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Film Distortion Compensation for Photogrammetric Use

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Technical Bulletin No. 14, describes the irregular distortion found in the film base used in aerial photography, the methods by which the errors from that cause can be reduced, and the mathematical derivation of the formulas of compensation.

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Film Distortion Compensation for Photogrammetric Use

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THE irregular distortion of aerial photographic film has been a subject of concern ever since the metrical use of aerial photographs began. The ease of handling film as compared to glass plates, results in a preponderant economical advantage for the use of film. A focal-plane reseau plate is currently being used successfully in Great Britain, but American photogrammetrists tend to avoid its use because of several inconvenient factors. In about 1940, manufacturers began supplying aerial film on a "topographic" base, which had improved stability over the former bases normally used in roll-film photography.

Not until 1959, were further changes made in the film base (except for some nonphotogrammetric applications), although certain polyester and other plastics having greatly improved stability had been used for about 10 years for drafting and cartographic materials. In March 1960, Mr. William T. Harmon of the U. S. Geological Survey reported, at the annual convention of the American Society of Photogrammetry, that tests made with a new plastic aerial film showed an instability of essentially one-fifth of that obtained for the conventional topographic (acetate) base.

MAGNITUDE OF INSTABILITY

The measurement of the quantitative amount of distortion has been made by various authors and is repeated here. For introductory purposes, it may be said that the maximum distortion for the topographic base film along one diagonal of a 9-by 9-inch photograph varies between 500 and 1,000 microns (0.5 to 1.0 millimeter; 0.02 to 0.04 inch). Not only does the magnitude of the radial distortion vary greatly from photograph to photograph in the same roll of aerial film, but also the tangential component varies from photograph to photograph as well as within a single photograph.

As a crude example, if a piece of film is laid over a master camera plate on a light table, one can see, without any viewing aid, a discrepancy of roughly 1 millimeter on one diagonal. In this study, however, measurements were made with a comparator which had a standard error of only 2 microns, where the least reading was 1 micron, and which printed out coordinate data.

The 1,000-micron variance may be considered for rough purposes as composed of four linear and two nonlinear constituents:

Linear components	*Microns*
1. Over-all uniform scale ..... (about)	585
2. A scale in the y-direction different from that in the x-direction	110
3. Uniform rotation of the coordinate system.....	90
4. A rotation of the y-axis different from that of the x-axis	40

(A straight line remains invariantly a straight line under all of the linear components.)

Nonlinear components	
1. A curvature component in the x-direction.....	60
2. An independent curvature component in the y-direction	40
Uncorrectable residual	75
Total	1,000

These rough figures represent maximum errors if a given component is neglected. As the discrepancies tend to be randomly distributed with respect to the direction and magnitude of the distortions, the standard deviations may be considered to be roughly one-third of these values. Thus, the standard residual error after all efforts are exerted to eliminate the effect of the distortion is about 25 microns, based on a total of 1,000 microns gross initial discrepancy. Actually, most maximum gross discrepancies are of the order of 500 to 750 rather than 1,000. Consequently, one frequently refers to 15, 20, or 25 microns as the discrepancy to be expected from film shrinkage.

COMPENSATION EQUATIONS

A primary objective of this study is the determination of a system for the adjustment of

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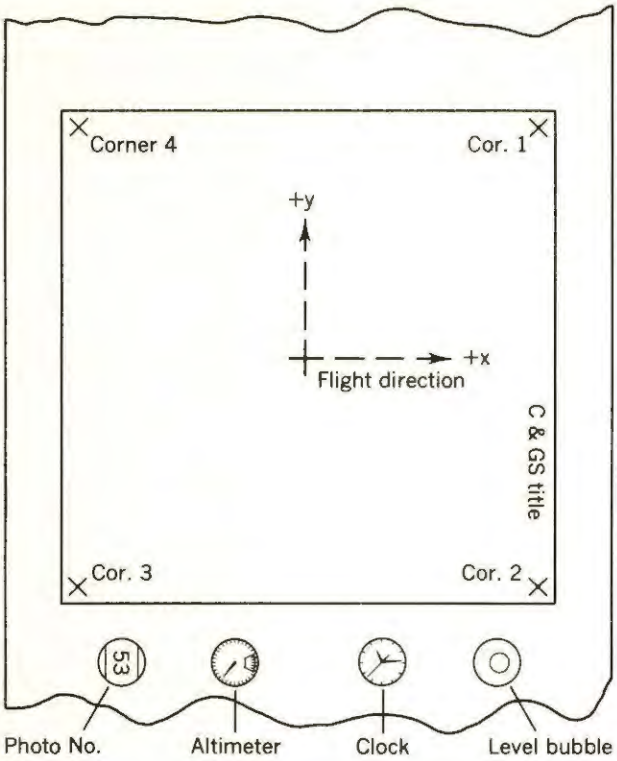

FIG 1.--Diagram of camera format indicating corner numbers.

image coordinates to compensate them for the effect of film distortion. As a basis for the study, a fiducial mark is located in each of the four corners of the aerial camera (fig. 1), and these marks are projected onto the film at the instant of photography. The specific orientation of the marks is assured by a group of data instruments photographed along one margin: a clock, a level bubble, an altimeter, and an exposure counter. A master photograph of the marks and data instruments was made on a thick glass plate in the laboratory to determine the correct locations of the fiducial marks.

After ordinary aerial film is exposed, developed, and dried, a contact print on glass (diapositive) is made to arrest any further distortion.

Consider two lists of coordinates of the four fiducial marks, one list measured on the master glass plate and the other on a conventional diapositive. To study the discrepancies, the coordinates need to be expressed in a comparable Cartesian or other system, whereas a measuring comparator yields coordinates in an arbitrary instrument system which can best be transformed into any other desirable system numerically by affine translation, rotation, and dilation techniques of analytic geometry. In figure 1, the corners are arbitrarily numbered 1, 2, 3, and 4. Corner 3 is made to agree in the two systems by a translation of the diapositive coordinates; then the diapositive coordinate system is rotated about Corner 3 so that Corner 2 agrees in the y-direction. Thereupon discrepancies may be evident in the values of the coordinates at Corners 1 and 4. By the use of different scales in the x and y-directions, Corner 4 can also be made to agree. To make the last two coordinates at Corner 1 agree, the following non-affine equations proposed by the writer were used:

$$x' = x + r\,xy$$
$$y' = y + s\,xy$$

where

$$r = \frac{x_1^* - x_1}{x_1 y_1}$$

$$s = \frac{y_1^* - y_1}{x_1 y_1}$$

in which

x_1, y_1 = diapositive coordinates for Corner 1

x_1^*, y_1^* = corresponding master plate coordinates.

Consequently, Corner 1 will thereafter also agree in both x and y.

Thus all four corner marks can be made agree in a systematic manner, and the distortion due to film instability is "compensated." The *degree* of compensation remains to be discussed.

RESULTS

Although this method of compensation may be a logical manner of solving the problem, it cannot be assumed that the kind of compensation will necessarily agree with the character of the film instability. The studies showed, however, that the character of instability was not constant and varied in an apparent random fashion, therefore this manner of compensation is considered to be appropriate until more knowledge is gained about this "character" of instability. Moreover, it is obvious from the equations that: (1) no bow or bending is allowed for points along either of the x or y axes through Corner 3; (2) the discrepancies are "squeezed" by the equations from the four corners into the midpoints of the four sides; (3) the corrections at the midpoints of the sides are about one-half of those at the corners; and (4) the compensation at the photo center is about one-fourth that of Corner 1 (see fig. 2.) Nevertheless, a gross discrepancy of 1,000 microns will probably result, before any of the

operations, in a standard discrepancy of about 25 microns after compensation; inasmuch as images are utilized only four-fifths of the distance from the center to the fiducial marks, this value may be considered as reduced to about 20 microns.

The value of 20 microns is too large for the best use of photography analytically where it is possible to measure and compensate for other systematic discrepancies (lens, earth, air, observations), with a residual of only 4 microns, and where glass plates and reseaus yield a value of zero. However, the value is considerably better than making no compensation at all for differential scale, differential rotation, and the non-affine effects, which the current map plotting apparatus tolerates. As previously indicated, these compensations account for 250 microns of distortion, leaving 75 uncompensated, where 675 microns have little or no serious effect on the photogrammetric solution, and where the total gross discrepancy is considered to be 1,000 microns.

NEW PLASTICS AND MORE FIDUCIAL MARKS

By the use of a special plastic stable film base, the value of 20 microns standard error can be reduced to less than 5 microns, which is very attractive indeed. This is a practical value, inasmuch as it may be some time before small values can be obtained in practice for the other systematic residuals.

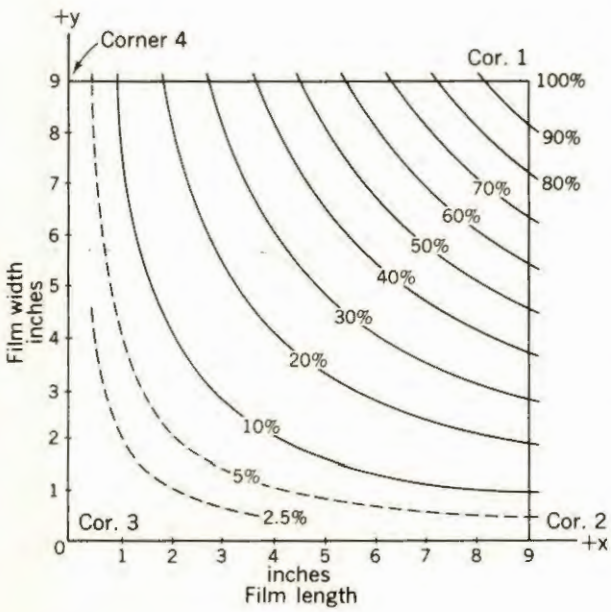

FIG. 2.—Distribution of nonlinear portion of film distortion over photographic area by employing the formulas $x' = x + r\ xy$ and $y' = y + s\ xy$, and corner 3 as origin.

But it is also possible to go a step further: install more than the four fiducial marks and proceed to "squeeze" the discrepancies into smaller confines; and also install one fiducial mark in the center. This has been proposed to the manufacturer of the aerial camera, and the firm is very attentive inasmuch as it will allow the camera to compete favorably with glass plate and reseau cameras for analytical photogrammetry. The marks near the midpoints of the sides of the format present no particular mechanical difficulty but the one in the center is not so easy. However, the center mark is *possible* inasmuch as the nine-lens camera of the Coast and Geodetic Survey has been functioning with one since 1944. The mark is projected from the rear through the antihalation backing of the film. The effect of using more fiducial marks is to reduce the residual error in proportion to the square of the distance between adjacent marks. Consequently, if five more fiducial marks are installed, reducing the distance from 9 to 4 1/2 inches, the residual errors from this source will be quartered; if the center point is omitted, the ratio will lie somewhere between 1/2 and 1.

CONCLUSIONS

Thus it is evidently possible, with the use of eight or nine fiducial marks and the new stable film base, to compensate for film distortion to the extent that standard relative residual errors of distances measured with a comparator can be reduced to the order of 2 microns, which is comparable to, or less than, the reliability of observations and image resolution. This is the goal toward which this study was instituted and for which the compensation equations are a necessary adjunct. The study has shown that without the latter two innovations (new film base and more marks), topographic-base film may not yield an accuracy consistent with the other accuracies of the system. A computational routine is given below.

These equations were adopted by the Photogrammetry Division of the Coast and Geodetic Survey—they were formulated ready for programing by computer along with the routines for lens distortion and atmospheric refraction corrections. To test these principles further, a roll of the new stable film was procurred, and experimental photography was recently completed over a special targeted test area along U. S. Route 11 in the Shenandoah Valley, Va. This test area is approximately 100 miles long and 5 miles wide. It contains 40 horizontal control points which can be marked with panels large enough to be imaged on 1:40,000 scale aerial photographs. Also, in the area are sufficient precise level lines from which the elevations of photogrammetric level points can be measured.

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APPENDIX 1

LIST OF COMPENSATION EQUATIONS

The equations for all the necessary computational steps are stated. Consider that a list of measured x and y coordinates are given for a score or so of images on a photograph, where the first four images are those of the corner points.

1. Translation

$$x_i' = x_i - x_3 \qquad y_i' = y_i - y_3$$

2. Rotation and scaling

$$x_i'' = mx_i' + ny_i' \qquad y_i'' = px_i' + qy_i'$$

let $\quad d = x_2'y_4' - x_4'y_2'$

then $\quad m = (X_2 y_4' - X_4 y_2')/d$

$$n = (X_4 x_2' - X_2 x_4')/d$$

$$p = Y_4 y_2'/d$$

$$q = Y_4 x_2'/d$$

3. The hyperbolic correction

$$X'_i{}'' = x_i''(1 + ry_i'') \qquad y'_i{}'' = y_i''((1 + sx_i'')$$

$$r = (X_1 - x_1'')/x_1''y_1'' \qquad s = (Y_1 - y_1'')/x_1''y_1''$$

Although this completes the film distortion compensation, a further operation is needed which, although entirely independent in purpose, must be incorporated. For the photogrammetric problem the origin of image coordinates is the principal point, or photo center. Consequently, a final translation is required which could not have been performed previously:

$$x^* = x_i''' - x_p \qquad y^* = y_i''' - y_p$$

where x_p and y_p must be known constants.

APPENDIX 2.

NUMERICAL EXAMPLE

Consider the following table of coordinates of the four corners of the master plate and a test plate (fictitious example), where it is possible to place the test plate on the measuring comparator in only an approximate position and orientation. The coordinates of the test plate corners have already been transformed into the indicated form by affine operations.

Corner mark	Master plate X mm.	Master plate Y mm.	Test plate X mm.	Test plate Y mm.
1	212.0718	211.9859	245.2260	251.4210
2	212.0129	0	245.2530	39.8000
3	0	0	33.5500	39.7350
4	0.0550	211.9771	33.6000	251.3920

The purpose of the affine transformation is to adjust the test plate coordinates so that zeros occur in the same three places indicated by the master values, and also the other cordinates, except for Corner 1, should be identical. It is accomplished by two computational steps: (1) translation, and (2) rotation combined with different scale factors in the x and y directions, where the rotations about the x and y axes are allowed to be different.

TRANSLATION

Obviously, the two zeros in Corner 3 can be obtained by subtracting the indicated values from each of the values in the columns of the coordinates of the test plate. In general, for any image on the photograph, the translated coordinates x', y' are:

$$x_i' = x_i - x_3$$

$$y_i' = y_i - y_3$$

If these equations are applied, the new coordinates for the test plate are:

Corner	x'	y'
1	211.6760	211.6860
2	211.7030	+ 0.0650
3	0	0
4	+0.0500	211.6570

ROTATING AND SCALING

The specific purpose of this transformation is to enforce the four master-plate values at Corners 2 and 4, allowing Corner 1 to take on whatever values it may. All four parameters are enforced simultaneously.

We recall the form of the standard rotation formulas:

$$x' = x\cos\theta - y\sin\theta, \text{ or } x' = ax - by$$

$$y' = x\sin\theta + y\cos\theta \text{ or } y' = bx + ay$$

Here the angle between the axes is 90°; and the scale factor is unity: $a^2 + b^2 = 1$. The 90° condition is enforced by having the same terms, a and b, in both equations.

But in our problem we do not want to enforce these restrictions, but allow a and b to take on other values:

$$x'' = mx' + ny'$$

$$y'' = px' + qy'$$

Now $m^2 + n^2 \neq p^2 + q^2 \neq 1$ necessarily, but values are found such that the four corner numbers will agree. This is done through the use of simultaneous linear equations for determining the values of $m, n, p. q$.

$$X_2 = x_2 m + y_2 n$$

$$X_4 = x_4 m + y_4 n$$

Inasmuch as all the x's and y's are known values from the master and test plates,

$$m = \frac{X_2 y_4 - X_4 y_2}{x_2 y_4 - x_4 y_2} \qquad n = \frac{X_4 x_2 - X_2 x_4}{x_2 y_4 = x_4 y_2}$$

Two similar equations can be written for y-coordinates of Corners 2 and 4:

$$Y_2 = x_2 p + y_2 q$$

$$Y_4 = x_4 p + y_4 q$$

from whence

$$p = \frac{-Y_4 y_2}{x_2 y_4 - x_4 y_2}$$

$$q = \frac{Y_4 x_2}{x_2 y_4 - x_4 y_2}$$

The values for m, n, p, q for this problem, therefore are:

$m = +1.00146383$

$n = +0.00002328$

$p = -0.00030751$

$q = +1.00151242$

and the new coordinates for the test plate after application of these equations are:

Corner	x mm.	y mm.
1	211.9908	211.9411
2	212.0129	0
3	0	0
4	+0.0550	211.9771

Thus the only values which are different from those of the master plate are the values for Corner1. The residual discrepancies after the foregoing linear transformations are:

X_1	212.0718 *mm.*	Y_1	211.9859 *mm.*
x_1	211.9908	y_1	211.9411
	+0.0810 *mm.*		+0.0448 *mm.*

or 81 microns in x and 45 microns in y.

Therefore, the discrepancies have been "squeezed" into Corner 1 by affine methods, that is, without applying any curve techniques, and preserving the collinearity of all lines on the photograph.

THE NON-AFFINE OR HYPERBOLIC TRANSFORMATION

The formulas are repeated in general form:

$$x' = x + r\,xy \qquad y' = y + s\,xy$$

Inasmuch as the algorithm is based solely on the two coordinates of Corner 1, after the linear transformations, the values for r and s can be solved directly by transposition:

$$r = (X_i - x_i)/x_1 y_1 \qquad s = (Y_i - y_i)/x_1 y_1.$$

In the numerical example,

$r = +1.8028 \times 10^{-6}$ $\qquad s = +0.9971 \times 10^{-6}$.

Then the compensated coordinates of the corner points are exactly identical to the master values given initially.

Now the coordinates of any other image on the photograph may be compensated for film distortion by the application of the foregoing formulas, using the values x_3, y_3, m, n, p, q, r, s as constants.

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