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E. LESTER JONES, DIRECTOR

VELOCITY OF SOUND IN SEA WATER

By

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and

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VELOCITY OF SOUND IN SEA WATER

By

Commander N. H. Heck and Ensign Jerry H. Service, *U. S. Coast and Geodetic Survey*

INTRODUCTION

While the subject of sound has always been recognized as one of the important divisions of physics and certain phases of it have been thoroughly investigated, other phases have remained almost untouched until recently. An especial example of this is the transmission of sound through sea water. Possible application in navigation was recognized just prior to the World War and some progress was made in the design of apparatus, but it was the development of the submarine as a menace to shipping and the consequent need for methods of counteracting its activities that led to concentrated investigation by the leading physicists of this and other countries. Suitable means of setting up sound waves capable of transmission through long distances and receivers capable of detecting faint sounds reaching them were among the results of this investigation.

After the war interest was not allowed to die, but on the contrary, every effort was made to find peace-time uses for this addition to knowledge. This is evidenced by the large number of organizations continuing in or taking up the work. In the United States the Navy Department developed the sonic depth finder; the War Department perfected methods for accurately determining the velocity of sound along the surface and made important determinations of velocity; the Coast and Geodetic Survey and the Bureau of Standards jointly developed the radio-acoustic method for use in hydrographic surveying.

The British Navy during the same period has been at work on acoustic methods for obtaining the depth of the water and has made determinations of velocity along the surface; the French Hydrographic Office has studied the velocity of sound along the surface; the German Hydrographic Office has studied the theoretical velocity of sound with special reference to use in obtaining depth. These statements are made on the basis of the latest available published information.[1]

[1] "Modern navigational devices," by F. E. Smith, Engineering, vol. 117, pp. 299–300, Mar. 17, 1924.
"Acoustical methods for depth sounding," Nature, vol. 113, pp. 463–65 Mar. 29, 1924.
"A radio-acoustic method of locating positions at sea: Application to navigation and to hydrographical surveys," by Dr. A. B. Wood and Capt. H. E. Browne, Proc. Phys. Soc. of London, vol. 35, part 3, pp. 183–194, Apr. 15, 1923.
"The sounding of the sea by sound," by P. Marti (hydrographic engineer of the French Navy), La Nature, Aug. 20, 1921, pp. 125–127.
"Les signaux sous-marins par ondes ultra-sonores," by A. Troller, La Nature, second half, 1923.
"The velocity of sound in sea water," La Nature, p. 117, Oct. 14, 1922.
"Über Echolotungen der nordamerikanischen Marine," by Dr. H. Maurer, Annalen der Hydrographie und maritimen Meteorologie, Apr., 1924, pp. 75–87.
"Hydrographische Bemerkungen und Hilfsmittel zur akustiken Tiefenmessung," by Dr. Arnold Schumacher, Deutsche Seewarte, Annalen der Hydrographie und maritimen Meterologie, Apr., 1924, pp. 87–95.

The sonic depth finder was developed by Dr. Harvey C. Hayes, research physicist, United States Navy. It is capable of measuring accurately the time required for sound to travel from the surface to the bottom and for the echo to return to the surface.[2]

The radio-acoustic apparatus was developed by Dr. E. A. Eckhardt, Bureau of Standards, for the use of the Coast and Geodetic Survey, in hydrographic work. The function of the apparatus is to measure accurately the time required for a sound wave to travel from a bomb explosion near the surveying vessel to a hydrophone whose position is known.[3]

In both these cases the function of the apparatus is to measure accurately the time interval. It is evident that to determine depth in one case and distance in the other it is necessary to know the velocity of sound in sea water under the existing conditions.

RÉSUMÉ OF EXISTING INFORMATION ON VELOCITY OF SOUND

The subaqueous sound-ranging section of the United States Army, under Col. R. S. Abernethy, Coast Artillery Corps, has made a very accurate determination of the velocity of sound along the surface in certain localities. The results are discussed by E. B. Stephenson, physicist, who was associated in this work.[4]

Work of the British Navy resulted in obtaining velocities of sound along the surface. An empirical formula based on their results expresses velocity as a function of temperature and salinity of the water.[5]

There is no evidence in existing publications to show that any organization, except the United States Coast and Geodetic Survey, has made experimental determinations of the velocity for vertical transmission to great depths.

From November 17 to December 29, 1923, the Coast and Geodetic Survey steamer *Guide* was engaged on an oceanographic cruise from New London, Conn., to San Diego, Calif., by way of Porto Rico and the Panama Canal. The wide range of conditions encountered is evident from inspection of the map. As the work included an investigation of Nares Deep, north of Porto Rico, the deepest part of the Atlantic Ocean, and also the development of a hitherto unexplored deep in the Pacific off the coasts of Central America and Mexico, the range in depth and the number of deep soundings was exceptional. The actual range in depth was from 185 fathoms to 4,617 fathoms.

The *Guide* was equipped with a sonic depth finder, and was also equipped with standard apparatus for taking wire soundings, temperatures, and water samples (for later determination of salinity) at any depth, and for taking specimens of the bottom. A definite scheme of soundings was laid out in advance. At every fourth or fifth sounding the depth was obtained by wire and the corresponding time interval for the transmission of sound was determined by the

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[2] "Measuring ocean depths by acoustical methods," by Dr. Harvey C. Hayes, Journal of the Franklin Institute, vol. 197, pp. 323-354, Mar., 1924.

[3] "Radio-acoustic method of position finding in hydrographic surveys," by N. H. Heck, E. A. Eckhardt, and M. Keiser, Special Publication No. 107, U. S. Coast and Geodetic Survey.

[4] "Velocity of sound in sea water," by E. B. Stephenson, Physical Review, vol. 21, pp. 182-185, February, 1923.

[5] "A radio-acoustic method of locating positions at sea; application to navigation and to hydrographical survey," by Dr. A. B. Wood and Capt. H. E. Browne, Proc. Phys. Soc. of London, vol. 35, part 3, pp. 183-194, Apr. 15, 1923.

sonic depth finder. Temperatures and water samples were obtained at the surface, at the depth of 200 fathoms, and at the bottom. In one case in the Atlantic and one in the Pacific serial temperatures and water samples were obtained from surface to bottom. On arrival

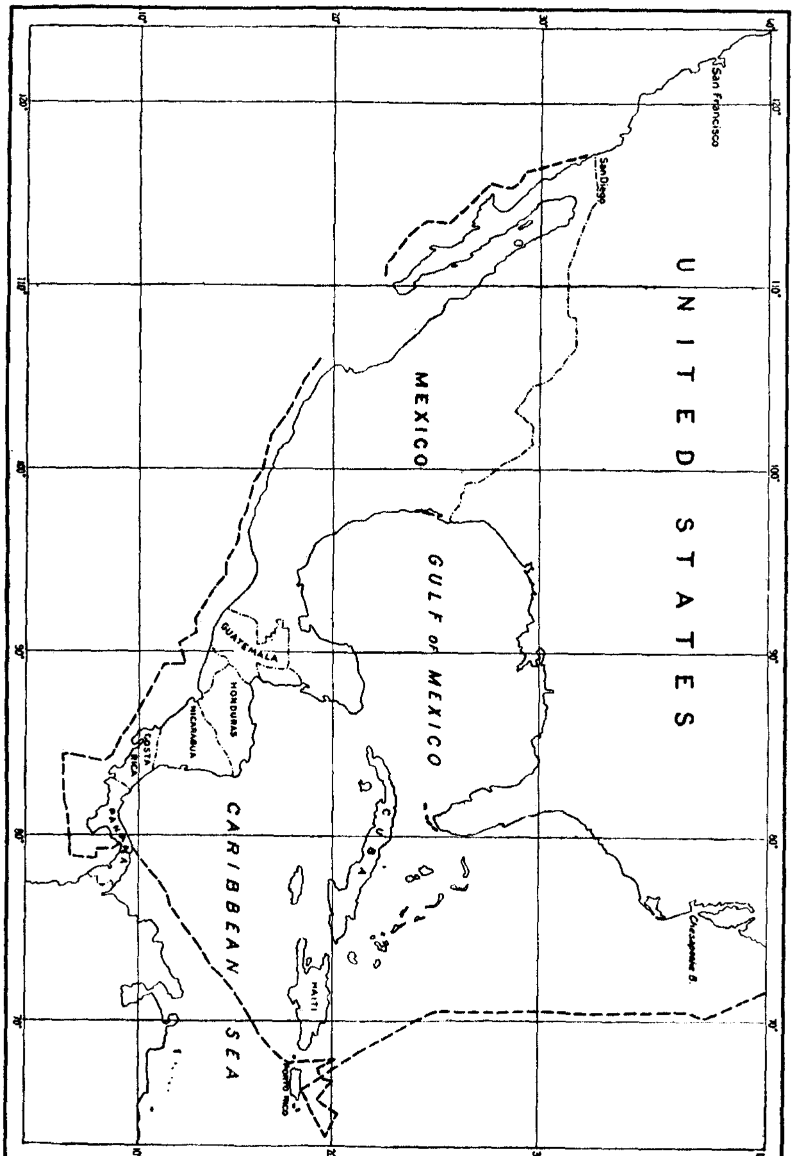

FIG. 1.—Cruise of the Steamer *Guide* in 1923.

at San Diego, the water samples were turned over to the Scripps Institution for Biological Research, at La Jolla, Calif., for determination of the salinities.

Intermediate soundings were taken by the sonic depth finder. The velocity of sound to be used in each case was not decided upon until the velocities obtained by simultaneous depth and time determinations had been studied and a rational basis for applying theoretical velocities had been developed.

Inasmuch as the piano-wire soundings, which were taken with special care in recognition of their importance in connection with the velocity of sound, were direct measurements of depth, and the observations with the sonic depth finder, taken with equal care, were direct measurements of time, it is evident that the work of the *Guide* made available a reliable series of measurements of the velocity of sound in sea water under a wide range of conditions. Owing to strong surface currents in a few places affecting the accuracy of the wire soundings, to faint echoes, to instrumental difficulties, and to other causes, a few of the determinations are less reliable than others, and such velocities are given less weight than those obtained under good conditions.

Early in the cruise of the *Guide* it became evident that the velocity increased with the depth in spite of the fact that the temperature fell and the salinity remained practically the same. This fact suggested that velocity is a function not only of temperature and salinity but also of pressure. Work was begun on the problem of finding the relation, based upon reliable theoretical grounds, of velocity to temperature, pressure, and salinity. The authors of this publication, Commander N. H. Heck, Coast and Geodetic Survey, who exercised general supervision over acoustic depth and position determination work of the *Guide*, and Ensign Jerry H. Service, United States Coast and Geodetic Survey, an officer of the *Guide*, who had had previous experience in physical research, succeeded in finding a solution of this problem. It is the purpose of this publication to present the results of this solution in a form convenient for practical use, as well as to show how the problem has been solved.

THEORY

The Newtonian equation for the velocity of sound in a given medium suggested itself as a logical and reliable foundation upon which to work. Sir Isaac Newton first showed rigorously, reasoning from fundamentals, that the velocity of transmission of sound through any given medium is given by the equation

$$V=\sqrt{\frac{\text{elasticity of the medium}}{\text{density of the medium}}}$$

By "elasticity of the medium" is meant the ratio:

$$\frac{\text{Increase of pressure applied to the medium}}{\text{Resulting decrease in volume expressed as a fraction of the original volume.}}$$

The "density of the medium" is, of course, the mass per unit volume, and the mass and volume must be expressed in units corresponding to those of the force and area, respectively, in the pressure. V will then be the velocity of transmission of sound through the medium, in units depending upon those used for pressure and density.

It has seemed most satisfactory to make use in the application of Newton's equation of the specific-volume data tabulated in Dynamic Meteorology and Hydrography, part 1, by V. Bjerknes and J. W. Sandström, published in 1910 by the Carnegie Institution of Washington. These specific volumes are based upon the very precise work of Knudsen, Ekman, and others. The use of these tables was suggested by Dr. George F. McEwen, of the Scripps Institution, who also gave other valuable advice and assistance. These specific volumes are probably nowhere in error by more than 1 part in 10,000, and for the most part are correct to 1 part in 100,000. The specific volume is, of course, the reciprocal of the density and can therefore be used directly in the application of Newton's equation.

The specific volumes tabulated by Bjerknes and Sandström are not directly measured but are built up as the sum of directly measured specific volumes and directly measured changes in specific volume due to pressure, temperature, and salinity changes. It is possible, therefore, by taking differences, to obtain from the tables satisfactory values of the elasticity of sea water, which elasticities are probably nowhere in error by as much as 1 per cent. It will now be shown how Bjerknes and Sandström's tables were used in the computation of velocity.

In the first place it should be stated that as unit of pressure the bar, which equals 10^6 dynes per cm^2, was used in this work. It was first necessary to reduce the depth for which velocities were to be computed from fathoms to meters, and thence to *dynamic meters* by means of Table 3H. The dynamic meter is a unit used to take into account the increase in the force of gravity with depth. By means of Table 15H the pressure in decibars obtaining at the various depths were then found.

It is desirable to explain at this point the form in which the specific-volume tables of Bjerknes and Sandström have been compiled. Seven tables are required which are as follows:

Table 8H gives the specific volumes of sea water in cm^3/gm at 0° C. temperature and 35‰ (35 parts per thousand) salinity for every 10 decibars pressure from 0 to 10,000 decibars.

Table 9H is a table of salinity corrections to specific volume and has a range from salinity 0‰ (pure water) to salinity 39‰.

Table 10H gives temperature corrections to specific volume and ranges from $-1°$ to 29° C.

Table 11H is a table of combined salinity-temperature corrections.

Table 12H is a table of combined salinity-pressure corrections.

Table 13H is a table of combined temperature-pressure corrections.

Table 14H is a table of combined salinity-temperature-pressure corrections.

It will be noted that each of these tables is designated by a number followed by H. In what follows it will be understood that tables so designated are Bjerknes and Sandström tables without mention of the names of those authors.

It should be understood that the corrections in Tables 9H and 10H are first-order corrections and that the corrections in Tables 11H, 12H, 13H, and 14H, are additional second-order corrections.

It was found advantageous to transform Newton's equation into a more convenient form that would be better adapted to Bjerknes and Sandström's tables. The definition of elasticity which has been given can be put into the form

$$\text{Elasticity} = \frac{\text{increase of pressure in dynes/cm}^2}{\dfrac{\text{resulting decrease in sp. vol. in cm}^3\text{/gm}}{\text{specific volume in cm}^3\text{/gm}}}$$

Increase of pressure is always taken as 10 decibars or 10^6 dynes/cm². "Resulting decrease in sp. vol." may be designated by dv. Specific volume may be designated by v.

The elasticity equation then becomes

$$\text{Elasticity} = \frac{10^6}{\frac{dv}{v}} = \frac{10^6 v}{dv}.$$

Furthermore, in order to have dv a whole number instead of a small decimal, it is found convenient to use $10^5 dv$ instead of dv, necessitating multiplying the numerator also by 10^5, which gives:

$$\text{Elasticity} = \frac{10^{11} v}{(10^5 dv)}.$$

Since density $= \frac{1}{v}$ we have from Newton's equation

$$V \text{ in cm/sec.} = \sqrt{\frac{10^{11} v}{(10^5 dv)} \div \frac{1}{v}} = \sqrt{\frac{10^{11} v^2}{10^5 dv}} = 10^5 v \sqrt{\frac{10}{(10^5 dv)}}$$

$$V \text{ in m/sec.} = 10^3 v \sqrt{\frac{10}{10^5 dv}} \qquad (1)$$

$$V \text{ in fathoms/sec.} = 5.468 \times 10^2 v \sqrt{\frac{10}{(10^5 dv)}} \qquad (2)$$

In addition to facility in entering tables, this form lends itself well to the use of reciprocal and square-root tables and multiplying machines, with consequent ease and speed in obtaining velocities.

METHOD OF PREPARING VELOCITY TABLES

In order that the velocity of sound may be obtained in accordance with equation (2) at any place, the water from the surface to the bottom is considered in 200-fathom layers and the mean temperature and salinity for each layer is obtained from the best available source of information. The velocity for the entire depth is then taken as the mean of the various layer velocities.

Accordingly, Velocity Table No. 13, pages 26–27 gives the velocity for the possible range of temperature and salinity for the surface and for the depth corresponding to the middle of each 200-fathom layer.

The formation of a table of values of v consisted simply of taking out from Table 8H the "base specific volume" (for 0° C. and 35‰ salinity) for the pressures corresponding to the depths at the middle of the 200-fathom layers and applying to these base specific volumes corrections for salinity and temperatures from Tables 9H to 14H, inclusive. The resulting corrected values of v are given in Table 1.

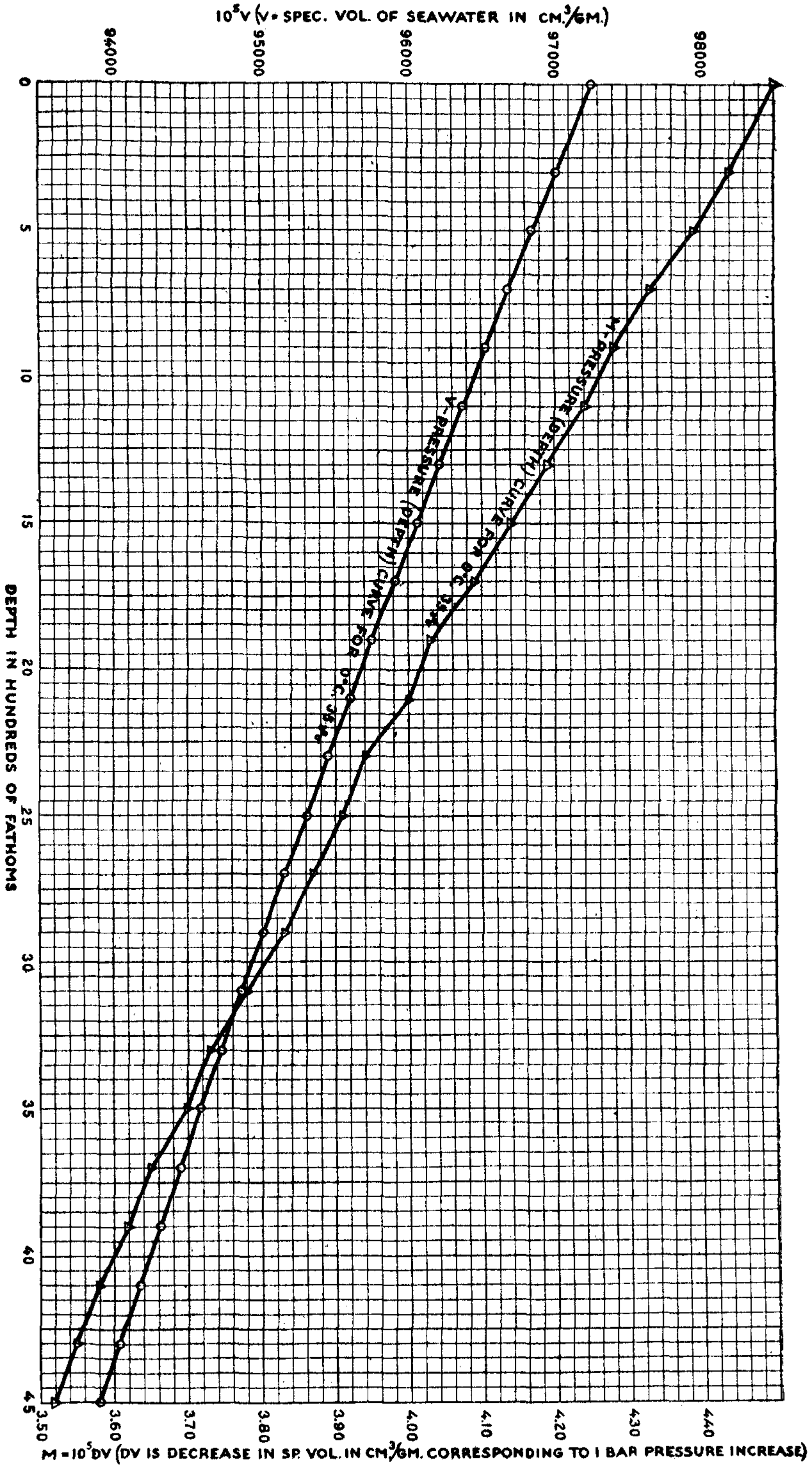
10^5V (V = SPEC. VOL. OF SEAWATER IN CM.^{3}/GM.)
94000
95000
96000
97000
98000
V-PRESSURE (DEPTH) CURVE FOR 0° C. 35‰
M-PRESSURE (DEPTH) CURVE FOR 0° C. 35‰
DEPTH IN HUNDREDS OF FATHOMS
0
5
10
15
20
25
30
35
40
45
3.50
3.60
3.70
3.80
3.90
4.00
4.10
4.20
4.30
4.40
M = 10^5DV (DV IS DECREASE IN SP. VOL. IN CM.^{3}/GM. CORRESPONDING TO 1 BAR PRESSURE INCREASE)

FIG. 2

TABLE 1

[$10^5 v$ (v=specific volume of sea water expressed in $\frac{cm^3}{gm}$)]

Depth (fathoms)	Salinity (0/00)	Temperature (degrees centigrade) 0	2	4	6	8	10	12	14	16	18	20	22
Surface and 100.	31	97, 570	97, 581	97, 596	97, 616	97, 641	97, 670	97, 703	97, 740	97, 781	97, 824	97, 873	97, 923
	32	97, 493	97, 504	97, 520	97, 540	97, 566	97, 595	97, 629	97, 666	97, 707	97, 751	97, 799	97, 850
	33	97, 417	97, 429	97 445	97, 466	97, 492	97, 521	97, 555	97, 593	97, 634	97, 679	97, 727	97, 778
	34	97, 340	97, 352	97, 369	97, 391	97, 417	97, 447	97, 481	97, 519	97, 561	97, 605	97, 654	97, 705
	35	97, 264	97, 277	97, 294	97, 316	97, 343	97, 373	97, 408	97, 446	97, 488	97, 533	97, 582	97, 633
	36	97, 188	97, 201	97, 219	97, 241	97, 269	97, 299	97, 335	97, 373	97, 415	97, 460	97, 510	97; 561
	37	97, 112	97, 126	97, 144	97, 167	97, 195	97, 226	97, 261	97, 300	97, 342	97, 388	97, 437	97, 489
300 (554 deci-bars).	31	97, 317	97, 331	97, 349	97, 371	97, 398	97, 429	97, 464	97, 503	97, 545	97, 589	97, 639	97, 690
	32	97, 241	97, 255	97, 274	97, 296	97, 324	97, 355	97, 391	97, 430	97, 472	97, 517	97, 566	97, 618
	33	97, 166	97, 181	97, 200	97, 223	97, 251	97, 282	97, 318	97, 358	97, 400	97, 446	97, 495	97, 547
	34	97, 090	97, 105	97, 125	97, 149	97, 177	97, 209	97, 245	97, 285	97, 328	97, 373	97, 423	97, 475
	35	97, 015	97, 031	97, 051	97, 075	97, 104	97, 136	97, 173	97, 213	97, 256	97, 302	97, 352	97, 404
	36	96, 940	96 956	96, 977	97, 001	97, 031	97, 063	97, 101	97, 141	97, 184	97, 230	97, 281	97, 333
	37	96, 865	96, 882	96, 903	96, 928	96, 958	96, 991	97, 028	97, 069	97, 112	97, 159	97, 209	97, 262
500 (923 deci-bars).	31	97, 154	97, 170	97, 189	97, 213	97, 242	97, 274	97, 310	97, 350	97, 393	97, 438	97, 489	97, 540
	32	97, 078	97, 094	97, 114	97, 138	97, 167	97, 200	97, 237	97, 277	97, 320	97, 366	97, 416	97, 468
	33	97, 003	97, 020	97, 040	97, 065	97, 094	97, 126	97, 163	97, 204	97, 247	97, 294	97, 344	97, 396
	34	96, 928	96, 945	96, 966	96, 992	97, 021	97, 054	97, 091	97, 132	97, 177	97, 223	97, 274	97, 326
	35	96, 853	96, 871	96, 892	96, 918	96, 948	96, 981	97, 019	97, 060	97, 104	97, 151	97, 202	97, 254
	36	96, 778	96, 796	96, 818	96, 844	96, 875	96, 908	96, 947	96, 988	97, 032	97, 079	97, 131	97, 183
	37	96, 704	96, 723	96, 745	96, 772	96, 803	96, 837	96, 875	96, 917	96, 960	97, 008	97, 059	97, 112
700 (1,293 deci-bars).	31	96, 989	97, 006	97, 027	97, 053	97, 084	97, 117	97, 154	97, 195	97, 240	97, 285	97, 337	------
	32	96, 914	96, 931	96, 953	96, 979	97, 010	97, 044	97, 082	97, 123	97, 168	97, 214	97, 265	------
	33	96, 840	96, 858	96, 878	96, 905	96, 936	96, 969	97, 007	97, 049	97, 095	97, 142	97, 193	------
	34	96, 765	96, 783	96, 806	96, 834	96, 865	96, 899	96, 937	96 979	97, 025	97, 071	97, 124	------
	35	96, 691	96, 710	96, 733	96, 761	96, 793	96, 827	96, 866	96, 908	96, 954	97, 001	97, 053	------
	36	96, 617	96, 636	96, 660	96, 688	96, 721	96, 755	96, 795	96, 837	96, 883	96, 930	96, 983	------
	37	96, 543	96, 563	96, 587	96, 616	96, 649	96, 684	96, 722	96, 765	96, 811	96, 859	96, 911	------
900 (1,665 deci-bars).	31	96, 827	96, 846	96, 869	96, 897	96, 929	96, 963	97, 002	------	------	------	------	------
	32	96, 753	96, 772	96, 796	96, 823	96, 857	96, 891	96, 931	------	------	------	------	------
	33	96, 679	96, 699	96, 723	96, 751	96, 784	96, 819	96, 859	------	------	------	------	------
	34	96, 605	96, 625	96, 650	96, 679	96, 712	96, 749	96, 787	------	------	------	------	------
	35	96, 531	96, 552	96, 577	96, 606	96, 640	96, 675	96, 716	------	------	------	------	------
	36	96, 457	96, 478	96, 504	96, 533	96, 568	96, 603	96, 645	------	------	------	------	------
	37	96, 384	96, 406	96, 432	96, 462	96, 497	96, 532	96, 573	------	------	------	------	------

Depth (fathoms)	Temperature 0	2	4	6	8	Salinity 0/00	Temperature 0	1	2	3	Depth (fathoms)
1,100 (2,036 deci-bars).	96, 667	96, 688	96, 713	96, 741	96, 773	31	96, 041	96, 055	96, 069	96, 085	1,900(3,527 deci-bars).
	96, 593	96, 614	96, 639	96, 668	96, 701	32	95, 969	95, 983	95, 996	96, 013	
	96, 520	96, 542	96, 567	95, 596	96, 630	33	95, 898	95, 913	95, 926	95, 943	
	96, 446	96, 468	96, 494	96, 524	96, 557	34	95, 826	95, 840	95, 854	95, 871	
	96, 373	96, 396	96, 422	96, 452	96, 486	35	95, 755	95, 770	95, 784	95, 801	
	96, 300	96, 323	96, 350	96, 380	96, 415	36	95, 684	95, 699	95, 713	95, 731	
	96, 227	96, 251	96, 278	96, 309	96, 343	37	95, 613	95, 629	95, 643	95, 661	
1,300 (2,406 deci-bars).	96, 509	96, 532	96, 558	96, 588	-------	31	95, 890	95, 905	95, 920	95, 936	2,100(3,902 deci-bars).
	96, 436	96, 459	96, 485	96, 516	-------	32	95, 819	95, 834	95, 848	95, 865	
	96, 363	96, 387	96, 413	96, 444	-------	33	95, 748	95, 764	95, 778	95, 795	
	96, 290	96, 314	96, 341	96, 373	-------	34	95, 677	95, 692	95, 707	95, 724	
	96, 217	96, 242	96, 269	96, 301	-------	35	95, 606	95, 622	95, 637	95, 654	
	96, 144	96, 169	96, 197	96, 229	-------	36	95, 535	95, 551	95, 566	95, 584	
	96, 072	96, 098	96, 126	96, 159	-------	37	95, 465	95, 482	95, 497	95, 515	
1,500 (2,780 deci-bars).	96, 351	96, 376	96, 404	-------	-------	31	95, 738	95, 754	95, 769	95, 787	2,300(4,275 deci-bars).
	96, 278	96, 302	96, 332	-------	-------	32	95, 667	95, 683	95, 697	95, 716	
	96, 206	96, 231	96, 260	-------	-------	33	95, 597	95, 614	95, 628	95, 647	
	96, 133	96, 158	96, 188	-------	-------	34	95, 526	95, 542	95, 557	95, 576	
	96, 061	96, 087	96, 117	-------	-------	35	95, 456	95, 473	95, 488	95, 507	
	95, 989	96, 015	96, 046	-------	-------	36	95, 386	95, 403	95, 418	95, 438	
	95, 917	95, 944	95, 975	-------	-------	37	95, 316	95, 334	95, 349	95, 369	
1,700 (3,154 deci-bars).	96, 195	96, 222	96, 251	-------	-------	31	95, 588	95, 606	95, 621	95, 639	2,500(4,652 deci-bars).
	96, 122	96, 148	96, 179	-------	-------	32	95, 518	95, 535	95, 551	95, 569	
	96, 051	96, 078	96, 108	-------	-------	33	95, 448	95, 466	95, 481	95, 501	
	95, 978	96, 005	96, 036	-------	-------	34	95, 378	95, 395	95, 411	95, 430	
	95, 907	95, 935	95, 966	-------	-------	35	95, 308	95, 326	95, 342	95, 361	
	95, 836	95, 864	95, 896	-------	-------	36	95, 238	95, 256	95, 272	95, 292	
	95, 764	95, 793	95, 825	-------	-------	37	95, 169	95, 188	95, 203	95, 223	

Table 1—Continued

Depth (fathoms)	Temperature (degrees centigrade) 0	1	2	3	Salinity 0/00	Temperature (degrees centigrade) 0	1	2	Depth (fathoms)
2,700 (5,026 decibars).	95,442	95,460	95,476	95,494	31	--------	--------	--------	
	95,371	95,388	95,405	95,423	32	--------	--------	--------	
	95,302	95,320	95,336	95,356	33	94,443	94,465	94,486	3,900 (7,308 decibars).
	95,232	95,249	95,266	95,285	34	94,376	94,397	94,418	
	95,163	95,181	95,198	95,217	35	94,309	94,331	94,352	
	95,094	95,112	95,129	95,149	36	94,242	94,264	94,285	
	95,025	95,044	95,060	95,080	37	94,176	94,199	94,219	
2,900 (5,404 decibars).	95,155	95,174	95,191	--------	33	94,305	94,328	94,349	4,100 (7,688 decibars).
	95,086	95,104	95,122	--------	34	94,238	94,260	94,281	
	95,017	95,036	95,054	--------	35	94,172	94,195	94,216	
	94,948	94,967	94,985	--------	36	94,106	94,129	94,150	
	94,879	94,899	94,916	--------	37	94,040	94,064	94,084	
3,100 (5,780 decibars).	95,011	95,031	95,048	--------	33	94,167	94,190	94,212	4,300 (8,070 decibars).
	94,942	94,961	94,979	--------	34	94,101	94,123	94,145	
	94,874	94,894	94,912	--------	35	94,035	94,058	94,080	
	94,806	94,826	94,844	--------	36	93,969	93,992	94,014	
	94,737	94,758	94,776	--------	37	93,903	93,927	93,948	
3,300 (6,167 decibars).	94,866	94,886	94,904	--------	33	94,033	94,057	94,079	4,500 (8,451 decibars).
	94,797	94,816	94,835	--------	34	93,966	93,989	94,011	
	94,729	94,749	94,768	--------	35	93,901	93,925	93,947	
	94,661	94,681	94,700	--------	36	93,836	93,860	93,882	
	94,593	94,614	94,633	--------	37	93,770	93,795	93,816	
3,500 (6,547 decibars).	94,723	94,744	94,762	--------	33	93,897	93,921	93,944	4,700 (8,834 decibars).
	94,654	94,674	94,693	--------	34	93,831	93,854	93,877	
	94,587	94,608	94,627	--------	35	93,766	93,790	93,813	
	94,520	94,541	94,560	--------	36	93,701	93,725	93,748	
	94,452	94,474	94,493	--------	37	93,636	93,661	93,683	
3,700 (6,927 decibars).	94,582	94,604	94,624	--------	33	--------	--------	--------	
	94,514	94,535	94,555	--------	34	--------	--------	--------	
	94,447	94,469	94,489	--------	35	--------	--------	--------	
	94,380	94,402	94,422	--------	36	--------	--------	--------	
	94,313	94,336	94,355	--------	37	--------	--------	--------	

Table 2

[$M_{35,0,P}$ ($M_{35,0,P}=10^6\,dv$. at 35 ‰ salinity, 0° C., and standard pressure, where dv=decrease in specific volume in cm^3/gm corresponding to 1 bar increase in pressure)]

Pressure (decibars)	$M_{35,0,P}$	Pressure (decibars)	$M_{35,0,P}$	Pressure (decibars)	$M_{35,0,P}$	Pressure (decibars)	$M_{35,0,P}$
0	4.50	2,500	4.17	5,000	3.87	7,500	3.60½
100	4.50	2,600	4.15	5,100	3.85	7,600	3.59
200	4.50	2,700	4.14	5,200	3.83½	7,700	3.58
300	4.49	2,800	4.13½	5,300	3.83½	7,800	3.57½
400	4.46½	2,900	4.12	5,400	3.82½	7,900	3.56½
500	4.44½	3,000	4.10½	5,500	3.80½	8,000	3.55
600	4.43	3,100	4.09½	5,600	3.80	8,100	3.54½
700	4.41½	3,200	4.08½	5,700	3.79	8,200	3.53½
800	4.40½	3,300	4.05½	5,800	3.77½	8,300	3.52
900	4.39½	3,400	4.06½	5,900	3.77	8,400	3.52½
1,000	4.37½	3,500	4.04½	6,000	3.76	8,500	3.51½
1,100	4.36	3,600	4.00	6,100	3.73½	8,600	3.50
1,200	4.35	3,700	4.00	6,200	3.73	8,700	3.48½
1,300	4.33	3,800	4.00	6,300	3.72½	8,800	3.47½
1,400	4.31½	3,900	4.00	6,400	3.71	8,900	3.47
1,500	4.30	4,000	4.00	6,500	3.70½		
1,600	4.29	4,100	3.99½	6,600	3.69		
1,700	4.27½	4,200	3.95	6,700	3.68½		
1,800	4.26	4,300	3.93½	6,800	3.68		
1,900	4.25	4,400	3.94½	6,900	3.65		
2,000	4.24	4,500	3.92	7,000	3.65		
2,100	4.22½	4,600	3.91	7,100	3.65		
2,200	4.20½	4,700	3.89½	7,200	3.62½		
2,300	4.19	4,800	3.88	7,300	3.62		
2,400	4.18½	4,900	3.88	7,400	3.61½		

$10^5 dv$, which will hereafter be referred to as M is obtained from the tables in a somewhat similar manner, that is "base M" (for 0° C. and $35^0/_{00}$) is taken out and then corrections are applied, but fortunately only two corrections are necessary. The base M is approximately the difference between successive values of the base specific volumes in Table 8H. As these simple differences yield only one significant figure, and M is required to three significant figures, a logical method of computation which would yield the desired accuracy was necessary. That suggested by D. L. Hazard, assistant chief, division of terrestrial magnetism, Coast and Geodetic Survey, was adopted. The process was as follows: A preliminary table (Table 2) of values of base M was first computed for every 100 decibars from 0 to 8,900 decibars. The method used in computing this table may be illustrated by computing one of the values, say base M for 8,300 decibars pressure.

TABLE 8H

[Specific volumes in $\frac{cm^3}{gm} \times 10^6$]

Decibars	0	10	20	30	40	50	60	70	80	90
8,200	93,989	93,986	93,982	93,979	93,975	93,971	93,968	93,964	93,961	93,957
8,300										
8,400	93,919	93,915	93,912	93,908	93,905	93,901	93,897	93,894	93,890	93,887
	70	71	70	71	70	70	71	70	71	70

Mean difference, 70.4.

The change per 10 decibars, which is the base M for 8,300 decibars, equals 70.4 divided by 20, or 3.52. The final table of M (Table 5) is then blocked out in the same manner as Table 1 and base values of M are inserted in their proper places.

Corrections computed from Tables 12H and 13H were necessary in order to obtain the values of M for other temperatures and salinities. The corrections in Tables 9H, 10H, and 11H do not change with pressure and therefore do not affect M, and the correction from Table 14H is negligible in so far as M is concerned. The salinity and temperature corrections used in the computations of Table 5 are tabulated in Tables 3 and 4, respectively.

TABLE 3.—*Salinity corrections to M*

Salinity (0/00)	Depths where applicable (fathoms)	Correction	Salinity (0/00)	Depths where applicable (fathoms)	Correction
31	0–1,300 1,500–2,300 2,500–2,700	+0.06 +.05½ +.05	34	0–2,500 2,700–3,500 3,700–4,700	+0.01½ +.01 +.00½
32	0–1,100 1,300–2,700	+.04½ +.04	36	0–2,500 2,700–3,500 3,700–4,700	−.01½ −.01 −.00½
33	0–1,300 1,500–3,900 4,100–4,700	+.03 +.02½ +.02	37	0–1,300 1,500–3,900 4,100–4,700	−.03 −.02½ −.02

TABLE 4.—*Temperature corrections to M*

Temperature (degrees centigrade)	Depths where applicable (fathoms)	Correction	Temperature (degrees centigrade)	Depths where applicable (fathoms)	Correction
1	1, 900–3, 300	−0. 02	12	0–300	−0. 25
	3, 500–4, 700	−. 01½		500	−. 24
2	0–1, 100	−. 05		700	−. 24
	1, 300–2, 300	−. 04½		900	−. 23
	2, 500–3, 300	−. 04	14	0–300	−. 27
	3, 500–4, 100	−. 03½		500	−. 26
	4, 300–4, 700	−. 03		700	−. 25
3	1, 900–2, 700	−. 05½	16	0–500	−. 30
				700	−. 29
4	0–1, 100	−. 09½	18	0–300	−. 33
	1, 300–1, 700	−. 09		500	−. 32
6	0–1, 300	−. 13		700	−. 31
8	0–500	−. 17½	20	0–100	−. 35
	700–1, 100	−. 17		300	−. 34
				500	−. 34
10	0–300	−. 22		700	−. 33
	500	−. 21½			
	700	−. 21	22	0	−. 38
	900	−. 20		100	−. 37
				300	−. 36
				500	−. 36
				700	−. 35

TABLE 5

[*M* (*M*=$10^5 dv$, where *dv*=decrease in specific volume in cm³/gm corresponding to 1 bar increase in pressure]

Depth (fathoms)	Salinity (0/00)	Temperature (degrees centigrade) 0	2	4	6	8	10	12	14	16	18	20	22
Surface and 100	31	4. 56	4. 51	4. 47	4. 43	4. 39	4. 34	4. 31	4. 29	4. 26	4. 23	4. 21	4. 18
	32	4. 55	4. 50	4. 46	4. 42	4. 38	4. 33	4. 30	4. 28	4. 25	4. 22	4. 20	4. 17
	33	4. 53	4. 48	4. 44	4. 40	4. 36	4. 31	4. 28	4. 26	4. 23	4. 20	4. 18	4. 15
	34	4. 52	4. 47	4. 43	4. 39	4. 35	4. 30	4. 27	4. 25	4. 22	4. 19	4. 17	4. 14
	35	4. 50	4. 45	4. 41	4. 37	4. 33	4. 28	4. 25	4. 23	4. 20	4. 17	4. 15	4. 12
	36	4. 49	4. 44	4. 40	4. 36	4. 32	4. 27	4. 24	4. 22	4. 19	4. 16	4. 14	4. 11
	37	4. 47	4. 42	4. 38	4. 34	4. 30	4. 25	4. 22	4. 20	4. 17	4. 14	4. 12	4. 09
300	31	4. 50	4. 45	4. 41	4. 37	4. 33	4. 28	4. 25	4. 23	4. 20	4. 17	4. 16	4. 14
	32	4. 48	4. 43	4. 39	4. 35	4. 31	4. 26	4. 23	4. 21	4. 18	4. 15	4. 14	4. 12
	33	4. 47	4. 42	4. 38	4. 34	4. 30	4. 25	4. 22	4. 20	4. 17	4. 14	4. 13	4. 11
	34	4. 45	4. 40	4. 36	4. 32	4. 28	4. 23	4. 20	4. 18	4. 15	4. 12	4. 11	4. 09
	35	4. 44	4. 39	4. 35	4. 31	4. 27	4. 22	4. 19	4. 17	4. 14	4. 11	4. 10	4. 08
	36	4. 42	4. 37	4. 33	4. 29	4. 25	4. 20	4. 17	4. 15	4. 12	4. 09	4. 08	4. 06
	37	4. 41	4. 36	4. 32	4. 28	4. 24	4. 19	4. 16	4. 14	4. 11	4. 08	4. 07	4. 05
500	31	4. 45	4. 40	4. 36	4. 32	4. 28	4. 24	4. 21	4. 19	4. 15	4. 13	4. 11	4. 09
	32	4. 44	4. 39	4. 35	4. 31	4. 27	4. 23	4. 20	4. 18	4. 14	4. 12	4. 10	4. 08
	33	4. 42	4. 37	4. 33	4. 29	4. 25	4. 21	4. 18	4. 16	4. 12	4. 10	4. 08	4. 06
	34	4. 41	4. 36	4. 32	4. 28	4. 24	4. 20	4. 17	4. 15	4. 11	4. 09	4. 07	4. 05
	35	4. 39	4. 34	4. 30	4. 26	4. 22	4. 18	4. 15	4. 13	4. 09	4. 07	4. 05	4. 03
	36	4. 38	4. 33	4. 29	4. 25	4. 21	4. 17	4. 14	4. 12	4. 08	4. 06	4. 04	4. 02
	37	4. 36	4. 31	4. 27	4. 23	4. 19	4. 15	4. 12	4. 10	4. 06	4. 04	4. 02	4. 00
700	31	4. 39	4. 34	4. 30	4. 26	4. 22	4. 18	4. 15	4. 14	4. 10	4. 08	4. 06	------
	32	4. 38	4. 33	4. 29	4. 25	4. 21	4. 17	4. 14	4. 13	4. 09	4. 07	4. 05	------
	33	4. 36	4. 31	4. 27	4. 23	4. 19	4. 15	4. 12	4. 11	4. 07	4. 05	4. 03	------
	34	4. 35	4. 30	4. 26	4. 22	4. 18	4. 14	4. 11	4. 10	4. 06	4. 04	4. 02	------
	35	4. 33	4. 28	4. 24	4. 20	4. 16	4. 12	4. 09	4. 08	4. 04	4. 02	4. 00	------
	36	4. 32	4. 27	4. 23	4. 19	4. 15	4. 11	4. 08	4. 07	4. 03	4. 01	3. 99	------
	37	4. 30	4. 25	4. 21	4. 17	4. 13	4. 09	4. 06	4. 05	4. 01	3. 99	3. 97	------
900	31	4. 34	4. 29	4. 25	4. 21	4. 17	4. 14	4. 11	------	------	------	------	------
	32	4. 33	4. 28	4. 24	4. 20	4. 16	4. 13	4. 10	------	------	------	------	------
	33	4. 31	4. 26	4. 22	4. 18	4. 14	4. 11	4. 08	------	------	------	------	------
	34	4. 30	4. 25	4. 21	4. 17	4. 13	4. 10	4. 07	------	------	------	------	------
	35	4. 28	4. 23	4. 19	4. 15	4. 11	4. 08	4. 05	------	------	------	------	------
	36	4. 27	4. 22	4. 18	4. 14	4. 10	4. 07	4. 04	------	------	------	------	------
	37	4. 25	4. 20	4. 16	4. 12	4. 08	4. 05	4. 02	------	------	------	------	------

TABLE 5—Continued

Depth (fathoms)	Temperature 0	2	4	6	8	Salinity 0/00	Temperature 0	1	2	3	Depth (fathoms)
1,100	4.30	4.25	4.21	4.17	4.13	31	4.09	4.07	4.05	4.04	1,900
	4.28	4.23	4.19	4.15	4.11	32	4.07	4.05	4.03	4.02	
	4.27	4.22	4.18	4.14	4.10	33	4.06	4.04	4.02	4.01	
	4.25	4.20	4.16	4.12	4.08	34	4.05	4.03	4.01	4.00	
	4.24	4.19	4.15	4.11	4.07	35	4.03	4.01	3.99	3.98	
	4.22	4.17	4.13	4.09	4.05	36	4.02	4.00	3.98	3.97	
	4.21	4.16	4.12	4.08	4.04	37	4.01	3.99	3.97	3.96	
1,300	4.25	4.21	4.16	4.12	--------	31	4.06	4.04	4.02	4.01	2,100
	4.23	4.19	4.14	4.10	--------	32	4.04	4.02	4.00	3.99	
	4.22	4.18	4.13	4.09	--------	33	4.03	4.01	3.99	3.98	
	4.20	4.16	4.11	4.07	--------	34	4.02	4.00	3.98	3.97	
	4.19	4.15	4.10	4.06	--------	35	4.00	3.98	3.96	3.95	
	4.17	4.13	4.08	4.04	--------	36	3.98	3.97	3.95	3.94	
	4.16	4.12	4.07	4.03	--------	37	3.98	3.96	3.94	3.93	
1,500	4.20	4.16	4.11	--------	--------	31	4.00	3.98	3.96	3.95	2,300
	4.18	4.14	4.09	--------	--------	32	3.98	3.96	3.94	3.93	
	4.17	4.13	4.08	--------	--------	33	3.97	3.95	3.93	3.92	
	4.15	4.11	4.06	--------	--------	34	3.96	3.94	3.92	3.91	
	4.14	4.10	4.05	--------	--------	35	3.94	3.92	3.90	3.89	
	4.12	4.08	4.03	--------	--------	36	3.93	3.91	3.89	3.88	
	4.11	4.07	4.02	--------	--------	37	3.92	3.90	3.88	3.87	
1,700	4.15	4.11	4.06	--------	--------	31	3.96	3.94	3.92	3.91	2,500
	4.14	4.10	4.05	--------	--------	32	3.95	3.93	3.91	3.90	
	4.12	4.08	4.03	--------	--------	33	3.94	3.92	3.90	3.89	
	4.11	4.07	4.02	--------	--------	34	3.93	3.91	3.89	3.88	
	4.09	4.05	4.00	--------	--------	35	3.91	3.89	3.87	3.86	
	4.08	4.04	3.99	--------	--------	36	3.90	3.88	3.86	3.85	
	4.06	4.02	3.97	--------	--------	37	3.89	3.87	3.85	3.84	

Depth (fathoms)	Temperature (degrees centigrade) 0	1	2	3	Salinity 0/00	Temperature (degrees centigrade) 0	1	2	Depth (fathoms)
,700	3.92	3.90	3.88	3.87	31	--------	--------	--------	
	3.91	3.89	3.87	3.86	32	--------	--------	--------	
	3.89	3.87	3.85	3.84	33	3.65	3.64	3.62	3,900
	3.88	3.86	3.84	3.83	34	3.63	3.62	3.60	
	3.87	3.85	3.83	3.82	35	3.62	3.61	3.59	
	3.86	3.84	3.82	3.81	36	3.61	3.60	3.58	
	3.84	3.82	3.80	3.79	37	3.60	3.59	3.57	
2,900	3.85	3.83	3.81	--------	33	3.60	3.59	3.57	4,100
	3.84	3.82	3.80	--------	34	3.59	3.58	3.56	
	3.83	3.81	3.79	--------	35	3.58	3.57	3.55	
	3.82	3.80	3.78	--------	36	3.57	3.56	3.54	
	3.80	3.78	3.76	--------	37	3.56	3.55	3.53	
3,100	3.80	3.78	3.76	--------	33	3.57	3.56	3.54	4,300
	3.79	3.77	3.75	--------	34	3.56	3.55	3.53	
	3.78	3.76	3.74	--------	35	3.55	3.54	3.52	
	3.77	3.75	3.73	--------	36	3.54	3.53	3.51	
	3.75	3.73	3.71	--------	37	3.53	3.52	3.50	
3,300	3.76	3.74	3.72	--------	33	3.54	3.53	3.51	4,500
	3.74	3.72	3.70	--------	34	3.53	3.52	3.50	
	3.73	3.71	3.69	--------	35	3.52	3.51	3.49	
	3.72	3.70	3.68	--------	36	3.51	3.50	3.48	
	3.71	3.69	3.67	--------	37	3.50	3.49	3.47	
3,500	3.72	3.71	3.69	--------	33	3.49	3.48	3.46	4,700
	3.71	3.70	3.68	--------	34	3.48	3.47	3.45	
	3.70	3.69	3.67	--------	35	3.47	3.46	3.44	
	3.69	3.68	3.66	--------	36	3.46	3.45	3.43	
	3.67	3.66	3.64	--------	37	3.45	3.44	3.42	
3,700	3.68	3.67	3.65	--------	33	--------	--------	--------	
	3.66	3.65	3.63	--------	34	--------	--------	--------	
	3.65	3.64	3.62	--------	35	--------	--------	--------	
	3.44	3.63	3.61	--------	36	--------	--------	--------	
	3.63	3.62	3.60	--------	37	--------	--------	--------	

The salinity corrections tabulated in Table 3 were computed from Table 12H as the rate of change per bar of the values in that table at the salinity and pressure in question. The method used in computing these values may be illustrated by computing one of the values, say the correction for $31\frac{0}{00}$ salinity applicable between depths 0 and 1,300 fathoms (pressures 0 and 2,400 decibars).

TABLE 12H

[10^6 × salinity-pressure correction in $\frac{cm^3}{gm}$ to specific volume]

Salinity 31	Decibars
0	0
−13	2,300
−14	2,400
−14	2,500
−15	2,600

From an inspection of the table one may fairly assume that the exact pressure to which −14 belongs is very approximately 2,450 decibars. Since the correction for 0 decibars is 0, the mean change per bar in the salinity-pressure correction is −14 divided by 245, or −0.06. Since this tends to make the specific volume less at the higher pressure, it is additive to *M*. Hence the salinity correction, as tabulated in Table 3, to the base *M* at $31\frac{0}{00}$ salinity between 0 and 1,300 fathoms is +0.06.

The temperature corrections tabulated in Table 4 were computed from Table 13H, in an exactly similar manner, as the rate of change per bar of the values in that table at the temperature and pressure in question.

of velocity in Table 13 were then computed directly by means of equations (1) and (2) from the values of v and M in Tables 1 and 5, respectively.

The curves on Plate 2 show how the two quantities from which V is computed, v and M, respectively, vary with the depth. The curves on Plate 3 show how velocity varies with depth, temperature, and salinity, respectively.

ADIABATIC CORRECTIONS TO VELOCITY

The values of velocity in Table 13 were computed using values of M experimentally determined under isothermal conditions. Granting that the condensations and rarefactions of the sea water during the transmission of sound take place under adiabatic conditions, then the velocities in Table 13 theoretically need to be increased by small corrections, which were neglected in computing that table. It was suggested by Dr. L. H. Adams, of the geophysical laboratory of the Carnegie Institution, that these corrections might increase the velocity by as much as 0.5 of 1 per cent, so it was decided to investigate the effect. The theory underlying the computation will now be given. The symbols used were as follows:

β_a is the adiabatic compressibility (pressure rate of change of specific volume) of the sea water. In cm^3/gm per $dyne/cm^2$;

β is the isothermal compressibility;

C_v is the specific heat at constant volume, in ergs per gram per degree centigrade;

C_p is the specific heat at constant pressure;

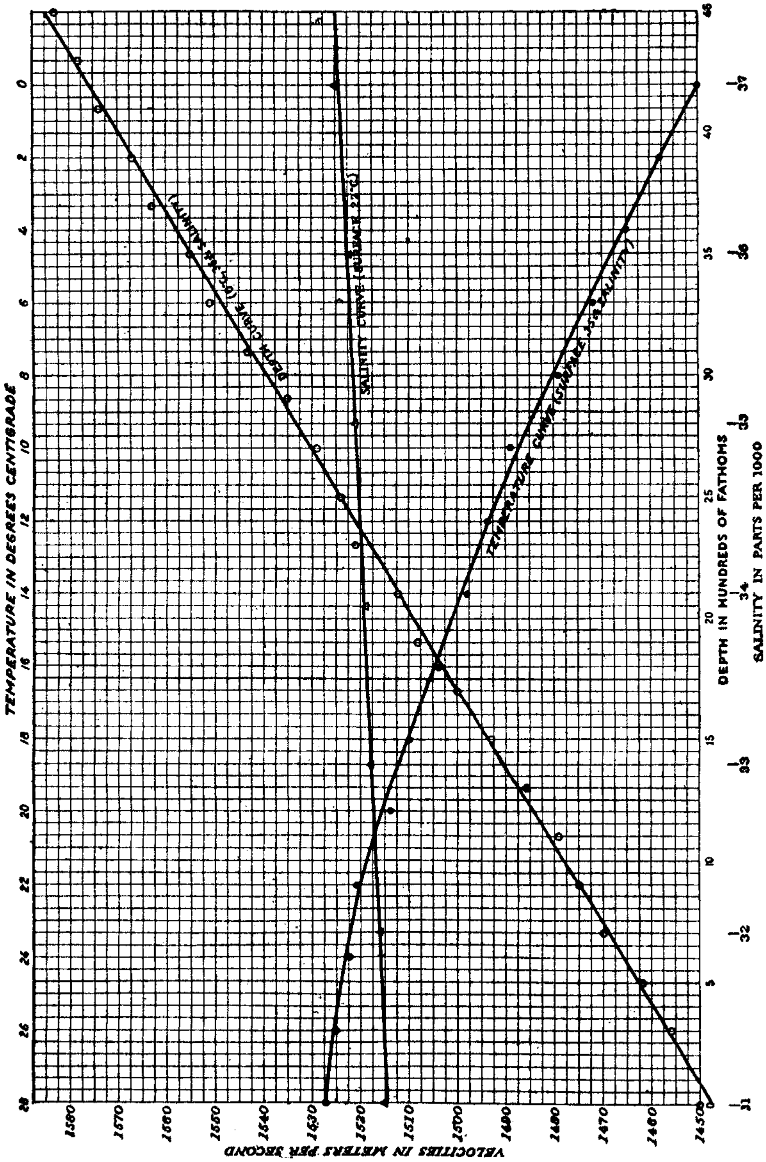

FIG. 3.

a is the thermal coefficient of expansion (temperature rate of change of specific volume), in cm³/gm per degree centigrade;

T is the absolute temperature on the centigrade scale.

It is shown in works on thermodynamics that

$$\frac{\beta_a}{\beta}=\frac{C_v}{C_p}$$

$$\text{and that } C_p - C_v = \frac{\alpha^2 T}{\beta}$$

$$\text{whence } C_v = C_p - \frac{\alpha^2 T}{\beta}$$

$$\text{and } \frac{\beta_a}{\beta} = \frac{C_v}{C_p} = 1 - \frac{\alpha^2 T}{\beta C_p}$$

In other words, the adiabatic compressibility, which probably obtains during sound transmission, is less than the isothermal compressibility, which is what Ekman measured and Bjerknes used in his tables, by a fraction equal to $\frac{\alpha^2 T}{\beta C_p}$. Now the elasticity is very nearly equal to the reciprocal of the compressibility, so that the adiabatic elasticity will be greater than the isothermal elasticity by the same fraction. And since the velocity is proportional to the square root of the elasticity, the velocity computed from adiabatic compressibility will be greater than the velocity computed from isothermal compressibility by approximately one-half this fraction. Tables are given herewith of $10^5\,\alpha$, $10^5\,\beta$, $\frac{C_p}{4.18}$, $\frac{\alpha^2 T}{\beta C_p}$, and of the adiabatic corrections to the velocities under various conditions. Since the unit of pressure used for β was the bar, or 10^6 dynes/cm², a unit of energy in C_p equal to 10^6 ergs, or 1 decijoule, was necessary.

TABLE 6

[$10^5\alpha$ $\alpha = \left(\frac{\delta v}{\delta t}\right)_p$ = temperature rate of change of specific volume, in $\frac{cm^3}{gm}$ per degree centigrade, of sea water of salinity 35 $\frac{0}{00}$]

Depth (fathoms)	Temperature in degrees centigrade				
	0	5	10	15	20
Surface	5	11	16	21	25
1,100	10	15	19		
2,100	15	19			
3,300	19				
4,300	22				

The above values were computed by means of Tables 10H and 13H.

TABLE 7

[$10^5\beta$ $\beta = \left(\frac{\delta v}{\delta p}\right)_T$ = isothermal pressure rate of change of specific volume, in $\frac{cm^3}{gm}$ per bar, of sea water of salinity 35 $\frac{0}{00}$]

Depth (fathoms)	Temperature in degrees centigrade				
	0	5	10	15	20
Surface	4.5	4.4	4.3	4.2	4.2
1,100	4.2	4.1	4.0		
2,100	4.0	3.9			
3,300	3.7				
4,300	3.6				

The above values were taken directly from Table 5 to two significant figures.

TABLE 8

[$\frac{C_p}{4.18}$ C_p=specific heat at constant pressure, in decijoules per gram per degree centigrade, of sea water of salinity 35 $\frac{0}{00}$]

Depth (fathoms)	Temperature in degrees centigrade				
	0	5	10	15	20
Surface	9.3	9.3	9.3	9.3	9.3
1,100	9.1	9.1	9.1		
2,100	9.0	9.0			
3,300	8.9				
4,300	8.8				

TABLE 9

[$\frac{\alpha^2 T}{\beta C_p}$]

Depth (fathoms)	Temperature in degrees centigrade				
	0	5	10	15	20
Surface	0.0004	0.002	0.004	0.008	0.111
1,100	.0017	.004	.0067		
2,100	.004	.007			
3,300	.007				
4,300	.010				

TABLE 10

[Adiabatic corrections to velocity, in fathoms per second. For the surface, corrections are also given in meters per second (lower line)]

Depth (fathoms)	Temperature in degrees centigrade				
	0	5	10	15	20
Surface	0.2 .3	0.8 1.5	1.6 3.0	3.2 6.0	4.4 8.3
1,100	.7	1.6	2.7		
2,100	1.6	2.8			
3,300	2.8				
4,300	4.0				

The authors are somewhat in doubt as to the advisability of applying this correction. The maximum effect is about 0.5 of 1 per cent and the average effect all through the tables is only about 0.2 or 0.3 of 1 per cent. Furthermore, in practice the depth obtained by wire under good conditions is accepted as the standard. It will be shown that the depth computed from the time interval measured with the sonic depth finder and the mean velocity obtained from Table 13 and known physical conditions agrees as closely as can be expected with the corresponding wire depth.

ACCURACY OF VELOCITY TABLE NO. 13

The accuracy of the velocities tabulated in Table 13 is controlled by the accuracy of the values of *M*. Judging from the records of the experimental work of Ekman, which is the ultimate source of the values of this quantity, no value of *M* will be in error by more than 1 per cent. Since *M* appears under the radical in the velocity equation this would indicate that no value of velocity will be in error by more than 0.5 of 1 per cent, which would amount to about 7 m./sec., or 4 fathoms/sec.

It is believed that the velocities of Table 13 are of the highest degree of accuracy possible with compressibility data available at the present time and that they are adequate for acoustic-sounding purposes. It is realized, however, that the accuracy depends upon whether the values of *M* used in the table are the true values. Further study is being given by one of the authors to the possibility of obtaining directly from the results of Ekman's compressibility experiments more precise values of *M*.

COMPARISON OF COMPUTED VELOCITIES WITH DIRECTLY MEASURED VELOCITIES

COMPARISON OF SURFACE VELOCITIES

At the surface, at $-0.3°$ C. and at salinity $33.5\frac{0}{00}$, E. B. Stephenson, working over a distance of about 15, 000 meters, and using very precise methods of measuring distance and time, found the velocity of sound to be 1,453 ± 1.5 m./sec. Table 13 gives 1,448 m./sec. for these conditions.

In connection with the tests of the radio-acoustic apparatus devised by Dr. E. A. Eckhardt, the subaqeous sound-ranging section of the Army and the steamer *Guide* cooperated in a long-distance test of the velocity of sound during November, 1923. Both time and distance were determined with great precision. The distance was about 100,000 meters. At the surface, at 13° C. and salinity $33.5\frac{0}{00}$ the velocity was found to be 1,492 m./sec. Table 13 gives for these conditions 1,494 m./sec.

On April 8, 1924, off Encinitas, Calif., the steamer *Guide*, as a test of the radio-acoustic apparatus aboard ship and at two hydrophone stations fired six detonators in the water near the ship. The position of the ship at each explosion was determined by sextant angles. The time required for the sound wave to travel from the ship to each hydrophone was measured by the radio-acoustic apparatus. The accompanying table gives the results of the test, and shows a mean measured velocity of 1,495 m./sec. at temperature 14°C. and salinity $33.5\frac{0}{00}$, the sound wave being assumed to pass close to the surface. Table 13 gives 1,496 m./sec. for these conditions.

Detonator No.	Time from hydrophone 1	Distance from hydrophone 1	Velocity	Time from hydrophone 2	Distance from hydrophone 2	Velocity
	Seconds	*Meters*	*M. seconds*	*Seconds*	*Meters*	*M. seconds*
1	12.38	18,439	1,489	12.69	19,018	1,499
2	12.40	18,456	1,488	12.72	19,001	1,494
3	12.38	18,451	1,490	12.71	19,010	1,496
4	12.34	18,455	1,496	12.66	19,000	1,501
5	12.33	18,450	1,496	12.66	19,006	1,501
6	12.35	18,439	1,493	12.73	19,013	1,494
Mean			1,492			1,497

COMPARISON OF VERTICAL VELOCITIES TO GREAT DEPTHS

For convenience in making comparisons, computed velocity derived from Table 13 will be designated by V_c and measured velocity by V_m. In every case that will be discussed V_m is determined by dividing a distance measured as accurately as conditions permit by a time interval determined with equal care. The percentage difference $\frac{V_c - V_m}{V_m} \times 100$ per cent can properly be regarded as the ultimate test of the reliablity of the method.

The results of the observations made during the oceanographic cruise of the *Guide* are given in detail in Tables 11 and 12, which follow. Table 11 also shows how V_c was computed for each sounding.

TABLE 11

Number of sounding	Observed temperatures centigrade and salinities			Adopted temperatures, salinities, and tabular velocities for depths expressed in hundreds of fathoms																							V_0
	Surface	200 fathoms	Bottom	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	
1	13.9 34.8	6.2 35.0	3.3 34.9	9 35 812	5 35 807	5 35 810	4 35 812	4 35 815	3 35 816																		812
2	15.0 34.9	6.4 34.9	2.4	10 35 814	5 35 807	5 35 810	4 35 812	4 35 815	4 35 816	3 35 819	3 35 823	3 35 826	3 35 830														817
3	22.6 36.3	17.6 36.4	2.6 35.1	20 36 829	15 36 825	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	4 35 829	4 35 831	3 35 832	3 35 837	3 35 839											828
4	21.1 36.4	17.7 36.4	35.1	19 36 828	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	4 35 829	4 35 831	3 35 832	3 35 837	3 35 839											828
5	21.3 36.4	17.7 36.4	3.0 35.2	19 36 828	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846									831
6	21.7 35.9	17.5 36.4	2.3 35.0	19 36 828	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846									831
7	23.5 36.4	17.9 36.5	1.5 35.1	21 36 831	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	4 35 829	4 35 831	5 35 836	5 35 841	4 35 841	3 35 843	2 35 844									831
8	25.1 36.3	17.4 36.5	2.5 35.2	21 36 831	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846									831
9	25.1 36.0	17.6 36.4	35.1	21 36 831	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846									831
10	26.3 35.8	17.2 36.3	35.1	22 36 833	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846									
11	26.8 36.0	18.2 36.3	−0.3 35.2	22 36 833	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	2 35 844	2 35 849	2 35 853	2 35 854	1 35 856	1 35 858	1 35 862	0 35 863	0 35 866	841
12	25.6 35.8	16.6 36.1	2.5 35.1	21 36 831	14 36 824	10 36 821	8 35 820	7 35 822	6 35 823	5 35 824	4 35 826	3 35 826	3 35 830														825
13	26.3 36.1	17.1 36.3	2.3 35.1	22 36 833	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	4 35 831	4 35 834	4 35 839	3 35 839	3 35 843	3 35 846	3 35 851	2 35 853	2 35 854	2 35 858	2 35 861				837
14	25.2 36.1	19.2 36.4	2.0 36.1	22 36 833	15 36 825	11 36 822	10 35 825	9 35 825	8 35 827	7 35 829	6 35 831	5 35 831	5 35 833	4 35 834	4 35 839	4 35 841	3 35 843	3 35 846	3 35 851								833
15	23.9 36.0	17.7 36.4	2.2 35.2	21 36 831	14 36 824	10 36 821	9 35 822	8 35 824	7 35 825	6 35 827	5 35 828	5 35 831	5 35 833	4 35 834	4 35 839	4 35 841	3 35 843	3 35 846	3 35 851	3 35 855	2 35 854	2 35 858	2 35 861	2 35 865	2 35 867	2 35 870	841
16	27.2 35.4	16.7 36.3	2.5	22 36 833	13 36 823	9 36 819	7 35 818	6 35 820	5 35 821	4 35 822	3 35 824	3 35 826															823
17	26.2 35.2	17.6 36.3	4.0 35.1	22 35 832	11 35 819	7 35 814	4 35 812																				819
18	25.7 34.9	17.6 35.5	4.5 35.3	22 35 832	13 35 821	10 35 820	9 35 822	8 35 824	7 35 825	6 35 827	6 35 831	6 35 834	6 35 835	5 35 836	5 35 840	5 35 843	5 35 847										831
19	25.7 35.0	17.0 36.2	4.1 35.1	21 35 830	13 35 821	10 35 820	8 35 820	7 35 822	6 35 823	6 35 827	6 35 831	5 35 831	5 35 833	5 35 836	4 35 838												828
20	26.8 35.4	13.4 36.0	4.0 35.2	20 36 829	9 36 816	7 35 815	6 35 816	6 35 820	5 35 821	5 35 824	5 35 828	4 35 829	4 35 832	4 35 834													824
21	26.8 36.0	12.5 36.3	4.0 35.3	19 36 827	9 36 816	7 35 815	6 35 816	6 35 820	5 35 821	5 35 824	5 35 828	4 35 829	4 35 832														823
22	27.3 36.0	13.5 36.3	3.9 35.2	20 36 829	9 36 816	7 35 815	5 35 816	4 35 815																			818
23	26.8 34.9	13.8 35.6		20 35 828																							828

TABLE 11—Continued

Number of sounding	Observed temperatures centigrade and salinities			Adopted temperatures, salinities, and tabular velocities for depths expressed in hundreds of fathoms																							V_o
	Surface	200 fathoms	Bottom	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	
24				20 32 825																							825
25	26.8 31.4	13.6 33.6	2.3 34.3	20 32 825	9 33 814	7 34 812	6 34 815	5 34 817	4 34 818	4 34 821	3 34 822	3 34 825															819
26	26.4 31.1	13.5 32.5	1.9 34.5	20 32 825	8 33 811	6 33 810	5 34 813	4 34 815	3 34 816	2 34 816																	815
27	26.6 32.7	9.9 33.1	2.6 34.5	18 33 824	7 33 809	5 34 808	4 34 811	3 34 812																			813
28	27.8 32.1	12.0 34.3	------ 34.6	20 33 826	8 34 812	6 34 810	4 34 811																				815
29	27.3 33.0	13.4 34.6	2.1 34.5	20 34 827	9 34 814	7 34 812	6 34 815	5 34 817	4 34 818	3 34 818	3 34 822	2 34 823															818
30	27.1 33.3	11.3 34.4	2.2 34.6	19 34 826	8 34 812	7 34 812	6 34 815	5 34 817	4 34 818	3 34 818	3 34 822	2 34 823															818
31	27.7 34.0	11.1 34.7	2.3 34.7	19 34 826	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828														818
32	25.7 33.5	12.7 34.6	2.4 34.6	19 34 826	9 34 814	7 34 812	6 34 815	5 34 817	4 34 818	3 34 818	3 34 822	2 34 823	2 34 828	2 34 830	2 34 835	2 34 837	2 34 840	2 34 844	2 34 848								827
33	26.8 33.5	11.6 34.4	2.1 34.6	19 34 826	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828														818
34	27.3 33.8	1.8 34.2	2.5 34.6	20 34 827	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828	2 34 830	2 34 835	2 34 837	2 34 840	2 34 844	2 34 848	2 34 852							828
35	27.3 33.8	12.0 34.7	2.4 34.6	20 34 827	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828	2 34 830	2 34 835	2 34 837	2 34 840	2 34 844	2 34 848	2 34 852							828
36	23.5 34.1	10.9 34.7	2.3 34.5	18 34 825	7 34 810	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828														818
37	27.2 34.0	10.6 34.7	1.8 34.6	19 34 826	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828	2 34 830													819
38	27.8 33.8	12.0 34.7	2.8 34.6	20 34 827	8 34 812	6 34 810	5 34 813	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828	2 34 830	2 34 835	2 34 837											822
39	28.0 33.8	13.7 34.7	2.9 ------	21 34 829	10 34 817	7 34 812	6 34 815	5 34 817	4 34 818	4 34 821	3 34 822	3 34 825	3 34 829	3 34 831	2 34 835	2 34 837											824
40	28.2 34.1	10.4 34.5	2.8 34.6	19 34 826	8 34 812	6 34 810	5 34 813	5 34 817	4 34 818	4 34 821	3 34 822	3 34 825	2 34 828	2 34 830													820
41	23.5 34.7	11.1 34.6	------ ------	17 34 823	8 34 812	6 34 810	5 34 813	5 34 817	4 34 818	4 34 821	3 34 822	3 34 825															818
42	19.1 34.2	9.6 34.0	2.4 34.5	14 34 818	7 34 810	6 34 810	5 34 813	4 34 815	4 34 818	3 34 818	2 34 820	2 34 823	2 34 828														817
43	18.5 34.3	6.1 34.4	2.4 34.5	12 34 816	6 34 808	5 34 808	4 34 811	4 34 815	3 34 816	3 34 818	2 34 820	2 34 823	2 34 828														816
44	16.8 33.9	7.9 34.0	2.4 34.6	12 34 816	6 34 808	5 34 808	5 34 813	4 34 815	4 34 818	4 34 821	3 34 822	3 34 825	2 34 828	2 34 830	2 34 835	2 34 837											821
45	16.8 33.9	7.4 34.0	1.5 34.5	12 34 816	6 34 808	5 34 808	4 34 811	3 34 812	3 34 816	2 34 816	2 34 820																813
46	15.7 33.9	8.0 34.1	2.4 34.3	12 34 816	6 34 808	4 34 807	3 34 809																				810

TABLE No. 12

NORTH ATLANTIC OCEAN

No.	Date 1923	Lat. N. °	Lat. N. ′	Long. W. °	Long. W. ′	Depth by wire	Time interval	V_m	V_c	$\frac{V_c - V_m}{V_m} \times 100\%$ Plus	Minus
						Fathoms	*Seconds*	*Fathoms/sec.*	*Fathoms/sec.*	*Plus*	*Minus*
1	11-21	39	33	71	30	1,167	1.464	797	812	1.9	
2	11-22	38	32	71	00	1,937	2.382	813	817	.5	
3	11-22	37	07	70	06			Strong surface current.			
4	11-23	35	03	70	29	2,625		Trouble with sonic.			
5	11-23	32	29	70	36	2,984	3.522	847	831		1.9
6	11-24	29	44	70	27	2,988	3.549	842	831		1.3
7	11-25	27	06	70	31	3,030	3.636	833	831		.2
8	11-26	24	46	70	18	3,060	3.704	826	831	.6	
9	11-26	23	09	69	12	3,027	3.639	832	831		.1
10	11-27	21	23	68	01	2,965	3.625	818	831	1.6	
11	11-28	19	40	66	52	4,515	5.440	829	841	1.4	
12	11-30	19	03	65	05	1,974	2.325	849	825		1.9
13	12-01	19	44	65	24	4,075	4.804	848	837		1.3
14	12-02	20	11	66	44	3,234	3.931	823	833	1.2	
15	12-03	19	36	67	32	4,617	5.565	830	841	1.3	
16	12-03	19	07	67	51	1,817	2.154	843	823		2.4
CARIBBEAN SEA											
17	12-03	17	50	67	41	857	1.059	809	819	1.2	
18	12-04	16	57	68	15	2,831	3.384	837	831		0.7
19	12-05	15	15	71	16	2,339	2.729	857	828		3.4
20	12-05	13	23	74	02	2,206	2.716	812	824	1.5	
21	12-06	11	35	77	05	1,919	2.350	817	823	.7	
22	12-07	10	02	79	08	1,084	1.329	816	818	.2	
23	12-07	9	36	79	49	185	.229	808	828	2.5	
PACIFIC OCEAN											
24	12-13	7	34	78	56	248	.302	821	825	0.4	
25	12-13	6	19	79	02	1,830	2.297	797	819	2.8	
26	12-14	6	10	81	11	1,358	1.716	791	815	3.0	
27	12-15	6	14	84	26	907	1.104	822	813		1.1
28	12-15	7	48	84	06	736	.895	827	815		1.4
29	12-16	8	47	84	55	1,763	2.227	792	818	3.3	
30	12-16	9	45	86	39	1,894	2.274	833	818		1.8
31	12-17	11	10	89	02	1,928	2.345	822	818		.5
32	12-17	12	10	89	40	3,161	3.796	833	827		.7
33	12-18	12	02	90	41	2,020	2.456	822	818		.5
34	12-18	12	50	91	23	3,472	4.135	840	828		1.4
35	12-19	13	43	93	12	3,426	3.990	859	828		3.6
36	12-19	14	46	95	36	2,051	2.440	841	818		2.7
37	12-20	15	21	95	54	2,171	2.631	828	819		1.1
38	12-20	16	17	99	45	2,522	3.057	825	822		.4
39	12-21	16	52	101	45	2,660	3.174	838	824		1.8
40	12-22	18	02	104	02	2,212	2.686	823	820		.4
41	12-25	22	20	110	50	1,783	2.237	797	818	2.6	
42	12-26	24	59	113	34	2,030	2.489	816	817	.1	
43	12-27	26	44	114	36	1,939	2.359	822	816		.7
44	12-27	28	57	115	56	2,497	3.002	830	821		1.1
45	12-28	30	03	116	34	1,569	1.864	842	813		3.5
46	12-28	32	06	117	09	702	.855	821	810		1.3

$$\begin{array}{r} -37.2 \\ +26.8 \\ \hline 44)\,-10.4 \\ \hline -0.24 \end{array}$$

It is seen from Table 12 that the average percentage difference between V_m and V_c for the entire 44 determinations is 0.2 of 1 per cent. Further, it has been computed that the probable error of a single value is 1.2 per cent, which indicates that the average percentage difference may properly be taken as a guide in estimating the accuracy of the method.

In determining V_m the assumption is made that the echo returns from a point vertically below the ship. It is of course true that, if the bottom is sloping, the reflected sound wave which is received will follow the line passing through the vessel normal to the slope. If many of the soundings are taken at places of considerable slope, the time intervals measured will be too small and V_m will be too great, and accordingly the average percentage difference could not possibly be as near zero as shown by the analysis of Table 12. The inevitable conclusion is that if a 6,500-mile cruise with all kinds of bottom conditions, including several deeps, fails to show the effect of slope, areas of steep slope are of relatively insignificant extent.

To test this conclusion further, slopes for the positions where soundings 24 to 46, inclusive (Pacific Ocean), were taken, were deduced from the best available information, including the work of the *Guide* and wire soundings by other vessels of the Coast and Geodetic Survey. These are given in the following table:

No.	Slope	No.	Slope	No.	Slope	No.	Slope
	° ′		° ′		° ′		° ′
24	2 00	30	0 30	36	1 00	42	0 50
25	1 00	31	0 20	37	0 30	43	1 48
26	0 03	32	2 00	38	0 50	44	0 00
27	0 50	33	0 08	39	0 40	45	0 55
28	0 17	34	6 00	40	0 30	46	0 12
29	0 20	35	0 15	41	0 05		

The mean slope is about 1° with a maximum of 6°.

It must not be inferred that steep slopes do not exist. They are important geographical features of interest to the geologist and of concern to the hydrographer and oceanographer. The purpose of the discussion has been to bring out the fact that by proper procedure the velocity of sound can be determined without appreciable error due to slope.

The agreement between V_o and V_m has been tested by determining the average percentage difference and the probable error of a single value for all the soundings of Table 12. A better test is the examination by the same method of characteristic groups, each of approximately the same physical conditions and general depth.

Soundings 5 to 10, inclusive, of general depth of 3,000 fathoms, taken over a level portion of the sea floor of the North Atlantic, give an average percentage difference of 0.2 of 1 per cent, with a probable error of a single value of 0.8 of 1 per cent. Serial temperatures and salinities were taken at sounding No. 7.

A particularly rigid test is the application to three soundings taken in Nares Deep, north of Porto Rico, ranging from 4,075 to 4,617 fathoms. For these the average percentage difference was 0.5 of 1 per cent, with a probable error for a single value of 1.1 per cent.

For a group of four soundings, Nos. 18 to 21, inclusive, in the Caribbean Sea, with depths from 1,900 to 2,800 fathoms, the average percentage difference was 0.5 of 1 per cent, with a probable error of a single value of 1.6 per cent.

For a group of nine soundings, Nos. 36 to 44, inclusive, in the Pacific Ocean, with depths from 2,000 to 2,500 fathoms, the average percentage difference was 0.6 of 1 per cent, with a probable error of a single value of 1 per cent.

With differing conditions it should follow that in different regions velocities for the same depth should vary. This is found to be the case. Soundings Nos. 3 in the Atlantic and 44 in the Pacific form an example of this kind, with velocities 828 and 821 fathoms/sec., respectively, the depth at each being approximately 2,500 fathoms. Also in some cases the velocities are the same for widely differing depths. Sounding No. 23, depth 185 fathoms, and sounding No. 34, depth 3,472 fathoms, both have a computed velocity of 828 fathoms/sec.

SOURCES OF ERROR

The agreement between V_m and V_o which has been shown by the above study of Tables 11 and 12 is seen to be remarkably good when it is considered how many elements enter into a comparison of these two quantities and what sources of error there are in the determination of each of these elements. These sources of error will be discussed in some detail.

Errors in the determination of V_m include errors in the determination of depth by wire sounding, and errors in the measurement of of the time interval with the sonic depth finder.

The accuracy of the determination of depth by wire sounding depends upon the skill with which the sounding is taken. The commanding officer of the steamer *Guide*, Lieut. Commander R. F. Luce, Coast and Geodetic Survey, showed exceptional skill in handling the vessel, and the wire was kept as nearly vertical as possible during every sounding. In only a few cases were the currents strong enough seriously to affect the accuracy of the wire measurement. The accuracy of the registering sheave was tested by running over it a measured length of wire, and the error was found to be negligible. Change in length of the wire with temperature was also found negligible. One common source of error, unfavorable weather conditions, fortunately was absent during most of the cruise of the *Guide*.

The question of the accuracy of time-interval determination under service conditions is of special interest because previously to the cruise of the *Guide* the apparatus had not been submitted to the test of continuous check against wire soundings in depths such as to make the time intervals large. The essential precaution is the maintenance of period of disk rotation at exactly 10 seconds, which the tuning-fork governor usually accomplishes. The depth finder used was of the first type developed by Doctor Hayes and had some operating defects that have been remedied in later types. One of these was the difficulty of reducing the loudness, as heard in the phones, of the original oscillator sound so as to be comparable with that of the echo. When the oscillator is operated at full power it is often extremely difficult to hear the echo and synchronize it with the original sound. The strength of echo also varies with the character of the bottom, so that in some cases the echo was faint in moderate depths and strong in great depths. Precision of synchronism depends very largely on the distinctness and strength of the echo. The personal equation of the observer affects to a certain extent the determination of a time interval with the sonic depth finder. The indications are that this is small for a skilled observer, but by no means negligible, and that it may be slightly different for two equally skilled observers. It lies chiefly in the synchronizing of outgoing signals and returning echoes.

In the studies so far made it seems to be important chiefly in depths less than 500 fathoms. It is therefore advisable in a given region of moderate depths to take the personal equation into account.

The analysis of all the results indicates that a satisfactory degree of consistency is obtained. On one occasion a special effort was made to determine the ultimate possibilities under exceptionally favorable conditions. For five soundings in depths ranging from 535 to 702 fathoms the maximum difference of any value of V_m from the mean was 1.5 fathoms/sec.

The accuracy of the determination of V_0 depends not only upon the fundamental corrections of the method but also upon the reliability of the adopted values of temperature and salinity. The fundamental correctness of the method has been fully discussed, and it has been brought out that there is a possibility of small errors in the tables themselves and in the method of deriving M from the tables. The reliability of the adopted values of temperature and salinity depends on whether they have been actually measured or interpolated between such measurements as in the case of the *Guide*, or whether they have been derived from less reliable sources.

APPLICABILITY OF COMPUTED VELOCITIES TO ACOUSTIC SOUNDING

During the cruise of the *Guide* depths were determined at 150 positions by the sonic depth finder alone, using computed velocities. Some of these determinations were in the vicinity of previous wire soundings obtained by various Coast Survey vessels and the agreement was found to be very satisfactory. This brings up the important question as to whether satisfactory soundings can be made with the sonic depth finder alone, using computed velocities, but without the control afforded by wire measurements and determinations of temperature and salinity. In this case it would be necessary to obtain the adopted temperatures and salinities from the best available published values. These are found in various publications.[6]

Such a procedure will give results much nearer the truth than the adoption of a single value of the velocity of sound for all conditions.

It would obviously be of advantage to have tables expressing velocity as a function of depth alone. It has been clearly brought out that such tables can not be of universal application, but it is probable that they can be prepared for regions of considerable extent provided that the physical conditions of the sea water are approximately the same throughout the region.

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[6] "A study of the salinity of the surface water in the North Pacific Ocean and in the adjacent inclosed seas," by A. H. Clark, Smithsonian Miscellaneous Collections, vol. 60, No. 13, Dec. 4, 1912.

"Das spezifische Gewicht des Meerwassers im Nordöst Pacifischen Ozean in Zusammenhange mit Temperaturund Strömungszuständen," by Adolph Lindenkohl, Dr. A. Petermann's Geogr. Mitteilungen —1897, Heft XII.

"Exploration of the United States Coast and Geodetic Survey steamer *Bache* in the western Atlantic, January–March, 1914, under the direction of the United States Bureau of Fisheries," "Oceanography," by Henry B. Bigelow, App. V to the Report of the U. S. Commission of Fisheries for 1915, Bureau of Fisheries Document No. 833, 1917.

"The temperatures, specific gravities, and salinities of the Weddell Sea and of the North and South Atlantic Ocean," by W. S. Bruce, Andrew King, and D. W. Wilton, Transactions of the Royal Society of Edinburgh, Vol. LI, Part I, p. 71, 1914–15.

"Physiographische Probleme, Salzgehalt und Temperatur des Pacifischen Ozeans betreffend," by A. Lindenkohl, U. S. Coast and Geodetic Survey, Dr. A. Petermann's Mitteilungen aus Justus Perthes' Geographischer Anstalt, herausgegeben by Prof. Dr. A. Supan, 45 Band, 1899, Gotha, Justus Perthes.

"Die Warme Verteilung in der Tiefen des stillen Ozeans," by Gerhardt Schott and Fritz Schu, Berlin, 1910.

"Atlantischer Ozean. Ein Atlas von 39 Karten, die physikalischen verhältnisse und die vehrkehrs—strassen darstellend, mit einer erläuternden einleitung und als beilage zum segelhandbuch für den Atlantischen Ozean." 2 aufl. Hrsg. von der direktion Hamburg, L. Friederichshen & Co., 1902.

Inspection of Table 12 shows that even under the best conditions it is difficult to obtain consistently accurate results. Unless wire measurements are made and physical conditions determined there is no way of knowing how accurate the results are. If expeditions are to continue the practice of sounding by acoustic methods alone, it is important that there should be further oceanographic work similar to that of the *Guide*. This vessel during a cruise of 6,500 miles encountered a temperature range of 28 degrees (0° to 28° C.), a salinity range of 5.5‰ (31 to 36.5‰), a depth range from 185 to 4,617 fathoms, and used computed velocities ranging from 810 to 841 fathoms/sec. While future expeditions can scarcely expect to have a wider range they can do much to provide control for acoustic sounding by determining physical conditions and making velocity measurements on all the oceans, and especially by fixing more accurately the places where physical conditions change.

TABLE 13

[Velocity of sound in sea water in fathoms per second. For the surface, velocities are also given in meters per second]

Depth (fathoms)	Salinity 0/00	Temperature (degrees centigrade)											
		0	2	4	6	8	10	12	14	16	18	20	22
Surface and 100	31	790 1,445	795 1,453	798 1,459	802 1,466	806 1,474	811 1,482	814 1,488	816 1,492	819 1,498	822 1,504	825 1,508	828 1,515
	32	791 1,446	795 1,454	799 1,461	803 1,467	806 1,474	811 1,483	814 1,489	816 1,493	820 1,499	823 1,505	825 1,509	829 1,516
	33	792 1,448	796 1,455	800 1,462	804 1,469	807 1,476	812 1,485	815 1,491	817 1,495	821 1,501	824 1,507	826 1,511	830 1,518
	34	792 1,449	796 1,456	800 1,463	804 1,471	808 1,477	813 1,486	816 1,492	818 1,496	821 1,502	825 1,508	827 1,513	831 1,519
	35	793 1,450	797 1,458	801 1,465	805 1,472	809 1,479	814 1,489	817 1,494	819 1,498	822 1,504	826 1,510	828 1,514	832 1,521
	36	793 1,451	798 1,459	802 1,466	806 1,473	809 1,480	815 1,490	818 1,495	820 1,499	823 1,505	826 1,511	829 1,515	833 1,522
	37	794 1,452	799 1,461	803 1,468	807 1,475	810 1,482	815 1,491	819 1,497	821 1,501	825 1,507	827 1,513	830 1,518	834 1,525
300	31	793	798	802	805	809	814	817	820	823	827	828	830
	32	794	799	803	806	810	815	818	821	824	827	829	831
	33	795	799	803	807	811	816	819	822	825	828	829	832
	34	796	801	804	808	812	817	820	823	826	829	830	833
	35	796	801	805	809	813	818	820	823	826	830	831	834
	36	797	802	806	809	814	819	822	824	827	831	832	835
	37	798	802	806	810	814	820	823	825	828	832	833	836
500	31	796	801	804	809	813	817	820	822	827	829	832	834
	32	797	802	805	809	814	818	821	823	827	830	832	835
	33	798	803	806	810	814	819	822	824	828	831	833	836
	34	798	803	807	810	815	819	822	825	829	832	834	837
	35	799	804	808	812	816	820	823	826	830	832	835	838
	36	800	804	809	813	817	821	824	827	831	833	836	838
	37	801	805	810	813	818	822	825	828	832	834	837	839
700	31	801	805	809	813	817	821	825	826	830	833	835	------
	32	801	806	809	814	818	822	825	826	831	833	836	------
	33	802	807	810	814	819	823	826	827	832	835	837	------
	34	803	807	811	815	819	824	827	828	833	835	838	------
	35	804	808	812	816	820	825	828	829	834	837	839	------
	36	804	809	813	817	821	826	829	830	835	837	840	------
	37	805	810	814	818	822	827	830	831	836	839	841	------

> For computing Pacific depths see Science, Dec 24, 1926, Vol. LXIV, No. 1669, p. 627

July, 1926

Approximate method for obtaining vel.

Use as arguments in Table 13

approx. depth { × 0.6 for Atlantic
{ × 0.55 for Caribbean & Pacific

Salinity of { 35 for Atlantic & Caribbean
{ 34 for Pacific

Temperature of { 5° for Atlantic
{ 6° for Caribbean
{ 3° for Pacific

(Com. Heck has recently devised a new scheme for obtaining velocity in Pacific)

TABLE 13—Continued

Depth (fathoms)	Temperature							Salinity 0/00	Temperature				Depth (fathoms)
	0	2	4	6	8	10	12		0	1	2	3	
900	803	808	812	816	820	824	827	31	821	823	825	826	1,900
	804	809	813	817	821	825	828	32	822	825	827	827	
	805	809	814	818	822	825	829	33	823	825	827	828	
	805	810	815	819	823	826	830	34	824	826	828	829	
	806	811	815	820	824	827	831	35	825	827	829	830	
	807	812	816	820	825	828	832	36	825	827	829	831	
	809	813	818	821	826	829	833	37	826	828	830	831	
1,100	806	811	815	819	823			31	823	825	827	828	2,100
	807	812	816	820	824			32	824	826	828	829	
	808	813	817	821	825			33	825	827	829	830	
	809	814	818	822	826			34	825	827	830	831	
	809	814	819	823	827			35	826	828	831	832	
	810	815	820	824	828			36	827	829	831	833	
	811	816	820	825	828			37	827	829	832	833	
1,300	809	814	819	822				31	828	830	832	833	2,300
	810	814	820	824				32	829	831	833	834	
	811	815	820	825				33	829	832	834	835	
	812	816	821	826				34	830	832	835	836	
	813	817	822	827				35	831	833	836	837	
	814	818	823	828				36	832	834	837	838	
	814	818	823	828				37	832	834	837	838	
1,500	813	817	822					31	830	833	835	836	2,500
	814	818	823					32	831	833	836	837	
	815	819	824					33	831	834	836	838	
	816	820	825					34	832	834	837	838	
	816	821	826					35	833	835	838	839	
	817	822	827					36	834	836	839	839	
	817	823	828					37	835	837	839	840	
1,700	816	820	826					31	833	836	838	839	2,700
	817	821	827					32	834	837	839	840	
	818	822	828					33	835	838	840	841	
	819	823	828					34	836	838	840	842	
	820	824	829					35	836	838	841	843	
	821	825	830					36	837	839	842	843	
	822	826	831					37	838	841	843	844	

Salinity 0/00	Depth (fathoms)	Temperature			Depth (fathoms)	Temperature			Depth (fathoms)	Temperature		
		0	1	2		0	1	2		0	1	2
33		838	841	843		852	854	856		864	866	868
34		839	842	844		854	856	858		865	867	869
35	2,900	839	842	844	3,700	855	856	858	4,500	866	867	870
36		840	843	845		855	857	859		866	867	870
37		842	844	846		856	858	860		867	868	871
33		843	845	848		855	856	859		869	871	873
34		843	845	848		856	857	860		869	872	874
35	3,100	844	846	849	3,900	857	858	861	4,700	870	872	875
36		844	847	850		857	859	862		871	873	875
37		845	848	851		858	860	862		872	873	876
33		846	848	851		860	861	863				
34		847	850	852		860	861	864				
35	3,300	848	851	853	4,100	861	862	865				
36		849	851	854		861	863	866				
37		849	851	854		862	863	866				
33		849	851	853		862	863	866				
34		850	851	854		862	863	866				
35	3,500	850	852	854	4,300	863	864	867				
36		851	852	855		864	865	868				
37		852	854	856		864	866	868				

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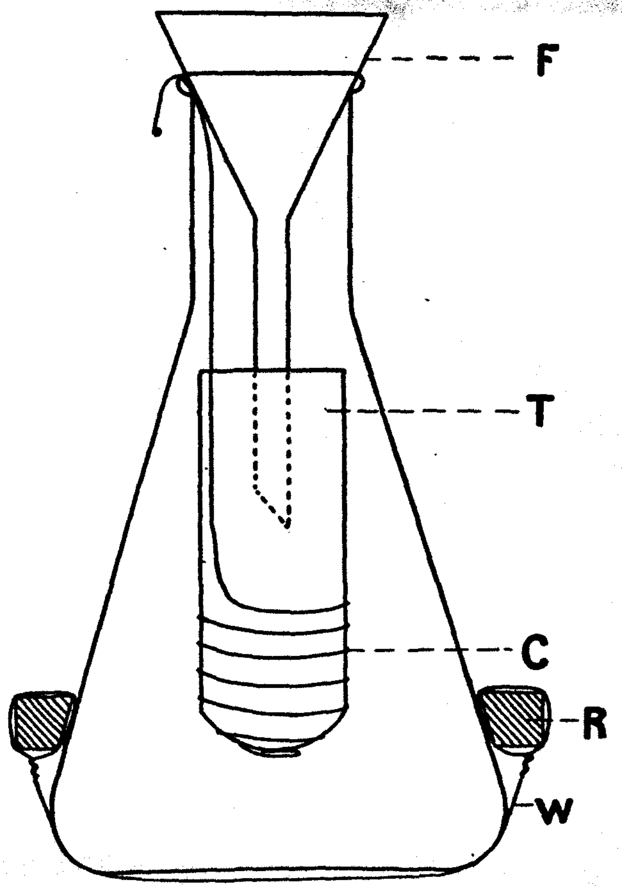

"F"—funnel. "T"—thimble. "C"—coil. "R"—ring. "W"—wire.

vessel for the reduction of Fehling's solution in a hot water bath. The handling of ordinary containers while hot during filtration is difficult and uncertain. Comfortable and safe manipulation of the flask by the ring "handle" is possible almost immediately after removal from the bath, if the flask is set into cold water, or held under the tap for a few seconds. Rinsing with hot water is accomplished with equal ease and safety. Nearly six hundred sugar determinations have been run without a single loss when this special "ringed flask" was used.

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SPECIAL ARTICLES

CORRECT VELOCITIES FOR ECHO SOUNDING IN THE PACIFIC OCEAN

SPECIAL Publication No. 108, United States Coast and Geodetic Survey, "Velocity of Sound in Sea Water," gives tables from which may be derived, by a method described in the publication, the proper velocity of sound to use in echo sounding, provided that the temperature and salinity of the water are known.

If we know the average annual conditions of temperature and salinity for any ocean for all depths, tables can be prepared which indicate regions throughout which the same velocities can be used for given depths without introducing material error. Determination of the extent of such regions and the velocities to be used in each is the purpose of this paper. The results show that the information can be given in a simple and easily used table for the entire Pacific Ocean.

Information as to mean annual temperature at the surface and at various depths was obtained from the publication "Die Warme Verteilung in den Tiefen des Stillen Ozeans" by Gerhardt Schott and Fritz Schu. The information was amplified from the Report of the Scientific Results of the Challenger Expedition, Physics and Chemistry, Vol. 1. Information regarding salinity was obtained from an atlas of thirty-one maps published by the Deutsche Seewarte in 1896 and from the Challenger report just mentioned.

For convenience, a map of the North Pacific was selected which gave the outlines of the land and meridians and parallels 10° apart. At each intersection the average temperature for each two hundred fathoms layer was entered and also the proper salinity to use, the same salinity being used for all depths. Seasonal changes of temperature were neglected, as these affect only the two upper layers and the second layer but slightly. While the variation of temperature at the surface during the season is considerable in the temperate zones, the average temperature of the upper layer does not vary much.

With the temperature and salinity fixed for each layer, the proper velocity for any desired depth was obtained from the tables of Special Publication No. 108, applying the adiabatic correction. The velocities were tabulated for 600 fathoms, 1,200, etc., to 5,400 fathoms, the greatest depth known.

The form of table adopted as most convenient was not a velocity table, but a percentage correction table. In practically all echo sounding now being carried on, all soundings are computed for a standard velocity and all that is needed is the correction to apply for the proper velocity. The percentage of correction to be applied is equal to the proper velocity divided by the standard velocity, minus one. Thus, if standard velocity is 800 fathoms per second and proper velocity is 824 fathoms per second, the percentage of correction is 3.0. The United States Navy and the Coast and Geodetic Survey have adopted the above named standard velocity of 800 fathoms (4,800 feet) per second. So long as the standard velocity is less than any adopted proper velocity, the correction is additive.

The percentages of correction were compiled on a similar map to that used for temperature and salinity. The next operation was to determine the extent of the region for which the same table might be used without introducing appreciable error.

Inspection of the map showed that for all intersections from and including 30° N. to the equator there was little departure from the mean value for the entire region. Mean percentages of correction were adopted for each layer and the rule was adopted that in general the range of the corrected sounding should not exceed five fathoms on either side of the mean. Further inspection showed that for all intersections on parallels 40° and 50° N. a second table would suffice. A still different table was found necessary for the north part of Bering Sea, deep portion approximately in Lat. 60° N. These three tables with interpolation between them cover the entire North Pacific except the adjacent seas on the west side and the shoal portion of Bering Sea.

The area of the South Pacific was then examined, selecting the extreme conditions in each region. It was found that the table for the tropical region could be extended to 40° S.; that the table for 40–50° N. applies to 50° S. and that a fourth table was needed for 60° S. The tables follow:

Fathoms	Table 1 60° N	Table 2 50–40° N, 50° S	Table 3 30° N to 40° S	Table 4 60° S
600	0.0	0.6	2.0	0.0
1,200	0.5	1.0	1.7	0.1
1,800	1.1	1.5	2.0	0.7
2,400	1.9	2.0	2.5	1.5
3,000	2.5	2.6	3.0	2.1
3,600	3.2	3.3	3.6	2.7
4.200	3.9	4.0	4.2	3.5
4,800	4.5	4.6	4.9	4.1
5,400	5.1	5.2	5.4	4.7

In order that interpolation may be easy, and only one table be needed, the information has been rearranged in Table 5.

Examples of use of table: At 35° N., 140° W. an echo sounding of 2,923 fathoms was obtained using standard velocity. Interpolating in the above table we get correction percentage of 2.7. Correction is $29.23 \times 2.7 = 79$ fathoms. Correct sounding is 3,002 fathoms.

24° N., 141° E. unreduced sounding 5,200 fathoms. Table gives 5.2. Correction is 270 fathoms and correct sounding 5,470.

TABLE 5

PERCENTAGES OF CORRECTION TO BE APPLIED TO ECHO SOUNDINGS

Lat.	Depth in fathoms 600	1,200	1,800	2,400	3,000	3,600	4,200	4,800	5,400
60° N	0.0	0.5	1.1	1.9	2.5	3.2	3.9	4.5	5.1
55	0.3	0.8	1.3	2.0	2.6	3.2	4.0	4.6	5.2
50–40	0.6	1.0	1.5	2.0	2.6	3.3	4.0	4.6	5.2
35	1.3	1.4	1.7	2.2	2.8	3.4	4.1	4.8	5.3
30° N–40° S	2.0	1.7	2.0	2.5	3.0	3.6	4.2	4.9	5.4
45° S	1.3	1.4	1.7	2.2	2.8	3.4	4.1	4.8	5.3
50	0.6	1.0	1.5	2.0	2.6	3.3	4.0	4.6	5.2
55	0.3	0.5	1.1	1.7	2.3	3.0	3.7	4.3	4.9
60	0.0	0.1	0.7	1.5	2.1	2.7	3.5	4.1	4.7

For the benefit of those who wish to use meters and a different standard value, Table 6 has been prepared, giving the computed velocities in fathoms and meters corresponding to the percentage corrections in Tables 1 to 4.

TABLE 6

Depth		Table 1		Table 2		Table 3		Table 4	
Fathoms	Meters	Fathoms	Meters	Fathoms	Meters	Fathoms	Meters	Fathoms	Meters
600	1,097	799	1,461	805	1,472	815	1,492	799	1,461
1,200	2,194	804	1,471	808	1,478	813	1,488	801	1,465
1,800	3,292	809	1,479	812	1,485	815	1,492	806	1,474
2,400	4,389	815	1,491	815	1,492	820	1,500	812	1,485
3,000	5,486	820	1,499	821	1,501	824	1,507	817	1,494
3,600	6,584	826	1,511	826	1,511	828	1,515	822	1,504
4,200	7,681	831	1,519	833	1,522	833	1,524	828	1,515
4,800	8,778	836	1,529	837	1,530	839	1,534	833	1,522
5,400	9,875	841	1,537	842	1,539	844	1,542	838	1,532

Salinities used: 34 was used from 30° N. to 40° S., except 33 at 20° N. from 160° to 130° W., and at 10° N. from 140° to 120° W. For all other areas, salinity 33 was adopted. Error introduced by this practice is small.

To show that the error involved in using the tables for the large regions adopted is not great, Table 7 was prepared. The equatorial region is selected as it has a greater range than the others. The reason that the departure from the mean is different for the maximum and minimum is that a much smaller region is affected by the minimum. The departure from the mean is much smaller than the extreme values given for all but a small part of the total area.