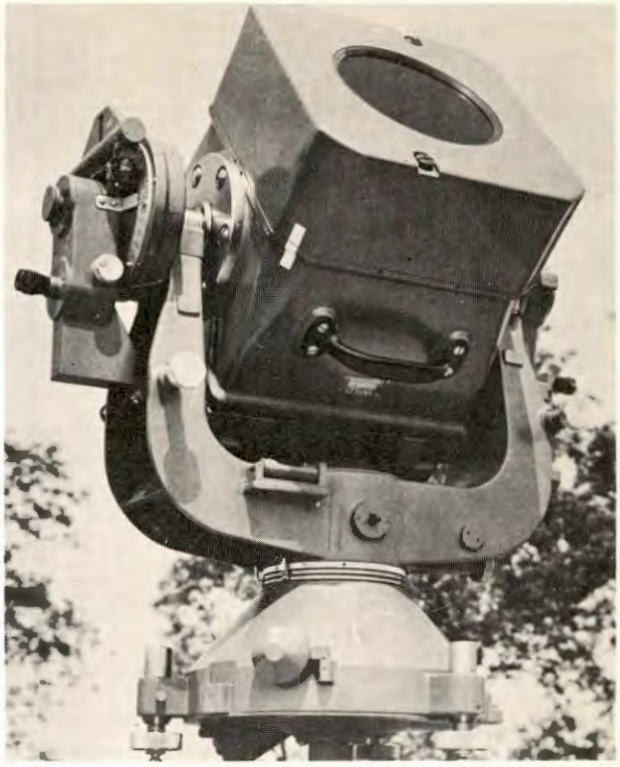
To: Mr. L. B. Odgers
Coast and Geodetic Survey
Post Office Box 4096
Patrick AFB, Fla.

Subject: *Instructions*—Reconnaissance for Special Surveys, Cape Canaveral, Fla., Project 40000-920

1. Upon your arrival at Patrick AFB, in accordance with telegraphic instructions dated 3 Feb. 1960, you will report to Capt. Franklin R. Gossett, our liaison officer. From him you will obtain details as to requirements of this assignment. Patrick AFB is requesting us to determine the horizontal positions of certain inland sites with an accuracy of 1:400,000 relative to sites at the base. This, as you know, exceeds any previous requirement we have had, and will call for extreme care in all of our operations and for extremely strong triangulation figures. You, assisted by Mr. Lonnie G. Shiplet, will plan the reconnaissance; Party 607, under the direction of Comdr. Glenn W. Moore, will observe the triangulation beginning about 1 March 1960. One or two Geodimeter teams will also be utilized.

2. The sites to be connected to Cape Canaveral are the following triangulation stations: ***Jackson***, ***Kenan***, and ***Coon*** in Osceola County, Fla.; ***Indrio*** in St. Lucie County; ***Clarkson*** and ***Brookside*** in Indian River County; ***Mait*** and ***Gene*** in Seminole County; and ***Padgett*** in Volusia County. Captain Gossett will identify the control at Cape Canaveral to which these sites are to be connected.

3. You will plan reconnaissance connecting the above sites by a complete coverage of the area with quadrilateral figures, calling for both diagonals to be observed. The quadrilaterals should be as strong as possible; that is, they should approach squares and shall be fairly uniform in size. They should be composed of lines as long as possible, rarely less than eight miles. Stations should be placed on firm ground to provide maximum stability for towers. Maps with a diagram illustrating the desired type of figure and spacing are being furnished to you.



BC-4 ballistic camera.

4. In providing this reconnaissance, you should not attempt to connect to all existing stations, but to use only those that will fit into your spacing. It will probably be well for you to make a preliminary survey of the area to determine the maximum uniform size of quadrilaterals that can be established. Tree heights will be the determining factor in this respect. You should determine not only the general heights of trees but also the locations of isolated groves of trees of unusual height in order that these may be bypassed wherever possible.

5. You will indicate on your reconnaissance sketch all lines that provide visibility from the ground at one station to a tower at the other end. This information will be needed in the selection of lines to be measured with a Geodimeter, which will be numerous.

Triangulation

16 February 1960

To: Comdr. Glenn W. Moore
Coast and Geodetic Survey
Post Office Box 309
Benton, Ill.

Subject: *Instructions*—Triangulation, Special Surveys, Cape Canaveral, Fla., Project 40000-920

1. *Purpose.* Patrick AFB has requested us to determine the positions of nine stations, in three groups, to an accuracy of 1:400,000 in relation to control at Cape Canaveral. The nine stations are: *Jackson*, *Kenan*, and *Coon* in Osceola County, Fla.; *Indrio* in St. Lucie County; *Clarkson* and *Brookside* in Indian River County; *Mait* and *Gene* in Seminole County; and *Padgett* in Volusia County. You will undertake the triangulation observations and should arrange to discontinue your present project about 1 March and move your party to the vicinity of Patrick AFB.

2. *Description.* It is intended to connect the four groups of stations by a network of quadrilateral figures of fairly uniform size. These figures will compose a network covering the area lying between the four groups of stations.

3. *Specifications and Special Precautions.* This is the most exacting horizontal control ever undertaken by the Bureau. Conscientious attention to details and a general tightening up all along the line will be indispensable requirements. Nothing valuable will be lost by avoiding haste. While economy is important to this as to all projects, precision of operations will be the over-riding rule.

Observations shall conform to specifications for First-Order, Class I, Triangulation, revised as follows:

(a) Each station shall be occupied on at least two nights. A set of observations on each night shall normally consist of 16 positions of the circle, never less than 12 positions. When the means of the two sets, taken on each of two nights, differ by more than one second, a third set shall be taken on a third night. When three sets are available, the values of directions to be used in the computations shall be taken as, in order of preference, (1) the mean of two sets which agree within one second, (2) the mean of the two sets in closest agreement, (3) the mean of all three sets, and (4) —as a last resort—any combination of the three sets including the use of any one set alone. Methods (3) and (4) shall not be employed unless it can be clearly demonstrated that either is required to best satisfy angle and side conditions.

(b) The average triangle closure shall not exceed 0".70, and the maximum triangle closure shall seldom exceed 2".0.

(c) For a regular quadrilateral in which the triangles have been closed by applying one-third the closure to each angle, the side check shall not exceed 1.5 times the tabular difference for one second of the log sine of the smallest angle entering into the computation. For more complicated figures, the factor of 1.5 may be replaced by one-third the number of triangles involved.

All instruments and signal lights shall be collimated immediately before the observing period. In case of any doubt of movement of the tower, the collimation should be checked immediately after the observing period. Special caution shall be employed in the collimation operations, such as resetting the collimator and redetermining the center of the instrument and the lights. This should be repeated until there is assurance that the collimation is correct within two or three millimeters.

Your records shall contain complete notes as to date, time, weather conditions, approximate direction and velocity of the wind, the name of the observer, and the number and type of the theodolite. Special care shall be exercised in leveling and maintaining the level of the theodolite. The observers should be cautioned not to hurry their observations, but to be particularly careful in obtaining precise pointings. Everything that may work against precision in operations shall be avoided and nothing reasonable that will be favorable to accuracy of results shall be overlooked.

Consideration is being given to a plan for you to telephone each night's results to the Washington Office. You will be informed if it is decid to have you do this.

4. *Reconnaissance.* The reconnaissance for this work will be undertaken by Mr. L. B. Odgers assisted by Mr. Lonnie G. Shiplet. Mr. Odgers is being transferred from New Mexico to Florida and should arrive about 18 February. Mr. Shiplet will begin preliminary work on reconnaissance on 15 February. In addition to this, Mr. John V. Crusemeyer, who is preceding your party to Cape Canaveral, may assist Mr. Odgers until such time as he may be needed for your operations.

5. *Geodimeter Measurements.* It is intended that numerous lines in the network will be observed with Geodimeters. If possible we will obtain the use of a Micro-Dist (Electro-Tape) instrument for length measurements also. Mr. Odgers has been directed to identify lines that will be intervisible station to station from the ground at one end. Ens. James S. Midgley is working on a platform to support the Geodimeter at various sections of the Bilby tower so that it may be possible to avoid the necessity for a ground station. The Geodimeter bases will be selected by this office, possibly through consultation with you and Mr. Odgers. You should extend whatever help is necessary to the Geodimeter party to keep all phases of the work progressing together.

Geodimeter

9 March 1960

To: Ens. James S. Midgley
Coast and Geodetic Survey
General Delivery
Cocoa, Fla.

Subject: *Instructions*—Geodimeter measurements, Cape Canaveral, Fla., Project 40000-920

1. *General.* Upon receipt of these Instructions, you will suspend operations on your present assignment in Missouri, Arkansas, and Illinois, proceed to the vicinity of Cocoa, Fla., and undertake Geodimeter measurements on Project 40000-920. This project consists of a 1:400,000 survey by triangulation, strengthened by numerous Geodimeter measurements. It will extend from Cape Canaveral, south to the vicinity of Stations *Jackson, Kenan,* and *Coon* in Osceola County and northwesterly from Cape Canaveral to the vicinity of Stations *Mait* and *Padgett.* Your party complement has been increased to provide for two Geodimeter teams. An additional Navy-owned Model 2A Geodimeter will be delivered about the middle of March by Mr. Austin C. Poling of this office. At a later date, your present Model 2 Geodimeter may be replaced by a second Model 2A converted from a Model 1. Two Kytoon balloons for elevating intermediate thermometers along the line between stations are being ordered. You will test one of these balloons as soon as possible and notify this office as to the results. If practical, additional balloons will be ordered. If not practical, EPI aluminum towers will be furnished, or a method of elevating the thermometers will be devised on the party. Capt. Franklin R. Gossett, Liaison Officer at Patrick AFB, has been requested to find a supply of helium gas. Additional sensitive thermometers for the end stations and reversing frames and thermometers for the intermediate stations are being procured by this office. Five additional radios will also be furnished in the near future.

2. *Description.* Approximately 35 to 40 lines will be measured on this project. The lines will be selected in this office after the reconnaissance sketch is received.

3. *Operations.* You will attempt to measure two Geodimeter lines each working night from each master station. Additional men for erecting the center towers, hoisting the Geodimeter, and manning the stations will be furnished by Commander Moore, Chief of Party 607, after his building operations are sufficiently advanced. A high degree of cooperation with Party 607 will be necessary. Until both Geodimeters are in operation, you will detail your excess personnel to assist Party 607.

4. *Specifications.* (a) Each line will be measured with at least two nights' observations with six complete sets each night. If the spread between the two nights' observations exceeds 2.5 cm., additional nights' observations will be made until any two nights' observations are within the above limit. All observations within a spread of 4.0 cm. (including the two accepted sets) will be meaned. Predetermined factors for computing the results on the stations will be prepared; wild sets approximately 5 cm. from the mean will be reobserved. Observations will not be taken on absolutely calm nights or when previous experience has shown that erroneous readings may result. Extreme care will be used in collimating the towers and measuring eccentricities. For example, lightkeeping or observing tarps will be placed on the outer tower before collimating; after the nights' observations, the towers will be re-collimated if the wind has increased and/or changed direction appreciably.

(b) For this project, additional temperature readings will be taken along the line between stations. At the beginning of observations one intermediate station will be used. The station will be placed within 0.1 mile of the line between the Geodimeter and mirror stations, if possible. It will be placed near the center point between stations if practicable, but any point along the center one-third segment will be acceptable. The topography will be taken into account, and the height of the center thermometer made as near the line of sight as possible. In the case of the Patrick Base, consideration will be given to anchoring a small boat along the line between the base ends. If this is impractical, the center thermometer station may be placed on the shore. For every determination, the location of the intermediate thermometer station/s will be spotted on the topographic map and an overlay showing the end and intermediate station/s will be furnished with the results. For field computations, the center thermometer readings will be given a weight of *four* with each end thermometer given a weight of one. Thermometer readings will be interpolated to 0.1°C. When the aluminum towers are used, a horizontal arm not less than 2 feet in length will be devised for hoisting the thermometer and protecting it from striking the tower. Thermometer readings at all stations will be taken before and after each set. Additional intermediate thermometer stations will be used along lines of very unlike terrain, or where tests may show that divergent temperatures will be encountered along the line.

(c) Additional notes on Geodimeter operation are attached to these Instructions.

5. *Calibrations.* Before beginning operations with the Model 2 Geodimeter, you will calibrate the instrument on a 50-meter range, using invar

tapes. The Model 2A Geodimeter is being calibrated in this office. All Geodimeters used will be re-calibrated after completion of use on this project. You will make frequency drift calibrations of the Geodimeters. At weekly intervals, you will make frequency checks on each instrument. Use of the frequency counter/s will be arranged through Capt. Franklin R. Gossett.

Leveling

3 March 1960

To: Mr. Spellman J. Diez
Supervisory Cartographer
Coast and Geodetic Survey
P. O. Box 631
Augusta, Ga.

Subject: *Instructions*—Leveling: Special Surveys, Cape Canaveral, Fla., Project 40000-920

1. About 14 March 1960, you will discontinue leveling in Georgia and South Carolina and transfer the personnel of your party now working in Georgia and South Carolina to the vicinity of Patrick AFB, Fla. In Florida you will undertake leveling requested by the Air Force Missile Test Center, Patrick AFB, Fla. Before leaving Georgia and South Carolina, you should bring your work to as satisfactory a conclusion as possible.

2. You will be furnished maps on which are shown nine triangulation stations as follows: ***Jackson***, ***Kenan***, and ***Coon*** in Osceola County; ***Indrio*** in St. Lucie County; ***Clarkson*** and ***Brookside*** in Indian River County; ***Mait*** and ***Gene*** in Seminole County; and ***Padgett*** in Volusia County, Fla. You will run sufficient second-order levels to determine the elevations of these nine triangulation stations. You will also determine the elevations of needed marks at Cape Canaveral. All spur lines will be double-run. Suggested routes of leveling will be shown on maps furnished with your tie data.

3. Upon your arrival at Patrick AFB, you will consult Capt. Franklin R. Gossett, C&GS, Air Force Missile Test Center, Post Office Box 4096, Patrick AFB, Fla., and Comdr. Glenn W. Moore. Captain Gossett will designate the marks to which you will run second-order leveling in the vicinity of Cape Canaveral. Commander Moore is in charge of a party observing triangulation on this Project.

Astronomic

6 April 1960

To: Lt. Comdr. Allen L. Powell
Coast and Geodetic Survey
Post Office Box 4096,
Patrick Air Force Base, Fla.

Subject: Astronomic observations for Special Surveys, Cape Canaveral, Fla.

1. Relative geoid heights are required at certain stations being observed by the party of Comdr. Glenn W. Moore under Project 40000-920. This will require modified first-order astronomic latitude and longitude observations at approximately 12 stations in the project. It has been determined practicable for your party to accomplish these observations at such times as can reasonably be spared from your other activities.

2. At this time, six stations have been selected for the astronomic observations. These are: ***Padgett***, Volusia County; ***Mait*** and ***Gene***, Seminole County; and ***Jackson***, ***Kenan***, and ***Coon***, Osceola County. The remaining stations will be selected by this office when additional information is available.

3. You will employ standard first-order astronomic equipment and procedures, except that latitude may be determined on one night with a minimum of eight star pairs and longitude may be determined on one night with a minimum of three star sets.

TRIANGULATION OBSERVATIONS

Towers

Special tower building precautions were taken. Anchor legs were reinforced by welding a reinforcement just above the anchor plate. All 4" 8" x 42" oak anchor plates were either new or in good condition. The tops of the inside anchor legs were securely clamped to the tower legs. A strap iron collar was welded around the bottom of the welded section. Adjustable heads were made true and level and clamped in place by the builders with two V-clamps on each leg. The legs on the adjustable heads were extended about a foot by welding on pieces. The building party leveled the light plate with shims. The collimation party clamped a tribrach on the adjustable head with C-clamps and the tribrach remained in place either until it was found eccentric or until the tower was dismantled. Lights were shown from the tribrach wherever possible. A plastic tribrach center was designed to center and hold the lights. All vertical braces were put on. Since there were not enough of these braces to go around, a special one was designed that was adjustable to four different lengths and several hundred were made in a machine shop in Cocoa. This vertical brace will work in any position, both on the inside and the outside towers.

There were a few towers that caused some difficulty. The trouble at ***Jim*** was found in the sand bags in the "V" of the diagonals. These touched both towers. The trouble at ***Sharpes*** was found to be in the adjustable head. Two V-clamps h

stripped threads. The trouble at *Oafb* was caused by using regular tower nuts on standard machine bolts. The tower bolt and nut are just a shade larger than a standard machine bolt and nut. After two nearly serious accidents caused by the heads popping off faulty tower bolts, the party resorted to the use of standard bolts and heavy-duty nuts in place of all regular tower tie bolts.

Vertical Collimation

Unusual collimation precautions were taken. A special party was assigned to develop a method to accomplish the collimation in accordance with paragraph 3(c) of the Instructions. A 4-foot stand was built over a stake about 125 feet below a 5 mm. hole in the catwalk of the Cocoa Water Tower. A tribrach was centered over the stake and this was used for standardizations. It was found that the standard Bureau collimator can be relied upon for an accuracy of about 5 mm. when collimating a 116-foot tower. A T-2 theodolite with a diagonal eyepiece would give an accuracy of better than 2 mm. A 7-inch repeater equipped with diagonal eyepiece and telescope level could also be used but with difficulty and awkwardness. Enough observations were taken to establish that the inner tower is about four times as stable as the outer tower and that the towers do not move more than a millimeter or so with the sun.

A 90-foot Bilby steel tower complete.

Vertical collimation with T-2 theodolite.

Since the T-2 proved to be the best, another was borrowed from the Air Force and two collimating parties were put in the field when necessary. It became unnecessary to collimate immediately before each observing period. After the tower had been up a week it proved to be quite stable. Every Monday all towers in use were collimated, then only new towers were collimated until near the end of the week when all were checked again. Using this procedure there was very little eccentricity to apply. Collimators were always used on tripods, and they were checked daily at the camp standard. Two or more independent setups were made to collimate each tower. The tribrach was clamped by the collimation party, thus marking the center for the theodolite and signal lights. The transparent plastic center facilitated showing lights from the tribrach. Lights between "O" parties were shown from the light plate. Lines close to grazing were shown from the outside tower. The extreme care and special precautions taken assure that there is very little error in the work due to eccentricity.

Signal Lights

Special precautions were taken with signal lights. Past experience with conventional lights ruled these out for work of this high caliber. An attempt was made to observe upon the filament of a Bureau 6-volt bulb. It was not bright enough. A very powerful 6-volt bulb was obtained that was bright enough to be seen around the horizon, but the current drain was too high. The same difficulty was encountered with a powerful 12-volt bulb. The use of 3-inch plastic reflector lamps with PR-13 bulbs was decided upon. These proved satisfactory. An experiment was made with a 40-watt, 4-foot fluorescent tube powered through an inverter by a 12-volt wet battery. The current drain was too great. A 20-watt tube can be operated through 130 feet of 18-gage wire from an inverter off two fully charged 12-volt batteries, 5 hours a day for a week. A 20-watt tube is feasible to power with wet batteries at the base of a tower. Tests show this to be an identifiable and good light to point on from distances up to 17 miles. Such a light would probably give better results than the plastic reflector lamps. But since the required precision was already achieved, it was thought inadvisable to experiment further.

Some experimentation was done with colored filters in an attempt to cut down flaring. There was some indication that a red filter helped somewhat, but again since the required precision was possible with the unfiltered lights, experimentation was discontinued.

T-3 theodolite used in all angle measurements.

There is no known way of correcting for horizontal refraction. Experience seemed to prove that the better a light appeared in the telescope, the better were the observing conditions. The problem is just about as simple as that. Good-appearing lights produced good results. Almost always, everywhere, the lights look the best just after sundown. Turning angles right after sundown, rather than calling, identifying, and communicating with lightkeepers, resulted in improved observations.

Automatic Lighters

Only through the use of automatic lighters was it possible to complete the observations with so few men and in such a short time. Most of the time only two men were available to show lights from the 10 to 12 stations of a full schedule. The posted lights ran into the dozens. A total of 19 automatic lighters were used. Most of the time the lighters were charged with 2 banks of 3 dry cells in parallel. These will power three PR-13 bulbs 5 hours per day for 5 days. The automatic lighters had to be recharged every week. Like any other instrument the automatic lighter has bugs and it takes some time to learn to live with them. For most of the lines in Florida the power needed for a PR-13 bulb was more than that of one dry cell and less than that of two. Proper intensity was attained by the installation of rheostats and the use of three dry cells on all lights.

Theodolites

All six T-3 theodolites were specially serviced in the field before observations were begun. The grease in the vertical clamps was worked out with watch oil. The lower clamps were removed, thoroughly cleaned and replaced with only a hint of fine spindle oil in them. Special attention was given to the horizontal slow-motion screw. All of the grease was removed during cleaning and only a light coating of oil was given to the threads and springs. The operation of the horizontal clamp and slow-motion screw of each instrument was thoroughly checked before it was used.

Horizontal Angle Observations

In accordance with paragraph 3 of the Instructions, special precautions were taken with the angle observations in addition to the specifications for first-order Class I Triangulation. Each station was occupied on at least two nights. In so far as possible, all directions were observed on the same run. Only in very unusual circumstances

were less than three directions observed at one me. Reobservations were taken whenever the ıean of the first eight positions diverged from the mean of the second eight by more than 1", also when the second night's observations diverged from the first by more than 1". The horizon was closed on the main runs. In general, the values used in the field triangle computations were the means of all observations. Only a very few triangles closed over 1" and the average closure was 0".44. No trouble was had in the quadrilaterals in meeting the side check of 1.5 times the tabular difference for 1" of the log sine of the smallest angle in the computation. Most side checks were less than half the allowable. Some difficulty was encountered with some of the poles common to four quadrilaterals.

Special precautions were taken with the theodolite not normally taken on first-order triangulation. Each tower was given the "bubble" and "twist" tests before observations were started. The theodolite was pointed on a signal, read, rotated several times in a clockwise direction, repointed and read to detect "drag" of the instrument base. The 40-power eyepiece was used exclusively. All final pointings were made clockwise against the slow-motion spring. Positions were rejected that had more than a 2" horizon closure. Directions were taken over if the collimation was in excess of 3" from the mean collimation for that position. Directions were rejected if the collimation diverged more than 4" from the mean. Directions obviously in error were rejected. Directions more than 2" from the mean were reobserved, but those 3" from the mean rejected. Observers were instructed to check the horizontal level frequently and relevel when necessary. They recorded L & R plate level readings in two directions, 90° apart after the 8th and 16th positions.

Connections to Nearby Points

Special precautions were taken at each station to insure the accuracy of the connections to nearby points. A ground setup was made at each station and the nearby marks turned in with the azimuth mark. Eight positions were taken to camera points. In these cases eight positions were taken upon the azimuth mark. Azimuth marks were set at least one-half mile from the station. At stations near camera positions, one direct observation was taken on the camera point from the top of the tower to check for correct degrees and minutes. Camera points were measured to by the triangulation party of Commander Moore and checked against traverses run by men from Commander Powell's party. In a few instances traverses were run by both parties.

Vertical Angle Observations

Vertical angles were taken in connection with the triangulation observations on this project to obtain slopes for the correction of Geodimeter measurements to the horizontal, and elevations for the correction of these lengths to sea level. Most of the area is covered by U. S. Geological Survey 1:24,000 topographic quadrangles, and the elevations of the stations could be determined from these maps within a foot or two. Loop closures carried through the vertical angle observations were of the order of 1 meter or so in elevation, but some intermediate stations failed to agree with the topographic map elevations by two meters or more. Generally, the map elevations were considered better than those derived through the vertical angle observations.

GEODIMETER OBSERVATIONS

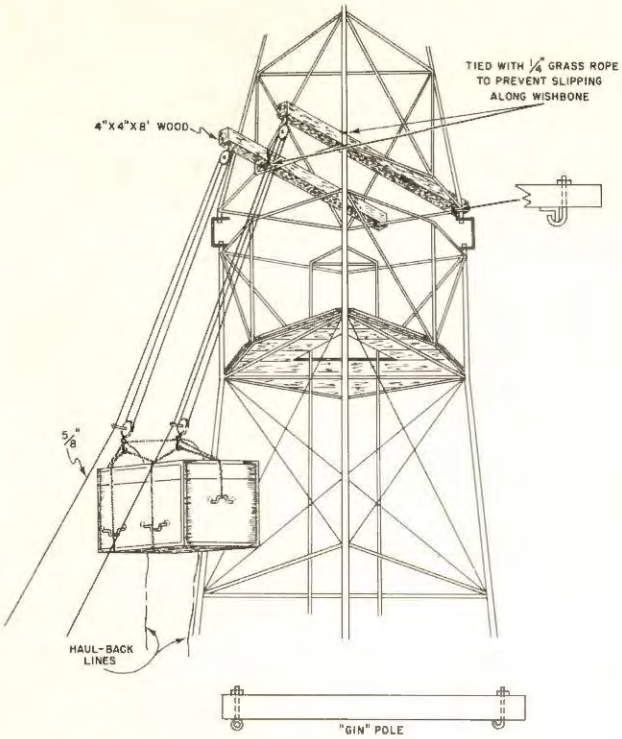
Progress Achieved

Of the 200 lines in the triangulation net, 43 were measured with the Geodimeter. This was possible only because it had been demonstrated, in a special test, that the Geodimeter could be operated from the top of a Bilby steel tower. There had been some question as to the stability of the Bilby tower in providing proper support for the operation of the Geodimeter. Such apprehensions, however, proved groundless. The Geodimeter was simply mounted on top of the inner tripod with no guys or extra braces and no trouble was encountered in its operation. Experience indicated that the Geodimeter could be successfully operated during periods of higher winds than the theodolite.

The Geodimeter Party started operations late in March 1960 and completed its work about the middle of July. During the early part of the project only one Geodimeter was employed, but later a second instrument was put into the schedule. At times, a hazy condition of the atmosphere would make the Geodimeter measurements impossible for days on end. Because of this, many of the longer lines called for in the Instructions were replaced by shorter lines, generally in the same quadrilateral. This deviation from the original requirements helped progress considerably. There were many nights, for example, on which a 7- or 8-mile line could be measured, when a 12- or 13-mile line would have been impossible.

Details of Tower Operations

Six men are required to operate the Geodimeter from a Bilby-tower setup when mid-point temperatures are taken. The observer and the recorder first climb the tower with a 1/4-inch haul line used later to pull up the block and tackle. The block is hooked to an upper horizontal member



Rigging for hauling Geodimeter up the Bilby tower.

of the observer's cage and then the gin poles are hauled up and put in place. The blocks are hooked into the eyebolts at the end of the gin poles. Next, the plywood head is hauled up and attached to the adjustable head of the inner tower by means of three special clamps made of strap iron. A cross, filed on the end of the bolt in the center of the head, is collimated over the station mark. This is used as a reference for small eccentricity measurements made later for each line measured by the Geodimeter.

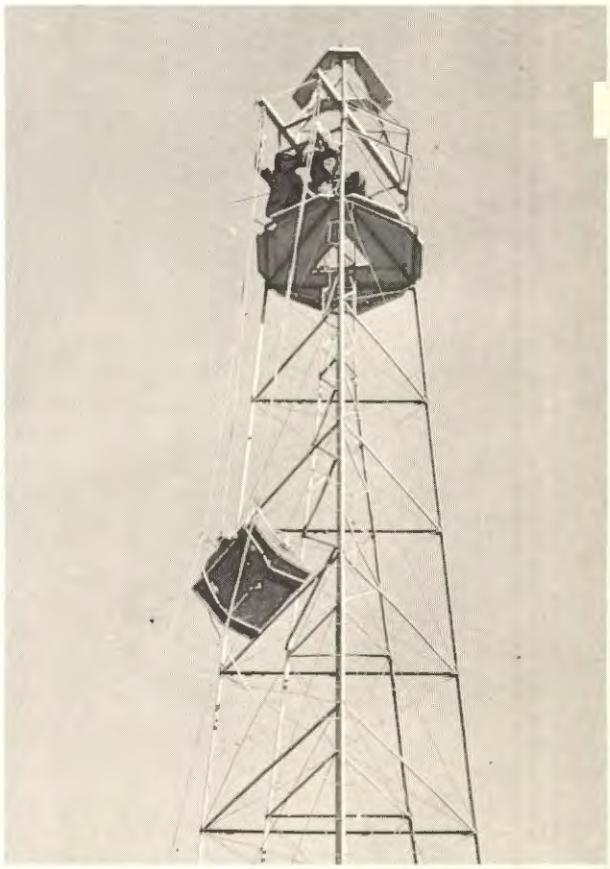
The four men remaining on the ground attach to the blocks the box containing the electronic section of the Geodimeter. This is hauled up to the top of the tower by two men on each of the two haul lines. The electronic section is taken out of its box and set in place on the tripod while the men on the ground haul back the blocks and attach the light conductor box. This second box is then hauled to the top of the tower, the equipment put in place and the tripod head is again collimated. The first empty box is placed on top of the gin poles to get it out of the way, while the light conductor box is left in the observer's cage to serve as a desk for the recorder. The remaining gear is then hauled up in a bag.

The haul lines are tied off at the bottom or are used as guys, if necessary. Power is supplied to the top of the tower by three 50-foot extension cords which run down the tower to the generator mounted in a truck. The height of the Geodimeter is then measured and everything is ready for Geodimeter operations.

Two of the four men on the ground go to thei respective mirror stations at which the necessary measurements and collimation had been completed earlier. The other two men drive the truck containing the mid-point temperature gear to a preselected position as close to the center of the line as the terrain permits. The reversing thermometer is elevated and 10 to 15 minutes are allowed to permit the thermometer and reversing frame to come to air temperature. The mid-point party then reports to the Geodimeter station by radio that observations may begin. After the observations over this line are completed, the mid-point party drives to the center of the next line to be measured. When this is completed, all men return to the Geodimeter station for dismantling the gear.

General Procedures

The Geodimeters used on this project were Coast and Geodetic Survey Model 2, No. 114, and a borrowed U. S. Navy Model 2A, No. 101. A third instrument was available, but it was not used



Geodimeter being hauled up the Bilby tower.

to any extent. Regular observational procedures were employed, but because of the high accuracy equirements a mid-point temperature was taken ı most cases. Although humidity has little effect on Geodimeter measurements, a change of 1° C. in the average temperature over the line affects the measured length about 1:1,000,000. The mid-point temperature was taken by hoisting to line height, with a 50-foot portable TV antenna, a deep-sea reversing thermometer, installed in a special reversing frame. When the thermometer could not be lifted high enough, a correction was applied to the reading by the application of a vertical temperature gradient determined on a Bilby tower at one end of the line.

It turned out that, on this project, the mid-point temperatures were probably not necessary. The range of the average temperatures taken at each end of the line and at the mid-point seldom exceeded 2° C. and usually was considerably less.

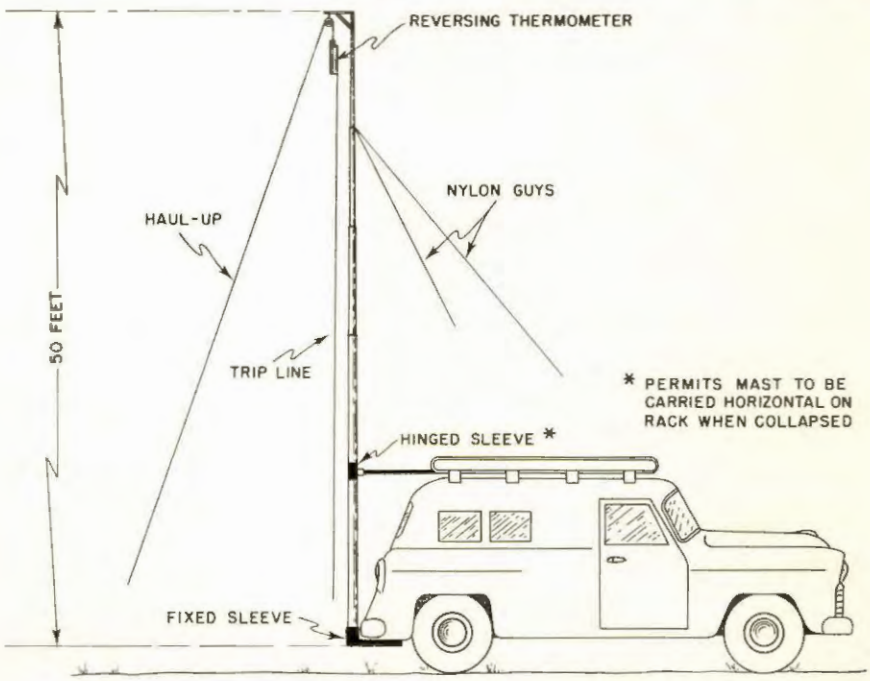
Usually an instrument was used to measure two lines on one night from the same tower when the plan called for adjacent lines to be measured. Of course a second night's observations were required over these lines before the measurements were complete. No trouble was encountered in obtaining the 25 mm. check between the two nights' observations.

There appeared to be a tendency, when the two Geodimeters were used to measure the same line, ›r the lengths obtained by No. 114 to be roughly ıe centimeter longer than those obtained by No. 101. The evidence was not conclusive. The difference is insensible to the work at hand and is just about the limitation of the Geodimeter itself.

Geodimeter used for all distance measurements.

Electro-Tape Observations

About the middle of May the Geodimeter measurements had fallen so far behind the triangulation that it was impossible to complete the lines without holding up tower dismantling. In an effort to speed up operations, an attempt was made to measure lines with the Electro-Tape. Eleven lines were measured with an average accuracy



Mid-temperature equipment. (Drawing not to scale.)

of about 1:90,000. Since the observations were not accurate enough for the project, observations were discontinued on June 10.

ASTRONOMICAL AND SPIRIT LEVEL OBSERVATIONS

A requirement of the survey was the determination, within about 3 feet, of the relative spheroidal elevations of the nine ballistic camera points. This involved spirit leveling to obtain the topographic elevations, and some astronomical observations to supply sufficient deflection information for an astro-geodetic determination of the spheroid-geoid relation.

Accordingly, some 130 miles of double-run second-order leveling were accomplished to tie the camera points to the leveling net and 15 modified first-order astronomical latitudes and longitudes were determined at points selected to best support the required astro-geodetic leveling. Since there were no unusual problems in those operations, details are omitted.

COMPUTATION OF GEOID HEIGHTS

A total of 24 deflection stations were employed to determine geoid heights of the nine BC-4 camera stations with respect to an assumed zero value for station *Hub* on Cape Canaveral. Gravimetric deflections were determined for two of the stations at which astronomic observations were not available. Of the 22 astronomic stations used, 15 were observed specifically for this project. The least-squares adjustment of the geoid height network, involving quadrilaterals, triangles, and a closed loop of 12 stations, indicated a probable error of ± 5.8 cm. in the geoid height of *Brick*, the station most distant from Cape Canaveral. The maximum range of geoid height throughout the network was 191.6 cm.

MANPOWER

The following data on manpower are only approximate, but should be of interest to those who have had anything to do with geodetic operations.

Field work started about the middle of February 1960 and was completed in a little more than five months.

Operation	*Man-months*
Reconnaissance	4
Triangulation observations	118
Geodimeter observations	29
Spirit level observations	16
Astronomic observations	6
Triangulation-Geodimeter adjustment	4*
Leveling adjustment	2
Astronomic computations	3

*Plus 15 hours of 650 computer time.

THE ADJUSTMENT OF OBSERVATIONS

General Statement

The adjustment of the direction and length observations was carried out simultaneously by the method of variation of coordinates. Geodimeter length measurements were introduced as observation equations with weights equal to those for directions. That is, a correction of 1:206,000 (arc 1") to a length is considered the same as a correction of one second to a direction. Only two stations were held fixed, *Central* and *Central S.E. Base*. These points are on Cape Canaveral and it was thought best not to upset the conformity of this line to the many other points already adjusted in the area, particularly since its azimuth and length had been well determined. Three Laplace azimuths were included in the adjustment, each given a weight of one half.

All length measurements were reduced to the geoid. Since the relative geoid heights ranged only about two meters over the area, no significant error was introduced.

As can be seen by the diagram of the project, there is a heavy redundancy of observations, both in distance and in direction. This is essential to high positional accuracy to be obtained by a least squares adjustment. Observations in this adjustment of 63 stations are comprised of 400 horizontal directions, 43 Geodimeter lengths, and 3 Laplace azimuths, or a total of 446 observations. The mathematical solution was carried out on the 650 intermediate electronic calculator. From the 446 observation equations, 189 normal equations were formed. These were solved by a square-root elimination technique, with an additional routine which determined the weight coefficients for the nine selected ballistic camera points.

Determination of the Probable Error of a Position

A measure of the strength of the figure is given by the weight coefficients of the stations in the figure. These weight coefficients are independent of the accuracy of the survey, but the probable error of the position of a station is a function of the accuracy of the survey and the strength of the figure. The larger the weight coefficients, the smaller the probable error of position will be.

Denote the probable error of a single observation by r_0, the weight coefficient by u, and the probable error of position r_p. There will be two weight coefficients for each station, one for lat

tude and one for longitude. Then $r_p = r_0\sqrt{u_\phi + u_\lambda}$*

here $r_0 = .6745\sqrt{\frac{\Sigma v^2}{f}}$

in which v is the correction to an observation and f is the number of observations minus the number of unknowns.

This gives the probable error of a position in seconds of arc which can be converted to feet. The accuracy of the position of a station is related to a fixed station which is assumed to be errorless. For example, if the probable error of a station 40 miles from the fixed station is found to be 0.25 foot, we say that the relative accuracy of this station is 1:845,000.

Statistics on Observational Accuracies

Average triangle closure (183 triangles)	0".44
Maximum triangle closure	1".93
Average correction to a direction	0".18
Maximum correction to a direction	0".77
Maximum correction to an angle	1".18
Average correction to a Geodimeter measurement	1:1,210,000
Maximum correction to a Geodimeter measurement	1:355,000
Average correction to a Laplace azimuth	1".07
Maximum correction to a Laplace azimuth	1".57
Probable error, r_0, of a single observation	0".21**
Speed of light employed	299,792.5 km./sec.

**For a Geodimeter observation this is equivalent to 1:980,000.

Probable error, r, is related to standard error, σ, by the equation $r = 0.6745\sigma$.

Statistics on Positional Accuracy of Ballistic Camera Stations, Relative to Station Central

Camera Station	Distance from *Central*	Probable error	
	Miles	*Foot*	*1 part in*
1	67	0.29	1,220,000
2	58	0.25	1,220,000
3	49	0.20	1,290,000
4	59	0.22	1,420,000
5	49	0.18	1,440,000
6	51	0.18	1,500,000
7	49	0.18	1,440,000
8	38	0.14	1,430,000
9	51	0.21	1,280,000

Probable Errors of Certain Unobserved Adjusted Lengths

Length	Probable error *1 part in*
Camera Station 9 to *Central*	2,080,000
Camera Station 8 to *Central*	1,530,000
Camera Station 8 to Camera Station 9	1,420,000

*This is not a rigorous expression for the probable error of a position; it does not define the axes of the error ellipse. Such a refinement was not deemed necessary to the problem at hand, since the accuracy attained far exceeded the specifications.

These probable errors were computed by means of functional equations written for the length differential as a function of latitude and longitude differentials. By means of a bordering technique, the weight coefficients of the lengths were found along with the solution of the normal equations, and from these the probable errors follow directly from the relation $r_S = r_0\sqrt{u_S}$.

CONCLUSIONS

This project has clearly demonstrated that, with careful planning and an assiduous application and attention to details in the field, a very high degree of accuracy is practicable. It was confirmed, in quite a lucid manner, that the requirement of at least two nights' observations at each triangulation station is indispensable if superior results are sought. The accuracy of the scale attained for the triangulation network was made possible only by the use of many Geodimeter measurements. It turns out, however, that the number of such measurements could probably have been substantially reduced in view of the low probable error of an observed direction. Those in responsible charge of the field work, particularly the triangulation and Geodimeter observations, deserve the approbation of geodesists everywhere.

It does not follow, as a result of this work, that comparable accuracies can be attained everywhere. The conditions in Florida are singularly favorable to high precision of operations. The

terrain is flat, thus no steep lines were encountered. All observations were made from the tops of Bilby towers and over ground covered with trees or other vegetation. This avoided the troubles that can arise from the use of 4-foot stands, and also reduced the adverse effect of a "boiling" atmosphere. Additionally, temperatures were quite uniform, making it possible to obtain accurate mean temperatures needed for superior Geodimeter measurements. There are many parts of the country in which none of these favorable conditions exist and where such high-order results might be attained only by unreasonable expenditure of funds and effort, if, indeed, they could be attained at all.

Favorable conditions notwithstanding, the results of this project are quite remarkable. The ultimate accuracy was not, as it never is, reached. But the attainment of a significantly higher accuracy, over this large an area, would require operation under the law of diminishing return—a grossly disproportionate expenditure of time and funds. The writer feels that a substantial portion of the ingenuity, care, and precision built into the instruments employed on this work have been wrung from them.

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