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NOAA Technical Memorandum NOS 9

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Survey

The Earth's Gravity Field Represented by a Simple Layer Potential from Doppler Tracking of Satellites

KARL-RUDOLF KOCH AND BERTOLD U. WITTE

ROCKVILLE, MD.

April 1971

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THE EARTH'S GRAVITY FIELD REPRESENTED BY A SIMPLE LAYER POTENTIAL FROM DOPPLER TRACKING OF SATELLITES

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ROCKVILLE, MARYLAND
April 1971

UDC 528.232.2:528.026:629.783

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The Earth's Gravity Field Represented by a Simple Layer Potential from Doppler Tracking of Satellites

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ABSTRACT. Ten weeks of Doppler tracking of the five satellites for which data are available at the National Space Science Data Center have been analyzed to determine the earth's gravity field as represented by the potential of a simple layer. Density values of this layer for 104 surface elements have been computed in a least squares adjustment and transformed into harmonic coefficients up to the 11th degree and order. Comparisons with other solutions show good agreement. The results for the equatorial radius of the earth, its flattening, and its gravity at the equator are 6,378,156m, 1/298.255, and 978,028.8mgal, respectively.

INTRODUCTION

The representation of the earth's gravity field by means of the potential of a simple layer, instead of an expansion into spherical harmonics, has been proposed by Koch [1968] and applied to optical satellite observations by Koch and Morrison [1970]. Although the number of observations processed in this application was small in comparison to solutions based on the expansion of the geopotential into spherical harmonics, the results showed good agreement with existing solutions. However,

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the station coordinates were held fixed in the analysis of Koch and Morrison [1970], and the effects of radiation pressure and air drag on the satellite were neglected.

For the analysis described here a greater number of satellite observations, in form of Doppler tracking, have been used. Instead of 48 density values of the potential of a simple layer, 104 density values are determined; this is equivalent to an expansion into spherical harmonics up to the 10th degree and order. To avoid loss of information in the conversion from density values to harmonic coefficients, an expansion up to the 11th degree and order is used. Radiation pressure and air drag have been added to the forces acting on the satellite. Coordinates for the tracking stations also have been determined; however, the results of these computations are not listed here for reasons explained later.

Many Doppler tracking sites lie close to the stations of the worldwide National Ocean Survey satellite triangulation network [Schmid, 1969] so they can be connected by local surveys. It is planned to use the results of the satellite triangulation as additional observation equations for the coordinates of the Doppler stations; however, such a data combination will not be possible before the end of 1972 when the coordinates of the satellite triangulation network will become available. Later, it is also planned to add a combination which includes gravity anomalies. Hence, the results given here must be regarded as preliminary values. But the determination of the geopotential, expressed by an expansion into spherical harmonics up to the 11th degree and order and based on satellite observations alone, should be of interest since the latest solution for the geopotential in the open literature is a combination solution by Gaposchkin and Lambeck [1970].

OBSERVATIONS

The U. S. Navy Doppler Network observations used in this study were obtained from the National Space Science Data Center of the National Aeronautics and Space Administration. Table 1 lists the satellites for which data were available, the number of arcs, the number of passes tracked, and the number of observations for each satellite. Each arc extends over seven days. Table 2 shows the location and number of the tracking stations and the satellites observed from these stations. Figures 1 and 2 show the locations and worldwide distribution of these stations.

Except for a gap in the South Pacific, the distribution of the stations is quite satisfactory. However, 55 percent of the stations have tracked only one satellite, and only one station has observed all five satellites. The arcs were chosen using the data available from as many tracking stations as possible. The antennas at seven Doppler sites were relocated between 1963, when the first observations were made, and 1968, when the last observations used in this study were made. The differences in coordinates between these sites were constrained by the differences obtained from local surveys.

The Doppler data of the Data Center is corrected for first order ionospheric refraction by analog combination of two coherent frequencies transmitted by the satellite. The data are filtered and aggregated [Anderle 1965]; about 20 observations, taken at about 32-second intervals, constitute a pass of a satellite over a station. To each observation a standard deviation obtained in the filtering process is assigned. The data are not corrected for tropospheric refraction. To apply this correction, a refraction model* based on a standard atmosphere has been used. This model agreed very well with different models for tropospheric refraction [Witte 1971a]. To avoid significant errors in the tropospheric refraction model, observations with elevation angles smaller than 5° and satellite passes lower than 10° in elevation are deleted. Furthermore, data obtained only during the time of the satellite's approach or retreat are excluded.

Preliminary values for the coordinates of the tracking stations of table 2 were taken from Lerch et al. [1969], from preliminary results for stations of the satellite triangulation net for which survey ties to Doppler tracking stations were available, or from preliminary adjustments of the Doppler observations.

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*This model is used at the U. S. Naval Weapons Laboratory.

Table 1.--Satellite observational data

No.	Satellite			Inclination (deg.)	Perigee ht. (km)	Apogee ht. (km)	No. of arcs	No. of passes	No. of observa-tions
1	1965	32A	BE-C	41.2	940	1320	2	353	6492
2	1962	$\beta\mu 1$	ANNA 1B	50.1	1080	1180	2	513	11938
3	1965	89A	GEOS I	59.4	1120	2270	2	705	23613
4	1964	64A	BE-B	79.7	880	1080	2	209	3796
5	1968	02A	GEOS II	105.8	1080	1570	2	257	7745
						Total:	10	2037	53584

Table 2.--Satellite tracking stations

Station number	Lat. (deg.)	Long. (deg.)	Location	Observed Satellites of table 1
8	-23	314	Brazil I	2,3
13	41	141	Japan	2,3
14	61	210	Alaska	2,3
18	76	291	Greenland	3,4,5
19	-78	167	Antarctica	4
92	30	262	Texas	2,3
100	22	202	Hawaii I	1,3
103	32	253	New Mexico	2,3
106	51	359	England	2,3
111	39	283	Maryland	1,2,3
112	-35	139	Australia	2,3
115	-26	28	South Africa	2,3
117	-14	189	Samoa	1,2,3,4,5
121	15	120	Philippines	2,3
200	34	241	California	2
203	38	284	Virginia	5
717	- 5	55	Seychelles	3
722	- 8	346	Ascension	1
723	-12	97	Cocos Islands	4
729	33	343	Madeira	3
738	47	241	Washington	1
809	-46	168	New Zealand	3
811	21	204	Hawaii II	3
817	36	60	Iran	5
820	-32	295	Argentina	5
822	12	15	Chad	5
837	- 6	324	Brazil II	5

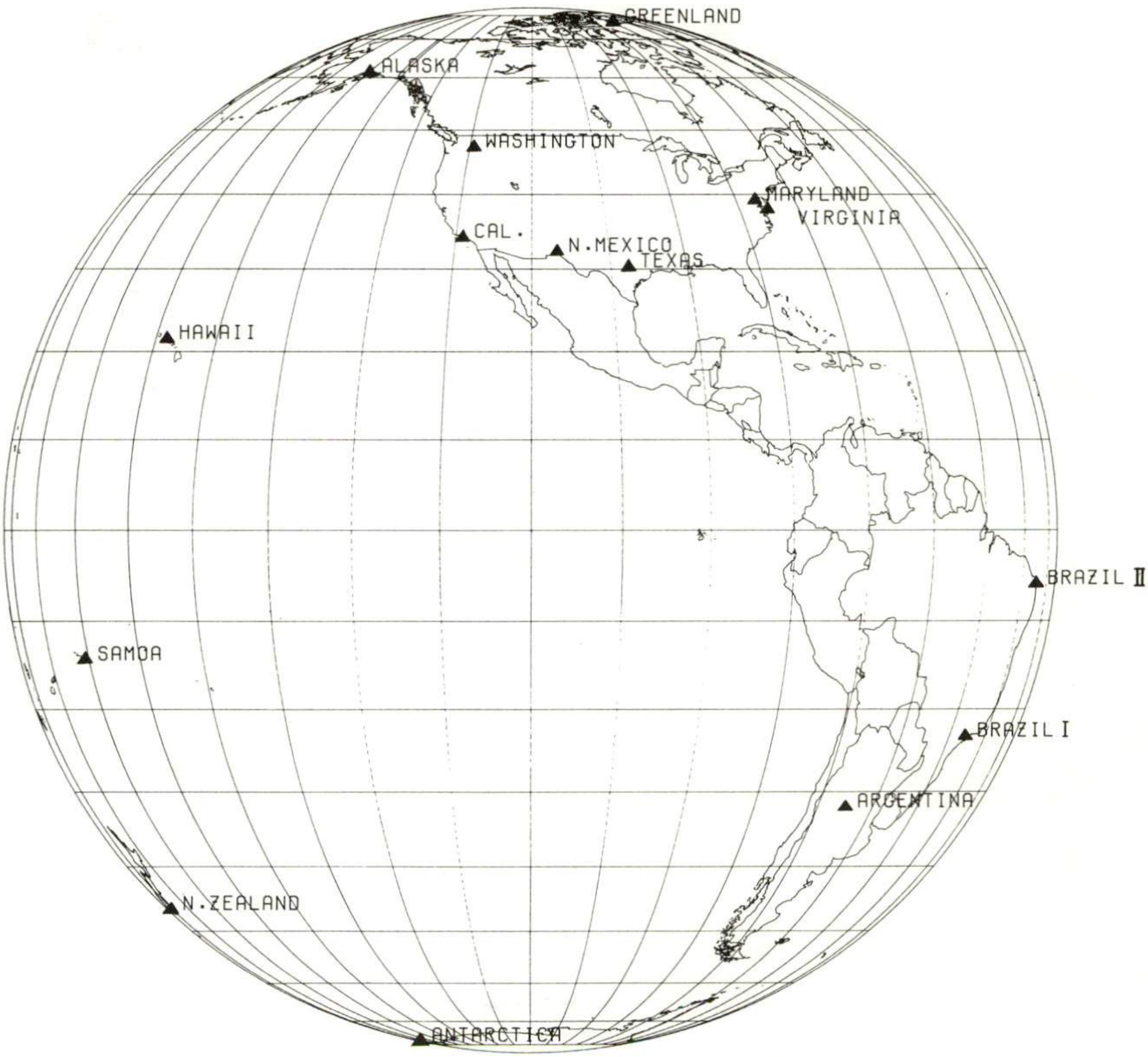

Figure 1.--Distribution of Doppler tracking stations, first view

Figure 2.--Distribution of Doppler tracking stations, second view

The coordinate system for the tracking stations is geocentric, with an orientation identical to that for the satellite triangulation network. Thus the 3-axis points towards the mean pole 1900-1905 and the 1-axis towards the intersection of the Greenwich meridian (the zero meridian of the Bureau International de l'Heure-UT1 System) with the equator.

REPRESENTATION OF THE GRAVITY FIELD

The geopotential W of the earth is divided into the known potential U and the potential T which must be determined. Hence

$$W = U + T \tag{1}$$

with

$$U = \frac{kM}{r} \left[1 + \sum_{n=2}^{7} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n \overline{P}_{nm} (\sin\phi) \times \right.$$

$$\left. (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda)\right] + 1/2\ \omega^2 r^2 \cos^2 \phi \tag{2}$$

r, ϕ, λ are spherical coordinates in the earth fixed coordinate system, k is the gravitational constant, M the mass of the earth, a the mean equatorial radius, $\overline{P}_{nm}$ the fully normalized associated Legendre function of degree n and order m, and ω the angular velocity of the earth. When computing U outside the earth, the centrifugal term in eq (2) is omitted. The fully normalized harmonic coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are taken up to the 7th degree and order from the solution of Anderle [1967]. However, the coefficients $\overline{C}_{21}$ and $\overline{S}_{21}$ have been omitted because of the choice of the orbital coordinate system defined below. The values for kM and a are chosen as

$$kM = 3.986013\times10^{14} m^3 sec^{-2},\ a = 6,378,145m \tag{3}$$

The potential of a simple layer distributed over the earth's surface represents the potential T. To evaluate the integral in this expression, the earth's surface is divided into elements in which the density of the simple layer is assumed constant. One hundred and four (104) surface elements ΔE_i are chosen here. They are bordered by parallels and meridians and approximate the size of a 20 degree latitude-longitude area centered on the equator. Hence, [Koch and Morrison 1970]

$$T = \sum_{i=1}^{104} \chi_i \iint_{\Delta E_i} \frac{dE}{\ell} \tag{4}$$

ℓ is the distance between the fixed point at which T is computed and the variable point at the surface of the earth over which the integration is taken. The 104 density values χ_i are the unknown parameters of the gravity field.

The integral over the surface element ΔE_i in eq (4) is computed numerically. To reduce the influence of errors in the numerical integration, the preliminary values for the density values χ_i are set equal to zero. Thus, the nominal orbits of the analyzed satellites are computed by means of the expansion eq (2) into spherical harmonics. For the determination of the derivatives of the observations with respect to the unknown parameters χ_i, the integral in eq (4) is computed by subdividing ΔE_i into four subdivisions, for whose midpoints the kernel of the integral is assumed to be constant. To determine the coordinates of these midpoints, the equipotential surface at sea level is defined by setting $U = U_o$ in eq (2) and the topographic heights above sea level are added. U_o is computed from the equation which holds for the surface of a level ellipsoid [Koch and Morrison 1970].

With a relative error of the order of the flattening of the earth, i.e. 1 part in 300, U_o may be approximated by

$$U_o = \frac{kM}{a}$$

so that

$$\Delta U_o = -\frac{kM}{a^2} \Delta a \tag{5}$$

The value for kM is well determined from space probes; the equatorial radius a is known only approximately. Hence, eq (5) may be applied to compute a new equatorial radius for the earth, if the densities, χ_i, are developed by means of a series of spherical harmonics containing the zero order and degree coefficient.

COMPUTATIONAL METHOD

The origin of the coordinate system for the orbit computation is the center of mass of the earth. Its 3-axis is identical with the instantaneous axis of the earth and its 1-axis points at an angle, east of the true vernal equinox, which equals the precession and nutation in right ascension since 1950.0. This system assures that no variations with time due to the coordinate system are introduced into the second zonal harmonic. Since the orbit computations extend over arcs not longer than 7 days, this coordinate system is a good approximation of an inertial system. By means of the polar motion as determined by the Bureau de l'Heure and the sidereal angle as defined by the Smithsonian Institution [Lundquist and Veis 1966] the coordinates of the tracking stations are rotated into this system. Since the time of the Doppler observations are given in UTC, the rotation is corrected for the difference between UTC and UT1 published by the Bureau de l'Heure. No jumps in UTC appear during the time of the orbit integration, so that UTC could be used instead of atomic time.

The model of the force field acting upon the satellite consists of the influence of the geopotential as defined by eq (1), the attraction of the sun and the moon, air drag, and radiation pressure. The orbits of the observed satellites are computed numerically with a 48-second time step using a 12th order Cowell-Störmer integration for positions, a 10th order Adams-Bashforth predictor and a 10th order Adams-Moulton corrector for velocities, and an 8th order Adams-Moulton integration for the variational equations to determine the derivatives of the observations with respect to the unknown parameters. To interpolate between the time steps, Lagrange's interpolation is applied.

The unknown parameters are the 104 density values, the coordinates of the tracking stations, a base frequency offset per pass of a satellite over a station, and for each arc, the six orbital elements. These parameters are determined by least squares adjustment based on a differential correction process. Hence, we have the observation equations

$$\left[\underline{A},\underline{B},\underline{C},\underline{D}\right] \begin{bmatrix} \Delta b \\ \Delta e \\ \Delta x \\ \Delta \chi \end{bmatrix} = \underline{\ell} + \underline{v} \tag{6}$$

where $\underline{A},\underline{B},\underline{C},\underline{D}$ are matrices of coefficients, and $\underline{\Delta b}$, $\underline{\Delta e}$, $\underline{\Delta x}$, $\underline{\Delta \chi}$ are vectors of corrections to the preliminary values of the base frequencies, the orbital elements, the station coordinates, and the density values, respectively. $\underline{\ell}$ and $\underline{v}$ are the vectors of the observations and the residuals. The normal equations of the least squares adjustment are obtained by means of the covariance matrix $\underline{\Sigma}_\ell$ of the observations, which is a diagonal matrix

$$\begin{bmatrix} \underline{A}^T\underline{\Sigma}_\ell^{-1}\underline{A} & \cdots & \underline{A}^T\underline{\Sigma}_\ell^{-1}\underline{D} \\ \vdots & & \vdots \\ \underline{D}^T\underline{\Sigma}_\ell^{-1}\underline{A} & \cdots & \underline{D}^T\underline{\Sigma}_\ell^{-1}\underline{D} \end{bmatrix} \begin{bmatrix} \underline{\Delta b} \\ \vdots \\ \underline{\Delta \chi} \end{bmatrix} = \begin{bmatrix} \underline{A}^T\underline{\Sigma}_\ell^{-1}\underline{\ell} \\ \vdots \\ \underline{D}^T\underline{\Sigma}_\ell^{-1}\underline{\ell} \end{bmatrix} \tag{7}$$

To avoid a singularity when solving the normal equations eq (7), the longitude of station 111, Maryland, is held fixed. The bias frequencies are eliminated whenever the contribution of one pass to the normal equations has been computed. The six orbital elements are eliminated after obtaining the contribution of one arc to the normal equations. The reduced normal equations are then added. They contain only the unknown parameters for the station coordinates and the density values.

Detailed information about the computational procedures involved in the determination of density values and station coordinates from Doppler observations are given by Witte [1971b].

CONSTRAINTS

The density values χ_i are converted into normalized harmonic coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ by the method described by Koch [1968]. Here

$$\overline{C}_{nm} = \overline{C}_{nmu} + \frac{1}{(2n+1)kM\,a^n} \sum_{i=1}^{104} \chi_i \iint_{\Delta E_i} r^n\, \overline{P}_{nm}(\sin\phi)\cos m\lambda\, dE$$
$$\overline{S}_{nm} = \overline{S}_{nmu} + \frac{1}{(2n+1)kM\,a^n} \sum_{i=1}^{104} \chi_i \iint_{\Delta E_i} r^n\, \overline{P}_{nm}(\sin\phi)\sin m\lambda\, dE \tag{8}$$

where $\overline{C}_{nmu}$ and $\overline{S}_{nmu}$ denote the harmonic coefficients introduced into eq (2) to define the potential U. The integral over the surface elements ΔE_i in eq (8) is solved numerically by dividing ΔE_i into 9 subregions.

Since the origin for both the earth-fixed coordinate system and the orbital system is the center of mass of the earth, the harmonic coefficients $\overline{C}_{10}$, $\overline{C}_{11}$, and $\overline{S}_{11}$ must equal zero.

Furthermore, $\overline{C}_{21}$ and $\overline{S}_{21}$ must be small in comparison to the rest of the harmonic coefficients since, for the orbit computations, the 3-axis coincides with the rotational axis of the earth. To insure this, constraints in the form of observation equations with small variances are set up according to eq (8), and their contribution is added to the normal equations [Koch and Pope 1969]. The same method is applied to constrain the coordinate differences obtained by local surveys between Doppler sites at which the antennas have been shifted.

RESULTS

The variance, σ^2, of an observation of unit weight is given by

$$\sigma^2 = \frac{\underline{v}^T \underline{\Sigma}_\ell^{-1} \underline{v}}{o - u} ,$$

where o is the number of observations and u the number of unknown parameters. From the least squares adjustment eq (7) σ^2 is 12.7. This indicates deficiencies in the model for the force field being applied, biases in the data which have not been accounted for, or standard deviations of the data which are too small. For example, the model of the force field could be improved by introducing resonant terms into eq (2) for the satellites being observed. This would permit orbit fitting with smaller residuals. The introduction of more bias parameters, such as the frequency drift, will reduce σ^2. However, such a procedure might weaken the information inherent in the data if the introduced bias parameters are physically irrelevant. The question, how much to increase the given standard deviations of the observations to get a reasonable estimate for the accuracy of the data, is not too important here since Doppler data alone are analyzed, but must be answered for the combination solutions. Judging from the value for σ^2, the standard deviations of the Doppler data obtained in the filtering process [Anderle 1965] should be increased by a factor between 2 and 3. The factor of 3 has been applied to obtain the standard deviations given below.

The standard deviations for the density values χ_i lie between ±0.66 mgal and ±0.14 mgal and the correlation coefficients for χ_i are less than 0.79. The correlation coefficients between the coordinates of the tracking stations are less than 0.69. The standard deviations of all three coordinates of station 117, Samoa, which tracked all five satellites, are less than ±6m, while the maximum standard deviation of the coordinates for the stations which tracked only one satellite (see table 2) is ±92m. The determination of the coordinates of these stations is poor. The additional information from the results of the geometric satellite triangulation will be needed for more accurate determinations. The station coordinates obtained from the Doppler solution are therefore not given here.

The results of the transformation eq (8) from density values to harmonic coefficients up to degree and order 11 are given in table 3. We obtain for $\overline{C}_{20}$ 484.1709×10^{-6}, which corresponds to a flattening of 298.256 and to a gravity of 978,032.2 mgal at the equator for the reference ellipsoid with the values for a and kM defined by eq (3). The geoid of Figure 3 (computed with the coefficients of table 3) refers to this ellipsoid.

The value for $\overline{C}_{00}$ from eq (8) equals -1.7871×10^{-6}, so that a new equatorial radius, a, is computed by eq (5). We obtain a = 6,378,156 m, which exceeds by 1m the value adopted by the Smithsonian Institution [Gaposchkin and Lambeck 1970]. With this value and $\overline{C}_{20}$ of table 3 we now obtain the flattening of 298.255 and the gravity of 978 028.8 mgal at the equator.

The results are compared to two solutions based on satellite data only and two combined solutions: the solution for the 1966 Standard Earth of the Smithsonian Institution [1966], the solution of Anderle [1967], the combined solution of Koch and Morrison [1970], and the combined solution of Gaposchkin and Lambeck [1970]. The rms discrepancies between the common coefficients of these solutions and the solution of table 3 are, per coefficient, $\pm0.19\times10^{-6}$, $\pm0.21\times10^{-6}$, $\pm0.30\times10^{-6}$, and $\pm0.17\times10^{-6}$, respectively. The rms discrepancies between the geoid heights computed at 10 degree intervals for the solutions mentioned above, and the solution of table 3, are ±12.4m, ±12.7m, ±17.3m, and ±12.0m respectively. Except for the combined solution of Koch and Morrison [1970] which suffers from a lack of data, the solution of table 3 agrees well with existing results.

Table 4 contains the zonal harmonics $\overline{C}_{no}$ of Kozai [1969], King-Hele et al. [1969], and table 3.

Table 3.--Potential coefficients to (11,11) from Doppler observations

n	m	$10^6\ \overline{C}_{nm}$	$10^6\ \overline{S}_{nm}$	n	m	$10^6\ \overline{C}_{nm}$	$10^6\ \overline{S}_{nm}$
2	0	-484.1709		8	5	-.2118	-.0733
2	2	2.4705	-1.4499	8	6	.1648	.0273
3	0	.9251		8	7	.0268	-.1800
3	1	2.1895	.1798	8	8	-.2215	-.0933
3	2	.8899	-.8307	9	0	-.0027	
3	3	.8105	1.3283	9	1	.0117	.1602
4	0	.5512		9	2	-.0129	-.0184
4	1	-.4220	-.4195	9	3	-.0143	-.4088
4	2	.4699	.2265	9	4	.1144	-.0293
4	3	.9786	-.1118	9	5	.2177	.1294
4	4	-.1695	.4799	9	6	.2854	.2459
5	0	.0858		9	7	-.1595	-.4245
5	1	-.2906	.1377	9	8	-.3270	.3078
5	2	.5869	-.4227	9	9	-.0395	.1499
5	3	-.5185	-.2054	10	0	-.0066	
5	4	-.3540	.0657	10	1	.0739	-.1139
5	5	.3292	-.4550	10	2	-.2851	.2077
6	0	-.1886		10	3	-.0131	-.0850
6	1	-.1809	-.0156	10	4	.0380	-.2368
6	2	-.1456	.0844	10	5	-.0946	-.0470
6	3	.1228	.2518	10	6	-.0029	-.3110
6	4	-.1275	-.4128	10	7	-.0047	-.1533
6	5	-.2264	-.3788	10	8	.0942	-.0667
6	6	-.1690	-.0924	10	9	.2025	.2276
7	0	.0952		10	10	.2122	-.1192
7	1	.4809	-.2867	11	0	.0081	
7	2	.3187	.1889	11	1	.0936	-.1526
7	3	.2223	-.0891	11	2	-.0039	-.2263
7	4	-.2061	-.4174	11	3	-.0204	.0559
7	5	.2676	.0358	11	4	.0539	-.0015
7	6	-.1569	-.4117	11	5	-.0148	-.2490
7	7	.1069	-.4366	11	6	.0371	.0508
8	0	.0337		11	7	.1113	-.2117
8	1	.0766	-.1544	11	8	-.1605	.0484
8	2	.4486	-.3639	11	9	-.0789	-.0600
8	3	-.0214	-.1668	11	10	-.0292	-.0462
8	4	-.3370	-.0421	11	11	-.1372	.0539

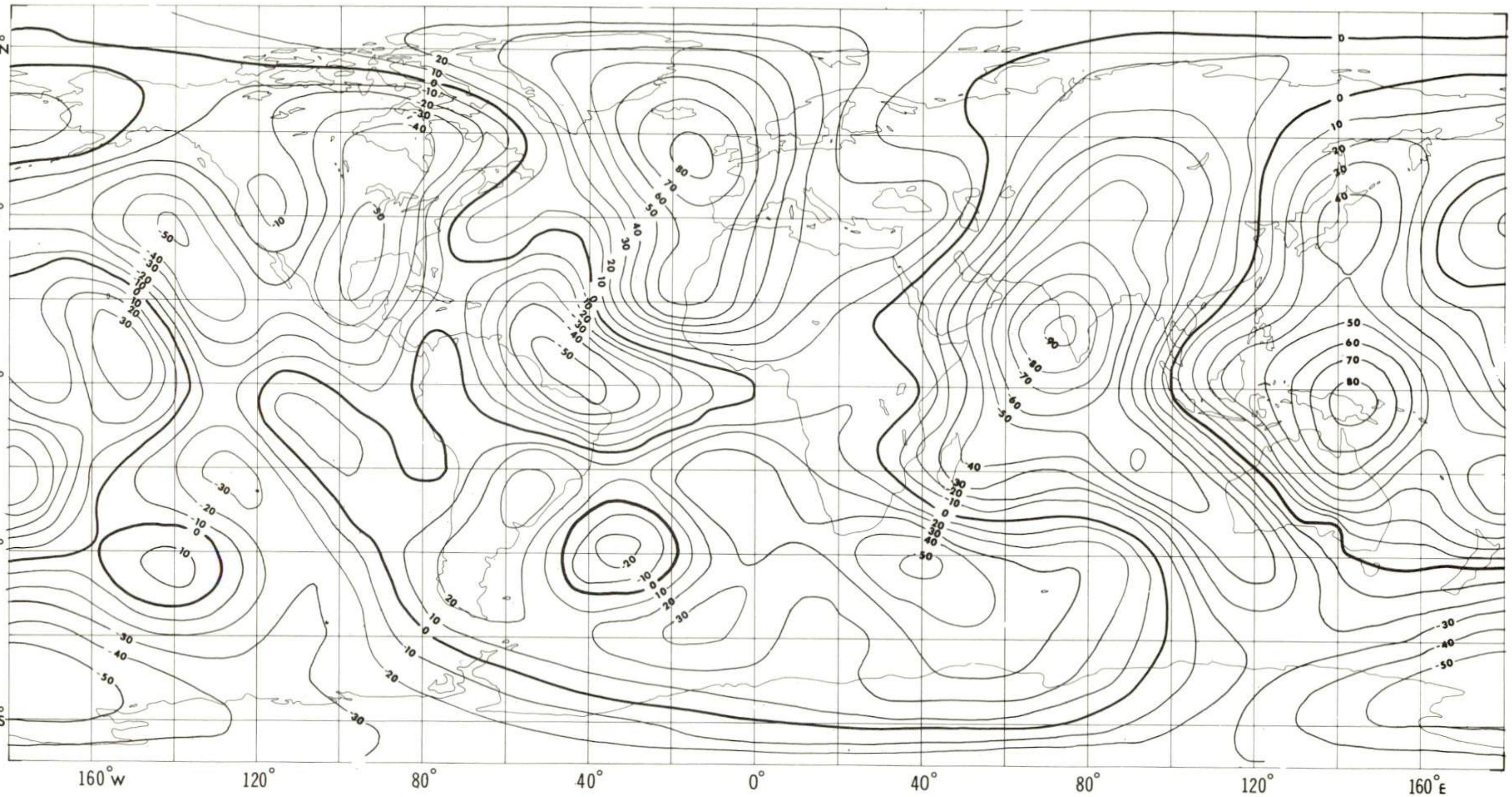

Figure 3.--Geoid heights in meters of the new Doppler solution corresponding to a reference ellipsoid of flattening f = 1/298.256.

Table 4.--Zonal harmonics $\overline{C}_{no}$

Coefficient	Kozai [1969]	King-Hele et al. [1969]	Table 3
$10^6\ \overline{C}_{20}$	-484.1659	-	-484.1709
$10^6\ \overline{C}_{30}$	0.9593	0.9615	0.9251
$10^6\ \overline{C}_{40}$	0.5310	-	0.5512
$10^6\ \overline{C}_{50}$	0.0693	0.0648	0.0858
$10^6\ \overline{C}_{60}$	-0.1392	-	-0.1886
$10^6\ \overline{C}_{70}$	0.0932	0.1030	0.0952
$10^6\ \overline{C}_{80}$	0.0286	-	0.0337
$10^6\ \overline{C}_{90}$	0.0229	0.0000	-0.0027
$10^6\ \overline{C}_{10\ 0}$	0.0772	-	-.0066
$10^6\ \overline{C}_{11\ 0}$	-0.0421	0.0000	0.0081

Table 5 compares the low order harmonics obtained by Wagner [1970], from resonant orbits, with those of table 3.

Table 5.--Low order harmonic coefficients

Solution	$10^6\ \overline{C}_{22}$ $10^6\ \overline{S}_{22}$	$10^6\ \overline{C}_{32}$ $10^6\ \overline{S}_{32}$	$10^6\ \overline{C}_{33}$ $10^6\ \overline{S}_{33}$	$10^6\ \overline{C}_{44}$ $10^6\ \overline{S}_{44}$
Wagner	2.4524	0.8871	0.6942	-0.0771
[1970]	-1.4082	-0.6060	1.4207	0.3569
Table	2.4705	0.8899	0.8105	-0.1695
3	-1.4499	-0.8307	1.3283	0.4799

Table 6.--Degree of variances

n	$10^6\ \sigma_n$	$10^6\ \sigma_d$	$\sigma^2_{\Delta g,n}$ in mgal²
2	216.53	2.50	7.9
3	1.17	1.11	36.8
4	0.49	0.62	16.5
5	0.36	0.40	21.3
6	0.21	0.28	14.3
7	0.28	0.20	41.6
8	0.20	0.16	31.9
9	0.21	0.12	51.9
10	0.16	0.10	39.2
11	0.11	0.08	26.5

Table 6 shows the positive square roots of the mean degree variances for the coefficients of table 3,

$$\sigma_n = \left[\frac{1}{2n+1} \sum_{m=0}^{n} (\overline{C}^2_{nm} + \overline{S}^2_{nm}) \right]^{1/2}$$

which indicate the decay of the harmonic coefficients in comparison with the rule of thumb,

$$\sigma_d = \frac{10^{-5}}{n^2}$$

and the anomaly degree variances,

$$\sigma^2_{\Delta g,n} = \gamma^2 (n-1)^2 \sum_{m=1}^{n} (\overline{C}^2_{nm} + \overline{S}^2_{nm})$$

where γ denotes the normal gravity. The last column indicates that the higher order coefficients of the solution presented here may be improved upon.

CONCLUSIONS

Although the worldwide distribution of the Doppler tracking stations for which data were available is good, each satellite was tracked by only a few stations. Furthermore, Doppler data could not be obtained for polar satellites and satellites with inclinations less than 41°. Despite these restrictions the results show good agreement with existing solutions for the earth's gravity field. These results again prove the feasibility of using a simple layer model for the geopotential in satellite geodesy. The poor determination of coordinates for the tracking stations will be overcome by a combination solution using the results of the worldwide satellite triangulation network. To strengthen results for the higher order harmonic coefficients, gravity anomalies will be introduced into the combined solution.

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