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COAST AND GEODETIC SURVEY

C&GS 32

Space Resection in Photogrammetry

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This series of Technical Reports replaces the former Technical Bulletins of the Coast and Geodetic Survey. They are primarily for the personnel of the Survey and are incidentally for others, presenting technical information related to the scientific and technical activities. Since many of these reports deal with new practices and new techniques, the views expressed are those of the authors and do not necessarily represent final Survey policy.

This Technical Report No. 32 describes a Fortran II program, prepared for an IBM-1620 computer, which derives the position and the orientation for an aerial photograph based on the focal length, the image coordinates, and corresponding object coordinates from 3 to 15 points. The program uses 27,824 digits of core memory and requires 3 minutes per photograph to compute, applying the principles of collineation, least squares, and successive approximations. The solution terminates when the angular corrections become less than 10^{-5} radian (approximately 2 arc seconds). The requirement for one point near the center of each photograph makes it unnecessary to enter any initial approximations for near-vertical photography.

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Space Resection in Photogrammetry

M. KELLER AND G. C. TEWINKEL

COAST AND GEODETIC SURVEY TECHNICAL BULLETIN NO. 32
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Space Resection in Photogrammetry

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INTRODUCTION

RESECTION in photogrammetry is the determination of the six parameters of the camera (three linear elements of position and three angular elements of orientation) from the positions of three or more objects and the positions of their corresponding images on the photograph. The word *space* indicates that the determination is three-dimensional as differentiated from the three-point resection problem in plane surveying. The following discussion includes a documented Fortran computer program with a sample solution.

Although the topic has had classical and academic consideration in the past,[1] *resection* did not draw practical attention until the more recent studies of analytic adjustment of a block of aerial photography. On the other hand, in conventional mapping work, the contemporary topic of *relative orientation* for two photographs received a great deal of practical and theoretical treatment because it applied directly to the compilation of maps using stereoscopic plotting instruments. Now that the electronic computer is available at a relatively small cost, the problem of resection has an important, direct application to analytic aerotriangulation.

The Coast and Geodetic Survey system for analytic aerotriangulation consists of three principal parts. The initial program combines image coordinate refinement[2] and three-photo relative orientation[3] of strips of photographs, and includes a scheme for detecting and eliminating blunders which may occur in the input card sequence and in the observed values of the image coordinates. The resulting data are adjusted[4] then to fit ground control through the application of a second program, which not only assigns provisional coordinates to all the objects (pass points), but also provides a means to detect and to eliminate any blunders that may occur in the control data. A third program, called block adjustment, adjusts several overlapping strips of photographs to form a unified block. Block adjustment consists of *simultaneous resection* solutions for all photographs of the system, adjusting the provisional coordinates to fit the control data and to minimize residual observational discrepancies.

The basic theory of the general resection problem consists of the collineation principle in which each image, its object, and the lens point lie on a common straight line.[5] Although resection is the photogrammetric principle applied in block adjustment (for which a separate report is planned), resection *per se* is treated separately here because (1) the solution may also be useful to others in its simplest form, and (2) resection tends to be obscured in the larger problem by the technique of handling the voluminous adjustment equations.

MATHEMATICAL BASIS

The equations of collinearity are stated, without derivation, as follows because they were established previously in Technical Bulletin No. 21:[6]

$$\frac{x}{z}=\frac{(X-X_0)a_{11}+(Y-Y_0)a_{12}+(Z-Z_0)a_{13}}{(X-X_0)a_{31}+(Y-Y_0)a_{32}+(Z-Z_0)a_{33}}$$

$$\frac{y}{z}=\frac{(X-X_0)a_{21}+(Y-Y_0)a_{22}+(Z-Z_0)a_{23}}{(X-X_0)a_{31}+(Y-Y_0)a_{32}+(Z-Z_0)a_{33}}. \quad (1)$$

Here x and y are image coordinates based on the principal point as origin, z is the calibrated focal length of the camera considered to have a negative sign, X, Y, Z are the ground rectangular coordinates of the corresponding object, X_0, Y_0, Z_0 are the initially unknown coordinates of the camera in the ground system, and the a's (also unknown initially) are the direction cosines that indicate

[1] *Photogrammetry, Collected Lectures and Essays* by O. von Gruber, Chapman & Hall, p. 9, 1932. *Manual of Photogrammetry* by the American Society of Photogrammetry, 3d ed., Section 2.2.3.3 (The Church Method), 1965. *Analytical Photogrammetry* by Everett L. Merritt, Pitman Publishing Corp., p. 66 ff., 1958. *Manual of Photogrammetry* by the American Society of Photogrammetry, 3d ed., Section 2.2.3.4 (The Morse Method), 1965.

[2] Aerotriangulation: Image coordinate refinement by M. Keller and G. C. Tewinkel, *Coast and Geodetic Survey Technical Bulletin No. 25*, March 1965.

[3] Aerotriangulation strip adjustment by M. Keller and G. C. Tewinkel, *Coast and Geodetic Survey Technical Bulletin No. 23*, August 1964.

[4] Three-photo aerotriangulation by M. Keller and G. C. Tewinkel, *Coast and Geodetic Survey Technical Bulletin No. 29*, February 1966.

[5] An analytical treatment of the orientation of a photogrammetric camera by Dr. Hellmut H. Schmid, *Ballistic Research Laboratories Report No. 880*, October 1953. Analytic aerotriangulation by W. D. Harris, G. C. Tewinkel, and C. A. Whitten, *Coast and Geodetic Survey Technical Bulletin No. 21*, corrected July 1963.

[6] Analytic aerotriangulation by W. D. Harris, G. C. Tewinkel, and C. A. Whitten, *Coast and Geodetic Survey Technical Bulletin No. 21*, corrected July 1963.

the relative angular orientation of the image and ground coordinate axes.

The values of x and y for a set of images are measured (observed) with a comparator, z is considered to be a known constant, and X, Y, Z are known ground data. X_0, Y_0, Z_0 constitute the three *linear* unknown parameters whose determination is required in the resection analysis.

Three angles ω, ϕ, κ constitute the three *angular* parameters whose values are required. These values are related to the a's as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix} \tag{2}$$

The solution for the six parameters is based on a set of initial approximations that are adjusted iteratively until the adjustments are of insignificant size. The criterion for the solution is the minimization of discrepancies between the observed x-, y-coordinates of images and their theoretically correct computed values based on Equation 1.

A pair of equations, Equation 1, is generated by each image-object set that is analyzed. As six unknown parameters exist, a minimum of six equations--or three images--must therefore be used. The minimum ground data that can result in a solution consist of three objects of the completely known X, Y, Z. The least-squares solution permits the use of an unlimited number of points above the minimum number, although this program is limited arbitrarily to 15 points.

Adjustments $\Delta\omega, \Delta\phi, \Delta\kappa, \Delta X_0, \Delta Y_0, \Delta Z_0$ are obtained by a simultaneous solution of a set of observation equations derived from Equation 1:

$$(p_{12}\,\Delta\omega + p_{13}\,\Delta\phi + p_{14}\,\Delta\kappa - p_{15}\,\Delta X_0 - p_{16}\,\Delta Y_0 - p_{17}\,\Delta Z_0)\,/\,A_3B = -p_{11}/A_3B$$

$$(p_{22}\,\Delta\omega + p_{23}\,\Delta\phi + p_{24}\,\Delta\kappa - p_{25}\,\Delta X_0 - p_{26}\,\Delta Y_0 - p_{27}\,\Delta Z_0)\,/\,A_3B = -p_{21}/A_3B, \tag{3}$$

in which the p-coefficients are defined in table 1. The terms A_3B represent the matrix product

$$A_3B = [a_{31} \quad a_{32} \quad a_{33}] \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix}. \tag{4}$$

The other AB terms are partial derivatives referring to Equation 2:

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{5}$$

$$\frac{\partial A}{\partial\omega} = \begin{bmatrix} 0 & -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa \\ 0 & -\sin\omega\cos\kappa - \cos\omega\sin\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa \\ 0 & -\cos\omega\cos\phi & -\sin\omega\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_{13} & a_{12} \\ 0 & -a_{23} & a_{22} \\ 0 & -a_{33} & a_{32} \end{bmatrix}, \tag{6}$$

$$\frac{\partial A}{\partial\phi} = \begin{bmatrix} -\sin\phi\cos\kappa & \sin\omega\cos\phi\cos\kappa & -\cos\omega\cos\phi\cos\kappa \\ \sin\phi\sin\kappa & -\sin\omega\cos\phi\sin\kappa & \cos\omega\cos\phi\sin\kappa \\ \cos\phi & \sin\omega\sin\phi & -\cos\omega\sin\phi \end{bmatrix}, \tag{7}$$

Table 1.--The coefficients for the observation equations.

$$p_{11} = \begin{vmatrix} x & z \\ A_1B & A_3B \end{vmatrix} \qquad p_{21} = \begin{vmatrix} y & z \\ A_2B & A_3B \end{vmatrix}$$

$$p_{12} = \begin{vmatrix} x & z \\ \frac{\partial A_1}{\partial\omega}B & \frac{\partial A_3}{\partial\omega}B \end{vmatrix} \qquad p_{22} = \begin{vmatrix} y & z \\ \frac{\partial A_2}{\partial\omega}B & \frac{\partial A_3}{\partial\omega}B \end{vmatrix}$$

$$p_{13} = \begin{vmatrix} x & z \\ \frac{\partial A_1}{\partial\phi}B & \frac{\partial A_3}{\partial\phi}B \end{vmatrix} \qquad p_{23} = \begin{vmatrix} y & z \\ \frac{\partial A_2}{\partial\phi}B & \frac{\partial A_3}{\partial\phi}B \end{vmatrix}$$

$$p_{14} = \begin{vmatrix} x & z \\ \frac{\partial A_1}{\partial\kappa}B & \frac{\partial A_3}{\partial\kappa}B \end{vmatrix} \qquad p_{24} = \begin{vmatrix} y & z \\ \frac{\partial A_2}{\partial\kappa}B & \frac{\partial A_3}{\partial\kappa}B \end{vmatrix}$$

$$p_{15} = \begin{vmatrix} x & z \\ a_{11} & a_{31} \end{vmatrix} \qquad p_{25} = \begin{vmatrix} y & z \\ a_{21} & a_{31} \end{vmatrix}$$

$$p_{16} = \begin{vmatrix} x & z \\ a_{12} & a_{32} \end{vmatrix} \qquad p_{26} = \begin{vmatrix} y & z \\ a_{22} & a_{32} \end{vmatrix}$$

$$p_{17} = \begin{vmatrix} x & z \\ a_{13} & a_{33} \end{vmatrix} \qquad p_{27} = \begin{vmatrix} y & z \\ a_{23} & a_{33} \end{vmatrix}$$

$$\frac{\partial \mathbf{A}}{\partial \kappa} = \begin{bmatrix} -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ -\cos\phi\cos\kappa & -\cos\omega\sin\kappa - \sin\omega\sin\phi\cos\kappa & -\sin\omega\sin\kappa + \cos\omega\sin\phi\cos\kappa \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ -a_{11} & -a_{12} & -a_{13} \\ 0 & 0 & 0 \end{bmatrix}. \tag{8}$$

INITIAL APPROXIMATIONS

The solution is based on a set of initial approximate values for the six unknown parameters. Their values are programed, requiring no attention from the user except that the first three image input cards must conform to a strict arrangement.

The angles ω and ϕ are initially set to zero, which is a logical approximation for photography that is intended to be vertical.

The position parameters X_0 and Y_0 of the camera station are set initially equal to the parameters X and Y of a ground point near the principal point of the photograph, a condition that always occurs in the application of the Coast and Geodetic Survey system. Should the condition not be valid for other applications, it would be necessary to revise the program, such as by reading into the computer appropriate approximations derived in some other manner. The ground point that serves this purpose must be introduced into the computation as the first data input card.

Initial values for the azimuth angle κ and the altitude Z_0 are obtained by a relatively simple calculation based on the image and object coordinates of the second and third points in the list on which the solution is based. If the image coordinates for these two points are represented by x_1, y_1, x_2, y_2, and if the corresponding object coordinates by X_1, Y_1, X_2, Y_2, then the following sequence of operations can be expressed:

$$\begin{aligned}
\Delta x &= x_2 - x_1 \\
\Delta y &= y_2 - y_1 \\
\Delta X &= X_2 - X_1 \\
\Delta Y &= Y_2 - Y_1 \\
D &= \Delta X^2 + \Delta Y^2 \\
a &= (\Delta X\ \Delta x + \Delta Y\ \Delta y)/D \\
b &= (\Delta Y\ \Delta x - \Delta X\ \Delta y)/D \\
S &= (a^2 + b^2)^{1/2} \\
\sin\kappa &= b/S \\
\cos\kappa &= a/S \\
Z_0 &= f/S.
\end{aligned} \tag{9}$$

The focal length of the camera is represented by f.

Because the foregoing formulation has not been presented previously, a brief derivation is shown. The solution is based on the standard formulas for the rotation of the axes from classical plane analytic geometry:

$$\begin{aligned}
x' &= ax + by \\
y' &= -bx + ay.
\end{aligned} \tag{10}$$

If the two sets of coordinate axes $(x, y; x', y')$ are rectangular and if the coordinates are expressed in the same units of measurement (scale), a is the cosine and b is the sine of the angle between and x and x' axes.

Four equations are obtained by substituting into Equation 10:

$$\begin{aligned}
x_1 &= aX_1 + bY_1 \\
x_2 &= aX_2 + bY_2 \\
y_1 &= aY_1 - bX_1 \\
y_2 &= aY_2 - bX_2.
\end{aligned} \tag{11}$$

By subtracting the first equation from the second, and the third equation from the fourth,

$$\begin{aligned}
\Delta x &= \Delta X a + \Delta Y b \\
\Delta y &= \Delta Y a - \Delta X b,
\end{aligned} \tag{12}$$

the simultaneous solution of Equation 12 results in the principal formulas of Equation 9.

The final part of the formulation expresses the approximate flight altitude in terms of focal length divided by the scale of the photograph--a basic photogrammetric relation. What may not be obvious is that in Equation 10 the value $(a^2 + b^2)$ is unity in the classical case, and any variance from unity indicates that a nonunit scale factor exists between the dimensions used in the ground and photograph coordinate systems.

To aid in accomplishing the task of assigning and of determining the first approximations during the computations, a strict order of input for the first three cards is required. The first card must represent a point near the center of the photograph to furnish the initial X_0-, Y_0-values, and the second and third cards must represent two widely separated points to furnish initial values for κ and Z_0. It is suggested that the first point be within 1 centimeter of the center, and that the second and third images be separated by at least 7 centimeters on the photograph.

SOLUTION OF EQUATIONS

The number of observation equations provided for in this program can be as great as 30, where the number of unknowns is 6. The observation equations are reduced to six normal equations which are solved for the values of the unknowns by Gaussian elimination. Details of the solution were presented earlier in Technical Bulletins Nos. 21[7] and 23[8].

---

[7]Harris, Tewinkel, and Whitten, app. 12, p. 38, see footnote 6.

[8]Keller and Tewinkel, p. 6, see footnote 3.

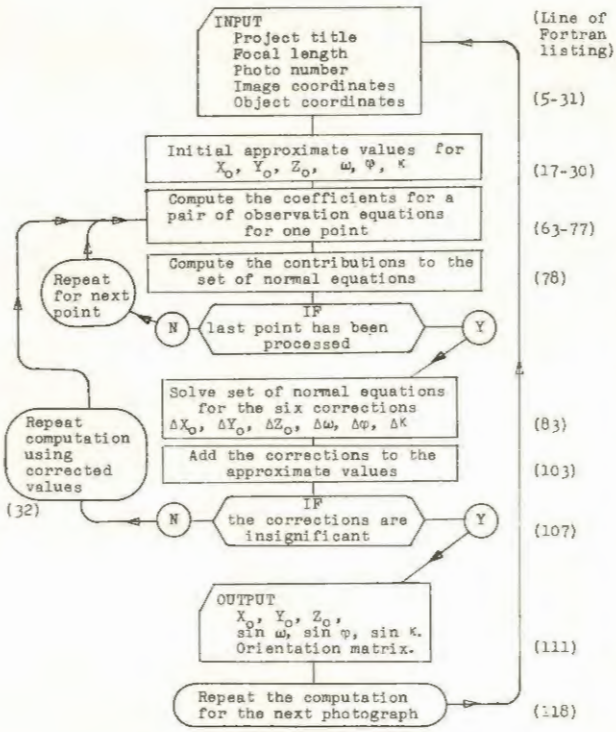

FIG. 1.--Generalized flow diagram.

SCHEME OF COMPUTATION

The generalized flow diagram (fig. 1) depicts the scheme of the computation. Numbers on the right serve to associate the steps of the diagram to the statements of the Fortran program shown in appendix 2. Details of the input and output card formats, as specified, are found in sections to follow.

The image and object coordinates for one point are contained on one input card. It is assumed that the image coordinates have already been corrected for distortions by a program such as given in Technical Bulletin No. 23.[9] All the input cards for one photograph are put into storage (the *B*-array) before the actual computation begins, the last card being identified by a 1 punched in column 80 (ITEST). As indicated in the division on "Input Formats," the first three input cards have strict specifications.

Based on data contained on the first three input coordinate cards, initial values for the six unknowns are derived only once during the computation, after which they are adjusted iteratively.

In the process of determining the coefficients of observation equations, auxiliary terms relating to the orientation elements are computed and are stored in the first 15 rows of the *C*-array. These values for a photograph are constant for all the points until the angular values are adjusted eventually at the completion of an iteration.

The coefficients of the linear terms (ΔX_0, ΔY_0, ΔZ_0) are multiplied by the flight altitude (C(1, 3) on line 76) as a numerical scaling measure which equalizes their magnitudes so that the linear coefficients are of the same general size as the angular terms. The effect is reversed later on line 102.

The coefficients (table 1) for the two equations associated with a point are computed and are stored in the *P*-array, after their contributions to the set of normal equations are computed and are added to the values already stored in the *D*-array. As soon as this is completed, the procedure is repeated for the next point until the ITEST signal is reached. Figure 1 shows this branch as an IF statement--actually, it is accomplished by a DO statement which is related to ITEST through IMAGE on line 64 of the program list.

After the last point is processed, the normal equations are solved by a modified Gaussian elimination[10] routine which has been applied throughout this series of bulletins. The terms "forward solution" (line 83) and "back solution" (line 93), which were applied to former desk calculator methods, serve to indicate the separate stages of the operation. All results are stored in the same *D*-array in which the normal equations were formed, and the "answers" or adjustments ($\Delta\omega \ldots \Delta Z_0$) to the approximate values are stored in the last column. The linear adjustments are added to the approximate values on line 104, and the angular adjustments are applied to the sines and cosines through the application of standard trigonometric formulas on lines 1062 and 1063.

The sizes of the angular adjustments are tested for termination of the iterative procedure on line 109. If any one of them ($\Delta\omega$, $\Delta\phi$, $\Delta\kappa$) is larger than 10^{-5} (which the user can alter by changing lines 109 and 127), the process is repeated; otherwise, the solution is considered as finished (no test is made on the linear adjustments). Three solutions are required ordinarily, and rarely a fourth is needed, but the number depends on the relative validity of the initial approximations. If the focal length is 6 inches, 10^{-5} corresponds to about 1.5 microns and to about 2 arc seconds.

It is probably significant, because of repeated rounding of numbers in computing, that new coefficients are recomputed for each iteration, which frees the solution from the dependence on the requirement that the observation equations be linear throughout the range of correction. It is also noted that the average sizes of the adjustments on any iteration are about one-tenth the size of those of the previous iteration. Consequently, the last adjustments are quite small; in fact, they are in the vicinity of 10^{-5}. As a result, the effect of rounding, which is always experienced during any system involving numerous arithmetic operations, is not significant, especially in this problem with a short solution. Even in larger applications, however, the solution would continue

[9]Keller and Tewinkel, see footnote 3.

[10]Harris, Tewinkel, and Whitten, see footnote 6.

to converge as long as the adjustments are accurate to only the first significant digit; that is, so long as no more than seven of the eight digits are damaged by the encroachment of rounding discrepancies.

The solution will obviously fail to converge geometrically if the camera station and the control points are on or near the surface of a common sphere, or if the control points are collinear.

THE STORAGE ARRAYS

Four active arrays (DIMENSION statement, app. 2, line 004) are used in core storage to facilitate the computational procedure. The *B*-array (15 rows of 6 columns) is used to store all the input data for one photograph, consisting of the image numbers, the two image coordinates, and the three object coordinates that are input by the READ statement on line 013 in the Fortran program. Any number of lines of the array may be used from 3 to 15, one line for each point used in the solution.

The *C*-array (20 rows of 3 columns) is used to store the auxiliary values needed to determine the values of the coefficients of the observation equations. The first row contains the current values of the coordinates of the camera station, and the second row has the sines of the three angular elements of the camera orientation--the six unknown parameters. The third row stores the cosines corresponding to the sines in the second row. Rows four through six contain the nine elements of the rotation matrix, and the following nine rows are their partial derivatives relative to ω, ϕ, κ. The first 15 rows remain unchanged during an iteration for all the points in the B-matrix, whereas the remaining five rows are changed for each point and are used for forming the A and B terms for the expressions shown in table 1.

The *P*-array (2 rows of 7 columns) is used to store the 14 *p*-coefficients (table 1) for the pair of observation equations for one point.

The *D*-array (6 rows of 7 columns) is used to store the normal equations formed from the observation equations in the *P*-array. After all of the 3 to 15 points for a photograph are processed, the normal equations are solved by a modified Gaussian elimination routine that stores the resulting intermediate terms in the same area. The six final results of the solution are located in the seventh or last column. The 15 locations below the main diagonal are never utilized.

It is assumed that the input data result from a previous computation, such as that shown in Technical Bulletins Nos. 23 and 29.[11] Consequently, the decimal numbers are shown in floating-point notation. If the floating-point notation is not convenient for the user, he can alter the format accordingly without having any effect on the computation itself.

[11] Keller and Tewinkel, see footnotes 3 and 4.

INPUT FORMATS

Three input formats are listed near the end of the program (app. 2); a sample input for four photographs is contained in appendix 1. Input card 1 occurs only once for a series of photographs. Card 2 is used once for each photograph; card 3 is used for each point. The minimum number of card 3 is 3 for any photograph; the maximum is 15. The line of the Fortran program (app. 2) is recorded in parenthesis on the right.

Input card 1.

```
      READ 531                                  (line 005)
  531 FORMAT (55H--title of
      project--)                                (126)
```

Input card 2.

```
  604 READ 525, IPLATE, FL                      (011)
  525 FORMAT (3X, I7, 3X, E14.8)                (120)
```

The number (called IPLATE) of the photograph is punched in columns 4 through 10 as an integer containing up to seven digits.

Input card 3.

```
  602 READ 526, (B (IMAGE, J),
      J = 1, 6), ITEST                          (013)
  526 FORMAT (1X,I7, 1X, 5E14.8, I1)            (121)
```

At least 3 cards are required; 15 is maximum. IMAGE is a card count number for the photograph. Referring to appendix 1, the first number is the point number stated as a seven-digit integer, the next two numbers are the image coordinates in floating-point meters, the last three numbers are object coordinates, also in floating-point meters, and finally a one is punched in column 80 of the last image card for the photograph. However, considerable freedom is permissible with the units of mensuration. The two image coordinates must be in the same units as the focal length of input card 2, but the unit may be inches, feet, or millimeters instead of meters. Also the three object coordinates can be in entirely different units from the image coordinates, such as feet, miles, or kilometers instead of meters in the example.

The first three input image coordinate cards must be in a prescribed order relative to their photographic locations.

The first card must pertain to an image near the center of the photograph because its object *X*- and *Y*-coordinates become the initial approximations for the location X_0, Y_0 of the camera station. It is suggested that the image be within 1 centimeter of the picture center. It is expected, however, that a larger separation will result only in an additional iteration or two before the termination limit of 10^{-5} is achieved.

The second and third images should be separated by at least 7 centimeters on the photograph, as these images are used to determine the initial approximations of the azimuth κ and altitude Z_0. Again, if the distance is smaller,

the effect is expected to be only an additional iteration or two.

OUTPUT FORMATS

Seven output punch formats are employed as shown in appendix 3. Five of the formats refer to headings, whereas only two of them refer to numerical output data.

Output card 1.

```
      PUNCH 531                          (line 006)
  531 FORMAT (55H--title of
      project--)                         (126)
```

The input project title is duplicated immediately as the first *output* card.

Output card 2.

```
      PUNCH 532                          (007)
  532 FORMAT (/50H ORIENTATION
      PARAMETER CORRECTION
      LIMIT IS 0.00001)                  (127)
```

The criterion for the termination of the iterative process is exhibited as a reminder of the precision of the system. The user can change this value by altering lines 109 and 127 accordingly.

Output card 3.

```
  763 PUNCH 527                          (112)
  527 FORMAT (/49H PLATE XO
      YO   ZO)                           (122)
```

The column headings for the photograph number (PLATE) and the three coordinates of the camera station are punched on one card, and are printed out on one line to match the numerical data to follow.

Output card 4.

```
      PUNCH 528, IPLATE,
      (C(1, J), J = 1, 3)                (113)
  528 FORMAT (I7, 3(2X, E14.8))          (123)
```

The photograph number and the three adjusted coordinates of the camera station are shown on a line. The coordinates are in floating-point meters in this example, but are always in the same units as the input object coordinates.

Output card 5.

```
      PUNCH 529                          (114)
  529 FORMAT (/50H   PLATE
      OMEGA   PHI   KAPPA)               (124)
```

The column headings for the photograph number (PLATE) and the sines of the three adjusted angular parameters are shown on one line in the same manner as the linear parameters in Output statement 4.

Output card 6.

```
      PUNCH 528, IPLATE,
      (C(2, J), J = 1, 3)                (115)
  528 FORMAT (I7, 3(2X, E14.8))          (123)
```

The sines of the three adjusted angular parameters of the camera are shown using exactly the same format as Output statement 4, expressed in floating-point notation, and the corresponding cosines are printed on the second line from a second card.

Output card 7.

```
      PUNCH 530, IPLATE                  (116)
  530 FORMAT (/30H ORIENTATION
      MATRIX FOR PLATE I7)               (125)
```

The pair of statements results in punching on one card the heading data plus the photograph number in its appropriate place.

Output card 8.

```
      PUNCH 533, ((C(I, J), J = 1, 3),
      I = 4, 6)                          (117)
  533 FORMAT (1X, 3E14.8)                (128)
```

The nine elements of the orientation matrix are punched as three cards, containing three numbers per card, and are printed in their correct matrix form (they constitute an orthogonal rotation matrix). Each of the nine elements is a direction cosine. The first term in the third row is $\sin \phi$, which is also printed as PHI in the output immediately preceding.

Depending on the mechanical design of the system, some elements of the matrix may be functions of angular rectifier settings for transforming the picture photographically.

THE FORTRAN PROGRAM

The program, shown as appendix 2, is written in the Fortran II language for an IBM-1620 computer, but the statements are also acceptable as Fortran IV. The program occupies 27,824 digits of core memory, and the computing time is 3 minutes per photograph.

The input data are considered to have resulted from a previous computer operation. The computation provides for an indefinite number of input cards ranging from 3 to 15, with 1 card per image point and with a number 1 punched in column 80 of the last card. The first input card must pertain to an image which is near the center of the photograph, and the second and third input cards should be for images which are widely separated on the photograph (preferably more than 7 centimeters), as these points are used to determine initial approximations.

Appendix 2 constitutes a listing of the source program. The lines (cards) of the program are numbered serially on the right, to which frequent references have been made throughout the preceding pages. Although frequent comment

statements indicate the separate phases of the computation, the listing also is annotated with more specific explanations. The procedure follows the generalized flow diagram shown as figure 1.

Comments referring to the details of the solution of the simultaneous equations are omitted because the subject is discussed at length in Technical Bulletins Nos. 23[12] and 21[13].

[12] Harris, Tewinkel, and Whitten, see footnote 6.
[13] Keller and Tewinkel, see footnote 3.

Appendixes

APPENDIX 1

SAMPLE INPUT DATA FOR FOUR PHOTOGRAPHS

```
TEST OF RESECTION PROGRAM   KELLER          (title of project)
       51   +.15000000E+00                  (photo number; focal length)
5151330 -.26635629E-02+.25725648E-02+.90205900E+03+.36003650E+04-.71500000E+00
5151320 -.42263599E-02+.92105142E-01+.90175700E+03+.45001810E+04-.27300000E+00
5151310 -.11326561E-02-.85133022E-01+.89721000E+03+.26996160E+04-.18820000E+01
5152320 +.84399750E-01+.92701783E-01+.17981310E+04+.44990890E+04-.21430000E+01
5152310 +.85671978E-01-.82758078E-01+.18003310E+04+.27003690E+04-.15240000E+011
       52   +.15000000E+00
5252330 -.40293974E-02+.38250679E-02+.18005390E+04+.36032660E+04-.28300000E+00
5252320 -.63870663E-02+.93860810E-01+.17981310E+04+.44990890E+04-.21430000E+01
5252310 -.17435367E-02-.83468432E-01+.18003310E+04+.27003690E+04-.15240000E+01
5253310 +.84207345E-01-.79965949E-01+.27019060E+04+.27003710E+04-.24720000E+01
5253320 +.82293945E-01+.94728451E-01+.27017420E+04+.44987440E+04-.34420000E+011
       53   +.15000000E+00
5353330 -.52902636E-02-.10800980E-02+.27007150E+04+.36003230E+04+.80300000E+00
5353310 -.91768655E-02-.90097919E-01+.27019060E+04+.27003710E+04-.24720000E+01
5353320 -.14435167E-02+.87024892E-01+.27017420E+04+.44987440E+04-.34420000E+01
5354320 +.84863263E-01+.81508285E-01+.35995050E+04+.44995000E+04+.74800000E+00
5354310 +.78163012E-01-.92073314E-01+.36014850E+04+.26980750E+04+.22090000E+011
       61   +.15229000E+00
6161330 -.20186159E-02-.27127389E-02+.64591710E+04+.14855611E+05+.85847100E+03
6161320 +.45915236E-02+.89821222E-01+.99725700E+04+.13231008E+05+.81297300E+03
6161310 -.19468599E-02-.96737575E-01+.28454360E+04+.16220375E+05+.90017600E+03
6163320 +.91071273E-01+.80960060E-01+.83450210E+04+.99950530E+04+.82767600E+03
6163310 +.96288021E-01-.95000960E-01+.14466600E+04+.12403341E+05+.85988000E+031
```

(point no.) (x) (y) (X) (Y) (Z)

(ITEST)

APPENDIX 2

LISTING OF THE FORTRAN II PROGRAM

```
**    R E S E C T I O N    KELLER
*0807                                    (precision: 8 decimal; 7 integer)
C     R E S E C T I O N    KELLER
      DIMENSION B(15,6),C(20,3),P(2,8),D(6,7)     (arrays of core storage)
      READ 531                                                (project title)
      PUNCH 531                                               (project title)
      PUNCH 532                 (orientation parameter convergence criterion)
C     P H O T O   R E S E C T I O N   P H A S E
C       READ IN REFINED PLATE AND GROUND COORDINATES OF RESECTION POINTS
  604 READ 525,IPLATE,FL                                          (focal length)
      IGO=1
      IMAGE=1                                                (photograph number)
  602 READ 526,(B(IMAGE,J),J=1,6),ITEST     (image and object coordinates)
      IF(ITEST)600,600,601                        (branch on last image card)
  600 IMAGE =IMAGE+1                                        (input card count)
      GO TO 602
C       INITIAL APPROXIMATIONS OF CAMERA PARAMETERS
  601 DO 608 J=1,2
      C(1,J)=B(1,J+3)                                   (approximate X0, Y0)
      C(2,J)=0.                                   (approximate omega and phi)
  608 C(3,J)=1.
C       ADJUST APPROXIMATE AZIMUTH PARAMETERS FOR SWING AND Z0
      DO 609 I=1,2
      D(1,I)=B(2,I+1)-B(3,I+1)                                   (Equation 9)
  609 D(2,I)=B(2,I+3)-B(3,I+3)
      D(2,4)=D(2,1)*D(2,1)+D(2,2)*D(2,2)
      D(1,3)=(D(2,1)*D(1,1)+D(2,2)*D(1,2))/D(2,4)
      D(2,3)=(D(2,2)*D(1,1)-D(2,1)*D(1,2))/D(2,4)
      SCALE=SQRT(D(1,3)*D(1,3)+D(2,3)*D(2,3))
      C(2,3)=D(2,3)/SCALE                          (approximate sine kappa)
      C(1,3)=FL/SCALE                                       (approximate Z0)
      C(3,3)=D(1,3)/SCALE                         (approximate cosine kappa)
C       ORIENTATION FACTORS IN C ARRAY
  610 C(4,1)=C(3,2)*C(3,3)
      C(5,1)=-C(3,2)*C(2,3)
      C(6,1)=C(2,2)
      C(10,1)=-C(2,2)*C(3,3)
      C(11,1)=C(2,2)*C(2,3)
      C(12,1)=C(3,2)                          (auxiliary  rotational
      C(10,2)=C(4,1)*C(2,1)                    functions, including
      C(11,2)=C(5,1)*C(2,1)                    orientation matrix and
      C(12,2)=C(2,1)*C(2,2)                    its partial derivatives;
      C(10,3)=-C(4,1)*C(3,1)                   Equations 2, 6, 7, and 8)
      C(11,3)=-C(5,1)*C(3,1)
      C(12,3)=-C(3,1)*C(2,2)
      C(4,2)=C(3,1)*C(2,3) + C(12,2)*C(3,3)
      C(5,2)=C(3,1)*C(3,3) - C(12,2)*C(2,3)
      C(6,2)=-C(2,1)*C(3,2)
      C(4,3)=C(2,1)*C(2,3) + C(10,1)*C(3,1)
      C(5,3)=C(2,1)*C(3,3) + C(11,1)*C(3,1)
      C(6,3)=C(3,1)*C(3,2)
      DO 612 I=7,9
      C(I,1)=0.
      C(I,2)=-C(I-3,3)
      C(I,3)=C(I-3,2)
      C(13,I-6)=C(5,I-6)
      C(14,I-6)=-C(4,I-6)
  612 C(15,I-6)=0.
      GO TO (613,763),IGO                                (763:  line 112)
C       CLEAR NORMAL EQUATION D ARRAY TO ZERO
  613 DO 614 I=1,6
      DO 614 J=I,7
  614 D(I,J)=0.
C       COMPUTE P TERMS FOR RESECTION PASS POINTS  (p-terms are coeffi-
      DO 618 NU=1,IMAGE                             cients of observation
      DO 619 K=1,3                                  equations: table 1)
```

```
  619 C(16,K)=B(NU,K+3)-C(1,K)                                          066
      K=4                                                               067
      DO 620 L=17,20                                                    068
      DO 620 I=1,3                                                      069
      C(L,I)=C(K,1)*C(16,1) + C(K,2)*C(16,2) + C(K,3)*C(16,3) (AB-terms;  070
  620 K=K+1                                                Equation 4)  071
      DO 621 I=1,2                          (constant term and coefficients  072
      DO 622 L=1,4                                 of omega, phi, and kappa)  073
  622 P(I,L)=(B(NU,I+1)*C(L+16,3)-(-FL)*C(L+16,I))/C(17,3)              074
      DO 623 L=5,7                          (coefficients of ΔX0, ΔY0, ΔZ0)  075
  623 P(I,L)=(-B(NU,I+1)*C(6,L-4)+(-FL)*C(I+3,L-4))*C(1,3)/C(17,3)      076
  621 P(I,8)=-P(I,1)                          (transposition; Equation 3)  077
C       CONTRIBUTION TO NORMAL EQUATIONS                                078
      DO 618 I=1,6                                                      079
      DO 618 J=I,7                  (for the two equations in the P-array)  080
      DO 618 K=1,2                                                      081
  618 D(I,J)=D(I,J)+P(K,I+1)*P(K,J+1)                                   082
C       FORWARD SOLUTION                                                083
  624 DO 699 I=1,6                                                      084
      SQR=SQRT(D(I,I))                                                  085
      DO 698 J=I,7                                                      086
  698 D(I,J)=D(I,J)/SQR                                                 087
      IF(I-6)697,696,696                                                088
  697 IP1=I+1                                                           089
      DO 699 L=IP1,6                                                    090
      DO 699 J=L,7                                                      091
  699 D(L,J)=D(L,J)-D(I,L)*D(I,J)                                       092
C     BACK SOLUTION                                                     093
  696 D(6,7)=D(6,7)/D(6,6)                                        (ΔZ0)  094
      DO 691 I=1,5                                                      095
      NMI=6-I                                                           096
      NMIP1=NMI+1                                                       097
      DO 690 J=NMIP1,6                                                  098
  690 D(NMI,7)=D(NMI,7)-D(J,7)*D(NMI,J)                                 099
  691 D(NMI,7)=D(NMI,7)/D(NMI,NMI)                 (the other five answers)  100
      DO 625 I=4,6                                                      101
  625 D(I,7)=D(I,7)*C(1,3)                       (numerical scaling reversal)  102
C       ADD LEAST SQUARES RESULTS TO CAMERA PARAMETERS IN C ARRAY       103
      DO 626 J=1,3                                  (the 3 linear parameters)  104
      C(1,J)=C(1,J)+D(J+3,7)                                            105
      C(4,J)=D(J,7)                                                     106
      C(5,J)=SQRT(1.-C(4,J)*C(4,J))                                     1061
      C(6,J)=C(2,J)*C(5,J)+C(3,J)*C(4,J)                   (sin (α + Δα))  1062
      C(7,J)=C(3,J)*C(5,J)-C(2,J)*C(4,J)                    (cos (α + Δα))  1063
      C(2,J)=C(6,J)                                                     1064
  626 C(3,J)=C(7,J)                                                     1065
C       TEST MAGNITUDE OF CORRECTIONS FOR ORIENTATION PARAMETERS        107
  627 DO 628 I=1,3                                                      108
      IF(ABS(D(I,7))-.00001)628,628,610          (.00001 ≈ 2 arc seconds;  109
  628 CONTINUE                                 statement 610 is on line  33)  110
      IGO=2                                                             1101
      GO TO 610                                   (output column headings)  1102
C       CAMERA PARAMETERS OUTPUT                (camera station coordinates)  111
  763 PUNCH 527                                                         112
      PUNCH 528,IPLATE,(C(1,J),J=1,3)             (output column headings)  113
      PUNCH 529                                                         114
      PUNCH 528,IPLATE,(C(2,J),J=1,3)                                   115
      PUNCH 528,IPLATE,(C(3,J),J=1,3)  (output camera orientation angles)  1151
      PUNCH 530,IPLATE                                (photograph number)  116
      PUNCH 533,((C(I,J),J=1,3),I=4,6) (9 elements of orientation matrix)  117
      GO TO 604                                                (604: line 011)  118
  525 FORMAT(3X,I7,3X,E14.8)    (for input of photo number, focal length)  120
  526 FORMAT(1X,I7,1X,5E14.8,I1) (for image and object coordinate input)  121
  527 FORMAT(/49H PLATE          XO                YO                ZO)  122
  528 FORMAT(I7,3(2X,E14.8))                (for output of camera parameters)  123
  529 FORMAT(/50H PLATE        OMEGA              PHI              KAPPA)  124
  530 FORMAT(/30H ORIENTATION MATRIX FOR PLATE I7)                      125
  531 FORMAT(55H                                                        )  126
  532 FORMAT(/50H ORIENTATION PARAMETER CORRECTION LIMIT IS 0.00001)    127
  533 FORMAT(1X,3E14.8)                   (for output of orientation matrix)  128
      END                                                               129
```

APPENDIX 3

SAMPLE OUTPUT FOR THE RESECTION SOLUTION FOR THE FOUR PHOTOGRAPHS INDICATED IN APPENDIX 1

```
TEST OF RESECTION PROGRAM   KELLER

ORIENTATION PARAMETER CORRECTION LIMIT IS 0.00001

PLATE          XO                YO                ZO
    51   .90474663E+03    .36064653E+04    .15234077E+04

PLATE        OMEGA             PHI              KAPPA
    51  -.20549264E-01  -.14475223E-01  -.18584619E-01    (sines)
    51   .99978890E+00   .99989522E+00   .99982732E+00    (cosines)

ORIENTATION MATRIX FOR PLATE       51
 .99972255E+00-.18283292E-01 .14851567E-01
 .18582671E-01 .99962177E+00-.20276756E-01
-.14475223E-01 .20547110E-01 .99968414E+00

PLATE          XO                YO                ZO
    52   .17995316E+04    .36058795E+04    .15210447E+04

PLATE        OMEGA             PHI              KAPPA
    52  -.29064517E-01  -.26724701E-01  -.26466726E-01    (sines)
    52   .99957756E+00   .99964285E+00   .99964977E+00    (cosines)

ORIENTATION MATRIX FOR PLATE       52
 .99929274E+00-.25679077E-01 .27473297E-01
 .26457273E-01 .99924802E+00-.28347321E-01
-.26724701E-01 .29054136E-01 .99922056E+00

PLATE          XO                YO                ZO
    53   .26908631E+04    .36048243E+04    .15195731E+04

PLATE        OMEGA             PHI              KAPPA
    53   .58306660E-02  -.41592527E-01   .43456981E-01    (sines)
    53   .99998302E+00   .99913470E+00   .99905536E+00    (cosines)

ORIENTATION MATRIX FOR PLATE       53
 .99819087E+00 .43213960E-01 .41805914E-01
-.43419377E-01 .99904892E+00 .40177032E-02
-.41592527E-01-.58256207E-02 .99911773E+00

PLATE          XO                YO                ZO
    61   .65289270E+04    .14746920E+05    .71634654E+04

PLATE        OMEGA             PHI              KAPPA
    61  -.14612753E-02  -.89694648E-03  -.93519250E+00    (sines)
    61   .99999894E+00   .99999959E+00  -.35413988E+00    (cosines)

ORIENTATION MATRIX FOR PLATE       61
-.35413973E+00-.93519196E+00 .10489296E-02
 .93519211E+00-.35413828E+00 .13563125E-02
-.89694648E-03 .14612747E-02 .99999853E+00
```