NOAA Technical Memorandum NOS NGS-45

EMPIRICAL CALIBRATION OF ZEISS NI-1 LEVEL INSTRUMENTS TO ACCOUNT FOR MAGNETIC ERRORS

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Rockville, MD
June 1986

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ABSTRACT. The existence of magnetic error in some automatic level instruments is well established. In particular, the Zeiss Ni-1 exhibited magnetic error reaching as high as 2 mm km^{-1} in the north-south direction. Some Ni-1 compensators have been calibrated in specially equipped laboratories by Whalen and Rumpf. However, 70 percent of the Ni-1 compensators used by the National Geodetic Survey (NGS) are not available for laboratory calibration, having been replaced, lost, or damaged. Only the last-used compensators could be calibrated in the NGS laboratory. Consequently, an empirical approach is required to estimate the scale of magnetic error of nonrecoverable Ni-1 compensators for time periods between repairs, and prior to loss, damage, or retirement of the instrument. All Ni-1 leveling measurements in the United States, outside of known areas of surface deformation, were matched with the most recent non-Ni-1 measurements to simultaneously estimate magnetic error correction constants for all Ni-1 instruments. The correction constants obtained empirically differ from those obtained from laboratory calibrations by a factor of about 2. The average of the magnetic error correction factors derived in the laboratory is -8.7 mm km^{-1} $gauss^{-1}$ whereas the empirically derived average is -4.5 for the same compensators. The empirically derived magnetic error correction constants perform well; the corrected data check previous spirit leveling and are also consistent around circuits, and with lake level and tide gauges. The empirical approach can be used to calibrate the magnetic sensitivity of most other automatic levels, provided a sufficient amount of magnetic-free data in nondeforming areas can be matched with the measurements observed with the problem instrument. The laboratory correction constants were usually not as successful except in one instance when correcting data obtained just prior to the laboratory calibration.

INTRODUCTION

Zeiss Ni-1 level instruments were used by the National Geodetic Survey (NGS) to perform 25,000 km of precise leveling during the period 1972-80. Because these measurements are recent, they should provide valuable support to the readjustment of the North American Vertical Datum. For this reason a special effort has been made to develop a correction to remove magnetic error from this data set.

Magnetic error of Zeiss Ni-1 level instruments has been estimated by laboratory calibrations (Rumpf and Meurisch 1980; Whalen 1983). The tests verified the sinusoidal variation in error with changing azimuth, and the linear increase in error as the intensity of a simulated magnetic field is increased. The error is caused by residual magnetic sensitivity of the compensator's invar-alloy suspension tapes (fig. 1), and is particularly disadvantageous for the Zeiss Ni-1 because of that instrument's high mechanical tilt amplification. The laboratory calibrations attempted to estimate the extent to which the instrument is influenced by the Earth's magnetic field at the time of calibration. However, attempts to use the laboratory calibration constants were usually not successful, resulting in overcorrection of the data. Table 1 shows the dates of Ni-1 compensator repairs and replacements. There is significant variation in the magnitude of magnetic error for compensators found in different instruments (Whalen 1983; Strange 1985; Rumpf and Meurisch 1981). Consequently it is not realistic to expect the laboratory estimate of Ni-1 magnetic error to be applicable to previous compensators installed in the same instrument. There is also uncertainty as to whether some repairs may have changed the magnetic characteristics of the Ni-1 compensators (Strange 1985). For these reasons, the laboratory estimates of magnetic error may not be valid for much of the period when a particular Ni-1 instrument was used.

Most Ni-1 compensators have been replaced or repaired, and thus the compensators installed previously in the instrument or the earlier characteristics of the last compensator cannot be evaluated in the laboratory. The alternative calibration method described in this paper is a computational approach wherein the magnetic sensitivity of a Ni-1 compensator is determined by comparison of the Ni-1 measurements with corresponding measurements by instruments considered to be essentially free of magnetic influence. The Fischer spirit level, principal instrument of the National Geodetic Survey (NGS) prior to 1962, was normally the control instrument. Other spirit levels were also used by NGS between 1962 and 1972. Ni-1 levels were used by NGS from 1972 to 1980. Since 1980, the Ni 002 has been used, and it has been demonstrated to be nearly free of magnetic error (Whalen 1983).

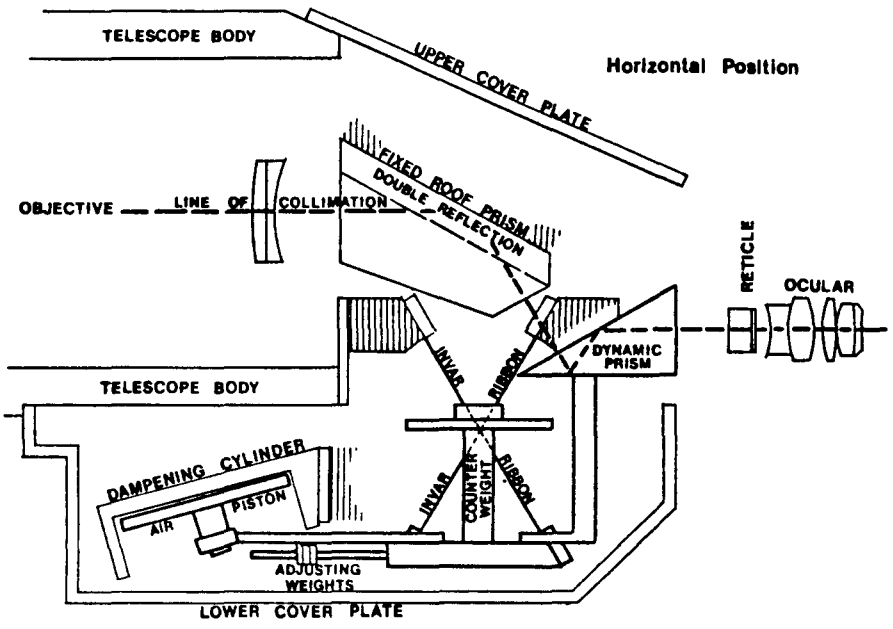

Figure 1.--Diagram of the Zeiss Ni-1 level instrument showing suspending wires of the compensator.

Table 1.--Repair dates for Ni-1 Instruments

Instrument Number	Purchase date	1975	1976	1977	1978	1979	1980	1981
78300	1972	9-11*	7-?	8-16	8-2*			
90760	8-73	9-23*	8-23 11-04	3-15	5-18 8-10 10-26	3-26* 6-22 8-28		9-11
90823	1972	6-01 10-25	5-03	1-05		4-10		
90824		9-26	10-12	7-27 11-15	10-12	4-02 6-22	loaned COE	
90825	1972	11-28	12-28		4-06*&	loaned		
90829	1974	7-01		3-09 10-25	4-07*+	4-02 10-79		
90834	9-72	1-27 4-08 11-75	11-04			1-18		
90856	1974	1-15	5-06	2-15	10-11*	10-22		
107286	7-75		5-06	5-03 1-15	7-20 10-12			10-26*
107293	7-75			8-16	7-? 12-08	9-17*		7-31
107302	7-75			9-03	5-06+			
107367	3-75				12-26	1-18	6-06	

* compensator replaced
\+ compensator went to instrument #107286
& compensator went to instrument #78300
Note: Repair records not available prior to 1975.

The empirical approach treats the measured height differences (dh values) between bench marks as observations and determines the scale of magnetic error which best explains discrepancies between the Ni-1 and control measurements. The non-Ni-1 leveling is presumed to be free of magnetic error or large amounts of other systematic errors which would preclude their use as control measurements. With the empirical method, most other compensators can be calibrated if they generated significant amounts of data along level lines previously or later leveled by instruments that are not significantly influenced by the Earth's magnetic field.

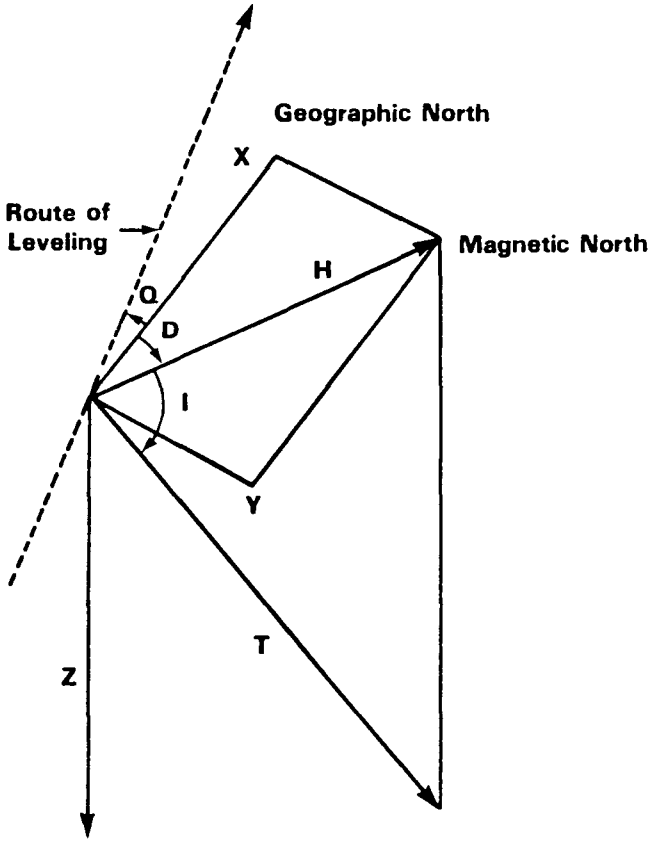

Figure 2.--Components of the Earth's magnetic vector.

MATHEMATICAL DEVELOPMENT

The influence of the Earth's magnetic vector on Ni-1 leveling is proportional to the magnitude of its horizontal component projected onto the direction of leveling (Leitz 1983). The vertical component may also raise or lower Ni-1 sight levels (Rumpf and Meurisch 1981) but that influence cancels when foresight is subtracted from backsight.

We can express the total intensity, T, of the Earth's local magnetic vector as a function of x, y, z components in a Cartesian coordinate system, where x is in the direction of geographic north, and Z points into the Earth in the direction of the local vertical. (See fig. 2.)

$$T = (x^2 + y^2 + z^2)^{1/2} \qquad (1)$$

The horizontal component H of the local magnetic vector points in the direction of magnetic north and has magnitude

$$H = (x^2 + y^2)^{1/2} \; ; \qquad (2)$$

or H can also be expressed as a function of inclination angle, I, and total intensity, T, of the Earth's magnetic vector.

$$H = T \cos(I). \tag{3}$$

The magnetic error varies as the cosine of the difference between the azimuth of leveling, Q, and the declination, D, of the Earth's magnetic vector. D is the azimuth of H. Knowing that instruments may differ in their response to the Earth's magnetic field we assign a correction constant, C, to each instrument. The total magnetic error, M, is a function of C and the distance leveled, S, projected onto the magnetic north-south axis, hence

$$M = C\ H \cos(Q-D)\ S. \tag{4}$$

T, I, and D can be evaluated directly from a model of the Earth's magnetic field. The model used by NGS was prepared by the U.S. Geological Survey in Denver, Colorado (Peddie 1981). The main geomagnetic reference field is described by 120 spherical harmonic coefficients and the secular variation is described by an additional 80 coefficients. S and Q can be evaluated from the known positions of the terminal bench marks of the section. The units of T are gauss, S is in km, M is to be expressed in mm, and C is in mm km^{-1} $gauss^{-1}$. If points P_1 and P_2 are connected by leveling on a spherical Earth

$$S = R\ d\phi/\cos(Q) \tag{5}$$

$$Q = \tan^{-1} [\cos(\phi)\ d\lambda/d\phi] \tag{6}$$

where R = 6371 km is the mean Earth's radius, ϕ is the mean latitude of P_1 and P_2, and $d\phi$ and $d\lambda$ are the latitude and longitude differences between the two points.

M can then be written as

$$M = C\ b \tag{7}$$

where b can be calculated from known quantities and C is unknown. C may be called the magnetic error correction constant.

A height difference between points P_i and P_j observed with a Ni-1 instrument can be expressed as

$$dh_{ij}^{k} = dh_{ij}^{*} + C_k b_{ij} + e \tag{8}$$

where k denotes the particular Ni-1 compensator involved, e is random error, and the true height difference is denoted by dh*. Here we recognize that one instrument housing may have had several different compensators, corresponding to several

different periods of use. The compensator used to make the observation is identified by the recorded serial number and date of measurement, so that the proper subscript k can be obtained. A height difference measured by an instrument not having magnetic error is expressed simply as

$$dh_{ij} = dh_{ij}^* + e \; ; \tag{9}$$

and the disagreement between measured height differences with a Ni-1 and a nonmagnetic instrument over the same section would be

$$ddh_{ij} = C_k b_{ij} + e' \; , \tag{10}$$

where e' is the difference of random errors of the two measurements. We could estimate C directly from two measurements,

$$C = (ddh_{ij} - e')/b_{ij} \; , \tag{11}$$

but the signal-to-noise ratio is poor because e' may be a significant percentage of the total error, possibly as large as the magnetic error itself for one section. Therefore we need to look at hundreds of such estimates of C if possible, knowing that the mean value of e' will tend toward zero. As Q-D approaches 90°, b approaches zero and C becomes indeterminant. To avoid this situation, data are excluded from the solution if $75^o < Q\text{-}D < 105^o$.

Consider figure 3 which corresponds to the first two sections of a level line. The first section was leveled with a nonmagnetic instrument, and also with two different Ni-1 instruments. The second section is leveled by only the first of the two Ni-1's. The first section contributes information to the solution as indicated before, but also constrains the difference between C_1 and C_2.

$$C_1 - C_2 = (dh_{12}{}^1 - dh_{12}{}^2 + e_1 - e_2)/b_{12}. \tag{12}$$

From relevelings such as shown in figure 3 we can form a system of observation equations and estimate the values of C_1, and C_2, and the unknown true height differences (dh_{ij}*) for the two sections. There is one equation such as (8) or (9) for each observed dh, and collectively they form a system:

$$[A_1 \; A_2] \; [X_1 \; X_2]^T = L \tag{13}$$

where L is a column of observed dh values; A_1 is a matrix of coefficients, which are partial derivatives of (8) with respect to the unknown magnetic error correction constants (C values); A_2 contains partial derivatives with respect to the true height differences; X_1 is a column of unknown magnetic error correction constants; and X_2 corresponds to the true height differences to be estimated.

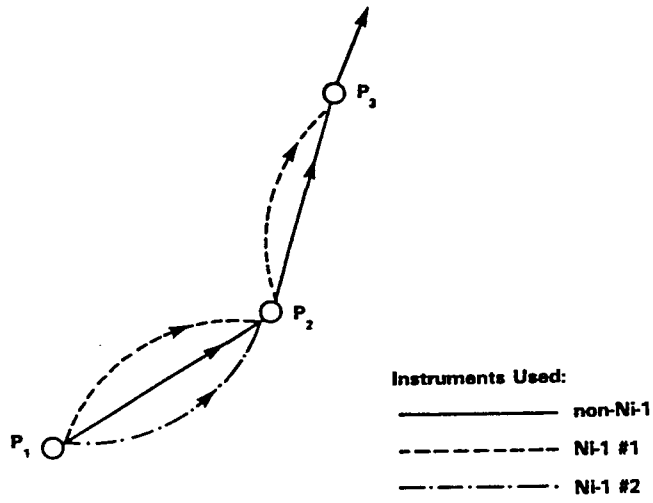

Figure 3.--Schematic example of Ni-1 releveling.

Denoting the weight matrix for the observations by P, we can form normal equations

$$\begin{bmatrix} A_1^T PA_1 & A_1^T PA_2 \\ A_2^T PA_1 & A_2^T PA_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} A_1^T PL \\ A_2^T PL \end{bmatrix}$$

or

$$N_{11}X_1 + N_{12}X_2 = U_1 \tag{14a}$$

$$N_{21}X_1 + N_{22}X_2 = U_2 \tag{14b}$$

or

$$NX = U \tag{14c}$$

From (14b) we have

$$X_2 = N_{22}^{-1} (U_2 - N_{21} X_1). \tag{15}$$

Substituting (15) into (14a),

$$(N_{11} - N_{12} N_{22}^{-1} N_{21}) X_1 = U_1 - N_{12} N_{22}^{-1} U_2 \tag{16}$$

or

$$MX_1 = W . \tag{17}$$

The magnetic error correction constants are thus easily obtained:

$$[C_1 \; C_2 \; \ldots C_k]^T = X_1 = M^{-1}W. \tag{18}$$

The full normal equations for this problem as in (14) are never actually formed. Instead the M and W matrices are formed directly in increments corresponding to

each observation. N_{22} is a diagonal matrix, and hence trivial to invert. The X_2 unknowns do not need to be solved by inverting N which would be awkward for solutions involving thousands of sections of leveling. Instead, an estimate of the true height difference for each section is obtained directly by (15), which looks complicated in matrix form but actually amounts to calculating the weighted mean, $\overline{dh}$, of the observed height differences for each section after applying the estimated magnetic error correcton constants from X_1.

$$dh = \frac{\Sigma\ p_i\ dh_i}{\Sigma\ p_i}\ . \tag{19}$$

The summations in (19) range over all observations dh_i' for the section. The weights, p_i, are discussed in the next section.

Having the estimated true height differences allows us to compute residuals

$$v_i = dh_i' - \overline{dh} \tag{20}$$

The size of the residuals can be used as a criterion for rejecting spurious data, and to compute the variance for an observation of unit weight

$$m^2 = \frac{\Sigma\ p_j\ v_j^2}{n-u} \tag{21}$$

where n is the total number of observations from all sections of leveling, and u is the number of unknowns, including the dh* unknowns.

The covariance matrix for the magnetic error correction constants is

$$\Sigma_c = m^2\ M^{-1} \tag{22}$$

OBSERVATION WEIGHTS

As previously mentioned, the potential of a particular section of releveling to yield information about the scale of magnetic error depends on the distance leveled in the magnetic north-south direction, and the magnitude of the horizontal component of the Earth's local magnetic vector. Long leveling segments that run in the magnetic north-south direction will contribute significantly, whereas lines in the magnetic east-west direction will contribute much less. If we assign weight $p = S^{-1}$ for each measured dh of a section, the variance for a single section estimate of C (see eq. 11) using one Ni-1 dh and one non-Ni-1 dh would be

$$\sigma_C^{\ 2} = 2S/b^2. \tag{23}$$

Our simultaneous solution for all C values based on the total leveling data is equivalent to determining weighted means from many estimates for each C while also considering correlations between C values when more than one Ni-1 was used to level a section. The influence of each section estimate is automatically propagated forward in a way that is inversely proportional to the individual variances (eq. 23), hence sections with small b-values have little influence on the solution.

The non-Ni-1 observations are most suitable for the calibration process if they were made at nearly the same time as the corresponding Ni-1 observations, because the negative influence of crustal movements or ground noise would be minimized. The weights of the non-Ni-1 observations should therefore reflect the time elapsed, dt, between the non-Ni-1 and the Ni-1 observations. If we let g denote a scale factor, the following expression can be used to downweight a non-Ni-1 observation based on the size of dt.

$$p = [S\ (1 + g\ dt)]^{-1}. \tag{24}$$

In our calibrations, g was set to 0.1. With downweighting, the variance of a single-section estimate of C using one non-Ni-1 dh becomes

$$\sigma_C^{\ 2} = (2S + g\ S\ dt)/b^2. \tag{25}$$

Levelings that are directed nearly $\pm$ 90° from magnetic north may confuse the solution, but will not strengthen it significantly. For this reason, observations with b values less than 0.045 were not included in the calibration adjustment.

Input Data

The entire NGS vertical data base was scanned to find matching Ni-1 and non-Ni-1 observations between the same bench marks. Where possible, observations from several Ni-1 instruments over the same section were used. Figure 4 shows the locations of level lines in the United States that were observed with the Zeiss

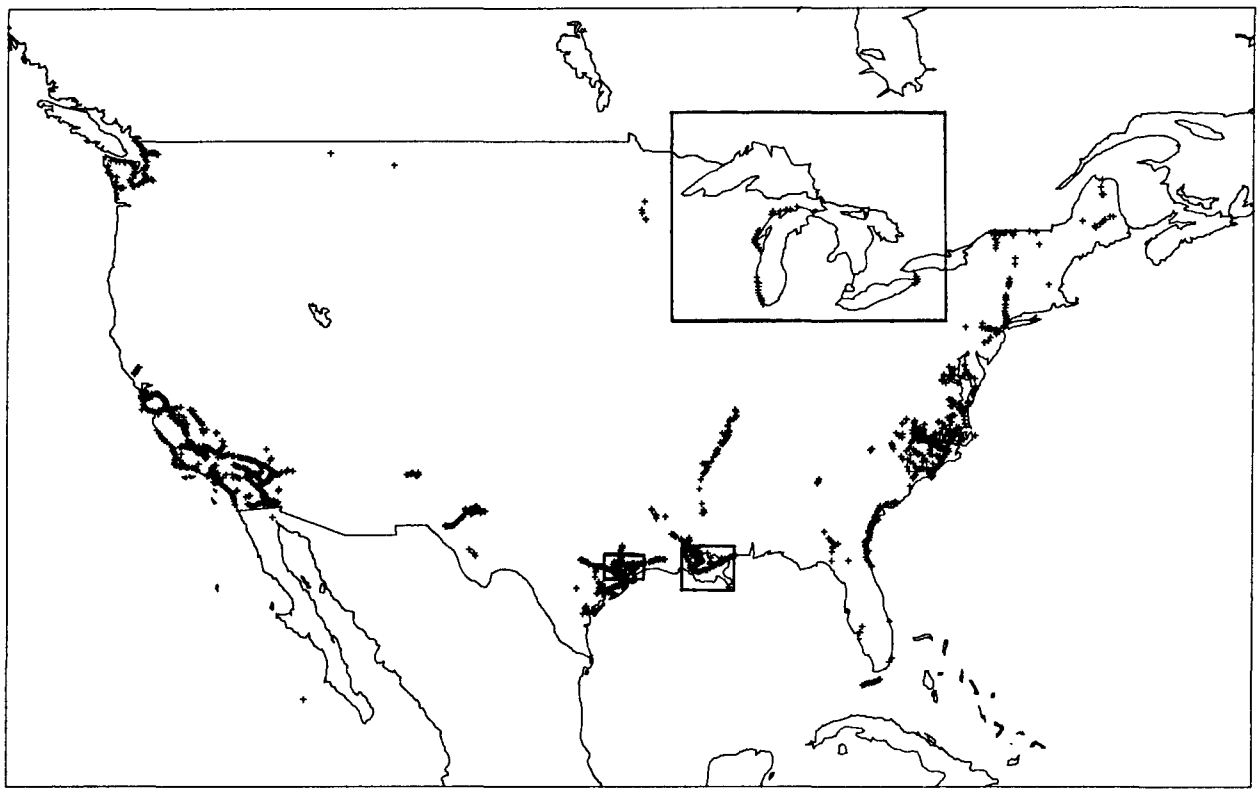

Figure 4.--Locations of Ni-1 leveling data. Conterminous U.S. blocked areas indicate where data were excluded from solution to avoid bias due to regional surface deformation.

Ni-1 instrument. Table 2 indicates the amount of leveling that was used to calibrate each Ni-1 compensator. Height differences between consecutive bench marks were used as observations whenever possible, rather than longer sequences. In this way the azimuth of leveling is more refined, and bad observations are more clearly isolated. Where necessary, longer links were formed from sums of consecutive sections until terminal marks were found that were common to both Ni-1 and the control leveling. Longer sequences were also used to span unstable bench marks and very local areas of deformation.

It was necessary to screen out some data that were likely to be contaminated by crustal motions. The elapsed time between Ni-1 leveling and the spirit leveling often exceeded 10 years in areas of known movement and thus precluded use of data from the vicinity of Houston, Texas, the Great Lakes area, and the Mississippi Delta. These zones are blocked out in figure 4. In all other areas, data with very large point-to-point discrepancies between Ni-1 and nonmagnetic observations

Table 2.--Results of empirical calibrations

Serial Number	C mm/km/gauss	Standard deviation	C laboratory	North-south distance (km)	Period of use Year	to year
78300	-3.28	0.75		789.4	1972.00	1975.70
78300	-10.18	2.08		101.1	1975.70	1978.59
78300	-3.64	2.61	-7.59	57.8	1978.59	1980.00
90760	-6.40	1.11		413.8	1973.67	1975.73
90760	-5.29	1.56		209.0	1975.73	1979.24
90760	-4.76	2.45	-11.51	104.2	1979.24	1980.00
90823	1.28	1.14		359.9	1972.00	1974.46
90823	-5.24	1.85		134.2	1974.46	1975.42
90823	-4.10	1.85	-6.38	182.8	1975.42	1980.00
90824	-4.98	.75	-9.04	922.6	1972.00	1980.00
90825	-2.13	1.12		342.4	1972.00	1974.46
90825	-5.83	1.39		282.7	1974.46	1975.91
90825	-4.68	1.81		93.2	1975.91	1978.27
90825	-8.48	2.13	-10.38	104.3	1978.27	1980.00
90829	-3.93	.76		681.4	1972.67	1978.27
90829	-5.57	1.31	-11.28	223.4	1978.27	1980.00
90834	.16	1.37		241.1	1972.67	1975.07
90834	-1.96	1.29		244.2	1975.07	1979.05
90834	-6.86	1.37	-10.65	454.2	1979.05	1980.00
90856	-4.36	1.75		191.9	1974.00	1975.04
90856	-9.87	.91		389.2	1975.04	1978.78
107286	-.42	1.38	-2.90	231.3	1975.50	1981.80
107293	-6.55	1.24		355.0	1975.50	1979.62
107367	-.67	.71	-7.96	854.0	1975.25	1980.00
71130	.48	1.58		347.2	1972.00	1980.00
78298	-5.37	1.46	-11.20	203.2	1972.00	1980.00
90822	-.86	2.61		84.2	1972.00	1980.00
90827	1.37	1.84		114.7	1972.00	1980.00
107298	-2.69	.99		263.3	1972.00	1980.00
107345	-1.52	2.55		57.1	1972.00	1980.00

were also excluded from the solution. Ground noise of up to ± 1 cm, plus a liberal amount of anticipated magnetic error (-8.74b), and up to the normal tolerance limit of random error (± 4 mm$\sqrt{S}$) were allowed. Non-Ni-1 leveling was assumed to be free of systematic errors.

The initial calibration strategy was to assume that a level compensator changed its characteristics each time it was repaired. As no repair records existed prior to 1975, it was assumed that no repairs of compensators occurred before 1975. Under these assumptions the 12 NGS instruments gave rise to 74 different possible unknown correction constants. Repair records were not available for the 23 NI-1's owned by other agencies, therefore only one unknown correction constant was associated with these instruments. Most of these latter instruments did not generate sufficient data to permit resolution of reliable correction constants, and were therefore dropped as unknowns from later solutions.

Results

The first solutions showed that successive correction constants for a single instrument usually tended to be close to one another if the instrument's compensator was not replaced. However, clear exceptions were noted. In later solutions, most minor repairs were ignored and repair intervals combined if successive C values changed insignificantly. It is possible that NGS compensators were replaced, or that major repairs were made to one or more compensators before service records were kept, starting in 1975. Several compensators required multiple correction constants. Ultimately, 30 magnetic correction constants were resolved for 23 compensators belonging to 17 instruments. These compensators are responsible for approximately 80 percent of the Ni-1 measurements in the NGS data holdings. For the remainder of the instruments, there was such a small amount of leveling data available that the derived constants had very large standard deviations and were clearly not meaningful.

Table 2 shows the results of the final solution. The correction constants ranged in value between +1.37 and -10.18 mm km^{-1} gauss^{-1}. The weighted average value for all correction constants is -3.68 $\pm$0.64 mm km^{-1} gauss^{-1}. This is a particularly important number because it is to be used to correct the magnetic error of instruments that could not be calibrated due to insufficient data. This value corresponds to 0.75 mm/km in the magnetic north-south direction at Corbin, Virginia. The simple mean of the 10 correction constants determined in the laboratory is -8.89. The mean of the empirical correction constants for the same compensators is -4.48. Because of the large discrepancy between the mean empirical correction constant and the laboratory mean, it is especially necessary to evaluate both results carefully. This was done by appying both types of corrections to Ni-1 observations in tectonically stable areas to see if the corrected data agree well with spirit leveling and with external standards.

Figure 5 is a comparison between Ni-1 levelings accomplished in the period 1974-80 and spirit levelings performed between 1954 and 1963 along a route beginning at Norfolk, Virginia, and terminating at St. Augustine, Florida. Without any correction the divergence between the two levelings (magnetic error) accumulates to more than 1 m. The divergence is reduced to 18 cm when the empirical correction constants are used, with most of the remaining divergence occurring in the last 200 km beginning at Savannah, Georgia. The constants derived in the laboratory

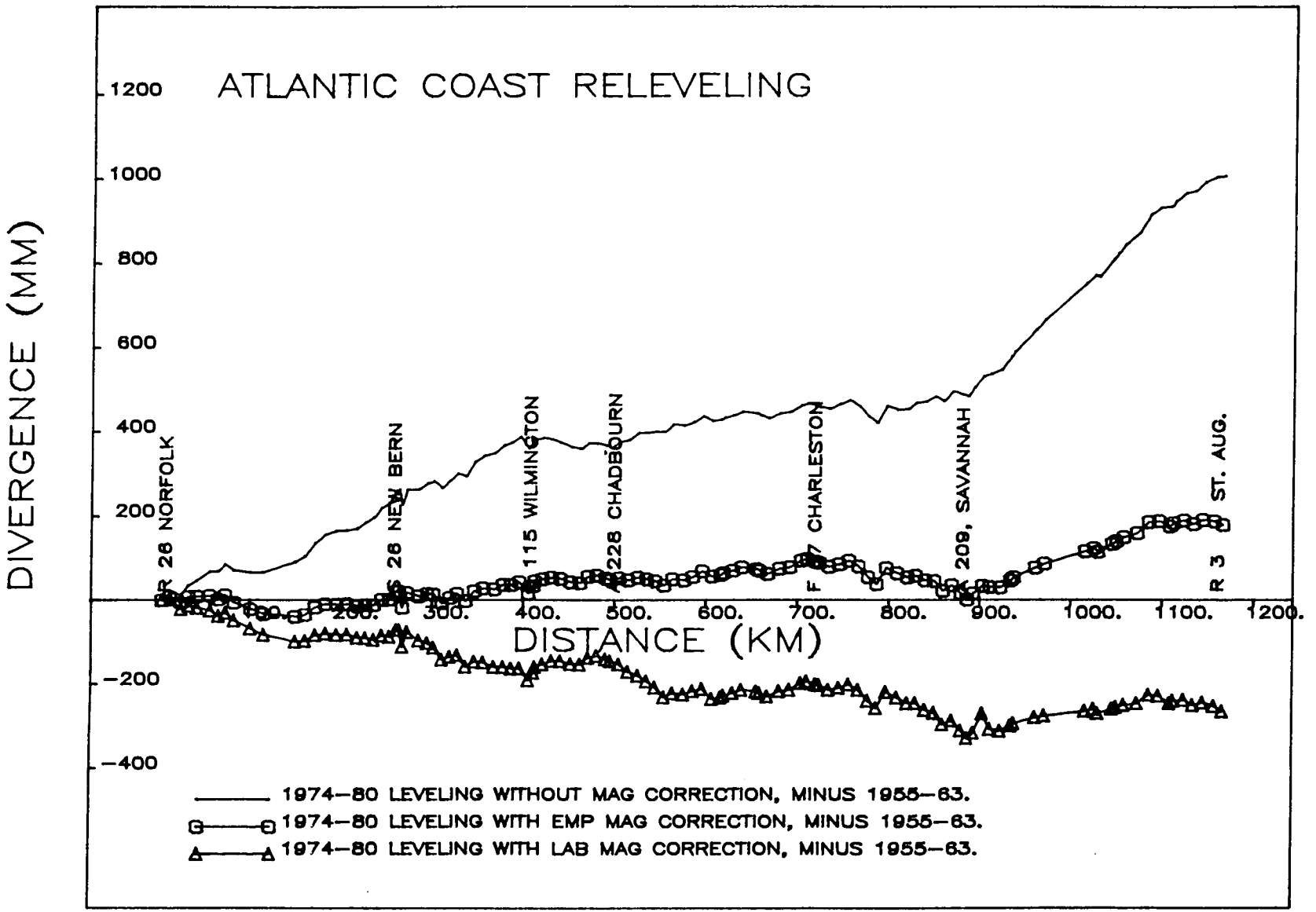

Figure 5.--Divergence of Ni-1 leveling and previous spirit leveling along the Atlantic Coast.

were also used to correct the Ni-1 data. They systematically overcorrect for the first 900 km where the accumulated divergence reaches a maximum of 32 cm. Clearly the empirical constants give better agreement with the spirit leveling. Since this leveling was used in deriving the empirical constants, this result is not entirely surprising. However, the empirical calibration constants for any instrument represent the fit to data in a number of different areas; therefore, it is reassuring to see how good the fit is along a single line.

To more accurately assess the relative merits of the empirical and laboratory calibration constants along this coastal route, comparison was made between the same levelings and local mean sea level (LMSL) values derived from tide gauge records. In this way, the slope of the sea, caused by ocean dynamics, is calculated relative to an equipotential surface. Sea slope near the western Atlantic coast was estimated independently by steric and geostrophic leveling to be 2.2 cm per degree of latitude between latitude 12° and 36°N, rising from north to south (Sturges 1973). This corresponds to about 33 cm of rise from Atlantic City (lat. 38.8^{o}N) to Key West (lat. 23.6^{o}N). Frank Chew (oral communication, March 24, 1986) maintains that a more complete computation, which accounts for ageostrophic dynamics, results in a sea slope that is down to the south by about 10 cm. Figure 6 shows the divergence, leveled height minus LMSL, from Eastport, Maine, to Key West, Florida (Zilkoski and Kammula 1986). Only the leveling south of Atlantic City involves Ni-1 observations. The data corrected with empirically derived magnetic error correction constants show a 10 cm fall in the sea between Atlantic City and Key West, in good agreement with Chew. The same leveling data, when corrected with constants from laboratory calibrations, shows a drop of sea level by 63.5 cm between the same locations. Thus the empirically determined correction constants agree better with the sea slope as determined by either Chew or Sturges. The laboratory constants overcorrect the data.

Another long north-south stretch of Ni-1 releveling exists along the west side of Lake Michigan. Lake levels have been monitored for decades at Chicago, Milwaukee, Sturgeon Bay, and Mackinaw City, which are located on or near this route. Figure 7 shows the divergence between Ni-1 surveys performed in 1972-75, and original spirit levelings accomplished 40 years earlier. The uncorrected leveling shows an agreement which is misleading. Postglacial uplift is well known in the Great Lakes region. Analysis of historical lake level gauge records yields a relative uplift rate of approximately +243 mm per century at Mackinaw City relative to Milwaukee (Tate and Balduc 1985). Figure 8 is a plot of the Mackinaw City water level relative to that at Milwaukee and clearly reveals the linear tilting trend. When the leveling is reduced using empirical correction constants, 47 mm of uplift is calculated for the 40 year period between levelings. This should be considered rough agreement with the 100 mm deduced from lake level gauges between Chicago and Mackinaw City, because the tolerance limit for a circuit misclosure for that distance would be 157 mm. The laboratory correction constants were used to compute apparent uplift also, and yielded an excessive uplift estimate of 429 mm. This is strong evidence that the laboratory calibration constants produce an overcorrection.

The Hudson River profile (fig. 9) starts at New York City and proceeds north to Rouses Point, New York, near the Canadian border. The 1973-74 Ni-1 leveling was accomplished as three connecting surveys: Battery to Rhinebeck, Rhinebeck to Saratoga Springs, and Saratoga Springs to Rouses Point. The southern two segments of the Hudson River profile were leveled in late 1974, before repair records were kept. Originally, these data were badly undercorrected, as were data from the same instruments in subsequent surveys in other locations. Prior data at other locations were overcorrected, indicating the magnetic sensitivity of two instruments

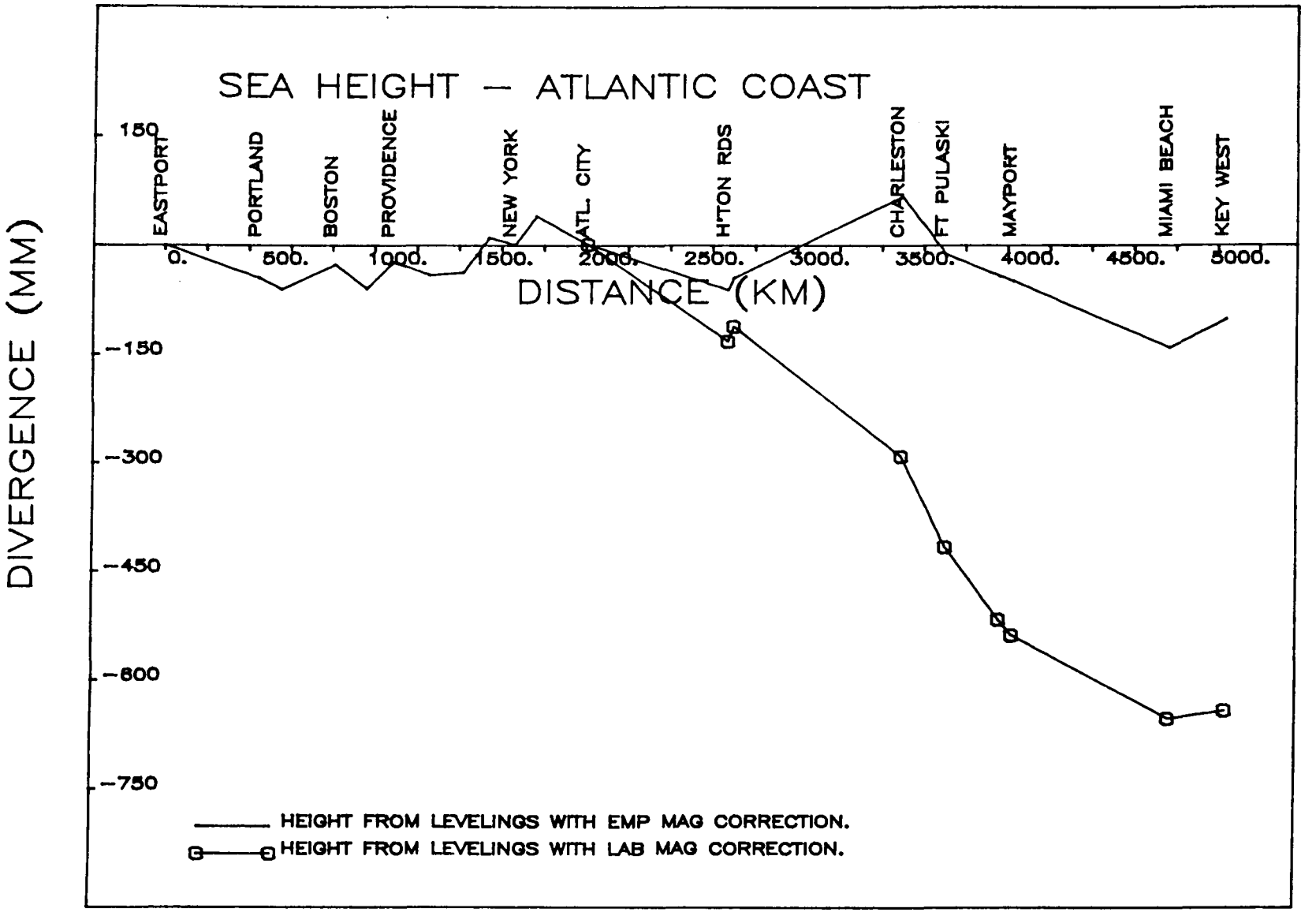

Figure 6.--Sea height along the Atlantic coast. Tide gauge bench marks (TGBM's) are connected by continuous leveling, starting at Eastport, Maine. Plotted values are obtained by comparing leveled heights with local mean sea level values at the TGBM's.

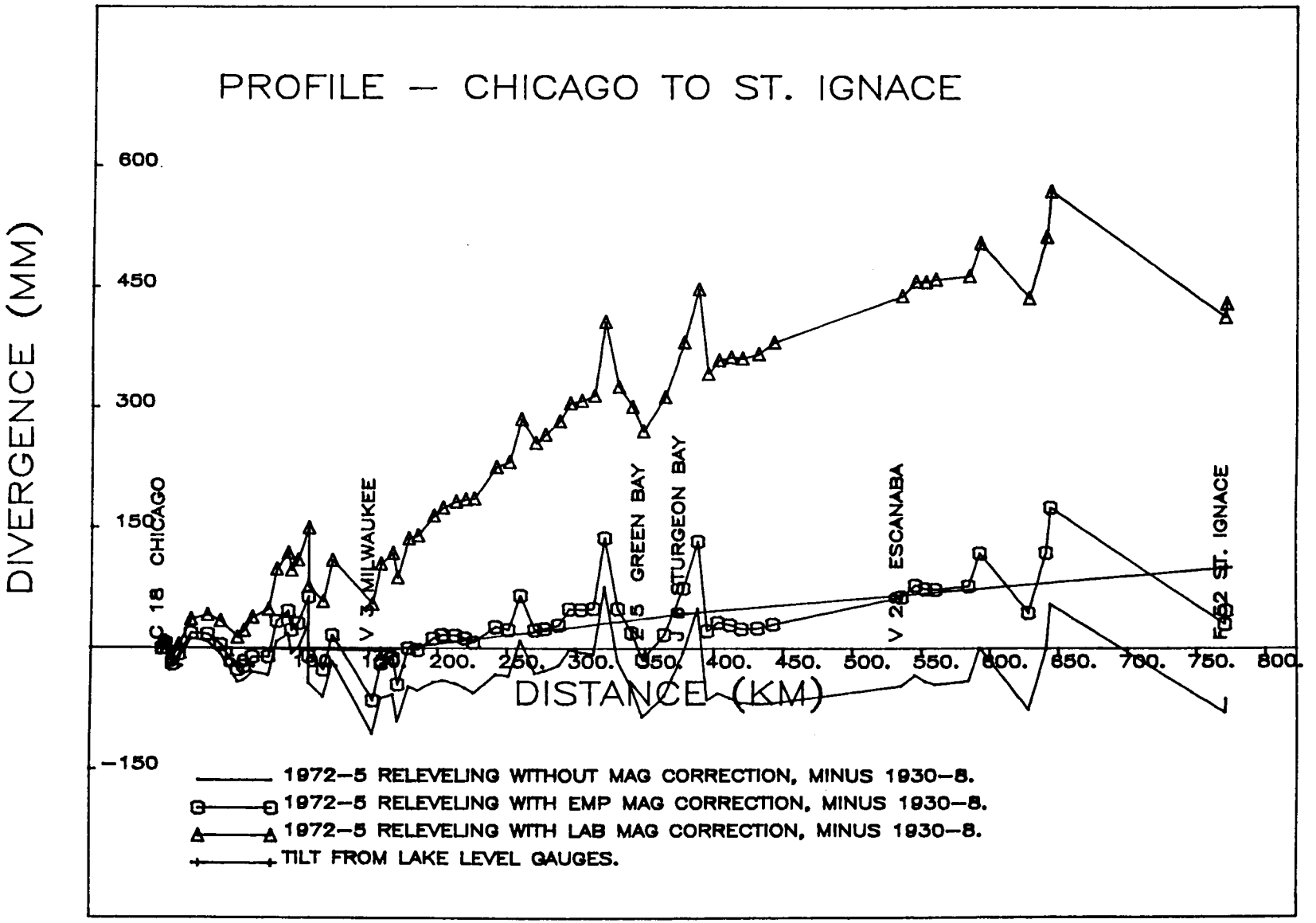

Figure 7.--Relative vertical motion from Chicago, Illinois, to St. Ignace, Michigan.

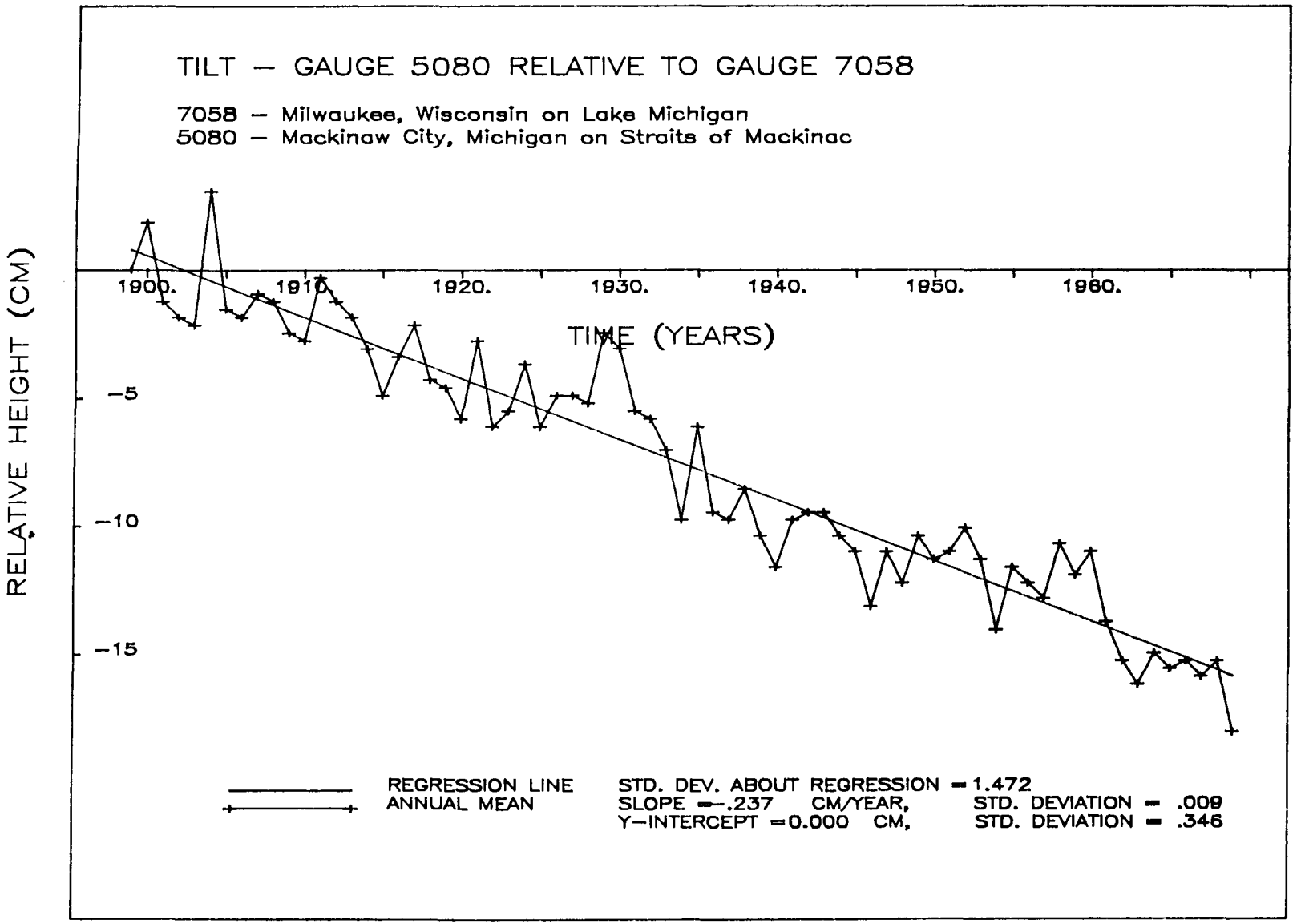

Figure 8.--Plot of annual mean difference of lake levels, Mackinaw City relative to Milwaukee.

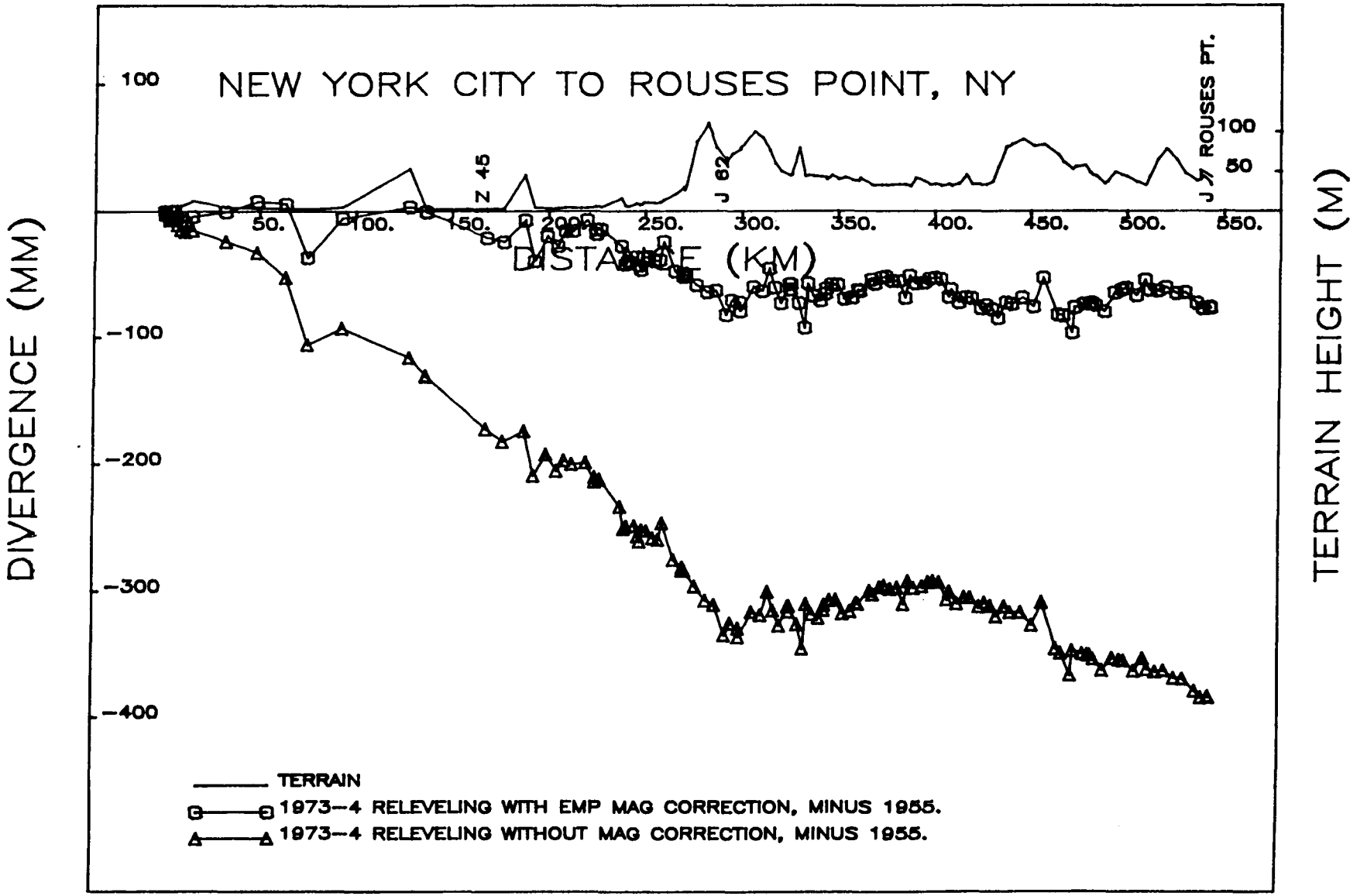

Figure 9.--Divergence of Ni-1 leveling from New York City, along the Hudson River to Rouses Point near the Canadian border.

had significantly increased at about the time that survey work began in New York City. Accordingly, the compensators of the two instruments were allowed to have a "before" and "after" constant in a later solution. For instrument #90823 the "before" and "after" correction constants became +1.28 and -5.24 respectively; and for instrument #90825, -2.13 and -5.83. Using the improved empirical correction constants the uncorrected 384 mm divergence (error) between the 1973-74 Ni-1 leveling and 1955 spirit leveling was reduced to 77 mm.

These large changes in magnetic sensitivity may be due to the exposure to strong man-made magnetic fields in New York City, although repairs cannot be ruled out as a cause because service records were not kept at that time. In New York City, the level line passed alongside large power transmission lines as well as electric train tracks. Some of the bench marks were set in the concrete bases of the power transmission towers. The Ni-1 instrument may have been in close proximity to the transmission tower grounding wires so that its residual magnetic sensitivity was significantly increased. The high level of sensitivity apparently lasted for the duration of the two consecutive surveys leading to Saratoga Springs and for several surveys after that.

Another test of the validity of the empirical correction constants was to recompute circuit misclosures within a network resulting from the Southern California Releveling Project (SCARP). This was one of the largest projects ever undertaken in the United States, involving both Federal and local agencies. Table 3 shows misclosures for 18 SCARP circuits that involve at least some Ni-1 leveling and are at least 10 km in length. Fourteen of the 18 misclosures improved while four became worse. The total of improvements was a substantial 418.6 mm, while the four circuit misclosures were enlarged by a total of only 51.2 mm. Without correction for magnetic error, five of the 18 circuit misclosures exceeded tolerance limits (4 mm $\sqrt{S}$) After correcting with empirical correction constants, only one circuit exceeded the tolerance limit. The same circuits were also corrected using the laboratory correction constants. Eight misclosures improved by a total of 313.4 mm, while 10 misclosures became worse by a total of 442.9 mm. As may be seen from table 3, application of the laboratory correction constants actually increases the total of misclosures for those circuits involving Ni-1 leveling.

A last example illustrates successful application of the laboratory calibration constants in a case where the calibration was performed soon after a test survey in Los Angeles County (Packard and MacNeil 1983). In that test, simultaneous measurements were made with the Zeiss Ni-1 #78298, and a wild N3 spirit level, using a common pair of rods. The survey line was generally directed north. The divergence between these two sets of measurements is 146 mm after leveling 72.5 km. In figure 10, the line segments have been projected onto the magnetic north direction, and thus total distance is only 61 km. The -11.20 laboratory correction constant produces a 171 mm correction over this distance, overcorrecting by only 24 mm. The tolerance limit for the 145 km circuit created by the spirit and Ni-1 leveling is 48 mm. The empirical correction constant (-5.37) produces a correction of 85 mm, undercorrecting by 65 mm. This is the best data set to evaluate the laboratory calibration process because the comparison leveling was simultaneous, thereby eliminating confusion caused by crustal motions, and avoiding bias due to systematic errors such as refraction or rod errors which cancel in the comparison. This result clearly allows the possibility that the laboratory correction constants are correct and might be validly applied to measurements from surveys closely preceding the calibration date. The empirical correction constant was derived from measurements made at least 4 years prior to the test survey, and did not utilize the test survey measurements. The repair record for instrument #78298 indicates it was cleaned and adjusted in

Table 3.--Misclosures of SCARP circuits*

Circuit distance (km)	Uncorrected (mm)	Empirically corrected (mm)	Change (mm) better	worse	Lab corrected (mm)	Change (mm) better	worse
146.99	-34.91	-30.88	4.03		-28.73	6.18	
265.98	-12.73	6.56	6.17		28.25		15.52
10.09	0.97	0.95	0.02		0.93	0.04	
287.31	-75.10	-21.64	53.70		-14.58	60.52	
565.29	-180.97	-91.62	89.35		-33.71	147.26	
372.18	-66.71	-20.27	46.44		-43.92	22.79	
284.64	71.64	5.38	66.26		-85.55		13.91
202.15	24.80	21.02	3.78		-28.21		3.41
292.68	20.95	13.89	7.06		8.01		
251.08	89.76	97.47		7.71	92.77	12.94	
232.49	-112.86	-38.40	74.46		-54.25	58.61	
359.89	-1.56	-1.72		0.16	-1.68		0.12
297.63	36.49	17.54	18.95		-31.42	5.07	
745.80	34.48	-8.59	25.89		116.59		82.11
415.21	-23.84	-52.01		28.17	-115.49		91.65
251.39	58.99	45.75	13.24		99.15		40.16
422.52	19.41	10.13	9.28		-90.95		71.54
401.07	9.17	-24.32		15.15	-130.65		121.48
		Total	418.63	51.19		313.41	442.91

* Circuit greater than 10 km

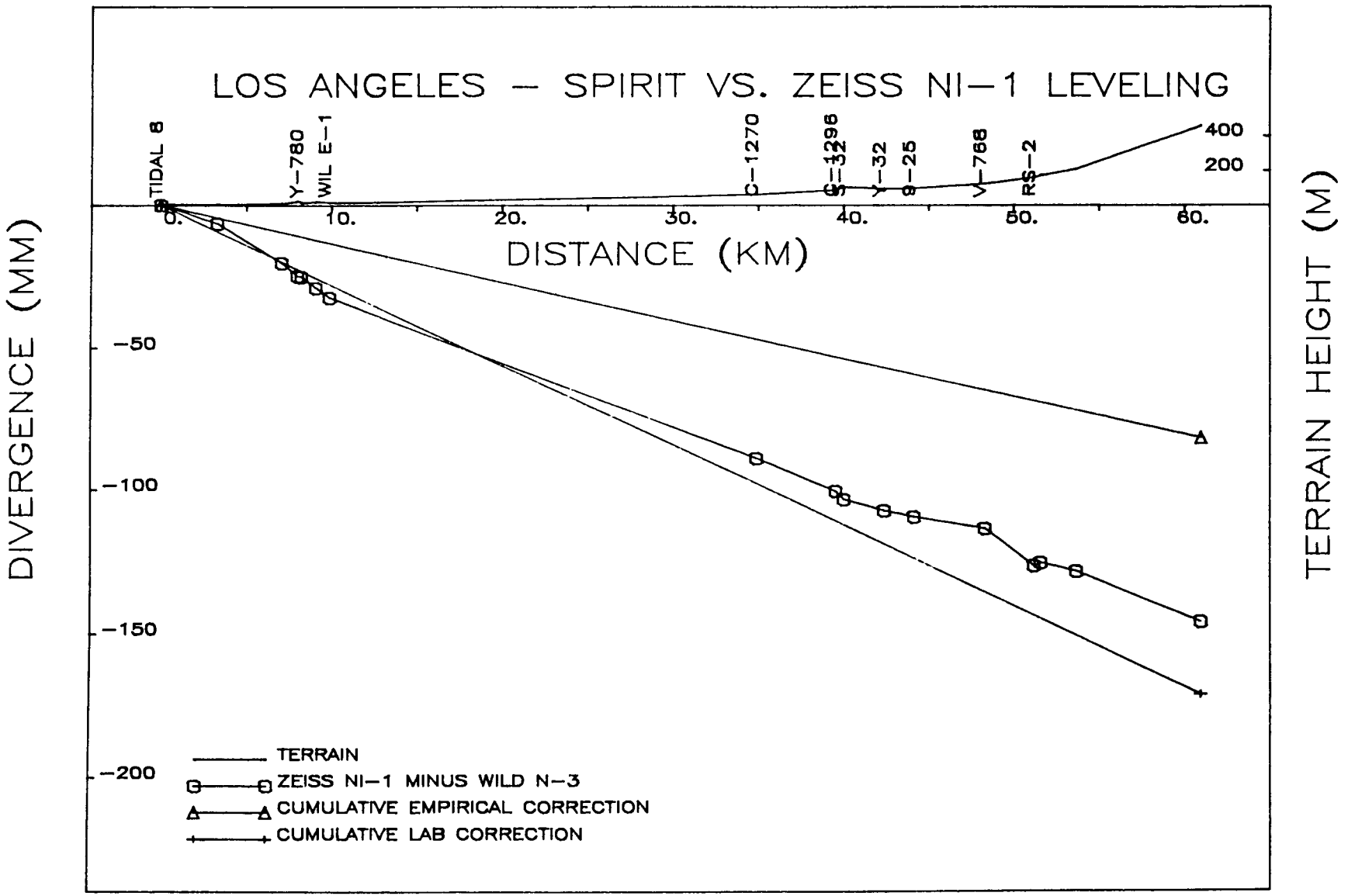

Figure 10.--Divergence of Zeiss Ni-1 and Wild N3 leveling along line extending north from Los Angeles. Accumulated empirical and laboratory corrections are estimated.

1978 following acquisition of the data used to derive the empirical correction constant (oral communication, Robert Packard, April 22, 1986). The instrument was used frequently during the four intervening years, 1978-82.

CONCLUSIONS

The empirically determined magnetic error correction constants worked well in nearly all applications attempted. We should expect empirically corrected Ni-1 levelings on the Atlantic coast and elsewhere to agree with former spirit leveling because of the method used to derive the magnetic error correction constants. However, the general agreement of the empirically corrected Ni-1 data with oceanographically determined sea heights provides an external check on the validity of the correction constants. The Ni-1 level data on the west side of Lake Michigan were not included in the calibration solution because of known crustal motion. Thus, the agreement between corrected Ni-1 data and tilts determined between lake level gauges on Lake Michigan is an external check. The substantial reduction in SCARP misclosures can also be considered an external check because the corrected Ni-1 measurements are essentially being compared to non-Ni-1 measurements in the remainder of the circuit (not included in the calibration solution) rather than other measurements between the same pairs of points.

The Ni-1 correction constants derived from laboratory calibrations are usually too large and produce a result which is not satisfactory when applied to data gathered several years prior to date of calibration. There may be several reasons why the laboratory calibrations were not often successful. The laboratory calibrations were performed on instruments that usually had been repaired at least once. During those repairs, magnetic screw drivers may have imparted some residual magnetic sensitivity to the instrument. This would cause the instrument to have a higher C-value in the latter part of its operational history, and during the laboratory calibration. Exposures to large man-made fields may also cause increases in the instrument's magnetic sensitivity. The laboratory calibration is simple in concept but complex to implement, requiring careful calibration of optical and electronic systems. The optics and mechanical design of the instrument must be understood well. Leitz (1983) warns against using excessive power when calibrating an instrument because its magnetic response characteristics can be changed. In the NGS laboratory, the horizontal component of the magnetic vector was at times 16x greater than the Earth's horizontal component. This is dangerously close to the 20x limit that Leitz states will produce drastic change in the instrument's magnetic sensitivity.

Table 2 indicates that the magnetic sensitivity of a Ni-1 compensator changes (becomes greater) as it becomes older, or is further used, transported, or repaired/adjusted. Instruments 90823, 90825, 90834, and 90856 all finished with empirical correction constants that were twice as large as their initial correction constants. None of these instruments had their compensators replaced. To determine the exact cause of changes requires further study. The empirical correction constants usually perform better than laboratory constants because the empirical calibration allows for the operating history of the instrument to be subdivided into epochs corresponding to differing magnetic sensitivity. The laboratory calibration may overcorrect because the data being corrected were obtained years earlier when the instrument's magnetic sensitivity was lower.

Former users of Ni-1 instruments who wish to correct their old measurements may use an average correction factor rather than derive individual correction constants as described above. A reasonable average value would be -3.68 ± 3.0 mm km^{-1} gauss^{-1}. Judging by the values in table 2, to do so would improve the measurements about

three-fourths of the time. However, because the constants range between +1.37 and -10.18 mm km^{-1} $gauss^{-1}$, the mean value might still leave substantial error.

Corrected Ni-1 leveling data should not be considered as reliable as spirit leveling data obtained with the same observing procedures. When network adjustments involve Ni-1 data, the weight, p, for a single Ni-1 measurement can be calculated as:

$$p = [b^2\sigma_C{}^2 + F\ S]^{-1}$$

where F is the variance per unit distance of a similar non-Ni-1 measurement. For long links in a network involving many Ni-1 measurements, a practical but even less rigorous weighting procedure is to assign half the weight that would be given to a non-Ni-1 measurement. We do not pretend that weighting is a rigorous way to eliminate the influence of systematic error. But for convenience, we assume that after applying corrections the residual magnetic error is random, and ignore that measurements in a network may be slightly correlated through the correction constants.

The empirical method can be used to calibrate other "compensator type" level instruments. The results will not be so conclusive, as with the Zeiss Ni-1, because the error of other instruments is smaller due to a lower mechanical tilt amplification than that of the Zeiss Ni-1 (15.5x). If we assume that the magnetic error of an instrument is proportional to its mechanical tilt amplification, G, we can use the average Ni-1 value (-3.68 mm km^{-1} $gauss^{-1}$) to obtain a crude estimate of its correction constant, C, by the ratio

$$C = -3.68\ G/15.5.$$

Mechanical tilt amplification for the Zeiss Ni-2 is 1.7x. This leads to an estimated average value for the Ni-2 correction constant of -0.404, and corresponds to about 0.1 mm km^{-1} in southern California, in the magnetic north-south direction.

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