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COAST AND GEODETIC SURVEY C&GS 34

Aerotriangulation: Transformation of Surveying and Mapping Coordinate Systems

Rockville, Md.
August 1967

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Aerotriangulation: Transformation of Surveying and Mapping Coordinate Systems

LIEUTENANT COMMANDER MELVIN J. UMBACH
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AEROTRIANGULATION is a photogrammetric technique for deriving the ground coordinates of numerous terrestrial objects from a set of overlapping aerial photographs. Ground coordinates for a few of these objects must be known and properly distributed throughout the photographed area. The Coast and Geodetic Survey technique for processing analytic aerotriangulation is divided into five distinct and separate operations: (1) photograph preparation and coordinate measurement;[1] (2) coordinate refinement[2] for the compensation of known systematic errors caused by film distortion, lens distortion and atmospheric refraction; (3) relative orientation,[3] by either the stereo-pair or stereo-triplet method, to compute a set of three-dimensional photogrammetric model coordinates that are independent of a ground coordinate system (if the stereo-pair relative orientation technique is used, a cantilever assembly program[4] is required for the mathematical attachment of successive models along the photographic strip); (4) strip adjustment[5] to remove the nonlinear accumulation of systematic errors that have propagated along the strip, to transform the photogrammetric strip model coordinates to a meaningful earth coordinate system, and if maximum precision is required, to provide provisional coordinates as input for a block adjustment solution; and (5) block adjustment[6] to perform a simultaneous photogrammetric solution of ground coordinates for all objects appearing on a block of overlapping aerial photographs.

This paper presents a documented computer program for the transformation of various mapping coordinate systems common to the United States into a three-dimensional orthogonal system particularly suitable for precise analytic aerotriangulation encompassing large areas and particularly mountainous terrain—specifically, the secant plane coordinate system. Secant plane coordinates for ground control are used as input for both the strip adjustment and block adjustment phases of the analytic system.

The author has drawn heavily upon the works of Charles N. Claire, Geodesy Division; and Rosemary E. Riordan, Electronic Computing Division, and expresses his sincere appreciation. Mr. Claire's recent work, on the development of modern algorithms for state plane coordinate conversion, has provided the necessary background for the most important portions of this documented program. Miss Riordan has been deeply involved in the programing of coordinate transformations; much of her work appears here in its original form.

INTRODUCTION

The secant plane coordinate system was adopted by the Coast and Geodetic Survey as an ideal coordinate system for the adjustment of photogrammetric data. It is essentially a local three-dimensional orthogonal system in which the Z coordinates depict the aggregate effects of both earth curvature and elevation, and in which the X and Y axes are related to the cardinal compass points at the origin of the system. Although a tangent plane would essentially achieve the same purpose, the secant plane generally avoids negative Z-coordinates. Classical geocentric coordinates are unwieldy to handle and extremely difficult to visualize in the data analysis phases of operational aerotriangulation.

The primary objective of this project was to develop a high-speed computer program for direct and inverse transformations between various earth systems and the secant plane, however, transformations between any of the coordinate

[1]Harris, W. D., Tewinkel, G. C., and Whitten, C. A., Analytic Aerotriangulation, *Coast and Geodetic Survey Technical Bulletin No. 21*, July 1962.

[2]Keller, M., and Tewinkel, G. C., Aerotriangulation: Image Coordinate Refinement, *Coast and Geodetic Survey Technical Bulletin No. 25*, March 1965.

[3]Keller, M., and Tewinkel, G. C., Three-Photo Aerotriangulation, *Coast and Geodetic Survey Technical Bulletin No. 29*, February 1966. Horsfall, C. T., Electronic Computer Programs for Analytical Aerial Triangulation, (Published by the *U.S. Coast and Geodetic Survey*) June 1965. Umbach, M. J., Supplement to: Electronic Computer Programs for Analytical Aerial Triangulation, and Aerotriangulation Strip Adjustment Using FORTRAN and the IBM 1620 Computer, (Published by the *U.S. Coast and Geodetic Survey*) February 1966.

[4]Horsfall, C. T., and Umbach, M. J., see footnote 3, above.

[5]Keller, M., and Tewinkel, G. C., Aerotriangulation Strip Adjustment, *Coast and Geodetic Survey Technical Bulletin No. 23*, August 1964. Horsfall, C. T., Aerotriangulation Strip Adjustment Using FORTRAN and the IBM 1620 Computer, (Published by the *U.S. Coast and Geodetic Survey*) March 1965. Umbach, M. J., see footnote 3, above.

[6]Keller, M., and Tewinkel, G. C., Block Analytic Aerotriangulation, *ESSA Technical Report C&GS No.* (in process).

systems acceptable as input are also possible. Systems acceptable by this documented program include the geographic system of positions and elevations, the secant plane system, the state plane systems based on either the transverse Mercator or Lambert projections, Alaska Zone 1 which is based on an oblique Mercator system, the Universal Transverse Mercator, and the classical geocentric coordinate system.

The Fortran IV program presented in this publication was written specifically for the IBM 7030 (Stretch), located at the Naval Weapons Laboratory, Dahlgren, Va., but can be easily adapted for other computers with few alterations. Each transformation is self-contained in a subprogram to facilitate a simple conversion to a different version or segmentation for a computer with a more limited core storage. The published program is self-initializing for batch processing.

This publication will be limited to a brief discussion of each program accepted transformation with its equations and parameters. References are given, so that a more detailed study may be made, if desired by the reader.

SECANT PLANE COORDINATE SYSTEM

As previously mentioned, the secant plane provides a local three-dimensional Cartesian coordinate system in which X and Y are comparable to horizontal grid coordinates, and in which Z is equivalent to the combination of elevation and earth curvature. The Z-axis is the extension of the ellipsoid normal of an arbitrary coordinate system origin usually selected near the center of the project area.

The initial step in the conversion of geographic positions and elevations to secant plane coordinates is the transformation to classical geocentric coordinates:[7]

$$X_G = (N + h) \cos \phi \cos \lambda$$

$$Y_G = (N + h) \cos \phi \sin \lambda \qquad (1)$$

$$Z_G = [N (1 - e^2) + h] \sin \phi$$

Where ϕ and λ are the latitude and longitude of the point, e is the ellipsoid eccentricity, h is the point elevation, and N is the length of the ellipsoid normal through ϕ and λ.

Since the geocentric system will be rotated about the intersection of the polar axis and the normal of the ellipsoid through the secant plane origin, a small increment, $N_o\ e^2\ \sin\phi_o$, is added to the Z_G coordinate to translate the $X-Y$ plane to this point (see figure 1):

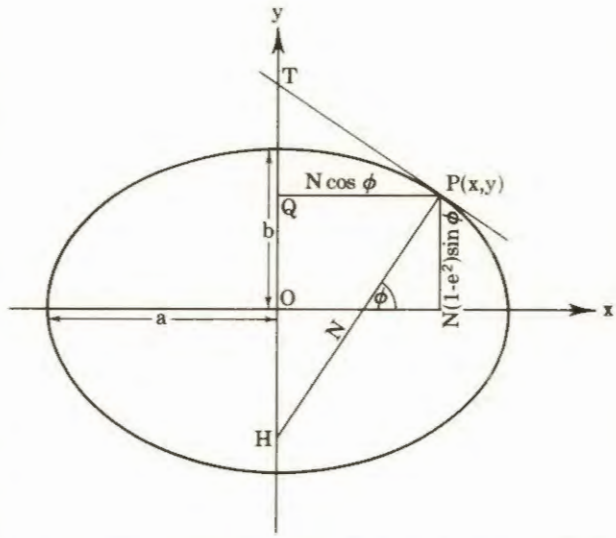

FIGURE 1.—Geometric elements used in the secant plane transformation.

$$Z'_G = [N (1 - e^2) + h] \sin \phi + N_o e^2 \sin \phi_o \qquad (2)$$

The modified geocentric system is then rotated to make the Z-axis coincide with the origin normal, with the Y-axis indicating north, and the X-axis indicating east:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos \lambda_o & -\sin \lambda_o & 0 \\ -\sin \phi_o \sin \lambda_o & -\sin \phi_o \cos \lambda_o & \cos \phi_o \\ \cos \phi \sin \lambda_o & \cos \phi_o \cos \lambda_o & \sin \phi_o \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z'_G \end{bmatrix} \qquad (3)$$

The $X-Y$ plane of this rotated system is then translated along the Z-axis to a "secant" position. The magnitude of translation was arbitrarily

[7]Schmid, Erwin, Transformation of Rectangular Space Coordinates, *Coast and Geodetic Survey, Technical Bulletin No. 15*, 1960.

chosen as the length of the origin normal rounded down to the nearest 10,000 meters. After this translation the coordinates are in the secant plane system required for either strip or block adjustment.

The inverse transform of the secant plane system to geographic positions first requires the translation of the $X-Y$ plane back to the intersection of the ellipsoid normal and the polar axis, followed by the rotation back into the modified geocentric system:

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} \cos\lambda_o & -\sin\phi_o \sin\lambda_o & \cos\phi_o \sin\lambda_o \\ -\sin\lambda_o & -\sin\phi_o \cos\lambda_o & \cos\phi_o \cos\lambda_o \\ 0 & \cos\phi_o & \sin\phi_o \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{4}$$

from equations (1), the longitude is computed:

$$\lambda = \tan^{-1} (X_G / Y_G) \tag{5}$$

from equations (1) and (2)

$$\phi \cong \tan^{-1} \left[Z_G / (X_G^2 + Y_G^2)^{1/2} \right] \tag{6}$$

By using the recursive relationship between equations (6) and (2), the latitude of the point converges rapidly in several iterations.

The elevation of the point can be computed from equations (1):

$$h = (X_G / \cos\phi \sin\lambda) - N, \text{ or}$$

$$h = (Y_G / \cos\phi \cos\lambda) - N \tag{7}$$

STATE PLANE COORDINATES BASED ON THE TRANSVERSE MERCATOR PROJECTION

The transverse Mercator is a conformal projection of the so-called cylindrical type in which the central meridian of the area under consideration is represented by a straight line of constant scale.[8] The projection may be visualized as a cylinder, tangent to the reference ellipsoid along the central meridian of the projected area. The vertical axis of the cylinder is normal to the plane of that meridian.

Since the scale of the transverse Mercator projection deteriorates with the distance from the central meridian, it is primarily applicable to states having a limited east-west dimension, or for those states or areas that can be conveniently subdivided in this manner. In order to balance the scale throughout the projection area, an arbitrary scale reduction is made at the central meridian. The scale then becomes true at a certain distance both east and west of the central meridian. Areas or states for which official state plane coordinate systems are based on the transverse Mercator projection include: Alabama, Arizona, Delaware, parts of Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Maine, Michigan, Mississippi, Missouri, Nevada, New Hampshire, New Jersey, New Mexico, New York, Rhode Island, Vermont, and Wyoming.

The algorithm for the transformation of geographic positions to state grid coordinates based on the transverse Mercator projection follows:[9]

$$\Delta\lambda_{secs} = T_2 - \lambda_{secs} \tag{8}$$

$$\Delta\lambda_1 = \Delta\lambda_{secs} - 3.9174 \left(\frac{\Delta\lambda_{secs}}{10^4}\right)^3 \tag{9}$$

$$S_1 = \frac{30.92241724\ \Delta\lambda_1 \cos\phi}{(1 - e^2 \sin^2\phi)^{1/2}} \tag{10}$$

$$S_m = S_1 + 4.0831 \left(\frac{S_1}{10^5}\right)^3 \tag{11}$$

$$S_g = 3.280833333\ T_5\ S_m \tag{12}$$

$$x' = S_g + T_6 \left(\frac{S_g}{10^5}\right)^3 \tag{13}$$

$$X = T_1 + x' \tag{14}$$

$$B = (25.52381)(10^{-10}) \tan\phi\, (1 - e^2 \sin^2\phi)^2 \tag{15}$$

$$\Delta\phi_{secs} = B\ S_m^2 \tag{16}$$

$$\phi'_{secs} = \phi_{secs} + \Delta\phi_{secs} \tag{17}$$

[8]Deetz, Charles S., and Adams, Oscar S., Elements of Map Projections, *Coast and Geodetic Survey Special Publication No. 68*, 5th Edition, 1945. Thomas, Paul, D., Conformal Projections in Geodesy and Cartography, *Coast and Geodetic Survey Special Publication No. 251*, 1952. Mitchell, Hugh C., Definition of Terms Used in Geodetic and other Surveys, *Coast and Geodetic Survey Special Publication No. 242*, 1948.

[9]Claire, Charles N., State Plane Coordinates by Automatic Data Processing, *Coast and Geodetic Survey Publication 62-4* (in press).

Iterate between equations (15) through (17) until $\Delta\phi$ converges. The term ϕ_{secs} in equation (17) is held constant as the original station latitude and is not updated in each iteration.

The Y-state grid may then be computed:

$$Y = 101.2794065\ T_5 \left\{ \phi'_{secs} - 60\ T_3 - T_4 - \sin\phi' \cos\phi' \left[1052.893882 - \cos^2\phi' \left(4.483344 - 0.023520 \cos^2\phi'\right) \right] \right\} \quad (18)$$

The equations that perform the inverse transformation of transverse Mercator state plane coordinates to geographic positions are:

$$x' = X - T_1 \quad (19)$$

$$S_{g_1} = x' - T_6 \left(\frac{x'}{10^5}\right)^3 \quad (20)$$

$$S_g = x' - T_6 \left(\frac{S_{g_1}}{10^5}\right)^3 \quad (21)$$

$$S_m = \frac{0.3048006099\ S_g}{T_5} \quad (22)$$

$$\omega_{secs} = 60\ T_3 + T_4 + \frac{0.009873675553\ Y}{T_5} \quad (23)$$

$$\phi'_{secs} = \omega_{secs} + \sin\omega \cos\omega \left[1047.546710 + \cos^2\omega \left(6.192760 + 0.050912 \cos^2\omega\right) \right] \quad (24)$$

$$\Delta\phi_{secs} = (25.52381)\ (10^{-10})\ (S_m^2 \tan\phi')\ (1 - e^2 \sin^2\phi')^2 \quad (25)$$

$$\phi_{secs} = \phi'_{secs} - \Delta\phi_{secs} \quad (26)$$

$$S_a = S_m - 4.0831 \left(\frac{S_m}{10^5}\right)^3 \quad (27)$$

$$S_1 = S_m - 4.0831 \left(\frac{S_a}{10^5}\right)^3 \quad (28)$$

$$\Delta\lambda_{1secs} = \frac{S_1 (1 - e^2 \sin^2\phi)^{1/2}}{30.92241724 \cos\phi} \quad (29)$$

$$\Delta\lambda_{a\ secs} = \Delta\lambda_{1secs} + 3.9174 \left(\frac{\Delta\lambda_{1secs}}{10^4}\right)^3 \quad (30)$$

$$\Delta\lambda_{secs} = \Delta\lambda_{1secs} + 3.9174 \left(\frac{\Delta\lambda_{a\ secs}}{10^4}\right)^3 \quad (31)$$

$$\lambda_{secs} = T_2 - \Delta\lambda_{secs} \quad (32)$$

where:

ϕ = Station latitude

λ = Station longitude

X = State Plane X- coordinate

Y = State Plane Y- coordinate

e = First ellipsoid eccentricity of ref. ellipsoid

$\left[T_i\right]_{i=1,6}$ = State zone constants

T_1 = False Easting

T_2 = Central meridian in seconds of longitude

T_3 = Minutes of latitude at zone origin

T_4 = Seconds of latitude at zone origin

T_5 = Scale factor at origin

$T_6 = 1/6\rho_o^2 R^2 \cdot 10^{15}$ T_5^2

The constants, T_i, are tabulated by Claire; see footnote 9.

STATE PLANE COORDINATES BASED ON THE LAMBERT PROJECTION

The Lambert conformal conic projection employs a "secant cone" in which all meridians are depicted as straight lines that meet at a common point and all parallels are concentric circles, the center of which lies at the point of meridian intersection. The secant cone intersects the spheroid at two standard parallels for the area to be projected and are normally spaced at one-sixth and five-sixths of the latitude range of the projection. The Lambert projection is especially well suited for areas where the east-west dimension is predominant.[10]

The states that have officially adopted the Lambert conformal conic projection include: Alaska Zone 10, Arkansas, California (7 zones), Colorado, Connecticut, Florida, Iowa, Kansas, Kentucky, Louisiana, Maryland, Massachusetts, Michigan, Minnesota, Montana, Nebraska, New York, North Carolina, North Dakota, Ohio, Oklahoma, Oregon, Pennsylvania, South Carolina, South Dakota, Tennessee, Texas, Utah, Virginia, Washington, West Virginia, and Wisconsin.

The algorithm for the transformation of geographic positions to Lambert state plane coordinates follows:[11]

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[10] Deetz, Charles S., and Adams, Oscar S., see footnote 8, above.

[11] Claire, Charles N., see footnote 9, above.

$$S = 101.2794065 \left\{ 60\, L_7 + L_8 - \phi_{secs} + \sin\phi \cos\phi \left[1052.893882 - \cos^2\phi\,(4.483344 - 0.023520 \cos^2\phi)\right] \right\} \quad (33)$$

$$R = L_3 + SL_5 \left\{ 1 + \left(\frac{S}{10^8}\right)^2 \cdot \left[L_9 - \frac{S}{10^8} L_{10} + \left(\frac{S}{10^8}\right)^2 L_{11} \right] \right\} \quad (34)$$

$$\theta_{secs} = L_6\,(L_2 - \lambda_{secs}) \quad (35)$$

$$X = R \sin\theta + L_1 \quad (36)$$

$$Y = L_4 - R + 2R \sin^2\frac{\theta}{2} \quad (37)$$

The algorithm for the inverse transform, or the conversion of Lambert state plane coordinates to geographic positions follows:

$$\tan\theta = \frac{X - L_1}{L_4 - Y} \quad (38)$$

$$\lambda_{secs} = L_2 - \frac{\theta\,secs}{L_6} \quad (39)$$

$$R = \frac{L_4 - Y}{\cos\theta} \quad (40)$$

$$y' = Y - 2R \sin^2\frac{\theta}{2} \quad (41)$$

$$S' = \frac{L_4 - L_3 - y'}{L_5} \quad (42)$$

$$S = \frac{S'}{1 + \left(\frac{S}{10^8}\right)^2 L_9 - \left(\frac{S}{10^8}\right)^3 L_{10} + \left(\frac{S}{10^8}\right)^4 L_{11}} \quad (43)$$

Iterate between equations (42) and (43) until S converges to the desired accuracy. The documented program in this publication considers $\Delta S \leq .01$ as the satisfactory convergence. For the first interation, substitute S' for S in the denominator of equation (43).

$$\omega_{secs} = 60\, L_7 + L_8 - 0.009873675553\, S \quad (44)$$

$$\phi_{secs} = \omega_{secs} + \sin\omega \cos\omega \left[1047.546710 + \cos^2\omega\,(6.192760 + 0.050912 \cos^2\omega)\right] \quad (45)$$

where:

L_i = State zone constants for i = 1, 11

L_1 = False Easting.

L_2 = Central meridian in seconds of longitude.

L_3 = Projection radius at the latitude of the origin.

L_4 = Projection radius at the latitude for which $Y = 0$.

L_5 = Scale reduction factor at the latitude of the origin.

L_6 = Projection constant.

L_7 = Minutes of latitude at zone origin.

L_8 = Seconds of latitude at zone origin.

L_9 = $1/6\, \rho_o^{\,2} \cdot 10^{16}$

L_{10} = $\tan\phi_o / 24\, \rho_o^{\,3} \cdot 10^{24}$

L_{11} = higher order term

The constants, L_i, are tabulated by Claire; see footnote 9.

STATE PLANE COORDINATES BASED ON THE OBLIQUE MERCATOR PROJECTION

The oblique Mercator projection, similar to other Mercator projections, may be visualized as a cylinder, tangent to the reference ellipsoid; but tangent along an oblique, noncardinal section of the ellipsoid rather than to a parallel or meridian. This projection is particularly well suited for narrow regions or zones whose primary axis is considerably skewed with respect to a cardinal direction. Alaska Zone 1, the southeastern portion of the state, is the only region in the United States employing this projection.

The center of projection for the Alaska Zone 1 grid system is defined as the geodesic passing through latitude = 57° N, and longitude = 133° 40' W, at azimuth $\tan^{-1}(-3/4)$.

The equations for the transformation of geographic positions to Alaska Zone 1 grid coordinates are:[12]

$$\mu = \ln \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \frac{e}{2} \ln\left(\frac{1 + e \sin\phi}{1 - e \sin\phi}\right) \quad (46)$$

$$\mu = \int_0^{\phi} \frac{R\,d\phi}{N\cos\phi} = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}}\right]$$

[12] Plane Coordinate Intersection Tables (2 1/2 - Minute), Alaska Zone 1, *Coast and Geodetic Survey Publication 65-1, Part 49*, 1961.

$$\tan(Bu/Ak_c) = \frac{\cos \alpha_o \sinh (B\mu + C) - \sin \alpha_o \sin B(\lambda - \lambda_o)}{\cos B(\lambda - \lambda_o)} \tag{47}$$

$$\tanh(Bv/Ak_c) = \frac{-\cos \alpha_o \sin B(\lambda - \lambda_o) - \sin \alpha_o \sinh (B\mu + C)}{\cosh (B\mu + C)} \tag{48}$$

$$X = -0.6\, u + 0.8 v + 5{,}000{,}000 \tag{49}$$

$$Y = 0.8\, u + 0.6 v - 5{,}000{,}000 \tag{50}$$

where:

ϕ = station latitude

λ = station longitude

X = state grid X-coordinate

Y = state grid Y-coordinate

e = ellipsoid eccentricity

Ak_c/B = 6,386,352.670 meters

$\sin \alpha_o$ = - 0.3270155172

$\cos \alpha_o$ = 0.9450189689

B = 1.0002997727

C = 0.00447599131

λ_o = 101° 30' 50".51319

Kc = scale factor = 0.9999

u, v = skewed coordinates

w

The inverse transformation from grid coordinates to geographic latitude and longitude follows:

$$u = -0.182880365761\, X + 0.243840487681\, Y + 700{,}000 \tag{51}$$

$$v = 0.243840487681\, X + 0.182880365761\, Y - 100{,}000 \tag{52}$$

$$\tanh(B\mu + C) = \frac{\cos\alpha_o \sin (Bu/Ak_c) - \sin\alpha_o \sinh (Bv/Ak_c)}{\cosh (Bv/Ak_c)} \tag{53}$$

$$\tan\left[B(\lambda - \lambda_o)\right] = \frac{-\cos\alpha_o \sinh (Bv/Ak_c) - \sin\alpha_o \sinh (Bu/Ak_c)}{\cos (Bu/Ak_c)} \tag{54}$$

A convenient algorithm for the inverse solution after computing the u, v skew coordinates is:

$$\mu = \frac{1}{2B} \ln \left[\frac{\cosh\left(\frac{Bv}{Ak_c}\right) - \sin \alpha_o \sinh\left(\frac{Bv}{Ak_c}\right) + \cos \alpha_o \sin\left(\frac{Bu}{Ak_c}\right)}{\cosh\left(\frac{Bv}{Ak_c}\right) + \sin \alpha_o \sinh\left(\frac{Bv}{Ak_c}\right) - \cos \alpha_o \sin\left(\frac{Bu}{Ak_c}\right)} \right] - \frac{C}{B} \tag{55}$$

$$X_R = 2 \tan^{-1} (e^{\mu}) - \frac{\pi}{2} \qquad \text{(radians)} \tag{56}$$

$$\phi_R = X_R + \sin X \cos X\, (0.006761032571 + 0.000053172205 \cos^2 X + 0.000000573027 \cos^4 X + 0.000000007128 \cos^6 X) \tag{57}$$

$$\phi_{secs} = 206{,}264.8062471\, \phi_R$$

$$\lambda_{secs} = \lambda_o + \frac{1}{B}\tan^{-1}\left[\frac{\sin\alpha_o \sin\left(\frac{Bu}{Ak_c}\right) - \cos\alpha_o \sinh\left(\frac{Bv}{Ak_c}\right)}{\cos\left(\frac{Bu}{Ak_c}\right)}\right] \quad (58)$$

THE UNIVERSAL TRANSVERSE MERCATOR GRID SYSTEM

The Universal Transverse Mercator (UTM) grid is primarily a military coordinate system in which the spheroid is divided into sixty equal bands of longitude. Each of these six-degree bands is assigned a zone number that can be related to the central meridian of the zone by the expression:

Central meridian = 6 x (Zone number) - 183

Alaska Zones 2 through 9 are special cases of the UTM, the main differences being the central meridian scale factor (see discussion in section *State Plane Coordinates Based on the Transverse Mercator Projection*) and the Y origin. The scale factor for the central meridian of the UTM system is 0.9996, whereas that of the Alaskan zones is 0.9999. The UTM system has its Y-axis origin on the equator where the origin for the Alaska Zones 2-9 Y-axis is almost six million meters north of the equator.

The equations for the transformation of geographic positions to UTM coordinates are as follows[13].

$$N_n = (\mathrm{I}) + (\mathrm{II})\,P^2 + (\mathrm{III})\,P^4 + A_6 \quad (59)$$

$$E = (\mathrm{IV})\,P + (\mathrm{V})\,P^3 + B_5 + FE \quad (60)$$

or

$$N_s = 10{,}000{,}000 - N_n \text{ for stations south of the equator.} \quad (61)$$

where:

$$P = 0.0001\Delta\lambda \text{ from central meridian} \quad (62)$$

$$v = \frac{a}{(1 - e^2\sin^2\phi)^{1/2}} \quad (63)$$

$$\mathrm{I} = 6{,}367{,}399.689 \text{ Scale}\left\{\phi_{\text{radians}} - \sin\phi\cos\phi\, 10^{-6}\left[5104.57388 - \cos^2\phi\,(21.73607 - 0.11422\cos^2\phi)\right]\right\} \quad (64)$$

$$\mathrm{II} = \frac{v\sin\phi\cos\phi\sin^2 1''}{2}\text{ Scale}\cdot 10^8 \quad (65)$$

$$\mathrm{III} = \frac{\sin^4 1''\, v\sin\phi\cos^3\phi}{24}\left(5 - \tan^2\phi + 9e'^2\cos^2\phi + 4e'^4\cos^4\phi\right)\cdot \text{ Scale}\cdot 10^{16} \quad (66)$$

$$\mathrm{IV} = v\cos\phi\sin 1''\cdot\text{Scale}\cdot 10^4 \quad (67)$$

$$\mathrm{V} = \frac{\sin^3 1''\, v\cos^3\phi}{6}\left(1 - \tan^2\phi + e'^2\cos^2\phi\right)\cdot \text{Scale}\cdot 10^{12} \quad (68)$$

$$A_6 = P^6\left(\frac{\sin^6 1''\, v\sin\phi\cos^5\phi}{720}\right)\left(61 - 58\tan^2\phi + \tan^4\phi + 270e'^2\cos^2\phi - 330e'^2\sin^2\phi\right)\cdot \text{Scale}\cdot 10^{24} \quad (69)$$

$$B_5 = P^5\left(\frac{\sin^5 1''\, v\cos^5\phi}{120}\right)\left(5 - 18\tan^2\phi + \tan^4\phi + 14e'^2\cos^2\phi - 58e'^2\sin^2\phi\right)\cdot \text{Scale}\cdot 10^{20} \quad (70)$$

[13]Universal Transverse Mercator Grid, *Department of the Army Technical Manual TM 5-241-8*, July, 1958. Latitude Functions Clarke 1880 Spheroid, *Department of the Army Technical Manual TM 5-241-20*, December, 1960.

ϕ' in all equations

where:

N = grid Northing coordinate

E = grid Easting coordinate

$$FE = \text{false easting} = \begin{cases} 500{,}000 \text{ meters for UTM} \\ 500{,}000 \text{ feet for Alaska zones } 2,3,4,5,6,8 \\ 700{,}000 \text{ feet for Alaska zone } 7 \\ 600{,}000 \text{ feet for Alaska zone } 9 \end{cases}$$

$$\text{Scale} = \begin{cases} 0.9999 \text{ for Alaska zones 2 through 9} \\ 0.9996 \text{ for UTM} \end{cases}$$

a = semimajor axis

b = semiminor axis

$$e' = \left(\frac{a^2 - b^2}{b^2}\right)^{1/2} = \left(\frac{e^2}{1 - e^2}\right)^{1/2}$$

The meridional arc for latitude 54°: 5,985,100.923 meters, is subtracted from N_n in equation (59) prior to the transformation computations for Alaska zones 2-9. All transformation computations are in terms of meters.

The equations for the inverse transformation of UTM grid coordinates to geographic positions are:

$$\phi = \phi' - (\text{VII})\, q^2 + (\text{VIII})\, q^4 - D_6 \qquad (71)$$

$$\Delta\lambda = (\text{IX})\, q - (\text{X})\, q^3 + E_5 \qquad (72)$$

$$\lambda = \lambda_{\text{central meridian}} + \Delta\lambda \qquad (73)$$

where:

$$q = 0.000001 \cdot (\text{grid distance from central meridian}) \qquad (74)$$

* $$\omega = \frac{0.1570499810 \cdot N \cdot 10^{-6}}{\text{Scale}} \text{ (in radians)} \qquad (75)$$

$$\phi' = \omega + \sin\omega \cos\omega\, 10^{-6} [5078.64977 + \cos^2\omega\,(30.02335 + 0.24682 \cos^2\omega)] \qquad (76)$$

$$\text{VII} = \frac{\tan\phi'}{2v^2 \sin 1''} (1 + e'^2 \cos^2\phi) \cdot (\text{Scale})^{-2} \cdot 10^{12} \qquad (77)$$

$$\text{VIII} = \frac{\tan\phi'}{24v^4 \sin 1''} (5 + 3\tan^2\phi + 6e'^2 \cos^2\phi - 6e'^2 \sin^2\phi - 3e'^4 \cos^4\phi - 9e'^4 \cos^2\phi \sin^2\phi) \cdot (\text{Scale})^{-4} \cdot 10^{24} \qquad (78)$$

$$\text{IX} = \frac{\sec\phi'}{v \sin 1''} \cdot (\text{Scale})^{-1} \cdot 10^6 \qquad (79)$$

$$\text{X} = \frac{\sec\phi'}{6v^3 \sin 1''} (1 + 2\tan^2\phi + e'^2 \cos^2\phi) \cdot (\text{Scale})^{-3} \cdot 10^{18} \qquad (80)$$

$$D_6 = q^6 \frac{\tan\phi'}{720v^6 \sin 1''} (61 + 90\tan^2\phi + 45\tan^4\phi + 107e'^2 \cos^2\phi - 162e'^2 \sin^2\phi - 45e'^2 \tan^2\phi \sin^2\phi) \cdot (\text{Scale})^{-6} \cdot 10^{36} \qquad (81)$$

$$E_5 = q^5 \frac{\sec\phi'}{120v^5 \sin 1''} (5 + 28\tan^2\phi + 24\tan^4\phi + 6e'^2 \cos^2\phi + 8e'^2 \sin^2\phi) \cdot (\text{Scale})^{-5} \cdot 10^{30} \qquad (82)$$

Constants appearing in equations (64), (75), and (76) are based on the parameters of the Clarke Spheroid of 1866. Reference is made to Claire (footnote 9) for the derivation of constants for other reference spheroids.

For Alaska zones 2-9, the latitude of the origin, 54° 00', is added to ϕ from equation (71).

THE GEOCENTRIC COORDINATE SYSTEM

The geocentric coordinate system is a right-hand coordinate system with its origin at the center of the reference ellipsoid. The X-Y axes form the plane of the ellipsoid equator, with the positive X- axis coincident with the Greenwich meridian. The positive Z- axis is in the direction of the North Pole.

* For Alaska zones 2-9 use
$\omega = \ldots [N + 5{,}985{,}100.923]$

The following equations transform geographic positions to X, Y, Z geocentric coordinates:

$$X = (N + h) \cos \phi \cos \lambda$$

$$Y = (N + h) \cos \phi \sin \lambda$$

$$Z = N(1 - e^2) \sin\phi + h \sin\phi \qquad (83)$$

The algorithm for the inverse transformation of geocentric coordinates to geographic positions is:

$$\lambda = \tan^{-1} Y/X \qquad (84)$$

Caution must be exercised to place the longitude into the proper quadrant.

To solve for ϕ and h, compute the reduced latitude of the station as the initial estimate of the geographic latitude:

$$\phi' = \tan^{-1} \left[Z/(X^2 + Y^2)^{1/2} \right] \qquad (85)$$

(for polar areas, invert the term in the brackets and use the cotangent function).

ϕ and h may then be computed by the recursive relationship:

$$h_{i+1} = (X^2 + Y^2 + Z^2)^{1/2} - a \left[1 - (1-e^2)e^2 \sin^2\phi_i \,/\, (1-e^2 \sin^2\phi_i) \right]^{1/2} \qquad (86)$$

$$\phi_{i+1} = \tan^{-1} \left[\frac{Z}{(X^2 + Y^2)^{1/2}} \cdot \frac{a^2 + ah_{i+1} (1-e^2 \sin^2\phi_i)^{1/2}}{b^2 + ah_{i+1} (1-e^2 \sin^2\phi_i)^{1/2}} \right] \qquad (87)$$

iterating between equations 86 and 87 until the desired accuracy in latitude is attained. The terms are defined as:

ϕ = station latitude

λ = station longitude

h = station elevation above reference ellipsoid

X = geocentric X coordinate

Y = geocentric Y coordinate

Z = geocentric Z coordinate

N = ellipsoid normal

a = ellipsoid major semidiameter

b = ellipsoid minor semidiameter

e = ellipsoid eccentricity

where west longitudes and south latitudes are considered negative.

THE FORTRAN PROGRAM

The coordinate transformation program was written in Fortran IV for the IBM 7030 (Stretch) computer located at the Naval Weapons Laboratory in Dahlgren, Va. Each program-accepted transformation is self-contained in a separate subroutine as an aid to reprograming for computers with a more limited core storage. The program accepted transformations include: secant plane-geographic position; state grids based on the transverse Mercator projection-geographic position; state grids based on the Lambert projection-geographic position; Alaska Zone 1 (oblique Mercator-geographic position; UTM and Alaska Zones 2 through 9 - geographic position; and geocentric-geographic position). Those familiar with the Fortran language should not encounter any problems in the event that this program must be adapted to fit other specific needs and equipment.

The flow diagram shown in figure 2 is presented to give the reader a general idea of the logic employed in the main calling program. The Fortran program itself (Appendix A) has been annotated to further assist the reader.

The Coast and Geodetic Survey strip adjustment and block adjustment phases of analytic aerotriangulation require that the ground control be expressed in a Cartesian coordinate system. Although the secant plane system fulfills this requirement and, in addition, accounts for earth curvature, it introduces a minor inconvenience to operational data handling. Normally, *only* latitude and longitude or state grid X and Y coordinates are known for horizontal control stations, and *only* elevation, or Z, is known for vertical control stations. The secant plane transformation computations require that both horizontal *and* vertical station data be known.

The following procedure is recommended for the determination of secant plane positions of partially known stations:

1. If the coordinate data is in geographic position format, transform to the appropriate state plane grid system.

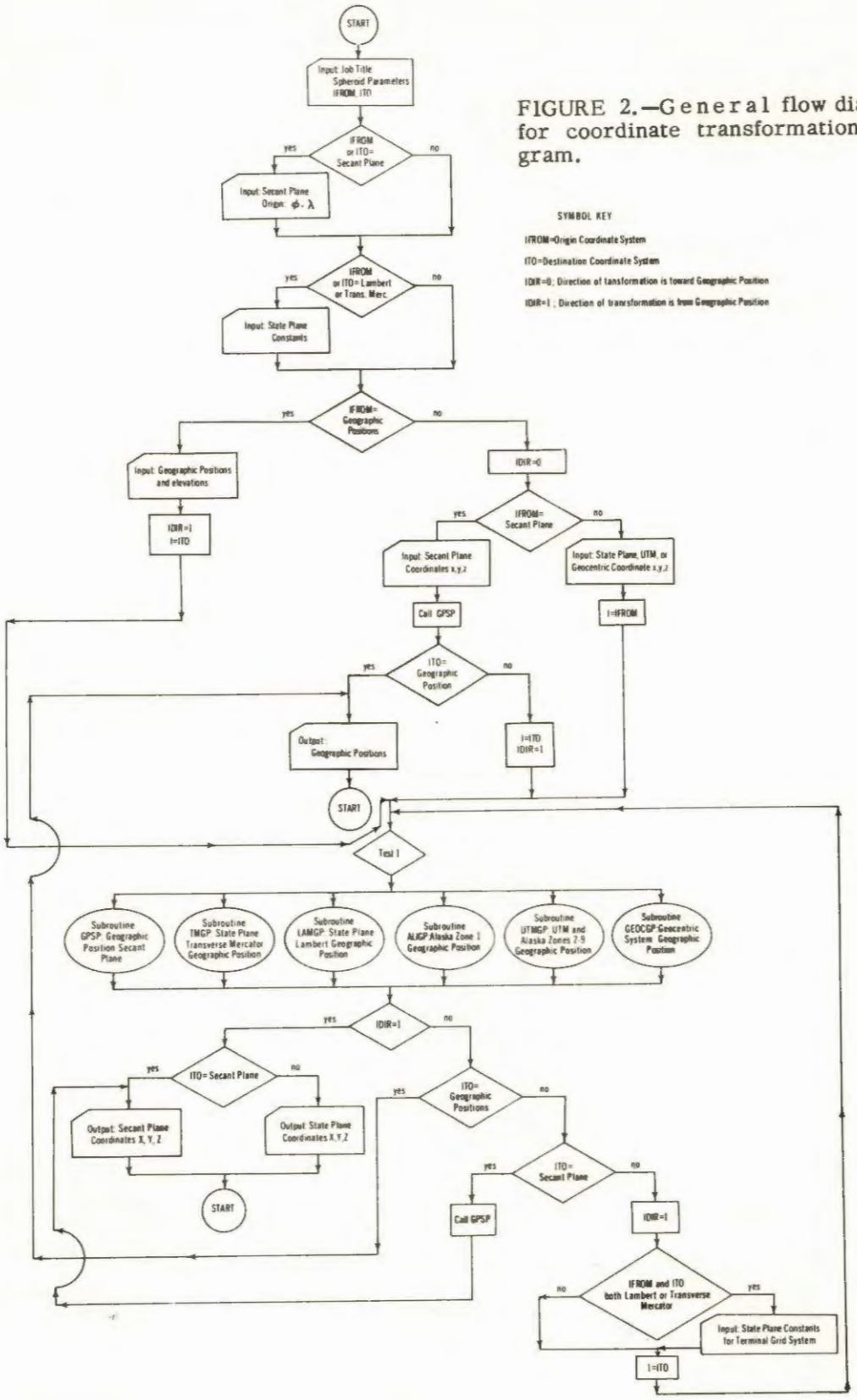

FIGURE 2.—General flow diagram for coordinate transformation program.

SYMBOL KEY

IFROM=Origin Coordinate System

ITO=Destination Coordinate System

IDIR=0; Direction of tansformation is toward Geographic Position

IDIR=1 ; Direction of transformation is from Geographic Position

2. Perform an initial strip or block adjustment with the ground control referenced to the state grid system. This step provides photogrammetric Z-coordinates for horizontal control points and photogrammetric X and Y coordinates for the vertical control points.

3. Transform the coordinates resulting from step 2 back to geographic positions.

4. Substitute the correct latitudes and longitudes in the horizontal control data, and the correct elevations in the vertical control data.

The above four steps provide photogrammetric elevations for the horizontal control, and photogrammetric latitudes and longitudes for the vertical control.

5. Finally, convert these data into the secant plane system used for a final more precise strip or block adjustment.

OPERATIONAL PROCEDURES

Both input and output formats for the coordinate transformation program are compatible with the published Strip Adjustment [14] and Block Adjustment [15] programs that are presently in operational use at the Coast and Geodetic Survey. The punched card program input is as follows:

Card 1.—Job title, project number, date, etc.

FORMAT (80 H).
This card contains 80 columns of alphameric data to identify the program output.

Card 2.—Origin and destination coordinate systems

FORMAT (2I5)
This card defines the two coordinate systems involved, IFROM and ITO, and the direction of transformation where IFROM equals the original system and ITO equals the final system. The following numbering key was used for the coordinate system identification.

ID Number	*System*
1	State plane grid systems based on the transverse Mercator projection.
2	State plane grid systems based on the Lambert projection.
3	Alaska Zone 1, based on the oblique Mercator projection.
4-11	Alaska Zones 2-9, based on the transverse Mercator projection. ID Number = Alaska zone number + 2.
12	Geographic position.
13	Secant plane.
14	Geocentric.
101-160	Universal Transverse Mercator Grid. ID Number = UTM zone number + 100.

For example; if the transformation from Alaska Zone 6 to secant plane coordinates is desired, the origin coordinate system identification is 8; and the destination coordinate system identification is 13.

Card 3.—Secant plane coordinate system origin latitude and longitude.

FORMAT (2 (F4.0, F2.0, F8.5, 6X))
The origin of the system is normally selected as an arbitrary point in the vicinity of the center of the project area. If ***neither*** IFROM ***nor*** ITO = 13, this card is omitted.

Cards 4, 5, and 6.—State plane grid systems constants

FORMAT (5E16.10)
The constants for each state zone are tabulated by Claire; see footnote 9.
If ***either*** IFROM ***or*** ITO defines a state plane grid system based on the transverse Mercator projection, only two cards are necessary to load the six constants. Card 6 is omitted. Three cards are necessary for the definition of the Lambert constants. If ***neither*** IFROM ***nor*** ITO designates coordinate systems that require definition of state plane constants, cards 4, 5, and 6 are omitted.

If ***both*** IFROM ***and*** ITO are coordinate systems requiring projection constants, a special case arises. Cards 4, 5, and, if necessary, 6 provide the state plane constants for the origin system as defined by IFROM. The constants for the destination coordinate system, ITO, immediately follow the station data.

The remainder of the input defines the identification number, horizontal position, and elevation of the stations whose coordinates are to be transformed. The input and output format for geographic position and elevation is:

FORMAT (1X, I6, 2X, I3, 1X, I2, 1X, F8.5, 5X, I4, 1X, I2, 1X, F8.5, 4X, F11.5, 19X, I1)

[14] Keller, M., and Tewinkel, G.C., see footnote 5.
[15] Keller, M., and Tewinkel, G.C., see footnote 6.

The input and output format for the station identification and the X, Y, and Z coordinates in the secant plane system is:

FORMAT (1X, I9, 3(3X, E14.8), 18X, I1),
where X, Y, and Z are expressed in meters.

The input and output format for the UTM, state plane, and geocentric coordinate systems is:

FORMAT (2X, I6, 3E14.8, 29X, I1)

Geocentric coordinates are expressed in meters, state plane coordinates in feet, and UTM coordinates in meters for X and Y. The Z coordinates in the UTM system are expressed in feet.

Vertical control data is not necessary as input for all transformations. Station elevations need be input only when either the origin or destination coordinate systems include either the secant plane or geocentric system.

Regardless of the type of coordinate input, a 1 punch must be placed in column 80 of the last station data card for each transformation to be performed. Additional transformation problems may be batch-processed in a single program run simply by repeating the outlined procedure from card 1.

Coordinates may be transformed between any two coordinate systems that have been defined as program acceptable. Sample input and output data listings of actual computer runs are shown in Appendix B.

Appendixes

APPENDIX A

FORTRAN IV PROGRAM

NOTE.—In this printout the small s is used for the equal to sign and the commas print as apostrophes.

```
B         JOB'K34-00B7   COORD TRANS SUBR  FOR BLOCK..UMBACH 6/14/67
*         *****     *****     *****     *****     *****     *****     ***
*         OPERATOR - THE RUNNING TIME SHOULD BE ABOUT 1 MINUTE          *
*         THIS PROGRAM IS TO BE CHARGED TO PROJECT NO 221104100
*         *****     *****     *****     *****     *****     *****     ***
B         TYPE'COMPILGO'FORTRAN'PM
T         SUBTYPE'FORTRAN'LMAP'PUNCH
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
      CALL SETIT
C SET ELLIPSOID PARAMETERS AND CONVERSION CONSTANTS
C FOR CLARKE 1866 SPHEROID
C
      ESQs.006768658
      As6378206.4
      PIs3.1415926535898
      FTMTs39.37/12.
      AMTFTs1./FTMT
      SECRDs648000./PI
      SNONSs1./SECRD
C READ AND PRINT JOB TITLE'ETC.
    1 PRINT 998
      READ 999
      PRINT 999
      PRINT  1020'ESQ'A
C READ AND PRINT ORIGIN AND DESTINATION COORDINATE SYSTEMS
      READ 1001'IFROM'ITO
      PRINT 1021'IFROM'ITO
      Ls0
C IF SECANT PLANE SYSTEM IS INVOLVED' READ AND PRINT THE LATITUDE AND
C LONGITUDE OF THE SYSTEM ORIGIN
      IF(IFROM.NE.13.AND.ITO.NE.13) GO TO 2
      READ 1000'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC
      PRINT 1022'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC
    2 IF(IFROM.EQ.1) GO TO 5
      IF(IFROM.EQ.2) GO TO 10
      IF(ITO.EQ.1) GO TO 5
      IF(ITO.EQ.2) GO TO 10
      GO TO 15
C READ AND PRINT STATE PLANE SYSTEMS CONSTANTS IF REQUIRED
    5 READ 1003'(T(M)'Ms1'6)
      PRINT 1023
      PRINT 1024'(T(M)'Ms1'6)
      IF(L.EQ.1) GO TO 20
      GO TO 15
   10 READ 1003'(T(M)'Ms1'11)
      PRINT 1023
      PRINT 1024'(T(M)'Ms1'11)
      IF(L.EQ.1) GO TO 20
```

```
   15 NNs0
      IF(IFROM.NE.12) GO TO 130
      PRINT 1026
   16 NNsNN+1
C READ AND PRINT  GEOGRAPHIC POSITION AND ELEVATION INPUT COORDINATES
      READ 1008'G(1'NN)'LAD'LAM'XS'LOD'LOM'YS'G(4'NN)'IEND
      PRINT 1008'G(1'NN)'LAD'LAM'XS'LOD'LOM'YS'G(4'NN)
C REDUCE GEOGRAPHIC POSITIONS TO SECONDS OF ARC
      IF(LAD.GT.0) GO TO 17
      LAMs-LAM
      XSs-XS
   17 G(2'NN)sLAD*3600+LAM*60
      G(2'NN)sG(2'NN)+XS
      IF(LOD.GT.0) GO TO 18
      LOMs-LOM
      YSs-YS
   18 G(3'NN)sLOD*3600+LOM*60
      G(3'NN)sG(3'NN)+YS
      IF(IEND.EQ.0) GO TO 16
   19 IsITO
      IDIRs1
C CALL APPROPRIATE TRANSFORMATION SUBROUTINE
   20 IF(IABS(I).GT.100) GO TO 80
      GO TO (50'60'70'80'80'80'80'80'80'80'80'1'90'100)'I
   50 CALL TMGP
      GO TO 110
   60 CALL LAMGP
      GO TO 110
   70 CALL AL1GP
      GO TO 110
   80 CALL UTMGP
      GO TO 110
   90 CALL GPSP
      GO TO 110
  100 CALL GEOCGP
  110 IF(IDIR.EQ.0) GO TO 160
      IF(ITO.EQ.13) GO TO 120
C PRINT STATE PLANE OUTPUT
      PRINT 1027
      PRINT 1016'((G(M'J)'Ms1'4)'Js1'NN)
      PUNCH 1016'((G(M'J)'Ms1'4)'Js1'NN)
      GO TO 900
C PRINT SECANT PLANE OUTPUT
  120 PRINT 1028
      PRINT 1017'((G(M'J)'Ms1'4)'Js1'NN)
      PUNCH 1017'((G(M'J)'Ms1'4)'Js1'NN)
      GO TO 900
  130 IDIRs0
      IF(IFROM.NE.13) GO TO 150
  132 PRINT 1029
  133 NNsNN+1
C READ AND PRINT SECANT PLANE COORDINATES IN METERS
      READ 1007'(G(M'NN)'Ms1'4)'IEND
      PRINT 1007'(G(M'NN)'Ms1'4)
      IF(IEND.EQ.0) GO TO 133
C TRANSFORM SECANT PLANE COORDINATES TO GEOGRAPHIC POSITIONS AND
C ELEVATIONS
      CALL GPSP
      IF(ITO.NE.12) GO TO 19
  135 PRINT 1030
```

```
C CONVERT SECONDS OF GEOGRAPHIC POSITIONS TO DEGREES'MINUTES'AND SECONDS
  136 DO 140 Js1'NN
      LADsG(2'J)/3600.
      XSsLAD
      G(2'J)sG(2'J)-3600.*XS
      LAMsG(2'J)/60.
      XSsLAM
      XSsG(2'J)-60.*XS
      LODsG(3'J)/3600.
      YSsLOD
      G(3'J)sG(3'J)-3600.*YS
      LOMsG(3'J)/60.
      YSsLOM
      YSsG(3'J)-60.*YS
      LOMsIABS(LOM)
      YSsABS(YS)
      LAMsIABS(LAM)
      XSsABS(XS)
C PRINT GEOGRAPHIC POSITION AND ELEVATION OUTPUT
      PRINT 1008'G(1'J)'LAD'LAM'XS'LOD'LOM'YS'G(4'J)
  140 PUNCH 1008'G(1'J)'LAD'LAM'XS'LOD'LOM'YS'G(4'J)
      GO TO 900
  150 PRINT 1025
  151 NNsNN+1
C READ AND PRINT INPUT STATE PLANE'UTM'OR GEOCENTRIC COORDINATES
      READ 1006'(G(M'NN)'Ms1'4)'IEND
      PRINT 1006'(G(M'NN)'Ms1'4)
      IF(IEND.EQ.0) GO TO 151
      IsIFROM
      GO TO 20
  160 IF(ITO.EQ.12) GO TO 135
      IF(ITO.NE.13) GO TO 170
      IDIRs1
      CALL GPSP
      GO TO 120
  170 IDIRs1
      IsITO
      Ls1
      KsIFROM+ITO
      IF(K.GT.3) GO TO 20
      GO TO (5'10)'ITO
C PRINT OUT PROGRAM RUN TIME *** SUBROUTINE TIME IS ON 7030 LIBRARY
  900 CALL TIME(CLOCK)
      PRINT 1009'CLOCK
      GO TO 1
  998 FORMAT(1H1)
C JOB TITLE'ETC.' FORMAT
  999 FORMAT(80H
     1                                )
C SECANT PLANE SYSTEM ORIGIN LATITUDE AND LONGITUDE FORMAT
 1000 FORMAT(2(F4.0'F2.0'F8.5'6X))
C ORIGIN AND DESTINATION SYSTEMS DEFINITION FORMAT
 1001 FORMAT(3I5)
C STATE PLANE CONSTANTS INPUT FORMAT
 1003 FORMAT(5E16.10)
C STATE PLANE'UTM'AND GEOCENTRIC COORDINATE INPUT FORMAT
 1006 FORMAT(2X'I6'3E14.8'29X'I1)
C SECANT PLANE COORDINATE INPUT FORMAT
 1007 FORMAT(1X'I9'3(3X'E14.8)'18X'I1)
C GEOGRAPHIC POSITION AND ELEVATION INPUT FORMAT
 1008 FORMAT(1X'I6'2X'I3'1X'I2'1X'F8.5'5X'I4'1X'I2'1X'F8.5'4X'F11.5'
     1        19X'I1)
 1009 FORMAT(////22H PROGRAM RUN TIME WAS 'F7.2'8H SECONDS///)
C STATE PLANE'UTM'OR GEOCENTRIC COORDINATE OUTPUT FORMAT
 1016 FORMAT(2X'I6'3E14.8)
C SECANT PLANE COORDINATE OUTPUT FORMAT
 1017 FORMAT(1X'I9'3X'E14.8'3X'E14.8'3X'E14.8)
```

```
C GEOGRAPHIC POSITION AND ELEVATION OUTPUT FORMAT
 1018 FORMAT(1X'I6'2X'I3'1X'I2'1X'F8.5'5X'I4'1X'I2'1X'F8.5'4X'F11.5)
 1020 FORMAT(/16H ECCENTRICITY s 'E14.8'23H MAJOR SEMI-DIAMETER s '
     1         E14.8/)
 1021 FORMAT(19H ORIGINAL SYSTEM s 'I3'16H FINAL SYSTEM s 'I3/)
 1022 FORMAT(20H SECANT ORIGIN LAT  'I3'1X'I2'F8.5'6H LONG 'I4'1X'I2'1X'
     1        F8.5/)
 1023 FORMAT(/22H STATE PLANE CONSTANTS/)
 1024 FORMAT(1X'E16.10)
 1025 FORMAT(///48H STATE PLANE'GEOCENTRIC'OR UTM INPUT COORDINATES/)
 1026 FORMAT(///40H GEOGRAPHIC POSITION AND ELEVATION INPUT/)
 1027 FORMAT(///49H STATE PLANE'GEOCENTRIC'OR UTM OUTPUT COORDINATES/)
 1028 FORMAT(///31H SECANT PLANE COORDINATE OUTPUT/)
 1029 FORMAT(///30H SECANT PLANE COORDINATE INPUT/)
 1030 FORMAT(///41H GEOGRAPHIC POSITION AND ELEVATION OUTPUT/)
      END
T        SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE GPSP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
      DATA PI(3.1415926535898)
C
C TRANSFORMATION AND INVERSE FOR GEOGRAPHIC POSITIONS AND SECANT PLANE
C
C SOLVE FOR THE TRANSFORMATION CONSTANTS DEPENDENT ON ELLIPSOID
C PARAMETERS AND SECANT PLANE SYSTEM ORIGIN
C
      ANMEs1.-ESQ
      IF(PHDG)3'4'4
    3 SGLATs-1.
      PHMINs-PHMIN
      PHSECs-PHSEC
      GO TO 11
    4 SGLATs1.0
   11 PHRADs(PHDG*3600.+PHMIN*60.+PHSEC)*SNONS
      IF(PLGDG)5'6'6
    5 SGLNGs-1.0
      PLGMNs-PLGMN
      PLSECs-PLSEC
      QNT1s0.
      QNT2s-PI
      QNT3s-PI/2.
      GO TO 13
    6 SGLNGs1.0
      QNT1sPI
      QNT2s0.
      QNT3sPI/2.
   13 PLRADs(PLGDG*3600.+PLGMN*60.+PLSEC)*SNONS
      SNPHOsSIN(PHRAD)
      CSPHOsCOS(PHRAD)
      SNLGOsSIN(PLRAD)
      CSLGOsCOS(PLRAD)
      EPHMsSQRT(1.-ESQ*SNPHO**2)
C COMPUTE LENGTH OF NORMAL THROUGH SYSTEM ORIGIN
      ENOsA/EPHM
      ENSYTsENO*ESQ*SNPHO
      SNPLsSNPHO*SNLGO
      CSPLsCSPHO*SNLGO
      SCPLAsSNPHO*CSLGO
      CSPLAsCSPHO*CSLGO
C COMPUTE MAGNITUDE OF TRANSLATION FOR X-Y PLANE
      IBASEsENO/1000.
      BASEsIBASE*1000
      IF(IDIR.EQ.1) GO TO 99
```

```
C SECANT PLANE TO GEOGRAPHIC POSITION
C
   50 DO 98 Ns1'NN
C TRANSFORM SECANT PLANE COORDINATES TO MODIFIED GEOCENTRIC COORDS'
C I.E.'Zs0 AT INTERSECTION OF NORMAL WITH MINOR AXIS
      ZPBsG(4'N)+BASE
      XsG(2'N)*CSLGO-G(3'N)*SNPL+ZPB*CSPL
      Ys-(G(2'N)*SNLGO+G(3'N)*SCPLA)+ZPB*CSPLA
      ZsG(3'N)*CSPHO+ZPB*SNPHO
C COMPUTE LATITUDE BY ITERATIVE PROCESS
      ZAPPsZ
      DEMsSQRT(X*X+Y*Y)
      DO 80 IXs1'5
      TNPHPsZ/DEM
      PHRsATAN(TNPHP)
      SNPHPsSIN(PHR)
      CSPHPsCOS(PHR)
      ENPHsSQRT(1.-ESQ*SNPHP**2)
      ENPTsA/ENPH
   80 Zs-ENSYT+(ENPT*ESQ*SNPHP)+ZAPP
C COMPUTE STATION LONGITUDE
   81 ALAMsATAN(X/Y)
      IF(ALAM)74'75'76
   74 ANGLEsALAM+QNT1
      GO TO 77
   75 ANGLEsQNT3
      GO TO 77
   76 ANGLEsALAM+QNT2
   77 G(3'N)sANGLE*SECRD
C COMPUTE STATION ELEVATION
      SINLsSIN(ANGLE)
      Hs(X/(CSPHP*SINL))-ENPT
      G(4'N)sH*FTMT
      G(2'N)sPHR*SECRD
   98 CONTINUE
      RETURN
C
C GEOGRAPHIC POSITION TO SECANT PLANE
C
   99 DO 199 Ns1'NN
      ELEVsG(4'N)
C CONVERT LATITUDE AND LONGITUDE TO RADIANS
      PPRADsG(2'N)*SNONS
      PLGRDsG(3'N)*SNONS
      SNPHIsSIN(PPRAD)
      CSPHIsCOS(PPRAD)
      SNLAMsSIN(PLGRD)
      CSLAMsCOS(PLGRD)
      HsG(4'N)*AMTFT
      AMSQsSQRT(1.-ESQ*SNPHI**2)
C COMPUTE NORMAL OF STATION
      RNsA/AMSQ
C COMPUTE CLASSICAL GEOCENTRIC COORDINATES
      SNHsRN+H
      XsSNLAM*CSPHI*SNH
      YsCSLAM*CSPHI*SNH
      Zs((RN*ANME+H)*SNPHI)+ENSYT
C COMPUTE SECANT PLANE COORDINATES
      G(2'N)sX*CSLGO-Y*SNLGO
      G(3'N)sZ*CSPHO-(X*SNPL+Y*SCPLA)
      G(4'N)sX*CSPL+Y*CSPLA+Z*SNPHO-BASE
  199 CONTINUE
  200 RETURN
      END
```

```
T         SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE TMGP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
C
C TRANSFORMATION AND INVERSE FOR TRANSVERSE MERCATOR AND
C                GEOGRAPHIC POSITION
C
C REFERENCE**SEE EQUATIONS 19 THROUGH 32 IN SECTION 3 OF MAIN TEXT
C
      IF(IDIR.EQ.1) GO TO 200
C
C TRANSVERSE MERCATOR TO GEOGRAPHIC POSITION
C
      DO 150 Ns1'NN
      XPRsG(2'N)-T(1)
      SG sXPR-T(6)*((XPR*1.0E-5)**3)
      SGsXPR-T(6)*((SG*1.0E-5)**3)
      SMsAMTFT*SG/T(5)
      Ws(60.*T(3))+T(4)+(.009873675553*G(3'N)/T(5))
      RsW/SECRD
      COSWsCOS(R)
      PHIPRsW+((SIN(R)*COSW)*(1047.54671+COSW*COSW*(6.19276 +
     1       .050912*COSW*COSW)))
      RsPHIPR/SECRD
      DELPHIs(25.52381*1.0E-10)*(SM*SM*SIN(R)/COS(R))*
     1        ((1.0-(ESQ*SIN(R)**2))**2)
      G(2'N)sPHIPR-DELPHI
      PHRsG(2'N)*SNONS
      SAsSM-(4.0831*((SM*1.0E-5)**3))
      SAsSM-(4.0831*((SA*1.0E-5)**3))
      DLAM1sSA*SQRT(1.-ESQ*SIN(PHR  )*SIN(PHR  ))/(30.92241724*
     1       COS(PHR  ))
      DLAMAsDLAM1+3.9174*((DLAM1*1.0E-4)**3)
      DLAMsDLAM1+3.9174*((DLAMA*1.0E-4)**3)
  150 G(3'N)sDLAM-T(2)
      RETURN
C
C GEOGRAPHIC POSITION TO TRANSVERSE MERCATOR
C
C REFERENCE**SEE EQUATIONS 8 THROUGH 18 IN SECTION 3 OF MAIN TEXT
C
  200 DO 250 Ns1'NN
      G(3'N)s-G(3'N)
      PHIsG(2'N)
      ANSECsG(3'N)
      PHRsG(2'N)*SNONS
      DLAMsT(2)-ANSEC
      DLAMsDLAM-3.9174*((DLAM*1.0E-4)**3)
      Rs1.-(ESQ*SIN(PHR)*SIN(PHR))
      SGs30.92241724*COS(PHR)*DLAM/SQRT(R)
      SMsSG+4.0831*((SG*1.0E-5)**3)
      SGsFTMT*T(5)*SM
      XsSG+(T(6)*((SG*1.0E-5)**3))
      G(2'N)sT(1)+X
C ITERATIVE SOLUTION FOR LATITUDE
      DO 210 JXs1'100
      Bs25.52381E-10*((1.-(ESQ*SIN(PHR)*SIN(PHR)))**2)*SIN(PHR)/COS(PHR)
      DELPHIsB*SM*SM
      PHIPRsPHI  +DELPHI
      IF(ABS(DELPHI)-1.0E-6)215'215'205
  205 PHRsPHIPR*SNONS
  210 CONTINUE
  215 COSPsCOS(PHR)
      Rs4.483344-.023520*COSP*COSP
      RsSIN(PHR  )*COSP*(1052.893882-(COSP*COSP*R))
  250 G( 3'N )s101.2794065*T(5)*(PHIPR-60.*T(3)-T(4)-R)
      RETURN
      END
```

```
T         SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE LAMGP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
C
C TRANSFORMATION AND INVERSE FOR LAMBERT AND GEOGRAPHIC POSITION
C
      IF(IDIR.EQ.1) GO TO 400
C
C LAMBERT TO GEOGRAPHIC POSITION
C
C REFERENCE**SEE EQUATIONS 38 THROUGH 43 IN SECTION 4 OF MAIN TEXT
C
      DO 350 Ns1'NN
      YsG(3'N)
      Fs(G(2'N)-T(1))/(T(4)-G(3'N))
      FsATAN(F)
      BsF*SECRD
      G( 3'N )sT(2)-(B/T(6))
      G(3'N)s-G(3'N)
      Rs(T(4)-Y)/COS(F)
      FsSIN(F/2.)**2
      FsY-2.*R*F
      SAs(T(4)-T(3)-F)/T(5)
      S1sSA
C ITERATIVE SOLUTION FOR S' SEE EQUATION 43
      DO 320 IXs1'100
      BsS1*1.0E-8
      Bs1.+B*B*T(9)-B*B*B*T(10)+B*B*B*B*T(11)
      SsSA/B
      IF(ABS(S-S1)-0.01)330'330'315
  315 S1sS
  320 CONTINUE
  330 Bs60.*T(7)+T(8)-.00987367553*S
      RsB/SECRD
      COSPsCOS(R)
      PHIPRs6.192760+.050912*COSP*COSP
      PHIPRs1047.54671+COSP*COSP*PHIPR
  350 G( 2'N )sB+SIN(R)*COSP*PHIPR
      RETURN
C
C GEOGRAPHIC POSITION TO LAMBERT
C
C REFERENCE** SEE EQUATIONS 33 THROUGH 37 IN SECTION 4 OF MAIN TEXT
C
  400 DO 450 Ns1'NN
      G(3'N)s-G(3'N)
      PHRsG(2'N)*SNONS
      COSPsCOS(PHR)
      Ss4.483344-.02352*COSP*COSP
      Ss1052.893882-COSP*COSP*S
      Ss101.2794065*(60.*T(7)+T(8)-G( 2'N )+SIN(PHR)*COSP*S)
      BsS*1.0E-8
      RsT(3)+(S*T(5)*(1.+(B*B*(T(9)-B*T(10)+B*B*T(11)))))
      FsT(6)*(T(2)-G(3'N))
      FsF*SNONS
      G( 2'N )s(R*SIN(F))+T(1)
      G( 3'N )sT(4)-R+(2.*R*(SIN(F/2.)**2))
  450 CONTINUE
      RETURN
      END
```

```
T         SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE AL1GP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SFCRD'ESQ'A'SNONS'ITO
      DATA D(6386352.67013)'C(.00447599131)'B(1.00029977273)'
     1  Z(.945018968871)'F(.327015517176)'FLAM(1.771754085630)'
     2  E(.822718542E-1)'BM1(.0002997727)'ALORG(365450.51319)'
     3  EC(1.0044860235)'AKB(6386352.67)'BASE(5.0E+06)'
     4  SNALZ(.3270155172)'CSALZ (.9450189689)'TNALZ(.3460412203)'
     5  PI(3.1415926535898)
C
C TRANSFORMATION AND INVERSE FOR ALASKA ZONE 1 AND
C                 GEOGRAPHIC POSITION
C
C REFERENCE** SEE EQUATIONS 51 THROUGH 58 IN SECTION 5 OF MAIN TEXT
C
      IF(IDIR.EQ.1) GO TO 600
C
C ALASKA ZONE 1 TO GEOGRAPHIC POSITION
C
      DO 550 Ns1'NN
      Us(-.182880365761*G( 2'N ))+(.243840487681*G( 3'N ))+7.0E+6
      Vs(.243840487681*G( 2'N ))+(.182880365761*G( 3'N ))-1.0E+6
      VsV/D
      VsEXP(V)
      VMs1./V
      UODsU/D
      UsSIN(UOD)
      Rs(V-VM)/2.
      Ss(V+VM)/2.
      Us(.5/B)*(ALOG((S+F*R+Z*U)/(S-F*R-Z*U)))-(C/B)
      Xs2.*ATAN(EXP(U))-(.5*PI)
      SXsSIN(X)
      CXsCOS(X)
      XsX+SX*CX*(.006761032571+.000053172205*CX*CX+
     1  .000000573027*(CX**4)+.000000007128*(CX**6))
      G(2'N)sX*SECRD
      UsSIN(UOD)
  550 G( 3'N )s(FLAM+(1./B)*ATAN((F*U-Z*R)/COS(UOD)))*(-SECRD)
      RETURN
C
C GEOGRAPHIC POSITION TO ALASKA ZONE 1
C
C REFERENCE**SEE EQUATIONS 46 THROUGH 50 IN SECTION 5 OF MAIN TEXT
C
  600 DO 650 Ns1'NN
      G(3'N)s-G(3'N)
      PHRsG(2'N)*SNONS
      SNLTsSIN(PHR)
      EOV2sE/2.
      ESLTsE*SNLT
      SMLATsPHR*.5+(PI/4.)
      TAUs(SIN(SMLAT)/COS(SMLAT))*((1.-ESLT)/(1.+ESLT))**EOV2
      EBMTCsEC*TAU*TAU**BM1
      REBMTCs1./EBMTC
      SINHs(EBMTC-REBMTC)*.5
      COSHs(EBMTC+REBMTC)*.5
      BLLOsB*(G( 3'N )-ALORG)
      BLLRsBLLO*SNONS
      SBLLOsSIN(BLLR)
      CBLLOsCOS(BLLR)
      TNBTs(SINH*CSALZ+SBLLO*SNALZ)/CBLLO
      ANGsATAN(TNBT)
      UsANG*AKB
      TNs(-CSALZ*SBLLO+SNALZ*SINH)/COSH
      TANHsTN+TN**3/3. + TN**5/5. + TN**7/7. + TN**9/9.
      VsTANH*AKB
      XMs-U*.6+V*.8+BASE
      YMsU*.8+V*.6-BASE
      G( 2'N )sXM*FTMT
```

```
  650 G( 3'N )sYM*FTMT
  699 RETURN
      END
T         SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE UTMGP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1        AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
      DATA CONS1(.24682)'CONS2(30.02335)'CONS3(5078.64977)'
     1      CONS4(.1570499810)'SEPQ(.0408887094)'Y1(.11422)'Y2(21.73607)'
     2      Y3(5104.57388)'Y4(6367399.689)'CNII(.2424068406E-1)'
     3      CNIII(.0195870255E-2)'CNV(.0391740509E-2)'
     4      CNA6(.0153460626E-6)'CNB5(.0460381879E-6)'CNI3(7.83481018)'
     5      CNC5(.36830550E-02)
C                                                                       ,
C
C TRANSFORMATION AND INVERSE FOR UNIVERSAL TRANSVERSE MERCATOR AND
C                 ALASKA ZONES 2 THROUGH 9 TO GEOGRAPHIC POSITION
C
C THE CONSTANTS ABOVE ARE APPLICABLE ONLY TO THE CLARKE 1866 SPHEROID
      IF(IABS(IFROM).LE.14.AND.IDIR.EQ.0) GO TO 703
      IF(IABS(IFROM).GT.14.AND.IDIR.EQ.0) GO TO 700
      IF(IABS(ITO).LE.14.AND.IDIR.EQ.1) GO TO 704
      IsITO
      GO TO 701
  700 IsIFROM
C SET SCALE AND ORIGIN PARAMETERS FOR UTM SYSTEM
  701  ALORGs0.
      SCALEs.9996
      FEs500000.*FTMT
      IXsIABS(  I  )-100
      XsIX
      CMs(6.*X-183.)*3600.
      GO TO 706
  703 IsIFROM
      GO TO 705
  704 IsITO
C SET SCALE AND ORIGIN PARAMETERS FOR ALASKA ZONES 2 THROUGH 9
  705 ALORGs5985100.923
      SCALEs.9999
      IXsI-2
      FEs500000.
      IF(IX.EQ.7)FEs700000.
      IF(IX.EQ.9)FEs600000.
      XsIX
      CMs482400.+X*14400.
  706 EPSQsESQ/(1.-ESQ)
      IF(IDIR.EQ.1) GO TO 800
C
C UTM AND ALASKA ZONES 2 - 9 TO GEOGRAPHIC POSITION
C
C REFERENCE**SEE EQUATIONS 71 THROUGH 82 IN SECTION  6 OF MAIN TEXT
C
      DO 750 Ns1'NN
      IF(  I  .LE.0) G(3'N)s10000000.-G(3'N)
C CONVERT FROM METERS TO FEET IF UTM CONVERSION
      IF(IABS(  I  ).LE.14)  GO TO 702
      G(2'N)sFTMT*G(2'N)
      G(3'N)sFTMT*G(3'N)
  702 PPRDs((G( 3'N )*AMTFT+ALORG)*CONS4*10.0E-7)/SCALE
      SNLTsSIN(PPRD)
      CSLTsCOS(PPRD)
      CSSQsCSLT*CSLT
      PHRDs((CONS1*CSSQ+CONS2)*CSSQ+CONS3)*(SNLT*CSLT)*10.0E-7+PPRD
      PHISsPHRD*SECRD
      Qs((G( 2'N )-FE)*AMTFT)*10.0E-7
      IF(Q)720'710'720
```

```
  710 DLAMs0.
      G(2'N)sPHIS
      GO TO 740
  720 SNLATsSIN(PHRD)
      CSLATsCOS(PHRD)
      SNSQsSNLAT*SNLAT
      CCSQsCSLAT*CSLAT
      TNLATsSNLAT/CSLAT
      TNSQsTNLAT*TNLAT
      ENUsA/SQRT(1.-ESQ*SNSQ)
      ENSNSsENU*SNONS
      EPCSsEPSQ*CCSQ
      EPCSQs1.+EPCS
      QSQsQ*Q
      QCUsQSQ*Q
      QFRsQCU*Q
      QFVsQFR*Q
      QSXsQFV*Q
      SCLATs1./CSLAT
      ENSN5sENU**4*ENSNS
      SVNs(((TNLAT/(2.*ENU*ENSNS))*EPCSQ)/SCALE**2)*10.E11
      EGHTs((TNLAT/(24.*ENU**3*ENSNS))*(5.+3.*TNSQ+SEPQ*(CCSQ-SNSQ)
     1      -(3.*EPSQ**2*CCSQ)*(CCSQ+3.*SNSQ))/SCALE**4)*10.E23
      D6s(QSX*(TNLAT/(720.*ENU**5*ENSNS))*(61.+(45.*TNSQ)*(2.+TNSQ-
     1   EPSQ*SNSQ)+EPSQ*(107.*CCSQ-162.*SNSQ))/SCALE**6)*10.0E35
      ANINEs(SCLAT/ENSNS)/SCALE *10.0E5
      TENs(SCLAT/(6.*ENU**2*ENSNS))*(1.+2.*TNSQ+EPCS)/SCALE**3*10.E17
      E5sQFV*(SCLAT/(120.*ENSN5))*(5.+(4.*TNSQ)*(7.+6.*TNSQ)
     1   +(2.*EPSQ)*(3.*CCSQ+4.*SNSQ))/SCALE**5*10.0E29
      G( 2'N )sPHIS-SVN*QSQ+EGHT*QFR-D6
      IF(IFROM.LT.0) G(2'N)s-G(2'N)
      DLAMsANINE*Q-TEN*QCU+E5
  740 G( 3'N )sCM-DLAM
      IF(IABS(IFROM).GE.15) G(3'N)sCM+DLAM
      IF(IABS(IFROM).LE.11) G(3'N)s-G(3'N)
  750 CONTINUE
      RETURN
C
C GEOGRAPHIC POSITION TO ALASKA ZONES 2 - 9 AND UTM
C
C REFERENCE**SEE EQUATIONS 59 THROUGH 70 IN SECTION 6 OF MAIN TEXT
C
  800 DO 850 Ns1'NN
      IF(IABS( ITO ).LE.11) G(3'N)s-G(3'N)
      PHRDsG(2'N)*SNONS
      SNLATsSIN(PHRD)
      CSLATsCOS(PHRD)
      SNSQsSNLAT*SNLAT
      CCSQsCSLAT*CSLAT
      TNLATsSNLAT/CSLAT
      TNSQsTNLAT*TNLAT
      ENUsA/SQRT(1.-ESQ*SNSQ)
      ENSNSsENU*SNONS
      EPCSsEPSQ*CCSQ
      EPCSQs1.+EPCS
      WONs(Y4*(PHRD-(SNLAT*CSLAT*10.E-7)*(Y3-CCSQ*(Y2-Y1*CCSQ))))*SCALE
      Ps(CM-G( 3'N ))*10.E-5
      IF(IABS( ITO ).GE.15) Ps-P
      IF(P)819'820'819
  820 G(2'N)sFE
      G( 3'N )s(WON-ALORG)*FTMT
      GO TO 825
```

```
  819 PSQsP*P
      PCUsPSQ*P
      PFRsPCU*P
      PFVsPFR*P
      PSXsPFV*P
      FOURsENSNS*CSLAT*SCALE*10.E3
      TWOsFOUR*SNLAT*CNII
      EPCTNsEPCSQ-TNSQ
      THREsTWO*CNIII*CCSQ*(4.*EPCSQ**2+EPCTN)
      FIVEsFOUR*CNV*CCSQ*EPCTN
      CSFRsCCSQ*CCSQ
      CSFRRs1./CSFR
      A6sPSX*TWO*CNA6*CSFR*(60.*EPCTN+CSFRR+540.*EPCS-330.*EPSQ)
      B5sPFV*FOUR*CNB5*CSFR*(20.*EPCTN+CSFRR+52.*EPCS-58.*EPSQ-16.)
      G( 2'N )s(P*FOUR+PCU*FIVE+B5)*FTMT+FE
      G( 3'N )s((PSQ*TWO+PFR*THRE+WON+A6)-ALORG)*FTMT
C CONVERT FEET TO METERS IF UTM CONVERSION
  825 IF(IABS( ITO ).LE.14) GO TO 850
      G(2'N)sG(2'N)*AMTFT
      G(3'N)sG(3'N)*AMTFT
  850 IF( ITO .LE.0) G(3'N)s10000000.-G(3'N)
      RETURN
      END
T        SUBTYPE'FORTRAN'LMAP'PUNCH
      SUBROUTINE GEOCGP
      DIMENSION G(4'2000)'T(11)
      COMMON G'T'PHDG'PHMIN'PHSEC'PLGDG'PLGMN'PLSEC'IFROM'IDIR'NN'
     1       AMTFT'FTMT'SECRD'ESQ'A'SNONS'ITO
C
C TRANSFORMATION AND INVERSE FOR GEOGRAPHIC POSITIONS AND
C RIGHT-HAND GEOCENTRIC SYSTEM IN METERS
C
      PIs3.1415926535898
      IF(IDIR.EQ.1) GO TO 800
C
C GEOCENTRIC TO GEOGRAPHIC POSITION
C
C REFERENCE**SEE EQUATIONS 84 THROUGH 87 IN SECTION 7 OF MAIN TEXT
C
      DO 750 Ns1'NN
      XsG(3'N)/G(2'N)
      IF(X.LT.10.) GO TO 710
      FLONGsATAN(G(2'N)/G(3'N))
      Xs0.5*PI-ABS(FLONG)
      IF(FLONG.LT.0.) FLONGs-X
      IF(FLONG.GE.0.) FLONGsX
      GO TO 715
  710 FLONGsATAN(X)
      IF(G(2'N).LT.0.0.AND.G(3'N).GT.0.0) FLONGsPI+FLONG
      IF(G(2'N).LT.0.0.AND.G(3'N).LT.0.0) FLONGs-PI+FLONG
  715 PHIESTsATAN(G(4'N)/SQRT(G(2'N)**2+G(3'N)**2))
      DO 720 Is1'10
      IF(I.EQ.10) PRINT 1000
      SsSIN(PHIEST)**2
      TEMPsA*SQRT(1.-((ESQ*S*(1.-ESQ))/(1.-ESQ*S)))
      HsSQRT(G(2'N)**2+G(3'N)**2+G(4'N)**2) - TEMP
      BSQsA*A*(1.-ESQ)
      TEMPsBSQ+A*H*SQRT(1.-ESQ*S)
      TEMPs(A*A+A*H*SQRT(1.-ESQ*S))/TEMP
      PHIsATAN(G(4'N)*TEMP/SQRT(G(2'N)**2+G(3'N)**2))
      IF(ABS(PHIEST-PHI).LT.1.E-15) GO TO 730
      PHIESTsPHI
  720 CONTINUE
  730 G(2'N)sPHI*SECRD
      G(3'N)sFLONG*SECRD
  750 G(4'N)sH*FTMT
      RETURN
```

```
C
C GEOGRAPHIC POSITION TO GEOCENTRIC
C
C REFERENCE**SEE EQUATIONS 83 IN SECTION 7 OF MAIN TEXT
C
  800 DO 850 Ns1'NN
      HsG(4'N)*AMTFT
      SPsSIN(G(2'N)*SNONS)
      CPsCOS(G(2'N)*SNONS)
      SLsSIN(G(3'N)*SNONS)
      CLsCOS(G(3'N)*SNONS)
      DsSQRT(1.-ESQ*SP*SP)
      G(2'N)s(A*CP*CL/D)+H*CP*CL
      G(3'N)s(A*CP*SL/D)+H*CP*SL
  850 G(4'N)s(A*(1.-ESQ)*SP/D)+H*SP
      RETURN
 1000 FORMAT(//36H 10 ITERATIONS ON LATITUDE NECESSARY//)
      END
T        SUBTYPE'DATA
```

APPENDIX B

SAMPLE PROGRAM INPUT AND OUTPUT LISTINGS

INPUT

```
TEST DATA FOR GEOGRAPHIC POSITION TO SECANT PLANE TRANSFORMATION
  12   13
+4119                  -8358
  1101   +41 20 51.75196        -84 00 31.65292        681.19943
  1102   +41 20 52.75196        -84 00 32.65292        682.19943
  1103   +41 20 53.75196        -84 00 33.65292        683.19943
  1104   +41 20 54.75196        -84 00 34.65292        684.19943
  1105   +41 20 55.75196        -84 00 35.65292        685.19943
```

OUTPUT

```
TEST DATA FOR GEOGRAPHIC POSITION TO SECANT PLANE TRANSFORMATION
ECCENTRICITY =  .67686580E-02 MAJOR SEMI-DIAMETER =  .63782064E+07
ORIGINAL SYSTEM =  12 FINAL SYSTEM =  13
SECANT ORIGIN LAT   41 19 -.00000 LONG  -83 58  -.00000

GEOGRAPHIC POSITION AND ELEVATION INPUT
  1101   41 20 51.75196        -84  0 31.65292        681.19943
  1102   41 20 52.75196        -84  0 32.65292        682.19943
  1103   41 20 53.75196        -84  0 33.65292        683.19943
  1104   41 20 54.75196        -84  0 34.65292        684.19943
  1105   41 20 55.75196        -84  0 35.65292        685.19943

SECANT PLANE COORDINATE OUTPUT      ( in meters)
     1101   -.35257815E+04    .34484741E+04    .84210824E+03
     1102   -.35490156E+04    .34793363E+04    .84238337E+03
     1103   -.35722495E+04    .35101986E+04    .84265827E+03
     1104   -.35954832E+04    .35410610E+04    .84293294E+03
     1105   -.36187167E+04    .35719234E+04    .84320737E+03

PROGRAM RUN TIME WAS     0.39 SECONDS
```

(elevations in feet)

INPUT

```
TEST DATA FOR TRANS MERC (INDIANA WEST) TO SECANT PLANE TRANSFORMATION
   1   13
+3932                -8635
+.5000000000E+06+.3135000000E+06+.2241000000E+04+.3284965000E+02+.9999666667E+00
+.3811064000E+00
    2001+.65407170E+06+.74565147E+06+.20010000E+04
    2002+.65407270E+06+.74565247E+06+.20020000E+04
    2003+.65407370E+06+.74565347E+06+.20030000E+04     ( in feet)
    2004+.65407470E+06+.74565447E+06+.20040000E+04
    2005+.65407570E+06+.74565547E+06+.20050000E+04                                1
```

OUTPUT

```
TEST DATA FOR TRANS MERC (INDIANA WEST) TO SECANT PLANE TRANSFORMATION

ECCENTRICITY =  .67686580E-02 MAJOR SEMI-DIAMETER =  .63782064E+07

ORIGINAL SYSTEM =   1 FINAL SYSTEM =  13

SECANT ORIGIN LAT   39 32 -.00000 LONG  -86 35  -.00000

STATE PLANE CONSTANTS

0.5000000000E+06
0.3135000000E+06    ( Indiana, west, state plane constants:
0.2241000000E+04      see lines 4 and 5 from input data
0.3284965000E+02      listing above)
0.9999666667E+00
0.3811064000E-00

STATE PLANE,GEOCENTRIC,OR UTM INPUT COORDINATES   ( in feet )

   2001 .65407170E+06 .74565147E+06 .20010000E+04
   2002 .65407270E+06 .74565247E+06 .20020000E+04
   2003 .65407370E+06 .74565347E+06 .20030000E+04
   2004 .65407470E+06 .74565447E+06 .20040000E+04
   2005 .65407570E+06 .74565547E+06 .20050000E+04

SECANT PLANE COORDINATE OUTPUT    ( in meters )

      2001     .39836362E+04     .14330252E+04     .15788391E+04
      2002     .39839429E+04     .14333284E+04     .15791437E+04
      2003     .39842496E+04     .14336316E+04     .15794482E+04
      2004     .39845564E+04     .14339348E+04     .15797527E+04
      2005     .39848631E+04     .14342380E+04     .15800573E+04


PROGRAM RUN TIME WAS    0.86 SECONDS
```

INPUT

```
TEST DATA FOR ALASKA ZONE 1 TO GEOGRAPHIC POSITION TRANSFORMATION
   3   12
   4001+.31242410E+07+.10357317E+07+.10010000E+04
   4002+.31242420E+07+.10357317E+07+.10020000E+04
   4003+.31242430E+07+.10357337E+07+.10030000E+04
   4004+.31242440E+07+.10357347E+07+.10040000E+04
   4005+.31242450E+07+.10357357E+07+.10050000E+04
```

1

OUTPUT

```
TEST DATA FOR ALASKA ZONE 1 TO GEOGRAPHIC POSITION TRANSFORMATION
ECCENTRICITY =  .67686580E-02 MAJOR SEMI-DIAMETER =  .63782064E+07
ORIGINAL SYSTEM =   3 FINAL SYSTEM =  12


STATE PLANE,GEOCENTRIC,OR UTM INPUT COORDINATES   ( in feet )

   4001 .31242410E+07 .10357317E+07 .10010000E+04
   4002 .31242420E+07 .10357317E+07 .10020000E+04
   4003 .31242430E+07 .10357337E+07 .10030000E+04
   4004 .31242440E+07 .10357347E+07 .10040000E+04
   4005 .31242450E+07 .10357357E+07 .10050000E+04


GEOGRAPHIC POSITION AND ELEVATION OUTPUT

   4001    54 39   2.65660      -131 35 45.55038      1001.00000
   4002    54 39   2.65631      -131 35 45.53339      1002.00000
   4003    54 39   2.67572      -131 35 45.51538      1003.00000
   4004    54 39   2.68527      -131 35 45.49787      1004.00000   (elevations in
   4005    54 39   2.69483      -131 35 45.48037      1005.00000    feet)


PROGRAM RUN TIME WAS     1.26 SECONDS
```

INPUT

```
TEST DATA FOR ALASKA ZONE 5 TO UTM TRANSFORMATION
   7  105
   5001+.77031161E+06+.25208511E+07+.10010000E+04
   5002+.77031261E+06+.25208521E+07+.10020000E+04
   5003+.77031361E+06+.25208531E+07+.10030000E+04
   5004+.77031461E+06+.25208541E+07+.10040000E+04
   5005+.77031561E+06+.25208551E+07+.10050000E+04
```

1

OUTPUT

```
TEST DATA FOR ALASKA ZONE 5 TO UTM TRANSFORMATION
ECCENTRICITY =   .67686580E-02 MAJOR SEMI-DIAMETER =   .63782064E+07
ORIGINAL SYSTEM =    7 FINAL SYSTEM = 105


STATE PLANE,GEOCENTRIC,OR UTM INPUT COORDINATES     ( in feet )

   5001 .77031161E+06 .25208511E+07 .10010000E+04
   5002 .77031261E+06 .25208521E+07 .10020000E+04
   5003 .77031361E+06 .25208531E+07 .10030000E+04
   5004 .77031461E+06 .25208541E+07 .10040000E+04
   5005 .77031561E+06 .25208551E+07 .10050000E+04


STATE PLANE,GEOCENTRIC,OR UTM OUTPUT COORDINATES

   5001 .52809864E+06 .67505893E+07 .10010000E+04     ( X and Y in meters;
   5002 .52809895E+06 .67505896E+07 .10020000E+04       Z in feet )
   5003 .52809926E+06 .67505899E+07 .10030000E+04
   5004 .52809957E+06 .67505902E+07 .10040000E+04
   5005 .52809988E+06 .67505905E+07 .10050000E+04


PROGRAM RUN TIME WAS     1.61 SECONDS
```

INPUT

```
TEST DATA FOR LAMBERT (NO.CAROLINA) TO GEOCENTRIC TRANSFORMATION
   2   14
+.2000000000E+07+.2844000000E+06+.2963705947E+08+.3018361125E+08+.9998725510E+00
+.5771707700E+00+.2106000000E+04+.5160353000E+02+.3814800000E+01+.3224830000E+01
+.0000000000E+00
    3001+.19520618E+07+.64461186E+06+.10010714E+04
    3002+.19520628E+07+.64461286E+06+.10020714E+04
    3003+.19520638E+07+.64461386E+06+.10030714E+04
    3004+.19520648E+07+.64461486E+06+.10040714E+04
    3005+.19520658E+07+.64461586E+06+.10050714E+04                                  1
```

OUTPUT

```
TEST DATA FOR LAMBERT (NO.CAROLINA) TO GEOCENTRIC TRANSFORMATION

ECCENTRICITY =  .67686580E-02 MAJOR SEMI-DIAMETER =  .63782064E+07

ORIGINAL SYSTEM =   2 FINAL SYSTEM =  14

STATE PLANE CONSTANTS

0.2000000000E+07
0.2844000000E+06
0.2963705947E+08     ( State plane constants for North Carolina;
0.3018361125E+08       see lines 3,4,and 5 from input data listing above )
0.9998725510E+00
0.5771707700E+00
0.2106000000E+04
0.5160353000E+02
0.3814800000E+01
0.3224830000E+01
 .0000000000E 00


STATE PLANE,GEOCENTRIC,OR UTM INPUT COORDINATES   ( in feet )

   3001 .19520618E+07 .64461186E+06 .10010714E+04
   3002 .19520628E+07 .64461286E+06 .10020714E+04
   3003 .19520638E+07 .64461386E+06 .10030714E+04
   3004 .19520648E+07 .64461486E+06 .10040714E+04
   3005 .19520658E+07 .64461586E+06 .10050714E+04


STATE PLANE,GEOCENTRIC,OR UTM OUTPUT COORDINATES   ( in meters )

   3001 .97736511E+06-.51046971E+07 .36850638E+07
   3002 .97736542E+06-.51046972E+07 .36850643E+07
   3003 .97736573E+06-.51046972E+07 .36850647E+07
   3004 .97736604E+06-.51046972E+07 .36850651E+07
   3005 .97736635E+06-.51046972E+07 .36850655E+07


PROGRAM RUN TIME WAS    2.05 SECONDS
```

INPUT

```
TEST DATA FOR LAMBERT(FLA.NORTH) TO TRANS MERC (FLA. EAST) TRANSFORMATION
   2    1
+.2000000000E+07+.3042000000E+06+.3603044305E+08+.3645492453E+08+.9999484343E+00
+.5025259000E+00+.1802000000E+04+.2611701000E+02+.3818980000E+01+.2656430000E+01
+.0000000000E+00
    7001+.27000010E+07+.40000100E+06+.10010000E+04
    7002+.27000020E+07+.40000200E+06+.10020000E+04
    7003+.27000030E+07+.40000300E+06+.10030000E+04
    7004+.27000040E+07+.40000400E+06+.10040000E+04
    7005+.27000050E+07+.40000500E+06+.10050000E+04                                  1
+.5000000000E+06+.2916000000E+06+.1453000000E+04+.2609287000E+02+.9999411765E+00
+.3821090000E+00
```

OUTPUT

```
TEST DATA FOR LAMBERT(FLA.NORTH) TO TRANS MERC (FLA. EAST) TRANSFORMATION

ECCENTRICITY =  .67686580E-02 MAJOR SEMI-DIAMETER =  .63782064E+07

ORIGINAL SYSTEM =   2 FINAL SYSTEM =   1

STATE PLANE CONSTANTS

0.2000000000E+07
0.3042000000E+06
0.3603044305E+08      ( Florida, North, state plane constants; see lines
0.3645492453E+08        3, 4, and 5 from input data listing above )
0.9999484343E+00
0.5025259000E+00
0.1802000000E+04
0.2611701000E+02
0.3818980000E+01
0.2656430000E+01
 .0000000000E 00

STATE PLANE,GEOCENTRIC,OR UTM INPUT COORDINATES  ( in feet )

   7001 .27000010E+07 .40000100E+06 .10010000E+04
   7002 .27000020E+07 .40000200E+06 .10020000E+04
   7003 .27000030E+07 .40000300E+06 .10030000E+04
   7004 .27000040E+07 .40000400E+06 .10040000E+04
   7005 .27000050E+07 .40000500E+06 .10050000E+04

STATE PLANE CONSTANTS

0.5000000000E+06
0.2916000000E+06
0.1453000000E+04      ( Florida, East, state plane constants; see lines
0.2609287000E+02        11 and 12 from input data listing above )
0.9999411765E+00
0.3821090000E-00

STATE PLANE,GEOCENTRIC,OR UTM OUTPUT COORDINATES    ( in feet )

   7001 .93024500E+05 .20916845E+07 .10010000E+04
   7002 .93025530E+05 .20916855E+07 .10020000E+04
   7003 .93026561E+05 .20916864E+07 .10030000E+04
   7004 .93027591E+05 .20916874E+07 .10040000E+04
   7005 .93028621E+05 .20916884E+07 .10050000E+04


PROGRAM RUN TIME WAS     2.78 SECONDS
```


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