NOAA Technical Memorandum NOS NGS 77

Geodetic Survey of NIST and JILA Clock Laboratories

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February 2019

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NOAA – National Geodetic Survey

Introduction

Einstein's theory of general relativity indicates that when any clock is operated at a location "higher" than another ("up" is measured away from the mass that generates the local gravity field) it will be observed to run faster. That is, it will appear to "tick" at a higher frequency to those observers "below." In our everyday experience this effect is unnoticeable, but groups at the National Institute for Standards and Technology and JILA (University of Colorado) in Boulder, Colorado are developing atomic clocks (so-called "optical" clocks and "optical lattice" clocks) with accuracies approaching a few parts in 10^{20}. [[1],[2],[3]] At this level, changes in a clock's height of even a few millimeters will cause a noticeable difference in its output frequency and must be accounted for.

Technically, it is changes in the potential of the Earth's gravity field – known as "geopotential" – to which the clocks are sensitive. To facilitate comparisons amongst clocks within a laboratory and/or between different laboratories, NOAA's National Geodetic Survey (NGS) agreed to measure geopotential differences at various locations between the NIST and JILA facilities. This includes both the laboratory for the existing time standard as well as the new, experimental clock laboratories. With geodetically-determined geopotential differences in hand, a prediction of the expected frequency difference between any two NIST-JILA laboratories will be possible immediately.

This is an update to a technical memorandum (NOS-NGS-73) describing the NIST-only survey of 2015[[4]]. In 2018, NGS extended the NIST geopotential network to new sites on the University of Colorado campus, including the lattice clock laboratories in JILA. Those results, along with the existing NIST results, will all be presented here for ease of use. The techniques and instruments used in 2018 were more or less identical to those of 2015, and any differences will be noted below.

Looking forward, the hope is that one day these clocks can then be linked across continental or even global scales. Once the difference in local geopotential values is taken into account, it will allow for the direct comparisons of clocks for metrological, time distribution purposes. Further, the process can also be reversed: observed differences in the frequencies of clocks operating at far-flung locations can be used to infer geopotential differences directly; so-called "Chronometric Leveling." [[5],[6]] This real-time "geo potentiometer" would revolutionize the field of geodesy.

Background and Nomenclature

Geopotential

All mass generates a gravitational field potential, V, which surrounds that mass and extends to infinity. As one considers points that lie further and further away from such a mass, the gravitational potential at those points decreases (linearly) as the inverse of the distance from the mass.* Further, if the body is rotating and the point under consideration is rotating at the same angular rate as the body, then the point also has centrifugal potential, Φ. The combination of both potential sources is known as the *gravity* potential (or "geopotential"), $W = V + \Phi$, and the combined gravitational and centrifugal fields are called the *gravity field* of that mass.

The gravity vector is given by the gradient of this potential: **g** = grad W. It is the force acting on a unit mass, has units of acceleration, and decreases as the inverse square of the distance from the source mass.

Gravity Units

Here we must pause and introduce the somewhat archaic units that geodesists use to quantify acceleration. The acceleration of a falling mass due to the Earth's gravity field at its surface, g, is typically measured in (the c-g-s) units of Gals (after Galileo):

$$1 \text{ Gal} \equiv 1 \text{ cm/s}^2$$
$$g \approx 980 \text{ Gal}$$
$$1 \ \mu\text{Gal} \approx 1\text{x}10^{-9}\text{g}$$
$$1 \ \mu\text{Gal} = 10 \text{ nm/s}^2$$

The current accuracy limit for state of the art gravity meters is on the order of 1 μGal.

The Geoid

Returning to a gravity field surrounding a mass, note that the equation, W = *const* describes a surface of constant potential surrounding this mass. There are an infinite number of these so-called "equipotential surfaces." Even for a complicated distribution like the Earth, these surfaces are continuous and do not intersect any other equipotential surface. However, they are not, in general, parallel to each other; the distance separating any two given equipotential surfaces is not constant. See Figure 1.

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* Here, and throughout this paper, we will follow the convention used in the geodetic community. This is the opposite of that used in the physics community, where bodies moved away from a mass are said to have *more* gravitational potential.

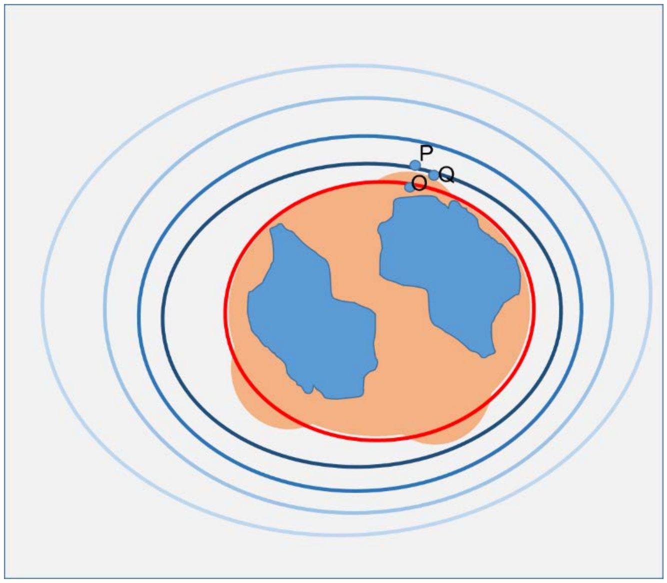

Figure 1. Two dimensional representation of equipotential surfaces around an irregular massive body. The surface depicted in red is defined as the geoid.

On the surface of the Earth, one can define a special equipotential surface that most closely coincides (in a least-squares sense) with mean sea level. This conceptual surface is referred to as the "geoid" (the red surface in Figure 1) and serves as a natural basis (or "datum") for a height system. Determining its shape is a fundamental problem of geodesy.

The "geopotential number," C (not to be confused with the speed of light, c), at a given location P, is defined as the difference between the geopotential at point P and at a point O on the geoid, measured along the plumb line:

$$C_P = -(W_P - W_0) = -\int_O^P g\,dH \approx H(g + 0.0424H). \qquad (1)$$

This last (Helmert) approximation takes into account the terrain between a point on the surface of the Earth and the geoid. In it the acceleration of gravity, g, is measured in Gals, and the height, H, is in kilometers. Because g is approximately 10 m/s^2, the result is that the geopotential number for a point on the surface of the earth is quite close to the value of its elevation above the geoid in meters.

By way of nomenclature, note that geopotential values are typically reported in "geopotential units":

$$1 \text{ gpu} = 10 \text{ m}^2/\text{s}^2 = 1 \text{ kGal-m}.$$

Finally, as was mentioned, the theory of general relativity describes how a clock's frequency changes with geopotential. The fractional change in frequency between two locations P and Q (each on a different equipotential surface) is given as:

$$(f_P - f_Q)/f_Q = (W_Q - W_P)/c^2 = -(C_Q - C_P)/c^2, \qquad (2)$$

where c is the speed of light.

It is interesting to note that – near the surface of the Earth, and for a given vertical displacement – the value of the geopotential number is actually about one thousand times more sensitive to the change in height than to the change in gravity. This is because (given the size and density of the Earth) a 1 meter change in height (1 part in 1.6×10^3 in Boulder) is only a 300 µGal change in gravity (3 parts in 1×10^7). For the purposes of the present work, this is important: when comparing the rates of two clocks at different locations, it is their height difference that dominates their geopotential difference. In fact, between two labs at the NIST-Boulder campus gravity can usually be taken as a constant, and the height difference used to directly estimate geopotential (and thus frequency) differences. That said, in the work described in this report the actual measured gravity values were used in all calculations of geopotential (unless otherwise stated).

Local versus Global Geopotential Reference

As will be described in detail below, the height measurements performed at NIST during this project have exceptionally high differential accuracy. However, when referring these height (and resulting geopotential) values from Boulder to the geoid (to determine absolute values of height and geopotential), some (temporary) complications arise. NGS is currently in the midst of the airborne campaign portion of the "Gravity for the Redefinition of the American Vertical Datum" (GRAV-D) program, [7], to model the geoid using gravity based measurements and update the most recent, classic-leveling based datum, NAVD88. [8] The project has many aspects, but the idea is to use short, spatial-wavelength gravity data from aircraft to supplement the long wavelength GRACE and GOCE space-based geoid models. By 2022, the goal is to have the geoid determined with 1 cm absolute accuracy throughout the United States.

So, as in 2015, the height and geopotential results will be rigorously quantified *relative* to a bench mark on the NIST campus (Q407; arbitrary but convenient). For quantities referenced to the geoid, this same rigor will not be possible until the airborne gravity portion of the GRAV-D project is complete (though some rule-of-thumb values are provided). The problem is that the current vertical datum, NAVD88, has both a known bias and a continent-wide slope relative to the geoid. When the new geoid is published it will be a simple matter to "tie" the results from this project to the new datum. At that point, continent-wide projects that require few-centimeter accuracy will be possible in the United States.

Survey: Instruments and Methods

The geopotential survey of 2018 was again divided into approximately four independent tasks: mark setting, leveling, absolute gravity measurements, and gravity gradient measurements.

Mark Setting

In general, mark setting involves the placement of permanent brass disks, each with a specific pinpoint in it, at various locations for the purpose of representing a unique, survey-able point in space. NGS maintains a database of these marks: some are used for horizontal location control, some vertical location control, some for gravity values, and some for a combination of the above. For the NIST survey (2015), NGS installed six new permanent gravity and height control marks:

- ATOMIC 1 on the floor of the F1 lab, Building 1, Room 2048.
- ATOMIC 1V on the “north” wall of the F1 lab, Building 1, Room 2048.
- NIST 101 on the floor near the foyer at the north end of the main hallway in Building 81
- NIST 102 on the superstructure on top of Building 81 (currently height control only)
- 1H116 on the floor of the “Aluminum” lab in Building 81, Room 1H116
- 811G104 on the floor of the “Ytterbium” lab in Building 81, Room 1G104

For the JILA survey (2018), NGS installed five new permanent gravity and height control marks:

- STS135V near the stairwell entrance to the JILA building on the west side
- JILA S1B60H in the floor of the Ye lab, Room #S1B60
- JILA S1B60V1 in the north wall of the Ye lab, Room #S1B60
- JILA S1B60V2 in the east wall of the Ye lab, Room #S1B60
- JILA X1B21 in the floor of the Thompson lab, Room #X1B21

Figure 2 and Figure 3 show the approximate locations of the newly installed bench marks.

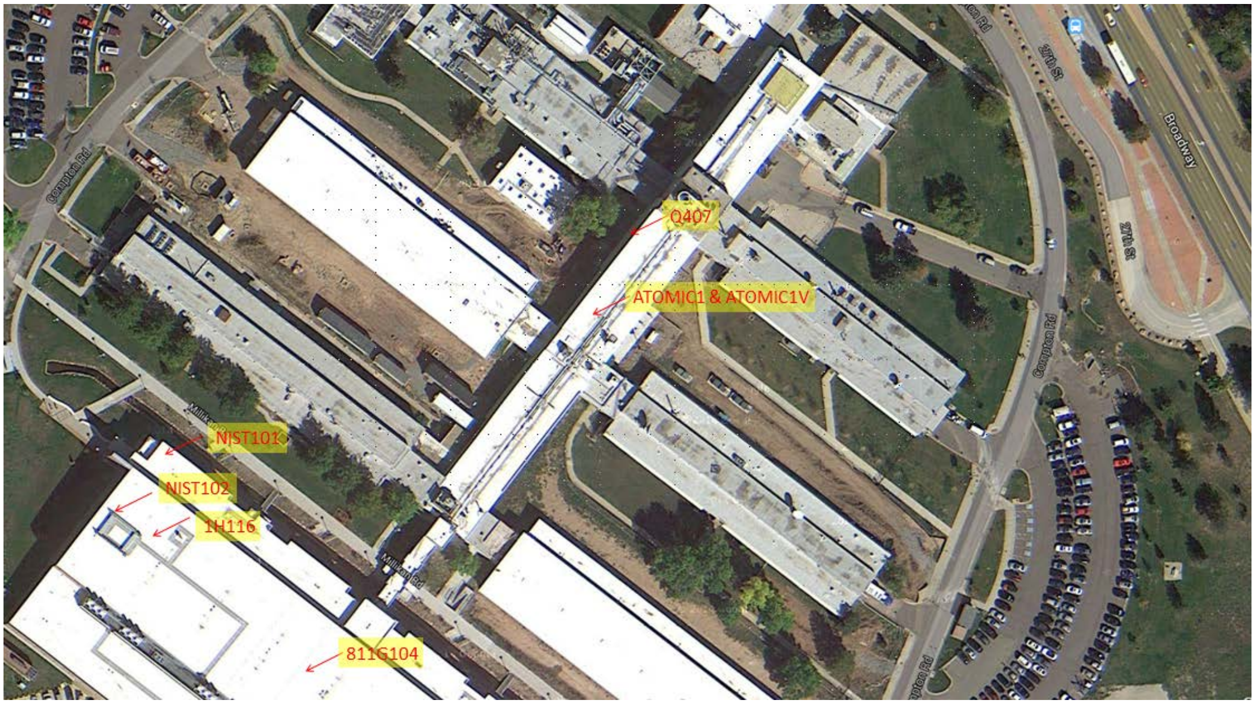

Figure 2. Approximate locations of the NIST (2015) bench marks. North is up.

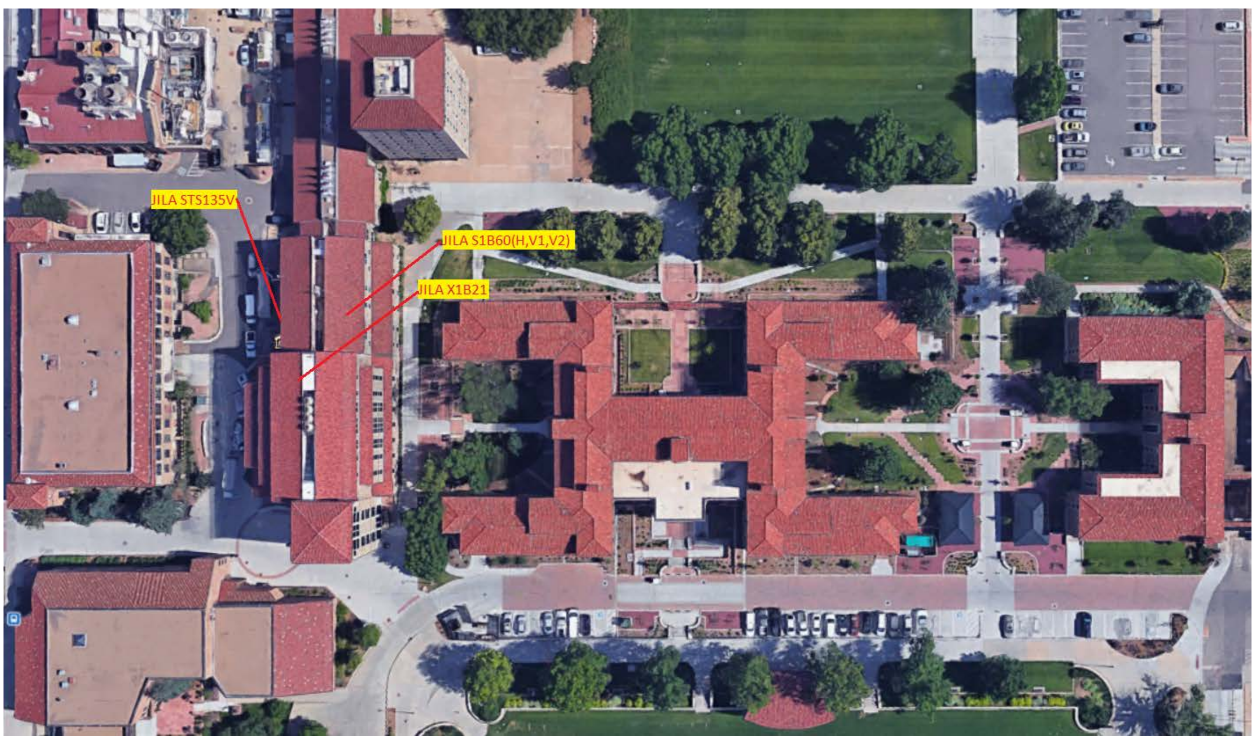

Figure 3. Approximate locations of the JILA (2018) bench marks. North is up.

In each case, the mark was stamped with a name (not to be confused with the unique, NGS database [9] point identification number, or "PID"), and epoxied into a hole in the ground (or wall). See Figure 4 and Figure 5 for examples of mark placement.

Figure 4. Examples of a floor and wall mark in the F1 clock laboratory (ATOMIC1 and ATOMIC1V).

Figure 5. Examples of a floor and wall mark in a JILA clock laboratory (JILA S1B60H and S1B60V1). The photo is taken facing north.

Finally, a few more "locations of convenience" were surveyed in a few of the laboratories: locations of optic tables, laser beam height for clocks, etc. These will be listed in Appendix A.

Geodetic Leveling

The (simplified) principle behind leveling between two ground-based bench marks, A and B, is this: A precision height measurement rod is placed on one of the marks in the ground (A), another rod is placed approximately 10 m towards the next mark (B) (the distance is not critical and depends on the slope of the ground), and a precision level is placed between them. The difference in height is established between the two rods. The first rod is then moved approximately 10 m on the "far side" of the second rod, in the direction of mark B. The precision level is again placed between them. The new height difference between the rods is then established, and so on. Eventually mark B is reached, and the difference in height between A and B is determined by the summation of all the intermediate relative heights. (An example of a typical set up is shown in Figure 6.) The final precision of the difference depends strongly on the distance between the marks and the number of steps taken to traverse this distance. Further, to meet a so-called "First Order, Class II" specification in the Federal Geodetic Control Subcommittee (FGCS) system, the height from B back to A must also be measured (referred to as a "double run"), and the discrepancy must be smaller than a defined value (dependent on distance and number of steps) [10]. Next, the leveling measurements are corrected for [11]:

- Rod Temperature
- Level Collimation (applied in the digital level instrument)
- Refraction
- Astronomic Correction
- Scale factor adjustment for each rod

Finally, due to the non-parallel nature of geopotential surfaces, this method also must take into account the actual changes in gravity along the route between A and B. This combination of geometric observations (rod readings) and gravity changes is ultimately used to compute geopotential differences between points A and B.

The data are then statistically analyzed and errors approximated through a least-squares adjustment (see the LOCUS software discussion below). This combination of careful field methods and data analysis can be shown to yield very repeatable differential height accuracies: After the least squares adjustment, First Order, Class II survey differential height accuracies are approximately 0.7 mm per square root of the traverse length in km. [12]

Figure 6. Leveling example. One rod is clearly visible in the foreground, the leveling unit is visible further down the street, and the far rod is just visible behind that.

Figure 7. Leveling down the stairs into the JILA basement.

Phase 1. Consistency Check of Existing First Order Network Bench Marks

A crew of three was assigned to establish precise heights for the newly established marks. This process begins by re-leveling between three previously established vertical control marks and confirming that their relative height differences are consistent with the information in the NGS database. The three marks recovered in 2015 were already considered First Order, Class II:

- Q407 (PID KK1350) on the north side of NIST Building 1
- J440 (PID KK1563) SW of the intersection of 17th and King St, NW of the NIST campus
- R405 (PID KK1351) NW of the intersection of Broadway and Dartmouth, SE of the NIST campus

In addition to the three marks recovered in 2015, two more were included in 2018 (and were also already considered First Order, Class II):

- Q407 (PID KK1350) on the north side of NIST Building 1
- J440 (PID KK1563) SW of the intersection of 17th and King St, NW of the NIST campus
- R405 (PID KK1351) NW of the intersection of Broadway and Dartmouth, SE of the NIST campus
- G321 RESET (PID LL0704) on the SW corner of CU campus, near the Law Building
- NIST 101 (PID DP9514) in the north foyer of NIST building 81.

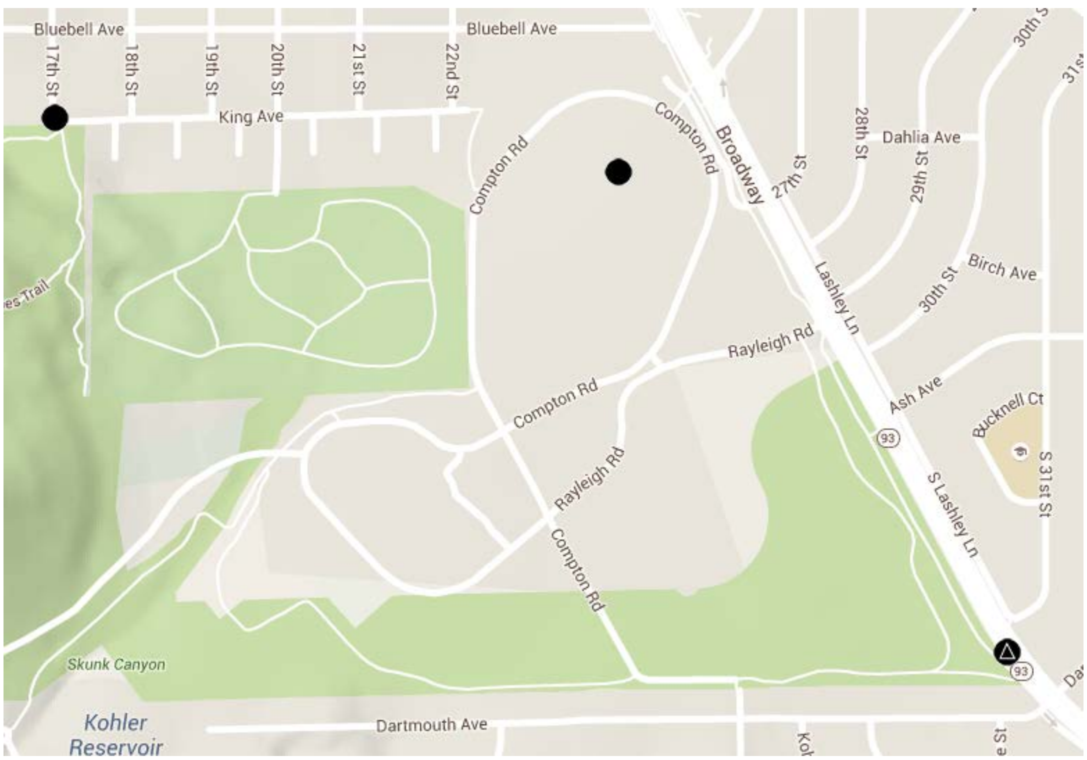

Figure 8. Locations of pre-existing vertical control bench marks (2015). J440 to the northwest, Q407 on the NIST building in the center, and R405 near Dartmourth and Broadway to the southeast. (As a scale, Dartmouth Avenue is approximately 600 m long.)

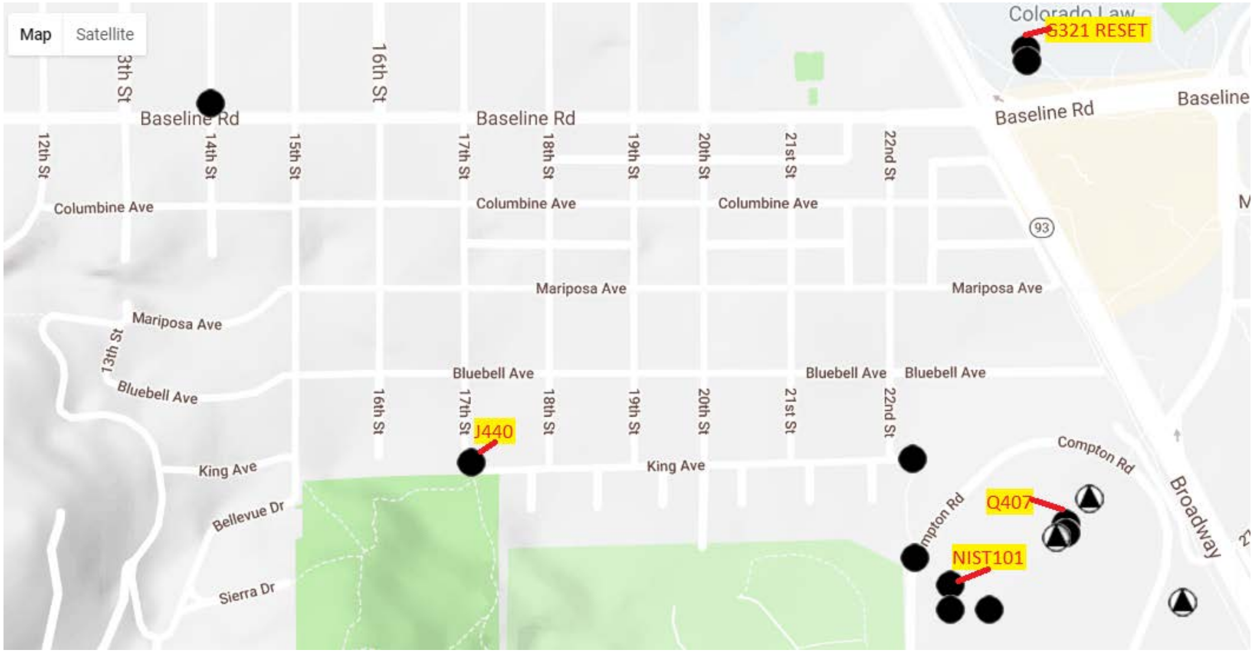

Figure 9. Locations of pre-existing vertical control bench marks (2018).

The re-leveling of these existing marks (2015 and 2018) confirmed the database height *differences* to within 2 mm. This indication of consistency then allows any of one the bench marks to be used as the basis for the creation of new First Order marks, provided that First Order, Class II field procedures are followed when establishing the new marks.

Phase 2. Determine Heights of New Marks. Because of its convenient location on the NIST campus, Q407 was the obvious choice as a base from which to establish precise heights for the new marks.

- 2015. From Q407, the heights of ATOMIC1 and ATOMIC1V were established via a double run. Then, from Q407 again, the heights of NIST 101, 1H116, and 811G104 were determined via a separate double run.
- 2018. From Q407, the heights of JILA STS135V, JILA S1B60H, JILA S1B60V1, JILA S1B60V2, and JILA X1B21 were established via a double run.

Figure 10. Leveling operations near Q407 (north side of Building 1). Q407 is visible on the wall just above the leveling unit.

Finally, because First Order leveling cannot take place up and down multiple flights of stairs (the stack up of errors becomes too large over a short distance), the height of NIST 102 (2015) was determined by "non-reciprocal trigonometric leveling." First, two "auxiliary" First Order vertical control marks were established on the ground (two small, unnamed brass plugs are mounted in the sidewalk north of Building 81). The heights of these marks were determined by double run from Q407. A "total station" was then used to determine the distance and angle to a corner cube retro-reflector mounted on the mark on the roof of the building. This method is "non-reciprocal" because the reverse procedure (mounting a total station at the roof mark and taking angle and distance measurements down to the ground points), was not performed due to time constraints and access issues. As such, certain atmospheric refraction errors will not cancel and the expected total error is larger in non-reciprocal, versus reciprocal, trigonometric leveling surveys.

LOCUS. For both phases of the leveling survey, a least-squares analysis is used to determine the relative height differences between the bench marks. This NGS software package, LOCUS[13], also provides geopotential numbers according to Equation (1), where it typically uses a modeled value for the gravity at each location (the software is intended for massive leveling projects in which an actual absolute gravity observation on each bench mark is not feasible).

For this project, at every location, the LOCUS-determined heights were actually identical (to better than 1 mm), whether or not modeled or measured gravity values were used. It is important to remember this if one retrieves bench mark information from the NGS database: the modeled gravity values listed will not match the measured gravity values (below), but the final height values will not be significantly affected.

Finally, note that for this report, the measured gravity values **are** used in the geopotential calculation. This is because the highest levels of accuracy were desired in this report and the gravity values provided by NGS in the LOCUS software are generally (a) decades old and (b) interpolated to our points of interest. Both of those issues did have the possibility of adding error to the study and so new gravity measurements were felt to be preferable from a purely scientific standpoint, although numerically it may not have had any significant impact.

Absolute Gravity Measurements

At each ground-based mark (2015: ATOMIC1, NIST101, 1H116, 811G104 and 2018: JILA S1B60H) absolute gravity values were determined using an interferometer-based, freefall gravimeter. NGS owns and operates FG5X-102, manufactured by Micro-g LaCoste. [[14,15]] The instrument operates by placing a retroreflector into freefall in a vacuum chamber, and its position is tracked with a frequency stabilized laser. The number of zero crossings in the interference signal is used to determine distance, and the time of the zero-crossings is recorded with a calibrated rubidium clock (nominal frequency 10 MHz). Proprietary software included with the gravimeter applies a least-squares fit of the data to the equation of motion. [[16]] This is used to determine the free parameter, g. Corrections are made for earth tides, ocean load, polar motion, and barometric pressure and an independently determined vertical gravity gradient (see below). Occupation times are usually in multiples of twelve hours to insure that any residual error in the tide model averages to zero (and usually acquired overnight to minimize ambient seismic noise).

Note that gravity was actually determined at the instrument height of approximately 140 cm that varied slightly between each setup. As described below, the vertical gravity gradient was measured separately and used to reduce the gravity value to the height of the mark. Finally, due to ease of access, the absolute value of gravity at site JILA X1B21 was determined by relative transfer (difference measurement) from JILA S1B60H using a Scintrex CG-6 portable relative gravimeter[[17]].

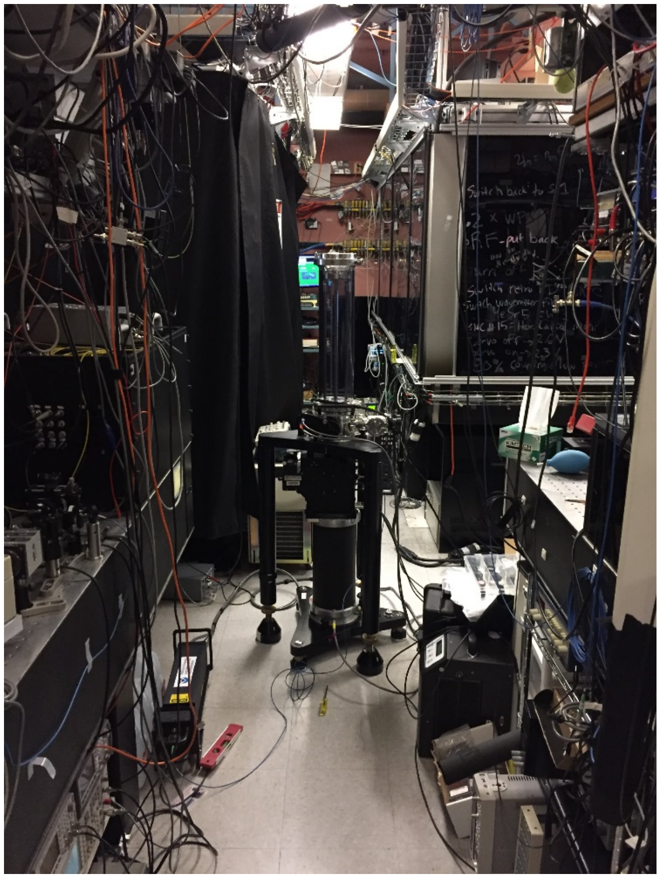

Figure 11. The FG5X absolute gravity meter was operated on each horizontally mounted mark overnight (with the exception of JILA X1B21 which was established by relative tie). It is shown here occupying JILA S1B60H, facing east. The glass vacuum chamber is visible on a tripod above the laser interferometer.

Relative Gravity Gradient Measurements

As one moves away from the center of the Earth, the acceleration due to gravity decreases in accordance with Newton's law. If the Earth were a perfect sphere, gravity would decrease at a nominal rate of 3.086 µGal/cm (near the surface). On the actual surface however, this value can fluctuate substantially: from <2 µGal/cm in a valley to >4 µGal/cm on a mountain top.

At each ground-based mark the vertical gravity gradient was measured using a LaCoste & Romberg g-Meter (2015)[18] and the Scintrex CG-6 (2018). These are small, spring-based relative gravity meters, each with a precision of a few µGal. The relative difference in gravity was measured between three, fixed height tiers on a rigid tripod, set up over each bench mark. A quadratic fit for gravity-as-a-function-of-height is then used to transfer the absolute value from the FG5 X measurement height down to the height of the top of the mark.[†]

[†] Due to limitations in the FG5X software, a linear approximation of the gradient is used to account for the increase of gravity during the freefall; approximately 90 µGal in 30 cm. This determines g at the top of the drop. The quadratic estimate of the gravity gradient is then used to separately transfer this measurement down to the bench mark.

In the case of vertical marks (2015: ATOMIC1V, 2018: JILA S1B6V1, JILA S1B60V2, and JILA STS135) the gradient was also used to transfer the FG5X value from ATOMIC1 to the height of ATOMIC1V, and similarly for S1B60H. (In all cases, the horizontal gravity gradient from the FG5X measurement height to ATOMIC1V was assumed to be negligible).

Finally, for multiple reasons, the FG5X cannot occupy the roof site, NIST102. The original idea was to use the relative gravity meter to “tie” the gravity value from NIST101 up to NIST102. Unfortunately, access to the roof was not available during this phase of the survey. To provide a decent estimate of the gravity value (compared with the global model used in LOCUS), the vertical gradient was used to transfer the NIST101 value up to the height of NIST102. Should roof access become available later, performing the relative instrument tie at a later date is a fast and straightforward process.

Data Reduction and Results

Height Results

LOCUS derived heights and uncertainties, relative to Q407, are tabulated below. Note that the four external control pints (Q407, J440, R405, and G325 RESET) were all held as fixed constraints. Their relative uncertainty is set to zero.

Table 1. Station heights relative to Q407 (meters).

Station	Height (m)	Uncertainty (m)
Q407	0.000	
J440	23.5180	
R405	-5.1072	
G325 RESET	-8.6668	
ATOMIC1	-1.3638	0.0008
ATOMIC1V	-0.0919	0.0012
NIST101	7.4462	0.0016
NIST102	21.8645	0.0036
1H116	7.4382	0.0018
811G104	7.4337	0.0018
JILA STS135V	-4.5718	0.0025
JILA S1B60H	-11.1790	0.0025
JILA S1B60V1	-9.7874	0.0025
JILA S1B60V2	-9.8227	0.0025
JILA X1B21	-12.4008	0.0025

Raw Gravity Values

Table **2** lists the absolute gravity values measured at the top of the freefall trajectory.

Table 2. Raw Gravity Values at measurement height (µGal)

Station	Acceleration of gravity (µGal)	Uncertainty (µGal)	Measurement Height above mark (cm)
ATOMIC1	979 603 970.0	2	139.3
NIST101	979 602 007.7	2	139.1
1H116	979 601 993.5	2	139.7
811G104	979 602 006.2	2	139.9
JILA S1B60H	979 607 512.6	2	139.6

Gravity Gradient Values

Below are the gravity gradient parameters used to determine the change in gravity as a function of height above the mark (note gravity decreases as z increases):

$$dg/dz = az^2 + bz + c$$

Table 3. Gravity Gradient parameters

Station	a (µGal/cm^2) ± 0.0005	b (µGal/cm) ± 0.01
ATOMIC1	0.0014	-2.69
NIST101	0.0005	-3.03
1H116	0.0011	-3.06
811G104	0.0002	-2.84
JILA S1B60H	0.0003	-2.55

Reduced Gravity Values

The gradient formula is then used to transfer the gravity value measured at the top of the freefall trajectory down to the bench mark height. Table 4 lists the reduced (at mark height), absolute gravity values The modeled values for the vertical control marks J440, R405, and Q407 are included for completeness (the FG5X was not operated on these sites).

As was mentioned, gravity was not measured on NIST102. As an estimate, the vertical gradient measured at NIST101 was used to transfer the gravity value of NIST101 to the LOCUS-determined height of NIST102. Finally, the tie to JILA X1B21 was performed via relative gravity meter.

Table 4. Gravity Values at mark height (µGal)

Station	Acceleration of gravity (µGal)	Uncertainty (µGal)
J440	979 595 400	Modeled
R405	979 602 100	Modeled
Q407	979 602 200	Modeled
G235 RESET	979 606 763.4	4 (relative to JILA S1B60H)
ATOMIC1	979 604 344.5	3
ATOMIC1V	979 604 026.1	4 (transferred from ATOMIC1 using dg/dz)
NIST101	979 602 429.4	3
NIST102	979 598 070	20 (transferred from NIST101 using dg/dz)
1H116	979 602 420.5	3
811G104	979 602 403.2	3
JILA STS135V	979 606 184.7	20 (transferred from S1B60H using dg/dz)
JILA S1B60H	979 607 868.4	3
JILA S1B60V1	979 607 513.8	4
JILA S1B60V2	979 607 522.8	4
JILA X1B21	979 608 032.2	4 (relative tie to JILA S1B60H)

Geopotential Numbers and Differences

Next, combining these height and gravity values according to Equation (1), we can compute geopotential differences. Table 5 lists the geopotential difference between each NIST-Boulder station and Q407. The difference between any other two marks can be derived from this table as well.

The uncertainties were calculated by expanding Equation (1) and using these values as representative total uncertainties for height and gravity values:

- Height uncertainty: 2 mm (4 mm for NIST-102)
- Gravity uncertainty: 4 µGals (20 µGals for NIST 102 and JILA STS135V)

Below are the geopotential values (at mark height), relative to Q407.

Table 5. Geopotential values (at mark height) relative to Q407.

Station	Geopotential (gpu)	Uncertainty (gpu)
ATOMIC1	-1.3326	0.002
ATOMIC1V	-0.0914	0.002
NIST101	7.2958	0.002
NIST102	21.418	0.004
1H116	7.2879	0.002
811G104	7.2834	0.002
JILA STS135V	-4.4726	0.002
JILA S1B60H	-10.9432	0.002
JILA S1B60V1	-9.5804	0.002
JILA S1B60V2	-9.6150	0.002
JILA X1B21	-12.140	0.002

As was mentioned earlier, actual geopotential numbers (referenced to the geoid) will have to wait until the GRAV-D project is completed in the new few years. But once an accurate geopotential number for Q407 is determined, the above differences can be used to immediately calculate geopotential numbers for all other stations.

Note, that as a rough estimate for "back of the envelope" purposes, the published geopotential value (referenced to NAVD88) for Q407 is approximately 1617 gpu.

Fractional Clock Frequency Differences

Finally, using Equation (2), we can predict the differences in clock output frequency given the geopotential differences in each laboratory (and the associated uncertainty, using 2.5 mm as a nominal height uncertainty). Again, we tabulate the differences from Q407. A more positive value means the clock runs faster.

Table 6. Clock frequency differences (at mark height) relative to Q407.

Station	$\Delta f/f$ x10^{16}	Uncertainty x10^{16}
ATOMIC1	-1.478	0.003
ATOMIC1V	-0.010	0.003
NIST101	8.123	0.003
NIST102	23.827	0.010

1H116	8.114	0.003
811G104	8.109	0.003
JILA STS135V	-4.976	0.003
JILA S1B60H	-12.176	0.003
JILA S1B60V1	-10.660	0.003
JILA S1B60V2	-10.698	0.003
JILA X1B21	-13.508	0.003

Conclusions and Outlook

NGS has established six (2015) and five (2018) new bench marks in and around various atomic clock laboratories at the Boulder campuses of NIST and the University of Colorado (JILA). Classical leveling and absolute gravity measurements were used to determine heights, gravity values, and geopotential differences between the bench marks. In all facets of the project, state of the art instrumentation and procedures were employed, resulting in the highest possible accuracy for such a survey.

Looking forward, it is expected that the fruits of these labors will manifest themselves over various time scales. The geopotential differences can be used directly – and immediately – to calculate the expected frequency shifts between the laboratories. NGS looks forward to hearing of confirmations of the predicted clock frequency differences as they become available.

After the GRAV-D airborne campaign is complete in 2022, NGS will define a new vertical datum for the United States. At that point it will be easy to provide a supplement to this report with geopotential numbers referenced to the geoid, accurate to the ~2 cm level. As continent-scale networks of linked optical clocks become feasible, these absolute geopotential values will be critical for direct clock comparisons (for example, a comparison of clocks between Boulder and NIST headquarters in Gaithersburg, Maryland).

Then, as a worldwide network of clocks becomes available, a consistent, global system for precise geoid determination will be necessary. Realization of such a system is the responsibility of the International Association of Geodesy (IAG) and efforts are already well underway. NGS is responsible for coordination of the U.S. datum into a global system and should be considered a resource when trying to navigate possibly tricky datum transformations.

Finally, from an NGS perspective, it is hoped that one day this whole process can be "reversed," allowing for so-called "Chronometric Leveling:" Networks of precise atomic clocks will be able to provide instant and direct measurements of geopotential differences. Such a real time geo-potentiometer would allow NGS to continue its mission to "…define, maintain and provide access to the National Spatial Reference System…" with unprecedented precision and accuracy. Further, if the clocks have precisions better than $\sim 10^{-19}$, as is expected, they will have surpassed classical surveying's ability to constrain them!

For all of these reasons and more, it is hoped that the relationship formed between NGS and NIST-Boulder will stay strong, and that collaborations such as this will occur regularly for the foreseeable future.

Appendix A

In 2015, heights and geopotential numbers were determined for not-so-permanent "locations of convenience" in a few laboratories. While not officially entered into the NGS database (which has stringent bench marking requirements), the results are listed here for reference:

Height Results

Table 7. Height differences

Station	Height (m)	Uncertainty (m)
NIST 811G104	0.0	0.0
NIST 811G104 Table Bottom	0.6134	0.001
NIST 811G104 Floor at Table Foot	0.0009	0.001
NIST 811G104 Clock Center	1.3892	0.001
NIST 1H116	0.0	0.0
NIST 1H116 Line on Wall	1.5938	0.001
JILA X1B21	0.0	0.0
JILA X1B21 Scribe Line on Door Jamb	1.2656	0.001

Geopotential Results

Table 8. Geopotential numbers (at mark height).

Station	Geopotential (gpu)	Uncertainty (gpu)
NIST 811G104	0.0	0.0
NIST 811G104 Table Bottom	0.602	0.001
NIST 811G104 Floor at Table Foot	0.001	0.001
NIST 811G104 Clock Center	0.362	0.001
NIST 1H116	0.0	0.0
NIST 1H116 Line on Wall	1.565	0.001
JILA X1B21	0.0	0.0
JILA X1B21 Scribe Line on Door Jamb	1.240	0.001

Fractional Clock Frequency Differences

Table 9. Clock frequency differences (at mark height).

Station	$\Delta f/f \times 10^{16}$	Uncertainty $\times 10^{16}$
NIST 811G104	0.0	0.0
NIST 811G104 Table Bottom	0.6688	0.001
NIST 811G104 Floor at Table Foot	0.0009	0.001
NIST 811G104 Clock Center	1.5144	0.001

NIST 1H116	0.0	0.0
NIST 1H116 Line on Wall	1.737	0.001
JILA X1B21	0.0	0.0
JILA X1B21 Scribe Line on Door Jamb	1.380	0.001

Acknowledgements

The project would not have been possible without the expertise and willingness of our NGS field crew: Roy Anderson, Steve Breidenbach, Charlie Geoghegan, Tim Wilkins (2015) and Jeff Kanney, Steve Breidenbach, Kevin Jordan, and Jim Harrington (2018). Dru Smith's tireless efforts on the report were an immense help as well. Thanks also to Neil Ashby, Marc Weiss, and Nikolaos Pavlis for their helpful discussions, along with Tom O'Brian, Jun Ye, and Eyvon Petty for their flexibility and patience. Finally, a special thanks to Andrew Novick at NIST for facilitating the initial introductions and discussions (way back in 2014…) that finally got this project off the ground!

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