NOAA Technical Report NOS NGS 60

NAD83 (NSR2007) National Readjustment Final Report

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Rockville, MD
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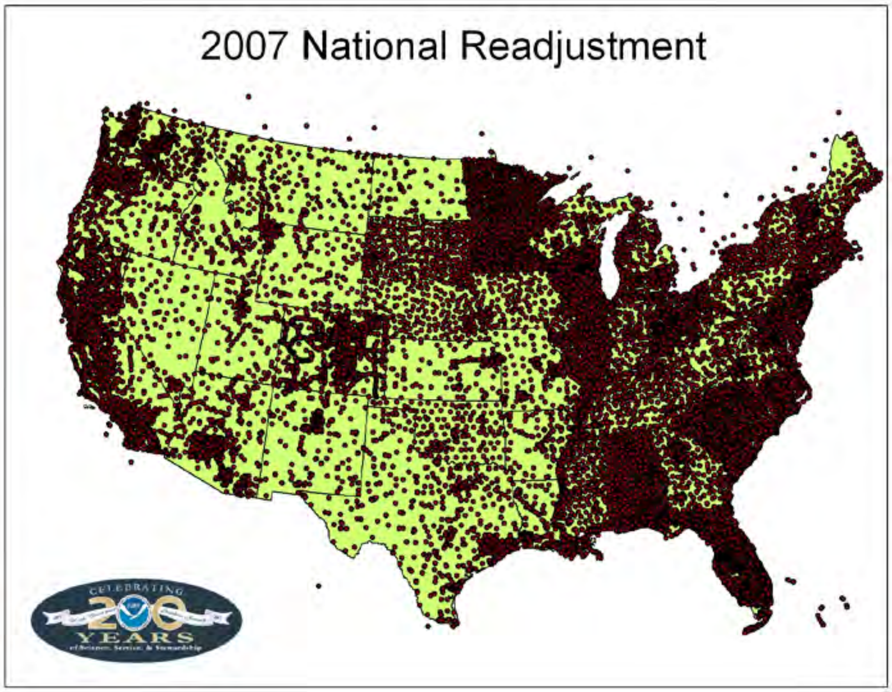

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U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Service

Contents

Overview

Dale Pursell and Chris Pearson

Purpose

A readjustment of all Global Positioning System (GPS) survey control in the United States was completed in 2007 by the National Geodetic Survey (NGS). The adjustment was undertaken to resolve inconsistencies between the existing statewide High Accuracy Reference Network (HARN) and/or Federal Base Network (FBN) adjustments and the nationwide Continuously Operating Reference Station (CORS) system, as well as between states, and to develop individual local and network accuracy estimates. For these reasons, on September 24, 2003, NGS' Executive Steering Committee approved a plan for the readjustment of horizontal positions and ellipsoid heights for GPS stations only in the contiguous United States. Classical surveys were not included in this readjustment.

Local and network accuracies are two measures which express to what accuracy the coordinates of a point are known. Network accuracies define how well the absolute coordinates are known, and local accuracies define how well the coordinates are defined relative to other points in the surrounding network. Both accuracies can be calculated from the appropriate elements of the coordinate covariance matrix which can be produced during a least squares adjustment. In general, a local accuracy can be determined between any two points, regardless of whether or not they were directly connected (share a single GPS vector). However, NGS will adhere to the Federal Geographic Data Committee (FGDC) guidelines and compute only local accuracies between directly connected stations.

Strategy

To prepare for the planned national readjustment, NGS began an analysis of every GPS project loaded into the NGS' Integrated Database (NGSIDB). The analysis began in early 2000, eventually including over 3,500 projects completed by November 15, 2005. A minimally constrained adjustment was performed on each project. Residual plots of the horizontal and vertical components for every vector were produced, and residual outliers greater than 5 cm were rejected in the NGSIDB. Connectivity to the HARN, Federal Base Network (FBN), and CORS Networks were checked and notes made if connections to the National Spatial Reference System (NSRS) were made through other projects. Based on this individual project analysis, it was determined that certain projects lacked the quality and/or connectivity to the NSRS required to be part of the national readjustment. Indeed, with the development of improved observing techniques and more advanced GPS equipment, a number of the earlier projects, such as the original Tennessee HARN and the Eastern Strain Network (ESN), were not included in the readjustment. Other identified projects included numerous third order Federal Aviation Administration (FAA) Projects from the 1980's and some projects that had no ties to the Network. A total of 170 projects were excluded.

A second important result of the individual project analysis was the development of a uniform set of weights reflecting the relative accuracies of the disparate survey data sources included in the national readjustment. These were used to identify and resolve deficiencies in the current weighting schemes of projects. Through the experience NGS gained in 20 years of project analysis, it is now known that the formal accuracy estimate of the GPS horizontal component is approximately three times smaller than the formal accuracy estimate of the vertical component. In order to properly weight the observations, software was developed to allow the re-scaling of weights by separate horizontal and vertical components. All individual projects underwent yet another minimally constrained adjustment to determine a separate horizontal and vertical weighting factor (variance factors) to be applied during the national readjustment (See chapter 8). These "variance factors" were designed to ensure a uniform set of weights when all projects were combined during the readjustment. Variance factors were not computed for projects located within California, because individual projects there had many rejections from a prior statewide readjustment, preventing them from adjusting separately upon database retrieval.

The readjustment involved two different datums. The first datum is NAD 83 (North American Datum of 1983), which is the U.S. national datum. The major advantage of NAD 83 is that the datum definition assumes a zero velocity in the motion of the North American Plate (which covers most of the 48 contiguous states). Points on the stable part of the plate can have coordinates fixed in time. The far west of the United States straddles two tectonic plates and a zone—a few hundred kilometers wide and including most of California, Nevada, Oregon Washington, and Alaska—which is deforming. The deformation causes the relative position of points on the Earth to change with time. Consequently, accurate surveying in the western United States requires a model describing crustal velocities and earthquakes, so that survey measurements can be corrected for differential movement if surveys conducted at different epochs are to be compared. This was accomplished using Horizontal Time Dependent Positioning (HTDP) [Snay 1999] software for transforming horizontal positional coordinates and/or geodetic observations across time and between spatial reference frames. Users may also apply HTDP to predict the velocities and displacements associated with crustal motion in any of several reference frames. The version of HTDP used for the national readjustment introduced dislocation models for two recent earthquakes: (1) the magnitude 6.5 San Simeon, CA earthquake that occurred in December 2003, and (2) the magnitude 6.0 Parkfield, CA earthquake that occurred in October 2004.

For the creation of the NAD 83(NSRS2007) reference frame in California, it was necessary to decide on a common epoch date for all adjusted stations in California. NGS, in conjunction with the California Spatial Reference Center (CSRC), decided on January 1, 2007 as the adjusted epoch date.

The second datum involved in the national readjustment was ITRF2000, which at the time was the most current realization of the International Terrestrial Reference Frame (ITRF). This frame is used for GPS processing and is thus the natural frame for CORS. These coordinates are later transformed into NAD83 (CORS96), which is currently the best defined realization of NAD 83. NGS adopted an alternative realization of NAD 83 called NAD 83(NSRS2007) for the distribution of coordinates at 67,693 passive geodetic control monuments. This realization approximates the more rigorously defined NAD 83(CORS96), but can never be equivalent to it.

NAD 83(NSRS2007) was created by adjusting GPS data collected during various campaign-style geodetic surveys performed between mid-1980 and 2005. The NAD 83(CORS96) positional coordinates for 685 CORS were held fixed (predominantly at the 2002.0 epoch for the stable North American plate, but 2003.0 in Alaska and 2007.0 in western CONUS). Derived NAD 83(NSRS2007) positional coordinates should be consistent with corresponding NAD 83(CORS96) positional coordinates to within the accuracy of the GPS data used in the adjustment.

In California, the NAD 83 epoch 2007.0 values for the California CORS (CGPS) were obtained through Scripps' Sector utility and are available through the CSRC website at: http://csrc.ucsd.edu.

Helmert Blocking

The national readjustment was conducted using the Helmert blocking technique. This technique allows for breaking up a least squares adjustment problem, which is too large to be managed as a single computation, into many smaller sub regions or blocks which are then reassembled to produce a solution equivalent to a single simultaneous solution.

Division of survey data into blocks is perhaps the key step to developing a successful adjustment using Helmert blocking. The Helmert blocking strategy used for the readjustment was based on NGS' knowledge that most of the projects submitted to NGS and located in the NGS integrated database were contained within state boundaries. Helmert blocks based on state boundaries would minimize the number of observations crossing block boundaries (junction baselines). Because of the large amount of survey data contained within the states of California, Florida, North Carolina, South Carolina and Minnesota, each of these States was further divided into two sub blocks.

The 2007 national readjustment was conducted using a Helmert blocking software suite developed for the purpose of multi-epoch processing of CORS data. This software, which continues to be used for the purpose of computing multi-year adjustments of CORS data, exists in two separate programs, GPSCOM and LLSOLV, and was modified for use in the national readjustment. These two programs were later incorporated into NETSTAT, a Helmert blocking network adjustment program developed explicitly for the readjustment. A detailed description of this software is included in the report.

Adjustments

The first stage of the national readjustment was a minimally constrained adjustment for the entire network in order to identify and remove large residuals and blunders in the observations when all observations were combined. This step was necessary because the minimally constrained adjustment of a single project did not combine all projects within a state together, resulting in some bad observations being missed. In certain cases, rejections in local areas would cause previously rejected observations to become very good. These observations were un-rejected when residuals fell below the 5 cm tolerance. This stage was also used to identify stations undergoing large positional shifts and to get an accurate value of the a-posteriori variance of unit weight used to determine more realistic network and local accuracies. This adjustment included 851,073 observations and produced a standard deviation of unit weight of 1.28. The variance of unit weight was still relatively high because of very weak stations purposely left in during the analysis phase to prevent these stations from being removed if any further rejections were made. The readjustment team decided to publish weak stations in an attempt to notify the user community (via the local and network accuracies) that stations—which surveyors might be using regularly—were poorly determined, because if stations were simply removed, the user would never know the true accuracy of that station.

The next phase was a series of constrained adjustments which held fixed all available CORS stations observed and loaded into the National Geodetic Survey's Integrated Database (NGSIDB) as of November 2005. Available CORS stations included 468 national CORS obtained from the NGSIDB, 213 California CORS (CGPS), 3 Canadian CORS, 1 Mexican CORS obtained from Scripps Orbit and Permanent Array Center (SOPAC). From the initial constrained adjustment, it was found that several CORS (when rigidly constrained) produced large residuals. Possible causes for the large residuals included misidentified antenna reference points, changes in the CORS configuration after the observations were originally observed, and low quality, or poorly reduced, observations. When no explanation could be found to explain the excessive residuals at the CORS stations, they were freed during a subsequent constrained adjustment. Out of 685 possible CORS constraints, 673 were totally constrained, 7 were freed (with 10 cm standard deviation), 5 were freed only in height (with 10 cm standard deviation), and three CORS were left completely free. The standard deviation of unit weight for the final adjustment was 1.38. A subsequent constrained adjustment was run, scaling the errors by the standard deviation of unit weight, so realistic local and network errors could be determined.

A comparison of the original published values to the readjusted values for each Helmert Block developed a list of maximum and average horizontal and vertical shifts for all stations participating in the national readjustment. In general, the average shifts for each block were fairly small, with values typically less than 2 cm, and with maximum shifts of less than 1 m. In certain cases, very large shifts were observed, caused by stations with no publishable ellipsoid heights or stations located in areas of known movement.

Publication

When the national readjustment was completed in February 2007, software for distributing the readjusted coordinates and their associated local and network accuracies through the NGS datasheet was not yet ready for public use. As a result, NGS decided to release the readjusted coordinates with 3-D variances and covariances in a simple text-based format called “Re-adjustment Distribution Format” or “RDF” as an interim measure until the readjustment was able to be distributed as datasheets.

After September 2007, the readjusted coordinates were loaded into the NGS database and were distributed as standard datasheets. Since then, the following modifications to the data sheets have been implemented:

- NGS has decided to use the “NAD 83(2007)” tag as the permanent identifier of points with an NSRS2007 coordinate.
- For survey control stations determined “NO CHECK” by the national readjustment, the published NAD 83 coordinate line has been designated “NO CHECK” (replacing “ADJUSTED”) and the ELLIP HEIGHT line has been designated “NO CHECK” (replacing “GPS OBS”).
- The ellipsoid height line has been designated “ADJUSTED” rather than “GPS OBS” (except for NO CHECK stations; see above).
- Network Accuracies have been published on the datasheet and Local Accuracies will be published as soon as software to do so is available.

Stations submitted after the 2005 cutoff date have not been readjusted and are still listed on the datasheets in whatever was the most recent adjustment for the state in which the mark is located. NGS has not made a commitment as to whether resources will be available to readjust projects, and it is suggested that the submitting agency readjust the project and submit the results to NGS for database entry.

Part I. Background

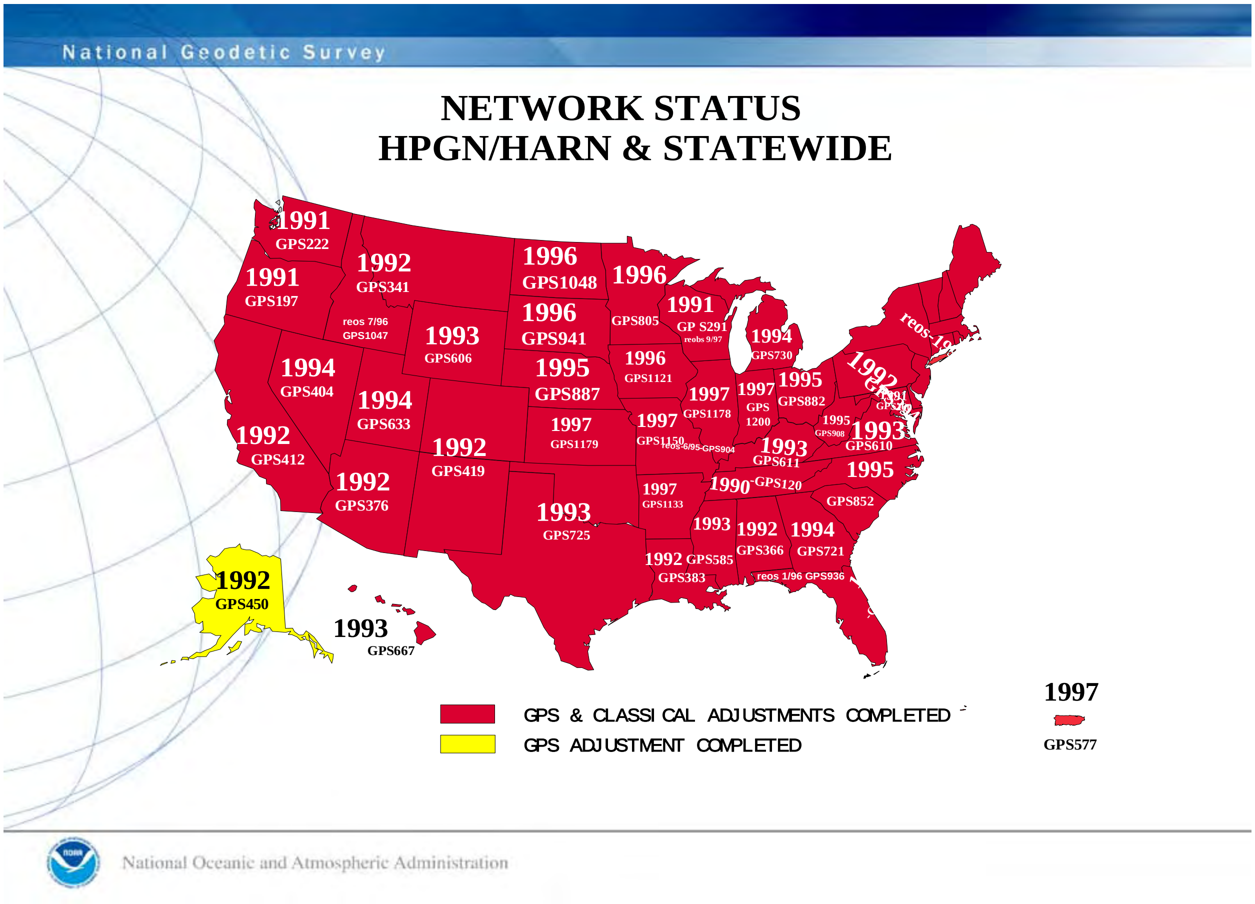
Maralyn Vorhauer, Kathy Milbert, and Dale Pursell

1. North American Datum of 1983 (1986)

The North American Datum of 1983 (NAD 83) described in [Schwarz 1989] was first published in 1986 and was known as NAD 83 (1986). The NAD 83 (1986) was the third horizontal geodetic datum of continental extent in North America, and it was intended to replace both the original United Standard Datum, later named North American Datum in 1913, and the North American Datum of 1927 (NAD 27). During the 1960s, electronic distance measuring equipment was introduced, and it quickly became clear that the substantially increased accuracy possible with these measurements was not supported by the existing NAD 27 control points. Also, significant local distortions had accumulated due to the piecemeal nature of expanding the control framework. The decision was made to not only re-compute the positions of all the existing survey points, but also to adopt a new ellipsoid, move the NAD 27 datum origin from its location on the earth's surface (Meade's Ranch) to the earth's center of mass, and to digitize the observational data to be used, allowing the use of computer technology for the computations. The establishment of the new datum was the result of an international project which included Canada, Mexico, and Greenland as parts of the North America continent. The Geodetic Reference System of 1980 (GRS 80) [Moritz 1984]—using an earth-centered ellipsoid determined from satellite-based computations—was chosen, and the mass center of the earth became the datum origin. Advances in technology, e.g. satellite observations, eliminated the need for the datum origin to coincide with the surface of the earth. Beginning in 1974 and continuing for the next 12 years, data from the observational surveys which had taken place from the 1800's (excluding some early surveys which were not of sufficient accuracy to be included), were digitized, checked, and analyzed. The project involved a team of more than 300 people, with a cost of more than $37 million to complete. At that point, NGS embarked on one of the largest computer tasks ever undertaken—the simultaneous algebraic solution of nearly 1,000,000 equations. The method used (called "Helmert blocking") had been proposed by F.R. Helmert [Helmert 1880], but was never applied by Helmert. Instead, it was used in the adjustment of the European survey network in 1950 and then, later, computer software was written for its application. Still, it had never been used on as massive a scale as this. It proved to be an ideal way of solving all the equations simultaneously by dividing the data into blocks to expedite the task. The project was completed in 1986, and new positions were published for the 300,000 included points. While NAD 83 provided a significant improvement over NAD 27, the basis for the readjustment was conventional surveying measurements; GPS was not yet a fully capable system.

2. High Accuracy Reference Networks (HARNs)

Although the NAD 83 (1986) adjustment provided significant improvements over NAD 27, changes were rapidly occurring in the way NGS established new control positions. GPS use rapidly increased soon after the adoption of the NAD 83 (1986) datum, because more satellites were added to the GPS constellation, greatly increasing the viability and productivity of GPS-based surveys. Then, a new issue with the datum was exposed: the accuracies of the new GPS surveys were significantly better than the positional accuracies of the available NAD 83 (1986) control stations (called the National Geodetic Reference System, or NGRS. A few years later the name National Spatial Reference System, or NSRS, was adopted). This basically meant that new high-accuracy GPS surveys had to be distorted to fit existing control stations, and this quickly became an issue of importance within NGS. NGS, in cooperation with many federal, state, and local government partners, as well as those in the private sector, conducted GPS surveys to increase the positional accuracies of the existing and new control stations. These GPS surveys formed the basis for the High Precision GPS Networks (HPGNs), later renamed the High Accuracy Reference Networks (HARNs). These surveys were conducted on a state-by-state basis, with field observations beginning in Tennessee in 1989 and concluding in Indiana in 1997. The HARN surveys then served as the basis for readjustments in each state, including all available surveying data—both conventional and GPS—in the determination of new positional values. The surveys also drove the creation of A-order and B-order control designations for GPS-based high-accuracy control points to express their superior accuracy relative to the existing First-, Second- and Third-Order designations found in the original NAD 83 (1986). State HARNs proved to be a significant improvement over the original datum realization and an important resource for all users of GPS positioning. Figure 2.1 shows the project source number (GPS number) and the year the state HARN was adjusted.


National Geodetic Survey

NETWORK STATUS
HPGN/HARN & STATEWIDE

🟥 GPS & CLASSICAL ADJUSTMENTS COMPLETED

🟨 GPS ADJUSTMENT COMPLETED

National Oceanic and Atmospheric Administration

Figure 2.1 Completion Dates and Project Code Numbers of State-by-State HARN/HPGN Surveys

3. Continuously Operating Reference Stations (CORS)

(http://www.ngs.noaa.gov/CORS/)
The Continuously Operating Reference Stations (CORS) network is a heterogeneous system of geodetic quality, permanently monumented Global Navigation Satellite System (GNSS) receivers—such as GPS and GLONASS—which collect data continuously.

Beginning with the installation of a permanently mounted, continuously operating GPS receiver at Gaithersburg, Maryland in 1994, the CORS network has grown through the partnerships of dozens of different organizations. Each organization installs a GNSS receiver for their own purposes, and then they join the CORS network, managed by NGS. Figure 3.1 depicts the CORS coverage as of November 2005. CORS provides an accurate three-dimensional coordinate and velocity in the NAD 83 and International Terrestrial Reference Frame (ITRF). Each CORS site also collects—and NGS distributes—GPS carrier phase and code range measurements in support of three-dimensional differential positioning activities throughout the United States and its territories. Surveyors, GIS/LIS professionals, engineers, scientists, and others may apply CORS data to position points where GPS data have been collected. The long-time series of data available for the CORS system enables positioning accuracies that approach a few centimeters both horizontally and vertically. It was the widespread use of differential positioning from CORS that showed that even the HARN-based coordinates had some state-by-state weaknesses and which ultimately led to a resurvey of the HARNs—this time tied to the CORS network. The resurveys were known as the Federal Base Networks, or FBNs.

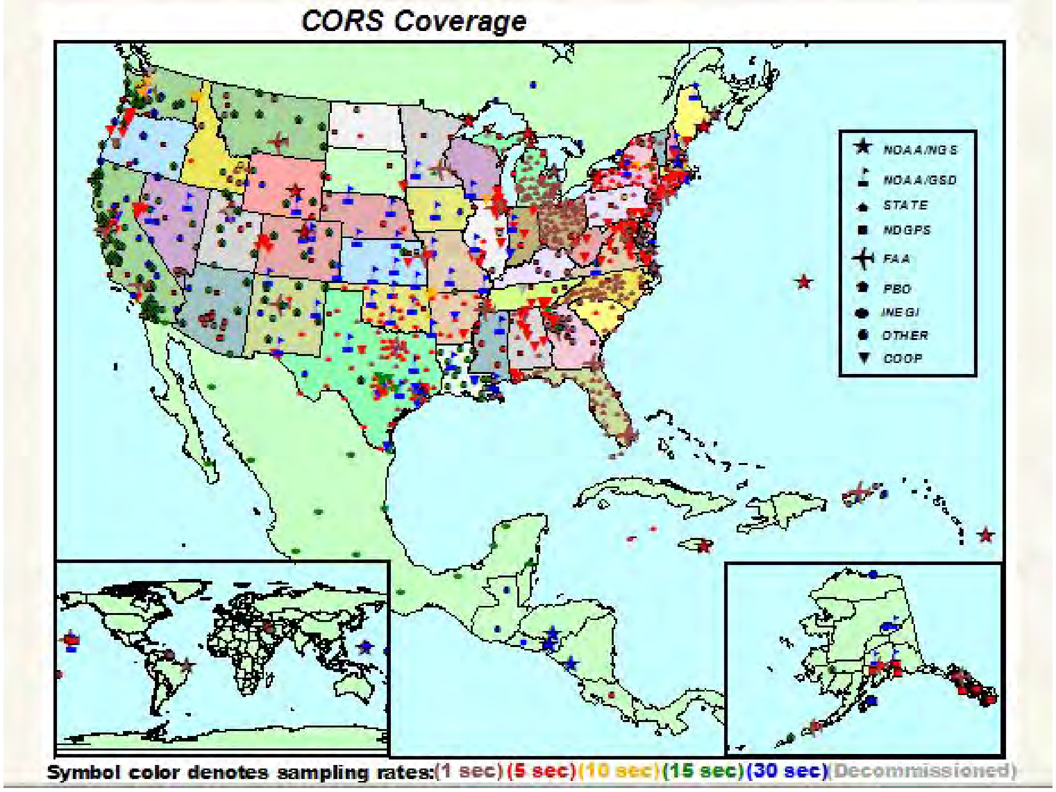


Figure 3.1 CORS Coverage as of November 2005

4. Federal Base Networks (FBNs)

Due to the improvement of GPS technology (e.g. more satellites and more robust software), newer HARNs were found to be more accurate than the older ones [Milbert 1994]. Also, previous HARN adjustments were initially conducted on a state-by-state basis using Very Long Baseline Interferometry (VLBI) stations as control, and then—as more states' HARN surveys were completed—using previously determined HARN coordinates as control. This allowed minor differences in HARN positions from one state to another [Milbert 1998]. Also, the growth in popularity and use of the CORS from 1994 until the present time had created a new issue. Early HARN surveys were completed prior to the establishment of a dense CORS, leaving the door open for minor inconsistencies between the growing national-based CORS and the state-based HARN systems. Therefore it was possible to find discrepancies of up to 6 or 7 cm, depending on whether a HARN or CORS was used as control. While both systems were highly accurate, they were also generally independent. Therefore, there was a need to re-observe passive control points in order to tie to the CORS and produce better ellipsoid heights. In order to remove the inconsistencies between the passive control HPGN/HARN stations and CORS, NGS conducted a second (and final) national observation campaign from 1997 to 2004, referred to as the Federal and Cooperative Base Network (FBN/CBN) surveys. The aim of the final national resurvey was to establish and maintain a network of high accuracy control stations, spaced at roughly 100 km, with a minimum relative accuracy of 1:1,000,000 horizontally, to provide accurate connections to the CORS and to ensure the integrity of the ellipsoid height component of the HPGN/HARN stations to no worse than 2 cm. Figure 4.1 shows the FBN project source number (GPS number) and the year the adjustment was completed.

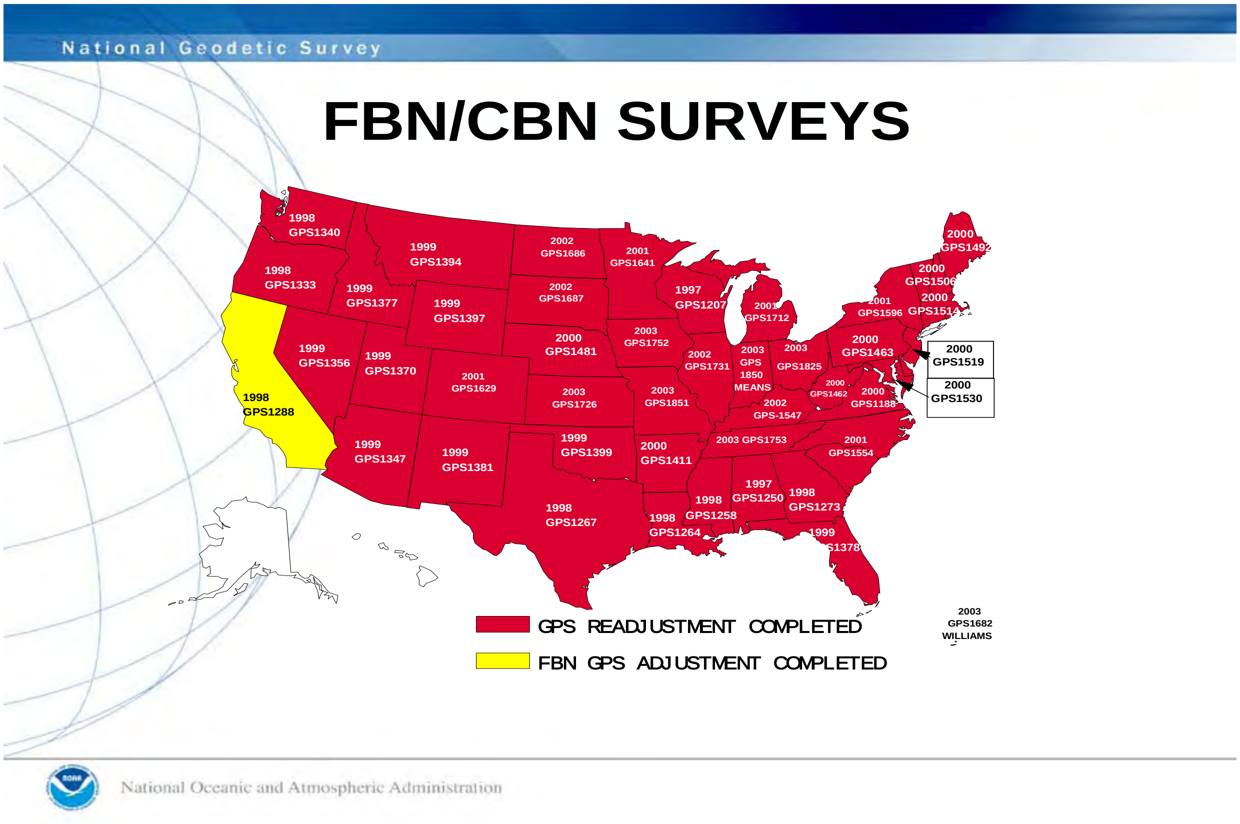


Figure 4.1 Completion Dates and Project Code Numbers of State-by-State FBN/CBN Surveys

5. National Readjustment

Although the FBN/CBN surveys were performed in order to reduce HARN/CORS discrepancies, they were nonetheless done on a state-by-state basis, with earlier states held fixed as control for later states. This inevitably led to some minor state-by-state biases relative to CORS and inconsistencies throughout the national FBN network itself.

Additionally, as the FBN surveys were ongoing, the Federal Geographic Data Committee issued a document [FGDC 1998] requiring that all points (including geodetic control) be assigned an appropriate “network accuracy” and “local accuracy,” defining a point’s positional accuracy relative to the network as a whole and relative to “directly connected” points, respectively.

For these two reasons, on September 24, 2003, NGS' Executive Steering Committee approved a plan for the readjustment of horizontal positions and ellipsoid heights for GPS stations (only in the contiguous United States). Classical surveys were not to be included in this readjustment. The remainder of this document describes the work done to arrive at a readjustment of all Global Positioning System (GPS) survey control in the United States, completed in 2007 by the National Geodetic Survey (NGS) and given the datum name of NAD 83 (NSRS2007).

Part II. Data Inventory, Assessment and Input

Kathy Milbert, Janie Hobson, and Gloria Edwards

6. Preliminary GPS Project Analysis

In preparation for the planned national readjustment, the NGS Observation and Analysis Division began an analysis of every GPS project loaded into the National Geodetic Survey's Integrated Database (NGSIDB) as of November 15, 2005. This analysis began in early 2000 and involved the following steps for over 3,500 projects:

1. GPS observations were retrieved from the NGSIDB on a project level basis.
2. A free adjustment was run on each project.
3. The differences between the observed and adjusted vectors (residuals) were plotted for both the horizontal and vertical components for every vector.
4. Residual outliers greater than 5 cm were rejected (vectors greatly downweighted), subject to guidelines concerning no checks. This information was recorded in NGS' integrated database (NGSIDB).
5. If rejections were made, steps one through three were repeated.
6. A log sheet was created, containing the date of observations and the largest horizontal residual and vertical residual spread.
7. Connectivity to the HARN, FBN, and CORS networks were checked, and notes were made if connections were through other projects.
8. A summary sheet of stations in the projects was produced. The summary contained information on the published survey order of the stations in the project and the state in which the stations were located. For example,

Table 6.1 shows that in project GPS1048/B in Minnesota, there are two A order stations and six B order stations:

Table 6.1 Summary Sheet Example

```
OBS_SOURCE    |A  |B  |1  |2  |3  |4  |?
 ------------ --- --- --- --- --- --- —
 GPS1048/B    |  2|  6|  0|  0|  0|  0|  0  MN
 GPS1048/B    |  2|  6|  0|  0|  0|  0|  0  SD
 GPS1048/B    |  5| 39|  0|  0|  0|  0|  0  ND
 GPS1048/B    |  6|  2|  0|  0|  0|  0|  0  MT
```

7. Master File

After the individual project analysis was completed, each project was categorized into specific layers of accuracy and connectivity to the NSRS. Initially, the national readjustment was to be accomplished through layers determined by their specific orders of accuracy. A master file (See Table 7.1) created for each state, identified:

1. Projects located entirely or predominately in the state.
2. Projects classified based on specific orders of accuracy. (This is directly related to the orders of accuracy of the points in the project.) Projects of poor quality were identified and excluded (see the following list of skipped projects).
3. Test data was retrieved from the NGSIDB by layer for select states and regions.
4. An adjustment analysis of each layer was performed. All problems were identified, documented in a report, and added to the appropriate project folder.

Table 7.1 Master File Example

```
OBS_SOURCE    |A  |B  |1  |2  |3  |4  |?
------------ --- --- --- --- --- --- —    ND   HARN-B Order-1996-Excellent residuals-3 cm
                                               or less
GPS1048/B    |  2|  6|  0|  0|  0|  0|  0  MN   No GPS readjustment performed in this state
GPS1048/B    |  2|  6|  0|  0|  0|  0|  0  SD
GPS1048/B    |  5| 39|  0|  0|  0|  0|  0  ND   Level 2 of 3 in ND GPS readjustment
GPS1048/B    |  6|  2|  0|  0|  0|  0|  0  MT   All either in FBN or in ID/MT HARN-Not
                                              included in ID/MT GPS
```

8. Projects Omitted from the Readjustment (Skipped Projects)

Based on the individual project analysis obtained during the master file creation phase, it was determined that certain projects lacked sufficient quality and/or connectivity to the NSRS to be valuable to a nationwide readjustment. Even some supposedly high accuracy projects, such as the original Tennessee HARN, were found to not be of sufficient quality to be part of the national readjustment. With the development of improved observing techniques and more advanced GPS equipment, many earlier projects were found to have insufficient quality to be included in the readjustment. A total of 170 projects were not included in the national readjustment. See Table 8.1

Table 8.1 Omitted Projects

Project ID	State
17221	KS
17282	AK
17299	MD
17310	WA
17407	TX
GPS013	AZ
GPS016	MS
GPS022	CA
GPS031	OR
GPS034	AR
GPS044	SC
GPS048	FL
GPS049	NC
GPS054	CO
GPS055/B	CA
GPS056	CO
GPS060/1	OR
GPS060/2	OR
GPS062	AK
GPS064	CO
GPS069	MT
GPS070	WA
GPS071	NH
GPS072	DE
GPS073	NY
GPS076	LA
GPS078	TX
GPS084	LA
GPS088	TX
GPS089	LA
GPS090	AL
GPS091	CA
GPS092	VA
GPS095	WA
GPS099	OH
GPS104	GA
GPS1049	MO
GPS105	VA
GPS106	VA
GPS1069	MT
GPS108	VA
GPS111	FL
GPS1138	TX
GPS115	GA
GPS116	MS
GPS117	SC
GPS1170/4	TX
GPS119	NJ
GPS120	TN
GPS1201/2	NE
GPS127	NJ
GPS129	GA
GPS131	MA
GPS133	IN
GPS138	MT
GPS139	NY
GPS144	PA
GPS145	WI
GPS146	TX
GPS157	NM
GPS158	TX
GPS163	LA
GPS166	OK
GPS169	NE
GPS172	KY
GPS175	FL
GPS177	MI
GPS179	IA
GPS180	KS
GPS183	NJ
GPS193	SC
GPS194	MN
GPS196	CA
GPS198	SC
GPS203	MO
GPS204	AR
GPS205	CA
GPS209	NJ
GPS218	TN
GPS225	WI
GPS231	SC
GPS232	CA
GPS247	ID
GPS251	IL
GPS256	IN
GPS256/C	IN
GPS258	RI
GPS261	CA
GPS264	NJ
GPS264/B	PA
GPS266	AL
GPS270	NC
GPS272	NC
GPS273	VA
GPS277	AK
GPS279	AR
GPS283	LA
GPS284	VA
GPS288	TX
GPS295	MO
GPS300	LA
GPS304	TX
GPS310	CA
GPS314	MI
GPS323	OR
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GPS348	OR
GPS353	NC
GPS355	CA
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GPS365	CA
GPS367	GA
GPS372	VA
GPS377	FL
GPS381	SD
GPS395	AK
GPS398	VA
GPS399	VA
GPS400	VA
GPS407	TX
GPS416	VA
GPS417	VA
GPS419/D	NM
GPS421/C	GA
GPS427	WA
GPS431	NM
GPS437	NV
GPS438	AR
GPS440	WY
GPS443	OR
GPS445	CA
GPS461	NV
GPS462	NV
GPS471	MD
GPS482	TX
GPS483	NC
GPS484	DE
GPS490	SD
GPS491	IA
GPS496	VA
GPS519	OH
GPS523	AR
GPS524	AR
GPS532	LA
GPS545	NC
GPS550	AR
GPS568	CA
GPS569	LA
GPS572	LA
GPS574	AR
GPS577	TX
GPS577/D	TX
GPS582	TX
GPS584	TX
GPS586	TX
GPS589	SD
GPS592	SD
GPS596	SD
GPS627	PA
GPS655	LA
GPS724	OH
GPS742	OH
GPS776	MI
GPS843	MT
GPS849/193	TX
GPS868	MT
GPS903	MD

9. Variance Factors

It was known that the sigmas of the GPS horizontal component are approximately three times smaller than the sigmas of the vertical component. In addition, it had been NGS policy to not scale higher order projects (A and B) by the standard deviation of unit weight, while lower order projects (first order or lower) were scaled. Initially, NGS believed higher order projects should carry more weight when projects of lower order were combined.

In order to properly weight the observations, software was developed to allow the re-scaling of weights by separate horizontal and vertical components. The software [Lucas 1985] described a variance component estimation method for sparse matrix applications. The method was incorporated into the ADJUST software [Milbert 1993]. These factors worked well for the adjustment, although Lucas‘s equations required the observations be uncorrelated—not the case for the national readjustment. Since the observations were correlated, the resulting variance factors must be considered as approximate. Because of this approximation, the variance factors for 93 out of the 3,411 projects were negative.

With ADJUST enhanced to produce the variance factor, all individual projects underwent yet another minimally constrained adjustment to determine a separate horizontal and vertical weighting factor to be applied during the national readjustment. These “variance factors” were designed to ensure a uniform set of weights when all projects were combined during the readjustment. The determined variance factors for each project were then loaded into the NGS database. The variance factors were later retrieved from the database and incorporated into the national readjustment through the individual Helmert block input files.

Variance factors were not computed for projects located within California, because California underwent a complete state readjustment prior to the computation of variance factors. During the state readjustment, individual projects had many rejections, preventing them from adjusting separately upon database retrieval, and therefore it was not possible to compute reliable variance factors. In most cases the rejections were valid since many of these observations were re-observed during later campaigns. Variance factors were computed for all subsequent California projects submitted after the state readjustment was complete.

10. HTDP

The NGS Horizontal Time Dependent Positioning (HTDP) software was used for transforming horizontal positional coordinates and observations from one epoch to another. For most of the continental United States, the NAD 83 horizontal velocities are zero, and there is no change in NAD 83 positions from one epoch to another. However, there is significant motion in the western states within a few hundred kilometers of the Pacific coast. These areas are subject to both a slow rotation, caused by tectonic plate movement, and episodic deformation due to earthquakes [Snay 1999].

The version of HTDP used for the national readjustment introduced dislocation models for two recent earthquakes: (1) the magnitude 6.5 San Simeon, CA earthquake that occurred in December 2003, and (2) the magnitude 6.0 Parkfield, CA earthquake that occurred in October 2004 [Johanson 2006; Pearson and Snay 2006; Pearson and Snay 2007].

For the creation of the NAD 83(NSRS2007) reference frame in California, it was necessary to decide on a common epoch date for all adjusted stations in California. NGS, in conjunction with the California Spatial Reference Center (CSRC), decided January 1, 2007 would be the adjustment epoch date. The positions of all NGS CORS stations in California were updated to January 1, 2007.

The GPS derived vectors used for the NSRS adjustment were in a number of different reference frames, mostly some version of ITRF, and were performed at a number of different times. For observations taken near the west coast, i.e., points in California, Nevada, Arizona, Oregon, Washington, and Alaska, HTDP was used to update the observed vectors from their respective dates of observation to the values that would have been observed on January 1, 2007. HTDP was not used to update observations in any other state.

11. Data Retrieval

All GPS projects loaded into the NGSIDB as of November 15th, 2005, with the exception of the 170 skipped projects listed in Table 8.1, were retrieved from the NGSIDB and included in the combined dataset for the national readjustment. Because Helmert Blocking had already been determined as the approach for this adjustment (see section 15), it was necessary to group the data into “data blocks”. Each data block was given a name (generally the name of a state). To determine the data block to which a project would be assigned, all projects were reviewed within the NGSIDB, and the stations were sorted by the state in which they were located. A single state code was then assigned to each project based on which state contained the highest number of stations in that project. The project’s state code then determined which data block the project was located in. Note that while the data blocks were identified by assigning the name of a state to each block, any particular block could (and did) have data from multiple states within it.

Because of the amount of data in the California, Florida, North Carolina, South Carolina, and Minnesota data blocks, these blocks were further divided into two sub-blocks. Note, during the process of splitting states into sub-blocks, projects (but not GPS sessions themselves) were also split between sub-blocks. All projects within each data block were combined into the standard NGS input bluebook formats (Bfile, Gfile and Afile). However, the actual Bfile retrieved was a modified version of the bluebook format which included NGS’ unique station identifier (PID) for each station in columns 1 through 6, and the ellipsoid height located in columns 15 through 23. Only the *ellipsoid* heights (not orthometric) were retrieved from the NGSIDB since NGS’ objective for the national readjustment was to readjust only the horizontal coordinates and ellipsoidal heights. In order to perform a simultaneous least squares adjustment of all retrieved vectors throughout the country, the retrieved vectors in the western states of California, Alaska, Washington, Oregon, Arizona, and Nevada were transformed into a common epoch (2007.0) through HTDP (though this did not include vertical velocities) and then combined with the rest of the country. Since the readjustment was performed in the NAD 83 system, NGS assumed all vectors in all other states were rigid, without any movement. (It was later discovered that some vectors crossing from the six “western” states into the other states did not have HTDP applied, due to those vectors being placed into Helmert blocks which did not have HTDP applied). In addition, the vectors within California were further scaled, based on the age and length of the vector. The reliability and accuracy of earlier vectors due to plate tectonic motion necessitated the need for down-weighting these observations. This scaling greatly aided analysis when all vectors over time were combined into a common epoch.

As a result of the retrieval, the following statistics were computed:

- A total of 3,411 projects were retrieved.
- 67,693 points (includes 685 CORS) and 236,239 sessions
- 313,477 vectors, total (283,691 vectors, un-rejected, 29,786 vectors, rejected)
- 851,073 non-rejected observations

Part III. Methodology

Dale Pursell

12. Datum Definitions

The readjustment involved both the NAD 83 and the ITRF00. The ITRF uses the center of mass of the entire Earth, including the oceans and the atmosphere as its origin. The ITRF approximates the NUVEL1-NNR model [DeMets et al. 1994], or no net rotation reference frame where plate motions average globally to zero. Plate tectonic movement is accommodated explicitly by giving each point a coordinate at a reference epoch and a velocity vector that reflects the future trajectory of the point with time. The ITRF is periodically updated.

NAD 83 has a center of mass origin *best known at the time the original NAD 83 parameters were defined* [Snay and Soler 2000]. We now know the original determination of the center of mass is approximately 2.2 meters away from the current location of the NAD 83 origin. Points that fell on the stable North American Plate (which covers most of the 48 contiguous states) have NAD 83 coordinates that are assumed to be fixed in time. Points in the far west of the United States, which lie on the boundary between the North American and Pacific plates, have velocities provided by the NGS utility HTDP. We also know that specific areas of the country have known vertical velocities due to subsidence and/or glacial uplift. Due to the lack of a vertical velocity model, vectors were not modified to account for any vertical movement.

The national readjustment was computed in the NAD 83 coordinate system. For CORS stations (whose defining coordinate is in the ITRF frame), the ITRF coordinates were transformed through a fourteen-parameter transformation to the NAD 83 coordinate system and designated as NAD 83(CORS96). This methodology was chosen, because performing the readjustment in the ITRF would have required velocity vectors for each passive point. Since the NAD 83 was referenced to the stable part of North America, NGS was able to assume little or no velocities on each passive point. HTDP provided the required horizontal velocity vectors for points on the far west of North America which straddle the divide between the North American and Pacific plates and was used to transform the vectors into a common epoch and produce a modified version of the G-file. The modified version of the G-file was used solely for the readjustment and was not loaded back into the database. The computed coordinates from the readjustment were produced and published as NAD 83(NSRS2007).

Because of the difference in how plate tectonic velocities are treated, the differences between the two systems are slowly changing. Transformations between different realizations of NAD 83 and ITRF are periodically updated [Craymer et al. 2001]. Table 12.1 shows which fourteen-parameter Helmert transformations are supported between different ITRF and NAD 83 realizations. Details are found at http://www.ngs.noaa.gov/CORS/coordinates.

Table 12.1 Various Supported Helmert Transformations in HTDP

ITRF93 ↔ NAD 83(CORS93)
ITRF94 ↔ NAD 83(CORS94)
ITRF96 ↔ NAD 83(CORS96)
ITRF97 ↔ NAD 83(CORS96)
ITRF00 ↔ NAD 83(CORS96)

13. Statement Regarding Control Used for NAD 83(NSRS2007)

When the national readjustment was complete, NGS adopted the realization name NAD 83, called NAD 83(NSRS2007) for the distribution of coordinates at the 67,693 passive geodetic control monuments that were part of the national readjustment. This realization *approximates* (but is not, and can never be, equivalent to) the more rigorously defined NAD 83(CORS96) realization in which Continuously Operating Reference Station (CORS) coordinates are distributed. NAD 83(NSRS2007) was created by adjusting GPS data collected during various campaign-style geodetic surveys between mid-1980 and 2005. For the adjustment, NAD 83(CORS96) positional coordinates for 685 CORS were held fixed (predominantly at the 2002.0 epoch for the stable North American plate, but 2003.0 in Alaska and 2007.0 in western CONUS) to obtain consistent positional coordinates for the 67,693 passive marks. Derived NAD 83(NSRS2007) positional coordinates should be consistent with corresponding NAD 83(CORS96) positional coordinates to within the accuracy of the GPS data used in the adjustment and the accuracy of the corrections applied to these data for systematic errors, such as refraction. In particular, there were no corrections made to the observations for vertical crustal motion when converting from the epoch of the GPS survey to the epoch of the adjustment, while the NAD 83(CORS96) coordinates do reflect motion in all three directions at CORS sites. For this reason alone, there can never be total equivalency between NAD 83(NSRS2007) and NAD 83(CORS96).

NGS has not computed NAD 83(NSRS2007) velocities for any of the 67,693 passive marks involved in the adjustment. Also, the positional coordinates of a passive mark will refer to an “epoch date.” Epoch dates are the date the positional coordinates were adjusted and are therefore considered “valid” (within the tolerance of not applying vertical crustal motion). Because a mark’s positional coordinates will change due to the dynamic nature of the earth’s crust, the coordinate of a mark on epochs different than the listed “epoch date” can only be accurately known if a three-dimensional velocity has been computed and applied to the mark.

In California, the NAD 83 values for the California CORS (CGPS) were obtained through Scripps’ Sector utility which stated that the NAD 83 coordinates were transformed from ITRF2000. It was later discovered that the NAD 83 coordinates were incorrectly labeled and were actually transformed from ITRF2005. The incorrect NAD 83 values in the 2007.0 epoch used as control for the readjustment are currently available through the California Spatial Reference Center (CSRC) website at: http://csrc.ucsd.edu.

In Alaska, the values used were the NAD 83(CORS96) 2003.0 values currently published by NGS. Although the HTDP model was used to transform the vectors to the 2007.0 epoch, this model was considered very poor in Alaska, due to the lack of data, and therefore the 2007.0 adjustment in Alaska will also produce poor quality results.

For Arizona, Oregon, Washington, and Nevada, HTDP was used to convert the currently published NAD 83 positions of the CORS to epoch 2007.0

14. Network and Local Accuracies

Local and network accuracies are measures which express to what accuracy the coordinates of a point are known. These measures are defined in the FGDC accuracy standards as follows:

“The network accuracy of a control point is a value that represents the uncertainty in the coordinates of the control point with respect to the geodetic datum at the 95-percent confidence level. For NSRS network accuracy classification, the datum is considered to be best expressed by the geodetic values at the CORS supported by NGS. By this definition, the network accuracy values at the CORS sites are considered to be infinitesimal, i.e., to approach zero.

“The local accuracy of a control point is a value that represents the uncertainty in the coordinates of the control point relative to the coordinates of other directly connected, adjacent control points at the 95-percent confidence level.”

The FGDC states that the reported local accuracy should be “an approximate average of the individual local accuracy values between this control point and other observed control points used to establish the coordinates of the control point". [FGDC 1998] At the time of this report, the NGS datasheets currently do not publish the local accuracies.

Both accuracies can be calculated from the elements of the coordinate covariance matrix produced during the national readjustment. The necessary elements were extracted and stored in the NGS database. In general, a "local accuracy" could be determined between any two points, regardless of whether they were, or were not, directly connected (share a single GPS vector). However, NGS will adhere to the FGDC guidelines and only compute local accuracies between directly connected stations. Note that, by this definition, a "local accuracy" might be reported for points spaced hundreds of km apart, while highly local pairs of points that aren't directly connected will have no local accuracy reported.

Local accuracies may also be computed for "no-check" stations participating in a session solution containing correlations between baselines. Also, it was possible for local accuracies to exceed network accuracies in rare cases where a "rejected" (by down weighting) vector corrupted the computations.

These accuracies have been implemented with the publication of the National Readjustment.

15. Helmert Blocking Strategy

Helmert blocking, proposed a little over 100 years ago by F. R. Helmert [Helmert 1880], is basically a technique for breaking up a least squares adjustment problem that is too large to be managed as a single computation into many smaller computational tasks, with potentially large savings in computer storage and CPU requirements. The main idea of Helmert blocking is to break the data into "blocks" which are partially solved independent of one-another, and then combining these partial solutions into a complete solution. Undertaking the analysis of smaller blocks becomes much easier than analyzing the entire computation at once. While several other strategies exist for dividing a large survey network into manageable sized pieces for adjustment, the method of Helmert blocking has the crucial advantage of producing a set of adjusted coordinates equivalent to a simultaneous least squares solution of all the data. This allows computation of the covariance matrices, relating errors in adjusted coordinates in the network to other adjusted coordinates.

The first step in Helmert Blocking is to divide the large network into smaller blocks. In the national readjustment, an attempt was made to generally break blocks up by state. The stations whose coordinates are to be adjusted are then associated within their geographic blocks. All observations for which the "from" station is inside the block are also associated with the block. Within each block, most of the stations are connected by observations to other stations only within the same block, and their coordinates are classified as *local parameters*. A number of the stations in each block are connected by observations to stations in other blocks. These are called "junction" stations, and their coordinates are called *global parameters*. There are two configurations to consider:

1. There is a GPS vector from a station inside the block to another station outside the block. The outside station is classified as an outside junction point, and its coordinates are added to the list of global parameters.

2. There is a GPS vector from a station outside the block to another station inside the block. The station inside the block is identified as a junction station, and its coordinates are added to the list of global parameters. It is not necessary to include the outside station as a junction station, because the observation belongs to the other block and it will be processed with that block. However, making this identification requires a global view of all the observations in the entire network, not just those in the block being processed.

The set of observations available for the readjustment contained many groups of correlated vectors. The Helmert blocking algorithm required that such a set of correlated observations be processed together in the same block. This was accomplished naturally in the national readjustment; because each observing session had a single hub station, all the observations were associated with that hub station, and they were all assigned to the block where the hub station fell. GPS projects were therefore kept intact and assigned to a specific Helmert block.

Division of survey data into blocks is, perhaps, the key step in developing a successful adjustment using Helmert blocking. Generally, blocks are based on some criterion, such as survey order (for example, FBN/CBN surveys, First order, etc.) or geographical location (for example, all surveys within an individual state). The Helmert blocking strategy used for the readjustment was based on the fact that most of the projects submitted to NGS, and stored in the NGS integrated database, were contained within state boundaries. Helmert blocks based on state boundaries would minimize the number of observations crossing block boundaries (junction baselines) and thus minimize the number of possible baselines which might cross between Helmert block boundaries. Within each block, the unknowns (ie. coordinates) were divided into global unknowns (ie. those that have some observation connection with neighboring blocks) and local unknowns (which have no observation connection outside the block). Constrained coordinates (such as CORS coordinates) were identified as global junctions and were computationally part of all Helmert block levels up to the top level where they are then constrained. Once all the normal equations of each block had been formed and adjusted, they were inversed and reassembled into lower-level, further inversed blocks, until finally the lowest level Helmert block normal equations were inversed.

The following schematic (Figure 15.1) details the Helmert blocking strategy developed for undertaking the national readjustment. A simple binary decomposition of the network was chosen based on the availability of existing Helmert blocking software and the simplification of analyzing problems during the block combination process.

HELMERT BLOCKING STRATEGY

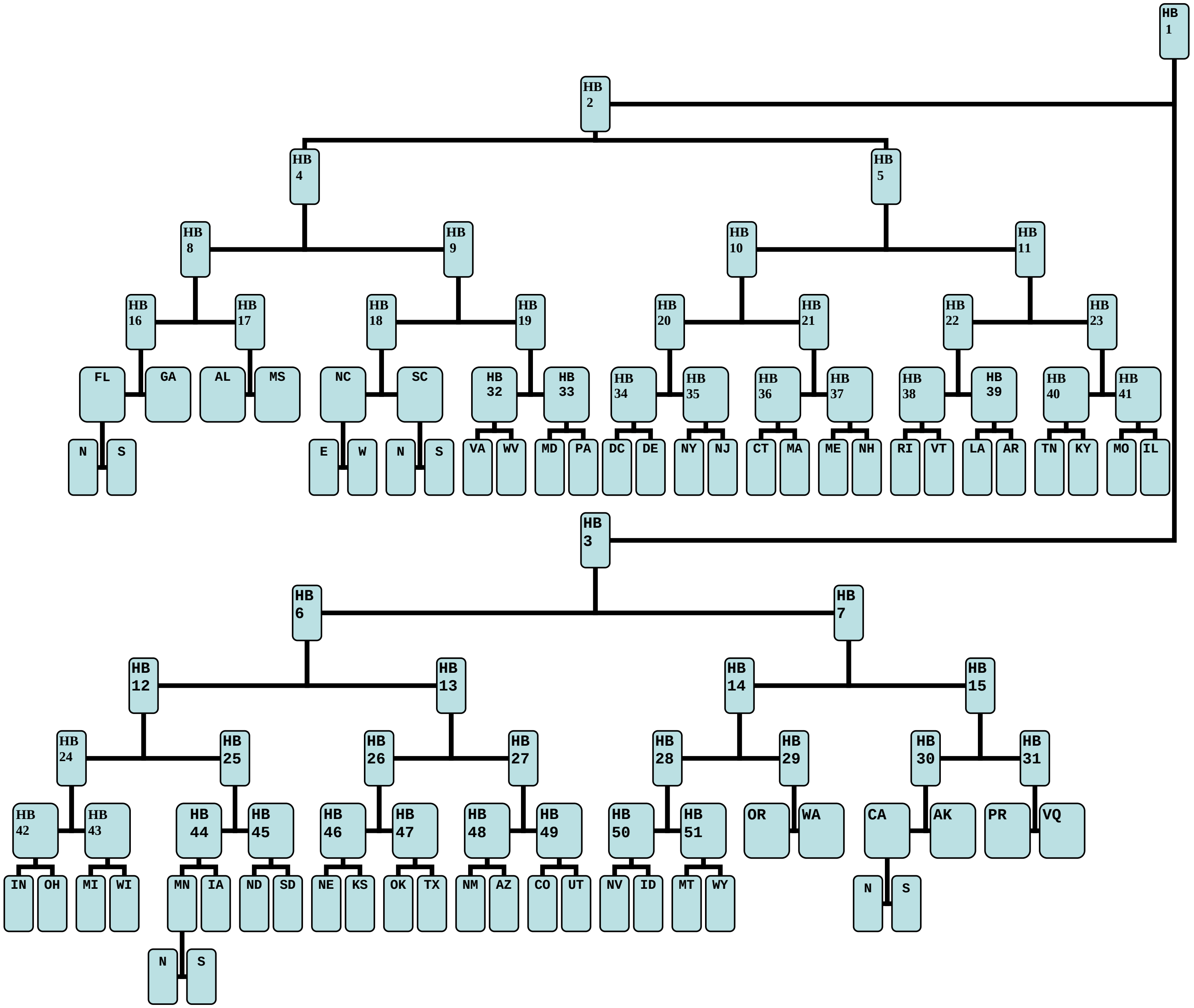


Figure 15.1 Helmert Block Strategy

Part IV. Computer Software

Mike Potterfield and Charles R. Schwarz

16. Introduction

In this section, we will describe the computer software used to compute the national readjustment, that culminated in NAD 83(NSRS2007), with a focus on NETSTAT, a Helmert blocking network adjustment program developed specifically for the readjustment. Other NGS computer programs involved in the readjustment will also be discussed. We will also describe the background considerations that led to the development of the new software, as well as the technical details and algorithms used to produce the final adjustment results.

17. Background

To briefly recap Part I of this report, after the 1983 adjustment of the North American Datum, NGS embarked on a series of High Accuracy Reference Networks (HARNs) in individual states. This created discontinuities at the state boundaries, and a number of approaches were devised to smooth the transition from one state HARN to another [Milbert and Milbert 1994]. Resurveying the HARNs with CORS ties (as Federal Base Networks or FBNs) did not fully remove state-by-state discontinuities. In 2003, NGS made a commitment and set a date for a comprehensive simultaneous readjustment of all these GPS surveys [Vorhauer 2007]. The goal was to complete the readjustment by February of 2007, coinciding with the 200th anniversary of the founding of the Coast Survey, the predecessor agency to the National Geodetic Survey.

It was widely understood that the national readjustment would produce formal error estimates for the adjusted coordinates, and these could be used to compute network and local accuracies, as required by the accuracy standards of the Federal Geographic Data Committee [FGDC 1998].

The initial plan (eventually abandoned) was to compute the readjustment by a “layered” approach using the existing NGS network adjustment software ADJUST [Milbert and Kass 1993]. This strategy involved defining seven layers of control stations, with the top level being the most accurate (the CORS network) and each layer below being less accurate. Each layer was to be adjusted to the next higher level, tightly constraining the higher level results. (The ADJUST software applies relative or stochastic weighting, rather than absolute constraints, to constrain estimable parameters.) The process would have begun with the most accurate layer being adjusted first, and each succeeding layer fixed to the layer above it.

The layered approach is the approach of classical geodesy—the first order networks were adjusted first. Second order densification surveys were adjusted to the first order networks. Quite possibly, third or lower order surveys could then be adjusted to the second order points. A major weakness of the classical approach was that it provided no formal mathematical method of error propagation from higher levels to lower levels. The classification of points as first, second, third, and lower order was approximate and intuitive, and did not always work as desired. A second weakness was that the classical approach was not equivalent to a simultaneous least squares adjustment of all the observations.

18. ADJUSTHB/GPSCOM/LLSOLV

Upon the abandonment of the layered adjustment approach, NGS determined the new simultaneous adjustment could only be feasibly computed using Helmert blocking.

NGS then needed to determine what existing software might be in the NGS software library that could compute a Helmert blocking adjustment, and what new software might be required in order to complete the task.

One possibility would have been to exploit the Helmert Blocking software developed by NGS for use in the NAD 83 adjustment completed in 1986. However, this software was developed for use with classical terrestrial observations and was never upgraded to accommodate GPS observations.

Additionally, it had fallen into disuse and no one at NGS had a good working knowledge of its operation. So, other possibilities were sought for network adjustment software using Helmert Blocking.

At that time (2004), NGS had available a Helmert blocking software suite developed for the purpose of multi-epoch processing of CORS data. The software, which continues to be used for the purpose of computing multi-year adjustments of CORS data, exists in two separate programs: GPSCOM and LLSOLV. GPSCOM is used to combine lower level Helmert blocks into higher level blocks and to solve the normal equations and unknowns at the highest level. The program LLSOLV is used to solve the lower level normal equations using the solutions to the levels immediately above them.

It was determined that the GPSCOM/LLSOLV programs provided almost all the functions necessary for computing the readjustment of the NSRS. However, some modifications to ADJUST and to GPSCOM/LLSOLVE were required.

The first modification to ADJSUST was to save the normal equations for GPS networks, which could then be passed to GPSCOM for the beginning of the Helmert blocking adjustment sequence. ADJUST does compute GPS normal equations as part of its usual application in adjusting project networks, so it was decided that ADJUST could be modified to write the computed normal equations to the normal equation files [Dillinger 1996] recognized by GPSCOM. Several additional modifications to ADJUST were also necessary, not the least of them being to transform ADJUST's normal equation format from its usage of local NEU coordinate systems for the unknowns into the usage of XYZ Cartesian coordinates, which is the format understood by GPSCOM/LLSOLV.

The resulting modified version of ADJUST is known as ADJUSTHB. Testing of this version proved it could successfully write the normal equations to the normal equation files. However, these normal equations were written to file in unreduced form, meaning that no block-diagonal partitioning was computed for the local and global parts of the normal equations. ("Local" and "global" parameters in Helmert blocks are defined in Section 22 of this report.) In order to complete the necessary partitions and reduction of the normal equations, GPSCOM was modified so that, in addition to its usual function of combining reduced and partially reduced lower level normal equations, it could also convert the unreduced normal equations produced by ADJUSTHB into the partitioned normals.

19. Development of NETSTAT

Once the application ADJUSTHB was developed at NGS, it was necessary to develop an entirely new application to be used to analyze the residuals for already solved lower level Helmert blocks computed by LLSOLV. The residual analysis was necessary in order to analyze adjusted observations and solved coordinate unknowns, complete with formal errors, and, not least, to compute the FGDC-supported network and local accuracies for the readjustment. The new software, which at the outset was devoted entirely to the residual analysis, was given the name NETSTAT. All versions of NETSTAT were developed specifically for the readjustment project.

19.1 NETSTAT 1.x

All versions 1.x of NETSTAT were devoted entirely to the residual analysis functions. At the time these versions were developed, the final LLSOLV solutions were being computed in approximately eight days using one computer working full time. NETSTAT 1.x did nothing to improve this performance in time, but it did produce the required new analysis output. As part of the development of version 1.x of NETSTAT, some errors—including incorrectly computed standard deviation of residuals in N, E, U, and incorrect vector counts in the observational summary when vectors were rejected—were found and corrected in ADJUST version 4.33, ADJUSTHB, and GPSCOM.

One of the primary requirements for NETSTAT 1.x was to parse the binary Level 1 solution files created by LLSOLV. Since these output files use unformatted FORTRAN records, it was necessary for NETSTAT (developed in C++) to parse the file descriptors appended and prepended to these records. The compatibility with Fortran-produced binary data files continued up to the development of version 5.02, when it was abandoned.

Although NETSTAT 1.x performed the required functions, the use of multiple programs, written in different programming languages, was a cumbersome and wasteful use of computer and human resources. Therefore, beginning in January of 2006, development of new and enhanced versions of NETSTAT was undertaken.

19.2 NETSTAT 2.x

The first major upgrade was to have NETSTAT take over all functions previously performed by ADJUSTHB, including the reading and parsing of all the input files, setting up all the internal indices, forming the observation equations, and computing the normal equations for a single block. This version is NETSTAT 2.x.

19.3 NETSTAT 3.x

During the development of version 2.x, it became clear that the normal equations file created by NETSTAT should be the same kind of binary file as produced by GPSCOM. This required implementing the partial and full reduction of the partitioned normal equations.

At the same time, in the early part of 2006, it also became clear that the processing of the readjustment Helmert block tree was taking far too long. Changes made to any Level 1 Helmert block could not be analyzed until a complete adjustment had been computed—a process that took eight days.

A major effort was undertaken to implement the reduced column profile (see Section 22.2.5) in the outgoing and incoming Level 1 normal equation files. This required both replacing the Gaussian factor with the Cholesky factor and also implementing the recursive partitioning of the Cholesky factor [Hanson 1978] in the Level 1 files. The literature describing this algorithm was not well developed, and this solution required considerable time and effort, but eventually the Cholesky factor under the reduced column profile was put into place. (The algorithm is further discussed in Part IV.)

However, because GPSCOM did not recognize the Cholesky factor, it was also necessary that NETSTAT version 3.x should be able to combine lower level blocks (those containing Cholesky factors) into higher level blocks, resulting in version 3.01. LLSOLV continued to be used to bring the highest level solutions down into the lower level solution files. Once the new feature was put into place, it became evident that NETSTAT could also replace LLSOLV in the downward solutions of the Helmert block files, and this, too, was put into place. As part of these developments, a new text output file, called BlockAdjust.out, was devised for NETSTAT. This file presents adjustment results from the solution computed for the highest-level Helmert block, and this is the first place the HB system standard error of unit weight, number of parameters, and various other global statistics are output, along with the adjustments for all parameters present in this highest level block. The results can also be found in the lowest-level adjustment results output files.

Version 3.2 implemented a change to GPSCOM's Problem Definition Files (typically named gpscom.pdf), so constraints could be applied individually to horizontal and vertical components, instead of only one constraining weight being applied to all three parameters.

The benefit of version 3.x was quickly realized; the processing time for the full Helmert block structure was reduced from 8 days to 32 hours. Version 3 of NETSTAT was used to process most of the first level blocks and to compute the initial adjustments.

19.4 NETSTAT 4.x

Version 4.x of NETSTAT added the capability to subdivide an existing Helmert block into two smaller blocks (see Section 25). This feature was used to subdivide the Helmert blocks for California, North Carolina, South Carolina, Minnesota, and Florida. The savings in computer time was striking; the time to compute a complete adjustment was reduced from 32 hours to 12 hours.

19.5 NETSTAT 5.x

Even though the reduced column profile was being employed, beginning with version 3.2, the size of the normal equation files remained very large, because the full normal equations and inverse were being written to disk, even though the computations were only taking place under the column profile. Version 5.01 implemented the storage of the normal equations only under the column profile. Implementing this development required modifying the order the normal equations were being stored internally in NETSTAT,

so that the previous row-order storage was replaced in all matrices by column-order storage.
Version 5.02 removed all dependencies upon Fortran-created binary data files, so from this version onwards it was no longer possible to pass the binary data files back and forth between NETSTAT and GPSCOM/LLSOLV.

Version 5.03 implemented the read/write of binary data files using the reduced column profile. In addition, this version also implemented a shortcut in the computation of the circular error [Leenhouts 1985], reducing the processing time from 12 hours to 9 hours. Version 5.03 is the version of NETSTAT used to process the final NSRS 2007 adjustment. During the readjustment project, the computer time required to perform a complete network adjustment was reduced from eight days with NETSTAT 1.x to nine hours with NETSTAT 5.03, suggesting that, should future network readjustments be necessary, the need for computer and human resources should not be a limiting factor.

Development of NETSTAT continued after the completion of the NSRS2007 adjustment in February 2007. New features were added so that NETSTAT can serve as a general purpose network adjustment program used for any desired network adjustments, both using and not using the Helmert blocking algorithm.

20. Datum Transformations

The 2007 Readjustment was computed in the NAD 83 reference frame, as defined by constrained published NAD 83 coordinates on 685 CORS sites. The GPS vectors were expressed in a number of different coordinate systems, usually the coordinate system of the precise orbit used in processing the GPS observations. NETSTAT preprocessed the vectors to transform them to NAD 83. The transformation algorithms used were identical to those used by ADJUST. The coordinate system of each GPS observing session was identified on the B record of the G-File. There are currently 24 of these satellite datums defined within NGS software packages, although not all of these have a defined transformation to NAD 83.

For most of the continental United States, the corrections for the rotation of the North American plate—implicit in the definition of the NAD 83 datum—takes care of most tectonic displacements. However, this is not the case in the Western States, where the geophysical scenario is more complex and points are affected by time-dependant secular and episodic motions. For these states, the NAD 83 constraints were first updated to the epoch 2007.00 (using HTDP if necessary), and likewise the GPS vectors connected to these stations were updated to NAD 83 2007.00 using HTDP.

A slightly different configuration of constraints was adopted in California. In California (with two Helmert blocks—CANorth and CASouth) the ITRF2000 coordinates for the constraints were obtained from the SECTOR utility provided by the Scripps Orbit and Permanent Array Center (SOPAC) (http://sopac.ucsd.edu/). They were then converted to NAD 83 using the exact transformation at http://www.ngs.noaa.gov/CORS/coordinates/. As mentioned earlier in Chapter 13, ITRF05 values were used, as they were incorrectly labeled as ITRF2000.

21. The NETSTAT Adjustment Model

NETSTAT performs a least squares adjustment of vectors between observing stations. NETSAT uses the Variation of Coordinates method, as described in many textbooks, e.g. [Leick 2004; Mikhail 1976]. It differs from other adjustment programs in its features for efficient handling of large sparse systems of equations.

The normal equations are written

$$\mathbf{N}\mathbf{X} = \mathbf{U} \tag{0.1}$$

The least squares solution is found formally by solving the normal equations

$$\mathbf{X} = \mathbf{N}^{-1}\mathbf{U} \tag{0.2}$$

NETSTAT is designed to treat problems with so large a number of unknown parameters that equation (0.2) cannot be applied directly.

In many texts (e.g. [Leick 2004; Mikhail 1976]) the inverse of the normal equation coefficient matrix

$$\mathbf{Q} = \mathbf{N}^{-1} \tag{0.3}$$

is called the cofactor matrix. The estimated error-covariance (also called “dispersion”) matrix of the estimated parameters is

$$\mathbf{\Sigma} = \hat{\sigma}_0^2 \mathbf{Q} \tag{0.4}$$

where $\hat{\sigma}_0^2$ is the variance of the observation of unit weight estimated from

$$\hat{\sigma}_0^2 = \frac{\mathbf{V}^{\mathbf{T}}\mathbf{PV}}{n-u} \tag{0.5}$$

Here V is the vector of residuals, P is the observational weights matrix, n is the number of observations, and u is the number of unknowns.

The unknown parameters in NETSTAT are the earth-centered, earth-fixed Cartesian coordinates of the GPS observing stations. Two types of observations are processed: vectors between GPS observing stations and constraints on coordinates.

The vectors between observing stations are handled as described in [Leick 2004]. The vector between stations k and m is modeled as

$$\begin{bmatrix} \Delta X_{km} \\ \Delta Y_{km} \\ \Delta Z_{km} \end{bmatrix} = \begin{bmatrix} X_k - X_m \\ Y_k - Y_m \\ Z_k - Z_m \end{bmatrix} \tag{0.6}$$

The linearized observation equations for a single vector observation are

$$\begin{aligned} \delta X_k - \delta X_m &= \Delta X^b_{km} - \Delta X^0_{km} + v_{x_{km}} \\ \delta Y_k - \delta Y_m &= \Delta Y^b_{km} - \Delta Y^0_{km} + v_{y_{km}} \\ \delta Z_k - \delta Z_m &= \Delta Z^b_{km} - \Delta Z^0_{km} + v_{z_{km}} \end{aligned} \tag{0.7}$$

Here δX_k, δY_k, and δZ_k as well as δX_m, δY_m, and δZ_m are corrections to a-priori values of the coordinates, ΔX^b_{km} is the X component of the observed vector, ΔX^0_{km} is the X component computed from a priori values, and $v_{x_{km}}$ is the residual to the observation with similar expressions for $\Delta Y^b_{km}, \Delta Z^b_{km} \Delta Y^0_{km} \Delta Z^0_{km} v_{y_{km}} v_{z_{km}}$. Because each of these observation equations relies only on two unknown parameters (either the X, Y or Z coordinates of the two points connected by the vector), then each observation contains only two non-zero coefficients (a +1 and a -1). All other coefficients are zero. Such a situation obviously tends to create sparseness in the normal equations as well.

As discussed in [Leick 2004, p. 302], vectors may come in groups corresponding to observing sessions. All the vectors observed in a session may be correlated with each other. Thus, if *R* receivers observe the same satellites simultaneously, there will be (*R*-1) independent vectors with a 3(*R*-1) ×3(*R*-1) covariance matrix.

Consider element n_{ij} of the normal equation coefficient matrix **N**. Suppose that unknown *i* is a coordinate of station *k* and unknown *j* is a coordinate of station *l*. Then, it can be shown that n_{ij} is non-zero only if there is at least one session involving stations *k* and *l*. In a large network, there will be many pairs of stations which do not observe together in any session. This means that there will be many zeroes in the normal equation coefficient matrix. Such matrices are called “sparse.”

NETSTAT processes the vectors and their covariance matrices presented to it in G-Files <http://www.ngs.noaa.gov/FGCS/BlueBook/pdf/Annex_N.pdf>, the same as the ADJUST program.

22. Processing Sparse Normal Equations.

NETSTAT brings together two sets of algorithms, both designed to produce efficiencies in the processing of sparse normal equations. The efficiencies are achieved in three ways:

1. by not storing elements already stored elsewhere (e.g., storing only the upper triangular part of a symmetric matrix),

2. by not storing elements whose value is already known to be zero (e.g., those elements outside the matrix profile), and
3. by not performing computations whose result is already known (e.g., skipping multiplications by elements whose value is already known to be zero).

The two sets of algorithms are:

1. Those applied within a single computer run. These are the Cholesky factorization and the refinements that allow storage of only the elements of the factor and the inverse under the matrix profile.

2. Those that allow a large project to be broken up into a set of separate computer runs so that the results are algebraically equivalent to a single simultaneous solution (Helmert Blocking).

22.1 Cholesky Factorization

The Cholesky factor **K** is that upper triangular matrix which uniquely fulfils the following property:

$$\mathbf{N} = \mathbf{K}^{\mathbf{T}}\mathbf{K} \tag{22.1}$$

There are two standard methods for computing the Cholesky factor: the “inner product” method and the “outer product” method. The inner product method lends itself to fast computation of both the solution and the inverse of the normal equations and is the method employed both in NETSTAT and in most NGS adjustment software.

Once the Cholesky factor has been computed, the inverse may be computed from the inverse of the Cholesky factor:

$$\mathbf{N}^{-1} = \mathbf{K}^{-1}\mathbf{K}^{-T} \tag{22.2}$$

Therefore the vector **X** of the unknowns is computed as

$$\mathbf{X} = \mathbf{K}^{-1}\mathbf{K}^{-T}\mathbf{U} \tag{22.3}$$

If the inverse matrix is not needed, the vector **X** is typically found in two steps:

$$\mathbf{Y} = \mathbf{K}^{-T}\mathbf{U} \qquad \text{(forward reduction)} \tag{22.4}$$

$$\mathbf{X} = \mathbf{K}^{-1}\mathbf{Y} \qquad \text{(back substitution)} \tag{22.5}$$

22.1.1 Cholesky Factorization by the Inner Product Method

Let **N** be a symmetric positive definite n x n matrix and define its upper left k x k submatrix as

$$\mathbf{N}_k = \mathbf{N}(1:\mathrm{k},1:\mathrm{k})$$

so that

$$\mathbf{N}_1 = (n_{11})$$

and

$$\mathbf{N}_n = \mathbf{N}$$

Let $\mathbf{K}_k$ be the Cholesky factor of $\mathbf{N}_k$, so that $\mathbf{K}_k^{\mathrm{T}}\mathbf{K}_k = \mathbf{N}_k$. Then $\mathbf{K}_1 = (\sqrt{n_{11}})$.

Suppose the Cholesky factor of $\mathbf{N}_k$ (K_k) is known and a method is sought to use this information to solve for the Cholesky factor of N_{k+1}.(K_{k+1}) Write the Cholesky factor of $\mathbf{N}_{k+1}$ as

$$\mathbf{K}_{k+1} = \begin{pmatrix} \mathbf{K}_k & \boldsymbol{\beta}_{k+1} \\ 0 & \boldsymbol{\gamma}_{k+1} \end{pmatrix} \tag{22.6}$$

where β_{k+1} is a k x 1 vector and γ_{k+1} is a scalar. By the definition of the Cholesky factor, one can sub-divide the N_{k+1} submatrix and write:

$$\mathbf{N}_{k+1} = \begin{pmatrix} \mathbf{K}_k^{\mathrm{T}} & 0 \\ \boldsymbol{\beta}_{k+1}^{\mathrm{T}} & \boldsymbol{\gamma}_{k+1} \end{pmatrix} \begin{pmatrix} \mathbf{K}_k & \boldsymbol{\beta}_{k+1} \\ 0 & \boldsymbol{\gamma}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{N}_k & \overline{\mathbf{N}}_{k+1} \\ \overline{\mathbf{N}}_{k+1}^{\mathrm{T}} & n_{k+1,k+1} \end{pmatrix}$$

where $\overline{\mathbf{N}}_{k+1}$ is **N**(1:k,k+1).

Then

$$\mathbf{K}_k^{\mathrm{T}}\boldsymbol{\beta}_{k+1} = \overline{\mathbf{N}}_{k+1}$$
$$\boldsymbol{\beta}_{\mathrm{k+1}}^{\mathrm{T}}\boldsymbol{\beta}_{k+1} + \boldsymbol{\gamma}_{k+1}^{2} = n_{k+1,k+1}$$

Solving

$$\boldsymbol{\beta}_{k+1} = \mathbf{K}_k^{-\mathrm{T}}\overline{\mathbf{N}}_{k+1}$$
$$\gamma_{k+1} = \sqrt{n_{k+1,k+1} - \boldsymbol{\beta}_{k+1}^{\mathrm{T}}\boldsymbol{\beta}_{k+1}}$$

so that now the two unknown components of $\mathbf{K}_{k+1}$ can be computed, and when combined with the known

K_k, the solution for the entire K_{k+1} matrix (the Cholesky factor of N_{k+1}) has been found.

Now consider the recursiveness of these solutions, and the efficiencies which present themselves. Note that $\mathbf{K}_k^T$ is lower triangular, so the equation $\mathbf{K}_k^T\boldsymbol{\beta}_{k+1} = \bar{\mathbf{N}}_{k+1}$ can be solved by back substitution.

The complete algorithm is thus:

```
1    for k=1,n {  // stage k
2        for i= 1,k {
3            $sum = \bar{\mathbf{N}}_{k+1}(i)$ ;  // =$n_{i,k+1}$
4            for j=1,i-1 {
5                sum-= $\mathbf{K}_k$(j,i)*$\boldsymbol{\beta}_{k+1}$(j);
6            }
7            if (i<k)  $\boldsymbol{\beta}_{k+1}$(i)=sum/$\mathbf{K}_k$(i,i);
8            else $\gamma_{k+1}$=sqrt(sum);
9        }
10   }
```

Several efficiencies are available here. First, the normal equation coefficient matrix **N** is symmetric, so it is only necessary to store the upper half of the matrix (including the diagonal) in order to have all of the terms. This is shown in the left diagram of Figure 22.1, where the elements in the grey portion of the diagram are not stored.

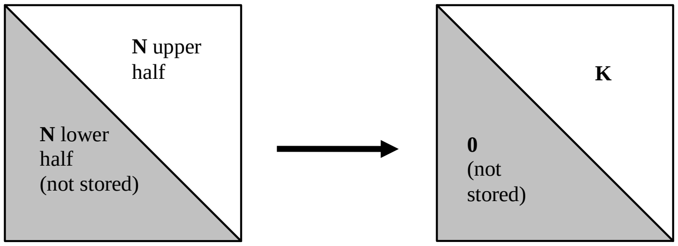


Figure 22.1: Decomposing the Normal Equations into the Cholesky Factor (Elements in Grey Area Are Not Stored)

The second efficiency is that once the element $n_{i,k+1}$ is used at line three, it is not needed again. Its storage slot can therefore be reused by $\beta_{k+1}(i)$=**K**(i,k+1), and the Cholesky factor **K** can be developed in the storage space previously used for the upper triangular part of **N**. The factorization algorithm can be rewritten in terms of storage locations as

```
1    for k=1,n {  // stage k
2        for i= 1,k {
3            sum=N(i,k);  // =$n_{i,k+1}$
4            for j=1,i-1 {
5                sum-= N(j,i)*N(j,k);
6            }
7            if (i<k)  N(i,k)=sum/N(i,i);
8            else N(k,k)=sqrt(sum)
9        }
10   }
```

22.1.2 The Matrix Profile

In most network adjustment problems, the coefficient matrix of the normal equations is sparse, meaning most of the elements in the matrix are zero. As such, it will be beneficial to make use of known sparse matrix manipulation tools.

For example, consider the column profile of a sparse matrix. The column profile is the column-by-column formation of only those parts of each column falling below the first non-zero element in each column. See Figure 22.2; the elements colored red and green are within the column profile. The term “column” profile may refer to the pictorial representation, to the set of elements within the profile, or to the total number of such elements.

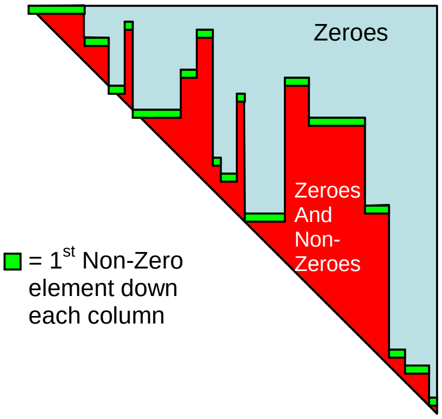


Figure 22.2. The Column Profile

The profile of a sparse matrix may be represented by a profile array. Let **P**(1:n) be the profile array for matrix **N**. **P**(j)=m will be taken to mean that the first m elements of column j of **N** are all zeros. The column height of column j, counting from the diagonal element, is j-**P**(j). If only the elements within the profile are stored column by column in a list, then the location within the list of element **N**(i,j) is

$$\sum_{k=1}^{j}(k-\mathbf{P}(k))+i-k$$

There are a number of algorithms to reorder the unknown parameters so that the matrix profile is small. NETSTAT uses the Banker's Algorithm [Snay 1976].

22.1.3 Factorization Under the Profile

By examining the algorithm of section 22.1.1, one notes:

(A) If $\bar{\mathbf{N}}_{k+1}(1)=0$, then $\boldsymbol{\beta}_{k+1}$ (1) =0,

(B) If $\bar{\mathbf{N}}_{k+1}(i)=0$ and $\boldsymbol{\beta}_{k+1}$(1:i-1) =0, then $\boldsymbol{\beta}_{k+1}$ (i) =0

Together, these mean that if $\bar{\mathbf{N}}_{k+1}(1:i)=0$, then $\boldsymbol{\beta}_{k+1}$ (1:i) =0, so we can say that if **P**(k+1)=m, then β_{k+1}(1:m) =0. This means that the Cholesky factor **K** has the same column profile as **N**. During factorization, there is no fill-in above the profile (although fill-in may occur below the profile).

The inner product factorization algorithm can therefore be further modified to skip computations involving terms outside the profile (already known to be zeroes). Given **N** (or at least its upper triangular part) and its profile array **P**, develop the Cholesky factor in the upper triangular part of **N** by

```
For k=1,n {  // stage k
        for i= P(k)+1,k {
                for j=max(P(i),P(k))+1,i-1 {
                        N(i,k)-=N(j.i)*N(j.k);
                }
                if (i<k) N(i,k)=N(i,k)/N(i,i);
                else N(i,i)=sqrt(N(i,i));
        }
}
```

Since the only elements addressed in the algorithm above are those in the upper triangular part and below the matrix profile, a compact storage structure containing only those elements may be used.

22.1.4 Matrix Inverse by Recursive Partitioning

Let **N** be symmetric and positive definite. Let **B**=**N**$^{-1}$ and let **K** be the upper Cholesky factor of **N**, so

$$\mathbf{N}=\mathbf{K}^{\mathrm{T}}\mathbf{K}$$
$$\mathbf{B}=\mathbf{N}^{-1}=\mathbf{K}^{-1}\mathbf{K}^{-\mathrm{T}}$$
$$\mathbf{K}\mathbf{B}\mathbf{K}^{\mathrm{T}}=\mathbf{I}$$

Partition

$$\mathbf{N}=\begin{pmatrix}\dot{\mathbf{N}}_k & \bar{\mathbf{N}}_k\\ \bar{\mathbf{N}}_k^T & \ddot{\mathbf{N}}_k\end{pmatrix}$$

where

$$\begin{aligned}\dot{\mathbf{N}}_k &= \mathbf{N}(1:k,1:k)\\ \bar{\mathbf{N}}_k &= \mathbf{N}(1:k,k+1:n)\\ \ddot{\mathbf{N}}_k &= \mathbf{N}(k+1:n,k+1:n)\end{aligned}$$

Partition **K** and **B** similarly. Then we have

$$\begin{pmatrix}\dot{\mathbf{K}}_k & \bar{\mathbf{K}}_k\\ 0 & \ddot{\mathbf{K}}_k\end{pmatrix}\begin{pmatrix}\dot{\mathbf{B}}_k & \bar{\mathbf{B}}_k\\ \bar{\mathbf{B}}_k^T & \ddot{\mathbf{B}}_k\end{pmatrix}\begin{pmatrix}\dot{\mathbf{K}}_k^{\mathrm{T}} & 0\\ \bar{\mathbf{K}}_k^{\mathrm{T}} & \ddot{\mathbf{K}}_k^{\mathrm{T}}\end{pmatrix}=\begin{pmatrix}\mathbf{I}_k & 0\\ 0 & \mathbf{I}_{n-k}\end{pmatrix}$$

Equating the bottom right partition of both sides gives

$$\ddot{\mathbf{K}}_k\ddot{\mathbf{B}}_k\ddot{\mathbf{K}}_k^{\mathrm{T}}=\mathbf{I}_{n-k}$$

In particular, for k=n-1,

$$\ddot{\mathbf{K}}_{n-1}=(k_{n,n})$$
$$\ddot{\mathbf{B}}_{n-1}=(b_{n,n})$$
$$\text{so}\quad \ddot{\mathbf{B}}_{n-1}=(b_{n,n})=(1/k_{n,n}^2)$$

Suppose $\ddot{\mathbf{B}}_k$ is known. The task is then to find

$$\begin{aligned}\bar{\mathbf{B}}_k &= -\dot{\mathbf{N}}_k^{-1}\bar{\mathbf{N}}_k\ddot{\mathbf{B}}_k \qquad \text{and}\\ \dot{\mathbf{B}}_k &= \dot{\mathbf{N}}_k^{-1}+\dot{\mathbf{N}}_k^{-1}\bar{\mathbf{N}}_k\ddot{\mathbf{B}}_k\bar{\mathbf{N}}_k^{\mathrm{T}}\dot{\mathbf{N}}_k^{-1}\end{aligned}\tag{22.7}$$

This is done one row and column at a time.

Let $\bar{\bar{\mathbf{K}}}_k=\mathbf{K}(k,k+1:n)$ (this is the part of row k to the right of the diagonal)

Write $\ddot{\mathbf{K}}_{k-1}\ddot{\mathbf{B}}_{k-1}\ddot{\mathbf{K}}_{k-1}^{\mathrm{T}} = \mathbf{I}_{n-k+1}$ as

$$\begin{pmatrix} k_{k,k} & \bar{\bar{\mathbf{K}}}_k \\ \mathbf{0} & \ddot{\mathbf{K}}_k \end{pmatrix}\begin{pmatrix} \gamma_k & \boldsymbol{\beta}_k^T \\ \boldsymbol{\beta}_k & \ddot{\mathbf{B}}_k \end{pmatrix}\begin{pmatrix} k_{k,k} & \mathbf{0} \\ \bar{\bar{\mathbf{K}}}_k^{\mathrm{T}} & \ddot{\mathbf{K}}_k^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n-k} \end{pmatrix}$$

This yields three independent equations

$$\boldsymbol{\gamma}_k k_{k,k}^2 + 2k_{k,k}\bar{\bar{\mathbf{K}}}_k\boldsymbol{\beta}_k + \bar{\bar{\mathbf{K}}}_k\ddot{\mathbf{B}}_k\bar{\bar{\mathbf{K}}}_k^{\mathrm{T}} = 1 \qquad (22.8)$$

$$k_{k,k}\ddot{\mathbf{K}}_k\boldsymbol{\beta}_k + \ddot{\mathbf{K}}_k\ddot{\mathbf{B}}_k\bar{\bar{\mathbf{K}}}_k^{\mathrm{T}} = \mathbf{0} \qquad (22.9)$$

$$\ddot{\mathbf{K}}_k\ddot{\mathbf{B}}_k\ddot{\mathbf{K}}_k^{\mathrm{T}} = \mathbf{I}_{n\text{-}k} \qquad (22.10)$$

Solving equation (22.9) for $\boldsymbol{\beta}_k$ yields

$$\boldsymbol{\beta}_k = -(\ddot{\mathbf{B}}_k\bar{\bar{\mathbf{K}}}_k^{\mathrm{T}})/k_{k,k} \qquad (22.11)$$

Substituting this into (22.8) yields

$$\gamma_k k_{k,k}^2 - \bar{\bar{\mathbf{K}}}_k\ddot{\mathbf{B}}_k\bar{\bar{\mathbf{K}}}_k^{\mathrm{T}} = 1$$

$$\gamma_k = (1 + \bar{\bar{\mathbf{K}}}_k\ddot{\mathbf{B}}_k\bar{\bar{\mathbf{K}}}_k^{\mathrm{T}})/k_{k,k}^2 \qquad (22.12)$$

Now all the pieces of $\ddot{\mathbf{B}}_{k-1}$ are known. Proceeding from *k=n-1* to *k=0,* we eventually reach $\ddot{\mathbf{B}}_0 = \mathbf{N}^{-1}$.

22.1.5 Special Properties of Recursive Partitioning

Once $\boldsymbol{\beta}_{\mathrm{k}}$ has been computed by equation (22.10), the partial row $\bar{\bar{\mathbf{K}}}_{k-1}$ is not used again. Its storage space can therefore be reused by $\boldsymbol{\beta}_k^{\mathrm{T}}$. Similarly, the space occupied by $k_{k\text{-}1,k\text{-}1}$ can be reused by γ_k. However, note that the elements of $\bar{\bar{\mathbf{K}}}_{k-1}$ cannot be replaced one by one by elements of $\boldsymbol{\beta}_k$, as is the case in the forward Cholesky algorithm. A vector of length *n-k+1* must be allocated to hold the elements of $\boldsymbol{\beta}_k$. Once they are all computed, $\bar{\bar{\mathbf{K}}}_{k-1}$ is not needed anymore and can be replaced by $\boldsymbol{\beta}_k$.

Given the Cholesky factor **K** of **N**, stored in the upper triangular part of a square matrix **A**, $\mathbf{N}^{-1}$ is computed by the algorithm

```
1       Allocate a work vector W of length n
3       for k=n,1,-1 {
4               for i=k+1,n {
5                       sum=0;
6                       for j=k+1,n {
7                               sum+= A (i,j)* A (k,j);
8                       }
9                       W(i)=sum;
10              }
11              gamma=1.0;
12              for j=k+1,n  {
13                      gamma+= A (k,j)*W(j);
14              }
15              for j=k+1,n {
16                      A (k,j)=-W(j)/ K (k,k);
                                // equation (22.11)
17              }
18              A (k,k)=gamma/( A (k,k)* A (k,k));
                                        //equation (22.12)
19      }
```

This algorithm develops the inverse in the space **A** originally occupied by the Cholesky factor **K**. Note that elements of the inverse below the main diagonal are referenced at line 7. However, these same elements are also available above the diagonal, so it is possible to use this algorithm with a storage scheme that stores only the upper triangular part of **A**.

22.1.6 Computation of Elements of the Inverse under the Profile

Suppose that element m of $\bar{\bar{\mathbf{K}}}_k$ vanishes. This is stored at **A**(k,m). Then, the term with *j=m* in the loop at lines 6-8 can be skipped, since this term is already known to be zero. Similarly, the term with *j=m* in the loop at lines 15-17 can be skipped. The elements in row m and column m of $\ddot{\mathbf{B}}_k$ are not referenced.

Also, if element m of $\bar{\bar{\mathbf{K}}}_k$ vanishes then element m of the loop at lines 12-14 may also be skipped. In this case, element m of **W** is not used, so the computation of element m of **W** may be skipped.

Suppose that element m of $\bar{\bar{\mathbf{K}}}_k$ and all elements above it vanish. This means that element m of $\bar{\bar{\mathbf{K}}}_k$ is outside the matrix profile of **N**. If the computation of elements of **B** corresponding to these locations is skipped, then none of these elements will be referenced by the algorithm.

The algorithm above may be modified to compute only the terms of **B** within the matrix profile:

```
Allocate a work vector W of length n
For k=n,1,-1 {
    for i=k+1,n {
        if(P(i)>=i) next;
        sum=0;

        for j=k+1,n {
            if(P(j)>=k) next;
            sum+= A(i,j)*A(k,j);
        }
        W(i)=sum
    }
    gamma=1.0
    for j=k+1,n  {
        if(P(j)>=k) next;
        gamma+=A(k,j)*W(j)
    }
    for j=k+1,n {
        if(P(j)>=k) next;
        A(k,j)=-W(j)/A(k,k)// eq (22.11)
    }
    A(k,k)=gamma/(A(k,k)*A(k,k));
            //equation (22.12)
}
```

22.2. Helmert Blocking

Helmert [1880] describes a procedure by which a large network adjustment problem can be broken up into a set of smaller problems so that the final result will be the same as if the large problem had been solve directly. The procedure can be used to advantage in large network adjustments, such as the national readjustment. The object is to design the smaller problems so they can be solved in a reasonable amount of computer time and so the analyst can manage the output, as described in Chapter 15.

22.2.1 Matrix Partitioning and Outgoing Helmert Blocks

The first step in Helmert Partitioning is to divide the large network into smaller geographic blocks. The parameters and observations associated with each lowest level block are determined by the method described in Chapter 15.

The unknown parameters are arranged so that the local parameters come first. The normal equation coefficient matrix **N** and right hand side **U** are partitioned as shown in Figure 22.3.

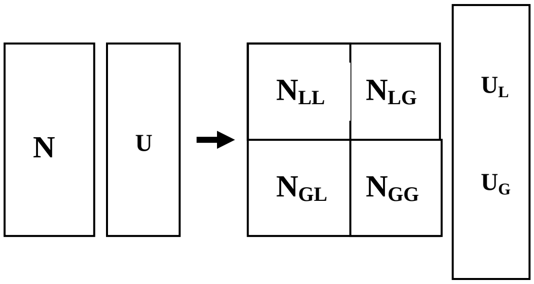


Figure 22.3: Partitioning a Helmert Block

In the diagram on the right, $\mathbf{N}_{LL}$ is the part of the normal equations devoted entirely to the local parameters, $\mathbf{N}_{GG}$ is the part of the normal equations devoted entirely to the global parameters, and $\mathbf{N}_{LG}$ and $\mathbf{N}_{GL}$ are the pseudo-cross-correlation matrices between the local and global parameters.

Based upon the discussion in Section 22.1.4, we know we can compute the inverse of **N** by first decomposing **N** into **K** (from the top row down) and then computing $\mathbf{N}^{-1}$ from the bottom row up. In other words, if we have the portion of $\mathbf{N}^{-1}$ in the space occupied by $\mathbf{N}_{GG}$, and the rest of the space occupied by the normal equations is filled with **K**, we can complete the inverse of **N** by beginning with the first row above $\mathbf{N}_{GG}$ and carrying the inverse all the way up to the top row. The inverse of $\mathbf{N}_{GG}$ is computed in higher level blocks, and then when this inverse is returned to the local block, the inverse of the local block is completed by carrying the back Cholesky solution up from $\mathbf{N}_{GG}$ to the rest of the normal equations.

We first compute the Cholesky factor $\mathbf{K}_{LL}$ of $\mathbf{N}_{LL}$. This is also the upper left partition of the Cholesky factor **K** of **N**, as may be seen from equation(22.6). We also compute the upper right partition

$$\mathbf{K}_{LG} = \mathbf{K}_{LL}^{-T}\mathbf{N}_{LG} \tag{22.13}$$

and the coefficient matrix and constant terms of the "partially reduced global normal equations"

$$\tilde{\mathbf{N}}_{\mathbf{GG}} = \mathbf{N}_{\mathbf{GG}} - \mathbf{N}_{\mathbf{GL}}\mathbf{N}_{\mathbf{LL}}^{-1}\mathbf{N}_{\mathbf{LG}} = \mathbf{N}_{\mathbf{GG}} - \mathbf{K}_{\mathbf{LG}}^{\mathbf{T}}\mathbf{K}_{\mathbf{LG}} \tag{22.14}$$

$$\tilde{\mathbf{U}}_{\mathbf{G}} = \mathbf{U}_{\mathbf{G}} - \mathbf{N}_{\mathbf{GL}}\mathbf{N}_{\mathbf{LL}}^{-1}\mathbf{U}_{\mathbf{L}}^{\mathbf{T}} = \mathbf{U}_{\mathbf{G}} - \mathbf{K}_{\mathbf{LG}}^{\mathbf{T}}\mathbf{K}_{\mathbf{LL}}^{-}\mathbf{U}_{\mathbf{L}} \tag{22.15}$$

These transformations may be described in a number of ways. The computation of $\mathbf{K}_{\mathbf{LL}}$ is done by the inner product factorization method described in section 22.1.1 (or 22.1.3). But, this computation may also be represented as the result of a series of row operations, or as the result of multiplying $\mathbf{N}_{\mathbf{LL}}$ on the left by $\mathbf{K}_{\mathbf{LL}}^{-\mathbf{T}}$, since $\mathbf{K}_{\mathbf{LL}}^{-\mathbf{T}}\mathbf{N}_{\mathbf{LL}} = \mathbf{K}_{\mathbf{LL}}$. If we apply the same row operations to $\mathbf{N}_{\mathbf{LG}}$ we obtain $\mathbf{K}_{\mathbf{LG}}$, the upper right partition of the Cholesky factor **K** of **N**. Thus $\mathbf{K}_{\mathbf{LG}}$ is not computed explicitly by equations (22.13), but by an extension of the inner product factorization algorithm. The result of this transformation is shown in Figure 22.4

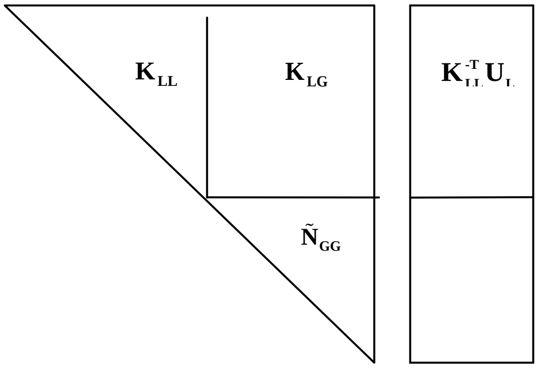


Figure 22.4: The Outgoing Helmert Block

The terms on the right in equations (22.14) and (22.15) also contains inner products—very similar to the inner products computed during the Cholesky factorization. All the terms in figure 21.4 can thus be computed by a single compact algorithm:

```
 1   for k=1,n+1 {  // column n+1 contains the
            constant terms U
 2          for i= 1, min(k,n) {
 3                 for j= 1,min(i-1,num_local) {
 4                        N(i,k)-=N(j,i)*N(j,k);
 5        }
 6             if(i>num_local) next;
 7             if (i<k)  N(i,k)/=N(i,i);
 8             else N(i,i)=sqrt(N(i,i));
 9          }
10   }
```

If we wish to compute only the terms in $\mathbf{K}_{\mathbf{LL}}$ and $\mathbf{K}_{\mathbf{LG}}$ under the matrix profile, line 3 is modified to

```
 3A              for j=max(P(i),P(k))+1, min(i-
                        1,num_local) {
```

22.2.2 Combine Lower Level Helmert Blocks into Higher Level Helmert Blocks

Higher level Helmert blocks are created when the outgoing "partially reduced" global normal equations from two lower level blocks are combined into a higher level block, as shown in Figure 22.5.

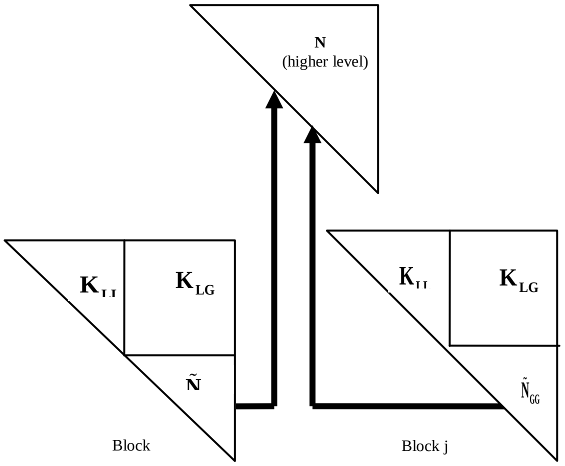


Figure 22.5 Combining Two Helmert Blocks

The constant columns $\tilde{\mathbf{U}}_{\mathbf{G}}$ from each lower level block are also carried up to the higher block. The higher block is associated with a geographic area which is the union of the geographic areas of the lower level blocks. The set of unknown parameters which appear in the combined block is the union of those which appeared in the lower blocks. These parameters can now be classified into local and global unknowns with respect to the geographic area of the combined block. This classification produces the partitioning shown in Figure 22.3.

If the combined block is not the highest level block, it can be prepared for combination with another block by the forward reduction described in 22.2.1, producing the decomposition shown in Figure 22.4.

The process is repeated until all of the lower level blocks have been reduced and accumulated into higher level blocks, and the highest level block is reached. At this stage, constraints are applied and a solution for the highest level block is computed.

22.2.3 Solving the Highest Level

The geographic area associated with the highest level is the entire project area. The unknowns solved for at the highest level are conventionally called global unknowns, because they are global to the entire project. However, they may also be formally classified as local with respect to the project boundary. The Cholesky factorization is carried out. The solution to the highest level unknowns is found by back substitution and the inverse of the normal equations is found by the algorithm in section 22.1.4.

The cofactor matrix $\mathbf{Q_{LL}}$ is the inverse of $\mathbf{N_{LL}}$, the coefficients of the normal equations at the highest level as shown in Figure 22.6. It contains the variances and covariances among the highest level unknowns from the total cofactor matrix in equation (0.3).

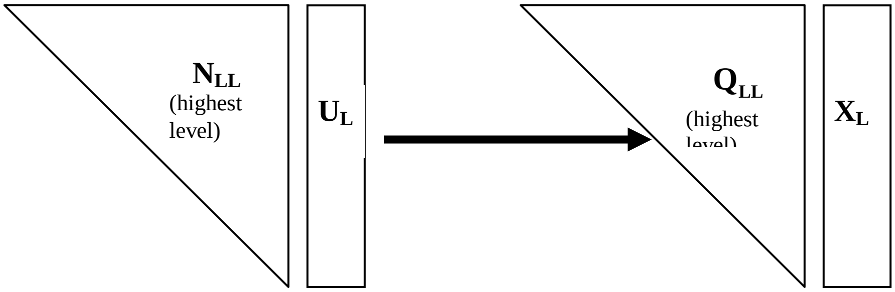


Figure 22.6 Transformations of the Highest Level of the Helmert Blocking Project

22.2.4 Back Substitution to Lower Level Blocks

Once the solution vector and matrix inverse for the unknowns at a higher level block are known, they can be propagated to lower level blocks, as shown in Figure 22.7.

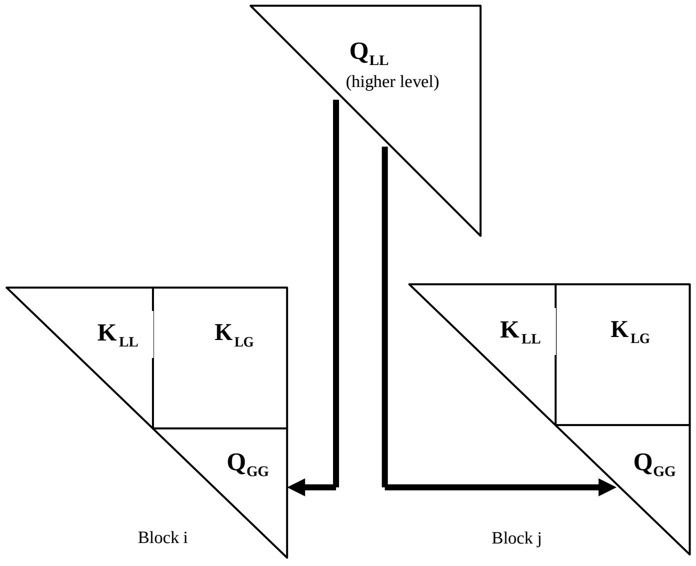


Figure 22.7. Propagation of the Solution and Inverse Matrix to Lower Level Blocks

The terms of the inverse $\mathbf{Q}_{LL}$ are picked out from the inverse at the higher level and placed in the lower right partition of each block. Similarly, the appropriate terms of the solution are picked out and placed in the lower partition of the constant column of each block. Note that the parameters that were local at the higher block are global at the lower block. The solution for the local unknowns are then computed by back substitution and the remaining partitions of the inverse matrix are computed by the recursive partitioning algorithm in section 22.1.4. These transformations are shown in Figure 22.8.

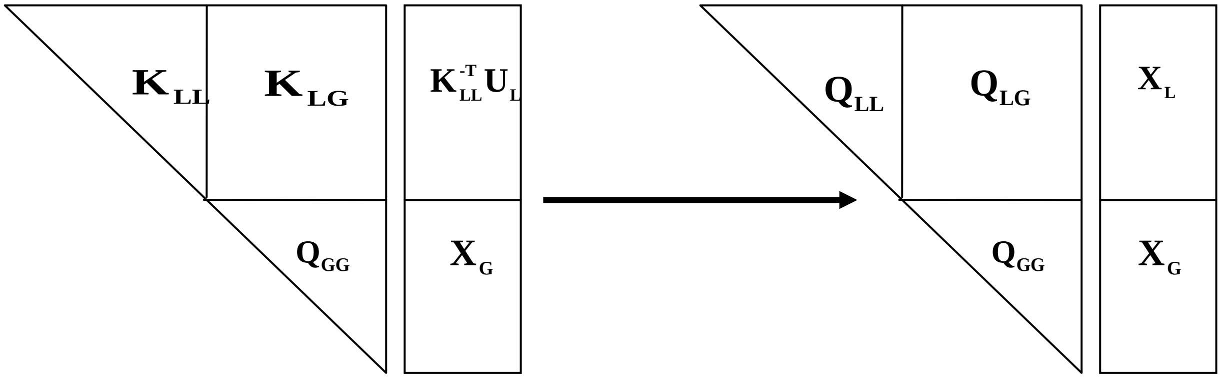


Figure 22.8. Transformation of a Lower Level Block During Back Substitution.

At the lowest level, the algorithm of section 22.1.6 is used to compute just the terms of the inverse under the profile. The absence of covariance terms above the profile does not impact the computations of network and local accuracies for the readjustment. The 3 x 3 covariance matrices for each station are always present (and therefore each station's Network Accuracy can be computed), and the only Local Accuracies computed for the Readjustment include only those pairs of stations either directly connected by observations, or participating in the same correlated GPS solution, and these terms are always present below the column profile.

22.2.5 The Reduced Matrix Profile

In Helmert blocking, reordering of the unknowns is performed only at the lowest level, and the reordering algorithm is applied only to the local unknowns. This is because we want the full inverse of the higher parameters to be computed. The partial application of the reordering algorithm is called the reduced column profile.

An example of a reduced column profile is shown below in Figure 22.9. The columns in this matrix assigned to the global parameters are moved to the far right-hand side of the image. It can be seen that most of the coefficients for the local parameters are zero, and are positioned above the reduced column profile.

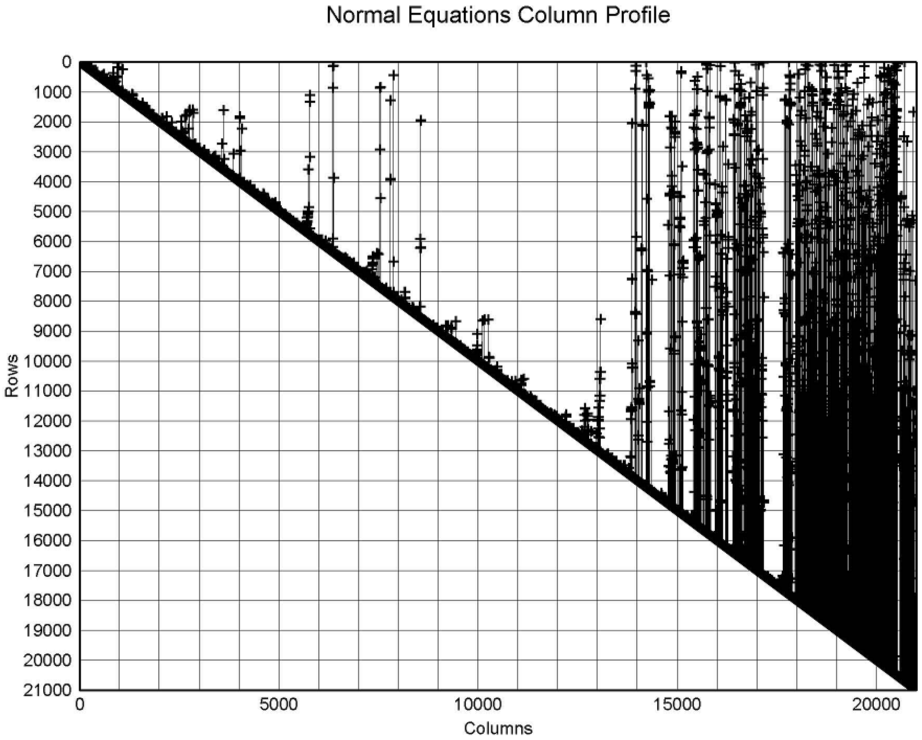


Figure 22.9: A Reduced Column Profile: The First 12801 Unknowns Are Local. The Remaining Unknowns are Global.

22.2.6 Computations of Additional Covariances.

The Helmert Block procedure described above will compute all terms of the covariance matrix of the parameters under the profile—sufficient for the national readjustment. However, there may be future needs to compute other terms of the covariance matrix.

At higher levels, NETSTAT always computes all terms of the matrix inverse. NETSTAT also provides an option to suppress consideration of the matrix profile at the lowest level blocks. This means that the covariances between the coordinates of two stations within a block can always be computed.

To find the covariances between parameters that are local to two different blocks, for example block i and block j, NETSTAT can compute

$$\begin{aligned}\mathbf{Q}_{\mathbf{L_iL_j}} &= \mathbf{N}^{-1}_{\mathbf{LL_i}}\mathbf{N}_{\mathbf{L_iG}}\mathbf{Q}_{\mathbf{GG}}\mathbf{N}_{\mathbf{GL_j}}\mathbf{N}^{-1}_{\mathbf{LL_j}} \\ &= \mathbf{K}_{\mathbf{L_i}}\mathbf{K}_{\mathbf{L_iG}}\mathbf{Q}_{\mathbf{GG}}\mathbf{K}_{\mathbf{GL_j}}\mathbf{K}_{\mathbf{L_j}}\end{aligned} \qquad (22.16)$$

This feature was not needed for the national readjustment, but is available should further analysis be required.

23. The Sum of Weighted Squares of Residuals

One of the requirements for computing network adjustments is the computation of the sum of the weighted squares of the residuals, which is needed for the computation of the estimated variance of unit weight in equation (0.5).

Consider the system of observation equations $\mathbf{AX} = \mathbf{L} + \mathbf{V}$, with weight matrix **P**, giving rise to the normal equations $\mathbf{NX} = \mathbf{U}$, where $\mathbf{N}=\mathbf{A}^T\mathbf{PA}$ and $\mathbf{U}=\mathbf{A}^T\mathbf{PL}$. We can compute

$$\begin{aligned}\mathbf{V}^T\mathbf{PV} &= \mathbf{V}^T\mathbf{P}(\mathbf{AX} - \mathbf{L}) \\ &= \mathbf{V}^T\mathbf{PAX} - \mathbf{V}^T\mathbf{PL} \\ &= -\mathbf{V}^T\mathbf{PL} \\ &= -(\mathbf{X}^T\mathbf{A}^T - \mathbf{L}^T)\mathbf{PL} \\ &= \mathbf{L}^T\mathbf{PL} - \mathbf{X}^T\mathbf{A}^T\mathbf{PL} \\ &= \mathbf{L}^T\mathbf{PL} - \mathbf{X}^T\mathbf{U}\end{aligned} \qquad (23.1)$$

Many adjustment programs use this identity to compute the sum of weighted squares of residuals without ever computing the residuals themselves.

It is also possible to obtain this term as a byproduct of the Cholesky factorization process. We accumulate the quantity $\mathbf{L}^T\mathbf{PL}$ as the observations are processed and append it under the constant column of the normal equations as shown in Figure 23.1:

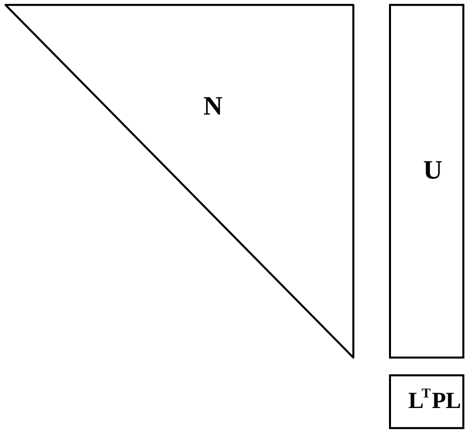


Figure 23.1 The Quantity $\mathbf{L}^T\mathbf{PL}$ Appended

The single element at the bottom, which initially holds the weighted sum of squares of the misclosures $\mathbf{L}^T\mathbf{PL}$, is colloquially called the basement window.

We compute the Cholesky factor **K** of **N** and apply the same row operations to **U** and apply the reduction algorithm to the last row. This transforms the term in the basement window into

$$\begin{aligned}\mathbf{L}^T\mathbf{PL} - \mathbf{U}^T\mathbf{K}^{-1}\mathbf{K}^{-T}\mathbf{U} &= \mathbf{L}^T\mathbf{PL} - \mathbf{U}^T\mathbf{N}^{-1}\mathbf{U} \\ &= \mathbf{L}^T\mathbf{PL} - \mathbf{U}^T\mathbf{X} \qquad (23.2) \\ &= \mathbf{V}^T\mathbf{PV}\end{aligned}$$

This scheme can be extended to an adjustment carried out by Helmert blocking. Consider the computation of the highest level block from two sub-blocks. The weight matrix **P** is block diagonal, since each GPS observing session may be assigned to only one block. We can write

$$\mathbf{L}^T\mathbf{PL} = \mathbf{L}_1^T\mathbf{P}_1\mathbf{L}_1 + \mathbf{L}_2^T\mathbf{P}_2\mathbf{L}_2$$

We also have

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{\mathbf{L}_1} \\ \mathbf{U}_{\mathbf{L}_2} \\ \mathbf{U}_{\mathbf{G}_1} + \mathbf{U}_{\mathbf{G}_2} \end{pmatrix} \quad \text{and} \quad \mathbf{X} = \begin{pmatrix} \mathbf{X}_{\mathbf{L}_1} \\ \mathbf{X}_{\mathbf{L}_2} \\ \mathbf{X}_{\mathbf{G}} \end{pmatrix}$$

so that

$$\begin{aligned} \mathbf{U}^{\mathbf{T}}\mathbf{X} &= \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{X}_{\mathbf{L}_1} + \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{X}_{\mathbf{L}_2} + (\mathbf{U}^{\mathbf{T}}_{\mathbf{G}_1} + \mathbf{U}^{\mathbf{T}}_{\mathbf{G}_2})\mathbf{X}_{\mathbf{G}} \\ &= \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{N}^{-1}_{\mathbf{LL}_1}(\mathbf{U}_{\mathbf{L}_1} - \mathbf{N}_{\mathbf{LG}_1}\mathbf{X}_{\mathbf{G}}) \\ &\quad + \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{N}^{-1}_{\mathbf{LL}_2}(\mathbf{U}_{\mathbf{L}_2} - \mathbf{N}_{\mathbf{LG}_2}\mathbf{X}_{\mathbf{G}}) \\ &\quad + (\mathbf{U}^{\mathbf{T}}_{\mathbf{G}_1} + \mathbf{U}^{\mathbf{T}}_{\mathbf{G}_2})\mathbf{X}_{\mathbf{G}} \\ &= \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{N}^{-1}_{\mathbf{LL}_1}\mathbf{U}_{\mathbf{L}_1} + \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{N}^{-1}_{\mathbf{LL}_2}\mathbf{U}_{\mathbf{L}_2} \qquad (23.3) \\ &\quad + ((\mathbf{U}^{\mathbf{T}}_{\mathbf{G}_1} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{N}^{-1}_{\mathbf{LL}_1}\mathbf{N}_{\mathbf{LG}_1}) \\ &\quad + (\mathbf{U}^{\mathbf{T}}_{\mathbf{G}_2} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{N}^{-1}_{\mathbf{LL}_2}\mathbf{N}_{\mathbf{LG}_2}))\mathbf{X}_{\mathbf{G}} \\ &= \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{N}^{-1}_{\mathbf{LL}_1}\mathbf{U}_{\mathbf{L}_1} + \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{N}^{-1}_{\mathbf{LL}_2}\mathbf{U}_{\mathbf{L}_2} \\ &\quad + (\tilde{\mathbf{U}}_{\mathbf{G}_1} + \tilde{\mathbf{U}}_{\mathbf{G}_2})\mathbf{X}_{\mathbf{G}} \end{aligned}$$

and

$$\begin{aligned} \mathbf{V}^{\mathbf{T}}\mathbf{PV} &= \mathbf{L}^{\mathbf{T}}_{1}\mathbf{P}_{1}\mathbf{L}_{1} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_1}\mathbf{N}^{-1}_{\mathbf{LL}_1}\mathbf{U}_{\mathbf{L}_1} \\ &\quad + \mathbf{L}^{\mathbf{T}}_{2}\mathbf{P}_{2}\mathbf{L}_{2} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_2}\mathbf{N}^{-1}_{\mathbf{LL}_2}\mathbf{U}_{\mathbf{L}_2} \qquad (23.4) \\ &\quad - (\tilde{\mathbf{U}}_{\mathbf{G}_1} + \tilde{\mathbf{U}}_{\mathbf{G}_2})\mathbf{X}_{\mathbf{G}} \end{aligned}$$

We create a basement window for each block containing the weighted sum of squares of misclosures, carry out the forward reduction described in section 22.2.1, and extend the reduction computations to the basement window. This can be easily done by storing the basement window in **N**(n+1,n+1) and modifying line 2 of the forward reduction algorithm in section 22.2.1 to read

```
2A        for i= 1, min(k,n+1) {
```

After the forward reduction algorithm is executed, the basement window for block i will contain $\mathbf{L}^{\mathbf{T}}_{i}\mathbf{P}_{i}\mathbf{L}_{i} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_i}\mathbf{N}^{-1}_{\mathbf{LL}_i}\mathbf{U}_{\mathbf{L}_i}$. When the two sub-blocks are combined into the highest level block, we add the values in the basement windows of the two sub-blocks and place the result in the basement window of the highest level block. As part of solving the highest level block, we apply the forward reduction algorithm. This transforms the basement window at the highest level into the value in (23.4).

The process can be extended recursively down to the lowest level blocks, so that

$$\begin{aligned} \mathbf{V}^{\mathbf{T}}\mathbf{PV} &= \sum_i (\mathbf{L}^{\mathbf{T}}_{i}\mathbf{P}_{i}\mathbf{L}_{i} - \mathbf{U}^{\mathbf{T}}_{\mathbf{L}_i}\mathbf{N}^{-1}_{\mathbf{LL}_i}\mathbf{U}_{\mathbf{L}_i}) \\ &\quad - \sum_i \tilde{\mathbf{U}}_{\mathbf{G}_i}\mathbf{X}_{\mathbf{G}} \end{aligned} \qquad (23.5)$$

24. Residual Analysis

Once the lowest level Helmert blocks have been solved, NETSTAT’s residual analysis is computed for every Level 1 block. The data available to this function includes only the vector of unknowns and the inverse of the normal equations below the reduced column profile. All the quantities that had been computed by the ADJUST program are also produced by NETSTAT. Completion of the adjustment computations requires that:

- The a-priori station coordinates are extracted from the Level 1 block B-file, and the adjusted coordinates are computed by adding the unknowns to these a priori coordinates.

- The G-file is opened and the GPS vectors are read into memory. The vectors are rotated into NAD 83 (see Section 20) and their covariance matrices are scaled according to the VS records in the Level 1 A-file.

- The adjusted vector components are computed by inversing between adjusted coordinates, and the vector residuals are computed by differencing the adjusted and unadjusted observations.

- The covariance matrix of the adjusted coordinates is computed, and vector component redundancy numbers are computed.

- The global statistics for the block (standard error of unit weight, degrees of freedom) are computed.

- The covariance matrix of the adjusted observations is computed.
- The covariance matrix of the observation residuals is computed.
- The standardized residuals are computed.
- The observational redundancy numbers are computed.
- The MDE (Minimum Detectable Error) statistic is computed.
- Local and network accuracies, including the circular error [Leenhouts 1985], are computed.

25. Subdividing Helmert Blocks

In the computational process involved in the 2007 Readjustment, the greatest amount of time by far was spent in processing the lowest level (Level 1) Helmert blocks. In particular, 5 blocks out of 52 took longer to process than the other 47 combined. These blocks were the states of Minnesota, California, Florida, North Carolina, and South Carolina.

In order to improve the processing times for the entire sequence, additional work was undertaken to subdivide these five blocks. The block subdivision alone reduced the processing time from 30 hours to 12 hours for the entire national readjustment.

The first step was to specify whether the block is to be subdivided into north and south sub-blocks, or east and west sub-blocks. The geometrical shape of the state original block is named for is the most important consideration in choosing a subdivision strategy. Of the five blocks mentioned above, all but North Carolina were subdivided into north and south sub-blocks; North Carolina was subdivided into east and west sub-blocks.

The algorithm will be explained using the subdivision into north and south sub-blocks. Each block to be subdivided already has a set of global parameters (mostly coordinates of junction stations, but also including some constraints). The purpose of the subdivision is to divide a big block into two sub-blocks as nearly equal in size as possible, and the goal is to minimize the number of new junction points.
Two stations are called "connected" if they participate together in any GPS observing session. In evaluating the station connections, the existing junction stations are ignored, as they are already global stations.

The first step is to choose an optimum percentage of local stations to be placed in each of the two new blocks. The optimum size was empirically determined to be 44%, although this can easily be modified. This means that 44% of the total local stations will be placed in the first sub-block, and 44% of the remaining local stations will be placed in the second sub-block, leaving 12% of the total number of local stations temporarily unassigned.

To build the first (e.g. southerly) block, the most southerly local station in the block is found and is added to the first block. All stations connected to this station are added sequentially to the new south sub-block, until all connected stations have been added. The algorithm then goes back through the local stations, finds the most southerly unused station, and repeats the process, until 44% of all of the local stations have been added to the southerly block (these are the stations shown in BLUE on the example for Minnesota, shown in
Figure 25.1 The northerly block is built the same way, by selecting the most northerly unused station, and adding stations connected to it to the northerly sub block until 44% of the local stations have been added. These stations are shown as GREEN, the existing junction stations are shown as RED, and the unassigned stations are shown as WHITE.

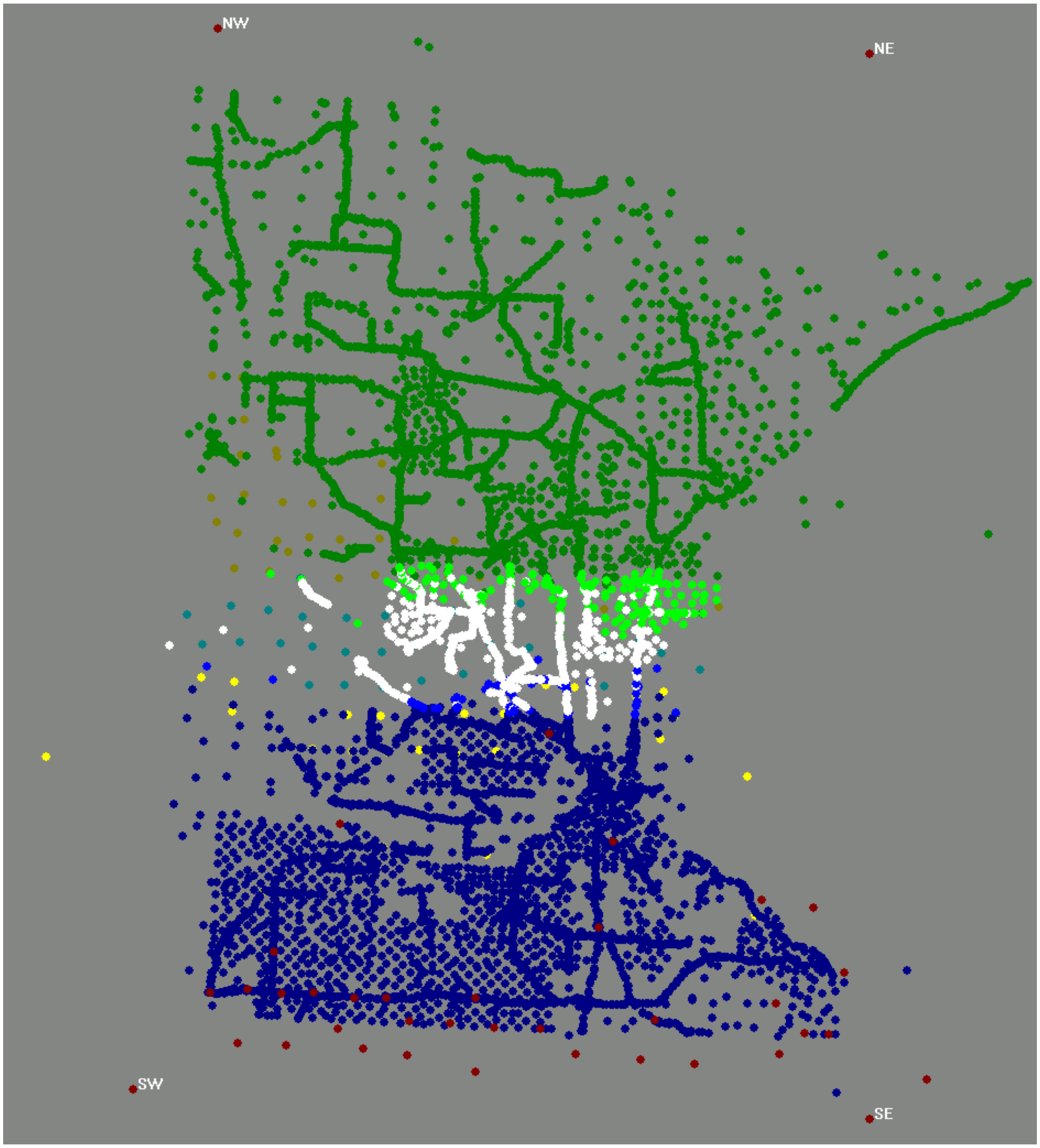


Figure 25.1: The First Step in Subdividing the Minnesota Block

All stations in Block 1 connected directly to Block 2 are shown as YELLOW (these will become new junction stations). All stations in Block 2 connected directly to Block 1 are shown as TAN (but these won't become new junction stations, because they are connected to the new junction stations just defined by Block 1). All unassigned stations connected to Block 1 are shown as LIGHT BLUE. All unassigned stations connected to Block 2 are shown as LIGHT GREEN. All unassigned stations connected to both Block 1 and Block 2 are shown as CYAN (these will become junction stations). If the unassigned stations have more stations connected to Block 1 than to Block 2, they are all merged into Block 1, and the connections to Block 2 will become junction stations. Conversely, if there are more unassigned stations connected to Block 2 than to Block 1, all unassigned stations will be added to Block 2 and the connections to Block 1 will become junction stations.

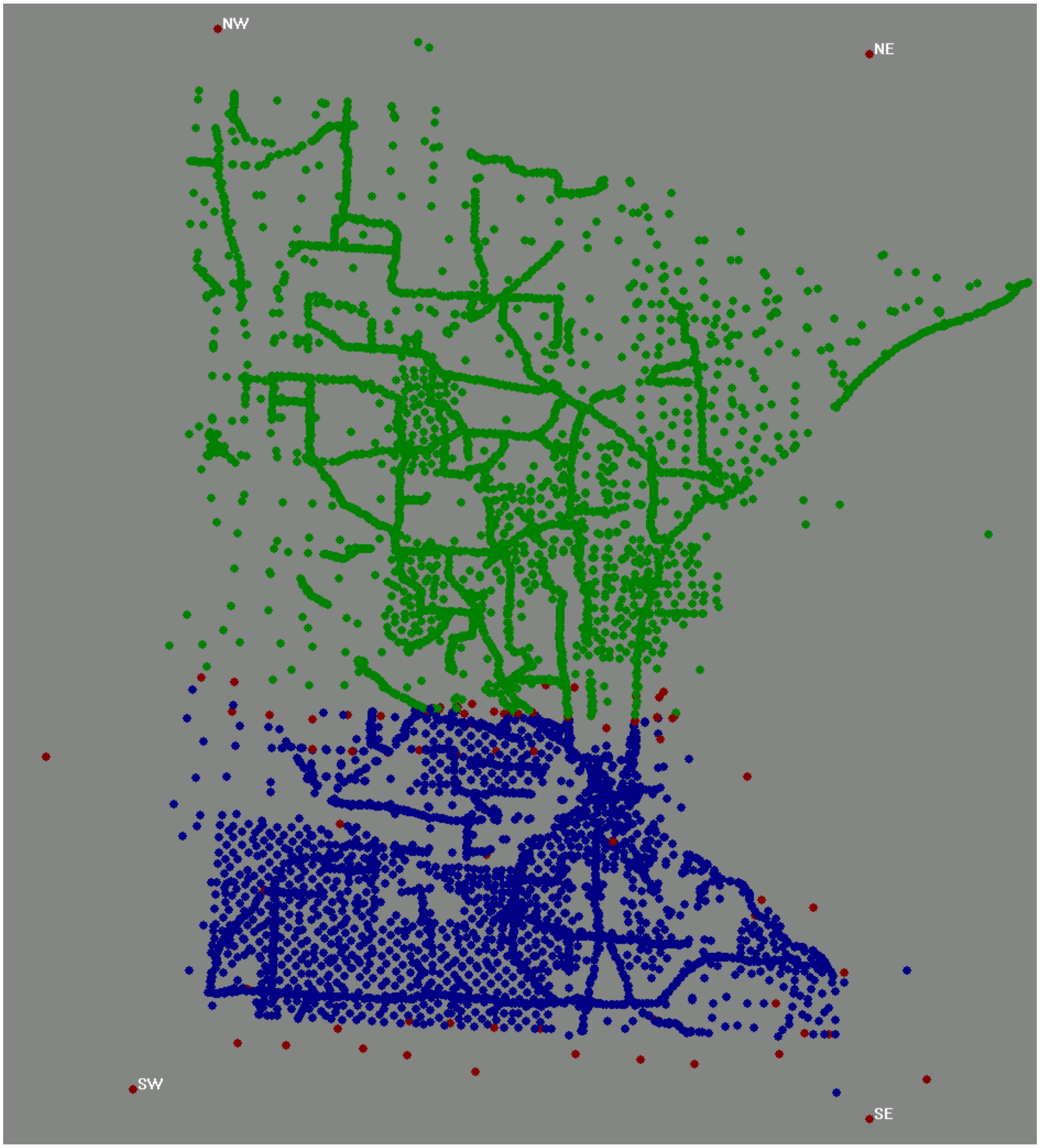


Figure 25.2: The Final Step in Subdividing the Minnesota Block

The final configuration of the subdivision is shown in Figure 25.2. The BLUE stations are in Block 1, the GREEN stations are in Block 2, and the RED stations are old and new junction stations. The NE, SE, SW, NW labeled points are reference points used in scaling the plots.

The ultimate test of this empirical algorithm was to see how many new junction stations had to be added. Minnesota was the ideal case, because only 1 percent of the local stations had to be converted into new junction stations. California required converting 4 percent of the local stations into new junction stations, and Florida required 2 percent.

26. Project Network Adjustments

Although NETSTAT was created in order to solve very large Helmert block systems, it is also targeted for use in blue-booked GPS projects similar to ADJUST, in that it will be used to solve both unconstrained and constrained adjustments of GPS networks intended to be added to the NSRS. In particular, NETSTAT is designed to compute Network Accuracies and Local Accuracies for project networks tightly constrained to the 2007.

Adding the capability of project adjustments to NETSTAT was quite simple, as the same files used to create outgoing Helmert blocks and to analyze incoming Helmert blocks, are used in project networks (see Section 34 on NETSTAT menus below). The only differences between NETSTAT and ADJUST for use in project adjustments are:

- NETSTAT presently does not recognize orthometric heights or geoid heights, but rather restricts height computations to ellipsoid heights.

- The G-file, A-file, and B-file used for project adjustments are not specified on the command line, in the same manner as ADJUST, but rather are specified in the NETSTAT input file format (see Section 34 below).

The NETSTAT adjustment output files are identical to the files created for Helmert block residual analysis. The primary adjustment results file is essentially identical to the output file from ADJUST. The updated B-file produced by NETSTAT project adjustments is also identical to the updated B-file produced by ADJUST, except that NETSTAT's updated B-file includes *91* (Network Accuracy) records and *92* (Local Accuracy) records. At the time of this report, these records are not official entries in the NGS BlueBook.

Part V. Helmert Block Analysis

D. Pursell, M. Vorhauer, and G. Edwards

27. Outlier Detection using Free Adjustments

Minimally constrained adjustments of the entire network were made in order to remove large residuals and blunders in the observations when all observations were combined. When all GPS observations contained in the NSRS were combined, all stations were found to be interconnected, resulting in one component requiring only one constrained station to minimally constrain the entire network. Many trial solutions were necessary to identify areas of inconsistency between stations observed over time and some misidentifications of observed stations.

The first trial solution of the minimally constrained adjustment was completed on December 19th, 2005. Residual plots for both the horizontal and up components were generated, along with output from program NETSTAT, giving complete residual information for every observation. The objective in the first trial solution was to analyze every high residual and to determine if rejections could be made based on better repeat observations or observations which better determined more accurate positions. This first trial solution also investigated and corrected for all singularities caused by stations which failed to tie to the NSRS. Upon completion of the residual analysis from the first trial solution, the NGS Data Base was updated to reflect all corrections made to each observation, and new blocks were retrieved to verify all corrections were properly made to the database. Subsequent trial solutions did not update the database. The database will be updated to reflect all corrections after the final readjustment is completed.

Trial solutions two and three resumed analysis of each block based on the corrections made in the first trial solution. The trial solutions resulted in a significant improvement over the results obtained from the first trial solution, although residuals greater than 5 cm still remained in many of the blocks. These trial solutions were also run to do analysis in weakly determined areas. Such areas were greatly affected by various rejections of particular observations, which either created larger residuals in other areas, or just shifted the residual to another area. The analysis of weakly determined areas were the most time consuming aspect of the minimally constrained adjustments.

In certain cases, rejections in local areas would cause previously rejected observations to become very good. These observations were un-rejected when residuals fell below the 5 cm tolerance. Trial solution four was the result after the analysis was made of all residuals to rejected observations.

Trial solution five was the result of analyzing the results, showing the stations which showed high positional shifts from the published values.

Trial solution six was the result of analyzing stations showing extremely large network accuracies.

Trial solution seven was the result of multiplying the square root of the variance of each parameter being estimated by the a-posteriori variance of unit weight. The purpose for this scaling was to determine more realistic network and local accuracies.

All trial solutions of the minimally constrained adjustments were performed constraining the National CORS NAD 83 published coordinate value for station GAITHERSBURG CORS ARP (AF9522). The variance of unit weight (unitless) for all seven trial solutions are shown in Table 27.1. The final statistics from the minimally constrained adjustment are shown in Table 27.2.

Table 27.1 Minimally Constrained Adjustment Statistics (Variance of Unit Weight)

		Minimally constrained Adjustments			Constrained Gaithersburg CORS ARP (AF9522)			
STATE	# Stations	Variance	Variance	Variance	Variance	Variance	Variance	Variance
		12/19/2005	4/26/2006	6/27/2006	9/11/2006	9/22/2006	10/24/2006	2/6/2007
		trial solution 1	trial solution 2	trial solution 3	trial solution 4	trial solution 5	trial solution 6	trial solution 7
ALASKA	792	4.94	1.28	1.28	1.25	1.30	1.30	0.80
ALABAMA	3668	1.73	1.31	1.29	1.29	1.87	1.87	1.14
ARKANSAS	396	2.23	1.97	1.78	1.77	1.77	1.76	1.09
ARIZONA	1389	1.74	1.80	1.76	1.53	1.54	1.53	0.94
CALIFORNIA	3532	0.48	1.31	0.90				
CALIFORNIA NORTH					1.18	1.18	1.18	0.72
CALIFORNIA SOUTH					1.01	1.01	0.99	0.54
COLORADO	1737	1.99	1.64	1.67	1.62	1.62	1.62	1.00
CONNECTICUT	103	1.27	1.26	1.25	1.25	1.25	1.25	0.77
DC	33	2.38	1.39	1.57	1.52	1.52	1.23	0.75
DELAWARE	91	2.90	2.45	2.35	1.86	1.84	1.74	1.30
FLORIDA	6608	1.47	1.29	1.29				
FLORIDA NORTH					1.26	1.26	1.26	0.77
FLORIDA SOUTH					1.31	1.31	1.31	0.81
GEORGIA	1529	2.34	1.34	1.31	1.30	1.30	1.31	0.90
IOWA	329	1.58	1.50	1.70	1.66	1.68	1.58	1.00
IDAHO	280	1.43	1.32	1.38	1.32	1.32	1.22	0.81
ILLINOIS	2515	1.50	1.33	1.33	1.33	1.36	1.36	0.84
INDIANA	270	1.76	1.66	1.72	1.71	1.73	1.68	1.03
KANSAS	469	2.39	2.09	1.99	1.47	1.52	1.52	1.37
KENTUCKY	1012	1.57	1.44	1.45	1.44	1.47	1.49	0.90
LOUISIANA	1158	1.84	1.18	1.16	1.16	1.16	1.16	0.71
MASSACHUSETTS	284	1.71	1.64	1.78	1.80	1.80	1.79	1.08
MARYLAND	2097	2.03	1.73	1.76	1.70	1.69	1.57	0.98
MAINE	466	1.72	1.74	1.65	1.65	1.81	1.81	1.03
MICHIGAN	1090	1.56	1.49	1.44	1.43	1.44	1.43	0.87
MINNESOTA	7003	2.27	1.96	1.94				
MINNESOTA NORTH					1.91	1.92	1.92	1.17
MINNESOTA SOUTH					1.98	1.98	1.98	1.21
MISSOURI	861	1.43	1.29	1.26	1.24	1.33	1.34	0.87
MISSISSIPPI	557	2.02	1.72	1.67	1.66	1.66	1.65	1.02
MONTANA	383	2.34	1.57	1.65	1.59	1.57	1.53	0.97
NORTH CAROLINA	5640	2.01	1.81	1.82				
N CAROLINA EAST					1.65	1.65	1.64	1.00
N CAROLINA WEST					1.98	1.98	1.98	1.21
NORTH DAKOTA	162	2.02	1.50	1.46	1.43	1.44	1.35	0.85
NEBRASKA	610	1.63	1.51	1.54	1.49	1.50	1.48	1.25
NEW HAMPSHIRE	63	1.46	1.47	1.78	1.77	1.77	1.77	1.08
NEW JERSEY	1449	1.71	1.63	1.61	1.60	1.61	1.60	0.98
NEW MEXICO	560	2.20	1.71	1.59	1.49	1.51	1.50	0.93
NEVADA	247	3.50	5.59	4.83	1.49	1.50	1.33	0.83
NEW YORK	1213	1.49	1.41	1.43	1.40	1.41	1.41	0.87
OHIO	3758	1.50	1.45	1.46	1.36	1.37	1.37	0.84
OKLAHOMA	132	1.85	1.82	1.82	0.94	0.94	0.87	0.58
OREGON	830	1.70	1.55	1.54	1.56	1.51	1.49	0.93
PENNSYLVANIA	670	1.38	1.31	1.31	1.31	1.31	1.32	0.81
PUERTO RICO	108	3.94	1.28	1.29	1.27	1.28	1.28	0.79
RHODE ISLAND	236	2.19	2.35	1.59	1.62	1.62	1.62	1.00
SOUTH CAROLINA	5013	1.90	1.83	1.85				
S CAROLINA NORTH					1.56	1.56	1.56	0.95
S CAROLINA SOUTH					1.86	1.80	1.81	1.14
SOUTH DAKOTA	490	1.68	1.45	1.47	1.46	1.60	1.60	0.98
TENNESSEE	740	1.56	1.20	1.21	1.20	1.20	1.19	0.73
TEXAS	2405	1.31	1.22	1.16	1.15	1.16	1.13	0.70
UTAH	310	1.79	2.04	1.99	1.44	1.44	1.29	0.82
VIRGINIA	2262	26.59	1.90	2.15	2.08	2.09	1.77	1.09
VIRGIN ISLANDS	125	1.13	0.99	1.05	1.16	1.13	1.13	0.71
VERMONT	751	1.27	1.24	1.27	1.26	1.26	1.26	0.77
WASHINGTON	1296	1.82	1.53	1.53	1.54	1.55	1.57	0.95
WISCONSIN	2385	2.14	2.13	2.12	2.12	2.19	2.19	1.35
WEST VIRGINIA	260	1.61	1.40	1.50	1.53	1.53	1.47	0.90
WYOMING	306	1.52	1.37	1.41	1.40	1.40	1.27	0.81
TOTAL	70673	2.20	1.66	1.62	1.60	1.63	1.62	1.00
Total Unique Stations	67693	Total # of unrejected Vectors = 283691						

Table 27.2 Final Statistics of Minimally Constrained Adjustment

No. of observations = 851,073
No. of auxiliary parameters = 0
No. of unknowns = 203,079
No. of rigid constraints (+/-10 micron) = 3
No. of weighted constraints (+/-10 micron) = 0
Degrees of freedom = 647,997 approximate
Variance sum ($\mathbf{V}^T\mathbf{PV}$) = 1,056,077.7
Variance of unit weight = 1.629757
Standard deviation of unit weight = 1.276619

The degrees of freedom are approximate, because NGS' guidelines for rejecting observations recommended down-weighting the observation, rather than removing the observation. The down-weighted observations—although they had very little impact on the solution—masked network singularities and resulted in rank defects. The observations were included during the degrees of freedom computation.

The variance of unit weight was relatively high, due to low quality observations that were purposely left in during the analysis phase. In most cases, the affected stations would become "no check," if any further rejections were made. NGS guidelines also recommend not to reject observations, if the rejection would cause a station to become "no check." The readjustment team adhered to NGS' strict guidelines. The readjustment team also felt it was important to publish weak stations in an attempt to notify the user community—via the local and network accuracies—that some regularly used stations were poorly determined. If stations were simply removed, the user would never know the true accuracy of that station.

28. Constrained Adjustments

All GPS data submitted to NGS in the bluebook format and loaded into the National Geodetic Survey's Integrated Database (NGSIDB) as of November 15, 2005 involved observational ties with a total of 468 national CORS, 3 CORS in Canada, 1 CORS in Mexico, and 213 California CORS (CGPS). Table 28.5 lists the CORS stations located outside of the USA. These CORS sites were all identified as possible constraints for the national readjustment.

The NSRS2007 readjustment incorporated both the LI Phase Centers and the ARPs, distinguished by having different PID's assigned to each reference point. The readjustment also distinguished between different configurations of the same CORS station by assigning a different PID when the configuration changed. Therefore, for one unique CORS site, there may have been one PID for the L1 Phase Center, another PID for the ARP, and multiple PID's if the station changed configurations over time. If the same CORS station also had a ground monument, the ground monument would receive yet another PID.

The following trial solutions were performed to analyze the results, based on constraining all observed National CORS and CGPS sites. Section 28.1 (Table 28.3) contains the statistics from all three solutions.

28.1 Constrained Adjustment Results

The following trial solutions were performed to analyze the results, based on constraining all observed National CORS and CGPS sites. Table 28.3 contains the variances of unit weight (unitless) from all three trial solutions.

Trial solution 1 contained the results obtained by rigidly constraining all 685 CORS.

Trial solution 2 contained the results obtained by freeing up CORS sites created by large residuals when rigidly constrained. Possible reasons for this include misidentified antenna reference points and changes in the CORS configuration after the observations were originally observed. In all cases, every attempt was made to identify the cause. In a few cases, additional corrections were made that incorrectly identified CORS observations.

Trial solution 3 resulted when all a-posteriori errors were scaled by the standard deviation of unit weight 1.375490 during the final constrained adjustment run. The final statistics from the constrained adjustment are shown in Table 28.1.

Table 28.1 Final Statistics of Constrained Adjustment

No. of observations = 851,073
No. of total constrained parameters = 2,055
No. of rigid constrained parameters (+/-10 micron) = 2,029
No. of weighted constrained parameters (+/-10 cm) = 26
No. of unknown parameters = 203,079
Degrees of Freedom = 650,049
Variance sum ($\mathbf{V}^T\mathbf{PV}$) = 1,229,874.4
Variance of unit weight = 1.8919718
Standard deviation of unit weight = 1.375490

Unconstrained Sites

During the analysis phase of the constrained adjustment runs, a few of the published CORS coordinates originally constrained, created excessively high residuals on the observations associated with them. The most likely cause for the high residuals on the observations to these CORS is due to either the change to the CORS site configuration or incorrect identification of the reference point during the field observation. Out of 685 possible CORS constraints, 673 were totally constrained, 7 stochastically freed (with 10 cm standard deviation) and 5 where heights were stochastically freed (with 10 cm standard deviation). Post-adjustment analysis [Milbert 2008] showed that 3 CORS were inadvertently left completely free. Table 28.2 identifies the unconstrained CORS parameters.

Table 28.2 Unconstrained CORS parameters (NGS bluebook format)

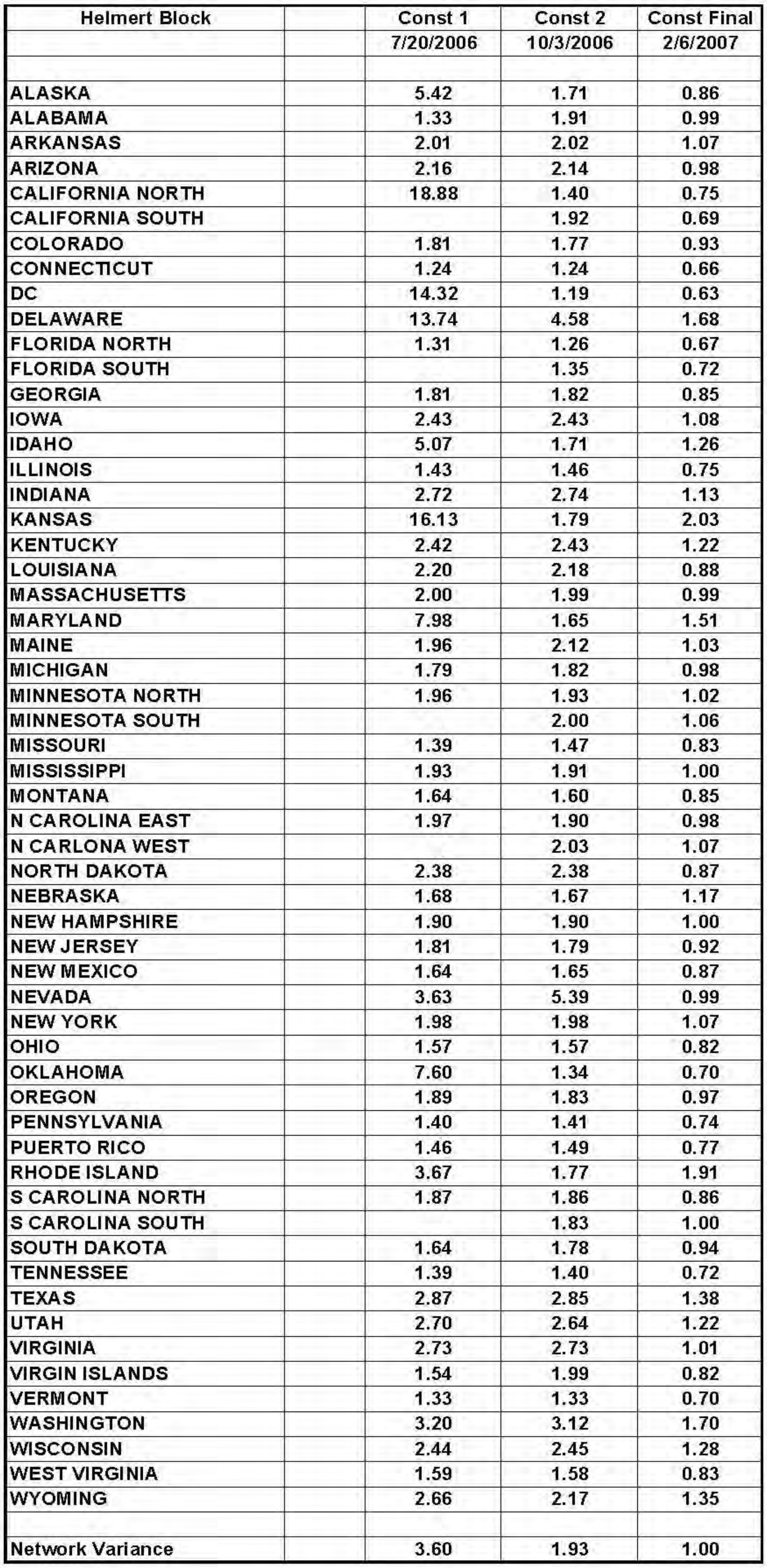
Unconstrained Position and Ellipsoid Height

```
AB6289*80*0170ST. LOUIS 2 CORS L1 PHASE CENT38364070837N089453202569W       IL
AB6289*86*0170                                166767
AB6387*80*1888EGMONT KEY 1 CORS L1 PHASE CEN27360148442N082453714458W       FL
AB6387*86*1888                                -16885
AF9543*80*0825BEAUMONT RRP CORS ARP         30094217916N094104693107W       TX
AF9543*86*0825                                -10555
AF9689*80*2058LEXI 1989                     35531805139N120255097198W       CA
AF9689*86*2058                                479332
AI4495*80*2067MUSB MUSICK MOUNTAIN GRP      37101177344N119183361057W       CA
AI4495*86*2067                               2043137
DG4677*80*2704BAKERSFIELD 1 CORS ARP        35075658127N119063406927W       CA
DG4677*86*2704                                 57432
DH6759*80*2910CARR HILL SITE 2 CORS GRP     35531816979N120255090524W       CA
DH6759*86*2910                                480870
```

Unconstrained Ellipsoid Height

```
DG8361*80*3053RAMAGERNCHCS2004 GRP          35380949301N120521099952W       CA
DG8361*86*3053                                417447
DG7413*80*3052CLEGGRANCHCS2004 GRP          35330629478N121001060353W       CA
DG7413*86*3052                                107602
AI5126*80*0162SHINN GPS BASE STATION ARP    40353002879N120133010329W       CA
AI5126*86*0162                               1377886
DF5886*80*0477PELLISSIPPI STATE CORS ARP    35565351748N084100037626W       TN
DF5886*86*0477                                281059
AF9563*80*0118VERMONT CAPITAL CORS ARP      44154310706N072345655559W       VT
AF9563*86*0118                                160550
```

Table 28.3 Constrained Adjustment Statistics



Helmert Block		Const 1	Const 2	Const Final
		7/20/2006	10/3/2006	2/6/2007
ALASKA		5.42	1.71	0.86
ALABAMA		1.33	1.91	0.99
ARKANSAS		2.01	2.02	1.07
ARIZONA		2.16	2.14	0.98
CALIFORNIA NORTH		18.88	1.40	0.75
CALIFORNIA SOUTH			1.92	0.69
COLORADO		1.81	1.77	0.93
CONNECTICUT		1.24	1.24	0.66
DC		14.32	1.19	0.63
DELAWARE		13.74	4.58	1.68
FLORIDA NORTH		1.31	1.26	0.67
FLORIDA SOUTH			1.35	0.72
GEORGIA		1.81	1.82	0.85
IOWA		2.43	2.43	1.08
IDAHO		5.07	1.71	1.26
ILLINOIS		1.43	1.46	0.75
INDIANA		2.72	2.74	1.13
KANSAS		16.13	1.79	2.03
KENTUCKY		2.42	2.43	1.22
LOUISIANA		2.20	2.18	0.88
MASSACHUSETTS		2.00	1.99	0.99
MARYLAND		7.98	1.65	1.51
MAINE		1.96	2.12	1.03
MICHIGAN		1.79	1.82	0.98
MINNESOTA NORTH		1.96	1.93	1.02
MINNESOTA SOUTH			2.00	1.06
MISSOURI		1.39	1.47	0.83
MISSISSIPPI		1.93	1.91	1.00
MONTANA		1.64	1.60	0.85
N CAROLINA EAST		1.97	1.90	0.98
N CARLONA WEST			2.03	1.07
NORTH DAKOTA		2.38	2.38	0.87
NEBRASKA		1.68	1.67	1.17
NEW HAMPSHIRE		1.90	1.90	1.00
NEW JERSEY		1.81	1.79	0.92
NEW MEXICO		1.64	1.65	0.87
NEVADA		3.63	5.39	0.99
NEW YORK		1.98	1.98	1.07
OHIO		1.57	1.57	0.82
OKLAHOMA		7.60	1.34	0.70
OREGON		1.89	1.83	0.97
PENNSYLVANIA		1.40	1.41	0.74
PUERTO RICO		1.46	1.49	0.77
RHODE ISLAND		3.67	1.77	1.91
S CAROLINA NORTH		1.87	1.86	0.86
S CAROLINA SOUTH			1.83	1.00
SOUTH DAKOTA		1.64	1.78	0.94
TENNESSEE		1.39	1.40	0.72
TEXAS		2.87	2.85	1.38
UTAH		2.70	2.64	1.22
VIRGINIA		2.73	2.73	1.01
VIRGIN ISLANDS		1.54	1.99	0.82
VERMONT		1.33	1.33	0.70
WASHINGTON		3.20	3.12	1.70
WISCONSIN		2.44	2.45	1.28
WEST VIRGINIA		1.59	1.58	0.83
WYOMING		2.66	2.17	1.35
Network Variance		3.60	1.93	1.00

Occasionally, stations were positioned and loaded into the database prior to their final determination and inclusion into the CORS database, resulting in multiple PIDS for the same station. Table 28.4 shows stations not identified as CORS during NGS' analysis of available constraints.

Table 28.4 Stations Not Identified as CORS During the Analysis Phase

Station Name	FBN campaign PID	CORS Network PID	4 char siteid
Brookfield	DH4463	DH5825	ctbr
Darien	DH4464	DH5827	ctda
East Granby	DH4465	DH5829	cteg
Groton	DH4466	DH5831	ctgr
Guilford	DH4467	DH5833	ctgu
Mansfield	DH4468	DH5835	ctma
Paquette (Newington)	AD9919	DH7113	ctne
Putnam	DH4469	DH5837	ctpu
Winchester	DH4470	DH5839	ctwi

A subsequent adjustment of the project was performed, and the newly determined CORS coordinates were held fixed for those stations, so that the positions of the new project marks would be consistent with the CORS published position.

Table 28.5 Stations Constrained Outside of USA (NGS Bluebook Format)

```
AA9185*80*0554ALGONQUIN MONUMENT            45572084776N078041690684W       CD
AA9185*86*0554                                201978
DE6592*80*0716OTTAWA NRC CORS ARP           45271495118N075372576577W       ON
DE6592*86*0716                                 83593
TP1405*80*0155PENTICTON 887006              49192141006N119372987577W       BC
TP1405*86*0155                                542234
AJ1850*80*2220CIC1 CICESE GRM               31521442894N116395669576W       MX
AJ1850*86*2220                                 65127
```

28.2 All National CORS and CGPS Sites Observed

A file of all National CORS and CGPS site's horizontal coordinate and ellipsoidal height values along with the PID and Designation for each station can be found on the national readjustment Web page at: http://www.ngs.noaa.gov/NationalReadjustment/. The list also contains the weight level placed on the constraint for each coordinate value. For example, (AI5126 1.000e+10 1.000e+10 1.000e+002) shows the PID of the constrained CORS site, the 1.000e+10 signifies a constraint value of 0.01 millimeters on the latitude and longitude while the 1.000e+002 places a constraint of 0.10 meters on the ellipsoid height. When only one constraint value is shown all three coordinate values are constrained at the same level.

28.3 Helmert Block Coordinate Shifts

Table 28.6 illustrates the maximum and average horizontal and vertical shifts from the published values to the readjusted values for each Helmert block. The list also contains the number of stations contained within each Helmert block. Note: This file shows the actual shifts from what currently existed in the NGS database as of November 15, 2005. In certain cases, stations had no publishable ellipsoid heights, and therefore statistics on “shifts” will not reflect these first-ever ellipsoid heights. In other cases, especially in areas of plate tectonic movement or subsidence, the shifts do not reflect velocity corrections prior to computing the shift. Large shifts in areas of known movement will also show abnormally high shifts.

The statistics page found at http://www.ngs.noaa.gov/NationalReadjustment/archives.html can be viewed to show the actual shift for every station involved in the readjustment.

Table 28.6 Helmert Block Coordinate Shifts

BLOCK	STATIONS	HORIZONTAL SHIFT (m)			VERTICAL SHIFT (m)	
		MAX	AVERAGE		MAX	AVERAGE
AK	792	1.094	0.109		0.631	0.073
AL	3668	0.287	0.035		0.224	0.033
AR	396	0.041	0.012		0.064	0.01
AZ	1388	0.148	0.03		0.201	0.025
CANorth	1688	0.845	0.12		0.382	0.029
CASouth	1950	1.429	0.288		1.931	0.058
CO	1737	0.138	0.02		0.147	0.016
CT	103	0.024	0.008		0.069	0.016
DC	33	0.039	0.011		0.029	0.008
DE	91	0.049	0.012		0.112	0.017
FLNorth	3117	0.093	0.013		0.139	0.022
FLSouth	3699	0.14	0.014		0.157	0.019
GA	1529	0.516	0.025		0.372	0.023
IA	329	0.035	0.011		0.084	0.018
ID	280	0.047	0.014		0.094	0.01
IL	2515	0.106	0.011		0.173	0.014
IN	270	0.032	0.008		0.075	0.019
KS	464	0.089	0.017		0.303	0.032
KY	1012	0.106	0.009		0.173	0.02
LA	1158	0.272	0.039		0.469	0.028
MA	284	0.065	0.013		0.107	0.01
MD	2097	0.768	0.017		0.526	0.013
ME	446	0.056	0.017		0.131	0.01
MI	1090	0.291	0.027		0.228	0.027
MNNorth	3910	0.247	0.01		0.083	0.022
MNSouth	3184	0.160	0.021		0.191	0.44
MO	861	5.254	0.031		0.303	0.017
MS	557	0.11	0.014		0.11	0.014
MT	383	0.062	0.018		0.133	0.019

NCEast	3200	0.42	0.018		0.051	0.008
NCWest	2633	0.338	0.011		0.136	0.011
ND	162	0.039	0.013		0.115	0.021
NE	610	0.119	0.015		0.303	0.022
NH	63	0.023	0.013		0.039	0.01
NJ	1450	0.056	0.01		0.131	0.014
NM	560	0.077	0.019		0.146	0.019
NV	247	0.194	0.031		0.692	0.026
NY	1213	0.08	0.012		0.085	0.009
OH	3758	0.348	0.016		0.503	0.013
OK	132	0.12	0.013		0.062	0.01
OR	830	0.165	0.069		0.172	0.016
PA	670	0.099	0.02		0.255	0.022
PR	108	0.076	0.011		0.078	0.028
RI	236	0.084	0.017		0.073	0.017
SCNorth	2451	0.421	0.01		0.387	0.017
SCSouth	2725	0.277	0.01		0.145	0.012
SD	490	0.041	0.018		0.081	0.014
TN	740	0.094	0.016		0.213	0.03
TX	2400	0.208	0.016		0.25	0.018
UT	310	0.069	0.03		0.159	0.019
VA	2262	0.384	0.024		0.312	0.033
VQ	125	0.175	0.079		0.098	0.029
VT	751	0.058	0.021		0.105	0.013
WA	1296	0.387	0.043		0.224	0.016
WI	2385	0.09	0.009		0.303	0.012
WV	260	0.089	0.017		0.067	0.016
WY	306	0.094	0.019		0.084	0.015

Part VI. Publication of Adjusted Results

29. Web Page (See Appendix 1)

http://www.ngs.noaa.gov/NationalReadjustment/

The national readjustment Web page was created during the initial stages of the analysis process to inform users on the progress of the readjustment. Complete statistics were created for each iteration result and placed into tables for easy access. Each iteration result listed the vectors contained within each Helmert block, along with the horizontal and ellipsoid height residual, the "from" and "to" station PID, and the designation for each vector. The horizontal and vertical plots of the residuals versus the vector length, a list of the rejected vectors, the residuals associated with each vector, and a summary of the number of vectors, were included for each station, as well as the computed position shifts from the published value to the readjusted value.

The Web page also contained critical announcements pertaining to the publication of the results, policy statements in reference to present and future project submittals, an explanation of the constrained control used for the readjustment, an overview of why the national readjustment was undertaken, existing power point presentations, a contact page, and finally, the adjusted coordinates in the Re-adjustment Distribution Format (RDF). Appendix 1 contains sample Web pages along with further documentation explaining the contents found within each page.

30. RDF Format (See Appendix 2)

NGS recognized that the primary method for accessing NAD 83 monumented coordinates is via the NGS datasheet format; however, the software for making the readjusted coordinates and their associated local and network accuracies was not yet ready for public use in February 2007. Rather than wait for the software to be complete, NGS decided to release the readjusted coordinates with variances in the latitude, longitude, and ellipsoid height, as well as the correlation coefficient between the variances in latitude and longitude. These values are related to—but not the same as—network and local accuracies. They were released in a simple text-based format for immediate use. The text format NGS chose for dissemination of the readjusted coordinates—"Re-adjustment Distribution Format" or "RDF"—was an internal-use format familiar to the programmers working on loading the data to the NGS database. While there are similarities between this format and the well-known NGS "Blue Book" format, it must be emphatically stated that ***RDF is not Blue Book format***.

31. Datasheets (See Appendix 3)

The following modifications to the data sheets have been implemented:

- NGS will use the "NAD 83(2007)" tag as the permanent identifier of points with an NSRS2007 coordinate.
- For survey control stations determined "NO CHECK" by the national readjustment, the published NAD 83 coordinate line has been designated "NO CHECK" (replacing "ADJUSTED") and the ELLIP HEIGHT line has been designated "NO CHECK" (replacing "GPS OBS").
- The ellipsoid height line has been designated "ADJUSTED" rather than "GPS OBS" (except for NO CHECK stations; see above).

Network Accuracies and Local Accuracies will eventually be published on the NGS datasheet.

Part VII. Implementation

32. National Readjustment Implementation Plan—Issues and Concerns

The National Readjustment Implementation Team was established to address current and future issues arising after the completion of the national readjustment. The team was comprised of the Chief Geodesist and representatives from the Observation and Analysis Division, Geodetic Services Division, Spatial Reference Systems Division, Remote Sensing Division, Systems Development Division, and the Geosciences Research Division. The goal of the team is to debate and decide on issues with respect to NGS' Ten-Year Plan.

Tasks and Topics to be decided upon include:

- Form an implementation team (including all divisions within NGS)
- Request questions/concerns from the user community on website
- Create FAQs based on above responses and respond to FAQs
- Publish adjusted results (datum name and epoch dates?) via:
    - NGS datasheets
    - OPUS results
- Determine how NGS will compute network and local accuracies:
    - for projects submitted after cutoff date, but before the readjustment
    - for projects submitted after the readjustment, but before accuracies were computed,
    - for OPUS (Can realistic accuracies be computed?)
    - for tolerances on accuracies (When should values change?)
- Determine positional tolerances (should NGS publish both unchanging and best values?) for:
    - NAD 83(CORS96) CORS values (should tolerance be same as ITRF?)
    - readjusted values determined from new projects
- Determine ITRF coordinates on passive stations:
    - transform and publish on datasheets
    - ITRF adjustments of future projects
- Create graphical representations of results (GIS applications) for:
    - shape files
    - error ellipses
    - analysis tools
- Develop time dependent programs (HTDP, VTDP, etc) for:
    - Alaska, Western states and throughout the US
- Develop one consistent datasheet (i.e. dynamic datasheet)

32.1 Policy Regarding the Readjustment of Database Projects Not Included in the National Readjustment

NGS has made no commitment regarding whether resources will be available to readjust projects submitted after the 2005 cutoff date. It is suggested that the submitting agency readjust the project and submit the results to NGS for database entry. Only the output of ADJUST, the G-file, the B-file with the final coordinates and ellipsoid heights, and a very short report addendum detailing any problems or comments need be submitted.

As of this time, NGS has not determined the mechanism for computing the local and network accuracies for individual projects, so there is no requirement for their submission.

32.2 Future National Readjustments

Since ITRF00 was not determined using absolute antenna calibrations, another realization of the CORS network based on ITRF00 will most likely not be performed. NGS, like the International GNSS Service (IGS), has decided to re-process all historic GNSS data in their archives to achieve a consistent set of coordinates in the ITRF05 reference frame. The effort is expected to last into 2010. Although it is possible another readjustment of all passive mark data may take place after that timeframe, it is also likely that (in alignment with the NGS Ten-Year Plan) resources for maintaining the passive network will be significantly reduced at NGS and alternative tools (such as OPUS-DB and OPUS-Projects) will become the norm for users whose work requires the use of passive marks.

Part VIII. References and Appendices

33. References

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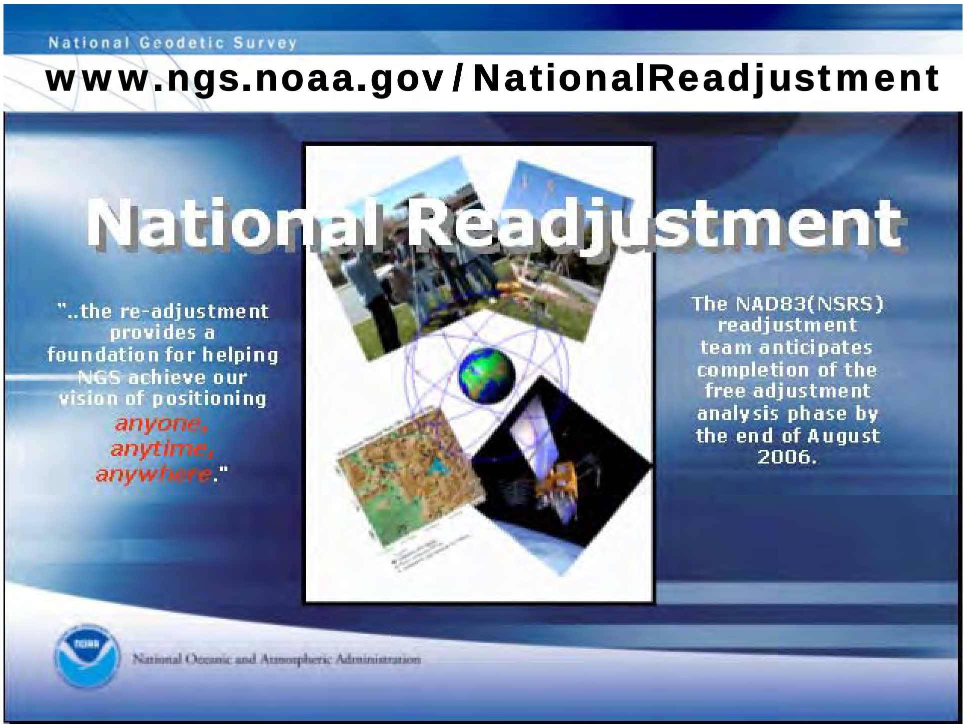
34. Appendix 1: Sample Web Pages



National Geodetic Survey

www.ngs.noaa.gov/NationalReadjustment

National Readjustment

"..the re-adjustment provides a foundation for helping NGS achieve our vision of positioning *anyone, anytime, anywhere*."

The NAD83(NSRS) readjustment team anticipates completion of the free adjustment analysis phase by the end of August 2006.

National Oceanic and Atmospheric Administration

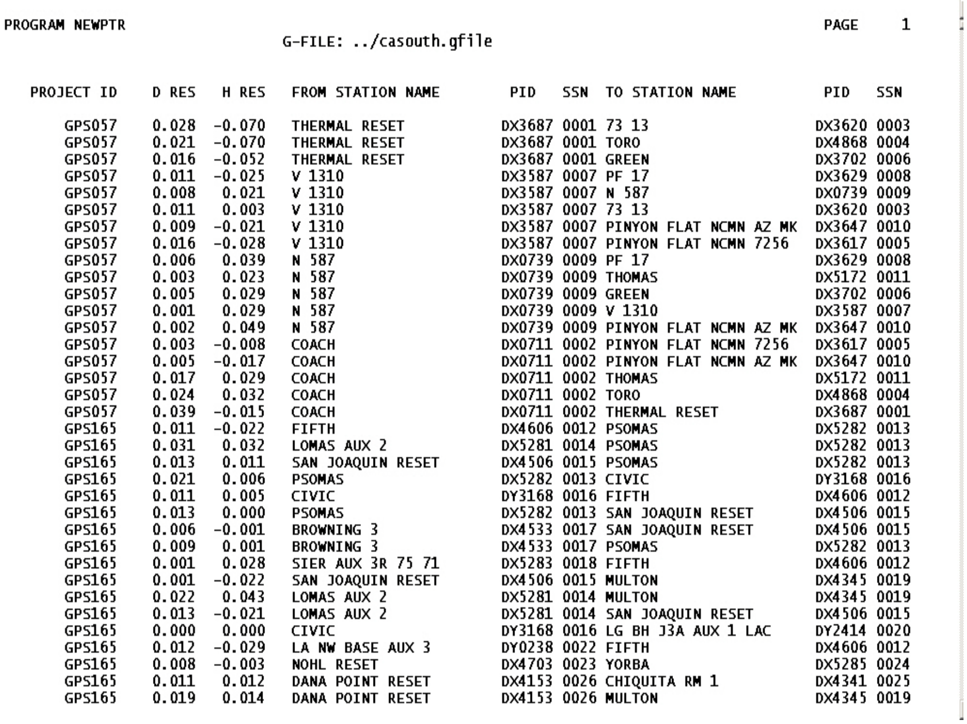
34.1 Overall Statistics Page

	FINAL FREE ADJUSTMENT						FINAL CONSTRAINED ADJUSTMENT			
Free	Constrained AF9522			Rejected	Station	FINAL	Constrained	Constrained 678 CORS/CGPS sites		
Statistics	Variance	Horizontal	Vertical	Vectors	Summary	SHIFTS	Statistics	Variance	Horizontal	Vertical
	2/6/2007	Plot 2/6/07	Plot 2/6/07					2/6/2007	Plot 2/6/07	Plot 2/6/07
ALASKA	0.80	yes	yes	644	792	yes	ALASKA	0.86	yes	yes
ALABAMA	1.14	yes	yes	3271	3668	yes	ALABAMA	0.99	yes	yes
ARKANSAS	1.09	yes	yes	81	396	yes	ARKANSAS	1.07	yes	yes
ARIZONA	0.94	yes	yes	316	1388	yes	ARIZONA	0.98	yes	yes
CALIFORNIA NORTH	0.72	yes	yes	1694	1688	yes	CALIFORNIA NORTH	0.75	yes	yes
CALIFORNIA SOUTH	0.54	yes	yes	6152	1950	yes	CALIFORNIA SOUTH	0.69	yes	yes
COLORADO	1.00	yes	yes	644	1737	yes	COLORADO	0.93	yes	yes
CONNECTICUT	0.77	yes	yes	73	103	yes	CONNECTICUT	0.66	yes	yes
DC	0.75	yes	yes	74	33	yes	DC	0.63	yes	yes
DELAWARE	1.30	yes	yes	63	91	yes	DELAWARE	1.68	yes	yes
FLORIDA NORTH	0.77	yes	yes	1211	3117	yes	FLORIDA NORTH	0.67	yes	yes
FLORIDA SOUTH	0.81	yes	yes	1422	3699	yes	FLORIDA SOUTH	0.72	yes	yes
GEORGIA	0.90	yes	yes	290	1529	yes	GEORGIA	0.85	yes	yes
IOWA	1.00	yes	yes	40	329	yes	IOWA	1.08	yes	yes
IDAHO	0.81	yes	yes	139	280	yes	IDAHO	1.26	yes	yes
ILLINOIS	0.84	yes	yes	317	2515	yes	ILLINOIS	0.75	yes	yes
INDIANA	1.03	yes	yes	43	270	yes	INDIANA	1.13	yes	yes
KANSAS	1.37	yes	yes	85	464	yes	KANSAS	2.03	yes	yes
KENTUCKY	0.90	yes	yes	156	1012	yes	KENTUCKY	1.22	yes	yes
LOUISIANA	0.71	yes	yes	452	1158	yes	LOUISIANA	0.88	yes	yes
MASSACHUSETTS	1.08	yes	yes	29	284	yes	MASSACHUSETTS	0.99	yes	yes
MARYLAND	0.98	yes	yes	784	2097	yes	MARYLAND	1.51	yes	yes
MAINE	1.03	yes	yes	139	446	yes	MAINE	1.03	yes	yes
MICHIGAN	0.87	yes	yes	446	1090	yes	MICHIGAN	0.98	yes	yes
MINNESOTA NORTH	1.17	yes	yes	885	3910	yes	MINNESOTA NORTH	1.02	yes	yes
MINNESOTA SOUTH	1.21	yes	yes	758	3184	yes	MINNESOTA SOUTH	1.06	yes	yes
MISSOURI	0.87	yes	yes	150	861	yes	MISSOURI	0.83	yes	yes
MISSISSIPPI	1.02	yes	yes	195	557	yes	MISSISSIPPI	1.00	yes	yes
MONTANA	0.97	yes	yes	177	383	yes	MONTANA	0.85	yes	yes

The overall statistics page shows all statistics generated for each Helmert block as a result of the final simultaneous least squares adjustment of the horizontal and ellipsoid height components of the national spatial reference system. The variance of unit weight is unitless.

Clicking on any of the highlighted links above opens a window containing specific information pertaining to each Helmert block.

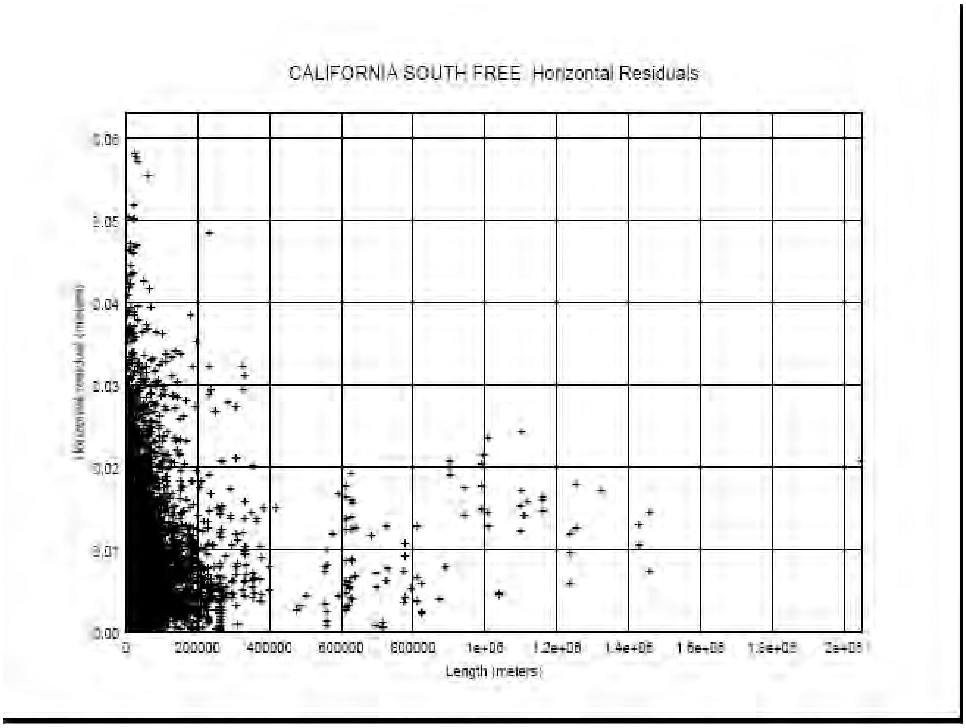
34.2 Helmert Block Statistics Page (All Residuals Are Given in Meters)



```
PROGRAM NEWPTR                                                                              PAGE     1
                                G-FILE: ../casouth.gfile

   PROJECT ID    D RES   H RES   FROM STATION NAME        PID   SSN  TO STATION NAME         PID   SSN

       GPS057    0.028  -0.070   THERMAL RESET          DX3687 0001 73 13                  DX3620 0003
       GPS057    0.021  -0.070   THERMAL RESET          DX3687 0001 TORO                   DX4868 0004
       GPS057    0.016  -0.052   THERMAL RESET          DX3687 0001 GREEN                  DX3702 0006
       GPS057    0.011  -0.025   V 1310                 DX3587 0007 PF 17                  DX3629 0008
       GPS057    0.008   0.021   V 1310                 DX3587 0007 N 587                  DX0739 0009
       GPS057    0.011   0.003   V 1310                 DX3587 0007 73 13                  DX3620 0003
       GPS057    0.009  -0.021   V 1310                 DX3587 0007 PINYON FLAT NCMN AZ MK DX3647 0010
       GPS057    0.016  -0.028   V 1310                 DX3587 0007 PINYON FLAT NCMN 7256  DX3617 0005
       GPS057    0.006   0.039   N 587                  DX0739 0009 PF 17                  DX3629 0008
       GPS057    0.003   0.023   N 587                  DX0739 0009 THOMAS                 DX5172 0011
       GPS057    0.005   0.029   N 587                  DX0739 0009 GREEN                  DX3702 0006
       GPS057    0.001   0.029   N 587                  DX0739 0009 V 1310                 DX3587 0007
       GPS057    0.002   0.049   N 587                  DX0739 0009 PINYON FLAT NCMN AZ MK DX3647 0010
       GPS057    0.003  -0.008   COACH                  DX0711 0002 PINYON FLAT NCMN 7256  DX3617 0005
       GPS057    0.005  -0.017   COACH                  DX0711 0002 PINYON FLAT NCMN AZ MK DX3647 0010
       GPS057    0.017   0.029   COACH                  DX0711 0002 THOMAS                 DX5172 0011
       GPS057    0.024   0.032   COACH                  DX0711 0002 TORO                   DX4868 0004
       GPS057    0.039  -0.015   COACH                  DX0711 0002 THERMAL RESET          DX3687 0001
       GPS165    0.011  -0.022   FIFTH                  DX4606 0012 PSOMAS                 DX5282 0013
       GPS165    0.031   0.032   LOMAS AUX 2            DX5281 0014 PSOMAS                 DX5282 0013
       GPS165    0.013   0.011   SAN JOAQUIN RESET      DX4506 0015 PSOMAS                 DX5282 0013
       GPS165    0.021   0.006   PSOMAS                 DX5282 0013 CIVIC                  DY3168 0016
       GPS165    0.011   0.005   CIVIC                  DY3168 0016 FIFTH                  DX4606 0012
       GPS165    0.013   0.000   PSOMAS                 DX5282 0013 SAN JOAQUIN RESET      DX4506 0015
       GPS165    0.006  -0.001   BROWNING 3             DX4533 0017 SAN JOAQUIN RESET      DX4506 0015
       GPS165    0.009   0.001   BROWNING 3             DX4533 0017 PSOMAS                 DX5282 0013
       GPS165    0.001   0.028   SIER AUX 3R 75 71      DX5283 0018 FIFTH                  DX4606 0012
       GPS165    0.001  -0.022   SAN JOAQUIN RESET      DX4506 0015 MULTON                 DX4345 0019
       GPS165    0.022   0.043   LOMAS AUX 2            DX5281 0014 MULTON                 DX4345 0019
       GPS165    0.013  -0.021   LOMAS AUX 2            DX5281 0014 SAN JOAQUIN RESET      DX4506 0015
       GPS165    0.000   0.000   CIVIC                  DY3168 0016 LG BH J3A AUX 1 LAC    DY2414 0020
       GPS165    0.012  -0.029   LA NW BASE AUX 3       DY0238 0022 FIFTH                  DX4606 0012
       GPS165    0.008  -0.003   NOHL RESET             DX4703 0023 YORBA                  DX5285 0024
       GPS165    0.011   0.012   DANA POINT RESET       DX4153 0026 CHIQUITA RM 1          DX4341 0025
       GPS165    0.019   0.014   DANA POINT RESET       DX4153 0026 MULTON                 DX4345 0019
```

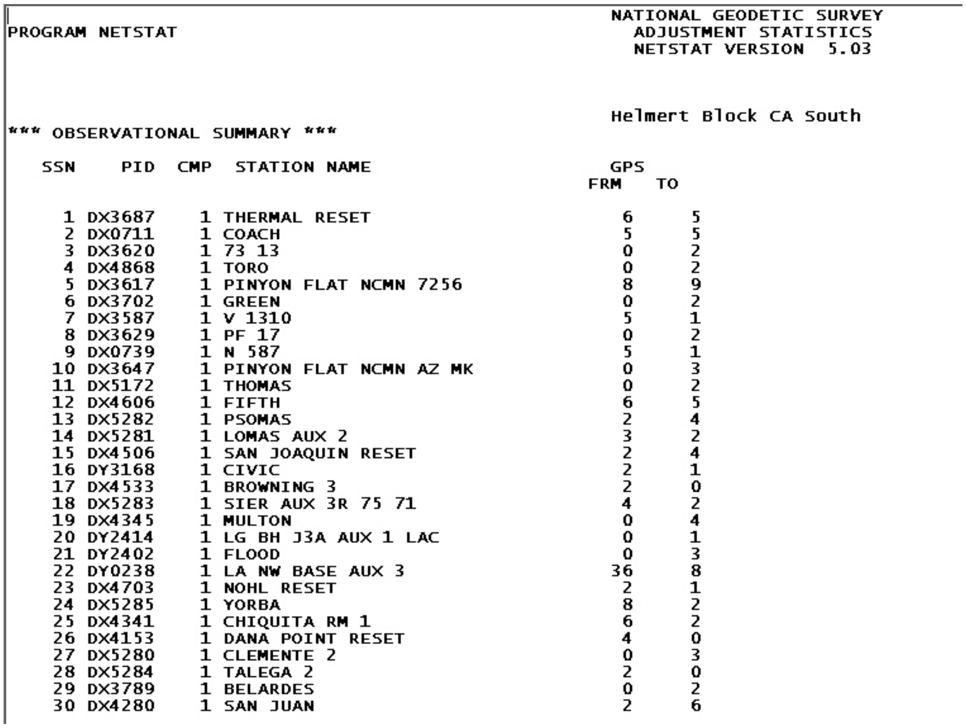
When the link to the Helmert block, designated by a state name, is entered, the statistics page is shown. The page lists the project source of the data, the horizontal (D RES) residual, and the ellipsoid height (H RES) residual for each vector contained within the block, as well as the PID and designation for both the "from" and "to" station. The file is text based and may be downloaded for further data analysis.

34.3 Residual Plot of a Helmert Block



When the highlighted link to any one of the horizontal or vertical plots is entered, the above image will appear. The plot is the graphical representation of the residuals shown on the previous statistics page. For each vector shown on the statistics page, the residual, along with the vector length, is plotted.

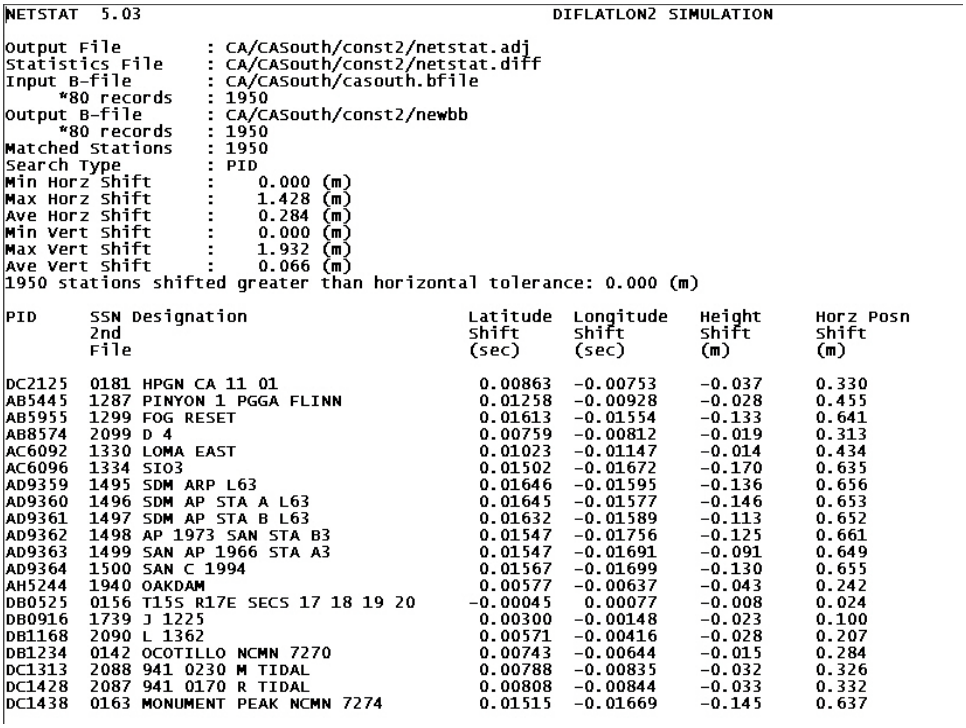
34.4 Station Summary for a Helmert Block



```
                                                     NATIONAL GEODETIC SURVEY
PROGRAM NETSTAT                                        ADJUSTMENT STATISTICS
                                                       NETSTAT VERSION  5.03


                                                     Helmert Block CA South
*** OBSERVATIONAL SUMMARY ***

   SSN    PID  CMP  STATION NAME                     GPS
                                                   FRM   TO

     1 DX3687    1 THERMAL RESET                      6     5
     2 DX0711    1 COACH                              5     5
     3 DX3620    1 73 13                              0     2
     4 DX4868    1 TORO                               0     2
     5 DX3617    1 PINYON FLAT NCMN 7256              8     9
     6 DX3702    1 GREEN                              0     2
     7 DX3587    1 V 1310                             5     1
     8 DX3629    1 PF 17                              0     2
     9 DX0739    1 N 587                              5     1
    10 DX3647    1 PINYON FLAT NCMN AZ MK             0     3
    11 DX5172    1 THOMAS                             0     2
    12 DX4606    1 FIFTH                              6     5
    13 DX5282    1 PSOMAS                             2     4
    14 DX5281    1 LOMAS AUX 2                        3     2
    15 DX4506    1 SAN JOAQUIN RESET                  2     4
    16 DY3168    1 CIVIC                              2     1
    17 DX4533    1 BROWNING 3                         2     0
    18 DX5283    1 SIER AUX 3R 75 71                  4     2
    19 DX4345    1 MULTON                             0     4
    20 DY2414    1 LG BH J3A AUX 1 LAC                0     1
    21 DY2402    1 FLOOD                              0     3
    22 DY0238    1 LA NW BASE AUX 3                  36     8
    23 DX4703    1 NOHL RESET                         2     1
    24 DX5285    1 YORBA                              8     2
    25 DX4341    1 CHIQUITA RM 1                      6     2
    26 DX4153    1 DANA POINT RESET                   4     0
    27 DX5280    1 CLEMENTE 2                         0     3
    28 DX5284    1 TALEGA 2                           2     0
    29 DX3789    1 BELARDES                           0     2
    30 DX4280    1 SAN JUAN                           2     6
```

This image shows the number of times each station is encountered as the “from” and “to” endpoints of all vectors contained in a Helmert block. The PID and designation are also shown. It is noteworthy that a few of the stations may show a station being observed only once or twice, and this normally indicates a station was determined as “no check.” Since the summary shows only the number of occurrences within each block, it is quite possible more observations to the station may be located within a neighboring block.

34.5 Coordinate Shifts for Each Station Within a Helmert Block



```
NETSTAT  5.03                                       DIFLATLON2 SIMULATION

Output File        : CA/CASouth/const2/netstat.adj
Statistics File    : CA/CASouth/const2/netstat.diff
Input B-file       : CA/CASouth/casouth.bfile
     *80 records   : 1950
Output B-file      : CA/CASouth/const2/newbb
     *80 records   : 1950
Matched Stations   : 1950
Search Type        : PID
Min Horz Shift     :    0.000 (m)
Max Horz Shift     :    1.428 (m)
Ave Horz Shift     :    0.284 (m)
Min Vert Shift     :    0.000 (m)
Max Vert Shift     :    1.932 (m)
Ave Vert Shift     :    0.066 (m)
1950 stations shifted greater than horizontal tolerance: 0.000 (m)

PID     SSN Designation                     Latitude  Longitude  Height     Horz Posn
        2nd                                 Shift     Shift      Shift      Shift
        File                                (sec)     (sec)      (m)        (m)

DC2125  0181 HPGN CA 11 01                   0.00863  -0.00753   -0.037     0.330
AB5445  1287 PINYON 1 PGGA FLINN             0.01258  -0.00928   -0.028     0.455
AB5955  1299 FOG RESET                       0.01613  -0.01554   -0.133     0.641
AB8574  2099 D 4                             0.00759  -0.00812   -0.019     0.313
AC6092  1330 LOMA EAST                       0.01023  -0.01147   -0.014     0.434
AC6096  1334 SIO3                            0.01502  -0.01672   -0.170     0.635
AD9359  1495 SDM ARP L63                     0.01646  -0.01595   -0.136     0.656
AD9360  1496 SDM AP STA A L63                0.01645  -0.01577   -0.146     0.653
AD9361  1497 SDM AP STA B L63                0.01632  -0.01589   -0.113     0.652
AD9362  1498 AP 1973 SAN STA B3              0.01547  -0.01756   -0.125     0.661
AD9363  1499 SAN AP 1966 STA A3              0.01547  -0.01691   -0.091     0.649
AD9364  1500 SAN C 1994                      0.01567  -0.01699   -0.130     0.655
AH5244  1940 OAKDAM                          0.00577  -0.00637   -0.043     0.242
DB0525  0156 T15S R17E SECS 17 18 19 20     -0.00045   0.00077   -0.008     0.024
DB0916  1739 J 1225                          0.00300  -0.00148   -0.023     0.100
DB1168  2090 L 1362                          0.00571  -0.00416   -0.028     0.207
DB1234  0142 OCOTILLO NCMN 7270              0.00743  -0.00644   -0.015     0.284
DC1313  2088 941 0230 M TIDAL                0.00788  -0.00835   -0.032     0.326
DC1428  2087 941 0170 R TIDAL                0.00808  -0.00844   -0.033     0.332
DC1438  0163 MONUMENT PEAK NCMN 7274         0.01515  -0.01669   -0.145     0.637
```

The final shifts column located on the overall statistics page will bring users to this Web page. The page shows the minimum and maximum horizontal and vertical shifts, as well as the average shifts for each Helmert block. In addition, the actual horizontal and ellipsoid height shift from the published value to the readjusted value is shown for every station located within each block.

35. Appendix 2: The Readjustment Distribution Format

File Format Information for Web-Distributed, Text-Based NAD 83 (NSRS2007) Coordinate + Accuracy Files

In order to facilitate immediate access to NAD 83 (NSRS2007) coordinates and their respective variances and covariances (related to, but not the same as, network and local accuracies), the data were distributed in a text format called the Readjustment Distribution Format (or “RDF”).

RDF is similar to, but in many ways quite different from, the well known NGS Blue Book format. Although RDF was used to quickly make the readjustment data available, users should be cautioned that no plans are in place to continue the widespread use of this format for any other NGS products. It was, quite simply, a convenient tool solving a temporary delay in datasheet availability of the readjustment data.

The description of RDF is below.

Record Descriptions in Readjustment Distribution Format

Each record (line) in RDF is an 80-column character field. There are only seven record types in RDF (A1, 10, 13, 80, 86, 91 and 92). Each record type is described in a separate section later in this document. Each RDF record contains a number of fields occupying specific columns in the record. The record descriptions are broken down by field, showing the columns, format, range, and description of each. Certain special symbols are used in the format and range specifications, and they are defined below.

Symbol	Definition
\n	Newline character.
A	Capitalized alphabetic characters only [A..Z].
9	Numeric, digits, sign and decimal point only [0..9, +, -, .] according to field format picture.
N	Capitalized alphabetic characters and numeric characters only [A..Z, 0..9].
X	Capitalized alphabetic characters, numeric characters [A..Z, 0..9], and special characters as specified in the field range.
.	Decimal point or period.
..	Denotes a range of characters such as [A..Z] or [1..9].
✦	Denotes a blank or the space character.
±	Denotes a plus sign "+', a minus sign '-', or a blank.

Any field may be defined in short- or long-field format notation. For example, the field format X(30) is equivalent to XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.

The format specification A(1-200) means the field is a variable length with 1 to 200 alphabetic characters.

In some cases the numeric fields, e.g. latitude, longitude, and elevation, have more detailed field formats. In these situations there will be an additional format specification to describe the breakdown of the numeric value. For example, the latitude field definition DDMMSSsssss means DD = degrees, MM = minutes and SSsssss = seconds (with the SS and sssss values separated by an implied decimal point). The range for degrees is 0 to 90, for minutes is 0 to 59, and for seconds is 0000000 to 5999999, with an implied precision of five places to the right of the decimal point. Separation of capital and lower case in such format descriptors always means an implied decimal point.

The individual types of records are described below.

A1 Record (First and Last Records of a file)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	Blanks	X(6)	✦	Intentionally Blank
07-10	Data Code	XA9X	*A1*	Record identifier
11-80	Blanks	X(70)	✦	Intentionally Blank

10 Record (Helmert Block Identifier)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	Blanks	X(6)	✦	Intentionally Blank
07-10	Data Code	X99X	*10*	Record identifier
11-80	Helmert Block Name	X(70)	[A..Z, 0..9, =, -, ., +, /, ✦]	Description identifying this Helmert Block (within the greater readjustment)

13 Record (Horizontal Datum)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	Blanks	X(6)	✦	Intentionally Blank
07-10	Data Code	X99X	*13*	Record identifier
11-34	Datum Name	X(24)	“NAD 83 (NSRS2007)”	The official name of the 2007 Readjustment of NAD 83.
35-80	Blanks	X(46)	✦	Intentionally Blank

80 Record (Latitude and Longitude of one control point)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	PID (Permanent Identifier)	AANNNN	AA0001 to ZZZZZZ	A unique identifier assigned to every recoverable survey point in the NGSIDB.
07-10	Record Type	X99X	*80*	Record identifier
11-14	SSN (Station Serial Number) of Control Point	9999	0001 to 9999	A number which uniquely identifies this control point within this Helmert Block.
15-44	Designation (a.k.a. Station Name)	X(30)	[A..Z, 0..9, =, -, ., +, /, ✦]	The name of the control point.
45-55	Latitude	99999999999 (DDMMSSsssss)	00000000000 to 90000000000	The latitude of the control point.
56	Latitude Hemisphere	A	“N” or “S”	A code representing the hemisphere (direction) of the latitude.
57-68	Longitude	999999999999 (DDDMMSSsssss)	000000000000 to 360000000000	The longitude of the control point.
69	Longitude Hemisphere (a.k.a. Longitude Direction)	A	“W” or “E”	A code representing the hemisphere (direction) of longitude.
70-76	Blank	X(6)	✦	Intentionally Blank
77-78	State Code	AA	Any of the valid two character state codes from the NGSIDB.	State where the control point is located.
79-80	Blank	AA	✦	Intentionally Blank

86 Record (Ellipsoid Height of one control point)

Column	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	PID (Permanent Identifier)	AANNNN	AA0001 to ZZZZZZ	A unique identifier assigned to every recoverable survey point in the NGSIDB.
07-10	Data Code	A99A	*86*	Record identifier.
11-14	SSN (Station Serial Number) of Control Point	9999	0001 to 9999	A number which uniquely identifies this control point within this Helmert Block.
15-45	Blanks	A(31)	✦	Intentionally Blank
46-52	Ellipsoid Height	9999999 (MMMMmmm).	-999999 to 9999999	Ellipsoid Height in meters (when implied decimal point is in place)
53-80	Blanks	A(28)	✦	Intentionally Blank

91 Record (Standard Deviation of one control point)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	PID (Permanent Identifier)	AANNNN	AA0001 to ZZZZZZ	A unique identifier assigned to every recoverable survey point in the NGSIDB.
07-10	Data Code	X99X	*91*	Record identifier.
11-14	SSN (Station Serial Number) of Control Point	9999	0001 to 9999	A number which uniquely identifies this control point within this Helmert Block.
15-20	Blanks	X(6)	✦	Intentionally Blank
21-30	Latitude standard deviation	9999999.99	0.00 to 9999999.99	Latitude component of the horizontal standard deviation, or “1 sigma” in the North-South Direction. In centimeters.
31-40	Longitude standard deviation	9999999.99	0.00 to 9999999.99	Longitude component of horizontal standard deviation, or “1 sigma” in the East-West Direction. In centimeters.
41-50	Horizontal Correlation Coefficient	±.99999999	-.99999999 to +.99999999	The correlation coefficient between the north-south (latitude) component and the east-west (longitude) component of horizontal network accuracy.
51-60	Ellipsoid Height standard deviation	9999999.99	0.00 to 9999999.99	Ellipsoid height standard deviation, or “1 sigma” in the direction normal to the ellipsoid. In centimeters.

61-64	Blanks	X(4)	✦	Intentionally Blank
65	Accuracy Scaled Code	A	“Y” or “N”	A code which indicates whether or not the horizontal network accuracy is computed using a-priori (N) or a posteriori (Y) standard deviation of unit weight.
66-80	Blanks	X(15)	✦	Intentionally Blank

92 Record (Covariance between two control points)

Columns	*Field Name*	*Field Format*	*Field Range*	*Field Description/Comments*
01-06	Blanks	X(6)	✦	Intentionally Blank
07-10	Data Code	X99X	*92*	Record identifier.
11-14	First Point SSN (Station Serial Number)	9999	0001 to 9999	A number which uniquely identifies the first (of two) control points within this Helmert Block.
15-16	Blanks	XX	✦	Intentionally Blank
17-20	Second Point SSN (Station Serial Number)	9999	0001 to 9999	A number which uniquely identifies the second (of two) control points within this Helmert Block.
21-22	Blanks	XX	✦	Intentionally Blank
23-32	Standard deviation of the relative latitude	9999999.99	0000000.00 to 9999999.99	Relative latitude (north-south) component of horizontal standard deviation of the two control points relative to one another. In centimeters.
33-42	Standard deviation of the relative longitude	9999999.99	0000000.00 to 9999999.99	Relative longitude (east-west) component of horizontal standard deviation of the two control points relative to one another. In centimeters.
43-52	Horizontal Correlation Coefficient	±.99999999	-.99999999 to +.99999999	The correlation coefficient between the north-south (relative latitude) component and the east-west (relative longitude) component of horizontal standard deviation between the two control points.
53-62	Standard deviation of the relative ellipsoid height	9999999.99	0000000.00 to 9999999.99	Ellipsoid height local accuracy (standard deviation or “1 sigma” in the direction normal to the ellipsoid of the two control points relative to one another. In

				centimeters.
63-66	Blanks	X(4)	✦	Intentionally Blank
67	Accuracy Scaled Code	A	“Y” or “N”	A code which indicates whether or not the horizontal local accuracy is computed using a-priori (N) or a posteriori (Y) standard deviation of unit weight.
68-80	Blanks	X(15)	✦	Intentionally Blank

36. Appendix 3: Sample Datasheet

See file dsdata.txt for more information about the datasheet.

```
DATABASE = Sybase ,PROGRAM = datasheet, VERSION = 7.58
1        National Geodetic Survey,   Retrieval Date = MARCH  6, 2008
 HC1143 ************************************************************************
 HC1143  DESIGNATION -  PH 03
 HC1143  PID         -  HC1143
 HC1143  STATE/COUNTY-  MO/PHELPS
 HC1143  USGS QUAD   -  ROLLA (1992)
 HC1143
 HC1143                         *CURRENT SURVEY CONTROL
 HC1143  ___________________________________________________________________
 HC1143* NAD 83(2007)-  37 55 07.48406(N)    091 46 44.67232(W)     ADJUSTED
 HC1143* NAVD 88     -       339.80   (+/-2cm)    1114.8    (feet)   VERTCON
 HC1143  ___________________________________________________________________
 HC1143  EPOCH DATE  -        2002.00
 HC1143  X           -    -156,415.282 (meters)                     COMP
 HC1143  Y           -  -5,035,792.132 (meters)                     COMP
 HC1143  Z           -   3,898,523.022 (meters)                     COMP
 HC1143  LAPLACE CORR-           1.26  (seconds)                    DEFLEC99
 HC1143  ELLIP HEIGHT-         309.128 (meters)        (02/10/07) ADJUSTED
 HC1143  GEOID HEIGHT-         -30.70  (meters)                     GEOID03
 HC1143
 HC1143  ------- Accuracy Estimates (at 95% Confidence Level in cm) --------
 HC1143  Type    PID    Designation                      North   East  Ellip
 HC1143  --------------------------------------------------------------------
 HC1143  NETWORK HC1143 PH 03                             1.45   1.16   2.29
 HC1143  --------------------------------------------------------------------
```

NGS plans to modify and publish the Network and Local Accuracies as horizontal and vertical only (as per FGDC)

```
 HC1143  VERT ORDER  -  THIRD (See Below)
 HC1143
 HC1143.The horizontal coordinates were established by GPS observations
 HC1143.and adjusted by the National Geodetic Survey in February 2007.
 HC1143
 HC1143.The datum tag of NAD 83(2007) is equivalent to NAD 83(NSRS2007).
 HC1143.See National Readjustment for more information.
 HC1143.The horizontal coordinates are valid at the epoch date displayed above.
 HC1143.The epoch date for horizontal control is a decimal equivalence
 HC1143.of Year/Month/Day.
 HC1143
 HC1143.The NAVD 88 height was computed by applying the VERTCON shift value to
 HC1143.the NGVD 29 height (displayed under SUPERSEDED SURVEY CONTROL.)
 HC1143.The vertical order pertains to the NGVD 29 superseded value.
 HC1143
 HC1143.The X, Y, and Z were computed from the position and the ellipsoidal ht.
 HC1143
 HC1143.The Laplace correction was computed from DEFLEC99 derived deflections.
 HC1143
 HC1143.The ellipsoidal height was determined by GPS observations
 HC1143.and is referenced to NAD 83.
 HC1143
```

```
HC1143.The geoid height was determined by GEOID03.
HC1143
HC1143;                    North         East     Units Scale Factor Converg.
HC1143;SPC MO C     -   231,659.012   563,386.550   MT  0.99998280   +0 26 35.0
HC1143;UTM  15      - 4,197,502.455   607,314.070   MT  0.99974184   +0 45 01.4
HC1143
HC1143!             -  Elev Factor  x  Scale Factor =   Combined Factor
HC1143!SPC MO C     -   0.99995150  x   0.99998280  =   0.99993430
HC1143!UTM  15      -   0.99995150  x   0.99974184  =   0.99969335
HC1143
HC1143                          SUPERSEDED SURVEY CONTROL
HC1143
HC1143  NAD 83(1997)-  37 55 07.48273(N)    091 46 44.67846(W) AD(       ) 1
HC1143  ELLIP H (02/17/00)  308.977  (m)                       GP(       ) 4 1
HC1143  NAD 83(1986)-  37 55 07.49851(N)    091 46 44.67365(W) AD(       ) 1
HC1143  NGVD 29 (07/10/92)  339.75   (m)         1114.7    (f) LEVELING    3
HC1143
HC1143.Superseded values are not recommended for survey control.
HC1143.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
HC1143.See file dsdata.txt to determine how the superseded data were derived.
HC1143
HC1143_U.S. NATIONAL GRID SPATIAL ADDRESS: 15SXB0731497502(NAD 83)
HC1143_MARKER: DD = SURVEY DISK
HC1143_SETTING: 7 = SET IN TOP OF CONCRETE MONUMENT
HC1143_SP_SET: CONCRETE POST
HC1143_STAMPING: PH-03 1990
HC1143_MARK LOGO: MODNR
HC1143_MAGNETIC: M = MARKER EQUIPPED WITH BAR MAGNET
HC1143_STABILITY: C = MAY HOLD, BUT OF TYPE COMMONLY SUBJECT TO
HC1143+STABILITY: SURFACE MOTION
HC1143_SATELLITE: THE SITE LOCATION WAS REPORTED AS SUITABLE FOR
HC1143+SATELLITE: SATELLITE OBSERVATIONS - January 28, 1991
HC1143
HC1143  HISTORY     - Date     Condition        Report By
HC1143  HISTORY     - 1990     MONUMENTED       MODNR
HC1143  HISTORY     - 19910128 GOOD
HC1143
HC1143                          STATION DESCRIPTION
HC1143
HC1143'DESCRIBED BY MO DEPT OF NAT RES 1990
HC1143'DATE OF REPORT 10-02-1991
HC1143'STATION, AZIMUTH MARKS AND REFERENCE TIES FOLLOWS
HC1143'THE STATION IS LOCATED ON THE SOUTH SIDE OF ROLLA ON THE EAST
HC1143'RIGHT-OF-WAY OF MISSOURI ROUTE 63 NEAR THE PROPERTY LINE BETWEEN
HC1143'OZARK MEMORIAL GARDENS AND TAPJAC HOME CENTER IN SECTION 23, T37N,
HC1143'R8W.  THE STATION IS 41.8 FT (12.7 M)  ENE OF THE CENTER LINE OF
HC1143'ROUTE 63, 41.4 FT (12.6 M)  SSW OF A NAIL AND SHINER IN A TELEPHONE
HC1143'POLE, 17.9 FT (5.5 M)  WNW OF A NAIL AND SHINER IN A FENCE POST,
HC1143'101.1 FT (30.8 M) NNW OF A NAIL AND SHINER IN A TELEPHONE POLE, 24 FT
HC1143'(7.3 M)  SOUTH OF THE PROJECTION OF HARTVILLE ROAD (COUNTY ROAD 250)
HC1143'WITH ROUTE 63 AND 17.7 FT (5.4 M)  NORTHWEST OF A CARSONITE WITNESS
HC1143'POST.
HC1143'THE AZIMUTH MARK IS LOCATED 0.31 MILES SOUTHEAST OF THE STATION ON THE
HC1143'EAST RIGHT-OF-WAY OF MISSOURI ROUTE 63 JUST SOUTH OF COUNTY ROAD
HC1143'145.  THE MONUMENT IS 48.8 FT (14.9 M)  EAST OF THE CENTER LINE OF
HC1143'ROUTE 63, 92.9 FT (28.3 M)  NORTH OF A NAIL AND SHINER IN A TELEPHONE
HC1143'POLE, 31.1 FT (9.5 M)  NORTHWEST OF A NAIL AND SHINER IN A 16 INCH
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HC1143'POST OAK, 37.4 FT (11.4 M)  SOUTH OF A NAIL AND SHINER IN A TELEPHONE
HC1143'POLE, 65.3 FT (19.9 M)  SOUTHEAST OF NGS BMV-34 RESET 1940, 129.1 FT
HC1143'(39.3 M)  SOUTH OF THE CENTER LINE OF COUNTY ROAD 145 AND 3.3 FT
HC1143'(1.0 M)  NORTHWEST OF A CARSONITE WITNESS POST.
HC1143'STATION AND AZIMUTH MARK TO REACH
HC1143'TO REACH THE STATION FROM THE INTERSECTION OF MISSOURI ROUTE 63 AND
HC1143'BUSINESS LOOP 44 (KINGSHIGHWAY) IN ROLLA, GO SOUTH ON MISSOURI ROUTE
HC1143'63 FOR 2.0 MILES AND PARK ON THE SHOULDER.  WALK EAST ACROSS THE
HC1143'HIGHWAY TO THE STATION AS DESCRIBED.
HC1143'TO REACH THE AZIMUTH MARK FROM THE STATION, GO SOUTHEAST ON MISSOURI
HC1143'ROUTE 63 FOR 0.31 MILES, TURN LEFT ON COUNTY ROAD 145 AND PARK.
HC1143'WALK SOUTH ALONG THE EAST RIGHT-OF-WAY FOR 129 FT (39.3 M) TO THE
HC1143'AZIMUTH MARK AS PER DESCRIPTION.
HC1143'SPECIAL INFORMATION
HC1143
HC1143                          STATION RECOVERY (1991)
HC1143
HC1143'RECOVERED 1991
HC1143'RECOVERED IN GOOD CONDITION.
```