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# On the Time Interval Between Two Consecutive Earthquakes

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Technical Bulletin No. 17 describes a statistical study made of the frequency distribution of the time interval between two consecutive earthquakes occurring near Guam. In the case of aftershock sequences, certain frequency distributions are observed for those shocks occurring in the whole affected area, but these distributions are almost explained by the decrease in aftershock activity with time.

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# On the Time Interval Between Two Consecutive Earthquakes

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AS A MEANS for investigating the effect of the occurrence of an earthquake on the occurrence of the others, a statistical study has been made of the frequency distribution of the time interval $\tau$ between two consecutive earthquakes occurring near Guam Island. The observed frequency distribution (see fig. 4) differs from the exponential distribution which is expected when earthquakes occur completely at random with respect to time. It also differs from the $\tau^{-q}$ or $(\tau+\gamma)^{-q}$ type distribution, but it can be derived, as a first approximation, from the superposition of the $(\tau+\gamma)^{-q}$ type distributions assumed to be applicable to each elemental region. In the case of aftershock sequences, frequency distributions of $\tau^{-q}$ or $(\tau+\gamma)^{-q}$ type are observed for aftershocks occurring in the whole aftershock area, but these distributions are almost explained by the decrease in aftershock activity with time.

## 1. INTRODUCTION

If $N$ earthquakes occur completely at random with respect to time during a period of $T$, the frequency distribution of the time interval $\tau$ between two consecutive earthquakes should be expressed by

$$f(\tau) = (N/\bar{\tau})e^{-\tau/\bar{\tau}}, \quad \bar{\tau} = T/N. \qquad (1)$$

This exponential distribution approximately agrees with observations made by several authors (see, for example, Terada[1]). An example of such an exponential distribution is also shown in figure 1, in which the frequency distribution of $\tau$ is plotted on a semilogarithmic diagram for earthquakes recorded by the Benioff short period vertical seismograph at College, Alaska, with maximum trace amplitude of 5 mm. and larger during

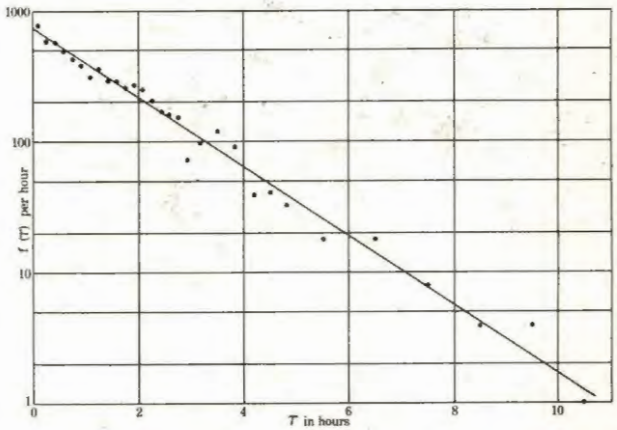


FIG. 1. Frequency distribution of the time interval between two consecutive earthquakes recorded at College. The straight line indicates the calculated frequency from equation (1) putting $N$ = 1,213 and $T$ = 2,012 hours.

[1]On the frequency of earthquakes and allied phenomena by T. Terada, *Proc. Tokyo Math. Phys. Soc.* 9: 515-522, 1918.

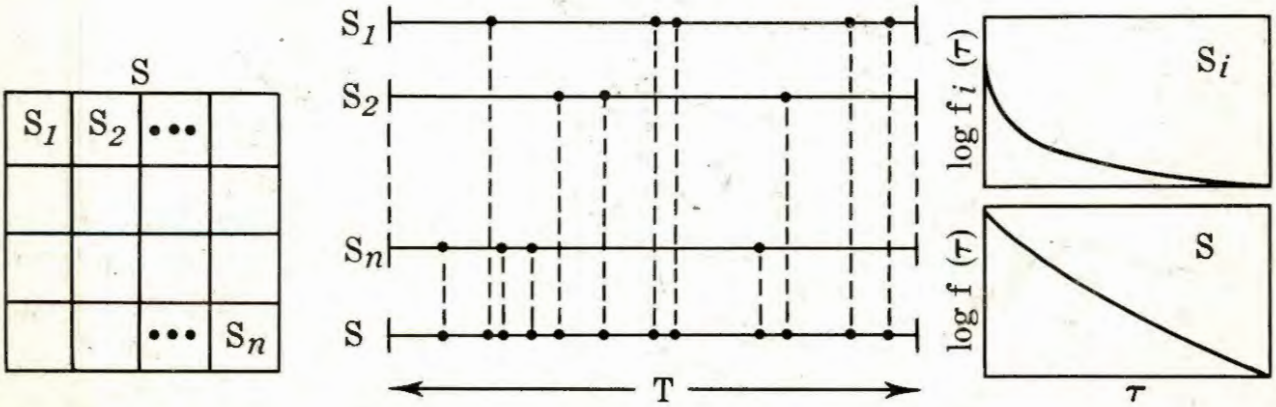


FIG. 2. Model I. A region $S$ consists of many elemental regions $S_i$ ($i$ = 1, 2, . . . , $n$). Each dot in central part indicates the time of occurrence of an earthquake in regions $S_i$ and $S$. Even if the frequency distribution of $\tau$ differs widely from the exponential for each elemental region (like the upper right curve), the distribution for the whole region $S$ becomes closer to the exponential (like the lower right curve).

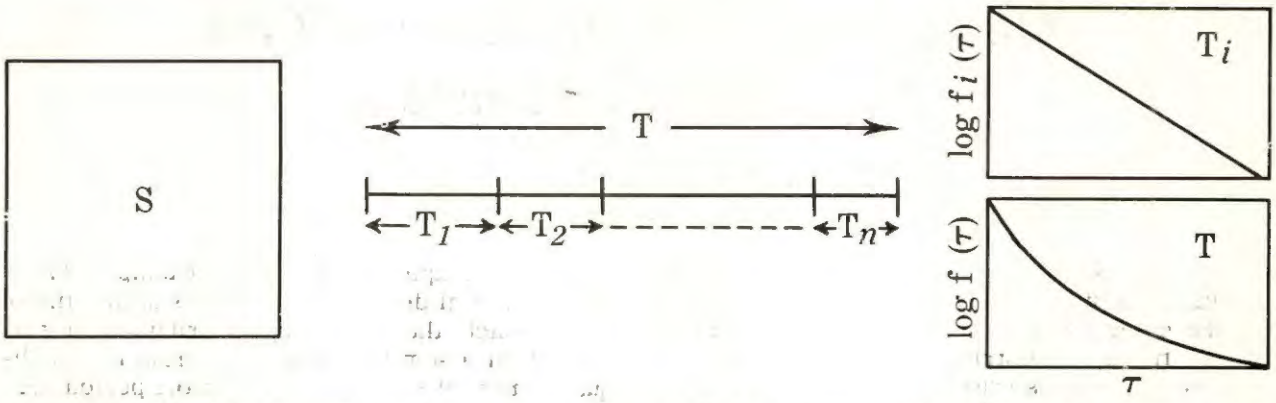


FIG. 3. Model II. If the frequency distribution of $\tau$ for each period $T_i$ ($i$ = 1, 2, ..., $n$) is exponential with different average time interval, the frequency distribution for total period $T$ becomes different from the exponential.

the period from October through December 1958. The plotted points fit closely a straight line computed from equation (1).

The fact that the observed frequency distribution of $\tau$ is expressed by equation (1) does not necessarily mean that all earthquakes concerned occur completely independently with each other. Suppose a model of earthquake occurrence shown in figure 2, in which a region marked with S consists of many elemental regions $S_1$, $S_2$, ..., $S_n$, and in each region the occurrence of an earthquake may have an effect on the occurrence of the other ones in that region but not on those in the other regions. The frequency distribution, $f_i(\tau)$, of $\tau$ for region $S_i$ ($i$ = 1, 2, ..., $n$) must be different from the exponential as is illustrated in upper right of figure 2. However, if we plot the frequency distribution, $f(\tau)$, for all earthquakes occurring in region $S$, its departure from the exponential may be smaller because the distribution of origin times of earthquakes may be more nearly random in region $S$ than in region $S_i$.

On the other hand, in another model shown in figure 3, a period of $T$ consists of many periods $T_1$, $T_2$, ..., $T_n$, and during the period $T_i$, $N_i$ earthquakes occur independently with each other in region $S$. The frequency distribution $f_i(\tau)$ for period $T_i$ is exponential, but the frequency distribution $f(\tau)$ for total period $T$ becomes

$$f(\tau) = \sum_i f_i(\tau) = \sum_i (N_i/\bar{\tau}_i)e^{-\tau/\bar{\tau}_i}, \quad \bar{\tau}_i = T_i/N_i. \quad (2)$$

This is no longer exponential unless all the mean time intervals $\tau_i$ are of the same value.

Y. Tomoda[2] found that the frequency distribution of the time interval between two consecutive earthquakes which took place at Volcano Asama, Japan, was expressed by the empirical equation

$$f(\tau) = k\tau^{-q}, \quad (3)$$

which is essentially different from the exponential distribution.

This equation is somewhat difficult to treat because the total number of earthquakes, $\int_0^\infty f(\tau)d\tau$, become infinity. Therefore, it has been modified here to add a small constant $\gamma$ to $\tau$, that is

$$f(\tau) = k(\tau + \gamma)^{-q}. \quad (4)$$

This is practically equal to equation (3) for $\tau \gg \gamma$.

In this paper, an attempt has been made to explain the observed frequency distribution of $\tau$ for earthquakes originating near Guam Island under the assumption that they occur in such a way that the distribution of $\tau$ follows equation (4) if we take statistics on earthquakes occurring in regions which are small enough to be regarded as an elemental region in the model shown in figure 2.

## 2. OBSERVATIONAL DATA FROM GUAM

Many local earthquakes were recorded by seismographs on Guam Island during IGY 1957-58, whose $S$ - $P$ time intervals ranged from 4 sec. to 60 sec. As the activity of these local shocks was fairly uniform with respect to time, they are not regarded as the aftershocks of a particular earthquake. Figure 4 shows the frequency distribution of $\tau$ plotted on a semilogarithmic diagram for these shocks recorded by the Benioff long period seismograph (N-S component) with maximum trace amplitude of 5 mm. and larger. This distribution does not fit a straight line nor any curve computed from equation (4).

Based on the assumption stated above, if a large region of seismic activity is divided into many small regions and the frequency distribution of $\tau$ for earthquakes occurring in each small region is plotted, this distribution may deviate from the exponential more widely than the distribution for all earthquakes in the large region.

[2]Statistical description of the time interval distribution of earthquakes and on its relations to the distribution of maximum amplitude by Y. Tomada, *Zisin (J. Seism. Soc. Japan)*, 7: 155-169, 1954.

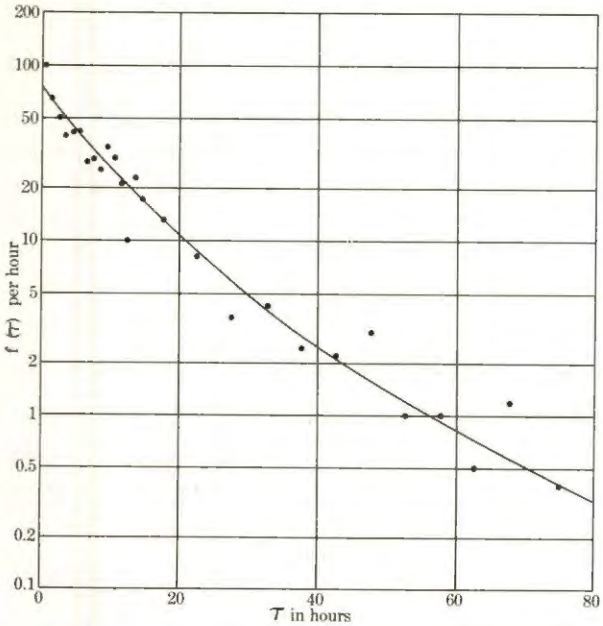


FIG. 4. Frequency distribution of the time interval between two consecutive earthquakes recorded on Guam. The curve represents the calculated frequency distribution for $\bar{\tau}/\gamma = 0.5$ by the method described in section 3. $\bar{\tau} = 13.4$ hours as $N = 783$ and $T = 10{,}512$ hours.

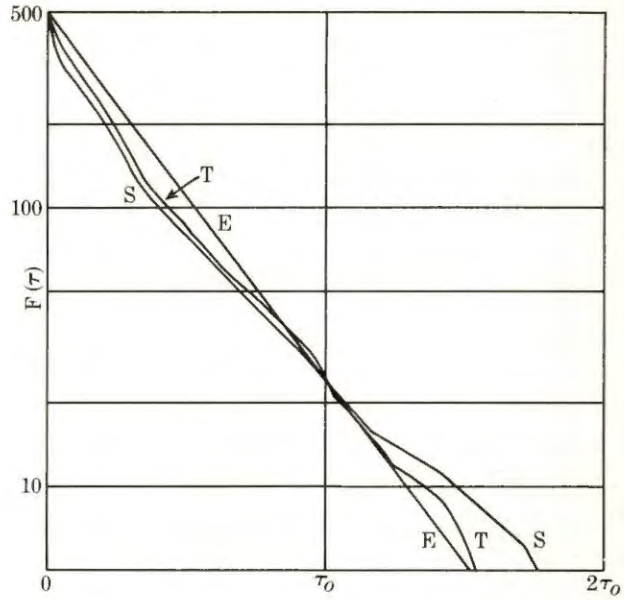


FIG. 5. Cumulative frequency distribution of $\tau$ for local shocks recorded on Guam. Curve $S$ shows the sum of the frequency for each group of shocks classified according to table 1. Curve $T$ indicates the frequency distribution for total shocks concerned. Straight line $E$ is the exponential distribution.

In order to confirm this, the shocks recorded on Guam were classified into several groups according to $S - P$ time intervals. The number of shocks which fall into each group is given in table 1. As the number of each group is approximately the same, the frequency distributions for every group were summed up and plotted on figure 5 in the form of the cumulative frequency distribution designated $S$. The curve $T$ is the cumulative frequency distribution for all earthquakes concerned. The abscissa for curves $S$ and $T$ is taken such that they meet at $\tau_0$ defined by $F(\tau_0) = F(0)/20$. A comparison with the exponential curve (the straight line marked with $E$) indicates that the departure from the exponential is larger for curve $S$ than curve $T$ as is expected.

*Table 1*

| $S - P$ (sec.) | Number of shocks |
|---|---|
| 5, 6, 7 | 100 |
| 8, 9 | 105 |
| 10, 11 | 99 |
| 12, 13 | 102 |
| 14, 15 | 93 |

## 3. COMPARISON BETWEEN OBSERVED AND CALCULATED DISTRIBUTIONS

If the frequency distribution of $\tau$ is expressed by equation (4) for small elemental regions, it is possible to obtain the frequency distribution of $\tau$ for all earthquakes occurring in a large region composed of many elemental regions by a method of statistical experiment. In this case it is not necessary to specify the value of $\gamma$ in equation (4), because if we put $\tau/\gamma = \tau'$, equation (4) becomes

$$f(\tau') = k'(\tau' + 1)^{-q}. \tag{5}$$

If the empirical relation described by Tomoda[3] between $q$ and the index $m$ of Ishimoto-Iida's formula, $f(a) \propto a^{-m}$, for frequency distribution of maximum amplitude $a$ is valid generally, that is, if

$$q = (m + 1)/2, \tag{6}$$

the value of $q$ may be between 4/3 and 3/2, as the value of $m$ falls between 1 2/3 and 2 for ordinary earthquakes.

The tables of random digits were used to make 2,000 imaginary origin times of earthquakes for

[3]Ibid.

which the frequency distribution of $\tau$ follows equation (5) with constant $q = 4/3$. This imaginary earthquake sequence is divided into many sections and they were superposed in a manner shown in central part of figure 2 in order to produce three earthquake sequences with average $\tau'$ of 50, 5, and 0.5. The results are shown in figure 6 in which the cumulative frequency distribution, $F(\tau)$, of $\tau$ is plotted for original distribution given by equation (5) (marked with $R$), three superposed cases, and the exponential distribution (marked with $E$). The abscissa is taken in the same manner as in figure 5.

It is seen from this figure that according to the decrease of $\bar{\tau}'$ or $\bar{\tau}/\gamma$, the frequency distribution curve approaches the exponential distribution. The frequency distribution of $\tau$ for Guam local shocks is again represented by a broken line in the same figure. This broken line seems to be in good agreement with the curve of $\bar{\tau}' = 0.5$.

## 4. TIME INTERVALS BETWEEN TWO CONSECUTIVE AFTERSHOCKS OF AN EARTHQUAKE

When earthquakes occur independently with each other in a certain region, but their frequency per unit time interval, $n(t)$, gradually varies with time $t$, the frequency distribution of $\tau$ from time $t_1$ to time $t_2$ is expressed by

$$f(\tau) = \int_{t_1}^{t_2} n(t)^2 e^{-n(t)\tau} dt \qquad (7)$$

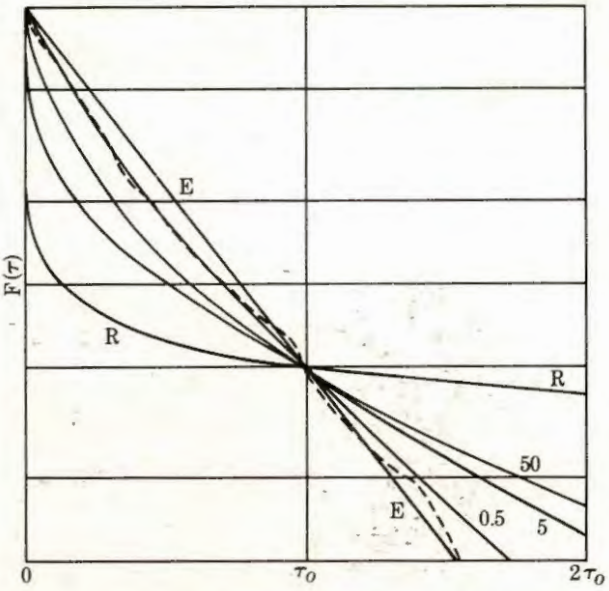


FIG. 6. Cumulative frequency distribution of $\tau$ calculated in the case of $\tau' = \tau/\gamma = 50$, 5, and 0.5. Curve $R$ indicates the original distribution expressed by equation (4). Broken curve is the same as curve $T$ in figure 5.

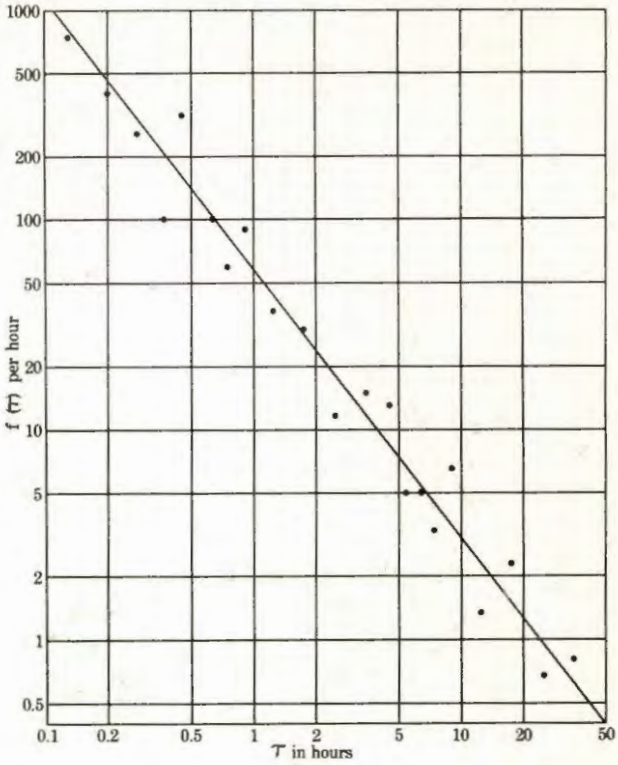


FIG. 7. Frequency distribution of the time interval between two consecutive aftershocks of the Alaskan earthquake of July 10, 1958.

in the same way as equation (2).

If the frequency of aftershocks of an earthquake decreases with time $t$ as given by

$$n(t) = A(t + c)^{-p}, \qquad (8)$$

the frequency distribution of $\tau$ from the origin time of the main shock to a long time later may be written

$$f(\tau) = \frac{\Gamma(2 - \frac{1}{p})}{A\left(1 - \frac{1}{p}\right)p} \tau^{-(2 - \frac{1}{p})} + \int_{c-p}^{\infty} \frac{A}{p} x\left(1 - \frac{1}{p}\right) e^{-(A\tau x)} dx. \qquad (9)$$

If $c = 0$, the second term of the right side of this equation vanishes and $f(\tau)$ takes the same functional form as equation (3).

For the aftershock sequence of the Alaskan earthquake of July 10, 1958,[4] the value of $p = 1.13$ and $c \doteqdot 0$ had been obtained. Figure 7 shows the

[4]On the nature of three Alaska aftershock sequences of 1957 and 1958 by T. Utsu, *Bull. Seism. Soc. Am.*, 52: (in press), 1962.

frequency distribution of $\tau$ for this aftershock sequence plotted on a logarithmic diagram. The plotted points fit a straight line with a slope of 1.37. From equation (9) this slope is calculated to be $2 - (1/p) = 1.12$. The small difference between observed and calculated slopes may be explained by the weak mutual dependence in the aftershock occurrence.

## 5. CONCLUSION

When the frequency distribution of time interval $\tau$ between two consecutive earthquakes are studied in many cases, we get nearly exponential distributions in some cases, and in other cases widely different ones from the exponential. An attempt has been made to explain this distribution assuming a model of earthquake occurrence shown in figure 2. Figure 6 shows a set of frequency distribution curves of $\tau$ calculated under this model with a frequency distribution given by equation (4) for each elemental region. As is seen from this figure, the frequency distribution curve gradually shifts from that given by equation (4) to the exponential distribution as the number of elemental regions concerned is increased. Accordingly, whether the frequency distribution observed is nearly exponential or widely different from the exponential depends on the size of the region concerned.

Of course this gives only a rough explanation of the problem, because the model adopted is too simple for actual seismic phenomena. Greater earthquakes may have a larger effect on the occurrence of the other ones in space and time than smaller ones do. Further study is needed taking magnitude and location of earthquakes into consideration, and this paper should be considered as a preliminary report on this complicated problem.

## 6. ACKNOWLEDGMENTS

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