NOAA Technical Report NOS 72 NGS 7

# Systems for the Determination of Polar Motion

Rockville, Md.
May 1977

**U.S. DEPARTMENT OF COMMERCE**
**National Oceanic and Atmospheric Administration**
National Ocean Survey

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Soren W. Henriksen
Geodetic Research and Development Laboratory
National Geodetic Survey

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**U.S. DEPARTMENT OF COMMERCE**
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**National Oceanic and Atmospheric Administration**
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# PREFACE

This report was undertaken by the National Geodetic Survey to provide basic information for planning the future of its two astronomical observatories. The study was initiated and completed in 1975. The conclusions cited, as well as the projected costs, are as of that date.

## LIST OF ACRONYMS AND ABBREVIATIONS

| | |
|---|---|
| APL | Applied Physics Laboratory (of Johns Hopkins University) |
| AT | Atomic Time |
| BIH | Bureau International de l'Heure |
| CIO | Conventional International Origin |
| CNES | Centre National d'Etudes Spatiales |
| DMATC | Defense Mapping Agency Topographic Center |
| DME | distance measuring equipment |
| DoD | Department of Defense |
| DPMS | Doppler Polar Motion Service |
| FAGS | Federation of Astronomical and Geophysical Services |
| FME | frequency measuring equipment |
| FZT | floating zenith telescope |
| GPS | Global Positioning System |
| GSFC | Goddard Space Flight Center |
| IAG | International Association of Geodesy |
| IAU | International Astronomical Union |
| IAR | instantaneous axes of rotation |
| IGA | International Geodetic Association (now IAG) |
| IGN | Institut Geographique National |
| ILS | International Latitude Service |
| IPMS | International Polar Motion Service |
| IUGG | International Union of Geodesy and Geophysics |
| JPL | Jet Propulsion Laboratory |
| Lageos | lunar geodetic observation satellite |
| LBI | long baseline interferometry |
| LURE | Lunar retrodirective reflector tracking network |
| MAR | mean axes of rotation |
| MIT | Massachusetts Institute of Technology |
| NASA | National Aeronautical and Space Administration |
| NGS | National Geodetic Survey |
| NOS | National Ocean Survey |
| PZT | photographic zenith telescope |
| rms | root mean square |
| SAO | Smithsonian Astrophysical Observatory |
| SBI | short baseline interferometer |
| USNO | United States Naval Observatory |
| UTC | Universal time coordinated |
| UTZ | Universal time zero |
| VLBI | very long baseline interferometry |
| VZT | visual zenith telescope |

# SYSTEMS FOR THE DETERMINATION OF POLAR MOTION

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ABSTRACT. The benefit of long-term observations of latitudinal variation is primarily the derivation of corrections to time and astronomical coordinates of control points. Other applications (those with which this report is concerned) include maintenance of an internationally accepted coordinate reference system, monitoring of the Earth's pole of rotation, and studies of the possible relationship of earthquakes to polar motion.

Data produced by classical methods, since 1899, are insufficient to permit accurate prediction of polar motion, to determine the amount or existence of secular drift of the pole, or to detect the possible effects of earthquakes on polar motion. For these and other reasons, the international participants in the program agree that the program should be continued for at least another 50 to 100 years and that more accurate observation methods should be developed. The National Geodetic Survey, in its role as the United States' participant in the program, suggests a number of alternatives for solution of the problems enumerated and for consideration by the international community.

## 1. BASIS FOR PRESENT STUDY

The National Geodetic Survey (NGS) of the National Ocean Survey (NOS) has operated, almost continuously since 1899, two astronomical observatories at Gaithersburg, Md., and at Ukiah, Calif., solely to obtain data on the variation of latitude. These observatories are part of a project, begun in 1900, involving three similar observatories in other countries with almost the same long history. Probably no other project in modern science has continued for so long unchanged in purpose, equipment, or technique and has managed to produce such a large volume of high-quality data. But even the most productive, long-lived project should undergo an occasional check of its health--not necessarily for signs of senility, but to find ways of continuing and improving the productivity, or even ending the project. Since the project is international in character, any definitive examination

must be made through international cooperation. However, a preliminary examination by a national participant, resulting in a tentative diagnosis and prescription, can be helpful in preparing for the international effort. This report describes such a study. It covers the project's history, its present condition, and possible ways of adapting the project to meet the greater demands being made upon it. Since the study was begun as an examination of the National Ocean Survey's present and future role in the project, this aspect, of course, receives special attention. Otherwise, conscientious efforts were made to perform an unbiased diagnosis and to suggest treatments applicable to the whole project.

This report presents preliminary proposals only. The other participating countries are also examining their present programs. Final solutions will depend on international agreements. These questions were discussed at the international level in August 1975 and will continue to be discussed for many years. Until a consensus is gained, any study from a single group must be considered tentative and its conclusions temporary.

## 2. HISTORICAL AND THEORETICAL BACKGROUND

Since 1899, the NOS, formerly the U.S. Coast and Geodetic Survey, has maintained two observatories for determining the motion of the Earth's pole of rotation. The reason for this long term program can best be understood by taking a brief look at its history.

### 2.1 Observational History

In the long history of the study of the Earth as a planet, probably two ideas have been held with more firmness than any others--that the Earth is spherical, and that the rotation of the Earth is perfectly uniform. The first idea was finally disproven in the eighteenth century as a result of the measurements of meridional arcs in Lappland and Peru. The second was questioned as early as 1765 by Euler and in 1820 by Brioche, who pointed out that Euler's equations of motion (see next section) implied that the Earth's instantaneous axis of rotation might not be fixed in the Earth. Various astronomers, Peters and Bessel among them, searched for the suggested motion without definite success. Definite proof that polar motion exists was first shown in about 1885 as a result of the investigations of Kustner (Dejaiffe 1972) at Berlin. He had been trying to find out if the aberration of light was a constant. His observations of stellar declinations and zenith distances seemed to indicate that the constant of aberration was variable. An extensive analysis of the data, however, showed that the variation detected in the data should have been attributed to changing positions of the Earth's axis of rotation rather than to aberration. This discovery raised considerable excitement, particularly when supported by additional observations made from the Potsdam Observatory, and was finally accepted in 1891 after a

year of "simultaneous" observations from observatories at Hawaii and Berlin.

About this time, S. C. Chandler published the results of his analysis of astronomical observations for the preceding two hundred years. He showed that not only is there a motion with a period of about one year, but also a variation with a period of 428 days. The inconsistency of this latter period with the 305-day period predicted by Euler's equations was shown by Newcomb in 1892 to be explainable by assuming the Earth to be elastic rather than rigid.

A direct and timely consequence of this work was the 1898 resolution of the Assembly of the International Geodetic Association (IGA) at Stuttgart to create an International Latitude Service (ILS) of six stations (Preston 1899). These stations were all to be located at approximately the latitude 39°08' and were to be used to determine polar motion by nightly observations of the zenith distances of 12 groups of 12 stars each. Plans were made for an initial observation period of five years. But since the pole seemed to follow a seven-year cycle, the U.S.A. delegate thought that at least three such cycles (21 years) would have to be observed for a satisfactory determination of the pole's motion* (Preston 1899).

The IGA provided funding of $10,000 per year. (At that time, each country appointed one official or delegate with voting power to the IGA. The American delegate was E. D. Preston, Executive Officer of the U. S. Coast and Geodetic Survey.)

Since inception of the ILS, two stations (Cincinnati and Tchardjui) have been dropped from the program. One station (Kitab) was added in 1922 to replace Tchardjui. The station at Gaithersburg interrupted service from about 1916 to 1932. The stations at Misuzawa, Ukiah, and Carloforte have given almost uninterrupted service up to the present time. (See table 1.)

All stations were equipped with similar instrumentation (VZT's) and used the same reduction method (Horrebow-Talcott). Responsibility for coordination and reduction of combined data was vested in the ILS. In 1962, the name was changed from International Latitude Service to International Polar Motion Service (IPMS). Along with the change in name came an increase in the number of observatories for the determination of polar motion. Together with the five existing ILS observatories, there are now about 50 observatories contributing data.

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*The two major periods in the polar motion are approximately 12 months and 14 months with complete cycles taking about 6 years. The directorship or headquarters has been changed six times since 1899.

Table 1.--International latitude service stations

| Name | Nominal longitude | Nominal latitude | Period of observation | Instrument |
|---|---|---|---|---|
| Misuzawa | 141°07'51" | (39°8') 3".602 | 1899-1975 | VZT |
| Carloforte | 8°18'44" | 8".941 | 1899-1942/ 1946-1975 | VZT |
| Tschardjui | 63°29' | 10".662 | 1899-1909.5 1909.5-1920 | VZT |
| Ukiah | -123°12'35" | 12".096 | 1899-1975 | VZT |
| Gaithersburg | -77°11'57" | 13".202 | 1899-1914/ 1932-1975 | VZT |
| Cincinnati | -84°25' | 19".364 | 1899-1915 | VZT |
| Kitab | 66°52'51" | 1".850 | 1922-1975 | VZT |

## 2.2 Theory of Polar Motion

The Earth is a rotating body. Its shape is nearly spherical and its composition is heterogeneous, with a lithosphere (predominant), a hydrosphere, and an atmosphere. Despite its deviation from the ideal homogeneous spherical body, the rotation of the Earth is governed by the same laws of mechanics as is every other material body. In particular, the Euler equations apply very closely. These equations are

$$\frac{d}{dt}[I_i\ \omega_i] - (I_j - I_k)\ \omega_j\ \omega_k = N_i; \quad i, j, k = 1, 2, 3.$$

The $I_i$ are the principal moments of inertia, $\omega_i$ are the components of angular velocity about the principal axes, and $N_i$ are external torques acting on the rotating body. (It would be more general to introduce forces instead of torques; but, since any set of forces can be resolved into a set of unbalanced forces and a set of torques, the resulting equations would also separate into two sets--one for translational motion, in which we are not interested, and the other (Euler's equations) for rotation.)

If the torques $N_i$ are set equal to zero, the equations become

$$\frac{d}{dt}\,[I_i\,\omega_i] = (I_j - I_k)\,\omega_j\,\omega_k.$$

Just how the rotational motion will appear depends on the relative values of the principal moments of inertia. When $I_1$ and $I_2$ are nearly equal, as is the case for the Earth, the equations become (assuming that the $I_i$ are constant)

$$I_1\,\dot{\omega}_1 = (I_1 - I_3)\,\omega_2\,\omega_3,$$

$$I_2\,\dot{\omega}_2 = -(I_1 - I_3)\,\omega_3\,\omega_1, \text{ and}$$

$$I_3\,\dot{\omega}_3 = 0\ .$$

These equations are easily solvable, leading first to the equation,

$$I_1\,\ddot{\omega}_1 = (I_1 - I_3)\,\dot{\omega}_2\,\omega_3,$$

by differentiating the first of the preceding equations and thence, by substitution from the second, to

$$\ddot{\omega}_1 = -\left[\frac{(I_1 - I_3)\,\omega_3}{I_1}\right]^2 \omega_1.$$

This is, of course, the differential equation for simple harmonic motion of angular frequency,

$$\Omega = \frac{(I_1 - I_3)}{I_1}\,\omega_3.$$

For the Earth, the ratio of the moments is about -0.0033 as determined from measurements of flattening. (The sign is negative because the polar axis is shorter than either equatorial

axis.) The period, therefore, about 300 days, or about 10 months, is Euler's period (Goldstein 1950).

Note that to obtain this simple result, we had to assume (among other things) that $dI_i/dt$ was zero, which implies that the Earth is rigid. Since the Earth is elastic, the moments of inertia generally will not be fixed. If one takes into account the elasticity, as Newcomb did in 1892 to explain Chandler's results, the period is lengthened. With a suitable value for the elasticity, one derives the Chandlerian period.* The modulus of elasticity is then adjusted to make the theoretical period agree with the observed period; hence, the long-held idea that the Earth is about as elastic as steel.

If we do not set the forces $N_i$ equal to zero, but substitute in their place known torques, we find that two different kinds of motion arise according to the sources of the torques. The largest torques arise from sources far outside the Earth--the Sun, the Moon, and the other planets. Their effects, the astronomical phenomena of precession and nutation (Woolard 1953), are not of great geodetic interest, except as they affect the stellar coordinates used in astrogeodetic work. The second kind of torque arises from phenomena on or inside the Earth, such as variations in atmospheric and oceanic circulation, earthquakes, continental drift, and changes in the Earth's interior. In this connection, the study of polar motion is particularly valuable since such a study, if based on enough sufficiently accurate data, can give information about geophysical processes otherwise not obtainable. Particularly outstanding examples of this close relationship between polar motion and geophysical phenomena are the series of investigations by Jeffreys (Munk and MacDonald 1960) on geophysical causes of the variations in polar motion (e.g., see reference for a description of these investigations). Another example is the very detailed set of equations prepared by E. Takagi (1970), which deal with phenomena in the atmosphere, hydrosphere, mantle, and outer and inner cores. Takagi's paper is discussed in appendix A. The paper is important because it is sufficiently detailed and extensive to be used as the basis for investigations into the relationships between the rotation of the Earth and other geophysical phenomena.

In dealing with the theory behind polar motion, one must be careful to differentiate between two very different but closely coupled forms of the motion. In one form, the Earth is considered to shift with respect to its axis of rotation; equivalently the axis of rotation shifts with respect to the Earth;

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*Proof is not difficult (appendix D), but the result is intuitively obvious. Elasticity makes the Earth flatter and increases its moment of inertia about the axis of rotation. To keep the total angular momentum constant, the rate of rotation must be decreased.

the result is the variation of astronomic latitudes and time. In the other form, the axis of rotation is assumed to stay fixed in the Earth, but to move with respect to the "fixed" stars. This last form of motion, known since the time of ancient Babylonia, results in precession and nutation. The actual motion is a combination of the two different forms, their separation being difficult when the amplitudes are small.

## 3. PROCEDURES USED FOR DETERMINING POLAR MOTION

It is convenient to discuss methods for determining polar motion in two parts: the classical methods where the stars are observed optically, and new methods (since 1960) where the objects observed are not stars but the Moon, artificial satellites of the Earth, or quasars. The newer methods of observation may involve use of the optical or the radio part of the spectrum. This division corresponds rather well to the relative costs of the methods; the most recent methods are considerably more costly by a factor of 10 to 1000. The gain in precision is not directly related to the increase in cost; a reduction of the rms error by a factor of 10 to 20 is the most that can be expected at present.

Table 2 lists the major characteristics of the methods; details are described later. The table lists the estimated rms error of a single measurement, the rms error in determination of the coordinates of the instantaneous pole of rotation, and the frequency with which the coordinates are observed can be determined. The (nominal) values given are dependent on unpredictable factors such as the number of observatories involved and weather conditions. The relative costs of acquiring and operating the systems are given in the last column; the low cost is less than $100,000, the medium cost is between $100,000 and $250,000, and the high cost is over $500,000. (See table 10.)

### 3.1 Classical Methods

Although the five principal stations of the IPMS have used VTZ's since the beginning, both PZT's and VZT's have been used in various observatories, particularly at Gaithersburg (1911 to 1914) and Misuzawa (1953 to present). Both instruments and the associated procedures, therefore, can be called classical and are considered so in this study. A third, hybrid form of instrument known as the floating zenith telescope (FZT), which resembles a VZT with a camera in place of an eyepiece and floats in a pool of mercury, has been used extensively for latitudinal observations at the Misuzawa Observatory. Extensive performance investigations of the FZT at Misuzawa indicate there is no real advantage over either the VZT or the PZT. The instrument, therefore, is not considered further. Details are described by Cookson (1901) and Hattori (1951).

Table 2.--Characteristics of various techniques used for finding polar motion

| Technique | rms error (cm) Single observations | Coordinates of pole | Interval between determinations | Relative Cost |
|---|---|---|---|---|
| Zenith telescope | | | | |
| Visual | 1000 | 30 | 1 mo. | low |
| Photographic | 300 | 10-20 | <1 mo. | medium |
| Satellite-tracking | | | | |
| Moon | | | | |
| Optical (laser) | <10 | <10 | 1 day | high |
| Electronic | --------- not in use at present --------- | | | |
| Artificial | | | | |
| Optical | <10 | <10 | 1 day | high |
| Elec. (Doppler) | Not applicable | 20-50 | 2 days | medium |
| Quasar-tracking | | | | |
| VLBI | <10 | <10 | 1-2 days | high |

### 3.1.1 The Visual Zenith Telescope

A VZT is typically a short (1.3 m) telescope attached near its midpoint to the top of a vertical column so the telescope is free to swing through a wide arc in the meridian but with practically no motion in any other direction. The eyepiece is perpendicular to the tube so that observations can be conveniently made about the zenith. Table 3 lists the principal characteristics of VZT's currently in use.

#### 3.1.1.1 Theory and Computational Procedure

The observational procedure, based on the Horrebow-Talcott method eliminates the largest refractional errors by observation of stars in pairs; the two stars in each pair lie on the meridian at opposite and nearly equal distances from the zenith (Dejaiffe 1972, Bomford, 1971, Mueller, 1969, Ramsayer, 1970a). The basic equation for the Horrebow-Talcott method is

$$\phi = \frac{1}{2} [\delta_N + \delta_S + \zeta_S - \zeta_N],$$

Table 3.--Principal characteristics of visual zenith telescopes used by ILS stations

| Item | Value |
|---|---|
| Focal length | 130 cm |
| Aperture | 108 mm |
| Vertical circle | |
| diameter | 24 cm |
| division | 10 in. |
| direct reading (vernier) | 10 in. |
| Level | |
| sensitivity | 1 in. per division |
| Micrometer | |
| range | 40 in. |
| direct reading to | 0.4 in. |
| estimation to | 0.04 in. |
| Construction by | |
| mechanical | Wanschaf |
| optical | Zeiss (Jena) |
| level | Reichel |

where the latitude $\phi$ is determined by the declination $\delta$ and zenith distances $\zeta$ of two stars, one north (N) of the zenith, the other south (S) of the zenith.

This equation is modified by the following considerations:

(1) Although refraction $\Delta\zeta$ is almost the same for both stars of a suitably chosen pair (called a Horrebow-Talcott pair), the two stars are not at exactly equal distances from the zenith. Hence, a correction term $C_1$ must be introduced:

$$C_1 = \Delta \zeta_S - \Delta \zeta_N.$$

$C_1$ will be very small and can be computed with great accuracy.

(2) The N-S horizontal axis of the telescope may change its inclination appreciably during the progress of several hours of observation. This lack of horizontality is corrected for by using the readings from two Horrebow levels fixed to the instrument. The correction $C_2$ is the product of the sensitivity L of the levels by the difference $(I_E - I_W)$ in readings of the levels.

(3) The exact zenith distances are measured with the micrometer, not on the vertical circle. The readings on the vertical circle cancel in the fundamental equation leaving only $C_3$, the difference $(M_E - M_W)$ multiplied by the scale factor R (the number of seconds of arc per turn of the micrometer screw).

(4) Finally, for all instruments equipped with reticles, a correction must be made for the curvature of the apparent path of the star through the field. This correction, expressed in terms of the correction dM to the micrometric readings, is $C_4 = R \cdot dM$ .

So the modified equation is

$$\Phi = \frac{1}{2} [(\delta_N + \delta_S) + C_1 + C_2 + C_3 + C_4] .$$

The Horrebow-Talcott method devised minimizes the effects of errors in declination and zenith distance, so the errors arising from the first three $C_i$ terms are small by design. The final term contributes very little to the total error. The most difficult sources of error to deal with are the levels and the micrometer readings. In fact, several leading ILS data analysts have said flatly that the secret of achieving good results lies in properly evaluating the value of R (Melchoir 1972).

Effects of other sources of error are diminished or removed by calibration or suitable observing procedures. Items listed below are such sources of error:

(5) bending of the telescope
(6) lack of collimation
(7) deviation from the meridian
(8) Earth tides
(9) nutational corrections made
(10) the constant of aberration, and
(11) the declinations used for the stars (Scott 1963).

The observing program is planned to minimize as many errors as possible. One hundred and forty-four stars are observed for determining latitudinal variation. (Some 48 additional stars are observed for other purposes such as determining refraction or the constant of aberration.) The 144 stars are divided into 12 groups of six pairs of stars each. Right ascensions are chosen so that three groups can be observed for about a month. During the succeeding month, stars in the first group gradually become unobservable, stars in the next three groups are all observable, and stars in the following group gradually become observable. Thus, during the course of one month, stars from three consecutive groups are observed; the first group drops out of view at the end of the month and another group comes into view. In the reduction, the x,y,z coordinates of the pole are

computed. The z-term, the so-called Kimura term, is assumed to contain all the declination errors, and the preliminary corrections to $\delta$ are obtained as the mean of the z-coordinate for each group for each year. A second method of correcting for errors in declination is to take the weighted mean misclosure (in latitude) of the ILS stations and distribute this equally among the 12 groups, assuming that the sum of the declinational errors in the group mean-latitudes is zero.

#### 3.1.1.2 Accuracy

The most recent estimates of the rms errors of the observations and location of the pole are given by S. Yumi (1972). These are 0".2 to 0".3 for the standard deviation (s.d.) of the observations, and 0".01 for the s.d. of a single coordinate of the instantaneous pole; these values are estimates of precision only. Values given by Melchior (1975) agree reasonably well with those of Yumi. (See also appendix G.)

### 3.1.2 The Photographic Zenith Telescope

While the VZT is simply a telescope which can be used to make observations at the zenith, the PZT proper (Ross 1915, Markowitz 1960) is designed for this purpose. It is a long ($\approx$ 4.5 m) tube fixed so that its optical axis points as closely as possible toward the zenith. The characteristics of the one designed by Ross for the Gaithersburg station in 1911 (and moved to the U. S. Naval Observatory in 1914) are given in table 4. Since the direction of the optical axis is fixed, stars appear to trail across the field of view. This motion is eliminated during observation by placing the photographic plate or film on a carriage that can be moved from west to east at the same rate as the image.

Table 4.--Principal characteristics of photographic zenith telescopes

| | Washington (PZT #3) | PZT #4 | Large PZT |
|---|---|---|---|
| Focal length | 4579 mm | 5167 mm | 130,000 mm |
| Aperture | 203 mm | 200 mm | 650 mm |
| Field of view | 27' | 24' | 60' (NS)<br>50' (EW) |
| Size of plate | 4.5x4.5 cm | 4.5x4.5 cm | 20x25 cm |
| Limiting magnitude | $8.5^m$ | $8.5^m$ | $11^m$ - $12^m$ |
| Diameter of reflector | 20 cm | 20 cm | 53.3 cm |

The Horrebow-Talcott method requires the observation of pairs of stars, each of which is at an equal distance from the zenith. This kind of pairing is not possible with PZT photographs because of the small (less than 35') field of view. The equations given in section 3.1.1.1, therefore, do not apply to PZT photographs. Fortunately, since all stars on the photographs are close to the zenith, refraction is not a problem. The equations used with measurements of PZT photography are called "the double zenith distance method." With this method, a star is observed before passage through the meridian, the observing apparatus is rotated through 180°, and the star is observed again after passage. The difference between the two measurements (corrected for curvature of the image-path) is twice the zenith distance of the star. Originally, stars were observed individually by a Newtonian kind of arrangement with the observations being made above the lens, i.e., after the light rays had passed for a second time through the lens. Ross (1915) substituted a photographic plate under the lens for the eyepiece above it, thus eliminating the need for the error-causing second traversal of the lens by the light rays. The formula for latitude is then given by

$$\phi = \delta - \zeta = \delta - RM/2,$$

where M is the separation of two images on the plate and R is the scale factor. In practice, the carriage is reversed three times, and four point-like images of each star are obtained.

The observation program is determined by the fact that the PZT is generally used for finding both astronomical time and the latitude. Stellar positions are determined relatively by the PZT itself, and then tied to a system of stars in the FK4 (Fricke et al. 1963). A typical program consists of observing eight groups of 10 stars each. Two groups are observed for 6.5 weeks; at the end of the period, one group is dropped and the next one introduced.

### 3.1.2.1 Accuracy

Extensive observations by the U. S. Naval Observatory show the standard deviations of time to be about $0^{s}.0045$ when more than nine stars are used and about $0^{s}.0050$ when four to eight stars are used. The standard deviations in latitude are about $0''.11$ for a single observation and about $0''.055$ for a full night's observations. Ramsayer (1970b) gives $\pm 0''.05$ for the PZT at Hamburg, which closely matches Markowitz's value.

### 3.1.3 The Impersonal Astrolabe of Danjon

Although Danjon's impersonal astrolabe is used at many observatories, it is unlikely it will be adopted either by the National Geodetic Survey or by the IPMS for polar motion (Hall 1975). The astrolabe is about as expensive as the PZT and, according to several investigators, slightly less precise.

## 3.2 Procedures under Development

Within the last decade, four procedures developed for finding polar motion promise lower errors or more frequent observations than the classical methods. These procedures are observation of artificial satellites by laser-type distance measuring equipment (DME), observation of artificial satellites by measuring the change in frequency of signal generators on the satellites, observation of corner-cube reflectors on the Moon using laser-type DME, and observation of quasars by pairs of radio telescopes. None, except perhaps the second, has so far proven its usefulness for determining polar motion as well as the classical methods. But each method has at least one desirable quality and could be developed further.

### 3.2.1 Determination from Variations of the Latitude of Artificial Satellites

The location of an artificial satellite can be specified by giving its longitude, latitude, and height (above the spheroid) at each instant. The variation in height will for all practical purposes be a result of the dynamics of the satellite motion. Variations in latitude and longitude will occur because of (1) the dynamics of the satellite, and (2) the rotation of the Earth. (The coordinate system rotates with the Earth.) However, the Earth's rotational contribution to the variation in longitude is composed of a large contribution from the Earth's daily rotation about the instantaneous axis and a small contribution from the rotation of the instantaneous axis about the average axis. The Earth's rotational contribution to the variation in latitude is composed primarily of the small contribution of the rotation of the instantaneous axis about the average axis. By removing the dynamic part of the variation from the variation in latitude, we get the rotational part--the polar motion. The magnitude of the polar motion is less than 10 m; its period is over one year. This is of the same order of magnitude as the variations from some other sources, so extraction of polar motion from the data is difficult and considerable computation is necessary. Two kinds of instruments have been used in finding polar motion with satellites: the laser-type DME and frequency-measuring equipment (FME), with typical radio frequencies of 150 and 400 MHz.

### 3.2.1.1 Observations with Laser-Type Distance-Measuring Equipment

A typical instrumentation system, less the computer (T.Johnston 1977), for determining polar motion by observation of satellites with laser-type DME consists of (1) the laser DME, which includes the laser, the sending and receiving telescopes, and the timing equipment which determines the interval between the pulse emission and return; (2) the electric power generator; and (3) the satellite. The National Aeronautic and Space Administration's Goddard Space Flight Center (GSFC) in 1970 used two sets of laser DME for this purpose, but one set was sufficient for determining polar motion. Items (1) and (2) are customarily housed in trailers, as is all auxiliary equipment. Table 5 gives some of the most important characteristics of the system.

This method and those following differ markedly from the classical methods previously described in that, practically, polar motion can be obtained by the newer methods only by using large-scale electronic data-processing machines to reduce the data. (Radio astronomy by interferometry, the last method to be discussed, can be accomplished with the use of a smaller, single-purpose calculating machine. Such machines ordinarily would be devoted primarily to other projects, with polar motion projects using only a small part of the machine capability.)

Table 5.--Characteristics of a system of laser-type DME for determining polar motion

| Feature | NASA/Goddard | Smithsonian* |
|---|---|---|
| Type of laser | Ruby (neodymium) | Ruby (neodymium) |
| Frequency | 0.6943 μm | 0.6943 μm |
| Pulse | | |
| Duration | 4 ns | 20 (5) ns |
| Energy | 0.5 joule | 5~7 (1) joules |
| Frequency | 60 pulses per min. | 4~10 pulses per min. |
| Telescope | | |
| Beam width | 70" | 100"~1000" (20") |
| Aperture (receiver) | 40 cm | 50.8 cm |
| Timing | | |
| Accuracy | ±5 cm | 10 (1) μs |
| Satellite | Lageos and Geos-3 | Lageos and Geos-3 |

*Values in parentheses refer to hypothetical system giving ±2 cm accuracy.

An interesting characteristic of the procedure described thus far for finding the polar motion is that the effects of variations in latitude arising directly from gravitational variations are minimized by locating the tracking station near the maximum latitude of the satellite. The procedure would be less accurate if a satellite in polar orbit were tracked. This factor may limit the applicability of the procedure solely to determining polar motion.

#### 3.2.1.1.1 The Computing Problem

Although placing the tracking station near the circle of the maximum latitude makes the effect of polar motion more noticeable, it does not, by any means, eliminate the need for precise computation of the orbit. The Goddard Space Flight Center, which has done most of the recent work on determining polar motion from distances measured with laser-type DME, computes the orbit by numerical integration of the differential equations of motion. The GEM 7 gravitational field inserted in the equations extends well past the 16th degree and order in Legendre coefficients. Among the corrections included are those for Earth tides, luni-solar perturbations (directly on the satellite), atmospheric drag, and solar pressure. Numerical integration is not the most economical method of solution. When the equations are long, the problem of finding polar motion becomes quite difficult computationally, so much so that the computer becomes equal in importance to the satellite and the tracking instrumentation.

To determine polar motion and other varying quantities such as Earth tides, GSFC computes, from observations, the orbit over a period of time sufficiently long to cover a cycle or more of the investigated variation. This period is then broken up into many smaller periods (the length depending on the temporal distribution of observations, and a separate orbit is computed for each of these shorter periods.) The assumption is that, while the effects of the variation sought for appear in the elements of all the orbits, they are largely averaged out over the long period but not over the much shorter periods (Smith 1974) and (Douglas et al. 1974). Comparison of the elements over the short periods with those over the long period should bring into evidence variations in the quantity (such as polar motion) concerned.

The basic equations used are Lagrange's equations which give the rates of variation of the elements as functions of the perturbing forces. These equations, by numerical integration, eliminate the short-period perturbations. The ones of long period, too long to be treated by numerical integration, are handled by analytical equations for the long-period variations (Wagner et al. 1974).

### 3.2.1.1.2 Accuracy

The precision of laser-type DME in 1970 was approximately 0.5 m (Berbert 1977). The precision is being improved by using lasers with higher power, telescopes of narrower beam width, pulses with steeper sides, and clocks of greater accuracy. Experiments on the tracking of satellites with equipment located near San Diego and Quincy, California, show precision better than 10 cm. The goal of NASA/GSFC for 1975 was an accuracy of 5 to 10 cm, which implies a goal of better than 5 cm in precision. Studies made by the Smithsonian Astrophysical Observatory of the accuracy with which orbits can be determined from laser-type-DME measurements assume an accuracy of ±5 cm for the measurements. Numerous NASA memoranda dealing with proposed programs for the employment of laser-type DME also cite the 5-cm value. If this value is accepted (NASA has also used the value ±2 cm at briefings*), a realistic value for the error in the latitude of the pole computed for one day's observations by one instrument would be about ±10 cm, the added 5 cm being the sum of contributions of errors in the orbit, errors in the contribution of Earth tides, etc.

Since the Earth's large moments of inertia prevent any sudden changes in the position of the pole, fitting to a month's observations should give considerably smaller errors. An rms error of ±2 cm seems reasonable. There are several reasons, however, for considering this value optimistic. In the first place, it must be remembered that the procedure used up to now gives only one component of the motion for each pass of the satellite. The circumstances of orbital inclination, conditions of observations, etc., are such that one component will be much more reliably determined than the other for long stretches of time. If two stations widely apart in the east-west direction were available, both components could be determined, of course. But for a single station, we can expect the error in one component to be considerably higher than that in the other. On this account alone, values of ±15 cm for one day's observations and ±3 cm for one month's observations would be more likely. In the second place, it must be remembered that the polar motion is determined from the orbit which, for all practical purposes, is independent of the polar motion. This means that the components of polar motion may be determinable with high precision, but that systematic errors may be present in the components determined, without appearing in the orbit. Another way of looking at the situation is to say that the accuracy of the polar coordinates will depend upon the accuracy of the coordinates to which the orbit-derived values are tied.

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*Recent studies make an rms error of ±10 cm in measuring distances to Lageos more likely.

Since the most accurate coordinates appear to be those of the IPMS, the accuracy of polar coordinates (not of polar motion) determined using laser-type DME will be limited by the accuracy of the coordinates determined by IPMS.

3.2.1.2 Observations with Frequency-Measuring Equipment

When the tracking of satellites with FME began with the launching of SPUTNIK I, the observation equation was set up to give the corrections to the orbital elements and tracking-station coordinates in terms of the correction to the time of closest approach of satellite to station (which occurs at the cross-over point of the curve of Doppler shift). After the TRANSIT program got underway, however, the Applied Physics Laboratory of Johns Hopkins University discovered that the mathematics was simpler and the equipment could be made more precise if the change in phase of the signal was measured rather than the frequency. The reason for the simplification is that, at a constant frequency, the change in phase between two times is directly proportional to the change in distance from satellite to station. The locus of points which are a known difference in distance from two known points (the ends of the orbital arc) is a hyperboloid. Given three such hyperboloids, the tracking station is located. The problem can be turned around, of course, to give the orbit rather than the locations of the stations; or (within limits) all unknowns can be solved for simultaneously. Since the relation of tracking stations to satellite is affected by the orientation of the Earth, the coordinates of the pole form part of the instantaneous coordinates of the stations and can be solved for.

When TRANSIT was first put in the field, the receiving equipment occupied a whole van. In the late 1960's, the equipment was successfully miniaturized. Table 6 gives the characteristics of a typical present-day receiver.

Table 6.--Characteristics of typical* FME for use with TRANSIT-type satellites

| | |
|---|---|
| Frequencies (MHz) | 149.988 ± 0.00375<br>399.968 ± 0.01000 |
| Oscillator stability | $<10 \times 10^{-12}$ or $< 5 \times 10^{-12}$ per 100s |
| Power | |
| Operating | 15 watts |
| Standby | < 5 watts for 10 hours |
| Weight | <16 kgm |
| Size | 60 x 20 x 35 cm |

*JMR-1 Doppler survey set

### 3.2.1.2.1 Computation

Field station data are sent to the Applied Physics Laboratory where they are collected, collated, arranged in standard format, and then sent to the Defense Mapping Agency Topographic Center (DMATC), Washington, D. C. (Yionoulis 1977). The data are processed in batches covering two days of tracking. Polar motion is computed as a part of the system of equations describing the satellite's motion. The DMATC procedure differs in this respect from the GSFC procedure described in 3.2.1, where polar motion is determined separately from the orbit and by a different set of equations.

The basic observational equation (Anderle 1977) is

$$df = \sum_k \frac{\partial f}{\partial \rho_k} d\rho_k ,$$

where the frequency $f$ is defined in terms of a basic frequency $f_b$, a correction $\delta f_b$ taking refraction into account, and a time $t_b$ at which the basic frequency was defined:

$$f \equiv f_s \left(1 - \frac{\vec{\rho}\,\dot{\vec{\rho}}}{c\,\rho}\right) + \delta f_b$$

$$f_s \equiv f_b + \dot{f}_b (t - t_b) .$$

The vector $\vec{\rho}$ is the distance traveled by the radio wave from satellite to receiver. It is, therefore, the difference between the location $\vec{r}_s$ of the satellite at the moment of emission and the location $\vec{r}_r$ of the receiver at the time of reception. The vector and its derivatives are found by numerical integration of the equations of motion and the corresponding variational equations. The vector $\vec{r}_s$ contains the corrections for polar motion, Earth tides, etc.

### 3.2.1.2.2 Accuracy

In the determination of polar motion, frequency measuring techniques are affected by most of the same kinds of errors as are techniques using laser-type DME on satellites. These are: error of a single measurement, error of the orbit, and errors not accounted for by theory. However, the Doppler technique involves not one instrument but about 23 and not one satellite but from one to five. Evaluation of the accuracy based on the accuracy of an individual station is, therefore, meaningless.

From data given by Anderle (1977), rms errors of as low as 13 cm were obtained. However, over a one-year period, the rms variation of differences from the coordinates was 0".029 and 0".022 in x and y, with differences on the average of 0".024 and 0".002 in x and y, respectively. These are slightly more than 0.5 m in each coordinate (except for the agreement with Bureau Internationale de l'Heure (BIH) in the y-coordinate). This would appear to be reasonable, given the Doppler technique's ability to determine geodetic locations to ±1 m, although somewhat better values should be expected over one-year's accumulation of data. A paper by Dr. M. Graber (1976) of GSFC seems to indicate a higher precision of polar coordinates derived by either the BIH or the IPMS. Since Graber used the maximum-entropy method for analyzing data on polar motion, and since this method gives results which in many cases are difficult to interpret, Graber's results can only be said to <u>indicate</u> the technique's general range of precision.

As in the laser-type DME technique, the accuracy of the derived polar coordinates depends on the accuracy of the IPMS coordinates. Agreement of Anderle's results with BIH results does not preclude the existence of large systematic errors, since such errors can also exist in the BIH coordinates.

### 3.2.2 Determination from Variation of the Latitude of the Moon

The procedure used at present for determining polar motion from measurements of distance to a corner-cube reflector on the Moon is similar to the procedure for determining polar motion using a similar reflector on artificial satellites (see section 3.2.1). The principal differences are the need for a larger telescope, since the amount of light returned from the Moon is considerably less (by a factor of about $10^6$), and the substitution of the Moon's orbit for that of an artificial satellite. The equipment, except for the scaling-up in size of the telescope, is much the same as that used for tracking artificial satellites. Characteristics are given in table 7.

#### 3.2.2.1 Computation

The theory used for error analysis purposes is basically that of the classical theory of the Moon (Hauser 1974, U. S. Naval Observatory 1952, Brown 1896, Eckhardt 1970, Arnold 1974) augmented by the theory of the physical librations of the Moon. In addition, corrections are introduced for the effects of Earth tides, atmospheric refraction, and calibration constants. The variational equation can be written

$$\Delta S \equiv S_{observed} - S_{assumed}$$
$$\equiv \sum_i B_i \, \Delta X_i \; .$$

Table 7.--Characteristics of laser-type DME for measuring distance to Moon

| Item | Type of telescope | |
|---|---|---|
| | McDonald Observatory | Proposed Mobile |
| Focal length (prime focus) | 10.8 m | ≈3.2 (?) |
| Aperture | 2.7 m | 0.8 m |
| Beam width | <1".5 | ≈1".5 |
| Pulse | | |
| Duration | 3 ns | 0.2 ~ 0.6 ns |
| Energy | 1.2 joules | 0.1 ~ 0.3 joules |
| Rate of repetition | 20 per minute | 180 ~ 600 per min. |
| Weight | Irrelevant | >2300 kg |

The quantities $B_i$ are calculable coefficients whose mathematical expressions are given in appendix B. This equation omits explicit mention of corrections for atmospheric refraction, relativistic differences, lunar libration, etc. These may be considered to be lumped into the terms that are shown. The $\Delta X_i$ are the corrections to the assumed values of p, r, $\delta$, $\xi$, $\eta$, $\kappa$, etc. (See appendix B.)

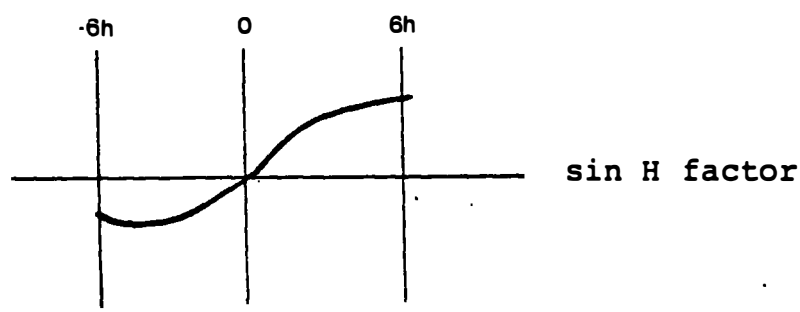
### 3.2.2.2 Accuracy

The values given by the LURE group in 1972 for the expected accuracy of the rotational period of the Earth and the position of the pole for one day's data are ± 0.1 msec and ± 0.005 m, respectively (Faller et al. 1973).

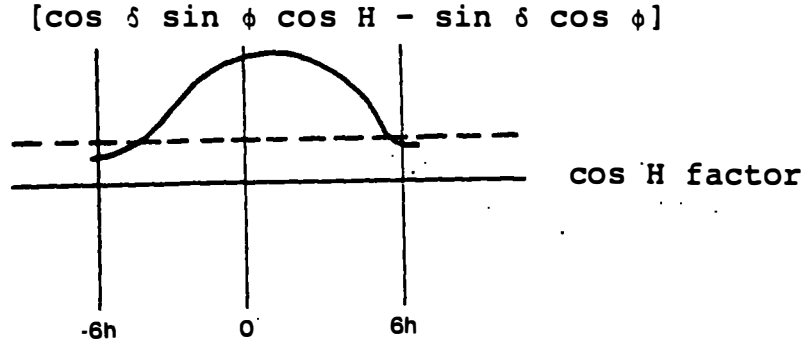
In evaluating the figures given for the potential accuracy of a system for determining the location of the pole, several factors should be considered. First, the locations of the tracking stations are very important. This is best seen by looking at the sensitivity of the measurable distance $\rho$ to the hour angle H and latitude $\phi$ of a tracking station. The pertinent equations are

$$\rho = r - R_E \cos\phi \cos\delta \cos H - R_E \sin\phi \sin\delta - X_M + \ldots$$

$$(\Delta\rho)_H = (R_E \cos\phi \cos\delta \cdot \Delta H) \cdot \sin H$$

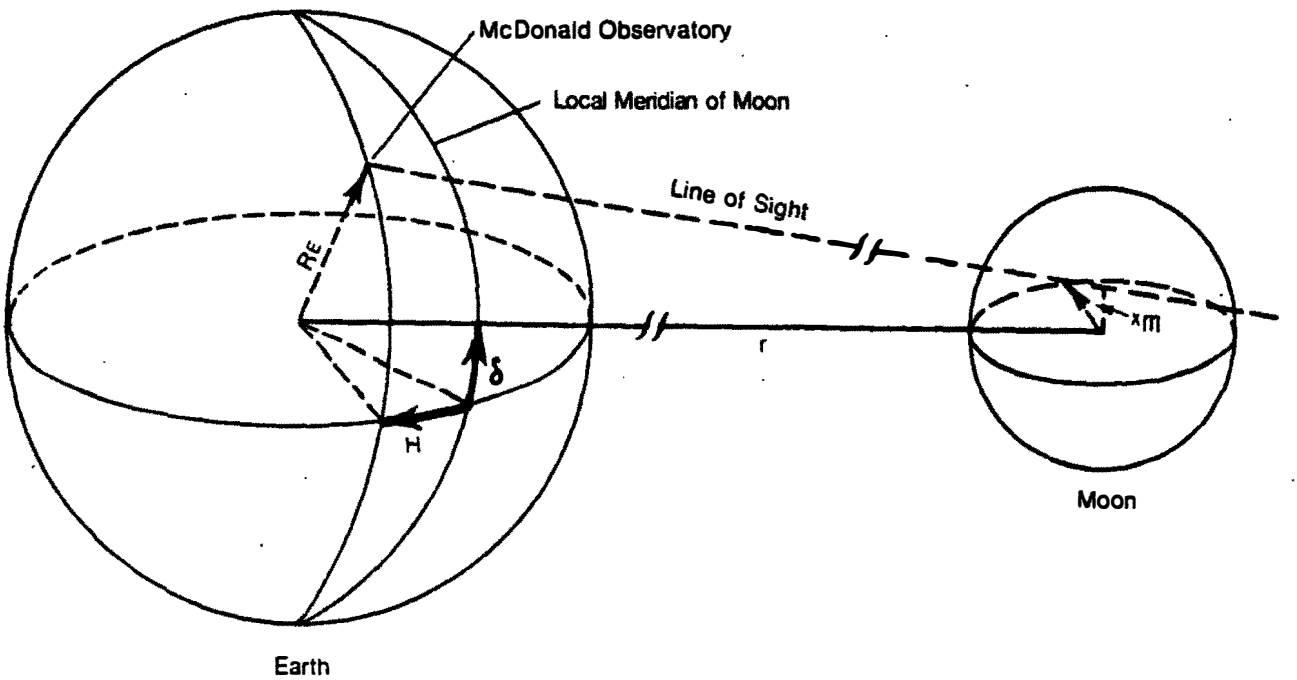



$$(\Delta\rho)_\phi = (R_E \Delta\phi)\ [\cos\delta \sin\phi \cos H - \sin\delta \cos\phi]$$




Here r is the distance from the center of the Earth to the center of the Moon; $R_E$ is the radius of the Earth; $\delta$ and H are declination and hour angle of the Moon; and $X_M$ the projection of the reflector's radius vector on the line of sight. From the equations and the corresponding graphs of $(\Delta\rho)_H$ and $(\Delta\rho)_\phi$ as functions of H, it is obvious that $(\Delta\rho)_\phi$ changes only slowly near passage through the meridian, while $(\Delta\rho)_H$ changes most rapidly and linearly there. Conditions for best determination are, therefore, somewhat opposite. Furthermore, it follows that one station can determine well only one component of the polar motion and that extreme latitudes are more suitable than the lower latitudes. It also follows, from the equations, that the

rotation can be determined somewhat better than the polar motion by a factor of about 2 (P. Bender 1975).




Another factor to be considered is the observing schedule. Data are sparse or lacking for a period of three to six days near new moon. This means there will be 13 gaps of three to six days each on daily polar motion during the year. Past records of polar motion do not appear to show any unpredictable departures of more than 0.1 m over six-day periods except in a few cases. These few exceptions, however, could well have resulted from errors in the data or in the method of analysis or both.

Another allied factor at present is the unreliability of the equipment. Dr. E. Silverberg of McDonald Observatory has remarked that the first few days of resumed tracking do not produce good data. It takes the operators some time to get the equipment operating again in a satisfactory manner.

### 3.2.3 Determination by Observation of Celestial Radio Sources

The pioneering work of H. Jansky and Groot Reber in the early part of this century showed that the Sun emits energy at radio frequencies and that, in fact, radio emission could be detected coming from all parts of the sky. This presumably rich field of astronomical research was left untouched until after World War II,

when the sudden availability of low-cost surplus radar and radio equipment created interest in using this equipment for radio-astronomy. The field gradually divided into interferometric astronomy and radar astronomy, according to the interests of the researchers. Almost all the early precise work was done with interferometers since these gave the best angular resolution (Fomalont and Wright 1974) and (K. Johnston 1975a). The technique of interferometry advanced somewhat slower than that of radar astronomy, however, because its accuracy was directly proportional to the linear distance between antennas. This meant that areas several kilometers in extent had to be devoted to the antennas, with the costs rising proportionally. (There are at least three varieties of radio-interferometric systems distinguished by the distances between telescopes as short base-line interferometers (SBI), less than 40 km; long base-line interferometers (LBI or ARIES), up to 1000 km; and very long base-line interferometers (VLBI), over 1000 km.) This obstacle was finally overcome in the 1960's with the introduction of highly accurate, relatively inexpensive, crystal and atomic clocks. Now, the ends of the interferometer could be separated as far as desired without astronomers having to rent or purchase the intervening land. At the present time, antennas which are thousands of kilometers apart have been used as ends of interferometric base lines, and there are discussions of eventually placing one end on an artificial satellite or on the Moon.

A radio interferometer consists basically of three components (Bolton 1960), (Fomalont and Wright 1974), (Clark and Erickson 1973): two sets of antennas for receiving radio signals from a source and a comparator for determining the difference in phase between the signals arriving at the two sets. (For convenience each set will be referred to from now on simply as an antenna.) The difference in phase between signals is directly proportional to the difference in distance from the two antennas to the source. (Note the similarity of this problem to that described in 3.2.1.2. A TRANSIT system can, in fact, be considered an inverted interferometer.) If the antennas are close enough together, the signals can be led to the comparator through ordinary coaxial cables (open wires have been used at lower frequencies). At greater distances, signals can be beamed from the antennas to a central receiver at the comparator (as has been done at the Greenbank Observatory, where the antennas were separated up to 30 km (K. Johnston 1975a). At still greater distances, the signals can be recorded at each antenna as functions of time. If the same time is kept at each antenna, the recordings can be recombined later to give almost the same result as if the signals had been sent directly to the comparator. (Because the process of recording and retranscribing into the comparator causes some signal distortion, there is a degradation from the direct method. However, any practical direct method also adds small amounts of distorting power to the signals. Probably, except for antennas very close together as those used

in PRIME MINITRACK and AZUSA, the direct and indirect ways of routing signal to the comparator are about equal in the amount of random distortion introduced.)

Table 8 gives the principal characteristics of several different types of radio interferometers that have been used for determining polar motion. Unfortunately, the data in the table cannot be considered particularly representative of equipment that might be used. One reason is that astronometric radio interferometry at its present stage of development will continue to be an experimental scientific technique for many years. Although the Massachusetts Institute of Technology (MIT) and the Jet Propulsion Laboratory (JPL) are the organizations doing most of the work where the technique is applied to geodetic problems, there are other organizations involved. As a consequence, experiments have been carried out between a large number of different antennas and at a large number of wavelengths, from the meter range down to the centimeter range. There are different ways of recording data at each antenna (e.g., in analog form as by the Canadian astronomers, or digitally as by MIT and JPL). Furthermore, there are interferometers planned, such as those proposed by K. Johnston (1975b) of the Naval Research Laboratory and a group at JPL (1975), whose physical characteristics are only conjectural. Table 8 gives a sampling of characteristics; there are no "representative" characteristics.

### 3.2.3.1 Computations

A radio interferometer may observe either the difference $\Delta\phi$ in phase of the signal at one antenna compared to the phase at the other antenna or the rate of change $\Delta\dot{\phi}$ of this phase difference with time. For discussion purposes, this quantity is called "phase." The equations frequently met with and given below are in terms of difference in time at the same phase. The two modes of formulation are actually equivalent, and no particular effort is made to distinguish between phase and time in this respect). The equations of observation and its variation with time are (Whitney 1974, Thomas 1972, Arnold 1974)

$$\tau = \tau_c + \frac{D}{c}\,[\sin\delta_B \sin\delta_S + \cos\delta_B \cos\delta_S \cos(H_B - H_S)] \qquad (1)$$

$$\frac{d\tau}{dt} = \frac{d\tau_c}{dt} + \frac{D}{c}\,\Omega \cos\delta_B \cos\delta_S \sin(H_B - H_S)\ . \qquad (2)$$

The observed quantity is assumed (for very long baseline interferometry) to be $\tau$, the difference in time of reception at the two antennas. D is the distance between antennas, $\Omega$ is the rate

Table 8.--Characteristics of radio-interferometric equipment

| Antenna | Diameter |
|---|---|
| | m |
| Arecibo, P. R. | 305 |
| Green Bank, W. Va. | 100 |
| Bonn, Germany | 100 |
| Jodrell Bank, Great Britain | 76 |
| Goldstone, Ca. | 64 |
| Algonquin Park, Ontario | 46 |
| Owens Valley, Ca. | 43 |
| Haystack, Ma. | 37 |
| Onsala, Sweden | 26 |

| Frequencies (MHz) | Reference |
|---|---|
| 8000 | Rogers (1973) |
| 7850 | Shapiro (1974) |
| 2695 | K. Johnston (1975b) |

| Clock | Type |
|---|---|
| MIT/GSFC | $H_2$ - maser |
| JPL | $H_2$ - maser |
| Owens Valley | Rb - maser + quartz clock |

| Recorder | Bandwidth (MHz) | Records | Used by |
|---|---|---|---|
| NRAO MI | 0.360 | $3^m$ | MIT/GSFC |
| NRAO MII | 2 | $480^m$ | Most others |
| JPL | 0.024 | $12^m$ | JPL |

of rotation of the Earth. $\delta$ and H are declination and hour-angle, respectively; while subscripts B and S refer to the baseline and radio source, respectively. The baseline is the line joining the electrical centers of the two antennas; its declination and hour angle are the $\delta$ and H of the point in the sky at which a specified end of the baseline points. $\tau_c$ is the difference in time shown by the two clocks for a common epoch. It is represented as a linear function of "true" time,

$$\tau_c = a + bT \tag{3}$$

The constants a and b are then also unknowns. Equation (2) is more commonly written in terms of frequency as

$$f_r \equiv \omega \frac{d\tau}{dt},$$

where $\omega$ is the frequency of the signal received.

Note that the hour angles of the baselines and sources occur only as a difference of $H_B - H_S$. Therefore, the hour angles, themselves, cannot be found but only their differences. For this reason, unaided radio astrometry cannot provide the right ascensions of radio sources or longitudes of antennas.

The velocity of the radio waves can be considered an unknown also, in which case one would write c as the sum of the velocity $c_o$ of light in a vacuum and an unknown variation $\delta_c$ caused by atmospheric refraction. It is the atmospheric refraction which now sets the limit on the accuracy of the technique.

If the baseline is not too long (less than 50 km), the signals may be conducted from the antennas to the comparator by cable or over microwave beams. In this case, the observables would be phase difference $\Delta\phi$ and rate of change $\Delta\dot{\phi}$ of phase difference. The quantity $\tau_c$ and its unknowns a and b (equation 3) would then be eliminated. (The advantage gained by this technique is greater than merely the dropping of one or two unknowns: several contributors of irrelevant random variations in amplitude and phase are also eliminated.)

The quantities $\tau$ and $d\tau/dt$ are found by comparing the signals arriving at the two antennas. The comparison is made by taking an interval of time in which the signal is known to have been received at the antennas. The two signals in this interval are broken up into subintervals with the subintervals from one antenna correlated with each subinterval from the other antenna. The exact procedure depends on finding the subintervals of maximum correlation (Whitney 1974, Thomas 1972).

### 3.2.3.2 Accuracy

A report (Shapiro 1974) on the results obtained from nine experiments in 1972 and 1973 gives the differences between the x-component of polar motion as found by the MIT/GSFC group and by the BIH. The actual discrepancies range from -4.4 m to +2.2 m. All experiments were carried out between the antenna at Goldstone, California, and the Haystack antenna at Westford, Massachusetts, a distance of about 3900 km. But the two antennas are separated in latitude by about 860 km, so the precision for variation of latitude would not be as great as for variation in time. The 860 km is still about 30 times as great as the 25 km

separation (in latitude) used by K. Johnston (1975b). Johnston believes that a 16-km baseline should give about 0.3-m accuracy (private conversation).

Another way of estimating accuracy is to list all known possible sources of error, estimate their contributions, and add the contributions statistically. Table 9 itemizes the major sources of error and their estimated contributions (JPL 1975, Moran 1974, Clark and Erickson 1973, Shapiro 1974.) There is obviously considerable variation among the various estimates. One reason for this variation is the difference in equipment and mode of operation. Jet Propulsion Laboratory's figures do not apply strictly to baselines much longer than 1000 km, while MIT/GSFC figures are for baselines as long as 5000 km. Another reason is in the mode of estimation. It seems that MIT/GSFC estimates that, with the existing equipment, individual errors (exclusive of the geophysical ones) contribute less than 20 cm to the total error, while equipment planned for 1980 would have individual errors which contribute less than 5 cm. The Jet Propulsion Laboratory's equipment for 1980 would give a total error (minus geophysical ones) of less than 10 cm. Considering the many assumptions made in arriving at the estimates, this is close agreement between the two groups' estimates.

Table 9.--Estimate of effects of various errors on length found for baseline

| Origin of error | Effect on length of baseline (cm) | | | |
|---|---|---|---|---|
| | MIT/GSFC* | | JPL Aries** | |
| | Now | 1980 | Now | 1980 |
| Correlation | 3 to 6 | 0.5 to 3 | 4 | 4 |
| Refraction | | | | |
| Charge-free | 11 | 3 | 6 | 1 |
| Ionosphere | 6 | 0.5 | 8 | 1 |
| Clock | 3 | 2 | 8 | 3 |
| Antenna & misc. | 5 | 2 | -- | -- |
| Cable | 10 | 0.5 | -- | -- |
| Miscellaneous | 7 | 2 | -- | 1 |
| Geophysical | 10 | 1 | 6 | 1 |

* For baselines ≈ 5000-km long
** Baselines less than 300-km long

The errors expected in data from present-day equipment have been estimated from the errors contributed by such sources as refraction, phase-measurement, etc. They are an order of magnitude smaller than the variations of VLBI's values from BIH's values, with which they have been compared. Furthermore, weighted averages of VLBI's values show agreement with BIH's values to 0.2 m. Finally, BIH's values differ at times from those of ILS by over 1 m, and a good part of this difference is likely to be systematic. These facts indicate, first, that the errors in VLBI estimated from errors in contributory sources are optimistic, since BIH's values are very unlikely to contain errors of 2-4 m. Secondly, the close agreement between BIH's values and averaged values from VLBI can indicate the presence of systematic errors in the VLBI's values if there are systematic errors in BIH's values. However, we do not yet know for sure whether the differences between BIH and ILS values are systematic or not or, if they are, with which service they originate. Further study of this matter is needed before we can find out how large the systematic errors in VLBI are.

## 4. RELATIONSHIP OF POLAR MOTION DATA TO THE CONVENTIONAL INTERNATIONAL ORIGIN (CIO)

A very important consideration in determining the future of the observatories is the necessity for preserving the past. The history of the pole's wanderings is contained in a set of data relating the instantaneous position of the pole since 1900 to the meridian of Greenwich and the CIO. Future observations must relate the pole's position to the same system of coordinates. Will this be possible if the instrumentation at the present observatories is changed or if observations are made from new locations? It is the opinion of some astronomers that in order to retain the CIO, the present locations must be continuously occupied. There are even a few who think that the present equipment must also be retained.

Neither requirement is, in general, necessary. The CIO is defined, not in terms of the locations of the observatories, but as the average position of the pole during the period 1900 to 1905. From the theoretical point of view, therefore, it makes no difference where the observatories are located. The reason for the opinion that the CIO is recoverable only from the original locations of the observatories is that the definition specifies the years 1900 to 1905, and the only useful data from these years come from the ILS observatories of that time. On the other hand, by referring the position of the present pole to the CIO, the ILS stations also allow the CIO pole to be located with respect to the present position of the pole and also to the present positions of the stations. That is, the equation $[x_o] = [A_o][\Delta\phi_o]$ or

$$x = -0.4359\ \Delta\phi_M + 0.1227\ \Delta\phi_K + 0.4483\ \Delta\phi_C + 0.1232\ \Delta\phi_G - 0.2583\ \Delta\phi_U$$

$$y = -0.2636\ \Delta\phi_M - 0.3133\ \Delta\phi_K + 0.0172\ \Delta\phi_C + 0.3382\ \Delta\phi_G + 0.2559\ \Delta\phi_U \quad (1)$$

$$z = +0.2305\ \Delta\phi_M + 0.2007\ \Delta\phi_K + 0.1755\ \Delta\phi_C + 0.1850\ \Delta\phi_G + 0.2082\ \Delta\phi_U$$

where the suffixes M, K, C, G and U of $\Delta\phi$ stand for Mizusawa, Kitab, Carloforte, Gaithersburg, and Ukiah respectively give the x, y, z coordinates of the present location of the pole and, in effect, the coordinates of the CIO. If the coordinates of the present pole are $x_p$, $y_p$, then the coordinates of the CIO are 0,0. Or, if we set the coordinates of the present pole as 0,0, then the coordinates of the CIO are $-x_p$, $-y_p$.

Suppose we have another set of observatories with different latitudes and instrumentation capable of determining astronomic latitudes. The equations for these stations are also

$$[x] = [A]\ [\Delta\phi] \quad (2)$$

where the elements of [A] and $[\Delta\phi]$ are different from the values in (1).

The values we get for x, y, z from these equations will differ from those gotten from the first set of equations and will depend on the coordinate system we relate to the second sets of observatories. But if the first and second sets of observatories are in operation over the same period of time, we can set up the condition that the coordinates x, y, z found by both sets be identical. Equation (2) can then be solved for the parameters [A], since the x, y, z are then known and would be completely suitable for determining x, y, z after the first set had ceased observing.

There are a few additional comments on the problem which may be interesting. First, if continuation of the original ILS stations were necessary for keeping the CIO, we would really be in trouble. One of the original observatories, that at Cincinnati, ceased observation many decades ago. Another, that at Tchardjui, was dropped for a while and then replaced by a different but nearby station at Kitab.

Also, we do not know for certain whether the geodetic and astronomic coordinates of the ILS stations have been changing or not. There is conflicting evidence on this point. Our definition of the CIO and our entire concept of the usefulness of the

observatories' data depend on the assumption that the changes in latitude of the stations are either due entirely to the motion of the pole or to changes that we can otherwise account for (such as changes in the stars' coordinates, etc.). If the observatories are drifting with respect to each other and this is not detectable, then a serious problem exists.

The IPMS, by its continued operation of the stations of the ILS, provides, correspondingly, a continuous and, therefore, valuable record of the motion of the instantaneous pole with respect to the CIO. This is another matter not strictly related to the CIO recoverability.

## 5. REQUIREMENTS FOR DATA ON POLAR MOTION

### 5.1 Organizations Producing Polar Motion Data

Two of the three major sources of information on polar motion have been listed previously. They are the International Polar Motion Service and the Bureau International de l'Heure. The third is the Doppler Polar Motion Service (DPMS). Only the IPMS has an officially acknowledged and worldwide responsibility for determining polar motion. The BIH produces polar-motion data from data of the IPMS and from data provided by a large number of independent observatories. The DPMS is a part of the U.S.A Department of Defense (DoD), and has no official commitments outside the Department.

In addition to these three, there are a few other sources of data on polar motion. These have no known official commitments or responsibilities for providing data on polar motion, but do so largely as the result of isolated scientific investigations. These are the NASA Goddard Space Flight Center, the Jet Propulsion Laboratory, the MIT/Goddard Space Flight Center, Lunar Retrodirective Reflector Tracking Team, and various other scientific groups outside the United States, particularly in Canada and Europe.

Also, many observatories provide the variations of latitude at the observatory. Most of this information is sent to the IPMS or the BIH where it is used in the solution for x and y.

Appendix C gives the names and addresses of the organizations and individuals who produce data on polar motion.

### 5.2 Users of Information on Polar Motion

At present, the two large demands for information on polar motion concern (1) time correction, and (2) correction of astronomic observations for longitude, latitude, and azimuth. There are also a number of small well-established sources of demand for polar motion data, almost all involved in the

reduction of data for the tracking of artificial satellites--but differing according to the use to which these data are put. They are (1) users of satellites for navigation--primarily DoD's TRANET system; (2) users of satellites for creating geodetic systems--primarily DoD, NASA, and NGS in the U.S., but also Institut Geographique National (IGN) and Centre National d'Etudes Spatiales (CNES) in France, and presumably various organizations in the U.S.S.R. and (3) users of satellites for scientific investigations of the moons and planets of the solar system--primarily JPL in the United States and spacecraft organizations in the U.S.S.R.

Lastly, we can identify a number of isolated scientists interested in any relationships between polar motion and (1) earthquakes (2) the Earth's interior, particularly the mantle and crust (3) continental drift, and (4) climate.

### 5.3 Identifying Organizations Using Information on Polar Motion

The best method of identifying those who have a need for information on polar motion is to list the recipients of such information.

The IMPS and the DPMS were asked for lists of persons and organizations receiving their data. So far, the DPMS has provided a list, which is attached as appendix E. No request was made of the BIH since the list of people and organizations receiving the BIH bulletins includes both those interested in time and polar motion, with no way to separate the two categories.

From the number of scientific papers being published on the analysis and use of polar motion data, it can be estimated that fewer than 1000 scientists in any one year are actually working with polar motion for scientific purposes. This is also based on fewer than 150 attendees at the NATO conference at London, Canada, in 1969, and fewer than 150 papers between about 1965 and 1973 quoted by Rochester (1973). In these works, the Russian effort is not adequately represented; but it and that of China and countries within the sphere of influence of Russia and China can be estimated to be about equal to that of the rest of the world. So we would expect about 300 scientists in all, many of whom would be working in applications such as determination of time, latitude, etc. Assuming that each scientist spends about $2,000 on research, i.e., is funded specifically for such research, we can estimate that less than $1,000,000 is spent annually on scientific research connected with polar motion per se. This does not include the amount spent on research in which polar motion is involved but not explicitly studied. This figure may be too high, by at least 50 percent, however, since most of the scientists involved with

polar motion are employed by universities or colleges primarily for other kinds of work.

## 5.4 Estimates of Allowed Error

As far as this study has been able to determine, there are no legally enforceable requirements for definite accuracies in polar motion. Nor do there appear to be any clear administrative directives fixing the accuracies with which polar motion should be obtained. Conclusions about allowable rms errors for various applications must be based mostly on reasonable estimates of what the requirements should be. The values given in the following sections, therefore, rely heavily on published reports of what is being done or, in the case of planetary-spacecraft tracking, on statements by workers in the field.

It is not possible to convert users' limits on allowable errors in location of the pole into standard limits in either the determination of the pole's position or the frequency with which it is given. At present, such conversion is based on assumptions of dubious validity. Nevertheless, such conversions will be for planning the exact manner of matching capabilities to users' requirements.

### 5.4.1 Errors Allowed for Planetary Exploration

A shift of 10 m in the position of the instantaneous pole corresponds to a shift of about $1.5 \times 10^{-6}$ radians in the orientation of a baseline under worst conditions. If a spacecraft location is determined by intersection from the two ends of a baseline and the shift is not known, an error is introduced into the calculated location of the spacecraft. In the worst case, this error amounts to about 100,000 km at the distance of Jupiter and about 6,000,000 km at the distance of Neptune. These errors, however, are much greater than those actually incurred since they do not take into account the constraints imposed by orbital mechanics or data from the craft themselves about their location.

### 5.4.2 Errors Allowed for Determination of Time

At present the unit of time and the epoch from which time is measured are, by international agreement, based on atomic clocks and frequency standards, not on the rotation of the Earth. There is, therefore, no requirement for information on the instantaneous axis of rotation in order to determine time (AT) in this sense. However, there are still in use several kinds of "time" which depend on the rotation of the Earth and location of the pole for their determination. These are UTO, UTI, UT2 and UTC. These "times" date back to when time was determined astronomically by the rotation of the Earth. The first three denote, respectively, rotation turned directly into time, rotation corrected for polar motion and then turned into time, and the

latter corrected rotation further corrected for irregular seasonal variations and then turned into time. Since time is no longer defined by the Earth's rotation, these three particular times are no longer as important as formerly. The corresponding rotations UTO and UTI, however, are astronomically important and are also important geodetically to the extent that geodesy makes use of astronomy. UT1 is derived from UTO and the coordinates of the instaneous pole through equation,

$$\Delta T = (-x \sin \lambda_{0,i} + y \cos \lambda_{0,i}) \tan \phi_{0,i}$$

$$\doteq UTO_i - UTI$$

where x and y are the coordinates of the instantaneous pole and $\lambda_{0,i}$, $\phi_{0,i}$ are the coordinates of the stations used in determining x and y.

Since people still run their lives by the rising and setting of the Sun, AT cannot be allowed to get too far out of step with UT. Hence, UTC, the time broadcast for ordinary use, is occasionally AT-adjusted to keep time determined by atomic clocks from differing too much from UTC.

### 5.4.3 Errors Allowed by Astronomical Requirements for Polar Motion

Astronomical measurements are usually referred to either the instantaneous axis of rotation or to the axis of figure. The latter is the usual referent. When measurements are referred to the instantaneous axis of rotation, the allowable error in location of this axis can be estimated from present requirements of precise astrometry. This would be the requirements of fundamental work with meridian transits and like instruments. The rms error of a single observation in fundamental work is on the order of 0".3 (Scott 1963). A requirement for the polar motion to be known to better than 0".03 (1 m) would, therefore, be quite conservative, with a requirement of 0".1 being reasonable.

Coordinates of fundamental stars are customarily given to 0".01. I do not know of any application which requires the instantaneous pole's coordinates to be known to the same precision. If such an application exists, it implies a need for the knowledge of x and y to better than 0".01 or to better than 30 cm.

### 5.4.4 Sampling Frequency

The practical approach to finding how often coordinates of the pole should be determined is to make the interval between

determinations sufficiently small that all known or guessed-at systematic variations will be detected. M. Graber (1976) has analyzed a particularly long set of data (15 years) from BIH, IPMS, and DPMS. He used, not Fourier analysis, but the method of maximum entropy (Burg 1972, Ulrych and Bishop 1975) to bring out the period constituents of the data. This method has been subjected to considerable theoretical criticism, particularly for its ability to create periods not in the data. By using Graber's results, we will (if we err) be erring on the safe side by choosing shorter intervals than are required. Graber's analyses show peaks at periods of about 365 days (the annual term attributed to atmospheric variations), 434 days (the Chandlerian motion) 275 days (unexplained), and at several other unexplained periods such as 145 days and ± 73 days. The shortest period here is 73 days. Two other periods, those at 145 days and 434 days, are nearly exact multiples of the shortest period. The other periods are not. Hence, if the interval between determinations is half the shortest period, all the variations found by Graber will be identifiable. This interval is about 37 days, or about 1 month. This is about the interval already in use by the IPMS. It seems prudent, however, to select an interval half as large, or about 20 days. This keeps the sampling interval from falling into step with the lunar month and introducing possible spurious frequencies arising from nutation and precession. Also, the 73-day period would be identifiable from smaller sets of data.

Another approach to selecting a sampling interval would be to take one-half or one-fourth the shortest period of scientific interest. This approach is not used because the term "of scientific interest" does not allow one to set a realistic bound on the shortest period. For example, it is believed possible that the Chandlerian motion and other components of polar motion are sustained by the energy set free by earthquakes. The shortest period involved is then the interval between earthquakes sufficiently large to maintain the motion. If we assume there are more than 4,000 earthquakes yearly of all sizes (Stacey 1969) and that 10 percent of these are sufficiently strong to affect the Chandlerian motion, this works out to an interval of less than 12 hours. The effects of such earthquakes on Chandlerian motion would, however, probably not be detectable from observations. An earthquake the size of the Alaskan earthquake of 1964 (Stacey 1969) would change the polar coordinates by less than 5 cm. Most earthquakes would have much smaller effects. Hence, designing a program to satisfy hypothetical scientific requirements is impracticable and unwarranted.

If we take 20 days as the largest interval between determinations of polar motion, the intervals between measurements will have to be correspondingly shorter, so that the average of the variances of the individual observation will come down to below the variance desired for the polar coordinates. Therefore, what the interval is will depend on the technique selected and

on the allowable variance in the pole coordinates. All techniques investigated have measurement intervals theoretically less than two days, except for the one (LURE) measuring distances to the Moon. This technique has a gap of three to six days once every lunar month. All the optical techniques, of course, have actual measurement-intervals randomly longer than the theoretical ones because of the effects of weather.

## 6. COMPARISON OF TECHNIQUES FOR DETERMINING POLAR MOTION

Six different kinds of instrumentation have been described in the previous sections: visual zenith telescopes, photographic zenith telescopes, frequency-measuring equipment of the TRANSIT-type, optical radar used on satellite LAGEOS, optical radar used on corner-reflectors on the Moon, and very-long-baseline-interferometry. While any one kind can be used alone for finding polar motion, an organization seriously interested in determining polar motion as a service to others may find that better results are obtained by combining information from several kinds of instrumentation. To aid in planning such a combination and to summarize some of the relevant characteristics of the methods, the methods and equipment are compared in table 10. In addition to the factors considered in the table, there are others, less clearly definable, which should be considered. Among these would be the capability for future improvement of the method, availability of the equipment on loan or on a cooperative basis, and expected continuing demand for the results. The last is perhaps the most important of all and the most difficult to define. (In fact, Parkinson's law would imply that the only possible conclusion is that there will be a permanent demand for results of ever-increasing accuracy). All the values in the table for methods involving DME and VLBI must be considered with the likelihood in mind that there will be considerable changes during the next years from the values given in the table. The figures given for VZT, PZT, and FME methods are not likely to change (unless we consider the Global Positioning System a variant of the FME), but costs and standard errors for the other methods (shown in the last three columns) are likely to decrease considerably, particularly for VLBI.

Table 10.--Comparison of characteristics relevant to planning a system

| | VZT | PZT | FME | DME + LAGEOS | DME + MOON | VLBI |
|---|---|---|---|---|---|---|
| Cost per set (in $1,000) | | | | | | |
| Equipment | N.A. | 150 | 100[b] | 1,000 | 1,000 | 2,000[c] |
| Operating (per annum)[a] | 50 | 45 | 40 | 175 | 175 | 270 |
| Number required | | | | | | |
| Minimum | 3 | 3 | 1[e] | 1 | 1 | 2 |
| Good | 5 | 5 | 20(?)[e] | ? | 3 | 4 |
| Standard error (m) | 3 | 1[d] | 0.5 | 0.1 | 0.05 | 0.01 |
| Smallest interval between positions (days) | 5 | 5 | 2 | 1 | 1 | 2 |
| Other results | | | | | | |
| Time | - | X | - | - | X | X |
| Tides | - | - | - | X | X | X |
| Lunar orbit | - | - | - | - | X | ?[f] |
| Special disadvantages | | | | | | |
| Finite life of system | - | - | X | - | - | - |
| Periods of no data | - | - | - | - | X | - |
| Weather dependent | X | X | - | X | X | - |
| Skilled personnel required | X | - | - | X | X | X |

[a]Including 10% amortization, $20,000 salary per person, $10,000 computation. Maintenance costs not included.

[b]Does not include satellites.

[c]This will probably come down considerably. By 1980, cost may be less than $1,000,000.

[d]Estimated by comparison with VZT.

[e]While data from one station can be used to determine the variation of latitude at that station, a precise orbit must first be available. At present, this precise orbit is the result of observations from over 20 stations. Realistically, about 20 stations are needed for accurately determining variation in latitude.

[f]If suitable relay is on the Moon.

## 7. CONCLUSIONS

The program of observations carried out by the NGS at the observatories of Ukiah and Gaithersburg is part of an international program of observation and reduction that has lasted for 75 years. The program was set up at an international conference at Stuttgart in 1898, was begun in 1899, and has continued virtually unchanged up to the present time. It provides the only available set of homogeneous data on polar motion over so long a period and defines, through the coordinates of the five basic observatories, the location of the Conventional International Origin.

The precision of individual observatories is generally considered to be equivalent to about ± 3 m in the coordinates of the pole; the precision of the coordinates at monthly intervals is about ± 30 cm. There is, however, some question as to the accuracy of these values. There is, furthermore, a need in some of the sciences for higher precision and smaller intervals between determinations of coordinates. These two problems can be solved by using some technique (or combination of techniques) other than the Horrebow-Talcott method now relied on by the program. The techniques that appear applicable to the problem are use of PZT, frequency-measuring equipment like the Geoceiver laser-type distance measuring equipment, and a reflector on artificial satellites (Lageos) or the Moon (LURE) and VLBI.

Some of these techniques make possible the attainment of accuracies of better than ± 10 cm at intervals as short as two days. Such values are well within the limits on results satisfying known requirements. But no single technique is likely to provide the best data for any application, particularly not for a program requiring homogeneous data, continuity of results, and economy of operation.

## APPENDIX A. DISCUSSION OF "NUMERICAL PROCESS OF THE ANALYSIS OF THE ROTATION OF THE EARTH: 1. FUNDAMENTAL EQUATIONS OF THE ROTATION OF THE EARTH," BY S. TAKAGI, PUBLICATION INTERNATIONAL OBSERVATORY MISUZAWA 7 (2), 149-172, 1970.

One of the simpler problems in mechanics is that of finding the equations governing the rotation of an isolated, rigid body. Once either of these qualifications is abandoned or modified, however, the problem is no longer simple. Klein and Sommerfeld, at the turn of the century, published a two-volume work on this problem; they considered principally the motion of rigid or elastic bodies in a gravitational field and still were far from exhausting the possibilities of the problem. The Earth is neither isolated nor rigid. Its motion is not only affected by the gravitational field of the Sun and Moon but its magnetic field interacts with the ionized gases of space, and the Earth is irradiated by solar radiation. The Earth itself is a source of energy that is responsible for a partially liquid interior, for sudden changes in the crust, and possibly for slow shifts in the crust and mantle. Hence, the Earth is so complex and is moving in an environment so much more complicated than that consisting of a single gravitational field, that no analytic solution is possible. The problem of determining the rotation of the Earth from theory must be tackled by first expanding the number of factors involved and then solving the resulting equations by numerical analysis rather than by looking for analytic expressions.

Dr. Takagi, in the paper cited, has begun work on the problem using this approach. He has gathered the primitive equations of motion of the principal mechanical components of the Earth - the core, the mantle, the oceans, and the atmosphere and combined them (by suitable choice of coordinated systems) into Lagrangian equations. It is these equations which are the basis for the subsequent numerical analysis. Because they are extremely lengthy, they will not be given here. I believe, however, that a few comments on the equations may be helpful in guiding further development.

(1) The approach taken by Dr. Takagi is to set up a completely analytical expression for the forces and motions involved. This may not be the most practical approach. In the case of the motions of the atmosphere and oceans in particular, any analytical description adequate for the problem will be costly to program and time-consuming in computation as compared with an empirical (numerical) description of the atmospheric and oceanic motions. While Takagi does, at the outset, state explicitly that he favors the numerical over the analytic approach, the equations given and apparently intended for use are not anywhere as simple as a numerical approach (in the strict sense) would give.

Furthermore, an important step in setting up the equations should be an analysis of the sensitivity of the polar motion to the quantities and terms involved in these equations. I assume that this analysis will be carried out after the equations have been programmed. By doing the analysis before programming, a considerable saving in time is possible.

(2) The equations do not explicitly cover movements in the crust. It is possible that, for the purposes of this work, Takagi considers the crust a part of the mantle. If this assumption is made, the crust will have to be treated as a layer of the mantle having different properties than the rest of the mantle. So nothing has been gained by lumping crust and mantle together conceptually; and a definite probability exists of excluding important phenomena peculiar to the crust. Also, perhaps, some confusion has been caused by going against general usage.

(3) Takagi considers an inner (solid?) part and an outer (liquid) part of the core. This is reasonable. However, he introduces a special set of axes for the outer part with the z-axis pointing in the direction of the magnetic pole. I do not think that our present knowledge of the interaction between core and magnetic field is extensive or sure enough to support a theory meaningful for polar motion. What is likely to happen is that introducing the magnetic field's interaction will merely introduce a few more degrees of freedom into the equations. This will allow a better fit of data to theory, but the theory itself will have no real meaning. Furthermore, Takagi apparently does not take account of the figure for the outer part of the core. The figure, it seems, may be more important for polar motion than the magnetic interaction.

(4) In summary, Dr. Takagi has taken a considerable forward step in setting up a comprehensive theory for the mechanics of polar motion. In its present state, however, the theory is probably too cumbersome to be usable on a large computer. Further development will have to be in the direction of slimming down the theory by removing those parts not significant for polar motion and by substituting, especially in the theory of atmospheric and oceanic motions, tabular values for analytic expressions.

## APPENDIX B. COEFFICIENTS IN OBSERVATIONAL EQUATION FOR TRACKING OF MOON

$$B_1 \equiv \left(\frac{1}{S}\right)\vec{D}\cdot\vec{P}_2 \qquad B_4 \equiv -\left(\frac{1}{rS}\right)\vec{D}\cdot(\vec{x}_M-\vec{y}_R)$$

$$B_2 \equiv \left(\frac{P}{S}\right)\vec{D}\cdot(\partial\vec{P}_2/\partial\lambda) \qquad B_5 \equiv -\left(\frac{r}{S}\right)\vec{D}\cdot\bar{\bar{R}}_1(\partial\vec{P}_1/\partial\alpha)$$

$$B_3 \equiv \left(\frac{P}{S}\right)\vec{D}\cdot(\partial\vec{P}_2/\partial\phi) \qquad B_6 \equiv -\left(\frac{r}{S}\right)\vec{D}\cdot\bar{\bar{R}}_1(\partial\vec{P}_1/\partial\delta)$$

$$B_7 \equiv -\left(\frac{r}{S}\right)\vec{D}(\partial\bar{\bar{R}}_2/\partial\xi)\bar{\bar{R}}_1\vec{P}_1$$

$$B_8 \equiv -\left(\frac{r}{S}\right)\vec{D}(\partial\bar{\bar{R}}_2/\partial\eta)$$

$$B_9 \equiv -\left(\frac{r}{S}\right)\vec{D}(\partial\bar{\bar{R}}_1/\partial\theta)\vec{P}_1$$

$$B_{10} \equiv B_9\cdot t$$

| | |
|---|---|
| $\vec{x}_E$ | location of tracking station |
| $\vec{x}_M$ | location of Moon |
| $\vec{x}_R$ | location of reflectors |
| $\vec{y}_R$ | vector from center of Moon to reflector |
| $\bar{\bar{R}}_1$ | dyadic rotating about Earth's mean axes of rotation (MAR) |
| $\bar{\bar{R}}_2$ | dyadic rotating from MAR to instantaneous axes of rotation (IAR) |
| $\vec{P}_1$ | direction of reflector |
| $\vec{P}_2$ | direction of station |

| | |
|---|---|
| $\rho$ | distance from telescope to reflector |
| $r$ | distance from Earth to Moon |
| $p$ | distance from Earth to telescope |
| $q$ | distance from Moon to reflector |
| $t,\theta$ | time (atomic), time, sidereal |
| $\vec{D}$ | $\vec{x}_E-\vec{x}_M+\vec{x}_R$ |
| $\xi,\eta$ | coordinates of IAR |
| $\alpha,\delta$ | right ascension & declination |

## APPENDIX C. ORGANIZATIONS PRODUCING DATA ON POLAR MOTION

1. Data produced regularly by:

International Polar Motion Service
International Latitude Observatory
Dr. Shigeru Yumi, Director
Misuzawa-shi
Japan

Bureau International de l'Heure
Dr. Bernard Guinot, Director
61 Avenue de l'Observatoire
Paris 14eme
France

Doppler Polar Motion Service
Defense Mapping Agency Topographic Center
6500 Brooks Lane
Washington, D. C. 20315

2. Data produced sporadically by:

National Aeronautics & Space Administration
Goddard Space Flight Center
Dr. David E. Smith
Greenbelt, Maryland 20771

Department of Earth and Planetary Sciences
Massachusetts Institute of Technology
Dr. I. I. Shapiro
Dr. C. C. Counselman, III
and associates
Cambridge, Massachusetts 02139

LURE
McDonald Observatory
Dr. Eric Silverberg
Fort Davis, Texas 79734

Jet Propulsion Laboratory
Dr. N. A. Mottinger
and associates
4800 Oak Grove Drive
Pasadena, California 91103

3. Data on variation of latitude produced by:

U. S. Naval Observatory
Washington, D. C. 20390

Royal Greenwich Observatory
Herstmonceaux
England

Pulkova Observatory
Leningrad
U.S.S.R.

## APPENDIX D. PERTURBATIONS OF THE CHANDLERIAN MOTION

Perturbations in the Chandlerian motion must come from forces within the Earth, since forces applied from outside the Earth would give rise to nutational perturbations. Hence, the extreme forces $N_i$ in the first equation of section 2.2 are zero. If we assume that the moments of inertia $I_1$ and $I_2$ are initially equal and are changed by small amounts $\delta I_1$, $\delta I_2$ and that $I_3$ is kept unchanged, we get

$$I_1\delta\dot{\omega}_1+(\dot{\omega}_1-\omega_2\omega_3)\delta I_1+(I_3-I_1)\omega_2\delta\omega_3+(I_3-I_1)\omega_3\delta\omega_2 = 0$$

$$I_1\delta\dot{\omega}_2+(\dot{\omega}_2-\omega_3\omega_1)\delta I_2+(I_1-I_3)\omega_1\delta\omega_3+(I_1-I_3)\omega_3\delta\omega_1 = 0$$

$$\omega_3 = K/I_3 \ ,$$

where K is a constant of integration. On assuming that $\delta\dot{\omega}_1$ and $\delta\dot{\omega}_2 << 1$ and tnat $\dot{\omega}_2$ are zero over the period we are interested in, we qet, after some algebraic manipulation,

$$\frac{\delta\omega_1}{\omega_1} = \frac{\delta I_2}{I_1-I_3} \quad ; \quad \frac{\delta\omega_2}{\omega_2} = \frac{\delta I_1}{I_3-I_1} \ .$$

Letting $\dot{\theta}_i = \omega_i$, we have

$$\delta\theta_1/\theta_1 = \delta I_2/(I_1-I_3);$$

$$\delta\theta_2/\theta_2 = \delta I_1/(I_3-I_1).$$

## APPENDIX E. WEEKLY AND MONTHLY DISTRIBUTION LISTS FOR POLAR MOTION

Weekly list - as of October 23, 1974

Dr. Glenn Hall
U. S. Naval Observatory
Washington, D. C. 20390

Dr. B. Guinot
Bureau International de l'Heure
61, Avenue de l'Observatoire
75014 Paris, France

Dr. Shigeru Yumi
Director
Central Bureau of the IPMS
International Latitude
Observatory
Mizusawa-shi, Iwate-Ken
Japan

Topographic Center Code 52322
Defense Mapping Agency
Washington, D. C. 20315

Dr. J. Popelar
Gravity Division
Earth Physics Branch
3 Observatory Circle
Ottowa, Ontario, Canada

Mrs. N. Capitaine
Observatoire de Meudon
92190 Meudon, France

U. S. Naval Surface Weapons Center
Dahlgren, Virginia

Dr. David E. Smith
National Aeronautics and Space Administration
Goddard Space Flight Center, code 921
Greenbelt, Maryland 20771

Aerospace Center
Defense Mapping Agency, code PRA
St. Louis, Missouri 63118

Monthly list - as of October 7, 1974

Dr. Chikara Sugawa
International Latitude
Observatory of Mizusawa
Mizusawa-shi, Iwate-Ken
Japan

Jet Propulsion Laboratory
4800 Oak Grove Drive
Mail Stop 156-220
Pasadena, California 91109

Dr. Ivan I. Mueller
Dept. of Geodetic Science
Ohio State University
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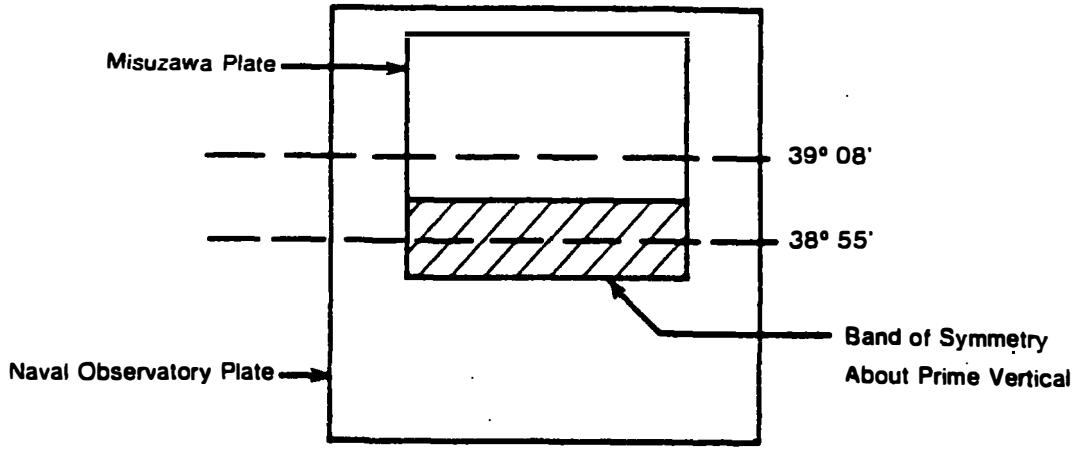
## APPENDIX F. SUBSTITUTION OF U. S. NAVAL OBSERVATORY PZT FOR GAITHERSBURG OBSERVATORY

The U. S. Naval Observatory has proposed replacing the VZT at Ukiah by a PZT donated by the U. S. Naval Observatory, and letting the new 62-cm PZT on the Observatory's grounds take over the work of observing now being done at Gaithersburg.

There is one problem that will have to be carefully investigated before the 62-cm PZT CAN finally take over the work of the Gaithersburg station. All observatories would be observing the same set of stars that are now used by the PZT at Misuzawa. The field of view of the Misuzawa's PZT is about 35 ft. The telescope is at latitude 39°08'03". The field of view of the 62-cm PZT is 60', but the telescope is located at latitude 38°55'17". Consequently, most of the stars of the Misuzawa catalog would be observed at Washington in the northern half of the field of view. Fewer than one-fifth of the Misuzawa stars would be paired about the zenith.




Field of Naval Observatory's PZT compared to field of Misuzawa's PZT

## APPENDIX G. DETERMINING PERSONAL ERRORS

Dr. Yumi has pointed out that, since the start of the program in 1900, there has been no well-designed and comprehensive investigation of the personal errors of the observers. Because there is the definite possibility that some of the data (as for instance from Ukiah) may be strongly biased through the entry of the observers' personal errors, it would be well to determine what these errors are. He suggests that the personal errors at all observatories be determined at the same time, which makes sense. The errors would be determined by a systematic and progressive substitution of observers. For example, one of the observers at Misuzawa would take the place of an observer at Kitab for a suitable period of time, say six months. The displaced observer at Kitab would substitute during that period for an observer at Carloforte. The displaced observer at Carloforte would replace an observer at Gaithersburg, who would move to Ukiah, and the person at Ukiah would move to Misuzawa. At the end of six months, should the personal errors still not be fully evaluated, the project could be extended for another six months with the substituting observers either remaining at their stations or moving on to the next observatory.

## APPENDIX H. SUMMARY OF DISCUSSIONS AT XVI GENERAL ASSEMBLY OF THE IAG

The essence of the meetings and the discussions which were held resulted in the following conclusions:

1. There was nearly unanimous agreement that the IPMS should be continued. It was agreed that the need for determination of polar motion is a permanent one and not one that will disappear in this century. The reasoning, which is convincing, is that the motion is too irregular by present standards to be predicted suitably and that satisfactory prediction will have to wait until a considerably larger, more accurate body of data is available.

2. Present procedures used by the IPMS are unsatisfactory and should be improved.

3. Thought should be given on how to improve the procedures in order that specific plans for studying the problem (not for solving it) can be presented at the IAU's congress at Grenoble in 1976.

4. The substitution of PZT's for the present VZT's appears to be both feasible and desirable.

5. Prof. Paul Melchior (Royal Observatory of Belgium) suggested using equipment of the TRANET type and DME with artificial satellites.

6. It is desirable to check the data coming from the IPMS stations by placing at or in the vicinity of these stations additional geodetic and geophysical equipment. In particular, the TRANET system should be used to check the geodetic coordinates of the stations. Variation of the astronomic coordinates should be checked with gravimeters and, if possible, tiltmeters (although tiltmeters are probably too sensitive for the job).

7. Prof. Robbins (University of Oxford) was asked to make a study of the problem and he agreed to do so.

## APPENDIX I. RETENTION OF ILS SITES

A belief exists among experts in polar motion and others that whatever method is ultimately selected for determining polar motion, the new equipment must be installed at or very close to the present ILS observatories. This is not necessary, and in most case it is undesirable. Except to the extent needed to decide whether or not the five ILS stations have been sliding around, the new equipment should be placed where it gives the most accuract determination of polar motion (and time). In general this will not be at the old ILS sites. The reasons are as follows:

1. The first responsibility of the service using the equipment is to provide information of current value on polar motion. If there must be a choice between providing current data of high quality or checking the past data, the former should be selected as the most important.

2. In any case, except for the PZT's, the quality of the data from all other systems would be compromised by placing the equipment at the ILS sites. This is immediately obvious for VLBI or radio interferometric equipment. It is also true for LURE equipment, which probably could not be moved to the ILS sites.

In the case of laser-type DME equipment for tracking artificial satellites, the situation is more complicated. First, assuming that the orbit is known, the best location of the polar motion equipment would depend on the latitude of the observer and on the inclination and semi-major axis of the satellite (or satellites) orbit (or orbits). Although the polar motion does affect a satellite orbit, because the satellite then sees a shift in the gravitational field, this effect is not as easily detectable as the effect on the observer. The observer's position shifts with respect to the satellite because of polar motion. A very precise measuring device, such as a laser-type DME, is needed. Since the satellite location along the orbit is more poorly determined than is the location of the orbit itself, the best location for a polar motion observatory would be a site where a change in latitude gives the maximum shift with respect to the orbit. This location, in general, would not be the best location for obtaining corrections to time. If the same station is to serve both purposes (obtaining data on polar motion and time), compromise will be necessary. But since the present locations of the ILS stations were selected with different criteria in mind, it will be very surprising if these locations turn out to be the best for satellite tracking.

It was assumed above that the orbit was known, i.e., that the orbit was determined to a satisfactory degree of precision from observations made at the observatory and other places. In

actuality, this assumption will have to be modified. To insure that the work of the observatories can be continued, regardless of changing policies in other organizations, the observatories will have to compute the orbit themselves. This means that the selection of sites will have to maximize a value function that depends on the errors in the orbit as well as on the errors in observation. This makes it even more unlikely that the present distribution of observatories will be optimum.

The case of TRANET-type equipment for measuring polar motion is somewhat analogous to that of the laser-type DME. However, the need for optimum placement of such equipment is not as imperative because the precision and accuracy expected for TRANET-type equipment are less. The fact that the TRANET satellites are in polar orbits is another reason why location of the equipment is not critical.

3. As shown in another part of this report, it is not necessary to keep the ILS stations in operation at their present sites in order to have continuous determination of polar motion, i.e., to retain the CIO. Since there is no practical reason for retaining the old ILS sites, one should look towards optimizing the placement of the new equipment. Optimization for accuracy has been discussed in the previous paragraph. From that discussion it is clear that the independent variable in the optimization is the latitude. This leaves the longitudes of the observatories available for optimization of other quantities such as economy, etc.

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