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National Ocean Survey

Complete Comparator Calibration

LAWRENCE W. FRITZ

WASHINGTON, D.C.
July 1973

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Laurence W. Fritz

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Complete Comparator Calibration

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ABSTRACT. The complete measurement and data reduction procedures used to perform routine comparator calibrations for quality control of all comparator measurements are presented. The techniques and mathematical models developed are applicable to all measuring comparators including those with rectangular formats. The mathematical model used for the isolation of linear systematic errors consists of a new exact solution of the general linear transformation problem, from which the calibration parameters are exactly derived functions. Polynomials are applied to define the systematic nonlinear errors inherent in the comparator. The adjustment residuals are presented in the coordinate systems of both the measurements and the grid in order to isolate grid errors from random errors. Rigorous statistical analyses of all system components and results are included. These statistics isolate the errors of the pointings, the grid, the comparator, and the residual system noise (random errors). The accompanying versatile FORTRAN-coded computer program provides numerous computational options and several graphic outputs to permit easy diagnostic evaluation of the complete measurement system.

I. INTRODUCTION

One of the primary components of the analytic photogrammetric system is the measuring comparator. To achieve the maximum accuracy in the mensuration process, a rigorous quality control practice must be pursued. This has been achieved in the analytical photogrammetry system at the National Ocean Survey (NOS). The backbone of this quality control practice is a complete comparator calibration computer program. The mathematical and statistical bases for this program are detailed in this report. This comparator calibration is considered complete inasmuch as the linear mathematical routines involved are exact and the statistical analysis is rigorous.

Before elaborating on the specific routines included in the computer program, a general outline of the reasons for calibration is in order. First, all comparators are mechanical and therefore are restricted in precision by the manufacturing processes that produced them. These mechanical imperfections result in the creation of systematic measuring errors that, by proper analysis, may be modeled mathematically and removed. Secondly, a calibration process, properly administered, will monitor the performance quality of the measuring machine and hence indicate when repair, readjustment, or maintenance are required. A complete calibration will also monitor the quality of the measuring performance of the instrument operator and the quality of the precision grid plate used for a standard.

In many analytical photogrammetry problems the calibration of the comparator may be included as a portion of the coordinate refinement needed for the reduction of other systematic errors such as film distortion. At NOS, this practice is discouraged as our policy is to isolate each individual source of systematic error. This policy provides an estimate of the magnitude of each contributing source of error and helps to prevent the modeling of quasi-systematic errors generated by false correlations of error sources in a combined adjustment.

The techniques and mathematical models provided in this report are valid for all brands of two-

coordinate measuring comparators now used in analytical photogrametric aerotriangulation. This includes all comparators that rely on precision lead screws, precision grid plates, precision linear scales, multiple measuring marks, or interferometric fringe counters. The techniques provided for the calibration of comparators designed for rectangular format photography (e.g., 9 by 18 in) vary from those used on the more common square format comparators, but the mathematical routines are the same. The computer coded routines given are written for comparators with a 1-micrometer (μm) least count, but are mathematically valid for more precise instruments.

II. THE MEASUREMENT PROCESS

A. The Grid Plate

The procedure outlined herein is designed to provide a thorough calibration for any two-axis measuring comparator. The method requires the use of a high-quality glass grid plate. The grid plate should contain well-defined gridline intersections symmetrically spaced in a square pattern. For square format comparators, the diagonal of the grid pattern must be long enough to extend across the maximum measuring area used on the comparator. An additional condition, imposed when the comparator axes are not mutually independent, requires that the maximum measuring area used be covered by the side length of the square grid pattern. For a comparator used to measure standard 9- by 9-in aerial photographs the ideal overall grid size is 9 by 9 in or larger. Experience has demonstrated that 25 grid intersections provide the needed redundancy for a complete comparator calibration. A 36-intersection grid is economically unfeasible because of the additional measuring and time involved. A 16-intersection grid provides limited redundancy.

A square, 25-intersection grid should also be used on rectangular format comparators. For example, on a 9- by 18-in comparator a 9- by 9-in grid is ideal. A smaller square grid can be used if the maximum measuring area can be spanned by several rotated placements of the grid.

The grid plate must be microflat and optically clear, and should be at least 1/4 in thick to ensure flatness stability. Ideally, the gridlines should be engraved. However, suitable high-quality grids which are marked with vacuum-deposited metallic chrome lines are now available. Lesser quality photographic grids are also available but not recommended. The gridline width is obviously dependent on the size of the comparator measuring mark. For least count of 1-micrometer (μm) comparators, which typically have 40-μm diameter measuring marks, a gridline width of 10μm is preferred.

Calibrated grid intersection X,Y coordinate values (Poetzschke 1967, Fritz 1972) are desirable for complete comparator calibration. However, as shown in this report, a linear comparator calibration is possible without calibrated grid values and a total (linear and nonlinear) comparator calibration is possible with minimal grid value information.

Each of the 25 grid intersections should be clearly marked by a finder circle and a number on the reverse side of the grid plate.

The accompanying computer program is designed for the numbering shown in figure 1 for an emulsion-up grid plate. The program will accept any transposition of this basic numbering scheme.

B. Measurement

The frequency of performing comparator calibrations is dependent upon the workload and on the inherent stability of the instrument. At NOS we find that lead screw and way comparators require single operator calibrations every 2 months during periods of constant work. This calibration interval provides a quality control check on the instrument performance and provides a means of anticipating any need for future repairs or adjustments. Under light workload conditions, an annual calibration would be sufficient. For the more stable glass grid, glass scale, and multiple measuring mark comparators, frequent initial calibrations are desirable. These initial calibrations will provide improved values of the manufacturer's built-in measuring standard values. After initial calibration these stable comparators need recalibration only after repairs have been made on their optical train, measuring standard, or encoder.

Preparation

A primary requisite for all comparators is environmental control, especially temperature and dust control. Temperature fluctuations can create havoc with measurements on any comparator in which the measurement standard is composed of a different material from the medium being measured. These fluctuations can cause internal stresses that will alter the calibration of the instrument even

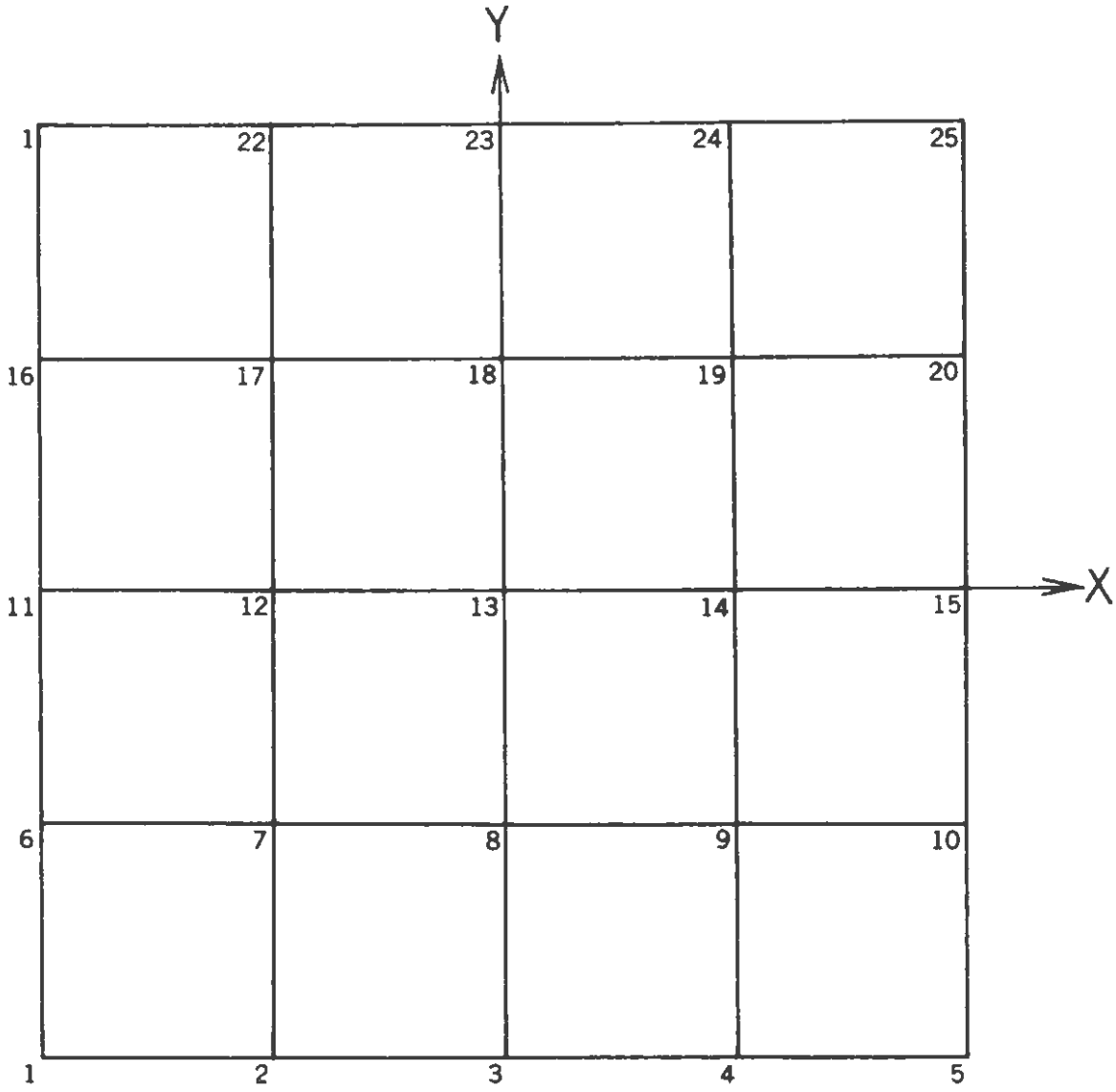


FIGURE 1.—The grid plate system, engraved side up.

when it is not in use. If a lead screw or other temperature sensitive comparator has a heat producing internal light source, it should be left on permanently. Otherwise, much time will be lost daily in warming up the comparator or, still worse, the daily measurements will be adversely affected by thermal gradients. Production measurements can provide an indication of the need for repairs but normally will not indicate a change in the basic calibration parameters.

Standard practice is to perform preventive maintenance, e.g., lubrication and lens cleaning, before calibration. A complete calibration *set* consists of single operator measurements of the grid plate in each of four different 90° rotations (*cases*) on the comparator. The designation of each rotation *case* is defined by the relationship of the corner grid intersection nearest to the comparator x, y origin (fig. 2).

The following outline assumes that the comparator is interfaced with a typewriter, an output numbering device, and tape or card output. The recommended setup and general preparatory guidelines for measurement are sequentially:

(1) Clean the comparator thoroughly and perform the preventive maintenance recommended by the manufacturer.

(2) Gently wipe off any dust on the grid plate with an antistatic brush. Keep fingers off of the grid plate!

(3) Place grid on comparator in the first case position. Make sure the grid is stable and flat on the comparator stage. The computer program allows the grid to be either emulsion up or down. The program can make a simultaneous adjustment of up to 20 cases measured by up to 5 operators. The case positions and numbers are:

Operator #	1	2	3	4	5	Grid corners nearest operator
	1	5	9	13	17	1 and 21
Case	2	6	10	14	18	1 and 5
numbers	3	7	11	15	19	5 and 25
	4	8	12	16	20	21 and 25

For comparators that allow plate rotation, the grid plate ideally should be oriented so that the grid lines are rotated ±11.3° from the comparator axes (fig. 20). It may be necessary for the grid plate to be rotated even more than 11.3° if the measuring area normally used exceeds this span of the grid plate.

(4) Allow ½ hour for the grid plate temperature to reach equilibrium with the comparator temperature. This is not necessary for comparators that have glass measuring standards. Also, it is good practice to store the grid overnight in the comparator room before the calibration (preferably on the comparator stage).

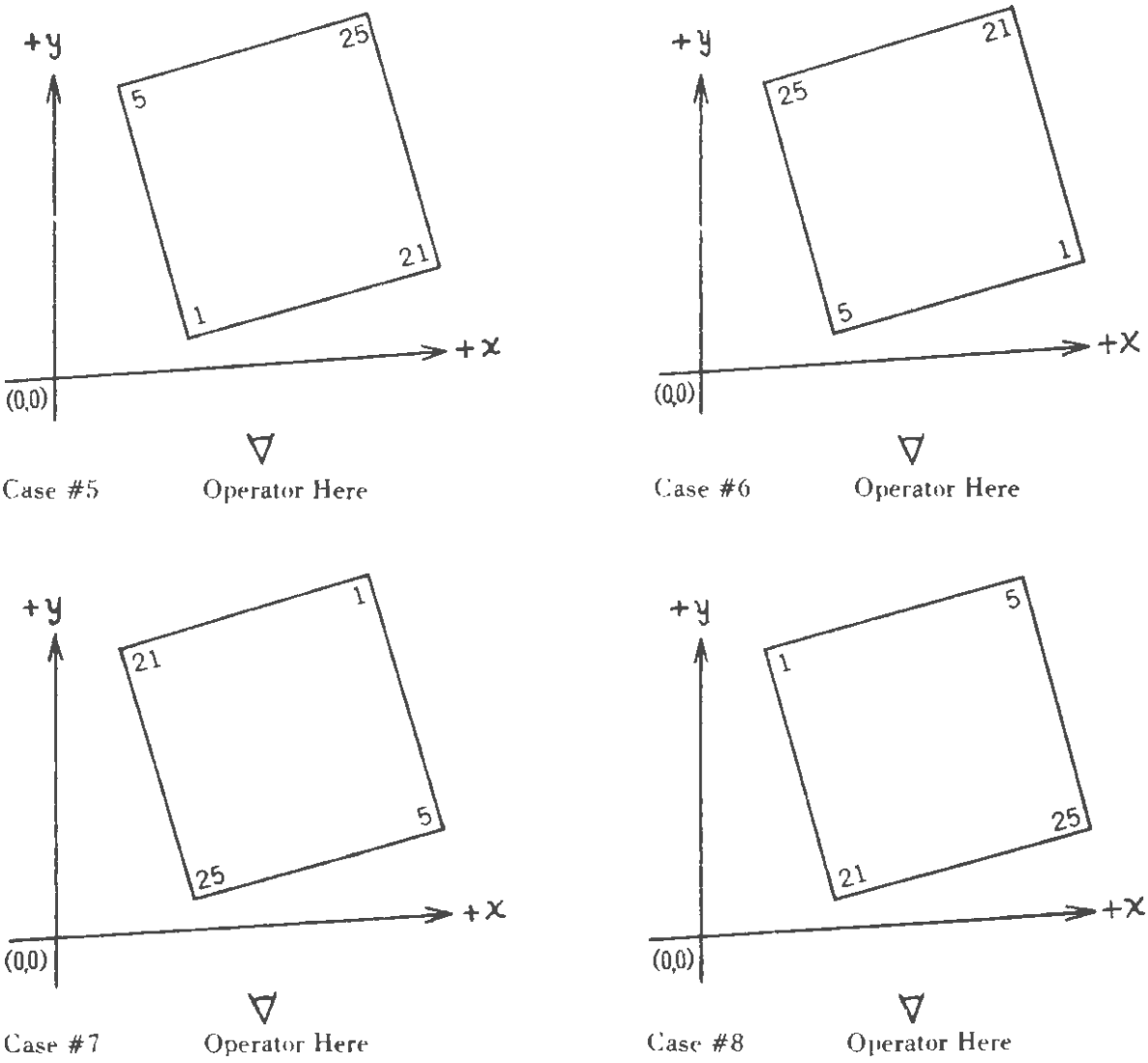


FIGURE 2.—Case numbers and grid orientations for second operator.

(5) Set up output numbering instrument. The program requirements for output and numbering of each measurement are:

Entry	Number of digits
grid plate number	2
case number	2
leg number	1
point number	2
x-coordinate	8
y-coordinate	8

If the comparator uses an external coordinate setting device, it is imperative that a key location on each axis be used to initialize *all* measurements. Otherwise, the nonlinear calibration parameters will be meaningless as they are defined for specific locations on the comparator range.

(6) Set up the typewriter sheet for an output record. Figure 3 is a sample typewriter output sheet. The output contains the following desired information listed by column: (with the data of the first line of fig. 3 given in parentheses) grid plate and case number (20 and 01), leg number (0= legs not read), point number (21), x-coordinate (008366 μm), y-coordinate (044929 μm), comparator and measurement sequence count number (comparator #7, measurement 1).

(7) Set magnification and focus settings. High magnification is recommended. Bring the measuring mark into focus by first approaching sharp focus from both extremes and converging to best focus. Then bring the grid intersection into focus. If binocular magnifiers are used, this procedure may be iterated until measuring mark parallax is eliminated.

(8) Use 5 minutes to warm up instrument and to condition eyes for measurement.

Technique

The ultimate goal of calibration grid measurement is to obtain measurements from a representative sampling of comparator locations throughout the usable format of the comparator. The measurements obtained at any location should have a reliability which is representative of that location. To obtain reliable coordinates, several pointings must be performed at each location. Unlike the standard practices of production measurements that normally provide two or three (sometimes only one!) pointings on a photo object, the calibration grid measurement requires many pointings on each grid intersection to provide absolute reliability of the mean of the measured coordinates.

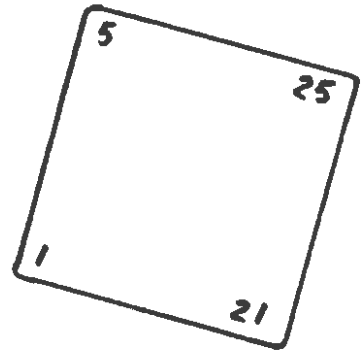
The standard number of pointings required on any particular comparator depends on several factors, the most dominant of which are the size and design of the measuring mark, the line width or dot diameter of the grid or reseau plate used, and the rotation of the gridlines as viewed by the operator. At NOS our experience has shown that, for an orthogonal view of a grid intersection, three pointings are sufficient. For a view of a rotated gridline intersection, five pointings are necessary.

In pointing, the most important criterion is that, for both production and calibration measurements, a mean of the pointings of an object is used for data reduction. N.B. The operator should strive to obtain measurements all of whose individual means are of equal quality. In other words, the operator should perform his minimum set of pointings on an object. Then, if he is not totally satisfied with his repeatability, he should continue to make pointings of the object until he feels that the mean of all the pointings is of the same quality as the mean of any other object he has pointed upon. Additional pointings should always be made in sets of two or three at a time.

Ideally, the most informative pointing precision statistics are obtained from repetitive, rather than replicative, (International Society for Photogrammetry 1964) pointings of each grid intersection. However, practical considerations, usually measuring time, generally outweigh use of this pointing method and replications are considered adequate.

The measuring sequence of the grid points should be so chosen that it randomizes the nonsystematic mechanical deficiencies of the instrument. This is especially important for lead screw instruments which depend on a fine film of lubricant between the nut and lead screw. In this case the accumulation of lubricant ahead of the traveling nut can cause sufficient back pressure to alter the coordinates of initial pointings of an object. Figure 4 shows several measuring sequence patterns.

It is good measuring policy to accept all pointings as read unless, at the time of pointing, the operator realizes that he has made a blunder. Pointings may also be rejected if an obvious blunder is detected in a scan of the measurement listing. Upon completion of the grid pointing circuit the operator should



Comparator Calibration
Comparator #7, 422F46
Plate #1320
Orientation 10 Degrees
Temperature 71 Degrees
June 20, 1972

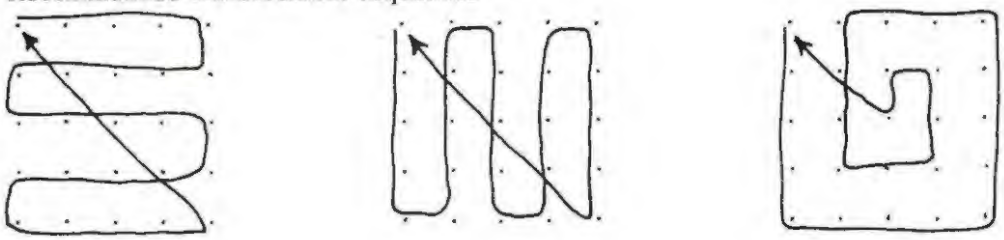
Lewis

Case #1

2001	0	21	008366	044929	700001
2001	0	21	008368	044930	700002
2001	0	16	017300	094126	700003
2001	0	16	017298	094127	700004
2001	0	16	017298	094125	700005
2001	0	11	026234	143321	700006
2001	0	11	026233	143321	700007
2001	0	11	026233	143321	700008
2001	0	06	035167	192518	700009
2001	0	06	035166	192518	700010
2001	0	06	035166	192519	700011
2001	0	01	044098	241716	700012
2001	0	01	044098	241714	700013
2001	0	01	044098	241716	700014
2001	0	02	093292	232783	700015
2001	0	02	093292	232783	700016
2001	0	02	093295	232783	700017
2001	0	02	093292	232783	700018
2001	0	02	093292	232782	700019
2001	0	07	084362	183586	700020
2001	0	07	084362	183586	700021
2001	0	07	084360	183587	700022
2001	0	12	075428	134388	700023
2001	0	12	075428	134387	700024
2001	0	12	075427	134387	700025
2001	0	17	066494	085191	700026
2001	0	17	066496	085192	700027
2001	0	17	066495	085193	700028
2001	0	22	057561	035998	700029
2001	0	22	057562	035998	700030
2001	0	22	057562	035996	700031
2001	0	23	106761	027063	700032
2001	0	23	106760	027062	700033

FIGURE 3.—Typical typewriter output.

Recommended measurement sequences



A measurement sequence that is not recommended

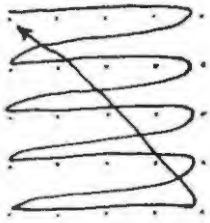


FIGURE 4. – Measurement sequence patterns.

perform a return pointing on the intial grid point measured, as shown in Figure 4. This will provide a measurement set stability check. This closeout pointing set should consist of three or more pointings. If the mean of these pointings differs from the mean of the initial pointings by more than 2 μm (assuming a least count of 1 μm comparator), then a plate shift is suspected and a closeout pointing set on the second intersection read is warranted. If the means differ by 4 μm or more, it is evident that a plate shift has occurred and the complete case must be remeasured. Obviously, remeasurement of a complete grid case should be rare; in such cases the operator should attempt to find the causal factor. The most common causes are a loose grid-plate on comparator stage, temperature fluctuation during the case, measuring mark parallax, or the need for mechanical adjustments or repair.

Evaluation studies have shown that the four radial legs of the grid intersection should be pointed upon only when the measuring mark and the grid intersection are not compatible in size and shape. Normally, and with the benefit of many less pointings required, direct pointings on the grid intersection will provide coordinates as accurate as those obtained with grid intersection leg pointings.

Each operator should read all 25 grid intersection locations in each of 4 cases of measurements. Each case must be rotated 90° from the previous case. The four cases by one operator constitute one *set* of calibration measurements. It is recommended that initial comparator calibrations comprise three sets of measurements performed by three operators. Subsequent calibrations will normally require only one set of measurements by one operator.

On rectangular format comparators, two or more overlapping sets of measurements necessary to cover the entire measurement range must be made by a *single* operator. The overlap area between sets may be minimal as the prime requirement is full coverage of the range of the axes.

All measurements by a single operator must be made with the same measuring mark. Also, regardless of the brand of comparator used, all operators should *always* perform all measurements with the same direction of approach to the object witb the measuring mark. Adherence to this rule will prevent measurement errors from any instrument backlash, will provide consistent operator bias, and will validate the comparator calibration correction parameters.

Computer Program Routines

There are several modes of data storage and retrieval on magnetic tape, many of which are incompatible with one another. Therefore, the computer program presented herein is restricted to punched card input.

Input data setup for the program consists of one header card, followed by all the data, followed by

one blank card. (See fig. 31 in the appendix.) Data within each case should be left in the sequence of measurement. Batch runs of the program are possible with each calibration data set arranged as above. One additional blank card to signify end of job is required after the last set of data.

The single header card must contain the following information:

Card column	*Content*
1	1 = Left-handed data. 0 = Right-handed data. NOTE: Right handedness is defined for the grid as viewed emulsion up.
2	1 = Input data of each case is grouped by point number or sorted. 0 = Input data in each case is in any order (as read).
3	0 = Least count of 1-μm comparator. 1 = Other comparators.
4	Blank
5	1 = Print plot of nonlinear corrections for each case and for the multiple case solution. 0 = Print plot of nonlinear corrections for only the multiple case solution.
6, 7	Comparator number 1–9 = D.M. Mann monocomparators 10, 11 = Wild-Heerbrug STK1 stereocomparators 12, 13 = Zeiss PSK stereocomparators left and right sides respectively. 14 = Other comparators or other input formats (usually requires a new program compilation).
8	R = Right side (secondary stage) of stereocomparators.
10, 69	Operator and case number: Place the operator number (right justified) under the appropriate case number card columns.

	Case No.
10–12	1
13–15	2
16–18	3
19–21	4
22–24	5
25–27	6
28–30	7
31–33	8
34–36	9
37–39	10
40–42	11
43–45	12
46–48	13
49–51	14
52–54	15
55–57	16
58–60	17
61–63	18
64–66*	19
67–69*	20

*Place gridplate number in cc 66–69 when cc 80 = 3

Card column	*Content*
72–79	Date of measurements, e.g., 12/27/69.
80	Master grid used for calibration. 0 = BRL #773 1 = C&GS #1320 2 = C&GS #778 3 = Other values—requires deck of 25 or less coordinate cards which pertain to grid measured.

Data cases must be set up in consecutive case number order. Data formats are given in the main Fortran IV program (see appendix). Point numbers include, sequentially:

Last two digits of grid number, e.g., 73 for grid #773.
Case number = 01–20.
Leg number. = Blank *or* if the four legs of a grid intersection are measured, each leg is assigned the numbers 1, 2, 3, and 4 from any starting point. Measurement sequence is arbitrary except leg #1 must be opposite leg #3. Multiple pointings are allowed on any or all legs.
Grid point number = 01–25.

All relevant data pertaining to the calibration event are printed on the output title page (fig. 5). Figure 6 is a printout of the input data and the close-out on the first point measured as computed by subroutine CLOSOUT.

If the comparator measurement system is left-handed, the calibrated X,Y coordinates of the master grid are interchanged by subroutine FLIPM. This flip option will enable all program computations to be performed in the measurement system.

III. THE MATHEMATICAL MODELS

A. Pointing Precision

Pointing precision is defined by a set of statistics which, when properly evaluated, can reveal several salient characteristics of the control state under which the calibration takes place. These statistics are a first approximation indicator of the quality of *any* measurements made on the comparator by the same operator. The following outline is the statistical approach used to determine the pointing precision in comparator calibration. Most of the equations are applicable to any measurements of coordinates made on photogrammetric instruments.

The pointing precision statistics are valuable in evaluating the three basic factors involved in a

COMPARATOR CALIBRATION - FINAL VERSION - SEP 1971

COORDINATES OF GRID NO.1320 - VALUES ADOPTED 5/ 6/1971

POINT	X	Y
1	-99998.96	-100000.17
2	-100000.12	-50000.44
3	-99999.74	.83
4	-100000.86	50001.11
5	-100001.46	100001.36
6	-49999.24	-99999.67
7	-50000.24	-50000.10
8	-50000.12	1.20
9	-50001.30	50000.81
10	-50002.57	99999.85
11	2.68	-99999.10
12	1.18	-49999.44
13	2.01	2.68
14	1.15	50001.10
15	.21	99999.94
16	50000.81	-100000.97
17	49999.75	-50001.46
18	50000.30	.73
19	49999.29	49999.09
20	49999.13	99997.92
21	99999.68	-100001.13
22	99999.29	-50001.07
23	100000.29	.48
24	99999.48	49998.52
25	99999.31	99997.86

*** ALL VALUES EXCEPT PARAMETERS ARE IN MICRONS ***

HEADER CARD SETUP

LEFT HANDED DATA	YES
INPUT DATA IS IN SORT	NO
LEAST COUNT OF ONE MICRON	YES
COMPARATOR NUMBER	3

CASE NUMBER	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
OPERATOR NUMBER	18	19	18	18	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0

MEASUREMENT DATE 7/17/72

DATE OF COMPUTER RUN IS 12/29/72

FIGURE 5.—Output title page.

complete comparator calibration. These basic factors are the operator, the comparator, and the grid plate.

Each pointing on a grid intersection produces a measurement of the x coordinate and a measurement of the y coordinate. The pointing precision of each of these coordinates is affected by different factors, even though both coordinates may have been pointed upon simultaneously. A typical example of these factors is that human visual acuity is generally considered 20 percent better in horizontal pointing than in vertical pointing. Another factor is that most comparator measurement systems have independent coordinate axes. Therefore, the statistics presented for pointing precision are computed separately for each coordinate.

In the ensuing equations the following notation is used.

Let:

i = the grid point identification number.

j = the measurement case identification number.

k = the identifying number of a single replication of a single coordinate.

l = the total number of grid points measured in the jth case.

n_i = the number of replicated or repeated pointings (measurements) of a single coordinate of grid point i in a single case (used when only one case is under discussion).

INPUT DATA CASE ORDER NUMBER 2

20020	0	1	240140	213261
20020	0	1	240140	213261
20020	0	1	240139	213262
20020	0	1	240139	213261
20020	0	1	240140	213262
20020	0	2	231446	164022
20020	0	2	231447	164023
20020	0	2	231446	164022
20020	0	2	231448	164022
20020	0	2	231447	164023
20020	0	3	222752	114783
20020	0	3	222750	114784
20020	0	3	222751	114783
20020	0	3	222750	114783
20020	0	3	222751	114782
20020	0	4	214056	65545
20020	0	4	214056	65543
20020	0	4	214057	65544
20020	0	4	214056	65545
20020	0	4	214057	65543
20020	0	5	205364	16304
20020	0	5	205364	16304
20020	0	5	205365	16304
20020	0	5	205364	16304
20020	0	5	205364	16305
20020	0	10	156127	25005
20020	0	10	156127	25004
20020	0	10	156127	25004
20020	0	10	156127	25005
20020	0	10	156128	25005
20020	0	15	106885	33703
20020	0	15	106884	33704
20020	0	15	106884	33704
20020	0	15	106884	33703
20020	0	15	106884	33702
20020	0	20	57647	42403
20020	0	20	57646	42403
20020	0	20	57647	42404
20020	0	20	57647	42403
20020	0	20	57647	42403
20020	0	25	8405	51103
20020	0	25	8405	51102
20020	0	25	8405	51102
20020	0	25	8405	51102
20020	0	25	8404	51101
20020	0	24	17100	100341
20020	0	24	17101	100338
20020	0	24	17099	100338
20020	0	24	17100	100339
20020	0	24	17100	100337
20020	0	24	17101	100338
20020	0	24	17100	100339
20020	0	24	17100	100337
20020	0	24	17099	100338
20020	0	24	17100	100338
20020	0	23	25797	149576
20020	0	23	25799	149575
20020	0	23	25797	149578
20020	0	23	25797	149577
20020	0	23	25798	149577
20020	0	22	34495	198819
20020	0	22	34494	198818
20020	0	22	34495	198819
20020	0	22	34493	198818
20020	0	22	34494	198818
20020	0	21	43191	248057
20020	0	21	43190	248056
20020	0	21	43189	248055
20020	0	21	43190	248056
20020	0	21	43190	248056
20020	0	16	92428	239357
20020	0	16	92427	239357
20020	0	16	92428	239357
20020	0	16	92427	239358
20020	0	16	92427	239357
20020	0	11	141666	230657
20020	0	11	141665	230657
20020	0	11	141665	230658
20020	0	11	141665	230658
20020	0	11	141665	230658
20020	0	6	190905	221961
20020	0	6	190905	221959
20020	0	6	190905	221960
20020	0	6	190904	221960
20020	0	6	190904	221958
20020	0	7	182212	172722
20020	0	7	182211	172721
20020	0	7	182213	172721
20020	0	7	182212	172720
20020	0	7	182210	172720
20020	0	8	173516	123482
20020	0	8	173514	123481
20020	0	8	173514	123480
20020	0	8	173513	123481
20020	0	8	173511	123482
20020	0	8	173516	123483
20020	0	8	173515	123481
20020	0	8	173515	123482
20020	0	8	173516	123481
20020	0	8	173511	123482
20020	0	8	173516	123483
20020	0	9	173515	123484
20020	0	8	173515	123481
20020	0	8	173515	123484
20020	0	8	173515	123482
20020	0	9	164820	74243
20020	0	9	164820	74243
20020	0	9	164820	74243
20020	0	9	164820	74244
20020	0	9	164818	74245
20020	0	14	115581	82943
20020	0	14	115578	82944
20020	0	14	115579	82942
20020	0	14	115579	82942
20020	0	14	115579	82942
20020	0	19	66343	91644
20020	0	19	66342	91644
20020	0	19	66342	91642
20020	0	19	66343	91642
20020	0	19	66341	91642
20020	0	18	75037	140878
20020	0	18	75037	140877
20020	0	18	75037	140878
20020	0	18	75037	140878
20020	0	18	75037	140879
20020	0	17	83734	190119
20020	0	17	83733	190120
20020	0	17	83735	190122
20020	0	17	83734	190121
20020	0	17	83733	190121
20020	0	12	132973	181420
20020	0	12	132971	181419
20020	0	12	132971	181420
20020	0	12	132971	181419
20020	0	12	132972	181418
20020	0	12	132971	181418
20020	0	13	124275	132180
20020	0	13	124273	132179
20020	0	13	124272	132179
20020	0	13	124273	132179
20020	0	13	124273	132179
20020	0	1	240140	213262
20020	0	1	240140	213262
20020	0	1	240138	213262
20020	0	1	240140	213261
20020	0	1	240139	213261

CHECKS AND CLOSEOUTS ON POINT NUMBER 1

SEQUENTIAL SHIFTS (XI+1)-(XI)

DELTA X	DELTA Y
-.2	.2

TOTAL SHIFT (LASTX - INITIALX)

-.2	.2

FIGURE 6. – Input data listing and closeout for a single case.

n_{ij} = the number of replicated or repeated pointings of a single coordinate of grid point i in the jth case (used when more than one case is considered).

q = the number of cases (may be for one operator or total number of cases for all operators).

x_{ik} = the value obtained in a single replication (measurement) of either the x or the y coordinate of the grid point i measured in a single case.

x_{ijk} = the value obtained in a single replication (measurement) of either the x or y coordinate of the grid point i measured in the jth case.

m, $\hat{\sigma}$ = estimate of standard deviation; m is used for individual grid points, $\hat{\sigma}$ for many grid points. The true value of the standard deviation, σ, is unknown but is needed in some of the derivations.

NOTE: The standard deviation is defined as the square root of the variance. The term "standard error" is not used.

The following standard assumption (a) underlies all the statistics developed in this chapter. (Other assumptions, (b), (c), and (d) are given later.) (a) All n_i individual measurements of the same grid point i in a single case are from the same population. That is:

$$\sigma^2_{x_{ik}} = \sigma^2_{x_i}.$$

Note that in the next section the subscript k is retained in $m^2_{x_{ik}}$, the estimate of $\sigma^2_{x_{ik}}$ under assumption (a). The notation identifies the quantity with which a standard deviation is to be associated (in this case a particular replication, x_{ik}). Standard assumption (a) tells us that $m^2_{x_{ik}}$, an estimate of $\sigma^2_{x_{ik}}$, has the same value for any k. Assumption (a) is always made, but is not always stated explicitly.

In subsequent sections the pooling of variance estimates is discussed. The pooling can be accomplished in different ways, depending on which random variables, measurements, or means of measurements are assumed to be from populations with the same variance, and depending on the intended purpose of the pooled variance. These different assumptions are stated explicitly immediately before the statistics to which they apply.

Statistics Computed for a Single Grid Point in a Single Case

Numerical examples of these statistics as computed by our program are shown in figure 7 under the heading "Individual Point Statistics". The following statistics are based on unbiased estimates of variances; however, they are biased estimators of standard deviations because of the small sample sizes involved. That is, the degree of freedom $n_i - 1$ should be modified slightly when dealing with sample sizes of five pointings. Several methods of computing unbiased estimates of standard deviations are given or referenced by Jarrett (1968).

The following equations are used to compute the statistics for a single grid point measured in a single case.

1. The mean measured coordinate $(\bar{x}_i, \bar{y}_i)$ is computed by the use of

$$\bar{x}_i = \frac{\sum_{k=1}^{n_i} x_{ik}}{n_i}. \tag{1}$$

Each mean coordinate is entered as a single observation in the least-squares adjustment fit of the grid measurements to the calibrated grid coordinates.

2. The standard deviation of a single coordinate measurement $(m_{x_{ik}}, m_{y_{ik}}, m_{pt_{ik}})$ is computed by the use of

$$m_{x_{ik}} = \sqrt{\frac{\sum_{k=1}^{n_i} (x_{ik} - \bar{x}_i)^2}{n_i - 1}}, \text{ and} \tag{2}$$

$$m_{pt_{ik}} = \sqrt{\frac{m^2_{x_{ik}} + m^2_{y_{ik}}}{2}}, \tag{3}$$

where the subscript pt_{ik} refers to the kth pointing of grid point i (as opposed to the kth pointing of a grid coordinate).

These statistics indicate the reliability of any single observation of a coordinate or pair of coordinates.

3. The standard deviation of the mean coordinate measurement ($m_{\bar{x}_i}$, $m_{\bar{y}_i}$, $m_{\overline{pl}_i}$) is computed by the use of

$$m_{\bar{x}_i}=\sqrt{\frac{\sum_{k=1}^{n_i}(x_{ik}-\bar{x}_i)^2}{n_i(n_i-1)}} \tag{4}$$

$$m_{\overline{pl}_i}=\sqrt{\frac{m^2_{\bar{x}_i}+m^2_{\bar{y}_i}}{2}} \tag{5}$$

These statistics indicate the precision of the mean coordinate determined from n_i pointings. These values describe the level of quality that the operator should strive to maintain for all means of all grid points measured.

4. The covariance of the coordinates of a single pointing ($m_{xy_{ik}}$) is computed by the use of

$$m_{xy_{ik}}=\frac{\sum_{k=1}^{n_i}\{(x_{ik}-\bar{x}_i)(y_{ik}-\bar{y}_i)\}}{n_i-1} \tag{6}$$

5. The correlation coefficient of the coordinates of a single pointing ($\rho_{xy_{ik}}$) is computed by the use of

$$\rho_{xy_{ik}}=\frac{m_{xy_{ik}}}{m_{x_{ik}}\cdot m_{y_{ik}}} \tag{7}$$

This statistic provides an indication of the mutual interdependence of the comparator axes, which are assumed to be totally independent for most comparators. Also, an insight into the measuring mode used by the operator is indicated; that is, whether the operator pointed with both coordinate axes simultaneously, or with one axis at a time.

NOTE: The covariance of the coordinates of a mean pointing ($m_{\bar{x}\bar{y}_{ik}}$) and the correlation coefficient of the coordinates of a mean pointing ($\rho_{\bar{x}\bar{y}_{ik}}$) are not computed by the program. It can be shown that these statistics of the mean differ from those given by equations (6) and (7) only by a factor of $\frac{1}{n_i}$.

Numerical examples of the statistics given by equations (1) through (7) are shown in figure 7 as they are printed out by the program. The columns of numbers shown in figure 7 are respectively denoted, from left to right, as: i, n_i, $m_{x_{ik}}$, $m_{y_{ik}}$, $m_{pl_{ik}}$, $m_{xy_{ik}}$, $\rho_{x_{ik}}$, $\bar{x}_i$, $\bar{y}_i$, $m_{\bar{x}_i}$, $m_{\bar{y}_i}$, $m_{\overline{pl}_i}$, i.

Statistics Computed for All Grid Points in a Single Case

These statistics are presented in figure 7 under the heading caption "Combined Point Statistics".

1. The pooled estimate of the variance of a single coordinate measurement (x_{ik}) of a single case ($(\hat{\sigma}^2_{x_{ik}})_p$, $(\hat{\sigma}^2_{y_{ik}})_p$, $(\hat{\sigma}^2_{pl_{ik}})_p$) is computed under the following additional assumption.

Assumption (b): All grid points of a single case provide the same appearance for measurement to the comparator operator. Therefore, any measurements of any grid point of the case may be considered to be made (pointed upon) with equal precision.

$$\sigma^2_{x_{ik}}=\sigma^2_x$$

That is, the true value of the variance of a single measurement from a single case (σ^2_x) is a constant for the case, of which the variances of the individual point measurements ($m^2_{x_{ik}}$) are estimates.

The statistic $(\hat{\sigma}^2_{x_{ik}})_p$, an estimator of $\sigma^2_{x_{ik}}$ under assumption (b), is defined as

$$(\hat{\sigma}^2_{x_{ik}})_p=\frac{\sum_{i=1}^{l}(w_i m^2_{x_{ik}})}{\sum_{i=1}^{l}(w_i)}, \tag{8}$$

where w_i, the weight assigned to an estimate of the variance of a measurement, is

$$w_i=\frac{1}{\sigma^2_{m^2_{x_{ik}}}}. \tag{9}$$

The variance of the estimated variance is

$$\sigma^2_{m^2_{x_{ik}}}=\frac{2\sigma^4_{x_i}}{\nu_i},$$

where ν_i = the degrees of freedom for the variance estimate and assumption (a), $\sigma^2_{x_{ik}}=\sigma^2_{x_i}$, has been used.

From eq (2)

$$\nu_i=n_i-1.$$

CASE NUMBER 2 POINTING PRECISION STATISTICS
OPERATOR NUMBER 18
COMPARATOR NUMBER 3
GRID PLATE NUMBER 1320
DATE OF MEASUREMENT 7/17/72

INDIVIDUAL POINT STATISTICS

POINT NUMBER	NUMBER OF TIMES READ	SINGLE OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	(MICRONS SQ) COVARIANCE XY	(PERCENTAGE) CORRELLATION COEFFICIENT	MEAN MEASURED COORDINATES X	Y	MEAN OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	POINT NUMBER
1	10	.707	.527	.624	-.056	-14.91	240139.50	213261.50	.224	.167	.197	1
3	5	.837	.707	.775	-.250	-42.26	222750.80	114783.00	.374	.316	.346	3
4	5	.548	1.000	.806	-.250	-45.64	214056.40	65544.00	.245	.447	.361	4
5	5	.447	.447	.447	-.050	-25.00	205364.20	16304.20	.200	.200	.200	5
10	5	.447	.548	.500	.100	40.82	156127.20	25004.60	.200	.245	.224	10
15	5	.447	.837	.671	-.050	-13.36	106884.20	33703.20	.200	.374	.300	15
20	5	.447	.447	.447	.050	25.00	57646.80	42403.20	.200	.200	.200	20
25	5	.447	.707	.592	.250	79.06	8404.80	51102.00	.200	.316	.265	25
24	10	.667	1.160	.946	.000	.00	17100.00	100338.30	.211	.367	.299	24
23	5	.894	1.140	1.025	-.700	-68.64	25797.60	149576.60	.400	.510	.458	23
22	5	.837	.548	.707	.400	87.29	34494.20	198818.40	.374	.245	.316	22
21	5	.707	.707	.707	.500	100.00	43190.00	248056.00	.316	.316	.316	21
16	5	.548	.447	.500	-.100	-40.82	92427.40	239357.20	.245	.200	.224	16
11	5	.447	.548	.500	-.150	-61.24	141665.20	230657.60	.200	.245	.224	11
6	5	.548	1.140	.894	.300	48.04	190904.60	221959.60	.245	.510	.400	6
7	5	1.140	.837	1.000	.400	41.93	182211.60	172720.80	.510	.374	.447	7
8	15	1.642	1.163	1.423	.390	20.45	173514.47	123481.93	.424	.300	.367	8
9	5	.894	.894	.894	-.700	-87.50	164819.60	74243.60	.400	.400	.400	9
14	5	1.095	.894	1.000	-.150	-15.31	115579.20	82942.60	.490	.400	.447	14
19	5	.837	1.095	.975	.300	32.73	66342.20	91642.80	.374	.490	.436	19
18	5	0.000	.447	.316	0.000	100.00	75037.00	140877.80	0.000	.200	.141	18
17	5	.837	1.140	1.000	.400	41.93	83733.80	190120.60	.374	.510	.447	17
12	6	.837	.894	.866	.200	26.73	132971.50	181419.00	.342	.365	.354	12
13	5	1.095	.447	.837	.450	91.86	124273.20	132179.20	.490	.200	.374	13
2	5	.837	.548	.707	.100	21.82	231446.80	164022.40	.374	.245	.316	2
					COMBINED POINT STATISTICS							
146	25	.888	.856	.872	.077	10.18	0.00	0.00	.329	.336	.332	

BASIS

25 POINTS 146 MEASUREMENTS 1 CASES 1 OPERATORS

FIGURE 7. – Pointing precision statistics for a single case.

Therefore

$$\sigma^2_{m^2_{x_{ik}}} = \frac{2\sigma^4_{x_i}}{n_i - 1}, \tag{10}$$

and the appropriate weights for eq (9) are

$$w_i = \frac{n_i - 1}{2\sigma^4_{x_i}}. \tag{11}$$

Substituting eq (11) into eq (8) we obtain:

$$(\hat{\sigma}^2_{x_{ik}})_p = \frac{\sum_{i=1}^{l}\left\{\frac{(n_i-1)}{2\sigma^4_{x_i}} \cdot m^2_{x_{ik}}\right\}}{\sum_{i=1}^{l}\left\{\frac{n_i-1}{2\sigma^4_{x_i}}\right\}}. \tag{12}$$

But according to assumption (b) the quantity $2\sigma^4_{x_i}$ is a constant. Therefore, rearranging terms and substituting eq (2) into eq (12) we obtain

$$(\hat{\sigma}^2_{x_{ik}})_p = \frac{\frac{l}{2\sigma^4_{x_i}} \sum_{i=1}^{l}\left\{(n_i-1)\frac{\sum_{k=1}^{n_i}(x_{ik}-\bar{x}_i)^2}{n_i-1}\right\}}{\frac{l}{2\sigma^4_{x_i}} \sum_{i=1}^{l}(n_i-1)}$$

which simplifies to

$$(\hat{\sigma}^2_{x_{ik}})_p = \frac{\sum_{i=1}^{l}\sum_{k=1}^{n_i}(x_{ik}-\bar{x}_i)^2}{N-l}. \tag{13}$$

Where

N = the total number of measurements of all the grid points of the case,

$$N = \sum_{i=1}^{l} n_i. \tag{14}$$

The pooled estimate of the variance of the pointing of any grid point of a single case is computed under assumption (a) by

$$(\hat{\sigma}^2_{pl_{ik}})_p = \tfrac{1}{2}\{(\hat{\sigma}^2_{x_{ik}})_p + (\hat{\sigma}^2_{y_{ik}})_p\}.$$

These statistics are indicators of the relative quality of the grid plate intersections.

2. The pooled estimate of the variance of a mean coordinate measurement ($\bar{x}_i$) of a single case

$$(\hat{\sigma}^2_{\bar{x}_i})_p,\ (\hat{\sigma}^2_{\bar{y}_i})_p,\ (\hat{\sigma}^2_{\bar{pl}_i})_p$$

is computed under the following assumption:

Assumption (c): The number of pointings (n_i) of a grid point (x_i) are determined subjectively by the measurer whose aim is to provide mean measurements ($\bar{x}_i$) of equal quality. This implies the following.

(1) The variance of single measurements of x_i varies from grid point to grid point, or

$$\sigma^2_{\bar{x}_i} \neq \sigma^2_{\bar{x}},$$

That is, the true value of the variance of a grid points are from populations with different variances.

(2) The mean measurements ($\bar{x}_i$) of different grid intersections are from populations with the same variance, or

$$\sigma^2_{\bar{x}_i} = \sigma^2_{\bar{x}},$$

that is, the true value of the variance of a mean ($\sigma^2_{\bar{x}}$) is a constant of which the variance of the mean at each individual grid point is an estimate ($m^2_{\bar{x}_i}$).

According to assumption (c) the statistic $(\hat{\sigma}^2_{\bar{x}})_p$ is defined as

$$(\hat{\sigma}^2_{\bar{x}_i})_p = \frac{\sum_{i=1}^{l}(w_i m^2_{\bar{x}_i})}{\sum_{i=1}^{l}(w_i)}. \tag{15}$$

where w_i is the weight determined for the estimate of the variance of a mean measurement,

$$w_i = \frac{1}{\sigma^2_{m^2_{\bar{x}_i}}}. \tag{16}$$

From an inspection of eq (2) and (4),

$$m^2_{\bar{x}_i} = \frac{m^2_{x_{ik}}}{n_i} \tag{17}$$

with the corresponding relation between true variances being

$$\sigma^2_{\bar{x}_i} = \frac{1}{n_i}\sigma^2_{x_{ik}}. \tag{18}$$

Propagating the variance through eq (17) we obtain:

$$\sigma^2_{m^2_{x_i}} = \left(\frac{1}{n_i}\right)^2 \sigma^2_{m^2_{x_{ik}}} .$$

By substitution from eq (10) aud eq (18) we obtain

$$\sigma^2_{m^2_{x_i}} = \frac{2\sigma^4_{x_i}}{n_i^2(n_i-1)} = \frac{2(\sigma^2_{x_i})^2}{n_i-1} \tag{19}$$

Therefore the appropriate weights for eq (16) are

$$w_i = \frac{n_i-1}{2(\sigma^2_{x_i})^2}. \tag{20}$$

Substituting eq (20) into eq (15) we obtain:

$$(\hat{\sigma}^2_{\bar{x}_i})_p = \frac{\sum_{i=1}^{l}\left\{\frac{(n_i-1)}{2(\sigma^2_{x_i})^2}\cdot m^2_{x_i}\right\}}{\sum_{i=1}^{l}\left\{\frac{(n_i-1)}{2(\sigma^2_{x_i})^2}\right\}}. \tag{21}$$

But according to assumption (c) the quantity $\sigma^2_{x_i} = \sigma^2_x$ is a constant. Therefore, rearranging the terms of eq (21), simplifying and then substituting in eq (4) and (14) we obtain

$$(\hat{\sigma}^2_{\bar{x}_i})_p = \frac{\sum_{i=1}^{l}\{(n_i-1)m^2_{x_i}\}}{\sum_{i=1}^{l}(n_i-1)} = \frac{\sum_{i=1}^{l}\left(\frac{1}{n_i}\right)\sum_{k=1}^{n_i}(x_{ik}-\bar{x}_i)^2}{N-l}. \tag{22}$$

The pooled estimate of the variance of a mean pointing of any grid point of a single case is computed under assumption (c) by

$$(\hat{\sigma}^2_{\bar{y}_i})_p = \tfrac{1}{2}\{(\sigma^2_{x_i})_p + (\sigma^2_{y_i})_p\}.$$

This is the appropriate statistic for evaluating the performance of the measuring operator.

3. The pooled estimate of the covariance between the coordinates of single replications (measurements) of a single case, $(\hat{\sigma}_{xy_{ik}})_p$, is computed by the use of

$$(\hat{\sigma}_{xy_{ik}})_p = \frac{\sum_{i=1}^{l}\sum_{k=1}^{n_i}\{(x_{ik}-\bar{x}_i)(y_{ik}-\bar{y}_i)\}}{N-l}. \tag{23}$$

4. The pooled correlation coefficient of the coordinates of single replications (measurements) of a single case, $(P_{xy_{ik}})_p$, is computed by the use of

$$(P_{xy_{ik}})_p = \frac{(\hat{\sigma}_{xy_{ik}})_p}{(\hat{\sigma}_{x_i})_p\cdot(\hat{\sigma}_{y_i})_p}. \tag{24}$$

The pooled estimate of the covariance between the coordinates of mean measurements of a single case $(\hat{\sigma}_{\overline{xy}_i})_p$ and the pooled correlation coefficient of the coordinates of mean measurements of a single case $(P^{\bar{}}_{xy_i})_p$ are not computed by the program.

A numerical example of the statistics given by equations (13), (14), (22), (23) and (24) is shown in figure 7 under the heading "Combined Point Statistics." The row of statistics are from left to right respectively, N, l, $(\hat{\sigma}_{x_{ik}})_p$, $(\hat{\sigma}_{y_{ik}})_p$, $(\hat{\sigma}_{pt_{ik}})_p$, $(\hat{\sigma}_{xy_{ik}})$, $(P_{xy_{ik}})_p$, $N.A.$, $N.A.$ $(\hat{\sigma}_{\bar{x}_i})_p$, $(\hat{\sigma}_{\bar{y}_i})_p$, and $(\hat{\sigma}_{\overline{pt}_i})_p$.

Statistics Computed for a Single Grid Point in Many Cases

In a typical comparator calibration each operator measures each of the 25 grid points 5 times (replications) in each of 4 rotated (90°) cases. A set of pointing precision statistics is pooled for each individual grid point from the approximate 20 pointings per grid point. An example of these statistics is given in figure 8 under the caption "Individual Point Statistics." These statistics provide a mechanism for comparing the relative quality of the different grid points.

When several operators measure the same grid in the same comparator calibration (typically four cases each), the pointing precision statistics are pooled for all measurements (replications) of each individual grid point. This pooling provides an even larger and less-biased sample for comparing the quality (pointability) of the different grid points. An example of the pooled statistics for several operators is similar in form to figure 8, however the number of times read is a multiple of those shown.

1. The pooled estimate of the variance of an individual grid point coordinate measured (replicated) for in many cases $(m^2_{x_{ijk}})_p$, $(m^2_{y_{ijk}})_p$, $(m^2_{pt_{ijk}})_p$, is computed under the following assumption:

Assumption (d). Each individual grid point provides the same appearance for measurement to the comparator operator regardless of any 90° rotation of the grid. Therefore, measurements of a single grid point from any case may be considered to be made (pointed upon) with equal precision.

Combining assumptions (a) and (d) we have

$$\sigma^2_{x_{ijk}} = \sigma^2_{x_i} \ (\neq \sigma^2_x).$$

Thus under assumption (d), the standard deviation of a single measurement is the same for all replications and for all cases at a given grid point, but may still vary from point to point. The variances of the individual point measurements in the individual cases $(m^2_{x_{ijk}})$ are estimates of $\sigma^2_{x_i}$.

It can be shown that under assumption (d) and by a similar development used to obtain eq (13) that

$$(m^2_{x_{ijk}})_p = \frac{\sum_{j=1}^{q} \sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij})^2}{\sum_{j=1}^{q} (n_{ij} - 1)}. \tag{25}$$

The pooled estimate of the variance of a single pointing on a grid point measured in many cases is computed under assumption (d) by

$$(m^2_{pt_{ijk}})_p = \tfrac{1}{2} \{ (m^2_{x_{ijk}})_p + (m^2_{y_{ijk}})_p \}.$$

2. The pooled estimate of the covariance between coordinates at a single measurement of an individual grid point coordinate, based on measurement in many cases $(m_{xy_{ijk}})_p$ is computed by the use of

$$(m_{xy_{ijk}})_p = \frac{\sum_{j=1}^{q} \sum_{k=1}^{n_{ij}} \{ (x_{ijk} - \bar{x}_{ij})(y_{ijk} - \bar{y}_{ij}) \}}{\sum_{j=1}^{q} (n_{ij} - 1)} \tag{26}$$

3. The pooled estimate of the correlation coefficient between coordinates at a single measurement of an individual grid point coordinate based on measurements in many cases $(\rho_{xy_{ijk}})_p$ is computed by the use of

$$(\rho_{xy_{ijk}})_p = \frac{(m_{xy_{ijk}})_p}{(m_{x_{ijk}})_p \cdot (m_{y_{ijk}})_p}. \tag{27}$$

The corresponding statistics for the pooled variances of the mean (using several cases) of an individual grid point for a single case (i.e., $(m_{x_{ij}})_p$, etc.) were not computed as their value is limited in evaluating any of the three primary factors of a complete calibration.

A numerical example, produced by the program, of the statistics given by eqs (25), (26), and (27) is shown in figure 8. The columns of numbers shown in figure 8, from left to right, are:

$$i, \sum_{j=1}^{q} n_{ij}, (m_{x_{ijk}})_p, (m_{y_{ijk}})_p, (m_{pt_{ijk}})_p,$$

$$(m_{xy_{ijk}})_p, \text{ and } (\rho_{xy_{ijk}})_p.$$

Statistics Computed for All Grid Points in Many Cases

A summary of all the pointing precision statistics is computed for all cases of all points measured by a single operator. A similar set of statistics is computed for all cases of all points measured by all operators in a complete comparator calibration. An example of these statistics is shown in figure 8 under the heading "Combined Point Statistics."

1. The pooled estimate of the variance of any single coordinate measurement measured (replicated) in many cases $(\hat{\sigma}^2_{x_{ijk}})_p$, $(\hat{\sigma}^2_{y_{ijk}})_p$, $(\hat{\sigma}^2_{pt_{ijk}})_p$, is computed under the following extension of assumption (b).

Assumption (b) extended: all grid points of all cases provide the same appearance for measurement to the comparator operator regardless of any 90° rotation of the grid. Therefore any measurements of any grid point of any case may be considered to be made (pointed upon) with equal precision, i.e.,

$$m^2_{x_{ijk}} = \sigma^2_{x_i}.$$

Using a development of equations similar to that used to obtain (13) it can be shown that

$$(\hat{\sigma}^2_{x_{ijk}})_p = \frac{\sum_{j=1}^{q} \sum_{i=1}^{l} \sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij})^2}{N_j - l_q}, \tag{28}$$

OPERATOR NUMBER 18 POINTING PRECISION STATISTICS
COMPARATOR NUMBER 3
GRID PLATE NUMBER 1320
DATE OF MEASUREMENT 7/17/72

INDIVIDUAL POINT STATISTICS

POINT NUMBER	NUMBER OF TIMES READ	SINGLE OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	(MICRONS SQ) COVARIANCE XY	(PERCENTAGE) CORRELATION COEFFICIENT	MEAN MEASURED COORDINATES X	Y	MEAN OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	POINT NUMBER
1	25	.698	.622	.661	.037	8.63						
2	20	.781	.616	.703	.021	4.38						
3	20	.725	.781	.754	-.032	-5.57						
4	20	.562	.632	.598	-.042	-11.85						
5	25	.849	.733	.793	.129	20.75						
6	20	.696	.795	.747	.232	41.88						
7	20	.962	.846	.906	.221	27.15						
8	30	1.256	.927	1.104	.154	13.23						
9	20	.951	.632	.808	-.126	-20.99						
10	25	.458	1.035	.867	.017	2.45						
11	20	.459	.711	.598	.032	9.68						
12	21	.718	.632	.676	.050	11.02						
13	20	.681	.665	.673	.021	4.65						
14	20	.858	.580	.733	-.084	-16.90						
15	20	.795	.795	.795	.084	13.33						
16	25	1.065	.487	.828	-.267	-51.40						
17	20	.918	.632	.788	.021	3.63						
18	20	.503	.481	.492	.011	4.35						
19	20	.696	.616	.657	.116	27.03						
20	20	.781	.871	.827	-.074	-10.83						
21	25	.661	.563	.614	.125	33.58						
22	25	.894	.636	.776	.275	48.36						
23	20	.918	.834	.877	-.358	-46.79						
24	35	1.114	.830	.982	-.162	-17.50						
25	30	1.017	.799	.915	-.276	-33.93						
					COMBINED POINT STATISTICS							
566	25	.852	.730	.793	.001	.11	0.00	0.00	.339	.297	.319	

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FIGURE 8.—Pointing precision statistics for a single operator.

where N_j is defined as the total number of replicated measurements made by a single operator, or made by all operators.

$$N_j = \sum_{j=1}^{q} \sum_{i=1}^{l} n_{ij} \tag{29}$$

The pooled estimate of the variance of any pointing of any grid point of any case is computed by

$$(\hat{\sigma}^2_{pt_{ijk}})_p = \tfrac{1}{2}\{(\hat{\sigma}^2_{x_{ijk}})_p + (\hat{\sigma}^2_{y_{ijk}})_p\}$$

2. The pooled estimate of the variance of any mean coordinate measurement measured in many cases

$$(\hat{\sigma}^2_{\bar{x}_{ij}})_p,\ (\hat{\sigma}^2_{\bar{y}_{ij}})_p,\ (\hat{\sigma}^2_{\bar{p}t_{ij}})_p.$$

is computed under assumption (c) and is obtained by a development similar to that used to obtain eq (22) and is computed by the use of

$$(\hat{\sigma}^2_{\bar{x}_{ij}})_p = \frac{\sum_{j=1}^{q} \sum_{i=1}^{l} \left(\frac{1}{n_{ij}}\right) \sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij})^2}{N_j - l_q} \tag{30}$$

The pooled estimate of the variance of any mean pointing of any grid point of any case is computed by

$$(\hat{\sigma}^2_{\bar{p}t_{ij}})_p = \tfrac{1}{2}\{(\hat{\sigma}^2_{\bar{x}_{ij}})_p + (\hat{\sigma}^2_{\bar{y}_{ij}})_p\}$$

3. The pooled estimate of the covariance between coordinates of single replications (measurements) of many cases $(\hat{\sigma}_{xy_{ijk}})_p$ is computed by the use of

$$(\hat{\sigma}_{xy_{ijk}})_p = \frac{\sum_{j=1}^{q} \sum_{i=1}^{l} \sum_{k=1}^{n_{ij}} \{(x_{ijk} - \bar{x}_{ij})(y_{ijk} - \bar{y}_{ij})\}}{N_j - l_q} \tag{31}$$

4. The pooled correlation coefficient between the coordinates of single measurements of many cases $(P_{xy_{ijk}})_p$ is computed by the use of

$$(P_{xy_{ijk}})_p = \frac{(\hat{\sigma}_{xy_{ijk}})_p}{(\hat{\sigma}_{x_{ijk}})_p \cdot (\hat{\sigma}_{y_{ijk}})_p} \tag{32}$$

A numerical example of the statistics given by eqs (28) through (32) is shown in figure 8 under the heading "Combined Point Statistics." The row of statistics shown from left to right are:

N_j, l, $(\hat{\sigma}_{x_{ijk}})_p$, $(\hat{\sigma}_{y_{ijk}})_p$, $(\hat{\sigma}_{pt_{ijk}})_p$, $(\hat{\sigma}_{xy_{ijk}})_p$, $(P_{xy_{ijk}})_p$, *N.A.*, *N.A.*, $(\hat{\sigma}_{\bar{x}_{ij}})_p$, $(\hat{\sigma}_{\bar{y}_{ij}})_p$, and $(\hat{\sigma}_{\bar{p}t_{ij}})_p$.

Summary

A summary of the pointing precision statistics and their value to a complete calibration is as follows.

Evaluation of the comparator operator:

1. From eq (1) the mean of the pointings of a grid point is computed. This mean is the value used as a single observation in all subsequent least-squares adjustments.
2. From eqs (4) and (5) the precision of the mean pointing used is determined. The inverse of this variance could be used as an appropriate precision weight associated with the mean pointing.
3. From eqs (22) and (30) a pooled estimate of the variance of a mean measurement $(\hat{\sigma}^2_{\bar{x}})_p$ is determined. Each of these statistics combines many pointing means into a single compatible value appropriate for comparing the pointing skills of different operators, under assumptions (a) and (c).
4. Monitoring of the correlation coefficients computed by eqs (7), (24), and (32) will indicate if any changes occur in the pointing technique used by an operator.

Evaluation of the grid plate used:

1. The standard deviation of single coordinate measurements $(m_{x_{ik}})$, computed by eqs (2) and (3), and the pooled standard deviations $(m_{x_{ijk}})_p$, computed by eq (25), normally will reveal any differences in the appearance of different grid points. A diffuse or irregular appearance of a grid point (to an operator) will be indicated by a consistently higher (from case to case) value of $m_{x_{ik}}$ for that grid point than $m_{x_{ik}}$ values of the other grid points. The pooling of $m_{x_{ik}}$ from several cases is accomplished by computing eq (25) for each grid point. After many comparator calibrations have been performed by several operators using the same grid, it is possible to determine different quality values for the pointability of each grid point.

2. The pooled estimate of the variance of a single measurement, $(\hat{\sigma}^2_{\bar{x}_{ijk}})_p$ computed by eq (28), for all measurements of all cases, is a single value indicator of the appearance or condition of all grid points on the grid plate. Many factors can cause a change in this value, such as different operators, a change in the viewing conditions (focus), etc.; therefore, its value as a grid quality statistic is limited.
3. The correlation coefficients of a single grid point pooled over several rotated cases, $(\rho_{xy_{ijk}})_p$ by eq (27), will reveal a fixed bias in the appearance of the grid point.

Evaluation of the comparator used:

1. When the standard deviation of single coordinate measurements, $(m_{x_{ik}})$ computed by eqs (2) and (3), is associated with the coordinates of the point, it can provide an indication of the less precise regions (zonal characteristics) of the comparator. This evaluation requires an inspection of the $m_{x_{ik}}$ of different grid points from different measurement cases, all measured in the same comparator coordinate region.
2. A comparison of the pooled correlation coefficients, $(P_{xy})_p$ computed by eqs (24) and (32), from several calibrations of the comparator, will indicate the degree of interdependence of the coordinate axes.
3. All of the computed statistics can be used in determining the relative merits of one reticle over another. This comparison should be made under controlled conditions.
4. The decision to perform direct pointings on a grid intersection, rather than point on the four legs of the intersection, was a direct outcome of an evaluation of the standard deviations of the means $(\hat{\sigma}_{\bar{x}})_p$ obtained.

Computer Program Routines

The pointing precision computations and the computations required for their graphic display are performed by three Fortran IV subroutines, entitled GROUP, STAT, and ELIPS. A sequential outline of the computer program flow is as follows:

The point number and x and y coordinates of each measured point of a single case are read and stored in the POINT, XMEAS, and YMEAS arrays respectively. If legs of a grid intersection were measured, the pointings on the four legs are reduced by intersection, in the LINLIN subroutine, to a single x, y coordinate set and then stored. After all measurements of a case have been stored, all multiple measurements of each point are assembled together. This grouping, performed by subroutine GROUP, rearranges the data in the POINT, XMEAS, and YMEAS arrays, but does not put the data in numerical order.

The arrays of grouped data are transferred to the STAT subroutine. The STAT routine computes and prints all of the pointing means and precision statistics according to the equations outlined in this chapter. Figure 7 shows the printed output for a single case of measurements.

The STAT routine also computes the parameters required to plot error ellipses of a single pointing. These parameters are computed by the following equations.

Let:

$$W=\sqrt{(m^2_{x_{ik}}-m^2_{y_{ik}})^2+4m^2_{xy_{ik}}}.$$

Then the semi-major axis (A) of the error ellipse is computed by

$$A=\sqrt{\frac{m^2_{x_{ik}}+m^2_{y_{ik}}+W}{2}},$$

and the semi-minor axis (B) is computed by

$$B=\sqrt{\frac{m^2_{x_{ik}}+m^2_{y_{ik}}-W}{2}}.$$

The angle between the major axis of the ellipse and the x coordinate axis is *ALPHA*.

$$ALPHA=\frac{1}{2}\tan^{-1}\left(\frac{2m_{xy_{ik}}}{m^2_{x_{ik}}+m^2_{y_{ik}}}\right),$$

where *ALPHA* is a positive angle measured from x to y.

The CDC 6600 computer used at NOS has Cathode Ray Tube (CRT) software routines incorporated in its system to generate microfilm output. The ELIPS subroutine generates annotated CRT plots of the pointing-precision error ellipses oriented in a center origin coordinate system that is parallel to the comparator measurement system. Figure 9 is the plot generated from the individual point statistics data of figure 7, that is $m_{x_{ik}}$, $m_{y_{ik}}$, and $m_{xy_{ik}}$. Options in the main program call statement provide a choice of four confidence limits, 39, 50, 95, or 99 percent that govern the scale of the ellipses. Figure 10 is the plot

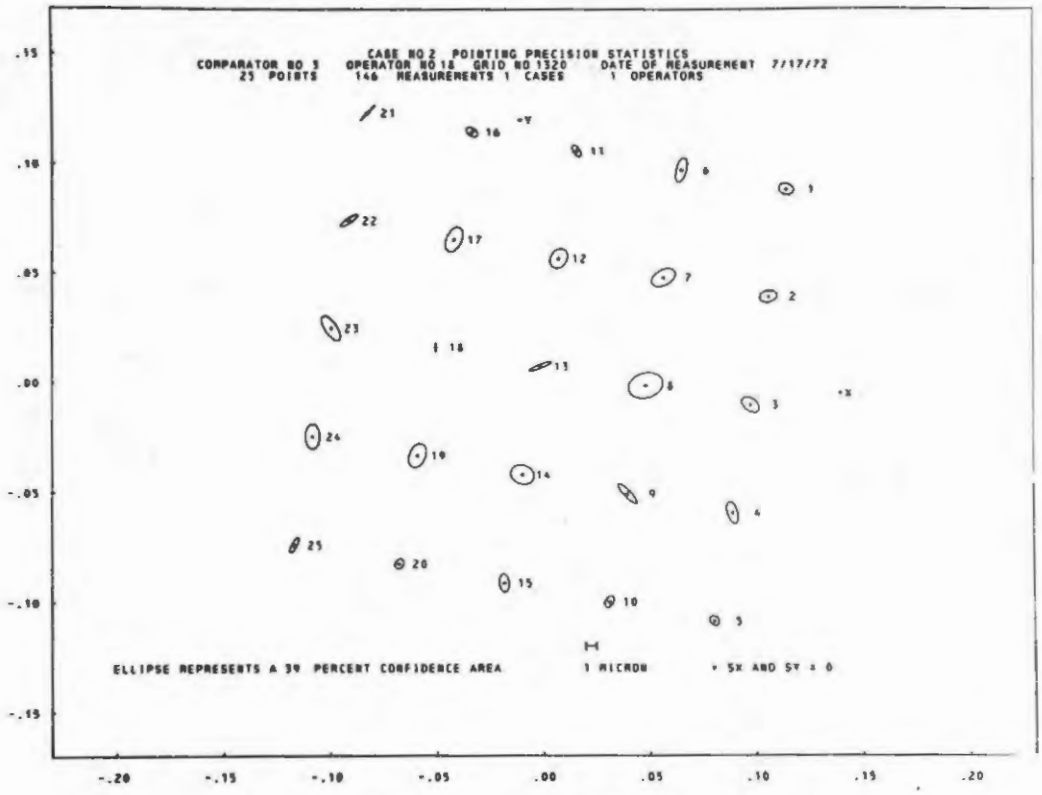

FIGURE 9. – Pointing precision error ellipses of a single case.

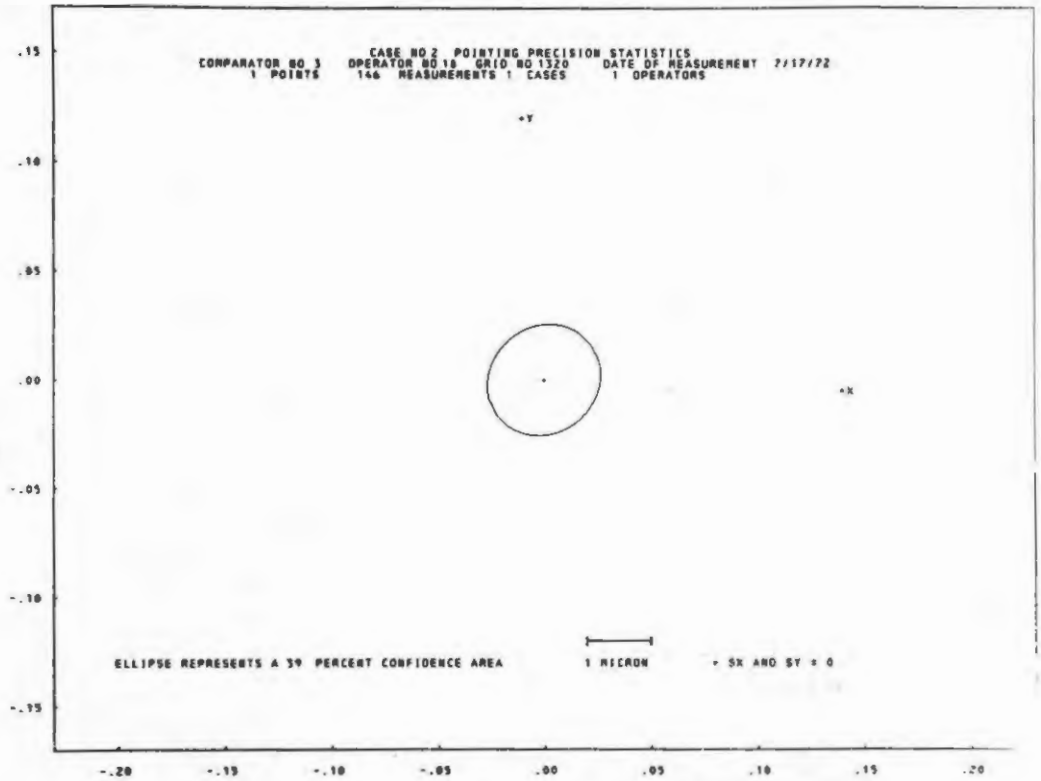

FIGURE 10. – Error ellipse for combined points of a single case.

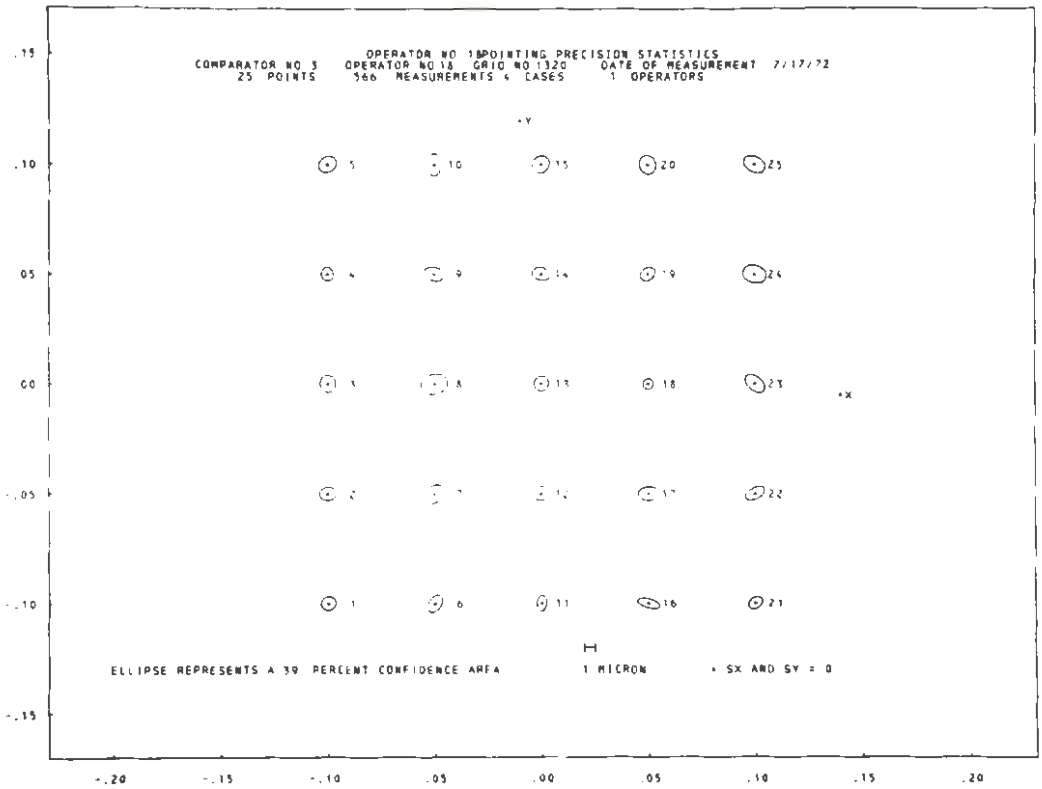

FIGURE 11. – Pointing precision error ellipses for a single operator.

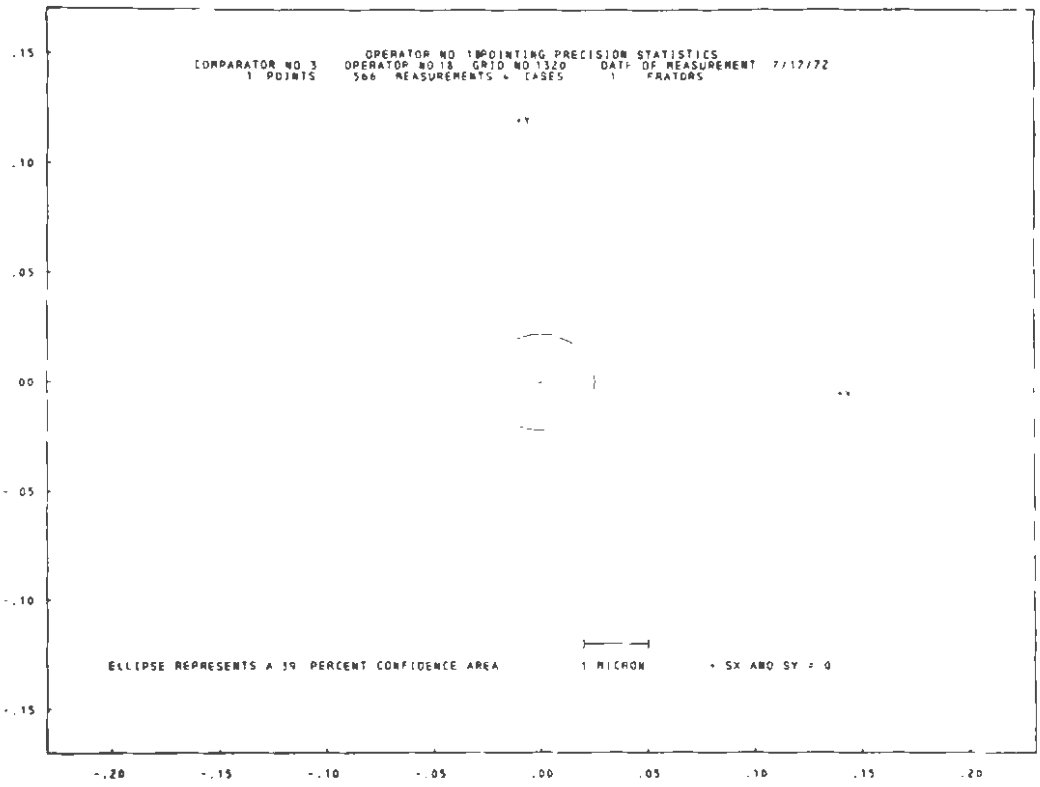

FIGURE 12. – Error ellipse for combined points for a single operator.

COMPARATOR NUMBER 3 POINTING PRECISION STATISTICS
GRID PLATE NUMBER 1320
DATE OF MEASUREMENT 7/17/72

INDIVIDUAL POINT STATISTICS

POINT NUMBER	NUMBER OF TIMES READ	SINGLE OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	(MICRONS SQ) COVARIANCE XY	(PERCENTAGE) CORRELLATION COEFFICIENT	MEAN MEASURED COORDINATES X	Y	MEAN OBSERVATIONS STANDARD DEVIATIONS X	Y	POINT	POINT NUMBER
1	25	.698	.622	.661	.037	8.63						
2	20	.781	.616	.703	.021	4.38						
3	20	.725	.781	.754	-.032	-5.57						
4	20	.562	.632	.598	-.042	-11.85						
5	25	.849	.733	.793	.129	20.75						
6	20	.698	.795	.747	.232	41.88						
7	20	.962	.846	.906	.221	27.15						
8	30	1.256	.927	1.104	.154	13.23						
9	20	.951	.632	.808	-.126	-20.99						
10	25	.658	1.035	.867	.017	2.45						
11	20	.459	.711	.598	.032	9.68						
12	21	.718	.632	.676	.050	11.02						
13	20	.681	.665	.673	.021	4.65						
14	20	.858	.580	.733	-.084	-16.90						
15	20	.795	.795	.795	.084	13.33						
16	25	1.065	.487	.828	-.267	-51.40						
17	20	.918	.632	.788	.021	3.63						
18	20	.503	.481	.492	.011	4.35						
19	20	.696	.616	.657	.116	27.03						
20	20	.781	.871	.827	-.074	-10.83						
21	25	.661	.563	.614	.125	33.58						
22	25	.894	.636	.776	.275	48.36						
23	20	.918	.834	.877	-.358	-46.79						
24	35	1.114	.830	.982	-.162	-17.50						
25	30	1.017	.799	.915	-.276	-33.93						
					COMBINED POINT STATISTICS							
566	25	.852	.730	.793	.001	.11	0.00	0.00	.339	.297	.319	

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FIGURE 13. – Pointing precision statistics for a complete calibration by several operators.

om the combined point statistics given in figure 7. This ellipse is generated from the combined (pooled) point statistics data of figure 7, that is $(\hat{\sigma}_{x\,ik})_p$, $(\hat{\sigma}_{yik})_p$, and $(\hat{\sigma}_{xyik})_p$.

As each case of data is cycled through the above routines, essential counts and summations are stored in the main program. Upon input completion of all cases measured by one operator, the STAT routine is called upon to compute the statistics for all grid points measured by the operator. Figure 8 shows the printed output for all of a single operator's measurements. Figures 11 and 12 are the corresponding error ellipse plots for the operator.

After all measurements of a calibration, possibly by several operators, have been read into the computer, the STAT routine is once again called to compute the total pointing precision statistics for the calibration. Figure 13 shows these statistics. Error ellipse plots also are generated for these statistics.

B. The General Linear Transformation

Comparator calibration is dependent on the measurement of a grid containing points whose calibrated positions are defined in an orthogonal coordinate system. The comparator measurement coordinate system is nonorthogonal because of inherent mechanical limitations. The relationship between these two coordinate systems is defined by an affine transformation. Consider the simplest relationship between these two systems as shown in figure 14.

Let:

$$\boldsymbol{X}=\begin{bmatrix}X\\Y\end{bmatrix}=\text{orthogonal (grid) system}$$

$$\boldsymbol{x}=\begin{bmatrix}x\\y\end{bmatrix}=\text{nonorthogonal (comparator) system}$$

α = lack-of-orthogonality angle of x, y system

For the figure 14 shown, the transformation between systems is

$$\begin{aligned}X&=x+y\sin\alpha\\Y&=y\cos\alpha\end{aligned}\qquad(33)$$

Let:

$$\boldsymbol{N}=\begin{bmatrix}1 & \sin\alpha\\0 & \cos\alpha\end{bmatrix}$$

The eq (33) in matrix form is

$$\boldsymbol{X}=\boldsymbol{N}\boldsymbol{x}$$

By definition the orthogonal grid system possesses uniform scale; however, the comparator system axes may each possess a unique scalar. These unique scalars are attributable to differences in comparator measuring standards, such as constant pitch or misalinement of the screws of lead screw comparators, or variation in measuring mark spacings of glass grid, multiple measuring mark or linear scale comparators. These differences may also be coupled with different thermal coefficients along each axis.

These different scalars *must* be applied to the comparator axes before orthogonalization to correspond with the true conditions existing during measurement. These scalars may be represented as

$$\boldsymbol{S}=\begin{bmatrix}Sx & 0\\0 & Sy\end{bmatrix}$$

The systems depicted in figure 14, with the differential scalars applied to x, y are related as

$$\boldsymbol{X}=\boldsymbol{N}\boldsymbol{S}\boldsymbol{x},$$

which in equation form is

$$\begin{aligned}X&=Sx\cdot x+Sy\cdot y\cdot\sin\alpha\\Y&=Sy\cdot y\cdot\cos\alpha\end{aligned}\qquad(34)$$

For small nonorthogonality angles (α), $\cos\alpha\approx 1$ which simplifies the **Y** equation. However, for completeness sake and for general error propagation, this derivation will not include any approximations.

Eq (34) would be sufficient to determine explicitly the linear comparator calibration parameters Sx, Sy, α, if the orthogonal grid plate could be placed on the comparator so that the X, x axes are exactly parallel. However, this is difficult.

Consider the two systems X, x which are rotated from one another by some angle θ. Let $\boldsymbol{M}$ represent an orthogonal rotation matrix.

$$\boldsymbol{M}=\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}\qquad(35)$$

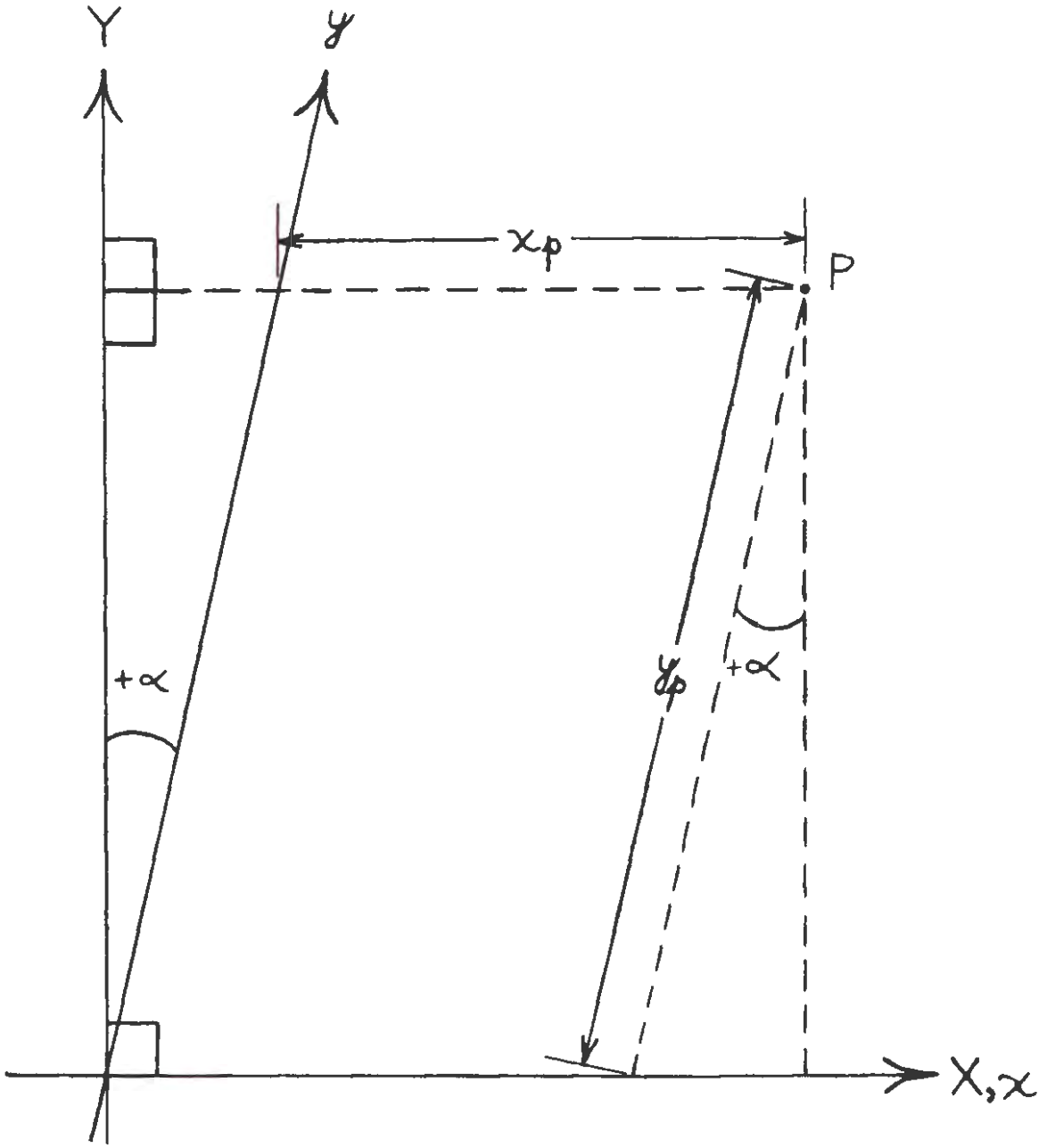

FIGURE 14. – Relationship of a nonorthogonal coordinate system with an orthogonal coordinate system.

Any rotation of a nonorthogonal coordinate system will significantly change the values of the shear parameters (Sx, Sy, α), as demonstrated by Pope (1966). Therefore, the rotation should not be applied until after the measurement system has been orthogonalized.

Thus

$$\boldsymbol{X} = \boldsymbol{MNSx}. \tag{36}$$

This equation represents the rotated relationship between an orthogonal coordinate system and a nonorthogonal coordinate system. An expanded matrix form of this transformation is

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & \sin\alpha \\ 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{36}$$

Collecting terms we have

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} Sx \cdot \cos\theta & Sy \cdot \sin\alpha \cdot \cos\theta + Sy \cdot \cos\alpha \cdot \sin\theta \\ -Sx \cdot \sin\theta & -Sy \cdot \sin\alpha \cdot \sin\theta + Sy \cdot \cos\alpha \cdot \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{36}$$

which is in the following form of a general linear transformation,

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} r & s \\ t & u \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{37}$$

Equating coefficients of eqs. (36) and (37) we have

$$r = Sx \cdot \cos\theta$$

$$s = -Sx \cdot \sin\theta$$

$$t = Sy \cdot \sin\alpha \cdot \cos\theta + Sy \cdot \cos\alpha \cdot \sin\theta$$

$$u = -Sy \cdot \sin\alpha \cdot \sin\theta + Sy \cdot \cos\alpha \cdot \cos\theta$$

Using algebraic manipulations and trignometric identities it can be shown that the explicit solutions for the physical parameters are:

$$\begin{aligned} Sx &= (r^2 + t^2)^{1/2} \\ Sy &= (s^2 + u^2)^{1/2} \\ \alpha &= \tan^{-1}\left(\frac{rs + tu}{-st + ru}\right) \\ \theta &= \tan^{-1}\left(\frac{-t}{r}\right). \end{aligned} \tag{38}$$

Therefore this has proved that there exists an exact solution for the general linear transformation from which differential scalars (Sx, Sy), non-orthogonality angle (α), and rotation angle (θ) are explicitly derived functions. This approach permits an exact linear least-squares solution for the parameters S_x, S_y, α, θ, whereas all previously published solutions to this problem have required iterative, nonlinear least-squares solutions (Hallert 1962, 1963, Brock 1966, Poetzschke 1967, Jeyapalan 1972).

C. Linear Parameters of Comparator Calibration—Individual Case Solutions

An exaggeration of the physical situation for which the comparator calibration routines in this report are derived is depicted in figure 15.

Derivation of the Exact Solution of the Linear Parameters

Equations (36), (37) and (38) describe the linear transformation between the grid (X, Y) and comparator (x, y) coordinate systems, except for the translation between their origins. Assuming the grid coordinates to be errorless and the comparator measurements to contain a variety of residual errors, the complete linear transformation between the systems is expressed as

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} r & s \\ t & u \end{bmatrix} \begin{bmatrix} x + v_x \\ y + v_y \end{bmatrix} + \begin{bmatrix} e' \\ f' \end{bmatrix}, \tag{39}$$

or

$$\boldsymbol{X} = \boldsymbol{R}(\boldsymbol{x} + \boldsymbol{v}) + \boldsymbol{T}' \tag{39}$$

where $\boldsymbol{R}$ is defined to include all coefficients of eq (36), i.e.,

$\boldsymbol{R} = \boldsymbol{MNS}$ = coefficient matrix of a general linear transformation.

$\boldsymbol{v} = v_x, v_y$ = residual errors in the comparator system.

$\boldsymbol{T}' = e', f'$ = translation in X and Y, respectively, between coordinate systems.

The measurement of more than three grid points will provide redundant observations which can be adjusted in a least-squares solution for the calibration parameters. However, eq (39) is nonlinear in the unknowns r, s, t, u, and v_x, v_y and must be linearized for a least-squares adjustment.

Instead of linearizing eq (39) by Taylor's series, consider the following equivalent set of equations which are in linear form:

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}, \tag{40}$$

or

$$\boldsymbol{x} + \boldsymbol{v} = \boldsymbol{AX} + \boldsymbol{T}.$$

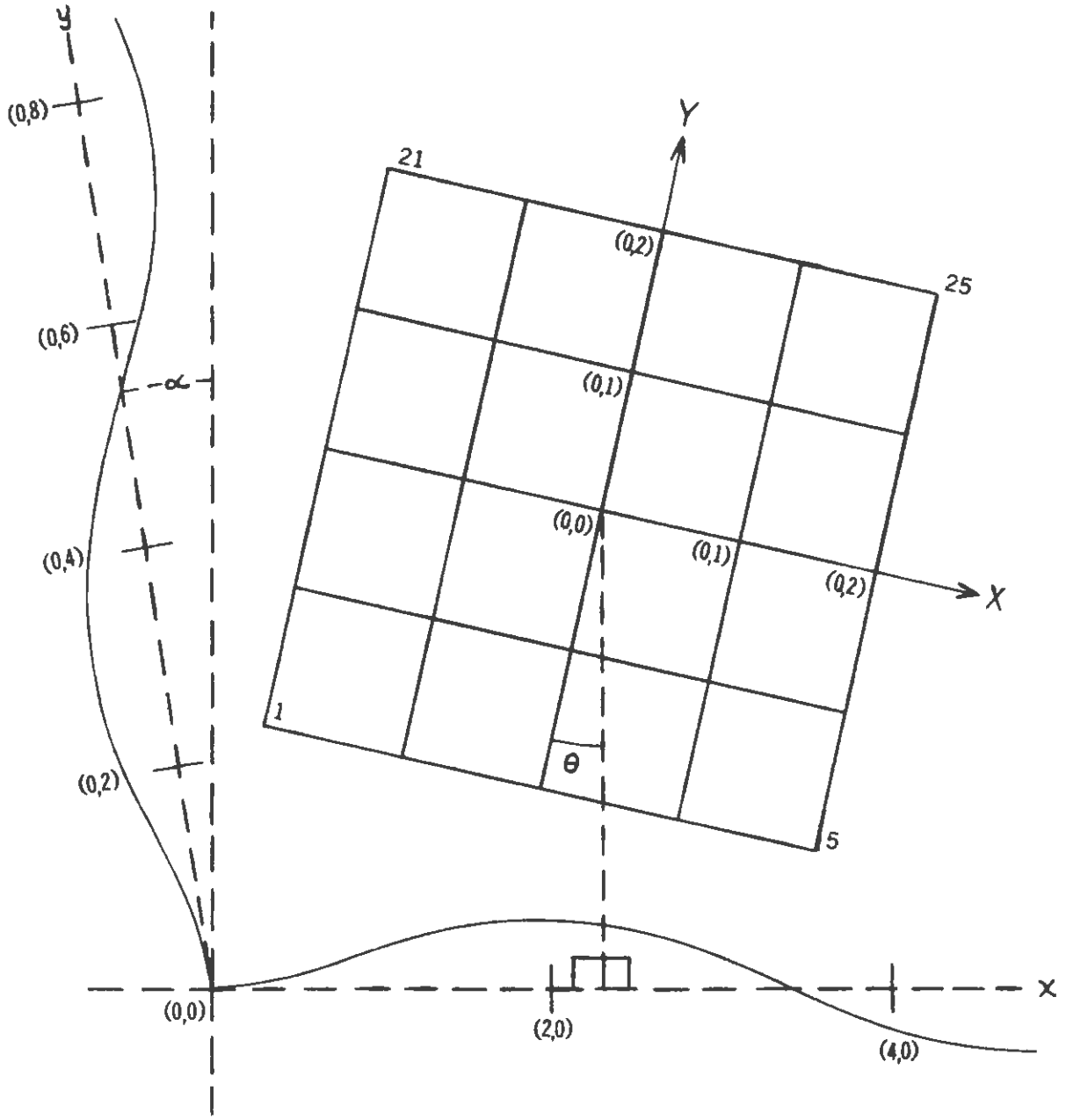

FIGURE 15.—An exaggerated grid-comparator relationship.

This set contains the same observations as equation (39) but different parameters. An interpretive comparison of eqs (39) and (40) reveals that in eq (39) the comparator measurement system is transformed to the grid system, whereas in eq (40) the grid system is transformed to the measurement system. Obviously, the transformation of eq (39) is more desirable for comparator calibrations as its parameters physically describe the comparator deformations. In both equations the residuals have purposely been defined in the measurement system so that they can be used later in the interpretations of systematic nonlinear comparator errors.

A rearrangement of eq (40) produces

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} x+v_x \\ y+v_y \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} e \\ f \end{bmatrix} \tag{41}$$

or

$$\boldsymbol{X} = \boldsymbol{A}^{-1}(\boldsymbol{x}+\boldsymbol{v}) - \boldsymbol{A}^{-1}\boldsymbol{T}. \tag{41}$$

A comparison of eqs (39) and (41) reveals the following identities:

(1) Transformation inverse identity

$$\begin{bmatrix} r & s \\ t & u \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \tag{42}$$

or

$$\boldsymbol{R} = \boldsymbol{A}^{-1}.$$

(2) Translation identity

$$\begin{bmatrix} e' \\ f' \end{bmatrix} = -\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} e \\ f \end{bmatrix} \tag{43}$$

or

$$\boldsymbol{T}' = -\boldsymbol{A}^{-1}\boldsymbol{T}.$$

An application of these inverse and translation identities to the linear least-squares solution for eq (40) produces an equivalent least-squares solution for the nonlinear equation set (39). Thus, an exact, linear, noniterative solution for the calibration with residuals given in the measurement system does exist. The mathematical model for this exact solution is

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix} \tag{40}$$

or

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{T} - \boldsymbol{x}. \tag{40}$$

The solution for the inverse identity eq (42) of the general linear transformation is as follows: From eqs (36), (37), and (40) we have solved for

$$\boldsymbol{A} = \boldsymbol{R}^{-1} = \begin{bmatrix} r & s \\ t & u \end{bmatrix}^{-1} = \begin{bmatrix} Sx \cdot \cos\theta & Sy \cdot \sin\alpha \cdot \cos\theta + Sy \cdot \cos\alpha \cdot \sin\theta \\ -Sx \cdot \sin\theta & -Sy \cdot \sin\alpha \cdot \sin\theta + Sy \cdot \cos\alpha \cdot \cos\theta \end{bmatrix}^{-1} \tag{45}$$

According to eq (42), $\boldsymbol{R}^{-1}$ corresponds to the elements of $\boldsymbol{A}$ of eq (40). The inversion of eq (45) in terms of $\boldsymbol{A}^{-1}$ produces

$$a = \frac{-\tan\alpha \cdot \sin\theta + \cos\theta}{Sx}$$

$$b = \frac{-\tan\alpha \cdot \cos\theta - \sin\theta}{Sx}$$

$$c = \frac{\sin\theta}{Sy \cdot \cos\alpha}$$

$$d = \frac{\cos\theta}{Sy \cdot \cos\alpha}. \tag{46}$$

Using algebraic manipulations and trignometric identities, it can be shown that

$$Sx = \frac{(c^2 + d^2)^{1/2}}{(ad - bc)}$$

$$Sy = \frac{(a^2 + b^2)^{1/2}}{(ad - bc)}$$

$$\alpha = \tan^{-1}\left(\frac{-ac - bd}{ad - bc}\right) \tag{47}$$

$$\theta = \tan^{-1}\left(\frac{c}{d}\right).$$

These are the physically identifiable comparator calibration parameters defined in the comparator coordinate system.

The Exact Least-Squares Solution of the General Linear Transformation

The following derivation was developed to provide the most economical and efficient method of computing the transformation.

The mathematical model is given by eq (40). The observation equations of this system are:

$$\begin{bmatrix} v_{x_i} \\ v_{y_i} \\ \vdots \end{bmatrix} = \begin{bmatrix} X_i & Y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_i & Y_i & 1 \\ \vdots & & & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ \vdots \end{bmatrix}$$

where the subscript i denotes the grid point identification number. Or, rewritten, in matrix form,

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{l}. \tag{44}$$

Note that the notation has been changed from the previous section.

The solution of the normal equations is

$$\boldsymbol{x} = -(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{l}$$

where

$$A^TA=\begin{bmatrix}\Sigma(X_i^2) & \Sigma(X_iY_i) & \Sigma(X_i) & 0 & 0 & 0\\ \Sigma(X_iY_i) & \Sigma(Y_i^2) & \Sigma(Y_i) & 0 & 0 & 0\\ \Sigma(X_i) & \Sigma(Y_i) & n & 0 & 0 & 0\\ 0 & 0 & 0 & \Sigma(X_i^2) & \Sigma(X_iY_i) & \Sigma(X_i)\\ 0 & 0 & 0 & \Sigma(X_iY_i) & \Sigma(Y_i^2) & \Sigma(Y_i)\\ 0 & 0 & 0 & \Sigma(X_i) & \Sigma(Y_i) & n\end{bmatrix},\quad -A^Tl=\begin{bmatrix}\Sigma(X_ix_i)\\ \Sigma(Y_ix_i)\\ \Sigma(x_i)\\ \Sigma(X_iy_i)\\ \Sigma(Y_iy_i)\\ \Sigma(y_i)\end{bmatrix}\text{ and }x=\begin{bmatrix}a\\ b\\ c\\ d\\ e\\ f\end{bmatrix},$$

NOTE: All summations (denoted Σ) are of grid point (i) coordinate values from 1 to n, where n is the number of grid points measured.

Inspection of the normal equations reveals that the solution can be determined more efficiently through elimination of the translation terms (e, f). This is accomplished by solving them implicitly from parameters a and b. That is, $A^TAx=A^Tl$. The third and sixth equations of the rearranged normals are

$$a\Sigma X_i+b\Sigma Y_i+ne=\Sigma x_i$$

$$c\Sigma X_i+d\Sigma Y_i+nf=\Sigma y_i$$

Divide each equation by n and rearrange terms to obtain the solution for the translations. Thus

$$e=\bar{x}-a\bar{X}-b\bar{Y}$$

$$f=\bar{y}-c\bar{X}-d\bar{Y},$$

where the overlined terms are the means of the observations, i.e.,

$$\bar{x}=\frac{\Sigma x_i}{n},\text{ etc.}$$

Since A^TA is block diagonal with equivalent symmetric blocks, the inverse of A^TA may readily be obtained from the inverse of one block. The solution of $(A^TA)^{-1}$ by the determinant adjoint method is as follows:

Let

$$D=A^TA\,|_{\text{block}} \tag{48}$$

Then

$$D=n[\Sigma X_i^2\Sigma Y_i^2-(\Sigma X_iY_i)^2$$

$$+\Sigma Y_i(2\Sigma X_iY_i\Sigma X_i-\Sigma Y_i\Sigma X_i^2)-\Sigma Y_i^2(\Sigma X_i)^2 \tag{48}$$

and

$$(A^TA)^{-1}_{\text{block}}=\frac{1}{D}=\begin{bmatrix}n\Sigma Y_i^2-(\Sigma Y_i)^2 & -n\Sigma X_iY_i+\Sigma X_i\Sigma Y_i & \Sigma Y_i\Sigma X_iY_i-\Sigma X_i\Sigma Y_i^2\\ \Sigma X_i\Sigma Y_i-n\Sigma X_iY_i & n\Sigma X_i^2-(\Sigma X_i)^2 & \Sigma X_i\Sigma X_iY_i-\Sigma Y_i\Sigma X_i^2\\ \Sigma Y_i\Sigma X_iY_i-\Sigma X_i\Sigma Y_i^2 & \Sigma X_i\Sigma X_iY_i-\Sigma Y_i\Sigma X_i^2 & \Sigma X_i^2\Sigma Y_i^2-(\Sigma X_iY_i)^2\end{bmatrix}. \tag{49}$$

Let

$$Q_p=(A^TA)^{-1}$$

$$=\frac{1}{D}\begin{bmatrix}Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0\\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0\\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{11} & Q_{12} & Q_{13}\\ 0 & 0 & 0 & Q_{12} & Q_{22} & Q_{23}\\ 0 & 0 & 0 & Q_{13} & Q_{23} & Q_{33}\end{bmatrix} \tag{50}$$

Q_p is the weight coefficient matrix of the adjusted parameters (a, b, c, d, e, f, respectively).

The solution of the parameters is

$$x=-(A^TA)^{-1}A^Tl$$

or

$$\begin{aligned}a&=(Q_{11}\Sigma X_ix_i+Q_{12}\Sigma Y_ix_i+Q_{13}\Sigma x_i)/D\\ b&=(Q_{12}\Sigma X_ix_i+Q_{22}\Sigma Y_ix_i+Q_{23}\Sigma x_i)/D\\ c&=(Q_{11}\Sigma X_iy_i+Q_{12}\Sigma Y_iy_i+Q_{13}\Sigma y_i)/D\\ d&=(Q_{12}\Sigma X_iy_i+Q_{22}\Sigma Y_iy_i+Q_{23}\Sigma y_i)/D\\ e&=\bar{x}-a\bar{X}-b\bar{Y}\\ f&=\bar{y}-c\bar{X}-d\bar{Y}.\end{aligned} \tag{51}$$

An efficient computer programmed solution for these parameters is given by Subroutine EXACT.

The Weight Coefficient Matrix of Functions of the Adjusted Parameters

To appreciate the physical meaning of the solved comparator calibration parameters, a complete error propagation through the adjustment is necessary. A weight of one is given to each measured coordinate, since care has been taken to provide mean observations of equal quality for the adjustment (viz., MEASUREMENT, Technique). The standard error of a single observation of unit weight (m_0) after adjustment of a single case is determined by

$$m_0=\sqrt{\frac{\sum_{i=1}^{n}(v_{xi}^2+v_{yi}^2)}{2n-u}} \tag{52}$$

where $u=6$ unknown parameters and $n=$ the number of grid points measured.

To determine the standard errors of the calibration parameters, the weight coefficient matrix of the functions of the adjusted parameters ($\boldsymbol{Q}_X$) must be determined. The portion of $\boldsymbol{Q}_X$ associated with the translations is of no interest to the calibration and will be left out of the following discussion.

The weight coefficient matrix $\boldsymbol{Q}_p$ (eq 50) rewritten for parameters a, b, c, d only is:

$$\boldsymbol{Q}_p=\frac{1}{\boldsymbol{D}}\cdot\begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 \\ 0 & 0 & Q_{11} & Q_{12} \\ 0 & 0 & Q_{12} & Q_{22} \end{bmatrix}$$

Let $\boldsymbol{Q}_X$ represent the weight coefficient matrix of the calibration parameters (S_x, S_y, α, θ). Then error propagation of the function X with the parameters p is computed by

$$\boldsymbol{Q}_X=\boldsymbol{G}\boldsymbol{Q}_p\boldsymbol{G}^T, \tag{53}$$

where $\boldsymbol{G}$ is a differential matrix relating functions of the parameters to the parameters, i.e.,

$$\boldsymbol{G}=\begin{bmatrix} \frac{\delta Sx}{\delta a} & \frac{\delta Sx}{\delta b} & \frac{\delta Sx}{\delta c} & \frac{\delta Sx}{\delta d} \\ \frac{\delta Sy}{\delta a} & \frac{\delta Sy}{\delta b} & \frac{\delta Sy}{\delta c} & \frac{\delta Sy}{\delta d} \\ \frac{\delta \alpha}{\delta a} & \frac{\delta \alpha}{\delta b} & \frac{\delta \alpha}{\delta c} & \frac{\delta \alpha}{\delta d} \\ \frac{\delta \theta}{\delta a} & \frac{\delta \theta}{\delta b} & \frac{\delta \theta}{\delta c} & \frac{\delta \theta}{\delta d} \end{bmatrix}$$

The solution for $\boldsymbol{G}$ using the relations given in eq (47) is

$$\boldsymbol{G}=\begin{bmatrix} \frac{-(c^2+d^2)^{1/2}\cdot d}{(ad-bc)^2} & \frac{c(c^2+d^2)^{1/2}}{(ad-bc)^2} & \frac{d(ac+bd)}{(ad-bc)^2(c^2+d^2)^{1/2}} & \frac{-c(ac+bd)}{(ad-bc)^2(c^2+d^2)^{1/2}} \\ \frac{-b(ac+bd)}{(ad-bc)^2(a^2+b^2)^{1/2}} & \frac{a(ac+bd)}{(ad-bc)^2(a^2+b^2)^{1/2}} & \frac{b(a^2+b^2)^{1/2}}{(ad-bc)^2} & \frac{-a(a^2+b^2)^{1/2}}{(ad-bc)^2} \\ \frac{b}{(a^2+b^2)} & \frac{-a}{(a^2+b^2)} & \frac{-d}{(c^2+d^2)} & \frac{c}{(c^2+d^2)} \\ 0 & 0 & \frac{d}{(c^2+d^2)} & \frac{-c}{(c^2+d^2)} \end{bmatrix} \tag{54}$$

The variance-covariance matrix of the adjusted calibration parameters ($\boldsymbol{\Sigma}_X$) is given by

$$\sum_X=m_0^2\boldsymbol{Q}_X. \tag{55}$$

Summary

The computations required to perform a complete linear adjustment of a single case of grid plate measurements on a two-axis comparator, including the determination of the comparator calibration linear parameters and their statistics, are:

1. Perform summations required by eqs (48) and (49) to obtain $\boldsymbol{D}$ and $\boldsymbol{Q}_p$.
2. solve for the adjustment parameters (a, b, c, d, e, f) by eq (51).
3. solve for the calibration parameters (Sx, Sy, α) by eq (47).

4. compute residuals (v_x, v_y) in the comparator coordinate system by substitution of the adjustment parameters and observations into the mathematical model, eq (40),
5. compute the standard error of a single observation of unit weight (m_0) after adjustment by eq (52),
6. compute the differential matrix G using the relations given by eq (54),
7. compute the weight coefficient matrix of the calibration parameters Q_X by eq (53), and
8. compute the variance-covariance matrix of the adjusted calibration parameters (Σ_X) by eq (55).

The correct application of the calibration parameters to any measurements performed on the comparator is given by eq (34).

It is useful to know the distribution of the adjustment residuals in the grid coordinate system. Computation of the inverse of the adjustment parameter (a, b, c, d) matrix, eq (42), will provide the parameters (r, s, t, u). It can be shown by derivation from eq (39) that the residuals in the grid system (v_X, v_Y) are computed by

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = -\begin{bmatrix} r & s \\ t & u \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}.$$

Computer Program Routines

The main computer program processes each case of measurements separately through a free solution, a general affine linear transformation solution, and then a nonlinear solution.

The free solution allows only rotation and translation, assuming orthogonality between the two coordinate systems. The residuals computed indicate the amount of total error present. Figure 16 is the printed output of a free solution of a single case of measurements of a 25-point grid as computed by Subroutine FIDFIT2.

The general affine linear transformation is computed by Subroutine EXACT. All values mentioned in the above summary are computed including the residuals in both coordinate systems. Figure 17 is the linear adjustment output of the same data presented in figures 6, 7, 9, 10, and 16.

The printed output of the adjusted observations and the residuals in both coordinate systems is set up by Subroutine VSPRINT.

D. The Generalized Weighted Mean Linear Solution—Multiple Case Solution

As previously stated, a complete calibration set consists of single operator measurements of the grid plate in four rotated cases. Up to 20 cases of measurements of a 25-point grid, with up to 300 pointings allowed per case, may be processed by this computer program. After all cases have undergone individual calibration solutions, they are processed through a combined linear calibration solution. Rather than form a large simultaneous least-squares adjustment of all cases, the equivalent adjustment is made by computing the generalized weighted mean solution. This alternative solution saves considerable computer storage and time without sacrificing computational or statistical rigor.

The Mathematical Models Used

A test evaluation of the variation of the variances of a single observation of unit weight (m_0^2), eq (52) from several case adjustments of a single linear comparator calibration was performed. The F_{Max} test (Hamilton 1964) was made to ascertain whether any significant variations between the $m_{0_j}^2$ were present (j refers to case number). The test, in which it is assumed that the degrees of freedom of all adjustments are equal, was made at the 95-percent significance level. Results indicated that $m_{0_j}^2$ were not significantly different. Therefore, the following generalized weighted mean (combined) adjustment was chosen, which proceeds under the assumption that all $m_{0_j}^2$ are from the same statistical population. Thus in eq (56) the product $\frac{m_{0_j}^2}{m_{0_l}^2} \cdot \Sigma_{X_j}^{-1}$ removes the $m_{0_j}^2$ already in $\Sigma_{X_j}^{-1}$ and replaces all $m_{0_j}^2$ by a constant $m_{0_l}^2$. This $m_{0_l}^2$ can be (and is) taken to be 1 without effect on the value of $\bar{X}$, given by eq (56). The unknown constant value of $m_{0_l}^2$ can then be estimated on the basis of $\bar{X}$ by equation (57) (Pope 1969).

The generalized weighted mean solution solves for a single set of linear calibration parameters while permitting each case of measurements to have a unique set of rotation and translation parameters. For example, with four cases of measurements, the combined solution will compute one set of calibration parameters (Sx, Sy, α) and four sets of rotations and translations, (Θ_j, T_{x_j}, T_{y_j}).

GRID AND CASE NUMBER 20020

COMPARATOR NUMBER 3 CALIBRATION
OPERATOR NUMBER 18
DATE OF MEASUREMENT 7/17/72

GRID POINT 25 IS NEAREST THE COMPARATOR ORIGIN

FREE ADJUSTMENT -
1 ROTATION (THETA), 2 TRANSLATIONS (TX,TY)

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
20020 1	240139.50	213261.50	4.210	1.574
20020 2	231446.80	164022.40	1.226	2.904
20020 3	222750.80	114783.00	-.242	3.284
20020 4	214056.40	65544.00	-1.661	3.979
20020 5	205364.20	16304.20	-5.787	5.593
20020 6	190904.60	221959.60	1.464	-.193
20020 7	182211.60	172720.80	-1.350	1.021
20020 8	173514.47	123481.93	-1.434	.794
20020 9	164819.60	74243.40	-2.211	1.471
2002010	156127.20	25004.60	-5.266	3.360
2002011	141665.20	230657.60	1.039	-1.547
2002012	132971.50	181419.00	-.598	-.708
2002013	124273.20	132179.20	-.357	-.687
2002014	115579.20	82942.60	-2.109	-.515
2002015	106884.20	33703.20	-2.855	2.029
2002016	92427.40	239357.20	3.171	-2.758
2002017	83733.80	190120.60	1.027	-3.694
2002018	75037.00	140877.80	.031	-.790
2002019	66342.20	91642.80	-.763	-2.186
2002020	57646.80	42403.20	-1.875	.703
2002021	43190.00	248056.00	3.877	-4.723
2002022	34494.20	198818.40	3.177	-5.085
2002023	25797.60	149576.60	1.649	-2.473
2002024	17100.00	100338.30	3.514	-.218
2002025	8404.80	51102.00	2.123	-1.133
		SUM VV =	148.2525	174.4794
NO. OF POINTS = 25		SIGMA =	2.6758	2.7248

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	3.872	2.283
2	.702	3.073
3	-.810	3.192
4	-2.328	3.629
5	-6.672	4.501
6	1.475	.064
7	-1.507	.771
8	-1.550	.532
9	-2.433	1.064
10	-5.771	2.393
11	1.292	-1.343
12	-.466	-.802
13	-.232	-.738
14	-1.987	-.874
15	-3.164	1.501
16	3.602	-2.164
17	1.654	-3.460
18	.168	-.773
19	-.371	-2.285
20	-1.969	.366
21	4.639	-3.977
22	4.013	-4.455
23	2.054	-2.148
24	3.499	.396
25	2.288	-.746
SUM VV =	207.1255	135.6187
SIGMA =	2.9688	2.4023

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = 2.7004

FIGURE 16.—Free adjustment-output of a single case.

LINEAR ADJUSTMENT -
2 SCALERS (SX,SY), 2 ROTATIONS (ALPHA,THETA), 2 TRANSLATIONS (TX,TY)

	PARAMETER VALUE	SIGMA
SCALER X =	9.9998712E-01	3.57947916E-06
SCALER Y =	9.9997711E-01	3.57938789E-06
NON-ORTHOG. ANGLE ALPHA =	0 0 11.253	0 0 1.0441
ROTATION ANGLE THETA =	169 58 53.781	0 0 .7383
SCALER RATIO SX/SY =	1.0000100E+00	

VARIANCE-COVARIANCE MATRIX

SCALER X	SCALER Y	ALPHA
1.28126710E-11	1.00638357E-19	1.62137300E-15
	1.28120176E-11	1.62134464E-15
		2.56256051E-11

SCALERS IN MICRONS/METER

SCALER X =	-12.8811	3.5795
SCALER Y =	-22.8881	3.5794
SX/SY =	10.0073	

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
20020 1	240148.04	213256.62	3.491	.270
20020 2	231452.77	164018.65	1.738	.709
20020 3	222754.19	114780.37	1.501	.200
20020 4	214057.22	65542.50	1.313	.004
20020 5	205362.44	16303.83	-1.582	.729
20020 6	190914.25	221954.52	-.127	.044
20020 7	182218.68	172716.85	-1.709	.369
20020 8	173518.97	123479.11	-.562	-.749
20020 9	164821.53	74241.90	-.108	-.961
2002010	156126.55	25004.03	-1.933	.038
2002011	141675.96	230652.32	-1.423	.232
2002012	132979.68	181414.85	-1.829	.182
2002013	124278.81	132176.17	-.357	-.687
2002014	115582.24	82940.70	-.878	-1.405
2002015	106884.66	33702.43	-.393	.249
2002016	92439.27	239351.72	-.163	.564
2002017	83743.09	190116.25	-1.076	-1.262
2002018	75043.72	140874.58	-.841	.752
2002019	66346.35	91640.70	-.403	-1.534
2002020	57648.37	42402.23	-.284	.466
2002021	43202.98	248050.32	-.329	.141
2002022	34504.60	198813.85	.203	-1.111
2002023	25805.43	149573.18	-.094	.612
2002024	17105.25	100336.00	3.002	1.976
2002025	8407.48	51100.83	2.842	.172
		SUM VV =	54.4978	15.9751
NO. OF POINTS = 25		SIGMA =	1.5739	.8521

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	3.391	.873
2	1.588	1.001
3	1.443	.458
4	1.292	.233
5	-1.685	.443
6	-.132	.021
7	-1.747	.066
8	-.424	-.835
9	.061	-.965
10	-1.910	-.298
11	-1.442	-.019
12	-1.833	-.139
13	-.232	-.738
14	-.620	-1.536
15	-.430	.177
16	-.258	.527
17	-.840	-1.430
18	-.959	.594
19	-.130	-1.580
20	-.361	.409
21	-.348	.081
22	[illegible]	-1.059
23	-	.586
[illegible]		2.468
25		.664
SUM VV =	50.2230	20.2488
SIGMA =	1.5109	.9594

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = 1.2656

FIGURE 17.—Linear adjustment-output of a single case.

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ADJUSTED COMPARATOR CALIBRATION PARAMETERS FROM THE SIMULTANEOUS SOLUTION OF  4 CASES

                                   LINEAR PARAMETERS

                                                                          VARIANCE-COVARIANCE MATRIX
                     PARAMETER VALUE          SIGMA                 SCALER X          SCALER Y          ALPHA
            SCALER X =  9.9999522E-01     1.77563146E-06     3.15286707E-12    2.66515238E-20    4.13617816E-16
            SCALER Y =  9.9997765E-01     1.77558250E-06                       3.15269323E-12    4.13610068E-16
NON-ORTHOG. ANGLE ALPHA =    0  0 10.218      0  0    .5180                                    6.30568105E-12
     SCALER RATIO SX/SY =  1.0000176E+00

                     SCALERS IN MICRONS/METER
            SCALER X =     -4.7809          1.7756
            SCALER Y =    -22.3478          1.7756
               SX/SY =     17.5674

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT =  1.2556
```

FIGURE 18. – Combined linear adjustment-output for many cases.

RESIDUALS AFTER APPLICATION OF FINAL LINEAR CORRECTIONS TO ORIGINAL DATA

GRID AND CASE NUMBER 20020

FREE ADJUSTMENT -
1 ROTATION (THETA), 2 TRANSLATIONS (TX,TY)

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
20020 1	240148.92	213256.73	2.756	.517
20020 2	231453.82	164018.73	.950	.961
20020 3	222755.42	114780.43	.660	.456
20020 4	214858.62	65542.54	.418	.266
20020 5	205364.03	16303.34	-2.529	.995
20020 6	190914.68	221954.64	-.441	.163
20020 7	182219.28	172716.94	-2.077	.492
20020 8	173519.75	123479.17	-.983	-.620
20020 9	164822.49	74241.94	-.582	-.828
2002010	156127.69	25004.04	-2.460	.176
2002011	141675.95	230652.45	-1.317	.223
2002012	132979.85	181414.95	-1.776	.177
2002013	124279.15	132176.25	-.357	-.687
2002014	115582.76	82940.75	-.931	-1.400
2002015	106885.36	33702.45	-.499	.259
2002016	92438.82	239351.85	.364	.426
2002017	83742.82	190116.35	-.602	-1.395
2002018	75043.62	140874.65	-.420	.624
2002019	66346.42	91640.75	-.036	-1.657
2002020	57648.62	42402.25	.030	.347
2002021	43202.08	248050.46	.619	-.125
2002022	34503.88	198813.96	1.097	-1.372
2002023	25804.89	149573.26	.747	.355
2002024	17104.89	100336.06	3.791	1.724
2002025	8407.29	51100.86	3.577	-.075
		SUM VV =	63.4866	16.7979
NO. OF POINTS = 25		SIGMA =	1.6569	.8523

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	2.624	.988
2	.768	1.111
3	.570	.564
4	.366	.334
5	-2.664	.540
6	-.463	.084
7	-2.131	.124
8	-.860	-.782
9	-.429	-.917
10	-2.453	-.254
11	-1.335	-.010
12	-1.780	-.135
13	-.232	-.738
14	-.673	-1.541
15	-.536	.168
16	.284	.483
17	-.350	-1.479
18	-.522	.541
19	.253	-1.638
20	-.031	.347
21	.631	-.016
22	1.319	-1.160
23	.674	.480
24	3.433	2.358
25	3.536	.548
SUM VV =	59.8943	20.3909
SIGMA =	1.6094	.9390

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = 1.3175

FIGURE 19.—Output of a single case—after combined linear adjustment.

The mean parameters are computed by

$$\bar{X}=\left[\sum_{j=1}^{q}\left(m_{0j}^{2}\sum_{X_j}^{-1}\right)\right]^{-1}\left[\sum_{j=1}^{q}\left(m_{0j}^{2}\sum_{X_j}^{-1}X_j\right)\right] \quad (56)$$

where:

$\bar{X}$ = the meaned calibration parameters Sx, Sy and α.

q = the total number of cases combined in the adjustment.

m_{0j}^2 = the variance of unit weight of the jth case. (Computed by eq (52).)

X_j = the calibration parameters Sx, Sy, and α of the jth case. (Computed by eq (47).)

$\sum_{Xj}$ = the variance covariance matrix of calibration parameters in the jth case. (A subsection of the entire matrix computed by eq (55).)

An estimate of the variance of a single observation of unit weight for tbe combined linear adjustment ($m_{0_l}^2$), computed under the assumption that all observations of all cases are from the same population, is given by

$$m_{0_l}^2=\frac{\sum_{j=1}^{q}(\nu_j m_{0j}^2)}{\nu}+\frac{1}{\nu}\sum_{j=1}^{q}\left[m_{0j}^2(X_j-\bar{X})^T\,\Sigma_{X_j}^{-1}(X_j-\bar{X})\right] \quad (57)$$

where:

$\nu_j = 2l_j - u_j$ = the degrees of freedom of the $j$tb case.

l_j = the total number of grid points measured in the jth case.

u_j = the number of unknowns (parameters) in the jth case. Note: $u_j = 6$.

ν = the total degrees of freedom for the combined adjustment.

$$\nu = 2\sum_{j=1}^{q}(l_j) - 3(q+1)$$

NOTE: After multiplying both sides of formula (57) by ν the right side of the formula is numerically equivalent to the sums of squares of residuals associated with the commonly used least-squares adjustment.

The variance-covarience matrix of the adjusted mean calibration parameters is computed by

$$\sum_{\bar{X}} = m_{0_l}^2\left[\sum_{j=1}^{q}\left(m_{0j}^2\,\Sigma_{X_j}^{-1}\right)\right]^{-1}. \quad (58)$$

After this combined adjustment has been performed, a new set of residuals is generated for each case of measurements. The combined solution does not produce the values of the new rotation and translations corresponding to each case. Thus, to compute the residuals, the mean parameters are applied to all raw measurements and a free adjustment, containing only rotation and translations (Subroutine FIDFIT2), of each case to the grid is performed.

Computer Program Routines

All computations of the combined adjustment are performed in the main (CALIB) program. The data from each individual case solution needed for this computation include: Sx_j, Sy_j, α_j, Σ_{Xj}, m_{0j}, and l_j. These values are stored for each case in the XILIN array. Figure 18 is a computer printout of the results of a combined linear adjustment. Figure 19 is a printout of a single case after treatment with the final linear calibration parameters. The residuals shown are for the same data shown in figure 17 after the combined solution. The standard error of a single observation of unit weight (m_{0j}) is recomputed for each case to allow for the additional degrees of freedom of the combined adjustment.

This is performed by an alteration of the count of unknowns in the degrees of freedom of each case, i.e.,

$$u_j = \frac{1}{2}\left(3+\frac{3}{q}\right)$$

is substituted into eq (52). Admittedly this standard error computation is superfluous. However, the sums of squares of residuals from all of the cases are identical to those computed by $\nu m_{0_l}^2$ of formula (57).

E. Nonlinear Parameters of Comparator Calibration

There are a variety of nonlinear systematic error sources which affect comparator measurements. They have been well documented (Bennett 1961, Rosenfield 1963, Gugel 1963, Loewen 1964, Brock 1966) and will only be summarized here. Obviously some of these errors are present only in certain brands or types of comparators.

Lead screws contain periodic errors which may be considered constant for a single turn of the screw.

This error, generated during the manufacturing process, should be independently determined by the glass scale method upon receipt of a new lead screw comparator. It is normally quite small ($\epsilon < 0.5\ \mu m$) in most modern comparators. Its form is generally considered to be periodic and correctable by Fourier series analysis.

Lead screws also contain larger ($\epsilon > 1\ \mu m$) secular errors initially created in the manufacturing process. With wear, these errors will often become larger and will assume new systematic forms.

Slide- or stage-guiding ways create nonlinear measurement errors. Any deviations in the planing of the ways process will cause the comparator stage to weave and curve in a nonlinear fashion.

Built-in scales, measuring marks, or reference measuring grids have inherent nonlinear errors that will affect measurements. However, if these standards have calibrated values for each of their reference marks, the nonlinear errors can be removed by tabular corrections for the discrete marks.

Other nonlinear systematic errors peculiar to individual comparators may also be present. These include time-variant error gradients, which are difficult to isolate, and errors attributable to wear or misuse.

Most previously published articles on this subject treat each of these nonlinear errors individually with some form of a Fourier series. Admittedly, this is the most elegant model for expressing the errors so that they are physically interpretable. However, the modeling is cumbersome when several of the aforementioned sources of error are present. During the early stages of modeling these errors at NOS, a decision was made to use low-degree polynomials to express their aggregate form. Initially, 3d degree polynomials were used for calibrated grid measurements. The 25-point grid was oriented parallel to the comparator axis to provide 5 discrete axis locations. Subsequent tests and refinements in the system have led to the more comprehensive treatment of these systematic errors as described in the following discussion.

The Polynomial Method – Individual Case Solutions

After the systematic linear errors of a single case have been removed, the residual errors at each grid point are computed in the measurement system. These residuals contain systematic nonlinear comparator errors, systematic nonlinear errors of the calibrated grid, residual nonlinear pointing errors, and random errors (noise). Investigations at NOS have demonstrated that nonlinear comparator errors can be removed effectively through the application of 4th degree polynomials to measurements from each axis. The 4th degree polynomials used, presented in a least-squares observation equation form, are

$$lx + v_x = Ax^4 + By^4 + Cx^3 + Dy^3 + Ex^2 + Fy^2 + Gx + Hy + J$$

$$ly + v_y = A'x^4 + B'y^4 + C'x^3 + D'y^3 + E'x^2 + F'y^2 + G'x + H'y + J', \tag{59}$$

or in the matrix rotational form introduced in equation (44)

$$-\boldsymbol{l} + \boldsymbol{v} = \boldsymbol{A}\boldsymbol{x}.$$

Where: $\boldsymbol{l} = lx, ly =$ residuals after linear calibration adjustment (previously denoted as v_x, v_y in eq (40)).

$\boldsymbol{v} = v_x, v_y =$ residuals after the least-squares nonlinear calibration adjustment

$\boldsymbol{x} = A - J, A' - J' =$ unknown linear coefficients of the polynomial.

$\boldsymbol{A} = x, y =$ comparator measurements after correction for systematic linear errors.

An inspection of the linear coefficients of eq (59) reveals a general linear transformation in the coefficients G, H, G′, H′, and a translation by terms J and J′. These coefficients are retained to absorb any additional general linear transformation created by the removal of higher order systematic nonlinear errors. This polynomial prompts the question of why is the removal of all comparator systematic errors not preformed with a single, simultaneous, mathematical model adjustment? The reason is threefold. First, to perform a simultaneous calibration adjustment on data obtained from multiple grid plate placements (cases), the polynomial must contain a variable number of rotation and translation parameters and only one set of linear (Sx, Sy, α) and nonlinear calibration parameters (A – F, A′ – F′). This is a difficult task and it precludes the explicit solution for the linear calibration parameters permitted by the generalized weighted mean linear solution. Second, calibration of many of the different brands

of comparators demonstrated that the nonlinear errors were of insignificant magnitude and so need not be corrected for. Third, at NOS, our research philosophy is to attempt to isolate mathematically each significant error source that can be identified physically. The program in its present form does allow a physical interpretation of the linear calibration parameters, whereas their physical interpretability would be masked in a polynomial. However, with small modification, eq (59) could be used to perform a single case calibration.

An extended linear polynomial containing cross-terms (xy, x^2y, xy^3, etc.) was developed and tested at NOS. This polynomial was designed specifically to remove any systematic nonlinear errors generated by dependent comparator axes. The least number of coefficients needed to completely describe the physical correlations between the x,y axes of a lead screw comparator was found to be 30. This polynomial was tested on the primary stages of two Wild STK1 stereocomparators and on four D. Mann monocomparators. The largest nonlinear error detected was 0.3 μm. Therefore, the mathematical model was abandoned.

Other tests determined that on these comparators the periodic errors of a single turn of the screw were negligible. If these errors are present, corrections for them must be determined independently by a method such as described by Bennett (1961). Then a precorrection to all data must be applied before using the NOS calibration program.

Figure 15 shows that the polynomials can be described more adequately with a rotated grid placement on the comparator, than from a squared-off grid placement. Since the errors are usually small ($\epsilon < 4\mu m$), the rotation of the non-linear errors through the small angle α during linear calibration ($\alpha <$ min. arc) will leave them unchanged. An 11.3° rotation of a square symmetric 25-point grid, as shown in figure 20, provides an even sampling of the nonlinear errors along the comparator axes. However, other rotations of the grid may be necessary so that the entire range of the comparator axes is covered.

The standard error of a single observation of unit weight after nonlinear least-squares adjustment of a single case is computed by

$$m_0 = \sqrt{\frac{\sum_{i=1}^{n} (v_{x_i} + v_{y_i})^2}{2l - \mu}} \tag{60}$$

where u = 18 unknown coefficients.

l = total number of grid points (i) measured in the case,

v_{x_i}, v_{y_i} = residuals in each coordinate of point i after nonlinear adjustment of the case.

The variance-covariance matrix of the adjusted coefficients is the same for both sets, A−J and A′−J′, because each set is computed from the same normal equation system ($(A^TA)^{-1}$) with separate constant columns (A^Tl). That is,

$$\Sigma_{A-J} = \Sigma_{A'-J'} = m_0^2\,(A^TA)^{-1}. \tag{61}$$

Multiple Case Solution

The validity of a descriptive nonlinear error polynomial is greatly enhanced by the increased sampling of different comparator axis locations made available by multiple case measurements. The recommended single set (4 cases) of calibration measurements, performed by one operator, provides a sampling of up to 100 discrete points along the comparator ways, screws, scales, etc. Under these circumstances the low-degree polynomial solution greatly reduces the chances of modeling random noise.

The combined nonlinear solution is computed by a simultaneous least-squares adjustment of eq (59). The observation equations for this multiple case adjustment are formed from the linearly corrected measurements and from the residuals computed from the generalized weighted mean linear solution. The standard error of a single observation of unit weight ($m_{0_{nl}}$) from the combined nonlinear adjustment is computed by

$$m_{0_{nl}} = \frac{\sum_{j=1}^{q} \sum_{i=1}^{l_j} (v_{x_{ij}}^2 + v_{y_{ij}}^2)}{2\left[\sum_{j=1}^{q} (l_j) - 9\right]}, \tag{62}$$

where

q = the total number of cases combined,

l_j = The total number of grid points measured in the jth case,

$v_{x_{ij}}, v_{y_{ij}}$ = residuals in each coordinate of point i of the jth case after multiple case nonlinear adjustment.

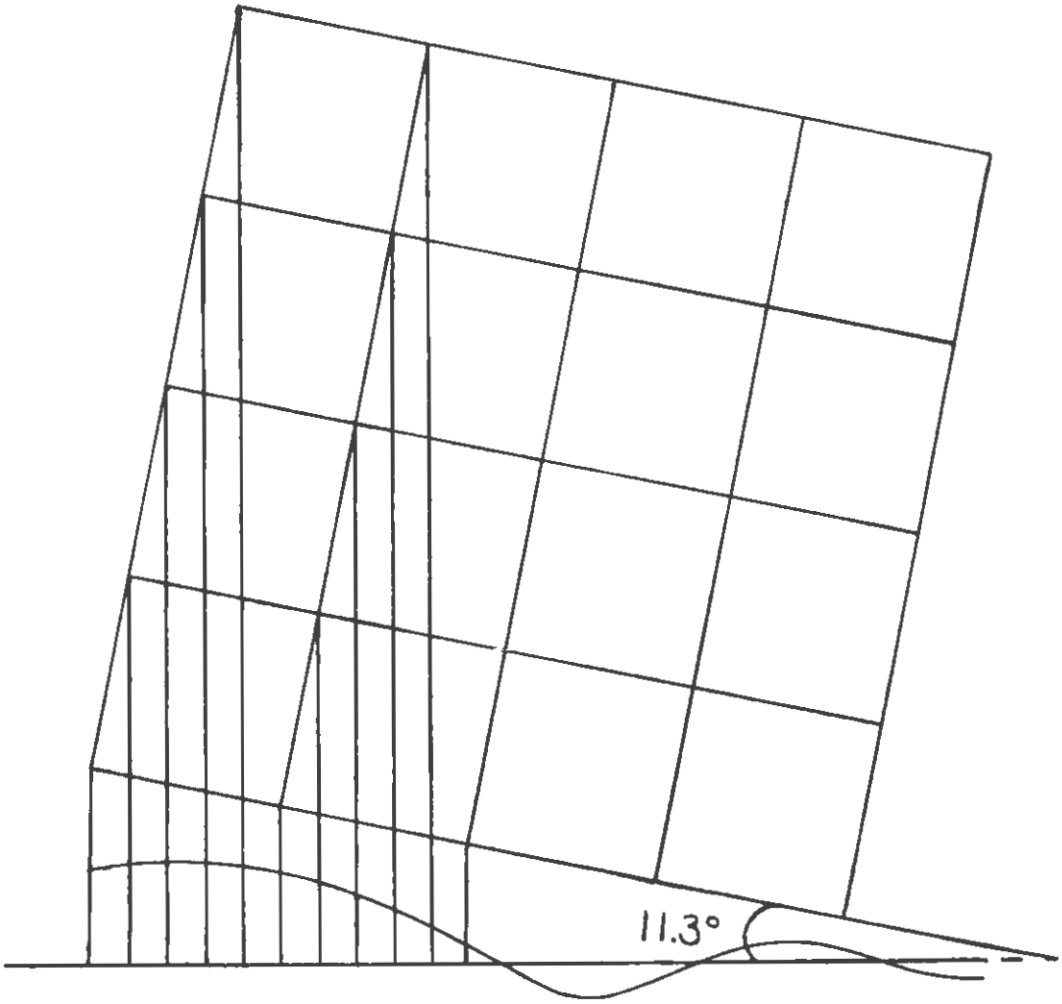

FIGURE 20.—Evenly-spaced samples of nonlinear errors.

The variance-covariance matrix of the adjusted coefficients of the combined nonlinear adjustment is computed from the inverse of the normal equations in the same manner as with the individual case adjustments (see eq 61).

The computation of the residuals after the combined case adjustment is similar to the computations performed after the combined case linear adjustment, i.e., the original measurements are corrected for linear and nonlinear systematic errors and then, case by case, are computed through a free fit adjustment (Subroutine FIDFIT2) to produce residuals.

The correct method of applying the nonlinear correction coefficients to linearly corrected observations is given by

$$x_a = x + Ax^4 + By^4 + Cx^3 + Dy^3 + Ex^2 + Fy^2 + Gx + Hy + J$$

$$y_a = y + A'x^4 + B'y^4 + C'x^3 + D'y^3 + E'x^2 + F'y^2 + G'x + H'y + J' \tag{63}$$

where

x_a, y_a = comparator measurements corrected for all systematic comparator errors.

x, y = comparator measurements corrected for all linear systematic comparator errors (Sx, Sy, α).

Computer Program Routines

Upon completion of the linear calibration adjustment of each case, the residuals and linearly corrected measurements are used to form the normal equations of the polynomial in array **A**. Subroutine INVSYM is called upon to invert and solve the normal equations. The adjustment residuals

NON-LINEAR ADJUSTMENT -

NON-LINEAR CORRECTION COEFFICIENTS FOR X

A	B	C	D	E	F	G	H	J
2.50930193E-02	-1.27816991E-02	-1.18887535E-02	8.52758723E-03	2.06751383E-03	-1.91313065E-03	-1.59036353E-04	1.60526688E-04	9.03337725E-08

NON-LINEAR CORRECTION COEFFICIENTS FOR Y

A	B	C	D	E	F	G	H	J
7.34793292E-03	1.71455837E-02	-4.03343894E-03	-9.46825574E-03	8.15792943E-04	1.78655314E-03	-6.93385214E-05	-1.30422507E-04	4.46489660E-06

VARIANCE-COVARIANCE MATRIX OF ADJUSTED COEFFICIENTS (SAME FOR BOTH X AND Y)

A	B	C	D	E	F	G	H	J
8.36250531E-05								
2.47990263E-05	8.36052595E-05							
-4.15711732E-05	-1.23281142E-05	2.09515809E-05						
-1.31116138E-05	-4.42029403E-05	6.51805722E-06	2.36564877E-05					
6.71551726E-06	2.01142954E-06	-3.44499310E-06	-1.06347244E-06	5.80002066E-07				
2.31284377E-06	7.73008227E-06	-1.14976289E-06	-4.20036007E-06	1.87669133E-07	7.60639371E-07			
-3.85042957E-07	-1.19131811E-07	2.02121073E-07	6.32858777E-08	-3.51528316E-08	-1.12482270E-08	2.23732607E-09		
-1.53265141E-07	-4.98955618E-07	7.58921240E-08	2.76250463E-07	-1.23398337E-08	-5.13226282E-08	7.43099222E-10	3.59066141E-09	
8.32615824E-09	1.07322006E-08	-4.33280901E-09	-6.03605022E-09	7.54165676E-10	1.14999177E-09	-4.97326133E-11	-8.38428127E-11	2.72963424E-12

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
20020 1	240151.48	213257.13	.057	-.238
20020 2	231454.65	164019.14	-.142	.213
20020 3	222755.72	114780.61	-.025	-.034
20020 4	214058.54	65542.15	-.013	.358
20020 5	205360.88	16304.90	-.015	-.346
20020 6	190913.55	221954.32	.576	.241
20020 7	182217.39	172716.70	-.420	.511
20020 8	173518.08	123478.99	.322	-.629
20020 9	164821.33	74241.27	.088	-.333
2002010	156124.82	25004.28	-.201	-.217
2002011	141675.03	230652.13	-.493	.419
2002012	132978.31	181414.51	-.450	.521
2002013	124277.82	132175.98	.634	-.487
2002014	115582.10	82940.02	-.742	-.719
2002015	106883.68	33702.13	.591	.548
2002016	92438.57	239351.67	.537	.613
2002017	83741.89	190115.74	.130	-.749
2002018	75042.74	140874.32	.135	1.004
2002019	66346.30	91640.10	-.363	-.928
2002020	57648.14	42401.73	-.053	.963
2002021	43203.05	248050.99	-.403	-.528
2002022	34504.55	198913.81	.252	-1.068
2002023	25805.95	149573.69	-.612	.099
2002024	17107.34	100336.57	.920	1.405
2002025	8410.63	51101.62	-.308	-.619
		SUM VV =	4.4300	10.2454
NO. OF POINTS = 25		SIGMA =	.5262	.8002

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	-.098	.225
2	.177	-.185
3	.019	.038
4	.075	-.351
5	-.045	.344
6	-.525	-.338
7	.503	-.430
8	-.426	.563
9	-.144	.313
10	.160	.248
11	.559	-.327
12	.534	-.435
13	-.709	.370
14	.606	.837
15	-.486	-.643
16	-.422	-.697
17	-.259	.715
18	.042	-1.012
19	.196	.977
20	.219	-.939
21	.305	.590
22	-.437	1.008
23	.620	.008
24	-.661	-1.543
25	.196	.664
SUM VV =	3.9826	10.6929
SIGMA =	.4989	.8175

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = .6772

FIGURE 21.—Nonlinear adjustment—output of a single case.

ADJUSTED COMPARATOR CALIBRATION PARAMETERS FROM THE SIMULTANEOUS SOLUTION OF 4 CASES

NON-LINEAR PARAMETERS

NON-LINEAR CORRECTION COEFFICIENTS FOR X

A	B	C	D	E	F	G	H	J
1.16561736E-02	-3.91230146E-03	-4.73651214E-03	2.49033847E-03	7.76495331E-04	-5.02147551E-04	-7.25123955E-05	3.34591998E-05	1.94338808E-06

NON-LINEAR CORRECTION COEFFICIENTS FOR Y

A	B	C	D	E	F	G	H	J
-5.59099411E-03	-5.97474242E-03	2.22254828E-03	3.34345790E-03	-1.87751317E-04	-6.27374361E-04	-8.29255365E-06	4.47734009E-05	-3.14622986E-07

VARIANCE-COVARIANCE MATRIX OF ADJUSTED COEFFICIENTS (SAME FOR BOTH X AND Y)

A	B	C	D	E	F	G	H	J
3.96751593E-05								
1.17307760E-05	3.96729154E-05							
-1.98613559E-05	-5.87246863E-06	1.00785645E-05						
-6.20992362E-06	-2.10015511E-05	3.10871015E-06	1.12535205E-05					
3.23763192E-06	9.66721957E-07	-1.67181141E-06	-5.11753796E-07	2.83827546E-07				
1.09706307E-06	3.67826719E-06	-5.49193676E-07	-2.00114358E-06	9.04442240E-08	3.62921520E-07			
-1.88221824E-07	-5.80140582E-08	9.94052710E-08	3.08531878E-08	-1.74193078E-08	-5.49106090E-09	1.12089027E-09		
-7.28479960E-08	-2.37919595E-07	3.63256704E-08	1.31883868E-07	-5.95952601E-09	-2.45367062E-08	3.63464174E-10	1.71991439E-09	
4.11644175E-09	5.17539943E-09	-2.15723306E-09	-2.91282288E-09	3.78597885E-10	5.55410510E-10	-2.52176919E-11	-4.05354299E-11	1.35023986E-12

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = .9347

FIGURE 22. – Combined nonlinear adjustment – multiple case solution.

COMPARATOR 3-PLOT OF NON LINEAR CORRECTIONS

X = .225000M. Y = .225000M. 1.307 .386	1.129 .350	1.014 .239	1.005 .161	1.105 .167	1.283 .251	1.472 .352	1.564 .353	X = .225000M. Y = .025000M. 1.420 .079
-.295 .436		-.587 .289		-.496 .217		-.130 .402		-.181 .129
-.972 .237	-1.150 .200	-1.264 .049	-1.274 .012	-1.174 .017	-.995 .102	-.807 .203	-.714 .203	-.859 -.071
-1.100 -.028		-1.393 -.175		-1.302 -.247		-.935 -.062		-.987 -.335
-.946 -.224	-1.124 -.261	-1.239 -.372	-1.248 -.449	-1.148 -.443	-.970 -.359	-.781 -.258	-.689 -.258	-.833 -.532
-.667 -.273		-.959 -.421		-.869 -.492		-.502 -.307		-.554 -.581
-.310 -.147	-.488 -.184	-.602 -.295	-.612 -.372	-.512 -.366	-.333 -.282	-.145 -.181	-.052 -.181	-.197 -.455
.187 .129		-.106 -.019		-.015 -.090		.351 .095		.300 -.179
X = .025000M. Y = .225000M. .993 .478	.815 .441	.701 .330	.691 .253	.792 .259	.970 .343	1.158 .444	1.251 .444	X = .025000M. Y = .025000M. 1.106 .170

FIGURE 23.—Correction values from nonlinear adjustment.

are computed in the measurement system for each point by the polynomial math model of eq (59). This polynomial computation is performed by arithmetic statement function *BX*. The residuals are then orthogonally rotated, through the angle θ (eq 47) of the case, to the grid coordinate system. Subroutine VSPRINT prints the adjusted measurements and residuals for the case. Figure 21 is a printout of a case after removal of all systematic errors from single case adjustment. A printed plot and graphic CRT plot of the systematic nonlinear correction model is available for each case according to the input header card option chosen. These plots are described below.

Upon completion of the combined linear solution, all original observations are corrected (orthogonalized) by the final linear calibration parameters and restored to the $\boldsymbol{B}$ array (columns 3–42, see Subroutine VSPRINT). Then each case is computed through a free fit adjustment (FIDFIT2) to compute the residuals of the combined linear adjustment. These residuals and linearly adjusted observations are collected in the $\boldsymbol{A}$ array to form the normal equations for the multiple case simultaneous polynomial adjustment. Subroutine INVSYM is called upon to invert and solve the normal equations.

The sums of the squares of the residuals needed for equation (62) are computed by

$$\sum_{j=1}^{q} \sum_{i=1}^{l_j} (v_{x_{ij}}^2) = \boldsymbol{X}_x^T(\boldsymbol{A}^T\boldsymbol{L}_x) + \boldsymbol{L}_x^T\boldsymbol{L}_x$$

$$\sum_{j=1}^{q} \sum_{i=1}^{l_j} (v_{y_{ij}}^2) = \boldsymbol{X}_y^T(\boldsymbol{A}^T\boldsymbol{L}_y) = \boldsymbol{L}_y^T\boldsymbol{L}_y.$$

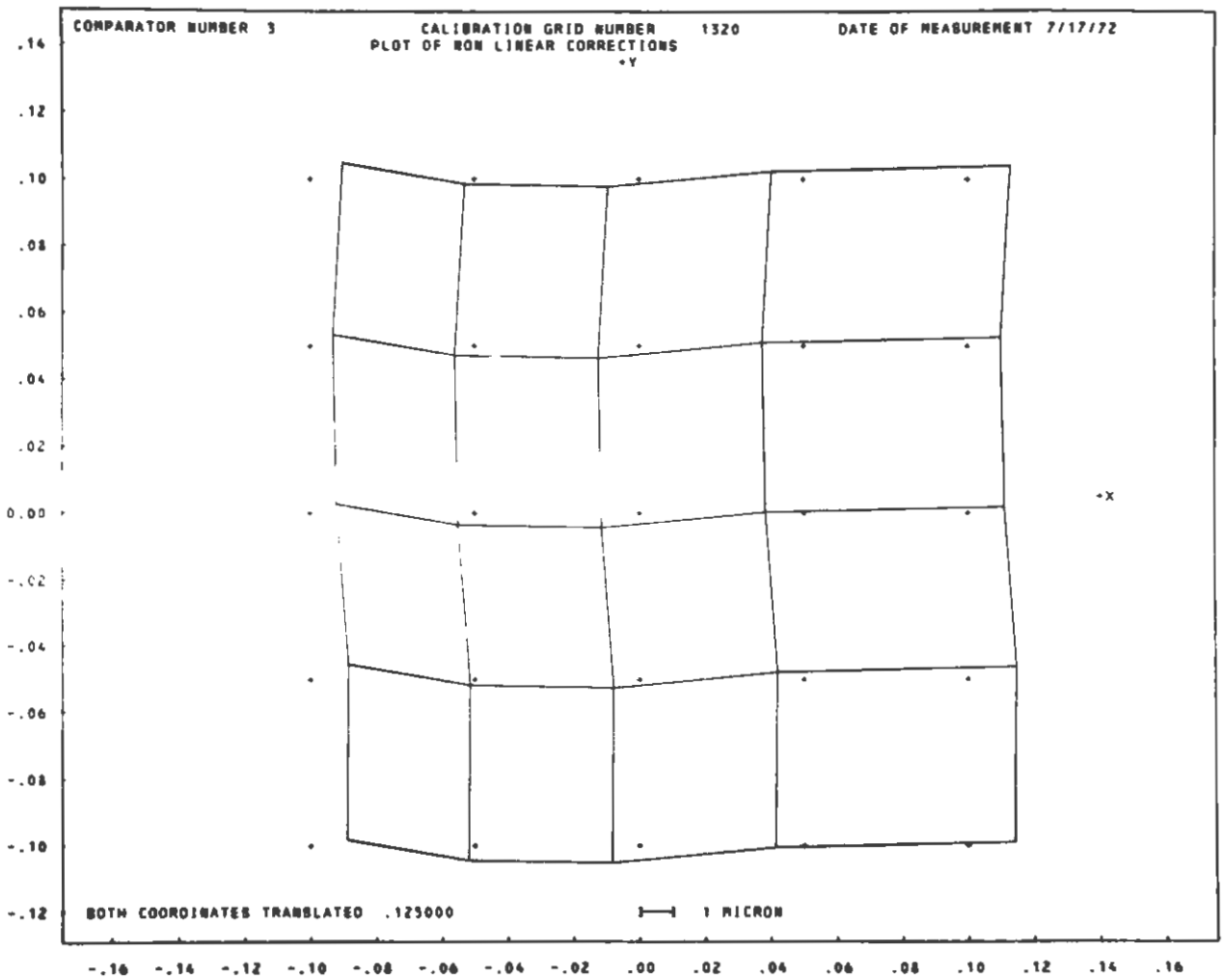

FIGURE 24.—The nonlinear correction model.

RESIDUALS AFTER APPLICATION OF FINAL LINEAR AND FINAL NON-LINEAR CORRECTIONS TO ORIGINAL DATA

GRID AND CASE NUMBER 20020

FREE ADJUSTMENT -
1 ROTATION (THETA), 2 TRANSLATIONS (TX,TY)

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
20020 1	240151.76	213256.87	-.090	.382
20020 2	231455.43	164018.85	-.673	.848
20020 3	222756.40	114780.65	-.325	.244
20020 4	214059.31	65542.97	-.269	-.170
20020 5	205363.96	16303.78	-2.467	1.051
20020 6	190914.03	221955.03	.201	-.220
20020 7	182218.14	172717.09	-.938	.344
20020 8	173518.57	123479.18	.195	-.623
20020 9	164821.59	74242.04	.315	-.922
2002010	156126.70	25003.77	-1.476	.451
2002011	141674.92	230652.32	-.294	.349
2002012	132978.56	181414.65	-.496	.473
2002013	124277.97	132175.79	.817	-.226
2002014	115582.01	82940.42	-.196	-1.073
2002015	106884.81	33702.00	.046	.706
2002016	92438.35	239351.52	.824	.755
2002017	83742.14	190116.06	.070	-1.103
2002018	75043.03	140874.27	.161	1.007
2002019	66346.31	91640.59	.075	-1.490
2002020	57648.88	42402.21	-.234	.396
2002021	43202.60	248050.51	.094	-.173
2002022	34504.33	198814.26	.641	-1.672
2002023	25805.54	149573.50	.082	.117
2002024	17106.22	100336.50	2.456	1.280
2002025	8409.38	51101.52	1.481	-.731
		SUM VV =	20.4248	15.9047
NO. OF POINTS = 25		SIGMA =	.9475	.8361

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	-.155	.360
2	-.810	.718
3	-.362	.184
4	-.236	-.215
5	-2.613	.606
6	.236	-.182
7	-.984	.176
8	.300	-.580
9	.470	-.853
10	-1.532	.188
11	-.351	.292
12	-.571	.380
13	.843	-.080
14	-.006	-1.091
15	-.078	.703
16	.680	.887
17	.261	-1.074
18	-.016	1.020
19	.333	-1.454
20	-.299	.349
21	.122	-.154
22	.922	-1.535
23	.061	.129
24	2.196	1.687
25	1.586	-.463
SUM VV =	21.4781	14.8517
SIGMA =	.9716	.8080

STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT = .8936

FIGURE 25.—Output of a single case—after combined nonlinear adjustment.

These equations correspond to a least-squares adjustment whose observation equation form is

$$AX + L = V$$

where

$A =$ is formed from the observations (x, y, x^2, y^2, x^3, etc.).

X_x^T, $X_y^T =$ the transposed vector of unknown coefficients A–J or A'–J'.

L_x, $L_y =$ the constant column = the x, y residuals from the combined linear calibration adjustment.

The variance-covariance matrix of the adjusted coefficients is computed from

$$\sum_{A-J} = \sum_{A'-J'} = m_{0_{nl}}^2 (A^T A)^{-1}$$

Figure 22 is a printout of the results of the multiple case polynomial adjustment for nonlinear systematic errors. Figure 23 is a graphic printout of the nonlinear correction model from the multiple case solution. The correction polynomial is generated through a range of typical comparator coordinates and the correction is printed in micrometers (μm) at discrete intervals. This computation is performed by Subroutine PLOT. These discrete corrections are also used to generate an annotated CRT plot of the correction polynomials. Figure 24 is the CRT plot generated displaying the corrections given by figure 23. This plot provides an historical record of the shape of the nonlinearities in the comparator. Comparison of these plots from several calibrations will demonstrate the effects of wear with time, or can indicate the presence of mechanical problems. The CRT plots are generated from Subroutine GRAFT.

Upon completion of the multiple case simultaneous polynomial adjustment, all linearly corrected observations are corrected by the final nonlinear coefficients and restored in the B array (columns 3–42). Then each case is computed through a free adjustment (FIDFIT2) to compute the residuals in both coordinate systems after all combined adjustments. Figure 25 displays the final residuals of a single case after all adjustments. As with the combined linear adjustment residuals, the standard error of a single observation of unit weight for each case (m_{0_j}) has been recomputed to allow for the additional degrees of freedom of the combined adjustment. This alteration is performed by the substitution of

$$\nu_j = 2\left(l_j - \frac{9}{q}\right)$$

for

$$\nu_j = 2(l - 9) \text{ of eq (60).}$$

F. Partial Grid Calibration

It is possible to perform a partial grid calibration if all the data recommended for a comparator calibration has been provided. The term partial is included since absolute overall scale normally is not available from modern comparators. An exception to this could be a comparator which measures with stable monochromatic laser interferometric fringe counters. The other reason a complete grid calibration can not be performed totally is due to certain similarities of the higher order nonlinear errors between the comparator axes with the higher order nonlinearities of the grid.

The minimum requirements for performing a partial grid calibration simultaneously with a comparator calibration are:

1. A symmetrically spaced, square-shaped pattern of grid points to be calibrated.
2. Four cases of data, each case measured with the grid rotated 90° from the previous case.
3. All grid points (up to 25 points for the given computer routines) to be calibrated must be measured in all 4 cases.
4. An arbitrary set of coordinates for the grid points is needed for orientation.
5. All four cases must be measured by the same operator.

Theory

Assume a calibrated grid has been measured in a single case and a comparator calibration has been performed. The residuals remaining after calibration are attributable to a combination of: residual pointing errors, random comparator errors (noise), errors in the calibrated grid coordinates, and possible residual systematic (unmodeled) comparator errors.

Now suppose the same grid has been measured in four cases, each rotated exactly 90° from the previous case, and all measured by the same operator. A combined case calibration produces residuals for each case. An analysis of these residuals with respect to their measured position in the comparator should reveal any uncompensated systematic comparator errors. Assume no systematic trends are

apparent and a comparison of the histogram of the residuals with their distribution curve reveals no significant statistical differences. Yet, systematic errors may still exist, as further analysis of these same residuals, rotated and displayed in the grid coordinate system, will demonstrate. These are the residual systematic errors present in the calibrated grid coordinates. Any unmodeled systematic linear (and most systematic nonlinear) comparator errors will nullify one another when rotated to and summed in the grid system. Residual pointing errors and random noise will usually persist when rotated to the grid system while pointing bias errors, which are self compensating with rotation, will disappear.

Theoretically, the average of the residuals summed from four 90° cases, computed in the grid system, should equal zero. Any differences from zero are attributable to random effects (grid coordinate errors) or to random errors (pointing errors and noise). The physical separation of these two error sources on the basis of a single experiment (calibration) is impossible. However, our experience at NOS has demonstrated that a significant portion of these average errors is caused by grid coordinate errors. The calibration of many different brands of comparators by several different operators, all using the same grid, will produce a large sample of grid corrections. Evaluation of this large sample will definitely exhibit persistent corrections to certain grid point coordinates.

Similar reasoning can be applied to measurements of an uncalibrated grid. In this case, the grid corrections will be made to the given approximate coordinates for the grid. The comparator calibration presented in this report is dependent on using the given grid coordinates to provide absolute scale. Therefore, any corrected grid coordinates will retain the inherent (incorrect) scale of the given approximate coordinates.

It is worth noting that, with the procedure recommended herein, the linear parameters of the comparator calibration will always be correct (except for an absolute scale factor), even though the grid is uncalibrated! However, the nonlinear parameters will seldom be correct, unless the given approximate coordinates are close to the true grid coordinates.

Figure 26 shows simplified examples of five possible systematic linear and nonlinear grid errors. The outlines represent some of the average ways in which actual grid coordinates could deviate from a regular square; i.e., these curves represent the form that manufacturing errors may take despite the manufacturer's intent to produce a square grid. This is a reasonable evaluation of engraved glass grids. Most manufacturers engrave all the parallel lines in one direction, then rotate the plate 90°, and engrave all the parallel lines in the other direction. Thus, any screw or way errors in their single axis engraving engines are repeated on both axes of the grid.

Consider each of the regular forms of figure 26 rotated through each of the four 90° rotations and superimposed on itself. The linear, quadratic, and quartic curves are self-nulling. That is, the average superimposed curve for each of these three cases is a square. The cubic and 5th degree forms are self-reinforcing and their average curve is the same as any one of their rotations. It can be shown that superimpositions of similar higher odd-degree curves are self-reinforcing whereas the higher even-degree curves are self-nulling. Note that *any* departure of these odd-degree curved forms from a regular figure will tend to nullify their average shape toward a square figure.

Now consider each of these regular forms alongside of and parallel to the two comparator axes. In the case of the self-nulling forms, any nonlinear errors of the comparator axes are distinguishable from the grid nonlinearities when the grid is measured in the four 90° rotation cases. In the case of the odd-degree figures, the nonlinear errors of the comparator axes will not be separable from the nonlinear errors of the grid.

This NOS comparator calibration program uses fourth degree polynomials to describe the nonlinear errors in the comparator axes. Therefore, its effectiveness in determining grid calibration corrections for uncalibrated grids is limited only when the grids are of regular cubic or fifth degree form. The fourth degree polynomial, in its computed form, will contain contributions from the comparator axis nonlinear errors, the average shape of the grid from four orientations, and any quasi-systematic polynomial present in the random errors.

In reality, most grids only approximate regular forms similar to those shown in figure 26. Another factor to be considered is that, when the grid placement for the four cases changes by some rotation angle from one comparator calibration to another, the nonlinear polynomial also will exhibit changes caused by new measurement range endpoints. The combining of several (4 case) calibrations, each

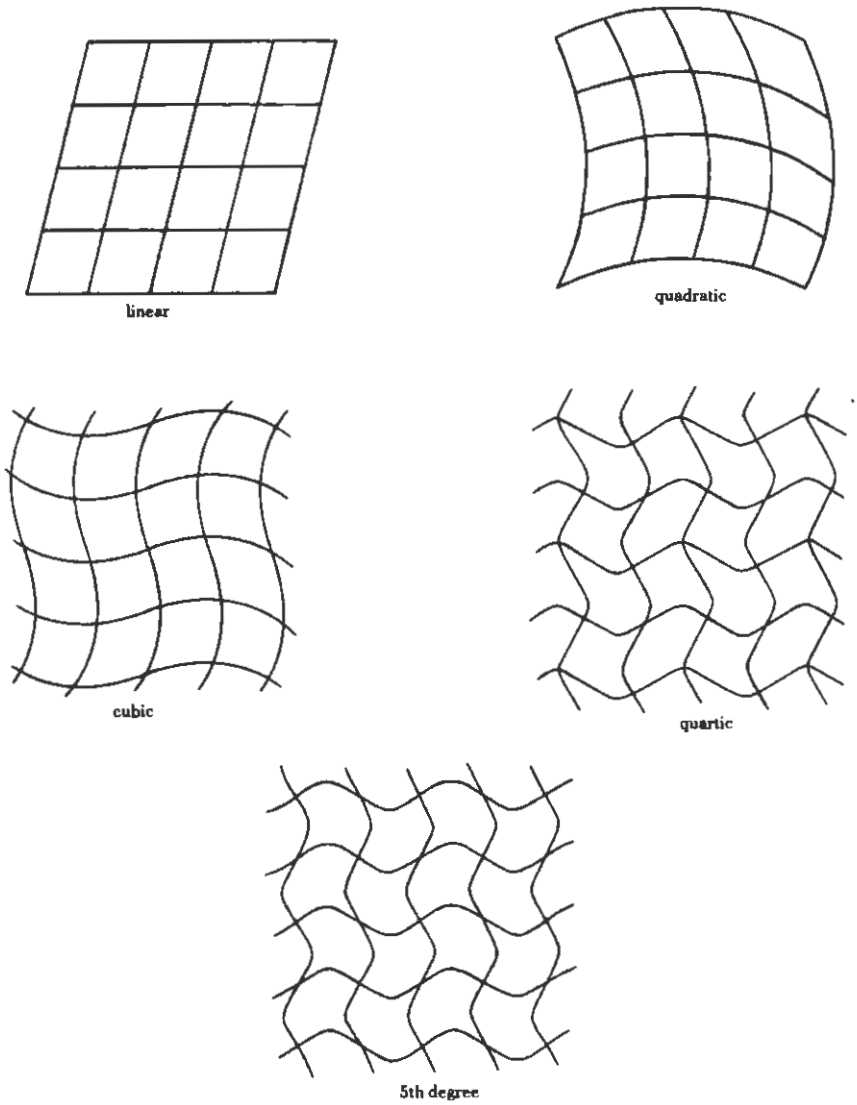

FIGURE 26. – Systematic error forms in grids.

performed with a different grid rotation, will tend to minimize correlations between the nonlinear errors of the grid and the comparator. Therefore the grid calibration capability of this program is quite acceptable.

If a regular cubic form does occur, the grid corrections determined will tend to alter the true spacing of the exterior border of a 25-point grid. Figure 27 shows an exaggerated example. The dashed line represents the incorrect (altered) border of the grid. The curved lines attempt to portray a three-dimensional view of the error-causing cubic polynomial. A regular fifth-degree form will not alter these spacings.

In summary, for an uncalibrated grid, the partial grid calibration will provide calibrated coordinates that describe the true shape of the grid. Exceptions are that the absolute scale is not determined, and the exterior border spacing may be in error because of regular cubic errors that are common to both comparator axes or are inherent in the grid.

For a grid for which the absolute scale and the average exterior border scale have been well determined, this program can provide calibrated coordinates for all grid points.

The grid calibration capabilities of this program are greatly enhanced when the same grid is used to calibrate many different brands of comparators. This procedure minimizes the effect of any quasi-systematic errors and other noise that may be superimposed on the calibration corrections produced from a single comparator calibration.

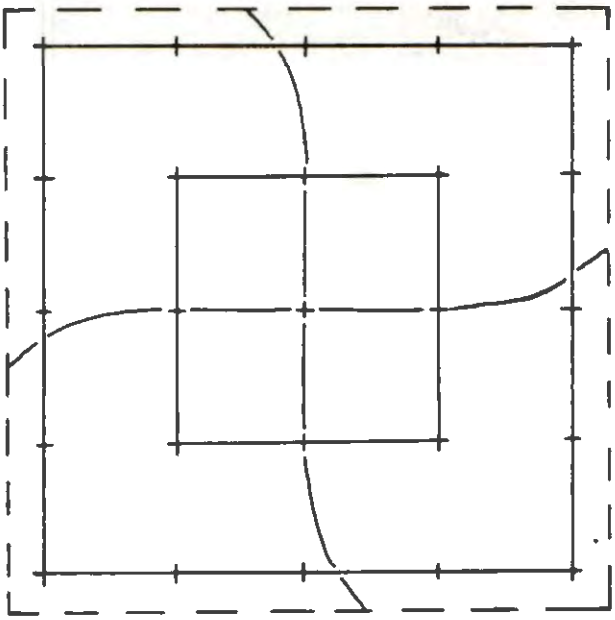

FIGURE 27. — The influence on gridline spacing due to a systematic cubic error present in both comparator axes.

Statistical Basis

The computation of grid coordinate corrections provides the basis for computing a statistical evaluation of the given grid coordinate set used for the comparator calibration. The given grid coordinate set has an a priori variance (σ_g^2) but is assumed errorless in the comparator calibration. The following is a derivation of an unbiased estimator of the given grid accuracy. The derivation neglects any correlations between the residuals after adjustment. The correlations may be obtained through computation of the variance-covariance matrix for the residuals after adjustment. Most of the statistics used can be found in Hamilton (1964).

Figure 28 is an example of the residuals (in the grid system) from four adjusted cases about the true grid location t_i of a single grid point i.

$$\bar{v}_i = \frac{\sum_{j=1}^{q} v_{ij}}{q}$$

where:

$\bar{v}_i$ = the mean residual (centroid) of grid point i after the combined adjustment of q cases. $\bar{v}_i$ represents either the $\bar{v}_{x_i}$ or the $\bar{v}_{y_i}$ coordinate of the residual centroid.

v_{ij} = represents either the $v_{x_{ij}}$ or the $v_{y_{ij}}$ residual from case j at grid point i after the combined multiple case adjustment.

q = the number of cases adjusted. (In figure 28, $q=4$ cases.)

d_i = the vector distance from the residual centroid location $\bar{v}_i$ and the true grid location t_i.

$$d_i = \bar{v}_i - t_i$$

If the residuals are attributable to given grid coordinate errors only, then the residual centroid will coincide with the true grid location and all d_i will equal zero. However, the presence of random errors (residual pointing errors and residual random errors) prevent this. If all $d_i = o$, then the variance of $\bar{v}$ (denoted hy $\hat{\sigma}^2_{\bar{v}}$) is a good between-classes estimator of the grid accuracy σ_g.

$$\hat{\sigma}^2_{\bar{v}} = \frac{\sum_{i=1}^{l} \sum_{j=1}^{q} (v_{ij} - \bar{v}_i)^2}{l(q-1)}$$

where $\bar{v}$ represents the average residual centroid for all grid points.

$$\bar{v} = \frac{\sum_{i=1}^{l} \bar{v}_i}{l} = o.$$

l equals the total number of grid points. Note, the value of $\bar{v}$ is zero because the residuals from the adjustment are transformed from the measurement system to a center origin system (the given grid coordinate system).

If all $d_i \neq o$, then $\hat{\sigma}^2_{\bar{v}}$ represents not only the grid accuracy estimator ($\hat{\sigma}^2_g$), which in Model II Analysis of Variance is the variance of a random effect, but also includes the variance of residual random error (σ^2_r). It can be shown for unequal sample sizes ($q_i \neq q$), i.e., more residuals computed for one grid point than for another, that

$$\hat{\sigma}^2_g = \left[\frac{N(l-1)}{N^2 - \sum_{i=1}^{l} (q_i^2)}\right] \cdot \left(\frac{S_b}{l-1} - \frac{S_w}{N-l}\right). \qquad (64)$$

Where:

q_i = the total number of residuals (transformed to a common grid system) meaned for grid point i.

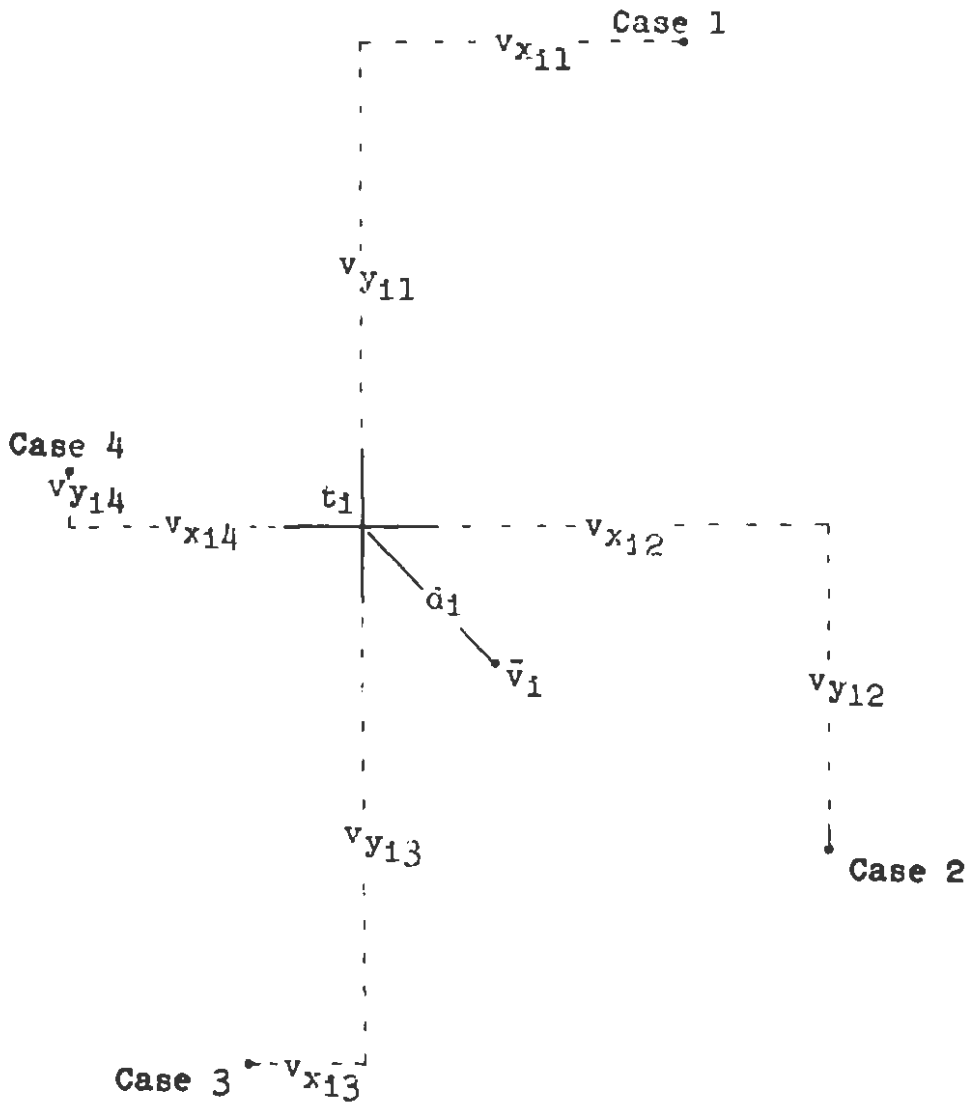

FIGURE 28.—Relation of residual centroid and true grid locations for grid point i.

N= the total number of residuals for all grid points,

$$N=\sum_{i=1}^{l} q_i$$

l= the total number of grid points.

S_b= the external or between classes sum of squares,

$$S_b=\sum_{i=1}^{l} q_i(\bar{v}_i-\bar{v})^2,$$

but

$\bar{v}$ = 0, therefore

$$S_b=\sum_{i=1}^{l}(q_i\bar{v}_i^2).$$

S_w= the internal or within-classes sum of squares

$$S_w=\sum_{i=1}^{l}\sum_{j=1}^{q_i}(v_{ij}-\bar{v}_i)^2$$

For the grid calibration to be meaningful, q_i must equal a constant which is a multiple of 4. Any other value for q_i will prevent the total nulling of residual systematic comparator errors or pointing biases. Therefore, for $q_i=q$, a constant multiple of 4

$$N=lq$$

and equation (64) becomes

$$\hat{\sigma}_g^2 = \frac{1}{q}\left(\frac{S_b}{l-1} - \frac{S_w}{N-l}\right).$$

Performing substitutions and collecting terms, we obtain

$$\hat{\sigma}_g^2 = \frac{\sum_{i=1}^{l} \bar{v}_i^2}{l-1} - \frac{\sum_{i=1}^{l}\sum_{j=1}^{q} (v_{ij}^2) - q\sum_{i=1}^{l} \bar{v}_i^2}{lq(q-1)} \qquad (65)$$

(variance of random effect) = (variance of total error) − (variance of random error)

This equation states that the unbiased estimate of the grid accuracy equals the total error variance minus the variance of the random error.

A comparison of $\hat{\sigma}_g^2$ with the a priori variance of the calibrated grid used in the calibration should show statistical agreement between them. Any significant difference between them indicates either an incorrect a priori variance or the presence of trouble in the comparator calibration.

Computer Program Routines

Counts, summations, and averages of residuals are required by eq (65). The counts and summations are accumulated in the ***B*** array. Subroutine AVGRES computes the average residuals and the grid accuracy statistics. The results are printed by Subroutine VSPRINT. Figure 29 is a sample printed output page. These computations are performed and printed twice: first with the residuals from the linear adjustment, and then with the residuals from the nonlinear adjustment.

A set of average residuals also is computed in the comparator system. To do this, the residuals in the measurement system are renumbered according to comparator coordinate location by Subroutine COMPVS. The averaging is done in Subroutine AVGRES. These average residuals show comparator zonal random errors that may indicate areas of wear or mechanical deficiency.

G. Statistical Summary

To evaluate the calibration under a rigorous state of statistical control, a Model II Analysis of Variance is performed. This evaluation assigns statistics to each of the primary error source contributors. An expression that contains all of the error sources determined in this report is

$$\sigma_t^2 = \sigma_l^2 + \sigma_{nl}^2 + \sigma_g^2 + \sigma_p^2 + \sigma_r^2, \qquad (66)$$

where

σ_t^2 = variance of all errors
= pooled RMS from initial free fit case adjustments,
σ_l^2 = variance of systematic linear errors,
σ_{nl}^2 = variance of systematic nonlinear errors,
σ_g^2 = a priori variance of the given master grid coordinates,
σ_p^2 = variance of mean pointing errors
= $(\hat{\sigma}_{\bar{x}_{ij}}^2)_p$ of eq (30), and
σ_r^2 = variance of random (noise) errors.

The variance of the systematic linear errors is obtained from

$$\sigma_l^2 = \sigma_t^2 - \sigma_{o_l}^2, \qquad (67)$$

where

$\sigma_{o_l}^2$ = the variance of a single observation of unit weight after generalized weighted mean linear adjustment,
= ($m_{0_l}^2$ of eq (57)).

Similarly, the variance of the systematic nonlinear errors is given by

$$\sigma_{nl}^2 = \sigma_{o_l}^2 - \sigma_{o_{nl}}^2, \qquad (68)$$

where

$\sigma_{o_{nl}}^2$ = the variance of a single observation of unit weight after combined polynomial adjustment,
= ($m_{o_{nl}}^2$ of eq (62)).

The variance of the random errors of the comparator system is determined from a rearrangement of eq (66). This statistic (σ_r) is the single quantity indicator of the accuracy limit of the comparator.

Each calibration provides an unbiased estimate of the grid accuracy $\hat{\sigma}_g^2$ (eq 65). This statistic should be compared with the a priori variance of the grid to ensure the validity of the initial independent calibration of the grid.

Figure 30 is a computer printout of the statistical and numerical summary of a complete comparator calibration.

AVERAGE RESIDUALS AFTER APPLICATION OF ALL FINAL CALIBRATION CORRECTIONS

RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM

PT. NO.	X ADJ	Y ADJ	VX	VY
0 1	10148.69	51432.09	-.573	-.090
0 2	18845.34	100668.17	.869	.457
0 3	27547.48	149905.67	-.286	-.264
0 4	36247.09	199143.83	.501	-.174
0 5	44946.36	248381.47	-.374	-.153
0 6	59385.86	42731.18	-.543	.375
0 7	68083.68	91969.76	.307	-.516
0 8	76783.21	141207.25	-.097	.032
0 9	85482.72	190442.65	-.130	.144
010	94181.14	239679.81	.246	.334
011	108624.00	34032.60	.476	.284
012	117323.00	83268.82	-.203	-.537
013	126023.36	132505.19	.126	.313
014	134722.46	181744.26	-.354	.016
015	143421.92	230978.33	-.010	.194
016	157859.52	25331.29	-.309	-.699
017	166558.80	74565.05	.443	.315
018	175259.73	123804.86	-.221	-.322
019	183957.99	173041.10	.076	.299
020	192656.30	222277.43	.045	-.342
021	207095.66	16630.19	.323	.184
022	215795.83	65867.26	-.088	.242
023	224496.95	115105.63	-.115	-.052
024	233194.83	164340.42	-.388	.200
025	241894.69	213577.26	.279	-.240
		SUM VV =	3.1415	2.4751
NO. OF POINTS = 25		SIGMA =	.3618	.3211
		STANDARD DEVIATION =	.3386	

GRID CALIBRATION CORRECTIONS

RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM

NO.	VX	VY
1	.112	1.020
2	-.450	.365
3	-.545	.409
4	-.291	-.097
5	-1.064	-.390
6	.750	-.207
7	-.242	.606
8	.076	-.132
9	-.235	-.424
10	-.710	.105
11	-.580	.592
12	-.042	.392
13	-.176	-.204
14	-.421	-.259
15	.060	-.309
16	.435	.649
17	.204	-.438
18	.270	-.386
19	.382	-1.129
20	.146	.048
21	.296	.655
22	.709	-.303
23	.398	.347
24	.173	.057
25	.746	-.968
SUM VV =	5.2722	6.4487
SIGMA =	.4687	.5184
STANDARD DEVIATION =	.4891	

RANDOM ERRORS

SIGMA =	.4252	.4392

UNBIASED GRID STANDARD ERROR = .2288

NOTE - GRID CALIBRATION CORRECTIONS ARE VALID ONLY IF ALL AVERAGE RESIDUALS ARE COMPUTED FROM A MULTIPLE OF FOUR EXACTLY 90 DEGREE ROTATION CASES.

FIGURE 29.—Results of the partial grid calibration.

```
                                   COMPARATOR NUMBER  3 CALIBRATION
                                         OPERATOR NUMBER  1
                                   DATE OF MEASUREMENT 7/17/72
                                   GRID AND CASE NUMBER   1320
                                         CALIBRATION SUMMARY

                ANALYSIS OF VARIANCE
   X      Y
2.355  2.620  2.491 = ROOT MEAN SQUARE ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT BEFORE CALIBRATION

1.481   .978  1.255 = STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT AFTER LINEAR ADJUSTMENT

 .934   .935   .935 = STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT AFTER LINEAR AND NON-LINEAR ADJUSTMENTS

 .339   .297   .319 = STANDARD ERROR OF MEAN POINTING MEASUREMENTS

                .249 = A PRIORI STANDARD ERROR OF MASTER GRID 1320

                .228 = COMPUTED UNBIASED STANDARD ERROR OF MASTER GRID 1320

               2.151 = STANDARD ERROR OF SYSTEMATIC LINEAR ERRORS

                .837 = STANDARD ERROR OF SYSTEMATIC NON-LINEAR ERRORS

                .843 = STANDARD ERROR OF RANDOM (IRREGULAR) COMPARATOR ERRORS


          ADJUSTED COMPARATOR CALIBRATION PARAMETERS FROM THE SIMULTANEOUS SOLUTION OF  4 C.SFS

                                                LINEAR PARAMETERS

                                                                        VARIANCE-COVARIANCE MATRIX
                        PARAMETER VALUE          SIGMA                 SCALER X          SCALER Y          ALPHA
                SCALER X =  9.9999522E-01     1.77563146E-06        3.15286707E-12    2.66515238E-20    4.13617816E-16
                SCALER Y =  9.9997765E-01     1.77558250E-06                          3.1526 9323E-12    4.13610068E-16
NON-ORTHOG. ANGLE ALPHA =     0  0 10.210       0  0    .5180                                          6.30568105E-12
     SCALER RATIO SX/SY =  1.0000176E+00

                        SCALERS IN MICRONS/METER
                SCALER X =     -4.7809          1.7756
                SCALER Y =    -22.3478          1.7756
                   SX/SY =     17.5674


                                                NON-LINEAR PARAMETERS

      NON-LINEAR CORRECTION COEFFICIENTS FOR X

      A              B              C              D              E              F              G              H              J
1.16561736E-02-3.91230146E-03-4.73651214E-03 2.49033847E-03 7.76495331E-04-5.02147551E-04-7.25123955E-05 3.34591998E-05 1.94338808E-06

      NON-LINEAR CORRECTION COEFFICIENTS FOR Y

      A              B              C              D              E              F              G              H              J
-5.59099411E-03-5.97474242E-03 2.22254828E-03 3.34345790E-03-1.87751317E-04-6.27374361E-04-8.29255365E-06 4.47734009E-05-3.14622986E-07
```

FIGURE 30.—Output summary of a complete comparator calibration.

```
      PROGRAM CALIB(INPUT,OUTPUT,TAPE10,TAPES1=OUTPUT,PUNCH)                CALIB      2
      DIMENSION A(9,11), B(25,52), C(4,2), ICASE(20), F(11), OPRATR(20),  CALIB      3
     1          OPVXSQ(25), OPVYSQ(25), CVXSQ(25), POINT(300), QX(25),    CALIB      4
     2          XMEAS(300), YMEAS(300), SVXSQ(25), SVYSQ(25), SVXVY(25),  CALIB      5
     4   PTING(25), QY(25), OPVXVY(25), OPTING(25), CVYSQ(25), CVXVY(25)  CALIB      6
     5         ,CPTING(25), ADUM(25), BDUM(25), CDUM(25), QPT(25),        CALIB      7
     6          S778(50),WM1320(50), W773(50), VX(25), VY(25)             CALIB      8
     7        ,VXM(25), VYM(25), XILIN(11,20), VXN(25), VYN(25),          CALIB      9
     8          SVCXI(6), SVCXIX(3), VCXI(6,20), XB(3), AX(25), AY(25),   CALIB     10
     9          ALX(9), ALY(9), TALLY(5,6), TALLYST(3), CAL(10), KRO(20)  CALIB     11
      INTEGER CPTING ,CASECT , DA     ,GRIDND ,GROUPD ,OLD    ,OPCT    ,  CALIB     12
     1         OPNCT  ,OPRATR ,OPTING ,POINT  ,PTING  ,ICASCT ,TNCT    ,  CALIB     13
     2        TOTOPS, YR, TALLY, CAL, UPDOWN                              CALIB     14
      COMMON ICASE, POINT, XMEAS, YMEAS, ADUM, BDUM, CDUM, TALLYST        CALIB     15
     1       , TRANS, B                                                   CALIB     16
      COMMON /PL4060/ Z(200)                                              CALIB     17
      COMMON /AFFINE/ VX, VY, VXM, VYM, THETA, VTH, SUMVX, SUMVY, SUMVXM  CALIB     18
     1                , SUMVYM, SUMVX2, SUMVY2, SUMVXM2, SUMVYM2, RT, N   CALIB     19
      DATA (OPTYES = 3HYES), (OPTNO = 3H NO), (STKRT = 1HR)               CALIB     20
      DATA W773/ -.1000016 ,-.0500020 ,-.0000007 ,+.0499991 ,+.0999999 ,  CALIB     21
     1-.1000010 ,-.0500009 ,+.0000001 ,+.0499997 ,+.1000008 ,-.1000003 ,  CALIB     22
     2-.0500001 ,+.0000009 ,+.0500004 ,+.1000016 ,-.1000003 ,-.0499998 ,  CALIB     23
     3+.0000017 ,+.0500007 ,+.1000020 ,-.1000032 ,-.0500021 ,+.0000001 ,  CALIB     24
     4+.0499995 ,+.1000009 ,                      -.0999991 ,-.1000003 ,  CALIB     25
     5-.1000015 ,-.1000019 ,-.1000000 ,-.0499980 ,-.0499992 ,-.0500003 ,  CALIB     26
     6-.0500006 ,-.0499985 ,+.0000007 ,-.0000006 ,-.0000019 ,-.0000021 ,  CALIB     27
     7+.0000001 ,+.0500015 ,+.0500005 ,+.0499994 ,+.0499993 ,+.0500018 ,  CALIB     28
     8+.1000006 ,+.0999999 ,+.0999989 ,+.0999990 ,+.1000019/             CALIB     29
      DATA WM1320/-.10000017,-.05000044,+.00000083,+.05000111,+.10000136 CALIB     30
     1,-.09999967,-.05000010,+.00000120,+.05000081,+.09999985,-.09999910 CALIB     31
     2,-.04999944,+.00000268,+.05000110,+.09999994,-.10000097,-.05000146 CALIB     32
     3,+.00000073,+.04999909,+.09999792,-.10000113,-.05000107,+.00000048 CALIB     33
     4,+.04999852,+.09999786,                     -.09999896,-.10000012 CALIB     34
     5,-.09999974,-.10000086,-.10000146,-.04999924,-.05000024,-.05000012 CALIB     35
     6,-.05000130,-.05000257,+.00000268,+.00000118,+.00000201,+.00000115 CALIB     36
     7,+.00000021,+.05000081,+.04999975,+.05000030,+.04999929,+.04999913 CALIB     37
     8,+.09999968,+.09999929,+.10000029,+.09999948,+.09999931/           CALIB     38
      DATA S778/-.09999961, -.04999883,+.00000035,+.05000144,+.10000131, CALIB     39
     1-.10000047,-.05000038,-.00000094,+.04999975,+.09999893,-.10000041, CALIB     40
     2-.05000014,-.00000102,+.04999980,+.09999893,-.10000010,-.04999981, CALIB     41
     3-.00000074,+.05000037,+.09999934,-.09999958,-.04999904,-.00000006, CALIB     42
     4+.05000077,+.10000017,                      -.10000003,-.10000062, CALIB     43
     5-.10000133,-.10000117,-.10000117,-.04999930,-.04999984,-.04999994, CALIB     44
     6-.04999974,-.04999957,+.00000023,-.00000053,-.00000068,-.00000110, CALIB     45
     7-.00000056,+.05000117,+.05000038,+.05000024,+.04999990,+.05000108, CALIB     46
     8+.10000044,+.09999976,+.10000021,+.10000020,+.10000197/            CALIB     47
      BX(A,M,DX,DY) = A(9,M) + DX*(A(7,M) + DX*(A(5,M) + DX*(A(3,M)      CALIB     48
     1                       + DX*A(1,M)))) + DY*(A(8,M) + DY*(A(6,M)    CALIB     49
     2                       + DY*(A(4,M) + DY*A(2,M))))                 CALIB     50
      CALL MODESG(Z,0)                                                    CALIB     51
C ******** CHANGE VALUE OF -UPDOWN- HERE ***********                      CALIB     52
      UPDOWN = 21                                                         CALIB     53
   61 PRINT 115                                                           CALIB     54
      PRINT 115                                                           CALIB     55
      PRINT 100                                                           CALIB     56
  100 FORMAT (2X, * COMPARATOR CALIBRATION - FINAL VERSION - SEP 1971 *,  CALIB     57
     1        ///)                                                        CALIB     58
C                                                                         CALIB     59
C      READ ONE OPTION HEADER CARD                                        CALIB     60
C                                                                         CALIB     61
      CALL CLEARV(OPRATR,20,1,20)                                         CALIB     62
      READ 110, LEFT, GROUPD, MIC, OLD, NL, NOCOMP, RT, OPRATR, DATE,     CALIB     63
     1     MASTER                                                         CALIB     64
  110 FORMAT (5I1, I2, A1, 1X, 2O13, 2X, A8, I1)                          CALIB     65
      IF (NOCOMP .EQ. 0) GO TO 39                                         CALIB     66
      MASTER = MASTER + 1                                                 CALIB     67
      GO TO (5,7,8,9),MASTER                                              CALIB     68
C                                                                         CALIB     69
C      BRL GRID 773 - AUGUST 1, 1966 VALUES                               CALIB     70
C                                                                         CALIB     71
    5 DO 51 I = 1,25                                                      CALIB     72
        B(I,1) = W773(I)                                                  CALIB     73
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 51      B(I,2) = W773(I+25)                                              CALIB
      GRIDNO = 773                                                        CALIB
      MO = 8                                                              CALIB
      DA = 1                                                              CALIB
      YR = 66                                                             CALIB
      SIGGD = 220.                                                        CALIB
      GO TO 10                                                            CALIB
C                                                                         CALIB
C     NOS GRID 1320 - WEIGHTED MEAN VALUES ADOPTED 5/6/71                 CALIB
C                                                                         CALIB
    7 DO 49 I= 1,25                                                       CALIB
          B(I,1) = WM1320(I)                                              CALIB
 49       B(I,2) = WM1320(I+25)                                           CALIB
      GRIDNO = 1320                                                       CALIB
      MO = 5                                                              CALIB
      DA = 6                                                              CALIB
      YR = 71                                                             CALIB
      SIGGD = 249.                                                        CALIB
      GO TO 10                                                            CALIB
C                                                                         CALIB
C       C+GS GRID 778 - OCTOBER 21,1969 VALUES                            CALIB
C                                                                         CALIB
    8 DO 12 I = 1,25                                                      CALIB
      B(I,1) = S778(I)                                                    CALIB
      B(I,2) = S778(I+25)                                                 CALIB
   12 CONTINUE                                                            CALIB
      GRIDNO = 778                                                        CALIB
      MO = 10                                                             CALIB
      DA = 21                                                             CALIB
      YR = 69                                                             CALIB
      SIGGD = 405.                                                        CALIB
      GO TO 10                                                            CALIB
C                                                                         CALIB
C       READ IN SPECIAL MASTER GRID VALUES                                CALIB
C                                                                         CALIB
    9 GRIDNO = OPRATR(20) + (OPRATR(19))*1000                             CALIB
      PRINT 118,GRIDNO                                                    CALIB
  118 FORMAT(41X,24HSPECIAL MASTER GRID NO. ,I4,/46X,16HCOORDINATES USED CALIB
     1)                                                                   CALIB
      SIGGD = 0.0                                                         CALIB
   71 READ122,KASE,LEG,IPT,B(IPT,1),B(IPT,2),ITEST                        CALIB
      IF (ITEST .EQ. 0) GO TO 71                                          CALIB
C                                                                         CALIB
C       FLIP MASTERGRID COORDINATES TO LEFT-HANDED VALUES FOR E-TO-E      CALIB
C          MEASUREMENTS (I.E. GRID EMULSION DOWN) OR FOR A LEFT-HANDED    CALIB
C          COMPARATOR.                                                    CALIB
C                                                                         CALIB
   10 IF (LEFT .EQ. 0) GO TO 32                                           CALIB
      CALL FLIPM(B)                                                       CALIB
C                                                                         CALIB
C       PRINT MASTER GRID VALUES USED                                     CALIB
C                                                                         CALIB
   32 IF(MASTER.EQ.4) GO TO 82                                            CALIB
      PRINT 104, GRIDNO,MO,DA,YR                                          CALIB
  104 FORMAT (40X,23HCOORDINATES OF GRID NO.,I4,18H - VALUES ADOPTED ,I2 CALIB
     1,1H/,I2,3H/19,I2)                                                   CALIB
   82 PRINT 120                                                           CALIB
  120 FORMAT ( 43X, 5HPOINT, 10X, 1HX, 28X, 1HY )                         CALIB
      DO  83 I = 1,25                                                     CALIB
 83   PRINT  121,I,B(I,1),B(I,2)                                          CALIB
  121 FORMAT ( 44X, I2, 7X, 6PF10.2, 19X, 6PF10.2 )                       CALIB
      OUTL = OPTNO                                                        CALIB
      OUTG = OPTNO                                                        CALIB
      OUTM = OPTNO                                                        CALIB
      IF(LEFT.EQ.1) OUTL = OPTYES                                         CALIB
      IF(GROUPD.EQ.1) OUTG = OPTYES                                       CALIB
      IF (MIC .NE. 1) OUTM = OPTYES                                       CALIB
      PRINT 139, OUTL, OUTG, OUTM, NOCOMP, RT, OPRATR, DATE               CALIB
  139 FORMAT ( 4(/),42X,51H*** ALL VALUES EXCEPT PARAMETERS ARE IN MICRO CALIB
     ANS ***,4(/),57X, *HEADER CARD SETUP*, //, 56X,  16HLEFT HANDED DAT CALIB
     1A,6X,A8,/51X,*INPUT DATA IS IN SOR1*,6X,A8,/47X,*LEAST COUNT OF ON CALIB
     2E MICRON*,6X,A8,/,55X,*COMPARATOR NUMBER*,6X,I3,A1,///,22X,91HCASE CALIB
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    3 NUMBER  01  02  03  04  05  06  07  08  09  10  11  12  13  14  1 CALIB     146
    45  16  17  18  19  20,/,18X,15HOPERATOR NUMBER,2014,//,56X,16HMEAS CALIB     147
    SUREMENT DATE,4X,A8,//)                                            CALIB     148
     CALL DATEC(DCR)                                                   CALIB     149
     PRINT 126,DCR                                                     CALIB     150
  126 FORMAT(///,51X,*DATE OF COMPUTER RUN IS*,A10)                    CALIB     151
     PRINT 115                                                         CALIB     152
C                                                                      CALIB     153
C      PREPARE TO READ DATA                                            CALIB     154
C                                                                      CALIB     155
     CALL CLEAR (C,4,2,1,4,1,2)                                        CALIB     156
     CALL CLEAR (B,25,52,1,25,3,52)                                    CALIB     157
     CALL CLEARV (OPVXSQ,25,1,25)                                      CALIB     158
     CALL CLEARV (OPVYSQ,25,1,25)                                      CALIB     159
     CALL CLEARV (OPVXVY,25,1,25)                                      CALIB     160
     CALL CLEARV (OPTING,25,1,25)                                      CALIB     161
     CALL CLEARV (CVXSQ,25,1,25)                                       CALIB     162
     CALL CLEARV (CVYSQ,25,1,25)                                       CALIB     163
     CALL CLEARV (CVXVY,25,1,25)                                       CALIB     164
     CALL CLEARV (CPTING,25,1,25)                                      CALIB     165
     CALL CLEARV (POINT,300,1,300)                                     CALIB     166
     CALL CLEARV (XMEAS,300,1,300)                                     CALIB     167
     CALL CLEARV (YMEAS,300,1,300)                                     CALIB     168
     OPTVXS = 0.0                                                      CALIB     169
     OPTVYS = 0.0                                                      CALIB     170
     OPTVXY = 0.0                                                      CALIB     171
     OPTVXMS = OPTVYMS = 0.0                                           CALIB     172
     OPNCT = 0                                                         CALIB     173
     TNCT = 0                                                          CALIB     174
     CTVXSQ = 0.0                                                      CALIB     175
     CTVYSQ = 0.0                                                      CALIB     176
     CTVXVY = 0.0                                                      CALIB     177
     CTVXMSQ = CTVYMSQ = 0.0                                           CALIB     178
     TCASCT=1                                                          CALIB     179
     TOTOPS = 0                                                        CALIB     180
     CASECT =0                                                         CALIB     181
     OPCT =0                                                           CALIB     182
    6 OPCT = OPCT + 1                                                  CALIB     183
     IF( OPRATR(OPCT) . EQ. 0 ) GO TO 6                                CALIB     184
     KOL = 3                                                           CALIB     185
     KOL1 = 4                                                          CALIB     186
     CNT = 0.0                                                         CALIB     187
     N = 0                                                             CALIB     188
     PRINT 138                                                         CALIB     189
     PRINT 140,OPCT                                                    CALIB     190
     IF ( OLD .EQ. 0) GO TO 4                                          CALIB     191
     READ 142, OLDMDR                                                  CALIB     192
  142 FORMAT(A8)                                                       CALIB     193
    4 IF (STKRT. EQ .RT) GO TO 27                                      CALIB     194
     GO TO (11,11,11,11,11,11,11,11,11,37,44,45,46,37,50),NOCOMP       CALIB     195
C                                                                      CALIB     196
C     READ IN DATA                                                     CALIB     197
C                                                                      CALIB     198
C        FIRST DATA CARD OF FIRST CASE                                 CALIB     199
C                                                                      CALIB     200
C      MANN COMPARATORS                                                CALIB     201
C                                                                      CALIB     202
   11 TRANS = .125                                                     CALIB     203
     READ 122,KASE,LEG,IPT,X,Y                                         CALIB     204
     IF (LEG)13,14,13                                                  CALIB     205
C                                                                      CALIB     206
C      STK1-A                                                          CALIB     207
C                                                                      CALIB     208
   37 TRANS = 1.0                                                      CALIB     209
     READ 122,KASE,LEG,IPT,X,Y                                         CALIB     210
     IF (LEG)13,14,13                                                  CALIB     211
C                                                                      CALIB     212
C      STK1-B                                                          CALIB     213
C                                                                      CALIB     214
   44 TRANS = 1.0                                                      CALIB     215
     IF ( MIC.EQ. 0 ) GO TO 20                                         CALIB     216
     TRANS = 1.5                                                       CALIB     217
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      CALL MICRON(KASE,LEG,IPT,X,Y)                                     CALIB    218
      GO TO 23                                                          CALIB    219
   20 READ 122, KASE, LEG, IPT, X, Y                                    CALIB    220
   23 IF (LEG) 13,14,13                                                 CALIB    221
C                                                                       CALIB    222
C      STK1 COMPARATORS - RIGHT SIDE                                    CALIB    223
C                                                                       CALIB    224
   27 TRANS = 1.0                                                       CALIB    225
      READ 146, KASE, LEG, IPT, X, Y                                    CALIB    226
  146 FORMAT (2X, I5, I1, I2, 21X, 2F10.8)                              CALIB    227
      IF (LEG) 13,14,13                                                 CALIB    228
C                                                                       CALIB    229
C      PSK - LEFT SIDE                                                  CALIB    230
C                                                                       CALIB    231
   45 TRANS = .3                                                        CALIB    232
      READ  123, KASE,LEG,IPT,ICALPT,X,Y                                CALIB    233
      GO TO 15                                                          CALIB    234
C                                                                       CALIB    235
C      PSK - RIGHT SIDE                                                 CALIB    236
C                                                                       CALIB    237
   46 TRANS=.3                                                          CALIB    238
      READ  124,ICALPT,KASE,LEG,IPT,X,Y                                 CALIB    239
   15 KASE = KASE + ICALPT * 100                                        CALIB    240
      IF (LEG) 13,14,13                                                 CALIB    241
C                                                                       CALIB    242
C      SPECIAL FORMATS AND/OR COMPARATORS                               CALIB    243
C                                                                       CALIB    244
   50 READ 143, KASE, LEG, IPT, X, Y, TRANS                             CALIB    245
   55 IF (LEG) 13,14,13                                                 CALIB    246
   14 N = N + 1                                                         CALIB    247
      PRINT 141,KASE,LEG,IPT,X,Y                                        CALIB    248
  141 FORMAT ( 8X, I6, 2X,I1, 2X, I2, 2X, 6PF10.0, 6PF10.0 )            CALIB    249
      POINT(N) = IPT                                                    CALIB    250
      XMEAS(N) = X                                                      CALIB    251
      YMEAS(N) = Y                                                      CALIB    252
C                                                                       CALIB    253
C        READ IN SUBSEQUENT POINTINGS                                   CALIB    254
C                                                                       CALIB    255
      GO TO (21,21,21,21,21,21,21,21,21,21,16,17,31,21,18),NOCOMP       CALIB    256
C                                                                       CALIB    257
C          MANN, STK1 COMPARATORS - LEFT SIDE                           CALIB    258
C                                                                       CALIB    259
   16 IF(MIC)19,21,19                                                   CALIB    260
   19 CALL MICRON (KAS,LEG,IPT,X,Y)                                     CALIB    261
      GO TO 22                                                          CALIB    262
   21 IF(RT .EQ. STKRT) GO TO 52                                        CALIB    263
      READ 122, KAS, LEG, IPT, X, Y                                     CALIB    264
      GO TO 22                                                          CALIB    265
C                                                                       CALIB    266
C         STK1 COMPARATORS - RIGHT SIDE                                 CALIB    267
C                                                                       CALIB    268
   52 READ 146, KAS, LEG, IPT, X, Y                                     CALIB    269
      GO TO 22                                                          CALIB    270
C                                                                       CALIB    271
C        PSK - LEFT SIDE                                                CALIB    272
C                                                                       CALIB    273
   17 READ 123,KAS,LEG,IPT,ICALP,X,Y                                    CALIB    274
      GO TO 2                                                           CALIB    275
C                                                                       CALIB    276
C        SPECIAL FORMATS AND/OR COMPARATORS                             CALIB    277
C                                                                       CALIB    278
   18 READ 143, KAS, LEG, IPT, X, Y                                     CALIB    279
      GO TO 22                                                          CALIB    280
C                                                                       CALIB    281
C        PSK - RIGHT SIDE                                               CALIB    282
C                                                                       CALIB    283
   31 READ 124, ICALP, KAS, LEG, IPT, X, Y                              CALIB    284
    2 KAS = KAS + ICALP * 100                                           CALIB    285
   22 IF (KASE - KAS) 87, 55, 87                                        CALIB    286
C                                                                       CALIB    287
C      REDUCTION OF GRID TICK LEG MEASUREMENTS                          CALIB    288
C                                                                       CALIB    289
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   13 PRINT 114,KASE,LEG,IPT,X,Y                                        CALIB     290
   78 C(LEG,1) = C(LEG,1) + X                                           CALIB     291
      C(LEG,2) = C(LEG,2) + Y                                           CALIB     292
      CT = 1.0                                                          CALIB     293
   75 GO TO (69, 69,69,69,69,69,69,69,69,65,65,66,67,65,57),NOCOMP      CALIB     294
C                                                                       CALIB     295
C        READ IN SUBSEQUENT POINTINGS OF GRID TICK LEGS                 CALIB     296
C                                                                       CALIB     297
   65 IF(RT.EQ.STKRT) GO TO 92                                          CALIB     298
      IF (MIC) 68,69,68                                                 CALIB     299
C                                                                       CALIB     300
C         MANN, STK1 COMPARATORS                                        CALIB     301
C                                                                       CALIB     302
   68 CALL MICRON (KAS,LG,IP,X2,Y2)                                     CALIB     303
      GO TO 70                                                          CALIB     304
   92 READ 146, KAS, LG, IP, X2, Y2                                     CALIB     305
      GO TO 70                                                          CALIB     306
   69 READ 122, KAS,LG,IP,X2,Y2                                         CALIB     307
      GO TO 70                                                          CALIB     308
C                                                                       CALIB     309
C        PSK - LEFT SIDE                                                CALIB     310
C                                                                       CALIB     311
   66 READ  123, KAS,LG,IP,ICALP,X2,Y2                                  CALIB     312
      GO TO 89                                                          CALIB     313
C                                                                       CALIB     314
C        PSK - RIGHT SIDE                                               CALIB     315
C                                                                       CALIB     316
   67 READ  124,ICALP,KAS,LG,IP,X2,Y2                                   CALIB     317
   89 KAS = KAS + ICALP*100                                             CALIB     318
      GO TO 70                                                          CALIB     319
C                                                                       CALIB     320
C        SPECIAL FORMATS AND/OR COMPARATORS                             CALIB     321
C                                                                       CALIB     322
   57 READ 141, KAS, LG, IP, X2, Y2                                     CALIB     323
   70 IF (IPT - IP)72,73,72                                             CALIB     324
   73 IF (LEG - LG)72,74,72                                             CALIB     325
   74 C(LEG,1) = C(LEG,1) + X2                                          CALIB     326
      C(LEG,2) = C(LEG,2) + Y2                                          CALIB     327
      CT = CT + 1.0                                                     CALIB     328
      GO TO 75                                                          CALIB     329
C                                                                       CALIB     330
C       MEAN MULTIPLE MEASUREMENTS OF TICK LEG                          CALIB     331
C                                                                       CALIB     332
   72 C(LEG,1) = C(LEG,1)/CT                                            CALIB     333
      C(LEG,2) = C(LEG,2)/CT                                            CALIB     334
      CNT = CNT + 1.0                                                   CALIB     335
      LEG = LG                                                          CALIB     336
      X = X2                                                            CALIB     337
      Y = Y2                                                            CALIB     338
      IF (CNT - 4.)76,77,76                                             CALIB     339
   76 IF(IPT .EQ. IP) GO TO 78                                          CALIB     340
      PRINT 125, IPT, CNT                                               CALIB     341
  125 FORMAT (11X,13HPOINT NUMBER ,I2,26H IS REJECTED BECAUSE ONLY ,I1, CALIB     342
     116H LEGS WERE READ.,/)                                            CALIB     343
      GO TO 80                                                          CALIB     344
   77 CALL LINEIN ( C(1,1), C(1,2), C(3,1), C(3,2), C(2,1), C(2,2),     CALIB     345
     1             C(4,1), C(4,2) )                                     CALIB     346
      N = N + 1                                                         CALIB     347
      POINT(N)= IPT                                                     CALIB     348
      XMEAS(N) = C(1,1)                                                 CALIB     349
      YMEAS(N) = C(1,2)                                                 CALIB     350
   80 IPT = IP                                                          CALIB     351
      I=0                                                               CALIB     352
      PRINT 114,KASE,I,POINT(N),XMEAS(N),YMEAS(N)                       CALIB     353
      CALL CLEAR (C,4,2,1,4,1,2)                                        CALIB     354
      CNT = 0.0                                                         CALIB     355
      GO TO 22                                                          CALIB     356
   87 CALL CLOSOUT (N)                                                  CALIB     357
C                                                                       CALIB     358
C       ALL MEASUREMENTS OF CASE HAVE BEEN STORED - NOW MULTIPLE        CALIB     359
C            MEASUREMENTS OF EACH POINT WILL BE GROUPED                 CALIB     360
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C                                                                       CALIB     361
      IF (GROUPD. NE. 0) GO TO 88                                       CALIB     362
      CALL GROUP (N)                                                    CALIB     363
C                                                                       CALIB     364
C      COMPUTE POINTING PRECISION AND PRODUCE CRT PLOT OF ERROR ELLIPSES CALIB     365
C           PRINT ALL PRECISION STATISTICS AND COMPUTE MEAN COORDINATES CALIB     366
C                                                                       CALIB     367
   88 CALL STAT (N, NPTS, 5000., SVXSQ, SVYSQ, SVXVY, PTING,TVXSQ,TVYSQ, CALIB     368
     1          TVXVY, 0, OPCT, OPRATR(OPCT), NOCOMP, GRIDNO, DATE, 1,  CALIB     369
     2          1, 1, 1, TVXM2, TVYM2 )                                 CALIB     370
      OPTVXMS = OPTVXMS + TVXM2                                         CALIB     371
      OPTVYMS = OPTVYMS + TVYM2                                         CALIB     372
      OPTVXS = TVXSQ +OPTVXS                                            CALIB     373
      OPTVYS = TVYSQ + OPTVYS                                           CALIB     374
      OPTVXY = TVXVY + OPTVXY                                           CALIB     375
C                                                                       CALIB     376
C      PLACE MEAN POINT VALUES AND THEIR ASSOCIATED STATISTICS INTO     CALIB     377
C           POINT NUMBER SORTED STORAGE ARRAYS                          CALIB     378
C                                                                       CALIB     379
      DO 84 I = 1,NPTS                                                  CALIB     380
      J = POINT (I)                                                     CALIB     381
      QPT(J) = POINT(I)                                                 CALIB     382
      QX(J) = XMEAS(I)                                                  CALIB     383
      QY(J) = YMEAS(I)                                                  CALIB     384
C                                                                       CALIB     385
C      ADD TRANS BACK ON HERE - IT WAS REMOVED IN STAT SUBROUTINE       CALIB     386
C          FOR PLOTTING PURPOSES.                                       CALIB     387
C                                                                       CALIB     388
          B(J,KOL) = XMEAS(I) + TRANS                                   CALIB     389
          B(J,KOL1) = YMEAS(I) + TRANS                                  CALIB     390
      OPVXSQ(J) = SVXSQ(I) + OPVXSQ(J)                                  CALIB     391
      OPVYSQ(J) = SVYSQ(I) + OPVYSQ(J)                                  CALIB     392
      OPVXVY(J) = SVXVY(I) + OPVXVY(J)                                  CALIB     393
          OPTING(J) = PTING(I) + OPTING(J)                              CALIB     394
   84 CONTINUE                                                          CALIB     395
      DO 3 I = 1,25                                                     CALIB     396
      XMEAS (I) = QX(I)                                                 CALIB     397
      YMEAS(I) = QY(I)                                                  CALIB     398
    3 POINT(I) = QPT(I)                                                 CALIB     399
      CASECT = CASECT + 1                                               CALIB     400
       OPNCT =  OPNCT + N                                               CALIB     401
      LOP=OPCT                                                          CALIB     402
   64 OPCT = OPCT + 1                                                   CALIB     403
      IF (OPRATR(OPCT) .EQ. 0) GO TO 64                                 CALIB     404
      IF (OPRATR(OPCT) .EQ. OPRATR(OPCT-1) ) GO TO 24                   CALIB     405
C                                                                       CALIB     406
C      PRINT AND PLOT OPERATOR POINTING PRECISION STATISTICS            CALIB     407
C                                                                       CALIB     408
      I25 = 25                                                          CALIB     409
      CALL STAT (OPNCT, I25, 5000., OPVXSQ, OPVYSQ, OPVXVY, OPTING,     CALIB     410
     1       OPTVXS,OPTVYS,OPTVXY,1,0,OPRATR(LOP),NOCOMP,GRIDNO,DATE,   CALIB     411
     2       CASECT, 1, 1, 1, OPTVXMS, OPTVYMS )                        CALIB     412
C                                                                       CALIB     413
C      STORE OPERATOR POINT STATISTICS INTO TOTAL COMPARATOR GRID       CALIB     414
C           POINTING STORAGE ARRAYS                                     CALIB     415
C                                                                       CALIB     416
      TOTOPS = TOTOPS+ 1                                                CALIB     417
      TALLY(TOTOPS,1) = CASECT                                          CALIB     418
      TALLY(TOTOPS,2) = TALLYST(1)*1.E+9                                CALIB     419
      TALLY(TOTOPS,3) = TALLYST(2)*1.E+9                                CALIB     420
      TALLY(TOTOPS,4) = TALLYST(3)*10.                                  CALIB     421
      TALLY(TOTOPS,5) = OPNCT                                           CALIB     422
      TALLY(TOTOPS,6) = OPRATR(LOP)                                     CALIB     423
      DO 85 I = 1,25                                                    CALIB     424
      CVXSQ(I) = CVXSQ(I) + OPVXSQ(I)                                   CALIB     425
      CVYSQ(I) = CVYSQ(I) + OPVYSQ(I)                                   CALIB     426
      CVXVY(I) = CVXVY(I) + OPVXVY(I)                                   CALIB     427
          CPTING(I) = CPTING(I) + OPTING(I)                             CALIB     428
   85 CONTINUE                                                          CALIB     429
      TNCT = TNCT + OPNCT                                               CALIB     430
      CTVXSQ = CTVXSQ + OPTVXS                                          CALIB     431
      CTVYSQ = CTVYSQ + OPTVYS                                          CALIB     432
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      CTVXVY = CTVXVY + OPTVXY                                          CALIB     433
      CTVXMSQ = CTVXMSQ + OPTVXMS                                       CALIB     434
      CTVYMSQ = CTVYMSQ + OPTVYMS                                       CALIB     435
C                                                                       CALIB     436
C      INITIALIZE FOR NEXT OPERATOR                                     CALIB     437
C                                                                       CALIB     438
      CALL CLEARV (OPVXSQ,25,1,25)                                      CALIB     439
      CALL CLEARV (OPVYSQ,25,1,25)                                      CALIB     440
      CALL CLEARV (OPVXVY,25,1,25)                                      CALIB     441
      CALL CLEARV (OPTING,25,1,25)                                      CALIB     442
      OPNCT = 0                                                         CALIB     443
      OPTVXS = 0.0                                                      CALIB     444
      OPTVYS = 0.0                                                      CALIB     445
      OPTVXY = 0.0                                                      CALIB     446
      OPTVXMS = OPTVYMS = 0.0                                           CALIB     447
      CASECT = 0                                                        CALIB     448
      ICASE(KOL/2) = KASE                                               CALIB     449
   24 IF (KAS.EQ.0) GO TO 86                                            CALIB     450
C                                                                       CALIB     451
C      INITIALIZE FOR NEXT CASE                                         CALIB     452
C                                                                       CALIB     453
      TCASCT = TCASCT +1                                                CALIB     454
      CALL CLEARV (SVXSQ,25,1,25)                                       CALIB     455
      CALL CLEARV (SVYSQ,25,1,25)                                       CALIB     456
      CALL CLEARV (SVXVY,25,1,25)                                       CALIB     457
      CALL CLEARV (PTING,25,1,25)                                       CALIB     458
      N = 0                                                             CALIB     459
      NPTS = 0                                                          CALIB     460
      ICASE(KOL/2) = KASE                                               CALIB     461
      KOL = KOL + 2                                                     CALIB     462
      KOL1 = KOL1 + 2                                                   CALIB     463
      KASE = KAS                                                        CALIB     464
      PRINT 138                                                         CALIB     465
      PRINT 115                                                         CALIB     466
      PRINT 140,OPCT                                                    CALIB     467
      GO TO 55                                                          CALIB     468
C                                                                       CALIB     469
C      PRINT AND PLOT TOTAL CALIBRATION POINTING PRECISION ST/  STICS   CALIB     470
C                                                                       CALIB     471
   86 CALL STAT (TNCT, I25,5000.,CVXSQ, CVYSQ, CVXVY, CPTING,     XSQ,  CALIB     472
     1          CTVYSQ, CTVXVY, 2, 0, 0, NOCOMP, GRIDNO, DATI     \SCT,  CALIB     473
     2          TOTOPS, 1, 1, CTVXMSQ, CTVYMSQ )                        CALIB     474
      OPCT = 0                                                          CALIB     475
      NCASE = 0                                                         CALIB     476
      TSVXM2L = TSVYM2L = TSVXM2N = TSVYM2N = 0.0                       CALIB     477
      RMSX = RMSY = RMSDF = 0.0                                         CALIB     478
C                                                                       CALIB     479
C     COMPUTE LEAST SQUARES SOLUTIONS FOR EACH CASE                     CALIB     480
C                                                                       CALIB     481
      DO 43 ICOL =3,KOL,2                                               CALIB     482
          ICOL1 = ICOL+1                                                CALIB     483
    1     OPCT = OPCT + 1                                               CALIB     484
          IF (OPRATR(OPCT) .EQ. 0) GO TO 1                              CALIB     485
      NCASE = NCASE + 1                                                 CALIB     486
      IF(NCASE.EQ.UPDOWN) CALL FLIPM(B)                                 CALIB     487
C                                                                       CALIB     488
C         COMPUTE ORIENTATION OF GRID PLATE ON THE COMPARATOR           CALIB     489
C                                                                       CALIB     490
          A1S = SQRT(B(8,ICOL)**2 + B(8,ICOL1)**2)                      CALIB     491
          A5S = SQRT(B(14,ICOL)**2 + B(14,ICOL1)**2)                    CALIB     492
          A21S = SQRT(B(12,ICOL)**2 + B(12,ICOL1)**2)                   CALIB     493
          A25S = SQRT(B(18,ICOL)**2 + B(18,ICOL1)**2)                   CALIB     494
          A1 = A1S+ A21S                                                CALIB     495
          A5 = A5S+ A1S                                                 CALIB     496
          A21 = A21S+ A25S                                              CALIB     497
          A25 = A25S+ A5S                                               CALIB     498
          P = AMIN1 (A1,A5,A21,A25 )                                    CALIB     499
         IF (P .NE. A1) GO TO 41                                        CALIB     500
          KROT = KROTC = 1                                              CALIB     501
          LCASCOL = ICOL                                                CALIB     502
          GO TO 42                                                      CALIB     503
   41     IF (P.NE. A5) GO TO 60                                        CALIB     504
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      KROT = 5                                                          CALIB    505
      KROTC = 4                                                         CALIB    506
      GO TO 42                                                          CALIB    507
60    IF (P .NE. A25) GO TO 79                                          CALIB    508
      KROT = 25                                                         CALIB    509
      KROTC = 3                                                         CALIB    510
      GO TO 42                                                          CALIB    511
79    KROT = 21                                                         CALIB    512
      KROTC = 2                                                         CALIB    513
42    PRINT 115                                                         CALIB    514
      KRO(NCASE) = KROTC                                                CALIB    515
   PRINT 101, ICASE(NCASE)                                              CALIB    516
      PRINT 127, NOCOMP, OPRATR(OPCT), DATE                             CALIB    517
      PRINT 136, KROT                                                   CALIB    518
      PRINT 161                                                         CALIB    519
C                                                                       CALIB    520
C     FREE ADJUSTMENT                                                   CALIB    521
C                                                                       CALIB    522
   CALL F1DF1T2 (B(1,ICOL), B(1,ICOL1), B(1,1), B(1,2), DUM, DUM)        CALIB    523
   UK = 1.5                                                             CALIB    524
   CALL VSPRINT (B(1,ICOL), B(1,ICOL1), N, ICASE(NCASE), UK)             CALIB    525
   SIGO = SQRT((SUMVX2 + SUMVY2)/(2.*(N-UK)))                           CALIB    526
   PRINT 108, SIGO                                                      CALIB    527
      RMSX = RMSX + SUMVX2                                              CALIB    528
      RMSY = RMSY + SUMVY2                                              CALIB    529
      RMSDF = RMSDF + N - UK                                            CALIB    530
   PRINT 115                                                            CALIB    531
   N = 0                                                                CALIB    532
C                                                                       CALIB    533
C     COMPUTE LINEAR PARAMETERS FOR CASE IN SUBROUTINE EXACT            CALIB    534
C                                                                       CALIB    535
   CALL EXACT (B(1,ICOL), B(1,ICOL1), B(1,1), B(1,2), XILIN(1,NCASE),    CALIB    536
  1            XILIN(2,NCASE), XILIN(3,NCASE), XILIN(4,NCASE),          CALIB    537
  2            XILIN(5,NCASE),XILIN(6,NCASE),  XILIN(7,NCASE),          CALIB    538
  3            XILIN(8,NCASE), XILIN(9,NCASE), N, XILIN(11,NCASE),      CALIB    539
  4            TSVXM2L, TSVYM2L)                                        CALIB    540
   XILIN(10,NCASE) = N                                                  CALIB    541
C                                                                       CALIB    542
C     PRINT CASE SUMMARY OF LINEAR COMPUTATION                          CALIB    543
C                                                                       CALIB    544
      PRINT 162                                                         CALIB    545
   CALL RADDG (XILIN(3,NCASE), IDEG, MIN, SEC)                          CALIB    546
   SIGAL = SQRT(XILIN(6,NCASE))                                         CALIB    547
   SIGTH = SQRT(VTH)                                                    CALIB    548
   SIGSY = SQRT(XILIN(5,NCASE))                                         CALIB    549
   SIGSX = SQRT(XILIN(4,NCASE))                                         CALIB    550
   PRINT 130, XILIN(1,NCASE), SIGSX, XILIN(4,NCASE), XILIN(11,NCASE),   CALIB    551
  1    XILIN(7,NCASE), XILIN(2,NCASE), SIGSY, XILIN(5,NCASE),          CALIB    552
  2    XILIN(8,NCASE), IDEG, MIN, SEC                                  CALIB    553
   CALL RADDG (SIGAL, IDEG, MIN, SEC)                                   CALIB    554
   PRINT 111, IDEG, MIN, SEC, XILIN(6,NCASE)                            CALIB    555
   CALL RADDG (THETA, IDEG, MIN, SEC)                                   CALIB    556
   PRINT 102, IDEG, MIN, SEC                                            CALIB    557
102 FORMAT ( /, 6X, *ROTATION ANGLE THETA = *, I4, I3, F7.3 )           CALIB    558
   CALL RADDG (SIGTH, IDEG, MIN, SEC)                                   CALIB    559
   PRINT 103, IDEG, MIN, SEC                                            CALIB    560
103 FORMAT (1H+, 46X, I4, I3, F8.4)                                     CALIB    561
   SXSY = XILIN(1,NCASE)/XILIN(2,NCASE)                                 CALIB    562
   PRINT 131, SXSY                                                      CALIB    563
   SXMM = XILIN(1,NCASE) - 1.0                                          CALIB    564
   SYMM = XILIN(2,NCASE) - 1.0                                          CALIB    565
   SXSYMM = SXSY - 1.0                                                  CALIB    566
   PRINT 112, SXMM, SIGSX, SYMM, SIGSY, SXSYMM                          CALIB    567
   SIGO = SQRT(XILIN(9,NCASE))                                          CALIB    568
      CALL CLEARV (AX,25,1,25)                                          CALIB    569
      CALL CLEARV (AY,25,1,25)                                          CALIB    570
   F(9) = 1.                                                            CALIB    571
   CALL CLEAR(A,9,11,1,9,1,11)                                          CALIB    572
   SYSINAL = XILIN(2,NCASE)*SIN(XILIN(3,NCASE))                         CALIB    573
   SYCOSAL = XILIN(2,NCASE)*COS(XILIN(3,NCASE))                         CALIB    574
   DO 54 I = 1,25                                                       CALIB    575
         IF (B(I,ICOL) .EQ. 0. .AND. B(I,ICOL1) .EQ. 0.) GO TO 54       CALIB    576
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C                                                                       CALIB    577
C     STORE SUMMATIONS OF RESIDUALS FROM LINEAR CASE SOLUTION.          CALIB    578
C                                                                       CALIB    579
          CALL COMPVS (KROTC, I, B, VX(I), VY(I), 43, J)                CALIB    580
      B(I,45) = B(I,45) + VXM(I)                                        CALIB    581
      B(I,46) = B(I,46) + VYM(I)                                        CALIB    582
      B(I,51) = B(I,51) + 1.                                            CALIB    583
          B(J,52) = B(J,52) + 1.                                        CALIB    584
C                                                                       CALIB    585
C     FORM NORMAL EQUATIONS FOR NON-LINEAR COMPUTATIONS AFTER ADJUSTING CALIB    586
C        ORIGINAL OBSERVATIONS WITH LINEAR CORRECTIONS                  CALIB    587
C                                                                       CALIB    588
      AX(I) = XILIN(1,NCASE)*B(I,ICOL) + SYSINAL*B(I,ICOL1)             CALIB    589
      AY(I) = SYCOSAL*B(I,ICOL1)                                        CALIB    590
      F(10) = VX(I)                                                     CALIB    591
      F(11) = VY(I)                                                     CALIB    592
      F(8) = AY(I)                                                      CALIB    593
      F(7) = AX(I)                                                      CALIB    594
      F(6) = F(8)*F(8)                                                  CALIB    595
      F(5) = F(7)*F(7)                                                  CALIB    596
      F(4) = F(6)*F(8)                                                  CALIB    597
      F(3) = F(5)*F(7)                                                  CALIB    598
      F(2) = F(6)*F(6)                                                  CALIB    599
      F(1) = F(5)*F(5)                                                  CALIB    600
      DO 33 K = 1,9                                                     CALIB    601
      DO 33 L = K,11                                                    CALIB    602
          A(K,L) = A(K,L) + F(K)*F(L)                                   CALIB    603
33    CONTINUE                                                          CALIB    604
54 CONTINUE                                                             CALIB    605
      CALL VSPRINT (AX(1), AY(1), N, ICASE(NCASE), 3.)                  CALIB    606
      PRINT 108, SIGO                                                   CALIB    607
      PRINT 115                                                         CALIB    608
C                                                                       CALIB    609
C     SOLVE FOR NON-LINEAR COEFFICIENTS IN MEASURED (LINEARLY           CALIB    610
C        CORRECTED) COORDINATE SYSTEM                                   CALIB    611
C                                                                       CALIB    612
      CALL INVSYM (A, 9, 11, 9, 11, DETA, SING)                         CALIB    613
C                                                                       CALIB    614
C     COMPUTE RESIDUALS, SIGO2 AND VARIANCE-COVARIANCE MATRIX           CALIB    615
C                                                                       CALIB    616
    COSTHE = COS(THETA)                                                 CALIB    617
    SINTHE = SIN(THETA)                                                 CALIB    618
    SUMVX = SUMVY = SUMVX2 = SUMVY2 = SUMVXM = SUMVYM = SUMVXM2 =       CALIB    619
   1    SUMVYM2 = 0.0                                                   CALIB    620
    CALL CLEARV (VXN,25,1,25)                                           CALIB    621
    CALL CLEARV (VYN,25,1,25)                                           CALIB    622
    DO 26 I = 1,25                                                      CALIB    623
          IF(AX(I) .EQ. 0. .AND. AY(I) .EQ. 0.0) GO TO 26               CALIB    624
          VXN(I) = BX(A,10,AX(I),AY(I))                                 CALIB    625
          VYN(I) = BX(A,11,AX(I),AY(I))                                 CALIB    626
          VX(I) = VX(I) - VXN(I)                                        CALIB    627
          VY(I) = VY(I) - VYN(I)                                        CALIB    628
      VXM(I) = VYM(I) = 0.0                                             CALIB    629
      VXM(I) = COSTHE*VX(I) + SINTHE*VY(I)                              CALIB    630
      VYM(I) = -SINTHE*VX(I) + COSTHE*VY(I)                             CALIB    631
      SUMVX = SUMVX + VX(I)                                             CALIB    632
      SUMVY = SUMVY + VY(I)                                             CALIB    633
      SUMVX2 = SUMVX2 + VX(I)*VX(I)                                     CALIB    634
      SUMVY2 = SUMVY2 + VY(I)*VY(I)                                     CALIB    635
      SUMVXM = SUMVXM + VXM(I)                                          CALIB    636
      SUMVYM = SUMVYM + VYM(I)                                          CALIB    637
      SUMVXM2 = SUMVXM2 + VXM(I)*VXM(I)                                 CALIB    638
      SUMVYM2 = SUMVYM2 + VYM(I)*VYM(I)                                 CALIB    639
          AX(I) = AX(I) + VXN(I)                                        CALIB    640
          AY(I) = AY(I) + VYN(I)                                        CALIB    641
C                                                                       CALIB    642
C     STORE SUMMATIONS OF RESIDUALS FROM NON-LINEAR CASE SOLUTIONS.     CALIB    643
C                                                                       CALIB    644
          CALL COMPVS (KROTC, I, B, VX(I), VY(I), 47, J)                CALIB    645
      B(I,49) = B(I,49) + VXM(I)                                        CALIB    646
      B(I,50) = B(I,50) + VYM(I)                                        CALIB    647
26 CONTINUE                                                             CALIB    648
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      TSVXM2N = TSVXM2N + SUMVXM2                                       CALIB     649
      TSVYM2N = TSVYM2N + SUMVYM2                                       CALIB     650
      SIGNL02 = (SUMVX2 + SUMVY2) / (2*N - 18)                          CALIB     651
    SIGNL0 = SQRT(SIGNL02)                                              CALIB     652
C                                                                       CALIB     653
    DO 25 I = 1,9                                                       CALIB     654
      DO 25 J = 1,I                                                     CALIB     655
         A(I,J) = SIGNL02*A(I,J)                                        CALIB     656
 25 CONTINUE                                                            CALIB     657
C                                                                       CALIB     658
C     PRINT SUMMARY PAGE FOR CASE SOLUTION OF NON-LINEAR COEFFICIENTS   CALIB     659
C                                                                       CALIB     660
    PRINT 163                                                           CALIB     661
    PRINT 116                                                           CALIB     662
    PRINT 117                                                           CALIB     663
    PRINT 105, ( A(I,10), I=1,9 )                                       CALIB     664
    PRINT 106                                                           CALIB     665
    PRINT 117                                                           CALIB     666
    PRINT 105, ( A(I,11), I=1,9 )                                       CALIB     667
    PRINT 107                                                           CALIB     668
    PRINT 117                                                           CALIB     669
    DO 28 I = 1,9                                                       CALIB     670
      PRINT 105, ( A(I,J), J=1,I )                                      CALIB     671
 28 CONTINUE                                                            CALIB     672
      CALL VSPRINT (AX, AY, N, ICASE(NCASE), 9.)                        CALIB     673
    PRINT 108, SIGNL0                                                   CALIB     674
      PRINT 153                                                         CALIB     675
    IF(NL)43,95,43                                                      CALIB     676
 95 ICSNO=MOD(ICASE(NCASE),100)                                         CALIB     677
    CALL PLOT (NOCOMP,GRIDNO,DATE,A,TRANS,ICSNO)                        CALIB     678
 43 CONTINUE                                                            CALIB     679
C                                                                       CALIB     680
C     ALL INDIVIDUAL CASE SOLUTIONS HAVE BEEN COMPLETED.                CALIB     681
C                                                                       CALIB     682
    IF(NCASE.GE.UPDOWN) CALL FLIPM(B)                                   CALIB     683
    PRINT 115                                                           CALIB     684
C                                                                       CALIB     685
C     COMPUTATION OF AVERAGE RESIDUALS FROM ALL CASES AFTER             CALIB     686
C        INDIVIDUAL ADJUSTMENTS.                                        CALIB     687
C                                                                       CALIB     688
    PRINT 109, NCASE                                                    CALIB     689
109 FORMAT(////,35X, *AVERAGE RESIDUALS AFTER INDIVIDUAL LINEAR ADJUS* CALIB     690
   1        , *TMENTS OF ALL*, I3, * CASES*, // )                       CALIB     691
    PRINT 134                                                           CALIB     692
    CALL AVGRES (43, B, CAL(8), NCASE, TSVXM2L, TSVYM2L, LCASCOL)       CALIB     693
    PRINT 135                                                           CALIB     694
    PRINT 115                                                           CALIB     695
    PRINT 128,NCASE                                                     CALIB     696
128 FORMAT(////, 27X, *AVERAGE RESIDUALS AFTER INDIVIDUAL LINEAR AND * CALIB     697
   1        , *NON-LINEAR ADJUSTMENTS OF ALL*, I3, * CASES*, // )      CALIB     698
    PRINT 134                                                           CALIB     699
    CALL AVGRES (47, B, CAL(9), NCASE, TSVXM2N, TSVYM2N, LCASCOL)       CALIB     700
    PRINT 135                                                           CALIB     701
C                                                                       CALIB     702
C     COMPUTATION OF GENERALIZED WEIGHTED MEAN LINEAR COMPARATOR        CALIB     703
C        CALIBRATION CORRECTION PARAMETERS.                             CALIB     704
C                                                                       CALIB     705
    CALL CLEARV (SVCXI  ,6,1,6)                                         CALIB     706
    CALL CLEARV (SVCXIX  ,3,1,3)                                        CALIB     707
    OFSUM = NSUM = DELX = 0.0                                           CALIB     708
    DO 29 I = 1,NCASE                                                   CALIB     709
C                                                                       CALIB     710
C     INVERT VARIANCE-COVARIANCE MATRIX AND PLACE IN VCXI ARRAY         CALIB     711
C                                                                       CALIB     712
      F1 = XILIN(5,NCASE)*XILIN(6,NCASE)                                CALIB     713
      F2 = -(XILIN(8,NCASE))**2                                         CALIB     714
      F3 = XILIN(11,NCASE)*XILIN(7,NCASE)                               CALIB     715
      F4 = -(XILIN(7,NCASE))**2                                         CALIB     716
      F5 = -(XILIN(11,NCASE))**2                                        CALIB     717
      DET = XILIN(9,NCASE) / ((F1+F2)*XILIN(4,NCASE) +                  CALIB     718
   1        2.0*F3*XILIN(8,NCASE) + XILIN(5,NCASE)*F4 + F5*XILIN(6,     CALIB     719
   2        NCASE))                                                    CALIB     720
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      VCXI(1,I) = DET*(F1+F2)                                           CALIB    721
      VCXI(2,I) = (XILIN(4,NCASE)*XILIN(6,NCASE) + F4)*DET              CALIB    722
      VCXI(3,I) = (XILIN(4,NCASE)*XILIN(5,NCASE) + F5)*DET              CALIB    723
      VCXI(4,I) = (-XILIN(11,NCASE)*XILIN(6,NCASE) + XILIN(7,NCASE)*    CALIB    724
     1            XILIN(8,NCASE))*DET                                   CALIB    725
      VCXI(5,I) = (XILIN(11,NCASE)*XILIN(8,NCASE) - XILIN(5,NCASE)*     CALIB    726
     1            XILIN(7,NCASE))*DET                                   CALIB    727
      VCXI(6,I) = (-XILIN(4,NCASE)*XILIN(8,NCASE) + F3)*DET             CALIB    728
C                                                                       CALIB    729
C     SUM INVERTED VAR-COV MATRICES INTO SVCXI VECTOR                   CALIB    730
C                                                                       CALIB    731
      DO 30 J = 1,6                                                     CALIB    732
         SVCXI(J) = SVCXI(J) + VCXI(J,I)                                CALIB    733
   30 CONTINUE                                                          CALIB    734
C                                                                       CALIB    735
C     SUM VAR-COV(I) TIMES X(I) IN SVCXIX VECTOR                        CALIB    736
C                                                                       CALIB    737
      SVCXIX(1) = SVCXIX(1) + VCXI(1,I)*XILIN(1,I) + VCXI(4,I)*         CALIB    738
     1            XILIN(2,I) +VCXI(5,I)*XILIN(3,I)                      CALIB    739
      SVCXIX(2) = SVCXIX(2) + VCXI(4,I)*XILIN(1,I) +                    CALIB    740
     1            VCXI(2,I)*XILIN(2,I) + VCXI(6,I)*XILIN(3,I)           CALIB    741
      SVCXIX(3) = SVCXIX(3) + VCXI(5,I)*XILIN(1,I) +                    CALIB    742
     1            VCXI(6,I)*XILIN(2,I)+ VCXI(3,I)*XILIN(3,I)            CALIB    743
      DFSUM = DFSUM + (2.*XILIN(10,I) - 6.)*XILIN(9,I)                  CALIB    744
      NSUM = NSUM + XILIN(10,I)                                         CALIB    745
   29 CONTINUE                                                          CALIB    746
C                                                                       CALIB    747
C     INVERT SUM OF VAR-COV MATRICES TO OBTAIN WI COEFFICIENT MATRIX    CALIB    748
C                                                                       CALIB    749
      F1 = SVCXI(2)*SVCXI(3)                                            CALIB    750
      F2 = SVCXI(5)*SVCXI(6)                                            CALIB    751
      F3 = SVCXI(5)**2                                                  CALIB    752
      F4 = SVCXI(6)**2                                                  CALIB    753
      F5 = SVCXI(4)**2                                                  CALIB    754
      DET = 1.0 / (SVCXI(1)*(F1-F4) + 2.0*SVCXI(4)*F2 - SVCXI(2)*F3 -   CALIB    755
     1            SVCXI(3)*F5 )                                         CALIB    756
      QX(1) = DET*(F1 - SVCXI(6)*SVCXI(6))                              CALIB    757
      QX(2) = DET*(SVCXI(1)*SVCXI(3) - F3)                              CALIB    758
      QX(3) = DET*(SVCXI(1)*SVCXI(2) - F5)                              CALIB    759
      QX(4) =-DET*(SVCXI(4)*SVCXI(3) - F2)                              CALIB    760
      QX(5) = DET*(SVCXI(4)*SVCXI(6) - SVCXI(2)*SVCXI(5))               CALIB    761
      QX(6) =-DET*(SVCXI(1)*SVCXI(6) - SVCXI(4)*SVCXI(5))               CALIB    762
C                                                                       CALIB    763
C     COMPUTE WEIGHTED MEAN LINEAR PARAMETERS, SIG0 AND VAR-COV MATRIX  CALIB    764
C                                                                       CALIB    765
      SX = QX(1)*SVCXIX(1) + QX(4)*SVCXIX(2) + QX(5)*SVCXIX(3)          CALIB    766
      SY = QX(4)*SVCXIX(1) + QX(2)*SVCXIX(2) + QX(6)*SVCXIX(3)          CALIB    767
      ALPHA = QX(5)*SVCXIX(1) + QX(6)*SVCXIX(2) + QX(3)*SVCXIX(3)       CALIB    768
      XB(1) = SX                                                        CALIB    769
      XB(2) = SY                                                        CALIB    770
      XB(3) = ALPHA                                                     CALIB    771
      DO 35 I = 1,NCASE                                                 CALIB    772
         DO 36 J = 1,3                                                  CALIB    773
            VX(J) = XILIN(J,I) - XB(J)                                  CALIB    774
   36    CONTINUE                                                       CALIB    775
         DELX = DELX +                                                  CALIB    776
     1      (VX(1)*(VX(1)*VCXI(1,I)+VX(2)*VCXI(4,I)+VX(3)*VCXI(5,I))    CALIB    777
     1    + VX(2)*(VX(1)*VCXI(4,I)+VX(2)*VCXI(2,I)+VX(3)*VCXI(6,I))     CALIB    778
     1    + VX(3)*(VX(1)*VCXI(5,I)+VX(2)*VCXI(6,I)+VX(3)*VCXI(3,I)))    CALIB    779
   35 CONTINUE                                                          CALIB    780
      DFTOT = 2.*NSUM - 3.*(NCASE + 1)                                  CALIB    781
      SIG02 = (DFSUM + DELX) / DFTOT                                    CALIB    782
      DO 38 I = 1,6                                                     CALIB    783
         QX(I) = QX(I)*SIG02                                            CALIB    784
   38 CONTINUE                                                          CALIB    785
C                                                                       CALIB    786
C     PRINT WEIGHTED MEAN, VAR-COV AND SIG0                             CALIB    787
C                                                                       CALIB    788
      PRINT 115                                                         CALIB    789
      PRINT 129, NCASE                                                  CALIB    790
      PRINT 154                                                         CALIB    791
      SIG0 = SQRT(SIG02)                                                CALIB    792
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      SIGSX = SQRT(QX(1))                                               CALIB     793
      SIGSY = SQRT(QX(2))                                               CALIB     794
      SIGAL = SQRT(QX(3))                                               CALIB     795
      CALL RADDG (ALPHA,IDEG,MIN,SEC)                                   CALIB     796
      PRINT 130, SX, SIGSX, QX(1), QX(4), QX(5), SY, SIGSY, QX(2), QX(6) CALIB    797
     1          , IDEG, MIN, SEC                                        CALIB     798
      CAL (3) = SEC*100.                                                CALIB     799
      CALL RADDG (SIGAL, IDEG, MIN, SEC)                                CALIB     800
      PRINT 111, IDEG, MIN, SEC, QX(3)                                  CALIB     801
      SXSY = SX/SY                                                      CALIB     802
      PRINT 131, SXSY                                                   CALIB     803
      SXMM = SX - 1.0                                                   CALIB     804
      SYMM = SY - 1.0                                                   CALIB     805
      CAL (1) = SXMM*1.E+8                                              CALIB     806
      CAL (2) = SYMM*1.E+8                                              CALIB     807
      SXSYMM = SXSY - 1.0                                               CALIB     808
      PRINT 112, SXMM, SIGSX, SYMM, SIGSY, SXSYMM                       CALIB     809
      PRINT 108, SIGO                                                   CALIB     810
C                                                                       CALIB     811
C      RECOMPUTE RESIDUALS OF EACH CASE TO TEST THE EFFECTIVENESS OF    CALIB     812
C         THE WEIGHTED MEAN PARAMETERS.  THESE ARE THE ADJUSTED         CALIB     813
C         RESIDUALS AS A RESULT OF A GRAND SIMULTANEOUS SOLUTION        CALIB     814
C         OF ALL OBSERVATIONS.                                          CALIB     815
C                                                                       CALIB     816
      CALL CLEAR (B,25,52,1,25,43,50)                                   CALIB     817
      CALL CLEAR (A,9,11,1,9,1,11)                                      CALIB     818
      SYSINAL = SY*SIN(ALPHA)                                           CALIB     819
      SYCOSAL = SY*COS(ALPHA)                                           CALIB     820
      MCASE = 0                                                         CALIB     821
      TSVXM2L = TSVYM2L = TSVXM2N = TSVYM2N = 0.0                       CALIB     822
      SUMLLX = SUMLLY = 0.0                                             CALIB     823
      DO 48 ICOL = 3,KOL,2                                              CALIB     824
         ICOL1 = ICOL + 1                                               CALIB     825
         PRINT 115                                                      CALIB     826
C                                                                       CALIB     827
C      APPLY FINAL LINEAR PARAMETERS TO RAW DATA AND FIT TO MASTER GRID. CALIB    828
C                                                                       CALIB     829
         MCASE = MCASE + 1                                              CALIB     830
      IF (MCASE.EQ.UPDOWN) CALL FLIPM(B)                                CALIB     831
         DO 53 I = 1,25                                                 CALIB     832
            IF(B(I,ICOL) .EQ. 0. .AND. B(I,ICOL1) .EQ. 0.) GO TO 53     CALIB     833
            B(I,ICOL) = SX*B(I,ICOL) + SYSINAL*B(I,ICOL1)               CALIB     834
            B(I,ICOL1) = SYCOSAL*B(I,ICOL1)                             CALIB     835
   53 CONTINUE                                                          CALIB     836
         CALL FIDFIT2 (B(1,ICOL), B(1,ICOL1), B(1,1), B(1,2),           CALIB     837
     1                 TSVXM2L, TSVYM2L)                                CALIB     838
C                                                                       CALIB     839
C      PRINT RESIDUALS AND SIGMA                                        CALIB     840
C                                                                       CALIB     841
      PRINT 119                                                         CALIB     842
  119 FORMAT ( 32X, *RESIDUALS AFTER APPLICATION OF FINAL LINEAR CORREC* CALIB    843
     1         , *TIONS TO ORIGINAL DATA*, 2(/) )                       CALIB     844
         PRINT 101, ICASE(MCASE)                                        CALIB     845
         PRINT 161                                                      CALIB     846
         N = XILIN(10,MCASE)                                            CALIB     847
         UK = .5*(3. + 3./NCASE)                                        CALIB     848
         CALL VSPRINT (B(1,ICOL), B(1,ICOL1), N, ICASE(MCASE), UK)      CALIB     849
         SIGO = SQRT( (SUMVX2+SUMVY2)/(2.*(N-UK)) )                     CALIB     850
      PRINT 108,SIGO                                                    CALIB     851
C                                                                       CALIB     852
C      FORM NORMAL EQUATIONS FOR GRAND SIMULTANEOUS SOLUTION OF         CALIB     853
C          NON-LINEAR COEFFICIENTS.                                     CALIB     854
C                                                                       CALIB     855
      DO 34 I = 1,25                                                    CALIB     856
            IF (B(I,ICOL) .EQ. 0. .AND. B(I,ICOL1) .EQ. 0.) GO TO 34    CALIB     857
            CALL COMPVS(KRO(MCASE), I, B, VX(I), VY(I), 43, J)          CALIB     858
            IF(MCASE.LT. UPDOWN) GO TO 1000                             CALIB     859
            B(I,45) = B(I,45 )+ VYM(I)                                  CALIB     860
            B(I,46) = B(I,46) + VXM(I)                                  CALIB     861
            GO TO 1001                                                  CALIB     862
 1000       B(I,45) = B(I,45) + VXM(I)                                  CALIB     863
         B(I,46) = B(I,46) + VYM(I)                                     CALIB     864
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1001          F(10) = VX(I)                                          CALIB    865
              F(11) = VY(I)                                          CALIB    866
              F(8) = B(I,ICOL1)                                      CALIB    867
              F(7) = B(I,ICOL)                                       CALIB    868
              F(6) = F(8)*F(8)                                       CALIB    869
              F(5) = F(7)*F(7)                                       CALIB    870
              F(4) = F(6)*F(8)                                       CALIB    871
              F(3) = F(5)*F(7)                                       CALIB    872
              F(2) = F(6)*F(6)                                       CALIB    873
              F(1) = F(5)*F(5)                                       CALIB    874
              SUMLLX = SUMLLX + VX(I)*VX(I)                          CALIB    875
              SUMLLY = SUMLLY + VY(I)*VY(I)                          CALIB    876
      DO 90 K = 1,9                                                  CALIB    877
      DO 90 L = K,11                                                 CALIB    878
                A(K,L) =  A(K,L) + F(K)*F(L)                         CALIB    879
   90 CONTINUE                                                       CALIB    880
   34 CONTINUE                                                       CALIB    881
   48 CONTINUE                                                       CALIB    882
      IF(MCASE.GE.UPDOWN) CALL FLIPM(B)                              CALIB    883
C                                                                    CALIB    884
C      COMPUTE GRAND ADJUSTMENT OF NON-LINEAR COEFFICIENTS, SIG0,    CALIB    885
C         AND THEIR VARIANCE-COVARIANCE MATRIX.                      CALIB    886
C                                                                    CALIB    887
      PRINT 115                                                      CALIB    888
      PRINT 129, NCASE                                               CALIB    889
      PRINT 113                                                      CALIB    890
      PRINT 116                                                      CALIB    891
      PRINT 117                                                      CALIB    892
      DELVVX = DELVVY = 0.0                                          CALIB    893
      DO 40 I = 1,9                                                  CALIB    894
          ALX(I) =  A(I,10)                                          CALIB    895
          ALY(I) =  A(I,11)                                          CALIB    896
   40 CONTINUE                                                       CALIB    897
          CALL INVSYM (A, 9, 11, 9, 11, DETA, SING)                  CALIB    898
      DO 56 I = 1,9                                                  CALIB    899
          DELVVX = DELVVX -  A(I,10)*ALX(I)                          CALIB    900
          DELVVY = DELVVY -  A(I,11)*ALY(I)                          CALIB    901
   56 CONTINUE                                                       CALIB    902
      SIG0X2 = ( (DELVVX + SUMLLX) / (NSUM -9.) )                    CALIB    903
      SIG0Y2 = ( (DELVVY + SUMLLY) / (NSUM -9.) )                    CALIB    904
      SIG02 = (SIG0X2 + SIG0Y2)/2.                                   CALIB    905
      DO 62 I = 1,9                                                  CALIB    906
      DO 62 J = 1,I                                                  CALIB    907
           A(I,J) = SIG02* A(I,J)                                    CALIB    908
   62 CONTINUE                                                       CALIB    909
C                                                                    CALIB    910
C      PRINT NON-LINEAR GRAND SOLUTION, SIGMA, AND VAR-COV MATRIX    CALIB    911
C                                                                    CALIB    912
      PRINT 105, (  A(I,10), I = 1,9 )                               CALIB    913
      PRINT 106                                                      CALIB    914
      PRINT 117                                                      CALIB    915
      PRINT 105, (  A(I,11), I = 1,9 )                               CALIB    916
      PRINT 107                                                      CALIB    917
      PRINT 117                                                      CALIB    918
      DO 47 I = 1,9                                                  CALIB    919
          PRINT 105, (  A(I,J), J = 1,I )                            CALIB    920
   47 CONTINUE                                                       CALIB    921
      SIG0X = SQRT(SIG0X2)                                           CALIB    922
      SIG0Y = SQRT(SIG0Y2)                                           CALIB    923
      SIG0 = SQRT(SIG02)                                             CALIB    924
      CAL(4) = (SQRT((2.*SUMLLX)/DFIOT))*1.E+9                       CALIB    925
      CAL(5) = (SQRT((2.*SUMLLY)/DFIOT))*1.E+9                       CALIB    926
      CAL(6) = SIG0X*1.E+9                                           CALIB    927
      CAL(7) = SIG0Y*1.E+9                                           CALIB    928
      PRINT 108, SIG0                                                CALIB    929
C                                                                    CALIB    930
C      PLOT NON-LINEAR CORRECTIONS FROM GRAND SOLUTION               CALIB    931
C                                                                    CALIB    932
      PRINT 153                                                      CALIB    933
      MCASE = 0                                                      CALIB    934
      CALL PLOT (NOCOMP,GRIDNO,DATE,A,TRANS,MCASE)                   CALIB    935
      DO 91 ICOL = 3,KOL,2                                           CALIB    936
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      ICOL1 = ICOL + 1                                                  CALIB    937
      PRINT 115                                                         CALIB    938
C                                                                       CALIB    939
C     APPLY FINAL NON-LINEAR PARAMETERS TO LINEARLY CORRECTED DATA      CALIB    940
C        AND FIT TO MASTER GRID TO TEST EFFECTIVENESS OF GRAND          CALIB    941
C        SIMULTANEOUS SOLUTION.                                         CALIB    942
C                                                                       CALIB    943
          MCASE = MCASE + 1                                             CALIB    944
      IF(MCASE.EQ.UPDOWN) CALL FLIPM(B)                                 CALIB    945
      DO 58 I = 1,25                                                    CALIB    946
          IF (B(I,ICOL) .EQ. 0. .AND. B(I,ICOL1) .EQ. 0.) GO TO 58      CALIB    947
      DX = B(I,ICOL)                                                    CALIB    948
          DY = B(I,ICOL1)                                               CALIB    949
          B(I,ICOL) = B(I,ICOL) + BX(A,10,DX,DY)                        CALIB    950
          B(I,ICOL1) = B(I,ICOL1) + BX(A,11,DX,DY)                      CALIB    951
   58 CONTINUE                                                          CALIB    952
      CALL FIDFIT2 (B(1,ICOL), B(1,ICOL1), B(1,1), B(1,2),              CALIB    953
     1                TSVXM2N, TSVYM2N)                                CALIB    954
C                                                                       CALIB    955
C        PRINT RESIDUALS AND SIGMA                                      CALIB    956
C                                                                       CALIB    957
      PRINT 133                                                         CALIB    958
  133 FORMAT ( 21X, *RESIDUALS AFTER APPLICATION OF FINAL LINEAR AND FI* CALIB    959
     1        , *NAL NON-LINEAR CORRECTIONS TO ORIGINAL DATA*, 2(/) )   CALIB    960
      PRINT 101, ICASE(MCASE)                                           CALIB    961
      PRINT 161                                                         CALIB    962
         UK = 9./NCASE                                                  CALIB    963
      CALL VSPRINT (B(1,ICOL), B(1,ICOL1), N, ICASE(MCASE), UK)         CALIB    964
      SIGO = SQRT( (SUMVX2 + SUMVY2)/(2.*(N-UK)) )                      CALIB    965
      PRINT 108,SIGO                                                    CALIB    966
      DO 59 I = 1,25                                                    CALIB    967
          IF (B(I,ICOL) .EQ. 0. .AND. B(I,ICOL1) .EQ. 0.) GO TO 59      CALIB    968
          CALL COMPVS (KRO(MCASE), I, B, VX(I), VY(I), 47, J)           CALIB    969
          IF(MCASE.LT. UPDOWN) GO TO 1002                               CALIB    970
          B(I,49) = B(I,49) + VYM(I)                                    CALIB    971
          B(I,50) = B(I,50) + VXM(I)                                    CALIB    972
          GO TO 59                                                      CALIB    973
 1002     B(I,49) = B(I,49) + VXM(I)                                    CALIB    974
      B(I,50) = B(I,50) + VYM(I)                                        CALIB    975
   59 CONTINUE                                                          CALIB    976
   91 CONTINUE                                                          CALIB    977
C                                                                       CALIB    978
C     ALL CASES HAVE BEEN POST TREATED WITH THE FINAL PARAMETERS.       CALIB    979
C                                                                       CALIB    980
      PRINT 115                                                         CALIB    981
C                                                                       CALIB    982
C     COMPUTE AVERAGE RESIDUALS FOR A PARTIAL GRID CALIBRATION.         CALIB    983
C                                                                       CALIB    984
      PRINT 132                                                         CALIB    985
  132 FORMAT(//,36X,*AVERAGE RESIDUALS AFTER APPLICATION OF FINAL LINEA* CALIB    986
     1        , *R CORRECTIONS*, // )                                   CALIB    987
      PRINT 134                                                         CALIB    988
      CALL AVGRES (43, B, CAL(8), NCASE, TSVXM2L, TSVYM2L, LCASCOL)     CALIB    989
      PRINT 135                                                         CALIB    990
      PRINT 115                                                         CALIB    991
      PRINT 145                                                         CALIB    992
  145 FORMAT (4(/), 32X, *AVERAGE RESIDUALS AFTER APPLICATION OF ALL *, CALIB    993
     1        *FINAL CALIBRATION CORRECTIONS*, // )                     CALIB    994
      PRINT 134                                                         CALIB    995
      CALL AVGRES (47, B, CAL(9), NCASE, TSVXM2N, TSVYM2N, LCASCOL)     CALIB    996
      PRINT 135                                                         CALIB    997
C                                                                       CALIB    998
C     CALIBRATION COMPLETED - PRINT AND PUNCH SUMMARY                   CALIB    999
C                                                                       CALIB   1000
      DO 63 J = 1, TOTOPS                                               CALIB   1001
      CAL(10) = SQRT((CAL(6)**2 + CAL(7)**2)/2 - (SIGGO**2)        -    CALIB   1002
     1              (TALLY(J,2)**2 + TALLY(J,3)**2)/2)                  CALIB   1003
      PUNCH 144, TALLY(J,6), NOCOMP, DATE, GRIDNO, MO, YR,              CALIB   1004
     1              (TALLY(J,I),I=1,5), (CAL(I),I=1,10)                 CALIB   1005
  144 FORMAT (I2, I2, A8, I4, I2, I2, I1, 4(I4), 3(I5), 7(I4))          CALIB   1006
   63 CONTINUE                                                          CALIB   1007
      PUNCH 147,(NOCOMP, DATE, NCASE, GRIDNO, (VXM(I),I=1,10))          CALIB   1008
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      PUNCH 148, (VXM(I),I=11,25), (VYM(I),I=1,25)                      CALIB    1009
  147 FORMAT (2X, I2, 2X, A8, 2X, I2, 2X, I4, 6X, 10(9PF5))             CALIB    1010
  148 FORMAT (16(9PF5))                                                 CALIB    1011
      IDUM = 0                                                          CALIB    1012
      DUM = 0.0                                                         CALIB    1013
      PRINT 115                                                         CALIB    1014
      PRINT 127, NOCOMP, TOTOPS, DATE                                   CALIB    1015
      PRINT 101, GRIDNO                                                 CALIB    1016
      SIGX = CAL(4)/1000.                                               CALIB    1017
      SIGY = CAL(5)/1000.                                               CALIB    1018
      SIGOL = SQRT((SIGX**2 + SIGY**2)/2.)                              CALIB    1019
      RMSX = SQRT(RMSX/RMSDF)                                           CALIB    1020
      RMSY = SQRT(RMSY/RMSDF)                                           CALIB    1021
      RMS = SQRT(( RMSX**2 + RMSY**2)/2.0)                              CALIB    1022
      PRINT 155, RMSX, RMSY, RMS, SIGX, SIGY, SIGOL                     CALIB    1023
  155 FORMAT (58X, *CALIBRATION SUMMARY*, // , 17X, *ANALYSIS OF VARIANC CALIB    1024
     1E*, /, 5X, *X*, 6X, *Y*,/,1X,3(6PF7.3),* = ROOT MEAN SQUARE ERROR  CALIB    1025
     2OF A SINGLE OBSERVATION OF UNIT WEIGHT BEFORE CALIBRATION*,//1X,3( CALIB    1026
     3OPF7.3),* = STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT WEIGHT  CALIB    1027
     4AFTER LINEAR ADJUSTMENT*, /)                                      CALIB    1028
      SIGOT = SQRT((SIGOX**2 + SIGOY**2)/2.)                            CALIB    1029
      PRINT 156, SIGOX, SIGOY, SIGOT                                    CALIB    1030
  156 FORMAT (1X, 3(6PF7.3), * = STANDARD ERROR OF A SINGLE OBSERVATION* CALIB    1031
     1    ,* OF UNIT WEIGHT AFTER LINEAR AND NON-LINEAR ADJUSTMENTS*,/) CALIB    1032
      SIGOP = SQRT((TALLYST(1)**2 + TALLYST(2)**2)/2.)                  CALIB    1033
      PRINT 157, TALLYST(1), TALLYST(2), SIGOP                          CALIB    1034
  157 FORMAT (1X, 3(6PF7.3), * = STANDARD ERROR OF MEAN POINTING MEASUR* CALIB    1035
     1        , *EMENTS*,/)                                             CALIB    1036
      SIGGD = SIGGD/1000.                                               CALIB    1037
      SIGO = CAL(9)/1000.                                               CALIB    1038
      PRINT 158, SIGGD, GRIDNO, SIGO, GRIDNO                            CALIB    1039
  158 FORMAT (15X, F7.3, * = A PRIORI STANDARD ERROR OF MASTER GRID*, I5 CALIB    1040
     1        ,//, 15X, F7.3, * = COMPUTED UNBIASED STANDARD ERROR OF *, CALIB    1041
     2        *MASTER GRID*, I5, /)                                     CALIB    1042
      SIGL = SQRT((RMS**2)*1.E+12- SIGOL**2)                            CALIB    1043
      PRINT 164, SIGL                                                   CALIB    1044
  164 FORMAT (15X, F7.3, * = STANDARD ERROR OF SYSTEMATIC LINEAR ERRORS* CALIB    1045
     1         ,/)                                                      CALIB    1046
      SIGO = SQRT( SIGOL**2 -(SIGOT*1.E+6)**2 )                         CALIB    1047
      PRINT 159, SIGO                                                   CALIB    1048
  159 FORMAT (15X, F7.3, * = STANDARD ERROR OF SYSTEMATIC NON-LINEAR *, CALIB    1049
     1        *ERRORS*, / )                                             CALIB    1050
      SIGO = SQRT ((SIGOT*1.E+6)**2 - SIGGD**2 - (SIGOP*1.E+6)**2)      CALIB    1051
      PRINT 160, SIGO                                                   CALIB    1052
  160 FORMAT (15X, F7.3, * = STANDARD ERROR OF RANDOM (IRREGULAR) COMP*, CALIB    1053
     1        *ARATOR ERRORS* )                                         CALIB    1054
      PRINT 129, NCASE                                                  CALIB    1055
      PRINT 154                                                         CALIB    1056
      ASEC = CAL(3)/100.                                                CALIB    1057
      PRINT 130, SX, SIGSX, QX(1), QX(4), QX(5), SY, SIGSY, QX(2), QX(6) CALIB    1058
     1       ,IDUM, IDUM, ASEC                                          CALIB    1059
      PRINT 111, IDEG, MIN, SEC, QX(3)                                  CALIB    1060
      PRINT 131, SXSY                                                   CALIB    1061
      PRINT 112, SXMM, SIGSX, SYMM, SIGSY, SXSYMM                       CALIB    1062
      PRINT 113                                                         CALIB    1063
      PRINT 116                                                         CALIB    1064
      PRINT 117                                                         CALIB    1065
      PRINT 105, (A(I,10),I=1,9)                                        CALIB    1066
      PRINT 106                                                         CALIB    1067
      PRINT 117                                                         CALIB    1068
      PRINT 105, (A(I,11), I=1,9)                                       CALIB    1069
      PRINT 115                                                         CALIB    1070
      PRINT 149                                                         CALIB    1071
      PUNCH 149                                                         CALIB    1072
  149 FORMAT (//, 17X,*HEADER CARDS FOR SATELLITE TRIANGULATION PROGRA*, CALIB    1073
     1        *MS*,/)                                                   CALIB    1074
      PRINT 150,NOCOMP                                                  CALIB    1075
      PUNCH 150,NOCOMP                                                  CALIB    1076
  150 FORMAT (10X,I2,4X,*+1.0000000E-11 +5.0000000E-06*,34X,*1*)        CALIB    1077
      C1 = 1                                                            CALIB    1078
      C2 = 2                                                            CALIB    1079
      C3 = 3                                                            CALIB    1080
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      C4 = 4                                                            CALIB    1081
      C5 = 5                                                            CALIB    1082
      C6 = 6                                                            CALIB    1083
      C7 = 7                                                            CALIB    1084
      WX = 1.0                                                          CALIB    1085
      TWO = +.24166097E-04                                              CALIB    1086
      APME = +.1E+13                                                    CALIB    1087
      TLIM = +.1E-7                                                     CALIB    1088
      ALDIF = +.3E-4                                                    CALIB    1089
      PRINT 151, (A(I,10),I=1,5), C2, DUM, WX, WX, DUM, SX, C3, SY, TWO, CALIB    1090
     1        ALPHA, APME, DUM, C4, (A(I,10), I=6,9), A(1,11), C5,      CALIB    1091
     2        (A(I,11),I=2,6),C6, (A(I,11),I=7,9), TLIM, ALDIF, C7      CALIB    1092
      PUNCH 151, (A(I,10),I=1,5), C2, DUM, WX, WX, DUM, SX, C3, SY, TWO, CALIB    1093
     1        ALPHA, APME, DUM, C4, (A(I,10), I=6,9), A(1,11), C5,      CALIB    1094
     2        (A(I,11),I=2,6),C6, (A(I,11),I=7,9), TLIM, ALDIF, C7      CALIB    1095
      IF(NOCOMP.LT.10) GO TO 61                                         CALIB    1096
      PRINT 152                                                         CALIB    1097
  152 FORMAT (//, 10X, *HEADER CARDS FOR PROGRAM ANALYTC*,/)            CALIB    1098
      PUNCH 152                                                         CALIB    1099
      SAG = -.30454545E-04                                              CALIB    1100
      IF (NOCOMP. NE. 10) SAG = 0.0                                     CALIB    1101
      PRINT 137, SX, SY, ALPHA, SAG, DUM, C1, (A(I,10), I=1,5), C2      CALIB    1102
     1          ,(A(I,10),I=6,9), DUM, C3, (A(I,11), I=1,5), C4,        CALIB    1103
     2          (A(I,11), I = 6,9), DUM, C5                             CALIB    1104
      PUNCH 137, SX, SY, ALPHA, SAG, DUM, C1, (A(I,10), I=1,5), C2      CALIB    1105
      PUNCH 137, (A(I,10),I=6,9), DUM, C3, (A(I,11),I=1,5), C4,         CALIB    1106
     1        (A(I,11), I=6,9), DUM, C5                                 CALIB    1107
      GO TO 61                                                          CALIB    1108
   39 CALL EXITG(2)                                                     CALIB    1109
  101 FORMAT (52X, *GRID AND CASE NUMBER*, I7)                          CALIB    1110
  105 FORMAT (1X, 9E15.8)                                               CALIB    1111
  106 FORMAT ( //, 10X, *NON-LINEAR CORRECTION COEFFICIENTS FOR Y*, / ) CALIB    1112
  107 FORMAT ( //, 10X, *VARIANCE-COVARIANCE MATRIX OF ADJUSTED COEFFIC* CALIB    1113
     1       ,*IENTS (SAME FOR BOTH X AND Y)* , / )                     CALIB    1114
  108 FORMAT ( //, 10X, *STANDARD ERROR OF A SINGLE OBSERVATION OF UNIT* CALIB    1115
     1       , * WEIGHT =*, 6PF8.4)                                     CALIB    1116
  111 FORMAT (1H*, 46X, I4, I3, F8.4, 58X, E15.8)                       CALIB    1117
  112 FORMAT (18X, *SCALER X = *, 6PF11.4, 6X, 6PF8.4, /, 18X, *SCALER* CALIB    1118
     1       , * Y = *, 6PF11.4, 6X, 6PF8.4, /, 21X, *SX/SY = *,        CALIB    1119
     2         6PF11.4 )                                                CALIB    1120
  113 FORMAT (///, 57X, *NON-LINEAR PARAMETERS*, /)                     CALIB    1121
  114 FORMAT(/3X,I4,I1,I2,2(2X,F8.7))                                   CALIB    1122
  115 FORMAT (1H1)                                                      CALIB    1123
  116 FORMAT ( 10X, *NON-LINEAR CORRECTION COEFFICIENTS FOR X*, / )     CALIB    1124
  117 FORMAT (7X, *A*, 14X, *B*, 14X, *C*, 14X, *D*, 14X, *E*, 14X, *F*, CALIB    1125
     1        14X, *G*, 14X, *H*, 14X, *J* )                            CALIB    1126
  122 FORMAT (2X,I5,I1,I2,1X,2F10.8,48X,I1)                             CALIB    1127
  123 FORMAT(1X,I2,I1,I2,1X,I3,9X, F6.6,4X,F6.6)                        CALIB    1128
  124 FORMAT(1X,I4,    I2,I1,I2,29X,F6.6,4X,F6.6)                       CALIB    1129
  127 FORMAT ( //, 50X, *COMPARATOR NUMBER*, I3, * CALIBRATION*, /, 57X, CALIB    1130
     1        *OPERATOR NUMBER*, I3, /, 52X, *DATE OF MEASUREMENT *, A8) CALIB    1131
  129 FORMAT(///, 25X, *ADJUSTED COMPARATOR CALIBRATION PARAMETERS FROM* CALIB    1132
     1       , * THE SIMULTANEOUS SOLUTION OF *, I2, * CASES*)          CALIB    1133
  130 FORMAT  (93X, *VARIANCE-COVARIANCE MATRIX*,/,27X, *PARAMETER VALUE CALIB    1134
     1         SIGMA*, 27X, *SCALER X*, 12X, *SCALER Y*, 13X, *ALPHA*, / CALIB    1135
     2         18X, *SCALER X = *,E14.7, 4X, E15.8, 13X, 3(5X, E15.8), / CALIB    1136
     3         18X, *SCALER Y = *,E14.7, 4X, E15.8, 33X, 2(5X, E15.8), / CALIB    1137
     4         3X, *NON-ORTHOG. ANGLE ALPHA = *,I4, 13, F7.3)           CALIB    1138
  131 FORMAT ( 8X, *SCALER RATIO SX/SY = *, E14.7, //, 27X, *SCALERS I* CALIB    1139
     1       , *N MICRONS/METER* )                                      CALIB    1140
  134 FORMAT (88X, *GRID CALIBRATION CORRECTIONS*)                      CALIB    1141
  135 FORMAT (4(/), 70X, *NOTE - GRID CALIBRATION CORRECTIONS ARE VALID* CALIB    1142
     1    , * ONLY IF ALL*,/,77X,*AVERAGE RESIDUALS ARE COMPUTED FROM *, CALIB    1143
     2  ,*A MULTIPLE OF*,/,77X,*FOUR EXACTLY 90 DEGREE ROTATION CASES.*) CALIB    1144
  136 FORMAT (//, 43X, *GRID POINT*, I3, * IS NEAREST THE COMPARATOR *, CALIB    1145
     1        *ORIGIN*, // )                                            CALIB    1146
  137 FORMAT (5E15.8, 4X, F1)                                           CALIB    1147
  138 FORMAT(1H1)                                                       CALIB    1148
  140 FORMAT ( 14X, 29HINPUT DATA CASE ORDER NUMBER , I2, // )          CALIB    1149
  143 FORMAT (2X,I5,I1,I2,1X,2F10.8,29X,F9.6)                           CALIB    1150
  151 FORMAT (5(1X,E14.7), 4X, F1)                                      CALIB    1151
  153 FORMAT (1H2)                                                      CALIB    1152
```

```
154 FORMAT (//, 59X, *LINEAR PARAMETERS*,/)                              CALIB    1153
161 FORMAT (3X, *FREE ADJUSTMENT - *,/, 6X, *1 ROTATION (THETA), 2 TRA CALIB    1154
   1NSLATIONS (TX,TY)*,//)                                               CALIB    1155
162 FORMAT (3X, *LINEAR ADJUSTMENT - *,/, 6X, *2 SCALERS (SX,SY), 2 RO CALIB    1156
   1TATIONS (ALPHA,THETA), 2 TRANSLATIONS (TX,TY)*, //)                  CALIB    1157
163 FORMAT (3X, *NON-LINEAR ADJUSTMENT - *)                              CALIB    1158
    STOP                                                                 CALIB    1159
    EN D                                                                 CALIB    1160
```

```
      SUBROUTINE CLOSOUT (NCT)                                          CLOSOUT    2
      DIMENSION ADUM(20), POINT(300), XMEAS(300), YMEAS(300)            CLOSOUT    3
      INTEGER  POINT  ,PS                                               CLOSOUT    4
      COMMON ADUM, POINT, XMEAS, YMEAS                                  CLOSOUT    5
      POINT (NCT + 1) = 0                                               CLOSOUT    6
      SIG = 0.0                                                         CLOSOUT    7
      N   = 0                                                           CLOSOUT    8
      L   = 1                                                           CLOSOUT    9
    1 N = N + 1                                                         CLOSOUT   10
    5 IF(POINT(N).EQ.POINT(N+1))GO TO 1                                 CLOSOUT   11
      X = 0.0                                                           CLOSOUT   12
      Y = 0.0                                                           CLOSOUT   13
      DO 2 I = L,N                                                      CLOSOUT   14
      X  = X  + XMEAS(I)                                                CLOSOUT   15
      Y  = Y  + YMEAS(I)                                                CLOSOUT   16
    2 CONTINUE                                                          CLOSOUT   17
      IF (SIG.GT.0.0) GO TO 6                                           CLOSOUT   18
      SIG= 1.0                                                          CLOSOUT   19
      XS = X /N                                                         CLOSOUT   20
      YS = Y /N                                                         CLOSOUT   21
      PS = POINT(N)                                                     CLOSOUT   22
      XC = XS                                                           CLOSOUT   23
      YC = YS                                                           CLOSOUT   24
    7 J = N + 1                                                         CLOSOUT   25
      DO 4 I = J,NCT                                                    CLOSOUT   26
      N = N + 1                                                         CLOSOUT   27
      L = I                                                             CLOSOUT   28
      IF (POINT(I).EQ.PS) GO TO 5                                       CLOSOUT   29
    4 CONTINUE                                                          CLOSOUT   30
      PRINT 100                                                         CLOSOUT   31
  100 FORMAT (10X,  *NO CLOSEOUT MEASUREMENTS ON INITIAL POINT MADE*)   CLOSOUT   32
      IF(NCT.LE.9)GO TO 3                                               CLOSOUT   33
      RETURN                                                            CLOSOUT   34
    3 PRINT 104                                                         CLOSOUT   35
  104 FORMAT (//////////,10X, *INSUFFICIENT MEASUREMENTS MADE - STOP* ) CLOSOUT   36
      STOP                                                              CLOSOUT   37
    6 I  = N - L + 1                                                    CLOSOUT   38
      X2 = X /I                                                         CLOSOUT   39
      Y2 = Y /I                                                         CLOSOUT   40
      DX = X2 - XC                                                      CLOSOUT   41
      DY = Y2 - YC                                                      CLOSOUT   42
      XC = X2                                                           CLOSOUT   43
      YC = Y2                                                           CLOSOUT   44
      PRINT 101,PS                                                      CLOSOUT   45
  101 FORMAT (10X,*CHECKS AND CLOSEOUTS ON POINT NUMBER *,I2,/,13X,*SEQU CLOSOUT   46
     1ENTIAL SHIFTS (XI+1)-(XI)*,/,16X,*DELTA X*, 5X,*DELTA Y*)         CLOSOUT   47
      PRINT 102,DX,DY                                                   CLOSOUT   48
  102 FORMAT (16X,6PF6.1,6X,6PF6.1)                                     CLOSOUT   49
      IF (N.NE.NCT) GO TO 7                                             CLOSOUT   50
C                                                                       CLOSOUT   51
C     COMPUTE AND PRINT TOTAL CLOSEOUT                                  CLOSOUT   52
C                                                                       CLOSOUT   53
      DXT = XC - XS                                                     CLOSOUT   54
      DYT = YC - YS                                                     CLOSOUT   55
      PRINT 103,DXT,DYT                                                 CLOSOUT   56
  103 FORMAT (/,13X,*TOTAL SHIFT (LASTX - INITIALX)*/,16X,6PF6.1,6X,    CLOSOUT   57
     16PF6.1)                                                           CLOSOUT   58
      RETURN                                                            CLOSOUT   59
      END                                                               CLOSOUT   60
```

```
      SUBROUTINE GROUP(NPT)                                             GROUP     2
C             GROUP BY POINT NUMBER                                     GROUP     3
      DIMENSION ADUM(20), IPT(300), X(300), Y(300)                      GROUP     4
      COMMON ADUM, IPT, X, Y                                            GROUP     5
      I=1                                                               GROUP     6
      J=1                                                               GROUP     7
    4 J=J+1                                                             GROUP     8
      IF(J-NPT)3,3,10                                                   GROUP     9
    3 IF(IPT(I)-IPT(J))4,2,4                                            GROUP    10
    2 K = I + 1                                                         GROUP    11
      ITPT = IPT(K)                                                     GROUP    12
      TEMPX = X(K)                                                      GROUP    13
      TEMPY = Y(K)                                                      GROUP    14
      IPT(K) = IPT(J)                                                   GROUP    15
      X(K) = X(J)                                                       GROUP    16
      Y(K) = Y(J)                                                       GROUP    17
      IPT(J)=ITPT                                                       GROUP    18
      X(J)=TEMPX                                                        GROUP    19
      Y(J)=TEMPY                                                        GROUP    20
      IF(J-NPT)5,10,10                                                  GROUP    21
    5 I=I+1                                                             GROUP    22
      GO TO 4                                                           GROUP    23
   10 IF(I-NPT-1)6,11,11                                                GROUP    24
    6 I=I+1                                                             GROUP    25
      J=I                                                               GROUP    26
      GO TO 4                                                           GROUP    27
   11 RETURN                                                            GROUP    28
      END                                                               GROUP    29
```

```
      SUBROUTINE STAT(NPT,K,SCAL,SVXX,SVYY,SVYX,NOPTS,TVX2,TVY2,TVXY,     STAT      2
     1                IP, KASE, NUMOP, NOCOMP, NUMGD, DATE, NUMCAS,       STAT      3
     2                NUMOPS, L1, L2, TVXM2, TVYM2 )                      STAT      4
      DIMENSION A(25), ADUM(20), ALPHA(25), B(25), IPT(300), NOPTS(25),   STAT      5
     1          SVXX(25), SVYX(25), SVYY(25), X(300), Y(300)              STAT      6
      COMMON ADUM, IPT, X, Y, A, B, ALPHA, SXM, SYM, PXY, TRANS,E(25,52) STAT      7
      PRINT 555                                                          STAT      8
  555 FORMAT(1H1)                                                        STAT      9
      PRINT 109                                                          STAT     10
  109 FORMAT (1H5)                                                       STAT     11
      IF(IP-1)60,61,62                                                   STAT     12
   60 PRINT 106,KASE                                                     STAT     13
  106 FORMAT ( 46X, *CASE NUMBER *, I2,* POINTING PRECISION STATISTICS*) STAT     14
      PRINT 107,NUMOP                                                    STAT     15
  107 FORMAT ( 59X, *OPERATOR NUMBER*, I3)                               STAT     16
      GO TO 63                                                           STAT     17
   61 PRINT 104,NUMOP                                                    STAT     18
  104 FORMAT (44X,*OPERATOR NUMBER*,I3,* POINTING PRECISION STATISTICS*) STAT     19
   63 PRINT 108,NOCOMP                                                   STAT     20
  108 FORMAT ( 58X, *COMPARATOR NUMBER*, I3 )                            STAT     21
      GO TO 64                                                           STAT     22
   62 PRINT 105,NOCOMP                                                   STAT     23
  105 FORMAT (43X, *COMPARATOR NUMBER*, I3, * POINTING PRECISION STATI*, STAT     24
     1        *STICS* )                                                  STAT     25
   64 PRINT 103,NUMGD,DATE                                               STAT     26
  103 FORMAT(57X,*GRID PLATE NUMBER*,I5/54X,*DATE OF MEASUREMENT *,A8,   STAT     27
     1        ///)                                                       STAT     28
      PRINT 100                                                          STAT     29
  100 FORMAT (        54X,    *INDIVIDUAL POINT STATISTICS*,//, 24X, *SI*, STAT     30
     1*NGLE OBSERVATIONS*,5X,*(MICRONS SQ) (PERCENTAGE)*,6X,*MEAN MEAS*, STAT     31
     2        *URED*, 13X, *MEAN OBSERVATIONS*, /, 3X, *POINT   NUMBER*, STAT     32
     3       * OF    STANDARD DEVIATIONS      COVARIANCE  CORRELLATION*, STAT     33
     4        7X, *COORDINATES*, 13X, *STANDARD DEVIATIONS       POINT*, STAT     34
     5        /, 2X, *NUMBER  TIMES READ    X*, 9X, *Y        POINT*, 6X, STAT     35
     6        *XY        COEFFICIENT     X*, 13X, *Y*, 11X, *X*, 9X, *Y*, STAT     36
     7        7X, *POINT  NUMBER*, / )                                   STAT     37
      IF(IP)31,30,31                                                     STAT     38
   30 TVX2=0.                                                            STAT     39
      TVY2=0.                                                            STAT     40
      TVXY=0.                                                            STAT     41
      TVXM2 = TVYM2 = 0.0                                                STAT     42
      K=0                                                                STAT     43
      I=1                                                                STAT     44
      J=I                                                                STAT     45
C                                                                        STAT     46
C             MEAN MULTIPLE READINGS                                     STAT     47
C                                                                        STAT     48
    2 J=J+1                                                              STAT     49
      IF(J-NPT)6,6,3                                                     STAT     50
    6 IF(IPT(I)-IPT(J))3,2,3                                             STAT     51
    3 L=J-1                                                              STAT     52
      IF(L-I)20,20,8                                                     STAT     53
    8 XM=0.                                                              STAT     54
      YM=0.                                                              STAT     55
      DO 4 M=I,L                                                         STAT     56
      XM=XM+X(M)                                                         STAT     57
    4 YM=YM+Y(M)                                                         STAT     58
      K=K+1                                                              STAT     59
      DL=L-I+1                                                           STAT     60
      XM=XM/DL                                                           STAT     61
      YM=YM/DL                                                           STAT     62
C             SUM DIFFERENCES AND SQUARES                                STAT     63
      SVXY=0.                                                            STAT     64
      SVX2=0.                                                            STAT     65
      SVY2=0.                                                            STAT     66
      DO 5 M=I,L                                                         STAT     67
      VX=X(M)-XM                                                         STAT     68
      VY=Y(M)-YM                                                         STAT     69
      VX2=VX**2                                                          STAT     70
      VY2=VY**2                                                          STAT     71
      SVX2=SVX2+VX2                                                      STAT     72
      SVY2=SVY2+VY2                                                      STAT     73
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    5 SVXY=SVXY+VX*VY                                                   STAT      74
      SVXX(K)=SVX2                                                      STAT      75
      SVYY(K)=SVY2                                                      STAT      76
      SVYX(K)=SVXY                                                      STAT      77
      NOPTS(K)=DL                                                       STAT      78
      TVX2=TVX2+SVX2                                                    STAT      79
      TVY2=TVY2+SVY2                                                    STAT      80
      TVXY=TVXY+SVXY                                                    STAT      81
      X(K) = XM - TRANS                                                 STAT      82
      Y(K) = YM - TRANS                                                 STAT      83
      IPT(K)=IPT(I)                                                     STAT      84
C             COMPUTE STATISTICS FOR EACH POINT                         STAT      85
      DIV=L-I                                                           STAT      86
      DIVN = 1.0/DL                                                     STAT      87
      TVXM2 = TVXM2 + DIVN*SVX2                                         STAT      88
      TVYM2 = TVYM2 + DIVN*SVY2                                         STAT      89
      SX2=SVX2/DIV                                                      STAT      90
      SY2=SVY2/DIV                                                      STAT      91
      SXM2 = SX2*DIVN                                                   STAT      92
      SYM2 = SY2*DIVN                                                   STAT      93
      IF(.10E-20-SX2)250,251,251                                        STAT      94
  251 SX=0.                                                             STAT      95
      SXM = 0.0                                                         STAT      96
      GO TO 260                                                         STAT      97
  250 SX=SQRT(SX2)                                                      STAT      98
      SXM = SQRT(SXM2)                                                  STAT      99
  260 IF(.10E-20-SY2)252,253,253                                        STAT     100
  253 SY=0.                                                             STAT     101
      SYM = 0.0                                                         STAT     102
      GO TO 261                                                         STAT     103
  252 SY=SQRT(SY2)                                                      STAT     104
      SYM = SQRT(SYM2)                                                  STAT     105
  261 IF(.10E-20-SX2-SY2)263,262,262                                    STAT     106
  262 SPT=0.                                                            STAT     107
      SPTM = 0.0                                                        STAT     108
      SXY=0.                                                            STAT     109
      PXY=0.                                                            STAT     110
      A(K)=0.                                                           STAT     111
      B(K)=0.                                                           STAT     112
      ALPHA(K)=0.                                                       STAT     113
      LL=DL                                                             STAT     114
      PRINT 101, IPT(K), LL, SX, SY, SPT, SXY, PXY, XM, YM, SXM, SYM,   STAT     115
     1        SPTM, IPT(K)                                              STAT     116
      PRINT 300,IPT(K)                                                  STAT     117
  300 FORMAT( 1X,26HALL READINGS FOR POINT NO ,I4,15H WERE IDENTICAL )  STAT     118
      GO TO 9                                                           STAT     119
  263 SPT=SQRT((SX2+SY2)/2.)                                            STAT     120
      SPTM = SQRT((SXM2 + SYM2)/2.)                                     STAT     121
      IF(SX)270,238,270                                                 STAT     122
  238 SXY=0.                                                            STAT     123
      PXY = 100.                                                        STAT     124
      GO TO 203                                                         STAT     125
  270 IF(SY)272,271,272                                                 STAT     126
  271 SXY=0.                                                            STAT     127
      PXY = 100.                                                        STAT     128
      GO TO 203                                                         STAT     129
  272 SXY = SVXY/DIV                                                    STAT     130
      PXY = (SXY/(SX*SY))*100.                                          STAT     131
      SXY = SXY* 1.E+12                                                 STAT     132
  203 LL=DL                                                             STAT     133
      PRINT 101 ,IPT(K), LL, SX, SY, SPT, SXY, PXY, XM, YM, SXM, SYM,   STAT     134
     1        SPTM, IPT(K)                                              STAT     135
      SXY = SXY/ 1.E+12                                                 STAT     136
C                                                                       STAT     137
C             COMPUTE PARAMETERS FOR ERROR ELIPSE (POINT)               STAT     138
C                                                                       STAT     139
      W=SQRT((SX2-SY2)**2+4.*SXY**2)                                    STAT     140
      A(K)=SQRT((SX2+SY2+W)/2.)                                         STAT     141
      IF(SX2+SY2-W)341,341,342                                          STAT     142
  341 B(K)=0.                                                           STAT     143
      GO TO 343                                                         STAT     144
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  342 B(K)=SQRT((SX2+SY2-W)/2.)                                          STAT      145
  343 IF(SX2-SY2)204,205,204                                             STAT      146
  205 IF(SXY)206,208,208                                                 STAT      147
  208 ALPHA(K)=.785398163                                                STAT      148
      GO TO 9                                                            STAT      149
  206 ALPHA(K)=-.785398163                                               STAT      150
      GO TO 9                                                            STAT      151
  204 ALPHA(K)=(ATAN(2.*SXY/(SX2-SY2)))/2.                               STAT      152
      IF(SX2-SY2)21,9,9                                                  STAT      153
   21 ALPHA(K)=ALPHA(K)+1.5707963267949                                  STAT      154
    9 IF(J-NPT)7,7,10                                                    STAT      155
    7 I=J                                                                STAT      156
      GO TO 2                                                            STAT      157
   10 IF(IP)151,152,151                                                  STAT      158
  151 KK=25                                                              STAT      159
      GO TO 153                                                          STAT      160
  152 KK=K                                                               STAT      161
  153 CALL ELIPS(KK,SCAL,KASE,NUMOP,NOCOMP,NUMGD,DATE,NUMCAS,NUMOPS,     STAT      162
     1            NPT,IP,49,L1,NOPTS)                                    STAT      163
C                                                                        STAT      164
C             COMPUTE STATISTICS FOR ALL POINTS                          STAT      165
C                                                                        STAT      166
      DIV=NPT-K                                                          STAT      167
      IF (DIV. EQ. 0.0) GO TO 77                                         STAT      168
      SX2=TVX2/DIV                                                       STAT      169
      SY2=TVY2/DIV                                                       STAT      170
      SX=SQRT(SX2)                                                       STAT      171
      SY=SQRT(SY2)                                                       STAT      172
      SPT=SQRT((SX2+SY2)/2.)                                             STAT      173
      SXM2 = TVXM2/DIV                                                   STAT      174
      SYM2 = TVYM2/DIV                                                   STAT      175
      SXM = SQRT(SXM2)                                                   STAT      176
      SYM = SQRT(SYM2)                                                   STAT      177
      SPTM = SQRT((SXM2 + SYM2)/2.)                                      STAT      178
      DUMX = 0.0                                                         STAT      179
      SXY = TVXY/DIV                                                     STAT      180
      IF (SX*SY. EQ. 0.0) GO TO 600                                      STAT      181
      PXY = (SXY/(SX*SY))*100.                                           STAT      182
      SXY = SXY* 1.E+12                                                  STAT      183
  600 PRINT 102                                                          STAT      184
  102 FORMAT ( //, 55X, *COMBINED POINT STATISTICS* )                    STAT      185
      PRINT 101, NPT, K, SX, SY, SPT, SXY, PXY, DUMX, DUMX, SXM, SYM,    STAT      186
     1         SPTM                                                      STAT      187
      SXY = SXY/ 1.E+12                                                  STAT      188
C                                                                        STAT      189
C             COMPUTE PARAMETERS FOR ERROR ELIPSE (TOTAL)                STAT      190
C                                                                        STAT      191
      W=SQRT((SX2-SY2)**2+4.*SXY**2)                                     STAT      192
      AT = SQRT((SX2+SY2+W)/2.)                                          STAT      193
      BT = SQRT((SX2+SY2-W)/2.)                                          STAT      194
      IF(SX2-SY2)434,435,434                                             STAT      195
  435 IF(SXY)436,438,438                                                 STAT      196
  438 AL=0.785398163                                                     STAT      197
      GO TO 70                                                           STAT      198
  436 AL=-0.785398163                                                    STAT      199
      GO TO 70                                                           STAT      200
  434 AL=(ATAN(2.*SXY/(SX2-SY2)))/2.                                     STAT      201
      IF(SX2-SY2)71,70,70                                                STAT      202
   71 AL=AL+1.5707963267949                                              STAT      203
   70 SSCAL=6.*SCAL                                                      STAT      204
      TTXX = E(1,1)                                                      STAT      205
      TTYY = E(1,2)                                                      STAT      206
      E(1,1) = 0.0                                                       STAT      207
      E(1,2) = 0.0                                                       STAT      208
      TX=X(1)                                                            STAT      209
      TY=Y(1)                                                            STAT      210
      X(1)=0.                                                            STAT      211
      Y(1)=0.                                                            STAT      212
      TAT=A(1)                                                           STAT      213
      TBT=B(1)                                                           STAT      214
      TA=ALPHA(1)                                                        STAT      215
      A(1)=AT                                                            STAT      216
```

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      B(1)=BT                                                           STAT      217
      ALPHA(1)=AL                                                       STAT      218
      CALL ELIPS( 1,SSCAL,KASE,NUMOP,NOCOMP,NOMGD,DATE,NOMCAS,NUMOPS,   STAT      219
     1            NPT,IP,B1,L2,NOPTS)                                   STAT      220
      A(1)=TAT                                                          STAT      221
      B(1)=TBT                                                          STAT      222
      ALPHA(1)=TA                                                       STAT      223
      E(1,1) = TIXX                                                     STAT      224
      E(1,2) = TIYY                                                     STAT      225
      X(1)=TX                                                           STAT      226
      Y(1)=TY                                                           STAT      227
      GO TO 77                                                          STAT      228
   31 K = 0                                                             STAT      229
      DO 50 N = 1,25                                                    STAT      230
          IF (NOPTS(N) - 1) 50,40,32                                    STAT      231
   32 DIV=NOPTS(N)-1                                                    STAT      232
      K = K + 1                                                         STAT      233
      SX2=SVXX(N)/DIV                                                   STAT      234
      SY2=SVYY(N)/DIV                                                   STAT      235
      SX=SQRT(SX2)                                                      STAT      236
      SY=SQRT(SY2)                                                      STAT      237
      SPT=SQRT((SX2+SY2)/2.)                                            STAT      238
      SXY = SVYX(N)/DIV                                                 STAT      239
      IF(SX.EQ.0..OR.SY.EQ.0.) GO TO 1                                  STAT      240
      PXY = (SXY/(SX*SY))*100.                                          STAT      241
      GO TO 11                                                          STAT      242
    1 PXY = 0.0                                                         STAT      243
   11 SXY = SXY* 1.E+12                                                 STAT      244
      PRINT 101,IPT(N),NOPTS(N),SX,SY,SPT,SXY,PXY                       STAT      245
      SXY = SXY/ 1.E+12                                                 STAT      246
      W=SQRT((SX2-SY2)**2+4.*SXY**2)                                    STAT      247
      A(N)=SQRT((SX2+SY2+W)/2.)                                         STAT      248
      B(N)=SQRT((SX2+SY2-W)/2.)                                         STAT      249
      IF(SX2-SY2) 12,13,12                                              STAT      250
   13 IF(SXY) 14,15,15                                                  STAT      251
   15 ALPHA(N) = 0.785398163                                            STAT      252
      GO TO 50                                                          STAT      253
   14 ALPHA(N) = -0.785398163                                           STAT      254
      GO TO 50                                                          STAT      255
   12 ALPHA(N) = (ATAN(2.*SXY/(SX2-SY2)))/2.                            STAT      256
      IF(SX2-SY2)41,50,50                                               STAT      257
   41 ALPHA(N)=ALPHA(N)+1.5707963267949                                 STAT      258
      GO TO 50                                                          STAT      259
   40 SX=0.                                                             STAT      260
      SY=0.                                                             STAT      261
      K = K + 1                                                         STAT      262
      SPT=0.                                                            STAT      263
      SXY=0.                                                            STAT      264
      PXY=0.                                                            STAT      265
      PRINT 101,IPT(N),NOPTS(N),SX,SY,SPT,SXY,PXY                       STAT      266
      A(N)=0.                                                           STAT      267
      B(N)=0.                                                           STAT      268
      ALPHA(N)=0.                                                       STAT      269
   50 CONTINUE                                                          STAT      270
      GO TO 10                                                          STAT      271
   20 K=K+1                                                             STAT      272
      X(K) = X(I) - TRANS                                               STAT      273
      Y(K) = Y(I) - TRANS                                               STAT      274
      IPT(K)=IPT(I)                                                     STAT      275
      SX=0.                                                             STAT      276
      SY=0.                                                             STAT      277
      SPT=0.                                                            STAT      278
      SXY=0.                                                            STAT      279
      PXY=0.                                                            STAT      280
      SVXX(K)=0.                                                        STAT      281
      SVYY(K)=0.                                                        STAT      282
      SVYX(K)=0.                                                        STAT      283
      NOPTS(K)=1                                                        STAT      284
      LLL=1                                                             STAT      285
      PRINT 101, IPT(K) ,LLL, SX, SY, SPT, SXY, PXY, X(I), Y(I), SX,    STAT      286
     1           SY, SPT, IPT(K)                                        STAT      287
```

```
      A(K)=0.                                                           STAT      288
      B(K)=0.                                                           STAT      289
      ALPHA(K)=0.                                                       STAT      290
      GO TO 9                                                           STAT      291
   77 CONTINUE                                                          STAT      292
      PRINT 110                                                         STAT      293
  110 FORMAT(//42X,5HBASIS/)                                            STAT      294
      PRINT 111,K,NPT,NUMCAS,NUMOPS                                     STAT      295
  111 FORMAT(8X,I2,7H POINTS, 8X,I5,13H MEASUREMENTS,10X,I2,6H CASES,   STAT      296
     1   10X,I2,10H OPERATORS)                                          STAT      297
  101 FORMAT ( 2X, I5, 6X, I3, 4X, 3(6PF7.3,3X), 0PF7.3, 4X, 0PF8.2,    STAT      298
     1         2(3X,6PF11.2), 3(3X,6PF7.3), 1X, I5 )                    STAT      299
      RETURN                                                            STAT      300
      END                                                               STAT      301
```

```
      SUBROUTINE ELIPS(NUPT,SCAL,KASE,NUMOP,NOCOMP,NUMGD,DATE,NUMCAS,
     1            NUMOPS,NORED,IP,NUPLT,ICONF,NOPTS)
      DIMENSION A(25), ADUM(20), ALPHA(25), B(25), IPT(300), X(300),
     1          XE(101), XF(101), Y(300), YE(101), YF(101), NOPTS(25)
      COMMON ADUM, IPT, X, Y, A, B, ALPHA, BDUM(4), E(25,52)
      COMMON /PL4060/ Z(200)
C         MAKE SURE NO OF PTS = 4*N+1
      NOPLOT=NUPLT
      L=NOPLOT-1
      LL=MOD(L,4)
      LL=4-LL
      NOPLOT=NOPLOT+LL
C         PRINT TITLE
      CALL SETSMG(Z,55,0.)
      CALL SETSMG(Z,84,1H.)
      CALL SETSMG(Z,30,.5)
      CALL SETSMG(Z,93,0.)
      CALL SETSMG(Z,90,0.)
      CALL SETSMG(Z,104,-1.5)
      CALL SETSMG(Z,105,-2.)
      CALL SUBJEG(Z,-.23,-.17,.23,.17)
      CALL OBJCTG(Z,.13,.1,1.333,1.0)
C     TITLE THE GRAPH
      IF (IP-1)70,71,72
   70 CALL LEGNDG (Z,-.08,.15,8,8HCASE NO )
      CALL NUMBRG(Z,-.055,.15,2,KASE)
      CALL LEGNDG (Z,-.04,.15,29,29HPOINTING PRECISION STATISTICS)
      GO TO 73
   71 CALL LEGNDG (Z,-.08,.15,11,11HOPERATOR NO)
      CALL NUMBRG(Z,-.037,.15,3,NUMOP)
      CALL LEGNDG(Z,-.03,.15,30,30H POINTING PRECISION STATISTICS)
      GO TO 73
   72 CALL LEGNDG (Z,-.08,.15,14,14HCOMPARATOR NO )
      CALL NUMBRG(Z,-.033,.15,2,NOCOMP)
      CALL LEGNDG(Z,-.029,.15,31,31H  POINTING PRECISION STATISTICS)
   73 CALL LEGNDG (Z,-.16,.145,14,14HCOMPARATOR NO )
      CALL NUMBRG(Z,-.110,.145,2,NOCOMP)
      CALL LEGNDG (Z,-.09,.145,12,12HOPERATOR NO )
      CALL NUMBRG(Z,-.05,.145,3,NUMOP)
      CALL LEGNDG (Z,-.03,.145,8,8HGRID NO )
      CALL NUMBRG(Z,-.001,.145,4,NUMGD)
      CALL LEGNDG (Z,.03,.145,20,20HDATE OF MEASUREMENT )
      CALL LEGNDG(Z,.11,.145,8,DATE)
      CALL NUMBRG(Z,-.14,.14,2,NUPT)
      CALL LEGNDG (Z,-.13,.14,7,7H POINTS)
      CALL NUMBRG(Z,-.09,.14,4,NORED)
      CALL LEGNDG (Z,-.07,.14,13,13H MEASUREMENTS)
      CALL NUMBRG(Z,-.02,.14,2,NUMCAS)
      CALL LEGNDG (Z,-.01,.14,6,6H CASES)
      CALL NUMBRG(Z,.03,.14,2,NUMOPS)
      CALL LEGNDG (Z,.04,.14,10,10H OPERATORS)
C       DRAW THE GRID
      CALL SETSMG(Z,100,3.)
      CALL GRIDG(Z,0,0,0,0)
C         PLOT AXIS AND LABEL
C         LABEL THE GRAPH, X AND Y AXES
      CALL SETSMG(Z,102,.005)
      CALL SETSMG(Z,103,.005)
      CALL LABELG(Z,0,.05,0,5.2)
      CALL LABELG(Z,1,.05,0,5.2)
C       GENERATE ELLIPSE AT ORIGIN
      DO 20 MM=1,NUPT
      IF(NUPT-1)305,306,305
  305 IF(NOPTS(MM) .LE. 0) GO TO 20
  306 IF(A(MM))2,3,2
    3 IF (B(MM))2,19,2
    2 A1=A(MM)*SCAL
      B1=B(MM)*SCAL
      A12=A1*A1
      B12=B1*B1
      IPLOT=(NOPLOT-1)/4
      PLOTNU=IPLOT
```

```
      AIN=A1/PLUTNU                                                     ELIPS    74
      I=1                                                               ELIPS    75
      XE(I)=-A1                                                         ELIPS    76
   10 YE(I)=SQRT(B12*(1.-XE(I)**2/A12))                                 ELIPS    77
      I=I+1                                                             ELIPS    78
      XE(I)=XE(I-1)+AIN                                                 ELIPS    79
      IF(I-IPLUT-1)10,11,11                                             ELIPS    80
   11 XE(1)=0.                                                          ELIPS    81
      YE(I)=B1                                                          ELIPS    82
      J=IPLUT+1                                                         ELIPS    83
      K=J+2*IPLUT                                                       ELIPS    84
      L=J+2*IPLUT                                                       ELIPS    85
   12 I=I-1                                                             ELIPS    86
      J=J+1                                                             ELIPS    87
      L=L+1                                                             ELIPS    88
      K=K-1                                                             ELIPS    89
      XE(J)=-XE(1)                                                      ELIPS    90
      YE(J)=YE(1)                                                       ELIPS    91
      XE(K)=-XE(I)                                                      ELIPS    92
      YE(K)=-YE(1)                                                      ELIPS    93
      XE(L)=XE(I)                                                       ELIPS    94
      YE(L)=-YE(I)                                                      ELIPS    95
      IF(I-1)13,13,12                                                   ELIPS    96
   13 IIII=3*IPLUT+1                                                    ELIPS    97
      XE(IIII)=0.                                                       ELIPS    98
      YE(IIII)=-B1                                                      ELIPS    99
C       ROTATE AND TRANSLATE                                            ELIPS   100
      SINAL=SIN(ALPHA(MM))                                              ELIPS   101
      COSAL=COS(ALPHA(MM))                                              ELIPS   102
      DO 15 I=1,NOPLOT                                                  ELIPS   103
            IF(IP. LE . 0) GO TO 301                                    ELIPS   104
            XF(I) = COSAL*XE(I) -SINAL*YE(I) + E(MM,1)                  ELIPS   105
            YF(I) = SINAL*XE(I) + COSAL*YE(I) + E(MM,2)                 ELIPS   106
            GO TO 15                                                    ELIPS   107
  301       XF(1) = COSAL*XE(I) - SINAL*YE(I) + X(MM)                   ELIPS   108
            YF(I) = SINAL*XE(I) + COSAL*YE(1) + Y(MM)                   ELIPS   109
   15 CONTINUE                                                          ELIPS   110
C       PLOT X AND Y                                                    ELIPS   111
      NPTS = NOPLOT + 1                                                 ELIPS   112
      XF(NPTS)=XF(1)                                                    ELIPS   113
      YF(NPTS)=YF(1)                                                    ELIPS   114
      CALL LINESG(Z,NPTS,XF,YF)                                         ELIPS   115
      CALL SETSMG(Z,84,1H+)                                             ELIPS   116
      IF(IP.LE.0) GO TO 302                                             ELIPS   117
      CALL POINTG(Z,1,E(MM,1),E(MM,2))                                  ELIPS   118
      CALL SETSMG(Z,84,1H.)                                             ELIPS   119
      GO TO 200                                                         ELIPS   120
  302 CALL POINTG(Z,1,X(MM),Y(MM))                                      ELIPS   121
      CALL SETSMG(Z,84,1H.)                                             ELIPS   122
      GO TO 200                                                         ELIPS   123
C       CHANGE PLOTTING SYMBOL                                          ELIPS   124
   19 CALL SETSMG(Z,84,1H*)                                             ELIPS   125
      CALL POINTG(Z,1,X(MM),Y(MM))                                      ELIPS   126
C       RESET PLOTTING CHAR TO A POINT                                  ELIPS   127
      CALL SETSMG(Z,84,1H.)                                             ELIPS   128
C       PRINT POINT NO                                                  ELIPS   129
  200 IF(NUPT-1)304,20,304                                              ELIPS   130
  304 IF(IP.LE.0) GO TO 303                                             ELIPS   131
      CALL NUMBRG(Z,E(MM,1),E(MM,2),4,IPT(MM))                          ELIPS   132
      GO TO 20                                                          ELIPS   133
  303 CALL NUMBRG(Z,X(MM),Y(MM),4,IPT(MM))                              ELIPS   134
   20 CONTINUE                                                          ELIPS   135
C     LABELS AND SCALE                                                  ELIPS   136
      CALL LEGNDG(Z,-.01,.12,2,2H+Y)                                    ELIPS   137
      CALL LEGNDG(Z,.14,-.005,2,2H+X)                                   ELIPS   138
      GO TO (250,251,252,253), ICONF                                    ELIPS   139
  250 CONLIM=1.                                                         ELIPS   140
      IPERC=39                                                          ELIPS   141
      GO TO 254                                                         ELIPS   142
  251 CONLIM=1.18                                                       ELIPS   143
      IPERC=50                                                          ELIPS   144
      GO TO 254                                                         ELIPS   145
```

```
  252 CONLIM=2.45                                                       ELIPS     146
      IPERC=95                                                          ELIPS     147
      GO TO 254                                                         ELIPS     148
  253 CONLIM=3.03                                                       ELIPS     149
      IPERC=99                                                          ELIPS     150
  254 XE(1)=.02                                                         ELIPS     151
      XE(2)=.02+SCAL*.000001/CONLIM                                     ELIPS     152
      YE(1)=-.12                                                        ELIPS     153
      YE(2)=-.12                                                        ELIPS     154
C     CHANGE PLOTTING SYMBOL                                            ELIPS     155
      CALL SETSMG(Z,84,1H1)                                             ELIPS     156
      CALL POINTG(Z,2,XE,YE)                                            ELIPS     157
      CALL SETSMG(Z,84,1H.)                                             ELIPS     158
      CALL LINESG(Z,2,XE,YE)                                            ELIPS     159
      CALL LEGNDG(Z,-.20,-.13,21,21HELLIPSE REPRESENTS A )              ELIPS     160
      CALL NUMBRG(Z,-.12,-.13,2,IPERC)                                  ELIPS     161
      CALL LEGNDG(Z,-.11,-.13,24,24H PERCENT CONFIDENCE AREA)           ELIPS     162
      CALL LEGNDG(Z,.02,-.13,8,8H1 MICRON)                              ELIPS     163
      CALL LEGNDG(Z,.08,-.13,15,15H* SX AND SY = 0)                     ELIPS     164
      CALL PAGEG(Z,0,0,1)                                               ELIPS     165
      RETURN                                                            ELIPS     166
      END                                                               ELIPS     167
```

```
      SUBROUTINE EXACT (X, Y, XM, YM, SX, SY, ALPHA, VSX, VSY, VAL,
     1                  CSXAL, CSYAL, SIG02, N, CSXSY, (SVXM2L, TSVYM2L)
      COMMON /AFFINE/ VX, VY, VXM,
     1               VYM, THETA ,VTH, SUMVX, SUMVY, SUMVXM, SUMVYM
     2               , SUMVX2, SUMVY2, SUMVXM2, SUMVYM2
      DIMENSION X(1), Y(1), XM(1), YM(1), VX(25), VY(25),VXM(25),VYM(25)
     1          ,QX(4,4), GQX(4,4), G(4,4), QP(4,4)
C
C      COMPUTE EXACT LINEAR LEAST SQUARES SOLUTION OF THE GENERAL AFFINE
C          LINEAR TRANSFORMATION.
C
      SG = SH = SI = SJ = SK = SL = SM = SO = SP = SQ = SR = 0.0
      N = 0
      DO 1 I = 1,25
         IF( X(I) .EQ. 0. .AND. Y(I) .EQ. 0. ) GO TO 1
         N = N + 1
         SG = SG + XM(I)
         SH = SH + YM(I)
         SI = SI + X(I)
         SJ = SJ + Y(I)
         SK = SK + X(I)*XM(I)
         SL = SL + Y(I)*XM(I)
         SM = SM + Y(I)*YM(I)
         SO = SO + X(I)*YM(I)
         SP = SP + XM(I)**2
         SQ = SQ + YM(I)**2
         SR = SR + XM(I)*YM(I)
    1 CONTINUE
      XBAR = SI/N
      YBAR = SJ/N
      XMBAR = SG/N
      YMBAR = SH/N
      H2 = SH*SH
      QN = SQ*N
      RN = SR*N
      GH = SG*SH
      HR = SH*SR
      GQ = SG*SQ
      PN = SP*N
      G2 = SG*SG
      HP = SH*SP
      GR = SG*SR
      PQ = SP*SQ
      R2 = SR*SR
      Q11 = QN - H2
      Q12 = GH - RN
      Q13 = HR - GQ
      Q22 = PN - G2
      Q23 = GR - HP
      Q33 = PQ - R2
      DET = 1./ (N*Q33 + 2.*GR*SH - H2*SP - G2*SQ)
      A = DET*(Q11*SK + Q12*SO + Q13*SI)
      B = DET*(Q12*SK + Q22*SO + Q23*SI)
      C = DET*(Q11*SL + Q12*SM + Q13*SJ)
      D = DET*(Q12*SL + Q22*SM + Q23*SJ)
      E = XBAR - A*XMBAR - B*YMBAR
      F = YBAR - C*XMBAR - D*YMBAR
C
C      COMPUTE COMPARATOR CALIBRATION LINEAR CORRECTION PARAMETERS
      CDD = C/D
      ADMBC = A*D - B*C
      C2PD2 = C*C + D*D
      A2PB2 = A*A + B*B
      ACPBD = A*C + B*D
      ADMBC2 = ADMBC*ADMBC
      C2PD2RT = SQRT(C2PD2)
      A2PB2RT = SQRT(A2PB2)
      AOB = A/B
      THETA = ATAN2(C,D)
      ALPHA = ATAN2(-ACPBD, ADMBC)
      SX = C2PD2RT/ADMBC
      SY = A2PB2RT/ADMBC
```

```
C                                                                       EXACT
C      COMPUTE PARAMETERS FOR TRANSFORMATION FROM MEASUREMENT SYSTEM TO  EXACT
C         MASTER GRID SYSTEM.                                           EXACT
C                                                                       EXACT
      DETM = -1./ADMBC                                                  EXACT
      R = DETM*D                                                        EXACT
      S =-DETM*B                                                        EXACT
      T =-DETM*C                                                        EXACT
      U = DETM*A                                                        EXACT
C                                                                       EXACT
C      COMPUTE RESIDUALS AFTER ADJUSTMENT (IN BOTH COORDINATE SYSTEMS)  EXACT
C                                                                       EXACT
      SUMVX = SUMVY = SUMVX2 = SUMVY2 = SUMVXM = SUMVYM = SUMVXM2 =     EXACT
     1    SUMVYM2 = 0.0                                                 EXACT
      DO 3 I = 1,25                                                     EXACT
         VX(I) = VY(I) = VXM(I) = VYM(I) = 0.0                          EXACT
    3 CONTINUE                                                          EXACT
      DO 2 I = 1,25                                                     EXACT
         IF (X(I) .EQ. 0. .AND. Y(I) .EQ. 0. ) GO TO 2                  EXACT
         VX(I) = A*XM(I) + B*YM(I) + E - X(I)                           EXACT
         VY(I) = C*XM(I) + D*YM(I) + F - Y(I)                           EXACT
         VXM(I) = R*VX(I) + S*VY(I)                                     EXACT
         VYM(I) = T*VX(I) + U*VY(I)                                     EXACT
         SUMVX = SUMVX + VX(I)                                          EXACT
         SUMVY = SUMVY + VY(I)                                          EXACT
         SUMVX2 = SUMVX2 + VX(I)*VX(I)                                  EXACT
         SUMVY2 = SUMVY2 + VY(I)*VY(I)                                  EXACT
         SUMVXM = SUMVXM + VXM(I)                                       EXACT
         SUMVYM = SUMVYM + VYM(I)                                       EXACT
         SUMVXM2= SUMVXM2+ VXM(I)*VXM(I)                                EXACT
         SUMVYM2= SUMVYM2+ VYM(I)*VYM(I)                                EXACT
    2 CONTINUE                                                          EXACT
      TSVXM2L = TSVXM2L + SUMVXM2                                       EXACT
      TSVYM2L = TSVYM2L + SUMVYM2                                       EXACT
C                                                                       EXACT
C      COMPUTE ADJUSTMENT STATISTICS                                    EXACT
C                                                                       EXACT
      SIG02 =(SUMVX2 + SUMVY2)/(2*N - 6)                                EXACT
C                                                                       EXACT
      G(1,1) = -SX*D/ADMBC                                              EXACT
      G(1,2) = -G(1,1)*CDD                                              EXACT
      G(1,3) = D*ACPBD/(ADMBC2*C2PD2RT)                                 EXACT
      G(1,4) = -G(1,3)*CDD                                              EXACT
      G(2,1) = -B*ACPBD/(ADMBC2*A2PB2RT)                                EXACT
      G(2,2) = -G(2,1)*ADB                                              EXACT
      G(2,3) = B*SY/ADMBC                                               EXACT
      G(2,4) = -G(2,3)*ADB                                              EXACT
      G(3,1) = B/A2PB2                                                  EXACT
      G(3,2) = -A/A2PB2                                                 EXACT
      G(3,3) = - D/C2PD2                                                EXACT
      G(4,3) = -G(3,3)                                                  EXACT
      G(3,4) = C/C2PD2                                                  EXACT
      G(4,4) = -G(3,4)                                                  EXACT
      G(4,1) = G(4,2) = 0.0                                             EXACT
      QX(1,1) = QX(3,3) = DET*Q11                                       EXACT
      QX(2,2) = QX(4,4) = DET*Q22                                       EXACT
      QX(1,2) = QX(2,1) = QX(3,4) = QX(4,3) = DET*Q12                   EXACT
      QX(1,3) = QX(1,4) = QX(2,3) = QX(2,4) = QX(3,1) = QX(3,2) =       EXACT
     1          QX(4,1) = QX(4,2) = 0.0                                 EXACT
      DO 4 I = 1,4                                                      EXACT
      DO 4 J = 1,4                                                      EXACT
         GQX(I,J) = 0.0                                                 EXACT
      DO 4 K = 1,4                                                      EXACT
         GQX(I,J) = GQX(I,J) + G(I,K)*QX(K,J)                           EXACT
    4 CONTINUE                                                          EXACT
      DO 5 I = 1,4                                                      EXACT
      DO 5 J = 1,4                                                      EXACT
         QP(I,J) = 0.0                                                  EXACT
      DO 5 K = 1,4                                                      EXACT
         QP(I,J) = QP(I,J) + GQX(I,K)*G(J,K)                            EXACT
    5 CONTINUE                                                          EXACT
```

```
C                                                                       EXACT     145
      VSX = SIG02*QP(1,1)                                               EXACT     146
      VSY = SIG02*QP(2,2)                                               EXACT     147
      VAL = SIG02*QP(3,3)                                               EXACT     148
      VTH = SIG02*QP(4,4)                                               EXACT     149
      CSXAL = SIG02*QP(1,3)                                             EXACT     150
      CSYAL = SIG02*QP(2,3)                                             EXACT     151
      CSXSY = SIG02*QP(1,2)                                             EXACT     152
      RETURN                                                            EXACT     153
      EN D                                                              EXACT     154
```

```
      SUBROUTINE VSPRINT (X, Y, N, ICASCOL, UK)                         VSPRINT     2
C                                                                       VSPRINT     3
C      THIS SUBROUTINE PRINTS RESIDUALS FOR TWO SYSTEMS - X AND Y COLUMN VSPRINT    4
C                                                        FOR EACH.      VSPRINT     5
C                                                                       VSPRINT     6
      COMMON /AFFINE/ VX, VY, VXM, VYM, BDUM(2), SUMVX, SUMVY,          VSPRINT     7
     1                SUMVXM, SUMVYM, SUMVX2, SUMVY2, SUMVXM2, SUMVYM2  VSPRINT     8
      DIMENSION VX(25), VY(25), VXM(25), VYM(25), X(1), Y(1)            VSPRINT     9
      DF = N - UK                                                       VSPRINT    10
      SIGMAX = SQRT(SUMVX2/DF)                                          VSPRINT    11
      SIGMAY = SQRT(SUMVY2/DF)                                          VSPRINT    12
      SIGMAXM = SQRT(SUMVXM2/DF)                                        VSPRINT    13
      SIGMAYM = SQRT(SUMVYM2/DF)                                        VSPRINT    14
      PRINT 100                                                         VSPRINT    15
      PRINT 101, (ICASCOL, I, X(I), Y(I), VX(I), VY(I), I, VXM(I),      VSPRINT    16
     1            VYM(I), I = 1,25 )                                    VSPRINT    17
      PRINT 102, SUMVX2, SUMVY2, SUMVXM2, SUMVYM2, N, SIGMAX, SIGMAY,   VSPRINT    18
     1       SIGMAXM, SIGMAYM                                           VSPRINT    19
  100 FORMAT(//,12X,*RESIDUALS AFTER ADJUSTMENT - IN MEASUREMENT SYSTEM* VSPRINT   20
     1        15X, *RESIDUALS AFTER ADJUSTMENT - IN MASTER GRID SYSTEM*, VSPRINT   21
     2        /,8X,*PT. NO.*,7X,*X ADJ*, 8X,*Y ADJ*, 10X, 2HVX,12X,2HVY, VSPRINT   22
     3        24X, *NO.*, 7X, 2HVX, 13X, 2HVY )                         VSPRINT    23
  101 FORMAT ( 8X, I5, I2, 2(2X, 6PF11.2), 2X, 6PF10.3, 4X, 6PF10.3,    VSPRINT    24
     1        22X, I2, 2X, 6PF10.3, 5X, 6PF10.3 )                       VSPRINT    25
  102 FORMAT(/,33X, *SUM VV =*,12PF13.4, 1X,12PF13.4, 15X, *SUM VV =*,  VSPRINT    26
     1        12PF13.4, 2X,12PF13.4, /, 11X, *NO. OF POINTS = *, I2, 3X, VSPRINT   27
     2        *  SIGMA =*, 6PF13.4, 1X,                                 VSPRINT    28
     3        6PF13.4, 14X, *  SIGMA =*, 6PF13.4, 2X, 6PF13.4 )         VSPRINT    29
      RETURN                                                            VSPRINT    30
      END                                                               VSPRINT    31
```

SUBROUTINE VSPRINT

Subroutine VSPRINT is a utility routine that prints the output data for each of the three types of least-squares adjustments, i.e., free adjustment, linear parameter adjustment, and nonlinear parameter adjustment. The adjusted observations and the residuals are printed in both coordinate systems.

All measurement and grid data essential for a complete calibration are stored in the ***B***(25, 52) array. A summary of the ***B*** array storage allocations is:

Rows 1–25 = Grid point number.

Columns 1–2 = X and Y master grid coordinates used.

Columns 3–42 = $\bar{x}$ and $\bar{y}$ mean measurements for each case, starting with case 1 in cols. 3, 4. Storage space allows for 20 cases.

Columns 43–44 = Sum of residuals in x, y measurement system at each grid point after linear adjustment.

Columns 45–46 = Sum of residuals in X, Y grid system at each grid point after linear adjustment.

Columns 47–48 = Sum of residuals in x, y measurement system at each grid point after nonlinear adjustment.

Columns 49–50 = Sum of residuals in X, Y grid system at each grid point after nonlinear adjustment.

Column 51 = Total number of meaned observations of each grid point from all cases (storage is according to grid point number).

Column 52 = Total number of meaned observations of each grid point from all cases (storage is according to measurement point locations as determined by Subroutine COMPVS.

```
      SUBROUTINE INVSYM(A,NRA,NCA,NR,NC,D,S)                            INVSYM     2
C      11/15/70 HANSEN                                                  INVSYM     3
      DIMENSION A(NRA,NCA)                                              INVSYM     4
C                                                                       INVSYM     5
C THIS SUBROUTINE INVERTS SYMMETRIC MATRICES. ALL OPERATIONS ARE        INVSYM     6
C CONFINED TO THE UPPER TRIANGULAR PART OF THE GIVEN MATRIX UNTIL THE   INVSYM     7
C INVERSE HAS BEEN OBTAINED. AS A FINAL STEP, SYMMETRY IS RESTORED BY   INVSYM     8
C SETTING ALL ELEMENTS A(J,I), J.GT.I, = TO ELEMENTS A(I,J).            INVSYM     9
C     THE ARGUMENTS ARE...                                              INVSYM    10
C     A....NAME OF ARRAY CONTAINING MATRIX TO BE INVERTED. UPON EXIT, A INVSYM    11
C          CONTAINS THE INVERSE OF THE INPUT MATRIX.                    INVSYM    12
C     D....UPON EXIT, D IS THE DETERMINANT OF THE INPUT MATRIX.         INVSYM    13
C     S....SINGULARITY INDICATOR. UPON EXIT, IF S = 1., THEN THE GIVEN  INVSYM    14
C          MATRIX WAS SINGULAR. IF S=0., MATRIX WAS NON-SINGULAR.       INVSYM    15
C     N....PORTION OF ARRAY A CONTAINING MATRIX TO BE INVERTED (FIRST   INVSYM    16
C          N ROWS AND COLUMNS)  A(NXN).                                 INVSYM    17
C     N1...DIMENSION OF ARRAY A IN CALLING ROUTINE  A(N1XN1), N1.GE.N   INVSYM    18
      D = 1.                                                            INVSYM    19
      S = 0.                                                            INVSYM    20
      DO 10 I = 1,NR                                                    INVSYM    21
      X = A(I,I)                                                        INVSYM    22
      D = D*X                                                           INVSYM    23
      IF(X.GT.0.) GO TO 1                                               INVSYM    24
      S = 1.                                                            INVSYM    25
      RETURN                                                            INVSYM    26
    1 A(I,I) = 1.                                                       INVSYM    27
      DO 2 J = I,NC                                                     INVSYM    28
    2 A(I,J) = A(I,J)/X                                                 INVSYM    29
      DO 10 K = 1,NR                                                    INVSYM    30
      IF(K-I)3,10,4                                                     INVSYM    31
    3 X = A(K,I)                                                        INVSYM    32
      GO TO 5                                                           INVSYM    33
    4 X = A(I,K)/A(I,I)                                                 INVSYM    34
    5 DO 9 J = K ,NC                                                    INVSYM    35
      IF(I-J)6,7,8                                                      INVSYM    36
    6 A(K,J) = A(K,J)-X*A(I,J)                                          INVSYM    37
      GO TO 9                                                           INVSYM    38
    7 A(K,J) = -X*A(I,I)                                                INVSYM    39
      GO TO 9                                                           INVSYM    40
    8 A(K,J) = A(K,J)+X*A(J,I)*A(I,I)                                   INVSYM    41
    9 CONTINUE                                                          INVSYM    42
   10 CONTINUE                                                          INVSYM    43
      DO 11 I= 2,NR                                                     INVSYM    44
      DO 11 J = I,NR                                                    INVSYM    45
   11 A(J,I-1) = A(I-1,J)                                               INVSYM    46
      RETURN                                                            INVSYM    47
      END                                                               INVSYM    48
```

```
      SUBROUTINE PLOT (NOCOMP,GRIDNO,DATE,A,TRANS,ICSNO)                PLOT      2
      DIMENSION PVX(5,5), PVY(5,5), PX(5,5), PY(5,5), G(9,2), VX(9),    PLOT      3
     1          VY(9), A(9,11)                                          PLOT      4
      INTEGER GRIDNO                                                    PLOT      5
      RESID(G,M,X,Y)= (G(9,M)+X*(G(7,M)+X*(G(5,M)+X*(G(3,M)+X*G(1,M))))  PLOT      6
     1                        +Y*(G(8,M)+Y*(G(6,M)+Y*(G(4,M)+Y*G(2,M))))) PLOT      7
      DO 1 I = 1,9                                                      PLOT      8
        DO 1 J = 1,2                                                    PLOT      9
            G(I,J) = A(I, J+9)                                          PLOT     10
    1 CONTINUE                                                          PLOT     11
      X = .125 + TRANS                                                  PLOT     12
      IF (NOCOMP.LT.10)PRINT 104,NOCOMP                                 PLOT     13
      IF (NOCOMP.EQ.10)PRINT 105                                        PLOT     14
      IF (NOCOMP.EQ.11)PRINT 106                                        PLOT     15
      IF (NOCOMP.GT.11) PRINT 107                                       PLOT     16
      TP = TRANS + .1                                                   PLOT     17
      TM = TRANS - .1                                                   PLOT     18
      PRINT 108, TP, TP, TP, TM                                         PLOT     19
      L=0                                                               PLOT     20
      DO 21 J = 1,9                                                     PLOT     21
      K=0                                                               PLOT     22
      X=X-.025                                                          PLOT     23
          IF (MOD(J,2)) 4,3,4                                           PLOT     24
    3     Y = .150 + TRANS                                              PLOT     25
      DO 30 I=1,5                                                       PLOT     26
      Y=Y-.050                                                          PLOT     27
          VX(I) = RESID(G,1,X,Y)                                        PLOT     28
          VY(I) = RESID(G,2,X,Y)                                        PLOT     29
   30 CONTINUE                                                          PLOT     30
      PRINT 100, (VX(IK), IK = 1,5)                                     PLOT     31
      PRINT 101,(VY(IK),IK=1,5)                                         PLOT     32
      GO TO 21                                                          PLOT     33
    4 L=L+1                                                             PLOT     34
          Y = .125 + TRANS                                              PLOT     35
      DO 20 I=1,9                                                       PLOT     36
      Y=Y-.025                                                          PLOT     37
          VX(I) = RESID(G,1,X,Y)                                        PLOT     38
          VY(I) = RESID(G,2,X,Y)                                        PLOT     39
      IF(MOD(I,2))50,20,50                                              PLOT     40
   50 K=K+1                                                             PLOT     41
      PVX(L,K)=VX(I)                                                    PLOT     42
      PVY(L,K)=VY(I)                                                    PLOT     43
              PX(L,K) = X - TRANS                                       PLOT     44
              PY(L,K) = Y - TRANS                                       PLOT     45
   20 CONTINUE                                                          PLOT     46
          IF(J .EQ. 9) PRINT 108,TM,TM,TP,TM                            PLOT     47
      PRINT 102, (VX(IK) , IK = 1,9)                                    PLOT     48
      PRINT 103,(VY(IK),IK=1,9)                                         PLOT     49
   21 CONTINUE                                                          PLOT     50
      CALL GRAFT (PVX,PVY,PX,PY,NOCOMP,GRIDNO,DATE,ICSNO,TRANS)         PLOT     51
      L=L+1                                                             PLOT     52
  100 FORMAT (1X,4(6PF10.3,20X),6PF10.3)                                PLOT     53
  101 FORMAT (1X, 4(6PF10.3, 20X), 6PF10.3, ////,)                      PLOT     54
  102 FORMAT (1X,8(6PF10.3,5X),6PF10.3)                                 PLOT     55
  103 FORMAT (1X, 8(6PF10.3, 5X), 6PF10.3, ///// )                      PLOT     56
  104 FORMAT (46X,11HCOMPARATOR ,I2,31H-PLOT OF NON LINEAR CORRECTIONS ) PLOT     57
  105 FORMAT (44X,48HCOMPARATOR STK-1A PLOT OF NON LINEAR CORRECTIONS  ) PLOT     58
  106 FORMAT (44X,48HCOMPARATOR STK-1B PLOT OF NON LINEAR CORRECTIONS  ) PLOT     59
  107 FORMAT (46X, 45HCOMPARATOR PSK PLOT OF NON LINEAR CORRECTIONS )   PLOT     60
  108 FORMAT (1X, *X =*, F9.6, *M.*, 106X,*X =*,F9.6,*M.*, /, 1X,*Y =*, PLOT     61
     1        F9.6, *M.*, 106X, *Y =*, F9.6, *M.* )                     PLOT     62
      RETURN                                                            PLOT     63
      EN D                                                              PLOT     64
```

```
      SUBROUTINE GRAFT(PVX,PVY,PX,PY,NOCOMP,GRIDNO,DATE,ICSNO,TRAN)
      DIMENSION PVX(5,5),PVY(5,5),PX(5,5),PY(5,5),XO(5),YO(5)
      DIMENSION XY(5,5,2)
      INTEGER GRIDNO
      COMMON/PL4060/Z(200)
C        INITIALIZE IGS
      CALL SETSMG(Z,55,0.)
      CALL SETSMG(Z,93,0.)
      CALL SETSMG(Z,90,0.)
      CALL SETSMG(Z,30,.5)
      CALL SUBJEG(Z,-.175,-.129,.175,.150)
      CALL OBJCTG(Z,.13,.1,1.259,1.0)
      CALL SETSMG(Z,104,-1.5)
      CALL SETSMG(Z,105,-2.)
C        TITLE THE GRAPH
      CALL LEGNDG(Z,-.17,.145,18,18HCOMPARATOR NUMBER )
      CALL NUMBRG(Z,-.115,.145,2,NOCOMP)
      CALL LEGNDG(Z,-.065,.145,24,24HCALIBRATION GRID NUMBER )
      CALL NUMBRG(Z,0.02,.145,4,GRIDNO)
      CALL LEGNDG(Z,.062,.145,20,20HDATE OF MEASUREMENT )
      CALL LEGNDG(Z,.125,.145,8,DATE)
      CALL LEGNDG(Z,-.08,.14,30,30HPLOT OF NON LINEAR CORRECTIONS)
C        NEXT DRAW THE GRID
      CALL SETSMG(Z,100,3.)
      CALL GRIDG(Z,0.,0.,0.,0.)
C        LABEL THE GRAPH, X AND Y AXES
      CALL SETSMG(Z,102,.005)
      CALL SETSMG(Z,103,.003)
      CALL LABELG(Z,0,.02,0,5.2)
      CALL LABELG(Z,1,.02,0,5.2)
      DO 1 J=1,5
      DO 2 K=1,5
      XY(J,K,1) = PX(J,K) + 10000. * PVX(J,K)
      XY(J,K,2) = PY(J,K) + 10000. * PVY(J,K)
    2 CONTINUE
    1 CONTINUE
      NPTS = 1
      CALL SETSMG(Z,84,1H+)
      DO 10 J=1,5
      DO 5 K=1,5
      CALL POINTG(Z,NPTS,PX(J,K),PY(J,K))
    5 CONTINUE
   10 CONTINUE
      CALL SETSMG(Z,84,1H.)
      NPTS = 5
      DO 30 J=1,5
      DO 20 K=1,5
      XO(K) = XY(J,K,1)
      YO(K) = XY(J,K,2)
   20 CONTINUE
      CALL LINESG(Z,NPTS,XO,YO)
      DO 25 L=1,5
      XO(L) = XY(L,J,1)
      YO(L) = XY(L,J,2)
   25 CONTINUE
      CALL LINESG(Z,NPTS,XO,YO)
   30 CONTINUE
```

```
      CALL LEGNDG(Z,-.005,.135,2,2H+Y)                                  GRAFT     59
      IF(ICSNO)40,41,40                                                 GRAFT     60
   40 CALL LEGNDG(Z,-0.17,0.14,12,12H CASE NUMBER)                      GRAFT     61
      CALL NUMBRG(Z,-0.1275,0.14,3,ICSNO)                               GRAFT     62
   41 CALL LEGNDG(Z,-0.17,-0.12,28,28H BOTH COORDINATES TRANSLATED)     GRAFT     63
      CALL NUMBRG(Z,-0.08,-0.12,8,6,TRAN)                               GRAFT     64
      NPTS = 2                                                          GRAFT     65
      XO(1) = .00                                                       GRAFT     66
      XO(2) = .01                                                       GRAFT     67
      YO(1) = -.12                                                      GRAFT     68
      YO(2) = -.12                                                      GRAFT     69
      CALL SETSMG (Z,84,1HI)                                            GRAFT     70
      CALL POINTG (Z,NPTS,XO,YO)                                        GRAFT     71
      CALL SETSMG (Z,84,1H.)                                            GRAFT     72
      CALL LINESG (Z,NPTS,XO,YO)                                        GRAFT     73
      CALL LEGNDG (Z,.02,-.12,8,8H1 MICRON)                             GRAFT     74
      CALL LEGNDG (Z,.14,.005,2,2H+X)                                   GRAFT     75
      CALL PAGEG(Z,0,0,1)                                               GRAFT     76
      RETURN                                                            GRAFT     77
      END                                                               GRAFT     78
```

SUBROUTINE FIDFIT2

This routine computes the rotation and translation between two coordinate systems without permitting a scale change. Derivation of the efficient programmed least squares solution using the notation developed in The Exact Least-Squares Solution of the General Linear Transformation is as follows:

The mathematical model is:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = k \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix},$$

where

k = a uniform overall scale factor,
θ = the rotation angle between the coordinate systems, and
c, d = translations in X and Y respectively between the systems.

Rewriting with linear parameters in least-squares form, it is

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} - \begin{bmatrix} X \\ Y \end{bmatrix}.$$

Observation equations are in form of $\boldsymbol{v} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{l}$, or

$$\begin{bmatrix} v_{X_i} \\ v_{Y_i} \\ \vdots \end{bmatrix} = \begin{bmatrix} x_i & y_i & 1 & 0 \\ y_i & -x_i & 0 & 1 \\ \vdots & & & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} - \begin{bmatrix} X_i \\ Y_i \\ \vdots \end{bmatrix}$$

Matrices of normal equations are

$$\mathbf{A}^T\mathbf{A} = \begin{bmatrix} (x_i^2+y_i^2) & 0 & (x_i) & (y_i) \\ 0 & (x_i^2+y_i^2) & (y_i) & -(x_i) \\ (x_i) & (y_i) & n & 0 \\ (y_i) & -(x_i) & 0 & n \end{bmatrix},$$

and

$$-\mathbf{A}^T\mathbf{l} = \begin{bmatrix} (x_iX_i)+(y_iY_i) \\ (y_iX_i)-(x_iY_i) \\ (X_i) \\ (Y_i) \end{bmatrix},$$

where quantities in parentheses represent the summation from $i=1$ to $i=n$, and n = number of observations (grid points).

The solution can be obtained more efficiently by eliminating c and d from the normal equations and solving them implicitly from a and b. That is

$$\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^T\boldsymbol{l}$$

The last two equations of this set are

$$a(x_i) + b(y_i) + cn = (Xi)$$

$$a(y_i) - b(x_i) + dn = (Yi).$$

Dividing by n and rearranging terms we obtain

$$c = \bar{X} - a\bar{x} - b\bar{y}$$

$$d = \bar{Y} - a\bar{y} + b\bar{x},$$

where

$$\bar{x} = \frac{x_i}{n}, \text{ etc.}$$

Solving for a and b:

Inversion of A^TA is best facilitated by partitioning into 2×2 matrices and utilizing the Householder identity

$$(A^TA)^{-1} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c} Q_{11} & Q_{12} \\ \hline Q_{21} & Q_{22} \end{array}\right] = \begin{bmatrix} A^{-1} - A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}, & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}A^{-1}B, & (D - CA^{-1}B)^{-1} \end{bmatrix},$$

by Householder identity

$$\left[\begin{array}{c|c} Q_{11} & Q_{12} \\ \hline Q_{21} & Q_{22} \end{array}\right] = \begin{bmatrix} (A - BD^{-1}C)^{-1}, & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1}, & D^{-1} + D^{-1}C(A - BD^{-1}C)^{-1}BD^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} (A - BD^{-1}C)^{-1}, & -Q_{11}BD^{-1} \\ -D^{-1}CQ_{11}, & D^{-1} + Q_{12}BD^{-1} \end{bmatrix}.$$

Solving for a and b requires only the first row inverse partitions of A^TA, i.e., Q_{11} and Q_{12}, where

$Q_{11} = (A - BD^{-1}C)^{-1}$ or

$$Q_{11} = \begin{bmatrix} \sum_{i=1}^{n} (x_i^2 + y_i^2) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2, & 0 \\ 0, & \sum_{i=1}^{n} (x_i^2 + y_i^2) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2 \end{bmatrix}^{-1}.$$

It can be shown that:

$$\sum_{i=1}^{n} (x_i^2 + y_i^2) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 + \sum_{i=1}^{n} (y_i - \bar{y})^2,$$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}.$$

Therefore:

$$Q_{11} = \frac{1}{\sum_{i=1}^{n} \{(x_i - \bar{x})^2 + (y_i - \bar{y})^2\}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where

$$\boldsymbol{R}=1\Big/\left\{\sum_{i=1}^{n}(x_i^2+y_i^2)-\bar{x}\sum_{i=1}^{n}(x_i)-\bar{y}\sum_{i=1}^{n}(y_i)\right\}.$$

$$\boldsymbol{Q}_{12}=-\boldsymbol{Q}_{11}\boldsymbol{B}\boldsymbol{D}^{-1}$$

$$\boldsymbol{Q}_{12}=-\boldsymbol{R}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\sum_{i=1}^{n}x_i & \sum_{i=1}^{n}y_i\\ \sum_{i=1}^{n}y_i & \sum_{i=1}^{n}x_i\end{bmatrix}\begin{bmatrix}\frac{1}{n} & 0\\ 0 & \frac{1}{n}\end{bmatrix}$$

$$=-\boldsymbol{R}\begin{bmatrix}\bar{x} & \bar{y}\\ \bar{y} & -\bar{x}\end{bmatrix}$$

The solution for a and b becomes

$$\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}\boldsymbol{R} & 0 & -\boldsymbol{R}\bar{x} & -\boldsymbol{R}\bar{y}\\ 0 & \boldsymbol{R} & -\boldsymbol{R}\bar{y} & +\boldsymbol{R}\bar{x}\end{bmatrix}\begin{bmatrix}\sum_{i=1}^{n}(x_iX_i)+\sum_{i=1}^{n}(y_iY_i)\\ \sum_{i=1}^{n}(y_iX_i)-\sum_{i=1}^{n}(x_iY_i)\\ \sum_{i=1}^{n}X_i\\ \sum_{i=1}^{n}Y_i\end{bmatrix}.$$

Collecting terms we obtain

$$a=\boldsymbol{R}\left\{\sum_{i=1}^{n}[(x_i-\bar{x})X_i]+\sum_{i=1}^{n}[(y_i-\bar{y})Y_i]\right\}$$

$$b=\boldsymbol{R}\left\{\sum_{i=1}^{n}[(y_i-\bar{y})X_i]-\sum_{i=1}^{n}[(x_i-\bar{x})Y_i]\right\}$$

It can be shown that

$$\sum_{i=1}^{n}\{(x_i-\bar{x})X_i\}=\sum_{i=1}^{n}\{(x_i-\bar{x})(X_i-\bar{X})\}=\sum_{i=1}^{n}(x_iX_i)-n\bar{x}\bar{X},$$

therefore,

$$a=\boldsymbol{R}\left\{\sum_{i=1}^{n}[(x_i-\bar{x})(X_i-\bar{X})]+\sum_{i=1}^{n}[(y_i-\bar{y})(Y_i-\bar{Y})]\right\},$$

and

$$b=\boldsymbol{R}\left\{\sum_{i=1}^{n}[(y_i-\bar{y})(X_i-\bar{X})]-\sum_{i=1}^{n}[(x_i-\bar{x})(Y_i-\bar{Y})]\right\}.$$

Thus

$$a = \boldsymbol{R}\left\{ \sum_{i=1}^{n} (x_i X_i) - n\bar{x}\bar{X} + \sum_{i=1}^{n} (y_i Y_i) - n\bar{y}\bar{Y} \right\},$$

$$b = \boldsymbol{R}\left\{ \sum_{i=1}^{n} (y_i X_i) - n\bar{y}\bar{X} - \sum_{i=1}^{n} (x_i Y_i) + n\bar{x}\bar{Y} \right\},$$

$c = \overline{\boldsymbol{X}} - a\bar{x} - b\bar{y}$, and

$d = \overline{\boldsymbol{Y}} - a\bar{y} + b\bar{x}$,

```
      SUBROUTINE FIDFIT2 (X, Y, XM, YM,          TSVXM2L, TSVYM2L)
      DIMENSION X(1), Y(1), XM(1), YM(1), VX(25), VY(25), VXM(25),
     1          VYM(25)
      COMMON /AFFINE/ VX, VY, VXM, VYM, BDUM(2)
     1                ,SUMVX, SUMVY, SUMVXM, SUMVYM
     2          , SUMVX2, SUMVY2, SUMVXM2, SUMVYM2, RT, N
      E = F = G = H = O = P = Q = V = W = 0.0
      N = 0
      DO 1 I = 1,25
         IF( X(I) .EQ. 0.0 .AND. Y(I) .EQ. 0.0 ) GO TO 1
         E = E + X(I)
         F = F + Y(I)
         G = G + X(I)*X(I) + Y(I)*Y(I)
         H = H + X(I)*XM(I)
         O = O + Y(I)*YM(I)
         P = P + X(I)*YM(I)
         Q = Q + Y(I)*XM(I)
         V = V + XM(I)
         W = W + YM(I)
         N = N + 1
    1 CONTINUE
      XSB = E/N
      YSB = F/N
      XMB = V/N
      YMB = W/N
      R = 1.0 / (G - XSB*E - YSB*F)
      A = R*(H - E*XMB + O - F*YMB)
      B = R*(Q - F*XMB - P + E*YMB)
      C = XMB - A*XSB - B*YSB
      D = YMB - A*YSB + B*XSB
      SCALE = 1.0 / SQRT(A*A + B*B)
C
C      COMPUTE RESIDUALS
C
      SUMVX = SUMVY = SUMVXM = SUMVYM = SUMVX2 = SUMVY2 = 0.0
      SUMVXM2 = SUMVYM2 = 0.0
      DO 3 I = 1,25
         VX(I) = VY(I) = VXM(I) = VYM(I) = 0.0
    3 CONTINUE
      DO 2 I = 1,25
         IF ( X(I) .EQ. 0.0 .AND. Y(I) .EQ. 0.0 ) GO TO 2
         VXM(I)=SCALE*( A*X(I) + B*Y(I) + C - XMB) + XMB - XM(I)
         VYM(I)=SCALE*(-B*X(I) + A*Y(I) + D - YMB) + YMB - YM(I)
         SUMVXM = SUMVXM + VXM(I)
         SUMVYM = SUMVYM + VYM(I)
         SUMVXM2 = SUMVXM2 + VXM(I)**2
         SUMVYM2 = SUMVYM2 + VYM(I)**2
          VX(I) = -A*VXM(I) + B*VYM(I)
          VY(I) = -B*VXM(I) - A*VYM(I)
         SUMVX = SUMVX + VX(I)
         SUMVY = SUMVY + VY(I)
         SUMVX2 = SUMVX2 + VX(I)*VX(I)
         SUMVY2 = SUMVY2 + VY(I)*VY(I)
    2 CONTINUE
      TSVXM2L = TSVXM2L + SUMVXM2
      TSVYM2L = TSVYM2L + SUMVYM2
      RETURN
      END
```

```
      SUBROUTINE COMPVS (KROTC, I, B, VX, VY, K, J)                     COMPVS     2
      DIMENSION B(25,52)                                                COMPVS     3
      L = K + 1                                                         COMPVS     4
      J = I                                                             COMPVS     5
      GO TO  (1, 2, 3, 4), KROTC                                        COMPVS     6
C                                                                       COMPVS     7
C      KAPPA ROTATION = 180 DEGREES.  GRID POINT 25 IS NEAREST ORIGIN.  COMPVS     8
C                                                                       COMPVS     9
    3 J = 26-I                                                          COMPVS    10
      GO TO 1                                                           COMPVS    11
C                                                                       COMPVS    12
C      KAPPA ROTATION =90 DEGREES.  GRID POINT 5 IS NEAREST ORIGIN.     COMPVS    13
C                                                                       COMPVS    14
    2 J = 26 - I                                                        COMPVS    15
C                                                                       COMPVS    16
C      KAPPA ROTATION = 270 DEGREES.  GRID POINT 21 IS NEAREST ORIGIN.  COMPVS    17
C                                                                       COMPVS    18
    4 J = 26 + (J-1) / 5 -5 *(J-((J-1)/5)*5)                            COMPVS    19
C                                                                       COMPVS    20
C      KAPPA ROTATION = 0 DEGREES.  GRID POINT 1 IS NEAREST ORIGIN.     COMPVS    21
C                                                                       COMPVS    22
    1 B(J,K) = B(J,K) + VX                                              COMPVS    23
      B(J,L) = B(J,L) + VY                                              COMPVS    24
      RETURN                                                            COMPVS    25
      EN D                                                              COMPVS    26
```

```
      SUBROUTINE AVGRES (K, B, ISIGMAG, NCASE, TSVXM2, TSVYM2, LCASCOL)  AVGRES      2
      DIMENSION VX(25), VY(25), VXM(25), VYM(25), B(25,52)               AVGRES      3
      COMMON /AFFINE/ VX, VY, VXM, VYM, BOUM(2), SUMVX, SUMVY, SUMVXM,   AVGRES      4
     1                SUMVYM, SUMVX2, SUMVY2, SUMVXM2, SUMVYM2, RT       AVGRES      5
      DATA (STKRT = 1HR)                                                 AVGRES      6
      SUMVX2 = SUMVY2 = 0.0                                              AVGRES      7
      SUMVX = SUMVY = SUMVXM2 = SUMVYM2 = SUMVXM = SUMVYM = 0.0          AVGRES      8
      K1 = K + 1                                                         AVGRES      9
      K2 = K + 2                                                         AVGRES     10
      K3 = K + 3                                                         AVGRES     11
      N = 0                                                              AVGRES     12
      M = 0                                                              AVGRES     13
      DO 1 I = 1,25                                                      AVGRES     14
          VX(I) = VY(I) = VXM(I) = VYM(I) = 0.0                          AVGRES     15
          IF (B(I,51) .EQ. 0.0) GO TO 1                                  AVGRES     16
          IF (B(I,51) .EQ. 0.0) GO TO 1                                  AVGRES     17
          IF (B(I,52) .EQ. 0.0) GO TO 4                                  AVGRES     18
          M = M + 1                                                      AVGRES     19
          VX(I) = B(I,K)/B(I,52)                                         AVGRES     20
          VY(I) = B(I,K1)/B(I,52)                                        AVGRES     21
          SUMVX = SUMVX + VX(I)                                          AVGRES     22
          SUMVY = SUMVY + VY(I)                                          AVGRES     23
          SUMVX2 = SUMVX2 + VX(I)*VX(I)                                  AVGRES     24
          SUMVY2 = SUMVY2 + VY(I)*VY(I)                                  AVGRES     25
    4     N = N + 1                                                      AVGRES     26
          VXM(I) = B(I,K2)/B(I,51)                                       AVGRES     27
          VYM(I) = B(I,K3)/B(I,51)                                       AVGRES     28
          SUMVXM = SUMVXM + VXM(I)                                       AVGRES     29
          SUMVYM = SUMVYM + VYM(I)                                       AVGRES     30
          SUMVXM2 = SUMVXM2 + VXM(I)*VXM(I)                              AVGRES     31
          SUMVYM2 = SUMVYM2 + VYM(I)*VYM(I)                              AVGRES     32
    1 CONTINUE                                                           AVGRES     33
      CALL VSPRINT (B(1,LCASCOL), B(1,LCASCOL+1), 25, 0, 1.)             AVGRES     34
      SIGMAS = SQRT( (SUMVX2 + SUMVY2)/(M*2-1) )                         AVGRES     35
      SIGMAM = SQRT( (SUMVXM2 + SUMVYM2) / (2*N - 1) )                   AVGRES     36
      PRINT 100, SIGMAS, SIGMAM                                          AVGRES     37
      IF (STKRT .EQ. RT) GO TO 3                                         AVGRES     38
      DF = N*NCASE*(NCASE-1)                                             AVGRES     39
      IF (DF .LE. 0.0) GO TO 3                                           B030673     1
      SIGMAXR = SQRT( (TSVXM2 - NCASE*SUMVXM2)/DF)                       AVGRES     40
      SIGMAYR = SQRT( (TSVYM2 - NCASE*SUMVYM2)/DF)                       AVGRES     41
      PRINT 102                                                          AVGRES     42
      PRINT 101, SIGMAXR, SIGMAYR                                        AVGRES     43
      SIGMAG = 0.0                                                       AVGRES     44
      SIGMAG2 = SIGMAM*SIGMAM - ((SIGMAXR**2 + SIGMAYR**2)/2.)           AVGRES     45
      IF (SIGMAG2 .LE. 0.0) GO TO 2                                      AVGRES     46
      SIGMAG = SQRT(SIGMAG2)                                             AVGRES     47
    2 PRINT 103, SIGMAG                                                  AVGRES     48
      ISIGMAG = SIGMAG*1.E+9                                             AVGRES     49
      RETURN                                                             AVGRES     50
    3 ISIGMAG = 0                                                        AVGRES     51
  100 FORMAT ( 2(29X, *STANDARD DEVIATION = *, 6PF8.4) )                 AVGRES     52
  101 FORMAT (84X, *SIGMA =*, 6PF13.4, 2X, 6PF13.4)                      AVGRES     53
  102 FORMAT (/, 78X, *RANDOM ERRORS* )                                  B030673     2
  103 FORMAT (/, 77X, *UNBIASED GRID STANDARD ERROR = *, 6PF8.4 )        AVGRES     55
      RETURN                                                             AVGRES     56
      END                                                                AVGRES     57
```

```
      SUBROUTINE LINLIN(XA,YA,XB,YB,XV,YV,XW,YW)                        LINLIN     2
C 02/16/71                                                              LINLIN     3
C COMPUTE THE INTERSECTION OF 2 LINES GIVEN THE ENDPOINT COORDINATES    LINLIN     4
      X1=XA-XB                                                          LINLIN     5
      Y1=YA-YB                                                          LINLIN     6
      X2=XV-XW                                                          LINLIN     7
      Y2=YV-YW                                                          LINLIN     8
      IF (Y1.EQ.0.) GO TO 103                                           LINLIN     9
      IF (Y2.EQ.0.) GO TO 104                                           LINLIN    10
C GENERAL CASE                                                          LINLIN    11
      S1=X1/Y1                                                          LINLIN    12
      S2=X2/Y2                                                          LINLIN    13
      IF (S1-S2) 101,102,101                                            LINLIN    14
  101 YA=(XW-S2*YW-XB+S1*YB)/(S1-S2)                                    LINLIN    15
      XA=S1*(YA-YB)+XB                                                  LINLIN    16
      RETURN                                                            LINLIN    17
C PARALLEL LINES                                                        LINLIN    18
  102 PRINT 801                                                         LINLIN    19
      RETURN                                                            LINLIN    20
C SPECIAL CASES                                                         LINLIN    21
  103 IF (Y2.EQ.0.) GO TO 102                                           LINLIN    22
      XA=(YA-YW)*X2/Y2+XW                                               LINLIN    23
      RETURN                                                            LINLIN    24
  104 XA=(YV-YB)*X1/Y1+XB                                               LINLIN    25
      YA=YV                                                             LINLIN    26
      RETURN                                                            LINLIN    27
  801 FORMAT (//1X,34HNO INTERSECTION FOR PARALLEL LINES//)             LINLIN    28
      END                                                               LINLIN    29
```

```
      SUBROUTINE MICRON (KASE,LEG,IPT,X,Y)                              MICRON     2
      DIMENSION MX(2),IX(2),LX(2)                                       MICRON     3
      READ 1,MX(1),MX(2),KASE,LEG,IPT,IX(1),IX(2)                       MICRON     4
      DO 8 I=1,2                                                        MICRON     5
      KX=(IX(I)/10)*10                                                  MICRON     6
      NX=IX(I)-KX                                                       MICRON     7
      IF (IABS(NX-MX(I))-5)3,3,4                                        MICRON     8
    3 LX(I)=KX+MX(I)                                                    MICRON     9
      GO TO 8                                                           MICRON    10
    4 IF (MX(I)-NX)5,3,6                                                MICRON    11
    5 LX(I)=KX+MX(I)+10                                                 MICRON    12
      GO TO 8                                                           MICRON    13
    6 LX(I)=KX+MX(I)-10                                                 MICRON    14
    8 CONTINUE                                                          MICRON    15
      X=LX(1)                                                           MICRON    16
      Y=LX(2)                                                           MICRON    17
      X=X*.000001                                                       MICRON    18
      Y=Y*.000001                                                       MICRON    19
      RETURN                                                            MICRON    20
    1 FORMAT (2X,2I1,I3,I1,I2,I9,I10)                                   MICRON    21
      END                                                               MICRON    22
```

```
      SUBROUTINE FLIPM(B)                                               CALIB    1161
      DIMENSION B(25,52)                                                CALIB    1162
      DO 1 I = 1,25                                                     CALIB    1163
          TEMP = B(I,1)                                                 CALIB    1164
          B(I,1) = B(I,2)                                               CALIB    1165
          B(I,2) = TEMP                                                 CALIB    1166
    1 CONTINUE                                                          CALIB    1167
      RETURN                                                            CALIB    1168
      EN D                                                              CALIB    1169


       SUBROUTINE RADDG (ARRAD, IDEG, IMIN, SEC )                       RADDG       2
      RS = .4848136811E-5                                               RADDG       3
      IDEG = ARRAD/ (RS*3600.)                                          RADDG       4
      DEG = IDEG                                                        RADDG       5
      IMIN = (ARRAD-(DEG*RS*3600.))/(RS*60.)                            RADDG       6
      XMIN = IMIN                                                       RADDG       7
      SEC = (ARRAD- ((DEG*RS*3600.)+(XMIN*RS*60.)))/RS                  RADDG       8
      RETURN                                                            RADDG       9
      EN D                                                              RADDG      10


      SUBROUTINE CLEAR (A,IR,IC,IRS,IRE,ICS,ICE)                        CLEAR       2
                                                                        CLEAR       3
       A = NAME OF ARRAY                                                CLEAR       4
       IR = ROW DIMENSION OF A ARRAY                                    CLEAR       5
       IC = COLUMN DIMENSION OF A ARRAY                                 CLEAR       6
       IRS = FIRST ROW ELEMENT TO BE CLEARED                            CLEAR       7
       IRE = LAST ROW ELEMENT TO BE CLEARED                             CLEAR       8
       ICS = FIRST COLUMN ELEMENT TO BE CLEARED                         CLEAR       9
       ICE = LAST COLUMN ELEMENT TO BE CLEARED                          CLEAR      10
C                                                                       CLEAR      11
      DIMENSION A(IR,IC)                                                CLEAR      12
      DO 1 I = IRS,IRE                                                  CLEAR      13
      DO 1 J = ICS ,ICE                                                 CLEAR      14
      A(I,J) = 0.0                                                      CLEAR      15
    1 CONTINUE                                                          CLEAR      16
      RETURN                                                            CLEAR      17
      EN D                                                              CLEAR      18


      SUBROUTINE CLEARV (A,IR,IRS,IRE)                                  CLEARV      2
C                                                                       CLEARV      3
C      A = VECTOR ARRAY NAME                                            CLEARV      4
C      IR = TOTAL LENGTH OF A ARRAY                                     CLEARV      5
C       IRS = FIRST VECTOR ELEMENT TO BE CLEARED                        CLEARV      6
C      IRE = LAST VECTOR ELEMENT TO BE CLEARED                          CLEARV      7
C                                                                       CLEARV      8
      DIMENSION A (IR)                                                  CLEARV      9
      DO 1 I = IRS,IRE                                                  CLEARV     10
      A (I) = 0.0                                                       CLEARV     11
    1 CONTINUE                                                          CLEARV     12
      RETURN                                                            CLEARV     13
      EN D                                                              CLEARV     14
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