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Luther H. Hodges, Secretary
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H. Arnold Karo, Director

Aerotriangulation Strip Adjustment

M. KELLER and G. C. TEWINKEL

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Technical Bulletin No. 23 comprises a documented computer program concerned only with the transformation and adjustment of the strip coordinates of points to fit ground control data. Technical Bulletins Nos. 1 and 10 presented the principal formulation still being applied in the Coast and Geodetic Survey, and No. 21 included an application of those ideas. This bulletin includes modifications which have been added since the dates of the original publications, combines the ideas of the three into a single operation, adapts the program for either instrumental or analytic aero triangulation, and embodies a systematic technique for correcting horizontal coordinates for the local inclinations of the strip.

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Aerotriangulation Strip Adjustment

M. KELLER and G. C. TEWINKEL

U. S. Coast and Geodetic Survey

AEROTRIANGULATION is a photogrammetric technique for deriving ground coordinates of objects from a set of overlapping aerial photographs that show images both of those objects and also of a relatively sparse distribution of other objects whose coordinates are known from previous classical measurements on the ground. Provisional photogrammetric coordinates of objects can be determined by at least two general methods: (1) through the use of a high-order photogrammetric plotting instrument, or (2) through analytic computations based on observed coordinates of images on the photographs. This discussion pertains specifically to both cases. In each instance, the photogrammetric strip coordinates of points comprise a thick, three-dimensional ribbon in space generally not referred specifically to any ground surveyed system of points.

Finally, the strip coordinates, in order for them to be useful, must be related to the ground system through the application of polynomial transformations in a curve-fitting procedure to adjust the photogrammetric strip coordinates to agree with known ground surveyed coordinates. This fitting technique is called ***Strip Adjustment.*** The polynomials are nonlinear because of the systematic accumulation of errors throughout the strip. The application of least squares provides a logical analysis of redundant data.

This paper comprises a documented computer program concerned only with the transformation and adjustment of the strip coordinates of points to fit ground control data.

Technical Bulletins No. 1[1] and No. 10[2] presented the principal formulation still being applied in the Coast and Geodetic Survey, and No. 21[3] included an application of those ideas. The present bulletin includes helpful modifications which have been added since the dates of the original releases, combines the ideas of the three former bulletins into a single operation, adapts the program for either instrumental or analytic aerotriangulation, and embodies a systematic technique for correcting horizontal coordinates for the local inclinations of the strip.

This program is considered to be the first of a series of two or three new programs for analytic aerotriangulation. Chronologically, this program will be used as the third step in the provisional adjustment of strips. The other two programs will consist of (1) the reduction of observed image coordinates and (2) relative orientation, including the assembly of the oriented data. The three programs will comprise a complete practical set for analytic strip aerotriangulation for use on a medium size computer.

INTRODUCTION

The adjustment of aerotriangulated strips has been the subject of numerous articles and publications for a few decades, as indicated presently by the references. The articles agree in general that the equations for transforming photogrammetric coordinates into ground coordinates can be expressed by polynomials which need to be at least second degree. Schut[4] and Mikhail[5] show that a conformal transformation in three dimensions is not possible if the degree is greater than one, although a conformal transformation in a *plane* is *not* limited as to degree. Inasmuch as an ideal solution of the three-dimensional problem does not seem to exist, photogrammetrists have been free to devise approximate, quasi-ideal, and impirical solutions that seem to give practical and usable solutions to their problems. Several examples are cited.

The Coast Survey formulas applied herein are:

$$
\begin{aligned}
x' &= x - \Delta z(3hx^2 + 2ix + j) + ax^3 + bx^2 + \\
&\quad cx - 2dxy - ey + f \\
y' &= y - \Delta z(kx^2 + lx + m) + 3ax^2y + 2bxy + \\
&\quad cy + dx^2 + ex + g \\
z' &= z\left[1 + (3hx^2 + 2ix + j)^2 + (kx^2 + lm + \right. \\
&\quad \left. m)^2\right]^{1/2} + hx^3 + ix^2 + jx + kx^2y + my + n.
\end{aligned}
\qquad (1)
$$

(The x, y coordinates refer to the axis-of-flight system after the application of Equations 22 and 23.) Schut[4] states the following conformal re-

[1]Aerotriangulation adjustment of instrument data by computational methods by W. D. Harris, *Technical Bulletin No. 1,* Coast and Geodetic Survey, January 1958.

[2]Vertical adjustment of instrument aerotriangulation by computational methods by W. D. Harris, *Technical Bulletin No. 10,* Coast and Geodetic Survey, September 1959.

[3]Analytic aerotriangulation by W. D. Harris, G. C. Tewinkel and C. A. Whitten, *Technical Bulletin No. 21,* Coast and Geodetic Survey, Corrected July 1963.

[4]Development of programs for strip and block adjustment by the National Research Council of Canada by G. H. Schut, *Photogrammetric Engineering,* Vol. 30, No. 2, page 284, 1964.

[5]Simultaneous three-dimensional transformation of higher degrees by E. M. Mikhail, *Photogrammetric Engineering,* Vol. 30, No. 4, page 588, 1964.

lations for *horizontal* coordinates to third degree:

$$x' = x + a_1 + a_3x - a_4y + a_5(x^2 - y^2) + 2a_6xy + a_7(x^3 - 3xy^2) - a_8(3x^2y - y^3) + \ldots$$
$$y' = y + a_2 + a_4x + a_3y + a_6(x^2 - y^2) + 2a_5xy + a_7(3x^2y - y^3) + a_8(x^3 - 3xy^2) + \ldots \quad (2)$$

Webb and Perry[6] used

$$x' = x + a_3x^2 + b_3x + c_3xy + d_3xz + e_3$$
$$y' = y + a_4x^2 + b_4xy + c_4y + d_4 \quad (3)$$
$$z' = z + a_5x^2 + b_5xy + c_5xy + d_5y + e_5x + f_5.$$

The following equations in a plane appear in Schwidefsky's textbook[7]:

$$x' = x + a_1x^3 + b_1x^2 - 3a_2x^2y - 2b_2xy \quad (4)$$
$$y' = y + a_2x^3 + b_2x^2 + 3a_1x^2y + 2b_1xy.$$

Arthur[8] of the Ordnance Survey of Britain published:

$$x' = x + a_1 + a_4x + a_6z - a_7y + \tfrac{1}{2}a_8x^2 + a_{10}xz - a_{11}xy$$
$$y' = y + a_2 + a_4y + a_5z + a_7x + a_8xy + a_9xz + \tfrac{1}{2}a_{11}x^2 \quad (5)$$
$$z' + z + a_3 + a_4z - a_5y - a_6x + a_8xy - \tfrac{1}{2}a_{10}x^2.$$

Norwicki and Born[9] suggest variable degree polynomials depending on the special conditions relative to the number and distribution of control points. The study included sixth degree.

The unusual terms in the Coast Survey formulas are designed to compensate for the local tilts of the strip: otherwise these formulas are not greatly different from the others. Justifications for their existence is given in the next section.

It may be appropriate at this stage to state the precision toward which this study is directed. Accuracies of a few feet have been experienced where the flight altitude is 20,000 feet; fractions of a foot are significant. Consequently, this program is prepared so as to preserve thousandths of a foot for round-off reasons even though the small distances are not ordinarily significant in themselves. Thus the fine precision being sought causes one to consider carefully the type of transformation being applied lest the transformation itself add systematic errors due to excessive constraint or relaxation.

The best root-mean-square accuracy that can be expected using the piecemeal, provisional Coast Survey analytic solution is probably in the neighborhood of 1/10,000 to 1/20,000 of the flight altitude where film is used in the aerial camera; that is, 1 foot if the altitude is 10,000 feet. If glass plates are used in the camera, and if the results are refined by a subsequent block adjustment technique, present results suggest that 1/50,000 can be approached; that is, about 2 1/2 inches if the altitude is 10,000 feet.

BASIS FOR THE FORMULATION

Analysis of the Curve Forms

The basis for the Coast Survey formulation is essentially that of Brandt[10] and Price[11] and is restated here for the sake of completeness.

Considering the abscissa direction first, the "new" or correct value x' (referred to the axis-of-flight coordinate system) for a point on the centerline (axis of flight) of a strip of aerotriangulation is considered to be composed of the "old" value plus a correction c_x:

$$x' = x + c_x$$

(which also serves to define $c_x = x' - x$). The correction is expressed by means of a polynomial of third degree in terms of the "old" coordinate:

$$x' = x + ax^3 + bx^2 + cx + f. \quad (6)$$

Similarly, the new y-coordinate of a point on the centerline is expressed using a quadratic polynomial:

$$y' = y + dx^2 + ex + g \quad (7)$$

also in terms of the abscissa x inasmuch as the magnitude of the correction is obviously related to the distance from the beginning of the strip.

Adequate theoretical and operational justification exists for assuming that the x and z equations need to be cubic whereas the y-equation does not need to be more than quadratic, but it was the experience of Coast Survey photogrammetrists based on having performed perhaps a hundred or more graphic solutions prior to the use of the computer that led to the final

[6]Forest Service procedure for stereotriangulation adjustment by electronic computer by S. E. Webb and O. R. Perry, *Photogrammetric Engineering*, Vol. 25, No. 3, page 404, 1959.

[7]An outline of photogrammetry by K. Schwidefsky, *Pitman Publishing Corp.*, p. 272, 1959.

[8]Recent developments in analytic aerial triangulation at the Ordnance Survey by D. W. G. Arthur, *Photogrammetric Record*, Vol. 3, No. 14, page 120, 1959.

[9]Improved stereotriangulation adjustments with electronic computers by A. L. Nowicki and C. J. Born, *Photogrammetric Engineering*, Vol. 26, No. 4, page 599, 1960.

[10]Resume of aerial triangulation adjustment at the Army Map Service by R. S. Brandt, *Photogrammetric Engineering* Vol. 17, No. 5, page 806, 1951.

[11]Some analysis and adjustment methods in planimetric aerial triangulation by C. W. Price, *Photogrammetric Engineering*, Vol. 19, No. 4, page 627, 1953.

conclusions. The x-curve was almost never symmetrical (a quadratic curve could be symmetrical) inasmuch as the second half invariably had a greater degree of curvature than the first half. A third degree correction polynomial adequately removed the discrepancy whereas the quadratic form left a residual error too large to be acceptable, although the situation could not be completely explained through a theoretical analysis. However, the y-curve was both smaller in magnitude and more nearly symmetrical. The graphic z-curves also were perceptably greater than second degree and the theoretical reasons seemed to be even more convincing.

Considering the y-curve for a moment, it is emphasized that Equation 7 applies to points on the centerline of the strip (fig. 1). The point m on the centerline needs to be corrected by moving it to n, and the magnitude of the correction is given by the equation. However, the point p on the edge of the strip needs to be corrected in two directions: (1) one component pq is equal to the correction mn at m, and (2) the second component is rq in the x-direction. In the right triangle mps, the angle θ at m is given by the first derivative of the equation of the y-curve, which is the centerline (Equation 7):

$$\tan \theta = (2d)\ x + e.$$

Then,

$$\overline{ps} = \overline{mp} \tan \theta.$$

But mp is the ordinate y of the point p:

$$\overline{qr} = \overline{ps} = y\ [\ (2d)\ x + e\ ]. \tag{8}$$

The complete x-equation is therefore composed of both Equations 6 and 8:

$$x' = x + ax^3 + bx^2 + cx - (2d))\ xy - ey + f \tag{9}$$

where the minus signs derive from the analytic definition of the direction of slope; i.e., the relative direction of rq.

The complete y-equation is formed through a comparable analysis:

$$y' = y + 3ax^2y + 2bxy + cy + dx^2 + ex + g. \tag{10}$$

Equation 6 depicts a lengthwise stretching or compressing of the strip. The term $(3ax^2y + 2bxy + cy)$ in Equation 10 is the effect of the local stretch or compression in the y-direction both as a function of the abscissa x of point in the strip and also as a function of the distance y that the point is off the centerline.

The vertical dimension is explained by a similar analysis. The basic equation is the cubic form

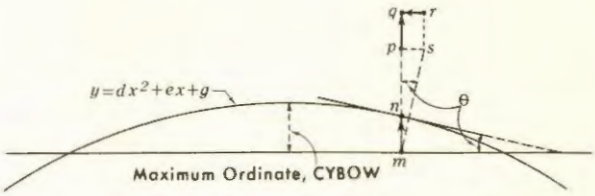

Fig. 1.—Sketch of the center line of the y or azimuth curve illustrating the derivation of a component x-correction for a point not on the center line. The definition of the term *maximum ordinate*, or CYBOW, is also indicated.

$$z' = z + hx^3 + ix^2 + jx + kx^2y + lxy + my + n \tag{11}$$

which can be considered as composed of the sum of the two principal geometric parts

$$hx^3 + ix^2 + jx \tag{12}$$

$$kx^2y + lxy + my. \tag{13}$$

The first part (Equation 12) is sometimes called the "BZ-curve". This curve is the projection of the center line of the strip onto the xz-plane. The second part (Equation 13) has been described as "twist" and as "cross tilt." The latter is considered to be quadratic, again based largely on experience gained from the graphic analyses of many strips, inasmuch as the graphic curves invariably were not straight lines. If they were linear, the effect would resemble a helix of constant pitch, like a screw thread, but the quadratic form fits most situations more closely. The terms in Equation 13 have as their common factor the ordinate y of the point so that the farther the point is off the axis of the strip, the greater is the correction.

The Slope Corrections

If y is factored out of Equation 13,

$$(ky^2 + lx + m)y, \tag{14}$$

it is obvious that the parenthetical expression represents the slope of the strip perpendicular to the centerline:

$$\tan \omega = kx^2 + lx + m \tag{15}$$

Moreover, the first derivative of Equation 12 is the instantaneous slope of the BZ-curve in the direction of the strip:

$$\tan \phi = d/dx\ (hx^3 + ix^2 + jx) = 3hx^2 + 2ix + j. \tag{16}$$

Thus Equations 15 and 16 depict the slopes of the strip "ribbon" in the x and y directions at any given abscissa x. Then the resultant tilt τ (deviation from the vertical) of the normal to the strip is given by

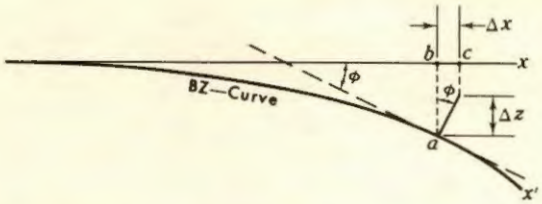

Fig. 2.—Sketch of the center line vertical BZ-curve illustrating how the local inclination of the curve causes a component horizontal correction in the x-direction.

$$\sec \tau = (1 + \tan^2 \phi + \tan^2 \omega + \tan^2 \phi \tan^2 \omega)^{1/2} \qquad (17)$$

which, inasmuch as ϕ and ω are both small angles, is approximated with sufficient accuracy for practical operations by

$$\sec \tau \approx (1 + \tan^2 \phi + \tan^2 \omega)^{1/2}. \qquad (18)$$

In aerotriangulation, both in instrumental and in the Coast Survey analytic systems, the observed or computed coordinates of points are related to the initial rectangular axes of the strip at the first model or first photograph rather than the curved centerline of the strip (fig. 2). Consequently, the base and top of an elevated object have different horizontal coordinates. In the figure, the base a has the abscissa of point b whereas the top has the abscissa of point c. The previous photogrammetric solution yields the abscissa of c and the elevation Δz, which introduce a discrepancy Δx equal to the distance from b to c:

$$\Delta x = \Delta z \tan \phi = \Delta z \ (3hx^2 + 2ix + j). \qquad (19)$$

The value is subtracted from the abscissa of c as a correction indicated in the first formula of Equation 1.

In a similar manner, the correction of the ordinate is:

$$\Delta y = \Delta z \tan \omega = \Delta z \ (ky^2 + lx + m) \qquad (20)$$

as indicated in the second formula of Equation 1.

The z-correction is probably of minor consequence; nevertheless it is also applied. The photogrammetric elevation is too small and needs to be increased by multiplying it by the secant of the resultant inclination τ of the line perpendicular to the surface of the strip. The term is applied in the third formula of Equation 1.

Two Preliminary Affine Transformations

In Equations 1 it was tacitly assumed both that (1) the directions of the x, y, z axes were essentially parallel to the x', y', z' axes, and also that (2) the x-axis represented the centerline or axis of flight of the photogrammetric strip. Both of these conditions are violated in practice; consequently, two preliminary affine transformations are utilized for rotation, translation, and scale change. Inasmuch as these conditions need not be exactly adhered to, unique transformations are utilized so as to impose as few fixed conditions as possible and to simplify the computations for determining the constants of the transformation equations.

Perhaps an explanation is in order as to the reasons for assuming that these conditions are necessary. The systematic errors depicted by the polynomials of Equation 1 are direction-sensitive inasmuch as they are propagated as functions of the length of the strip, or the number of photographs in the strip, or simply the abscissa of a point in the strip. However, the photogrammetric direction of the strip (axis of flight) is not known with sufficient accuracy until after the strip has been aerotriangulated, at which time the easiest way to obtain the desired coordinates is to transform them by means of a computer rather than to reobserve them.

Secondly, whereas the photogrammetric strip (model) coordinates may progress in any direction of the compass, the ground coordinates are oriented with $+X$ eastward. But the X-coordinates must also be reoriented into the axis-of-flight direction, which possibly would be unnecessary if all strips were flown north-south or east-west. Again, it is fairly easy to rotate, scale and translate the ground coordinate system into the axis-of-flight system with an electronic computer.

Finally, after the correction Equations 1 have all been applied to the coordinates of a point, it is necessary to convert the coordinates back into the ground system by applying the inverse of the second affine transformation above so that the resulting coordinates are meaningful and useful in surveying and mapping work.

Model Coordinates to Axis-of-Flight System

By "model" coordinates is meant the form of the data from a stereoplanigraph bridge. A comparable form results from the preliminary computer solution of the Coast Survey analytic aerotriangulation.

Let the model coordinates of a point near the center of the initial model be x_1, y_1, and near the center of the terminal model be x_2, y_2. The axis of flight is arbitrarily defined as passing through these two points. This axis of flight is to be the new x-axis: the new ordinates of both the above points are therefore zeros. The origin of the axis of flight is defined as midway between these initial and terminal points in order to reduce the numerical magnitude of the x-coordinates, which has added importance inasmuch as the x-coordinates are squared and cubed as indicated in Equation 1. The distance D between the initial

and terminal points is given by analytic geometry to be

$$D = (\Delta x^2 + \Delta y^2)^{1/2}, \qquad (21)$$
$$\Delta x = x_1 - x_2, \ \Delta y = y_1 - y_2,$$

Consequently, the coordinates of the initial and terminal points in the axis-of-flight system are

$$x_1' = -\tfrac{1}{2} D, \ y_1' = 0$$
$$x_2' = +\tfrac{1}{2} D, \ y_2' = 0.$$

The axis-of-flight coordinates of other points can be computed using the following set of affine transformation formulas (which comprise a special form of the more general Equation 25 discussed later):

$$x' = a_1 \ x - b_1 \ y + c_1$$
$$y' = b_1 \ x + a_1 \ y + d_1. \qquad (22)$$

$$a_1 = - \Delta x \ / D$$
$$b_1 = \Delta y \ / D \qquad (23)$$
$$c_1 = - a_1 \ x_1 + b_1 \ y_1 - \tfrac{1}{2} D$$
$$d_1 = - a_1 \ x_1 - a_1 \ y_1.$$

Equations 22 constitute simply a rotation and translation of the "model" coordinates into the "axis-of-flight" coordinates maintaining the same original model scale. The coefficients $a_1 \ . \ . \ . \ d_1$ are the constants for the transformation: their values are determined once only.

Ground Horizontal Coordinates Into the Axis-of-Flight System, and the Inverse

The coordinates of horizontal ground control stations also need to be transformed into the same axis-of-flight system. The transformation is based on only two of the stations, one near the initial end of the strip and one near the terminal end. In the program, the coordinates of the two stations are listed as the first and the last ones used in the adjustment. Two sets of coordinates are given for each point: one set is in the form of model coordinates $x_1 \ldots y_2$ and the other set in the ground survey system as $X_1 \ldots Y_2$.

The first step is to transform the model values x_1, etc., into the axis-of-flight system by applying Equations 22 and then applying the slope corrections as indicated by Equations 19 and 20. If $x_1' \ . \ . \ . \ y_2'$ are the axis-of-flight coordinates of the initial and terminal control stations, the following differences can be expressed:

$$\Delta x' = x_1' - x_2', \ \Delta y' = y_1' - y_2'$$
$$\Delta X = X_1 - X_2, \Delta Y = Y_1 - Y_2. \qquad (24)$$

(ΔX is called DGX in the Fortran program Statements 12+4 and 80+3.) The square of the axis-of-flight distance D^2 between the terminal model control points is

$$D^2 = \Delta x'^2 + \Delta y'^2.$$

The same type of affine transformation as Equation 22 is applied to the axis-of-flight coordinates to convert them into the ground system of coordinates (Statements 70+1 and 70+2 in the Fortran program) which is applied to all new (bridge) points as the last stage of the computation after implementing the corrections of Equation 1:

$$X = a_{20} \ x' - b_{20} \ y' + c_{20}$$
$$Y = b_{20} \ x' + a_{20} \ y' + d_{20}. \qquad (25)$$

If the values of x_1', etc., and X_1, etc., are substituted into Equations 25, one obtains four simultaneous linear equations in which the four constant coefficients $a_{20} \ . \ . \ . \ d_{20}$ are the only unknowns. The solution of the equations gives

$$a_{20} = (\Delta X \ . \ \Delta x' + \Delta Y . \ \Delta y') \ / D^2$$
$$b_{20} = (\Delta Y \ . \ \Delta x' - \Delta X \ . \ \Delta y') / D^2 \qquad (26)$$
$$c_{20} = X_1 - a_{20} \ x_1' + b_{20} \ y_1'$$
$$d_{20} = Y_1 - b_{20} \ x_1' - a_{20} \ y_1'.$$

Equations 25 embody a change in scale in addition to rotation and translation.

The inverse form of Equations 25 shows the corresponding transformation of ground coordinates into axis-of-flight values:

$$x' = a_{21} \ X + b_{21} \ Y - c_{21} \qquad (27)$$
$$y' = - b_{21} \ X + a_{21} \ Y - d_{21}.$$

The values of the four new constants can be computed from those of Equations 25 by applying the following relations:

$$d^* = 1/(a_{20}^2 + b_{20}^2)$$
$$a_{21} = a_{20} \ . \ d^*$$
$$b_{21} = b_{20} \ . \ d^*$$
$$c_{21} = (a_{20} \ c_{20} \ + b_{20} \ d_{20} \ ) \ d^* \qquad (28)$$
$$d_{21} = (a_{20} \ d_{20} - b_{20} \ c_{20}) \ d^*.$$

The evaluation of all eight of the constants of Equations 25 and 27 are indicated after Statement 80 of the Fortran program.

It is noted that $(a_{20}^2 + b_{20}^2)^{1/2}$ expresses the scale change included in the transformation.

Maximum Ordinate CXBOW

It should be noted that the application of transformation Equations 27 is such that no discrepancy c_x or c_y exists or remains at the two end horizontal control points on which the transformation is based. Consequently, if the two polynomial curves are plotted, they will both cross the zero line, or x-axis, at the location of these two control stations. (Fig. 1.)

Normally each curve describes a sweeping arc joining these two end points. Midway between the end points the y-curve will be at its greatest distance from the x-axis, indicating that the correction is maximum. One is assured of this feature because the equation is quadratic and is symmetrical with respect to the y-axis. Moreover Equations 22 have already been applied so that the origin of the observed coordinate system is at the center of the strip. Consequently the maximum height of the curve is where it crosses the y-axis which is where $x = 0$. If in Equation 10 x is set equal to zero, then the height of the curve is given simply by $c_y = g$. The value g may be called the "maximum ordinate" which is symbolized in the program as CYBOW. This quantity is a diagnostic term whose magnitude as determined through experience normally does not exceed certain practical limits unless a blunder occurs in some of the data.

Similarly, from Equation 9 the CXBOW is $c_x = f$. Inasmuch as the x-curve is cubic, it is not symmetrical and the value f is not the maximum. Nevertheless, f is sufficiently near the maximum that it still is useful for detecting a gross blunder.

Preliminary Vertical Affine Transform

The value of z is derived from the photogrammetric analysis and its exact relation to the ground system of measurement is not known until later. The following transformation and its inverse relating the two systems are stated and explained:

$$z' = z_0 + Z/s$$
$$Z = s(z' - z_0 . \qquad (29)$$

(In Fortran notation, z_0 is designated as EINDEX. These equations occur in the Fortran program as Statements 31+1 and 70+3.) The instrument or photogrammetric model elevation of a point in the axis-of-flight system is symbolized as z' whereas Z refers to the ground elevation, s is the horizontal (as well as vertical) scale factor that relates the magnitudes of z' and Z, and z_0 is an index elevation, or translation term. Thus Equation 29 implies simply dilation (scale) and translation. The scale factor is determined after Equation 24 above:

$$s = [(\Delta X^2 + \Delta Y^2) / (\Delta x'^2 + \Delta y'^2)]^{1/2}. \qquad (30)$$

(Statement 12+5 in the Fortran program.) Obviously the scale factor is normally a number greater than 1, such as 10,000, and is the number for multiplying instrument or model dimensions to convert them into equivalent ground dimensions.

The index elevation z_0 is defined as an average value for a list of control points:

$$z_0 = z'_a - Z_a/s.$$

(Fortran statement 14+2.) Thus the index is the difference between the average of the instrument elevations and the average of the corresponding ground elevations that have been scaled into the instrument units. The index serves not only as a translational element between the two systems, but also confines the adjustment to the actual zone of elevations rather than simply applying the adjustment to a set of abstract numbers which in practice would each ordinarily be of considerably greater magnitude than its corresponding elevation.

The Solution of Simultaneous Equations

The first two formulas of Equation 1 give rise to two simultaneous observation equations for each horizontal control points in which the unknowns are the seven constants $a \ldots g$. (The third formula gives rise to a single observation equation of a different set for each vertical control point where the unknowns consist of seven additional constants $h \ldots n$.) Four or more horizontal stations result in eight or more observation equations in seven unknowns. A least-squares solution applies in which seven normal equations are first formed from the set of observation equations by classical methods and then solved uniquely using a routine commonly applied in Coast Survey geodetic work. The routine consists principally of Gaussian elimination incorporating the square-root technique. The system is described in detail in Technical Bulletin 21.[12] (See also Hildebrand.[13])

If a set of m observation equations have the form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = a_{n+1}, \qquad (31)$$

each coefficient of the set of corresponding normal equations can be stated as

$$b_{ij} = \sum_{k=1}^{m} a_i \, a_j, \quad i = 1 \ldots n, \quad j = i \ldots (n+1) \qquad (32)$$

[13] Introduction to Numerical Analysis by F. B. Hildebrand, *McGraw-Hill Book Company, Inc.* page 431, 1956.

[12] See note 3 above.

in which i, j are the row and column numbers of the coefficient, m is the number of observation equations and n is the number of unknowns (as well as the number of normal equations). (Statement 23 of the Fortran program shown in Table 8.) Inasmuch as the normal equation system is symmetric with respect to its principal diagonal, the terms below the diagonal are not computed nor are the spaces in the matrix used.

An auxiliary c-matrix is formed from the normal equation b-matrix in the *forward* concept of the solution. For the terms in the first row

$$C_{ij} = b_{ij} / (b_{ii})^{1/2}, \quad j = 1 \ldots (n+1) \tag{33}$$

The formula for the diagonal terms is stated explicitly although it is essentially identical to that for the general terms:

$$c_{ii} = \left[b_{ii} - \sum_{k=1}^{(i-1)} (c_{ki})^2 \right]^{1/2}, \quad i = 2 \ldots n. \tag{34}$$

The general term is

$$c_{ij} = \left(b_{ij} - \sum_{k=1}^{(i-1)} c_{ik}\, c_{ij} \right) / (c_{ii})^{1/2}, \quad i = 2 \ldots n, \quad j = (i+1) \ldots (n+1) \tag{35}$$

This completes the forward solution. It is accomplished by the Fortran Statements following number 23 through 99, which apply the formulas in a different order than stated here in order to exploit the facilities of the DO and IF instructions of the program language. Again, terms below the diagonal are not computed. The terms are stored in the same spaces as the normal equations, that is, c_{ij} is stored in the space formerly occupied by b_{ij} inasmuch as the latter is no longer needed.

The final *back* concept of the solution d is given by:

$$d_{n,n+1} = c_{n,n+1} / c_{nn} \tag{36}$$

$$d_{i,n+1} = \left(c_{i,n+1} - \sum_{k=1}^{(n-1)} c_{i+1,\,n+1}\, c_{in} \right) / c_{ii}, \quad i = n-1,\ n-2,\ n-3 \ldots 1. \tag{37}$$

(Statements 96 through 91 of Table 8). These are the new values in the last column that comprise the "answers" to the solution. Again they are stored in the same matrix space as the corresponding b and c terms. The coefficients of the observation equations are symbolized as A in the program whereas the b, c, and d terms have the common designation EN.

FLOW DIAGRAMS

The accompanying Generalized Flow Diagram (fig. 3) is supplied to give the reader a visual conception of the order of the major steps of the computation. A comparison of this diagram with the Detailed Flow Diagram (fig. 4), together with the Fortran program itself (Table 8) may aid one in studying the system as a whole. Although numerous Comment Statements in the program are not indicated in the diagrams, they are considered to be self evident to one who has only a superficial familiarity with the Fortran notation. A further comparison of the diagrams with the Basis for the Formulation discussed earlier may also serve to relate the various formulations to the operational flow as well as to the program itself.

A summary of the formulation is not repeated here inasmuch as it seems that a distinct relationship has been established in each step by keying each formula in the Basis for the Formulation to the corresponding statement number. Moreover, a repetition of the formulation might lead to some confusion inasmuch as the symbolization in the program has been modified so as to exploit the opportunity of making multiple uses of common data memory areas.

The Detailed Flow Diagram is presented to give an idea of the actual order of computing, decision-making and the re-use of some of the algebraic functions. The Fortran program itself (Table 8) has also been annotated to assist the reader.

DISCUSSION OF THE FEATURES OF THE PROGRAM

This program includes several features that are considered to be significant:

1. Integrated Horizontal and Vertical adjustment programs combined into a single interrelated system.
2. Variable degree facility.
3. Incorporation of corrections for the slope of the strip.
4. Output display to aid in diagnosis of control data validity.
5. Acceptance of both analog and analytic data.

The idea that the horizontal and vertical phases should be computed simultaneously was indicated above in the Basis for the Formulation and in the references thereto. Nevertheless, several contemporary programs do not include the combination, and it is new in Coast Survey practice. The reasons for the combination are rather obvious. The single approach to the computer might at first seem trivial, but it has practical advantages where the computer is heavily scheduled. The reason of greater significance applies to the geometric integration of the two types of

corrections. It has already been pointed out that an ideal conformal adjustment of a three-dimensional field using higher degree cannot exist. To the contrary, the problem being confronted is in three dimensions and of third degree. This one-step iteration offers a practical solution of the dilemma. The "integration" is offered by (1) applying vertical-oriented slope corrections to horizontal coordinates, and (2) horizontal-oriented (scale) corrections to vertical coordinates.

The variable-degree facility of this program consists of provisions that allow one to select either the third, second, or first degree polynomial correction in the horizontal direction, and also in the vertical direction independent from the horizontal choice. This facility is needed and useful for two situations: (1) short strips (or inadequate control), and (2) the detection of faulty control data.

For example, the third degree correction in the horizontal applies nicely if five or more control stations are about evenly distributed throughout the length of the strip. (If several control stations occur in a group, only one of them is ordinarily used in the adjustment whereas the others are used as check points, referred to in the program as Other Horizontal Control.) If only four points are used, offering a unique solution, and the data for one of them is in error, the diagnostic data may indicate that a large discrepancy exists but may not isolate the erroneous point, whereas the use of five points helps to indicate the specific troublesome data. Further, if only four points are available, it is frequently preferrable to reduce the degree of the polynomial from third to second. This choice is offered by the initial input card of the program. (This also allows one to produce a shorter version of this program by eliminating this feature, which is quite easy to do.) Similarly in the vertical direction: seven control points are required for a unique third-degree adjustment, but fewer points can be utilized by selecting a lower degree.

This variable facility allows this same program to be applied to very short strips as well as long ones inasmuch as short strips ordinarily require only a linear adjustment. In this sense, the program might also be regarded as "absolute orientation" whereas the input data are "relative", in photogrammetric terminology.

The diagnostic feature has already been indicated above. It is ordinarily helpful, if the number of control points is insufficient or marginal, to select the degree of polynomial that is one less than the unique solution as an aid in detecting erroneous control data. A further application of this feature is to select arbitrarily the first degree as an initial computation, plot manually the output CX, CY, CZ as ordinates against the abscissa x. The resultant curves should be

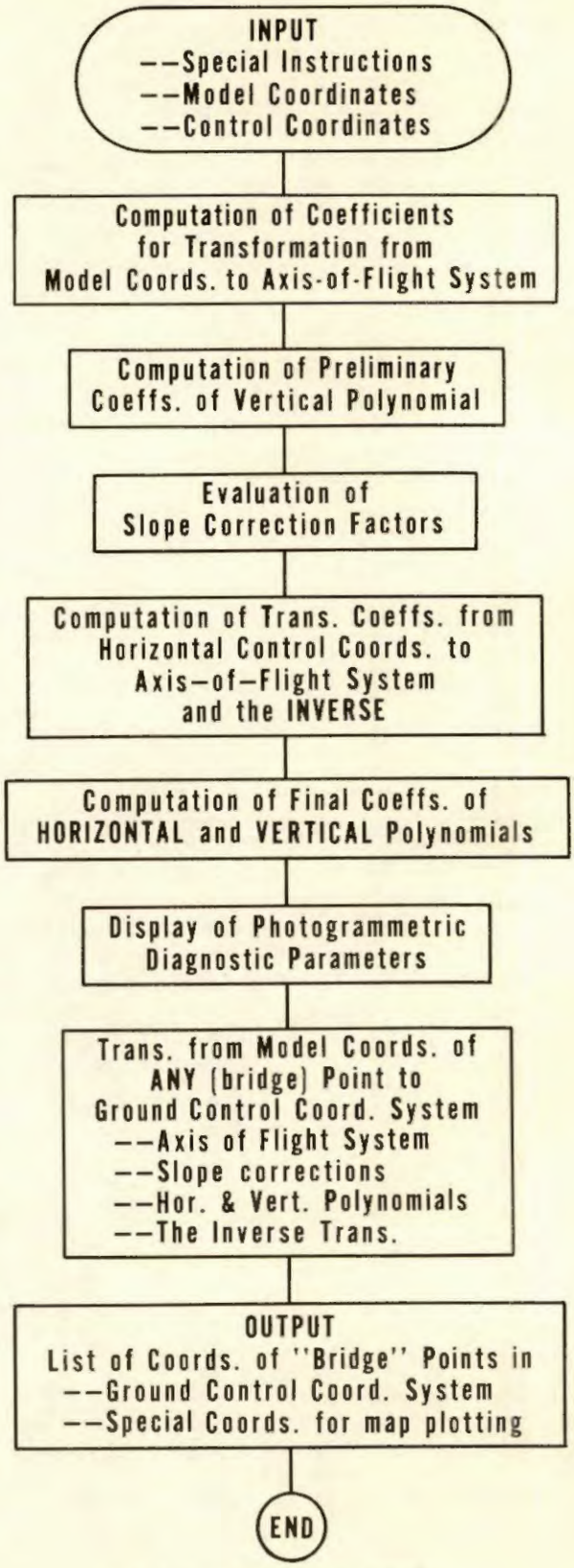

Fig. 3.—Generalized Flow Diagram.

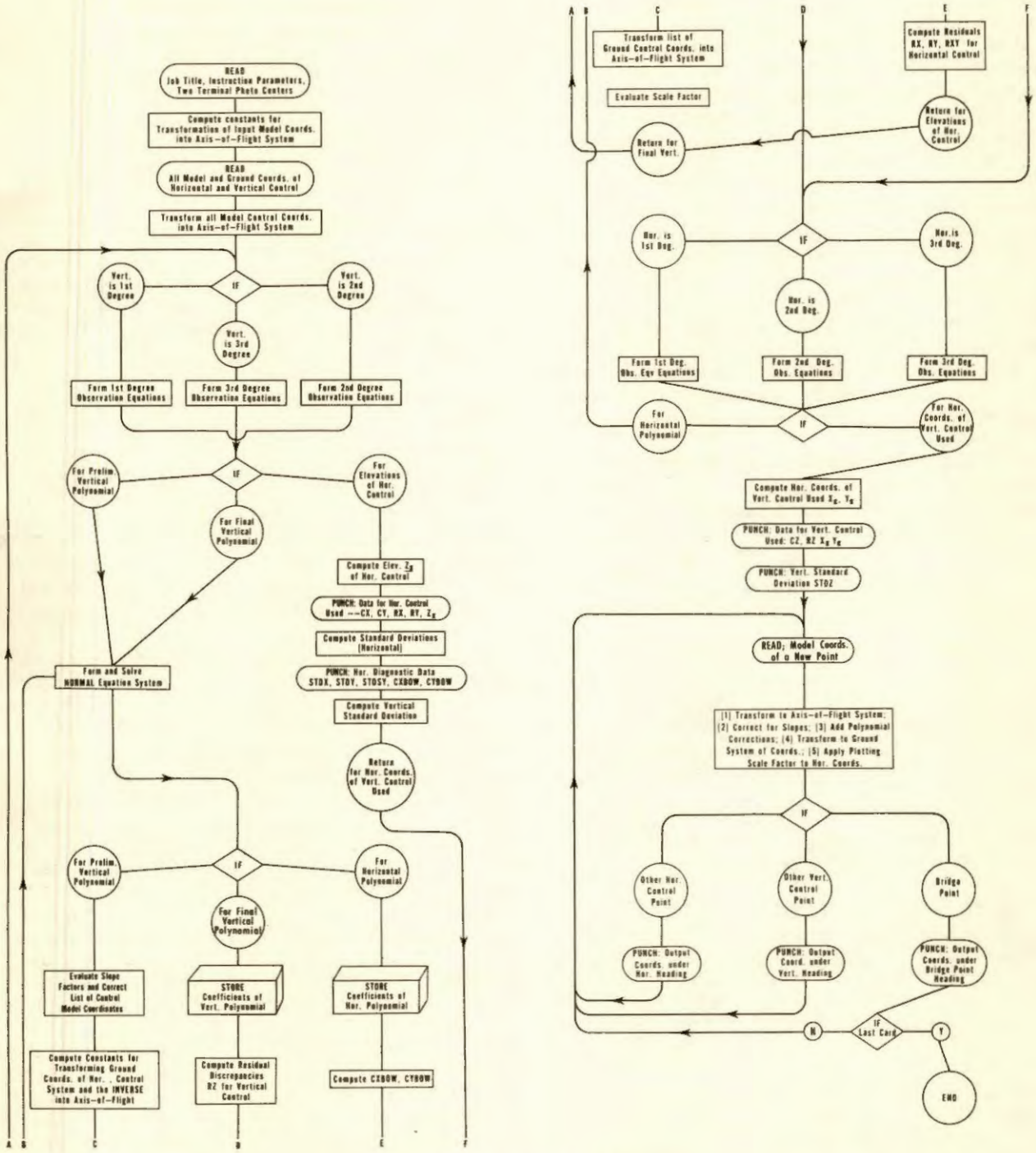

Fig. 4.—Detailed Flow Diagram.

smooth, and any radical deviation from smoothness is an indication of a discrepancy. The solution can then be recomputed a second and final time, utilizing the highest degree and the largest number of control points applicable, having deleted or corrected any discrepancies.

The importance of this diagnostic feature may not be readily appreciated until one realizes the unexplainably high incidence of gross mistakes in control data encountered in practice. Although published control lists are essentially faultless, and the practice of premarking control points before aerial photography essentially eliminates misidentification errors, still identification blunders occur where premarking is not practiced and measuring blunders occur in surveying short traverses from the station monuments to sites that are more photogenic. Control points are so costly that a hare minimum is normally specified. Consequently, human judgement, rather than statistical chance and least squares, must be relied on to accept and reject control data in those applications where the very hest possible adjustment is important.

The correction of the horizontal coordinates for the slope of the strip has already been discussed at length in the Basis for the Formulation on page.

The diagnostic features mentioned above are facilitated by the form and content of the output data. For each horizontal control point used in the adjustment are listed:

1. The *CX* and *CY* corrections that the control coordinates differ from the photogrammetric values;
2. The residual deviations *RX* and *RY* that the control coordinates differ from the polynomial-corrected model coordinates;
3. The standard deviations (root-mean-square deviation) *STDX*, *STDY* of all the horizontal control used, and the standard deviation vector *STDXY;* and
4. The maximum ordinates of the polynomial curves *(CXBOW, CYBOW* correction or bow error at the axis-of-flight origin.)

It has already been pointed out how the *CX* and *CY* discrepancies can be plotted graphically to give a visual depiction of the relative smoothness of the control data for the purpose of isolating blunders. The standard deviations can be helpful confidence criteria relative to the validity of the adjustment as a whole: e.g., 68 percent of the errors are within the *STDXY* value; 90 percent are within 1.67 times that value. The Bow values take on significance with experience: unreasonable values, related to the length of the strip, tend to indicate faulty data. The same type of data are printed out for the vertical adjustment.

Other control points consist simply of points adjusted in the same manner as *bridge points* or *pass points*, but grouped together so that one can quickly compare the results with published control lists. Separate listings are provided for the horizontal and vertical control points. It will be noted that the format is identical to that for Bridge Points. The "Other" control may also be contained again among the Bridge Points in their positional sequence in the strip.

A *switch* is provided (the third item, seventh column of the second input card) to inform the computer whether the input model data have been furnished by the analytic method (0) or by the analog (stereoplanigraph) method (1). The difference has to do with the format of the input data: if analytic, the data cards are the output of a prior computer program and are in floating point mode; if analog, the data (ordinarily listed at the stereoplanigraph source) need to be key punched--the key punching is conceived as being *fixed point* mode. After the input step, the subsequent operations are identical.

INPUT CARD FORMATS

References are made to (1) the accompanying input list (Table 2) containing a sample stereoplanigraph (analog) solution; (2) a similar list (Table 3) for the analytic solution; (3) the listing of the Fortran Program (Table 8); (4) the input data forms for the sample solution (Tables 9 to 20); and (5) the following list of Input Data Formats (Table 1). Seven different card formats are descrihed in Table 1. They are depicted graphically by the Input Data Forms (Tables 9 to 20).

The *first* card is simply the job title punched in alphanumeric characters heginning in column 2 and ending on or before column 64, as shown near the top of the Input Data Form 1/6 (Table 9) and also on the first line of the Input Listing (Table 2) for the sample solution.

The *second* card contains the Instruction Parameters and includes only the first 22 columns. The 04 in the first two columns indicates that 4 *horizontal* control points are used in the adjustment; 09 in columns 4 and 5 indicates that 9 *vertical* control points are to be used in the adjustment; 1 in column 7 indicates that the source of the data is the stereoplanigraph (analog) instrument; the two 3's indicate that both the horizontal and vertical adjustments, respectively, are to be of third degree (either digit might have been 1 or 2 instead of 3); and columns 14 through 22 indicate that the plotting constant value is 0.5 (or any number of the form x.xxx xxx xxx). The plotting scale is provided to assist plotting the horizontal positions of the points on a map sheet with a coordinatograph without changing its indicators. If one is not interested in the plotting

Table 1.—List of formats for input data formats.

Input Format No.	Fortran Description	Description of Data
122	(64H . . . . . .)	Job Title
100	(2(I2, 1X), 3(I1, 1X), F10.9)	Instruction Parameters.
101	(I2, 1X, I7, 2X, E14.8, 2X, E14.8, 2X, E14.8)	Model coordinates, analytic, Photo Centers, Horizontal and Vertical Control.
106	(I2, 1X, I7, 3F16.2)	Same as 101 for analog (stereoplanigraph).
105	(3X, I7, 3F16.3)	Ground Control data, both Horizontal and Vertical.
115	(3X, I7, 3(2X, E14.8), 20X, 2I1)	Model Coords., analytic, OTHER Horizontal, Vertical Control, Bridge Points.
116	(3X, I7, 3F16.2, 20X, 2I1)	Same as 115 for analog.

Table 2.—List of input data in stereoplanigraph mode for sample solution with annotations.

```
AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 3 DEG     (Job Title)
4 09 1 3 3 0500000000                                   (Instruction Parameters)
1     5300            50174          292355        (Model coordinates of
2     7700            68399           69455         Axis-of-Flight centers)
1    54101            46375          281504           51870  (Mod. coords. of
2    57101            57788          254666           52052   Horizontal
3    71101            79987          125091           52359   Control
4    75101            72721           84398           52597   Points)
1    54203            69791          281942           52061
2    58201            40687          244916           51915
3    58203            73528          246350           51891  (Mod. coords. of
4    64201            43668          189869           52349   Vertical
5    64203            77991          196762           51846   Control
6    69201            50039          140210           52362   Points)
7    69203            83948          144918           52056
8    75201            86412           87736           52378
9    75203            49278           85927           52903
1 3 54101       1877196900      258023400         1215000  (Ground coords.
2    57101      1873898400      238488100         1336400   Horizontal
3    71101      1839162600      156265700         1528500   Control
4    75101      1820146900      135671100         1678700   Stations)
1    54203      1890751020      249694220         1345900
2    58201      1860542870      239172030         1239600
3    58203      1879854350      227967260         1226400  (Ground coords.
4    64201      1842092450      206585530         1513100   Vertical
5    64203      1864261480      197965800         1180000   Control
6    69201      1827581220      175853980         1506100   Stations)
7    69203      1848679430      166143630         1311300
8    75201      1829187400      132578140         1532900
9    75203      1807312470      145105950         1874700
1    61101            64946          219943           52434  (Mod. coords. of
2    66101            67388          171209           52249   Other Hor.
3    73101            75398          107940           52377   Control)
```

Table 2.—List of input data in stereoplanigraph mode for sample solution with annotations.—Con.

1	54202	63540	285601	52182	(Mod. coords. Other Vert. Control) ①
2	58202	56914	247467	51787	
3	64202	61971	194049	52231	
4	69202	65299	142217	52289	
5	75202	70096	85141	52498	
1	54205	28451	280679	51848	(Mod. coords. Bridge Points) ②
2	57102	46070	249844	52096	
3	67101	50510	80259	53250	①

(Category signal punches)

Table 3.—List of input data in analytic mode for sample solution of same problem as Table 2.

```
AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 3 DEG
4 09 0 3 3  500000000
1     5300  +.50174000E+00  +.29235500E+01
2     7700  +.68399000E+00  +.69455000E+00
1 3 54101   +.46375000E+00  +.28150400E+01  +.51870000E+00
2   57101   +.57788000E+00  +.25466600E+01  +.52052000E+00
3   71101   +.79987000E+00  +.12509100E+01  +.52359000E+00
4   75101   +.72721000E+00  +.84398000E+00  +.52597000E+00
1   54203   +.49791000E+00  +.28194200E+01  +.52061000E+00
2   58201   +.40687000E+00  +.24491600E+01  +.51915000E+00
3   58203   +.73528000E+00  +.24635000E+01  +.51891000E+00
4   64201   +.43668000E+00  +.18986900E+01  +.52349000E+00
5   64203   +.77991000E+00  +.19676200E+01  +.51846000E+00
6   69201   +.50039000E+00  +.14021000E+01  +.52362000E+00
7   69203   +.83948000E+00  +.14491800E+01  +.52056000E+00
8   75201   +.86412000E+00  +.87736000E+00  +.52378000E+00
9   75203   +.49278000E+00  +.85927000E+00  +.52903000E+00
1 3 54101       1877196900       258023400         1215000
2   57101       1873898400       238488100         1336400
3   71101       1839162600       156265700         1528500
4   75101       1820146900       135671100         1678700
1   54203       1890751020       249694220         1345900
2   58201       1860542870       239172030         1239600
3   58203       1879854350       227967260         1226400
4   64201       1842092450       206585530         1513100
5   64203       1864261480       197965800         1180000
6   69201       1827581220       175853980         1506100
7   69203       1848679430       166143630         1311300
8   75201       1829187400       132578140         1532900
9   75203       1807312470       145105950         1874700
1   61101   +.64946000E+00  +.21994300E+01  +.52434000E+00
2   66101   +.67388000E+00  +.17120900E+01  +.52249000E+00
3   73101   +.75398000E+00  +.10794000E+01  +.52377000E+00
1   54202   +.63540000E+00  +.28560100E+01  +.52182000E+00                1
2   58202   +.56914000E+00  +.24746700E+01  +.51787000E+00
3   64202   +.61971000E+00  +.19404900E+01  +.52231000E+00
4   69202   +.65299000E+00  +.14221700E+01  +.52289000E+00
5   75202   +.70096000E+00  +.85141000E+00  +.52498000E+00
    54205   +.28451000E+00  +.28067900E+01  +.51848000E+00                2
    57102   +.46070000E+00  +.24984400E+01  +.52096000E+00
    67101   +.50510000E+00  +.80259000E+00  +.53250000E+00                 1
```

values, it is suggested that a dummy plotting scale number be used, such as 1.000000000. The dummy is perhaps of trivial importance: it only serves to avoid two columns of "entirely" meaningless output data later on.

The *third* and *fourth* cards identify the photo centers that have been selected to define the axis-of-flight coordinate system (format 106 in the sample). The first two columns 01 and 02, called LINE 1 and LINE 2 in the program, define the "initial" and "final" points. Columns 4 to 10 consist of a 7-digit (maximum) identification number assigned to each photogrammetric point. Columns 13 to 26 contain the x-coordinate of the photo center stated in hundredths of a millimeter without a decimal point. Thus, the number 50174 in the sample is actually 501.74 millimeters. The y-coordinate is treated in the same manner, whereas the z-coordinate is not utilized and not punched.

The *fifth* card (Input Data Form 1/6 (Table 9) and Format 106) contains the line number, image identification number, and the x, y, and z instrument (model) coordinates of a *horizontal* control point used in the adjustment and stated in the same mode as the previous fourth card except that the z-coordinate is included. (This same format is also used for the next category of points, Vertical Control Used in the Adjustment.) The program in its present form accommodates up to 20 of these cards—the actual number is specified by the initial number in the second card, the Instruction Parameter card.

The next series of cards (Table 10) consists of the *vertical* control points used in the adjustment. The format (106 or 101) is identical to the previous series. The number of cards can be as many as 20 and the actual number must be indicated in columns 4 and 5 of the second, or Instruction Parameter, card.

The *minimum* number of cards in the last two categories is related to the degrees of the adjustment specified in the Instruction parameter card, and is shown in the following table:

Degree of adjustment	Minimum number of control points	
	Horizontal	Vertical
1st degree	2	4
2d degree	3	5
3d degree	4	7

The minimum numbers are not automatic and must be methodically designated for each computation by means of the Instruction Parameter card.

The *ground* coordinates of *horizontal* control stations are shown both on the Input Data Form 3/6 (Table 11), and also on the Input List (Table 2) for the sample problem, and are designated as Format 105 in the program. The coordinates are stated to 1/1000 of a unit of length (foot or meter) in fixed point mode in which the decimal point is omitted entirely. For example, the number 1877196900 in the sample solution is actually 1,877,196.900 feet. The third or Z-coordinate is normally unknown and may be left blank.

The next category of input data consists of the *ground* coordinates of the *vertical* control points (Table 12), listed in exactly the same format 105 as the previous category. Inasmuch as the X and Y-coordinates are normally not known for vertical control stations, the spaces may be left blank.

If a station is to serve both as horizontal and vertical control, identical cards are entered in both categories, together with corresponding model coordinate cards in the previous two categories, formats 106 (or 101).

The order of the cards in foregoing four categories is critical inasmuch as the horizontal points shown on the Input Data Forms 1/6 and 3/6 (Tables 9 and 11) must be in exactly the same order and number, and the same must be true for the data on Forms 2/6 and 4/6 (Tables 10 and 12). Tests for the agreement of the point identification numbers is made at the beginning of the computation: a discrepancy stops the computation and outputs a diagnostic statement.

To the contrary, the order of the cards in the following three categories is relatively unimportant.

The format 116 (or 115) is used for all of the last three categories of points (Tables 13, 14 and 15), and is also very similar to Format 106 (or 101) used earlier, the only differences being occasional flag or signal punches in columns 79 and 80 of three of the cards. The computations of the last three categories are identical, but the output prints column headings appropriate to each category. The first of the final three categories is OTHER Horizontal Control points, which requires no signal punches. The second category is OTHER Vertical Control requiring a "1" signal punch in the 79th column of only the first card of the series. This term is called ITYPE in the program. The next signal punch does not occur until the first card of the next series of Bridge Points (pass points; points whose coordinates are sought) which contains a "2" in the 79th column. Again no more signal punches occur until the very last card of the deck which contains a "1" in column 80 (called ITEST in the program).

The Bridge Point series of cards normally comprises by far the largest number of input data cards. An exception occurs if one uses the program simply to study the diagnostic terms and the OTHER control point values. In this application where no Bridge Points at all are

used, a dummy final card is added having only a "2" in column 79 and a "1" in column 80.

Two card formats have not yet been discussed: 101 and 115. These are used in place of 106 and 116, respectively, where the solution follows analytic aerotriangulation data in place of stereoplanigraph aerotriangulation data. The fact must be indicated in the Instruction Parameter card, column 7. The basic difference in the two sets of Formats is that the analytic data cards are the output of a previous computation and are left in floating point mode, whereas the stereoplanigraph data are normally key punched and are in fixed point mode. A copy of Input Data Form 1/6 (Table 16) for the analytic solution is also included here to show how the two sets of data differ. The model coordinates are expressed for the analytic in *meters* instead of millimeters, requiring a decimal point and the "E" notation. For example, .68000000E-05 represents 6.8 microns, E-05 representing the exponential factor 10^{-5}.

Table 2 is the list of all the input data for the sample solution in stereoplanigraph mode. The list is anotated to designate the separate parts. Table 3 is a similar listing of the same data in the analytic mode. Corresponding lines of numbers in the two tables contain identical information for the computer.

THE OUTPUT FORMATS

The output formats are listed in Table 4 and the Output Listing for the Sample Solution (Tables 5, 6, 7) constitute vivid examples.

All the output data are displayed in floating point notation for both stereoplanigraph and analytic modes. All the items marked GROUND are in the same units of measurement as the Ground Control Data Input, the Input Data Form 3/6 (Table 9), and as expressed in Input Format 105. The values PLOT X, etc., are simply the ground horizontal units of measurement multiplied by the Plotting Constant.

All the other output data, CX, RX, STDX, etc., are in "model" units and depend on whether the input data is analytic or stereoplanigraph. If analytic, the unit is one meter; if stereoplanigraph, it is one millimeter at the "model" scale. The significance of the units in turn depends on the relation of the "model" to the "plate" scale, or the scale of the original photographic negative. In analytic work, the relation depends on the internal programming of the previous computation—normally it is identical, or nearly so, to the negative scale. In stereoplanigraph work, it is ordinarily approximately 2 to 3 times the negative scale.

Table 5 constitutes a typical output for a third degree adjustment based on the input data shown in both Tables 2 and 3. Table 6 is for a second degree adjustment; Table 7 for first degree; and all three tables have identical input data.

THE SOURCE PROGRAM IN FORTRAN LANGUAGE

Table 8 is a listing of the Fortran statements of the source program with annotations. It has been compiled and used an IBM-1620 having a core memory of 100 K digits. A modified form has also been compiled and tested on the IBM-7030 STRETCH computer. The input data of Table 2 yielded the output data of Tables 5, 6, and 7 through the use of the 1620 program.

Inasmuch as this program was developed to serve the various needs and organizational structures within the Coast and Geodetic Survey, numerous instances occur where the program can be shortened to meet the specific needs of a given office.

Table 4.—List of formats for output data.

Input Format No.	Fortran Description	Description of Data
122	(64H . . . . . .)	Job Title--same as input format.
107	(39H HORIZONTAL CONTROL USED FOR ADJUSTMENT/)	Point Category Title.
108	(77H POINT CX CY RX RY COMP Z GROUND	Column headings for Horizontal Control used in the adjustment.
110	(7H STDX = E14.8, 7H STDY + E14.8, 8H STDXY = E14.8)	Standard deviations of Horizontal control used.

Table 4.—List of formats for output data.—Continued.

Input Format No.	Fortran Description	Description of Data
111	(41H BOW ERROR AT AXIS OF FLIGHT ORIGIN. CX = E14.8, 5H CY = E14.8)	Horizontal bow errors.
112	(//37H VERTICAL CONTROL USED FOR ADJUSTMENT /)	Point Category Title.
113	(63H POINT CZ RZ COMP X GROUND COMP Y GROUND)	Column headings for Vertical control used in the adjustment.
114	(7H STDZ = E14.8)	Standard deviation in vertical.
109	(1X, I7, 5E14.8)	Principal numerical output used for (1) Horizontal control used; (2) OTHER Horizontal Control; (3) OTHER Vertical control; and (4) Bridge Points.
123	(1X, I7, 4E14.8)	Alternate numerical output form used only for Vertical Control Used for Adjustment.
117	(//25H OTHER HORIZONTAL CONTROL/)	Category Title.
118	(49H POINT COMP X GROUND COMP Y GROUND COMP Z GROUND PLOT X PLOT Y)	Column headings used for all three final categories of points: (1) OTHER Horizontal Control; (2) OTHER Vertical Control; and (3) Bridge Points.
119	(//23H OTHER VERTICAL CONTROL/)	Point Category Title.
120	(//14H BRIDGE POINTS/)	Point Category Title.
	Diagnostic Statements	
102	(54H INITIAL-TERMINAL CARDS REVERSED. REMOVE FROM COMPUTER).	
103	(48H PRESS START TO INITIALIZE PROGRAM FOR NEXT CASE).	
104	(49H CONTROL CARDS DO NOT MATCH. REMOVE FROM COMPUTER).	
121	(56H CASE COMPLETED. PRESS START TO INITIALIZE FOR NEXT CASE).	

Table 5.—List of output data for sample problem based on either Table 2 or 3 using a third degree adjustment.

```
AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 3 DEG
HORIZONTAL CONTROL USED FOR ADJUSTMENT

 POINT       CX              CY              RX              RY          COMP Z GROUND
3054101 .20000000E-05-.56000000E-06 .15599619E-01-.28306202E-01 .12174521E+04
  57101 .20229490E-00 .12894930E-01-.18525122E-01 .42225648E-01 .13363810E+04
  71101 .40881830E-00-.24595850E-00-.16407493E-02-.35978804E-01 .15128077E+04
  75101 .68000000E-05-.70700000E-05 .45662531E-02 .22059359E-01 .16771218E+04
STDXs  .14260329E-01 STDYs  .38145931E-01 STDXYs  .40724305E-01
BOW ERROR AT AXIS OF FLIGHT ORIGIN. CXs  .57678467E-00 CYs -.19243384E-00
```

Table 5.—List of output data for sample problem based on either Table 2 or 3 using a third degree adjustment.—Continued.

VERTICAL CONTROL USED FOR ADJUSTMENT

POINT	CZ	RZ	COMP X GROUND	COMP Y GROUND
54203	.41029080E-00	-.13900388E-02	.18907548E+07	.24969391E+06
58201	.30440740E-00	-.23466440E-02	.18605400E+07	.23917466E+06
58203	.34996450E-00	.66242083E-02	.18798539E+07	.22796531E+06
64201	-.68947000E-02	.72374354E-02	.18420906E+07	.20658801E+06
64203	.11642220E-00	-.12025443E-01	.18642469E+07	.19796792E+06
69201	-.23997960E-00	-.73733391E-02	.18275720E+07	.17585375E+06
69203	-.49455300E-01	.96339443E-02	.18486799E+07	.16614739E+06
75201	-.52834000E-02	-.25526863E-02	.18291843E+07	.13258059E+06
75203	-.22043990E-00	.21925632E-02	.18073136E+07	.14510327E+06

STDZs .71310360E-02

OTHER HORIZONTAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
61101	.18652722E+07	.21600250E+06	.15857909E+04	.93263612E+06	.10800125E+06
66101	.18488294E+07	.18723481E+06	.14433995E+04	.92441471E+06	.93617405E+05
73101	.18302735E+07	.14814163E+06	.15237276E+04	.91513676E+06	.74070815E+05

OTHER VERTICAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54202	.18885195E+07	.25407738E+06	.14284590E+04	.94425978E+06	.12703869E+06
58202	.18707581E+07	.23469007E+06	.11545784E+04	.93537907E+06	.11734503E+06
64202	.18540900E+07	.20227989E+06	.14379343E+04	.92704500E+06	.10113994E+06
69202	.18370301E+07	.17142187E+06	.14623441E+04	.91851509E+06	.85710939E+05
75202	.18189177E+07	.13705452E+06	.16085551E+04	.90945886E+06	.68527262E+05

BRIDGE POINTS

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54205	.18666433E+07	.26411927E+06	.12034750E+04	.93332166E+06	.13205963E+06
57102	.18654248E+07	.24002227E+06	.13644793E+04	.93271244E+06	.12001113E+06
67101	.18059495E+07	.14141542E+06	.21125142E+04	.90297479E+06	.70707711E+05

Table 6.—List of output data similar to Table 5 but using a second degree adjustment.

AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 2 DEG
HORIZONTAL CONTROL USED FOR ADJUSTMENT

POINT	CX	CY	RX	RY	COMP Z GROUND
3054101	.20000000E-05	-.29670000E-04	.52189944E-01	-.83382171E-02	.12235593E+04
57101	.20120910E-00	.12943830E-01	-.73083358E-01	.18558920E-01	.13342907E+04
71101	.40995480E-00	-.24612440E-00	.44753213E-01	-.38286998E-01	.15170201E+04
75101	.44000000E-05	-.32560000E-04	-.23859798E-01	.28066294E-01	.16748079E+04

STDXs .59545802E-01 STDYs .29819291E-01 STDXYs .66594990E-01
BOW ERROR AT AXIS OF FLIGHT ORIGIN. CXs .59020883E-00 CYs -.17178114E-00

Table 6.—List of output data similar to Table 5 but using a second degree adjustment.—Continued.

VERTICAL CONTROL USED FOR ADJUSTMENT

POINT	CZ	RZ	COMP X GROUND	COMP Y GROUND
54203	.41020250E-00	-.66719342E-01	.18907618E+07	.24969326E+06
58201	.30440970E-00	.24447108E-01	.18605379E+07	.23917026E+06
58203	.34997010E-00	.72475665E-01	.18798541E+07	.22796028E+06
64201	-.67915000E-02	.35459392E-02	.18420925E+07	.20658466E+06
64203	.11651360E-00	.32840816E-01	.18642449E+07	.19796582E+06
69201	-.23991420E-00	-.76767702E-01	.18275749E+07	.17585635E+06
69203	-.49393600E-01	-.38848103E-01	.18486804E+07	.16615000E+06
75201	-.51983000E-02	.81687014E-02	.18291862E+07	.13257729E+06
75203	-.22034850E-00	.40856916E-01	.18073082E+07	.14510630E+06

STDZs .50659916E-01

OTHER HORIZONTAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
61101	.18652710E+07	.21599783E+06	.15816193E+04	.93263550E+06	.10799891E+06
66101	.18488299E+07	.18723485E+06	.14444651E+04	.92441495E+06	.93617429E+05
73101	.18302749E+07	.14814355E+06	.15268249E+04	.91513746E+06	.74071777E+05

OTHER VERTICAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54202	.18885257E+07	.25407858E+06	.14349555E+04	.94426285E+06	.12703929E+06
58202	.18707573E+07	.23468547E+06	.11516324E+04	.93537865E+06	.11734273E+06
64202	.18540898E+07	.20227717E+06	.14363760E+04	.92704491E+06	.10113858E+06
69202	.18370320E+07	.17142446E+06	.14664454E+04	.91851602E+06	.85712230E+05
75202	.18189160E+07	.13705371E+06	.16063861E+04	.90945803E+06	.68526857E+05

BRIDGE POINTS

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54205	.18666375E+07	.26412471E+06	.12106331E+04	.93331876E+06	.13206235E+06
57102	.18654231E+07	.24001821E+06	.13627124E+04	.93271157E+06	.12000910E+06
67101	.18059425E+07	.14141739E+06	.21077537E+04	.90297128E+06	.70708696E+05

Table 7.—List of output data similar to Tables 5 and 6 but using a first degree adjustment.

AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 1 DEG
HORIZONTAL CONTROL USED FOR ADJUSTMENT

POINT	CX	CY	RX	RY	COMP Z GROUND
3054101	.14000000E-04	-.12120000E-04	-.11524665E-00	-.78212313E-02	.12126280E+04
57101	.20214420E-00	.13503610E-01	.71418954E-01	.22612840E-01	.13317649E+04
71101	.41104500E-00	-.24559390E-00	.23004934E-00	-.14621604E-00	.15185106E+04
75101	.62000000E-05	-.17680000E-04	-.18622164E-00	.13142443E-00	.16689796E+04

STDXs .18795714E-00 STDYs .11434454E-00 STDXYs .22000582E-00
BOW ERROR AT AXIS OF FLIGHT ORIGIN. CXs .15108435E-00 CYs -.63401524E-01

Table 7.—List of output data similar to Tables 5 and 6 but using a first degree adjustment.—Con.

VERTICAL CONTROL USED FOR ADJUSTMENT

POINT	CZ	RZ	COMP X GROUND	COMP Y GROUND
54203	.41030620E-00	.21272153E-01	.18907439E+07	.24969819E+06
58201	.30445870E-00	.76435203E-01	.18605526E+07	.23917542E+06
58203	.35001700E-00	.51006446E-01	.18798569E+07	.22797811E+06
64201	-.67957000E-02	-.52242734E-01	.18421141E+07	.20660695E+06
64203	.11651220E-00	-.56785152E-01	.18642636E+07	.19798900E+06
69201	-.23992930E-00	-.13290604E-00	.18275858E+07	.17587906E+06
69203	-.49408700E-01	-.97494431E-01	.18486990E+07	.16616313E+06
75201	-.51837000E-02	.10238841E-00	.18291835E+07	.13256301E+06
75203	-.22033550E-00	.88326151E-01	.18072870E+07	.14511680E+06
STDZs	.86821970E-01			

OTHER HORIZONTAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
61101	.18652864E+07	.21601807E+06	.15850866E+04	.93264320E+06	.10800903E+06
66101	.18488499E+07	.18725685E+06	.14499303E+04	.92442495E+06	.93628429E+05
73101	.18302785E+07	.14814659E+06	.15256901E+04	.91513928E+06	.74073295E+05

OTHER VERTICAL CONTROL

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54202	.18885083E+07	.25407790E+06	.14267313E+04	.94425416E+06	.12703895E+06
58202	.18707653E+07	.23469633E+06	.11502659E+04	.93538265E+06	.11734816E+06
64202	.18541100E+07	.20229970E+06	.14413243E+04	.92705500E+06	.10114985E+06
69202	.18370464E+07	.17144273E+06	.14703018E+04	.91852323E+06	.85721365E+05
75202	.18189039E+07	.13704953E+06	.16009931E+04	.90945198E+06	.68524766E+05

BRIDGE POINTS

POINT	COMP X GROUND	COMP Y GROUND	COMP Z GROUND	PLOT X	PLOT Y
54205	.18666423E+07	.26410330E+06	.11959836E+04	.93332115E+06	.13205165E+06
57102	.18654340E+07	.24002362E+06	.13591123E+04	.93271704E+06	.12001181E+06
67101	.18059173E+07	.14142484E+06	.21032714E+04	.90295865E+06	.70712422E+05

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory. See Glossary of Mnemonic Terms.

```
*1007                                          (precision statement)
C     HORIZONTAL AND VERTICAL ADJUSTMENT OF STRIP AEROTRIANGULATION
      DIMENSION C(40,14),A(40,8),EN(7,8),VAC(7),HAC(7)
    1 READ 122
      READ 100,NHC,NVC,IS1A0,IDHA,IDVA,PCONST (instruction parameters)
      IF(IS1A0)2,2,3          (stereoplanigraph (1); analytic (0))
    2 READ 101,LINE1,IPN,X1,Y1
      READ 101,LINE2,IPN,X2,Y2    (terminal photo centers--analytic)
      GO TO 4
    3 READ 106,LINE1,IPN,X1,Y1
      READ 106,LINE2,IPN,X2,Y2    (terminal centers--stereoplanigraph)
    4 IF(LINE2-LINE1)5,5,6        (centers in proper order?)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
    5 PRINT 102
      PRINT 103
      PAUSE
      GO TO 1
C     CONSTANTS FOR TRANSFORMATION OF MODEL COORDINATES TO AXIS OF
C     FLIGHT SYSTEM
    6 XDIF=X1-X2
      YDIF=Y1-Y2
      DIST=SQRTF(XDIF**2+YDIF**2)
      A1=-XDIF/DIST                                   (equations 23)
      B1=YDIF/DIST
      C1=(-DIST/2.)-(A1*X1)+(B1*Y1)
      D1=(-B1*X1)-(A1*Y1)
C     READING CONTROL CARDS
      ITNC=NHC+NVC      (total no. of control pts. used in adjustment)
      IF(IS1A0)7,7,8
    7 READ 101,(C(I,8),(C(I,J),J=1,4),I=1,ITNC)  (input all hor. &
      GO TO 9                                     vert. control pts.
    8 READ 106,(C(I,8),(C(I,J),J=1,4),I=1,ITNC)   used in the
    9 READ 105,(C(I,8),(C(I,J),J=5,7),I=1,ITNC)   adjustment)
      DO 10 I=1,ITNC
      IF(C(I,1)-C(I,8))11,10,11          (cards in proper sequence?)
   11 PRINT 104
      PRINT 103
      PAUSE
      GO TO 1
   10 CONTINUE
      IF(SENSE SWITCH 1)75,76
   75 PAUSE 00001
C     TRANSFORMATION OF MODEL COORDINATES TO AXIS OF FLIGHT SYSTEM
   76 DO 12 I=1,ITNC
      SUM1=A1*C(I,2)-B1*C(I,3)+C1
      SUM2=B1*C(I,2)+A1*C(I,3)+D1
      C(I,2)=SUM1
   12 C(I,3)=SUM2                                     (equations 22)
C     VERTICAL ADJUSTMENT FOR SLOPE FACTORS
      XDIF=C(1,2)-C(NHC,2)
      YDIF=C(1,3)-C(NHC,3)
      DGX=C(1,5)-C(NHC,5)
      DGY=C(1,6)-C(NHC,6)
      SCALE=SQRTF((DGX**2+DGY**2)/(XDIF**2+YDIF**2)) (e.g., 20,000)
      SUM2=0.
      FTNC=ITNC
      DO 13 I=1,ITNC
   13 SUM2=SUM2+C(I,4)                          (sum of elevations)
      AVIZ=SUM2/FTNC      (average vert. instrument (model) elev.)
      NHCP1=NHC+1
      FNVC=NVC
      SUM1=0.
      DO 14 I=NHCP1,ITNC
   14 SUM1=SUM1+C(I,7)
      SUM1=SUM1/FNVC
      EINDEX=AVIZ-(SUM1/SCALE)                     (equation 29)
      IGO=1
   31 DO 15 I=NHCP1,ITNC
      C(I,8)=C(I,7)/SCALE+EINDEX                   (equation 29)
   15 C(I,9)=C(I,8)-C(I,4)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
      LINE1=NHCP1
      LINE2=ITNC
      ITRANS=1
   45 I=1
      IF(IDVA-2)16,17,18        (1st, 2nd, 3rd deg. vert. adjustm't?)
C     3RD DEGREE VERTICAL OBSERVATION EQUATIONS
   18 DO 19 J=LINE1,LINE2
      A(I,3)=C(J,2)                          (coefficients of observation
      A(I,6)=C(J,3)                           equations.
      A(I,5)=A(I,3)*A(I,6)
      A(I,2)=A(I,3)*A(I,3)                    equations 11 and 31 )
      A(I,4)=A(I,2)*A(I,6)
      A(I,1)=A(I,3)*A(I,2)
      A(I,7)=1.
      A(I,8)=C(J,9)
   19 I=I+1
      N=7
      GO TO (77,74),ITRANS
C     2ND DEGREE VERTICAL OBSERVATION EQUATIONS
   17 DO 20 J=LINE1,LINE2
      A(I,2)=C(J,2)
      A(I,4)=C(J,3)
      A(I,3)=A(I,2)*A(I,4)
      A(I,1)=A(I,2)*A(I,2)
      A(I,5)=1.
      A(I,6)=C(J,9)
   20 I=I+1
      N=5
      GO TO (77,74),ITRANS
C     1ST DEGREE VERTICAL OBSERVATION EQUATIONS
   16 DO 21 J=LINE1,LINE2
      A(I,1)=C(J,2)
      A(I,3)=C(J,3)
      A(I,2)=A(I,1)*A(I,3)
      A(I,4)=1.
      A(I,5)=C(J,9)
   21 I=I+1
      N=4
      GO TO (77,74),ITRANS
   77 IF(SENSE SWITCH 1)78,22
   78 PAUSE 00002
C     FORMATION AND SOLUTION OF NORMAL EQUATIONS
   22 LINE=NVC
   40 NP1=N+1
      NM1=N-1
      DO 23 I=1,N
      DO 23 J=I,NP1
      EN(I,J)=0.
      DO 23 K=1,LINE                         (normal equations)
   23 EN(I,J)=EN(I,J)+A(K,I)*A(K,J)                (equation 32)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
      DO 99 I=1,N
      SQR=SQRTF(EN(I,I))
      DO 98 J=I,NP1
   98 EN(I,J)=EN(I,J)/SQR
      IF(I-N)97,96,96                                  (forward
   97 IP1=I+1                                             solution)
      DO 99 L=IP1,N
      DO 99 J=L,NP1
   99 EN(L,J)=EN(L,J)-EN(I,L)*EN(I,J)                      (equation 35)
   96 EN(N,NP1)=EN(N,NP1)/EN(N,N)                          (equation 36)
      DO 91 I=1,NM1
      NMI=N-I
      NMIP1=NMI+1                                      (back solution)
      DO 90 J=NMIP1,N
   90 EN(NMI,NP1)=EN(NMI,NP1)-EN(J,NP1)*EN(NMI,J)
   91 EN(NMI,NP1)=EN(NMI,NP1)/EN(NMI,NMI)                  (equation 37)
      GO TO (24,32,41),IGO
C     SLOPE FACTORS                        (application of slope factors
   24 IF(IDVA-2)25,26,27                    to list of hor. & vert.
   27 DO 28 I=1,ITNC                        control points)
      XS=3.*EN(1,8)*C(I,2)**2 + 2.*EN(2,8)*C(I,2) + EN(3,8)  (eq. 15)
      YS=EN(4,8)*C(I,2)**2 + EN(5,8)*C(I,2) + EN(6,8)        (eq. 16)
      ZS=SQRTF(1.+XS**2+YS**2)                               (eq. 18)
      C(I,2)=C(I,2)-(C(I,4)-AVIZ)*XS                      (equation 19)
      C(I,3)=C(I,3)-(C(I,4)-AVIZ)*YS                      (equation 20)
   28 C(I,4)=C(I,4)*ZS
      GO TO 201
   26 DO 29 I=1,ITNC
      XS=2.*EN(1,6)*C(I,2) + EN(2,6)
      YS=EN(3,6)*C(I,2) + EN(4,6)
      ZS=SQRTF(1.+XS**2+YS**2)
      C(I,2)=C(I,2)-(C(I,4)-AVIZ)*XS
      C(I,3)=C(I,3)-(C(I,4)-AVIZ)*YS
   29 C(I,4)=C(I,4)*ZS
      GO TO 201
   25 DO 200 I=1,ITNC
      XS=EN(1,5)
      YS=EN(2,5)*C(I,2) + EN(3,5)
      ZS=SQRTF(1.+XS**2+YS**2)
      C(I,2)=C(I,2)-(C(I,4)-AVIZ)*XS
      C(I,3)=C(I,3)-(C(I,4)-AVIZ)*YS
  200 C(I,4)=C(I,4)*ZS
  201 IF(SENSE SWITCH 1)79,80
   79 PAUSE 00003
C     CONSTANTS FOR TRANSFORMATION OF GROUND PLANE COORDINATES TO AXIS
C     OF FLIGHT SYSTEM
   80 XDIF=C(1,2)-C(NHC,2)
      YDIF=C(1,3)-C(NHC,3)
      DIST=XDIF**2+YDIF**2
      A2=(DGX * XDIF + DGY * YDIF)/DIST                   (equations 26)
      B2=(XDIF * DGY - YDIF * DGX)/DIST
      C2=C(1,5)-A2*C(1,2)+B2*C(1,3)
      D2=C(1,6)-B2*C(1,2)-A2*C(1,3)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
      DIST=1./(A2**2 + B2**2)
      A21=A2*DIST
      B21=B2*DIST                                  (inverse;  equations 28)
      C21=(A2*C2 + B2*D2)*DIST
      D21=(A2*D2 - B2*C2)*DIST
C     CONTROL TRANSFORMED INTO AXIS OF FLIGHT SYSTEM
      DO 30 I=1,NHC
      C(I,8)=A21*C(I,5) + B21*C(I,6) - C21        (inverse transform.
      C(I,9)=-B21*C(I,5) + A21*C(I,6) - D21            equation 27)
      C(I,10)=C(I,8)-C(I,2)                       (cx discrepancy)
   30 C(I,11)=C(I,9)-C(I,3)                       (cy discrepancy)
      SCALE=SQRTF((DGX**2 + DGY**2)/(XDIF**2 + YDIF**2))
      IF(SENSE SWITCH 2)81,82
   81 PAUSE 00004
   82 IGO=2
      GO TO 31
C     VERTICAL ADJUSTMENT COEFFICIENTS            (final z polynomial)
   32 DO 33 I=1,N
   33 VAC(I)=EN(I,NP1)
      DO 47 I=1,NVC
      SUM1=0.
      DO 48 J=1,N
   48 SUM1=SUM1+VAC(J)*A(I,J)
      K=I+NHC
   47 C(K,10)=C(K,9)-SUM1
C     HORIZONTAL ADJUSTMENT COEFFICIENTS          (x & y polynomials)
      LINE=NHC*2
      LINE1=1
      LINE2=NHC
      ITRANS=1
      IGO=3
   52 I=1
      IF(IDHA-2)34,35,36
C     3RD DEGREE HORIZONTAL OBSERVATION EQUATIONS
   36 N=7
      DO 37 J=LINE1,LINE2
      A(I,3)=C(J,2)
      A(I,5)=-C(J,3)
      A(I,2)=A(I,3)*A(I,3)                        (coefficients of
      A(I,1)=A(I,3)*A(I,2)                         observation equations.
      A(I,4)=2.*A(I,3)*A(I,5)                      equations 9, 10 & 31)
      A(I,6)=1.
      A(I,7)=0.
      A(I,8)=C(J,10)
      I=I+1
      A(I,3)=-A(I-1,5)
      A(I,5)=A(I-1,3)
      A(I,4)=A(I-1,2)
      A(I,2)=-A(I-1,4)
      A(I,6)=0.
      A(I,7)=1.
      A(I,1)=3.*A(I,4)*A(I,3)
      A(I,8)=C(J,11)
   37 I=I+1
      GO TO (83,54),ITRANS
C     2ND DEGREE HORIZONTAL OBSERVATION EQUATIONS
   35 N=6
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.--Continued.

```
      DO 38 J=LINE1,LINE2
      A(I,2)=C(J,2)
      A(I,4)=-C(J,3)
      A(I,1)=A(I,2)*A(I,2)
      A(I,3)=2.*A(I,2)*A(I,4)
      A(I,5)=1.
      A(I,6)=0.
      A(I,7)=C(J,10)
      I=I+1
      A(I,2)=-A(I-1,4)
      A(I,4)=A(I-1,2)
      A(I,3)=A(I,4)*A(I,4)
      A(I,1)=-A(I-1,3)
      A(I,5)=0.
      A(I,6)=1.
      A(I,7)=C(J,11)
   38 I=I+1
      GO TO(83,54),ITRANS
C     1ST DEGREE HORIZONTAL OBSERVATION EQUATIONS
   34 N=4
      DO 39 J=LINE1,LINE2
      A(I,1)=C(J,2)
      A(I,2)=-C(J,3)
      A(I,3)=1.
      A(I,4)=0.
      A(I,5)=C(J,10)
      I=I+1
      A(I,1)=-A(I-1,2)
      A(I,2)=A(I-1,1)
      A(I,3)=0.
      A(I,4)=1.
      A(I,5)=C(J,11)
   39 I=I+1
      GO TO (83,54),ITRANS
   83 IF(SENSE SWITCH 2)84,40
   84 PAUSE 00005
      GO TO 40                                  (equation solution)
   41 DO 42 I=1,N                  (return from  equation solution)
   42 HAC(I)=EN(I,NP1)
      CXBOW=HAC(N-1)                                     (equation 9)
      CYBOW=HAC(N)
C     DATA FOR PUNCH OUT OF HORIZONTAL CONTROL USED FOR ADJUSTMENT
      PUNCH 122                                            (job title)
      PUNCH 107                                ("hor. control used")
      PUNCH 108                                     (column headings)
   53 L=1
      DO 43 I=1,NHC
      SUM1=0.
      SUM2=0.
      DO 44 J=1,N
      SUM1=SUM1+HAC(J)*A(L,J)
   44 SUM2=SUM2+HAC(J)*A(L+1,J)
      L=L+2
      C(I,12)=C(I,10)-SUM1                          (residual Rx)
   43 C(I,13)=C(I,11)-SUM2                          (residual Ry)
      ITRANS=2
      GO TO 45                     (back to vert. observation equs.)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
 74 DO 49 I=1,NHC
    SUM1=0.
    DO 46 J=1,N
 46 SUM1=SUM1+VAC(J)*A(I,J)
    C(I,14)=C(I,4)+SUM1
 49 C(I,14)=SCALE*(C(I,14)-EINDEX)     (computed ground elevation)
    PUNCH 109,(C(I,1),(C(I,J),J=10,14),I=1,NHC) (hor. control used)
    SUM1=0.
    SUM2=0.
    DO 50 I=1,NHC
    SUM1=SUM1+C(I,12)**2
 50 SUM2=SUM2+C(I,13)**2
    DIST=NHC-1
    STDX=SQRTF(SUM1/DIST)
    STDY=SQRTF(SUM2/DIST)          (photogrammetric diagnostic data)
    STDXY=SQRTF(STDX**2 + STDY**2)
    PUNCH 110,STDX,STDY,STDXY
    PUNCH 111,CXBOW,CYBOW
    IF(SENSE SWITCH 2)85,86
 85 PAUSE 00006
C   DATA FOR PUNCH OUT OF VERTICAL CONTROL USED IN ADJUSTMENT
 86 SUM1=0.
    DO 51 I=NHCP1,ITNC
    C(I,11)=0.
 51 SUM1=SUM1+C(I,10)**2
    DIST=NVC-1
    STDZ=SQRTF(SUM1/DIST)
    LINE1=NHCP1
    LINE2=ITNC
    GO TO 52                      (back to hor. observation equs.)
 54 L=1
    DO 55 I=NHCP1,ITNC
    SUM1=0.
    SUM2=0.
    DO 73 J=1,N
    SUM1=SUM1+HAC(J)*A(L,J)
 73 SUM2=SUM2+HAC(J)*A(L+1,J)
    L=L+2
    C(I,11)=SUM1+C(I,2)
    C(I,12)=SUM2+C(I,3)
    SUM1=A2*C(I,11)-B2*C(I,12)+C2
    SUM2=B2*C(I,11)+A2*C(I,12)+D2
    C(I,11)=SUM1
 55 C(I,12)=SUM2
    PUNCH 112                          ("vert. control used")
    PUNCH 113                          (column headings)
    PUNCH 123,(C(I,1),(C(I,J),J=9,12),I=NHCP1,ITNC) (vert. used)
    PUNCH 114,STDZ                     ("other horizontal used")
    PUNCH 117
    PUNCH 118                          (column headings)
    IF(SENSE SWITCH 3)87,56
 87 PAUSE 00007
C   DATA FOR PUNCH OUT OF OTHER CONTROL AND BRIDGE POINTS
 56 IF(IS1A0)60,60,61
 60 READ 115,IPN,X,Y,Z,ITYPE,ITEST
    GO TO 57
```

Tab! ...ng of source Fortran program for IBM 1620 with 100K core memory.--Continued.

```
 61 READ 116,IPN,X,Y,Z,ITYPE,ITEST
 57 IF(ITYPE-1)62,58,59
 58 PUNCH 119                                  ("other vertical used")
    PUNCH 118                                  (column headings)
    GO TO 62
 59 PUNCH 120                                  ("bridge points")
    PUNCH 118                                  (column headings)
 62 SUM1 =A1*X - B1*Y + C1                 (axis-of-flight transform.
    SUM2 =B1*X + A1*Y + D1                          equations 22)
    IF(IDVA-2)210,211,212
212 XS=3.*VAC(1)*SUM1**2 + 2.*VAC(2)*SUM1 + VAC(3)  (slope factors
    YS=VAC(4)*SUM1**2 + VAC(5)*SUM1 + VAC(6)          3rd degree)
    GO TO 213                                        (equ. 15 & 16)
211 XS=2.*VAC(1)*SUM1 + VAC(2)                  (slope factors,
    YS=VAC(3)*SUM1 + VAC(4)                        2nd degree)
    GO TO 213
210 XS=VAC(1)                                   (slope factors,
    YS=VAC(2)*SUM1 + VAC(3)                        1st degree)
213 ZS=SQRTF(1.+XS**2+YS**2)                    (application
    X= SUM1-(Z-AVIZ)*XS                          of slope
    Y= SUM2-(Z-AVIZ)*YS                          corrections)
    Z=Z*ZS
    IF(IDHA-2)63,64,65
 63 SUM1=HAC(1)*X + HAC(2)*(-Y) + HAC(3)*1.   (polynomial transforms
    SUM2=HAC(1)*Y + HAC(2)*X + HAC(4)*1.              equations 1)
    GO TO 66
 64 SUM1=HAC(1)*X**2 + HAC(2)*X + HAC(3)*(-2.)*X*Y + HAC(4)*(-Y) +
   1HAC(5)*1.                                         (2nd degree)
    SUM2=HAC(1)*2.*X*Y + HAC(2)*Y + HAC(3)*X**2 + HAC(4)*X + HAC(6)*1.
    GO TO 66
 65 SUM1=HAC(1)*X**3 + HAC(2)*X**2 + HAC(3)*X + HAC(4)*(-2.)*X*Y +
   1HAC(5)*(-Y) + HAC(6)*1.                   (equ. 9)(3rd degree)
    SUM2=HAC(1)*3.*X**2*Y + HAC(2)*2.*X*Y + HAC(3)*Y + HAC(4)*X**2 +
   1HAC(5)*X + HAC(7)*1.                       (equation 10)
 66 TRUEX=SUM1+X
    TRUEY=SUM2+Y
    IF(IDVA-2)67,68,69
 69 SUM1=VAC(1)*X**3 + VAC(2)*X**2 + VAC(3)*X + VAC(4)*X**2*Y + VAC(5)
   1*X*Y + VAC(6)*Y + VAC(7)*1.        (equation 11; 3rd deg. in z)
    GO TO 70
 68 SUM1=VAC(1)*X**2 + VAC(2)*X + VAC(3)*X*Y + VAC(4)*Y +VAC(5)*1.
    GO TO 70                                        (2nd deg. in z)
 67 SUM1=VAC(1)*X + VAC(2)*X*Y + VAC(3)*Y + VAC(4)*1.  (1st deg.)
 70 TRUEZ=SUM1+Z
    X=A2*TRUEX-B2*TRUEY+C2                          (equations 25)
    Y=B2*TRUEX+A2*TRUEY+D2
    Z=SCALE*(TRUEZ-EINDEX)                          (equation  29)
    PLOTX=PCONST * X
    PLOTY=PCONST * Y
    PUNCH 109,IPN,X,Y,Z,PLOTX,PLOTY              (principal output)
    IF(ITEST)56,56,71
 71 PRINT 121
    PAUSE
    GO TO 1
100 FORMAT(2(I2,1X),3(I1,1X),F10.9)      (instruction parameters)
101 FORMAT(I2,1X,I7,2X,E14.8,2X,E14.8,2X,E14.8)    (analytic data)
```

Table 8.—Listing of source Fortran program for IBM 1620 with 100K core memory.—Continued.

```
106 FORMAT(I2,1X,I7,3F16.2)                    (stereoplanigraph data)
102 FORMAT(54H INITIAL-TERMINAL CARDS REVERSED. REMOVE FROM COMPUTER)
103 FORMAT(48H PRESS START TO INITIALIZE PROGRAM FOR NEXT CASE)
104 FORMAT(49H CONTROL CARDS DO NOT MATCH. REMOVE FROM COMPUTER)
105 FORMAT(3X,I7,3F16.3)                      (ground control data)
107 FORMAT(39H HORIZONTAL CONTROL USED FOR ADJUSTMENT/)
108 FORMAT(77H  POINT       CX              CY              RX              RY
   1         COMP Z GROUND)
122 FORMAT(64H
   1         )
109 FORMAT(1X,I7,5E14.8)                   (principal output list)
110 FORMAT(7H STDX= E14.8,7H STDY= E14.8,8H STDXY= E14.8)
111 FORMAT(41H BOW ERROR AT AXIS OF FLIGHT ORIGIN. CX= E14.8,5H CY= E1
   14.8)
112 FORMAT(//37H VERTICAL CONTROL USED FOR ADJUSTMENT/)
113 FORMAT(63H  POINT       CZ              RZ        COMP X GROUND COMP Y
   1 GROUND)
114 FORMAT(7H STDZ= E14.8)
117 FORMAT(//25H OTHER HORIZONTAL CONTROL/)
118 FORMAT(73H  POINT COMP X GROUND COMP Y GROUND COMP Z GROUND     PLO
   1T X          PLOT Y)
115 FORMAT(3X,I7,3(2X,E14.8),20X,2I1)  (bridge pt. analytic input)
116 FORMAT(3X,I7,3F16.2,20X,2I1)       (bridge pt. stereopl. input)
119 FORMAT(//23H OTHER VERTICAL CONTROL/)
120 FORMAT(//14H BRIDGE POINTS/)
121 FORMAT(56H CASE COMPLETED. PRESS START TO INITIALIZE FOR NEXT CASE
   1)
123 FORMAT(1X,I7,4E14.8)                   (vertical control used)
    END
```

Glossary of Mnemonic Terms Used in Fortran Program

A.—List of coefficients of observation equations.

A1, B1, C1, D1.—Coefficients for linear transformation of input model (instrument) coordinates into axisof-flight system.

A2, B2, C2, D2.—Coefficients for Linear transformation of axis-of-flight coordinates into the ground survey system.

A21, B21, C21, D21.—Inverse of A2, B2, C2, D2.

AVIZ.—Average vertical instrument (model) elevation Z.

C.—List of coordinates of control points.

CXBOW.—Essentially maximum x-discrepancy (ordinate) of x-polynomial curve when the discrepancies at the two ends are made zero.

DGX.—Difference in ground X-coordinates of two points.

DIST.—Distance between two points. Also used as distance-squared and as a general temporary storage item.

EINDEX.—Elevation Index—vertical translation term.

EN.—Equation (Normal)

FTNC.—Floating point total number of horizontal and vertical control used in the adjustment.

HAC.—Horizontal adjustment coefficients (of the polynomial).

IDHA.—Degree of horizontal adjustment.

IDVA.—Degree of vertical adjustment.

IGO.—Transfer decision number.

IPN.—Point number.

ISIAO.—Stereoplanigraph (1) or analytic (0)

ITEST.—"1" in the 80th column of the last card of deck of bridge points.

ITYPE.—A "1" placed in the 79th column of the first card of the deck of other vertical control points, or a "2" in the first card of the deck of bridge points.

ITNC.—(Integer) Total number of horizontal and vertical control points used.

ITRANS.—Transfer decision number.

LINE 1.—First photograph center. Also the first line of an array of stored data.

NHC.—Number of horizontal control points used in the adjustment.

NHCP1.—Number of horizontal control points used Plus 1.

NM1.—"N" Minus 1.

NP1.—"N" Plus 1.

NVC.—Number of vertical control points used in the adjustment.

PCONST.—Plotting constant scale factor.

SCALE.—Scale factor—ground distance divided by model distance.

STDX.—Standard deviation of the x-coordinates of control used.

SUMI.—Auxiliary temporary storage term.

VAC.—Vertical adjustment coefficients (polynomial).

XDIF.—Difference in x-coordinates of two points.

XS.—X-slope correction factor.

X1.—x-coordinate of first of two points.

SAMPLE MANUAL INPUT DATA FORMS

Tables 9 through 15 constitute a complete set of input data forms for the stereoplanigraph mode as would need to be furnished to the key punch operator by the photogrammetrist. The formats were explained above in the section on Input Data Format.

For the analytic mode, only Tables 11 and 12 would be needed by the key punch operator, and these cards would need to be inserted into their proper place in the deck of other cards already punched from the previous program. However, the formats for that deck are shown in Tables 16 to 20 for the purpose of furnishing the reader with test data for the analytic mode.

Table 9.—Manual input data form including (1) job title statement, (2) instruction parameter statement, (3) instrument (model) coordinates of terminal photo centers, and (4) list of instrument (model) coordinates of horizontal control images, all in stereoplanigraph mode.

HORIZONTAL AND VERTICAL ADJUSTMENT
OF STRIP AEROTRIANGULATION

REMARKS: *Stereoplanigraph Mode* 1/6

Charge No. *34304* Project No. *Test*

AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 3 DEG

HORIZONTAL CONTROL	VERTICAL CONTROL	*	DEGREES HOR.	DEGREES VERT.	PLOTTING CONSTANT
04	09	1	33		500000000

(yellow top cards)
10 digits 9 decimals
Decimal point not punched

* 1 -- Stereoplanigraph mode
0 -- Analytic mode

Model Coordinates of Initial and Terminal photo centers for Axis-of-Flight System (blue-top cards)

Point No.	X (instrument)	Y (instrument)
01	5300	50174	29235
02	7700	68399	69455

HORIZONTAL CONTROL USED FOR ADJUSTMENT (orange-top cards)

Point No.	X (instrument)	Y (instrument)	Z (instrument)
01	54101	46375	281504	51870
02	57101	57788	254666	52052
03	71101	79987	125091	52359
04	75101	72721	84398	52597
05				
06				
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19				
20				

Table 10.—Manual data input form for list of instrument (model) coordinates of vertical control images in stereoplanigraph mode.

VERTICAL CONTROL USED FOR ADJUSTMENT (green top cards)

REMARKS: Stereoplanigraph Mode

2/6

	POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01	54203	49791	281942	52061
02	58201	40687	244916	51915
03	58203	73528	246350	51891
04	64201	43668	189869	52349
05	64203	77991	196762	51846
06	69201	50039	140210	52362
07	69203	83948	144918	52056
08	75201	86412	87736	52378
09	75203	49278	85927	52903
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Table 11.—Manual data input form for list of ground coordinates of horizontal control stations for both modes of operation. The Z coordinates are irrelevant.

HORIZONTAL CONTROL USED FOR ADJUSTMENT (red top cards)

REMARKS: Stereoplanigraph Mode

3/6

	POINT NO.	X (ground)	Y (ground)	Z (ground)
01	54101	187719690 00	258023400	12150000
02	57101	187389840 0	238488100	13364000
03	71101	183916260 0	156265700	15285000
04	75101	182014690 0	135671100	16787000
05				
06				
07				
08				
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10				
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Table 12.—Manual data input form for list of elevations of vertical ground control stations for both modes. The X and Y coordinates are irrelevant.

VERTICAL CONTROL USED FOR ADJUSTMENT (brown top cards)

REMARKS: Stereoplanigraph Mode

4/6

	POINT NO.	X (ground)	Y (ground)	Z (ground)
01	54203	1890751020	2496942220	1345900
02	58201	1860542870	2391720300	1239600
03	58203	1879854350	2279672600	1226400
04	64201	1842092450	2065855300	1513100
05	64203	1864261480	1979658000	1180000
06	69201	1827581220	1758539800	1506100
07	69203	1848679430	1661436300	1311300
08	75201	1829187400	1325781400	1532900
09	75203	1807312470	1451059500	1874700
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Table 13.—Manual data input form for list of instrument (model) coordinates for OTHER *horizontal control stations not listed in Table 9, in stereoplanigraph mode.*

OTHER HORIZONTAL CONTROL (solid orange cards)

REMARKS: Stereoplanigraph Mode

5/6

	POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01	61101	64946	21943	52434
02	66101	67388	17209	52249
03	73101	75398	10740	52377
04				
05				
06				
07				
08				
09				
10				
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19				
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Table 14.—Manual data input form for list of instrument (model) coordinates of OTHER *vertical control stations not listed in Table 10 in stereoplanigraph mode.*

OTHER VERTICAL CONTROL (solid green cards)

REMARKS: *Stereoplanigraph Mode* 6/6

	POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01	54202	63540	285601	52182
02	58202	56914	247467	51787
03	64202	61971	194049	52231
04	69202	65299	142217	52289
05	75202	70096	85141	52498
06				
07				
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18				
19				
20				

Table 15.—Manual data input form for list of instrument (model) coordinates of bridged points (pass points and general points whose coordinates are sought) in stereoplanigraph mode. The bulk of the input data is of this form.

DATE | PROGRAMMER
PROJECT NO. | PROJECT TITLE
RUN NO. | RUN TITLE

Stereoplanigraph Mode
BRIDGE POINTS (solid white cards) require a "2" in column 79 of the first card and a "1" in column 80 of the last card.

01	54205	28451	280679	51848
02	57102	46070	249844	52096
03	67101	50510	80259	53250

Table 16.—Manual input data form for the analytic mode containing the same information as Table 9.

HORIZONTAL AND VERTICAL ADJUSTMENT
OF STRIP AEROTRIANGULATION

REMARKS: Analytic Mode

1/6

Charge No. 34 304 Project No. Test

AEROTRIANGULATION STRIP ADJUSTMENT SHENANDOAH VALLEY TEST 3 DEG

HORIZONTAL CONTROL	VERTICAL CONTROL	*	DEGREE HOR.	DEGREE VERT.	PLOTTING CONSTANT
04	09	0	3	3	5000000000

(yellow top cards)
10 digits 9 decimals
Decimal point not punched

* 1 -- Stereoplanigraph mode
0 -- Analytic mode

Model Coordinates of Initial and Terminal photo centers for Axis-of-Flight System (blue-top cards)

Point No.	X (instrument)	Y (instrument)
01 5300	+.50174000E+00	+.29235500E+01
02 7700	+.68399000E+00	+.69455000E+00

HORIZONTAL CONTROL USED FOR ADJUSTMENT (orange-top cards)

Point No.	X (instrument)	Y (instrument)	Z (instrument)
01 54101	+.46375000E+00	+.28150400E+01	+.51870000E+00
02 57101	+.57788000E+00	+.25466600E+01	+.52052000E+00
03 71101	+.79987000E+00	+.12509100E+01	+.52359000E+00
04 75101	+.72721000E+00	+.84398000E+00	+.52597000E+00
05			
06			
07			
08			
18			
19			
20			

Table 17.—Manual input data form for the analytic mode containing the same information as Table 10.

VERTICAL CONTROL USED FOR ADJUSTMENT (green top cards)

REMARKS: Analytic Mode

2/6

POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01 54203	+.49791000E+00	+.28194200E+01	+.52061000E+00
02 58201	+.40687000E+00	+.24491600E+01	+.51915000E+00
03 58203	+.73528000E+00	+.24635000E+01	+.51891000E+00
04 64201	+.43668000E+00	+.18986900E+01	+.52349000E+00
05 64203	+.77991000E+00	+.19676200E+01	+.51846000E+00
06 69201	+.50039000E+00	+.14021000E+01	+.52362000E+00
07 69203	+.83948000E+00	+.14491800E+01	+.52056000E+00
08 75201	+.86412000E+00	+.87736000E+00	+.52378000E+00
09 75203	+.49278000E+00	+.85927000E+00	+.52903000E+00
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Table 18.—Manual input data form for the analytic mode containing the same information as Table 13.

OTHER HORIZONTAL CONTROL (solid orange cards)

REMARKS: Analytic Mode

5/6

	POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01	61101	+.6494600 0E+00	+.2199430 0E+01	+.5243400 0E+00
02	66101	+.6738800 0E+00	+.1712090 0E+01	+.5224900 0E+00
03	73101	+.7539800 0E+00	+.1079400 0E+01	+.5237700 0E+00
04				
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Table 19.—Manual input data form for the analytic mode containing the same information as Table 14.

OTHER VERTICAL CONTROL (solid green cards)

REMARKS: Analytic Mode

6/6

	POINT NO.	X (instrument)	Y (instrument)	Z (instrument)
01	54202	+.6354000 0E+00	+.2856010 0E+01	+.5219200 0E+00
02	58202	+.5691400 0E+00	+.2474670 0E+01	+.5178700 0E+00
03	64202	+.6197100 0E+00	+.1940490 0E+01	+.5223100 0E+00
04	69202	+.6529900 0E+00	+.1422170 0E+01	+.5228900 0E+00
05	75202	+.7009600 0E+00	+.8514100 0E+00	+.5249800 0E+00
06				
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Table 20.—Manual input data form for the analytic mode containing the model coordinates of bridged points, the same information as shown in Table 15.

DATE	PROGRAMMER	Analytic Mode
PROJECT NO.	PROJECT TITLE	BRIDGE POINTS (solid white cards) require a "2" in column 79 of the first card and a "1" in column 80 of the last card.
RUN NO.	RUN TITLE	

Columns 1–5	Columns 6–10	Columns 11–12	Columns 13–26	Columns 27–28	Columns 29–42	Columns 43–44	Columns 45–58	Columns 59–64
	54205		+.28451000E+00		+.28067900E+01		+.51848000E+00	
	57102		+.46070000E+00		+.24984400E+01		+.52096000E+00	
	67101		+.50510000E+00		+.80259000E+00		+.53250000E+00	

SUGGESTIONS FOR SHORTENING THE PROGRAM

It is realized that this program may be too long to be compiled and operated on many computers inasmuch as no effort has been directed here toward restricting its length. Numerous parts can be eliminated quite readily, however, with only minor inconveniences.

Whereas this program has the facility of being used with three different degrees of polynomials, one might well restrict it to only one of the degrees, or possibly have separate programs for the other degrees if they are considered useful and important.

It is perhaps obvious that the program need not provide a choice of "analytic" or "analog." An office conceivably may have only one of the applications. If an office has both applications, a single routine can be arranged by merely providing identical input formats for the two modes of operation. For example, the analytic data from a prior computer program could be punched out in the same format as stereoplanigraph data; or else the stereoplanigraph data can be key punched in the analytic format. If such a change is made, however, the subject of relative precision becomes important inasmuch as the program was developed to recognize analytic data as being correct to a fraction of a micron and stereoplanigraph data as correct to ten microns. The problem can be dealt with correctly in various ways as long as it is not overlooked. A reason for providing the choice originally derives from the fact that two separate practices had already evolved independently from different groups of photogrammetrists. The transformation of one form of the data into the other form with the computer was less troublesome than to retrain a group of individuals who already had a well established working routine.

All the PAUSE and IF(SENSE SWITCH) statements can be removed. These were used in the original program to facilitate debugging. Now that the program has been tested, no need exists for these provisions.

At least two of the DIMENSION statements can be reduced in size. The program is written

for a maximum of 20 horizontal and 20 vertical control points. Each number might be cut in half. The A-table is of sufficient size that all the observation equations can be stored. This feature can be eliminated by recomputing the needed elements if they are needed later as a trade-off of computing time for memory space. This is entirely feasible inasmuch as the contribution to the normal equation system can be computed for one observation equation at a time. For example, an alternate form of Equation 32 is

$$b_{ij} = b_{ij} + a_i \; a_j \, .$$

All the alphabetic printouts can be eliminated, including the diagnostic comments and the column headings for the output data.

All the photogrammetric diagnostic output can be eliminated, including the computation of the standard errors. As a substitute, one may visually compare listed computed values with other lists of control data.

The program can be broken into two parts, one for the horizontal phase and one for the vertical, by repeating the vertical solution to retain the essence of a simultaneous solution.

The slope corrections can be eliminated. In smooth terrain they are usually not significant inasmuch as that function is served by the various linear terms in Equation 1. In steroplanigraph work, initial levelling of the strip reduced the need for the slope correction. Also, one can resort to the use of a separate short program to "rectify" the strip before adjustment, as was actually done in the Coast Survey for a short period before the present program evolved.

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