A UNITED STATES
DEPARTMENT OF
COMMERCE
PUBLICATION

ESSA Technical Memorandum C&GSTM 7

U.S. DEPARTMENT OF COMMERCE
ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION
Coast & Geodetic Survey

Error Study for Determination of Center of Mass of the Earth From Pageos Observations

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ROCKVILLE, MARYLAND

January 1970

ESSA TECHNICAL MEMORANDA

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January 1970

UDC 528.28:629.19

528.28 **Astronomical satellite**
629.19 **Geodesy, triangulation (coordinates)**

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Abstract

The least squares adjustment for the station coordinates of the worldwide satellite triangulation net gives, in addition to the station positions, the positions of the satellites observed in a simultaneous event from two or more stations. Through successive events short arcs can be fitted and the coordinates of the center of mass of the earth can be determined. It is shown that the center of mass will be obtained with an accuracy consistent with the accuracy of the station positions, if well distributed Pageos arcs of about one quarter of a revolution are taken and if three events per arc are given.

1. Introduction

The worldwide satellite triangulation network being established by the Coast & Geodetic Survey (U.S. Department of Commerce) and the U.S. Army Topographic Command (Department of Defense) will give the coordinates of 44 stations distributed rather uniformly over the world in an earth fixed reference system whose orientation is derived from the astronomical right ascension and declination system [Schmid, 1969]. The origin of the coordinate system is situated in an arbitrary position; it cannot be related to the mass center of the earth, since geometric methods are employed to establish the satellite triangulation net.

However, the least squares adjustment for the station coordinates of the satellite triangulation net gives as additional results the coordinates for the positions of the satellites photographed simultaneously from two or more stations. In the world net mainly Pageos has been observed and in a few cases Echo II. From the satellite trail on the photographic plate seven images are generated so that the adjustment for the station coordinates gives seven satellite positions per simultaneous event together with the covariance matrix associated with the satellite positions. Through these positions an orbit can be fitted and the origin of the coordinate system for the orbit

computation, which is the center of mass of the earth, gives the desired relationship between the center of mass and the coordinate system of the satellite triangulation. The orbits so obtained extend only over about 5 minutes. As we shall see, these arcs are too short to obtain the center of mass of the earth with an accuracy which is consistent with the accuracy of the stations in the triangulation network. In quite a number of cases, however, Pageos and Echo II have been photographed by adjacent pairs of stations in two successive simultaneous events. The time difference between the successive events varies between 8 minutes and 46 minutes, so that orbits of corresponding length can be fitted through the observations.

The purpose of this study is to find out how accurately these arcs determine the center of mass of the earth if two or three events per arc are given. Furthermore, the extent to which errors in the gravity field distort the results is investigated. These studies are based on generated data for Pageos.

2. Coordinate Systems

Satellite orbits are computed in an inertial coordinate system. As mentioned, the time span of one arc is less than 50 minutes. During 50 minutes the movement of the instantaneous pole of the earth and the movement of the

true vernal equinox are negligibly small. Hence, each satellite position computed in the rectangular system, whose origin is the mass center of the earth, whose x-axis points towards the true vernal equinox and whose z-axis towards the instantaneous pole at the time of observation, can be assumed as given in an inertial coordinate system. The position vector $\underline{r}$ of the satellite in this inertial system is given by

$$\underline{r}^T = [x,y,z]. \quad (1)$$ *)

The position vector $\underline{v}$ of the satellite

$$\underline{v}^T = [u,v,w], \quad (2)$$

which is obtained from the adjustment for the station coordinates of the satellite triangulation net, is given in the earth fixed coordinate system whose w-axis points towards the mean pole 1900 - 1905 and whose u-axis points towards the intersection of the Greenwich meridian, i.e. the zero meridian of the BIH-UT1 System, with the equator. Together with the satellite positions $\underline{v}$ the covariance matrix $\underline{\Sigma}_v$ of the positions $\underline{v}$ of each event are obtained from the adjustment. The covariances between positions of successive events can be assumed to be zero. The position vector $\underline{v}_0$ of the center of mass of the earth in the earth

*) Underlined letters denote vectors or matrices.

fixed system

$$\underline{v}_o^T = [u_o, v_o, w_o] \tag{3}$$

has to be determined.

The satellite position with respect to the mass center of the earth is given in the earth fixed system by

$$\underline{v} - \underline{v}_o . \tag{4}$$

By means of the rotation matrix $\underline{R}_{xu}$ which transforms the coordinates of the earth fixed system into the inertial system, the satellite position $\underline{r}$ in the inertial system is given by

$$\underline{r} = \underline{r}' - \underline{r}_o . \tag{5}$$

with

$$\underline{r}' = \underline{R}_{xu}\underline{v}, \quad \underline{r}_o = \underline{R}_{xu}\underline{v}_o \tag{6}$$

and the covariance matrix $\underline{\Sigma}_{r'}$ of $\underline{r}'$ by

$$\underline{\Sigma}_{r'} = \underline{R}_{xu}\underline{\Sigma}_v \underline{R}_{xu}^T .$$

3. Model of the Force Field

The satellite position $\underline{r}$ at the time t is a function of the orbital elements $\underline{e}_o$ at the epoch t_o, the so-called initial conditions, and a function of the force acting upon

it. The force is a combination of the earth's gravitational effect, the sun's and the moon's attraction, the atmospheric drag effect, and the effect due to the radiation pressure. Since the time over which the orbit has to be integrated is less than 50 minutes, the earth's gravitational field will be regarded as known. Because of the height of Pageos and Echo II the atmospheric drag effect can be neglected over this short time period. However, radiation pressure has to be taken into account. Because its effect on balloon satellites is predominant over the other effects which do not stem from the earth's gravity, a variable parameter is introduced into the model for the radiation pressure. Hence,

$$\underline{r} = \underline{r}\,(\underline{e}_0, \underline{p}_a) \tag{7}$$

where $\underline{p}_a$ denotes the vector with the parameter p_a of the radition pressure model as its element.

The gravitational potential W of the earth is expressed by an expansion into spherical harmonics

$$W = \frac{kM}{r}\left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n \overline{P}_{nm} (\sin \varphi) \right.$$

$$\left. (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda)\right] \tag{8}$$

where r, φ, λ are spherical coordinates in the earth fixed coordinate system, k is the gravitational constant, M the

mass of the earth, a the mean equatorial radius, $\overline{P}_{nm}$ the fully normalized associated Legendre function of degree n and order m, and $\overline{C}_{nm}$ and $\overline{S}_{nm}$ the fully normalized harmonic coefficients. The potential W is assumed to be known. The values for kM, a, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ can be taken, for instance, from the parameters of the 1966 Standard Earth of the Smithsonian Institution [1966].

The perturbations caused by the sun or the moon on a satellite orbit are computed by defining a perturbing function R_i which can be treated as the gravitational potential W [Danby, 1962, p. 209]

$$R_i = kM_i \left(\frac{1}{|\underline{r}-\underline{r}_i|} - \frac{\underline{r}\cdot\underline{r}_i}{|r_i|^3} \right) \tag{9}$$

where M_i is the mass of the sun or the moon, $\underline{r}$ the position vector of the satellite, and $\underline{r}_i$ the position vector of the sun or the moon. The position of the sun and the moon is obtained from the mean orbital elements of the sun and the moon published in the Explanatory Supplement to the Astronomical Ephemeris [1961].

The perturbing function R_r caused by radiation pressure of the sun is represented by

$$R_r = -p_a K \frac{\underline{r}\cdot\underline{r}_s}{|\underline{r}_s|} \tag{10}$$

where $\underline{r}_s$ is the position vector of the sun. The parameter

p_a depends on the reflection characteristics of the satellite, p_a is unknown. The constant K is given by $K = (A/m)(I/c)$ where A/m is the cross-sectional area-to-mass ratio of the satellite, I the solar flux in the vicinity of the earth, and c the speed of light [Shapiro, 1963]. For Pageos it is $K = 6.045\times10^{-3}\,cm/sec^2$. The acceleration of a satellite due to radiation pressure is a discontinuous function since it equals zero in the earth's shadow.

4. Method of Analysis

The satellite position $\underline{r}'$ in (5) is the observed quantity

$$\underline{r}' = \underline{r} + \underline{r}_o \tag{11}$$

and with (6) and (7)

$$\underline{r}' = \underline{r}(\underline{e}_o, \underline{p}_a) + \underline{R}_{xu}\underline{v}_o \,. \tag{12}$$

By means of approximate values for the initial conditions $\underline{e}_o$, the radiation pressure parameter $\underline{p}_a$, and the coordinates $\underline{v}_o$ of the center of mass in the earth fixed system we compute an approximate value $\underline{r}_n'$ for $\underline{r}'$. Using Taylor's series we obtain from (12)

$$\underline{r}' - \underline{r}_n' = \frac{\partial \underline{r}}{\partial \underline{e}_o}\Delta\underline{e}_o + \frac{\partial \underline{r}}{\partial \underline{p}_a}\Delta\underline{p}_a + \underline{R}_{xu}\Delta\underline{v}_o \tag{13}$$

where $\Delta\underline{e}_o$, $\Delta\underline{p}_a$, and $\Delta\underline{v}_o$ denote the corrections to the approximate values for $\underline{e}_o$, $\underline{p}_a$, $\underline{v}_o$, respectively. With

$$\frac{\partial \underline{r}}{\partial \underline{e}_o} = A, \quad \frac{\partial \underline{r}}{\partial \underline{p}_a} = \underline{B}, \quad \underline{R}_{xu} = \underline{C}, \quad \underline{r}'-\underline{r}_n' = \underline{\ell}, \quad \underline{\Sigma}_{r'} = \underline{\Sigma}_{\ell} \tag{14}$$

we obtain the observation equations where $\underline{m}$ is the vector of the residuals of the least squares adjustment

$$\underline{A}\ \Delta\underline{e}_o + \underline{B}\ \Delta\underline{p}_a + \underline{C}\ \Delta\underline{v}_o = \underline{\ell} + \underline{m}\,. \tag{15}$$

If the subscript i denotes the i th event, the normal equations for one arc with n events are given by

$$\begin{vmatrix} \sum_{i=1}^{n} \underline{A}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{A}_i & \sum_{i=1}^{n} \underline{A}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{B}_i & \sum_{i=1}^{n} \underline{A}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{C}_i \\ \sum_{i=1}^{n} \underline{B}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{A}_i & \sum_{i=1}^{n} \underline{B}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{B}_i & \sum_{i=1}^{n} \underline{B}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{C}_i \\ \sum_{i=1}^{n} \underline{C}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{A}_i & \sum_{i=1}^{n} \underline{C}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{B}_i & \sum_{i=1}^{n} \underline{C}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{C}_i \end{vmatrix} \begin{vmatrix} \Delta\underline{e}_o \\ \Delta\underline{p}_a \\ \Delta\underline{v}_o \end{vmatrix} = \begin{vmatrix} \sum_{i=1}^{n} \underline{A}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{\ell}_i \\ \sum_{i=1}^{n} \underline{B}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{\ell}_i \\ \sum_{i=1}^{n} \underline{C}_i^T \underline{\Sigma}_{\ell_i}^{-1} \underline{\ell}_i \end{vmatrix} \tag{16}$$

or abbreviated

$$\begin{vmatrix} \underline{D}_j & \underline{F}_j \\ \underline{F}_j^T & \underline{E}_j \end{vmatrix} \begin{vmatrix} \Delta\underline{f}_j \\ \Delta\underline{v}_o \end{vmatrix} = \begin{vmatrix} \overline{\underline{\ell}}_j \\ \overline{\underline{\ell}}_{oj} \end{vmatrix} \tag{17}$$

where the subscript j denotes the j th arc. With m arcs we find

$$\begin{vmatrix} \underline{D}_1 & 0 & \cdot & \cdot & \cdot & 0 & & \underline{F}_1 \\ 0 & \underline{D}_2 & \cdot & \cdot & \cdot & 0 & & \underline{F}_2 \\ \cdot & \cdot & & & & \cdot & & \cdot \\ \cdot & \cdot & & & & \cdot & & \cdot \\ \cdot & \cdot & & & & \cdot & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \underline{D}_m & & \underline{F}_m \\ \underline{F}_1^T & \underline{F}_2^T & \cdot & \cdot & \cdot & \underline{F}_m^T & \sum_{j=1}^{m} & \underline{E}_j \end{vmatrix} \begin{vmatrix} \Delta\underline{f}_1 \\ \Delta\underline{f}_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta\underline{f}_m \\ \Delta\underline{v}_o \end{vmatrix} = \begin{vmatrix} \bar{\underline{\ell}}_1 \\ \bar{\underline{\ell}}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{\underline{\ell}}_m \\ \sum_{j=1}^{m} \bar{\underline{\ell}}_{oj} \end{vmatrix} . \quad (18)$$

We eliminate $\Delta\underline{f}_j$ and obtain

$$\left[\sum_{j=1}^{m} \left(\underline{E}_j - \underline{F}_j^T \underline{D}_j^{-1} \underline{F}_j \right) \right] \left[\Delta\underline{v}_o \right] = \left[\sum_{j=1}^{m} \left(\bar{\underline{\ell}}_{oj} - \underline{F}_j^T \underline{D}_j^{-1} \bar{\underline{\ell}}_j \right) \right] . \quad (19)$$

The solution of (19) gives the corrections $\Delta\underline{v}_o$ to the approximate coordinates of the center of mass of the earth in the earth fixed coordinate system. The inversion of the left side of (19) yields the covariance matrix $\underline{\Sigma}_{vo}$ of $\Delta\underline{v}_o$.

The equations (19) are built up successively with each arc. The storage requirement for (19) in a computer is therefore independent of the number of arcs. The contributions of one single arc to the determination of the center of mass is obtained by the solution of (16).

For the error study it is helpful to rewrite (19). If $\Delta \underline{v}_{oj}$ denotes the result for the center of mass obtained from the j th arc by the solution of (17) and $\underline{\Sigma}_{voj}$ its covariance matrix, it holds [Wolf, 1968, p. 187]

$$\underline{\Sigma}_{voj}^{-1} = \underline{E}_j - \underline{F}_j^T \underline{D}_j^{-1} \underline{F}_j$$

and

$$\underline{\Sigma}_{voj}^{-1} \Delta \underline{v}_{oj} = \overline{\underline{\ell}}_{oj} - \underline{F}_j^T \underline{D}_j^{-1} \overline{\underline{\ell}}_j$$

Hence, instead of (19) we get

$$(\underline{\Sigma}_{vo1}^{-1} + \underline{\Sigma}_{vo2}^{-1} + \ldots + \underline{\Sigma}_{vom}^{-1}) \Delta v_o = (\underline{\Sigma}_{vo1}^{-1} \Delta \underline{v}_{o1} + \underline{\Sigma}_{vo2}^{-1} \Delta \underline{v}_{o2} + \ldots + \underline{\Sigma}_{vom}^{-1} \Delta \underline{v}_{om}), \tag{20}$$

which gives the coordinates $\Delta \underline{v}_o$ of the mass center of the earth as the general arithmetic mean of the results for the center of mass of the single arcs.

5. Orbit Integration and Data Generator

The equations of motion of Pageos were integrated numerically using the Cowell-Störmer multistep method. For the starting process and the interpolation between the time steps, a modified Runge-Kutta-Gill integration was applied. The derivatives of the position vector in (13) with respect to the initial conditions and the parameter

of the model for the radiation pressure were obtained by numerical integration of the variational equations. The computer program employed is a modification of the program by which density values were determined for a simple layer model of the geopotential [Koch and Morrison, 1970].

The modified program was also used to generate fictitious data for Pageos. Approximate orbital elements for Pageos were taken at the epochs

$$
\begin{aligned}
t_{o1} &= 1969,\ \text{Sept. } 12,\ 0^h\ 50^{min}\ 0\overset{s}{.}000; \\
t_{o2} &= 1968,\ \text{June } 26,\ 0^h\ 0^{min}\ 0\overset{s}{.}000; \\
t_{o3} &= 1967,\ \text{Jan. } 5,\ 23^h\ 40^{min}\ 0\overset{s}{.}000;
\end{aligned}
\tag{21}
$$

and used as initial elements for generating positions $\underline{r}$ at 22 second time intervals over a time period of 50 minutes and for the arc with epoch t_{o2} over one revolution, that is 180 minutes. The values for kM and a, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ up to the 4 th order and degree were taken from the parameters of the 1966 Standard Earth of the Smithsonian Institution [1966]. It was $p_a = 1$ in (10) and a 12 th order Cowell-Störmer process of integration was applied. The selection of the three epochs guarantees an even distribution of the arcs with respect to the inertial coordinate system.

When generating the positions $\underline{r}$, it was assumed that the coordinates of the center of mass of the earth in the

earth fixed system are given by

$$\underline{v}_o^T = [10m,\ 10m,\ 10m]. \tag{22}$$

According to (6) and (11) the observed quantities r' are obtained from

$$\underline{r}' = \underline{r} + \underline{R}_{xu}\underline{v}_o \tag{23}$$

with

$$\underline{R}_{xu} = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

where a rotation around the z-axis through θ, the Greenwich Sidereal time, was assumed to be sufficient to rotate the earth fixed system into the inertial system.

6. Results

Seven positions $\underline{r}'$ at 44 second time intervals comprise one event. With each event a covariance matrix $\Sigma_{r'}$ is associated which is assumed to be equal for all events. The matrix $\underline{\Sigma}_{r'}$ chosen here is characteristic for the accuracy of the satellite positions $\underline{r}'$ obtained by the geometric satellite triangulation method. The square roots of the variances, i.e. the standard deviations, for x, y,

and z lie between ±5m and ±14m with the exception of one value which equals ±30m. Arcs of different lengths with different numbers of events were processed, see Table 1. The six events of the arc of 180.4 minutes length begin at 0.3667, 33.0, 70.033, 106.333, 142.633, and 176.0 minutes, respectively, after the chosen epoch. The three events of the arcs of 49.5 minutes length begin at 0.3667, 22.0, and 45.1 minutes after the epochs defined by (21). In the case of two events the event after 22.0 minutes is omitted. The three events of the arcs of 26.4 minutes length begin at 0.3667, 11.733, and 22.0 minutes after the epochs given by (21). In the case of two events the event after 11.733 is omitted. Finally, the arcs of 4.8 minutes length with one event begin 0.3667 minutes after the epochs (21).

With the processing of the three arcs with the initial conditions at the epochs (21), the coordinates of the mass center and its accuracy were determined according to (19) and the contribution of a single arc was computed from (16). It was found that the arc at the epoch t_{0_2} gives the least accurate results. These results are shown in Table 1 in the case of one arc.

To generate the data, a 12 th order Cowell-Störmer process with a 22 second step size was used, as already mentioned. For the processing of the generated data a 10 th order process with a 45 second step size was applied.

Table 1. Determination of the Center of Mass and Its Accuracy

No.	Length of Arc [minutes]	No. of Arcs	No. of Events	Distorted Gravity Field	Coordinates of the Mass Center and Their Standard Deviations x [m]	y [m]	z [m]
1	180.4	1	6	no	10.03 ±3.1	9.90 ±3.3	9.98 ±4.5
2	49.5	3	3	no	9.93 ±9.0	10.01 ±10.9	9.99 ±8.5
3	49.5	1	3	no	10.02 ±128	9.95 ±41	10.01 ±97
4	49.5	3	3	yes	8.27 ±9.0	11.48 ±10.9	9.54 ±8.5
5	49.5	1	3	yes	11.34 ±128	6.87 ±41	12.14 ±97
6	49.5	3	2	no	9.94 ±29	10.00 ±34	9.99 ±30
7	49.5	1	2	no	10.02 ±146	9.94 ±74	10.01 ±106
8	26.4	3	3	no	9.98 ±29	9.99 ±33	9.99 ±31
9	26.4	1	3	no	10.01 ±547	9.99 ±76	10.00 ±185
10	26.4	3	3	yes	10.74 ±29	9.80 ±33	10.00 ±31
11	26.4	1	3	yes	11.38 ±547	4.11 ±76	13.81 ±185
12	26.4	3	2	no	9.96 ±58	9.99 ±69	10.00 ±63
13	26.4	1	2	no	10.01 ±593	9.98 ±132	10.00 ±200
14	4.8	3	1	no	10.06 ±1770	9.98 ±1790	10.00 ±1130
15	4.8	1	1	no	10.03 ±94700	9.95 ±3140	10.00 ±2970

Furthermore, the initial conditions at the epochs t_{01}, t_{02}, and t_{03} were changed arbitrarily by ±100m in position and by ±200 m/hour in velocity before determining the mass center. Thus it was necessary, first to get a good orbit fit and then to solve for the parameter of the radiation pressure model and the coordinates of the mass center.

To investigate errors in the expansion (8) for the geopotential W, the gravity field was distorted by using an expansion up to the 3rd degree and order, instead of an expansion up to the 4th degree and order with which the data was generated. This omission of the harmonic coefficients of the 4th degree introduces an error which probably has a greater effect on the orbit of Pageos than that due to the lack of knowledge of the gravity field. The results for the mass center obtained with the distorted gravity field are also given in Table 1. The absolute amount of the correction to the parameter p_a of the radiation pressure model is less than 0.07 for each arc in the case of the distorted gravity field, and less than 0.002 for each of the remaining arcs.

7. Conclusions

With an errorless gravity field the coordinates of the mass center are recovered with differences of less than 10 cm. These deviations are caused by round-off errors

since a different step size and integration order has been used for generating and processing the data. Errors from introducing a distorted gravity field affect the results for the mass center by less than 6 m in the case of single arcs and by less than 2 m in the case of three arcs.

Of special interest are the square roots of the variances, i.e. the standard deviations, computed for the coordinates of the mass center. These values give a clear picture of the geometric strength of the solution. While the results of one single arc of less than 50 minutes length are very discouraging, see No. 3 and 9 in Table 1, the picture completely changes if the same arc is extended over 180.4 minutes, that is one revolution of Pageos, see No. 1 in Table 1. The results demonstrate the restrictions of short arc methods, see also Hartwell [1968]. However, three well distributed arcs of less than 50 minutes length, that is about one quarter of a revolution of Pageos, with three events each give an accuracy for the center of mass which is compatible with the accuracy of ±6m for the latitude and longitude components and of ±11m in the direction of the height, with which the station positions of the satellite triangulation net are determined [Schmid, 1969].

As mentioned in the introduction, Pageos has been observed several times in two successive events. Up to

October 1969 observations of two successive events were obtained forming two arcs extending over a time period from 40 to 46 minutes, four arcs from 30 to 39 minutes, 13 arcs from 20 to 29 minutes, and several shorter arcs. By assuming that the accuracy computed for three arcs in Table 1 is representative of the accuracy to be obtained from real data and by neglecting the covariances in (20) and considering only the variances, we obtain a standard deviation for all three coordinates of the center of mass of the earth of about ±40m from the two longer arcs, ±40m from the four arcs of 30 to 39 minutes length, and ±30m from the 13 arcs of 20 to 29 minutes length. The 19 arcs together give a standard deviation of about ±20m in all three coordinates of the center of mass of the earth. After the observations in the worldwide satellite triangulation network have been completed, more than 1100 simultaneous events forming arcs of about 5 minutes length will be available. These arcs will determine the center of mass to about ±60m, so that they cannot contribute significantly to the accuracy obtained with the longer arcs.

With the completion of the observations in the satellite triangulation network in the middle of 1970 more short arcs will be available to determine the center of mass of the earth. However, it is improbable that there

is enough data to obtain the center of mass with an accuracy consistent with the accuracy of the station positions of the world net. It is therefore planned to apply long arc methods using Doppler satellite tracking from stations of the worldwide triangulation net. The results will be combined with gravity measurements by applying the simple layer model of the geopotential [Koch, 1968] and the coordinates for the tracking stations obtained by the satellite triangulation method.

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