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A Mathematical Model for the Simulation of a Photogrammetric Camera Using Stellar Control

CHESTER C SLAMA

ROCKVILLE, MD.
December 1972

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A Mathematical Model for the Simulation of a Photogrammetric Camera Using Stellar Control

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A MATHEMATICAL MODEL FOR THE SIMULATION OF A PHOTOGRAMMETRIC CAMERA USING STELLAR CONTROL

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ABSTRACT. A mathematical model that simulates a camera and its orientation in inertial space is described. Also, a computer program that utilizes this mathematical model and least squares techniques to estimate camera orientation parameters from measured coordinates of star images is presented. Although the primary purpose of the computer program is for application in a world-wide geometric satellite triangulation system, sufficient flexibility has been incorporated to allow its use as a photogrammetrist tool for precision camera calibration.

1. INTRODUCTION

The National Ocean Survey has technical responsibility for the execution of and the data reduction and evaluation for a worldwide satellite triangulation program. In general, this program employs simultaneous, high-precision photographs of a satellite trail against a background of stars to determine geometric positions of widely spaced points on the earth's surface. One step of the many required to accomplish this task is the computation of a mathematical simulation of the camera used to photograph the satellite and stars. That is, comparator-measured coordinates of star images on the photographs are processed in a computer program, along with directional coordinates of the stars, to define the mathematical orientation of the camera in inertial space. The computer program is based on a geometrical simulation of the camera central perspective augmented with all known physical perturbations present during the exposure or associated with the subsequent measuring process. This report describes one such computer program developed by the NOS specifically for operational application to the worldwide triangulation system; the program has sufficient additional flexibility for use in photogrammetric error studies and in camera calibrations.

2. THE MATHEMATICAL MODEL

A general approach to the problem of analytically determining the orientation of a precision photogrammetric camera in space has been well documented by many authors, particularly Brown (1964) and Schmid (1953, 1959), and updated and programmed by Butler (1964). A complete

description of the NOS satellite triangulation system is in print in German (Schmid 1972) and will be rewritten in English. The computer program described in this report employs the work of Schmid (1959) as a foundation for a system tailored specifically for application to satellite triangulation. For minimizing repetition, only those changes or additions necessary for adaptation will be discussed in detail; supplemental basic formulas of the unchanged portions are given with specific reference to their origin. It is assumed, therefore, that the reader is familiar with the basic formulations of analytical photogrammetry and the notations as used in the referenced texts.

Incorporating the geometrical optics concept of light rays, figure 1 depicts a light source from a star passing through the atmosphere and a camera lens (shown as an idealized point) onto a film plane to form an image.

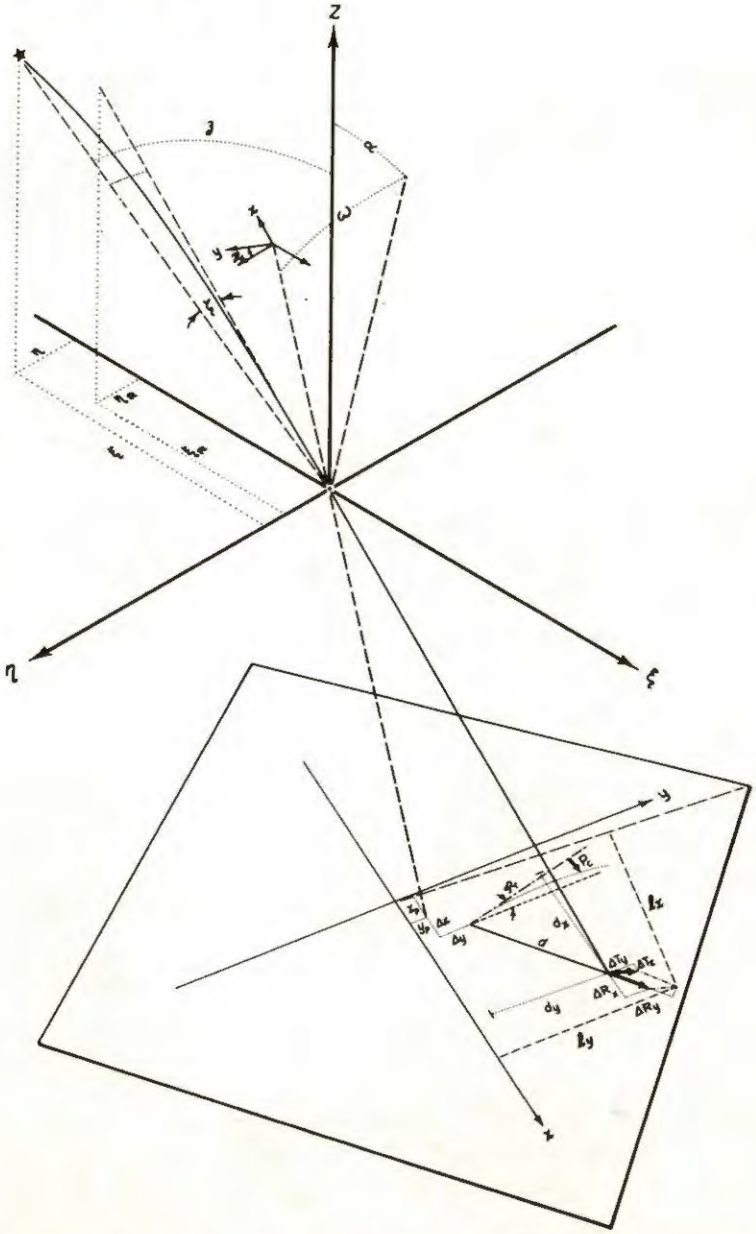

Figure 1.--Geometry of the photogrammetric reconstruction

The description, in mathematical terms, of the path of such a ray constitutes the mathematical model. For continuity, the description of the mathematical model in this section starts at the star (object space), quantifies the atmospheric refraction correction followed by the projective transformation to image space, and covers the deviations of the transformed ray from a true central perspective.

A. Object Space Coordinate Reference System

To date, all computer programs for the one-camera orientation problem, as conceived by Schmid, have employed XYZ Cartesian coordinates as the basic reference system for spatial control. This does not imply that the control must always be given in a rectangular system, as pointed out by Schmid (1951). In that report, he derives a camera orientation by defining directions to stars by spherical coordinates on a unit sphere (right ascension and declination). To apply these directions to the central perspective formulas, however, he further introduces expressions for the transformation of right ascension and declination to standard coordinates. The same approach is used by Schmid when using terrestrial points given in terms of geodetic coordinates φ, λ, and h. Here, he transforms the geodetic coordinates to a rectangular $X\ Y\ Z$ system with an origin and orientation convenient for the problem being solved (Wooten 1961). The orientation of a camera can be determined from directional control given in any system that can be transformed to rectangular coordinates. Also, the optimum program for computing this orientation for the general case will have the observation equations written as functions of rectangular coordinates. In this way, the program will not be restricted to one type of control; in addition, various types can be mixed in the same solution. One disadvantage to this approach is that, when the control is given by coordinates in systems other than rectangular, the transformation function must be used to propagate the variance-covariance matrix associated with the given control point into the XYZ system for weighting purposes. In most cases, however, this normalization can be performed as a preprocessing step and, as such, will not add steps to the orientation program. Although this system is well suited for terrestrial applications where generality is necessary because of the variety of control data used, it was judged to be an unnecessary refinement in satellite triangulation where the star field alone is the basic source of control. As a result, the right ascension-declination reference system was selected as the coordinates for object space; and the observation equations, in terms of rectangular coordinates as presented by Schmid (1959), were modified accordingly.

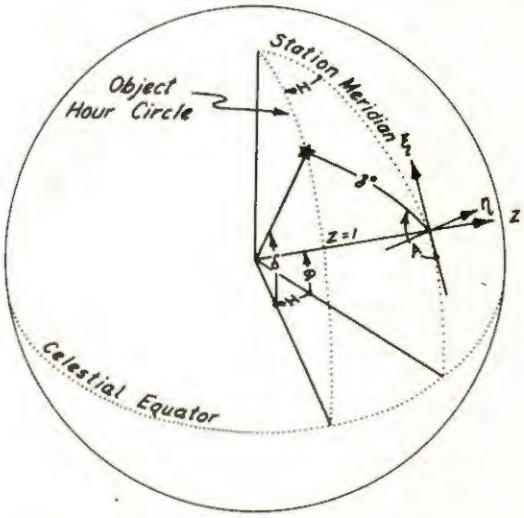

Figure 2.--Local rectangular, geodetic, and astronomic coordinate systems

Figure 2, incorporating the concept of a unit sphere centered on the observer, illustrates

three methods of describing the direction of a star relative to an observer's zenith. The direction can be defined first in terms of three angles or arcs: declination δ (along the hour circle of the object), latitude of the station φ (along the station meridian), and local hour angle H (the angle between the station meridian and the object hour circle, measured along the Celestial Equator). A second description is in terms of rectangular coordinates (ξ, η, Z) with ξ positive north parallel to the station meridian, η positive east, and Z in the direction of the zenith. (Note in fig. 2 that the ξ and η axes are shown at $Z = 1$ for clarity.) Finally, the direction can be defined by the star's zenith distance (z°) and the azimuth from astronomic south (A). Since these three systems are used frequently in the mathematical model, expressions for transforming from one system into the other will be derived here.

The successive rotation of a coordinate system through the angles δ, H, and φ, respectively, from the direction of the object to the local zenith can be written as the product of three matrices:

$$\begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos H & -\sin H \\ 0 & \sin H & \cos H \end{bmatrix} \begin{bmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{bmatrix} = \mathbf{R}_{\varphi, H, \delta}. \tag{1}$$

Applying eq (1) to Cartesian coordinates of the object (XYZ), we have

$$\begin{bmatrix} \xi \\ \eta \\ Z \end{bmatrix} = \mathbf{R}_{\varphi, H, \delta} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad X = Y = 0, \; Z = 1$$

and

$$\begin{bmatrix} \xi \\ \eta \\ Z \end{bmatrix} = \begin{bmatrix} \cos\varphi \sin\delta - \cos\delta \sin\varphi \cos H \\ -\cos\delta \sin H \\ \sin\varphi \sin\delta + \cos\delta \cos\varphi \cos H \end{bmatrix}. \tag{2}$$

Re-establishing a unit distance for Z, we arrive at expressions for ξ and η of the object in terms of φ, δ, and H:

$$\left.\begin{aligned} \xi &= \frac{\cos\varphi \sin\delta - \cos\delta \sin\varphi \cos H}{\sin\varphi \sin\delta + \cos\delta \cos\varphi \cos H} \\ \eta &= -\frac{\cos\delta \sin H}{\sin\varphi \sin\delta + \cos\delta \cos\varphi \cos H} \end{aligned}\right\}. \tag{3}$$

In another convenient form, the same relationships are given by

$$\left.\begin{aligned} \xi &= \frac{\tan\delta - \tan\varphi\cos H}{\cos H + \tan\varphi\tan\delta} \\ \eta &= -\frac{\sec\varphi\sin H}{\cos H + \tan\varphi\tan\delta} \end{aligned}\right\} \tag{4}$$

where H = local sidereal time minus right ascension α.

The coordinates (α, δ) used above are the apparent position of the star (i.e., the "geocentric" position, referred to the true equinox and Equator of date, in which the star is observed). This position differs from the position of the star in which the observation is affected by atmospheric refraction and diurnal aberration. A complete description of the procedures used to "update" star positions is given by Bush (1972).

The inverse relations (i.e., given ξ and η, compute H and δ) can be established in terms of intermediate coordinates azimuth (A) and zenith distance (z°). From figure 2, it can be seen that these quantities are (with a quadrant check on A):

$$\tan A = \eta/\xi$$

and

$$\tan z^\circ = (\xi^2 + \eta^2)^{1/2}.$$

Also from figure 2, when one applies the laws of sines and cosines of spherical trigonometry,

$$\sin H = \frac{\sin z^\circ \sin(180^\circ - A)}{\sin(90^\circ - \delta)},$$

$$\cos H = \frac{\cos z^\circ \cos\varphi + \sin z^\circ \sin\varphi \cos A}{\cos\delta},$$

and

$$\cos(90^\circ - \delta) = \cos z^\circ \cos(90^\circ - \varphi) + \sin(90^\circ - \varphi)\cos(180^\circ - A).$$

With these expressions and the identity

$$\tan H = \sin H / \cos H,$$

the coordinates H and δ in terms of z°, A, and φ are

$$\left.\begin{aligned} \tan H &= \frac{\sin z^\circ \sin A}{\cos z^\circ \cos\varphi + \sin z^\circ \sin\varphi \cos A} \\ \sin\delta &= \cos z^\circ \sin\varphi - \sin z^\circ \cos\varphi \cos A \end{aligned}\right\}. \tag{5}$$

Finally, by definition, the right ascension (α) is given by $\alpha = \angle ST - H$ where $\angle ST$ is the local sidereal time.

B. Atmospheric Refraction and Diurnal Aberration

A large amount of literature dealing with the problem of refraction or the bending of light as it passes through the atmosphere is available. The most recent, a theory published by Garfinkel (1944), uses a polytropic model of the atmosphere to develop an expression for refraction (R) in a power series as a function of the observed zenith distance, pressure, and temperature at the observing station. Here,

$$R = T_0^{1/2} \sum_0^{\infty} \mathcal{B}_i W^{i+1} = T_0^{1/2} W(\mathcal{B}_0 + \mathcal{B}_1 W + \mathcal{B}_2 W^2 + \cdots) \tag{6}$$

where T_0 and W are functions of the station and standard temperature and pressure. The $\mathcal{B}_i$ are tabular values given in the reference (i =0,4) with an argument θ (a function of the observed zenith distance and weather). That is, given the zenith distance of an observation ($\leq 90°$) together with the temperature and pressure at the station, one can compute the angle θ and the "weather factor" W, enter the tables using θ as an argument, and find values for the $\mathcal{B}_i$. These values, together with W and the station temperature and pressure, are used in eq (6) to compute refraction for the observed ray. For zenith distances greater than 90°, other tabular values (C_i) together with small corrections (ε_i) are given in Garfinkel (1944) to be used in a form similar to zenith distances $\leq 90°$.

With the advent of the large-scale electronic computer and a widespread interest in astronomical refraction for zenith distances greater than 93°, Garfinkel (1967) published a modification to his 1944 theory. He emphasizes an improvement in accuracy, an improved algorithm for computation of refraction, and (most important) includes a reference to a FORTRAN-coded computer program to replace the earlier tables. He further states that "The new theory is valid in the entire domain $0 \leq z \leq 180°$, $0 \leq h < \infty$ of the arguments z and h and satisfies the most crucial internal checks with an accuracy of better than 1 s."

Garfinkel's model was adopted for use for the computation of atmospheric refraction in this program. However, the referenced FORTRAN program was not introduced directly to avoid complexity. Instead, values of refraction for zenith distances of 0° to 90°, at 0.1° increments and standard temperature and pressure conditions (i.e., T = 0°C and P = 760 mm of mercury), were computed using Garfinkel's rigorous function. These values in turn were introduced as observations in a least squares fit of the data to a more simple function of the form

$$\zeta'' = T^{1/2} W(\tau_1 \tan\frac{\beta}{2} + \tau_2 \tan^3\frac{\beta}{2} + \tau_3 \tan^5\frac{\beta}{2} + \tau_4 \tan^7\frac{\beta}{2}) \tag{7}$$

where $T = T/T_0$, $W = P/T^2$, and $P = P/P_0$ (with T as station temperature, P as station pressure, T_0 as standard temperature, and P_0 as standard pressure) and the angle β is a function of weather parameters and observed zenith distance of the object given by

$$\tan\beta = \frac{T^{1/2}}{\gamma} \tan z \tag{8}$$

where γ = 8.7137 (a constant given by Garfinkel 1967).

The resulting coefficients (τ_i) were

$$\tau_1 = 1050.6103,$$
$$\tau_2 = 706.11502,$$
and
$$\tau_3 = 262.06086, \qquad (9)$$
$$\tau_4 = 142.67293.$$

For testing the validity of these computed coefficients, comparisons were made between the rigorous and the fitted function over a wide range of temperature and pressure changes. These comparisons are shown graphically in figure 3.

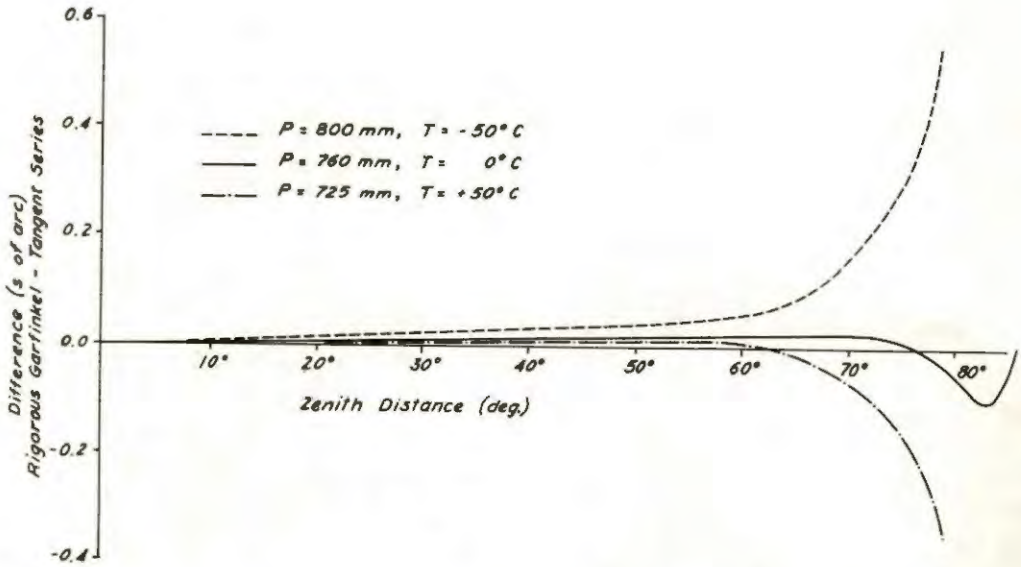

Figure 3.--Residuals after the "fit" to the Garfinkel refraction

Although the comparison was computed for many temperature and pressure combinations, only the outer bounds of the family of curves are shown on the graph. The solid center line is a plot of the differences at standard pressure and temperature and, therefore, represents the residuals of the curve fit solution. The two remaining lines are plots of the differences for extremes of pressure and temperature. From them, it is apparent that the adopted function essentially fits the rigorous data to within a maximum of 0.1 s of arc for zenith distances 0° to 70° over temperature and pressure ranges of 100C° and 75 mm of mercury, respectively. Since the deviations at normal operating conditions proved to fall well within the error budget for camera orientation, the generalized function [eq (7)] with coefficients [eq (9)] was selected for the computation of atmospheric refraction in the program.

Atmospheric refraction is computed individually for each star direction; adjustments are made in the form of corrections to the star's apparent coordinates (hour angle and declination). In figure 4B, the observer's zenith pierces the unit sphere at O. Likewise, directions to a star pierce the sphere at S' (refracted) and S (apparent). The direction difference OS is then defined as the unrefracted zenith distance z, and OS' is defined as the refracted

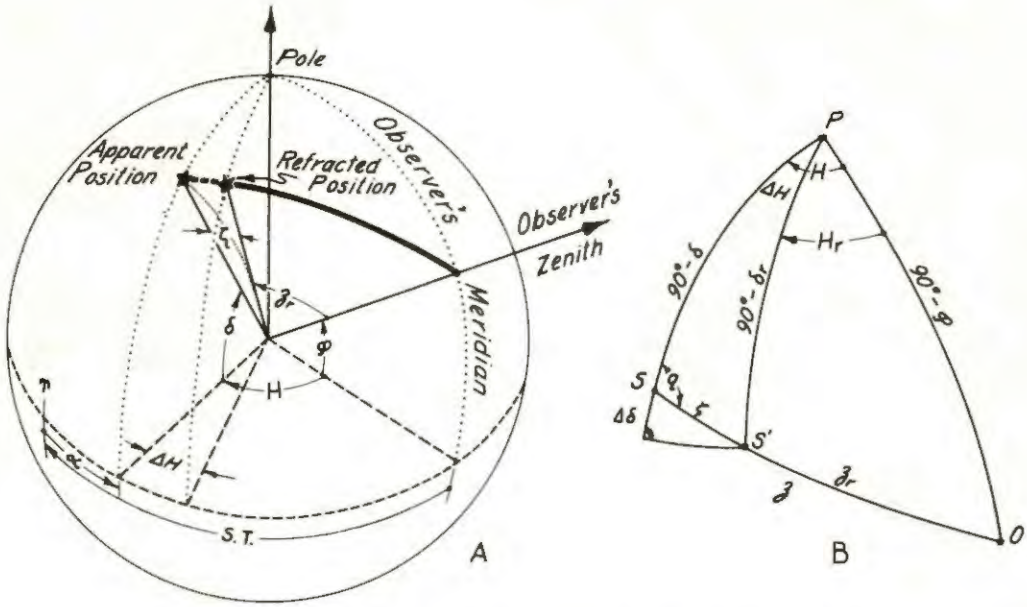

Figure 4.--Corrections for atmospheric refraction

zenith distance z_r. From the triangle OPS, an expression for the unrefracted zenith distance is

$$\cos z = \cos(90° - \delta)\cos(90° - \varphi) + \sin(90° - \delta)\sin(90° - \varphi)\cos H \tag{10}$$

where φ is the observer's latitude, δ is the star's apparent declination, and H is the apparent hour angle. Zenith distance is always positive; therefore, the tangent of the angle follows directly from the identity $\tan z = \left[(1-\cos^2 z)^{1/2}/\cos z\right]$ from which $\tan\beta$ can be computed using expression (8). Atmospheric refraction ζ for the given ray is then found by evaluating eq (7) for the argument $\tan\beta/2$. However, as stated earlier, the τ_i in eq (9) were derived for a refraction function that was evaluated for observed directions to stars (i.e., refracted directions). The star coordinates referred to above, on the other hand, are catalog values updated to apparent positions and as such are unrefracted directions. Since refraction corrections are required for inputs of both observed and apparent zenith distances, the problem could have been resolved by introducing two sets of τ_i and applying them to expression (7) at the appropriate places in the computer program. Another solution, the one used here, is to maintain one set of τ_i and solve for refraction of apparent directions using successive approximation. This is done by successively adjusting the apparent zenith distance first computed by eq (10) and using

$$z_{r_i} = z - \zeta_{i-1} \tag{11}$$

where z_{r_i} is the current adjusted value of zenith distance, z is the apparent zenith distance, and ζ_{i-1} is refraction evaluated at the previously adjusted z_r. This scheme is repeated until the change between successive ζ_i converge to a predetermined tolerance level. The final value for refraction is then the average of the last two determinations of ζ.

By definition, atmospheric refraction results in the bending of light rays toward the observer's zenith; its correction lies in the direction OS (fig. 4B). For one to apply this

correction to star coordinates, ζ must be projected into the H, δ system. From figure 4B, we have for the indicated auxiliary angle q

$$\cos q = \frac{\cos(90^\circ - \varphi) - \cos z \cos(90^\circ - \delta)}{\sin z \sin(90^\circ - \delta)}$$

with which an expression for the sine of the refracted declination coordinate (δ_r) can be written directly in terms of the unrefracted declination (δ) and the refraction angle ζ . Thus,

$$\sin \delta_r = \cos(90^\circ - \delta)\cos\zeta + \sin(90^\circ - \delta)\sin\zeta\cos q . \tag{12}$$

Again from figure 4B, the angle q is

$$\sin q = \frac{\sin(90^\circ - \varphi)\sin H}{\sin z} ;$$

and in the triangle PSS', an increment ΔH to the unrefracted hour angle is given by

$$\sin \Delta H = \frac{\sin q \sin\zeta}{(1 - \sin^2\delta_r)^{1/2}} . \tag{13}$$

The refracted hour angle coordinate follows as

$$H_r = H - \Delta H . \tag{14}$$

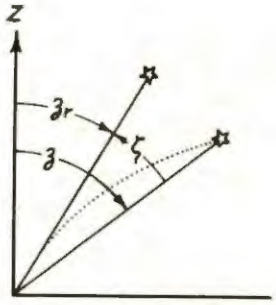

Figure 5.--Refracted and unrefracted zenith distances

The inverse of the problem just discussed (i.e., given a set of coordinates of a refracted star image, compute corrections for refraction) is more straightforward. As before, the zenith distance to the star is computed using either eq (10) (if the direction is given by H_r and δ_r) or by using (from fig. 2)

$$\tan z_r = (\xi_r^2 + \eta_r^2)^{1/2} \tag{15}$$

where the refracted direction is defined by Cartesian coordinates ξ_r and η_r . In either case, since the directions are "observed," expression (7) is evaluated directly with no iterations; the unrefracted (or corrected) zenith distance becomes

$$z = z_r + \zeta \tag{16}$$

as shown by figure 5.

Since the azimuth to the star remains the same for either refracted or unrefracted directions, expressions (5) in the form

$$\left.\begin{aligned} \tan H &= \frac{\sin z \sin A_r}{\cos z \cos\varphi + \sin z \sin\varphi \cos A_r} \\ \sin\delta &= \cos z \sin\varphi - \sin z \cos\varphi \cos A_r \end{aligned}\right\}$$

can be used to compute unrefracted coordinates H and δ where the azimuth is given by

$$\tan A_r = \eta_r / \xi_r .$$

Diurnal aberration corrections must be taken into account to complete the adjustment of the input spatial coordinates into directions seen by the camera. Diurnal aberration is the apparent displacement of a star image due to the relative velocities of the light ray and the observer on the earth's surface. Because of the earth's rotation, light rays from a star appear to an observer to be displaced in the direction of rotation by an amount proportional to the observer's linear velocity (because of rotation) and the direction and velocity of the incoming ray. From the "Explanatory Supplement to the Ephemeris" (Her Majesty's Stationery Office 1961), this displacement (in s of arc) is given by

$$\Delta\alpha = 0.320 \cos\varphi \cos H \sec\delta$$

and

$$\Delta\delta = 0.320 \cos\varphi \sin H \sin\delta ; \tag{17}$$

by definition, the corrections to the input coordinates (apparent places) are made using

$$\alpha' = \alpha + \Delta\alpha$$

and

$$\delta' = \delta + \Delta\delta . \tag{18}$$

It is obvious from eq (17) that the effect of the diurnal aberration correction is small as is the correction for atmospheric refraction. Consequently, the star coordinates in expression (18) can be either refracted or unrefracted. For continuity, the corrections for diurnal aberration in this program are always applied to unrefracted coordinates.

In summary, coordinate systems for input spatial control have been defined; and transformations of these coordinates into the Cartesian system more amenable to atmospheric refraction computation have been developed. In addition, corrections for refraction and diurnal aberration to adjust the directions to those seen by the observer (camera) were developed both for a ray starting at the camera (refracted) and for a ray described by star coordinates (unrefracted). The ray in object space is thereby completely defined.

C. Projective Transformation

The derivation and principles concerning projective transformations have been discussed adequately in literature preceding this report (e.g., Schut 1958, Schmid 1953, and Brown 1957). It will be sufficient, therefore, merely to define the input parameters used in this mathematical model (adopted from Schmid 1953) to provide a basis for their interpretation.

The relationship between a spatial coordinate system and its corresponding photographic image coordinate system is shown graphically in figure 6. In the figure, the coordinate axes $X, Y,$ and Z are associated with the spatial control (stars) and are identical to the rectangular system defined in figure 2 (where $X \equiv \xi$ and $Y \equiv \eta$). The x and y are coordinate axes in the image plane of the photograph that, when used with the camera principal distance c, define the point P in image space.

In the illustration, the image plane is shown as a diapositive (i.e., falling between the center of projection O and the spatial control). Also, the x and y axes are translated to

the image plane for clarity. The relationship between the two systems is given by three angles of rotation--a primary rotation α of the X-Z plane about the Y axis (Z toward X), a secondary rotation ω of the rotated Y-Z plane about the rotated X axis (Z toward Y), and a rotation $\varkappa$ of the X-Y plane about the Z axis (now coincident with the c axis) followed by a change in direction of the resulting x axis. The corresponding rotation matrices for the angles $\alpha, \omega, \varkappa$ and the reflection are

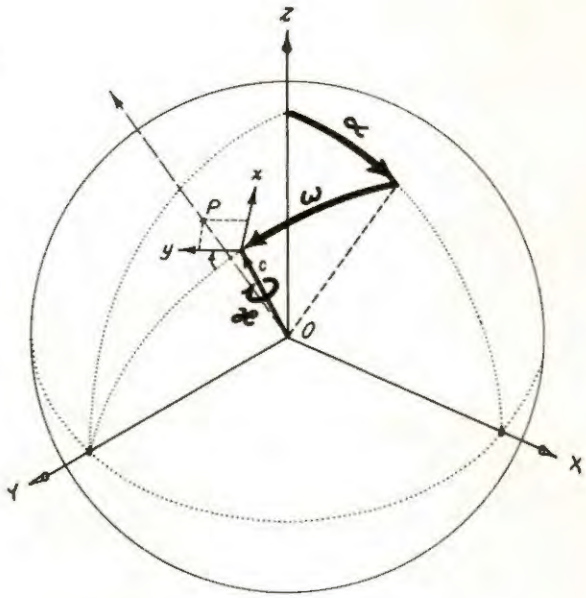

Figure 6.--Camera orientation angles

$$R_\alpha = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix},$$

$$R_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix},$$

and

$$R_\varkappa = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varkappa & \sin\varkappa & 0 \\ -\sin\varkappa & \cos\varkappa & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\cos\varkappa & -\sin\varkappa & 0 \\ -\sin\varkappa & \cos\varkappa & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The matrix product $R_\varkappa R_\omega R_\alpha$ results in the orthogonal direction cosine matrix

$$\begin{bmatrix} (-\cos\alpha\cos\varkappa + \sin\alpha\sin\omega\sin\varkappa) & (-\cos\omega\sin\varkappa) & (\sin\alpha\cos\varkappa + \cos\alpha\sin\omega\sin\varkappa) \\ (-\cos\alpha\sin\varkappa - \sin\alpha\sin\omega\cos\varkappa) & (\cos\omega\cos\varkappa) & (\sin\alpha\sin\varkappa - \cos\alpha\sin\omega\cos\varkappa) \\ (\sin\alpha\cos\omega) & (\sin\omega) & (\cos\alpha\cos\omega) \end{bmatrix}$$

that we designate as

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ D & E & F \end{bmatrix} = M. \tag{19}$$

The transformation from spatial to image coordinates may now be written in standard matrix notation as

$$\begin{bmatrix} x \\ y \\ c \end{bmatrix} = \frac{1}{\mu} M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{20}$$

where μ is a proportionality or scale factor. The inverse relation follows directly, since $M^{-1}=M^{T}$, as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mu M^{T} \begin{bmatrix} x \\ y \\ c \end{bmatrix}. \tag{21}$$

For convenience in notation and later use in the report, the right-hand sides of expressions (20) and (21) are written without the proportionality factor as

$$\begin{bmatrix} m \\ n \\ q \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{22}$$

and

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = M^{T} \begin{bmatrix} x \\ y \\ c \end{bmatrix}. \tag{23}$$

Expressions (20) through (23) represent the relationships between image and object space for an ideal projective transformation. As such, they alone would be adequate for an ideal camera. In reality, however, an ideal camera does not exist; therefore, the following discussion deals with the modifications that are necessary to account for known or postulated perturbations in image space.

D. Deviations From an Ideal Projective Transformation

Perturbations of light rays as they pass through a camera lens and are recorded on an emulsion in the image plane of the camera can take many forms. Those of most interest to a photogrammetrist are the perturbations that result in a physical displacement of the resulting image from its ideal projective position. Also of interest are the positional distortions that take place as a result of the recording medium and its processing and the errors in positional data attributable to inconsistencies in the measuring system.

Figure 7 is a graphic depiction of the perturbations, more commonly called inner orientation parameters, postulated for this mathematical model. In the illustration, the observer is at the center of projection O looking toward object space through a diapositive as depicted in figure 6. Each of the distortions shown will be discussed in the following paragraphs.

Every effort is made by the camera manufacturer to provide a precise reference system by which the photogrammetrist can recover the central ray of the projective transformation. The intersection of a series of marks or dots marked on the bordering edge of the film plane describes the point of intersection of the central ray and the focal plane of the camera.

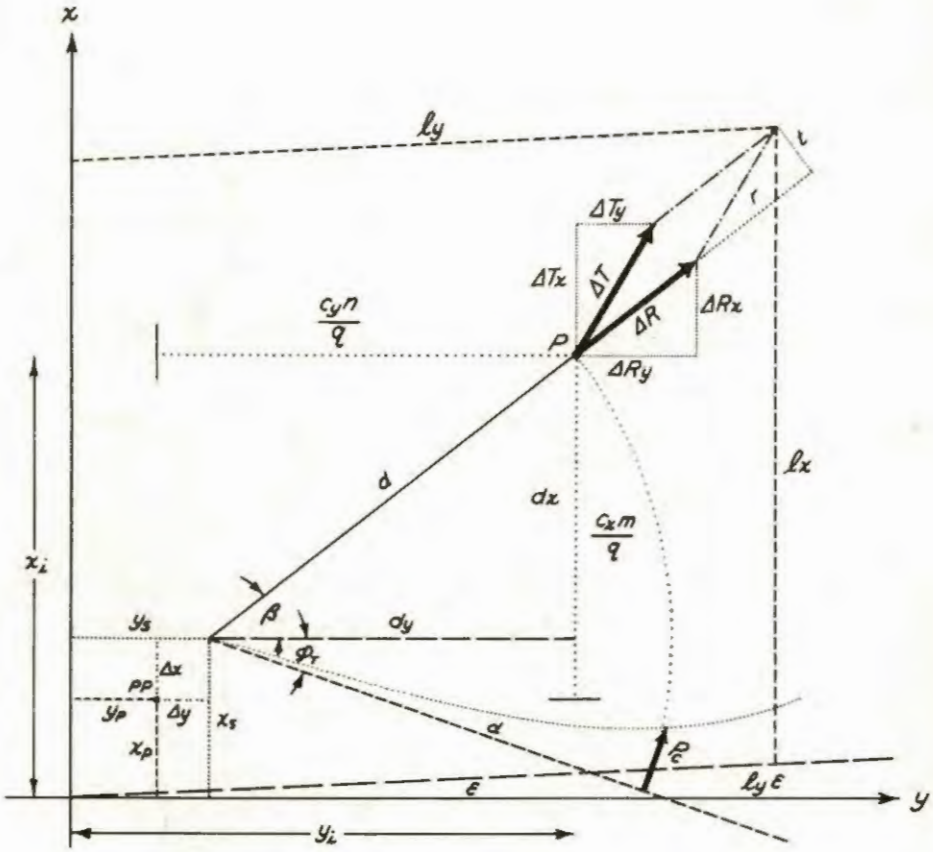

Figure 7.--Perturbed plate coordinates

These marks, in turn, are used by the photogrammetrist as an origin for the measured plate coordinate system to provide a means of relating the exterior orientation angles and interior orientation parameters to physically describable directions in and of the camera. Since the position of the fiducial intersection cannot be made to coincide precisely with the central ray (principal point), an adjustment must be made within the mathematical model. The adjustment can take the form of a simple translation only since the rotational component will be absorbed by the $\varkappa$ angle described in the previous subsection. The translation of the coordinates from eq (20) takes the form

$$\begin{bmatrix} x_i \\ y_i \\ c \end{bmatrix} = \begin{bmatrix} x \\ y \\ c \end{bmatrix} + \begin{bmatrix} x_p \\ y_p \\ 0 \end{bmatrix} \tag{24}$$

where x_p and y_p are coordinates of the principal point in the measured plate coordinate system.

A lens made of centered spherical elements will produce an image subject to five distinct aberrations (Seidel 1856). These aberrations, usually referred to as the Seidel sums, are ordinary spherical aberration, coma, astigmatism, curvature of field, and distortion. In most cases, a good lens will be designed and fabricated to minimize these conditions; however, they can never be eliminated completely. The most undesirable of the five, from the viewpoint

of satellite triangulation, is distortion that results in a dislocation of the image from an ideal projective transformation. Conrady (1918), using the law of lateral deviation of a marginal ray and restricting his discussion to aberrations of the third order, showed that the magnitude of this displacement is proportional to the cube of the angle which the extra-axial object point subtends and that it is present in undiminished magnitude however small the diaphragm opening may be. He states further that his derivation is based on curved images and that it can be shown to lead to additional higher order terms to secure a numerically correct value of distortion.

Brown (1956) first suggested that radial distortion be determined as part of the least squares solution to the orientation of a camera; both he and Schmid (1959) adopted an odd-power polynomial, carrying four terms as adequate to represent the distortion profile. That is, distortion is assumed to be representable by a polynomial of odd powers of radial distance from some point on the image plane and hence to be symmetrical about that point with a positive displacement directed outward. The polynomial is assumed to represent the amount of distortion evaluated for a point as a function of its undistorted position on the plate. The reason for this will become evident later in this report in the discussion of the formation of the observation equations.

From figure 8 and assuming terms up to seventh order as adequate, one obtains the radial symmetric distortion ΔR by

$$\Delta R = K_1 d^3 + K_2 d^5 + K_3 d^7 \tag{25}$$

where the K_i are unknown coefficients to be determined in our calibration and d is the radial distance from the point of symmetry of the distortion profile. Allowing for a translation of the point of symmetry from the principal point of projection (Δx and Δy) results in the expression for radial distance:

$$d = \left[(x_i - x_s)^2 + (y_i - y_s)^2\right]^{1/2} \tag{26}$$

where

$$x_s = x_p + \Delta x$$

and

$$y_s = y_p + \Delta y . \tag{27}$$

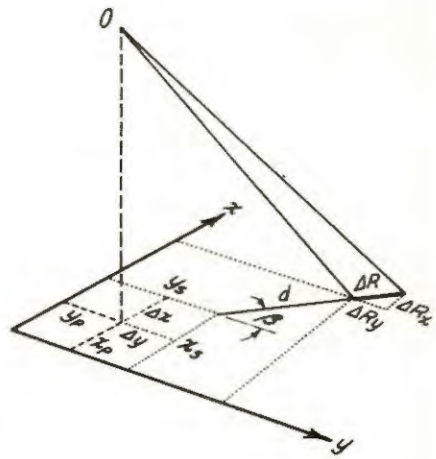

Figure 8.--Radial lens distortion

Finally, to resolve radial distortion (ΔR) into components of the plate system x and y, we compute the projections using the angle β (fig. 8) from

$$\Delta R_x = \Delta R \sin\beta = \frac{\Delta R}{d}(x_i - x_s)$$

and

$$\Delta R_y = \Delta R \cos\beta = \frac{\Delta R}{d}(y_i - y_s) . \tag{28}$$

The discussion of lens distortion thus far has been confined to image displacements arising from lenses made of centered spherical elements. That is, the assumption is made that the centers of curvature lie on a straight line. Early studies of analog methods by Pennington (1947), however, showed that unusual deformations in multiplex models could be attributed to tangential distortion resulting from misalinement of lens elements. Washer (1957) showed

that the effects of lens decentering could be simulated by a thin prism in combination with a centered system and coined the phrase "prism effect." No attempt was made to correct for decentering defects in applications of analog instruments; but with the advent of numerical methods, the subject was revived. Naturally, the adopted mathematical simulation took the form of a prism or wedge; the resulting equations were written as

$$\Delta x_\omega = P_\omega \cos \varphi_T$$

and

$$\Delta y_\omega = P_\omega \sin \varphi_T$$

where P_ω is the distortion profile that defines the amount of displacement as a function of radial distance and φ_T is the angle of the line along which the displacement has no radial component. A graphic example of the displacements resulting from the wedge model for one radial distance is shown in figure 9.

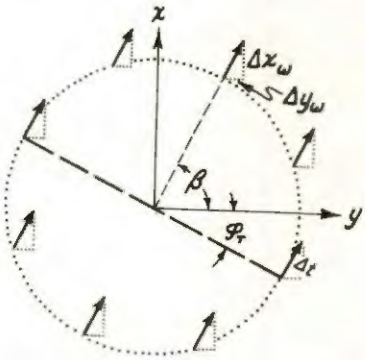

Figure 9.--Wedge distortion

Brown (1966) suggested the use of a decentering model (developed by Conrady 1919) that had gone unnoticed by most photogrammetrists. Using the same methods he used to develop the Seidel sums, Conrady established algebraic expressions for displacements resulting from decentrations of lens elements. In his notation, these expressions are

$$r = 3p_3 V^2 \cos \chi$$

and

$$t = p_3 V^2 \sin \chi \tag{29}$$

where V is the half angle of view of the image, p_3 is a constant peculiar to the lens system, and χ is an angle indicating the orientation of the image relative to the direction of decentration. (In this case, decentration is understood to mean the resultant direction when more than one element is involved.) The component r in the expression is taken as positive away from the origin along the radial line of the field, and the component t is perpendicular to

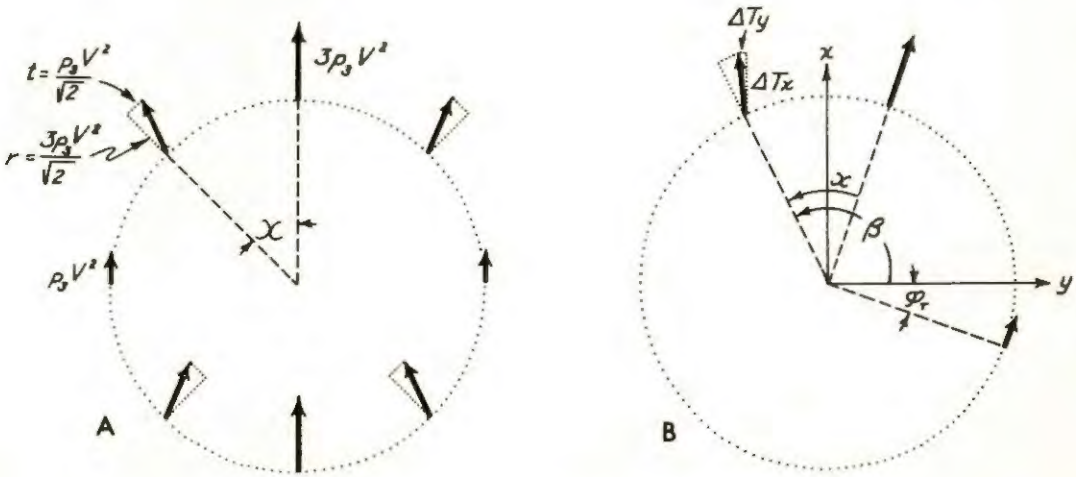

Figure 10.--Conrady decentered lens distortion

this radial line forming a right-handed system. Evaluating eq (29) for any given distance from the center of the field, we see that the displacement of the image would be radial and equal to $3p_3V^2$ in the direction χ =0° and equal to $-3p_3V^2$ at χ =180°. All intermediate values of χ result in displacements in both the radial and tangential directions (as indicated in fig. 10A) with the maximum tangential effects occurring at χ =90° and 270°.

To relate this distortion model to our plate coordinate system, we arbitrarily assign one of the directions of maximum tangential distortion (χ =270°) to our plate positive y axis and introduce a new variable φ_r as the angle of deviation from this standard position. From figure 10B, then, the angle χ of Conrady takes on the meaning

$$\chi = \varphi_r + \beta - 90° ;$$

and the expressions (29) in terms of φ_r and β become

$$r = 3p_3V^2 \sin(\varphi_r + \beta)$$

and

$$t = -p_3V^2 \cos(\varphi_r + \beta). \tag{30}$$

Further, to resolve the Conrady displacements into components in our plate system, we premultiply expressions (30) by a rotation matrix (rotate the r, t system through an angle β -90°, r toward t) and replace the distortion profile p_3V^2 by a polynomial in distance d given by

$$p_3V^2 = P_c = K_4 d^2 + K_5 d^4$$

resulting in the final form for decentered lens distortion (ΔT):

$$\begin{bmatrix} \Delta T_x \\ \Delta T_y \end{bmatrix} = \begin{bmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{bmatrix} \begin{bmatrix} r \\ t \end{bmatrix} = P_c \begin{bmatrix} (2\sin^2\beta + 1)\cos\varphi_r + (2\sin\beta\cos\beta)\sin\varphi_r \\ (2\sin\beta\cos\beta)\cos\varphi_r + (2\cos^2\beta + 1)\sin\varphi_r \end{bmatrix}. \tag{31}$$

Finally, from figure 7 and substitution of the relationships

$$\cos\beta = \frac{dy}{d}$$

and

$$\sin\beta = \frac{dx}{d}$$

where

$$dx = (x_i - x_s)$$

and

$$dy = (y_i - y_s) ,$$

expressions (30) reduce to the more convenient form

$$\Delta T_x = (x_i - x_s)\left[(K_4 + K_5 d^2)\left(\frac{3dx^2 + dy^2}{dx}\cos\varphi_r + 2\,dy \sin\varphi_r\right)\right] = (x_i - x_s)\, D_x$$

and

$$\Delta T_y = (y_i - y_s)\left[(K_4 + K_5 d^2)\left(\frac{3dy^2 + dx^2}{dy}\sin\varphi_r + 2\,dx \cos\varphi_r\right)\right] = (y_i - y_s) D_y . \tag{32}$$

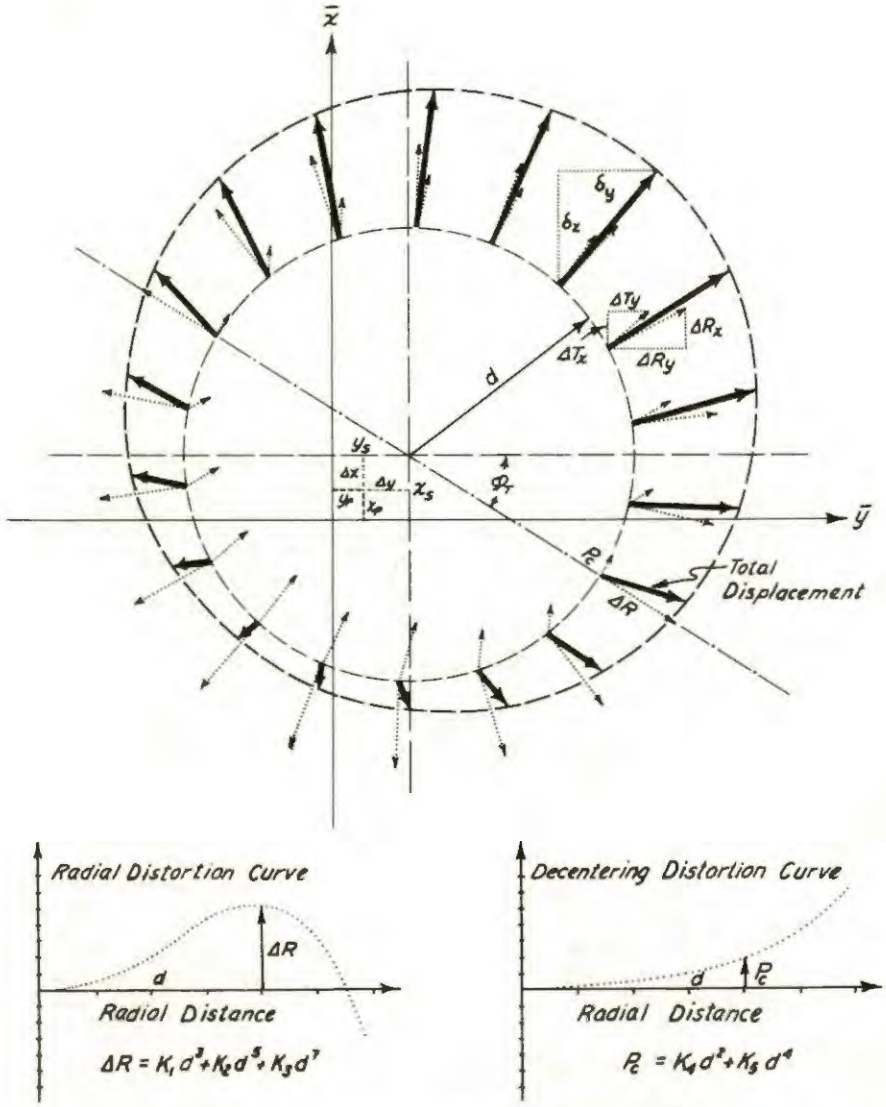

Figure 11.--Combined radial and decentered lens distortion

Figure 11 is a graphic example of the displacements resulting from a combination of both radial and lens decentering distortion evaluated at a distance d from the point of origin of the distortion profiles. As formulated by Conrady, the displacements from each distortion source can be considered as independent vectors in which the resultant is the sum of their individual x and y components. In matrix notation, then, the resultant shifts δ_x and δ_y of any point i can be written as

$$\begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} = \begin{bmatrix} \frac{\Delta R}{d} + D_x & 0 \\ 0 & \frac{\Delta R}{d} + D_y \end{bmatrix} \begin{bmatrix} x_i - x_s \\ y_i - y_s \end{bmatrix} \tag{33}$$

where the coordinates of the displaced image are given by

$$\bar{x}_i = x_i + \delta_x$$

and

$$\bar{y}_i = y_i + \delta_y . \tag{34}$$

To complete the mathematical model, one must allow for two additional deformations of the imaged point. That is, we introduce the provision for a differential scale change along the x and y plate axes and a second component that expresses a possible nonperpendicularity of the measured plate axes. The scale change provision can be brought about by premultiplying the coordinates from eq (20) by a diagonal matrix of the form

$$\begin{bmatrix} x_i \\ y_i \\ c \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ c \end{bmatrix} . \tag{35}$$

This effects a multiplication of both the x and y distances by individual scalers. However, by definition, we already have introduced a scale into our system with the principal distance c and, therefore, cannot leave eq (35) in its present form. To remove the redundancy, we assign the projective transformation component c to one of the axes and compute a departure of the other axis from this scale. This is accomplished by setting

$$S_x = \frac{c_x}{c} = 1$$

and

$$S_y = \frac{c_y}{c}$$

whereby eq (35) becomes

$$\begin{bmatrix} x_i \\ y_i \\ c_x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_y/c_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ c \end{bmatrix} . \tag{36}$$

Figure 12 shows a nonperpendicular system x, y with an angle of nonperpendicularity ε. From the figure, we can see that the transformation to coordinates with nonperpendicular axes, where ε is small (i.e., $\sin\varepsilon = \varepsilon$ and $\cos\varepsilon = 1$), has the matrix

$$\begin{bmatrix} 1 & -\varepsilon & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

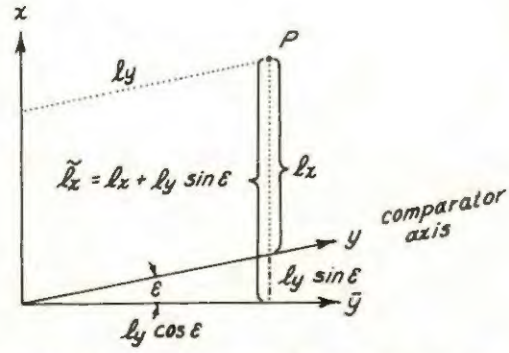

Figure 12.--Nonperpendicularity of the comparator axis

Combining this with eq (36), we have for the total effects of scale change and nonperpendicularity an expression of the form

$$\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ c_x \end{bmatrix} = \begin{bmatrix} 1 & (-c_y/c_x)\varepsilon & 0 \\ 0 & c_y/c_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ c \end{bmatrix} . \tag{37}$$

This completes the formulation of the mathematical model for the projective transformation from object space (star coordinates) to image space (measured plate coordinates). Combining the expressions from the previous subsection with those of the perturbations just discussed results in a matrix expression for the total operation:

$$\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ c_x \end{bmatrix} = \begin{bmatrix} 1 & -\varepsilon & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{c_y}{c_x} & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{\mu} \begin{bmatrix} m \\ n \\ q \end{bmatrix} + \begin{bmatrix} x_p \\ y_p \\ 0 \end{bmatrix} + \left(\begin{bmatrix} \frac{\Delta R}{d} + D_x & 0 & 0 \\ 0 & \frac{\Delta R}{d} + D_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{c_y}{c_x} & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{\mu} \begin{bmatrix} m \\ n \\ q \end{bmatrix} - \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix} \right\} \right) \right\} . \quad (38)$$

After carrying out the indicated multiplications and dividing the first and second equations by the third, expression (38) reduces to

$$\left.\begin{aligned} \tilde{x}_i &= \left(\frac{c_x m}{q} + x_p\right)\left(1 + \frac{\Delta R}{d} + D_x\right) - x_s\left(\frac{\Delta R}{d} + D_x\right) - \tilde{y}_i\,\varepsilon \\ \tilde{y}_i &= \left(\frac{c_y n}{q} + y_p\right)\left(1 + \frac{\Delta R}{d} + D_y\right) - y_s\left(\frac{\Delta R}{d} + D_y\right) \end{aligned}\right\} \quad (39)$$

that express the image coordinates $\tilde{x}_i$ and $\tilde{y}_i$ in terms of star coordinates and 16 additional unknown parameters.

It is apparent from close inspection of expression (38) and figure 7 that the evolutionary process leading to the present single-camera mathematical model resulted in a sequence of operations not easily interpreted in physical reality. That is, the distortion model is applied to projectively transformed coordinates that are assumed to be unequal in scale. This implies a scale change as part of the projective transformation. A more realistic form would have assumed that the lens distortions are independent of the scale deformations and that scale changes take place <u>after</u> the image is formed (either by differential emulsion shifts during processing or unequal scales of the comparator axes). A model of this type would take the form

$$\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \\ c_x \end{bmatrix} = \begin{bmatrix} 1 & -\frac{c_y}{c_x}\varepsilon & 0 \\ 0 & \frac{c_y}{c_x} & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \frac{1}{\mu} \begin{bmatrix} m \\ n \\ q \end{bmatrix} + \begin{bmatrix} x_p \\ y_p \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\Delta R}{d} + D_x & 0 & 0 \\ 0 & \frac{\Delta R}{d} + D_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \left(\frac{1}{\mu} \begin{bmatrix} m \\ n \\ q \end{bmatrix} - \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix} \right) \right\} \quad (40)$$

with associated observation equations

$$\tilde{x}_i = \left(\frac{c_x m}{q} + x_p\right)\left(1 + \frac{\Delta R}{d} + D_x\right) - x_s\left(\frac{\Delta R}{d} + D_x\right) - \tilde{y}_i\,\varepsilon$$

and (41)

$$\tilde{y}_i = c_y\left[\left(\frac{n}{q} + \frac{y_p}{c_x}\right)\left(1 + \frac{\Delta R}{d} + D_y\right) - \frac{y_s}{c_x}\left(\frac{\Delta R}{d} + D_y\right)\right] .$$

In satellite triangulation, the choice of expression (38) has no adverse effects on the final accuracies of the system since the ultimate goal is to interpolate a selected direction into the right ascension-declination system of the stars. In camera calibration, on the other hand, the purpose is to determine certain camera characteristics to which physical interpretation can be attached; therefore, the form (40) would be more desirable.

A further note of caution is now appropriate to those who may attempt to attach physical significance to the computed parameters when performing a camera calibration with this program. Because of the strong correlation that exists between some of the parameters carried in the solution, the resulting values in some cases tend toward interpolation parameters rather than physically interpretable quantities. Also, all transformations of the observational data prior to their introduction into the problem must be taken into account in evaluating the resulting model. A case in point is that of the scalars c_x and c_y along with the nonperpendicularity angle ε. In most measurement procedures, the photographic plate is usually placed on the comparator stage in an arbitrary orientation. After recording the individual coordinates of each image point together with measurements of the camera fiducial markers, the entire system is translated to an origin determined by the intersection of the camera fiducials and then is rotated so that one of the coordinate axes is coincident with a line through two of the fiducials. In this way, various plates from the same camera are all related to one physical orientation in the focal plane of the camera. This provides a common origin and direction for the exterior and interior orientation parameters of the camera. Unfortunately, it also carries with it a new definition of the computed scalars c_x and c_y and the nonperpendicularity parameter ε. That is, these parameters no longer refer to the original axes of the comparator, but are functionally related to the transformed system shown in figure 7 and expressed in eq (40). As a result, to relate these parameters to the original coordinate system, we cannot merely rotate the value ε alone; we must operate on the transformation matrix of (37) in total (Pope 1966). That is, given a transformation matrix $\boldsymbol{A}$ as in (37) such that

$$\boldsymbol{X}' = \boldsymbol{A}\boldsymbol{X}$$

where

$$\boldsymbol{A} = \begin{bmatrix} S_x & -S_y\varepsilon \\ 0 & S_y \end{bmatrix},$$

then the same transformation matrix in the rotated system is

$$\boldsymbol{A}_r = \boldsymbol{R}\boldsymbol{A}\boldsymbol{R}^T$$

where

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

and θ is the angle measured from x toward y between the original measured system and the

computed system. The resulting expression is

$$A_r = \begin{bmatrix} \left(\cos^2\theta + \frac{c_y}{c_x}\sin^2\theta - \frac{c_y}{c_x}\varepsilon\sin\theta\cos\theta\right) & \left(\left(\frac{c_y}{c_x}-1\right)(\sin\theta\cos\theta) - \frac{c_y}{c_x}\varepsilon\cos^2\theta\right) \\ \left(\left(\frac{c_y}{c_x}-1\right)(\sin\theta\cos\theta) + \frac{c_y}{c_x}\varepsilon\sin^2\theta\right) & \left(\sin^2\theta + \frac{c_y}{c_x}\cos^2\theta + \frac{c_y}{c_x}\varepsilon\sin\theta\cos\theta\right) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

Finally, from Pope, the relations

$$Sx_r = a_{11}\,,$$
$$Sy_r = a_{22}\,,$$

and

$$\varepsilon_r = a_{12} + a_{21}$$

result in the final rotated parameters

$$Sx_r = \cos^2\theta + \frac{c_y}{c_x}\sin^2\theta - \frac{c_y}{c_x}\varepsilon\sin\theta\cos\theta\,,$$
$$Sy_r = \sin^2\theta + \frac{c_y}{c_x}\cos^2\theta + \frac{c_y}{c_x}\varepsilon\sin\theta\cos\theta\,,$$

and

$$\varepsilon_r = \frac{c_y}{c_x}\varepsilon(\sin^2\theta - \cos^2\theta) + 2\left(\frac{c_y}{c_x}-1\right)\sin\theta\cos\theta\,. \tag{42}$$

Expressions (42) can be used to transform the parameters from the camera calibration computation directly to physical characteristics of the film and comparator.

In the discussion of lens distortion, expressions (34) were given as the ones to be used to apply displacements to coordinates from the projective transformation. The need arises, however, in the course of the problem to proceed in the opposite sense (i.e., to start with measured plate coordinates and apply the inner orientation parameters to arrive at projected coordinates). For accomplishing this, distortion must be computed by successive approximation. First, for the distorted plate coordinates (ℓ_x, ℓ_y) of an image point, a distance d' is computed using expression (26):

$$d' = \left[(\tilde{\ell}_x - x_s)^2 + (\ell_y - y_s)^2\right]^{1/2}$$

where $\tilde{\ell}_x$ is the measured ℓ_x coordinate corrected for nonperpendicularity by

$$\tilde{\ell}_x = \ell_x + \ell_y\varepsilon\,.$$

Expression (33) is then evaluated for this d'; the result is applied to $\tilde{\ell}_x$ and ℓ_y to arrive at a new d''. Thus,

$$d'' = \left[(\tilde{\ell}_x - \delta_x' - x_s)^2 + (\ell_y - \delta_y' - y_s)^2\right]^{1/2}$$

from which new values δ_x'' and δ_y'' are computed using (33). The process is repeated, each time

applying the new δ_x and δ_y corrections to the original given $\tilde{\ell}_x$ and ℓ_y until the changes in δ_x and δ_y are less than some predetermined tolerance. The final values of distortion are the averages of the last two determinations of δ_x and δ_y, respectively. With well-corrected cameras, the computation will converge with less than four iterations.

Another way to solve the problem is to compute for the distortion profiles new coefficients that are functions of the distorted distance. For example, when one uses the reversion of series, an expression for radial distortion in terms of the distorted distance d^* and the coefficients K_i from eq (25) is for moderate profiles

$$\Delta R = K_1 D^{*3} + K_2 D^{*5} + K_3 D^{*7}$$

where D^* is given as

$$D^* = d^* - K_1 d^{*3} + (3K_1^2 - K_2) d^{*5} + (8K_1K_2 - K_3 - 12K_1^3) d^{*7}.$$

Or, a simpler expression can be developed using Taylor series expansion when the distortions are small. Thus,

$$\Delta R = (K_1 d^{*3} + K_2 d^{*5} + K_3 d^{*7})(1 - 3K_1 d^{*2} - 5K_2 d^{*4} - 7K_3 d^{*6}).$$

Successive approximation was incorporated in the computer program for satellite triangulation.

To complete the transformation from plate image coordinates to spatial coordinates, we refer to expressions (21), (22), and (38). After inverting and rearranging terms from the right side of eq (38) and substituting expression (22), an expression of the form (21) results that expresses spatial coordinates ξ and η in terms of the transformation matrix $\boldsymbol{M}^T$ and corrected image coordinates:

$$\begin{bmatrix} \xi \\ \eta \\ 1 \end{bmatrix} = \mu \boldsymbol{M}^T \begin{bmatrix} (\tilde{x}_i + \tilde{y}_i \varepsilon - x_p) - \left(\frac{\Delta R}{d} + D_x\right)(\tilde{x}_i + \tilde{y}_i \varepsilon - x_s - \delta_x) \\ \frac{c_y}{c_x}(\tilde{y}_i - y_p) - \left(\frac{\Delta R}{d} + D_y\right)\frac{c_y}{c_x}(\tilde{y}_i - y_s - \delta_y) \\ c_x \end{bmatrix}. \tag{43}$$

Dividing the first and second equations by the third to eliminate the proportionality factor reduces the system to two equations of the form

$$\left.\begin{aligned} \xi &= \frac{(\ell_x^{**} - x_p)A_1 + \frac{c_x}{c_y}(\ell_y^{**} - y_p)A_2 + c_x D}{(\ell_x^{**} - x_p)C_1 + \frac{c_x}{c_y}(\ell_y^{**} - y_p)C_2 + c_x F} \\ \eta &= \frac{(\ell_x^{**} - x_p)B_1 + \frac{c_x}{c_y}(\ell_y^{**} - y_p)B_2 + c_x E}{(\ell_x^{**} - x_p)C_1 + \frac{c_x}{c_y}(\ell_y^{**} - y_p)C_2 + c_x F} \end{aligned}\right\} \tag{44}$$

where ℓ_x and ℓ_y are corrected coordinates from the preceding paragraph. Here,

$$\ell_x^{**} = \ell_x + \ell_y \varepsilon - \delta_x$$

and

$$\ell_y^{**} = \ell_y - \delta_y . \tag{45}$$

E. Linearized Observation Equations

Equations (39) with $\tilde{x}_i$ and $\tilde{y}_i$ replaced by the measured coordinates ℓ_{x_i} and ℓ_{y_i} plus their residuals become the observation equations for our system. They express the two observations ℓ_{x_i} and ℓ_{y_i} in terms of the given spatial control and as a function of 16 unknown parameters to be determined in the solution. For a unique solution, then, eight observational pairs are necessary. The problem in satellite triangulation becomes one of estimating the unknown parameters from an overdetermined set of observations (i.e., more than eight). This estimation is accomplished by applying the method of least squares as outlined in extended form by Schmid (1959) and by Schmid and Schmid (1965).

The observation eq (39) are not linear in the unknowns; they are, therefore, replaced by the first two terms of their Taylor series expansion. Thus, eq (39) become

$$\ell_x + v_x = \tilde{x}_i^\circ + \frac{\partial \tilde{x}_i^\circ}{\partial \mu^\circ} \Delta\mu^\circ$$

and

$$\ell_y + v_y = \tilde{y}_i^\circ + \frac{\partial \tilde{y}_i^\circ}{\partial \mu^\circ} \Delta\mu^\circ \tag{46}$$

where ℓ_x and ℓ_y are the measured plate observations with unknown residuals v_x and v_y. Expressions $\tilde{x}_i^\circ$ and $\tilde{y}_i^\circ$ are the functions [eq (39)] evaluated with the approximations μ° for the unknown parameters from which the estimates (adjusted values) of the unknowns are given by

$$\hat{\mu} = \mu^\circ + \Delta\mu^\circ \tag{47}$$

The $\partial\tilde{x}_i^\circ/\partial\mu^\circ$ and $\partial\tilde{y}_i^\circ/\partial\mu^\circ$ are partial derivatives of the expressions $\tilde{x}_i^\circ$ and $\tilde{y}_i^\circ$ with respect to each of the unknowns and are denoted in matrix form as

$$B = \begin{bmatrix} \frac{\partial\tilde{x}_i^\circ}{\partial\alpha^{*\circ}} & \frac{\partial\tilde{x}_i^\circ}{\partial\delta^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\alpha^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\omega^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\varkappa^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\varepsilon^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial x_p^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial y_p^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial c_x^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial c_y^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\Delta x^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\Delta y^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial K_1^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial K_2^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial K_3^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial K_4^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial K_5^\circ} & \frac{\partial\tilde{x}_i^\circ}{\partial\varphi_r^\circ} \\ \frac{\partial\tilde{y}_i^\circ}{\partial\alpha^{*\circ}} & \frac{\partial\tilde{y}_i^\circ}{\partial\delta^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\alpha^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\omega^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\varkappa^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\varepsilon^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial x_p^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial y_p^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial c_x^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial c_y^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\Delta x^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\Delta y^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial K_1^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial K_2^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial K_3^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial K_4^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial K_5^\circ} & \frac{\partial\tilde{y}_i^\circ}{\partial\varphi_r^\circ} \end{bmatrix} \tag{48a}$$

and

$$B = \left[\begin{array}{cc|ccccccccccccccc} J_{x_\alpha} & K_{x_\delta} & A_x & B_x & C_x & U_x & H_x & I_x & G_x & P_x & Q_x & M_x & N_x & O_x & R_x & S_x & T_x \\ J_{y_\alpha} & K_{y_\delta} & A_y & B_y & C_y & U_y & H_y & I_y & G_y & P_y & Q_y & M_y & N_y & O_y & R_y & S_y & T_y \end{array}\right] . \tag{48b}$$

For simplifying the notation for later reduction of the system, the matrix is partitioned as indicated by the dashed line in eq (48b) into coefficients associated with stars and orientation parameters, respectively. The eq (48b) can be written more compactly as

$$B = \left[B_x \,\vdots\, B_o \right].$$

To determine the coefficients in B_x, we refer to Schmid (1959, pp. 28 and 29). The following expressions are given as partial derivatives of the nonlinear observation equations with respect to the rectangular coordinates X_o and Y_o of the center of projection:

$$\begin{aligned} \frac{\partial F_1}{\partial X_o} &= D_x = \frac{c^\circ}{q}③ = -J_x , & \frac{\partial F_2}{\partial X_o} &= D_y = \frac{c^\circ}{q}⑥ = -J_y , \\ \frac{\partial F_1}{\partial Y_o} &= E_x = \frac{c^\circ}{q}④ = -K_x , \text{ and} & \frac{\partial F_2}{\partial Y_o} &= E_y = \frac{c^\circ}{q}⑦ = -K_y . \end{aligned} \tag{49}$$

The F_1 and F_2 are analogous to the functions $\tilde{x}_i^\circ$ and $\tilde{y}_i^\circ$ herein; and ③, ④, ⑥, and ⑦ are computational auxillaries defined as

$$\begin{aligned} ③ &= ① D - A_1 , & ⑥ &= ② D - A_2 , \\ ④ &= ① E - B_1 , \text{ and} & ⑦ &= ② E - B_2 \end{aligned}$$

where

$$① = x_i^\circ - x_p \text{ and } ② = y_i^\circ - y_p .$$

The quantities A_1, B_1, A_2, B_2, D, and E are elements of the transformation matrix (19). The right side of each expression demonstrates that these coefficients are the negatives of those for spatial control. When one uses the chain rule, coefficients in terms of α and δ (after substitution of ξ° for X° and η° for Y°) are

$$\begin{aligned} \frac{\partial F_1}{\partial \alpha^\circ} &= \frac{\partial F_1}{\partial \xi^\circ} \cdot \frac{\partial \xi^\circ}{\partial \alpha^\circ} + \frac{\partial F_1}{\partial \eta^\circ} \cdot \frac{\partial \eta^\circ}{\partial \alpha^\circ} , & \frac{\partial F_2}{\partial \alpha^\circ} &= \frac{\partial F_2}{\partial \xi^\circ} \cdot \frac{\partial \xi^\circ}{\partial \alpha^\circ} + \frac{\partial F_2}{\partial \eta^\circ} \cdot \frac{\partial \eta^\circ}{\partial \alpha^\circ} , \\ \frac{\partial F_1}{\partial \delta^\circ} &= \frac{\partial F_1}{\partial \xi^\circ} \cdot \frac{\partial \xi^\circ}{\partial \delta^\circ} + \frac{\partial F_1}{\partial \eta^\circ} \cdot \frac{\partial \eta^\circ}{\partial \delta^\circ} , \text{ and} & \frac{\partial F_2}{\partial \delta^\circ} &= \frac{\partial F_2}{\partial \xi^\circ} \cdot \frac{\partial \xi^\circ}{\partial \delta^\circ} + \frac{\partial F_2}{\partial \eta^\circ} \cdot \frac{\partial \eta^\circ}{\partial \delta^\circ} \end{aligned} \tag{50}$$

where from eq (3) and (4)

$$\begin{aligned} \frac{\partial \xi^\circ}{\partial \alpha^\circ} &= \eta^\circ \frac{\sin \delta^\circ}{\cos z^\circ} , & \frac{\partial \xi^\circ}{\partial \delta^\circ} &= \frac{\cos H^\circ}{\cos^2 z^\circ} , \\ \frac{\partial \eta^\circ}{\partial \alpha^\circ} &= \left(\frac{\cos \varphi}{\cos^2 z^\circ} - \xi^\circ \frac{\sin \delta^\circ}{\cos z^\circ} \right), \text{ and} & \frac{\partial \eta^\circ}{\partial \delta^\circ} &= \frac{\sin \varphi \sin H^\circ}{\cos^2 z^\circ} . \end{aligned} \tag{51}$$

From eq (49), (50), and (51) coefficients of α and δ in the expanded observation equations are (taking into account that the functions F_1 and F_2 have been extended to include separate scalars c_x° and c_y°)

$$J_{x_\alpha} = -\frac{c_x^\circ}{q\cos z^\circ}\left[③\,\eta^\circ \sin\delta^\circ + ④\left(\frac{\cos\varphi}{\cos z^\circ} - \xi^\circ \sin\delta^\circ\right)\right],$$

$$K_{x_\delta} = -\frac{c_x^\circ}{q\cos^2 z^\circ}\left[③\cos H^\circ + ④\sin\varphi \sin H^\circ\right],$$

$$J_{y_\alpha} = -\frac{c_y^\circ}{q\cos z^\circ}\left[⑥\,\eta^\circ \sin\delta^\circ + ⑦\left(\frac{\cos\varphi}{\cos z^\circ} - \xi^\circ \sin\delta^\circ\right)\right],$$

and (52)

$$K_{y_\delta} = -\frac{c_y^\circ}{q\cos^2 z^\circ}\left[⑥\cos H^\circ + ⑦\sin\varphi \sin H^\circ\right]$$

that are denoted for observation i on star j as

$$B_{x_{ji}} = \begin{bmatrix} J_{x_\alpha} & K_{x_\delta} \\ J_{y_\alpha} & K_{y_\delta} \end{bmatrix}_{ji}. \tag{53}$$

The coefficients for the rotational parameters $\Delta\alpha$, $\Delta\omega$, and $\Delta\varkappa$ are identical with those of Schmid (1959) since the direction and order of rotation and rectangular coordinate systems remained unchanged. For reference, they are repeated:

$$\frac{\partial F_1}{\partial \alpha^\circ} = A_x = -c_x^\circ\left(①\cdot⑨ + ⑦\right), \qquad \frac{\partial F_2}{\partial \alpha^\circ} = A_y = -c_y\left(②\cdot⑨ - ④\right),$$

$$\frac{\partial F_1}{\partial \omega^\circ} = B_x = c_x^\circ\left[\left(1 + ①^2\right)\sin\varkappa^\circ - ①\cdot②\cos\varkappa^\circ\right], \quad \frac{\partial F_2}{\partial \omega^\circ} = B_y = -c_y^\circ\left[\left(1 + ②^2\right)\cos\varkappa^\circ - ①\cdot②\sin\varkappa^\circ\right],$$

$$\frac{\partial F_1}{\partial \varkappa^\circ} = C_x = -c_x^\circ\,②, \qquad \text{and} \qquad \frac{\partial F_2}{\partial \varkappa^\circ} = C_y = c_y\,① \tag{54}$$

where

$$⑨ = ②B_1 - ①B_2,$$

again modified to incorporate c_x° and c_y° .

For the nonperpendicularity term ε ,

$$\frac{\partial F_1}{\partial \varepsilon^\circ} = U_x = -\tilde{y}_i^\circ \quad \text{and} \quad \frac{\partial F_2}{\partial \varepsilon^\circ} = U_y = 0. \tag{55}$$

To develop the new coefficients for the distortion terms, we again refer to figure 7 and establish the relationships

$$d_x = \ell_x + \ell_y \epsilon - \Delta R_x - \Delta T_x - x_s$$

and (56)

$$d_y = \ell_y - \Delta R_y - \Delta T_y - y_s .$$

From these, the radial distance of an observation can be written as

$$d^2 = d_x^2 + d_y^2 . \tag{57}$$

Also, using expressions (28) and (57), we have

$$\Delta R_x = \left[K_1 (d_x^2 + d_y^2) + K_2 (d_x^2 + d_y^2)^2 + K_3 (d_x^2 + d_y^2)^3 \right] d_x$$

and (58)

$$\Delta R_y = \left[K_1 (d_x^2 + d_y^2) + K_2 (d_x^2 + d_y^2)^2 + K_3 (d_x^2 + d_y^2)^3 \right] d_y$$

from which we can determine the coefficients for ΔK_1°, ΔK_2°, and ΔK_3° (the coefficients in the radial distortion profile):

$$\frac{\partial F_1}{\partial K_1} = M_x = d_x d^2, \qquad \frac{\partial F_2}{\partial K_1} = M_y = d_y d^2,$$

$$\frac{\partial F_1}{\partial K_2} = N_x = d_x d^4, \qquad \frac{\partial F_2}{\partial K_2} = N_y = d_y d^4, \tag{59}$$

$$\frac{\partial F_1}{\partial K_3} = O_x = d_x d^6, \quad \text{and} \quad \frac{\partial F_2}{\partial K_3} = O_y = d_y d^6 .$$

It will be noted that the above coefficients are not exact in that they are derived from the observations ℓ_x and ℓ_y that are subject to errors (v's); also, the quantities dx and dy are treated as constants in the expansion. However, experience with problems using fictitious data both with and without random noise has shown that the convergence of the problem is not hindered and that the effects of these approximations are smaller than computer significance in the final solution.

For decentering distortion, we again use expressions (57) and (31) to write

$$\Delta T_x = \left[K_4 (d_x^2 + d_y^2) + K_5 (d_x^2 + d_y^2)^2 \right] \left[\left(\frac{2 d_x^2}{d^2} + 1 \right) \cos\varphi_T + \frac{2 d_x d_y}{d^2} \sin\varphi_T \right]$$

and (60)

$$\Delta T_y = \left[K_4 (d_x^2 + d_y^2) + K_5 (d_x^2 + d_y^2)^2 \right] \left[\frac{2 d_x d_y}{d^2} \cos\varphi_T + \left(\frac{2 d_y^2}{d^2} + 1 \right) \sin\varphi_T \right] .$$

The coefficients for ΔK_4°, ΔK_5°, and $\Delta\varphi_r^\circ$ follow directly as

$$\frac{\partial F_1}{\partial K_4^\circ} = R_x = d^2\cos\varphi_r^\circ + 2(d_x^2\cos\varphi_r^\circ + d_x d_y \sin\varphi_r^\circ), \qquad \frac{\partial F_1}{\partial K_5^\circ} = S_x = d^4\cos\varphi_r^\circ + 2d^2(d_x^2\cos\varphi_r^\circ + d_x d_y \sin\varphi_r^\circ),$$

$$\frac{\partial F_2}{\partial K_4^\circ} = R_y = d^2\sin\varphi_r^\circ + 2(d_y^2\sin\varphi_r^\circ + d_x d_y \cos\varphi_r^\circ), \qquad \frac{\partial F_2}{\partial K_5^\circ} = S_y = d^4\sin\varphi_r^\circ + 2d^2(d_y^2\sin\varphi_r^\circ + d_x d_y \cos\varphi_r^\circ),$$

$$\frac{\partial F_1}{\partial \varphi_r^\circ} = T_x = -\sin\varphi_r^\circ(K_4^\circ d^2 + K_5^\circ d^4) + 2(K_4^\circ + K_5^\circ d^2)(d_x d_y \cos\varphi_r^\circ - d_x^2 \sin\varphi_r^\circ),$$

and (61)

$$\frac{\partial F_2}{\partial \varphi_r^\circ} = T_y = \cos\varphi_r^\circ(K_4^\circ d^2 + K_5^\circ d^4) + 2(K_4^\circ + K_5^\circ d^2)(d_y^2 \cos\varphi_r^\circ - d_x d_y \sin\varphi_r^\circ).$$

Finally, when using eq (56), (58), and (60), the coefficients for Δx°, Δy°, x_p°, and y_p° are found to be

$$\frac{\partial F_1}{\partial \Delta x^\circ} = P_x = -\frac{\Delta R}{d} - d_x^2 D_r - d_x D_w \cos\varphi_r^\circ - 4d_x\left[K_4^\circ + (2d_x^2 + d_y^2)K_5^\circ\right]\cos\varphi_r^\circ - 2d_y\left[K_4^\circ + (d_x^2 + 3d_y^2)K_5^\circ\right]\sin\varphi_r^\circ,$$

$$\frac{\partial F_2}{\partial \Delta x^\circ} = P_y = -d_x d_y D_r - d_x D_w \sin\varphi_r^\circ - 2d_y\left\{\left[K_4^\circ + (3d_x^2 + d_y^2)K_5^\circ\right]\cos\varphi_r^\circ + 2d_y K_5^\circ \sin\varphi_r^\circ\right\},$$

$$\frac{\partial F_1}{\partial \Delta y^\circ} = Q_x = -d_x d_y D_r - d_y D_w \cos\varphi_r^\circ - 2d_x\left\{\left[K_4^\circ + (d_x^2 + 3d_y^2)K_5^\circ\right]\sin\varphi_r^\circ + 2d_y K_5^\circ \cos\varphi_r^\circ\right\}, \tag{62}$$

$$\frac{\partial F_2}{\partial \Delta y^\circ} = Q_y = -\frac{\Delta R}{d} - d_y^2 D_r - d_y D_w \sin\varphi_r^\circ - 2d_x\left[K_4^\circ + (d_x^2 + 3d_y^2)K_5^\circ\right]\cos\varphi_r^\circ - 4d_y\left[K_4^\circ + (d_x^2 + 2d_y^2)K_5^\circ\right]\sin\varphi_r^\circ,$$

$$\frac{\partial F_2}{\partial x_p^\circ} = H_y = P_y, \quad \frac{\partial F_1}{\partial x_p^\circ} = H_x = 1 + P_x, \quad \frac{\partial F_1}{\partial y_p^\circ} = I_x = Q_x, \quad \text{and} \quad \frac{\partial F_2}{\partial y_p^\circ} = I_y = 1 + Q_y$$

where

$$D_r = \left[2K_1^\circ + 4K_2^\circ d^2 + 6K_3^\circ d^4\right] \quad \text{and} \quad D_w = \left[2K_4 + 4K_5 d^2\right].$$

F. Multidirections

In the preceding discussion of the mathematical model, we assume one orientation of the camera and, therefore, one set of $\alpha\omega\kappa$ at each exposure station. This assumption is generally valid since, in aerial photogrammetry, each exposure normally consists of <u>one</u> opening of the shutter, the duration of which is minimized for optimum image content and least image motion. On the other hand, satellite triangulation requires a multiple image exposure to provide optimum targets for measurement and a sufficient amount of data for camera calibrations immediately preceding (pre) and following (post) the satellite passage (event). Therefore, one

plate consists of exposures over an extended period of time, introducing the possibility of camera movement during the exposure sequence. If the exposed plate is rigidly clamped, the translational motions of the camera are negligible; and all the movement can be resolved into three rotations, $\alpha_1 - \alpha_2$, $\omega_1 - \omega_2$, and $\varkappa_1 - \varkappa_2$. If the change in direction of the camera axis is constant over the exposure period and the event is equally spaced in time between the pre and post calibrations, the bias error between the two calibrated directions would be absorbed in the least squares adjustment in the form of a mean ($\alpha\ \omega\ \varkappa$) resulting in a near correct result. Unfortunately, we cannot assume the motion to be uniform over the entire period; however, we can assume that the camera will maintain a constant orientation during each of the pre, post, and event exposure sets. The solution, therefore, can be designed to compute one set of interior orientation parameters for the entire plate and three sets of exterior parameters, $\alpha\ \omega\ \varkappa$. This is done by merely adding two additional independent sets of coefficients [eq (54)] to our observation equations and evaluating them with their associated image (ℓ_x, ℓ_y) using separate rotation matrices for each pre, post, or event star. An example of the form of the matrix of coefficients of the observation equations for one star during the pre, post, and event calibrations is

$$
\begin{array}{c}
\qquad\qquad\qquad\qquad \textit{Pre} \qquad\qquad \textit{Post} \qquad\qquad \textit{Event} \\
B_o = \begin{bmatrix} B_{o_1} \\ \hline B_{o_2} \\ \hline B_{o_3} \end{bmatrix} = \left[\begin{array}{ccc|ccc|ccc|ccccccccccc}
A_x & B_x & C_x & 0 & 0 & 0 & 0 & 0 & 0 & U_x & H_x & 0 & G_x & 0 & P_x & Q_x & M_x & N_x & O_x & R_x & S_x & T_x \\
A_y & B_y & C_y & 0 & 0 & 0 & 0 & 0 & 0 & U_y & H_y & I_y & 0 & G_y & P_y & Q_y & M_y & N_y & O_y & R_y & S_y & T_y \\
\hline
0 & 0 & 0 & A_x & B_x & C_x & 0 & 0 & 0 & U_x & H_x & 0 & G_x & 0 & P_x & Q_x & M_x & N_x & O_x & R_x & S_x & T_x \\
0 & 0 & 0 & A_y & B_y & C_y & 0 & 0 & 0 & U_y & H_y & I_y & 0 & G_y & P_y & Q_y & M_y & N_y & O_y & R_y & S_y & T_y \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & A_x & B_x & C_x & U_x & H_x & 0 & G_x & 0 & P_x & Q_x & M_x & N_x & O_x & R_x & S_x & T_x \\
0 & 0 & 0 & 0 & 0 & 0 & A_y & B_y & C_y & U_y & H_y & I_y & 0 & G_y & P_y & Q_y & M_y & N_y & O_y & R_y & S_y & T_y
\end{array}\right]
\end{array}
\tag{63}
$$

where the subscripts 1, 2, and 3 indicate observation equations resulting from pre, post, and event images, respectively.

As in the case of a single set of $\alpha\ \omega\ \varkappa$, the solution is iterated for successive corrections to the initial approximations of the $(\alpha\ \omega\ \varkappa)_{pre}$, $(\alpha\ \omega\ \varkappa)_{post}$, and $(\alpha\ \omega\ \varkappa)_{event}$ to arrive at final values of each and for one set of associated interior parameters: ε, x_p, y_p, c_x, c_y, Δx, Δy, K_1, K_2, K_3, K_4, K_5, and φ_r.

G. Formation, Reduction, and Solution of the Normal Equations

As pointed out be Schmid (1959) and by Schmid and Schmid (1965), the resulting j equations from eq (53) and eq (63), formed for all the stars in the problem, take the form

$$
\left[\begin{array}{ccccc|c}
(1)_1 & 0 & 0 & \cdots & 0 & (2)_1 \\
0 & (1)_2 & 0 & \cdots & 0 & (2)_2 \\
0 & 0 & (1)_3 & \cdots & 0 & (2)_3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & (1)_j & (2)_j \\
\hline
(2)_1^T & (2)_2^T & (2)_3^T & \cdots & (2)_j^T & N
\end{array}\right]
\Delta \mu =
\left[\begin{array}{c}
(8)_1 \\ (8)_2 \\ (8)_3 \\ \vdots \\ (8)_j \\ \hline \ell
\end{array}\right]
\tag{64}
$$

where the $(1)_j$ are the results of evaluating eq (53) for n observations of a given star in the form

$$(1)_j = \left(\left[\boldsymbol{B}_{x_1}^T \boldsymbol{P}_{\ell_1} \boldsymbol{B}_{x_1}\right] + \left[\boldsymbol{B}_{x_2}^T \boldsymbol{P}_{\ell_2} \boldsymbol{B}_{x_2}\right] + \left[\boldsymbol{B}_{x_3}^T \boldsymbol{P}_{\ell_3} \boldsymbol{B}_{x_3}\right] + \cdots \left[\boldsymbol{B}_{x_n}^T \boldsymbol{P}_{\ell_n} \boldsymbol{B}_{x_n}\right]\right)_j + \boldsymbol{P}_{\alpha\delta_j} \ . \tag{65}$$

The $\boldsymbol{P}_{\ell_n}$ are the (2x2) normalized weight matrices assigned to the nth observational pair given by

$$\boldsymbol{P}_{\ell_n} = \begin{bmatrix} P_{\ell_x} & P_{\ell_{xy}} \\ P_{\ell_{xy}} & P_{\ell_y} \end{bmatrix}_n = m_o^2 \begin{bmatrix} m_{\ell_x}^2 & m_{\ell_{xy}} \\ m_{\ell_{xy}} & m_{\ell_y}^2 \end{bmatrix}_n^{-1}$$

where

$$\begin{bmatrix} m_{\ell_x}^2 & m_{\ell_{xy}} \\ m_{\ell_{xy}} & m_{\ell_y}^2 \end{bmatrix}_n$$

is the covariance matrix for the observations and m_o^2 is the assumed unit variance for the problem. Likewise, $\boldsymbol{P}_{\alpha\delta_j}$ is the normalized weight matrix assigned to the given star coordinates $(\alpha,\delta)_j$. Thus,

$$\boldsymbol{P}_{\alpha\delta_j} = \begin{bmatrix} P_\alpha & P_{\alpha\delta} \\ P_{\alpha\delta} & P_\delta \end{bmatrix}_j = m_o^2 \begin{bmatrix} m_\alpha^2 & m_{\alpha\delta} \\ m_{\alpha\delta} & m_\delta^2 \end{bmatrix}_j^{-1} \ .$$

The matrices $(2)_j$ are the summations of the matrix products of eq (53) and (63) evaluated for the n individual observations of a star (where n can be different for each star):

$$(2)_j = \left(\left[\boldsymbol{B}_{x_1}^T \boldsymbol{P}_{\ell_1} \boldsymbol{B}_{o_1}\right] + \left[\boldsymbol{B}_{x_2}^T \boldsymbol{P}_{\ell_2} \boldsymbol{B}_{o_2}\right] + \left[\boldsymbol{B}_{x_3}^T \boldsymbol{P}_{\ell_3} \boldsymbol{B}_{o_3}\right] + \cdots \left[\boldsymbol{B}_{x_n} \boldsymbol{P}_{\ell_n} \boldsymbol{B}_{o_n}\right]\right)_j \ . \tag{66}$$

The matrices $(8)_j$ are

$$(8)_j = \left(\left[\boldsymbol{B}_{x_1} \boldsymbol{P}_{\ell_1} \Delta\boldsymbol{\ell}_1\right] + \left[\boldsymbol{B}_{x_2} \boldsymbol{P}_{\ell_2} \Delta\boldsymbol{\ell}_2\right] + \left[\boldsymbol{B}_{x_3} \boldsymbol{P}_{\ell_3} \Delta\boldsymbol{\ell}_3\right] + \cdots \left[\boldsymbol{B}_{x_n} \boldsymbol{P}_{\ell_n} \Delta\boldsymbol{\ell}_n\right]\right)_j \tag{67}$$

where $\Delta\boldsymbol{\ell}_n$ is the discrepancy vector associated with the nth observation given by

$$\Delta\boldsymbol{\ell}_n = \begin{bmatrix} \Delta\ell_{x_n} \\ \Delta\ell_{y_n} \end{bmatrix} = \begin{bmatrix} \ell_{x_n} - \tilde{x}_n \\ \ell_{y_n} - \tilde{y}_n \end{bmatrix} .$$

Finally, $\boldsymbol{N}$ is the summation, over all n observations of all j stars in the problem, of

$$\boldsymbol{N} = N_1 + N_2 + \cdots N_j + \boldsymbol{P}_o = \left[\sum_{i=1}^{i=n} \boldsymbol{B}_{o_i}^T \boldsymbol{P}_{\ell_i} \boldsymbol{B}_{o_i}\right]_1 + \left[\sum_{i=1}^{i=n} \boldsymbol{B}_{o_i}^T \boldsymbol{P}_{\ell_i} \boldsymbol{B}_{o_i}\right]_2 + \cdots \left[\sum_{i=1}^{i=n} \boldsymbol{B}_{o_i}^T \boldsymbol{P}_{\ell_i} \boldsymbol{B}_{o_i}\right]_j + \boldsymbol{P}_o \tag{68}$$

where $\boldsymbol{P}_o$ is the normalized weight matrix associated with the given orientation parameters.

Likewise, $\boldsymbol{\ell}$ is the summation, over all n observations of all j stars, of

$$\boldsymbol{\ell} = \ell_1 + \ell_2 + \cdots \ell_j + P_o \Delta \boldsymbol{\ell}_o = \left[\sum_{i=1}^{i=n} B_{o_i}^T P_{\ell_i} \Delta \boldsymbol{\ell}_i\right]_1 + \left[\sum_{i=1}^{i=n} B_{o_i}^T P_{\ell_i} \Delta \boldsymbol{\ell}_i\right]_2 + \cdots \left[\sum_{i=1}^{i=n} B_{o_i}^T P_{\ell_i} \Delta \boldsymbol{\ell}_i\right]_j + P_o \Delta \boldsymbol{\ell}_o \tag{69}$$

where

$$\Delta \boldsymbol{\ell}_o = O^\circ - O_a^\circ$$

where O° are the given orientation parameters associated with the weight P_o and O_a°, the updated orientation parameters used in the current iteration. For the first iteration, $\Delta \boldsymbol{\ell}_o = 0$.

Since the individual $(1)_i$ of eq (64) are independent, the partitioning and associated stepwise reduction, as suggested in Schmid (1959), provides the most economical approach to the problem from a standpoint of computer storage and manipulation. That is, the individual observations are sequentially processed through the summations indicated by eq (65) through (69) for all data associated with one star α, δ. The star is then eliminated from the solution using the algorithm

$$N_1 - (2)_1^T (1)_1^{-1} (2)_1 = \ell_1 - (2)_1^T (1)_1^{-1} (8)_1 \tag{70}$$

where N_1 and ℓ_1 are the sums [eq (68) and (69)] for star one only. Applying the same algorithm for star two produces

$$\left[N_1 - (2)_1^T (1)_1^{-1} (2)_1\right] + N_2 - (2)_2^T (1)_2^{-1} (2) = \left[\ell_1 - (2)_1^T (1)_1^{-1} (8)\right] + \ell_2 - (2)_2^T (1)_2^{-1} (8) . \tag{71}$$

After all stars are processed, the weight matrix P_o is added to the left side of eq (71); and $P_o \Delta \boldsymbol{\ell}_o$ is added to the right side. The reduced normal equation system is then of the form

$$\underset{(22\times 22)}{N_{reduced}} \; \underset{(22\times 1)}{\Delta \boldsymbol{\mu}} = \underset{(22\times 1)}{\boldsymbol{\ell}_{reduced}}$$

that has a solution for the corrections $\Delta \boldsymbol{\mu}$:

$$\Delta \boldsymbol{\mu} = N_{reduced}^{-1} \boldsymbol{\ell}_{reduced} .$$

3. THE COMPUTER PROGRAM

Several diagrams included as figures near the end of this report describe in graphic form a computer program designed to solve for the orientation of a single camera. Figure 17, a flow diagram of only the major steps, shows the general logic of the program. The diagrams in figure 18 list all the mathematical expressions used and their sequences of application. A flow diagram of FORTRAN statements is included as figure 19, followed by a listing of the

program in figure 20. The FORTRAN statements of figure 20 are supplemented by equivalent mathematical expressions. Figure 21 shows the organization of the problem input cards, and figure 22 describes the format and variable names of the input data.

A. General Problem Flow

The general flow of the problem (outlined in fig. 17) is relatively straightforward; it can be broken into eight major steps:

1. The input data, such as problem descriptors, constants, options, approximations, observations, and weights are assembled and ordered; the various internal controls are set accordingly. The title page for output is printed; this includes a complete description of the problem being computed and the various options selected.

2. After computing the direction cosine matrix associated with the current approximations for the rotations $\alpha\omega\varkappa$ of pre, post, and event exposures, the measurements ℓ_x and ℓ_y (with sidereal time and weight matrix P_ℓ for one observation) and the star coordinates α, δ and weight $P_{\alpha\delta}$ are read from storage. The plate measurements are corrected for nonperpendicularity and distortion using expressions from figure 7 and eq (33) and (37). At this point, if the plate coordinates are the first observations for the star in question and the star is of type 8 (a star with unknown α and δ coordinates), the program computes approximations for α, δ for use with the remaining observations of the star. To do this, one uses the $\tilde{\ell}_x, \tilde{\ell}_y$ and approximations to the orientation parameters with eq (21) to compute rectangular coordinates of the star direction. These coordinates are corrected for atmospheric refraction by using eq (15), (7), and (16) and are used in eq (5) to compute approximate right ascension and declination. A correction for diurnal aberration is made with eq (17), after which the coordinates are stored and given zero weight to signify them as unknowns. All star coordinates are then handled identically (i.e., they are treated as apparent places and adjusted for diurnal aberration and atmospheric refraction). They are then converted to rectangular coordinates and applied to the observation eq (39), the results of which, when compared to the adjusted plate coordinates, constitute the discrepancy vector $\Delta\ell$.

3. Coefficients for the linearized observation equations are computed and summed for all observations of one star, after which the weight matrix for that star is added and the star is eliminated from the normals. The results are added to the reduced normal equations. Steps 2 and 3 are repeated for all stars in the problem.

4. The solution is checked to determine if all criteria are met; if so, iteration is stopped. If not, the normals are inverted; and corrections to the orientation parameters are solved. The whole process, steps 2 through 4, is repeated.

5. After stopping the iterations, final values for the parameters are used to compute and print residuals for each observation.

6. Differences in direction between all combinations of pre, post, and event are computed across the diagonal of the plate. Based on the magnitudes of these angular differences and their mean errors, a decision is made for combining the various sets.

7. Results of the computation are printed and treated according to the options set in the beginning.

8. If the decision of step 6 was to combine orientations, the inverted normals are modified accordingly, new corrections to the orientation parameters are computed, and steps 5 and 7 are repeated.

B. Program Options

Moving to figure 18 (page 1 of 11), we have a list of 14 problem options. These options serve the purpose of modifying the general problem to require certain special computations. The following is a discussion of these options in terms of their meaning, use, and the modifications they impart to the internal structure of the program.

Option 1 asks if common scales should be computed in the problem. If the indicator is set to 0, the problem will proceed as outlined in the previous subsection and compute a differential scale for the x and y plate axes. If, however, the indicator is 1, the matrix (35) becomes a unit matrix $c_y/c = c_x/c = 1$; and only one principal distance is computed. This is accomplished in the program by carrying a common scale in the observation eq (39) and by placing the coefficient G_y directly under G_x in the coefficient matrix (48b). This leaves zeros in column 14 of (48b) that result in a zero element at location (14,14) of the reduced normals. Before inversion, all zeros on the diagonal of the normals are replaced by a large number (10^{20}) that has the effect of eliminating that particular unknown from the solution. As a result, no correction is computed for c_y° .

Option 2 is a control that allows a directional constraint to be applied to the camera orientation. If the indicator is set to 1, the program will read in as input a given azimuth and elevation of the central ray, the plate coordinates of the intersection of this ray with the camera focal plane, and the associated weight matrix. The given azimuth and elevation are first converted to rectangular coordinates using (from fig. 2)

$$\xi_{AE} = -\cot E \cos A ,$$

and

$$\eta_{AE} = -\cot E \sin A . \tag{72}$$

The direction is then treated the same as that of a given star with the exception that the refraction and distortion corrections are bypassed. Also, since the observation is considered to be independent of the camera lens distortion, coefficients in the observation equation are computed for the central perspective parameters only ($\alpha, \omega, \varkappa, \varepsilon, x_p, y_p, c_x$, and c_y). The variance of the observation is introduced by computing an equivalent weight matrix for the coordinates of the camera fiducial marker intersections ℓ_{x_f} and ℓ_{y_f} .

Options 3 and 4 are introduced to simplify input requirements. In the event that the input star or plate coordinates are all of equal weight, an indicator of 1 will cycle the program

to read the weight once and apply it to all observations thereafter.

In the discussion concerning atmospheric refraction, it was pointed out that the correction is computed from a tangent series using four coefficients τ_i determined from a "fit" to Garfinkel data. Option 5 allows the program to deviate from these given τ_i in special cases and compute corrections to the refraction model as an after treatment to the initial orientation. That is, if option 5 is 1, the solution will use the given refraction model to iterate to an acceptable level of residuals to arrive at an orientation of the camera. The program will then recycle through the entire computation for one more interation, this time carrying four additional unknowns to solve for corrections to the given τ_i. This is accomplished by introducing the final values of orientation as approximations and proceeding through the solution as usual with the exception that the matrix (63) is expanded to include the eight terms for $\Delta\tau_i$. (In reality, the reduced normals $\boldsymbol{N}$ are always of dimension 26 x 26 for programming simplicity; corrections are made to the τ_i only when coefficients are computed and the computer finds values other than zero on the diagonals.) The added coefficients are

$$\begin{aligned}
\frac{\partial F_1}{\partial \tau_1} &= V_x = r_x \tan\frac{\beta}{2}, & \frac{\partial F_2}{\partial \tau_1} &= V_y = r_y \tan\frac{\beta}{2}, \\
\frac{\partial F_1}{\partial \tau_2} &= W_x = r_x \tan^3\frac{\beta}{2}, & \frac{\partial F_2}{\partial \tau_2} &= W_y = r_y \tan^3\frac{\beta}{2}, \\
\frac{\partial F_1}{\partial \tau_3} &= X_x = r_x \tan^5\frac{\beta}{2}, & \frac{\partial F_2}{\partial \tau_3} &= X_y = r_y \tan^5\frac{\beta}{2}, \\
\frac{\partial F_1}{\partial \tau_4} &= Y_x = r_x \tan^7\frac{\beta}{2}, \text{ and } & \frac{\partial F_2}{\partial \tau_4} &= Y_y = r_y \tan^7\frac{\beta}{2}
\end{aligned} \tag{73}$$

where

$$r_x = J_x Z_\xi + K_x Z_\eta \quad \text{and} \quad r_y = J_y Z_\xi + K_y Z_\eta$$

and Z_ξ and Z_η are computational auxiliaries given by

$$Z_\xi = -T^{1/2} W \xi_R \left(\frac{1}{\tan z} + \tan z\right), \quad Z_\eta = -T^{1/2} W \eta_R \left(\frac{1}{\tan z} + \tan z\right), \tag{74}$$

and

$$\tan z = (\xi_R^2 + \eta_R^2)^{1/2}.$$

The corrected refraction model is then expression (7) evaluated with the new τ_i where

$$new\ \tau_i = \tau_i^\circ + \Delta\tau_i^\circ \qquad i = 1,4. \tag{75}$$

Since the corrections to the coefficients τ_i° by themselves are not readily interpreted, the program was extended, in the case of refraction computation, to compute and print values for refraction at 1° intervals over a range of 20°, centered at the camera zenith distance using both the given and computed refraction models.

In addition, the mean error for the corrected refraction model is computed for each ray using

$$m_\zeta = m\left(\mathbf{f}_\tau \mathbf{Q}_\tau \mathbf{f}_\tau^T\right)^{1/2} \tag{76}$$

where m is the mean error of unit weight for the problem and $\mathbf{f}_\tau$ is a vector of partial derivatives of eq (7) with respect to the τ_i^*. Thus,

$$\mathbf{f}_\tau = T^{1/2} W \left[\tan\frac{\beta}{2} \quad \tan^3\frac{\beta}{2} \quad \tan^5\frac{\beta}{2} \quad \tan^7\frac{\beta}{2}\right]. \tag{77}$$

The $\mathbf{Q}_\tau$ is the 4 x 4 matrix for the coefficients τ_i from the inverted reduced normal equations:

$$\mathbf{Q}_\tau = \begin{bmatrix} Q_{\tau_1\tau_1} & Q_{\tau_1\tau_2} & Q_{\tau_1\tau_3} & Q_{\tau_1\tau_4} \\ & Q_{\tau_2\tau_2} & Q_{\tau_2\tau_3} & Q_{\tau_2\tau_4} \\ \text{Symmetrical} & & Q_{\tau_3\tau_3} & Q_{\tau_3\tau_4} \\ & & & Q_{\tau_4\tau_4} \end{bmatrix}. \tag{78}$$

Options 6 and 7 are related in that they both were introduced for data preparation purposes (i.e., one for finding close approximations to the orientation parameters and the other for blunder detection). Option 6 (compute minimum solution) will cause the program to select a set number of stars from the total input data (usually a unique solution) to solve for $\alpha, \omega, \varkappa, \varepsilon, c_x, c_y, x_p$, and y_p. (For this type of solution, all stars will be treated as event type; and only one orientation will be computed.) If option 7 is not elected, the program will proceed to a maximum solution using all the star data together with the approximations from option 6. The minimum solution in this case has been used to arrive at close approximations to the unknowns from a minimum amount of data to conserve computer time. (The approximations to $\alpha\omega\varkappa$ pre and post are set equal to event.) On the other hand, if option 7 (pre-run) is elected with option 6, the computer will process all the remaining star data through the residual cycle using the orientation from the minimum solution to detect large blunders in the measured data.

Option 8 was added to the program for two reasons. First, the computed parameters that define lens distortion are (in the case of refraction) coefficients of an expansion which by themselves are difficult to interpret. To provide a clearer interpretation of distortion, then, an election of option 8 along with input parameters d_K, d_Δ, and d_{max} will cause the program to print out an evaluation of expression (25) for radial distances ranging from $d = 0$ to $d = d_{max}$ with increments d_Δ. Since radial distortion is symmetrical about the point x_s, y_s, this data provides a representation of the computed radial distortion over the full

format of the film plane. Decentering lens distortion, on the other hand, is not symmetrical about the point x_s, y_s ; but it has the characteristic of a common distortion profile P_c. Consequently, the program was designed to print an evaluation of P_c from expression (31) for the same limits and intervals as radial distortion to present the curve of maximum tangential distortion along the radial line $\varphi_T = 0°$. The second reason for option 8 has no application in satellite triangulation but is incorporated for presenting radial distortion in a form more familiar to the photogrammetrist than the foregoing. That is, borrowing terminology from Pestrecov (1959), we can consider the curve data from the computed parameters as the camera's characteristic distortion and the curve data normally provided to the photogrammetrist as calibrated distortion. The difference between the two curves is a mathematical transformation of the characteristic distortion to provide a distortion pattern over the entire image format that will minimize relative positional errors of the individual images. This transformation takes the form of a linear scale change.

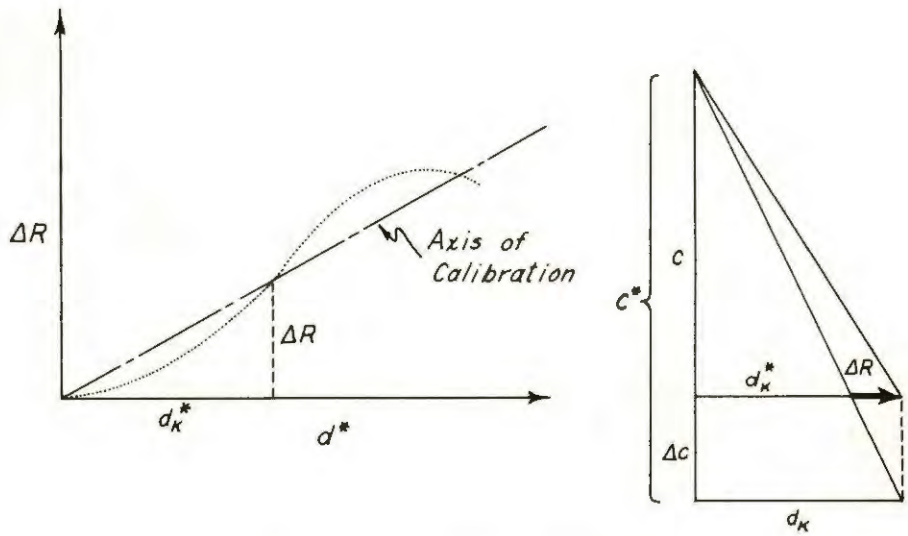

Figure 13.--Calibrated distortion

From figure 13, it is obvious that a different distortion curve can be presented mathematically for any chosen axis of calibration. For example, if one chooses to "balance" the distortion curve, an oblique axis of calibration can be established that equalizes the areas above and below this axis for some limiting angle of view. This defines the radial distance d_K^* where the oblique axis intercepts the computed distortion curve. Introducing a linear term $K_0 d^*$ into expression (25) results in the expression for calibrated distortion:

$$\Delta R_c = K_0 d^* + K_1 d^{*3} + K_2 d^{*5} + K_3 d^{*7}$$

where the distance argument is given by

$$d^* = d_K - \Delta R .$$

From an evaluation of the characteristic distortion curve, the quantity ΔR is at distance d^* and d_K an input parameter for option 8. In the case of figure 13, it defines the radial distance on the plate having zero calibrated distortion and establishes the coefficient K_0 as

$$K_0 = -\left(K_1 d^{*2} + K_2 d^{*4} + K_3 d^{*6}\right). \tag{79}$$

Similarly, the principal distance associated with the calibrated-distortion curve (c^*) is, from figure 13,

$$\frac{\Delta R}{d_K^*} = \frac{\Delta c}{c}$$

where

$$\Delta c = c^* - c\ ,$$

$$\Delta R = -K_0 d_K^*,$$

and

$$c^* = c\left(1 - K_0\right) \tag{80}$$

where c is the principal distance computed in the problem. In problems where separate scalers c_x and c_y are computed, their mean is used in expressions (80); and c^* is evaluated as

$$c^* = \frac{c_x + c_y}{2}\left(1 - K_0\right).$$

Also, to aid in the interpretation of the statistical significance of the printed curves, we compute the mean errors associated with each of the curves from

$$m_{\Delta R} = m\left(\mathbf{f}_R \mathbf{Q}_R \mathbf{f}_R^T\right)^{1/2},$$

$$m_{\Delta R_T} = m\left(\mathbf{f}_{R_T} \mathbf{Q}_R \mathbf{f}_{R_T}^T\right)^{1/2}, \tag{81}$$

and

$$m_{\Delta T} = m\left(\mathbf{f}_T \mathbf{Q}_T \mathbf{f}_T^T\right)^{1/2}$$

where $m_{\Delta R}$, $m_{\Delta R_T}$, and $m_{\Delta T}$ are the mean errors of radial distortion, calibrated distortion, and decentering distortion, respectively. The vectors $\mathbf{f}$ are obtained by taking the partial derivatives with respect to the coefficients K_i of the functions that describe the distortion; that is,

$$\mathbf{f}_R = \left[d^3 \quad d^5 \quad d^7\right],$$

$$\mathbf{f}_{R_T} = \left[d(d^2 - d_K^2) \quad d(d^4 - d_K^4) \quad d(d^6 - d_K^6)\right],$$

and

$$\mathbf{f}_T = \left[d^2 \quad d^4\right]. \tag{82}$$

Finally, the Q's are the matrices for the coefficients K_i from the inversion of the reduced normal equations

$$\left.\begin{aligned} Q_R &= \begin{bmatrix} Q_{K_1K_1} & Q_{K_1K_2} & Q_{K_1K_3} \\ Q_{K_1K_2} & Q_{K_2K_2} & Q_{K_2K_3} \\ Q_{K_1K_3} & Q_{K_2K_3} & Q_{K_3K_3} \end{bmatrix} \\ Q_T &= \begin{bmatrix} Q_{K_4K_4} & Q_{K_4K_5} \\ Q_{K_4K_5} & Q_{K_5K_5} \end{bmatrix} \end{aligned}\right\} . \tag{83}$$

In addition, the mean error of the principal distance associated with the transformed radial distortion curve (c^*) is

$$m_{c^*} = m\left(f\, Q_{c^*} f^T\right)^{1/2} \tag{84}$$

where f is the vector of partial derivatives with respect to the K_i and computed principal distance (c_x) of expressions (80) given by

$$f = \left[\, 1-K_0 \quad 1-K_0 \quad -c_x d_K^2 \quad -c_x d_K^4 \quad -c_x d_K^6 \,\right]; \tag{85}$$

when separate scalers c_x and c_y are computed,

$$f = \left[\, \frac{1-K_0}{2} \quad \frac{1-K_0}{2} \quad -\frac{c_x+c_y}{2} d_K^2 \quad -\frac{c_x+c_y}{2} d_K^4 \quad -\frac{c_x+c_y}{2} d_K^6 \,\right]. \tag{86}$$

The matrix Q_{c^*} is

$$Q_{c^*} = \begin{bmatrix} Q_{c_xc_x} & Q_{c_xc_y} & Q_{c_xK_1} & Q_{c_xK_2} & Q_{c_xK_3} \\ & Q_{c_yc_y} & Q_{c_yK_1} & Q_{c_yK_2} & Q_{c_yK_3} \\ & \text{Symmetrical} & Q_{K_1K_1} & Q_{K_1K_2} & Q_{K_1K_3} \\ & & & Q_{K_2K_2} & Q_{K_2K_3} \\ & & & & Q_{K_3K_3} \end{bmatrix} . \tag{87}$$

Option 9 is a control that permits the transformation of the final angular orientation of the camera. Because of the refraction correction, it is most convenient to compute the camera orientation angles $(\alpha,\omega,\varkappa)$ relative to a Cartesian coordinate system oriented at the camera station, with the Z axis alined along the station zenith. This system is shown in figure 1. A triangulation of a set of multistation directions, however, requires that the individual camera directions be given relative to a common origin (Hanson 1972). To accomplish this, we introduced option 9 into the program. When the option is yes, accompanied by reference station coordinates φ_o, λ_o, and H_o, the program will operate on the final $\alpha\,\omega\,\varkappa$ in the following manner. First, the system is transformed (fig 14) from the camera station system to the pole by rotating through the angle $(90°-\varphi_i)$ about the η axis, Z toward ξ. Thus,

$$r_1 = \begin{bmatrix} \sin\varphi_i & 0 & -\cos\varphi_i \\ 0 & 1 & 0 \\ \cos\varphi_i & 0 & \sin\varphi_i \end{bmatrix} \tag{88}$$

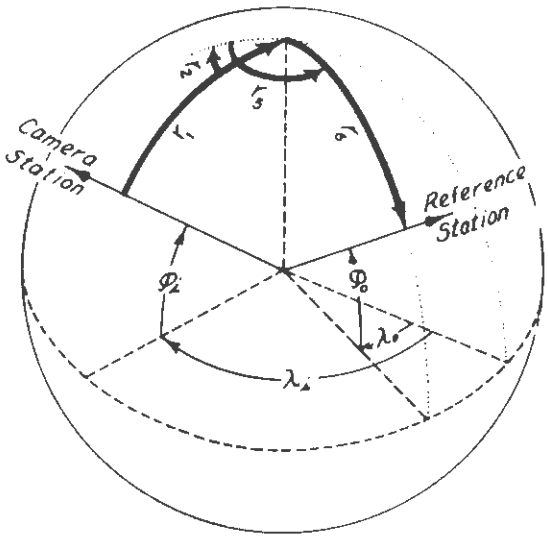

Figure 14.--Camera station transformation

where φ_i is the camera station latitude. This system is then rotated $180°-\lambda_i$ about the new Z axis, ξ toward η, to bring the direction of the rotated ξ axis coincident with the prime meridian with the positive direction toward the Equator. Thus,

$$r_2 = \begin{bmatrix} -\cos\lambda_i & \sin\lambda_i & 0 \\ -\sin\lambda_i & -\cos\lambda_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{89}$$

where λ_i is the camera station longitude. Combining r_1 and r_2 gives

$$R_{\varphi_i\lambda_i} = r_2 r_1 = \begin{bmatrix} -\cos\lambda_i \sin\varphi_i & \sin\lambda_i & \cos\lambda_i \cos\varphi_i \\ -\sin\lambda_i \sin\varphi_i & -\cos\lambda_i & \sin\lambda_i \cos\varphi_i \\ \cos\varphi_i & 0 & \sin\varphi_i \end{bmatrix}. \tag{90}$$

These two steps are then reversed (this time using the input reference station coordinates) to transform the system to the reference station. These rotations are

$$r_3 = \begin{bmatrix} -\cos\lambda_o & -\sin\lambda_o & 0 \\ \sin\lambda_o & -\cos\lambda_o & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{91}$$

and

$$r_6 = \begin{bmatrix} \sin\varphi_0 & 0 & \cos\varphi_0 \\ 0 & 1 & 0 \\ -\cos\varphi_0 & 0 & \sin\varphi_0 \end{bmatrix} \tag{92}$$

where λ_0 and φ_0 are the input reference station coordinates. Rotations r_5 and r_6 combined are

$$R_{\varphi_0\lambda_0} = r_6 r_5 = \begin{bmatrix} -\cos\lambda_0 \sin\varphi_0 & -\sin\lambda_0 \sin\varphi_0 & \cos\varphi_0 \\ \sin\lambda_0 & -\cos\lambda_0 & 0 \\ \cos\lambda_0 \cos\varphi_0 & \sin\lambda_0 \cos\varphi_0 & \sin\varphi_0 \end{bmatrix}. \tag{93}$$

The product of these two rotation matrices would be all that is necessary to bring the individual camera orientations to a common system were it not for the periodic motion of the pole. This motion, monitored by the International Polar Motion Service and described in the form of coordinates of the pole of rotation (instantaneous pole) relative to a mean pole position, is shown graphically in figure 15. For compensating for this apparent motion, the system above (rotated through $r_2 r_1$) must undergo a correction x, y before applying the transformation $r_6 r_5$. From figure 15, we first rotate our system about η for an angle a in the direction ξ toward Z, resulting in the matrix

$$r_3 = \begin{bmatrix} \cos a & 0 & \sin a \\ 0 & 1 & 0 \\ -\sin a & 0 & \cos a \end{bmatrix}. \tag{94}$$

Next, the rotation about ξ for an amount y (η toward Z) is given by

$$r_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos y & \sin y \\ 0 & -\sin y & \cos y \end{bmatrix}. \tag{95}$$

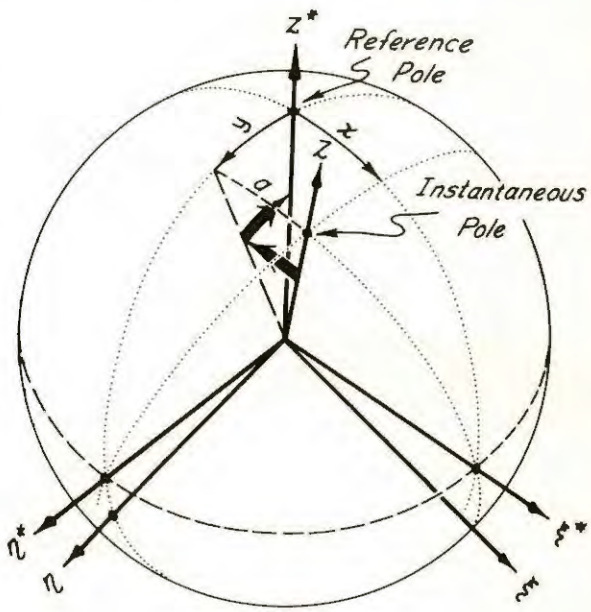

Figure 15.--Pole motion

The product of these two matrices, considering that x and y are small angles and therefore $x \approx a$, $\cos x \approx \cos y \approx 1$, $\sin y \approx y$, and $\sin x \approx x$ results in the total correction for polar motion as

$$R_{xy} = r_4 r_3 = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ -x & -y & 1 \end{bmatrix}. \tag{96}$$

To compute the transformed camera orientation angles, we combine transformations 1 through 6 and apply them to the matrix of direction cosines described in eq (19):

$$M^* = \begin{bmatrix} A_1^* & B_1^* & C_1^* \\ A_2^* & B_2^* & C_2^* \\ D^* & E^* & F^* \end{bmatrix} = R_{\varphi_0\lambda_0} R_{zy} R_{\varphi_i\lambda_i} M . \tag{97}$$

From the transformed direction cosines, we have (by definition) the transformed orientation angles given by

$$\sin\omega^* = E^* ,$$

$$\sin\varkappa^* = {-B_1^*}/{\cos\omega^*} ,$$

and

$$\cos\alpha^* = {F^*}/{\cos\omega^*} . \tag{98}$$

The program was designed to allow for the incorporation of a weight matrix for all input approximations to the orientation parameters including the coefficients of the refraction model in the refraction computation. Option 10 controls the input of this matrix. In the event that the matrix is not full (some parameters are complete unknowns), option 10 is set at the number of input cards containing weights (five weight numbers per card plus indexes). The program then reads the cards and assigns the weight numbers to the matrix according to their associated indexes (row and column numbers). On the other hand, for a full matrix, the option indicator contains a number greater than 52; and the computer reads an ordered set of weight numbers (six per card) and assigns them to their proper location sequentially. In either case, because of symmetry, it is only necessary to input the upper diagonal portion of the matrix. The computer program restores the lower portion.

Option 11 controls punching of output data other than that normally required in the satellite triangulation. Namely, when option 11 is elected, the residuals (v_{ℓ_x}, v_{ℓ_y}), the identifying number for each plate point, and the input measured coordinates (ℓ_x, ℓ_y) will be punched, one observation per card. These data are normally used in plot routines for a visual study of the residual patterns.

Options 12 and 13 override the internal iteration controls and allow the user to enforce the maximum number of iteration cycles to be computed for either the minimum or the maximum solution or both.

Option 14, useful for error studies of various camera and plate point configurations, is used to input any desired mean error of unit weight for the problem. With this control, flawless fictitious data can be introduced for the orientation solution; and the statistical significance of the computed parameters will be printed for any desired noise level. In other words, it acts as an arbitrary scaler for the covariance matrices of the solution.

C. Program Constants

On figure 18 (page 1 of 11), there are a series of problem constants. The problem iteration tolerance ($\Delta\Sigma\ell$) is, as the name implies, a control of the iteration cycles based on the rate of change of the level of the residuals ($\Sigma\ell$). The value ($\Sigma\ell$) is, more or less, a measure of the level of the discrepancy vector given by

$$\Sigma\ell = \left(\frac{\Sigma\,(\Delta\ell_x^2 + \Delta\ell_y^2)}{n-u}\right)^{1/2}$$

where n-u is the degrees of freedom for the problem. The absolute value of $\Sigma\ell$ by itself is not too significant, but the change between successive values is an indication of the rate of convergence or divergence of the problem and of the significance of computing an additional iteration. Therefore, iterations of the solution will be stopped when the absolute change between successive values of $\Sigma\ell$ are less than the input constant $\Delta\Sigma\ell$ (i.e., when

$$\Delta\Sigma\ell \geq \left|\Sigma\ell_a - \Sigma\ell_{(a+1)}\right|$$

where the subscript a indicates the last solution through solving of the normal equations). As shown on figure 18 (page 7 of 11), this iteration check and the set numbers from options 12 and 13 will be overruled by four divergent steps, given by

$$\Sigma\ell_a < \Sigma\ell_{(a+1)} .$$

These four divergent steps need not be successive to stop the iterations.

The refraction iteration tolerance ($\Delta\zeta$) and the distortion iteration tolerance (Δd) are input constants used to control the successive approximation loops in the correction for refraction and distortion, respectively. These iterative schemes are described previously in this report. The successive approximation for refraction is stopped when the difference between successive values of refraction computed for a given ray is less than the tolerance ($\Delta\zeta$), where

$$\Delta\zeta \geq \left|\zeta_{n+1} - \zeta_n\right| .$$

The distortion cycle is stopped, on the other hand, when the changes in both radial distortion and decentering distortion are less than the input constant (Δd). That is, when for radial distortion

$$\Delta d \geq \left|(\Delta R)_n - (\Delta R)_{n-1}\right|$$

and for, decentering distortion

$$\Delta d \geq \left|(\Delta\tilde{T})_n - (\Delta\tilde{T})_{n-1}\right| .$$

Here, ΔR and $\widetilde{\Delta T}$ are expressions (25) and (32) evaluated at the distance of the point in question.

The time correction (ΔST) is a factor in units of sidereal time that adds directly to the input time coordinates for all observations. This option was included in the program for operational convenience.

Finally, we have the constant k , a factor used to control the combining of pre, post, and event orientations (or combinations thereof) into a single value. That is, k is an input parameter (selected by the user) that multiplies the mean error of the computed direction difference to establish a value from which a test of the significance of the computed angular separation of the camera central rays can be judged. Specifically, this test for camera movement is accomplished in the following manner.

After the program has iterated to a satisfactory "level of residuals" and completed the "v-cycle" (step 5 of the general problem flow), direction differences for all combinations of pre, post, and event orientations are computed for selected points along the diagonal of the plate (step 6). This is done by successively introducing plate coordinate values of 0,0 1,1 ... 8,8 cm into the solution and treating them as plate measurements of type 8 stars. This results in a correction of the coordinates for lens distortion and nonperpendicularity. The corrected coordinates are then applied to eq (21) together with the final values of orientation (successively for pre, post, and event) to compute rectangular coordinates in object space. The resulting coordinates together with the station latitude are then applied to eq (5) to convert the directions into coordinates of right ascension and declination.

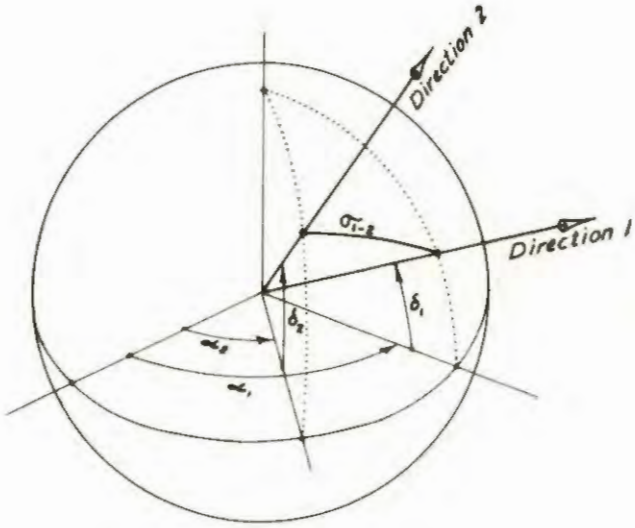

Figure 16--Camera movement

Figure 16 depicts two directions given by α_1, δ_1 and α_2, δ_2 on a unit sphere. We use the figure and the law of cosines of spherical trigonometry. Thus, the angle between directions 1 and 2 is given by

$$\cos \sigma_{1-2} = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1 - \alpha_2) \; ; \tag{99}$$

or since the angle is normally small, an expression in terms of the sine of the angle is desirable. Forming the square of the vector cross product between the unit vectors 1 and 2 results in

$$\sin^2 \sigma_{1-2} = \left\{\cos\delta_1 \sin(\alpha_1 - \alpha_2)\right\}^2 + \left\{\sin\delta_1 \cos\delta_2 - \cos\delta_1 \sin\delta_2 \cos(\alpha_1 - \alpha_2)\right\}^2 . \tag{100}$$

Expression (100) is evaluated for every combination of directions for which star data exist in the problem. For convenience in notation, the three possible combinations are identified on figure 18 (page 9 of 11) as

$$\cos\sigma_{i-j} = \begin{bmatrix} \cos\sigma_{2-1} \\ \cos\sigma_{3-2} \\ \cos\sigma_{3-1} \end{bmatrix} = \begin{bmatrix} Post - Pre \\ Event - Post \\ Event - Pre \end{bmatrix}.$$

The statistical significance and magnitude of each direction difference provide a basis for the decision to recompute the problem carrying two or all of the orientations as common. The inverse of the normal equation system $\boldsymbol{N}^{-1}$ is used to propagate the "errors" of the unknowns in the problem into the direction differences by

$$\boldsymbol{m}_{\sigma} = \begin{bmatrix} m_{\sigma_{2-1}} \\ m_{\sigma_{3-2}} \\ m_{\sigma_{3-1}} \end{bmatrix} = m\left(\boldsymbol{f}\boldsymbol{Q}\boldsymbol{f}^{T}\right) \tag{101}$$

where $\boldsymbol{f}$ is the row vector of partial derivatives of expression (100) with respect to right ascension and declination. That is,

$$\boldsymbol{f} = \begin{bmatrix} f_{2-1} \\ f_{3-2} \\ f_{3-1} \end{bmatrix} = \begin{array}{c} \begin{array}{cccccc} Pre & & Post & & Event & \end{array} \\ \begin{bmatrix} X & X & X & X & 0 & 0 \\ 0 & 0 & X & X & X & X \\ X & X & 0 & 0 & X & X \end{bmatrix} \end{array}$$

where

$$\boldsymbol{f}^{T}_{i-j} = \frac{1}{\sin\sigma_{i-j}} \begin{bmatrix} \cos\delta_j \cos\delta_i \sin(\alpha_j - \alpha_i) \\ -\cos\delta_j \sin\delta_i + \sin\delta_j \cos\delta_i \cos(\alpha_j - \alpha_i) \\ \hline -\cos\delta_j \cos\delta_i \sin(\alpha_j - \alpha_i) \\ -\sin\delta_j \cos\delta_i + \cos\delta_j \sin\delta_i \cos(\alpha_j - \alpha_i) \end{bmatrix}. \tag{102}$$

The matrix $\boldsymbol{Q}$ is the matrix of weight coefficients for the direction coordinates (α, δ) from the solution given by

$$\boldsymbol{Q} = \left[(1)^{*}\right]\left[(2)^{*}\right] \boldsymbol{N}^{-1} \left[(3)^{*}\right]\left[(1)^{*}\right] \tag{103}$$

where

$$(1)^{*} = \begin{array}{c} \begin{array}{ccc} Pre & Post & Event \end{array} \\ \begin{bmatrix} \left[\boldsymbol{B}_x^T \boldsymbol{P}_2 \boldsymbol{B}_x\right]^{-1} & 0 & 0 \\ 0 & \left[\boldsymbol{B}_x^T \boldsymbol{P}_2 \boldsymbol{B}_x\right]^{-1} & 0 \\ 0 & 0 & \left[\boldsymbol{B}_x^T \boldsymbol{P}_2 \boldsymbol{B}_x\right]^{-1} \end{bmatrix} \end{array},$$

$$(2)^* = \begin{bmatrix} B_x^T P_\ell B_o \\ B_x^T P_\ell B_o \\ B_x^T P_\ell B_o \end{bmatrix} \begin{matrix} Pre \\ Post \\ Event \end{matrix} ,$$

and

$$(3)^* = (2)^{*T}.$$

The logic for combining orientations follows the computation of directions for the central ray ($\ell_x = \ell_y = 0$) and is based on the three inequalities

$$\sigma_{2-1} < k\, m_{\sigma_{2-1}} \sim \text{Combine Pre \& Post},$$

$$\sigma_{3-2} < k\, m_{\sigma_{3-2}} \sim \text{Combine Event \& Post},$$

and

$$\sigma_{3-1} < k\, m_{3-1} \sim \text{Combine Event \& Pre}.$$

The new solution, based on common orientations, can be obtained simply by using the method of modification. This eliminates the need to recompute the entire problem. That is, corrections to the solution vector are computed from

$$\Delta\bar{O} = \left\{N^{-1} - N^{-1}C^T(CN^{-1}C^T)^{-1}CN^{-1}\right\}\ell + N^{-1}C^T(CN^{-1}C^T)^{-1}\ell^*$$

and (104)

$$O^\circ_{a+1} = O^\circ_a + \Delta\bar{O}$$

where N and ℓ are the inverse of the reduced normal equation and constant column, respectively, from the last iteration of the solution. The vector ℓ^* is a column of differences of estimated orientation angles from the solution in the form

$$\ell^* = \begin{bmatrix} \alpha^\circ_{EVENT} - \alpha^\circ_{PRE} \\ \omega^\circ_{EVENT} - \omega^\circ_{PRE} \\ \varkappa^\circ_{EVENT} - \varkappa^\circ_{PRE} \\ \alpha^\circ_{EVENT} - \alpha^\circ_{POST} \\ \omega^\circ_{EVENT} - \omega^\circ_{POST} \\ \varkappa^\circ_{EVENT} - \varkappa^\circ_{POST} \end{bmatrix}_a .$$

The matrix C contains the condition equations that express the quantities to be combined and so varies with each new problem. As an example, a C matrix that will cause all three orientations to combine into one set of orientation angles is written as

$$C = \left[\begin{array}{ccccccccc:c} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & \end{array}\right] O \cdot \text{(Columns 10 → 26 contain zeros.)}$$

The associated inverse to the reduced normal equation matrix is computed from

$$N^{-1^*} = N^{-1}\left[I - C^T\left(CN^{-1}C^T\right)^{-1}CN^{-1}\right]. \tag{105}$$

D. Program Input

Figure 21 (page 1 of 5) shows the input cards as they are assembled to follow the program deck. As indicated, the input data for each problem are made up of a series of problem control cards followed by the coordinate data. A more detailed description of the individual input cards is shown on pages 2 through 5 of figure 21. Here, the data contained on each card is identified as to field location and format followed by the variable name used in the program. In most cases, the data shown are self-explanatory; no further discussion is necessary. The following is a discussion of those parts of the input that are not obvious.

All descriptive data such as problem numbers, dates, and titles are introduced with an alphabetic field. The information is then printed out as an array (in the case of the title), and centering of the block of data must be done on the input card.

The "last problem indicator" on card 1 is a signal that terminates computations on batch runs. When more than one problem is being run, cc 80 should be zero for all but the last. A 1 punch will signal the program to terminate after the current problem.

A nonzero in cc 77 of card 3 will alert the program to eliminate certain stars or observations of stars from the input list of coordinate data. Those stars or observations to be deleted from the computation are identified by number on cards immediately following the input coordinates (see page 5 of figure 21). All deleted data will be printed on the output list.

The entry on card 6 at cc 26 is the sidereal time (in radians) used to compute directions for fictitious diagonal points. A common time is used for all orientations and can be any number from 0 to 2π. For consistency, however, common practice is to use the average between the earliest and latest exposure times in the observation period.

The entry titled "trail identification" in cc 72 of card 7 is a set of three three-digit numbers used to identify the orientation associated with each plate measurement. The program compares the leading digits of the plate point identification number with those given as "trail identification" for identification as to pre, post, or event and processes the measurement accordingly.

Cards 9 and 10 are included in the deck if, and only if, option 9 (transform $\alpha, \omega, \varkappa$) is other than zero. If one wishes to correct for pole motion only, card 9 should be identical to 8 (i.e., the reference station is the same as the camera station). Likewise, if one wishes to transform the orientation with no pole motion correction, card 9 will contain the reference station data; and card 10 will be blank. Equation (96) will become a unit matrix and effect no change to the pole position.

Similar to the transformation of $\alpha, \omega, \varkappa$, cards 11 and 12 are included in the deck only if option 2 is elected. Should no azimuth and elevation data be available, option 2 is zero; and cards 11 and 12 are omitted. The same applies to cards 20 and 21 (common weights for

stars and plate measurements). These cards are included only if options 3 and 4 so indicate.

Finally, the coordinate data cards are appended with the group of plate measurements to be deleted (if so indicated on card 3) and with the numbers of those stars selected for the minimum solution (again indicated on option card as IND 6). The remaining problem decks follow in sequence to the last problem with a 1 indicator in cc 80, card 1.

E. Program Output

Sample pages from a test problem selected at random are included as figure 22. The samples do not, by far, provide an example of each and every variation of output that can be generated from the program. They do, however, present a general view of the form of the output to be expected and show that all items are labeled adequately to aid the user in their interpretation. The sample is a problem made up of fictitious data designed to test the convergence of a total orientation using stars of pre, post, and event origin. Since the data are exact, a fictitious mean error of unit weight is introduced for computation of the problem statistics. Also note that the problem was designed to have exceedingly large separations between the directions of pre, post, and event, for purely academic reasons.

Pages 1 through 3 of figure 22 describe the problem being computed as dictated by the problem control cards of figure 21. These data are appended with a count of the observational data (star and plate coordinates) and a printout of the level of discrepancies after each iteration to show the trend of convergence. Also printed on page 1 of figure 22 is the criteria within the problem that determined the halt of iteration. Page 2 of figure 22 is a selected coordinate printout list to illustrate one form of observational data after solution of the problem (type 2 stars with common weights). This is followed by page 3 of figure 22 that shows the typical form for presenting the directional differences (also typical of a star having unknown coordinates). Finally, the remaining pages, which are self-explanatory, show the normal form in which the computed orientation data and associated statistics are presented.

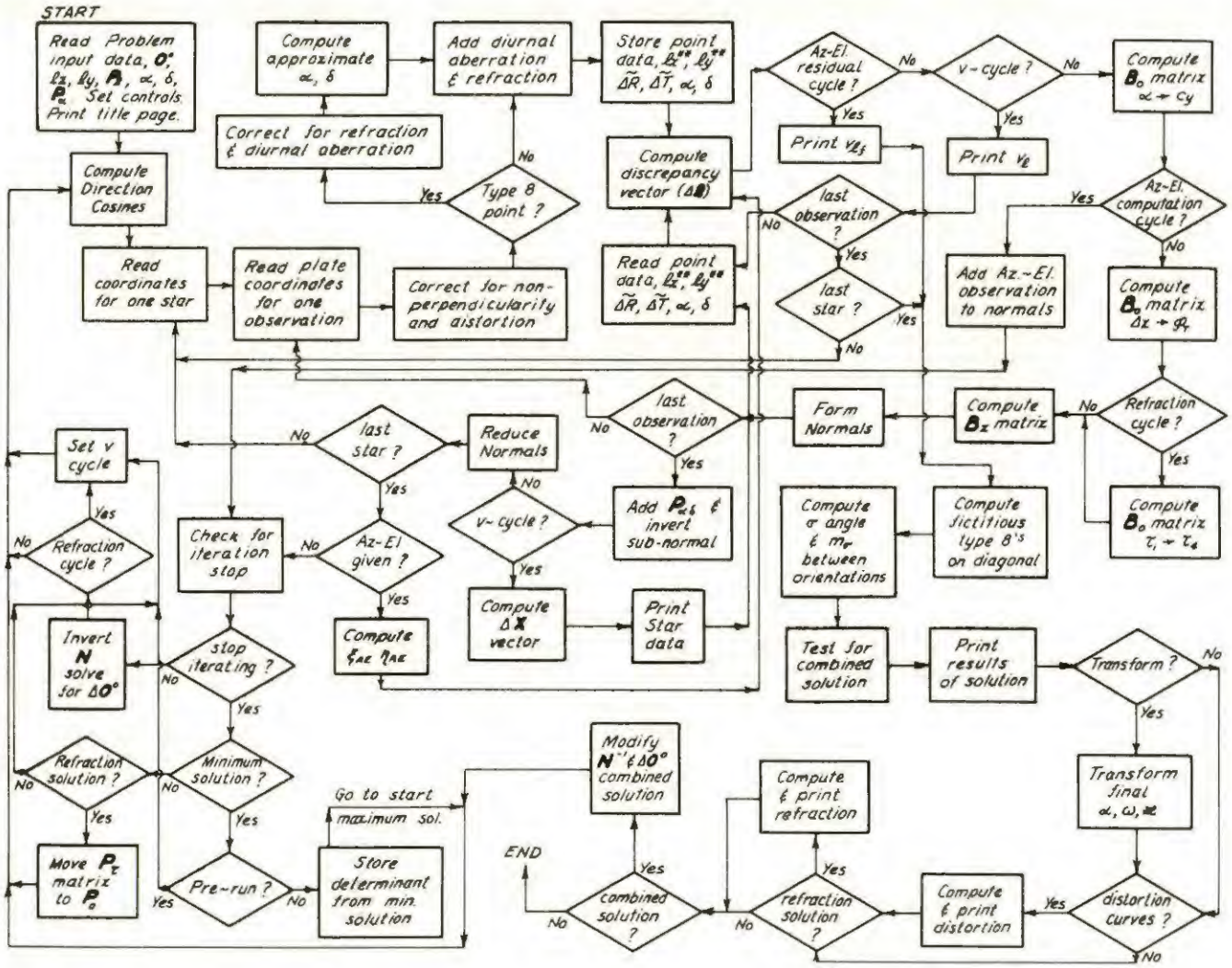

Figure 17.--Flow diagram of the major steps

SINGLE CAMERA ~ STELLAR

Problem Flow Diagram ~ 20 Sept. 1967

Read problem data:

PROBLEM DESCRIPTORS:

Problem number, Date of computation, Last problem indicator, Problem name.

PROBLEM OPTIONS:

1. $C_x = C_y$? 0 = No, 1 = Yes.
2. Camera azimuth & elevation given? 0 = No, 1 = Yes.
3. Common weight matrices for all stars with known α, δ? 0 = No, 1 = Yes.
4. Common weight matrices for all plate points? 0 = No, 1 = Yes.
5. Compute corrections to given refraction coefficients? 0 = No, 1 = Yes.
6. Compute minimum solution? 0 = No, # = Yes ~ use first # stars in solution.
7. Compute pre-run? 0 = No, 1 = Yes
8. Compute & print distortion curves? 0 = No, 1 = Yes ~ both radial & decentered lens.
9. Transform final $\alpha, \omega, \varkappa$? 0 = No, 1 = Yes ~ to reference station "mean coordinates".
10. Orientation weight matrix given? 0 = No, # = Yes ~ read # cards, (# ≥ 52 = 22x22 + 4x4).
11. Punch output data? 0 = No, 1 = Yes.
12. Number of iterations ~ Maximum solution? # } used to control maximum number of iterations
13. Number of iterations ~ Minimum solution? # }
14. Input mean error of unit weight? 0 = No, # = mean error } used for statistical studies.

PROBLEM CONSTANTS:

Problem iteration tolerance ($\Delta\xi\ell$); Refraction iteration tolerance ($\Delta\zeta$); Station Temp. (T); Station Pressure (P); Distortion curve crossover distance (d_k); Distortion curve interval (d_Δ); Distortion curve maximum distance ($d_{max.}$); Time correction (Δ_{ST}); Distortion iteration tolerance (Δd); Event number; Date of obs.; Plate, camera, tracking unit and comparator numbers; Name of comparator operator; Camera station number, name, latitude, longitude & elev.; Factor for combined solution (k).

If option 9 is yes:
Read ~ Reference Station name, number, Latitude, Longitude & elev.
Read ~ Instantaneous Pole coordinates, x & y.

If option 2 is yes:
Read ~ Azimuth, elevation data;

$$\boldsymbol{\ell}_f = \begin{bmatrix} \ell_{x_f} \\ \ell_{y_f} \end{bmatrix} \quad \begin{bmatrix} A \\ E \end{bmatrix} \quad \mathbf{P}_{\ell_f} = \begin{bmatrix} P_{\ell_{x_f}} & P_{\ell_{x_f}\ell_{y_f}} \\ & P_{\ell_{y_f}} \end{bmatrix}$$

Write appropriate output headings and set program controls as dictated by PROBLEM OPTIONS and PROBLEM CONSTANTS.

to page 2 to read 0°

1

Note: Broken lines on diagonals indicate symmetric matrices.

Figure 18.--Flow diagram of the mathematical expressions (page 1 of 11)

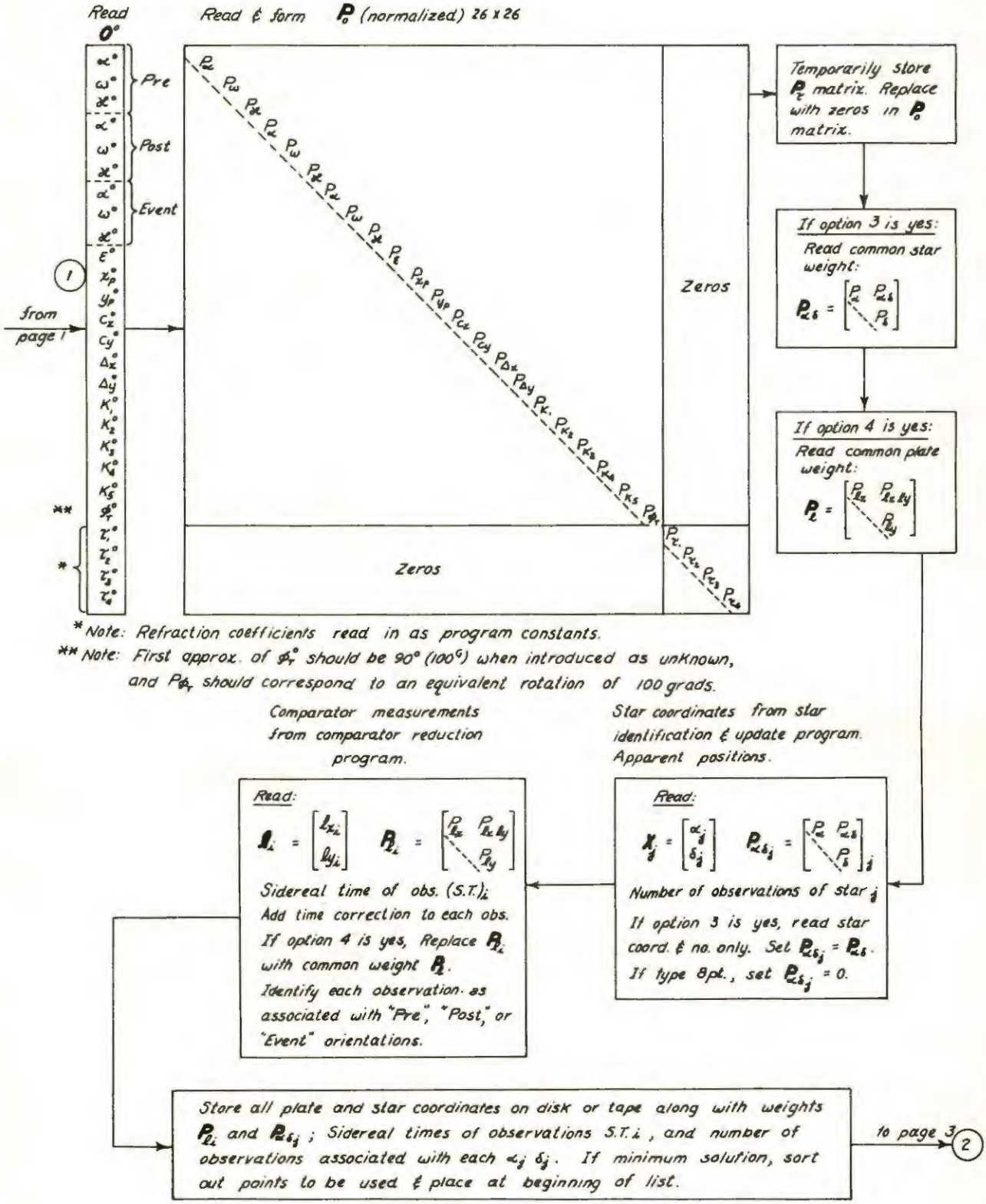

Figure 18.--Continued (page 2 of 11)

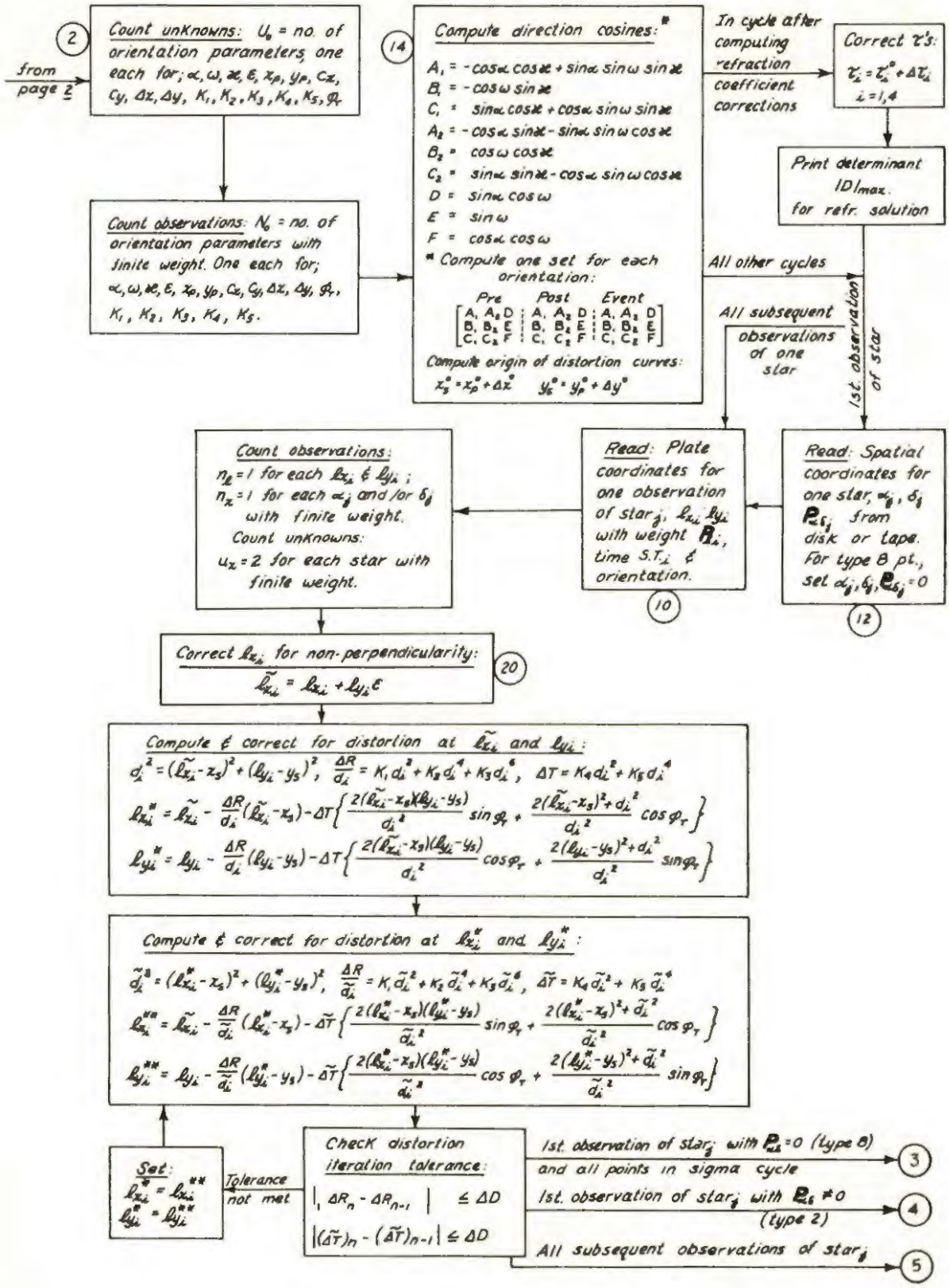

Figure 18.--Continued (page 3 of 11)

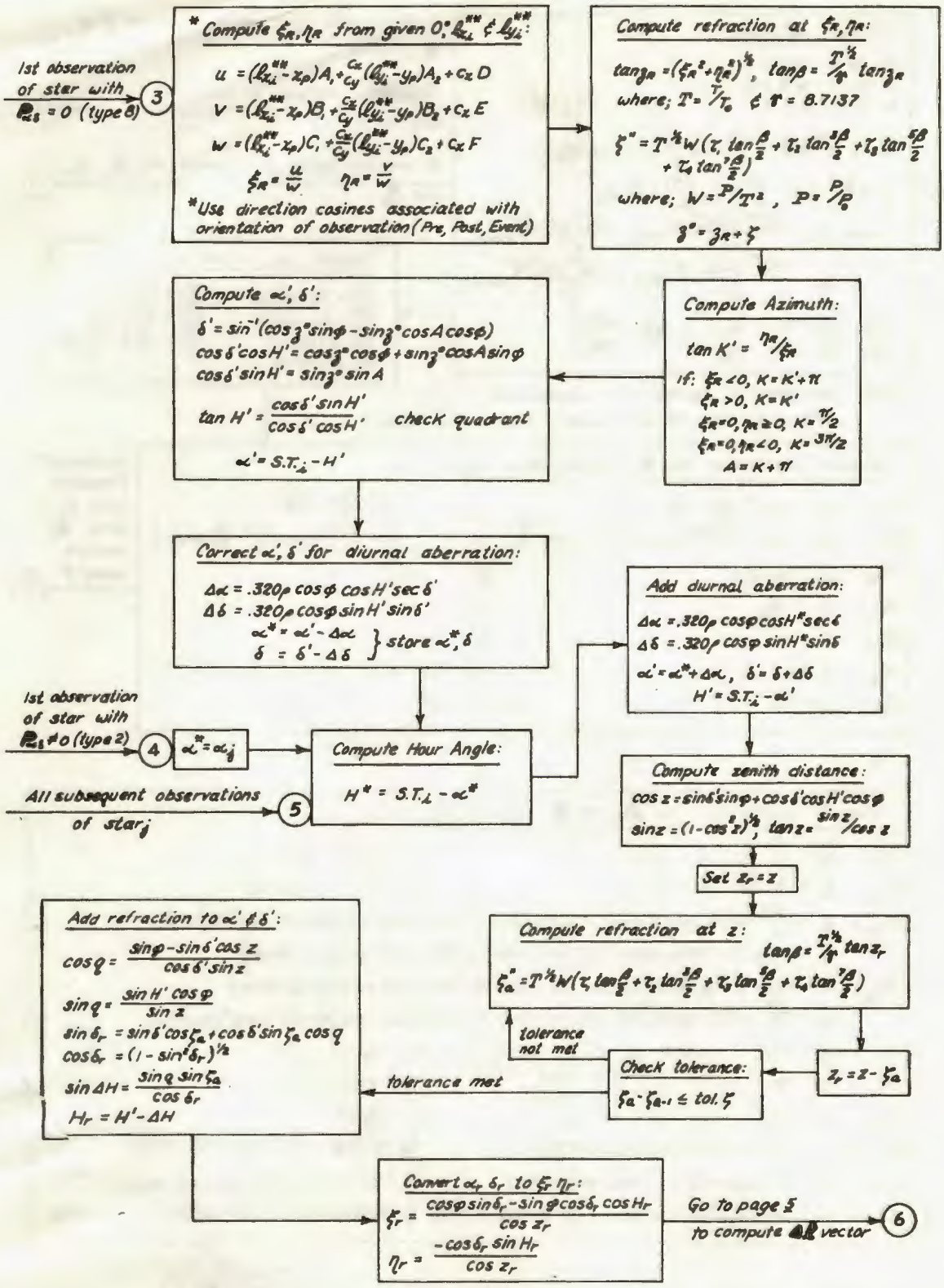

Figure 18.--Continued (page 4 of 11)

from page 4 → (6)

*** Compute $\Delta\ell$ vector:**

$m = \xi_R A_1 + \eta_R B_1 + C_1$

$n = \xi_R A_2 + \eta_R B_2 + C_2$

$q = \xi_R D + \eta_R E + F$

$$x_i = \left(\frac{c_x m}{q} + x_p\right)\left(1 + \frac{\Delta R}{\tilde{d}_i}\right) - \left(x_s \frac{\Delta R}{\tilde{d}_i}\right) + \Delta\tilde{T} \cdot D_{x_c} - \ell_{y_i}\,\varepsilon$$

$$y_i = \left(\frac{c_y n}{q} + y_p\right)\left(1 + \frac{\Delta R}{\tilde{d}_i}\right) - \left(y_s \frac{\Delta R}{\tilde{d}_i}\right) + \Delta\tilde{T} \cdot D_{y_c}$$

where: $$D_{x_c} = \frac{2(\ell_{x_i}^{**} - x_s)(\ell_{y_i}^{**} - y_s)}{\tilde{d}_i^{\,2}} \sin\varphi_r + \frac{2(\ell_{x_i}^{**} - x_s)^2 + \tilde{d}_i^{\,2}}{\tilde{d}_i^{\,2}} \cos\varphi_r$$

$$D_{y_c} = \frac{2(\ell_{x_i}^{**} - x_s)(\ell_{y_i}^{**} - y_s)}{\tilde{d}_i^{\,2}} \cos\varphi_r + \frac{2(\ell_{y_i}^{**} - y_s)^2 + \tilde{d}_i^{\,2}}{\tilde{d}_i^{\,2}} \sin\varphi_r$$

$\Delta\ell_{x_i} = \ell_{x_i} - x_i = -v_{\ell_x}$

$\Delta\ell_{y_i} = \ell_{y_i} - y_i = -v_{\ell_y}$

$$\Delta\ell_i = \begin{bmatrix} \Delta\ell_{x_i} \\ \Delta\ell_{y_i} \end{bmatrix}$$

In AE residual cycle, go to add $w_{\ell_i}^T P_{\ell_i} w_{\ell_i}$ to $\Sigma W_\ell^T P_\ell V_\ell$ → (15)

In v-cycle, return to sum and print v's. → (16)

In pre-run cycle, go to write v's, ℓ_{x_i}, ℓ_{y_i}, and P_{ℓ_i}. → (17)

*** Compute auxiliaries:**

(1) $= (x_i - x_p)/c_x$

(2) $= (y_i - y_p)/c_y$

(3) $=$ (1) $D - A_1$

(4) $=$ (1) $E - B_1$

(6) $=$ (2) $D - A_2$

(7) $=$ (2) $E - B_2$

(9) $=$ (2) $B_1 -$ (1) B_2

* Note: Use direction cosines & $\varkappa$ associated with orientation of observation ("Pre", "Post", or "Event")

Compute coefficients for B_o matrix ($Bo_{\omega} \rightarrow Bo_{c_y}$):

$(Bo_{\varkappa})$	$A_x = -c_x((1)\cdot(9) + (7))$	$A_y = -c_y((2)\cdot(9) - (4))$
*(Bo_{ω})	$B_x = c_x[(1 + (1)^2)\sin\varkappa - (1)\,(2)\cos\varkappa]$	$B_y = -c_y[(1 + (2)^2)\cos\varkappa - (1)\cdot(2)\sin\varkappa]$
$(Bo_{\varkappa})$	$C_x = -c_x\,(2)$	$C_y = c_y\,(1)$
(Bo_{c_x})	$U_x = -\ell_{y_i}$	$U_y = 0$
(Bo_{x_p})	$H_x = 1$	$H_y = 0$
(Bo_{y_p})	$I_x = 0$	$I_y = 1$
(Bo_{c_x})	$G_x = (1)$	(Bo_{c_y}) $G_y = (2)$

In Azimuth-Elevation cycle, go to form B_o matrix page 6 → (8)

Compute coefficients for B_x and B_o matrices ($Bo_{\omega_x} \rightarrow Bo_{\varphi_r}$):

(Bx_{ξ})	$D_x = \frac{c_x}{q}(3)$ $\quad J_x = -D_x$	$D_y = \frac{c_y}{q}(6)$ $\quad J_y = -D_y$
(Bx_{η})	$E_x = \frac{c_x}{q}(4)$ $\quad K_x = -E_x$	$E_y = \frac{c_y}{q}(7)$ $\quad K_y = -E_y$
(Bo_{ω_x})	$P_x = -\frac{\Delta R}{\tilde{d}_i} - dx^2 D_r - dx\,D_w \cos\varphi_r - 4dx\,[K_4 + (2dx^2 + dy^2)K_5]\cos\varphi_r - 2dy\,[K_4 + (dx^2 + 3dy^2)K_5]\sin\varphi_r$	
	$P_y = -dx\,dy\,D_r - dx\,D_w \sin\varphi_r - 2dy\left\{[K_4 + (3dx^2 + dy^2)K_5]\cos\varphi_r + 2dy\,K_5 \sin\varphi_r\right\}$	
(Bo_{ω_y})	$Q_x = -dx\,dy\,D_r - dy\,D_w \cos\varphi_r - 2dx\left\{[K_4 + (dx^2 + 3dy^2)K_5]\sin\varphi_r + 2dx\,dy\,K_5 \cos\varphi_r\right\}$	
	$Q_y = -\frac{\Delta R}{\tilde{d}_i} - dy^2 D_r - dy\,D_w \sin\varphi_r - 2dx\,[K_4 + (dx^2 + 3dy^2)K_5]\cos\varphi_r - 4dy\,[K_4 + (dx^2 + 2dy^2)K_5]\sin\varphi_r$	
(Bo_{K_1})	$M_x = dx\,d_i^2$ where: $dx = \ell_{x_i}^{**} - x_s$ $\quad dy = \ell_{y_i}^{**} - y_s$	$M_y = dy\,d_i^2$
(Bo_{K_2})	$N_x = dx\,d_i^4$ $\quad D_r = [2K_1 + 4K_2 d_i^2 + 6K_3 d_i^4]$	$N_y = dy\,d_i^4$
	$D_w = [2K_4 + 4K_5 d_i^2]$	
(Bo_{K_3})	$O_x = dx\,d_i^6$ $\quad d_i^2 = dx^2 + dy^2$	$O_y = dy\,d_i^6$
(Bo_{K_4})	$R_x = d_i^2 \cos\varphi_r + 2(dx^2 \cos\varphi_r + dx\,dy \sin\varphi_r)$	$R_y = d_i^2 \sin\varphi_r + 2(dy^2 \sin\varphi_r + dx\,dy \cos\varphi_r)$
(Bo_{K_5})	$S_x = d_i^4 \cos\varphi_r + 2d_i^2(dx^2 \cos\varphi_r + dx\,dy \sin\varphi_r)$	$S_y = d_i^4 \sin\varphi_r + 2d_i^2(dy^2 \sin\varphi_r + dx\,dy \cos\varphi_r)$
(Bo_{φ_r})	$T_x = -\sin\varphi_r(K_4 d_i^2 + K_5 d_i^4) + 2(K_4 + K_5 d_i^2)(dx\,dy \cos\varphi_r - dx^2 \sin\varphi_r)$	$T_y = \cos\varphi_r(K_4 d_i^2 + K_5 d_i^4) + 2(K_4 + K_5 d_i^2)(dy^2 \cos\varphi_r - dx\,dy \sin\varphi_r)$

to page 6 → (7)

Figure 18.--Continued (page 5 of 11)

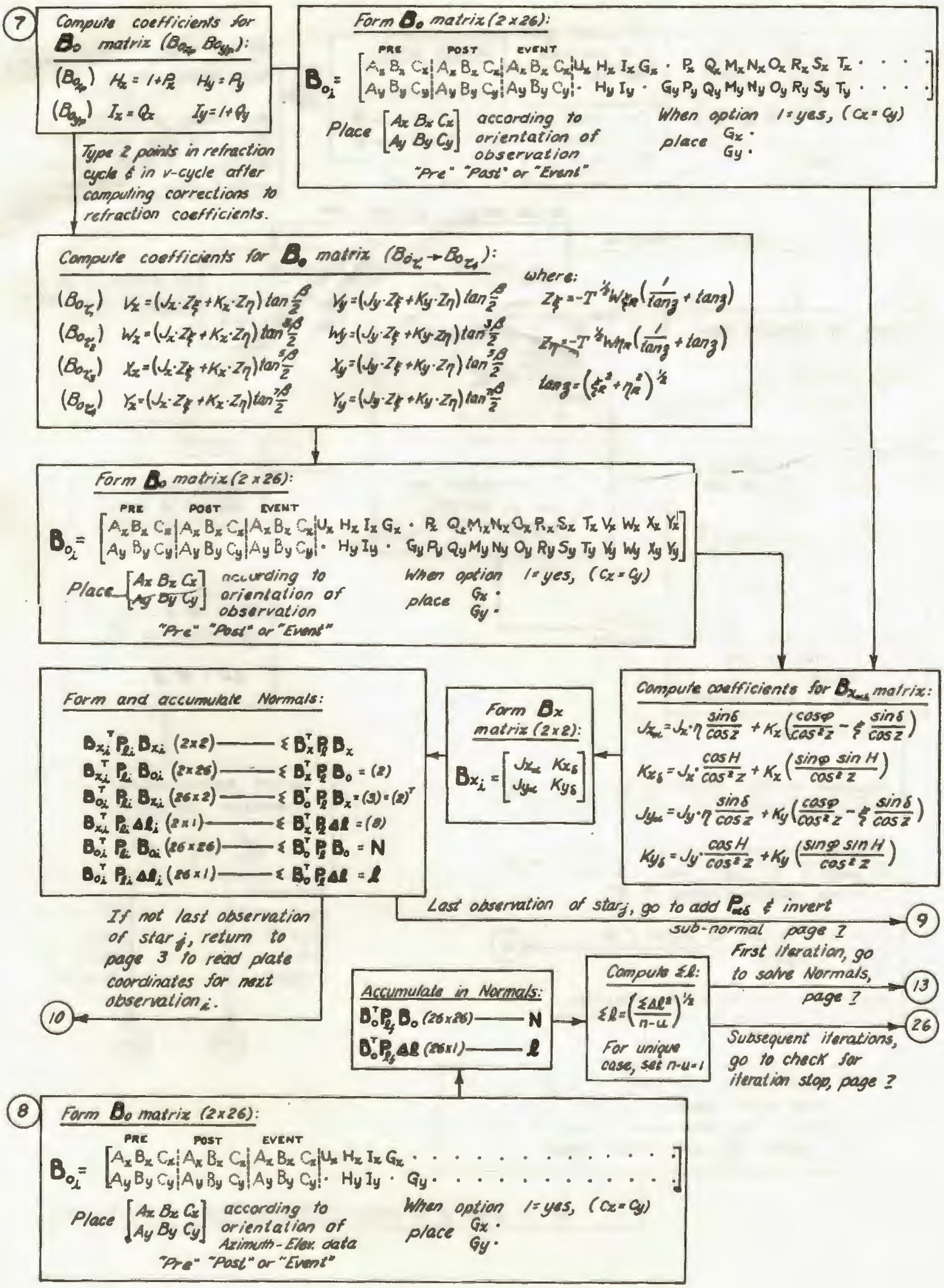

Figure 18.--Continued (page 6 of 11)

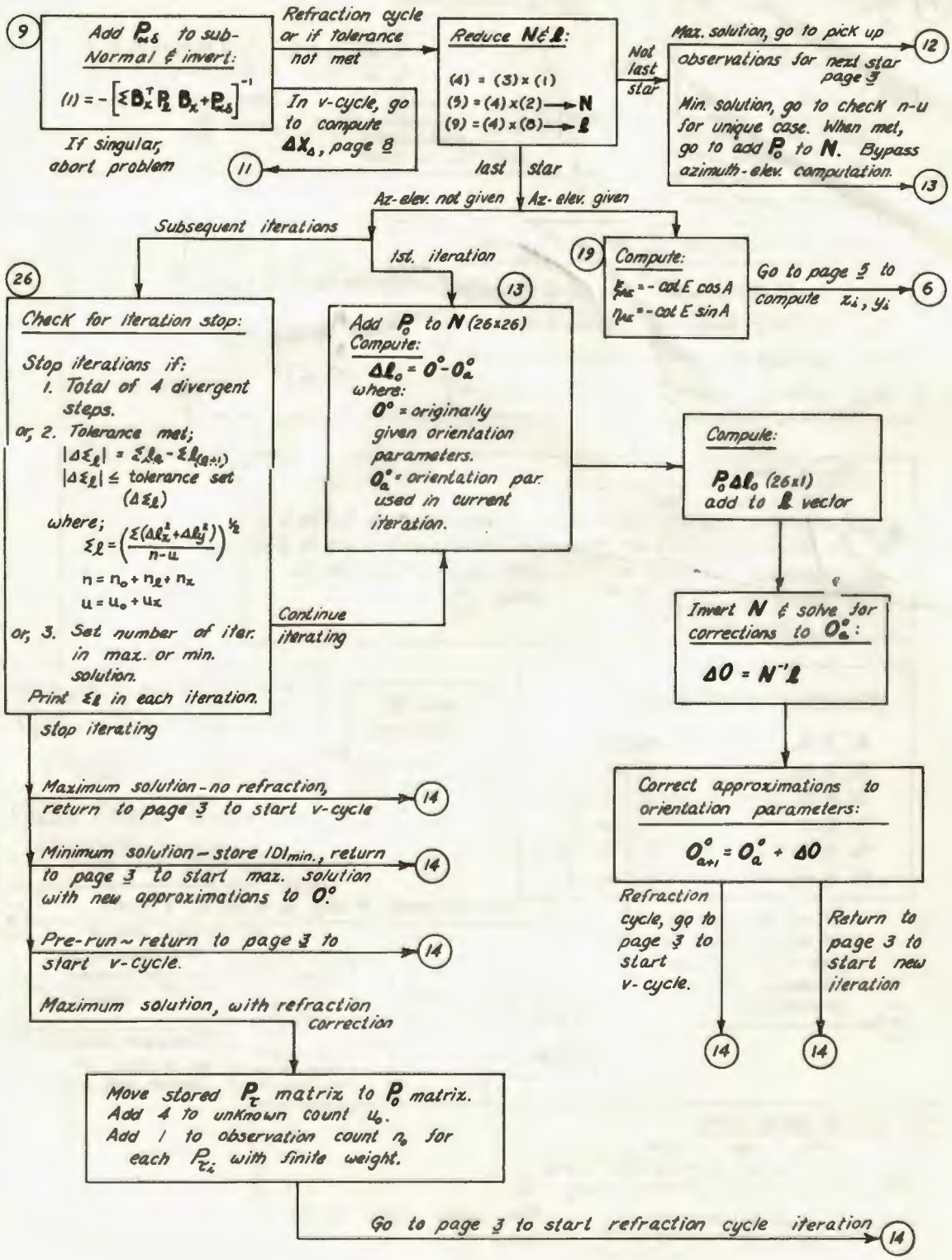

Figure 18.--Continued (page 7 of 11)

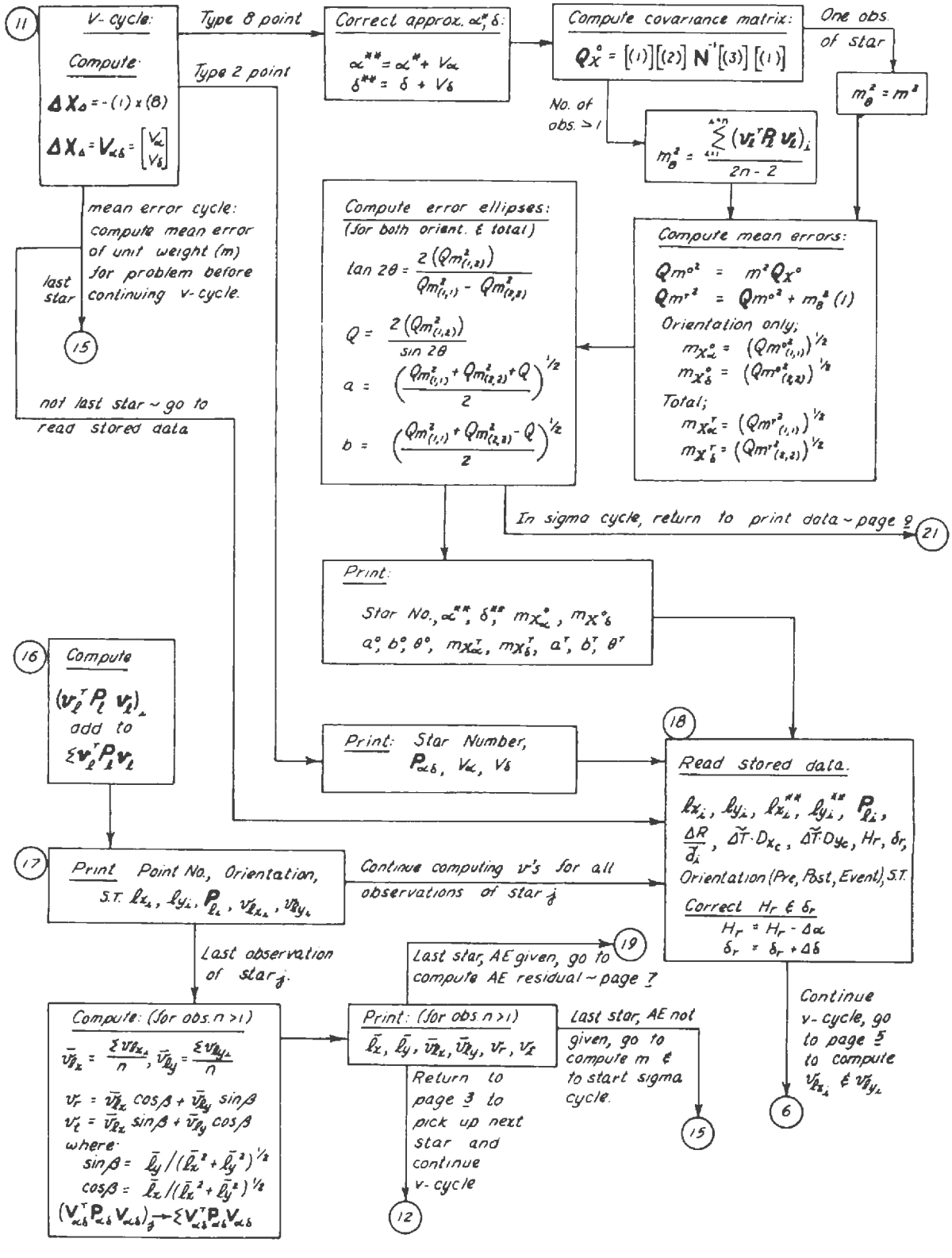

Figure 18.--Continued (page 8 of 11)

(15) *Compute:*

$$\Sigma V_o^T P_o V_o$$

$$\Sigma V^T P V = \Sigma v_\ell^T P_\ell v_\ell + \Sigma V_{\alpha\delta}^T P_{\alpha\delta} V_{\alpha\delta} + \Sigma V_o^T P_o V_o$$

$$m = \left(\frac{\Sigma V^T P V}{n-u}\right)^{1/2}$$

Start sigma cycle.

Compute fictitious coordinates ℓ_x & ℓ_y with unit weight $\left(P_\ell = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$ *for 10 points at 1 c.m. intervals along diagonal of plate* $(\ell_x = \ell_y)$. *Repeat for all orientations (Pre ~ Post ~ or Event) solved for in problem. Identify points as type 8 by setting* $P_{\alpha\delta} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Return to compute directions (α, δ) for fictitious points & their mean errors (20)

(21) *Print: Diagonal distance, Orientation of point (Pre~Post~Event),* α^{**}, δ^{**}, *Azimuth, Elev.,* $m_{X_\alpha^\circ}$, $m_{X_\delta^\circ}$, a°, b°, θ°, $m_{X_\alpha^T}$, $m_{X_\delta^T}$, a^T, b^T, θ^T.

Compute & Print difference in direction between Pre~Post~Event orientations at each diagonal point,

$$\cos\sigma_{i-j} = \sin\delta_j \sin\delta_i + \cos\delta_j \cos\delta_i \cos(\alpha_j - \alpha_i)$$

where

$$(i-j) = \begin{Bmatrix} \text{Post - Pre} \\ \text{Event - Post} \\ \text{Event - Pre} \end{Bmatrix}$$

After 1st. diagonal point $(\ell_x = \ell_y = 0)$, *test for combining orientations:*

If;

$\sigma_{2-1} < k\, m_{\sigma_{2-1}}$ ~ *Combine Pre & Post*

$\sigma_{3-2} < k\, m_{\sigma_{3-2}}$ ~ *Combine Event & Post*

$\sigma_{3-1} < k\, m_{\sigma_{3-1}}$ ~ *Combine Event & Pre*

All diagonal points computed go to print orientation parameter data page 10 (22)

Compute & Print mean error of direction differences:

$$m_\sigma = \begin{bmatrix} m_{\sigma_{2-1}} \\ m_{\sigma_{3-2}} \\ m_{\sigma_{3-1}} \end{bmatrix} = m\,(f\,Q\,f^T)^{1/2} \quad \text{where;} \quad f = \begin{bmatrix} f_{2-1} \\ f_{3-2} \\ f_{3-1} \end{bmatrix} = \begin{bmatrix} x & x & x & x & 0 & 0 \\ 0 & 0 & x & x & x & x \\ x & x & 0 & 0 & x & x \end{bmatrix}$$

$$f_{i-j}^T = \frac{1}{\sin\sigma_{i-j}} \begin{bmatrix} \cos\delta_j \cos\delta_i \sin(\alpha_j - \alpha_i) \\ -\cos\delta_j \sin\delta_i + \sin\delta_j \cos\delta_i \cos(\alpha_j - \alpha_i) \\ \hline -\cos\delta_j \cos\delta_i \sin(\alpha_j - \alpha_i) \\ -\sin\delta_j \cos\delta_i + \cos\delta_j \sin\delta_i \cos(\alpha_j - \alpha_i) \end{bmatrix}$$

$$Q = \left[(1)^*\right]\left[(2)^*\right] N^{-1} \left[(3)^*\right]\left[(1)^*\right]$$

$$(3)^* = (2)^{*T}$$

$$(1)^* = \begin{array}{c} \begin{array}{ccc} \text{Pre} & \text{Post} & \text{Event} \end{array} \\ \begin{bmatrix} \left[B_x^T P_\ell B_x\right]^{-1} & 0 & 0 \\ 0 & \left[B_x^T P_\ell B_x\right]^{-1} & 0 \\ 0 & 0 & \left[B_x^T P_\ell B_x\right]^{-1} \end{bmatrix} \end{array}$$

$$(2)^* = \begin{bmatrix} B_x^T & P_\ell & B_o \\ B_x^T & P_\ell & B_o \\ B_x^T & P_\ell & B_o \end{bmatrix} \begin{array}{l} \text{Pre} \\ \text{Post} \\ \text{Event} \end{array}$$

Figure 18.--Continued (page 9 of 11)

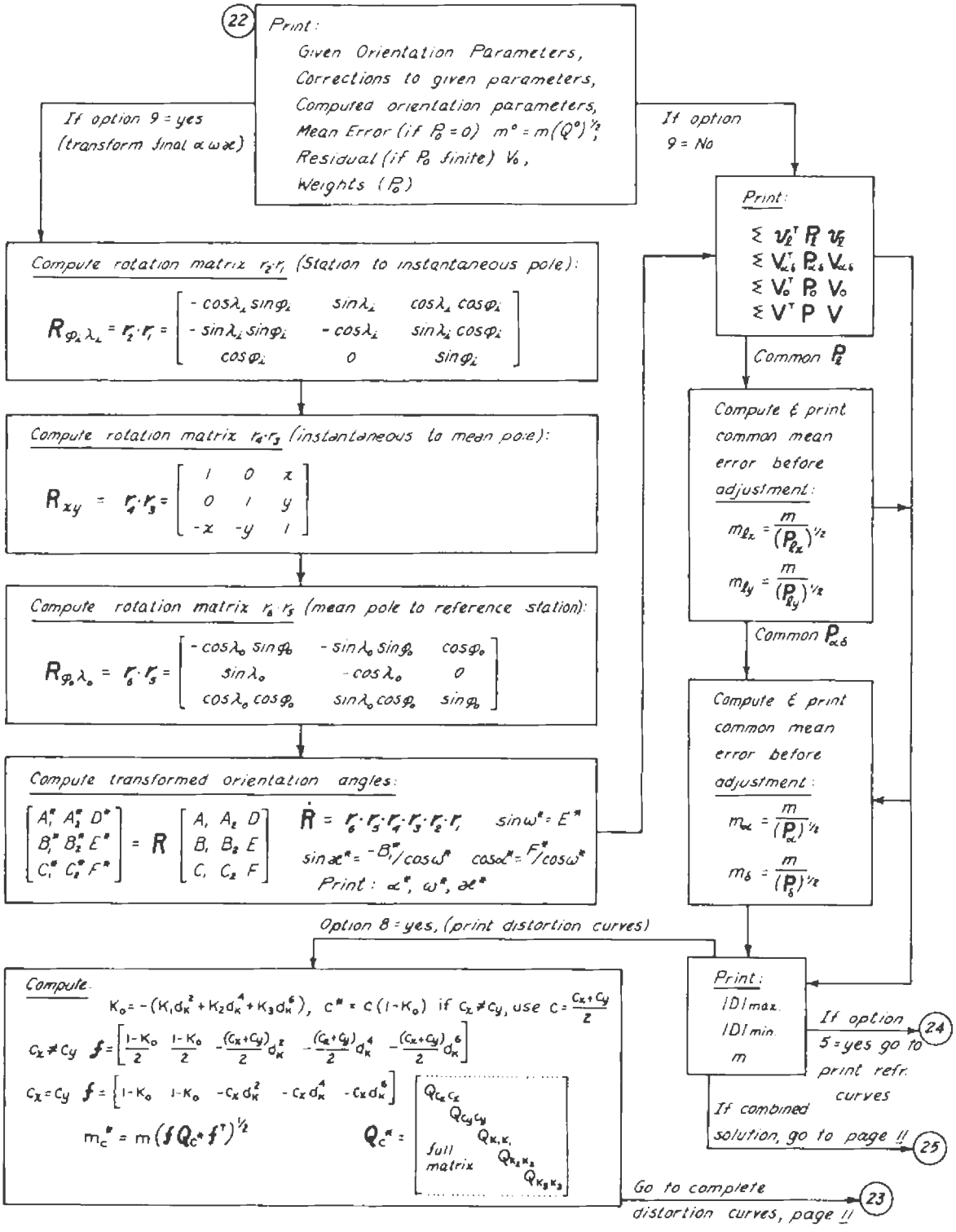

Figure 18.--Continued (page 10 of 11)

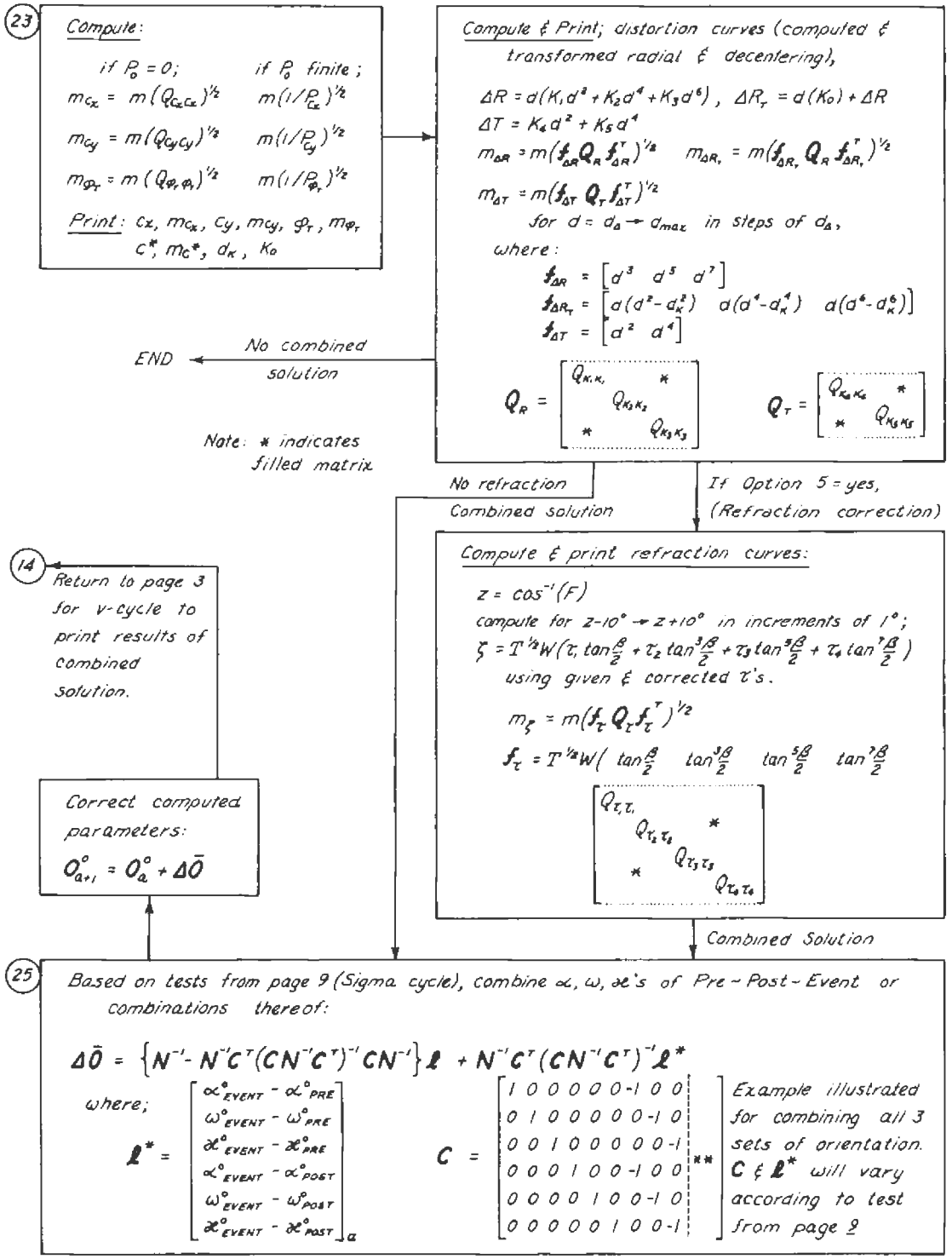

Figure 18.--Concluded (page 11 of 11)

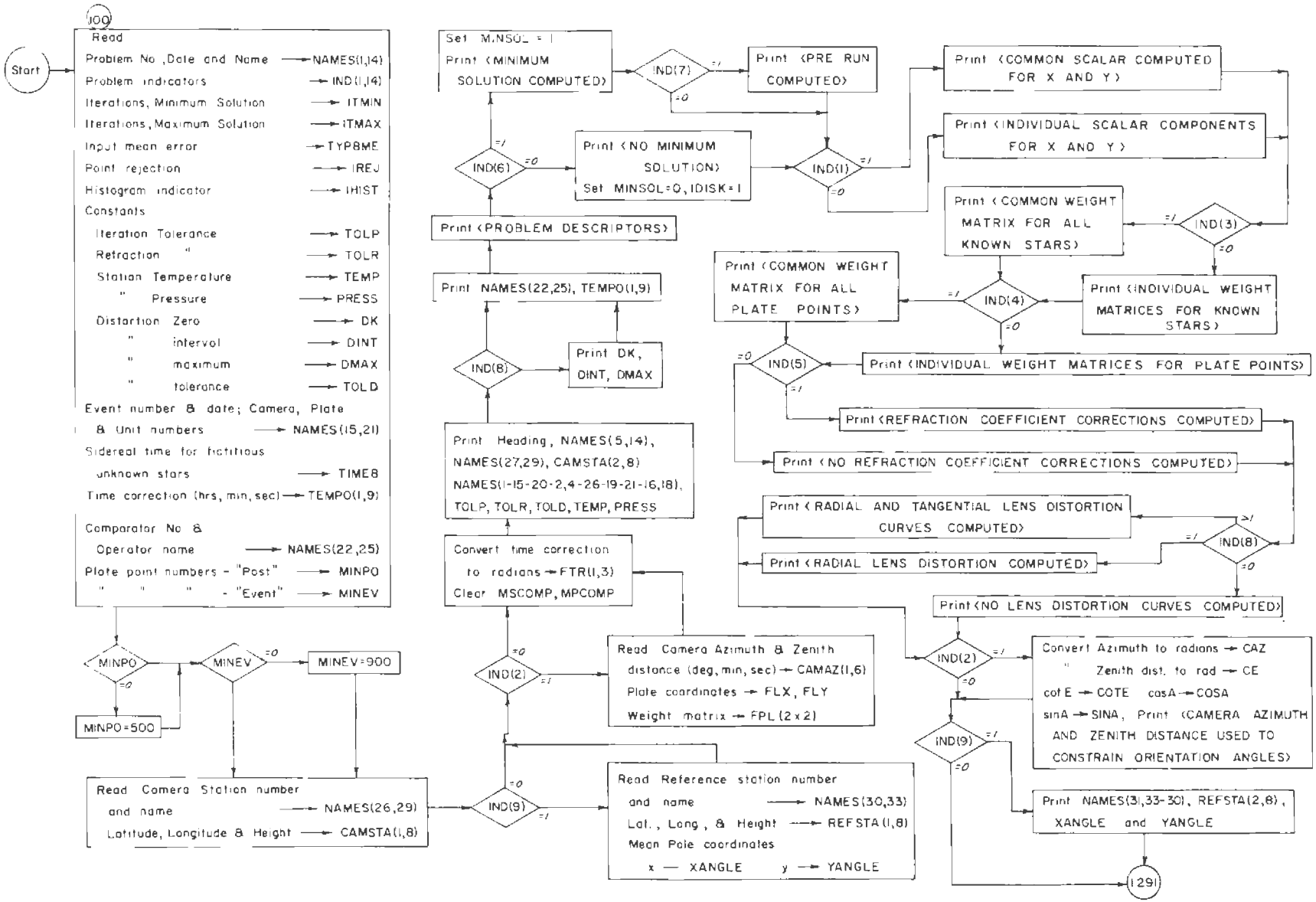

Figure 19.--Flow diagram of the FORTRAN statements (page 1 of 10)

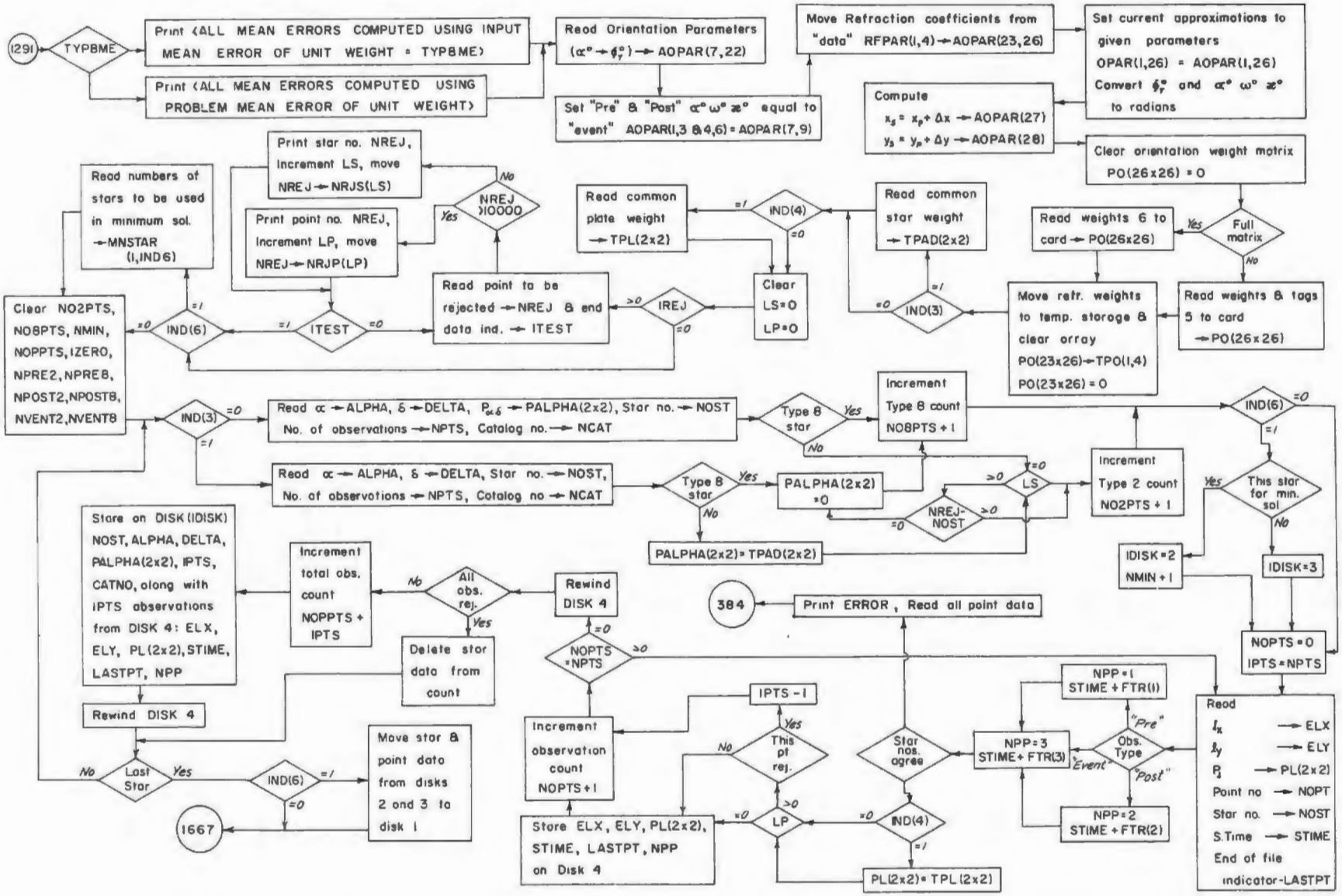

Figure 19.--Continued (page 2 of 10)

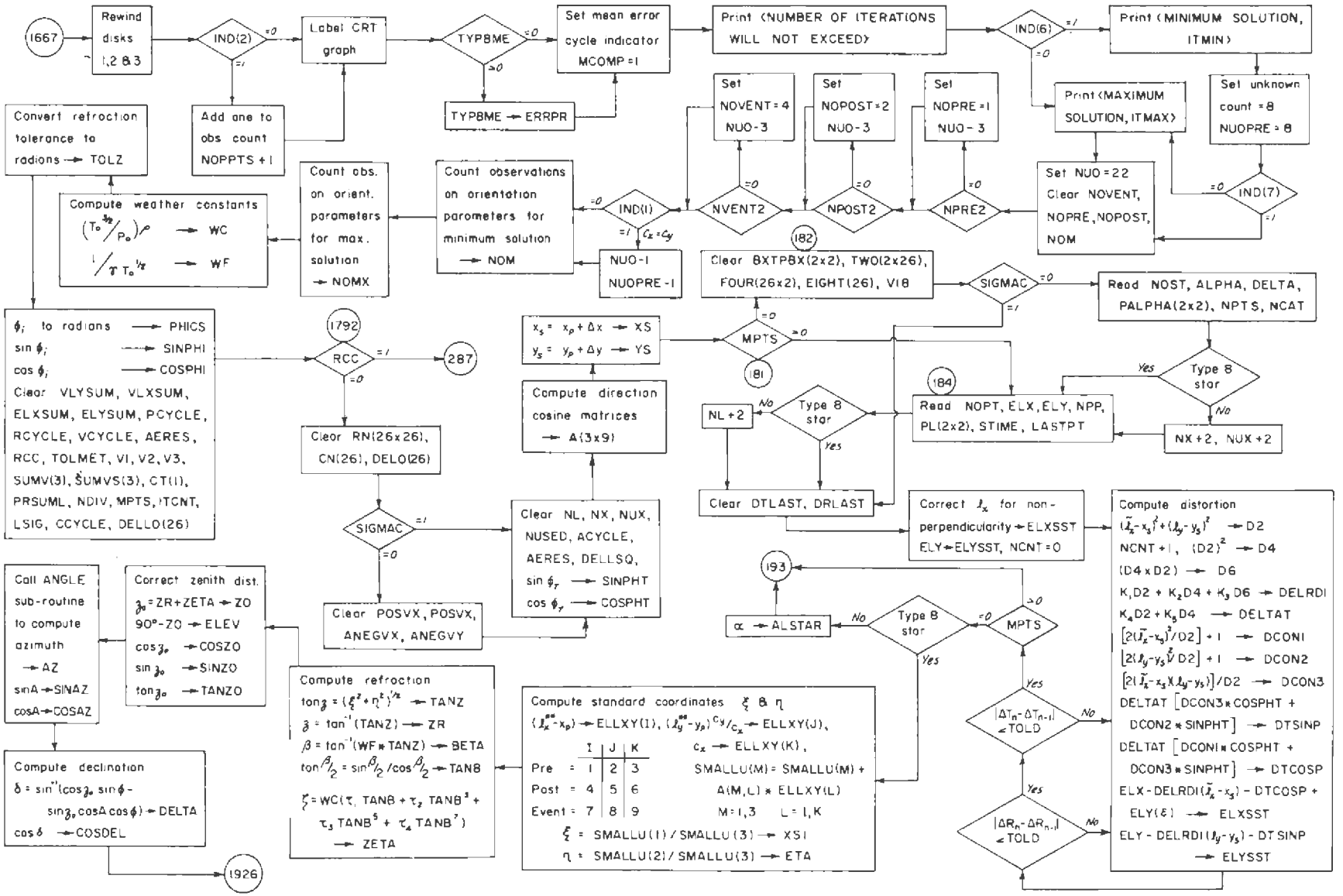

Figure 19.--Continued (page 3 of 10)

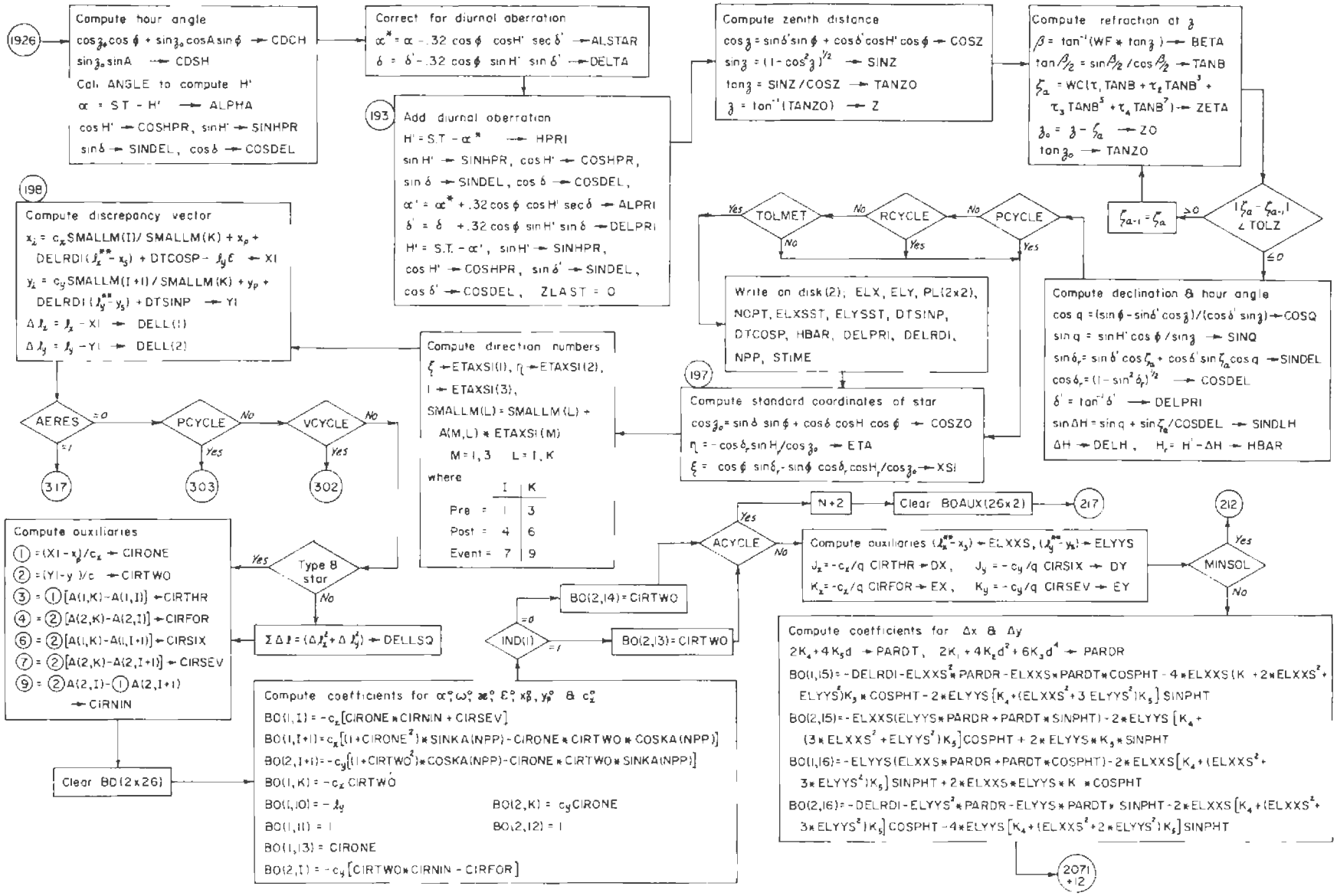

Figure 19.--Continued (page 4 of 10)

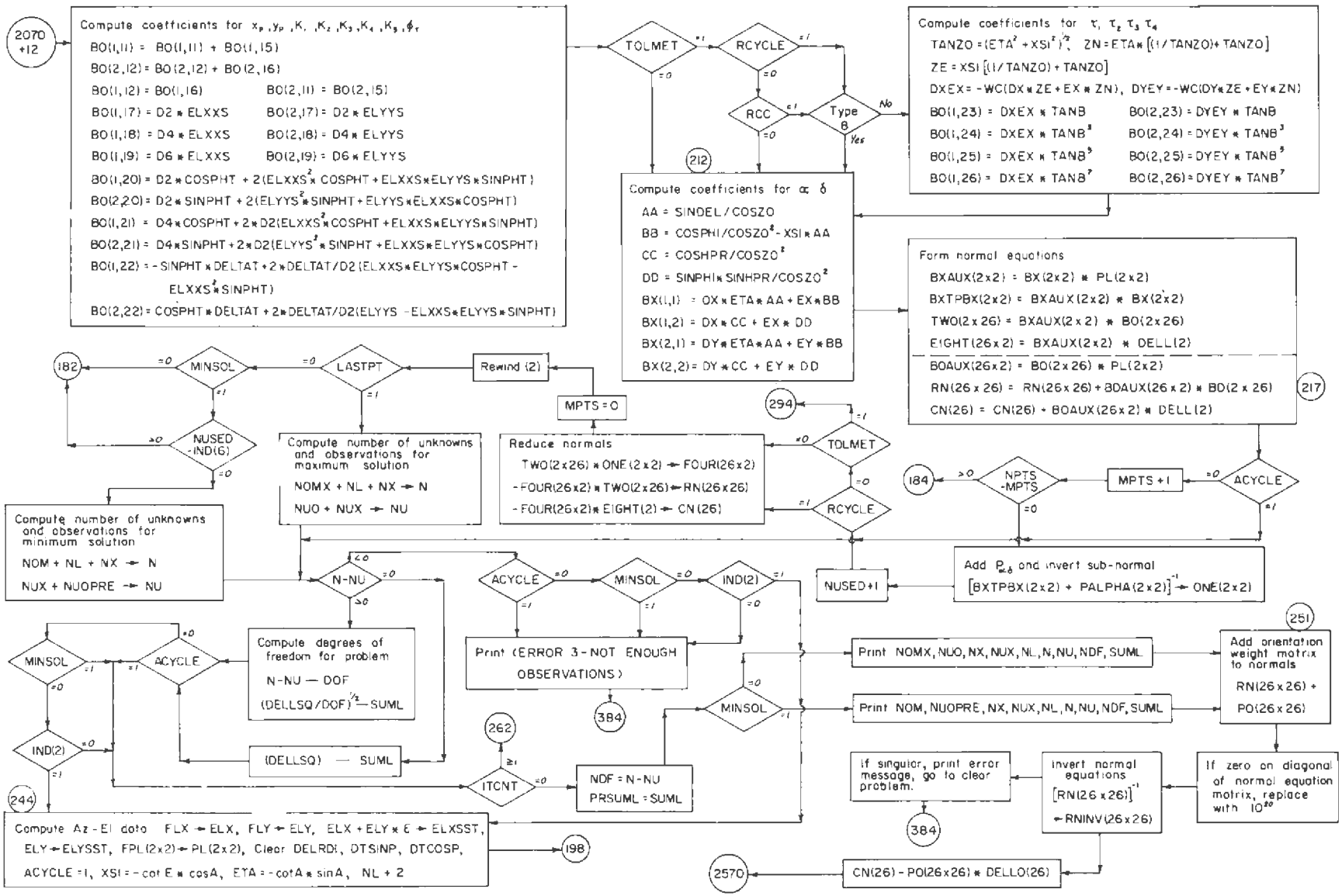

Figure 19.--Continued (page 5 of 10)

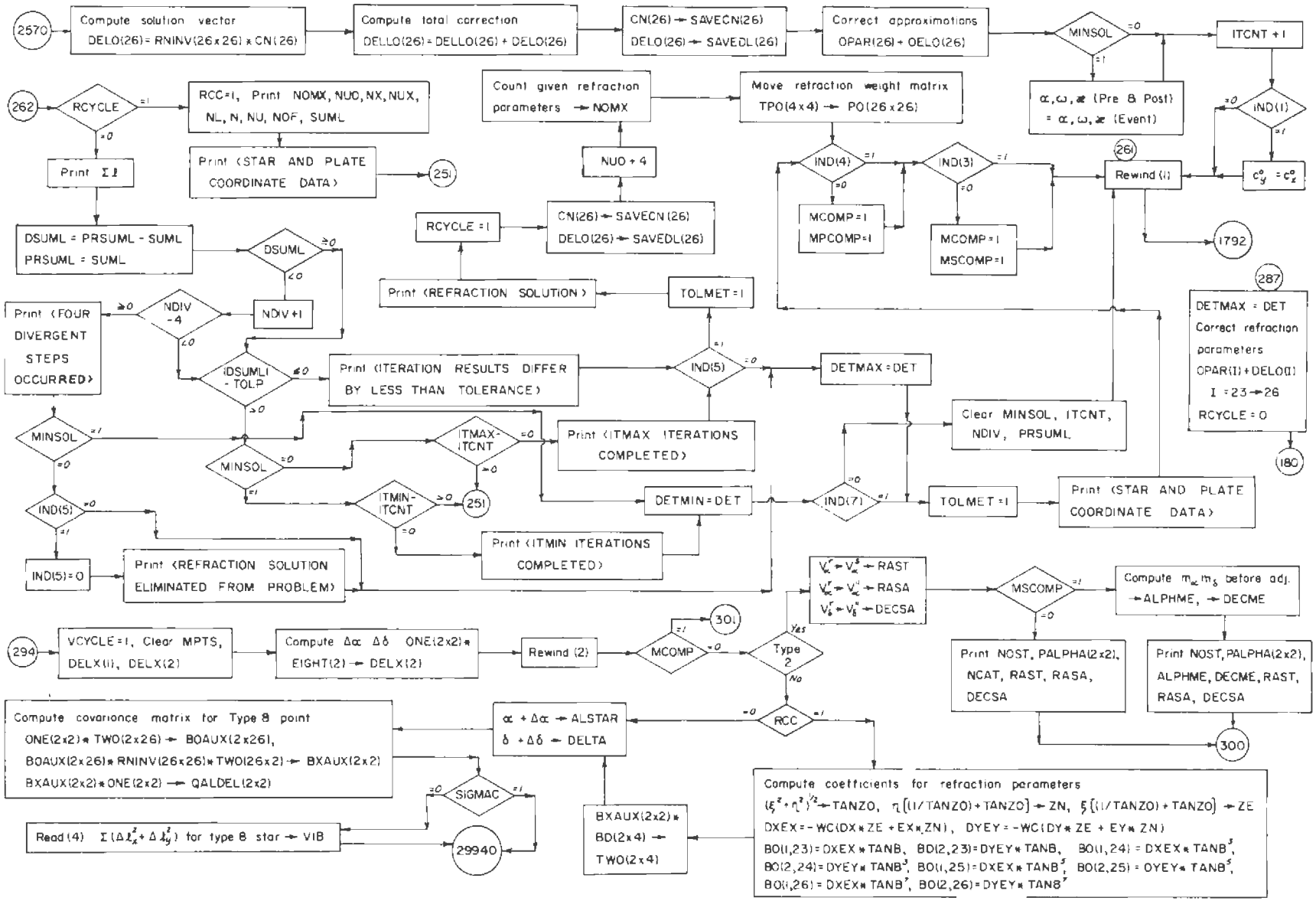

Figure 19.--Continued (page 6 of 10)

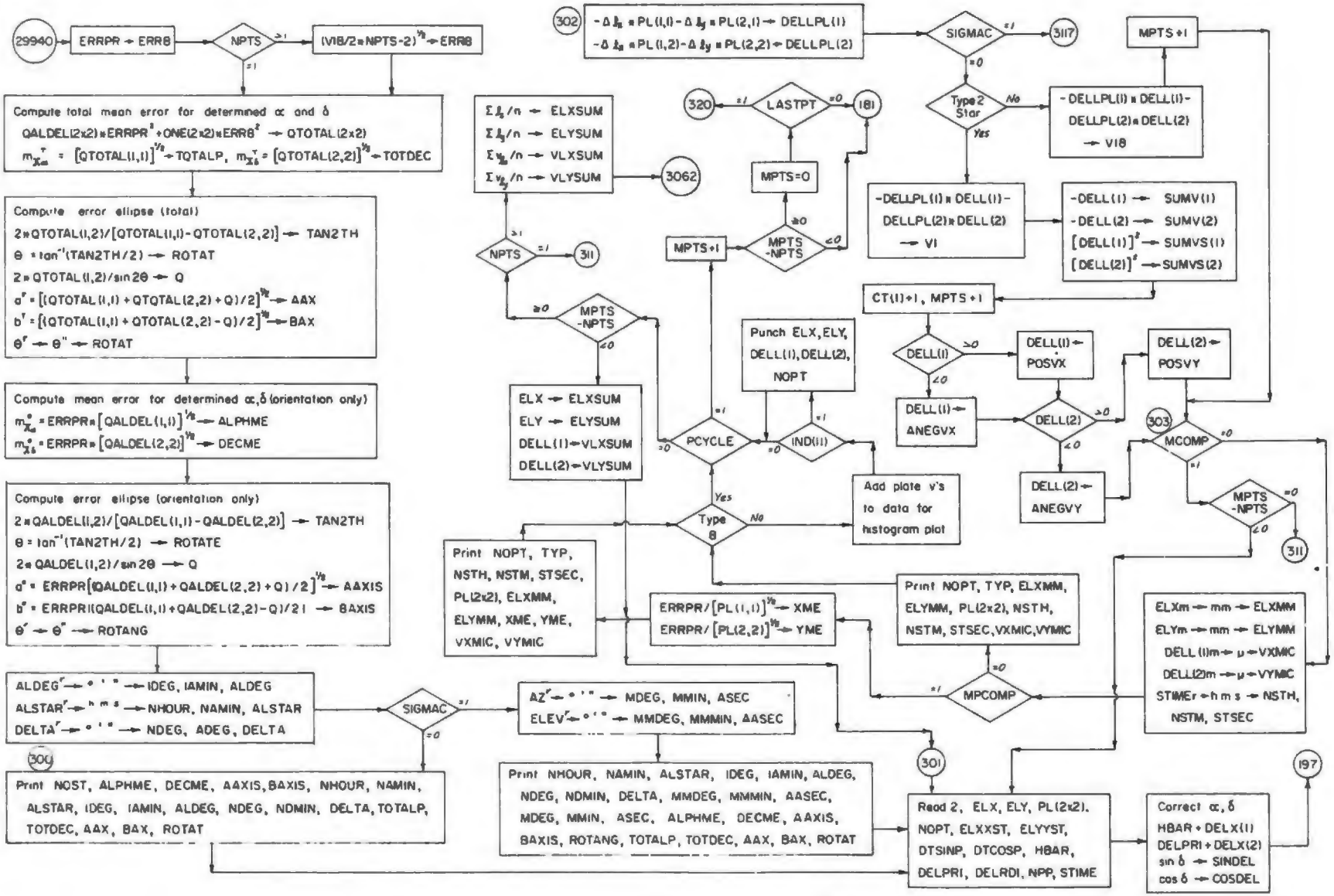

Figure 19.--Continued (page 7 of 10)

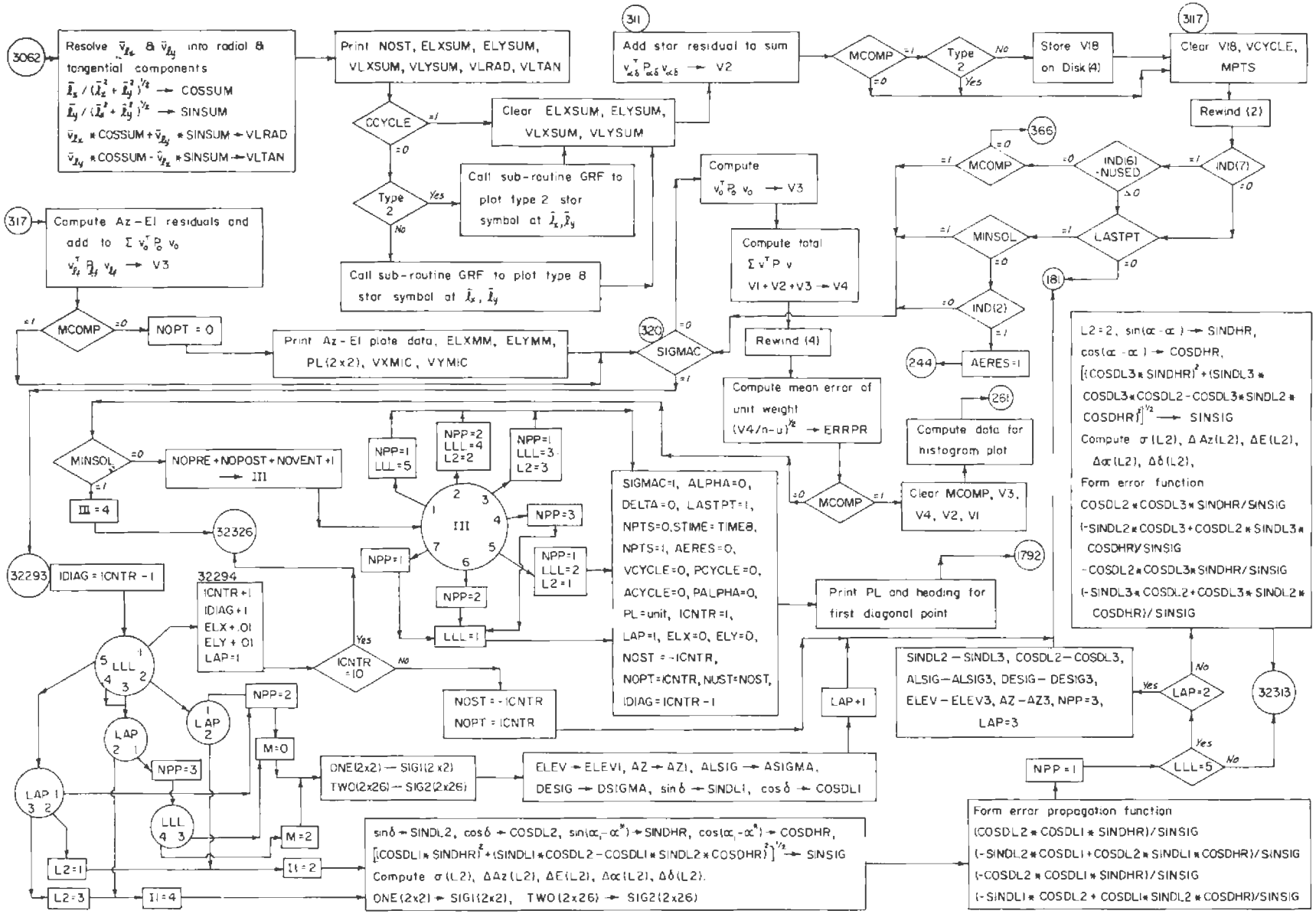

Figure 19.--Continued (page 8 of 10)

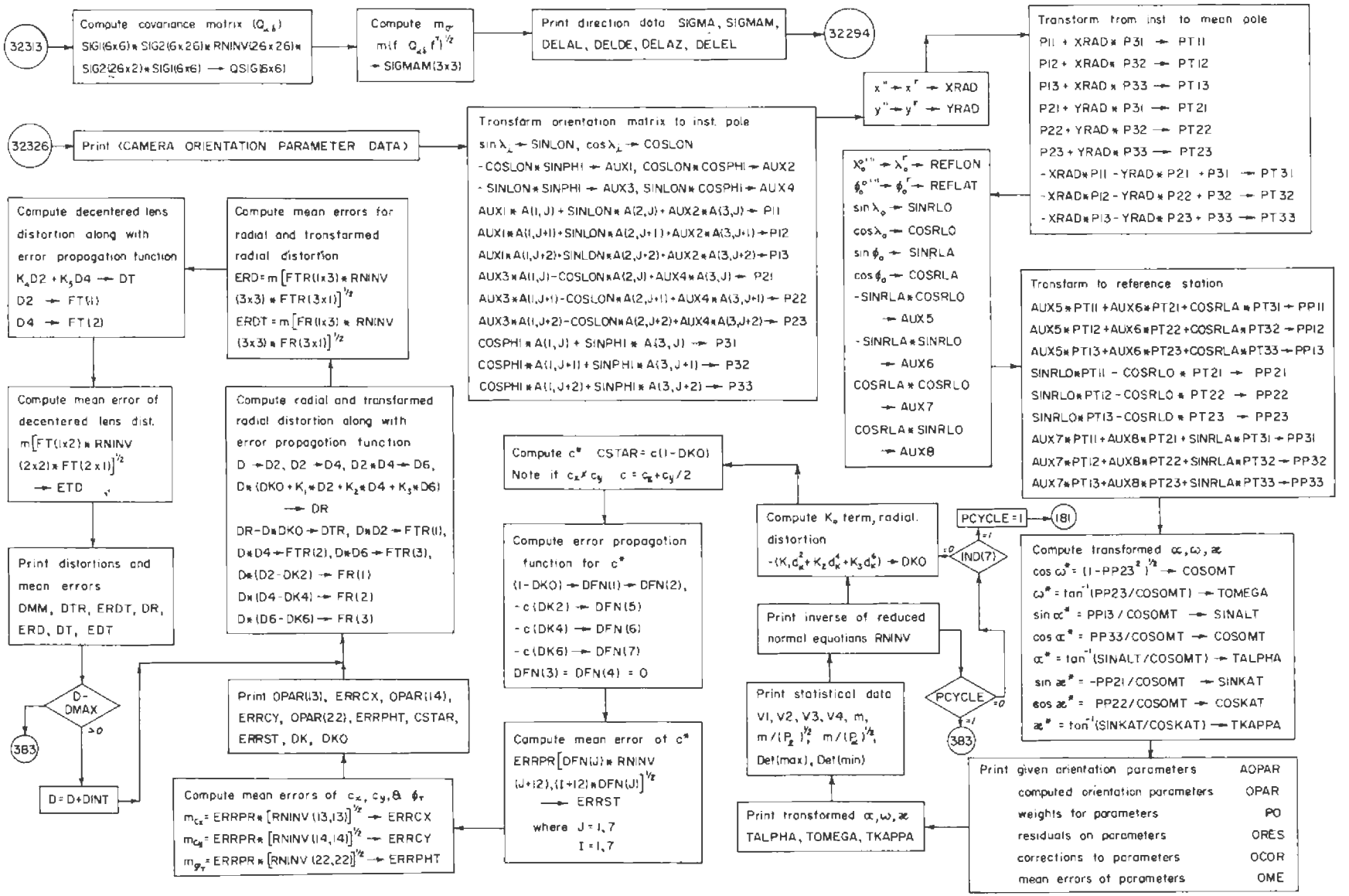

Figure 19.--Continued (page 9 of 10)

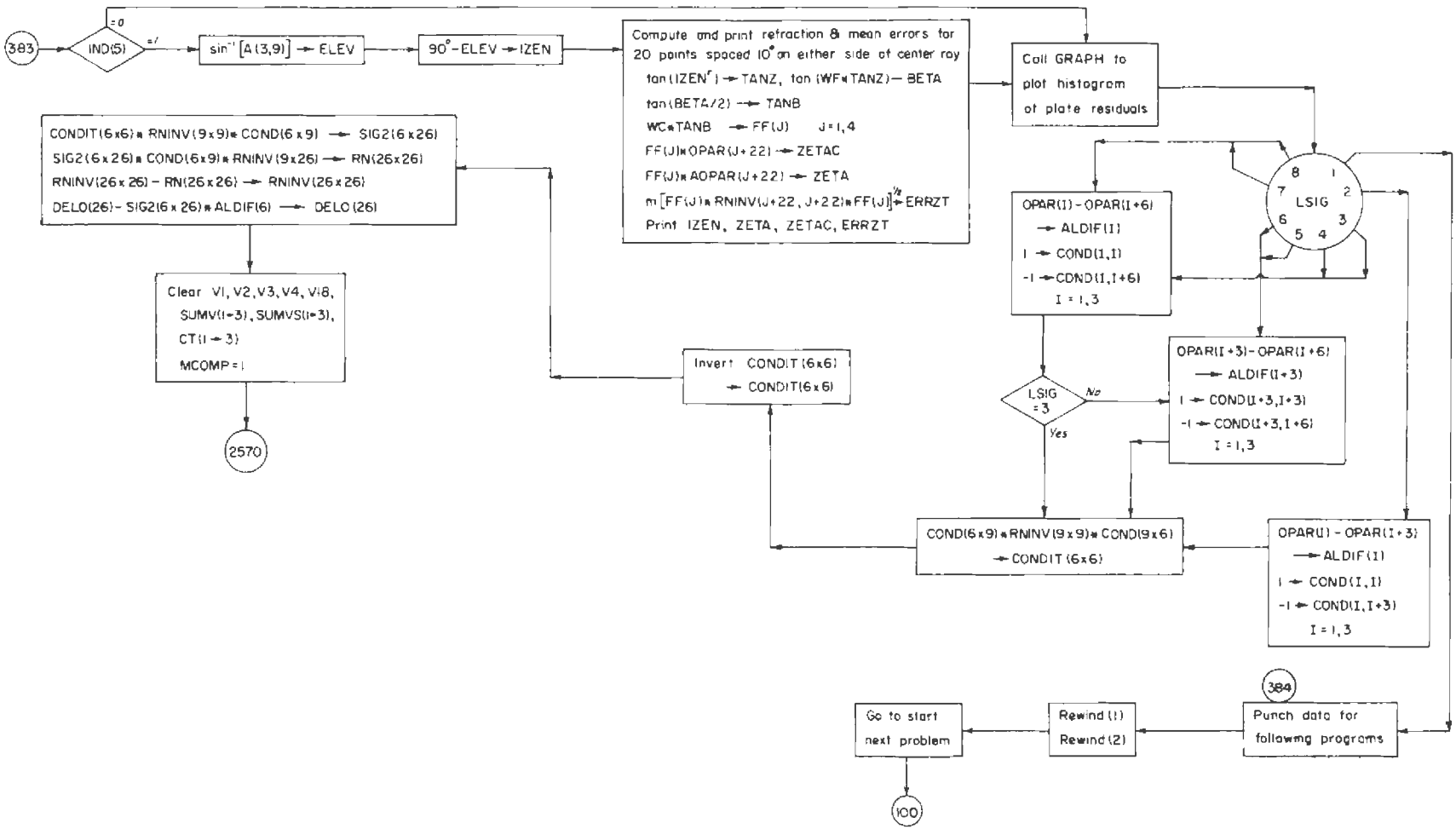

Figure 19.--Concluded (page 10 of 10)

```
B       JOB,K34-2487  SINGLE CAMERA ORIENTATION 9645  10/13/67
*       *****     *****     *****     *****     *****     *****     ***
*       OPERATOR - THE RUNNING TIME SHOULD BE ABOUT    MINUTES        *
*            THIS PROGRAM IS TO BE CHARGED TO PROJECT D 21501
*       *****     *****     *****     *****     *****     *****     ***
B       TYPE,COMPILGO,FORTRAN,PM
T       SUBTYPE,FIOD
B1      IOD,DISK,,,1400
B2      IOD,DISK,,,20
B3      IOD,DISK,,,1400
B4      IOD,DISK,,,5
        END
T       SUBTYPE,FORTRAN,LMAP,PUNCH
C
C                          *****
C               SINGLE CAMERA ORIENTATION - STELLAR
C                     FOR IBM 7030 STRETCH
C                     PROGRAM NUMBER 9643
C                          *****
C
      DIMENSION NAMES(33),IND(11),CAMSTA(8),REFSTA(8),CAMAZ(6),FPL(2,2),
     1  AOPAR(26),OPAR(26),RFPAR(4),PO(26,26),TPO(4,4),TPAO(2,2),TPL(2,2
     2  ),PALPHA(2,2),PL(2,2),DELLO(26),RN(26,28),CN(26),BXTPBX(2,2),TWO
     3  (2,26),FOUR(26,2),EIGHT(2),DELLPL(2),DELL(2),BO(2,26),BOAUX(26,
     4 6),BXAUX(6,6),BX(2,2),ONE(2,2),SMALLU(3),RNINV(26,26),DELO(26),
     5 DELX(2),FT(2),ORES(9),OCOR(9),OME(9),TEM(26),FA(3),PALPH(2,2),
     6  FF(4),RWORK(4),QALDEL(2,2),DFN(7),DFNN(7),QTOTAL(2,2),ETR(3),
     7  TEMT(3),MNSTAR(30),NRJS(100),NRJP(200),ETAXSI(3),SMALLM(9),ELLXY
     8(9),TYP(3)  ,A(3,9),QSIG(6,6),ERRFUN(3,6),SINAL(3),COSAL(3),SINOM(
     93),COSOM(3),SINKA(3),COSKA(3),SIG1(6,6),SIG2(6,26),SIGMA(3)
      DIMENSION SIGRAM(3,3),DELAZ(3),DELEL(3),DELAL(3),DELDE(3)
     1  ,CONO(6,9),ALDIF(6),TEMPO(9),CONOIT(6,6),SAVECN(26),SAVEDL(26),
     2 ANOC(3),VM(3),SDV(3),SUMV(3),SUMVS(3),CT(3),CLASS(3,32),NOC(3)
     3 ,TALPH(9),B(210)
      EQUIVALENCE IRN(1),B(1)
      DATA C3(.72722052166430E-04),C4(.17453292519943E-01),
     1  C5(.29088820866572E-03),C6(.48481368110954E-05),
     2  C7(.15707963267949E-01),C8(.28799631 44E-04),C9(.69436663 4E-02),
     3  C10(3.1415926535898),RFPAR(1)(1050.61031),RFPAR(2)(706.11502),
     4  RFPAR(3)(262.06086),(TYP(I),I=1,3)(5H PRE ,5HPOST ,5HEVENT),
     5  RFPAR(4)(142.67293)
      COMMON /AP/NAMES
C
C  READ PROBLEM DATA
C
C    PROBLEM DESCRIPTORS
C
  100 READ 1,(NAMES(I),I=1,4)
      READ 2,(NAMES(I),I=5,14)
C
C    PROBLEM OPTIONS
C
      READ 3,(IND(I),I=1,11),ITMIN,ITMAX,TYPBME,IREJ,IHIST
C
```

Dimensions of arrays.

Program constants, radians per second of time, radians per degree, radians per minute, radians per second, radians per grad,

Problem number, date of computation and problem name.

Options (11), Number of iterations for minimum and maximum solutions, Input mean error, and option to plot histogram.

Figure 20.--Program listing (page 1 of 57)

```
C     PROBLEM CONSTANTS                                                 Problem and refraction tolerances, temperature,
C                                                                       pressure, distortion crossover distance,
      READ 4,TOLP,TOLR,TEMP,PRESS,OK,DINT,DMAX,TOLD                     distortion curve interval, distortion curve
      READ 5,(NAMES(I),I=15,21),TIME8,(TEMPO(I),I=1,9),(NAMES(I),I=22,25  maximum distance, distortion iteration tolerance,
     1),MINPO,MINEV,FACT                                                Event number and date, Sidereal time for
      IF(MINPO.EQ.0) MINPO=500                                          fictitious points and time corrections, plate
      IF(MINEV.EQ.0) MINEV = 900                                        camera and unit nos., point identification for
C                                                                       "Pre" "Post" "Event", and factor for combining
C     CAMERA STATION NAME, NUMBER, LATITUDE, LONGITUDE AND ELEVATION    solutions.
C
      READ 6,(NAMES(I),I=26,29),(CAMSTA(I),I=1,8)
      IF(IND(9)-1)103,101,101
C
C  READ REFERENCE STATION NAME, NUMBER, LATITUDE, LONGITUDE AND ELEV.
C
  101 READ 6,(NAMES(I),I=30,33),(REFSTA(I),I=1,8)
C
C  READ INSTANTANEOUS POLE COORDINATES, X AND Y.
C
      READ 7,XANGLE,YANGLE
  103 IF(IND(2))104,105,104
C
C  READ AZIMUTH, ELEVATION DATA
C
  104 READ 8,(CAMAZ(I),I=1,6)                                           Camera Azimuth and zenith distance (deg.,min.,
      READ 9,FLX,FLY,FPL(1,1),FPL(2,2),FPL(1,2)                         sec.), plate coordinates and weights. Restore
  105 FPL(2,1) = FPL(1,2)                                               symmetry to weight matrix.
      MPCOMP = 0
      MSCOMP = 0                                                        Clear mean error computation indicators.
      DO 38429 I=1,3
      J=3*I-2
38429 FTR(I) = TEMPO(J)*.2617993878 + TEMPO(J+1)*.0043633231 + TEMPO(J+2   Convert time corrections from hours min and
     1 )*.0000727221                                                    seconds for each orientation (Pre, Post & Event
C
C  WRITE APPROPRIATE OUTPUT HEADINGS AND SET PROGRAM CONTROLS AS
C     DICTATED BY PROBLEM OPTIONS AND PROBLEM CONSTANTS.
C
      PRINT 1001                                                        Print SINGLE CAMERA ORIENTATION WITH CELESTIAL
                                                                              OBSERVATIONS.
      PRINT 1002,(NAMES(I),I=5,14)                                      Print Input problem name (80 col of data)
      PRINT 1003,(NAMES(I),I=27,29),(CAMSTA(I),I=2,8)                   Print Station Name, Latitude and Longitude.
      PRINT 1004,NAMES(1),NAMES(15),NAMES(20),(NAMES(I),I=2,4),NAMES(26)  Print Problem No., Event No., Camera No., Date
     1  ,NAMES(19),NAMES(21),(NAMES(I),I=16,18)                         of observation and computation, Station, Plate
      PRINT 1005,TOLP,TOLR                                              and tracking unit numbers.
      PRINT1006,TOLD                                                    Print Problem Constants; Problem, Refraction and
      PRINT 1008,TEMP                                                   Distortion tolerances, Station temperature and
      PRINT 1009,PRESS                                                  pressure.
      IF(IND(8))3051,3052,3051                                          If option 8 is yes, Print Distortion curve cross-
 3051 PRINT 1010,OK                                                     over distance, interval and maximum distance.
      PRINT 1011,DINT
      PRINT 1012,DMAX
 3052 PRINT 1013,(NAMES(I),I=22,25)                                     Print Comparator Number and Name of Operator.
 3053 PRINT 1126,(TEMPO(I),I=1,9)                                       Print time correction.
 3054 PRINT 1014                                                        Print PROBLEM DESCRIPTORS
      IF(IND(6))106,108,106
```

Figure 20.--Continued (page 2 of 57)

```
 106 PRINT 1015,IND(6)                                  79 If Option 6=1, Print MINIMUM SOLUTION COMPUTED
                                                                WITH XX STARS.
     MINSOL = 1                                         80 Set Minimum Solution Indicator.
     IF(IND(7))107,109,107                              81
 107 PRINT 1017                                         82 If Option 7=1, Print PRE-RUN COMPUTED.
     GO TO 109                                          83
 108 PRINT 1016                                         84 Print NO MINIMUM SOLUTION COMPUTED
     MINSOL = 0                                         85 Clear Minimum Solution Indicator.
     IDISK = 1                                          86
 109 IF(IND(1))110,111,110                              87 If Option 1=1, Print COMMON SCALAR COMPUTED FOR
 110 PRINT 1018                                         88                      X AND Y (CX EQUALS CY)
     GO TO 112                                          89
 111 PRINT 1019                                         90 Print INDIVIDUAL SCALAR COMPONENTS COMPUTED FOR
 112 IF(IND(3))113,114,113                                     X AND Y (CX DOES NOT EQUAL CY)
 113 PRINT 1020                                         91 If Option 3=1, Print COMMON WEIGHT MATRIX FOR ALL
     GO TO 115                                                 KNOWN STARS
 114 PRINT 1021                                         92
 115 IF(IND(4))116,117,116                              93
 116 PRINT 1022                                         94 Print INDIVIDUAL WEIGHT MATRICES FOR KNOWN STARS
     GO TO 118                                          95 If Option 4=1, Print COMMON WEIGHT MATRIX FOR ALL
 117 PRINT 1023                                                PLATE POINTS
 118 IF(IND(5))119,120,119                              96
 119 PRINT 1024                                         97
     GO TO 121                                          98 Print INDIVIDUAL WEIGHT MATRICES FOR PLATE POINTS
 120 PRINT 1025                                         99 If Option 5=1, Print REFRACTION COEFFICIENT
 121 IF(IND(8)-1)122,123,124                                   CORRECTIONS COMPUTED
 122 PRINT 1027                                        100
     GO TO 125                                         101
 123 PRINT 1028                                        102 Print NO REFRACTION COEFFICIENT  CORRECTIONS
     GO TO 125                                                 COMPUTED
 124 PRINT 1026                                        103 If Option 8=1, Print RADIAL AND DECENTERED LENS
 125 IF(IND(2))126,127,126                                     DISTORTION CURVES COMPUTED.
 126 CAZ = CAMAZ(1)*C4 + CAMAZ(2)*C5 + CAMAZ(3)*C6     104
     CE = C1D/2. - CAMAZ(4)*C4 - CAMAZ(5)*C5 - CAMAZ(6)*C6  105
     COTE = COS(CE)/SIN(CE)                            106
     COSA = COS(CAZ)                                   107
     SINA = SIN(CAZ)                                   108 Print NO LENS DISTORTION CURVES COMPUTED
     PRINT 1029,(CAMAZ(I),I=1,6)                       109
 127 IF(IND(9)-1)128,129,129                           110 Convert camera Azimuth-Elevation to radians.
 128 PRINT 1034                                        111
     GO TO 1291                                        112
 129 PRINT 1031,(NAMES(I),I=31,33),NAMES(30),(REFSTA(I),I=2,8)  113
     PRINT 1033,KANGLE,YANGLE                          114
1291 IF(TYPBME)130,1293,130                            115 Print CAMERA AZIMUTH AND ZENITH DISTANCE USED TO
1293 PRINT 1119                                              CONSTRAIN ORIENTATION ANGLES
     GO TO 131                                         116 If Option 9=1, Print FINAL CAMERA ORIENTATION
 130 PRINT 1120,TYPBME                                       ANGLES TRANSFORMED TO A REFERENCE STATION SYSTEM
C                                                      117
C  READ APPROXIMATIONS TO ORIENTATION PARAMETERS       118
C                                                      119 Print Reference Station Name, Number, Latitude,
 131 READ 10,(AOPAR(I),I=7,22)                                Longitude and Elevation.
     DO 1310 I=1,3                                     120 Print Mean Pole Coordinates (x and y).
       AOPAR(I) = AOPAR(I+6)                           121
1310   AOPAR(I+3) = AOPAR(I)                           122 Print ALL MEAN ERRORS COMPUTED USING PROBLEM
     DO 132 I=1,4                                              MEAN ERROR OF UNIT WEIGHT
                                                       123
                                                       124 Print ALL MEAN ERRORS COMPUTED USING INPUT MEAN
                                                                ERROR OF UNIT WEIGHT
                                                       125
                                                       126
                                                       127
                                                       128 Set Pre and Post first approximations of α, ω, κ
                                                                equal to Event values.
                                                       129
                                                       130
```

Figure 20.--Continued (page 3 of 57)

```
  132 AOPAR(I+22) = RFPAR(I)                                    131 Move refraction parameters to orientation
      DO 133 I=1,26                                                 parameter matrix.
  133   OPAR(I) = AOPAR(I)                                      132
      DO 134 I=1,10                                             133
  134   OPAR(I) = OPAR(I)*C7                                    134
      OPAR(22) = OPAR(22)*C7                                    135 Convert grads to radians (α, ω, κ, φ, ).
      AOPAR(27) = AOPAR(11)+AOPAR(15)                           136
      AOPAR(28) = AOPAR(12)+AOPAR(16)                           137 x_s = x_p + Δx
                                                                138 y_s = y_p + Δy
C
C  READ WEIGHT MATRIX FOR GIVEN ORIENTATION PARAMETERS
C
  135 DO 1351 I=1,26                                            139
      DO 1351 J=1,26                                            140
 1351     PO(I,J) = 0.                                          141 Clear Orientation Weight Matrix (PO).
      IF(IND(10)-52)139,136,136                                 142
  136 READ 11,((PO(I,J),J=1,22),I=1,22)                         143 If Option 10 is greater than 52, read in entire
      DO 137 I=1,22                                                 upper triangular part of PO Matrix (22 x 22).
         DO 137 J=1,22                                          144 (Note: Elements of input PO matrix must be
  137     PO(J,I) = PO(I,J)                                         arranged in ascending order eg. (1,1), 1,2),(1,3)
      READ 11,((PO(I,J),J=1,26),I=23,26)                        145 ... (1,16),(2,2),(2,3),(2,4),...(2,16).....
      DO 138 I=23,26                                                Read in upper triangular part of refraction
      DO 138 J=1,26                                             146 parameter weight matrix.
  138     PO(J,I) = PO(I,J)                                         Restore symmetry to both matrices.
      GO TO 141                                                 147
  139 IF(IND(10))1391,141,1391                                  148
 1391 NCO = 0                                                   149
  140 READ 12,I1,J1,PO(I1,J1),I2,J2,PO(I2,J2),I3,J3,PO(I3,J3),I4,J4,PO  150
     1  (I4,J4),I5,J5,PO(I5,J5)                                 151
      PO(J1,I1) = PO(I1,J1)                                     152 If less than full PO matrix, read in number of
      PO(J2,I2) = PO(I2,J2)                                         cards indicated by option 10.  all cards must
      PO(J3,I3) = PO(I3,J3)                                     153 be full.
      PO(J4,I4) = PO(I4,J4)                                     154
      PO(J5,I5) = PO(I5,J5)                                     155
      NCO = NCO + 1                                             156
      IF(NCO-IND(10))140,141,141                                157
C                                                               158
C  TEMPORARILY STORE REFRACTION WEIGHT MATRIX.  REPLACE WITH ZEROS.  159
C                                                               160
  141 DO 142 I= 23,26                                           161
      DO 142 J=23,26                                            162
      TPO(I-22,J-22) = PO(I,J)                                  163
  142     PO(I,J) = 0.                                          164
      IF(IND(3))143,144,143                                     165
C                                                               166
C  READ COMMON STAR WEIGHT MATRIX.                              167
C                                                               168
  143 READ 11,TPAD(1,1),TPAD(2,2),TPAD(1,2)                     169
      TPAD(2,1) = TPAD(1,2)                                     170
  144 IF(IND(4))145,1452,145                                    171
C                                                               172
C  READ COMMON PLATE WEIGHT MATRIX.                             173
C                                                               174
  145 READ 11,TPL(1,1),TPL(2,2),TPL(1,2)                        175
      TPL(2,1) = TPL(1,2)                                       176
```

Figure 20.--Continued (page 4 of 57)

```
C
C  READ NUMBERS FOR STARS AND OR PLATE POINTS TO BE REJECTED.
C
 1452 LS = 0
      LP = 0
      IF(IREJ.NE.0) PRINT 1120
      IF(IREJ)1453,1467,1453
 1453 READ 1135,NREJ,ITEST
      IF(NREJ-1000)1455,1463,1463
 1455 LS = LS + 1
      IF(LS-100)1462,1462,1456
 1456 PRINT 1136
 1457 READ 16,ITEST
      IF(ITEST)158,1457,158
 1462 NRJS(LS) = NREJ
      PRINT 1138,NREJ
      GO TO 1466
 1463 LP = LP + 1
      IF(LP-200)1464,1464,1456
 1464 NRJP(LP) = NREJ
      PRINT 1139,NREJ
 1466 IF(ITEST)1467,1453,1467
 1467 IF(IND(6))1468,1469,1468
C
C  READ STARS TO BE USED IN MINIMUM SOLUTION.
C
 1468 MINS = IND(6)
      READ (133,INNSTAR(I),I=1,MINS)
 1469 NO2PTS = 0
      NO3PTS = 0
      NOPPTS = 0
      NMIN = 0
      IZERO = 0
      NPRE2=0
      NPRE8=0
      NPOST2=0
      NPOST8=0
      NVENT2=0
      NVENT8=0
  147 IF(IND(3))150,148,150
C
C  READ AND STORE STAR COORDINATES FROM STAR IDENTIFICATION AND UPDATE
C    PROGRAM. (APPARENT POSITIONS)
C
  148 READ (3,ALPHA,DELTA,PALPHA(1,1),PALPHA(2,2),PALPHA(1,2),NOST,NPTS,
     1 NCAT
      PALPHA(2,1) = PALPHA(1,2)
      IF(NOST)1521,1521,1541
  150 READ (4,ALPHA,DELTA,NOST,NPTS,NCAT
      IF(NOST)151,151,153
  151 DO 152 I=1,2
        DO 152 J=1,2
  152     PALPHA(I,J) = 0.
 1521 NO8PTS = NO8PTS + 1
```

Figure 20.--Continued (page 5 of 57)

```
      GO TO 1546                                                        221
  153 DO 154 I=1,2
        DO 154 J=1,2
  154     PALPHA(I,J) = TPAD(I,J)                                       224 Set star weight equal to common weight.
 1541 IF(LS)1542,1540,1542                                              225
 1542 DO 1545 I=1,LS                                                    226
        IF(NUST-NRJS(I))1545,1543,1545                                  227
 1543   DO 1544 J=1,2                                                   228
          DO 1544 K=1,2                                                 229 Type 8 points identified by negative star number.
                                                                            If type 8 point, set star weight matrix to zeros
 1544       PALPHA(J,K) = 0.                                            230 and add one to type 8 point count.
        NO8PTS = NO8PTS + 1                                             231
      NUST = -NUST
        GO TO 1546                                                      232
 1545   CONTINUE                                                        233
 1540 NO2PTS = NO2PTS + 1                                               234 Add one to type 2 star count.
 1546 IF(IND(6))1547,1550,1547                                          235
 1547 DO 1548 I=1,MINS                                                  236
        IF(NUST-MNSTAR(I))1548,1549,1549                                237 Check if star to be used in minimum solution.
 1548   CONTINUE                                                        238
      IDISK = 3                                                         239
      GO TO 1550                                                        240
 1549 IDISK = 2                                                         241
      NMIN = NMIN + 1                                                   242
 1550 NOPTS = 0                                                         243 Clear plate point counter.
      IPTS = NPTS                                                       244
C
C  READ PLATE MEASUREMENTS FROM COMPARATOR REDUCTION PROGRAM.
C
 1551 READ 15,ELX,ELY,PL(1,1),PL(2,2),PL(1,2),NOPT,NUST,STIME,LASTPT    246
      NPP=MOD(NOPT,1000)                                                247
      IF(IABS(NPP)-MINPO) 1554,1552,1552                                    Determine orientation of plate observation,
                                                                            (Pre, Post or Event). Add identification to
 1552 IF(IABS(NPP)-MINEV) 1555,1553,1553                                    plate data.
 1553 NPP = 3                                                           250
      GO TO 1556                                                        251
 1554 NPP = 1                                                           252
      GO TO 1556                                                        253
 1555 NPP = 2                                                           254
 1556 STIME = STIME+FTR(NPP)                                                Add time correction to sidereal time (radians)
      IF(IABS(NUST)-IABS(NUST))156,159,156                                  If observational data is out of order, print
                                                                            error message ERROR 1 - STAR NUMBERS DO NOT AGREE
  156 PRINT 1103,NUST,NOPT                                              257 STAR NUMBER XXX, POINT NUMBER XXX. Clear input
                                                                            and go to next problem.
  157 IF(LASTPT)1384,158,384                                            258
  158 READ 16,LASTPT                                                    259
      GO TO 157                                                         260
  159 IF(IND(4))161,160,161                                             261
  160 PL(2,1) = PL(1,2)                                                 262 Restore symmetry.
      GO TO 1625                                                        263
  161 DO 162 I=1,2                                                      264
        DO 162 J=1,2                                                    265
  162     PL(I,J) = TPL(I,J)                                            266 Set plate weight equal to common weight.
 1625 IF(LP)163,1632,163                                                267
  163 DO 1631 I=1,LP                                                    268
        IF(NOPT-NRJP(I))1631,1630,1631                                  269
 1630   IPTS = IPTS - 1                                                 270
        GO TO 1633                                                      271
```

Figure 20.--Continued (page 6 of 57)

```
 1631   CONTINUE
C
C  STORE ALL PLATE AND STAR COORDINATES IN DISK ALONG WITH WEIGHTS,
C     SIDEREAL TIMES OF OBSERVATIONS AND NUMBER OF OBSERVATIONS
C     ASSOCIATED WITH EACH ALPHA AND DELTA.
C     IF MINIMUM SOLUTION, SORT OUT POINTS TO BE USED AND PLACE AT
C     BEGINNING OF LIST.
C
 1632 WRITE(4) NOPT,ELX,ELY,((PL(I,J),J=1,2),I=1,2),STIME,LASTPT,NPP
 1633 NOPTS = NOPTS + 1
      IF(NPTS-NOPTS)164,164,1551
  164 REWIND 4
      IF(IPTS.EQ.0) GO TO 165
      NOPPTS = NOPPTS + IPTS
      WRITE (IDISK) NOST,ALPHA,DELTA,((PALPHA(I,J),J=1,2),I=1,2),IPTS,
     1  NCAT
      DO 165 I=1,IPTS
        READ (4) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LSTSIG,NPP
        IF(LASTPT.EQ.1.AND.1.EQ.IPTS) LSTSIG = 1
      IF(NOST)1641,1641,1642
 1641 IF(NPP.EQ.3)NVENT8 = NVENT8 + 1
      IF(NPP.EQ.2)NPOST8 = NPOST8 + 1
      IF(NPP.EQ.1)NPRE8 = NPRE8 + 1
      GO TO 165
 1642 IF(NPP.EQ.3)NVENT2 = NVENT2 + 1
      IF(NPP.EQ.2)NPOST2 = NPOST2 + 1
      IF(NPP.EQ.1)NPRE2 = NPRE2 + 1
  165   WRITE (IDISK) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LSTSIG,
     1 NPP
      REWIND 4
      IF(LASTPT)166,147,166
 1651 PRINT 1137,NOST
      IF(NOST)1653,1651,1652
 1652 NO2PTS = NO2PTS - 1
      GO TO 1654
 1653 NO3PTS = NO3PTS - 1
 1654 IF(LASTPT.EQ.1) GO TO 147
      BACKSPACE IDISK
      READ (IDISK) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LSTSIG,NPP
      BACKSPACE IDISK
      WRITE (IDISK) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LASTPT,
     1 NPP
  166 IF(IND(6))1663,1667,1660
 1660 IF(MIN -MINS)1661,1662,1661
 1661 PRINT 1134
      GO TO 100
 1662 BACKSPACE 3
      READ (3) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LSTSIG,NPP
      BACKSPACE 3
      WRITE (3) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LASTPT,NPP
      REWIND 2
      REWIND 3
      DO 1663 I=1,MINS
        READ (2) NOST,ALPHA,DELTA,((PALPHA(K,L),L=1,2),K=1,2),NPTS,NCAT
```

Add plate observation to orientation type count separately for types 2 and 3.

Figure 20.--Continued (page 7 of 57)

```
      WRITE (1)NOST,ALPHA,DELTA,((PALPHA(K,L),L=1,2),K=1,2),NPTS,NCAT
      DO 1663 J=1,NPTS
        READ (2) NOPT,ELX,ELY,((PLCK,L),L=1,2),K=1,2),STIME,LASTPT,NPP
 1663   WRITE(1) NOPT,ELX,ELY,((PL(K,L),L=1,2),K=1,2),STIME,IZERO,NPP
 1664 READ (3) NOST,ALPHA,DELTA,((PALPHA(I,J),J=1,2),I=1,2),NPTS,NCAT
      WRITE(1) NOST,ALPHA,DELTA,((PALPHA(I,J),J=1,2),I=1,2),NPTS,NCAT
      DO 1666 I=1,NPTS
        READ (3) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LASTPT,NPP
 1666   WRITE(1) NOPT,ELX,ELY,((PL(J,K),K=1,2),J=1,2),STIME,LASTPT,NPP
      IF(LASTPT)1667,1664,1667
 1667 REWIND 1
      REWIND 2
      REWIND 3
      IF(IND(2))167,163,167
  167 NOPPTS = NOPPTS + 1
  163 NPPE2 = NPRE2 + NPOST2 + NVENT2
      NPPER = NPRER + NPOSTR + NVENTR
      PRINT 1035,NO2PTS,NOPPTS,NPPE2,NPRER,NPOST2,NPOSTR,NVENT2,NVENTR,
     1 NPPE2,NPPER
C     PREPARATORY STEPS
      CALL CRTIO (4HSATR)
      PRINT 1140
      PRINT 1141,(NAMES(I),I=5,14)
      PRINT 1142,NAMES(15),NAMES(19)
      CALL GRFE(-.1,-.1,.1,.1,0.,0.,.01,.01,2,2,0.1,0.,0.,1.1,2H75,-1,0)
      IF(TYPBME.GT.0.)ERRPR = TYPBME
 1682 MCOMP = 1
 1684 PRINT 1036
      IF(IND(6))169,170,169
  169 PRINT 1037,ITMIN
      NOOPRE=8
      IF(IND(7))171,170,171
  170 PRINT 1038,ITMAX
C
C  COUNT NUMBER OF UNKNOWNS ASSOCIATED WITH ORIENTATION PARAMETERS.
C
  171 NUO = 22
      NOVENT=0
      NOPRE=0
      NOPOST=0
      NCM = 0
      IF(NPRE2.GT.1)GO TO 1712
 1711 NUO=NUO-3
      NOPRE=1
 1712 IF(NPOST2.GT.1)GO TO 1714
 1713 NUO=NUO-3
      NOPOST=2
 1714 IF(NVENT2.GT.1)GO TO 1716
 1715 NUO=NUO-3
      NOVENT = 4
 1716 IF(IND(1))172,173,172
  172 NUO = NUO-1
      NOOPRE=NOOPRE-1
C
```

If azimuth-elevation given, add one to plate point count.

Print point and star counts.

If mean error of unit weight given for problem, set m=to given value.

Set indicator to compute m before starting residual cycle.

Print number of iterations minimum solution will be restricted to in the problem.

Print number of iterations maximum solution will be restricted to in the problem.

Figure 20.--Continued (page 8 of 57)

```
C  COUNT NUMBER OF OBSERVATIONS ASSOCIATED WITH ORIENTATION PARAMETERS.
C
  173 DO 175 I=7,14
        IF(PO(I,I))175,175,174
  174   NOM = NOM + 1
  175   CONTINUE
      NOMX = NOM
      DO 1752 I=1,6
        IF(PO(I,I))1752,1752,1751
 1751   NOMX = NOMX + 1
 1752   CONTINUE
      DO 1754 I=15,22
      IF(PO(I,I))1754,1754,1753
 1753 NOMX = NOMX+1
 1754 CONTINUE
      WC = (C8*PRESS)/SQRT(TEMP**3)                    Compute weather constants for refraction
      WF = C9 * SQRT(TEMP)                             function; WC = C8 P/T^3/2, WF = T^1/2.
      TOLZ = TOLR * C6                                 Convert to radians (refraction tolerance and
      PHICS = ABS(CAMSTA(2))*C4 + CAMSTA(3)*C5 + CAMSTA(4)*C6   Station coordinates).
      IF(CAMSTA(1))178,179,178
  178 PHICS = -PHICS
  179 SINPHI = SIN(PHICS)
C
C  CLEAR ARRAYS AND CYCLE INDICATORS.
C
      SIGMAC = 0.                                      Sigma cycle indicator.
      COSPHI = COS(PHICS)
      VLYSUM=0.
      VLXSUM=0.                                        Plate residual sums
      ELXSUM=0.
      ELYSUM=0.                                        Plate measurement sums.
      PCYCLE = 0.                                      Pre-run cycle indicator.
      RCYCLE = 0.                                      Refraction computation cycle indicator.
      VCYCLE = 0.                                      Residual cycle indicator.
      AERES = 0.                                       Azimuth-Elevation cycle indicator.
      RCC = 0.                                         Refraction coefficients computed cycle indicator.
      TOLMET = 0.                                      Problem tolerance met indicator.
      V1 = 0.                                          Plate measurement pvv sum.
      V2 = 0.                                          Star coordinate pvv sum.
      V3 = 0.                                          Orientation parameter pvv sum.
      DO 1790 J=1,3
        SUMV(J) = 0.
 1790   SUMVS(J)= 0.
      CT(1) = 0.
      PRSUML = 0.                                      Previous level of residuals.
      NDIV = 0                                         Number of divergent steps counter.
      NPTS = 0                                         Number of observations used for star j.
      ITCNT = 0                                        Number of iterations.
      LSIG = 1
      CCYCLE = 0.                                      Combined solution cycle indicator.
      DO 1791 I=1,26
 1791   DELLO(I) = 0.                                  Total correction vector, ΔL, (26x1)
 1792 IF(RCC)180,180,287
  180 DO 1802 I=1,26
```

Figure 20.--Continued (page 9 of 57)

```
      DO 1801 J=1,26
 1801   RN(I,J) = 0.
      CN(I) = 0.
 1802 DELO(I) = 0.
      IF(SIGMAC.EQ.1.) GO TO 1810
      POSVX=0.
      POSVY=0.
      ANEGVX=0.
      ANEGVY=0.
 1810 NL = 0
      NX = 0
      NUX = 0
      NUSED = 0
      ACYCLE = 0.
      AERES = 0.
      DELLSQ = 0.
      SINPHT = SIN(OPAR(22))
      COSPHT = COS(OPAR(22))
      J= 1
C
C  COMPUTE DIRECTION COSINES.  COMPUTE ONE SET FOR EACH ORIENTATION PRE,
C    POST AND EVENT.
C
      DO 1803 I=1,3
       SINAL(I) = SIN(OPAR(J))
       COSAL(I) = COS(OPAR(J))
       SINOM(I) = SIN(OPAR(J+1))
       COSOM(I) = COS(OPAR(J+1))
       SINKA(I) = SIN(OPAR(J+2))
       COSKA(I) = COS(OPAR(J+2))
       A(1,J)   =-COSAL(I)*COSKA(I)+SINAL(I)*SINOM(I)*SINKA(I)
       A(2,J)   =-COSOM(I)*SINKA(I)
       A(3,J)   = SINAL(I)*COSKA(I)+COSAL(I)*SINOM(I)*SINKA(I)
       A(1,J+1) =-COSAL(I)*SINKA(I)-SINAL(I)*SINOM(I)*COSKA(I)
       A(2,J+1) = COSOM(I)*COSKA(I)
       A(3,J+1) = SINAL(I)*SINKA(I)-COSAL(I)*SINOM(I)*COSKA(I)
       A(1,J+2) = SINAL(I)*COSOM(I)
       A(2,J+2) = SINOM(I)
       A(3,J+2) = COSAL(I)*COSOM(I)
 1803 J= J+3
C
C  COMPUTE ORIGIN OF DISTORTION CURVES.
C
      XS = OPAR(15)+OPAR(1)
      YS = OPAR(16)+OPAR(12)
  181 IF(NPTS)184,182,184
  182 DO 1823 I=1,2
        DO 1821 J=1,2
 1821     BXTPBX(I,J) = 0.
      DO 1822 J=1,26
         TWO(I,J) = 0.
 1822     FOUR(J,I) = 0.
 1823   EIGHT(I) = 0.
      V18 = 0
```

Reduced Normal Equation Matrix.

Reduced Discrepancy Vector.

Correction Vector.

Sums of positive and negative plate residuals.

Observation counters.

Azimuth-Elevation cycle indicator.

Azimuth-Elevation residual cycle indicator.

Sum of squared plate residuals.

$A_1 = -\cos\alpha \cos\kappa + \sin\alpha \sin\omega \sin\kappa$

$B_1 = -\cos\omega \sin\kappa$

$C_1 = \sin\alpha \cos\kappa + \cos\alpha \sin\omega \sin\kappa$

$A_2 = -\cos\alpha \sin\kappa - \sin\alpha \sin\omega \cos\kappa$

$B_2 = \cos\omega \cos\kappa$

$C_2 = \sin\alpha \sin\kappa - \cos\alpha \sin\omega \cos\kappa$

$D = \sin\alpha \cos\omega$

$E = \sin\omega$

$F = \cos\alpha \cos\omega$

$$A = \begin{array}{c} \begin{array}{ccc|ccc|ccc} \text{Pre} & & & \text{Post} & & & \text{Event} & & \\ \end{array} \\ \left[\begin{array}{ccc|ccc|ccc} A_1 & A_2 & D & A_1 & A_2 & D & A_1 & A_2 & D \\ B_1 & B_2 & E & B_1 & B_2 & E & B_1 & B_2 & E \\ C_1 & C_2 & F & C_1 & C_2 & F & C_1 & C_2 & F \end{array}\right] \end{array}$$

$x_s = x_p + \Delta x$

$y_s = y_p + \Delta y$

If first observation of star, clear arrays; $B_x^T P_2 B_x$ (2×2), $B_x^T P_2 B_0$ (2×26), $(B_0^T P_2 B_x)(B_x^T P_2 B_x + P_{\Delta\ell})^{-1}$ (26×2), $B_x^T P_2 \Delta\ell$ (2×1).

Figure 20.--Continued (page 10 of 57)

```
      IF(SIGNAC.EQ.1.) GO TO 1842                                       467
C
C  READ DATA FOR ONE STAR FROM STORAGE.  ADD TO OBSERVATION AND UNKNOWN
C    COUNT.
      READ (1) NOST,ALPHA,DELTA,((PALPHA(I,J),J=1,2),I=1,2),NPTS,NCAT   469
      IF(PALPHA(1,1))184,184,183                                        470
  183 NX = NX + 2                                                       471
      NUX = NUX + 2                                                     472
C
C  READ PLATE COORDINATES FOR ONE OBSERVATION OF STAR ALONG WITH WEIGHT,
C    SIDEREAL TIME AND ORIENTATION (PRE, POST, OR EVENT).
C
  184 READ (1) NOPT,ELX,ELY,((PL(I,J),J=1,2),I=1,2),STIME,LASTPT,NPP    473
      IF(PALPHA(1,1))1842,1842,1841                                     474
C
C  ADD TO OBSERVATION COUNT.
C
 1841 NL = NL+2                                                         475
C
C  CORRECT LX MEASUREMENT FOR NON-PERPENDICULARITY.
C
 1842 DTLAST = 0.                                                       477
      DRLAST=0.                                                         478
      ELXSST = ELX + ELY*OPAR(10)                                       479
      ELYSST = ELY                                                      480
C
C  COMPUTE AND CORRECT FOR DISTORTION AT LX AND LY.
C
      NCNT = 0                                                          481
  185 D2 = (ELXSST - XS)**2 + (ELYSST - YS)**2                          482
      NCNT = NCNT+1                                                     483
      D4 = D2**2                                                        484
      D6 = D4*D2                                                        485
      DELRDI = OPAR(17)*D2 + OPAR(18)*D4 + OPAR(19)*D6                  486
      DELTAT = OPAR(20)*D2 + OPAR(21)*D4                                487
      DCON1 = (2.*(ELXSST-XS)**2/D2)+1.                                 488
      DCON2 = (2.*(ELYSST-YS)**2/D2)+1.                                 489
      DCON3 = (2.*(ELXSST-XS)*(ELYSST-YS))/D2                           490
      DTSINP = DELTAT*(DCON3*COSPHT+DCON2*SINPHT)                       491
      DTCOSP = DELTAT*(DCON1*COSPHT+DCON3*SINPHT)                       492
      ELXSST = ELX - DELRDI*(ELXSST-XS) - DTCOSP + ELY*OPAR(10)         493
      ELYSST = ELY - DELRDI*(ELYSST-YS)-DTSINP                          494
C
C  CHECK DISTORTION ITERATION TOLERANCE.
C
      IF(ABS(DELRDI*SQRT(D2)-DRLAST)-TOLD)1861,1861,1862                495
 1861 IF(ABS(DELTAT-DTLAST)-TOLD)186,186,1862                           496
C
C  SET LX AND LY EQUAL TO DISTORTED DISTANCES AND GO TO RECOMPUTE
C    DISTORTION.
C
 1862 DTLAST=DELTAT                                                     497
      DRLAST=DELRDI*SQRT(D2)                                            498
      IF(NCNT-5)185,1863,1863                                           499
```

Clear distortion tolerance check.

$\tilde{\ell}_{x_i} = \ell_{x_i} + \ell_{y_i}\,\varepsilon$

Clear distortion iteration counter.

$d_i^2 = (\tilde{\ell}_{x_i} - x_s)^2 + (\ell_{y_i} - y_s)^2$

Increment distortion iteration counter.

$\Delta R/d_i = K_1 d_i^2 + K_2 d_i^4 + K_3 d_i^6$

$\Delta T = K_4 d_i^2 + K_5 d_i^4$

$DCON1 = 2(\tilde{\ell}_{x_i} - x_s)^2/d_i^2 + 1$

$DCON2 = 2(\ell_{y_i} - y_s)^2/d_i^2 + 1$

$DCON3 = 2(\tilde{\ell}_{x_i} - x_s)(\ell_{y_i} - y_s)/d_i^2$

$\Delta T_y = \Delta T\,(DCON3 \cos\varphi_r + DCON2 \sin\varphi_r)$

$\Delta T_x = \Delta T\,(DCON1 \cos\varphi_r + DCON3 \sin\varphi_r)$

$\ell_{x_i}^{*} = \tilde{\ell}_{x_i} - \Delta R/d_i(\tilde{\ell}_{x_i} - x_s) - \Delta T_x + \ell_{y_i}\,\varepsilon$

$\ell_{y_i}^{*} = \ell_{y_i} - \Delta R/d_i(\ell_{y_i} - y_s) - \Delta T_y$

Check if radial distortion change is less than tolerance.

Check if decentered lens distortion change less than tolerance.

If more than 5 iterations, print point number and continue problem.

Figure 20.--Continued (page 11 of 57)

```
 1863 PRINT 1127,NOPT,DELRUI,DELTAI                                    500
  186 IF(MPTS)193,187,193                                              501  Check for first observation of star.
C
C  FOR ALL OBSERVATIONS OF STAR J AFTER THE FIRST, BYPASS COMPUTATION
C    OF APPROXIMATIONS TO ALPHA AND DELTA.
C
  187 IF(PALPHA(1,1))189,189,188                                       502  Check star type.
C
C  IF FIRST OBSERVATION OF TYPE TWO STAR, BYPASS COMPUTATION OF
C    APPROXIMATIONS TO ALPHA AND DELTA.  SET ALPHA STAR EQUAL TO GIVEN
C    ALPHA AND GO TO 193.
C
  188 ALSTAR = ALPHA                                                   503  α* = α
      GO TO 193                                                        504
  189 IF(MINSOL.EQ.1.OR.NPP.EQ.3)GO TO 1891                            505  IF MINIMUM Solution, treat all unknown stars
      IF(NPP.EQ.2)GO TO 1892                                           506  as PRE orientation.
C
C  COMPUTE APPROXIMATIONS TO RIGHT ASCENSION AND DECLINATION FOR UNKNOWN
C    STAR.
C
      II = 1                                                           508 }
      JJ = 3                                                           509 } PRE star.
      GO TO 1893                                                       510
 1891 II=7                                                             511 }
      JJ=9                                                             512 } EVENT star.
      GO TO 1893                                                       513
 1892 II=4                                                             514 }
      JJ=6                                                             515 } POST star.
C
C  COMPUTE STANDARD COORDINATES USING APPROXIMATIONS TO ORIENTATION
C    PARAMETERS.
C
 1893 ELLXY(II) = ELXSST-OPAR(11)                                      516
      ELLXY(II+1) = (OPAR(13)/OPAR(14))*(ELYSST-OPAR(12))              517
      ELLXY(JJ) = OPAR(13)                                             518
 1894 DO 1895 I=1,3                                                    519
      SMALLU(I) = 0                                                    520  u = (l_xi^* - x_p)A_1 + c_x/c_y (l_yi^* - y_p)A_2 + c_x D
      DO 1895 J= II,JJ                                                 521  v = (l_xi^* - x_p)B_1 + c_x/c_y (l_yi^* - y_p)B_2 + c_x E
 1895 SMALLU(I) = SMALLU(I)+A(I,J)*ELLXY(J)                            522  w = (l_xi^* - x_p)C_1 + c_x/c_y (l_yi^* - y_p)C_2 + c_x F
      XSI = SMALLU(1)/SMALLU(3)                                        523  ξ_R = u/w
      ETA = SMALLU(2)/SMALLU(3)                                        524  η_R = v/w
C
C  COMPUTE ATMOSPHERIC REFRACTION AT XSI AND ETA.
C
      TANZ = SQRT(XSI**2 +.ETA**2)                                     525  tan z_R = (ξ_R^2 + η_R^2)^1/2
      ZR = ATAN(TANZ)                                                  526
      BETA = ATAN(WF*TANZ)                                             527
      TANB = SIN(BETA/2.)/COS(BETA/2.)                                 528  tan β = (T^1/2/ρ) tan z_R
      ZETA = WC*(OPAR(23)*TANB + OPAR(24)*TANB**3 + OPAR(25)*TANB**5 +  529  ζ^* = T^1/2 w(τ_1 tan β/2 + τ_2 tan^3 β/2 + τ_3 tan^5 β/2
     1  OPAR(26)*TANB**7)                                              530      + τ_4 tan^7 β/2)
      ZO = ZR + ZETA                                                   531  z_o = z_R + ζ
      ELEV = (C10/2.1-ZO)/C4                                           532
      COSZO = COS(ZO)                                                  533
      SINZO = SIN(ZO)                                                  534
```

Figure 20.--Continued (page 12 of 57)

```
      TANZ0 = SINZ0/COSZ0                                                535
C
C  COMPUTE AZIMUTH.
C
      CALL ANGLE(ETA,XSI,AZ)                                             536
      AZ = AZ + C10                                                      537
      IF(AZ-2.*C10)1921,1920,1920                                        538
 1920 AZ = AZ - 2.*C10                                                   539
 1921 SINAZ = SIN(AZ)                                                    540
      COSAZ = COS(AZ)                                                    541
C
C  COMPUTE RIGHT ASCENSION AND DECLINATION.
C
      SINDEL = COSZ0*SINPHI - SINZ0*COSAZ*COSPHI                         542   δ' = sin⁻¹(cos z₀ sin φ - sin z₀ cos A cos φ)
      IF(SINDEL-.0111922)1922,1922,1925                                  543
 1922 CALL ASIN(SINDEL,DELTA)                                            544
      COSDEL = COS(DELTA)                                                545
      GO TO 1926                                                         546
 1925 COSDEL = SQRT(1.-SINDEL**2)                                        547
      DELTA = ATAN(SINDEL/COSDEL)                                        548
 1926 COCH = COSZ0*COSPHI + SINZ0*COSAZ*SINPHI                           549   cos δ' cos H' = cos z₀ cos φ + sin z₀ cos A sin φ
      COSH = SINZ0*SINAZ                                                 550   cos δ' sin H' = sin z₀ sin A
      CALL ANGLE(COSH,COCH,HPRI)                                         551   tan H' = cos δ' sin H' / cos δ' cos H'
      ALPHA = STIME - HPRI                                               552   α' = S.T.ₐ - H'
      IF(ALPHA)1927,1928,1928                                            553
 1927 ALPHA = ALPHA + 2.*C10                                             554
 1928 COSHPR = COS(HPRI)                                                 555
C
C  COMPUTE AND CORRECT FOR DIURNAL ABERRATION.
C
      SINHPR = SIN(HPRI)                                                 556
      ALSTAR = ALPHA - (.32*COSPHI*C6*COSHPR/COSDEL)                     557   α* = α' - .32ρ cos φ cos H' sec δ'
      DELTA = DELTA - (.32*COSPHI*C6*SINHPR*SINDEL)                      558   δ = δ' - .32ρ cos φ sin H' sin δ'
C
C  COMPUTE HOUR ANGLE.
C
  193 HPRI = STIME - ALSTAR                                              560   H* = S.T.ₐ - α*
      SINHPR = SIN(HPRI)                                                 561
      COSHPR = COS(HPRI)                                                 562
      SINDEL = SIN(DELTA)                                                563
      COSDEL = COS(DELTA)                                                564
C
C  ADD DIURNAL ABERRATION TO RIGHT ASCENSION AND DECLINATION.
C
      ALPRI = ALSTAR + (.32*COSPHI*C6*COSHPR/COSDEL)                     565   α' = α* + .32ρ cos φ cos H' sec δ'
      DELPRI = DELTA + (.32*COSPHI*C6*SINHPR*SINDEL)                     566   δ' = δ + .32ρ cos φ sin H' sin δ'
      HPRI = STIME - ALPRI                                               567   H' = S.T.ₐ - α'
      SINHPR = SIN(HPRI)                                                 568
      COSHPR = COS(HPRI)                                                 569
      SINDEL = SIN(DELPRI)                                               570
      COSDEL = COS(DELPRI)                                               571
      ZLAST = 0.                                                         572   Clear refraction tolerance check.
C
C  COMPUTE ZENITH DISTANCE.
```

Figure 20.--Continued (page 13 of 57)

```
      COSZ = SINDEL*SINPHI + COSDEL*COSHPR*COSPHI                       573  cos z = sin δ' sin φ + cos δ' cos H' cos φ
      SINZ = SQRT(1. - COSZ**2)                                         574  sin z = (1 - cos² z)^½
      TANZO = SINZ/COSZ                                                 575  tan z = sin z / cos z
      Z = ATAN(TANZO)                                                   576
C
C  COMPUTE ATMOSPHERIC REFRACTION AT GIVEN ZENITH DISTANCE.
C
  190 BETA = ATAN(WF*TANZO)                                             577
      TANB = SIN(BETA/2.)/COS(BETA/2.)                                  578  tan β = (T^½/T) tan z
      ZETA = WC*(OPAR(23)*TANB + OPAR(24)*TANB**3 + OPAR(25)*TANB**5 +  579  ζ'' = T^½ W(τ₁ tan β/2 + τ₂ tan³ β/2 + τ₃ tan⁵ β/2
     1  OPAR(26)*TANB**7)                                              580      + τ₄ tan⁷ β/2)
      ZO = Z - ZETA                                                     581  z° = z - ζ
C
C  CHECK REFRACTION TOLERANCE.
C
      TANZO = SIN(ZO)/COS(ZO)                                           582
      DZR = ABS(ZETA - ZLAST)                                           583
      IF(DZR-TOLZ)192,192,191                                           584  ζ_a - ζ_a-1 ≤ tol. ζ ?
  191 ZLAST = ZETA                                                      585  ζ_a → ζ_a-1
C
C  IF TOLERANCE NOT MET GO TO RECOMPUTE REFRACTION WITH CORRECTED ZENITH
C    DISTANCE
C
      GO TO 190                                                         586
C
C  ADD REFRACTION CORRECTION TO RIGHT ASCENSION AND DECLINATION.
C
  192 COSQ = (SINPHI - SINDEL*COSZ)/(COSDEL*SINZ)                       587  cos q = (sin φ - sin δ' cos z)/cos δ' sin z
      SINQ = SINHPR *COSPHI/SINZ                                        588  sin q = (sin H' cos φ)/sin z
      SINDEL = SIN(DELPRI)*COS(ZETA) + COS(DELPRI)*SIN(ZETA)*COSQ       589  sin δ_r = sin δ' cos ζ_a + cos δ' sin ζ_a cos q
      COSDEL = SQRT(1.-SINDEL**2)                                       590  cos δ_r = (1 - sin² δ_r)^½
      DELPRI = ATAN(SINDEL/COSDEL)                                      591
      SINDLM = SINQ*SIN(ZETA)/COSDEL                                    592  sin ΔH = (sin q sin ζ_a)/cos δ_r
      CALL ASIN(SINDLM,DELH)                                            593
      HBAR = HPRI - DELH                                                594  H_r = H' - ΔH
      IF(PCYCLE)194,194,197                                             595  Check for PRE-RUN cycle.
  194 IF(RCYCLE)195,195,197                                             596  Check for REFRACTION computation cycle.
  195 IF(TOLMET)197,197,196                                             597  Check if problem tolerance has been met.
C
C  IF PROBLEM ITERATION TOLERANCE HAS BEEN MET, STORE COMPUTED DATA ON
C    DISK FOR LATER USE IN COMPUTING VS.
C
  196 WRITE(2) ELX,ELY,((PL(I,J),J=1,2),I=1,2),NOPT,ELXSST,ELYSST,DTSINP 598
     1,DTCOSP,HBAR,DELPRI,DELRDI,NPP,STIME                              599
C
C  CONVERT RIGHT ASCENSION AND DECLINATION TO STANDARD COORDINATES.
C
  197 SINHPR = SIN(HBAR)                                                601
      COSHPR = COS(HBAR)                                                602
      COSZO = SINDEL*SINPHI + COSDEL*COSHPR*COSPHI                      603  cos z_r = sin δ_r sin φ + cos δ_r cos H_r cos φ
      ETA = -COSDEL*SINHPR/COSZO                                        604  η_r = (-cos δ_r sin H_r)/cos z_r
      XSI = (COSPHI*SINDEL - SINPHI*COSDEL*COSHPR)/COSZO                605  ξ_r = (cos φ sin δ_r - sin φ cos δ_r cos H_r)/cos z_r
      ETAXSI(1) = XSI                                                   606
```

Figure 20.--Continued (page 14 of 57)

```
      ETAXSI(2) = ETA
      ETAXSI(3) = 1.
C
C  COMPUTE DISCREPANCY VECTOR.
C
  198 IF(MINSOL.EQ.1.OR.NPP.EQ.3) GO TO 1981
      IF(NPP.EQ.2) GO TO 1982
      II = 1
      JJ = 3
      GO TO 1983
 1981 II = 7
      JJ = 9
      GO TO 1983
 1982 II = 4
      JJ = 6
 1983 DO 1984 J=II,JJ
      SMALLM(J) = 0.
      DO 1984 I=1,3
 1984 SMALLM(J) = SMALLM(J)+A(I,J)*ETAXSI(I)
      XI = OPAR(13)*SMALLM(II)/SMALLM(JJ)+OPAR(11)+DELRDI*(ELXSST-XS)+DT
     1COSP-ELY*OPAR(10)
      YI = OPAR(14)*SMALLM(II+1)/SMALLM(JJ)+OPAR(12)+DELRDI*(ELYSST-YS)+
     1DTSINP
C     DIFFERENCE BETWEEN MEASURED AND COMPUTED PLATE COORDINATES
      DELL(1) = ELX - XI
      DELL(2) = ELY - YI
C
C  IF AZ ELEV RESIDUAL CYCLE, GO TO ADD OBSERVATION TO PVV SUM.
C
      IF(AERES)199,199,317
C
C  IF IN PRE-RUN CYCLE, GO TO WRITE STATISTICAL DATA.
C
  199 IF(PCYCLE)200,200,303
C
C  IF IN V-CYCLE, GO TO SUM PVV AND PRINT STATISTICAL DATA.
C
  200 IF(VCYCLE)202,202,302
  202 IF(PALPHA(1,1))2022,2022,2023
 2023 DELLSQ = DELLSQ + DELL(1)**2 + DELL(2)**2
C
C  COMPUTE AUXILIARIES FOR OBSERVATION EQUATIONS.
C
 2022 CIRONE = (XI - OPAR(11))/OPAR(13)
      CIRTWO = (YI - OPAR(12))/OPAR(14)
      CIRTHR = CIRONE*A(1,JJ)-A(1,II)
      CIRFOR = CIRONE*A(2,JJ)-A(2,II)
      CIRSIX = CIRTWO*A(1,JJ)-A(1,II+1)
      CIRSEV = CIRTWO*A(2,JJ)-A(2,II+1)
      CIRNIN = CIRTWO*A(2,II)-CIRONE*A(2,II+1)
C
C  COMPUTE COEFFICIENTS FOR OBSERVATION EQUATION MATRIX FOR ALPHA THRU
C    CY.
C
```

If MINIMUM solution, treat all stars as PRE orientation.

PRE star.

EVENT star.

POST star.

$m = \xi_r A_1 + \eta_r B_1 + C_1$

$n = \xi_r A_2 + \eta_r B_2 + C_2$

$q = \xi_r D + \eta_r E + F$

$x_i = [(c_x m/q) + x_p][1 + (\Delta R/d_i)] - [x_s(\Delta R/d_i)] + \Delta T_x - \ell y_i \varepsilon$

$y_i = [(c_y n/q) + y_p][1 + (\Delta R/d_i)] - [y_s(\Delta R/d_i)] + \Delta T_y$

Check for AZIMUTH-ELEVATION computation cycle.

Check for PRE-RUN cycle.

Check for V-cycle.

If known star, add to sum of squared residuals.

$\Delta \ell x_i^2 + \Delta \ell y_i^2 \rightarrow \Sigma \Delta \ell^2$

$① = (x_i - x_p)/c_x$

$② = (y_i - y_p)/c_y$

$③ = ① D - A_1$

$④ = ① E - B_1$

$⑥ = ② D - A_2$

$⑦ = ② E - B_2$

$⑨ = ② B - ① B_2$

Figure 20.--Continued (page 15 of 57)

```
      DO 2021 I=1,2
      DO 2021 J=1,26
 2021     BO(I,J) = 0.
      BO(1,II) = -OPAR(13)*(CIRONE*CIRNIN + CIRSEY)
      BO(1,II+1) = OPAR(13)*((1.+CIRONE**2)*SINKA(NPP)-CIRONE*CIRTWO*COS
     1KA(NPP))
      BO(2,II+1) = -OPAR(14)*((1.+CIRTWO**2)*COSKA(NPP)-CIRONE*CIRTWO*SI
     1NKA(NPP))
      BO(1,JJ) = -OPAR(13)*CIRTWO
      BO(1,10) = -ELY
      BO(1,11) = 1.
      BO(1,13) = CIRONE
      BO(2,II) = -OPAR(14)*(CIRTWO*CIRNIN - CIRFOR)
      BO(2,JJ) = OPAR(14)*CIRONE
      BO(2,12) = 1.
      IF(IND(1))204,203,204
  203 BO(2,14) = CIRTWO
      GO TO 205
  204 BO(2,13) = CIRTWO
C
C  IF IN AZIMUTH-ELEVATION CYCLE, GO TO FORM OBSERVATION EQUATION MATRIX
C
  205 IF(ACYCLE)207,207,206
  206       N = N + 2
            DO 2061 I=1,26
            DO 2061 J=1,2
 2061          BOAUX(I,J) = 0.
           GO TO 217
C
C  COMPUTE COEFFICIENTS FOR OBSERVATION EQUATION MATRIX FOR DELTA X
C    THRU PHI-T.
C
  207 ELXXS = ELXSST-XS
      ELYYS = ELYSST-YS
      DX = -OPAR(13)/SMALLM(JJ)*CIRTHR
      EX = -OPAR(13)/SMALLM(JJ)*CIRFOR
      DY = -OPAR(14)/SMALLM(JJ)*CIRSIX
      EY = -OPAR(14)/SMALLM(JJ)*CIRSEY
      IF(MINSOL)212,2071,212
 2071 PARDT = 2.*OPAR(20)+4.*OPAR(21)*D2
      PAROR = 2.*OPAR(17)+4.*OPAR(18)*D2+6.*OPAR(19)*D4
      BO(1,15)=-DELROI-ELXXS**2*PARDR-ELXXS*PARDT*COSPHT-4.*ELXXS*(OPAR(
     120)+(2.*ELXXS**2+ELYYS**2)*OPAR(21))*COSPHT-2.*ELYYS*(OPAR(20)+(EL
     2XXS**2+3.*ELYYS**2)*OPAR(21))*SINPHT
      BO(2,15) = -ELXXS*(ELYYS*PARDR+PARDT*SINPHT)-2.*ELYYS*((OPAR(20)+
     1(3.*ELXXS**2+ELYYS**2)*OPAR(21))*COSPHT+2.*ELYYS*OPAR(21)*SINPHT)
      BO(1,16) = -ELYYS*(ELXXS*PAROR+PARDT*COSPHT)-2.*ELXXS*((OPAR(20)+
     1(ELXXS**2+3.*ELYYS**2)*OPAR(21))*SINPHT+2.*ELXXS*ELYYS*OPAR(21)*
     2COSPHT)
      BO(2,16) = -DELROI-ELYYS**2*PAROR-ELYYS*PARDT*SINPHT-2.*ELXXS*
     1(OPAR(20)+(ELXXS**2+3.*ELYYS**2)*OPAR(21))*COSPHT-4.*ELYYS*(OPAR(2
     20)+(ELXXS**2+2.*ELYYS**2)*OPAR(21))*SINPHT
      BO(1,11) = BO(1,11)+BO(1,15)
      BO(2,12) = BO(2,12)+BO(2,16)
```

Clear array B_o (2x26)

$B_{o_{\omega x}}$ $A_x = -c_x\left(①\cdot⑨+⑦\right)$

$B_{o_{\omega x}}$ $B_x = c_x\left[\left(1+①^2\right)\sin\varkappa - ①\cdot②\cos\varkappa\right]$

$B_{o_{\omega y}}$ $B_y = -c_y\left[\left(1+②^2\right)\cos\varkappa - ①\cdot②\sin\varkappa\right]$

$B_{o_{\phi x}}$ $C_x = -c_x ②$

$B_{o_{\kappa x}}$ $U_x = -\ell_{y_s}$

$B_{o_{\kappa y}}$ $H_x = 1$

$B_{o_{\phi x}}$ $G_x = ①$

$B_{o_{\omega y}}$ $A_y = -c_y\left(②\cdot⑨-④\right)$

$B_{o_{\phi y}}$ $C_y = c_y ①$

$B_{o_{\kappa y}}$ $I_y = 1$

Check if Cx equals Cy in solution

$B_{o_{\phi y}}$ $G_y = ②$

If $c_x = c_y$ set $\frac{G_x}{G_y}$; If $c_x \neq c_y$ set $\frac{G_x}{G_y}$

Check if AZIMUTH-ELEVATION computation cycle

Add 2 to observation count

Clear array $B_o^T P_j$ (26x2)

B_{x_ξ} $D_x = (c_x/q)①$ $= -J_x$

B_{x_η} $E_x = (c_x/q)④$ $= -K_x$

B_{x_ξ} $D_y = (c_y/q)②$ $= -J_y$

B_{x_η} $E_y = (c_y/q)⑦$ $= -K_y$

Check for MINIMUM solution

$d_x = \ell_{x_s}^{\prime\prime} - x_s$

$d_y = \ell_{y_s}^{\prime\prime} - y_s$

$B_{o_{\phi x}}$ $P_x = -(\Delta R/d_x) - d_x^2 D_r - d_x D_t \cos\varphi_r - 4 d_x\left[K_s + (2d_x^2 + d_y^2)K_s\right]\cos\varphi_r - 2 d_y\left[K_s + (d_x^2+3d_y^2)K_s\right]\sin\varphi_r$

$B_{o_{\phi x}}$ $P_y = -d_x(d_y D_r + D_t \sin\varphi_r) - 2 d_y\left[(K_s + \{3d_x^2 + d_y^2\}K_s)\cos\varphi_r + 2 d_y K_s \sin\varphi_r\right]$

$B_{o_{\phi y}}$ $Q_x = -d_y(d_x D_r + D_t \cos\varphi_r) - 2 d_x\left[(K_s + (d_x^2+3d_y^2)K_s)\sin\varphi_r + 2 d_x d_y K_s \cos\varphi_r\right]$

$B_{o_{\phi y}}$ $Q_y = -(\Delta R/d_x) - d_y^2 D_r - d_y D_t \cos\varphi_r - 2 d_x\left[K_s + (d_x^2+3d_y^2)K_s\right]\sin\varphi_r - 4 d_y\left[K_s + (d_x^2+2d_y^2)K_s\right]\sin\varphi_r$

$B_{o_{\kappa x}}$ $H_x = 1 + P_x$

$B_{o_{\kappa y}}$ $I_y = 1 + Q_y$

Figure 20.--Continued (page 16 of 57)

```
      BO(1,12) = BO(1,16)                                                691  Bo_Mx   Ix = Qx
      BO(2,11) = BO(2,15)                                                692  Bo_My   Hy = Py
      BO(1,17) = D2*ELXXS                                                693  Bo_K1   Mx = dx d_s^2
      BO(2,17) = D2*ELYYS                                                694  Bo_K2   My = dy d_s^2
      BO(1,18) = D4*ELXXS                                                695  Bo_K3   Nx = dx d_s^4
      BO(2,18) = D4*ELYYS                                                696  Bo_K4   Ny = dy d_s^4
      BO(1,19) = D6*ELXXS                                                697  Bo_K5   Ox = dx d_s^6
      BO(2,19) = D6*ELYYS                                                698  Bo_K6   Oy = dy d_s^6
      BO(1,20) = D2*COSPHT+2.*(ELXXS**2*COSPHT+ELXXS*ELYYS*SINPHT)       699  Bo_K7   Rx = d_s^2 cos φr + 2(dx^2 cos φr + dx dy sin φr)
      BO(2,20) = D2*SINPHT+2.*(ELYYS**2*SINPHT+ELXXS*ELYYS*COSPHT)       700  Bo_K8   Ry = d_s^2 sin φr + 2(dy^2 sin φr + dx dy cos φr)
      BO(1,21) = D4*COSPHT+2.*D2*(ELXXS**2*COSPHT+ELXXS*ELYYS*SINPHT)    701  Bo_K9   Sx = d_s^4 cos φr + 2 d_s^2 (dx^2 cos φr + dx dy sin φr)
      BO(2,21) = D4*SINPHT+2.*D2*(ELYYS**2*SINPHT+ELXXS*ELYYS*COSPHT)    702  Bo_K10  Sy = d_s^4 sin φr + 2 d_s^2 (dy^2 sin φr + dx dy cos φr)
      BO(1,22)=-SINPHT*(DPAR(20)*D2+DPAR(21)*D4)+(2.*DELTAT/D2)*(ELXXS*E 703  Bo_φr   Tx = -sin φr (K4 d_s^2 + K5 d_s^4) + 2(K4 + K5 d_s^2)(dx dy cos φr -
     1LYYS*COSPHT-ELXXS**2*SINPHT)                                       704                dx^2 sin φr)
      BO(2,22)=COSPHT*(DPAR(20)*D2+DPAR(21)*D4)+(2.*DELTAT/D2)*(ELYYS**2 705  Bo_φr   Ty = cos φr (K4 d_s^2 + K5 d_s^4) + 2(K4 + K5 d_s^2)(dy^2 cos φr -
     1*COSPHT-ELXXS*ELYYS*SINPHT)                                        706                dx dy sin φr)
      IF(TOLMET)212,212,208                                              707  Check if Problem Tolerance has been met.
  208         IF(RCYCLE)209,209,210                                      708  Check for REFRACTION computation cycle.
  209         IF(RCC)212,212,210                                         709  Check if corrections to refraction coefficients
                                                                               have been computed.
  210         IF(PALPHA(1,1))212,212,211                                 710  Check for point type.  If type 2, compute
                                                                               Refraction correction coefficients.
C
C  FOR TYPE 2 POINTS IN REFRACTION CYCLE AND IN V-CYCLE AFTER COMPUTING
C    CORRECTIONS TO REFRACTION COEFFICIENTS, COMPUTE COEFFICIENTS FOR
C    TADS IN OBSERVATION EQUATIONS.
C
  211         TANZD = SQRT(ETA**2 + XSI**2)                              711  tan zr = (ξr^2 + ηr^2)^1/2
              ZN = ETA*(1./TANZD + TANZD)                                712  Zη = ηr [(1/tan zr) + tan zr]
              ZE = XSI*(1./TANZD + TANZD)                                713  Zξ = ξr [(1/tan zr) + tan zr]
              DXEX = -WC*(DX*ZE + EX*ZN)                                 714  DXEX = -T^1/2 W (Jx·Zξ + Kx·Zη)
              DYEY = -WC*(DY*ZE + EY*ZN)                                 715  DYEY = -T^1/2 W (Jy Zξ + Ky·Zη)
      BO(1,23) = DXEX*TANB                                               716  Bo_τ1   Vx = DXEX·tan β
      BO(2,23) = DYEY*TANB                                               717  Bo_τ1   Vy = DYEY·tan β
      BO(1,24) = DXEX*TANB**3                                            718  Bo_τ2   Wx = DXEX·tan^3 β
      BO(2,24) = DYEY*TANB**3                                            719  Bo_τ2   Wy = DYEY·tan^3 β
              BO(1,25) = DXEX*TANB**5                                    720  Bo_τ3   Xx = DXEX·tan^5 β
              BO(2,25) = DYEY*TANB**5                                    721  Bo_τ3   Xy = DYEY·tan^5 β
              BO(1,26) = DXEX*TANB**7                                    722  Bo_τ4   Yx = DXEX·tan^7 β
              BO(2,26) = DYEY*TANB**7                                    723  Bo_τ4   Yy = DYEY·tan^7 β
C
C  COMPUTE COEFFICIENTS FOR ALPHA AND DELTA AND FORM BX MATRIX.
C
  212 AA = SINDEL/COSZD                                                  724  AA = sin δr / cos zr
      BB = COSPHI/COSZD**2 - XSI*AA                                      725  BB = (cos φ / cos^2 zr) - ξ·AA
      CC = COSHPR/COSZD**2                                               726  CC = (cos H'/cos^2 zr)
      DD = SINPHI*SINHPR/COSZD**2                                        727  DD = sin φ sin H'/cos^2 zr
      BX(1,1) = DX*ETA*AA + EX*BB                                        728  Bx_α   Jx_α = Jx·η·AA + Kx·BB
      BX(1,2) = DX*CC + EX*DD                                            729  Bx_δ   Kx_δ = Jx·CC + Kx·DD
      BX(2,1) = DY*ETA*AA + EY*BB                                        730  Bx_α   Jy_α = Jy·η·AA + Ky·BB
      BX(2,2) = DY*CC + EY*DD                                            731  Bx_δ   Ky_δ = Jy·CC + Ky·DD
C
C  FORM AND ACCUMULATE NORMAL EQUATIONS.
C
      DO 2122 I=1,2                                                      733
      DO 2121 J=1,26                                                     734
```

Figure 20.--Continued (page 17 of 57)

```
2121      BOAUX(J,I) = 0.                                          735 Clear arrays  B_o^T P_l (26x2), B_x^T P_l (2x2).
        DO 2122 J=1,2                                              736
2122      BXAUX(I,J) = 0.                                          737
      DO 213 I=1,2                                                 738
        DO 213 J=1,2                                               739
          DO 213 K=1,2                                             740
 213        BXAUX(I,K) = BXAUX(I,K) + BX(J,I)*PL(J,K)              741 [B_x^T P_l]_i
      DO 216 I=1,2                                                 742
        DO 216 J=1,2                                               743
          DO 214 K=1,2                                             744
 214        BXTPBX(I,K) = BXTPBX(I,K) + BXAUX(I,J)*BX(J,K)         745 [B_x^T P_l B_x]_i ---> [Σ B_x^T P_l B_x]_j
      DO 215 L=1,26                                                746
 215        TWO(I,L) = TWO(I,L) + BXAUX(I,J)*BO(J,L)               747 (t) = [B_x^T P_l B_o]_i ---> [Σ B_x^T P_l B_o]_j
 216      EIGHT(I) = EIGHT(I) + BXAUX(I,J)*DELL(J)                 748 (8) = [B_x^T P_l Δℓ]_i ---> [Σ B_x^T P_l Δℓ]_j
 217 DO 218 I=1,26                                                 749
        DO 218 J=1,2                                               750
          DO 218 K=1,2                                             751
 218        BOAUX(I,K) = BOAUX(I,K) + BO(J,I)*PL(J,K)              752 [B_o^T P_l]_i
      DO 220 I=1,26                                                753
        DO 220 J=1,2                                               754
      DO 219 K=1,26                                                755
 219        RN(I,K) = RN(I,K) + BOAUX(I,J)*BO(J,K)                 756 N = [B_o^T P_l B_o]_i ---> Σ B_o^T P_l B_o
 220      CN(I) = CN(I) + BOAUX(I,J)*DELL(J)                       757 ℓ = [B_o^T P_l Δℓ]_i ---> Σ B_o^T P_l Δℓ
      IF(ACYCLE)221,221,234                                        758 Check for AZIMUTH-ELEVATION computation cycle.
 221  MPTS = MPTS + 1                                              759 Add 1 to observation count.
C
C  IF NOT LAST OBSERVATION OF STAR J, RETURN TO READ PLATE COORDINATES
C    FOR NEXT OBSERVATION.
C
      IF(NPTS-MPTS)222,222,184                                     760 Check for last observation of star.
C
C  WHEN ALL OBSERVATIONS OF STAR J HAVE BEEN ACCUMULATED, ADD ALPHA
C    -DELTA WEIGHT MATRIX TO ACCUMULATED BX MATRIX AND INVERT.
C
 222  DO 223 I=1,2                                                 762
        DO 223 J=1,2                                               763
 223      BXTPBX(I,J) = BXTPBX(I,J) + PALPHA(I,J)                  764 [Σ B_x^T P_l B_x]_j + P_αδ
      DETB = BXTPBX(1,1)*BXTPBX(2,2) - BXTPBX(1,2)*BXTPBX(2,1)     765
      IF(ABS(DETB)-1.E-300)224,224,225                             766
 224  PRINT 1104,NOST                                              767 If singular matrix, print error message.
      GO TO 384                                                    768
 225  ONE(1,1) = BXTPBX(2,2)/DETB                                  769
      ONE(1,2) = -BXTPBX(1,2)/DETB                                 770
      ONE(2,1) = -BXTPBX(2,1)/DETB                                 771
      ONE(2,2) = BXTPBX(1,1)/DETB                                  772 (1) = {[Σ B_x^T P_l B_x]_j + P_αδ}^-1
      NUSED = NUSED + 1                                            773 Add 1 to point count.
      IF(RCYCLE)226,226,227                                        774 Check for REFRACTION computation cycle.
C
C  IF PROBLEM TOLERANCE HAS BEEN MET (IN V-CYCLE), GO TO COMPUTE VS FOR
C    ALPHA AND DELTA.
C
 226  IF(TOLMET)227,227,294                                        775 Check if problem tolerance has been met.
C
C  IF PROBLEM TOLERANCE HAS NOT BEEN MET OR IN REFRACTION CYCLE, REDUCE
```

Figure 20.--Continued (page 18 of 57)

```
C    NORMAL EQUATION SYSTEM BY ELIMINATING STAR J.
C
  227 DO 228 I=1,26                                                  776
        DO 228 J=1,2                                                 777
          DO 228 K=1,2                                               778
  228       FOUR(I,K) = FOUR(I,K) + TWO(J,I)*ONE(J,K)                779  (4) = (2)^T·(1)
      DO 230 I=1,26                                                  780
        DO 230 J=1,2                                                 781
      DO 229 K=1,26                                                  782
  229       RN(I,K) = RN(I,K) - FOUR(I,J)*TWO(J,K)                   783  (4)·(2) → N
  230     CN(I) = CN(I) - FOUR(I,J)*EIGHT(J)                         784  (4)·(8) → ℓ
      MPTS = 0                                                       785  Clear observation counter.
      REWIND 2                                                       786  Rewind point data storage tape or disk.
C
C  IF NOT LAST STAR AND PROBLEM IS A MAXIMUM SOLUTION, GO TO READ DATA
C    AND FIRST OBSERVATION FOR NEXT STAR.
C
      IF(LASTPT)233,231,233                                          787  Check for last star in problem.
C
C  IF NOT LAST STAR AND PROBLEM IS A MINIMUM SOLUTION , CHECK DEGREES OF
C    FREEDOM FOR UNIQUE CASE.  WHEN MET, GO TO ADD PO TO NORMALS.
C    BY-PASS AZIMUTH-ELEVATION COMPUTATION.
C
  231 IF(MINSOL)182,182,232                                          788  Check for MINIMUM solution.
  232 IF(NUSED-IND(6))182,2331,2331                                  789  Check if all points used,as set for problem.
C
C  COMPUTE DEGREES OF FREEDOM FOR PROBLEM.
C
 2331 N = NOM + NL + NX                                              791  n = n_o + n_ℓ + n_x
      NU=NUOPRE+NUX                                                  792  u = u_opre + u_x
      GO TO 234                                                      793
  233 N = NOMX + NL + NX                                             794  n = n_o + n_ℓ + n_x
      NU = NUO + NUX                                                 795  u = u_o + u_x
  234 IF(N-NU)235,240,239                                            796  Check for unique case.
C
C  NOT SUFFICIENT DATA, GO TO CHECK FOR ADDITIONAL AZIMUTH ELEV. DATA.
C
  235 IF(ACYCLE)236,236,238                                          798  Check for AZIMUTH-ELEVATION computation cycle.
  236 IF(MINSOL)237,237,238                                          799  Check for MINIMUM solution.
  237 IF(IND(2))244,238,244                                          800  Check if AZIMUTH-ELEVATION data given.
  238 PRINT 1105                                                     801  Print heading for AZIMUTH-ELEVATION data.
      GO TO 384                                                      802  Go to compute standard coordinates for
                                                                          AZIMUTH-ELEVATION data.
C
C  ADEQUATE AMOUNT OF DATA FOR SOLUTION.
C
  239 DOF = N - NU                                                   804  Compute degrees of freedom.
      NDF = DOF + 1.E-10                                             805
      SUML = SQRT(DELLSQ/DOF)                                        806  Σℓ = [Σ(δℓ_x² + δℓ_y²)/n-u]^½
      GO TO 241                                                      807
C
C  UNIQUE SOLUTION, SET DEGREES OF FREEDOM EQUAL TO ONE.
C
  240 SUML = SQRT(DELLSQ)                                            808  Σℓ = [Σ(δℓ_x² + δℓ_y²)]^½
  241 IF(ACYCLE)242,242,246                                          809  Check for AZIMUTH-ELEVATION cycle.
```

Figure 20.--Continued (page 19 of 57)

```
  242 IF(MINSOL)243,243,246                                  810 Check if MINIMUM solution.
  243 IF(NO(2))244,246,244                                   811 Check if AZIMUTH-ELEVATION data given.
C
C  CAMERA AZIMUTH AND ELEVATION GIVEN, START A CYCLE.
C
  244       ELX = FLX                                        813 ℓx_a = ℓx_f
            ELY = FLY                                        814 ℓy_a = ℓy_f
            ELXSST = ELX + ELY*(PARI(10)                     815 ℓ̃x_a = ℓx_a + ℓy_a ε
            ELYSST = ELY                                     816
            DO 245 I=1,2                                     817
              DO 245 J=1,2                                   818
  245         PL(I,J) = FPL(I,J)                             819 P_ℓ = P_ℓf
            DELROI = 0.                                      820
            DTSINP = 0.                                      821
            DTCOSP = 0.                                      822 Clear distortion and set AZIMUTH-ELEVATION
            ACYCLE = 1.                                      823 cycle indicator.
            XSI = -COTE * COSA                               824 ξ_ae = - cot E cos A
            ETA = -COTE * SINA                               825 η_ae = - cot E sin A
            NL = NL + 2                                      826 Add 2 to plate observation count.
            GO TO 198                                        827
  246 IF(ITCNT)262,247,262                                   828 Check for first iteration.
C
C  IF FIRST ITERATION, GO TO SOLVE NORMALS.
C
  247 NDF = N - NU                                           830 Compute degrees of freedom.
      PASUML = SUML                                          831 Σℓ(a++) → Σℓa
  248 IF(MINSOL)249,249,250                                  832 Check for MINIMUM solution
  249 PRINT 1040                                             833 Print MAXIMUM SOLUTION.
      PRINT 1041,NOMX,NUO,NX,NUX,NL,N,NU,NDF,SUML            834 Print number of observations and unknowns for
                                                               orientation parameters, stars, and plate.
      GO TO 251                                              835 Print degrees of freedom and level of
                                                               residuals.
  250 PRINT 1039                                             836 Print MINIMUM SOLUTION.
      PRINT 1041,NOM,NUOPKE,NX,NUX,NL,N,NU,NDF,SUML          837 Print number of observations and unknowns for
                                                               orientation parameters, stars, and plate.
                                                               Print degrees of freedom and level of
                                                               residuals.
C
C    ADD ORIENTATION WEIGHT MATRIX TO NORMALS.
C
  251 DO 254 I=1,26                                          839
        DO 252 J=1,26                                        840
          RN(I,J) = RN(I,J) + PU(I,J)                        841 P_o → N
  252 RNINV(I,J) = RN(I,J)                                   842
        IF(RN(I,I))254,253,254                               843 Check for zero on diagonal.
  253 RNINV(I,I) = 1.E+20                                    844 If zero, change to large number.
  254   CONTINUE                                             845
C
C  INVERT NORMAL EQUATION MATRIX.
C
      DET = 1.0                                              846
      DO 2546 K=1,26                                         847
      ELMT = RNINV(K,K)                                      848
      DET = DET*ELMT                                         849
      IF(ELMT)2542,2541,2542                                 850
 2541 PRINT 1106                                             851 If matrix is singular, print error message
                                                               and go to terminate problem.
      GO TO 384                                              852
 2542 RNINV(K,K) = 1.                                        853
      DO 2543 J=1,26                                         854
```

Figure 20.--Continued (page 20 of 57)

```
 2543 RNINV(K,J) = RNINV(K,J)/ELMT                                  855
      DO 2546 I=1,26                                                856
      IF(I-K)2544,2546,2544                                         857
 2544 ELMT = RNINV(I,K)                                             858
      RNINV(I,K) = 0.                                               859
      DO 2545 J=1,26                                                860
 2545 RNINV(I,J) = RNINV(I,J)-ELMT*RNINV(K,J)                       861
 2546 CONTINUE                                                      862
C
C  ADJUST DISCREPANCY VECTOR FOR DIFFERENCES BETWEEN APPROXIMATIONS AND
C  WEIGHTED GIVEN PARAMETERS.
C
  256 DO 257 I=1,26                                                 863
      DO 257 J=1,26                                                 864
  257     CN(I) = CN(I) - PO(I,J)*DELLO(J)                          865  -P_0 Δℓ_0 → ℓ
C
C  COMPUTE CORRECTION VECTOR.
C
 2570 DO 259 I=1,26                                                 866
      DO 258 J=1,26                                                 867
  258     DELO(I) = DELO(I) + RNINV(I,J)*CN(J)                      868  ΔO = N^-1 ℓ
  259   DELLO(I) = DELLO(I) + DELO(I)                               869  ΔO → Δℓ_0
      DO 2941 I=1,26                                                870
        SAVECN(I) = CN(I)                                           871
 2941  SAVEDL(I) = DELO(I)                                          872
      DO 2591 I=1,22                                                873
C
C  ADD CORRECTION VECTOR TO GIVEN ORIENTATION PARAMETERS.
C
 2591   OPAR(I) = OPAR(I) + DELO(I)                                 874  O_a^* = O_{a-1}^* + ΔO
      IF(OPAR(22).GT.(2.*C10))OPAR(22) = OPAR(22)-2.*C10            875  Restrict φ_r to ± 2π.
      IF(MINSOL.EQ.0) GO TO 2593                                    876  Check for MINIMUM solution.
      DO 2592 I=1,3                                                 877
        OPAR(I) = OPAR (I+6)                                        878
 2592   OPAR(I+3) = OPAR(I)                                         879
 2593 ITCNT = ITCNT + 1                                             880  Add 1 to iteration count.
      IF(INO(1))260,261,260                                         881  If c_x = c_y, set corrected c_y equal to c_x.
  260 OPAR(14) = OPAR(13)                                           882
  261 REWIND 1                                                      883  Rewind observation data tape.
C
C  GO TO START NEXT ITERATION.
C
      GO TO 1792                                                    884
C
C  IF REFRACTION CYCLE, PRINT STATISTICS AND GO TO START V-CYCLE.
C
  262 IF(RCYCLE)264,264,263                                         886  Check for REFRACTION computation cycle.
  263           RCC = 1.                                            887  Set "Refraction coefficient corrections
                                                                         computed" indicator.
                PRINT 1041,NOMX,NUO,NX,NUX,NL,N,NU,NOF,SUML         888  Print number of observations and unknowns for
                                                                         orientation parameters, stars, and plate.
                PRINT 1050                                          889  Print degrees of freedom and level of
                                                                         residuals.
                LINCNT = 1                                          890  Print heading STAR AND PLATE COORDINATE DATA.
                GO TO 251                                           891  Set line counter and go to start V-cycle.
C
C  CHECK FOR ITERATION STOP.
```

Figure 20.--Continued (page 21 of 57)

```
C
  264 PRINT 1045,SUML                                   892 Print level of residuals for current iteration
      DSUML = PRSUML - SUML                             893  ΔΣℓ = Σℓ_a - Σℓ_(a+1)
      PRSUML = SUML                                     894
      IF(DSUML)265,271,271                              896 Check for divergence.
  265 NDIV = NDIV + 1                                   897 Add 1 to divergence count.
      IF(NDIV-4)271,266,266                             898 Check for 4 divergent steps.
  266 PRINT 1047                                        899 Print FOUR DIVERGENT STEPS OCCURRED.
      IF(MINSOL)268,268,284                             900 Check for MINIMUM solution.
  268 IF(IND(5))269,286,269                             901 Check for REFRACTION solution.
  269 IND(5) = 0                                        902 Clear REFRACTION solution indicator.
      PRINT 1049                                        903 Print REFRACTION SOLUTION ELIMINATED FROM
      GO TO 286                                             PROBLEM.
  271 IF(ABS(DSUML)-TOLP)272,272,274                    904
  272 PRINT 1048                                        905 Check if problem tolerance met.
      IF(MINSOL)279,279,284                             906 Print ITERATION RESULTS DIFFER BY LESS THAN
  274 IF(MINSOL)277,277,275                                 TOLERANCE SETTING.
  275 IF(ITMIN-ITCNT)276,276,251                        907 Check for MINIMUM solution.
  276 PRINT 1046,ITMIN                                  908 Check for MINIMUM solution.
      GO TO 284                                         909 Check number of iterations against number set.
  277 IF(ITMAX-ITCNT)278,278,251                        910 Print XX ITERATIONS COMPLETED.
  278 PRINT 1046,ITMAX                                  911
  279 IF(IND(5))289,286,289                             912 Check number of iterations against number set.
C                                                       913 Print XX ITERATIONS COMPLETED.
C  ITERATIONS STOPPED, MINIMUM SOLUTION, STORE DET. AND RETURN TO START   914 Check for REFRACTION solution.
C    MAXIMUM SOLUTION WITH NEW APPROXIMATIONS TO ORIENTATION PARAMETERS.
C
  284 DETMIN = DET                                      916
      IF(IND(7))2861,285,2861                           917 Check for PRE-RUN.
  285 MINSOL = 0                                        918 Clear MINIMUM SOLUTION indicator.
      ITCNT = 0                                         919 Clear iteration counter.
      NDIV = 0                                          920 Clear divergence counter.
      PRSUML = 0.                                       921 Clear residual level.
      GO TO 261                                         922 Go to start MAXIMUM SOLUTION.
C
C  ITERATIONS STOPPED, MAXIMUM SOLUTION, NO REFRACTION COMPUTATION,
C    RETURN TO START V-CYCLE.
C
  286 DETMAX = DET                                      923
 2861 TOLMET = 1.                                       924 Set tolerance met indicator.
      PRINT 1050                                        925 Print heading STAR AND PLATE COORDINATE DATA.
      LINCNT = 1                                        926 Set line counter.
      GO TO 2931                                        927 Go to start V-cycle.
C
C  REFRACTION CYCLE COMPLETE, ADD CORRECTIONS TO TAUS AND GO TO START
C    V-CYCLE.
C
  287           DETMAX = DET                            928
                DO 288 I=23,26                          929
  288             DPAR(I) = DPAR(I) + DELO(I)           930  τ_i = τ_i^0 + Δτ_i
                RCYCLE = 0.                             931 Clear REFRACTION cycle indicator.
                GO TO 180                               932 Go to start V-cycle.
C
C  ITERATIONS STOPPED, MAXIMUM SOLUTION WITH REFRACTION CORRECTIONS,
```

Figure 20.--Continued (page 22 of 57)

```
C    GO TO START REFRACTION CYCLE.
C
  289 TOLMET = 1.                                          934 Set tolerance met indicator.
      PRINT 1112                                           935 Print REFRACTION SOLUTION.
  290           RCYCLE = 1.                                936 Set REFRACTION cycle indicator.
      DO 2901 I=1,26                                       937
        SAVECN(I) = CN(I)                                  938
 2901   SAVEOL(I) = DELO(I)                                939
                NUO = NUO + 4                              940
                DO 292 I=1,4                               941
                  IF(TPO(I,I))292,292,291                  942
  291             NORX = NORX + 1                          943 Add to observation count for each refraction
  292             CONTINUE                                 944 coefficient with finite weight.
C
C  MOVE REFRACTION COEFFICIENTS WEIGHTS TO PO MATRIX.
C
                DO 293 I=1,4                               945
                  DO 293 J=1,4                             946
  293             PO(I+22,J+22) = TPO(I,J.                 947
 2931           IF(IND(4))2932,2932,2933                   948 Set mean error cycle indicator if plate
                                                              coordinates have individual weights.
 2932           RCOMP = 1                                  949
                RPCOMP = 1                                 950
 2933           IF(IND(3))2934,2934,261                    951 Set mean error cycle indicator if star
                                                              coordinates have individual weights.
 2934           MCOMP = 1                                  952
                MSCOMP = 1                                 953
                GO TO 261                                  954 Go to start REFRACTION cycle.
C
C  START COMPUTATION OF RESIDUALS (V-CYCLE).
C
  294 VCYCLE = 1.                                          956 Set V-cycle indicator.
      NPTS = 0                                             957 Clear observation counter.
      DELX(1) = 0.                                         958 Clear star residual array.
      DELX(2) = 0.                                         959
      DO 295 I=1,2                                         960
C
C  COMPUTE DELTA-X VECTOR (CORRECTIONS OR VS FOR STARS).
C
        DO 295 J=1,2                                       961
  295     DELX(I) = DELX(I) + ONE(I,J)*EIGHT(J)            962 ΔX = (1)·(B)
      REWIND 2                                             963 Rewind plate data storage tape or disk.
      IF(LINCNT.LT.55.OR.SIGMAC.EQ.1.) GO TO 2961          964 If at bottom of page, print new heading.
  296 PRINT 1053                                           965
      LINCNT = 2                                           966
 2961 IF(MCOMP)297,297,301                                 967 Check for mean error computation cycle.
  297 IF(PALPHA(1,1))299,299,298                           968 Check if star coordinates are known or
                                                              unknown.
  298 RAST = DELX(1)/C3                                    969 Right ascension residual, radians to seconds.
      RASA = DELX(1)/C6                                    970 Right ascension residual, radians to sec (arc).
      DECSA = DELX(2)/C6                                   971 Declination residual, radians to seconds (arc).
      IF(MSCOMP)2982,2981,2982                             972 Check for individual weights on stars.
 2982 ALPHME = (ERRPR/SQRT(PALPHA(1,1)))/C6                973 m_α = m/(P_α)^½
      DECME = (ERRPR/SQRT(PALPHA(2,2)))/C6                 974 m_δ = m/(P_δ)^½
C
C  PRINT STAR DATA.
C
```

Figure 20.--Continued (page 23 of 57)

```
      PRINT 1113,NOST,PALPHA(1,1),PALPHA(2,2),PALPHA(1,2),ALPHME,DECME,
     1  RAST,RASA,DECSA
      LINCNT = LINCNT + 8
      GO TO 300
 2981 PRINT 1052,NOST,PALPHA(1,1),PALPHA(2,2),PALPHA(1,2),NCAT,RAST,RASA
     1,DECSA
      LINCNT = LINCNT + 8
      GO TO 300
  299 IF(RCC)2983,2990,2983
 2983           TANZO = SQRT(ETA**2 + XSI**2)
                ZN = ETA*(1./TANZO + TANZO)
                ZE = XSI*(1./TANZO + TANZO)
                DXEX = -WC*(DX*ZE + EX*ZN)
                DYEY = -WC*(DY*ZE + EY*ZN)
                BO(1,23) = DXEX*TANB
                BO(2,23) = DYEY*TANB
                BO(1,24) = DXEX*TANB**3
                BO(2,24) = DYEY*TANB**3
                BO(1,25) = DXEX*TANB**5
                BO(2,25) = DYEY*TANB**5
                BO(1,26) = DXEX*TANB**7
                BO(2,26) = DYEY*TANB**7
                DO 2984 I=1,2
                  DO 2984 J=23,26
                    DO 2984 K=1,2
 2984                 TWO(I,J) = TWO(I,J) + BXAUX(I,K)*BO(K,J)
C
C  CORRECT APPROXIMATIONS TO COORDINATES FOR UNKNOWN STAR.
C
 2990 ALSTAR = ALSTAR + DELX(1)
      DELTA = DELTA + DELX(2)
      ALSIG = ALSTAR
      DESIG = DELTA
      DO 2991 I=1,2
      DO 2991 J=1,26
          BOAUX(J,I) = 0.
C
C  COMPUTE CO-VARIANCE MATRIX FOR UNKNOWN STAR.
C
          DO 2991 K=1,2
 2991       BOAUX(J,I) = BOAUX(J,I) + DME(I,K)*TWO(K,J)
      DO 2993 I=1,2
      DO 2992 J=1,26
          TEM(J) = 0
      DO 2992 K=1,26
 2992       TEM(J) = TEM(J) + BOAUX(K,I)*RNINV(K,J)
        DO 2993 J=1,2
          BXAUX(I,J) = 0.
      DO 2993 K=1,26
 2993       BXAUX(I,J) = BXAUX(I,J) + TEM(K)*TWO(J,K)
      DO 2994 I=1,2
        DO 2994 J=1,2
C
C  COMPUTE MEAN ERRORS AND ERROR ELLIPSES FOR UNKNOWN STAR.
```

Print star number, weight and mean error.

Add 8 to line counter.

Print star number, weight, catalog number and mean errors.

Add 8 to line counter.

Check if corrections have been computed for refraction coefficients

$\tan z_r = (\eta_r^2 + \xi_r^2)^{1/2}$

$Z_\eta = \eta_r \left[(1/\tan z_r) + \tan z_r\right]$

$Z_\xi = \xi_r \left[(1/\tan z_r) + \tan z_r\right]$

$DXEX = -T^{1/2} W (J_x \cdot Z_\xi + K_x Z_\eta)$

$DYEY = -T^{1/2} W (J_y \cdot Z_\xi + K_y Z_\eta)$

$B_{o_{\tau_1}} \quad V_x = DXEX \cdot \tan\beta$

$B_{o_{\tau_1}} \quad V_y = DYEY \cdot \tan\beta$

$B_{o_{\tau_2}} \quad W_x = DXEX \cdot \tan^3\beta$

$B_{o_{\tau_2}} \quad W_y = DYEY \cdot \tan^3\beta$

$B_{o_{\tau_3}} \quad X_x = DXEX \cdot \tan^5\beta$

$B_{o_{\tau_3}} \quad X_y = DYEY \cdot \tan^5\beta$

$B_{o_{\tau_4}} \quad Y_x = DXEX \cdot \tan^7\beta$

$B_{o_{\tau_4}} \quad Y_y = DYEY \cdot \tan^7\beta$

$(2) = B_x^T P_l B_o$

$\alpha^{oo} = \alpha^o + V_\alpha$

$\delta^{oo} = \delta + V_\delta$

$[(1)]\,[(2)]$

$[(1)]\,[(2)]\,N^{-1}$

$[(1)]\,[(2)]\,N^{-1}[(2)]$

Figure 20.--Continued (page 24 of 57)

```
C
      QALDEL(I,J) = 0.
      DO 2994 K=1,2
 2994   QALDEL(I,J) = QALDEL(I,J) + BKAUX(I,K)*ONE(K,J)
      IF(SIGMAC.EQ.1.) GO TO 29940
      READ (4F V18
29940 ERRB = ERRPR
      IF(NPTS.GT.1) ERRB = SQRT(V19/(2.*FLOAT(NPTS)-2.))
      DO 3016 I=1,2
        DO 3016 J=1,2
 3016 QTOTAL(I,J) = QALDEL(I,J)*ERRPR**2+ONE(I,J)*ERRB**2
 3084 TOTALP = (SQRT(QTOTAL(1,1))/C6*COS(DELTA)
      TOTDEC = (SQRT(QTOTAL(2,2))/C6
      SAM2=0.
      QTOTAL(1,1)=QTOTAL(1,1)*COS(DELTA)**2
      QTOTAL(1,2)=QTOTAL(1,2)*COS(DELTA)
      IF(QTOTAL(1,1)-QTOTAL(2,2))3019,3019,3017
 3017 IF(QTOTAL(1,2))3026,3024,3028
 3018 IF(QTOTAL(1,2))3020,3023,3021
 3019 IF(QTOTAL(1,2))3027,3024,3027
 3020 ROTAT=-.78539816339745
      Q=-2.*QTOTAL(1,2)
      GO TO 3030
 3021 ROTAT=.78539816339745
 3022 Q=2.*QTOTAL(1,2)
      GO TO 3030
 3023 SAM2=1.
      GO TO 3025
 3024 ROTAT=0.
 3025 Q=0.
      GO TO 3030
 3026 TAN2TH=2.*QTOTAL(1,2)/(QTOTAL(1,1)-QTOTAL(2,2))
      ROTAT=(ATAN(TAN2TH)+6.2831853071796)/2.
      GO TO 3029
 3027 TAN2TH=2.*QTOTAL(1,2)/(QTOTAL(1,1)-QTOTAL(2,2))
      ROTAT=(ATAN(TAN2TH)+3.1415926535898)/2.
      GO TO 3029
 3028 TAN2TH=2.*QTOTAL(1,2)/(QTOTAL(1,1)-QTOTAL(2,2))
      ROTAT=ATAN(TAN2TH)/2.
 3029 Q=2.*QTOTAL(1,2)/(SIN(ROTAT*2.))
 3030 AAX = (SQRT((QTOTAL(1,1)+QTOTAL(2,2)+Q)/2.))/C6
      BAX = (SQRT((QTOTAL(1,1)+QTOTAL(2,2)-Q)/2.))/C6
      ROTAT=ROTAT/C4
 2995 ALPHME = (ERRPR * SQRT(QALDEL(1,1)))/C6 * COS(DELTA)
      DECME = (ERRPR * SQRT(QALDEL(2,2)))/C6
      SAM = 0.
      QALDEL(1,1)=QALDEL(1,1)*COS(DELTA)**2
      QALDEL(1,2)=QALDEL(1,2)*COS(DELTA)
      IF(QALDEL(1,1)-QALDEL(2,2))3007,2997,2996
 2996 IF(QALDEL(1,2))3009,3004,3005
 2997 IF(QALDEL(1,2))2998,3011,2999
 3007 IF(QALDEL(1,2))3010,3004,3010
 2998 ROTATE=-.78539816339745
      Q=-2.*QALDEL(1,2)
```

$[\alpha][\alpha]^T N^{-1}[\alpha][\alpha] = Q_X^\circ$

If in sigma cycle, return to continue computing space angles.

$m^2 Q_X^r = m^2 Q_X^\circ + m_s^2(I)$

$m_{X_a}^r = (m^2 Q_{X_{(1,1)}}^r)^{1/2}$

$m_{X_b}^r = (m^2 Q_{X_{(2,2)}}^r)^{1/2}$

$\theta = -\pi/2$

$Q = -2(m^2 Q_{X_{(1,2)}}^r)$

$\theta = \pi/2$

$Q = 2(m^2 Q_{X_{(1,2)}}^r)$

$\theta = 0.0$

$Q = 0.0$

$\tan 2\theta = 2(m^2 Q_{X_{(1,2)}}^r)/(m^2 Q_{X_{(1,1)}}^r - m^2 Q_{X_{(2,2)}}^r)$

$\theta = (\tan^{-1} 2\theta + 2\pi)/2$

$\theta = (\tan^{-1} 2\theta + \pi)/2$

$\theta = \tan^{-1}\theta/2$

$Q = 2(m^2 Q_{X_{(1,2)}}^r)/\sin 2\theta$

$a^r = \left[(m^2 Q_{X_{(1,1)}}^r + m^2 Q_{X_{(2,2)}}^r + Q)/2\right]^{1/2}$

$b^r = \left[(m^2 Q_{X_{(1,1)}}^r + m^2 Q_{X_{(2,2)}}^r - Q)/2\right]^{1/2}$

$m_{X_a}^\circ = m(Q_{X_{(1,1)}}^\circ)^{1/2}$

$m_{X_b}^\circ = m(Q_{X_{(2,2)}}^\circ)^{1/2}$

$\theta = -\pi/2$

$Q = -2\, Q_{X_{(1,2)}}^\circ$

Figure 20.--Continued (page 25 of 57)

```
      GO TO 3006                                                        1074
 2999 ROTATE= .785398163397945                                          1075   θ = π/2
 3003 Q=2.*QALDEL(1,2)                                                  1076   Q = 2 Q*X(1,2)
      GO TO 3006                                                        1077
 3011 SAM=1.                                                            1078
      GO TO 3012                                                        1079
 3004 ROTATE=0.                                                         1080   θ = 0.0
 3012 Q= 0.                                                             1081   Q = 0.0
      GO TO 3006                                                        1082
 3009 TAN2TH=2.*QALDEL(1,2)/(QALDEL(1,1)-QALDEL(2,2))                   1083   tan 2θ = 2 Q*X(1,2) / (Q*X(1,1) - Q*X(2,2))
      ROTATE = (ATAN(TAN2TH)+6.283185307179612/2.                       1084   θ = (tan⁻¹2θ + 2π)/2
      GO TO 3008                                                        1085
 3010 TAN2TH=2.*QALDEL(1,2)/(QALDEL(1,1)-QALDEL(2,2))                   1086
      ROTATE = (ATAN(TAN2TH)+3.141592653589793/2.                       1087   θ = (tan⁻¹2θ + π)/2
      GO TO 3008                                                        1088
 3005 TAN2TH=2.*QALDEL(1,2)/(QALDEL(1,1)-QALDEL(2,2))                   1089
      ROTATE=ATAN(TAN2TH)/2.                                            1090   θ = tan⁻¹2θ/2
 3008 Q=2.*QALDEL(1,2)/(SIN(ROTATE*2.))                                 1091   Q = 2 Q*X(1,2) / sin 2θ
 3006 AAXIS=(ERRPR*SQRT((QALDEL(1,1)+QALDEL(2,2)+Q)/2.))/C6             1092   a* = m [(Q*X(1,1) + Q*X(2,2) + Q)/2]^½
      BAXIS=(ERRPR*SQRT((QALDEL(1,1)+QALDEL(2,2)-Q)/2.))/C6             1093   b* = m [(Q*X(1,1) + Q*X(2,2) - Q)/2]^½
      ROTANG=ROTATE/C4                                                  1094
      ALDEG = ALSTAR/C4                                                 1095
      IDEG = ALDEG + .1E-14                                             1096
      ADEG = (ALDEG-FLOAT(IDEG))*60.                                    1097
      IAMIN = ADEG + .1E-14                                             1098
      ALDEG = (ADEG-FLOAT(IAMIN))*60.                                   1099
      ALSTAR = ALSTAR/(C4*15.)                                          1100
      NHOUR = ALSTAR + .1E-14                                           1101
      ADEG = (ALSTAR-FLOAT(NHOUR))*60.                                  1102
      NAMIN = ADEG + .1E-14                                             1103
      ALSTAR = (ADEG-FLOAT(NAMIN))*60.                                  1104
      DELTA = DELTA/C4                                                  1105
      NDEG = DELTA + .1E-14                                             1106
      ADEG = (DELTA-FLOAT(NDEG))*60.                                    1107
      NOMIN = ADEG+.1E-14                                               1108
      DELTA = (ADEG-FLOAT(NOMIN))*60.                                   1109
C
C  IN SIGMA CYCLE, RETURN TO PRINT DATA.
C
      IF(SIGMAC.EQ.1.) GO TO 30000                                      1110  Check for sigma cycle.
      IF(SAM)3013,3013,3014                                             1111
 3013 IF(SAM2)3042,3042,3043                                            1112
 3014 IF(SAM2)3041,3041,3040                                            1113
C
C  PRINT STAR DATA FOR UNKNOWN STAR.
C
 3040 PRINT 1129,NOST,ALPHME,DECME,AAXIS,BAXIS,NHOUR,NAMIN,ALSTAR,      1114
     1 IDEG,IAMIN,ALDEG,NDEG,NOMIN,DELTA,TOTALP,TOTDEC,AAX,BAX          1115
      GO TO 3015                                                        1116
 3041 PRINT 1122,NOST,ALPHME,DECME,AAXIS,BAXIS,NHOUR,NAMIN,ALSTAR,      1117
     2 IDEG,IAMIN,ALDEG,NDEG,NOMIN,DELTA,TOTALP,TOTDEC,AAX,BAX,ROTAT    1118
      GO TO 3015                                                        1119
 3042 PRINT 1059,NOST,ALPHME,DECME,AAXIS,BAXIS,ROTANG,NHOUR,NAMIN,ALSTAR 1120
     1,IDEG, IAMIN,ALDEG,NDEG,NOMIN,DELTA,TOTALP,TOTDEC,AAX,BAX,ROTAT   1121
```

Figure 20.--Continued (page 26 of 57)

```
      GO TO 3015                                                        1122
 3043 PRINT 1130,NOST,ALPHME,DECME,AAXIS,BAXIS,ROTANG,NHOUR,NAMIN,ALSTAR  1123
     1,IDEG, IAMIN,ALDEG,NDEG,NOMIN,DELTA,TOTALP,TOTDEC,AAX,BAX          1124
 3015 LINCNT=LINCNT+11                                                   1125  Add 11 to line counter.
  300 IF(MPCOMP)3002,3002,3001                                           1126
 3001 PRINT 1114                                                         1127
      LINCNT = LINCNT + 2                                                1128  Add 2 to line counter.
      GO TO 301                                                          1129
 3002 PRINT 1055                                                         1130
      LINCNT = LINCNT + 3                                                1131  Add 3 to line counter.
      GO TO 301                                                          1132
30000 AZ = AZ/C4                                                         1133
      MDEG = ELEV + .1E-14                                               1134
      ADEG = (ELEV-FLOAT(MDEG))*60.                                      1135
      MMIN = ADEG+.1E-14                                                 1136
      ASEC = (ADEG-FLOAT(MMIN))*60.                                      1137
      MMDEG = AZ + .1E-14                                                1138
      ADEG = (AZ-FLOAT(MMDEG))*60.                                       1139
      MMMIN = ADEG + .1E-14                                              1140
      AASEC= (ADEG-FLOAT(MMMIN))*60.                                     1141
      IF(NPP.EQ.1) PRINT 1032,NHOUR,NAMIN,ALSTAR,IDEG,IAMIN,ALDEG,NDEG,N 1142  For unknown star, Print computed coordinates
                                                                               and associated statistics.
     1OMIN,DELTA,MMDEG,MMMIN,AASEC,MDEG,MMIN,ASEC,ALPHME,DECME,AAXIS,BAX 1143
     2IS,ROTANG,TOTALP,TOTDEC,AAX,BAX,ROTAT                              1144  PRE Star.
      IF(NPP.EQ.2) PRINT 1042,NHOUR,NAMIN,ALSTAR,IDEG,IAMIN,ALDEG,NDEG,N 1145
     1OMIN,DELTA,MMDEG,MMMIN,AASEC,MDEG,MMIN,ASEC,ALPHME,DECME,AAXIS,BAX 1146  POST Star.
     2IS,ROTANG,TOTALP,TOTDEC,AAX,BAX,ROTAT                              1147
      IF(NPP.EQ.3) PRINT 1043,NHOUR,NAMIN,ALSTAR,IDEG,IAMIN,ALDEG,NDEG,N 1148
     1OMIN,DELTA,MMDEG,MMMIN,AASEC,MDEG,MMIN,ASEC,ALPHME,DECME,AAXIS,BAX 1149  EVENT Star.
     2IS,ROTANG,TOTALP,TOTDEC,AAX,BAX,ROTAT                              1150
      LINCNT = LINCNT + 2                                                1151  Add 2 to line counter.
C
C  READ STORED DATA FOR ONE OBSERVATION OF STAR.
C
  301 READ (2) ELX,ELY,((PL(I,J),J=1,2),I=1,2),NOPT,ELKSST,ELYSST,DTS(NP 1152
     1,DTCOSP,HBAR,DELPRI,DELROT,NPP,STIME                               1153
C
C  CORRECT APPROXIMATIONS TO STAR COORDINATES AND GO TO COMPUTE
C    DISCREPANCY VECTOR.
C
      HBAR = HBAR - DELX(1)                                              1154  $H_r = H_r - \Delta\alpha$
      DELPRI = DELPRI + DELX(2)                                          1155  $\delta_r = \delta_r + \Delta\delta$
      SINDEL = SIN(DELPRI)                                               1156
      COSDEL = COS(DELPRI)                                               1157
      GO TO 197                                                          1158
C
C  COMPUTE PVV SUM FOR OBSERVATION I AND ADD TO TOTAL SUM OF ALL PVV.
C
  302 DELLPL(1) = -DELL(1)*PL(1,1) - DELL(2)*PL(2,1)                     1159
      DELLPL(2) = -DELL(1)*PL(1,2) - DELL(2)*PL(2,2)                     1160
      IF(SIGMAC.EQ.1.) GO TO 3117                                        1161  Check for sigma cycle.
      IF(PALPHA(2,2))3037,3037,3038                                      1162  Check for unknown star.
 3037 V18 = V18-DELLPL(1)*DELL(1)-DELLPL(2)*DELL(2)                      1163  $V_1^T P_2 V_2 \rightarrow V18$
      MPTS = MPTS + 1                                                    1164  Add 1 to point counter.
      GO TO 303                                                          1165
```

Figure 20.--Continued (page 27 of 57)

```
 3033 V1 = V1 - DELLPL(1)*DELL(1) - DELLPL(2)*DELL(2)                    1166  u_i^T P_i u_i → V1
      SUMV(1) = SUMV(1) - DELL(1)                                        1167
      SUMV(2) = SUMV(2) - DELL(2)                                        1168
      SUMVS(1) = SUMVS(1) + DELL(1)*DELL(1)                              1169
      SUMVS(2) = SUMVS(2) + DELL(2)*DELL(2)                              1170
      CT(1) = CT(1) + 1.                                                 1171
 3033 NPTS = NPTS + 1                                                    1172
      IF(DELL(1))3044,3044,3045                                          1173
 3044 POSVX=POSVX-DELL(1)                                                1174
      GO TO 3046                                                         1175
 3045 ANEGVX=ANEGVX-DELL(1)                                              1176
 3046 IF(DELL(2))3047,3047,3048                                          1177
 3047 POSVY=POSVY-DELL(2)                                                1178
      GO TO 303                                                          1179
 3048 ANEGVY=ANEGVY-DELL(2)                                              1180
  303 DELL(1) = -DELL(1)                                                 1181
      DELL(2) = -DELL(2)                                                 1182
      IF(MCOMP)3031,3031,3034                                            1183
 3031 ELXMM = ELX*.1E+4                                                  1184
      ELYMM = ELY*.1E+4                                                  1185
      VXMIC = DELL(1)*.1E+7                                              1186
      VYMIC = DELL(2)*.1E+7                                              1187
      STHRS = STIME/(C4*15.)                                             1188
      NSTH = STHRS + .1E-14                                              1189
      STMIN = (STHRS-FLOAT(NSTH))*60.                                    1190
      NSTM = STMIN+.1E-14                                                1191
      STSEC = (STMIN-FLOAT(NSTM))*60.                                    1192
      IF(MPCOMP)3032,3032,3033                                           1193
 3034 IF(NPTS-NPTS)301,311,311                                           1194
C
C  PRINT PLATE OBSERVATION DATA.
C
 3032 PRINT 1058,NOPT,TYPINPP1,ELXMM,ELYMM,PL(1,1),PL(2,2),PL(1,2),NSTH,  1195
     1NSTM,STSEC,VXMIC,VYMIC                                             1196
      LINCNT = LINCNT + 1                                                1197
      GO TO 3035                                                         1198
 3033 XME = (ERKPR/SQRT(PL(1,1)))*.1E+7                                  1199  m_{ℓx} = m/(P_{ℓx})^{1/2}
      YME = (ERKPR/SQRT(PL(2,2)))*.1E+7                                  1200  m_{ℓy} = m/(P_{ℓy})^{1/2}
      PRINT 1115,NOPT,TYPINPP1,NSTH,NSTM,STSEC,PL(1,1),PL(2,2),PL(1,2),  1201
     1 ELXMM,XME,VXMIC,ELYMM,YME,VYMIC                                   1202
      LINCNT = LINCNT + 3                                                1203
 3035 IF(PALPHA(I,1).EQ.0.)GO TO 304                                     1204
      I = 0                                                              1205
C
C  ADD OBSERVATION TO HISTOGRAM PLOT.
C
 30350 I = I+1                                                           1206
        XM = (DELL(I) - VM(I))/SOV(I)                                    1207
        MM = (NOC(I) + 1)/2                                              1208
        DO 30355 K=1,MM                                                  1209
          AA = 2*K-1                                                     1210
          AA = 3.25*AA/ANOC(I)                                           1211
          IF(XM.GE.0.) GO TO 30352                                       1212
          IF(AA+XM.GE.0.) GO TO 30351                                    1213
```

Figure 20.--Continued (page 28 of 57)

```
          IF(K.LT.MM) GO TO 30355                                  1214
30351     L1 = -K+MM+1                                             1215
          GO TO 30354                                              1216
30352     IF(AA-XH.GE.0.) GO TO 30353                              1217
          IF(K.LT.MM) GO TO 30355                                  1218
30353     L1 = K+MM-1                                              1219
30354     CLASS(I,L1) = CLASS(I,L1) + 1.                           1220
          GO TO 30356                                              1221
30355     CONTINUE                                                 1222
30356 IF(I.EQ.1) GO TO 30350                                       1223
      DO 30362 J= 1,2                                              1224
        XH = (DELL(J) -VM(3))/SDV(3)                               1225
        MM = (NOC(3)+1)/2                                          1226
        DO 30361 K=1,MM                                            1227
          AA = 2*K-1                                               1228
          AA = 3.25*AA/ANOC(3)                                     1229
          IF(XH.GE.0.) GO TO 30358                                 1230
          IF(AA+XH.GE.0.) GO TO 30357                              1231
          IF(K.LT.MM) GO TO 30361                                  1232
30357     L1 = -K+MM+1                                             1233
          GO TO 30360                                              1234
30358     IF(AA-XH.GE.0.) GO TO 30359                              1235
          IF(K.LT.MM) GO TO 30361                                  1236
30359     L1 = K+MM-1                                              1237
30360     CLASS(3,L1) = CLASS(3,L1) + 1.                           1238
          GO TO 30362                                              1239
30361     CONTINUE                                                 1240
30362 CONTINUE                                                     1241
      IF(IND(11))3036,304,3036                                     1242  If indicator 11 is one, punch plate residuals.
C
C  PUNCH PLATE RESIDUALS.
C
 3036 PUNCH 19,ELX,ELY,DELL(1),DELL(2),NOPT,NAMES(19)              1243
  304 IF(PCYCLE)305,305,308                                        1244
  305 IF(MPTS-NPTS)306,3060,3060                                   1245
  306 ELXSUM=ELXSUM+ELX                                            1246  $\Sigma l_x \rightarrow \Sigma l_x$
      ELYSUM=ELYSUM+ELY                                            1247  $\Sigma l_y \rightarrow \Sigma l_y$
      VLXSUM=VLXSUM+DELL(1)                                        1248  $\Sigma v_{l_x} \rightarrow \Sigma v_{l_x}$
      VLYSUM=VLYSUM+DELL(2)                                        1249  $\Sigma v_{l_y} \rightarrow \Sigma v_{l_y}$
      IF(LINCNT-69)301,301,307                                     1250  If bottom of page, go to print new heading.
  307 PRINT 1062                                                   1251
      PRINT 1053                                                   1252
      IF(MCOMP)3072,3072,3071                                      1253  Check for mean error computation cycle.
 3071 PRINT 1114                                                   1254
      GO TO 3073                                                   1255
 3072 PRINT 1055                                                   1256
 3073 LINCNT = 7                                                   1257  Set line counter to 7.
C
C  RETURN TO READ STORED DATA FOR NEXT OBSERVATION OF STAR J.
C
      GO TO 301                                                    1258
  308 MPTS = MPTS + 1                                              1259  Add one to observation count.
      IF( MPTS-NPTS)309,3081,3081                                  1260  Check for last observation of star.
 3081 MPTS = 0                                                     1261
```

Figure 20.--Continued (page 29 of 57)

```
C
C  IF NOT LAST STAR IN PROBLEM GO TO READ DATA FOR NEXT STAR.
C
      IF(LASTPT)320,309,320                                          1262  Check for last point in print line.
  309 IF(LINCNT-69)EAL,310,310                                       1263  Check for bottom of page.
  310 PRINT 1062                                                     1264  If new page, print heading.
      PRINT 1053                                                     1265
      IF(MCOMP)3102,3102,3101                                        1266
 3101 PRINT 1114                                                     1267
      GO TO 3103                                                     1268
 3102 PRINT 1055                                                     1269
 3103 LINCNT = 7                                                     1270  Set line counter to 7.
      GO TO 181                                                      1271
 3060 IF(NPTS-1)3074,3074,3061                                       1272
 3074 GO TO 311                                                      1273
 3061 ENPTS=NPTS                                                     1274
      ELXSUM=(ELXSUM+ELX)/ENPTS                                      1275  $\bar{\ell}_x = \Sigma \ell_x / n$
      ELYSUM=(ELYSUM+ELY)/ENPTS                                      1276  $\bar{\ell}_y = \Sigma \ell_y / n$
      VLXSUM=(VLXSUM+DELL(1))/ENPTS                                  1277  $\bar{v}_{\ell_x} = \Sigma v_{\ell_x} / n$
      VLYSUM=(VLYSUM+DELL(2))/ENPTS                                  1278  $\bar{v}_{\ell_y} = \Sigma v_{\ell_y} / n$
      IF(ELXSUM)3063,3062,3063                                       1279
 3062 SINSUM=0.                                                      1280
      IF(ELXSUM)3069,3067,3069                                       1281
 3063 IF(ELXSUM)3065,3064,3065                                       1282
 3064 COSSUM=0.                                                      1283
      GO TO 3066                                                     1284
 3065 COSSUM=ELXSUM/SQRT(ELXSUM**2+ELYSUM**2)                        1285
 3066 SINSUM=ELYSUM/SQRT(ELXSUM**2+ELYSUM**2)                        1286
      GO TO 3069                                                     1287
 3067 COSSUM=1.                                                      1288
      GO TO 3069                                                     1289
 3068 COSSUM=ELXSUM/ABS(ELXSUM)                                      1290
 3069 VLRAD=VLXSUM*COSSUM+VLYSUM*SINSUM                              1291  $v_r = \bar{v}_{\ell_x} \cos\beta + \bar{v}_{\ell_y} \sin\beta$
      VLTAN=VLYSUM*COSSUM-VLXSUM*SINSUM                              1292  $v_t = -\bar{v}_{\ell_x} \sin\beta + \bar{v}_{\ell_y} \cos\beta$
      PRINT 1132,NISF,ELXSUM,ELYSUM,VLXSUM,VLYSUM,VLRAD,VLTAN        1293  Print star number, average plate coordinates,
                                                                           and average of residuals.
      LINCNT = LINCNT+5                                              1294  Increment line counter.
      IF(CCYCLE.EQ.1.) GO TO 3116                                    1295
      IF(PALPHA(1,1).EQ.0.) GO TO 3115                               1296  Check for type 2 star.
C
C  PLOT AVERAGE PLATE POSITION FOR STAR J ON STAR PLOT.
C
      CALL GRF(-.1,-.1,.1,.1,0.,0.,.01,.01,2,2,1,1,ELXSUM,ELYSUM,    1297
     1 1,1,2H20,-1,3)                                                1298
      GO TO 3116                                                     1299
 3115 CALL GRF(-.1,-.1,.1,.1,0.,0.,.01,.01,2,2,1,1,ELXSUM,ELYSUM,    1300
     1 1,1,2H12,-1,0)                                                1301
 3116 ELYSUM=0.                                                      1302
      ELXSUM=0.                                                      1303
      VLXSUM=0.                                                      1304
      VLYSUM=0.                                                      1305
C
C  COMPUTE PVV SUMS FOR STARS AND ORIENTATION PARAMETERS.
C
  311 DELLPL(1) = DELX(1)*PALPHA(1,1) + DELX(2)*PALPHA(2,1)          1306
```

Figure 20.--Continued (page 30 of 57)

```
      DELLPL(2) = DELX(1)*PALPHA(1,2) + DELX(2)*PALPHA(2,2)
 3110 V2 = V2+DELLPL(1)*DELX(1)+DELLPL(2)*DELX(2)
      IF(MCOMP.EQ.0) GO TO 3117
      IF(PALPHA(1,1).NE.0.) GO TO 3117
      WRITE (4) V18
 3117 V18 = 0.
      REWIND 2
      VCYCLE = 0.
      NPTS = 0
      IF(IND(7))312,313,312
  312 IF(IND(6)-NUSED)3121,3121,313
 3121 IF(MCOMP)366,366,320
  313 IF(LASTPT)314,181,314
  314 IF(MINSOL)315,315,320
  315 IF(IND(2))316,320,316
  316 AERES = 1.
      GO TO 244
  317 DELLPL(1) = DELL(1)*PL(1,1) + DELL(2)*PL(2,1)
      DELLPL(2) = DELL(1)*PL(1,2) + DELL(2)*PL(2,2)
      V3 = V3 + DELLPL(1)*DELL(1) + DELLPL(2)*DELL(2)
      IF(MCOMP)3171,3171,320
 3171 NOPT = 0
  318 IF(LINCNT-61)3191,3191,319
  319 PRINT 1053
 3191 PRINT 1060
      DELL(1) = -DELL(1)
      DELL(2) = -DELL(2)
      ELXMM = ELX*.1E+4
      ELYMM = ELY*.1E+4
      VXMIC = DELL(1) *.1E+7
      VYMIC = DELL(2) *.1E+7
      PRINT 1171,ELXMM,ELYMM,PL(1,1),PL(2,2),PL(1,2),VXMIC,VYMIC
  320 IF(SIGMAC.EQ.1.)GO TO 32293
      DO 3201 I=1,26
 3201   TEM(I) = 0.
      DO 322 I=1,26
      DO 321 J=1,26
  321     TEM(I) = TEM(I) + DELLO(J)*PO(J,I)
  322   V3 = V3 + TEM (I)*DELLO(I)
      V4 = V1 + V2 + V3
      REWIND 4
      IF(NOF)3221,3221,3222
 3221 DF = 1.
      GO TO 3223
 3222 DF = NOF
 3223 ERRPR1 = SQRT(V4/DF)
      IF(TYPBME)3224,3224,3226
 3224 ERRPR = ERRPR1
 3226 IF(MCOMP)3228,3228,3227
 3227 MCOMP = 0
      V3 = 0.
      V4 = 0.
      V1 = 0.
      V2 = 0.
```

1307
1308 $(V_{ks}^T P_{ks} V_{ks})_j \rightarrow \Sigma V_{ks}^T P_{ks} V_{ks}$
1309
1310 Check for type 2 star.
1311
1312
1313 Rewind data storage tape.
1314 Clear V-cycle indicator.
1315 Clear point counter.
1316 Check for PRE-RUN.
1317 If PRE-RUN,check if residuals computed for all points used in MINIMUM SOLUTION.
1318
1319 Check for last point in solution.
1320 Check for MINIMUM SOLUTION.
1321 Check if AZIMUTH-ELEVATION data given.
1322 Set AZIMUTH-ELEVATION residual cycle indicator.
1323
1324 Compute Pvv sum for azimuth-elevation observation.
1325
1326 $V_{Ej}^T P_{Ej} V_{Ej} \rightarrow \Sigma V_o^T P_o V_o$
1327 Check for mean error computation cycle.
1328 Set point number equal to zero.
1329 Check for bottom of page.
1330 Print new page heading.
1331
1332
1333
1334
1335
1336
1337
1338 Print azimuth-elevation plate coordinates, weight matrix and residuals.
1339 Check for Sigma-cycle.
1340
1341
1342
1343
1344
1345 $V_o^T P_o V_o \rightarrow \Sigma V_o^T P_o V_o$
1346 $\Sigma V^T P V = \Sigma V_E^T P_E V_E + \Sigma V_{ks}^T P_{ks} V_{ks} + \Sigma V_o^T P_o V_o$
1347
1348 If unique case, set degrees of freedom equal to one.
1349
1350
1351
1352 $m = (\Sigma V^T P V/n-u)^{1/2}$
1353
1354
1355
1356 Clear mean error computation indicator.
1357 Clear residual sums.
1358
1359
1360

Figure 20.--Continued (page 31 of 57)

```
      I = 0
      CT(2) = CT(1)
 3202 I = I+1
      CC = 31.
      SS = CT(I)*(CT(I)-1.)
      DO 3203 K = 1,32
 3203   CLASS(I,K) = 0.
      VM(I) = SUMV(I)/CT(I)
      SDV(I) = SQRT((CT(I)*SUMVS(I)-SUMV(I)**2)/SS)
 3204 AA = 6.5*CT(I)*EXP(-.5*(3.25*(1.-1./CC))**2)/CC/SQRT(2.*C10)
      IF(AA.GT.1.) GO TO 3205
      CC = CC-2.
      GO TO 3204
 3205 NOC(I) = CC
      ANOC(I) = NOC(I)
      IF(I.EQ.3) GO TO 3206
      IF(I.EQ.1) GO TO 3202
      SUMV(3) = SUMV(1)+SUMV(2)
      SUMVS(3) = SUMVS(1)+SUMVS(2)
      CT(3) = 2.*CT(1)
      GO TO 3202
 3206 DO 3207 I=1,3
      SUMV(I) = 0.
      SUMVS(I) = 0.
 3207 CT(I) = 0.
      GO TO 261
C
C  START SIGMA CYCLE. COMPUTE FICTITIOUS COORDINATES WITH UNIT WEIGHT
C    FOR 10 POINTS AT 1 CM INTERVALS ALONG DIAGONAL OF PLATE.
C    REPEAT FOR ALL ORIENTATIONS (PRE, POST OR EVENT) SOLVED FOR IN
C    PROBLEM.  IDENTIFY POINTS AS TYPE EIGHT BY SETTING P-ALPHA EQUAL
C    TO ZERO.
C
 3228 CT(2) = CT(1)
      CT(3) = CT(1)*2.
      III = NOPRE+NOPOST+NOVENT+1
      IF(MINSOL.EQ.1) GO TO 38418
      DO 32327 I=1,3
      SIGMA(I)=0.
        DO 32327 J=1,6
32327     ERRFUN(I,J) = 0.
      DO 32330 I=1,6
        DO 32330 J=1,6
32330     SIG(I,J) = 0.
      GO TO (32280,32281,32282,32283,32284,32285,32286,32326),III
32280 NPP = 1
      LLL = 5
      GO TO 32288
32281 NPP = 2
      LLL = 4
      L2 = 2
      GO TO 32288
32282 NPP = 1
      LLL = 3
```

If MINIMUM SOLUTION, bypass sigma cycle.

Figure 20.--Continued (page 32 of 57)

```
      L2 = 3                                                            1410
      GO TO 32288                                                       1411
32283 NPP = 3                                                           1412
      GO TO 32287                                                       1413
32284 NPP = 1                                                           1414
      LLL = 2                                                           1415
      L2 = 1                                                            1416
      GO TO 32288                                                       1417
32285 NPP = 2                                                           1418
      GO TO 32287                                                       1419
32286 NPP = 1                                                           1420
32287 LLL = 1                                                           1421
32288 SIGMAC = 1.                                                       1422
      ALPHA = 0.                                                        1423
      DELTA = 0.                                                        1424
      LASTPT = 1                                                        1425
      MPTS = 0                                                          1426
      STIME = TIME8                                                     1427
      NPTS = 1                                                          1428
      AERES = 0.                                                        1429
      VCYCLE = 0.                                                       1430
      PCYCLE = 0.                                                       1431
      ACYCLE = 0.                                                       1432
      DO 32289 I = 1,2                                                  1433
        DO 32289 J=1,2                                                  1434
32289       PALPHA(I,J) = 0.                                            1435
      PL(1,1) = 1.                                                      1436
      PL(2,2) = 1.                                                      1437
      PL(2,1) = 0.                                                      1438
      PL(1,2) = 0.                                                      1439
      ICNTR = 1                                                         1440
      LAP = 1                                                           1441
      ELX = 0.                                                          1442
      ELY = 0.                                                          1443
      NOST = -ICNTR                                                     1444
      NOPT = ICNTR                                                      1445
      NUST = NOST                                                       1446
      IDIAG = ICNTR - 1                                                 1447
      PRINT 1007                                                        1448
      PRINT 1054,PL(1,1),PL(2,2),PL(1,2)                                1449
C
C  PRINT DIAGONAL DISTANCE.
C
      PRINT 1044,IDIAG,IDIAG                                            1450
      PRINT 1030                                                        1451
      LINCNT = 17                                                       1452
      GO TO 1792                                                        1453
32293 IDIAG = ICNTR - 1                                                 1454
      GO TO (38417,32295,32297,32297,322999),LLL                        1455
32294 DO 38405 I=1,3                                                    1456
C
C  AFTER FIRST DIAGONAL POINT, TEST FOR COMBINING ORIENTATIONS.
C
        SIGMAN(I,I) = SIGMAN(I,I)*FACT                                  1457
```

Figure 20.--Continued (page 33 of 57)

```
        IF(ICNTR.EQ.1.AND.SIGMAM(I,I).GT.SIGMA(I)) LSIG = LSIG + I
38405 CONTINUE
      IF(ICNTR.EQ.1.AND.SIGMAM(3,3).GT.SIGMA(3)LSIG=LSIG-1
      IF(ICNTR.EQ.1.AND.SIGMAM(2,2).GT.SIGMA(2)LSIG=LSIG+2
38417 ICNTR = ICNTR + 1
      IDIAG = ICNTR - 1
      ELX = ELX + .01
      ELY = ELY + .01
      LAP = 1
      IF(ICNTR.EQ.10) GO TO 32326
      NOST = -ICNTR
      NOPT = ICNTR
      NUST = NOST
      IF(LINCNT.LT.60) GO TO 32331
      PRINT 1007
      LINCNT = 4
32331 PRINT 1044,IDIAG,IDIAG
      PRINT 1030
      LINCNT = LINCNT + 9
      GO TO 181
C
C  COMPUTE AND PRINT DIFFERENCE IN DIRECTION BETWEEN PRE-POST-EVENT
C    ORIENTATIONS AT EACH DIAGONAL POINT.
C
32295 GO TO (32296,32292),LAP
32296 NPP = 2
      GO TO 32290
32297 GO TO (32298,32307),LAP
32298 NPP = 3
      GO TO 32300
32299 GO TO (32296,32291,32306),LAP
32290 M=0
      GO TO 32328
32300 IF(LLL.EQ.3) GO TO 32290
      M=2
32328 DO 32305 I=1,2
        DO 32304 J=1,2
          J1 = I+M
          J2 = J+M
32304 SIG1(J1,J2) = ONE(I,J)
        DO 32305 K=1,26
32305 SIG2(J1,K) = TWO(I,K)
      ELEVI = ELEV
      AZ1 = AZ
      ASIGMA = ALSIG
      DSIGMA = DESIG
      SINDL1 = SIN(DESIG)
      COSDL1 = COS(DESIG)
32301 LAP = LAP + 1
      GO TO 181
32291 L2 = 1
32292 I1 = 2
      GO TO 32302
32306 L2 = 3
```

$$(1) \rightarrow (1)^* = \begin{array}{c} \begin{array}{ccc} Pre & Post & Event \end{array} \\ \begin{bmatrix} \left[B_x^T P_1 B_x\right]^{-1} & 0 & 0 \\ 0 & \left[B_x^T P_1 B_x\right]^{-1} & 0 \\ 0 & 0 & \left[B_x P_1 B_x\right]^{-1} \end{bmatrix} \end{array}$$

$$(2) \rightarrow (2)^* = \begin{bmatrix} B_x^T P_1 B_o \\ B_x^T P_1 B_o \\ B_x^T P_1 B_o \end{bmatrix} \begin{array}{l} Pre \\ Post \\ Event \end{array}$$

Figure 20.--Continued (page 34 of 57)

```
32307 II = 4
32302 SINDL2 = SIN(DESIG)
      COSDL2 = COS(DESIG)
      SINDHR = SIN(ASIGMA-ALSIG)
      COSDHR = COS(ASIGMA-ALSIG)
      SNSQSG = (COSDL1*SINDHR)**2+(SINDL1*COSDL2-COSDL1*SINDL2*COSDHR)**
     12
      SINSIG = SQRT(SNSQSG)
      CALL ASIN(SINSIG,SIGM)
      SINSIG = SIN(SIGM)
      SIGMA(L2)= SIGM/C6
      DELAZ(L2)= (AZ-AZ1)*3600.*SINZO
      DELEL(L2)= (ELEV-ELEV1)*3600.
      DELAL(L2)= ((ALSIG-ASIGMA)/C6)*COSDL2
      DELDE(L2)= (DESIG-DSIGMA)/C6
      DO 32309 I=1,2
        DO 32308 J=1,2
          J1 = I+II
C
C  COMPUTE AND PRINT MEAN ERROR OF DIRECTION DIFFERENCES.
C
          J2 = J+II
32308 SIG1(J1,J2) = ONE(I,J)
        DO 32309 K=1,26
32309 SIG2(J1,K) = TWO(I,K)
      IF(SIGM.LT..5E-7) GO TO 32310
      ERRFUN(L2,M+1)= ( COSDL2*COSDL1*SINDHR)/SINSIG
      ERRFUN(L2,M+2)=(-SINDL2*COSDL1+COSDL2*SINDL1*COSDHR)/SINSIG
      ERRFUN(L2,II+1)= -ERRFUN(L2,M+1)
      ERRFUN(L2,II+2)= (-SINDL1*COSDL2+COSDL1*SINDL2*COSDHR)/SINSIG
      GO TO 32311
32310 ERRFUN(L2,M+1)= COSDL1**2
      ERRFUN(L2,M+2)=-1
      ERRFUN(L2,II+1)= -ERRFUN(L2,M+1)
      ERRFUN(L2,II+2)= 1.
32311 NPP = 1
      IF(LLL.LT.5) GO TO 32313
      IF(LAP.EQ.2) GO TO 32312
      L2 = 2
      SINDHR = SIN(ALSIG3-ALSIG)
      COSDHR = COS(ALSIG3-ALSIG)
      SNSQSG = (COSDL3*SINDHR)**2+(SINDL3*COSDL2-COSDL3*SINDL2*COSDHR)**
     12
      SINSIG = SQRT(SNSQSG)
      CALL ASIN(SINSIG,SIGM)
      SINSIG = SIN(SIGM)
      SIGMA(L2)= SIGM/C6
      DELAZ(L2)= (AZ3-AZ)*3600.*SINZO
      DELEL(L2)= (ELEV3-ELEV)*3600.
      DELAL(L2)= ((ALSIG-ALSIG3)/C6)*COSDL2
      DELDE(L2)= (DESIG-DESIG3)/C6
      IF(SIGM.LT..5E-7) GO TO 32314
      ERRFUN(2,3)=(+COSDL2*COSDL3*SINDHR)/SINSIG
      ERRFUN(2,4)=(-SINDL2*COSDL3+COSDL2*SINDL3*COSDHR)/SINSIG
```

$$\sin^2\sigma_{ij} = (\cos\delta_i \sin\Delta H)^2 + (\sin\delta_i \cos\delta_j - \cos\delta_i \sin\delta_j \cos\Delta H)^2$$

$$\sigma_{ij} = \sin^{-1}(\sin^2\sigma_{ij})^{1/2}$$

$$f_{ij}^T = \begin{bmatrix} (\cos\delta_j \cos\delta_i \sin\Delta\alpha)/\sin\sigma_{ij} \\ -(\cos\delta_j \sin\delta_i + \sin\delta_j \cos\delta_i \cos\Delta\alpha)/\sin\sigma_{ij} \\ -(\cos\delta_j \cos\delta_i \sin\Delta\alpha)/\sin\sigma_{ij} \\ -(\sin\delta_j \cos\delta_i + \cos\delta_j \sin\delta_i \cos\Delta\alpha)/\sin\sigma_{ij} \end{bmatrix}$$

Figure 20.--Continued (page 35 of 57)

```
      ERRFUN(2,5) = -ERRFUN(2,3)
      ERRFUN(2,6) = (-SINDL3*CDSDL2+CDSDL3*SINDL2*COSDHR)/SINSIG
      GO TO 32313
32314 ERRFUN(2,3) = CDSDL2**2
      ERRFUN(2,4) = -1.
      ERRFUN(2,5) = -ERRFUN(2,3)
      ERRFUN(2,6) = 1.
32313 DO 32315 I=1,6
      DO 32315 J=1,26
        BOAUX(J,I) = 0.
        DO 32315 K=1,6
32315       BOAUX(J,I) = BOAUX(J,I) + SIG1(I,K)*SIG2(K,J)
      DO 32317 I=1,6
      DO 32316 J=1,26
        TEM(J) = 0.
      DO 32316 K=1,26
32316       TEM(J) = TEM(J) + BOAUX(K,I)*RNINV(K,J)
        DO 32317 J= 1,6
          BXAUX(I,J) = 0.
      DO 32317 K= 1,26
32317         BXAUX(I,J) = BXAUX(I,J) + TEM(K)*SIG2(J,K)
      DO 32318 I=1,6
        DO 32318 J=1,6
          QSIG(I,J) = 0.
          DO 32318 K=1,6
32318         QSIG(I,J) = QSIG(I,J) + BXAUX(I,K)*SIG1(K,J)
      DO 32320 I=1,3
        DO 32319 J=1,6
          TEM(J)=0.
      DO 32319 K=1,6
32319         TEM(J) = TEM(J) + ERRFUN(I,K)*QSIG(K,J)
        DO 32320 J=1,3
          SIGMAM(I,J) = 0.
        DO 32320 K=1,6
32320         SIGMAM(I,J) = SIGMAM(I,J) + TEM(K)*ERRFUN(J,K)
      DO 32321 I=1,3
      IF(SIGMAM(I,I).GT.0.)SIGMAM(I,I)=(ERRPR*(SQRT(SIGMAM(I,I))))/C6
32321 CONTINUE
      GO TO (32322,32323,32324,32325,32325),III
32322 PRINT 1056
C
C  PRINT DIFFERENCES AND MEAN ERRORS.
C
      PRINT 1151,SIGMA(1),SIGMAM(1,1),DELAL(1),DELDE(1),DELAZ(1),DELEL(1
     1)
      PRINT 1153,SIGMA(3),SIGMAM(3,3),DELAL(3),DELDE(3),DELAZ(3),DELEL(3
     1)
      PRINT 1152,SIGMA(2),SIGMAM(2,2),DELAL(2),DELDE(2),DELAZ(2),DELEL(2
     1)
      LINCNT = LINCNT + 7
      GO TO 32294
32324 PRINT 1056
      PRINT 1153,SIGMA(3),SIGMAM(3,3),DELAL(3),DELDE(3),DELAZ(3),DELEL(3
     1)
```

$[11]^{*}[12]^{*}$

$[11]^{*}[12]^{*}N^{-1}$

$[11]^{*}[12]^{*}N^{-1}[12]^{*T}$

$[11]^{*}[12]^{*}N^{-1}[12]^{*T}[11]^{*} = Q$

$$m_{\sigma} = \begin{bmatrix} m_{\sigma_{x-1}} \\ m_{\sigma_{x-2}} \\ m_{\sigma_{x-3}} \end{bmatrix} = m(\mathcal{J}Q\mathcal{J}^{T})^{1/2}$$

Figure 20.--Continued (page 36 of 57)

```
      LINCNT = LINCNT + 5                                               1610
      GO TO 32294                                                       1611
32323 PRINT 1056                                                        1612
      PRINT 1152,SIGMA(2),SIGMAM(2,2),DELAL(2),DELDE(2),DELAZ(2),DELEL(2 1613
     1)                                                                 1614
      NPP = 2
      LINCNT = LINCNT + 5                                               1615
      GO TO 32294                                                       1616
32325 PRINT 1056                                                        1617
      PRINT 1151,SIGMA(1),SIGMAM(1,1),DELAL(1),DELDE(1),DELAZ(1),DELEL(1 1618
     1)                                                                 1619
      LINCNT = LINCNT + 5                                               1620
      GO TO 32294                                                       1621
32312 SINDL3 = SINDL2                                                   1622
      COSDL3 = COSDL2                                                   1623
      ALSIG3 = ALSIG                                                    1624
      DESIG3 = DESIG                                                    1625
      ELEV3 = ELEV                                                      1626
      AZ3 = AZ                                                          1627
      NPP = 3                                                           1628
      LAP = 3                                                           1629
      GO TO 181                                                         1630
C
C  COMPUTE ROTATION MATRICES, STATION TO INSTANTANEOUS POLE,
C    INSTANTANEOUS POLE TO MEAN POLE, AND MEAN POLE TO REFERENCE STATION
C    COMPUTE TRANSFORMED ORIENTATION ANGLES.
C
30418 III = 4                                                           1632
32326 PRINT 1063                                                        1633
  346 CSLON = CAMSTA(5)*C4 + CAMSTA(6)*C5 + CAMSTA(7)*C6                1634
      SINLON = SIN(CSLON)                                               1635
      COSLON = COS(CSLON)                                               1636
      AUX1 = -COSLON*SINPHI                                             1637
      AUX2 =  COSLON*COSPHI                                             1638
      AUX3 = -SINLON*SINPHI                                             1639
      AUX4 =  SINLON*COSPHI                                             1640
      GO TO (326,3261,3262,3263,3264,3265,3266,3267),III                1641
  326 L=1                                                               1642
 3260 J=1                                                               1643
      PRINT 1154                                                        1644
      GO TO 3268                                                        1645
 3261 K=1                                                               1646
      J=4                                                               1647
      L=3                                                               1648
      PRINT 1149                                                        1649
 3269 LL=1                                                              1650
      GO TO 3268                                                        1651
 3262 L=3                                                               1652
      J=1                                                               1653
      K=3                                                               1654
      LL=2                                                              1655
      PRINT 1148                                                        1656
      GO TO 3268                                                        1657
 3263 J=7                                                               1658
```

Figure 20.--Continued (page 37 of 57)

```
      PRINT 1154
      GO TO 3268
 3264 K=2
      L=3
      J=1
      PRINT 1147
      GO TO 3269
 3265 J=4
      L=2
      PRINT 1155
      GO TO 3268
 3266 L=2
      GO TO 3260
 3267 PRINT 1157
      GO TO 384
 3268 J1=J+2
      IF(IIND(9).EQ.0) GO TO 3516
      P11=AUX1*A(1,J)+SINLON*A(2,J)+AUX2*A(3,J)
      P12=AUX1*A(1,J+1)+SINLON*A(2,J+1)+AUX2*A(3,J+1)
      P13=AUX1*A(1,J+2)+SINLON*A(2,J+2)+AUX2*A(3,J+2)
      P21=AUX3*A(1,J)-COSLON*A(2,J)+AUX4*A(3,J)
      P22=AUX3*A(1,J+1)-COSLON*A(2,J+1)+AUX4*A(3,J+1)
      P23=AUX3*A(1,J+2)-COSLON*A(2,J+2)+AUX4*A(3,J+2)
      P31=COSPHI*A(1,J)+SINPHI*A(3,J)
      P32=COSPHI*A(1,J+1)+SINPHI*A(3,J+1)
      P33=COSPHI*A(1,J+2)+SINPHI*A(3,J+2)
      XRAD = XANGLE*C6
      YRAD = YANGLE*C6
      PT11 = P11 + XRAD*P31
      PT12 = P12 + XRAD*P32
      PT13 = P13 + XRAD*P33
      PT21 = P21 + YRAD*P31
      PT22 = P22 + YRAD*P32
      PT23 = P23 + YRAD*P33
      PT31 = -XRAD*P11 - YRAD*P21 + P31
      PT32 = -XRAD*P12 - YRAD*P22 + P32
      PT33 = -XRAD*P13 - YRAD*P23 + P33
  347 REFLAT = ABS(REFSTA(2))*C4 + REFSTA(3)*C5 + REFSTA(4)*C6
      REFLON = REFSTA(5)*C4 + REFSTA(6)*C5 + REFSTA(7)*C6
      IF(REFSTA(1))348,349,348
  348 REFLAT = -REFLAT
  349 SINRLA = SIN(REFLAT)
      COSRLA = COS(REFLAT)
      SINRLO = SIN(REFLON)
      COSRLO = COS(REFLON)
      AUX5 = -SINRLA*COSRLO
      AUX6 = -SINRLA*SINRLO
      AUX7 =  COSRLA*COSRLO
      AUX8 =  COSRLA*SINRLO
      PP11 = AUX5*PT11 + AUX6*PT21 + COSRLA*PT31
      PP12 = AUX5*PT12 + AUX6*PT22 + COSRLA*PT32
      PP13 = AUX5*PT13 + AUX6*PT23 + COSRLA*PT33
      PP21 = SINRLO*PT11 - COSRLO*PT21
      PP22 = SINRLO*PT12 - COSRLO*PT22
```

Check for transformation of α, ω, κ.

$$R_{\varphi_i \lambda_i} = r_2 \cdot r_1 = \begin{bmatrix} -\cos\lambda_i \sin\varphi_i & \sin\lambda_i & \cos\lambda_i \cos\varphi_i \\ -\sin\lambda_i \sin\varphi_i & -\cos\lambda_i & \sin\lambda_i \cos\varphi_i \\ \cos\varphi_i & 0 & \sin\varphi_i \end{bmatrix}$$

$$R_{xy} : r_6 \cdot r_5 = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ -x & -y & 1 \end{bmatrix}$$

$$R_{\varphi_o \lambda_o} = r_6 \cdot r_5 = \begin{bmatrix} -\cos\lambda_o \sin\varphi_o & \sin\lambda_o \sin\varphi_o & \cos\varphi_o \\ \sin\lambda_o & -\cos\lambda_o & 0 \\ \cos\lambda_o \cos\varphi_o & \sin\lambda_o \cos\varphi_o & \sin\varphi_o \end{bmatrix}$$

Figure 20.--Continued (page 38 of 57)

```
      PP23 = SINRLO*PT13 - COSRLO*PT23
      PP31 = AUX7*PT11 + AUX8*PT21 + SINRLA*PT31
      PP32 = AUX7*PT12 + AUX8*PT22 + SINRLA*PT32
      PP33 = AUX7*PT13 + AUX8*PT23 + SINRLA*PT33
 351  IF(ABS(PP23)-1.)3510,3511,3510
3510  COSOMT = SQRT(1.-PP23**2)
      TOMEGA = ATAN(PP23/COSOMT)/C7
      SINALT = PP13/COSOMT
      COSALT = PP33/COSOMT
      CALL ANGLE (SINALT,COSALT,TALPHA)
      TALPHA = TALPHA/C7
      SINKAT = -PP21/COSOMT
      COSKAT =  PP22/COSOMT
      CALL ANGLE (SINKAT,COSKAT,TKAPPA)
      TKAPPA = TKAPPA/C7
      GO TO 3514
3511  TKAPPA = 0.
      IF(PP23)3513,3512,3512
3512  TOMEGA = 1.5707963267949
      SINOMT = 1.
      GO TO 3514
3513  TOMEGA = -1.5707963267949
      SINOMT = -1.
3514  SINALT = -PP12/SINOMT
      COSALT = -PP32/SINOMT
      CALL ANGLE (SINALT,COSALT,TALPHA)
      TALPHA = TALPHA/C7
3516  TALPH(J) = TALPHA
      TALPH (J+1) = TOMEGA
      TALPH (J+2) = TKAPPA
C
C  PRINT GIVEN, CORRECTIONS TO AND COMPUTED ORIENTATION PARAMETERS, MEAN
C    ERRORS, RESIDUALS AND WEIGHTS.
C
      DO 3273 I=J,J1
        IF(PO(I,I))3271,3271,3272
 3271 ORES(I)=0.
      OCOR(I)=DELLO(I)/C7
      OME(I)=(ERRPR*SQRT(RNINV(I,I)))/C7
      GO TO 3273
 3272 ORES(I)=DELLO(I)/C7
      OCOR(I)=0.
      OME(I)=(ERRPR/SQRT(PO(I,I)))/C7
 3273 OPAR(I) = OPAR(I)/C7
      IF(L.EQ.3) GO TO 3274
      J1=J+2
      PRINT 1146,(AOPAR(I),I=J,J1), (OPAR(I),I=J,J1), (PO(I,I),I=J,J1),
     1 (ORES(I),I=J,J1), (OCOR(I),I=J,J1), (OME(I),I=J,J1)
      IF(J.EQ.7.OR.L.EQ.2) GO TO 3283
      J=J+3
      IF(J.EQ.4) PRINT 1155
      IF(J.EQ.7)PRINT 1156
      GO TO 3268
 3274 GO TO (3275,3276,3278),LL
```

$\cos\omega^* = (1-E^{*2})^{1/2}$

$\sin\alpha^* = D^*/\cos\omega^*$

$\cos\alpha^* = F^*/\cos\omega^*$

$\sin\kappa^* = -B_1^*/\cos\omega^*$

$\cos\kappa^* = B_2^*/\cos\omega^*$

Figure 20.--Continued (page 39 of 57)

```
 3275 J=J+3
      GO TO 3277
 3276 J=J+6
 3277 LL=3
      GO TO 3268
 3278 GO TO (3279,3280,3281),K
 3279 PRINT 1150, (AOPAR(I),I=4,6),(AOPAR(I),I=7,9),(OPAR(I),I=4,6),
     1 (OPAR(I),I=7,9),(PO(I,I),I=4,6),(PO(I,I),I=7,9),(ORES(I),I=4,6),
     2 (ORES(I),I=7,9),(OCOR(I),I=4,6),(OCOR(I),I=7,9),(OME(I),I=4,6),
     3 (OME(I),I=7,9)
      GO TO 3283
 3280 PRINT 1150,(AOPAR(I),I=1,3),(AOPAR(I),I=4,6),(OPAR(I),I=1,3),
     1 (OPAR(I),I=4,6),(PO(I,I),I=1,3),(PO(I,I),I=4,6),(ORES(I),I=1,3),
     2 (ORES(I),I=4,6),(OCOR(I),I=1,3),(OCOR(I),I=4,6),(OME(I),I=1,3),
     3 (OME(I),I=4,6)
      GO TO 3283
 3281 PRINT 1150,(AOPAR(I),I=1,3),(AOPAR(I),I=7,9),(OPAR(I),I=1,3),
     1 (OPAR(I),I=7,9),(PO(I,I),I=1,3),(PO(I,I),I=7,9),(ORES(I),I=1,3),
     2 (ORES(I),I=7,9),(OCOR(I),I=1,3),(OCOR(I),I=7,9),(OME(I),I=1,3),
     3 (OME(I),I=7,9)
 3283 I=2
  323 I=I+1
      DO 3235 J=1,5
         K=5*I+J-6
           IF(K.EQ.23) GO TO 330
      IF(K.GT.24) K=5*I+J-8
         IF(PO(K,K))3231,3231,3232
 3231 ORES(J)=0.
      OCOR(J)=DELLO(K)
      OME(J)=ERRPR*SQRT(RNINV(K,K))
      GO TO 32329
 3232 ORES(J)=DELLO(K)
      OCOR(J)=0.
      OME(J)=ERRPR/SQRT(PO(K,K))
32329 IF(K.EQ.10.OR.K.EQ.22) GO TO 3233
      GO TO 3234
 3233 ORES(J)=ORES(J)/C7
      OCOR(J)=OCOR(J)/C7
      OME(J)=OME(J)/C7
      OPAR(K)=OPAR(K)/C7
      IF(K.EQ.10)PRINT1070
      GO TO 3234
  330 PRINT 1072
      IF(PO(11,11))3362,3361,3362
 3361 IF(PO(15,15))3365,3366,3365
 3362 IF(PO(15,15))3364,3363,3364
 3363 OME(4)=ERRPR*(SQRT(RNINV(15,15)+1./PO(11,11)))
      GO TO 3367
 3364 DDD=PO(11,11)*PO(15,15)-PO(11,15)**2
      OME(4)=ERRPR*SQRT(PO(11,11)-2.*PO(11,15)+PO(15,15))/SQRT(DDD)
      ORES(4)=XS-AOPAR(27)
      OCOR(4)=0.
      PO27 = PO(11,11)+PO(15,15)+2.*PO(11,15)
      GO TO 3368
```

Figure 20.--Continued (page 40 of 57)

```
3365 OME(4)=(ERRPR*(SQRT(RNINV(11,11)+1./PO(15,15))))
     GO TO 3367
3366 OME(4)=ERRPR*(SQRT(RNINV(11,11)+2.*RNINV(11,15)+RNINV(15,15)))
3367 ORES(4)=0.
     PO27=0.
     OCOR(4)=XS-AOPAR(27)
3368 IF(PO(12,12))3370,3369,3370
3369 IF(PO(16,16))3373,3374,3373
3370 IF(PO(16,16))3372,3371,3372
3371 OME(5)=ERRPR*(SQRT(RNINV(16,16)+1./PO(12,12)))
     GO TO 3451
3372 DDD=PO(12,12)*PO(16,16)-PO(12,16)**2
     OME(5)=ERRPR*SQRT(PO(12,12)-2.*PO(12,16)+PO(16,16))/SQRT(DDD)
     ORES(5)=YS-AOPAR(28)
     OCOR(5)=0.
     PO28 = PO(12,12)+PO(16,16)+2.*PO(12,16)
     GO TO 3452
3373 OME(5)=ERRPR*(SQRT(RNINV(12,12)+1./PO(16,16)))
     GO TO 3451
3374 OME(5)=ERRPR*(SQRT(RNINV(12,12)+2.*RNINV(12,16)+RNINV(16,16)))
3451 ORES(5)=0.
     OCOR(5)=YS-AOPAR(28)
     PO28=0.
     GO TO 3452
3234 IF(K.EQ.13.AND.INO(1).EQ.1) GO TO 3236
     IF(K.EQ.28) GO TO 3452
3235 CONTINUE
3452 K=5*I-5
     IF(I.EQ.4) PRINT 1071
     IF(I.EQ.5) GO TO 3237
     IF(I.EQ.6) GO TO 3239
     PRINT 1158,AOPAR(K),AOPAR(K+1),AOPAR(K+2),AOPAR(K+3),AOPAR(K+4)
     PRINT 1159,OPAR(K),OPAR(K+1),OPAR(K+2),OPAR(K+3),OPAR(K+4)
     PRINT 1160,PO(K,K),PO(K+1,K+1),PO(K+2,K+2),PO(K+3,K+3),PO(K+4,K+4)
3238 PRINT 1161,(ORES(J),J=1,5)
     PRINT 1162, (OCOR(J),J=1,5)
     PRINT 1163,(OME(J),J=1,5)
     IF(I.LT.5) GO TO 323
     IF(I.EQ.5.AND.INO(5).EQ.1) GO TO 323
     GO TO 335
3236 PO(14,14)=PO(13,13)
     AOPAR(14)=AOPAR(13)
     ORES(5)=ORES(4)
     OCOR(5)=OCOR(4)
     OME(5)=OME(4)
     GO TO 3452
3237 PRINT 1158,AOPAR(K),AOPAR(K+1),AOPAR(K+2),AOPAR(27),AOPAR(28)
     PRINT 1159,OPAR(K),OPAR(K+1),OPAR(K+2),XS,YS
     PRINT 1160,PO(K,K),PO(K+1,K+1),PO(K+2,K+2),PO27,PO28
     GO TO 3238
3239 IF(MINSOL.EQ.1) GO TO 335                          Check for MINIMUM SOLUTION.
     PRINT 1073
     PRINT 1064,AOPAR(23),AOPAR(24),AOPAR(25),AOPAR(26)
     PRINT 1065,OPAR(23),OPAR(24),OPAR(25),OPAR(26)
```

Figure 20.--Continued (page 41 of 57)

```
      PRINT 1066,PO(23,23),PO(24,24),PO(25,25),PO(26,26)
      PRINT 1067,(DRES(J),J=1,4)
      PRINT 1068,(DCOR(J),J=1,4)
      PRINT 1069,(DME(J),J= 1,4)
  335 IF(INO(9).EQ.0) GO TO 352
C
C  PRINT TRANSFORMED ORIENTATION ANGLES.
C
 3515 IF(NOPRE.EQ.0) PRINT 1164,TALPH(1), TALPH(2), TALPH(3)
      IF(NOPOST.EQ.0) PRINT 1165,TALPH(4), TALPH(5), TALPH(6)
      IF(NOVENT.EQ.0) PRINT 1096,TALPH(7),TALPH(8),TALPH(9)
C
C  PRINT MEAN ERROR OF UNIT WEIGHT FOR THE PROBLEM
C
  352 PRINT 1076,V1,V2,V3,V4
      ALLVS=POSVX+POSVY+ANEGVX+ANEGVY
      PRINT 1128,POSVX,ANEGVX,POSVY,ANEGVY,ALLVS
      IF(MPCOMP)3520,3520,3521
 3520 XME = ERRPR/SQRT(PL(1,1))
      YME = ERRPR/SQRT(PL(2,2))
      PRINT 1117,XME,YME
 3521 IF(MSCOMP)3522,3522,3523
 3522 ALPHME = (ERRPR/SQRT(TPAD(1,1)))/C6
      DECME = (ERRPR/SQRT(TPAD(2,2)))/C6
      PRINT 1118,ALPHME,DECME
 3523 PRINT 1079,ERRPR1
      PRINT 1080
      IF(CCYCLE.EQ.1.)GO TO 356
      IF(IND(6))353,354,353
  353 PRINT 1081,DETMIN
  354 IF(IND(7))356,355,356
  355 PRINT 1082,DETMAX
C
C  PRINT INVERSE FOR REDUCED NORMAL EQUATION SYSTEM.
C
  356 PRINT 1083
      PRINT 1084
      J = 1
  357 K = J+1
      L = J+2
      M = J+3
      N = J+4
      IF(J.EQ.21)GO TO 359
      IF(J-6)359,358,361
  358 PRINT 1085
  359 DO 360 I=1,J
  360   PRINT 1088,I,J,RNINV(I,J),I,K,RNINV(I,K),I,L,RNINV(I,L),I,M,
     1    RNINV(I,M),I,N,RNINV(I,N)
      PRINT 1089,K,K,RNINV(K,K),K,L,RNINV(K,L),K,M,RNINV(K,M),K,N,
     1  RNINV(K,N)
      PRINT 1090,L,L,RNINV(L,L),L,M,RNINV(L,M),L,N,RNINV(L,N)
      PRINT 1091,M,M,RNINV(M,M),M,N,RNINV(M,N)
      PRINT 1092,N,N,RNINV(N,N)
      J = J + 5
```

Check for MINIMUM SOLUTION.

Check for PRE-RUN.

$m_{\ell_x} = m/(P_{\ell_x})^{1/2}$

$m_{\ell_y} = m/(P_{\ell_y})^{1/2}$

$m_x = m/(P_x)^{1/2}$

$m_y = m/(P_y)^{1/2}$

Figure 20.--Continued (page 42 of 57)

```
      IF(J.EQ.26) GO TO 3662                                    1919
      IF(J-16)357,357,3661                                      1920
  361 IF(J-11)364,362,364                                       1921
  362 PRINT 1086                                                1922
      GO TO 359                                                 1923
  364 PRINT 1087                                                1924
      GO TO 359                                                 1925
 3661 PRINT 1083                                                1926
      PRINT 1143                                                1927
      GO TO 357                                                 1928
 3662 PRINT 1144                                                1929
      DO 3663 I=1,26                                            1930
 3663 PRINT 1145,I,J,RNINV(I,J)                                 1931
  366 IF(PCYCLE)383,367,383                                     1932  Check for PRE-RUN cycle.
  367 IF(IND(7))368,371,368                                     1933  Check for PRE-RUN.
  368 PCYCLE = 1.                                               1935  Set PRE-RUN cycle indicator.
      IF(LINCNT-59)370,370,369                                  1936  Check for bottom of page.
  369 PRINT 1053                                                1937  Print new page heading.
      LINCNT = 2                                                1938  Set line counter.
  370 PRINT 1061                                                1939
      IF(MCOMP)3702,3702,3701                                   1940  Check for mean error computation cycle.
 3701 PRINT 1114                                                1941
      GO TO 3703                                                1942
 3702 PRINT 1055                                                1943
 3703 LINCNT = LINCNT + 9                                       1944  Increment line counter.
      GO TO 181                                                 1945
  371 IF(IND(8))372,383,372                                     1946  Check if distortion curves to be printed.
C
C  COMPUTE AND PRINT MEAN ERROR OF CX AND CY, BOTH TRANSFORMED AND AS
C     COMPUTED.
C
  372 DK2 = DK**2                                               1948
      DK4 = DK2**2                                              1949
      DK6 = DK2*DK4                                             1950
      DKO = -OPAR(17)*DK2 - OPAR(18)*DK4 - OPAR(19)*DK6         1951  K_o = -(K_1 d_x^2 + K_2 d_x^4 + K_3 d_x^6)
      DO 3722 I=1,7                                             1952
 3722 DFN(I)=0.                                                 1953
      IF(IND(1)) 3721,373,3721                                  1954  C_x = C_y
 3721 CSTAR=OPAR(13)                                            1955      C = C_x
      DFN(1)=1.-DKO                                             1956      f_1 = 1 - K_o
      COEF=OPAR(13)                                             1957
      GO TO 374                                                 1958  C_x ≠ C_y
  373 CSTAR=(OPAR(13)+OPAR(14))/2.                              1959      C = (C_x + C_y)/2
      DFN(1)=(1.-DKO)/2.                                        1960      f_1 = (1 - K_o)/2
      DFN(2)=DFN(1)                                             1961      f_2 = f_1
      COEF=CSTAR                                                1962
  374 CSTAR=CSTAR*(1.-DKO)                                      1963      C' = C(1 - K_o)
      DFN(5)=-COEF*DK2                                          1964      f_5 = -C d_x^2
      DFN(6)=-COEF*DK4                                          1965      f_6 = -C d_x^4
      DFN(7)=-COEF*DK6                                          1966      f_7 = -C d_x^6
      DO 3742 I=1,7                                             1967
        DFNN(I)=0.                                              1968
          DO 3742 J=1,7                                         1969
 3742 DFNN(J)=DFNN(J)+DFN(I)*RNINV(J+12,I+12)                   1970
```

Figure 20.--Continued (page 43 of 57)

```
      QCST=0.
      DO 3743 I=1,7
 3743 QCST=QCST+DFNN(I)*DFN(I)
      ERRST=ERRPR*SQRT(QCST)
      D=0.
      MM=0
      IF(PO(13,13))3744,3745,3744
 3744 ERRCX = ERRPR/SQRT(PO(13,13))
      GO TO 3746
 3745 ERRCX = ERRPR*SQRT(RNINV(13,13))
 3746 IF(PO(14,14))3748,3749,3748
 3748 ERRCY = ERRPR/SQRT(PO(14,14))
      GO TO 3752
 3749 ERRCY = ERRPR*SQRT(RNINV(14,14))
 3752 IF(PO(22,22))3753,3754,3753
 3753 ERRPHT = ERRPR/SQRT(PO(22,22))/C7
      GO TO 3755
 3754 ERRPHT = ERRPR*SQRT(RNINV(22,22))/C7
 3755 PRINT 1098,OPAR(13),ERRCX,OPAR(14),ERRCY,OPAR(22),ERRPHT,CSTAR,
     1ERRST,DK,DKO
      LINCNT = 17
 3741 PRINT 1099
      LINCNT = LINCNT+6
C
C  COMPUTE AND PRINT RADIAL, TRANSFORMED RADIAL, AND DECENTERING
C   DISTORTION FOR POINTS ALONG THE DIAGONAL OF THE PLATE AT INTERVALS
C    AND OVER THE DISTANCE SPECIFIED IN THE PROBLEM OPTIONS.  COMPUTE
C    MEAN ERRORS FOR EACH OF THE POINTS.
C
      IF(MM)382,375,382
  375 D = D + DINT
      D2 = D**2
      D4 = D2**2
      D6 = D2*D4
      DR = D*(DKO+OPAR(17)*D2+OPAR(18)*D4+OPAR(19)*D6)
      DTR = (DR-D*DKO)*.1E+7
      DR = DR*.1E+7
      FTR(1) = D*D2
      FTR(2) = D*D4
      FTR(3) = D*D6
      FR(1) = D*(D2 - DK2)
      FR(2) = D*(D4 - DK4)
      FR(3) = D*(D6 - DK6)
      DO 3751 I=1,3
        TEMT(I) = 0.
3751    TEM(I) = 0.
      DO 376 I=1,3
        DO 376 J=1,3
          TEMT(J) = TEMT(J) + FTR(I)*RNINV(I+16,J+16)
 376      TEM(J) = TEM(J) + FR(I)*RNINV(I+16,J+16)
      ERD = ERRPR*SQRT(TEM(1)*FR(1) + TEM(2)*FR(2) + TEM(3)*FR(3))*.1E+7
      ERDT = ERRPR*SQRT(TEMT(1)*FTR(1)+TEMT(2)*FTR(2)+TEMT(3)*FTR(3))
     1*.1E+7
      IF(IND(8)-1)377,377,378
```

$$Q_{c^*} = f \begin{bmatrix} Q_{c_x c_x} & & & & & \\ & Q_{c_y c_y} & & & & \\ & & Q_{c_z c_z} & & & \\ & & & Q_{c_y c_y} & & \\ & \text{full matrix} & & & Q_{K_1 K_1} & \\ & & & & & Q_{K_2 K_2} \\ & & & & & & Q_{K_3 K_3} \end{bmatrix} f^T$$

$$m_{c^*} = m(Q_{c^*})^{1/2}$$

$$m_{c_x} = m/(P_{c_x})^{1/2}$$

$$m_{c_z} = m/(Q_{c_z c_z})^{1/2}$$

$$m_{c_y} = m/(P_{c_y})^{1/2}$$

$$m_{c_y} = m\,(Q_{c_y c_y})^{1/2}$$

$$m_{\varphi_r} = m/(P_{\varphi_r})^{1/2}$$

$$m_{\varphi_r} = m(Q_{\varphi_r \varphi_r})^{1/2}$$

$$\Delta R_T = K_0 d + K_1 d^3 + K_2 d^5 + K_3 d^7$$

$$\Delta R = \Delta R_T - d K_0$$

$$f_{\Delta R_1} = d^3$$

$$f_{\Delta R_2} = d^5$$

$$f_{\Delta R_3} = d^7$$

$$f_{\Delta R_{T_1}} = d(d^2 - d_x^2)$$

$$f_{\Delta R_{T_2}} = d(d^4 - d_x^4)$$

$$f_{\Delta R_{T_3}} = d(d^6 - d_x^6)$$

$$Q_K = \begin{bmatrix} Q_{K_1 K_1} & * \\ & Q_{K_2 K_2} \\ & & Q_{K_3 K_3} \end{bmatrix} \quad * \text{ full matrix}$$

$$m_{\Delta R_T} = m(f_{\Delta R_T}\, Q_K\, f_{\Delta R_T}^T)^{1/2}$$

$$m_{\Delta R} = m(f_{\Delta R}\, Q_K\, f_{\Delta R}^T)^{1/2}$$

Figure 20.--Continued (page 44 of 57)

```
  377 DT = 0.                                                          2019
      ETD = 0.                                                         2020
      GO TO 380                                                        2021
  378 DT = (DPAR(20)*D2 + DPAR(21)*D4)*.1E+7                           2022   ΔT = K4 d^2 + K5 d^4
      FT(1) = D2                                                       2023   f_ΔT1 = d^2
      FT(2) = D4                                                       2024   f_ΔT2 = d^4
      TEM(1) = 0.                                                      2025
      TEM(2) = 0.                                                      2026
      DO 379 I=1,2                                                     2027
        DO 379 J=1,2                                                   2028   Q_ΔT = [Q_K4K4 *; Q_K5K5] * full matrix
  379     TEM(J) = TEM(J) + FT(I)*RMINV(I+19,J+19)                     2029
      ETD = ERRPR*SQRT(TEM(1)*FT(1) + TEM(2)*FT(2))*.1E+7              2030   m_ΔT = m (f_ΔT Q_ΔT f_ΔT^T)^1/2
  380 IF(LINCNT-69)382,381,381                                         2031
  381 PRINT 1102                                                       2032
      PRINT 1097                                                       2033
      PRINT 1051                                                       2034
      LINCNT = 2                                                       2035
      MM = 1                                                           2036
      GO TO 3741                                                       2037
  382 DMM = 0*.1E+4 + .1E-06                                           2038
      PRINT 1101,DMM,DTR,EROT,DR,ERD,DT,ETD                            2039
      LINCNT = LINCNT + 2                                              2040
      IF(D-DMAX+.1E-06)375,383,383                                     2041
C
C  IF REFRACTION SOLUTION, COMPUTE AND PRINT REFRACTION AT INTERVALS OF
C    ONE DEGREE OVER A RANGE OF 10 DEGREES ON EITHER SIDE OF CAMERA
C   CENTRAL RAY.  PRINT MEAN ERROR OF DETERMINATION FOR EACH POINT.
C
  383 IF(INO(5))3831,3837,3831                                         2042
 3831 CALL ASIN(A(3,9),ELEV)                                           2043   E = sin^-1 F
      IZEN = 90. - ELEV/C4 + .5                                        2044
      PRINT 1110,IZEN                                                  2045
      IZEN = IZEN - 10                                                 2046
      DO 3836 I=1,21                                                   2047
        ZEN = (ABS(IZEN)                                               2048
        ZEN = ZEN*C4                                                   2049
        TANZ = TAN(ZEN)                                                2050
        BETA = ATAN(WF*TANZ)                                           2051   β = tan^-1 [(T^1/2/T) tan ζ]
        TANB = SIN(BETA/2.)/COS(BETA/2.)                               2052
        DO 3832 J=1,4                                                  2053
 3832     FF(J) = WC*TANB**(2*J-1)/C6                                  2054
        ZETA = 0.                                                      2055
        ZETAC = 0.                                                     2056
        DO 3833 J=1,4                                                  2057
 3833     RWORK(J) = 0.                                                2058
        ERRZT = 0.                                                     2059
        DO 3835 J=1,4                                                  2060   ζ_given = T^1/2 w(τ1° tanβ/2 + τ2° tan^3β/2 + τ3° tan^5β/2 +
        ZETA = ZETA + FF(J)*AOPAR(J+22)                                2061                τ4° tan^7β/2)
      ZETAC = ZETAC + FF(J)*DPAR(J+22)                                 2062   ζ_computed = T^1/2 w(τ1 tanβ/2 + τ2 tan^3β/2 + τ3 tan^5β/2 +
        DO 3834 K=1,4                                                  2063                τ4 tan^7β/2)
 3834     RWORK(J) = RWORK(J) + FF(K)*RMINV(K+22,J+22)                 2064   f_τ = T^1/2 w  tanβ/2  tan^3β/2  tan^5β/2  tan^7β/2
 3835     ERRZT = ERRZT + RWORK(J)*FF(J)                               2065
        ERRZT = SQRT(ERRZT)*ERRPR                                      2066   Q_τ = [Q_τ1τ1  *; Q_τ2τ1 Q_τ2τ2; Q_τ3τ1 Q_τ3τ2 Q_τ3τ3; Q_τ4τ1 Q_τ4τ2 Q_τ4τ3 Q_τ4τ4] * full matrix
        PRINT 1111,IZEN,ZETA,ZETAC,ERRZT                               2067   m_ζ = m (f_τ Q_τ f_τ^T)^1/2
```

Figure 20.--Continued (page 45 of 57)

```
 3836   IZEN = IZEN + 1
 3837 IF(IND(7)) 3841,3838,3841
C
C  PLOT HISTOGRAM OF RESIDUALS.
C
 3838 CALL GRAPH(CT,ANOC,CLASS,SOV,VM,NOC)
 3841 IF(CCYCLE.EQ.1.) GO TO 384
      IF(LSIG.EQ.1) GO TO 38419
38380 DO 38381 I=1,26
      IF(I.LT.11) OPAR(I) = OPAR(I)*C7
      IF(I.EQ.22) OPAR(22) = OPAR(22)*C7
        CN(I) = SAVECN(I)
        OPAR(I) = OPAR(I) - SAVEDL(I)
38381   DELLO(I) = DELLO(I) - SAVEDL(I)
      SIGMAC = 0.
      CCYCLE = 1.
      DO 38382 I= 1,6
        ALDIF(I) = 0.
        DO 38382 J=1,9
38382      COND(I,J)=0.
      PRINT 1075
C
C  BASED ON TESTS MADE IN SIGMA CYCLE, COMBINE ORIENTATION ANGLES OF
C    PRE-POST-EVENT OR COMBINATIONS THEREOF.
C
38419 PRINT 1095,FACT
      GO TO (38406,38387,38383,38383,38385,38385,38383,38383),LSIG
38406 PRINT 1077
      GO TO 384
38383 DO 38384 I=1,3
        ALDIF(I) = OPAR(I) - OPAR(I+6)
        COND(I,I) = 1.
38384     COND(I,I+6) = -1.
      PRINT 1078
      IF(LSIG.LT.6) GO TO 38389
38385 DO 38386 I=1,3
        ALDIF(I+3) = OPAR(I+3)-OPAR(I+6)
        COND(I+3,I+3) = 1.
38386     COND(I+3,I+6) = -1.
      PRINT 1094
      GO TO 38389
38387 DO 38388 I=1,3
        ALDIF(I) = OPAR(I)-OPAR(I+3)
        COND(I,I)=1.
38388     COND(I,I+3) = -1.
      PRINT 1093
38389 DO 38391 I=1,6
        DO 38390 J=1,9
          TEMPO(J) = 0.
            DO 38390 K=1,9
38390       TEMPO(J) = TEMPO(J)+COND(I,K)*RNINV(K,J)
            DO 38391 J=1,6
              CONDIT(I,J)=0.
                DO 38391 K=1,9
```

$$\Gamma^{\circ} = \begin{bmatrix} \alpha^{\circ}_{Event} - \alpha^{\circ}_{Pre} \\ \omega^{\circ}_{Event} - \omega^{\circ}_{Pre} \\ \kappa^{\circ}_{Event} - \kappa^{\circ}_{Pre} \\ \alpha^{\circ}_{Event} - \alpha^{\circ}_{Post} \\ \omega^{\circ}_{Event} - \omega^{\circ}_{Post} \\ \kappa^{\circ}_{Event} - \kappa^{\circ}_{Post} \end{bmatrix}_{a}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

$$N^{-1} = \begin{bmatrix} N^{a^{-1}} & \\ & \end{bmatrix}$$

$$CN^{-1}$$

Figure 20.--Continued (page 46 of 57)

```
38391     CONDIT(I,J) = CONDIT(I,J)+TEMPO(K)*COND(J,K)                  2115  CN^-1C^T
      DO 38392 I=1,5                                                    2116
        IF(CONDIT(I,I).EQ.0.) CONDIT(I,I) = 1.E+20                      2117
38392 CONTINUE                                                          2118
      DO 38398 K=1,6                                                    2119
        ELMT = CONDIT(K,K)                                              2120
        IF(ELMT)38394,38393,39394                                       2121
38393     PRINT 1057                                                    2122
          GO TO 384                                                     2123
38394     CONDIT(K,K) = 1.                                              2124
            DO 38395 J=1,6                                              2125
38395         CONDIT(K,J) = CONDIT(K,J)/ELMT                            2126
      DO 38398 I=1,6                                                    2127
        IF(I-K) 38396,38398,39396                                       2128
38396 ELMT = CONDIT(I,K)                                                2129
      CONDIT(I,K) = 0.                                                  2130
        DO 38397 J= 1,6                                                 2131
38397     CONDIT(I,J) = CONDIT(I,J)-ELMT*CONDIT(K,J)                    2132  (CN^-1C^T)^-1
38398 CONTINUE                                                          2133
      DO 38400 I=1,26                                                   2134
        DO 38399 J=1,6                                                  2135
          TEMPO(J) = 0.                                                 2136
            DO 38399 K=1,9                                              2137
38399         TEMPO(J) = TEMPO(J)+RNINV(I,K)*COND(J,K)                  2138  N^-1C^T
      DO 38400 J=1,6                                                    2139
        SIG2(J,I) = 0.                                                  2140
          DO 38400 K=1,6                                                2141
38400       SIG2(J,I) = SIG2(J,I)+TEMPO(K)*CONDIT(K,J)                  2142  N^-1C^T(CN^-1C^T)^-1
      DO 38402 I=1,26                                                   2143
        DO 38401 J=1,9                                                  2144
          TEMPO(J) = 0.                                                 2145
            DO 38401 K=1,6                                              2146
38401         TEMPO(J)=TEMPO(J)+SIG2(K,I)*COND(K,J)                     2147  N^-1C^T(CN^-1C^T)^-1C
          DO 38402 J=1,26                                               2148
            RN(I,J) = 0.                                                2149
              DO 38402 K=1,9                                            2150
38402           RN(I,J)=RN(I,J)+TEMPO(K)*RNINV(K,J)                     2151  N^-1C^T(CN^-1C^T)^-1CN^-1
      DO 38403 I=1,26                                                   2152
        DO 38403 J=1,26                                                 2153
38403     RNINV(I,J) = RNINV(I,J)-RN(I,J)                               2154  N_c^-1 = N^-1 - N^-1C^T(CN^-1C^T)^-1CN^-1
      DO 38404 I=1,26                                                   2155
        DELO(I) = 0.                                                    2156
          DO 38404 J=1,6                                                2157
38404       DELO(I) = DELO(I) - SIG2(J,I)*ALDIF(J)                      2158  Δℓ_c = Δℓ - N^-1C^T(CN^-1C^T)^-1ℓ*
      PRINT 1050                                                        2159
      V1 = 0.                                                           2160  Clear residual sums.
      V2 = 0.                                                           2161
      V3 = 0.                                                           2162
      V4 = 0.                                                           2163
      V18 = 0.                                                          2164
      MCOMP = 1                                                         2165  Clear mean error computation cycle
      DO 39000 I=1,3                                                    2166  indicator.
        SUMV(I) = 0.                                                    2167
        SUMVS(I) = 0.                                                   2168
```

Figure 20.--Continued (page 47 of 57)

```
39000   CT(I) = 0.
      GO TO 2570
C
C  PUNCH DATA FOR DIRECTION AND TRIANGULATION PROGRAMS.
C
  384 PUNCH 1124,(NAMES(J),J=15,18),(NAMES(J),J=26,29),(CAMSTA(J),J=1,8)
     1 ,NAMES(1),(NAMES(J),J=19,25),EHRPR1,TOLD,TEMP,PRESS,WC,WF
      GO TO (38407,38408,38409,38409,38409,38409,38409,38409),LSIG
38407 GO TO (38409,38409,38409,38409,38408,38408,38410),III
38408 PUNCH 1125,(OPAR(K),K=4,6),OPAR(10)
      GO TO 38411
38409 PUNCH 1125,(OPAR(K),K=7,10)
      GO TO 38411
38410 PUNCH 1125,(OPAR(K),K=1,3),OPAR(10)
38411 PUNCH 1125,(OPAR(K),K=11,26),XS,YS
      IPROG = 21
      IF(INO(9).EQ.0) GO TO 38412
      IF(NOPRE.EQ.0) PUNCH 1131,TALPH(1),TALPH(2),TALPH(3),NAMES(19),
     1 NAMES(15),TYP(1),IPROG
      IF(NOPOST.EQ.0) PUNCH 1131,TALPH(4),TALPH(5),TALPH(6),NAMES(19),
     1 NAMES(15),TYP(2),IPROG
      IF(NOVENT.EQ.0) PUNCH 1131,TALPH(7),TALPH(8),TALPH(9),NAMES(19),
     1 NAMES(15),TYP(3),IPROG
      PUNCH 1074,(OPAR(K),K=11,14),IPROG,LASTPT
      GO TO 38416
38412 GO TO (38413,38413,38413,38413,38414,38414,38415),III
38413 PUNCH 1131,(OPAR(K),K=4,6),NAMES(19),NAMES(15),NAMES(10),IPROG
      PUNCH 1074,(OPAR(K),K=11,14),IPROG,LASTPT
      GO TO 38416
38414 PUNCH 1131,(OPAR(K),K=4,6),NAMES(19),NAMES(15),NAMES(10),IPROG
      PUNCH 1074,(OPAR(K),K=11,14),IPROG,LASTPT
      GO TO 38416
38415 PUNCH 1131,(OPAR(K),K=4,6),NAMES(19),NAMES(15),NAMES(10),IPROG
      PUNCH 1074,(OPAR(K),K=11,14),IPROG,LASTPT
38416 IF(LSIG.EQ.1) GO TO 38422
       GO TO (38420,38420,38420,38420,38421,38421,38423),III
38420 I=3
       GO TO 38424
38421 I=2
       GO TO 38424
38422 GO TO (38423,38421,38423,38420,38423,38421,38423),III
38423 I=1
38424 N=0
        DO 38426 J=1,3
          J2=3*I
          J1=J2-3+J
            DO 38425 K=J1,J2
              N=N+1
38425         B(N)=RNINV(J1,K)
            DO 38426 K=10,26
              N=N+1
38426         B(N)=RNINV(J1,K)
        DO 38427 K=10,26
          DO 38427 L=K,26
```

Figure 20.--Continued (page 48 of 57)

```
          N=N+1                                                          2220
38427     B(N)=RNINV(K,L)                                                2221
      PUNCH (166,NAMES(1),NAMES(26),NAMES(15),(NAMES(J),J=19,25),ERRPR   2222
      PUNCH 1170,(NAMES(J),J=26,29),(CAMSTA(J),J=1,8)                    2223
      PUNCH 7,XANGLE,YANGLE                                              2224
      PUNCH 1167,OPAR(J2-2),OPAR(J2-1),OPAR(J2),(OPAR(J),J=10,26),XS,YS  2225
      PUNCH 1168,TYP(I)  ,TALPH(J2-2),TALPH(J2-1),TALPH(J2)              2226
      PUNCH 1169,TOLD,TEMP,PRESS,WC,WF                                   2227
      PUNCH 1172,(B(K),K=1,207)
      PUNCH 1173,(B(K),K=208,210),LASTPT
       IF(I.EQ.1.AND.III.EQ.7) GO TO 38428                               2229
       IF(I.EQ.2.AND.III.EQ.5) GO TO 38428                               2230
       IF(I.EQ.2.AND.III.EQ.6) GO TO 38428                               2231
       IF(I.EQ.3) GO TO 38428                                            2232
       IF(I.EQ.1.AND.III.EQ.3) I=I+1                                     2233
       I=I+1                                                             2234
        GO TO 38424                                                      2235
38428 REWIND 1                                                           2236
      REWIND 2                                                           2237
      GO TO 100                                                          2238
    1 FORMAT(A4,2X,2A8,A1,56X,I1)                                        2239
    2 FORMAT(10A8)                                                       2240
    3 FORMAT(5(I1,4X),I2,3X,3(I1,4X),I2,3X,I1,2(3X,I2),1X,E14.8,I1,      2241
     1  1X,I1)                                                           2242
    4 FORMAT (8E10.6)                                                    2243
    5 FORMAT( A4,2X,2A8,A1/A4,2X,A8,6X,A3,2X,F11.8,2X,3(2F3.0,F8.4)/A8,
     1 2X,3A8,2(2X,I3),14X,F8.2)
    6 FORMAT(A4,2X,3A8,2X,F1.0,2X,2(F3.0,1X,F2.0,1X,F7.4,3X),F11.4)      2246
    7 FORMAT(4X,2(2X,F8.4))                                              2247
    8 FORMAT(29X,2(6X,F3.0,1X,F2.0,1X,F4.1))                             2248
    9 FORMAT(12X,5E12.8)                                                 2249
   10 FORMAT(1X,E14.8,1X,E14.8,1X,E14.8)                                 2250
   11 FORMAT(6E12.8)                                                     2251
   12 FORMAT(5(2I2,E12.8))                                               2252
   13 FORMAT(2E14.8,3E10.4,2X,I6,2X,I3,1X,A6)                            2253
   14 FORMAT(2E14.8,32X,I6,2X,I3,1X,A6)                                  2254
   15 FORMAT(2F12.10,3E10.4,2I6,F11.8,2X,I1)                             2255
   16 FORMAT(79X,I1)                                                     2256
   17 FORMAT(3(1X,E14.8),2(2X,A4),2X,A8,4X,I2/3(1X,E14.8),1X,3A8,1X,I2/  2257
     1 3(1X,E14.8),2(2X,A4),2X,A8,4X,I2/3(1X,E14.8),2(2X,A4),2X,A8,4X,   2258
     2 I2/3(1X,E14.8),2(2X,A4),2X,A8,4X,I2/1X,E14.8,32X,2(A4,2X),A8,4X,  2259
     3 I2/3(1X,E14.8),2(2X,A4),2X,A8,4X,I2)                              2260
   18 FORMAT(4(1X,E14.8))                                                2261
   19 FORMAT(4(1X,E14.8),1X,I6,1X,A4)                                    2262
 1001 FORMAT(1H1/32X,54H SINGLE CAMERA ORIENTATION WITH CELESTIAL OBSERV 2263
     1ATIONS/////)                                                       2264
 1002 FORMAT(20X,10A8/////)                                              2265
 1003 FORMAT(25X,13H STATION NAME,15X,9H LATITUDE,9X,10H LONGITUDE,8X,   2266
     110H ELEVATION//20X,3A8,7X,2(2I3,F8.4,5X),F11.4/////)               2267
 1004 FORMAT(16H PROBLEM NUMBER ,A4,6X,14H EVENT NUMBER ,A4,6X,17H CAMER 2268
     1A NUMBER   ,A8,6X,21H DATE OF COMPUTATION ,2A8,A1//16H STATION NUM 2269
     2BER ,A4,6X,14H PLATE NUMBER ,A4,6X,22H TRACKING UNIT NUMBER ,A3,   2270
     3 6X,21H DATE OF OBSERVATION ,2A8,A1/////)
 1005 FORMAT(50X,18H PROBLEM CONSTANTS//33X,18H PROBLEM TOLERANCE,21X,   2272
```

Figure 20.--Continued (page 49 of 57)

```
     1E14.8//33X,31H REFRACTION TOLERANCE (SEC ARC),8X,E14.8/)
1006 FORMAT(33X,21H DISTORTION TOLERANCE,18X,E14.8/)
1007 FORMAT(1H1////40X,37H FICTITIOUS DIAGONAL POINT DIRECTIONS)
1008 FORMAT(33X,37H STATION TEMPERATURE (DEGREES KELVIN),2X,E14.8/)
1009 FORMAT(33X,30H STATION PRESSURE (MM MERCURY),9X,E14.8/)
1010 FORMAT(33X,36H DISTORTION CURVE CROSSOVER DISTANCE,3X,E14.8/)
1011 FORMAT(33X,26H DISTORTION CURVE INTERVAL,13X,E14.8/)
1012 FORMAT(33X,34H DISTORTION CURVE MAXIMUM DISTANCE,5X,E14.8/////)
1013 FORMAT(33X,11H COMPARATOR/35X,7H NUMBER,23X,17H NAME OF OPERATOR//
     135X,A8,19X,3A8)
1014 FORMAT(1H1/49X,20H PROBLEM DESCRIPTORS/////)
1015 FORMAT(39X,32H MINIMUM SOLUTION COMPUTED WITH ,I2,6H STARS///)
1016 FORMAT(45X,29H NO MINIMUM SOLUTION COMPUTED///)
1017 FORMAT(51X,17H PRE-RUN COMPUTED///)
1018 FORMAT(34X,50H COMMON SCALAR COMPUTED FOR X AND Y (CX EQUALS CY)//
     1/)
1019 FORMAT(23X,73H INDIVIDUAL SCALAR COMPONENTS COMPUTED FOR X AND Y (
     1CX DOES NOT EQUAL CY)///)
1020 FORMAT(39X,41H COMMON WEIGHT MATRIX FOR ALL KNOWN STARS///)
1021 FORMAT(38X,43H INDIVIDUAL WEIGHT MATRICES FOR KNOWN STARS///)
1022 FORMAT(38X,42H COMMON WEIGHT MATRIX FOR ALL PLATE POINTS///)
1023 FORMAT(37X,44H INDIVIDUAL WEIGHT MATRICES FOR PLATE POINTS///)
1024 FORMAT(37X,44H REFRACTION COEFFICIENT CORRECTIONS COMPUTED///)
1025 FORMAT(36X,47H NO REFRACTION COEFFICIENT CORRECTIONS COMPUTED///)
1026 FORMAT(32X,54H RADIAL AND TANGENTIAL LENS DISTORTION CURVES COMPUT
     1ED///)
1027 FORMAT(42X,35H NO LENS DISTORTION CURVES COMPUTED///)
1028 FORMAT(40X,32H RADIAL LENS DISTORTION COMPUTED///)
1029 FORMAT(23X,72H CAMERA AZIMUTH AND ZENITH DISTANCE USED TO CONSTRAI
     1N ORIENTATION ANGLES//57X,14H DEG  MIN  SEC/47X,11H AZIMUTH   ,I3,
     2I4,F6.1/47X,11H ZEN DIS   ,I3,I4,F6.1///)
1030 FORMAT(/85X,12H MEAN ERRORS,6X,15H ERROR ELLIPSES/
     16X,13H RT ASCENSION,1X,13H RT ASCENSION,
     2     1X,12H DECLINATION,3X,8H AZIMUTH,5X,10H ELEVATION,7X,12H RA*C
     3OS(DEC),2X,5H DECL,5X,2H A,6X,2H B,1X,6H THETA/6X,12H HR MIN  SEC,
     42X,2(12H DEG MIN SEC,2X),2(12H DEG MIN SEC,1X),9X,2(8H SEC ARC,
     51X),3X,8H SEC ARC,4X,4H DEG/)
1031 FORMAT(22X,76H FINAL CAMERA ORIENTATION ANGLES TRANSFORMED TO A RE
     1FERERENCE STATION SYSTEM//43X,8H STATION/15X,23H REFERENCE STATION
     2 NAME,5X,7H NUMBER,8X,9H LATITUDE,9X,10H LONGITUDE,8X,10H ELEVATIO
     3N//15X,3A8,6X,A4,2X,2(5X,2I3,F8.4),5X,F11.4///)
1032 FORMAT(4H PRE,2X,2(1X,I2),F7.3,2X,I3,1X,I2,   F6.2,1X,2(1X,I2),F6.
     12,2X,I3,1X,   I2,F6.2,1X,2(1X,I2),F6.2,1X,7H ORIENT,2(2X,F7.3),2(1
     2X,F7.3),1X,F5.0/75X,6H TOTAL,2(2X,F7.3),2(1X,F7.3),1X,F5.0)
1033 FORMAT(///26X,66H FINAL CAMERA ORIENTATION ANGLES TRANSFORMED TO A
     1 MEAN POLE SYSTEM//21X,28H X-CORRECTION ANGLE (SEC) = ,F8.4,5X,28H
     2 Y-CORRECTION ANGLE (SEC) = ,F8.4)
1034 FORMAT(39X,48H FINAL CAMERA ORIENTATION ANGLES NOT TRANSFORMED)
1035 FORMAT(/////42X,33H TOTAL NUMBER OF STARS IN PROBLEM//38X,12H KNOW
     1N STARS,I5,5X,14H UNKNOWN STARS,I5///37X,44H TOTAL NUMBER OF PLATE
     2 OBSERVATIONS OF STARS//36X,47H ORIENTATION   OF KNOWN STARS  OF U
     3NKNOWN STARS//40X,4H PRE,11X,I4,13X,I4/40X,5H POST,10X,I4,13X,I4/
     4 39X,6H EVENT,10X,I4,13X,I4//48X,21H TOTAL,15,6X))
1036 FORMAT(1H(/40X,38H NUMBER OF ITERATIONS WILL NOT EXCEED /)
```

Figure 20.--Continued (page 50 of 57)

```
1037 FORMAT(48X,20H MINIMUM SOLUTION - ,I2)                                2325
1038 FORMAT(48X,20H MAXIMUM SOLUTION - ,I2)                                2326
1039 FORMAT(///51X,17H MINIMUM SOLUTION)                                   2327
1040 FORMAT(///51X,17H MAXIMUM SOLUTION)                                   2328
1041 FORMAT(//60X,23H NUMBER OF     NUMBER OF/59X,23H OBSERVATIONS  UNKN   2329
    1OWNS//37X,23H ORIENTATION PARAMETERS,6X,I2,10X,I2/37X,13H STAR  CO   2330
    2ORDINATES ,21X,I4/37X,18H PLATE COORDINATES,9X,I4//37X,7H TOTALS     2331
    3,12X,21X,I4//47X,20H DEGREES OF FREEDOM ,I4////48X,23H LEVEL OF      2332
    4DISCREPANCIES//53X,E14.8/)                                            2333
1042 FORMAT(5H POST,1X,2(1X,I2),F7.3,2X,I3,1X,I2,   F6.2,1X,2(1X,I2),F6   2334
    1.2,2X,I3,1X,  I2,F6.2,1X,2(1X,I2),F6.2,1X,7H ORIENT,2(2X,F7.3),2(1   2335
    2X,F7.3),1X,F5.0/75X,6H TOTAL,2(2X,F7.3),2(1X,F7.3),1X,F5.0)           2336
1043 FORMAT(6H EVENT,   2(1X,I2),F7.3,2X,I3,1X,I2,   F6.2,1X,2(1X,I2),F   2337
    16.2,2X,I3,1X, I2,F6.2,1X,2(1X,I2),F6.2,1X,7H ORIENT,2(2X,F7.3),2(1   2338
    2X,F7.3),1X,F5.0/75X,6H TOTAL,2(2X,F7.3),2(1X,F7.3),1X,F5.0)           2339
1044 FORMAT(//6X,17H DIAGONAL POINT (,I1,1H, I1,13H) CENTIMETERS)          2340
1045 FORMAT(53X,E14.8)                                                     2341
1046 FORMAT(///48X,I2,21H ITERATIONS COMPLETED)                            2342
1047 FORMAT(///45X,30H FOUR DIVERGENT STEPS OCCURRED)                      2343
1048 FORMAT(///31X,56H ITERATION RESULTS DIFFER BY LESS THAN TOLERANCE     2344
    1SETTING)                                                              2345
1049 FORMAT(///37X,44H REFRACTION SOLUTION ELIMINATED FROM PROBLEM)        2346
1050 FORMAT(1H1/44X,31H STAR AND PLATE COORDINATE DATA)                    2347
1051 FORMAT(53X,12H (CONTINUED))                                           2348
1052 FORMAT(///19X,24H NORMALIZED STAR WEIGHTS,16X,12H CATALOG NO.,18X,    2349
    115H STAR RESIDUALS/6H  STAR/55H NUMBER  RIGHT ASCENSION    DECLINA   2350
    2TION     CORRELATION,18X,47H R A (SEC TIME)   R A (SEC ARC)  DECL    2351
    3(SEC ARC)//1X,I5,2X,3(2X,E14.8),6X,A6,4X,3(2X,E14.8))                 2352
1053 FORMAT(1H1/44X,31H STAR AND PLATE COORDINATE DATA/53X,12H (CONTINU   2353
    1ED))                                                                  2354
1054 FORMAT(/71X,4H PLX,9X,4H PLY,8X,5H PLXY/15X,51H ALL DIAGONAL POINT   2355
    1S COMPUTED WITH PLATE WEIGHTS OF,2X,3(E10.4,3X)/)                     2356
1055 FORMAT(/6H POINT,2X,8H ORIENT-,2X,23H PLATE COORDINATES (MM),7X,     2357
    1 25H NORMALIZED PLATE WEIGHTS,7X,14H SIDEREAL TIME,5X,20H PLATE RE   2358
    2S (MICRONS)/7H NUMBER,2X,6H ATION,7X,3H LX,9X,3H LY,10X,4H PLX,7X,   2359
    3 4H PLY,7X,5H PLXY,6X,12H HR MIN  SEC,9X,4H VLX,9X,4H VLY)           2360
1056 FORMAT(/20X,24H SPACE ANGLE  MEAN ERROR,11X,36H COMPONENTS OF SPAC   2361
    1E ANGLE (ARC SEC)/21X,2(10H (ARC SEC),2X),    3X,51H R.A.*COS DEC    2362
    2  DECLINATION  AZ*COS ELEV  (ELEVATION/)                             2363
1057 FORMAT(/////35X,49H NO INVERSE FOR REDUCED CONDITION EQUATION MATR   2364
    1IX)                                                                   2365
1058 FORMAT(1X,I6,3X,A5,2X,F12.5,F12.5,2X,3(2X,E9.3),4X,2(1X,I2),F9.4,    2366
    12(F13.5))                                                             2367
1059 FORMAT(///15X,82H PLATE COORDINATE DATA AND COMPUTED STAR POSITION   2368
    1 FOR UNKNOWN STAR - STAR NUMBER  ,I6,//72X,12H MEAN ERRORS,18X,15H   2369
    2 ERROR ELLIPSES/65X,15H RA * COS(DECL),2X,12H DECLINATION,6X,2H A,   2370
    37X,2H B,2X,6H THETA/2(16H RIGHT ASCENSION,3X),2X,12H DECLINATION,    2371
    *10X,3(6X,10H (SEC ARC)),4X,6H (DEG)/3H HR,1X,4H MIN,2X,4H SEC,1X,    2372
    52(4X,4H DEG,1X,4H MIN,1X,4H SEC),8X,7H ORIENT/60X,5H ONLY,4X,F7.3,   2373
    68X,F7.3,6X,2(F7.3,2X),F4.0/1X,2(I2,2X),F7.3,4X,I3,2X,I2,2X,F6.2,     2374
    74X,2(I2,2X),F6.2,6X,6H TOTAL,4X,F7.3,8X,F7.3,6X,2(F7.3,2X),F4.0)     2375
1060 FORMAT(///36X,46H AZIMUTH-ZENITH DISTANCE PLATE COORDINATE DATA)     2376
1061 FORMAT(///28X,62H PLATE COORDINATE DATA FOR POINTS NOT USED IN PRE   2377
    1-RUN SOLUTION)                                                        2378
```

Figure 20.--Continued (page 51 of 57)

```
1062 FORMAT(/36X,47H (PLATE COORDINATE DATA CONTINUED ON NEXT PAGE))
1063 FORMAT(1H1/42X,34H CAMERA ORIENTATION PARAMETER DATA)
1064 FORMAT(10X,20H ORIGINAL PARAMETERS,3X,4(5X,E14.8))
1065 FORMAT(10X,20H COMPUTED PARAMETERS,3X,4(5X,E14.8))
1066 FORMAT(10X,20H WEIGHTS  (DIAGONAL),3X,4(5X,E14.8))
1067 FORMAT(10X,10H RESIDUALS,13X,4(5X,E14.8))
1068 FORMAT(10X,12H CORRECTIONS,11X,4(5X,E14.8))
1069 FORMAT(10X,12H MEAN ERRORS,11X,4(5X,E14.8))
1070 FORMAT(//29X,16H EPSILON (GRADS),9X,3H XP,16X,3H YP,16X,3H CX,16X,
     1 3H CY)
1071 FORMAT(//32X,8H DELTA X,11X,8H DELTA Y,13X,3H K1,16X,3H K2,16X,3H
     1K3)
1072 FORMAT(//35X,3H K4,16X,3H K5,11X,15H PHI-T (GRADS),10X,3H XS,16X,3
     1H YS)
1073 FORMAT(//42X,6H TAU-1,13X,6H TAU-2,13X,6H TAU-3,13X,6H TAU-4)
1074 FORMAT(4(1X,E14.8),11X,I2,6X,I1)
1075 FORMAT(1H1/////50X,18H COMBINED.SOLUTION)
1076 FORMAT(1H1/40X,38H SUMS OF WEIGHTED SQUARES OF RESIDUALS////45X,6H
     1 PLATE,9X,E14.8/45X,6H STARS,9X,E14.8/45X,12H ORIENTATION,3X,E14.8
     2//45X,12H GRAND TOTAL,3X,E14.8//////)
1077 FORMAT(1H1/////49X,21H NO COMBINED SOLUTION)
1078 FORMAT(//26X,67H ORIENTATIONS FROM PRE AND EVENT STARS CONSTRAINED
     1 TO COMMON VALUES)
1079 FORMAT(32X,41H MEAN ERROR OF UNIT WEIGHT FOR PROBLEM   ,E14.8/////
     1//////)
1080 FORMAT(36X,46H DETERMINANT OF REDUCED NORMAL EQUATION SYSTEM//)
1081 FORMAT(43X,19H MINIMUM SOLUTION  ,E14.8/)
1082 FORMAT(43X,19H MAXIMUM SOLUTION  ,E14.8)
1083 FORMAT(1H1/38X,42H INVERSE OF REDUCED NORMAL EQUATION SYSTEM)
1084 FORMAT(/5X,12H ALPHA (PRE),12X,12H OMEGA (PRE),12X,12H KAPPA (PRE)
     112X,13H ALPHA (POST),11X,13H OMEGA (POST)/)
1085 FORMAT (/5X,13H KAPPA (POST),10X,14H ALPHA (EVENT)10X,14H OMEGA (E
     1VENT),10X,14H KAPPA (EVENT),14X,9H EPSILON/)
1086 FORMAT(/10X,3H XP21X,3H YP21X,3H CX21X,3H CY19X,8H DELTA X/)
1087 FORMAT(/7X,8H DELTA Y,19X,3H K1,21X,3H K2,21X,3H K3,22X,3H K4/)
1088 FORMAT(2H ( I2,1H, I2,1H) E14.8,4(2X,2H ( I2,1H, I2,1H) E14.8))
1089 FORMAT(22X,4(2X,2H ( I2,1H, I2,1H) E14.8))
1090 FORMAT(46X,3(2X,2H ( I2,1H, I2,1H) E14.8))
1091 FORMAT(70X,2(2X,2H ( I2,1H, I2,1H) E14.8))
1092 FORMAT(96X,2H ( I2,1H, I2,1H) E14.8)
1093 FORMAT(//26X,66H ORIENTATIONS FROM PRE AND POST STARS CONSTRAINED
     1TO COMMON VALUES)
1094 FORMAT(//26X,68H ORIENTATIONS FROM POST AND EVENT STARS CONSTRAINE
     1D TO COMMON VALUES)
1095 FORMAT(//32X,54H DECISION FOR COMBINING SEPARATE ORIENTATIONS BASE
     1D ON/33X,11H FACTOR OF ,F8.2,32H TIMES MEAN ERROR OF SPACE ANGLE)
1096 FORMAT(/////33X,59H TRANSFORMED CAMERA ORIENTATION ANGLES FOR EVEN
     1T STARS ONLY//38X,14H ALPHA (GRADS),5X,14H OMEGA (GRADS),5X,14H KA
     2PPA (GRADS)/33X,3(5X,E14.8))
1097 FORMAT(1H1/48X,22H DISTORTION CURVE DATA)
1098 FORMAT(1H1/47X,22H DISTORTION CURVE DATA//17X,32H COMPUTED FOCAL L
     1ENGTHS (METERS) ,15X, 52H ORIENTATION ANGLE FOR DECENTERED DISTORT
     2ION (GRADS)//9X,3H CX,7X,11H MEAN ERROR,7X,3H CY,7X,11H MEAN ERROR
     319X,6H PHI-T,12X,11H MEAN ERROR//3X,2(2X,F10.7,2X,E14.8),6X,2(7X,
```

Figure 20.--Continued (page 52 of 57)

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     4E14.8)///6X,  56H FOCAL LENGTH FOR TRANSFORMED RADIAL DISTORTION I
     5METERS)/ 74X,17H DISTORTION CURVE/20X,7H C-STAR,12X,11H MEAN ERROR
     619X,26H CROSS-OVER POINT (METERS),6X,12H K-ZERO TERM//1X,F10.7,
     7I0X,E14.8,26X,F6.3,16X,E14.8)
1099 FORMAT(//50X,14H RADIAL CURVES,34X, 17H DECENTERED CURVE//2X,16H R
     1ADIAL DISTANCE,3X,20H COMPUTED DISTORTION,12H  MEAN ERROR, 3X,23H
     2TRANSFORMED DISTORTION,12H  MEAN ERROR,3X,11H DISTORTION, 12H  MEA
     3N ERROR/9X,5H (MM),6X,2(7X,10H (MICRONS)),2X,2(8X,10H (MICRONS)),
     42X,2(2X,10H (MICRONS)))
1101 FORMAT(/10X,I3,8X,2(8X,F8.3),2X,2(10X,F8.3),2X,2(4X,F8.3))
1102 FORMAT(48X,22H (CONTINUED NEXT PAGE))
1103 FORMAT(/////40X,36H ERROR 1 - STAR NUMBERS DO NOT AGREE//37X,13H S
     1TAR NUMBER ,I5,5X,14H POINT NUMBER ,I6)
1104 FORMAT(/////23X,67H ERROR 2 - NO INVERSE FOR STAR COORDINATE SUB-M
     1ATRIX - STAR NUMBER ,I5)
1105 FORMAT(/////42X,34H ERROR 3 - NOT ENOUGH OBSERVATIONS)
1106 FORMAT(/////31X,56H ERROR 4 - NO INVERSE FOR REDUCED NORMAL EQUATI
     1ON MATRIX)
1107 FORMAT(///43X,3H XS,16X,3H YS)
1108 FORMAT(10X,20H ORIGINAL PARAMETERS,3X,2(5X,E14.8))
1109 FORMAT(10X,20H COMPUTED PARAMETERS,3X,2(5X,E14.8))
1110 FORMAT(1H1/48X,22H REFRACTION CURVE DATA////32X,41H ZENITH DISTANC
     1E OF CAMERA CENTRAL RAY   I2,8H DEGREES////20X,16H ZENITH DISTANCE
     2 4X,17H GIVEN REFRACTION,3X,20H COMPUTED REFRACTION,5X,11H MEAN ER
     3ROR/23X,10H (DEGREES),3X,3(4X,17H (SECONDS OF ARC)))
1111 FORMAT(/28X,I2,2X,3(13X,F8.3))
1112 FORMAT(///50X,20H REFRACTION SOLUTION)
1113 FORMAT(///12X,24H NORMALIZED STAR WEIGHTS,6X,30H MEAN ERRORS BEFOR
     1E ADJUSTMENT,17X,15H STAR RESIDUALS/6H  STAR/12OH NUMBER     R A
     2     DECL        CORR       R A (SEC ARC)  DECL (SEC ARC)  R A (SEC
     3TIME)   R A (SEC ARC)  DECL (SEC ARC)//1X,I5,1X,3(1X,E10.4),5(2X,
     4E14.8))
1114 FORMAT(/6H POINT,2X,8H ORIENT-,3X,14H SIDEREAL TIME,7X,25H NORMALI
     1ZED PLATE WEIGHTS,15X,12H PLATE COORD,3X,12H M.E. BEFORE,3X,10H PL
     2ATE RES/7H NUMBER,2X,6H ATION,4X,12H HR MIN  SEC,8X,4H PLX,7X,4H P
     3LY,7X,5H PLXY,17X,5H (MM),6X,14H ADJ (MICRONS),2X,10H (MICRONS))
1115 FORMAT(/1X,I6,3X,A5,4X,2(1X,I2),F8.4,2X,3(2X,E9.3),7X,3H LX,3(F14.
     15)/75X,3H LY,3(F14.5))
1116 FORMAT(10X,24H MEAN ERRORS OF UNKNOWNS,4X,E14.8,5X,E14.8/10X,23H M
     1EAN ERRORS BEFORE ADJ,2(5X,E14.8)/10X,24H APPROXIMATE MEAN ERRORS,
     24X,E14.8,5X,E14.8)
1117 FORMAT(15X,56H COMMON MEAN ERROR OF PLATE MEASUREMENTS BEFORE ADJ
     1  LX ,E14.8,5H  LY ,E14.8/)
1118 FORMAT(15X,53H COMMON MEAN ERROR OF KNOWN STARS BEFORE ADJ      RA
     1 ,E14.8,8H   DECL ,E14.8/)
1119 FORMAT(///30X,65H ALL MEAN ERRORS COMPUTED USING PROBLEM MEAN ERRO
     1R OF UNIT WEIGHT)
1120 FORMAT(///23X,65H ALL MEAN ERRORS COMPUTED USING INPUT MEAN ERROR
     1OF UNIT WEIGHT =,E14.8)
1121 FORMAT(////29X,66H THE FOLLOWING PLATE POINTS OR STARS HAVE BEEN R
     1EJECTED IN ADVANCE/)
1122 FORMAT(///15X,82H PLATE COORDINATE DATA AND COMPUTED STAR POSITION
     1 FOR UNKNOWN STAR - STAR NUMBER  ,I6,//72X,12H MEAN ERRORS,18X,15H
     2 ERROR ELLIPSES/65X,15H RA * COS(DECL),2X,12H DECLINATION,6X,2H A,
```

Figure 20.--Continued (page 53 of 57)

```
     37X,2H B,2X,6H THETA/2(16H RIGHT ASCENSION,3X),2X,12H DECLINATION,     2487
     410X,3(6X,10H (SEC ARC)),4X,6H (DEG)/3H HR,1X,4H MIN,2X,4H SEC,1X,     2488
     52(4X,4H DEG,1X,4H MIN,1X,4H SEC),8X,7H ORIENT/60X,5H ONLY,4X,F7.3,    2489
     68X,F7.3,6X,2(F7.3,2X),4H NO /1X,2(I2,2X),F7.3,4X,I3,2X,I2,2X,F6.2,    2490
     74X,2(I2,2X),F6.2,6X,6H TOTAL,4X,F7.3,8X,F7.3,6X,2(F7.3,2X),F4.0)      2491
1123 FORMAT(/7X,46H  MEAN ERROR OF SELECTED FOCAL LENGTH(METERS) E14.8)    2492
1124 FORMAT(3X,A4,3X,2A8,A1/3X,A4,3A8,I1,2(F5.0,F4.0,F9.4),E14.8/1X,A4,    2493
     1  3X,A4,3X,A8,1X,A3,4X,A8,1X,3A8,2X,E14.8/5X,5(1X,E14.8))             2494
1125 FORMAT(8X,E16.10,1X,E16.10,1X,E16.10,1X,E16.10)                        2495
1126 FORMAT(/36X,47H TIME CORRECTIONS APPLIED TO PLATE OBSERVATIONS//53
     1X,16H HRS  MIN    SEC,/46X,4H PRE,4X,2(F3.0,2X),F8.4/46X,5H POST,3
     2X,2(F3.0,2X),F8.4/46X,6H EVENT,2X,2(F3.0,2X),F8.4/////)
1127 FORMAT(/35X,52H ERROR - DISTORTION NOT CONVERGING FOR POINT NUMBER    2497
     1 ,I6/48X,10H DELRDI = ,E14.8,14H      DELTAT = ,E14.8/)               2498
1128 FORMAT(34X,52H SUMS OF PLATE RESIDUALS FOR POINTS USED IN SOLUTION    2499
     1//25X,3H VX,40X,3H VY,25X,14H ALGEBRAIC SUM/12X,9H POSITIVE,10X,     2500
     29H NEGATIVE,15X,9H POSITIVE,10X,9H NEGATIVE,15X,10H VX AND VY//      2501
     3 2(10X,E14.8,5X,E14.8),10X,E14.8///)                                 2502
1129 FORMAT(///15X,82H PLATE COORDINATE DATA AND COMPUTED STAR POSITION    2503
     1 FOR UNKNOWN STAR - STAR NUMBER  ,I6.//72X,12H MEAN ERRORS,18X,15H   2504
     2 ERROR ELLIPSES/65X,15H RA * COS(DECL),2X,12H DECLINATION,6X,2H A,   2505
     37X,2H B,2X,6H THETA/2(16H RIGHT ASCENSION,3X),2X,12H DECLINATION,    2506
     410X,3(6X,10H (SEC ARC)),4X,6H (DEG)/3H HR,1X,4H MIN,2X,4H SEC,1X,    2507
     52(4X,4H DEG,1X,4H MIN,1X,4H SEC),8X,7H ORIENT/60X,5H ONLY,4X,F7.3,   2508
     68X,F7.3,6X,2(F7.3,2X),4H NO /1X,2(I2,2X),F7.3,4X,I3,2X,I2,2X,F6.2,   2509
     74X,2(I2,2X),F6.2,6X,6H TOTAL,4X,F7.3,8X,F7.3,6X,2(F7.3,2X),4H NO )   2510
1130 FORMAT(///15X,82H PLATE COORDINATE DATA AND COMPUTED STAR POSITION    2511
     1 FOR UNKNOWN STAR - STAR NUMBER  ,I6.//72X,12H MEAN ERRORS,18X,15H   2512
     2 ERROR ELLIPSES/65X,15H RA * COS(DECL),2X,12H DECLINATION,6X,2H A,   2513
     37X,2H B,2X,6H THETA/2(16H RIGHT ASCENSION,3X),2X,12H DECLINATION,    2514
     410X,3(6X,10H (SEC ARC)),4X,6H (DEG)/3H HR,1X,4H MIN,2X,4H SEC,1X,    2515
     52(4X,4H DEG,1X,4H MIN,1X,4H SEC),8X,7H ORIENT/60X,5H ONLY,4X,F7.3,   2516
     68X,F7.3,6X,2(F7.3,2X),F4.0/1X,2(I2,2X),F7.3,4X,I3,2X,I2,2X,F6.2,     2517
     74X,2(I2,2X),F6.2,6X,6H TOTAL,4X,F7.3,8X,F7.3,6X,2(F7.3,2X),4H NO )   2518
1131 FORMAT(3(1X,E14.8),2X,A4,2X,A4,2X,A5,7X,I2)                            2519
1132 FORMAT(/1X,5H STAR,10X,23H MEAN PLATE COORDINATES,37X,21H MEAN PLA    2520
     1TE RESIDUALS,/7H NUMBER,10X,3H LX,14X,3H LY,25X,4H VLX,12X,4H VLY,   2521
     27X,12H RADIAL COMP,2X,16H TANGENTIAL COMP//1X,I5,3X,2(3X,E14.8),12   2522
     3X,4(2X,E14.8))                                                       2523
1133 FORMAT(20I4)                                                          2524
1134 FORMAT(63H INCORRECT STAR(S) HAVE BEEN SPECIFIED FOR THE MINIMUM S    2525
     1OLUTION)                                                             2526
1135 FORMAT(16,73X,I1)                                                     2527
1136 FORMAT(/34H TOO MANY REJECTED STARS OR POINTS)                        2528
1137 FORMAT(/27X,36H ALL PLATE OBSERVATIONS FOR STAR NO.I5,14H (CATALOG
     1 NO. I6,6H) HAVE/27X,66H BEEN REJECTED. SO THE STAR HAS BEEN ELIMI
     2NATED FROM THIS SOLUTION)
1138 FORMAT(50X,14H STAR NUMBER  ,I6)
1139 FORMAT(50X,14H PLATE POINT  ,I6)
1140 FORMAT(2H$Z,30X,66H MEAN PLATE COORDINATES OF STARS USED IN SINGLE    2533
     1 CAMERA ORIENTATION)                                                 2534
1141 FORMAT(2H8 ,23X,10A8)                                                 2535
1142 FORMAT(2H8 ,46X,8H EVENT  ,A4,10X,8H PLATE  ,A4)                      2536
1143 FORMAT(/10X,3H K5,19X,6H PHI-T18X,6H TAU-1,18X,6H TAU-2,19X,6H TAU    2537
```

Figure 20.--Continued (page 54 of 57)

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    1-3/)
1144 FORMAT(/8X,6H TAU-4)
1145 FORMAT(2H ( I7,1H, I2,1H) E14.8)
1146 FORMAT( / 47X,14H ALPHA (GRADS),5X,14H OMEGA (GRADS),5X,14H KAPPA
    1(GRADS)/19X,20H ORIGINAL PARAMETERS,8X,3(E14.8,5X)/19X,20H COMPUTE
    2D PARAMETERS,8X,3(E14.8,5X)/19X,19H WEIGHTS (DIAGONAL),9X,3(E14.8,
    35X)/19X,10H RESIDUALS,18X,3(E14.8,5X)/19X,12H CORRECTIONS,16X,3(E1
    44.8,5X)/19X,24H MEAN ERRORS OF UNKNOWNS,4X,3(E14.8,5X))
1147 FORMAT(//25X,34H ORIENTATION ANGLES FROM PRE STARS,19X,35H ORIENTA
    1TION ANGLES FROM POST STARS)
1148 FORMAT(//25X,34H ORIENTATION ANGLES FROM PRE STARS,18X,36H ORIENTA
    1TION ANGLES FROM EVENT STARS)
1149 FORMAT(//24X,35H ORIENTATION ANGLES FROM POST STARS,18X,36H ORIENT
    1ATION ANGLES FROM EVENT STARS)
1150 FORMAT(/19X,14H ALPHA (GRADS)3X,14H OMEGA (GRADS),3X,14H KAPPA (GR
    1ADS),5X,14H ALPHA (GRADS),3X,14H OMEGA (GRADS),3X,14H KAPPA (GRADS
    2)/1X,13H ORIGINAL PAR,5X,3(E14.8,3X),2X,3(E14.8,3X)/1X,13H COMPUTE
    3D PAR,5X,3(E14.8,3X),2X,3(E14.8,3X)/1X,13H WEIGHTS-DIAG,5X,3(E14.8
    4,3X),2X,3(E14.8,3X)/1X,10H RESIDUALS,8X,3(E14.8,3X),2X,3(E14.8,3X)
    5/1X,12H CORRECTIONS,6X,3(E14.8,3X),2X,3(E14.8,3X)/1X,12H MEAN ERRO
    6RS,6X,3(E14.8,3X),2X,3(E14.8,3X))
1151 FORMAT(1X,15H POST MINUS PRE,   6X,2(F8.2,4X),4X,2(F8.2,6X),2(F8.2
    1,3X))
1152 FORMAT(1X,17H EVENT MINUS POST,4X, 2(F8.2,4X),4X,2(F8.2,6X),2(F8.2
    1,3X))
1153 FORMAT(1X,16H EVENT MINUS PRE,5X,  2(F8.2,4X),4X,2(F8.2,6X),2(F8.2
    1,3X))

1154 FORMAT(//55X,34H ORIENTATION ANGLES FROM PRE STARS)
1155 FORMAT(//55X,35H ORIENTATION ANGLES FROM POST STARS)
1156 FORMAT(//54X,36H ORIENTATION ANGLES FROM EVENT STARS)
1157 FORMAT(21X 74H INSUFFICIENT TYPE 2 STARS TO COMPUTE INDIVIDUAL CAM
    1ERA ORIENTATION ANGLES)
1158 FORMAT(1X,20H ORIGINAL PARAMETERS,9X,5(E14.8,5X))
1159 FORMAT(1X,20H COMPUTED PARAMETERS,9X,5(E14.8,5X))
1160 FORMAT(1X,19H WEIGHTS (DIAGONAL),10X,5(E14.8,5X))
1161 FORMAT(1X,10H RESIDUALS,19X,5(E14.8,5X))
1162 FORMAT(1X,12H CORRECTIONS,17X,5(E14.8,5X))
1163 FORMAT(1X,24H MEAN ERRORS OF UNKNOWNS 5X,5(E14.8,5X))
1164 FORMAT(/////33X,57H TRANSFORMED CAMERA ORIENTATION ANGLES FOR PRE
    1STARS ONLY//38X,14H ALPHA (GRADS),5X,14H OMEGA (GRADS),5X,14H KAPP
    2A (GRADS)/33X,3(5X,E14.8))
1165 FORMAT(/////33X,58H TRANSFORMED CAMERA ORIENTATION ANGLES FOR POST
    1 STARS ONLY//38X,14H ALPHA (GRADS),5X,14H OMEGA (GRADS),5X,14H KAP
    2PA (GRADS)/33X,3(5X,E14.8))
1166 FORMAT(1X,A4,2X,A4,1X,A5,1X,A4,A8,A3,4(A8),E14.8)
1167 FORMAT(7X,4E17.10)
1168 FORMAT(A5,2X,3(E17.10))
1169 FORMAT(4X,5(E15.8))
1170 FORMAT(2X,A4,3A8,I1,2(F5.0,F4.0,F8.4),E14.8)
1171 FORMAT(/10X,23H PLATE COORDINATES (MM),11X,25H NORMALIZED PLATE WE
    1IGHTS,13X,20H PLATE RES (MICRONS)/13X,3H LX,9X,3H LY,16X,4H PLX,6X
    2,4H PLY,6X,5H PLXY,14X,4H VLX,9X,4H VLY/8X,2(F12.5),9X,3(1X,E9.3),
    3 5X,2(F13.5))
1172 FORMAT(7X,3E20.14)
```

Figure 20.--Continued (page 55 of 57)

```
 1173 FORMAT(7X,3E20.14,12X,I1)
 9999 FORMAT(5(1X,E14.8))                                          2591
      END                                                          2592
T        SUBTYPE,FORTRAN,PUNCH                                     2593
      SUBROUTINE ASIN(SINX,X)                                      2594
      S = -1.                                                      2595
      T = SINX**2                                                  2596
      X = 0.                                                       2597
    2 X = X + SINX                                                 2598
      S = S + 2.                                                   2599
      SINX = SINX*T*S**2/((S+1.)*(S+2.))                           2600
      IF(ABS(SINX)-1.E-18)3,2,2                                    2601
    3 RETURN                                                       2602
      END                                                          2603
T        SUBTYPE,FORTRAN,PUNCH                                     2604
      SUBROUTINE ANGLE(Y,X,ANG)                                    2605
      IF(X)30,20,10                                                2606
   10 IF(Y)13,11,11                                                2607
   11 ANG = ATAN (Y/X)                                             2608
      GO TO 40                                                     2609
   13 ANG = 6.2831853071796 + ATAN (Y/X)                           2610
      GO TO 40                                                     2611
   20 IF(Y)23,22,21                                                2612
   21 ANG = 1.5707963267949                                        2613
      GO TO 40                                                     2614
   22 ANG = 0.                                                     2615
      GO TO 40                                                     2616
   23 ANG = 4.7123889803847                                        2617
      GO TO 40                                                     2618
   30 ANG = 3.1415926535898 + ATAN (Y/X)                           2619
   40 RETURN                                                       2620
      END                                                          2621
T        SUBTYPE,FORTRAN,LMAP,PUNCH                                2622
      SUBROUTINE GRAPH (CT,ANOC,CLASS,SDV,VM,NOC)                  2623
      DIMENSION CT(3),ANOC(3),CLASS(3,32),SDV(3),VM(3),NAMES(33),  2624
     1  TITLE(5,3),DX(64),DY(65),NOC(3)                            2625
      COMMON /AP/ NAMES                                            2626
      DATA (TITLE(I,1),I=1,5)(40H TYPE 2 POINTS (KNOWN STARS) - X VALUES  2627
     1 ),(TITLE(I,2),I=1,5)(40H TYPE 2 POINTS (KNOWN STARS) - Y VALUES )  2628
     2 ,(TITLE(I,3),I=1,5)(40H TYPE 2 POINTS (KNOWN STARS) X+Y VALUES )  2629
      DATA PI(3.1415926535898)                                     2630
      DO 502 I=1,3                                                 2631
        PRINT 1000,(NAMES(L),L=5,14)                               2632
        PRINT 1001,(TITLE(L,I),L=1,5)                              2633
        PRINT 1002,NAMES(19),NAMES(15),SDV(I),CT(I)                2634
        SDV(I) = SDV(I)*1.E+6                                      2635
        VM(I) = VM(I)*1.E+6                                        2636
        AK = -3.25*SDV(I)+VM(I)                                    2637
        BK = 6.5*SDV(I)/ANOC(I)                                    2638
        K = NOC(I) + 1                                             2639
        DY(1) = 0.                                                 2640
        DO 500 L = 1,K                                             2641
          CK = L-1                                                 2642
          DX(2*L-1) = AK+CK*BK                                     2643
```

Figure 20.--Continued (page 56 of 57)

```
        OX(2*L) = OX(2*L-1)
        OY(2*L) = CLASS(I,L)
  500   OY(2*L+1) = OY(2*L)
      YMAX = 6.5*CT(I)/ANCC(I)/SQRT(2.*PI)
      CON = YMAX
      N = K-1
      DO 501 L=1,N
        IF(YMAX.LT.CLASS(I,L)) YMAX=CLASS(I,L)
  501   CONTINUE
      LL = YMAX/20.+1.
      OY = LL
      DX = BK
      KK = 2*K
      XMAX = OX(KK) + BK/2.
      IF(NOC(I).GT.15) DX = 2.*DX
      CALL GRF (OX(1),OY(1),XMAX,YMAX,OX(1),OY(1),DX,DY,2,0,0,KK,
     1  OX(1),OY(1),1,1,2H75,1,0)
      OK = 3.25*SOV(I)/75.
      DO 502 L=1,151
        CK = L-1
        XNC = AK + CK*OK
        YNC = CON*EXP(-.5*((XNC-VM(I))/SOV(I))**2)
        CALL GRF (OX(1),OY(1),XMAX,YMAX,OX(1),OY(1),DX,DY,2,0,1,1,
     1    XNC,YNC,1,1,2H33,-1,0)
  502   CONTINUE
      CALL CRTID(4HSATR)
      RETURN
 1000 FORMAT(2H$2,10A8)
 1001 FORMAT(2H$ ,5A4)
 1002 FORMAT(2H$ ,5HPLATF,1X,A4,5X,5HEVENT,1X,A4,5X,10HMEAN ERROR,1X,
     1 E14.8,5X,16HNUMBER OF POINTS,1X,I4)
      END
T       SUBTYPE,BIN
```

Figure 20.--Concluded (page 57 of 57)

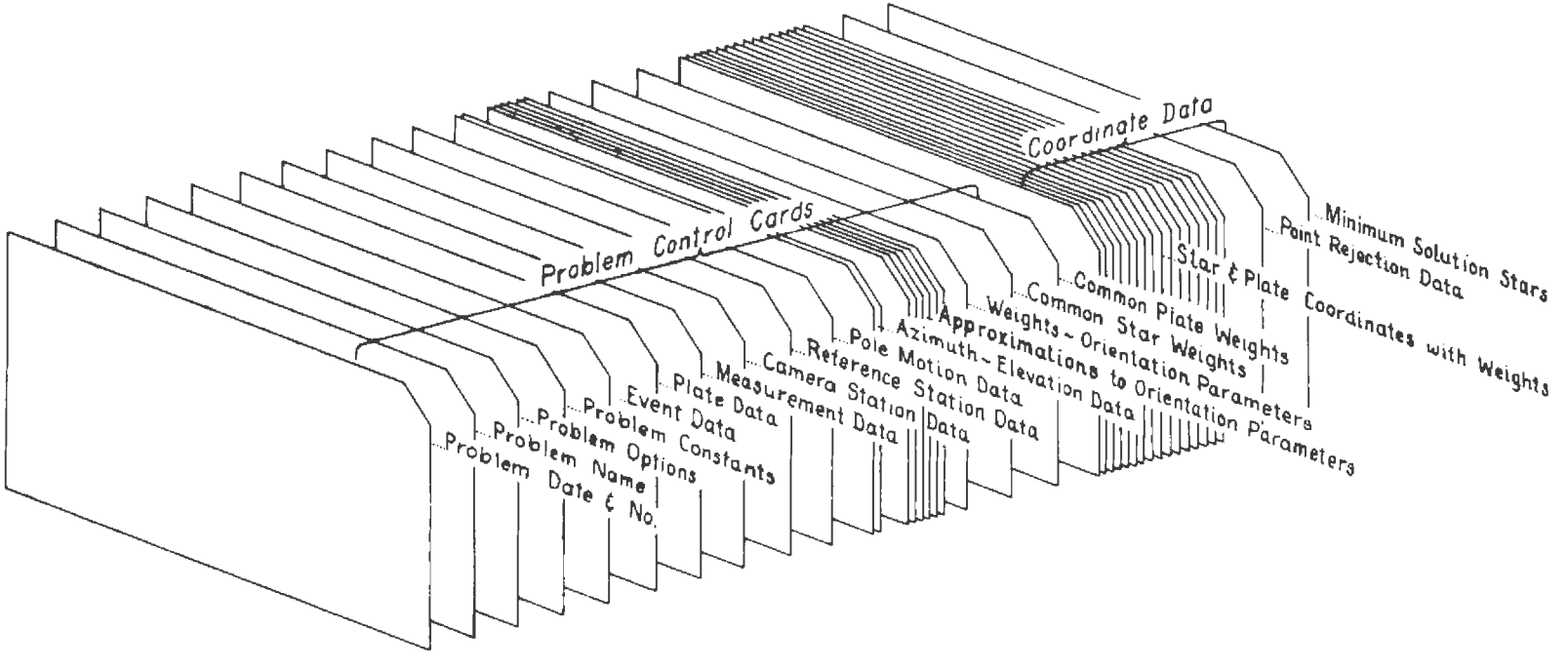

Figure 21.--Input formats (page 1 of 5)

PROBLEM CARDS FOR SINGLE CAMERA ORIENTATION (STELLAR)

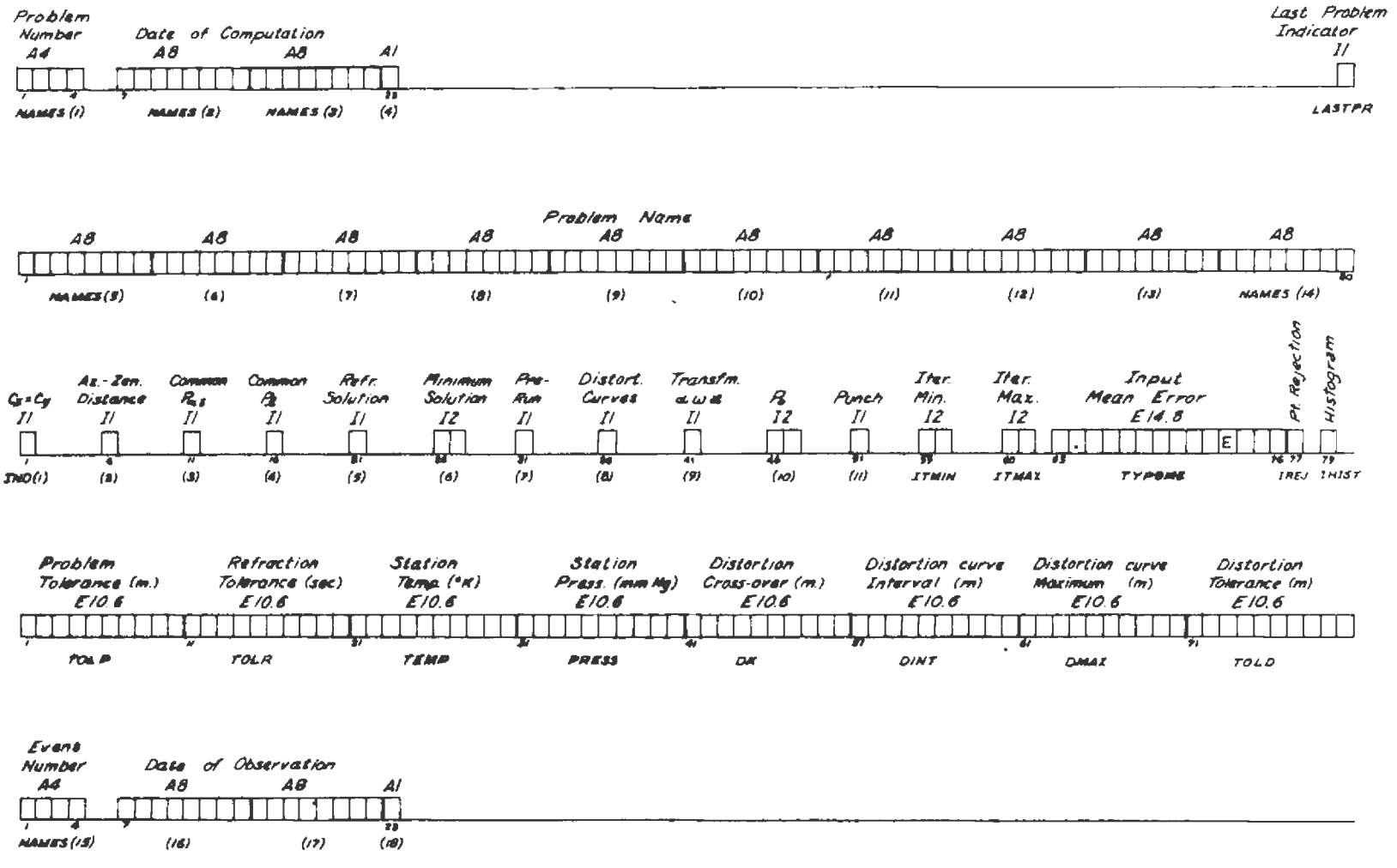

Figure 21.--Continued (page 2 of 5)

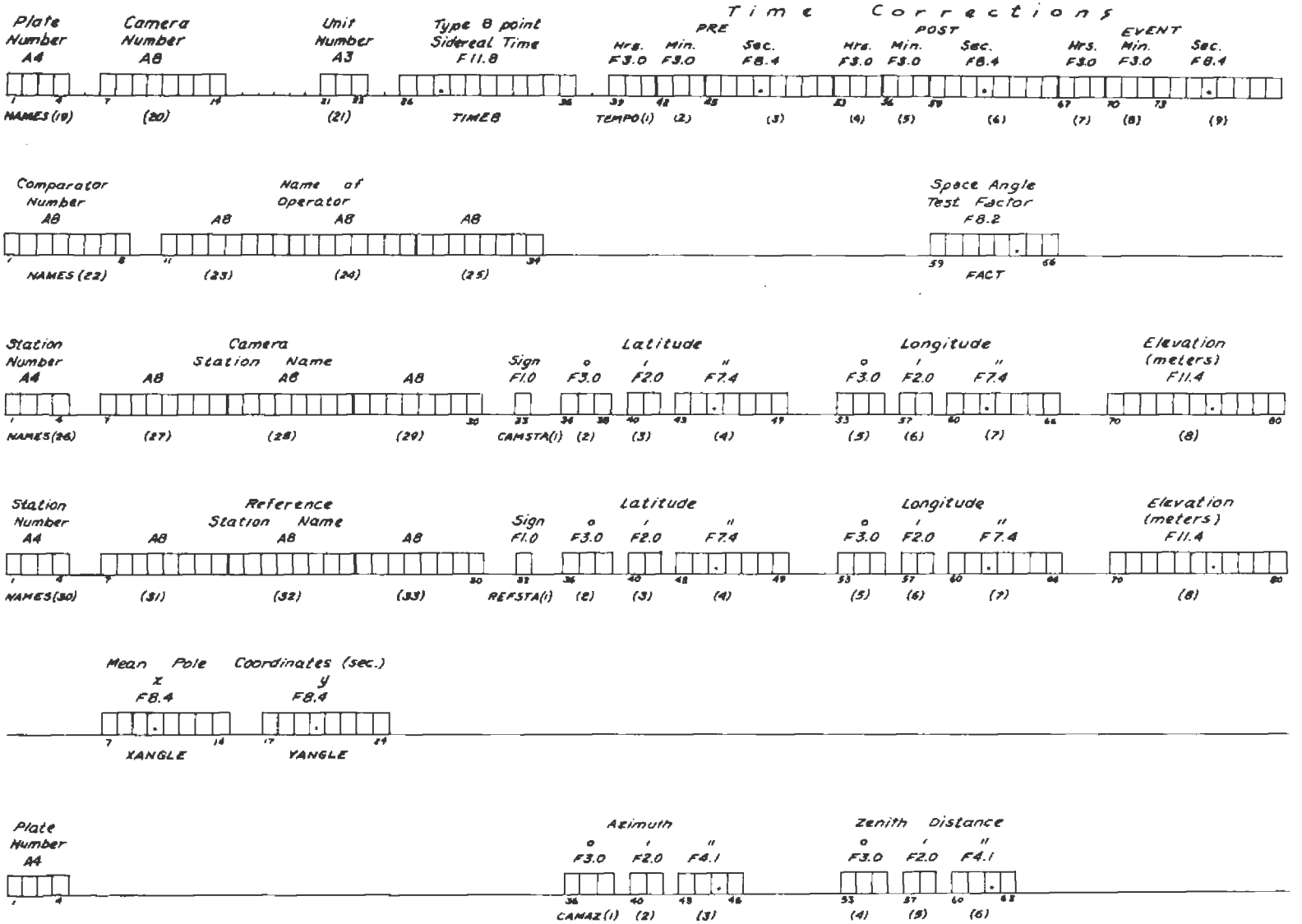

Figure 21.--Continued (page 3 of 5)

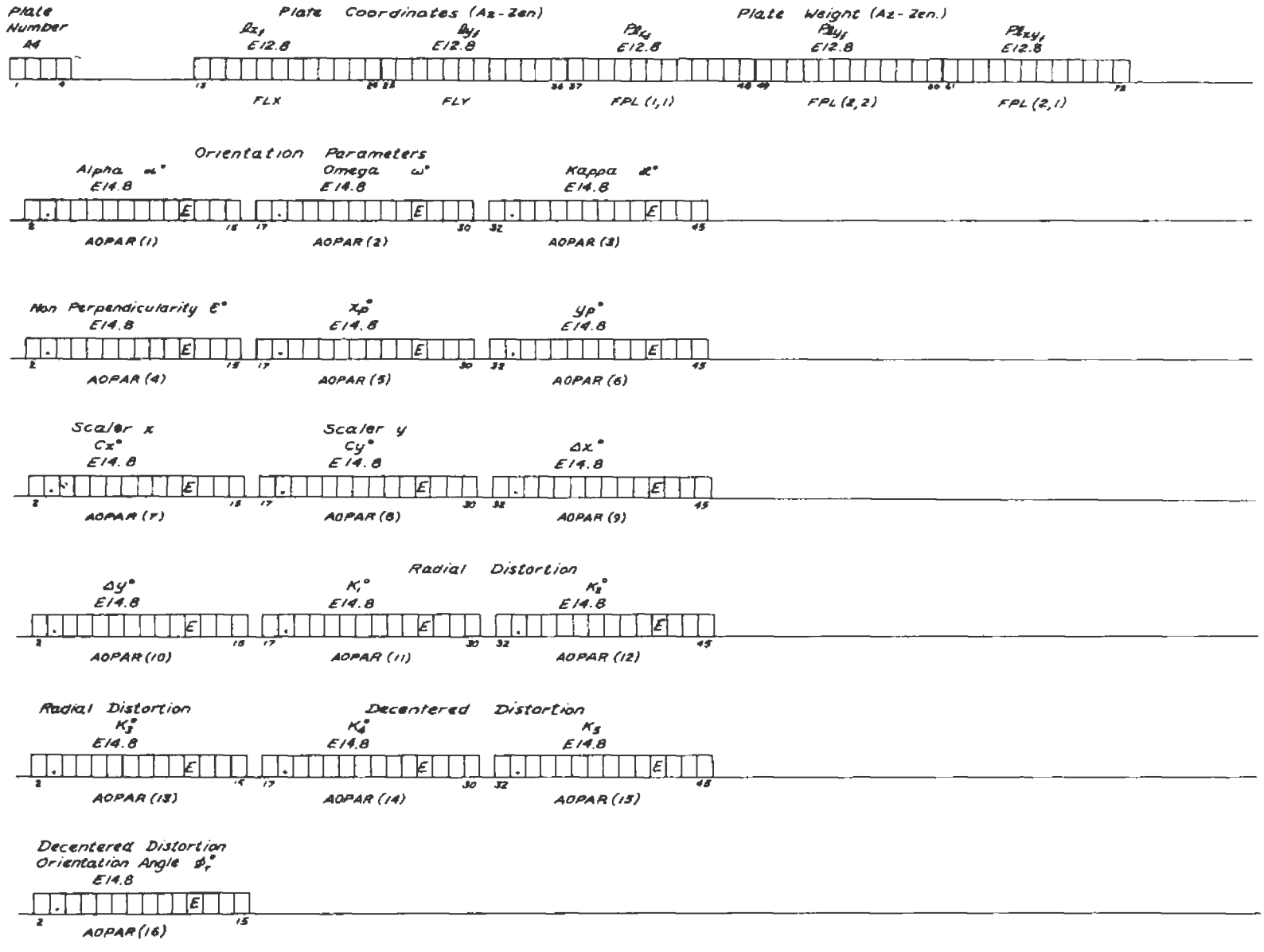

Figure 21.--Continued (page 4 of 5)

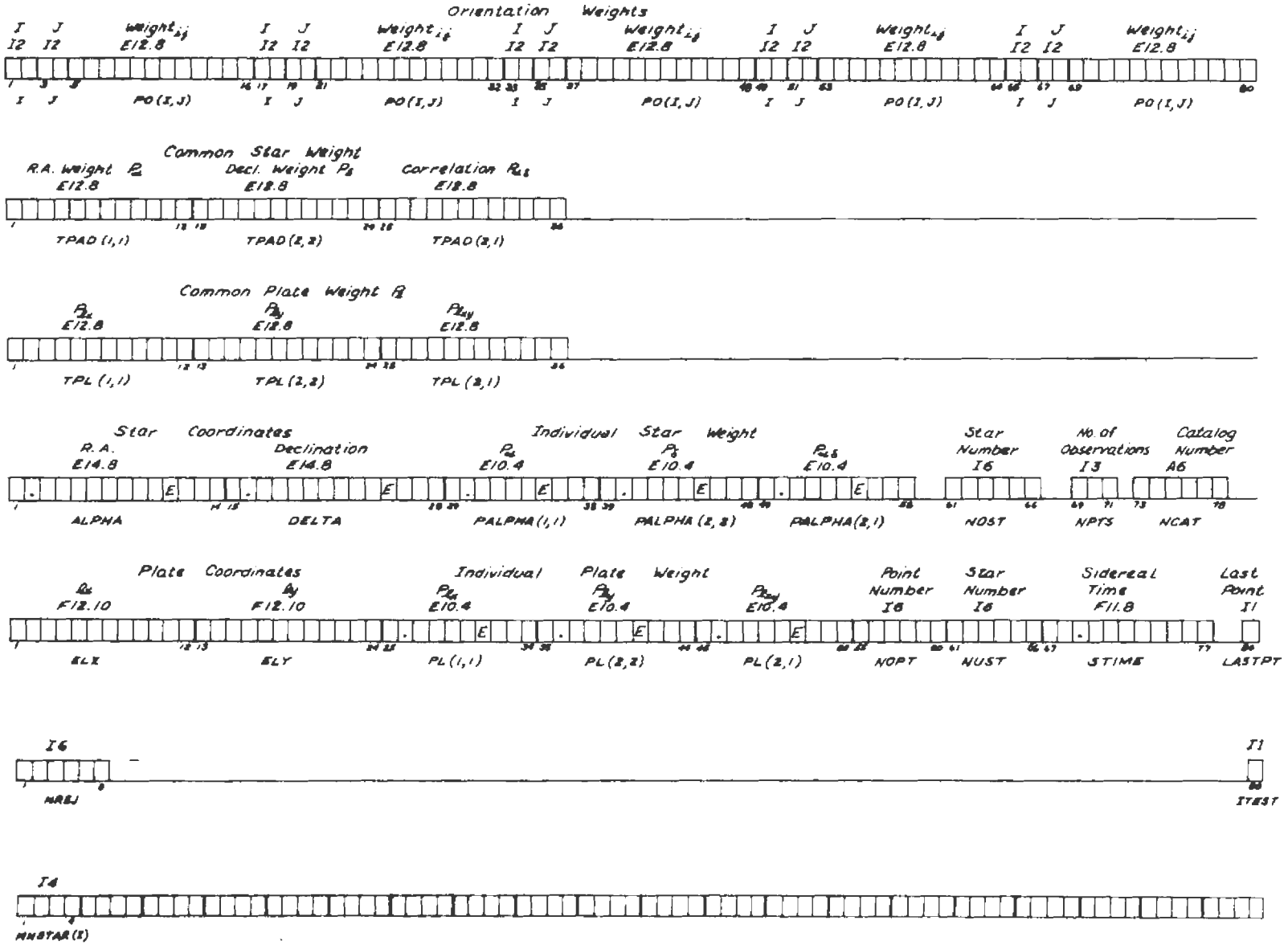

Figure 21.--Concluded (page 5 of 5)

```
                    SINGLE CAMERA ORIENTATION WITH CELESTIAL OBSERVATIONS

              CHECK OF CENTRAL PERSPECTIVE WITH RADIAL AND DECENTERED-LENS DISTORTIONS

                STATION NAME                LATITUDE          LONGITUDE          ELEVATION
                    CHET                  30  0  0.0000      80  0  0.0000        1376.4518

ROBLEM NUMBER    3      EVENT NUMBER    2      CAMERA NUMBER   PC1000/6      DATE OF COMPUTATION    26 JULY 1967
TATION NUMBER   46      PLATE NUMBER   53      TRACKING UNIT NUMBER AUK      DATE OF OBSERVATION  6 DECEMBER 1931

                                  PROBLEM CONSTANTS
                      PROBLEM TOLERANCE                          .10000000E-10
                      REFRACTION TOLERANCE (SEC ARC)             .10000000E-01
                      DISTORTION TOLERANCE                       .10000000E-06
                      STATION TEMPERATURE (DEGREES KELVIN)       .29500000E+03
                      STATION PRESSURE (MM MERCURY)              .76000000E+03
                      DISTORTION CURVE CROSSOVER DISTANCE        .65000000E-01
                      DISTORTION CURVE INTERVAL                  .30000000E-02
                      DISTORTION CURVE MAXIMUM DISTANCE          .10000000E-00

                      COMPARATOR
                        NUMBER                        NAME OF OPERATOR
                      MANN 372                        MYRON LAWRENCE

                                  TIME CORRECTION
                                  0.  0.   0.0040

                                PROBLEM DESCRIPTORS

                              NO MINIMUM SOLUTION COMPUTED

                   INDIVIDUAL SCALAR COMPONENTS COMPUTED FOR X AND Y (CX DOES NOT EQUAL CY)

                          COMMON WEIGHT MATRIX FOR ALL KNOWN STARS

                          COMMON WEIGHT MATRIX FOR ALL PLATE POINTS

                       NO REFRACTION COEFFICIENT CORRECTIONS COMPUTED

                     RADIAL AND TANGENTIAL LENS DISTORTION CURVES COMPUTED

              FINAL CAMERA ORIENTATION ANGLES TRANSFORMED TO A REFERENCE STATION SYSTEM
                                   STATION
          REFERENCE STATION NAME   NUMBER        LATITUDE          LONGITUDE          ELEVATION
               CHIN CHILLER          48        32  0  0.0000      82  0  0.0000         27.0763

              FINAL CAMERA ORIENTATION ANGLES TRANSFORMED TO A MEAN POLE SYSTEM
            X-CORRECTION ANGLE (SEC) =    0.3260      Y-CORRECTION ANGLE (SEC) =    -.4719

            ALL MEAN ERRORS COMPUTED USING INPUT MEAN ERROR OF UNIT WEIGHT = .30000000E-05

                     TOTAL NUMBER OF STARS AND PLATE POINTS IN PROBLEM
                 KNOWN STARS   75      UNKNOWN STARS    0      PLATE POINTS  375

                       NUMBER OF ITERATIONS WILL NOT EXCEED
                              MAXIMUM SOLUTION - 10

                                  MAXIMUM SOLUTION

                                      NUMBER OF     NUMBER OF
                                     OBSERVATIONS  UNKNOWNS
                 ORIENTATION PARAMETERS      2          27
                 STAR  COORDINATES         150         150
                 PLATE COORDINATES         750

                 TOTALS                    902         177

                          DEGREES OF FREEDOM  725

                           LEVEL OF DISCREPANCIES
                                .19447773E-01

                                .64189533E-03
                                .48671885E-04
                                .53660397E-06
                                .36796333E-06
                                .15110395E-07
                                .58446451E-08
                                .58427052E-08

            ITERATION RESULTS DIFFER BY LESS THAN TOLERANCE SETTING
```

Figure 22.--Sample output (page 1 of 6)

STAR AND PLATE COORDINATE DATA

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
1	.10000000E+04	.10000000E+04	.00000000E 00		-.95054395E-07	-.14258159E-05	.31230059E-06

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	-80.00000	-80.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	-.00587	0.00221
502	POST	-72.79345	-82.25371	.100E+01	.100E+01	.000E 00	2 2 36.7821	-.00583	0.00271
503	POST	-65.69435	-84.07812	.100E+01	.100E+01	.000E 00	2 10 38.0666	-.00570	0.00319
504	POST	-58.69811	-87.27571	.100E+01	.100E+01	.000E 00	2 18 39.3512	-.00557	0.00367
505	POST	-51.79936	-90.04900	.100E+01	.100E+01	.000E 00	2 26 40.6357	-.00548	0.00414

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
1	-.65797054E-01	-.84851309E-01	-.56900114E-08	.31851114E-08	.56974988E-09	-.64463136E-08

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
2	.10000000E+04	.10000000E+04	.00000000E 00		.24948177E-06	.37422265E-05	-.16711006E-06

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	-70.00000	70.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	0.01154	-.00417
502	POST	-59.94032	66.68749	.100E+01	.100E+01	.000E 00	2 2 36.7821	0.01129	-.00381
503	POST	-50.07008	63.28918	.100E+01	.100E+01	.000E 00	2 10 38.0666	0.01139	-.00386
504	POST	-40.37647	59.80765	.100E+01	.100E+01	.000E 00	2 18 39.3512	0.01154	-.00421
505	POST	-30.84393	56.24264	.100E+01	.100E+01	.000E 00	2 26 40.6357	0.01173	-.00470

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
2	-.50246158E-01	.63205391E-01	.11497122E-07	-.41488961E-08	-.10402241E-07	-.64179854E-08

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
3	.10000000E+04	.10000000E+04	.00000000E 00		-.23836353E-06	-.35745590E-05	-.50368177E-06

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	60.00000	60.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	-.00951	0.00500
502	POST	69.52429	55.25757	.100E+01	.100E+01	.000E 00	2 2 36.7821	-.00971	0.00464
503	POST	79.07041	50.38960	.100E+01	.100E+01	.000E 00	2 10 38.0666	-.01014	0.00424
504	POST	88.65061	45.38434	.100E+01	.100E+01	.000E 00	2 18 39.3512	-.01088	0.00379
505	POST	98.27318	40.22816	.100E+01	.100E+01	.000E 00	2 26 40.6357	-.01212	0.00319

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
3	.79103699E-01	.50251973E-01	-.10470037E-07	.41735717E-08	-.05964344E-08	.51412572E-08

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
4	.10000000E+04	.10000000E+04	.00000000E 00		.18402088E-06	.27603131E-05	-.52906973E-07

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	50.00000	-70.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	0.00912	-.00599
502	POST	57.50818	-75.32242	.100E+01	.100E+01	.000E 00	2 2 36.7821	0.00885	-.00607
503	POST	65.01982	-80.66926	.100E+01	.100E+01	.000E 00	2 10 38.0666	0.00821	-.00595
504	POST	72.53128	-86.04039	.100E+01	.100E+01	.000E 00	2 18 39.3512	0.00737	-.00551
505	POST	80.03162	-92.62749	.100E+01	.100E+01	.000E 00	2 26 40.6357	0.00636	-.00538

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
4	.65018182E-01	-.81092312E-01	.79860421E-08	-.58025205E-08	.95226741E-08	.26009211E-08

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
5	.10000000E+04	.10000000E+04	.00000000E 00		.63607167E-07	.95410751E-06	-.41759230E-06

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	-60.00000	-50.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	0.00316	-.00332
502	POST	-52.36120	-52.71833	.100E+01	.100E+01	.000E 00	2 2 36.7821	0.00324	-.00299
503	POST	-44.83310	-55.60166	.100E+01	.100E+01	.000E 00	2 10 38.0666	0.00336	-.00269
504	POST	-37.40644	-58.65287	.100E+01	.100E+01	.000E 00	2 18 39.3512	0.00327	-.00238
505	POST	-30.07195	-61.87529	.100E+01	.100E+01	.000E 00	2 26 40.6357	0.00327	-.00221

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
5	-.44934539E-01	-.55769631E-01	.32614667E-08	-.27188710E-08	.70900133E-10	.42455165E-08

STAR NUMBER	NORMALIZED STAR WEIGHTS RIGHT ASCENSION	DECLINATION	CORRELATION	CATALOG NO.	STAR RESIDUALS R A (SEC TIME)	R A (SEC ARC)	DECL (SEC ARC)
6	.10000000E+04	.10000000E+04	.00000000E 00		-.19479372E-06	-.92190359E-05	.57854009E-06

POINT NUMBER	ORIENT-ATION	PLATE COORDINATES (MM) LX	LY	NORMALIZED PLATE WEIGHTS PLX	PLY	PLXY	SIDEREAL TIME HR MIN SEC	PLATE RES (MICRONS) VLX	VLY
501	POST	-50.00000	60.00000	.100E+01	.100E+01	.000E 00	1 54 35.4975	-.00972	0.00558
502	POST	-40.36704	56.54697	.100E+01	.100E+01	.000E 00	2 2 36.7821	-.00913	0.00532
503	POST	-30.89464	53.00795	.100E+01	.100E+01	.000E 00	2 10 38.0666	-.00848	0.00504
504	POST	-21.56400	49.38040	.100E+01	.100E+01	.000E 00	2 18 39.3512	-.00806	0.00460
505	POST	-12.35824	45.66040	.100E+01	.100E+01	.000E 00	2 26 40.6357	-.00779	0.00426

STAR NUMBER	MEAN PLATE COORDINATES LX	LY	VLX	MEAN PLATE RESIDUALS VLY	RADIAL COMP	TANGENTIAL COMP
6	-.31036905E-01	.52419147E-01	-.86349437E-08	.49015666E-08	.86482580E-08	.49383225E-08

Figure 22.--Continued (page 2 of 6)

FICTITIOUS DIAGONAL POINT DIRECTIONS

ALL DIAGONAL POINTS COMPUTED WITH PLATE WEIGHTS OF PLX .1000E+01 PLY .1000E+01 PLXY .0000E 00

DIAGONAL POINT (0,0) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 25 29.459	321 22 22.03	29 52 25.17	108 15 31.78	32 37 31.58	ORIENT	0.272	0.268	0.309	0.247	53.
						TOTAL	2.080	2.077	2.085	2.072	39.
POST	21 16 30.693	319 7 40.40	29 7 55.02	108 11 3.22	30 32 11.41	ORIENT	0.271	0.288	0.309	0.247	53.
						TOTAL	2.079	2.077	2.085	2.072	40.
EVENT	21 20 57.220	320 14 18.31	29 31 2.82	108 14 4.49	31 34 39.91	ORIENT	0.271	0.268	0.309	0.247	53.
						TOTAL	2.079	2.077	2.085	2.072	40.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7524.03	0.26	-7059.32	-2671.15	-231.31	-7520.16
EVENT MINUS PRE	3772.39	0.26	-3553.68	-1283.35	-74.37	-3771.67
EVENT MINUS POST	3751.71	0.25	3479.01	1387.80	-154.43	-3748.50

DIAGONAL POINT (1,1) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 21 5.436	320 16 21.12	27 21 24.64	105 50 36.19	30 52 43.80	ORIENT	0.269	0.301	0.324	0.240	56.
						TOTAL	2.077	2.075	2.083	2.066	41.
POST	21 12 17.339	318 4 20.08	26 36 8.69	105 49 56.04	28 46 40.61	ORIENT	0.268	0.301	0.324	0.240	57.
						TOTAL	2.076	2.075	2.083	2.066	42.
EVENT	21 16 38.557	319 9 38.35	26 59 38.42	105 51 5.67	29 49 30.97	ORIENT	0.268	0.301	0.324	0.240	57.
						TOTAL	2.077	2.075	2.083	2.066	41.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7563.26	0.26	-7082.48	-2715.94	-35.19	-7563.19
EVENT MINUS PRE	3792.91	0.26	-3556.68	-1306.22	25.57	-3792.83
EVENT MINUS POST	3770.35	0.25	3491.39	1409.72	-60.41	-3770.37

DIAGONAL POINT (2,2) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 16 54.184	319 13 32.75	24 50 35.06	103 31 33.71	24 5 49.47	ORIENT	0.259	0.301	0.320	0.235	60.
						TOTAL	2.068	2.062	2.070	2.059	30.
POST	21 6 35.385	317 3 59.76	24 4 37.23	103 34 9.84	26 59 17.05	ORIENT	0.258	0.302	0.320	0.235	61.
						TOTAL	2.068	2.062	2.070	2.059	30.
EVENT	21 12 32.314	318 8 4.71	24 28 27.47	103 33 43.50	26 2 22.48	ORIENT	0.259	0.302	0.320	0.235	60.
						TOTAL	2.068	2.062	2.070	2.059	30.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7593.66	0.28	-7096.71	-2757.78	134.13	-7592.42
EVENT MINUS PRE	3808.09	0.28	-3575.10	-1327.59	114.55	-3806.59
EVENT MINUS POST	3785.50	0.27	3499.45	1430.19	23.25	-3785.43

DIAGONAL POINT (3,3) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 12 55.063	318 13 45.95	22 20 42.62	101 19 30.28	27 17 35.84	ORIENT	0.268	0.265	0.276	0.236	57.
						TOTAL	2.054	2.036	2.054	2.036	176.
POST	21 4 25.960	316 6 29.40	21 34 5.69	101 23 52.03	25 10 46.52	ORIENT	0.246	0.265	0.276	0.236	58.
						TOTAL	2.054	2.036	2.054	2.036	176.
EVENT	21 8 37.789	317 9 26.84	21 58 15.04	101 22 4.66	26 14 0.93	ORIENT	0.248	0.265	0.276	0.236	58.
						TOTAL	2.054	2.036	2.054	2.036	176.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7614.77	0.29	-7101.85	-2796.93	291.18	-7609.31
EVENT MINUS PRE	3815.70	0.29	-3578.84	-1347.59	192.32	-3814.90
EVENT MINUS POST	3795.64	0.29	3503.10	1449.34	96.30	-3794.41

DIAGONAL POINT (4,4) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 9 7.317	317 16 49.76	19 52 28.70	99 11 25.80	25 28 47.94	ORIENT	0.259	0.249	0.260	0.247	20.
						TOTAL	2.039	2.006	2.040	2.004	168.
POST	21 0 46.586	315 11 38.80	19 5 15.07	99 19 4.46	23 21 52.58	ORIENT	0.259	0.249	0.261	0.248	20.
						TOTAL	2.039	2.006	2.040	2.005	168.
EVENT	21 4 54.279	316 13 34.19	19 29 42.37	99 16 10.14	24 25 10.60	ORIENT	0.259	0.249	0.261	0.248	20.
						TOTAL	2.039	2.006	2.040	2.005	168.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7626.79	0.31	-7098.01	-2833.63	421.06	-7615.36
EVENT MINUS PRE	3826.03	0.31	-3577.97	-1366.34	258.90	-3817.34
EVENT MINUS POST	3801.36	0.31	3502.39	1467.29	158.73	-3798.02

DIAGONAL POINT (5,5) CENTIMETERS

	RT ASCENSION HR MIN SEC	RT ASCENSION DEG MIN SEC	DECLINATION DEG MIN SEC	AZIMUTH DEG MIN SEC	ELEVATION DEG MIN SEC		MEAN ERRORS RA*COS(DEC) SEC ARC	MEAN ERRORS DECL SEC ARC	ERROR ELLIPSES A SEC ARC	ERROR ELLIPSES B SEC ARC	ERROR ELLIPSES THETA DEG
PRE	21 5 30.189	316 22 32.84	17 26 28.68	97 10 14.04	23 40 6.31	ORIENT	0.301	0.287	0.303	0.285	19.
						TOTAL	2.024	1.976	2.027	1.973	166.
POST	20 57 17.163	314 19 17.44	16 38 40.55	97 19 43.17	21 33 14.29	ORIENT	0.302	0.288	0.303	0.286	19.
						TOTAL	2.024	1.976	2.027	1.973	167.
EVENT	21 1 21.055	315 20 15.82	17 3 24.72	97 15 54.81	22 36 31.27	ORIENT	0.301	0.289	0.303	0.286	19.
						TOTAL	2.024	1.976	2.027	1.973	166.

	SPACE ANGLE (ARC SEC)	MEAN ERROR (ARC SEC)	COMPONENTS OF SPACE ANGLE (ARC SEC) R.A.*COS DEC	DECLINATION	AZ*COS ELEV	ELEVATION
POST MINUS PRE	7630.11	0.34	-7085.53	-2868.13	529.33	-7612.01
EVENT MINUS PRE	3827.88	0.34	-3572.64	-1383.96	314.58	-3815.03
EVENT MINUS POST	3602.86	0.33	3497.46	1484.17	210.81	-3796.98

Figure 22.--Continued (page 3 of 6)

CAMERA ORIENTATION PARAMETER DATA

ORIENTATION ANGLES FROM PRE STARS

	ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
ORIGINAL PARAMETERS	.38000000E+02	-.63000000E+02	.13500000E+03
COMPUTED PARAMETERS	.29000051E+02	-.59000016E+02	.12900005E+03
WEIGHTS (DIAGONAL)	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	-.89999494E+01	.39999842E+01	-.59999504E+01
MEAN ERRORS OF UNKNOWNS	.50577845E-02	.35654113E-02	.41228046E-02

ORIENTATION ANGLES FROM POST STARS

	ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
ORIGINAL PARAMETERS	.38000000E+02	-.63000000E+02	.13500000E+03
COMPUTED PARAMETERS	.31000054E+02	-.61000015E+02	.13100005E+03
WEIGHTS (DIAGONAL)	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	-.69999456E+01	.19999851E+01	-.39999462E+01
MEAN ERRORS OF UNKNOWNS	.52865092E-02	.35539018E-02	.43989571E-02

ORIENTATION ANGLES FROM EVENT STARS

	ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
ORIGINAL PARAMETERS	.38000000E+02	-.63000000E+02	.13500000E+03
COMPUTED PARAMETERS	.30000053E+02	-.60000016E+02	.13000005E+03
WEIGHTS (DIAGONAL)	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	-.79999475E+01	.29999845E+01	-.49999470E+01
MEAN ERRORS OF UNKNOWNS	.51688008E-02	.35553410E-02	.42576424E-02

	EPSILON (GRADS)	XP	YP	CX	CY
ORIGINAL PARAMETERS	.00000000E 00	.00000000E 00	.00000000E 00	.30200000E-00	.30200000E-00
COMPUTED PARAMETERS	.30658159E-02	.10006051E-03	-.10006536E-03	.29999999E-00	.30099998E-00
WEIGHTS (DIAGONAL)	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	.30658159E-02	.10006051E-03	-.10006536E-03	-.20000147E-02	-.10000156E-02
MEAN ERRORS OF UNKNOWNS	.32649907E-03	.16192720E-04	.14551782E-04	.45624878E-05	.45929662E-05

	DELTA X	DELTA Y	K1	K2	K3
ORIGINAL PARAMETERS	.10000000E-03	-.10000000E-03	.00000000E 00	.00000000E 00	.00000000E 00
COMPUTED PARAMETERS	.10000000E-03	-.10000000E-03	.20601987E-00	-.94419639E+01	-.87094782E+03
WEIGHTS (DIAGONAL)	.10000000E+21	.10000000E+21	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	-.14746751E-30	.22919642E-30	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	.00000000E 00	.00000000E 00	.20601987E-00	-.94419639E+01	-.87094782E+03
MEAN ERRORS OF UNKNOWNS	.30000000E-15	.30000000E-15	.71497778E-02	.99277779E+00	.41326980E+02

	K4	K5	PHI-T (GRADS)	X5	Y5
ORIGINAL PARAMETERS	.00000000E 00	.00000000E 00	.00000000E 00	.10000000E-03	-.10000000E-03
COMPUTED PARAMETERS	-.39962316E-03	.79142137E-03	.30039916E+02	.20006051E-03	-.20006536E-03
WEIGHTS (DIAGONAL)	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
RESIDUALS	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00	.00000000E 00
CORRECTIONS	-.39962316E-03	.79142137E-03	.30039916E+02	.10006051E-03	-.10006536E-03
MEAN ERRORS OF UNKNOWNS	.11515287E-03	.59241136E-02	.98669552E+01	.16192720E-04	.14551782E-04

TRANSFORMED CAMERA ORIENTATION ANGLES FOR PRE STARS ONLY

ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
.24402169E+02	-.57725973E+02	.12600685E+03

TRANSFORMED CAMERA ORIENTATION ANGLES FOR POST STARS ONLY

ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
.26186934E+02	-.59780630E+02	.12781169E+03

TRANSFORMED CAMERA ORIENTATION ANGLES FOR EVENT STARS ONLY

ALPHA (GRADS)	OMEGA (GRADS)	KAPPA (GRADS)
.25296858E+02	-.58753200E+02	.12691148E+03

SUMS OF WEIGHTED SQUARES OF RESIDUALS

PLATE	.24740707E-13
STARS	.93811950E-17
ORIENTATION	.74283564E-41
GRAND TOTAL	.24740089E-13

SUMS OF PLATE RESIDUALS FOR POINTS USED IN SOLUTION

VX		VY		
POSITIVE	NEGATIVE	POSITIVE	NEGATIVE	ALGEBRAIC SUM VX AND VY
.11665838E-05	-.11667923E-05	.59714477E-06	-.59690789E-06	.28307768E-10

COMMON MEAN ERROR OF PLATE MEASUREMENTS BEFORE ADJ LX .30000000E-05 LY .30000000E-05

COMMON MEAN ERROR OF KNOWN STARS BEFORE ADJ RA .19567998E-01 DECL .19567998E-01

MEAN ERROR OF UNIT WEIGHT FOR PROBLEM .58415974E-08

DETERMINANT OF REDUCED NORMAL EQUATION SYSTEM

MAXIMUM SOLUTION .10494677E+79

Figure 22.--Continued (page 4 of 6)

INVERSE OF REDUCED NORMAL EQUATION SYSTEM

ALPHA (PRE)	OMEGA (PRE)	KAPPA (PRE)	ALPHA (POST)	OMEGA (POST)
(1, 1) .70132269E+03	(1, 2)-.10814449E+02 (2, 2) .34677913E+03	(1, 3) .57013565E+03 (2, 3)-.77405115E+00 (3, 3) .46559601E+03	(1, 4) .73229519E+03 (2, 4)-.30239376E+02 (3, 4) .59497736E+03 (4, 4) .76618778E+03	(1, 5) .24136968E+01 (2, 5) .34631031E+03 (3, 5) .10040072E+02 (4, 5)-.16404412E+02 (5, 5) .34626461E+03

KAPPA (POST)	ALPHA (EVENT)	OMEGA (EVENT)	KAPPA (EVENT)	EPSILON
(1, 6) .60943507E+03 (2, 6)-.10693361E+02 (3, 6) .49512165E+03 (4, 6) .63603393E+03 (5, 6)-.52675957E+01 (6, 6) .53051381E+03	(1, 7) .71035287E+03 (2, 7)-.20319011E+02 (3, 7) .58224063E+03 (4, 7) .74874880E+03 (5, 7)-.67940056E+01 (6, 7) .62168019E+03 (7, 7) .73244805E+03	(1, 8)-.42027526E+01 (2, 8) .34654432E+03 (3, 8) .46631656E+01 (4, 8)-.23326707E+02 (5, 8) .34628690E+03 (6, 8)-.10950770E+02 (7, 8)-.13554049E+02 (8, 8) .34654509E+03	(1, 9) .58889210E+03 (2, 9)-.84469251E+01 (3, 9) .47941678E+03 (4, 9) .61507230E+03 (5, 9) .26678140E+01 (6, 9) .51163675E+03 (7, 9) .60180238E+03 (8, 9)-.29502579E+01 (9, 9) .49697622E+03	(1,10)-.13393374E+02 (2,10)-.23396220E+01 (3,10)-.12160075E+02 (4,10)-.13653141E+02 (5,10)-.24920194E+01 (6,10)-.12763749E+02 (7,10)-.13619283E+02 (8,10)-.23661256E+01 (9,10)-.12467102E+02 (10,10) .29225446E+01

XP	YP	CX	CY	DELTA X
(1,11) .58011091E+02 (2,11)-.92711456E+02 (3,11) .45021215E+02 (4,11) .65611756E+02 (5,11)-.91569400E+02 (6,11) .52374450E+02 (7,11) .61729279E+02 (8,11)-.92151815E+02 (9,11) .48352372E+02 (10,11)-.45701465E-00 (11,11) .29133798E+02	(1,12)-.11117088E+03 (2,12)-.43481025E+02 (3,12)-.91451190E+02 (4,12)-.11364900E+03 (5,12)-.45550029E+02 (6,12)-.95532300E+02 (7,12)-.11238340E+03 (8,12)-.44524052E+02 (9,12)-.93469625E+02 (10,12) .24445575E+01 (11,12) .27775133E+01 (12,12) .23528263E+02	(1,13)-.16741844E+01 (2,13) .26156699E+01 (3,13)-.13487967E+01 (4,13)-.18926572E+01 (5,13) .25825931E+01 (6,13)-.15575995E+01 (7,13)-.17810444E+01 (8,13) .25946604E+01 (9,13)-.14534434E+01 (10,13) .74179468E-01 (11,13)-.81944120E+00 (12,13)-.75023489E-01 (13,13) .23129210E+01	(1,14)-.44453490E+01 (2,14) .10697495E+01 (3,14)-.36017379E+01 (4,14)-.46398455E+01 (5,14) .48538108E+00 (6,14)-.38910444E+01 (7,14)-.45098253E+01 (8,14) .10276965E+01 (9,14)-.37434238E+01 (10,14) .56836056E-01 (11,14)-.60282445E+00 (12,14) .56693903E+00 (13,14) .22018796E+01 (14,14) .23439265E+01	(1,15)-.25356616E-21 (2,15)-.18618376E-20 (3,15)-.18319214E-21 (4,15)-.16245692E-21 (5,15)-.18636671E-20 (6,15)-.11342856E-21 (7,15)-.20923999E-21 (8,15)-.16039929E-20 (9,15)-.14934374E-21 (10,15)-.27349033E-22 (11,15) .47921884E-21 (12,15) .28017592E-21 (13,15) .54269957E-24 (14,15)-.52545630E-23 (15,15) .10000000E-19

DELTA Y	K1	K2	K3	K4
(1,16) .44941628E-21 (2,16)-.67776772E-21 (3,16) .33637568E-21 (4,16) .50602176E-21 (5,16)-.66895374E-21 (6,16) .39097410E-21 (7,16) .47710535E-21 (8,16)-.67344258E-21 (9,16) .36296453E-21 (10,16) .59414330E-22 (11,16) .21150819E-21 (12,16) .25922119E-22 (13,16)-.34381792E-23 (14,16)-.17281197E-24 (15,16) .12526575E-43 (16,16) .10000000E-19	(1,17) .32301975E+04 (2,17)-.33118632E+04 (3,17) .20614850E+04 (4,17) .36089190E+04 (5,17)-.32483640E+04 (6,17) .29893235E+04 (7,17) .34409606E+04 (8,17)-.32605319E+04 (9,17) .28213197E+04 (10,17)-.11829880E+03 (11,17) .11287540E+04 (12,17)-.89630195E+02 (13,17)-.33579340E+04 (14,17)-.34045631E+04 (15,17)-.30771568E-19 (16,17) .32816155E-19 (17,17) .56799247E+07	(1,18)-.13283026E+06 (2,18) .51607363E+06 (3,18)-.10869002E+06 (4,18)-.16757761E+06 (5,18) .51354042E+06 (6,18)-.13936309E+06 (7,18)-.14962033E+06 (8,18) .51537393E+06 (9,18)-.12357111E+06 (10,18) .11051660E+05 (11,18)-.14594590E+06 (12,18)-.46235271E+05 (13,18) .42335514E+06 (14,18) .43054254E+06 (15,18) .10210791E-15 (16,18)-.12724201E-16 (17,18)-.76780676E+09 (18,18) .10951197E+12	(1,19)-.80791389E+07 (2,19)-.22753473E+08 (3,19)-.65150331E+07 (4,19)-.71552770E+07 (5,19)-.22896084E+08 (6,19)-.59158182E+07 (7,19)-.76275808E+07 (8,19)-.22827705E+08 (9,19)-.62288044E+07 (10,19)-.15781956E+06 (11,19) .53289503E+07 (12,19) .42246848E+07 (13,19)-.15961605E+08 (14,19)-.16217637E+08 (15,19)-.75423932E-15 (16,19) .10553142E-14 (17,19) .30252477E+11 (18,19)-.44651630E+13 (19,19) .18976880E+15	(1,20)-.48237838E+02 (2,20)-.62627912E+03 (3,20)-.53163340E+02 (4,20)-.16000886E+02 (5,20)-.62875831E+03 (6,20)-.27814218E+02 (7,20)-.32467611E+02 (8,20)-.62856658E+03 (9,20)-.40922591E+02 (10,20) .87279160E+01 (11,20) .16153725E+03 (12,20) .89072542E+02 (13,20)-.55198425E+01 (14,20)-.16833777E+01 (15,20) .55147745E-20 (16,20) .22743573E-20 (17,20) .59804478E+04 (18,20)-.11675135E+07 (19,20) .66921549E+08 (20,20) .14733538E+04

INVERSE OF REDUCED NORMAL EQUATION SYSTEM

K5	PHI-T	TAU-1	TAU-2	TAU-3
(1,21) .47421425E+04 (2,21) .21590952E+05 (3,21) .43728000E+04 (4,21) .37084694E+04 (5,21) .21668739E+05 (6,21) .36645994E+04 (7,21) .42658198E+04 (8,21) .21632535E+05 (9,21) .40321444E+04 (10,21)-.75987097E+03 (11,21)-.52719898E+04 (12,21)-.35409252E+04 (13,21) .26077276E+03 (14,21) .16133273E+03 (15,21)-.49509017E-18 (16,21)-.20469023E-18 (17,21)-.35685003E+06 (18,21) .73560071E+08 (19,21)-.39449531E+10 (20,21)-.66424604E+05 (21,21) .38994581E+07	(1,22) .12035691E+07 (2,22)-.55408361E+05 (3,22) .10263154E+07 (4,22) .13217664E+07 (5,22)-.31545687E+05 (6,22) .10991035E+07 (7,22) .12920387E+07 (8,22)-.43482219E+05 (9,22) .10630582E+07 (10,22)-.28707515E+05 (11,22) .11355840E+06 (12,22)-.19452894E+06 (13,22)-.27064530E+04 (14,22)-.86157168E+04 (15,22)-.14376603E-17 (16,22) .28623290E-17 (17,22) .65869969E+07 (18,22)-.45719517E+09 (19,22)-.14226345E+11 (20,22)-.26221951E+05 (21,22) .53449852E+07 (22,22) .26690921E+10	(1,23)-.00000000E 00 (2,23) .00000000E 00 (3,23)-.00000000E 00 (4,23)-.00000000E 00 (5,23) .00000000E 00 (6,23)-.00000000E 00 (7,23)-.00000000E 00 (8,23) .00000000E 00 (9,23)-.00000000E 00 (10,23) .00000000E 00 (11,23)-.00000000E 00 (12,23) .00000000E 00 (13,23) .00000000E 00 (14,23) .00000000E 00 (15,23) .00000000E 00 (16,23)-.00000000E 00 (17,23)-.00000000E 00 (18,23) .00000000E 00 (19,23) .00000000E 00 (20,23) .00000000E 00 (21,23)-.00000000E 00 (22,23)-.00000000E 00 (23,23) .10000000E-19	(1,24)-.00000000E 00 (2,24) .00000000E 00 (3,24)-.00000000E 00 (4,24)-.00000000E 00 (5,24) .00000000E 00 (6,24)-.00000000E 00 (7,24)-.00000000E 00 (8,24) .00000000E 00 (9,24)-.00000000E 00 (10,24) .00000000E 00 (11,24)-.00000000E 00 (12,24) .00000000E 00 (13,24) .00000000E 00 (14,24) .00000000E 00 (15,24) .00000000E 00 (16,24)-.00000000E 00 (17,24)-.00000000E 00 (18,24) .00000000E 00 (19,24) .00000000E 00 (20,24) .00000000E 00 (21,24)-.00000000E 00 (22,24)-.00000000E 00 (23,24) .00000000E 00 (24,24) .10000000E-19	(1,25)-.00000000E 00 (2,25) .00000000E 00 (3,25)-.00000000E 00 (4,25)-.00000000E 00 (5,25) .00000000E 00 (6,25)-.00000000E 00 (7,25)-.00000000E 00 (8,25) .00000000E 00 (9,25)-.00000000E 00 (10,25) .00000000E 00 (11,25)-.00000000E 00 (12,25) .00000000E 00 (13,25) .00000000E 00 (14,25) .00000000E 00 (15,25) .00000000E 00 (16,25)-.00000000E 00 (17,25)-.00000000E 00 (18,25) .00000000E 00 (19,25) .00000000E 00 (20,25) .00000000E 00 (21,25)-.00000000E 00 (22,25)-.00000000E 00 (23,25) .00000000E 00 (24,25) .00000000E 00 (25,25) .10000000E-19

TAU-4
(1,26)-.00000000E 00 (2,26) .00000000E 00 (3,26)-.00000000E 00 (4,26)-.00000000E 00 (5,26) .00000000E 00 (6,26)-.00000000E 00 (7,26)-.00000000E 00 (8,26) .00000000E 00 (9,26)-.00000000E 00 (10,26) .00000000E 00 (11,26)-.00000000E 00 (12,26) .00000000E 00 (13,26) .00000000E 00 (14,26) .00000000E 00 (15,26) .00000000E 00 (16,26)-.00000000E 00 (17,26)-.00000000E 00 (18,26) .00000000E 00 (19,26) .00000000E 00 (20,26) .00000000E 00 (21,26)-.00000000E 00 (22,26)-.00000000E 00 (23,26) .00000000E 00 (24,26) .00000000E 00 (25,26) .00000000E 00 (26,26) .10000000E-19

Figure 22.--Continued (page 5 of 6)

DISTORTION CURVE DATA

COMPUTED FOCAL LENGTHS (METERS)				ORIENTATION ANGLE FOR DECENTERED DISTORTION (GRADS)	
CX	MEAN ERROR	CY	MEAN ERROR	PHI-T	MEAN ERROR
0.3000000	.45624878E-05	0.3010000	.45929662E-05	.30039916E+02	.98669552E+01

FOCAL LENGTH FOR TRANSFORMED RADIAL DISTORTION (METERS)		DISTORTION CURVE	
C-STAR	MEAN ERROR	CROSS-OVER POINT (METERS)	K-ZERO TERM
0.3006912	.92961811E-05	0.065	-.63620306E-03

	RADIAL CURVES				DECENTERED CURVE	
RADIAL DISTANCE (MM)	COMPUTED DISTORTION (MICRONS)	MEAN ERROR (MICRONS)	TRANSFORMED DISTORTION (MICRONS)	MEAN ERROR (MICRONS)	DISTORTION (MICRONS)	MEAN ERROR (MICRONS)
3	0.006	0.000	-1.903	0.048	-.004	0.001
6	0.044	0.002	-3.773	0.095	-.014	0.004
9	0.150	0.005	-5.576	0.140	-.032	0.009
12	0.354	0.012	-7.281	0.181	-.058	0.016
15	0.688	0.023	-8.855	0.218	-.090	0.026
18	1.183	0.040	-10.269	0.250	-.129	0.037
21	1.868	0.062	-11.492	0.277	-.176	0.050
24	2.769	0.091	-12.500	0.296	-.230	0.065
27	3.910	0.127	-13.267	0.309	-.291	0.081
30	5.314	0.170	-13.772	0.315	-.359	0.099
33	6.997	0.221	-13.998	0.314	-.434	0.119
36	8.973	0.278	-13.930	0.306	-.517	0.141
39	11.249	0.343	-13.562	0.291	-.606	0.163
42	13.829	0.413	-12.892	0.270	-.703	0.187
45	16.706	0.489	-11.923	0.244	-.806	0.212
48	19.667	0.569	-10.671	0.213	-.917	0.238
51	23.289	0.652	-9.157	0.178	-1.035	0.265
54	26.939	0.738	-7.416	0.141	-1.159	0.293
57	30.770	0.824	-5.494	0.103	-1.291	0.321
60	34.720	0.909	-3.452	0.064	-1.429	0.349
63	38.713	0.992	-1.367	0.025	-1.574	0.378
66	42.654	1.072	0.665	0.013	-1.727	0.407
69	46.426	1.146	2.528	0.050	-1.886	0.435
72	49.851	1.214	4.084	0.087	-2.051	0.464
75	52.983	1.275	5.167	0.125	-2.224	0.492
78	55.208	1.328	5.584	0.165	-2.403	0.519
81	56.641	1.373	5.108	0.209	-2.589	0.546
84	56.921	1.410	3.479	0.257	-2.782	0.572
87	55.747	1.438	0.397	0.312	-2.981	0.597
90	52.777	1.460	-4.481	0.372	-3.187	0.621
93	47.622	1.477	-11.545	0.437	-3.399	0.644
96	39.839	1.491	-21.237	0.505	-3.618	0.666
99	28.931	1.504	-34.053	0.572	-3.843	0.687
102	14.338	1.520	-50.554	0.636	-4.074	0.707

DECISION FOR COMBINING SEPARATE ORIENTATIONS BASED ON
FACTOR OF 1003.00 TIMES MEAN ERROR OF SPACE ANGLE

Figure 22.--Concluded (page 6 of 6)

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