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Aerotriangulation: Image Coordinate Refinement

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Technical Bulletin No. 25 comprises a documented computer program concerned with the refinement of the image coordinate values that have been measured with a comparator. This program is considered to be the second of a series of three or more new programs for analytic aerotriangulation. Technical Bulletin No. 23 was the first program published and the next publication will deal with the orientation of the aerial camera and the assembly of independent models.

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Errata in Technical Bulletin No. 25

The errata pertain to Technical Bulletin No. 25, Aerotriangulation: Image Coordinate Refinement, by M. Keller and G. C. Tewinkel. The following items should be changed to read as follows:

Page 6, line 35 should read

```
A(I, 1) = 1.
```

Page 7, lines 43, 44, and 45 should read

```
242  DO  157  I = 1, N
          A(I, NP1) = A(9, I)
157  A(I, NP2) = A(10, I)    .
```

Aerotriangulation: Image Coordinate Refinement

M. KELLER and G. C. TEWINKEL
U.S. Coast and Geodetic Survey

AEROTRIANGULATION is a photogrammetric technique for deriving ground coordinates of objects from a set of overlapping aerial photographs which show images both of those objects and also of the relatively sparse distribution of other objects whose coordinates are known from previous classical measurements on the ground. Provisional photogrammetric coordinates of objects can be determined by at least two general methods: (1) through the use of conventional high-order photogrammetric plotting instruments which yield three dimensional coordinates of the objects, or (2) through the use of a comparator which yields only the two rectangular coordinates of an image on one photograph. This discussion pertains only to the latter case.

This paper comprises a documented computer program concerned with the refinement of the image coordinate values that have been measured with a comparator. The refinement consists of corrections for the calibration of the comparator, the distortion of the aerial camera film, both symmetrical and asymmetrical distortion of the aerial camera lens, atmospheric refraction, and, if desired, a correction for earth curvature. The present program is designed to be operable for cameras having either four or eight fiducial marks. This program can be used as a separate program or it may also be combined with the next program in the series called camera orientation.

Technical Bulletin No. 21, "Analytic Aerotriangulation," presented the principal formulation contained in this bulletin. The present bulletin, however, includes helpful and useful modifications which have been added since the date of the original publication.

This program is considered to be the second of a series of three or more new programs for analytic aerotriangulation. The first program was published as Technical Bulletin No. 23, "Aerotriangulation Strip Adjustment," and the next publication will deal with the orientation of the aerial camera and the assembly of independent models. Actually, Technical Bulletin No. 23 is chronologically the third program that one would use in computing a strip of analytic aerotriangulation. However, it was programmed first because of its dual application for both analogical and analytic aerotriangulation.

This program constitutes one of the most effective places in the series of computations where the accuracy potential of the analytic aerotriangulation system can be exploited.

INTRODUCTION

In analytic aerotriangulation, rectangular x, y-image coordinates are observed to the nearest micron using a measuring machine called a comparator. Each reading is repeated ordinarily three or five times in an effort to detect and avoid large blunders as well as to improve accuracy through the use of the average of several observations. The photograph (glass diapositive) is not zeroed into position; consequently, the arbitrary zeros of the comparator constitute the origin of the coordinate system. Moreover, the coordinate axes of the comparator are not parallel to the axes of the photograph. In addition, a calibration study of a comparator frequently indicates scale corrections that apply to one or both of the coordinate directions, as well as an angular correction to one coordinate so as to improve the right-angle relationship of the axes.

However, the principal point of the photograph constitutes the required origin of the image coordinates for their eventual use as data for the analytic perspective transformation, and the fiducial axes of the photograph define a logical orientation. All of these items are administered in conjunction with film distortion.

Inasmuch as most aerial photography today is taken on film (rather than glass plates) which is not completely stable dimensionally, provision is included for applying corrections to image coordinates for the effect of film distortion as indicated by the coordinates of the fiducial marks and as described in the next section. The computation is based on the correct coordinates of either four or eight camera fiducial marks.

In correcting image coordinates for the radial distortion of the lens of the aerial camera (no attempt is made to correct for tangential distor-

tion), the character of the correction falls into two rather distinct categories because the radial distortion is not equal along each of the radii. Consequently, a symmetric radial distortion correction is applied for the average radial distortion, which is the predominant portion. The remaining differences (called asymmetric distortion) along the separate radii are simulated by a false tilting of the focal plane. The latter asymmetric correction is applied prior to the symmetric type in this program for operational reasons. The symmetric distortion is then corrected in a final step in combination with atmospheric refraction (and earth curvature if desired) inasmuch as they can all be corrected by the same type of algebraic formulation.

The asymmetric correction is based on the values of three input constants, the symmetric distortion, is based on a list of 150 values at 1 millimeter intervals along a radius, and the atmospheric refraction correction for a strip or area is based on two input constants that one obtains from tables or graphs. If earth curvature is corrected, an appropriate value is added to the second refraction constant.

All of these items are treated in Technical Bulletin No. **21**. They are also clearly identified in the attached Fortran program by "Comment" statements.

FILM DISTORTION, ETC.

Two sets of coordinates of the fiducial marks are involved. One set consists of the correct values that have been measured on a glass photographic flash plate exposed in the aerial camera in the laboratory. These fiducial mark images are measured a relatively large number of times, and the coordinates are corrected for the calibration idiosyncrasies of the comparator and for the camera calibration constants that relate to the principal point location. Consequently, the coordinates represent the correct coordinates of the fiducial marks with respect to the principal point.

The other set of coordinates of the fiducial marks are those observed on the subject photograph that is being corrected. These marks are each measured five times as shown in the Input Data list included herein, and the average value of each coordinate is accepted as the valid observation.

The relationship between the two sets of coordinates is stated in polynomial form. If four fiducial marks are used,

$$\begin{aligned} x' &= x + a_1 + b_1 x + c_1 y + d_1 xy \\ y' &= y + a_2 + b_2 x + c_2 y + d_2 xy \end{aligned} \qquad (1)$$

where x', y' are correct coordinates, x, y are the observed values, and $a_1 \ldots d_2$ are coefficients to be determined. The four fiducial marks yield four simultaneous linear equations for x' in $a_1 \ldots d_1$ and four more equations for y' in $a_2 \ldots d_2$. The solutions of these equations for the eight coefficients enables one to compute correct x', y' coordinates for any given image whose observed coordinates are x, y.

It is pointed out that these two equations correct for several influences at the same time. The coefficients a_1, a_2 provide the translation values so that the origin of coordinates is moved from the comparator origin to the principal point. The terms b_1, c_1, b_2, c_2 (1) accomplish the rotation of the comparator axes into the fiducial axis system, (2) account for any nonperpendicularity of the comparator axes inasmuch as no restriction is placed on their relative values, (3) correct for any skewness caused by film distortion, and (4) help to correct for scale differences in x and y regardless of whether the error was caused by the screws of the comparator or by differential film distortion. The d_1, d_2 coefficients provide a quadratic or curvilinear correction for film distortion. Scale differences in the two dimensions are actually embodied in all four terms $a_2 \ldots d_2$.

Although the four simultaneous equations are unique and can be solved in a direct manner, they are nevertheless solved by the same least-squares technique used throughout the analytic triangulation system inasmuch as the same results are derived while avoiding an introduction of a different set of program steps.

If eight fiducial marks are utilized, the correction equations used are merely longer and somewhat more complex:

$$\begin{aligned} x' &= x + a_3 + b_3 x + c_3 y + d_3 xy + e_3 x^2 \\ &\quad + f_3 y^2 + g_3 xy^2 + h_3 x^2 y \\ y' &= y + a_4 + b_4 x + c_4 y + d_4 xy + e_4 x^2 \\ &\quad + f_4 y^2 + g_4 xy^2 + h_4 x^2 y. \end{aligned} \qquad (2)$$

Here eight simultaneous linear equations in eight unknowns are involved for both x' and for y'.

The computer program provides for *both* four and eight fiducial marks based on the number 0004 or 0008 that is entered as the first word on the second input card.

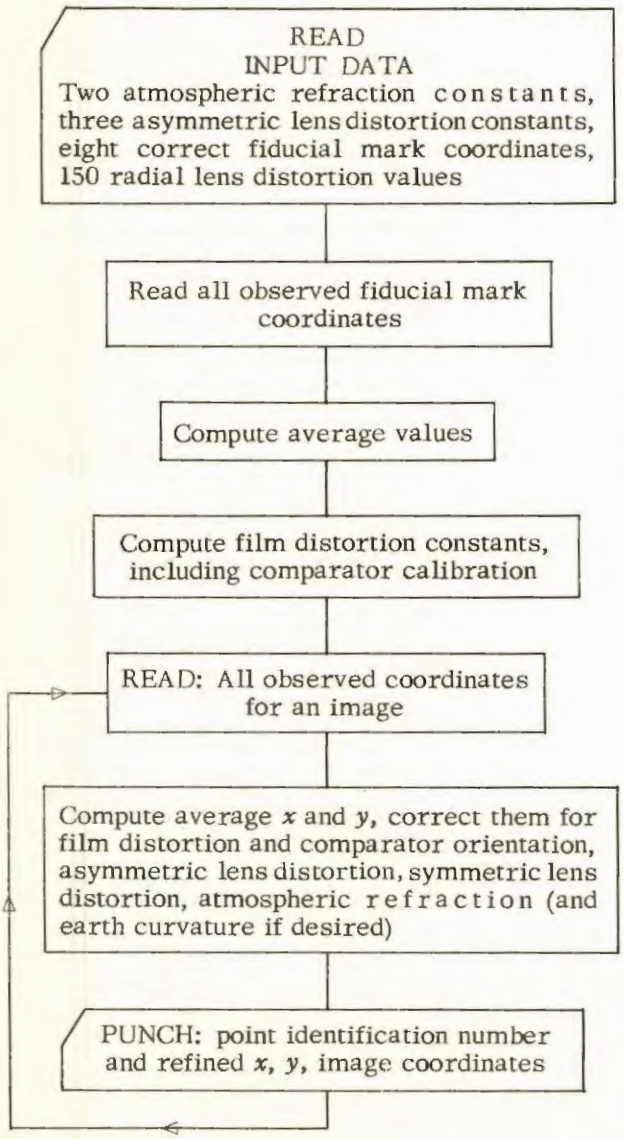
The attached Fortran program accomplishes the evaluation of the coefficients $a \ldots d$ by the series of steps ending on the second page at the comment statement "C Processing Nonfiducial . . . ."

The general procedure for the complete solution is illustrated by the attached generalized Flow Diagram. The coefficients $a \ldots d$ are first determined. Then the two coordinates of

a general image are read into the computer, they are (1) corrected for film distortion by the process above, (2) corrected for asymmetric radial lens distortion, (3) corrected for symmetric radial lens distortion combined with atmospheric refraction (and possibly earth curvature also), and (4) the corrected values are punched out, whereupon the next point is read and the steps repeated.

It seems somewhat paradoxical that the set of correct fiducial-mark coordinates constitute the all-important role as correction criteria independent from the comparator as a calibrated measuring engine. Consequently, an uncalibrated comparator could presumably be used for routine operations provided that (1) the comparator errors were "smooth," (2) the camera fiducial marks remain fixed, and (3) a truly calibrated instrument is used to determine the eight (or 16) correct fiducial coordinate values.

GENERALIZED FLOW DIAGRAM



ASYMMETRIC RADIAL LENS DISTORTION

As explained in Appendix 2 of Technical Bulletin 21, page 24, the correction for asymmetric radial lens distortion is accomplished by a simulated tilt of the focal plane. The three constants are given on the third input card as +43129860+00, etc., stored as $A(11,3)$, $A(11,4)$, $A(11,5)$. The first two numbers constitute the cosine and sine of a rotation angle. After the coordinates are rotated by

$$\begin{aligned} X &= A(11,3)*X1 + A(11,4)*Y1 \\ Y &= -A(11,4)*X1 + A(11,3)*Y1, \end{aligned} \tag{3}$$

they are corrected by introducing the third factor

$$\begin{aligned} X1 &= X + A(11,5)*X*X \\ Y1 &= Y + A(11,5)*X*Y \end{aligned} \tag{4}$$

and rotated back through the same angle using

$$\begin{aligned} X &= A(11,3)*X1 - A(11,4)*Y1 \\ Y &= A(11,4)*X1 + A(11,3)*Y1, \end{aligned} \tag{5}$$

which completes the sequence.

If one should wish to avoid this computation he could give $A(11,3)$ the value one, let $A(11,4)$ be zero and set $A(11,5)$ equal to zero. The program would still perform the computation but would have no effect on the coordinates.

SYMMETRIC LENS DISTORTION, ATMOSPHERIC REFRACTION (AND EARTH CURVATURE)

The general equation for this procedure is:

$$x' = x(1 + t_r + k_1 + k_2 r^2). \tag{6}$$

In this program the entire parenthetical expression is denoted by D. The value of t_r is obtained from the input table of distortion values and interpolated linearly using the expression

((1000.* FK−F) * (T(K+2) − T(K+1)) + T(K+1))* .000001

The term T represents the tabular value. K is a fixed-point subscript derived from the radial distance FK or r for the image

$$r = (x^2 + y^2)^{1/2}$$

and

$$K = FK * 1000.$$

which converts the floating-point radius to a fixed-point, truncated subscript. The numbers such as 1000 are needed for decimal reasons.

As explained in Technical Bulletin 21, page 27, the tabular values are actually d/r to avoid any possibility of the computer dividing by zero. Inasmuch as this constitutes a ***correction***, the algebraic sign of d/r is opposite from the sign of the observed distortional error.

The k_1 and k_2 constants for atmospheric refraction are called $A(11,1)$ and $A(11,2)$ in the program. As explained in the Bulletin on page 30, these values are obtained from graphs or tables which have been derived from the Leijonhvfvud publication. Both constants are of negative sign.

This completes the procedure normally used in the Coast Survey where earth curvature is recognized through the application of transformations that convert the ground coordinates of control points into a true three-dimensional cartesian system. This is not applied during the refinement of image coordinates but is at a later stage when the photogrammetric solution is oriented to the ground.

If one wishes to avoid this step, which includes at least two separate computer transformations, he may apply a false earth curvature term, adding it to the k_2 value. Although this will "flatten" the strip to fit sea level elevations, it is not precisely correct and will slightly increase residual parallaxes.

The value to be added to k_2 is $H/2Rf^2$ where H is the flight altitude, R is the earth radius in the same units as H, f is the camera focal length in the same units as the image-coordinate x, y values. The term is normally larger than k_2, is in the opposite direction, changing the sign of the term. This scheme has been used by others as a practical solution to the problem without any noticeable degradation of results.

CARD INPUT ORDER

The order of the card input is shown on the Input Data list shown on page 8. Each line on the list corresponds to one input card. As indicated by the annotations, the significance of the individual cards is as follows:

Card 1. Alpha-numeric project identification.

Card 2. The two atmospheric refraction constants K_1, K_2 effective for the project in columns 3-10 and 13-20. The first number is interpreted as -.0000352.

Card 3. The first of 16 cards (20 cards if eight fiducial marks are used) comprising the Camera Constants Deck designated as M1 in Columns 77 and 78 which identify the specific aerial camera. The first number specifies the number of fiducial marks (either 0004 or 0008) in card column 2 through 4. The next three numbers are the values of the A, B, C constants that refer to the asymmetric lens distortion (the terms $A(11,3)$, $A(11,4)$, and $A(11,5)$ in Equations 3, 4, and 5). These terms are placed in columns 7-18, 21-32, and 35-46, respectively, and are designated in floating point notation.

Cards 4 through 7. The correct coordinates of the fiducial marks. The first number is the identification of the specific fiducial mark in columns 4-10 and the final two numbers are the x- and y-coordinates stated as meters to seven decimals, e.g., the number +1060059 is interpreted as +.1060059 meters, punched in columns 13-20 and 23-30. If eight fiducial marks are employed, four more cards are added here.

Cards 8 through 18 (12 through 22 if eight fiducial marks are used). Symmetric lens distortion table. Each card (except the last one) contains 15 five-digit d/r values as explained heretofore, one value for each millimeter of image radial distance from the principal point, beginning with zero and ending with 150. The numbers are interpreted as having one decimal: 3000.0; 3000.0; 2850.0; etc. The values without algebraic sign are plus. Eventually the values become smaller so that space is available for an algebraic sign, e.g., +933.3. It should be noted that the algebraic signs are such that the tabulated values constitute ***correction*** factors which are opposite in sign from what is ordinarily regarded as distortional ***error***.

Card 19 (or 23). Photograph identification number. For example, the number 1637 shown in the input data sheet is a fixed point digit that indicates the particular photograph. The number is not processed, but it appears in a diagnostic statement (505) if a given difficulty-criterion is sensed. It is punched in columns 4-10.

Card 20, etc. (Card 24 if eight fiducial marks are utilized). Coordinate cards containing: (1) identification number, (2) x- coordinate, (3) y-coordinate punched in columns 4-10, 13-20, 23-30, respectively. The first number in an integer whereas the other two are interpreted as decimals, e.g., 0.2310520 meters. As many as ten cards having the same identification number ordinarily occur, constituting several observations on the same point. The specific number of repeated observations is not defined and may differ from point to point indiscriminately. The first set of points (20 in this sample) are the fiducial marks which must be listed in the specified numerical order. A 1 is punched in column 80 of the last fiducial mark card as a test signal. The other cards constitute any indefinite number of coordinates of photographic images. Although the repeat cards for a given point must be adjacent

to one another, the image sequence for points that are not fiducial marks may be in any order whatsoever insofar as this program is concerned. (The triangulation program to follow later contains restrictions in this respect).

Trailer card. After the last image coordinate card, a trailer card containing only a 1 in column 80 is used to signify that the computation for one photograph is complete and the entire computation is to be repeated for the next photograph.

OPERATIONAL NOTES

The Camera Constants Input Deck includes values that apply to a specific camera and ordinarily are not subject to any change unless the camera is recalibrated or readjusted. This deck of cards is used with all applicable photographs taken with the specific camera, and different decks are provided for the several individual cameras. Even supposedly duplicate cameras of the same lens design and manufacture require distinct individual input decks.

The correct coordinates of the fiducial marks are desk-calculated adjusted values of the observations made with a comparator. Ordinarily a very large number of observations are made on at least two "flash plates" exposed in the aerial camera in the laboratory. The adjustment includes: (1) averaging the numerous observations; (2) applying scale and angular corrections to the comparator observations in accordance with a prior complex calibration study of the comparator; and (3) expressing the coordinates with reference to the principal point as the origin of the system. As a consequence of using these "correct" or "adjusted" coordinates, the computation that follows simultaneously accounts for: (1) film distortion; (2) comparator calibration; (3) the indiscriminate angular orientation of the diapositive relative to the axes of the comparator; and (4) the indiscriminate location of the diapositive relative to the arbitrary zeros of the measuring counters of the comparator.

The 11 cards that contain the symmetric radial distortion factors comprise a table of t_r-values as shown in Equation 6, which terms have also been designated herein as d/r. Here d is the radial lens distortion at the radius r, and t_r is simply the quotient obtained by dividing d by r. In this application, the manufacturer's lens distortion data, given at relatively large intervals of radius, have been interpolated manually and graphically to determine a distortion value at each millimeter interval, from which the tabular values were calculated.

It seems pertinent to point out that, inasmuch as the *first* value in the table is for a radius of *zero*, the *second* value is for a radius of *one* millimeter, etc., and in general n-th value is for the radius of $(n-1)$ millimeters. Also, inasmuch as no tabular value exists at zero radius, the same value at a radius of 1 is arbitrarily adopted. This action has sufficient justification: it is equivalent to assuming that the change in distortion from 0 to 1 is linear.

This program does not include any provision for editing the input data for the elimination of questionable observations, although this idea is incorporated in the longer program that is prepared primarily for Coast Survey utilization.

The compiled program needs a core storage of somewhat less than 32,000 digits on the IBM-1620. The running time after the program is loaded is approximately 5 minutes per photograph.

Fortran Program.

```
**      REFINEMENT OF IMAGE COORDINATES MEASURED ON A COMPARATOR
*0807
C       REFINEMENT OF IMAGE COORDINATES MEASURED ON A COMPARATOR
        DIMENSION A(11,13),T(151),EN(8,10)
        READ 500                                        (project name)
        PUNCH 500
        PUNCH 505
        READ 508,(A(11,J),J=1,2)            (atmospheric refraction)
        READ 501,N,(A(11,J),J=3,5)          (asymmetric  distortion)
        NP1=N+1                          (N is no. of fiducial marks)
        NP2=N+2                            (correct coordinates of
        READ 502,(A(I,1),A(I,NP1),A(I,NP2),ITEST,I=1,N) fiducials)
        READ 503,(T(I),I=1,151)            (symmetric lens distortion)
        DO 156 I=1,N
        A(9,I) =A(I,NP1)
  156   A(10,I)=A(I,NP2)
C         MEANING COMPARATOR READING OF FIDUCIAL MARKS
  248   ITRANS=1
```

```
      READ 502,IPLATE
  238 LINE=1
      I=1
  204 READ 502,(A(I,J),J=11,13),ITEST      (observed fid. coords.)
  241 GO TO (222,239),ITRANS
  222 IF(A(1,11)-A(I,11))151,150,203       (test for proper order)
  203 PRINT 504, IPLATE
      STOP
  150 IF (ITEST)202,202,199            (test for last observation)
  202 I=I+1
      GO TO 204
  199 I=I+1
  151 LROW=I-1
      FK=LROW
      A(LINE,2)=0.
      A(LINE,3)=0.
      DO 205 I=1,LROW
      A(LINE,2) = A(LINE,2) + A(I,12)
  205 A(LINE,3) = A(LINE,3) + A(I,13)
      A(LINE,2) = A(LINE,2)/FK                                  (mean x)
      A(LINE,3) = A(LINE,3)/FK                                  (mean y)
  220 GO TO (198,240),ITRANS
  198 LINE=LINE+1
      IF(N-LINE)221,197,197                          (last fiducial?)
  197 A(1,11) = A(LROW+1,11)                       (image ident. no.)
      A(1,12) = A(LROW+1,12)                                      (x)
      A(1,13) = A(LROW+1,13)                                      (y)
      I=1
      GO TO 241
C       MATRIX FOR COMPUTATION OF CONSTANTS TO CORRECT FOR COMPARATOR
C       ERROR, FILM SHRINKAGE, AND ORIENT PLATE
  221 DO 223 I=1,N
      A(I,NP1) = A(I,NP1) - A(I,2)                     (x-correction)
      A(I,NP2) = A(I,NP2) - A(I,3)                     (y-correction)
      A(I,1) = 1.
      A(I,4) = A(I,2)*A(I,3)                                    (x.y)
      IF(N-4)223,223,224
  224 A(I,5) = A(I,2)*A(I,2)                                   (x^2)
      A(I,6) = A(I,3)*A(I,3)                                   (y^2)
      A(I,7) = A(I,4)*A(I,3)                                  (xy^2)
      A(I,8) = A(I,5)*A(I,3)                                  (x^2y)
  223 CONTINUE
C       FORMATION AND SOLUTION OF NORMAL EQUATIONS
  253 NM1 = N-1
      DO 225 I=1,N
      DO 225 J=I,NP2
      EN(I,J)=0.
      DO 225 K=1,N
  225 EN(I,J)=EN(I,J)+A(K,I)*A(K,J)     (formation of norm. eqs.)
      DO 227 I=1,N                                (forward solution)
      SQR = SQRT (EN(I,I))
      DO 228 J=I,NP2
  228 EN(I,J) = EN(I,J)/SQR
      IF(I-N)229,230,230
  229 IP1=I+1
      DO 227 L=IP1,N
      DO 227 J=L,NP2
  227 EN(L,J)=EN(L,J)-EN(I,L)*EN(I,J)                (back solution)
  230 EN(N,NP1)=EN(N,NP1)/EN(N,N)
      EN(N,NP2)=EN(N,NP2)/EN(N,N)
      DO 231 I=1,NM1
      NMI=N-I
```

```
      NMIP1=NMI+1
      DO 232 J=NMIP1,N
      EN(NMI,NP1) = EN(NMI,NP1)-EN(J,NP1)*EN(NMI,J)
  232 EN(NMI,NP2) = EN(NMI,NP2)-EN(J,NP2)*EN(NMI,J)
      EN(NMI,NP1) = EN(NMI,NP1)/EN(NMI,NMI)                    (answers)
  231 EN(NMI,NP2) = EN(NMI,NP2)/EN(NMI,NMI)
C       PROCESSING NONFIDUCIAL IMAGE READINGS FROM COMPARATOR
  237 ITRANS=2
      GO TO 238
  239 IF(A(1,11)-A(I,11))151,150,151
  240 PN=A(1,11)                                  (image ident. no.)
      X=A(1,2)                                              (mean x)
      Y=A(1,3)                                              (mean y)
C       CORRECTION FOR COMPARATOR ERROR, FILM SHRINKAGE AND ORIENTATION
      DO 243 K=NP1,NP2
      IF(N-4)244,244,245
  244 A(1,1)=EN(1,K) + EN(2,K)*X + EN(3,K)*Y + EN(4,K)*X*Y
      GO TO 246
  245 A(1,1)=EN(1,K) + EN(2,K)*X + EN(3,K)*Y + EN(4,K)*X*Y + EN(5,K)*X*X
     1+ EN(6,K)*Y*Y + EN(7,K)*X*Y*Y + EN(8,K)*X*X*Y
  246 IF(K-NP1)261,261,262
  261 X1=A(1,1)+X                          (x-correction applied)
      GO TO 243
  262 Y1=A(1,1)+Y                          (y-correction applied)
  243 CONTINUE
C       CORRECTION FOR FOCAL PLANE TILT
  257 X = A(11,3)*X1 + A(11,4)*Y1                        (rotation)
      Y=-A(11,4)*X1 + A(11,3)*Y1                         (rotation)
      X1=X+A(11,5)*X*X
      Y1 = Y +A(11,5)*X*Y                               (corrections)
      X=A(11,3)*X1 - A(11,4)*Y1                        (counter
      Y=A(11,4)*X1 + A(11,3)*Y1                             rotation)
C       CORRECTION FOR LENS DISTORTION AND ATMOSPHERIC REFRACTION
      FK=SQRT(X*X + Y*Y)                   (image radial distance)
      K=FK*1000.
      F=K
      D=1.+((1000.*FK-F)*(T(K+2)-T(K+1)) + T(K+1))*.CC0001 + A(11,1) +
     1FK*FK*A(11,2)        (correction factor for lens and atmos.)
      X1=X*D                              (correction factor applied)
      Y1=Y*D
      PUNCH 512,PN,X1,Y1                  (final refined coordinates)
      IF(ITEST)197,197,242
  242 DO 157 I=1,N
      A(1,NP1)=A(9,I)                    (final refined coordinates)
  157 A(1,NP2)=A(10,I)
      PRINT 513
      PAUSE
      GO TO 248
  500 FORMAT(55H                                                    )
  501 FORMAT(1X,I3,3(2X,E12.8))                (asymmetric constants)
  502 FORMAT(3X,I7,2F10.7,49X,I1)             (correct fid. coords. )
  503 FORMAT(15F5.1)                   (150 symmetric lens constants)
  504 FORMAT(50H MEASURED FIDUCIAL MARK CARDS NOT IN SEQUENCE FOR I4)
  505 FORMAT(/42H POINT NUMBER    REFINED X        REFINED Y)
  508 FORMAT(2F10.7)                  (2 atmos. refraction constants)
  512 FORMAT(1X,I7,7X,E14.8,2X,E14.8)
  513 FORMAT(56H CASE COMPLETED. PRESS START TO INITIALIZE FOR NEXT CASE
     1)
      END
```

Input Data—For Test Computation Indicating Card Formats.

```
 MIDLAND, TEXAS - STRIP 1 - COORDINATE REFINEMENT TEST
  -0000352  -0046400                                (Atmospheric refraction)
0004  +43129860+00  +90220920+00  +10024000-02 (Asymmetric lens distortion)  M1
         1  +1060059  +1060001 )                                              M1
         2  +1059869  -1059960 |                                              M1
         3  -1060017  -1059960 >  Correct Coordinates                         M1
         4  -1059927  +1060017 )  of fiducial marks                           M1
3000030000285002733323750218002033319000177501677816C001518214417138461328 6 M1 1
1266712063115291100010474100 00+9333+8682+8043+7417+6760+6077+5296+4464+3345 M1 2
+2000+0258-1219-2273-3353-4286-5000-5595-6132-6590-7000-7366-7690-7977-8182 M1 3
-8378-8565-8638-8688-8776-8780-8784-8769-8736-8704-8655-8607-8509-8414-8305 M1 4
-8200-8049-7903-7746-7578-7415-7227-7060-6882-6696-6500-6310-6125-5945-5770 M1 5
-5600-5408-5221-5038-4861-4675-4494-4317-4133-3976-3788-3616-3448-3261-3090 M1 6
-2867-2736-2554-2366-2160-1947-1740-1546-1408-1293-1160-1089-1000-0932-0865 M1 7
-0810-0755-0720-0685-0661-0636-0631-0634-0637-0632-0626-0603-0581-0551-0529 M1 8
-0483-0430-0385-0325-0258-0184-0119-0031+0039+0155+0254+0351+0455+0564+0672 M1 9
+0778+0904+1022+1138+1259+1393+1525+1641+1762+1896+2000+2110+2224+2338+2450 M110
+2560                                                                       M111
     1637
   3700001  02310520  02389760 )
   3700001  02310520  02389730 |
   3700001  02310510  02389740 |
   3700001  02310530  02389720 |
   3700001  02310520  02389730 |
   3700002  02310500  00274720 |
   3700002  02310490  00274750 |
   3700002  02310530  00274710 |
   3700002  02310530  00274730 |
   3700002  02310490  00274720 |
   3700003  00195110  00274850 |
   3700003  00195130  00274850 |
   3700003  00195130  00274840 |
   3700003  00195110  00274830 |
   3700003  00195120  00274850 |
   3700004  00195400  02390000 |
   3700004  00195430  02389990 |
   3700004  00195420  02389970 |
   3700004  00195400  02389980 |
   3700004  00195420  02390020 )                                            1
   3736310  00460810  00374360 )
   3736310  00460790  00374370 |
   3736310  00460790  00374390 |
   3736311  00475860  00372180 |
   3736311  00475860  00372150 |
   3736311  00475850  00372160 |
   3736320  00467740  02183780 |
   3736320  00467730  02183770 |
   3736320  00467730  02183770 |
   3736321  00483290  02184080 |
   3736321  00483280  02184050 |
   3736321  00483270  02184050 |
   3736330  00443660  01300020 |
   3736330  00443660  01299990 |
   3736330  00443650  01300000 |
   3736331  00456190  01300160 |
   3736331  00456210  01300180 |
   3736331  00456210  01300190 |
   3737310  01229930  00453020 |
   3737310  01229920  00453010 |
   3737310  01229930  00453020 |
   3737311  01275070  00466240 |
   3737311  01275060  00466220 |
```

symmetric radial lens distortion

observed coordinates of fiducial marks

observed coordinates of photographic images

Input Data—For Test Computation Indicating Card Formats.—Continued.

```
3737311  01275050  00466240
3737320  01260150  02266820
3737320  01260140  02266820
3737320  01260150  02266810
3737321  01274070  02272620
3737321  01274060  02272600
3737321  01274040  02272620
3737330  01250740  01344880
3737330  01250740  01344870
3737330  01250750  01344890
3737331  01268770  01344160
3737331  01268770  01344150
3737331  01268750  01344160
3738310  02085570  00400000
3738310  02085560  00399980
3738310  02085560  00400000
3738311  02100940  00394340
3738311  02100950  00394350
3738311  02100940  00394340
3738320  02029190  02250780
3738320  02029170  02250780
3738320  02029160  02250760
3738321  02014680  02252070
3738321  02014680  02252050
3738321  02014660  02252070
3738330  02070850  01340580
3738330  02070850  01340590
3738330  02070850  01340590
3738331  02086650  01339300
3738331  02086640  01339270
3738331  02086640  01339280
3737101  01186940  00876440
3737101  01186930  00876460
3737101  01186950  00876440
3737102  01199670  00868290
3737102  01199690  00868280
3737102  01199660  00868280
3737401  00817070  01268440
3737401  00817050  01268460
3737401  00817060  01268440
                                                                    1
   1638
3800001  02312150  02396250
3800001  02312180  02396250
3800001  02312150  02396220
3800001  02312170  02396240
3800001  02312180  02396240
3800002  02311900  00281120
3800002  02311940  00281130
3800002  02311910  00281160
3800002  02311900  00281130
3800002  02311900  00281170
3800003  00196410  00281550
3800003  00196430  00281550
3800003  00196420  00281530
3800003  00196410  00281530
3800003  00196430  00281540
3800004  00196940  02396440
3800004  00196930  02396400
3800004  00196930  02396400
3800004  00196950  02396430
3800004  00196940  02396420                                         1
```

```
3837310  00395620  00447410
3837310  00395590  00447410
3837310  00395620  00447410
3837311  00441290  00460750
3837311  00441250  00460740
3837311  00441280  00460750
3837320  00446600  02280730
3837320  00446590  02280730
3837320  00446590  02280730
3837321  00460850  02286630
3837321  00460840  02286620
3837321  00460880  02286610
3837330  00425060  01339130
3837330  00425050  01339110
3837330  00425060  01339120
3837331  00443080  01338260
3837331  00443070  01338260
3837331  00443080  01338270
3838310  01250470  00402380
3838310  01250450  00402350
3838310  01250460  00402360
3838311  01265580  00396920
3838311  01265570  00396960
3838311  01265580  00396940
3838320  01229510  02255720
3838320  01229510  02255730
3838320  01229500  02255740
3838321  01214800  02256940
3838321  01214810  02256940
3838321  01214790  02256930
3838330  01250380  01333720
3838330  01250400  01333700
3838330  01250400  01333700
3838331  01266140  01332390
3838331  01266140  01332420
3838331  01266150  01332380
3839310  02083960  00414450
3839310  02083960  00414430
3839310  02083990  00414440
3839311  02116660  00418540
3839311  02116670  00418500
3839311  02116680  00418510
3839320  02041040  02295710
3839320  02041030  02295690
3839320  02041050  02295720
3839321  02027120  02274570
3839321  02027150  02274560
3839321  02027140  02274570
3839330  02030190  01417160
3839330  02030170  01417180
3839330  02030170  01417160
3839331  02020390  01417190
3839331  02020360  01417210
3839331  02020380  01417210
                                                                    1
   1639
3900001  02310300  02396510
3900001  02310320  02396520
3900001  02310330  02396500
3900001  02310290  02396550
3900001  02310300  02396530
3900002  02309220  00280930
```

3900002	02309230	00280920	
3900002	02309230	00280940	
3900002	02309190	00280940	
3900002	02309210	00280920	8
3900003	00193720	00281100	
3900003	00193740	00281120	
3900003	00193710	00281080	
3900003	00193730	00281100	
3900003	00193720	00281110	
3900004	00194570	02396640	
3900004	00194540	02396620	
3900004	00194570	02396620	
3900004	00194530	02396630	
3900004	00194540	02396640	1
3938310	00406040	00412880	
3938310	00406030	00412870	
3938310	00406050	00412880	
3938311	00420800	00407090	
3938311	00420780	00407090	
3938311	00420810	00407090	
3938320	00443640	02262140	
3938320	00443640	02262130	
3938320	00443640	02262150	
3938321	00428970	02263530	
3938321	00428990	02263540	
3938321	00429000	02263550	
3938330	00431650	01335180	
3938330	00431670	01335210	
3938330	00431670	01335210	
3938331	00447370	01333600	
3938331	00447350	01333610	
3938331	00447380	01333590	
3939310	01239850	00389910	
3939310	01239830	00389910	
3939310	01239860	00389880	
3939311	01273070	00392500	
3939311	01273080	00392510	
3939311	01273090	00392500	
3939320	01264110	02289030	
3939320	01264100	02289050	
3939320	01264130	02289020	
3939321	01249030	02267640	
3939321	01249030	02267620	
3939321	01249030	02267630	
3939330	01220550	01396210	
3939330	01220570	01396200	
3939330	01220540	01396240	
3939331	01210550	01396570	
3939331	01210560	01396550	
3939331	01210560	01396550	
3940310	02152170	00463170	
3940310	02152180	00463160	
3940310	02152150	00463190	
3940311	02124430	00462990	
3940311	02124440	00462970	
3940311	02124430	00462970	
3940320	02078230	02275690	
3940320	02078220	02275670	
3940320	02078250	02275660	
3940321	02044730	02271820	
3940321	02044740	02271830	
3940321	02044750	02271800	

Input Data—For Test Computation Indicating Card Formats.—Continued

```
3940330  02108120  01380340
3940330  02108120  01380340
3940330  02108100  01380360
3940331  02108180  01388940
3940331  02108170  01388950
3940331  02108170  01388930
```

1

Output Data—Using the Preceding Input Data Indicating Output Card Formats.

MIDLAND, TEXAS - STRIP 1 - COORDINATE REFINEMENT TEST

POINT NUMBER	REFINED X	REFINED Y
3736310	-.79358455E-01	-.95994815E-01
3736311	-.77849258E-01	-.96215744E-01
3736320	-.78687369E-01	.85325910E-01
3736321	-.77129276E-01	.85354752E-01
3736330	-.81058304E-01	-.32421546E-02
3736331	-.79799856E-01	-.32246250E-02
3737310	-.23000649E-02	-.88089881E-01
3737311	.22207388E-02	-.86763777E-01
3737320	.72613417E-03	.93647779E-01
3737321	.21200635E-02	.94230071E-01
3737330	-.21624041E-03	.12645665E-02
3737331	.15948724E-02	.11918798E-02
3738310	.83430962E-01	-.93425094E-01
3738311	.84972779E-01	-.93992137E-01
3738320	.77803055E-01	.92065328E-01
3738321	.76349501E-01	.92194118E-01
3738330	.81939041E-01	.83714166E-03
3738331	.83523362E-01	.70671027E-03
3737101	-.66032593E-02	-.45647949E-01
3737102	-.53283407E-02	-.46465064E-01
3737401	-.43640905E-01	-.63980403E-02
3837310	-.86022891E-01	-.89343086E-01
3837311	-.81447155E-01	-.88004400E-01
3837320	-.80962443E-01	.94399560E-01
3837321	-.79532984E-01	.94989868E-01
3837330	-.83067242E-01	.19999418E-04
3837331	-.81259788E-01	-.65593539E-04
3838310	-.37994589E-03	-.93829663E-01
3838311	.11343609E-02	-.94373861E-01
3838320	-.25012665E-02	.91884813E-01
3838321	-.39748801E-02	.92005834E-01
3838330	-.39846890E-03	-.51262630E-03
3838331	.11841393E-02	-.64462229E-03
3839310	.83130150E-01	-.92623276E-01
3839311	.86407517E-01	-.92215387E-01
3839320	.78832028E-01	.95920389E-01
3839321	.77437516E-01	.93799993E-01
3839330	.77708105E-01	.78599085E-02
3839331	.76725146E-01	.78633190E-02
3938310	-.84710097E-01	-.92766591E-01
3938311	-.83231294E-01	-.93345955E-01
3938320	-.81021856E-01	.92517297E-01
3938321	-.82490514E-01	.92658021E-01
3938330	-.82152428E-01	-.36283643E-03
3938331	-.80577547E-01	-.52298809E-03
3939310	-.11759693E-02	-.95051100E-01

Output Data—Using the Preceding Input Data Indicating Output Card Formats.—Continued.

3939311	.21529757E-02	-.94789731E-01
3939320	.11813265E-02	.95207093E-01
3939321	-.32890397E-03	.93058366E-01
3939330	-.31547069E-02	.57673713E-02
3939331	-.41580477E-02	.58010256E-02
3940310	.90227046E-01	-.87721047E-01
3940311	.87447324E-01	-.87739859E-01
3940320	.82756742E-01	.93888773E-01
3940321	.79392587E-01	.93500651E-01
3940330	.85755053E-01	.41698333E-02
3940331	.85760850E-01	.50306239E-02

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