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MANUAL OF FIRST-ORDER TRIANGULATION

By

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FOREWORD

The purpose of this manual is to summarize the methods prevailing at present in the United States Coast and Geodetic Survey in executing first-order triangulation and base measurement. Little or no attention has been paid to historical developments or to theoretical discussions.

The credit for the material contained herein is due primarily to the geodesists and physicists in this survey who for a century or more have gradually improved the instruments and methods of geodetic surveying. The last two decades especially have seen remarkable progress toward standardization of methods and increased speed and economy of execution in geodetic work. It lies beyond the scope of this publication to pay the tribute of individual mention to those who have made special contributions to the science and art of geodesy in the United States.

Many of the engineers and mathematicians working in the division of geodesy have helped materially in the preparation of this manuscript by suggesting changes or additions, and by preparing tables and type forms, and their willing assistance is gratefully acknowledged. In particular, credit is due Dr. William Bowie, chief of the division of geodesy, for his careful review of the manuscript and for his many valuable suggestions.

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MANUAL OF FIRST-ORDER TRIANGULATION

By C. V. Hodgson, *Assistant Chief, Division of Geodesy, United States Coast and Geodetic Survey*

GENERAL STATEMENT

For some years the need has been apparent in the Coast and Geodetic Survey for a more complete description of the methods employed in its first and second order triangulation because of the great development in speed and economy during the past 20 years. This publication is designed to meet this need as far as possible. The aim has been to provide a manual of approved practices rather than to discuss in detail the theoretical principles of triangulation and base measurement. Only a sufficient amount of discussion of principles is given as will enable an engineer, either in or out of the survey, who may be engaged on this class of work, to execute triangulation of the class desired. The sources of error inherent in the various operations are pointed out and means suggested by which their relative magnitudes may be estimated. With that done the engineer can modify the methods given herein for first-order triangulation in order to meet the requirements for any less accurate class.

Since this publication is written primarily for the guidance of officers of the Coast and Geodetic Survey, where a party may extend triangulation several hundred miles in a single season, the description of party organization and of observing routine will be given in considerable detail. Sections of the text treating of those subjects will naturally not be of especial value or interest to an engineer who may be working in a limited area.

DEFINITIONS OF FIRST AND SECOND ORDER TRIANGULATION

Until very recently much confusion was caused by the conflicting terms used by different Federal organizations in describing the various classes of triangulation which they execute. Usually the more accurate class of work was called "primary" triangulation, irrespective of the degree of accuracy actually obtained, while the term "secondary" was applied to subsidiary schemes. In order to secure more uniform specifications and nomenclature, representatives of the various Federal map-making and map-using organizations in Washington agreed in 1921 upon a uniform classification. Under this agreement triangulation of the highest accuracy was termed "precise," the next lower "primary," and the third grade was called "secondary," corresponding respectively to what had been previously known in the Coast and Geodetic Survey as primary, secondary, and tertiary.

This terminology did not find wide favor or acceptance, and in May, 1925, the Federal Board of Surveys and Maps, after referring the matter to the various Federal map-making bureaus represented on the board, recommended that the four grades of vertical and horizontal control ordinarily used be designated as first order, second order, third order, and fourth order, respectively, the first named being the most accurate. These terms have been authorized by the Director of the Coast and Geodetic Survey and made applicable to the various grades of control executed by this organization.

The criteria adopted for the various grades of triangulation were first the closure of the triangle and second the agreement in length obtained by carrying the length from a measured base through the triangulation to the next measured base. First-order triangulation must have an average triangle closure of about 1 second or less and a maximum closure of not to exceed 3 seconds, while the closure in length upon a measured base or a line of adjusted triangulation must not exceed that represented by an error of 1/25000 after the angle and side equations have been satisfied in the adjustment. Similarly, second-order triangulation must have an average triangle closure of not to exceed 3 seconds, with a maximum of 6 seconds, and a closure in length represented by an error of not more than 1/10000 after the angle and side equations in the adjustment have been satisfied.

Under certain conditions the proportionate error in the length of a line as specified above may be found to be exceeded in either class of triangulation. Where two points are comparatively close together, as compared with the size of the triangulation scheme, the distance between those points may be in error in excess of that indicated by the class of triangulation of the scheme. The accuracy of the computed length of any line can be estimated by computing the ΣR_1 in accordance with the formula for the strength of figures as given on page 4. In any class of triangulation the subsidiary stations will be located with a less degree of accuracy than the main-scheme stations.

The specifications for the various grades of horizontal control now in use by the Federal map-making organizations are given in tabulated form below:

Specifications for horizontal control

	First order	Second order	Third order	Fourth order
Triangulation	Average triangle closure 1″, check on base 1/25000.	Average triangle closure 3″, check on base 1/10000.	Average triangle closure 5″, check on base 1/5000.	Graphic or transit angles.
Traverse	Position check 1/25000.	Position check 1/10000.	Position check 1/5000.	Stadia, transit and tape or wheel.

According to the plan adopted by the Federal Board of Surveys and Maps for the completion of the 1-inch-to-the-mile standard topographic map of the United States, first-order triangulation or traverse would be executed in belts about 100 miles apart. Second-order triangulation or traverse would subdivide the intervening areas until no considerable area would be farther than 25 miles from a horizontal-control point of the first or second order. Horizontal control of the third order would then be established, with a density of distribution of points depending upon the requirements of the topography.

The result of the adoption of this nomenclature and the corresponding specifications will be very beneficial, for the reason that any piece of triangulation will be classified in accordance with the accuracy obtained, irrespective of what the instructions for executing the work may have been, and much confusion will be avoided.

It should, however, be pointed out that these two standards, that of triangle closure and of closure in length, are not necessarily the only ones which should be applied. It is possible by a lucky balancing of errors to secure small triangle closures in a short scheme of triangulation even when a small theodolite is used. It is also possible by eliminating outstanding values to greatly reduce triangle closures. It may also happen that a similar balancing of errors in computing a chain of triangles will result in a very small discrepancy in length on the next fixed line. The accuracy of triangulation is perhaps best indicated by the probable error of a direction, but since this gauge of the work is not available until after the adjustment is made, the triangle closure and the closure in length, as given by the preliminary computations, are the best available field criteria.

To insure that the requisite accuracy is maintained throughout the triangulation, it is essential to give great consideration to the instruments and methods in order that both systematic and accidental errors may be made as small as possible, and that no part of the triangulation will exhibit undue weakness.

Chapter 1.—RECONNAISSANCE

SPECIFICATIONS FOR RECONNAISSANCE FOR FIRST-ORDER TRIANGULATION [1]

The reconnaissance specifications given below are practically the same as those in Coast and Geodetic Survey Special Publication No. 93, Reconnaissance and Signal Búilding. A detailed discussion of reconnaissance methods and descriptions of various types of signal towers are given in that publication.

1. **Character of figures.**—The chain of triangulation between base nets shall be made up of completed quadrilaterals and of central-point figures, with all stations occupied. It must not be allowed to degenerate even for a single figure to single triangles, except by specific permission by the Director. There must be two ways of computing the lengths through each figure. On the other hand, there must be no overlapping of figures and no excess of observed lines beyond those necessary to secure a double determination of every length, except as follows: In a four-sided central-point figure one of the diagonals may be observed; a figure used in expanding from a base often requires the observation of additional lines; and a network of triangulation over a city or other wide area may very properly contain a few overlapping figures to meet special conditions.

2. **Strength of figures.**—In the chain of triangulation between base nets the value of the quantity $R=\left(\frac{D-C}{D}\right)\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ for any one figure must not in the selected best chain (call it R_1) exceed 25, nor in the second best (call it R_2) exceed 80, in units of the sixth place of logarithms. These are extreme limits never to be exceeded. Keep the quantities R_1 and R_2 down to the limits 15 and 50 for the best and second-best chains, respectively, whenever the estimated total cost does not exceed that for a chain barely within the extreme limits by more than 25 per cent. The values of R may be readily obtained by the use of the "Table for determining relative strength of figures in triangulation," on page 6.

In the above formula the two terms $\frac{D-C}{D}$ and $\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ depend entirely upon the figures chosen and are independent of the accuracy with which the angles are measured. The product of these two terms is therefore a measure of the strength of the figures with respect to length, in so far as the strength depends upon the selection of stations and of lines to be observed over. The method of computing the strength of figure is explained on page 5.

3. **Lengths of lines.**—It is best that no line of the first-order triangulation outside of the base nets be less than 5 kilometers in length, if it is to be used directly in carrying the length forward through the scheme; that is, if it is opposite a distance angle used in computing R. So far as accuracy of observations is concerned, there is little advantage in having the lines longer than this.

---

[1] Detailed instructions for second and third order triangulation are given in Coast and Geodetic Survey Special Publication No. 26, and for first and second order traverse in Special Publication No. 58.

Above this minimum length two main considerations affect the size of the scheme: First, the combined cost per mile of progress of reconnaissance, building, and observing; and second, the number and accessibility of the points determined. When these two factors are opposed, the compromise scheme selected should have the stations close enough together to be used by engineers without special instruments and signal lamps. In general, lines of the main scheme in first-order triangulation should not exceed 50 miles in length.

4. **Frequency of bases.**—If the character of the country is such that a base site can be found near any desired location, ΣR_1 between base lines should be made about 80. This will be found to correspond to a chain of from 10 to 25 triangles, according to the strength of the figures secured. With strong figures but few base lines will be needed, and a corresponding saving will be made on this part of the work. If topographic conditions make it difficult to secure a base site at the desired location, ΣR_1 may be allowed to approach but not exceed 110.[1] There will be danger when this larger limit is used that an intervening base may be necessary, for if in any case the discrepancy between adjacent bases is found to exceed 1 part in 25,000 an intervening base must be measured.

5. **Base sites and base nets.**—In selecting base sites keep in mind that a base can be measured with the required degree of accuracy on any site where the grade on any 50-meter tape length does not exceed 10 per cent, and that narrow valleys or ravines less than 50 meters wide in the direction of the base are not insuperable obstacles to measurement. The length of each base is to be not less than 4 kilometers. In each base net great care should be taken to secure as good geometric conditions as possible. There should be no hesitancy in placing the base on rough ground, provided the roughness is not greater than that indicated above, if by doing so the geometric conditions in the base net are improved. Each base net should not be longer than two ordinary figures of the main chain between bases. The base net may also be strengthened by observing over as many lines between stations of the net as can be made intervisible without excessive cost for building or cutting. Caution is necessary in thus strengthening a base net by observing extra lines to avoid making the figure so complicated as to be excessively difficult and costly to adjust.

COMPUTATION OF STRENGTH OF FIGURE

In the following table the values tabulated are $[\delta_A^2+\delta_A\delta_B+\delta_B^2]$. The unit is one in the sixth place of logarithms. The two arguments of the table are the distance angles in degrees, the smaller distance angle being given at the top of the table. The distance angles are the angles in each triangle opposite the known side and the side required. δ_A and δ_B are the logarithmic differences corresponding to 1 second for the distance angles A and B of a triangle.

[1] These limits supersede the former limits of 100 and 130, respectively, as given in Special Publication No. 93 and in other publications issued previous to that one.

Table for determining relative strength of figures in triangulation

°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

HOW TO USE THE TABLE

To compare with each other two alternative figures, either quadrilaterals or central-point figures, in so far as the strength with which the length is carried is concerned, proceed as follows:

(*a*) For each figure take out the distance angles, to the nearest degree if possible, for the best and second-best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

(*b*) For each triangle in each chain enter the table with the distance angles as the two arguments and take out the tabular value.

(*c*) For each chain, the best and second best, through each figure, take the sum of the tabular values.

(*d*) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where D is the number of directions observed and C is the number of conditions to be satisfied in the figure. The quantities so obtained, namely, $\frac{D-C}{D}\ \Sigma\ [\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$, will for convenience be called R_1 and R_2 for the best and second-best chains, respectively.

(*e*) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the R_1 the greater the strength of the figure. The second-best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between figures they should be classed according to their best chains, unless said best chains are very nearly of equal strength and their second-best chains differ greatly.

SOME VALUES OF THE QUANTITY $\frac{D-C}{D}$

The starting line is supposed to be completely fixed.

For a single triangle, $\frac{4-1}{4}=0.75$.

For a completed quadrilateral, $\frac{10-4}{10}=0.60$.

For a quadrilateral with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

For a quadrilateral with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

For a three-sided, central-point figure, $\frac{10-4}{10}=0.60$.

For a three-sided, central-point figure with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

For a three-sided, central-point figure with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

For a four-sided, central-point figure, $\frac{14-5}{14}=0.64$.

For a four-sided, central-point figure with one corner station on the fixed line unoccupied, $\frac{12-3}{12}=0.75$.

For a four-sided, central-point figure with one corner station not on the fixed line unoccupied, $\frac{11-3}{11}=0.73$.

For a four-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{10-2}{10}=0.80$.

For a four-sided, central-point figure with one diagnonal also observed, $\frac{16-7}{16}=0.56$.

For a four-sided, central-point figure with the central station not on the fixed line unoccupied and one diagonal observed, $\frac{12-4}{12}=0.67$.

For a five-sided, central-point figure, $\frac{18-6}{18}=0.67$.

For a five-sided, central-point figure with a station on a fixed outside line unoccupied, $\frac{16-4}{16}=0.75$.

For a five-sided, central-point figure with an outside station not on the fixed line unoccupied, $\frac{15-4}{15}=0.73$.

For a five-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{13-2}{13}=0.85$.

For a six-sided, central-point figure, $\frac{22-7}{22}=0.68$.

For a six-sided, central-point figure with one outside station on the fixed line unoccupied, $\frac{20-5}{20}=0.75$.

For a six-sided, central-point figure with one outside station not on the fixed line unoccupied, $\frac{19-5}{19}=0.74$.

For a six-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{16-2}{16}=0.88$.

To illustrate the application of the preceding strength table the R_1 and R_2 for Figure 14, page 13, will be considered. Let it be assumed that the direction of progress is from the bottom line toward the top line. It will be found that the smallest R, called R_1, for this figure will be obtained by computing through the three best-shaped triangles around the central point. The next best R, called R_2, will be obtained by computing through the two triangles formed by the diagonal. The R_2 is easily computed as follows: From the known side to the diagonal the distance angles are 89° and 27°. Using these angles as arguments in the preceding strength table, the factor 17.5 is obtained. Similarly, from the diagonal to the top line the distance angles are 91° and 26° and the corresponding factor is 18.8. The sum of the two factors is 36.3. If the central point of the figure is an occupied station, $\frac{D-C}{D}=0.56$ (see above) and $R_2=36.3\times 0.56=20$.

If the central point is unoccupied, as shown in Figure 14, $\frac{D-C}{D}=0.67$ (see p. 8) and $R_2=36.3\times0.67=24$, as given opposite the figure.

The R_1 may be computed in a similar manner by using the distance angles in the three best-shaped triangles around the central point.

EXAMPLES OF VARIOUS TRIANGULATION FIGURES

The following 14 figures are given to illustrate some of the principles involved in the selection of strong figures and to illustrate the use of the preceding strength table:

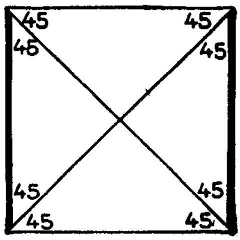

FIG. 1.—All stations occupied. $R_1=5$, $R_2=5$

Same, any one station not occupied. $R_1=6$, $R_2=6$

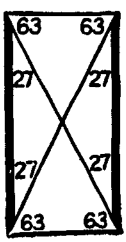

FIG. 2.—All stations occupied. $R_1=1$, $R_2=1$

Same, any one station not occupied. $R_1=2$, $R_2=2$

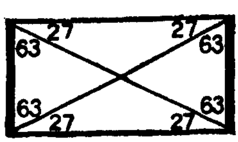

FIG. 3.—All stations occupied. $R_1=22$, $R_2=22$

Same, one station on fixed line not occupied $R_1=27$, $R_2=27$

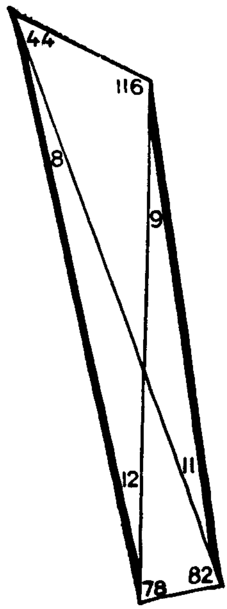

FIG. 4.—All stations occupied. $R_1=1$ $R_2=2$

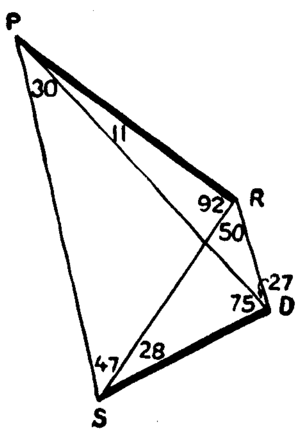

FIG. 5.—All stations occupied. $R_1=10$ $R_2=12$

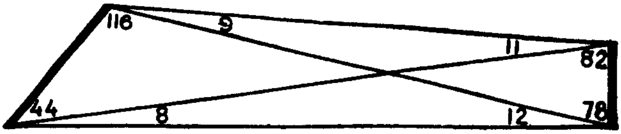

FIG 6.—All stations occupied. R_1—164 (approx.) R_2—176 (approx.)

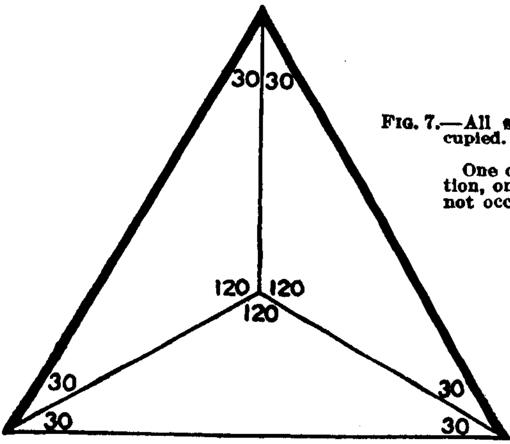

FIG. 7.—All stations occupied. R_1— 2 R_2—12

One outside station, on fixed line, not occupied. R_1— 3 R_2—15

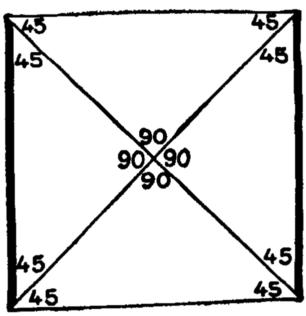

FIG. 8.—All stations occupied. R_1—18 R_2—18

Same, one corner station not occupied. R_1—16 R_2—16

Same, central station not occupied. R_1—17 R_2—17

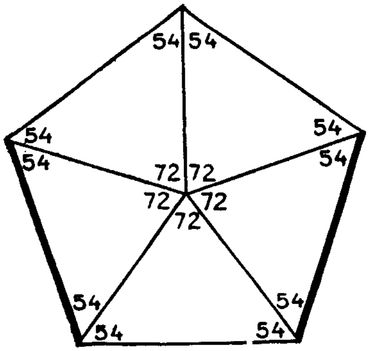

FIG. 9.—All stations occupied. $R_1=10$ $R_2=15$

Same, any one outside station not occupied. $R_1=11$ $R_2=16$

Same, central station not occupied. $R_1=13$ $R_2=19$

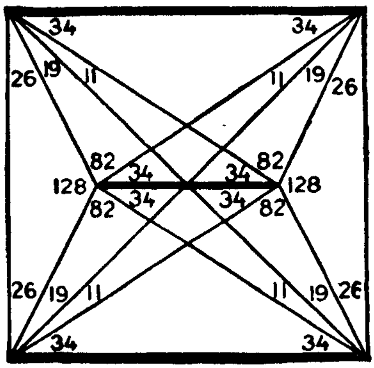

FIG. 10.—All stations occupied. $R_1=5$ $R_2=5$

$$\frac{D-C}{D}=\frac{28-16}{28}=0.43$$

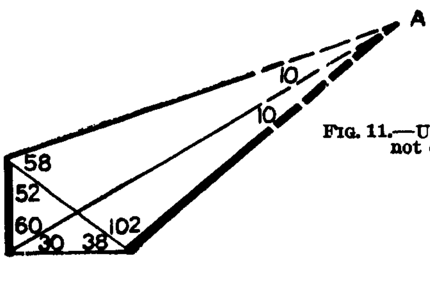

FIG. 11.—Unoccupied station not on fixed line. $R_1=36$ $R_2=102$

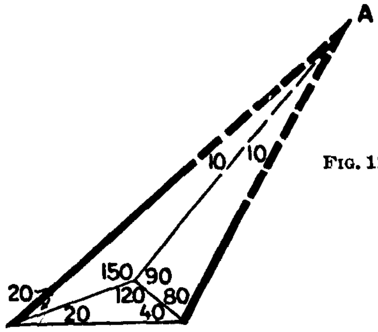

FIG. 12.—Unoccupied station at intersection of fixed line and line to be determined. R_1— 4 R_2—20

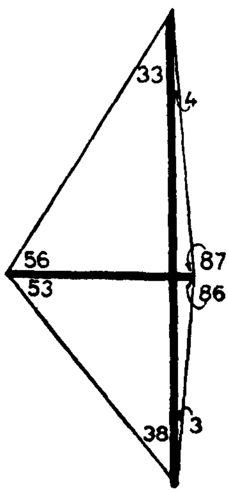

FIG. 13.—All stations occupied. (A strong and quick expansion figure.) R_1—9 R_2—9

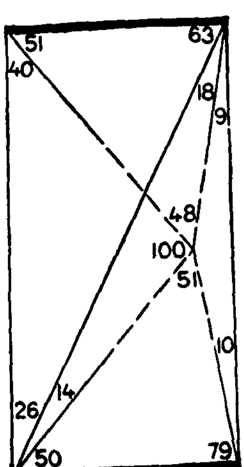

FIG. 14.—Central station not occupied. R_1—18 R_2—24

In every figure the line which is supposed to be fixed in length and the line of which the length is required are represented by heavy lines. Either of these two heavy lines may be considered to be the fixed line and the other the required line. Opposite each figure R_1 and R_2, as given by the table on page 6, are shown. The smaller the value of R_1 the greater the strength of the figure. R_2 need not be considered in comparing two figures unless the two values of R_1 are equal, or nearly so.

Compare Figures 1, 2, and 3. Figure 1 is a square quadrilateral; Figure 2 is a rectangular quadrilateral which is one-half as long in the direction of progress as it is wide; Figure 3 is a rectangular quadrilateral twice as long in the direction of progress as it is wide. The comparison of the values of R_1 in Figures 1 and 2 shows that shortening a rectangular quadrilateral in the direction of progress increases its strength. A comparison of Figures 1 and 3 shows that extending a rectangular quadrilateral in the direction of progress weakens it.

Figure 4, like Figure 2, is short in the direction of progress. Such short quadrilaterals are in general very strong, even though badly distorted from the rectangular shape, but they are not economical, as progress with them is slow.

Figure 5 is badly distorted from a rectangular shape, but is still a moderately strong figure. The best pair of triangles for carrying the length through this figure are $D\ S\ R$ and $R\ S\ P$. As a rule, one diagonal of the quadrilateral is common to the two triangles forming the best pair, and the other diagonal is common to the second-best pair. In the unusual case illustrated in Figure 5 a side line of the quadrilateral is common to the second-best pair of triangles.

Figure 6 is an example of a quadrilateral so much elongated, and therefore so weak, that it is not allowable in any class of triangulation.

Figure 7 is the regular three-sided, central-point figure. It is extremely strong.

Figure 8 is the regular four-sided, central-point figure. It is very much weaker than Figure 1, the corresponding quadrilateral.

Figure 9 is the regular five-sided, central-point figure. Note that it is much weaker than any of the quadrilaterals shown in Figures 1, 2, or 4.

Figure 10 is a good example of a strong, quick expansion from a base. The expansion is in the ratio of 1 to 2.

Figures 11 and 12 are given as a suggestion of the manner in which, in second and third order triangulation, a point (A), difficult or impossible to occupy, mav be used as a concluded point common to several figures.

Many of the figures given on the preceding pages are too weak to be used on first-order triangulation, but for convenience of reference and to illustrate the principles involved they are included with the figures which it is permissible to use.

Modifications.—To interpret correctly the foregoing reconnaissance specifications it must be borne in mind that they were drawn to meet particularly one single requirement, viz, to enable the triangulator to determine the lengths of successive lines of the triangulation as economically as possible consistent with the maintenance of a specified accuracy. In other words, they meet the requirements for long belts of triangulation better than those for the development of an area. It is very easy, however, to modify the specifications to meet other conditions. For instance, on short arcs it will be permissible to use a chain of triangles instead of figures which afford two or more ways of computing lengths through the scheme. If a more intensive development of an area is desired, instead of being concerned with distance angles only, and the ΣR_1 and ΣR_2 through the scheme, the observer should give attention to leaving lines of proper strength so located as to be of greatest value to topographers and other engineers.

CONNECTIONS TO EXISTING TRIANGULATION

In starting from or connecting with either first or second order triangulation, it is essential that the connection be made to a line of proper strength, and also that observations be made from the two ends of that line upon a third point of the existing triangulation. If the new and old angles agree closely the exact recovery of the old stations is assured. Even when connecting with triangulation of the third order it is better to connect with a line rather than a point, for the comparison of the lengths of the line common to the two systems of triangulation may give information of great value in adjusting the weaker scheme. Connection in position alone, namely, to a single point, or in position and azimuth, namely, to a single point but with a direction observed from that point to another old station, may sometimes be advantageously made at intervals between the connections in length made on lines of the old triangulation. If a line joined onto is opposite a weak angle in the old triangulation the comparison in length will have little value.

Chapter 2.—FIRST-ORDER TRIANGULATION

GENERAL INSTRUCTIONS FOR FIRST-ORDER TRIANGULATION

The condensed instructions for first-order triangulation given below were approved by the Director of the Coast and Geodetic Survey on September 21, 1925, although they differ but little in general principles from those approved by the superintendent of the survey in 1905, as first printed on pages 170–174 of Appendix 4 of the annual report for 1911.

The following instructions for first-order triangulation supersede all previous instructions for work of this character.

1. **Instruments.**—In general, direction instruments of the highest grade should be used in triangulation of this class. Repeating theodolites are to be used only when the station to be occupied is in such a position as to be difficult of occupation with a direction instrument or when there is doubt that the instrument support is of such a character as to insure that the movement of the observer about the instrument does not disturb it in azimuth. Such stations usually occur on lighthouses and buildings.

2. **Position settings, direction theodolite.**—One measure of the horizontal direction to each station, from some one selected initial station, consists of a reading with telescope direct and a reading with telescope reversed with the graduated circle oriented to give a reading on the initial station corresponding approximately to one of the settings given in one of the tables below, the choice of table depending upon the number of measures to be taken and the number of reading microscopes on the theodolite. These initial settings are so chosen as to space the micrometer readings uniformly around the graduated circle for a complete set of observations.

Theodolite settings when 16 positions of the circle are used

With a 3-micrometer theodolite		With a 2-micrometer theodolite	
Position No.	Setting	Position No.	Setting
	° ′ ″		° ′ ″
1	0 00 40	1	0 00 40
2	15 01 50	2	11 01 50
3	30 03 10	3	22 03 10
4	45 04 20	4	33 04 20
5	64 00 40	5	45 00 40
6	79 01 50	6	56 01 50
7	94 03 10	7	67 03 10
8	109 04 20	8	78 04 20
9	128 00 40	9	90 00 40
10	143 01 50	10	101 01 50
11	158 03 10	11	112 03 10
12	173 04 20	12	123 04 20
13	192 00 40	13	135 00 40
14	207 01 50	14	146 01 50
15	222 03 10	15	157 03 10
16	237 04 20	16	168 04 20

Theodolite settings when 8 positions of the circle are used

With a 3-micrometer theodolite		With a 2-micrometer theodolite	
Position No.	Setting	Position No.	Setting
	° ′ ″		° ′ ″
1	0 00 40	1	0 00 40
2	15 01 50	2	22 01 50
3	30 03 10	3	45 03 10
4	45 04 20	4	67 04 20
5	52 00 40	5	90 00 40
6	67 01 50	6	112 01 50
7	82 03 10	7	135 03 10
8	97 04 20	8	157 04 20

3. When a broken set is observed the missing stations are to be observed upon later in connection with the chosen initial or with some other one, and only one, of the stations already observed in that set. With this system of observing no local adjustment is necessary. Little time should be spent in waiting for a station light to show. If it is not showing within, say, one minute of when wanted, turn to the next station. A saving of time results from observing upon many or all of the stations in one set, provided there are no long waits for signals to show, but not otherwise.

4. When a new initial is used for observations upon one or more stations in all of the positions of the circle the settings given in one of the tables above may be used, but when a new initial is used to complete a set on a station the angle between the old and the new initial should be added to the setting for each position given in the table, in order that the readings of the directions to each station may be properly distributed about the circle.

5. The settings given in the table are those actually made with the finder microscope in a 3-micrometer theodolite, or with the A-microscope of a 2-micrometer instrument. Since the telescope is reversed between the settings for any two positions, the given settings require the changing of the circle almost 180° in orientation for each new position, and this tends to maintain the circle at a uniform temperature, even though temperature conditions may not be the same on all sides of the instrument.

6. **Number of observations, main scheme; direction instrument.**—Sixteen measures (usually called "positions," because of the different positions of the circle with reference to the initial station) will, in general, be made of each direction in the main scheme, though the number may be reduced to 12 on one or more directions at a station if the triangle closures have been satisfactory and it is necessary to do so to avoid a delay of a day or more at the station to obtain the missing measurements.

7. **Number of observations on second-order stations, direction instrument.**—On stations to be located with second-order accuracy from four to eight positions only need be used. When four positions are used the settings should be the first four given in the table for the type of instrument employed. Second-order accuracy should be obtained in connections to lines of third-order triangulation or in locating stations to be used as starting points for schemes of third-order triangulation, also in connecting to State boundary monuments.

8. **Number of observations on intersection stations, direction instrument.**—An intersection station is one located by observations from two or more stations but is not occupied. In these instructions the term is used in a restricted sense and means an unoccupied point determined with third-order

accuracy. The direction to an intersection station will be measured in any two positions of the circle, using a station of the main scheme as the initial. It is important to have three lines to each intersection station in order to secure a check upon its position, but a possible intersection station should not be ignored because only two lines to it can be secured.

9. **Observing conditions.**—In selecting the conditions under which to observe when executing triangulation, proceed upon the assumption that what is desired is the maximum progress consistent with the requirements for triangle closure. The standard of accuracy prescribed by the allowable triangle closures, used in connection with other portions of these instructions defining the necessary strength of figures and frequency of bases, will in general insure that the probable error of any base line, as computed through the triangulation from an adjacent base, is less than 1 part in 100,000 on first-order triangulation, and that the actual discrepancy between the true and the adjusted length of any line between bases is always less than 1 part in 25,000.

10. **Marking of stations.**—Every triangulation station which is not in itself a permanent mark, such as a lighthouse, church spire, or building, shall be marked in accordance with the specifications on pages 20–24. The chief of party shall satisfy himself that the stations are being marked in accordance with those specifications.

11. **Method of observing, repeating theodolite.**—When it is necessary to use a repeating theodolite on triangulation of first-order accuracy the observations should be made in sets, each consisting of six repetitions with the telescope direct (or reversed) upon the angle, followed immediately by six repetitions with the telescope reversed (or direct) upon its explement. The initial reading for each set should be on a different part of the circle. The angle between the initial settings should be obtained by dividing 180° by the number of sets to be observed for each angle, since a 2-vernier or a 2-micrometer instrument will usually be used. With a vernier instrument an increment represented by the value of one division of the circle divided by the number of sets to be observed should be added to the difference in degrees between settings in order to eliminate the error of graduation of the verniers. Thus with a circle graduated to 5 minutes and five sets observed on an angle the settings would be 0° 00′ 00″, 36° 01′ 00″, 72° 02′ 00″, 108° 03′ 00″, and 144° 04′ 00″. Measure only the single angle between adjacent lines of the main scheme and the angle necessary to close the horizon. With this scheme of observing no local adjustment is necessary except to distribute the horizon closing error uniformly among the angles measured. If it is necessary to measure double angles, due to the failure of one or more signals to show at the time of measurement of the other angles, the abstract of directions should be made out as described on pages 89–93.

Angles between stations to be located with second-order accuracy should be measured in the same way as that indicated above and should be connected with some one, and only one, line of the main scheme. No measures introducing station conditions other than closure of horizon should be made on or at stations located with second-order accuracy.

12. **Number of observations, repeating theodolite.**—Because of the variability in accuracy obtained by using different types of repeating theodolites, it is not possible to specify in general instructions the number of sets to be observed to secure first or second order accuracy. When such an instrument is to be used special instructions will be given regarding the number of sets to be observed with the particular type and size of theodolite, and the rejection limit will also be stated.

13. **Rejections, direction instrument.**—The limit of rejection for a direction measured with a direction instrument in one position ordinarily shall be

4 seconds from the mean. (Directions for combining two or more measures of a direction are given on pages 89 to 92.)

14. **Value of intersection stations.**—In selecting intersection stations it should be kept in mind that the geographic value of a scheme of triangulation depends upon the number of points determined, their distribution over the area, and their permanency. The geographic value of triangulation in any region is lost to a very great extent when points can not be recovered within the area. The possibility of recovering points is increased by increasing the number of points as well as by thorough marking. These considerations should lead to the determination by intersection methods of many artificial objects of a permanent character, such as lighthouses, church spires, cupolas, towers, and large chimneys, and should also lead to the determination of additional specially marked stations, where such are necessary to insure that a sufficient number of control points are provided for the ordinary needs of engineers. During unfavorable weather conditions it may be necessary to make special efforts to secure these intersection points by taking advantage of times of greatest visibility, such as the early morning or late afternoon. The distribution of points in an area should be sufficiently dense that engineers will not find it necessary to use special instruments in order to connect their surveys to control points. Where the scheme of triangulation is large it may be necessary to use supplemental points which may be occupied in order to locate the proper number of intersection points. It is especially desirable to increase the area effectively covered by triangulation by selecting intersection stations which are outside the area covered by the main scheme as well as inside.

15. **Land-section corners and other survey marks.**—It is desirable to make frequent connections to section corners of the General Land Office and to triangulation marks of other surveys, both Federal and private. These connections should be made with third-order accuracy, and where possible with a check. Special arrangements will sometimes be made for the recovery and identification by cadastral engineers from the General Land Office of land-section corners to be connected, but where that is not done, as much care as possible should be taken to make sure of the recovery of the section mark. The description of station should state the exact condition of the mark as found. Connections to such survey marks may be made either by triangulation or by a measured distance and direction.

16. **Trigonometric leveling.**—The principal purpose of measuring zenith distances in our triangulation is to provide data to be used in the reduction to sea level of the directions of the main-scheme lines of the triangulation, or to compute from the published sea-level length of a line its length at the mean elevation of its extremities,[1] since the mapping of the country has progressed to such an extent that the approximate elevations obtained by trigonometric leveling are usually of little geographic value. For this reason only such measures of zenith distances will be made as will fulfill the purposes indicated above.

It is desirable to have the zenith distances measured with an instrument having a vertical circle at least 6 inches in diameter which can be read to 10 seconds, although if such an instrument is not available one of less accuracy may be used. With an inferior instrument an increased number of observations over that prescribed below should be made. At least two measurements, each consisting of an observation with the circle right and an observation with the circle left, should be made on each of two days on all stations of the main scheme, provided the party is at the station that length of time. If the two measures on a single day do not agree within 15 seconds after the level correction has been applied, one additional measure should be made. Observations upon stations

[1] See p. 146.

not in the main scheme need be made only on a single day, but the same agreement between observations should be secured as on main-scheme stations.

It is desirable that the observations be made between 12 noon and 4 p. m., since the refraction is smaller and is more constant during that part of the day. If necessary to avoid delay to the party, however, observations may be made at any time during the day, or even on lights at night; provided, however, that if zenith distances are measured on lamps, night observations should also be made on one, or preferably two, stations which have been observed during the day. This procedure will make it possible to form a fair estimate of the difference in refraction, which difference would be applied to the night observations as a correction. If the ground at a distant station is sufficiently visible to be observed upon during the hours of relatively constant refraction, it is much more accurate to observe upon the ground then than to make observations at night, since the refraction at night is very erratic.

Observations for zenith distances may be omitted over one line in each figure, provided it is possible to obtain a check from the remaining lines upon the determinations of elevation carried through the figure. A line observed over in only one direction is of little value. Because of the rather large accumulation of errors in trigonometric leveling, connections to bench marks should be provided at intervals of from 150 to 300 miles along the axis of the scheme.

MARKS FOR HORIZONTAL CONTROL STATIONS

The essential parts of the specifications for station and reference marks now in use are given below:

Metal tablets.—Each station which has been located with first, second, or third order accuracy (see page 2), should be marked by a standard tablet of copper alloy, so fastened in the rock or concrete as to effectively resist extraction, change of elevation, or rotation. (See figs. 15 and 16.) The name of the station and the year established should be stamped upon the mark, preferably before it is set in the rock or concrete.

Setting of tablets.—Stations for horizontal control must often be located where the permanent marking of them is difficult, and for that reason a great variety of settings for the tablets must be permitted. The location of the station, depth of soil, or presence of rock ledges, and the availability of materials will usually control the choice of the mark to be used. The precautions to be taken in establishing each kind of mark are briefly stated below.

(1) *In rock outcrop.*—Care should be taken that the rock in which a mark is set is hard and is part of the main ledge, not a detached fragment. The tablet should be countersunk and well cemented in.

(2) *In bowlders.*—When a tablet is set in a bowlder, the latter should be of durable material and of cross section, area, and depth below the surface not less than the standard concrete mark as described below.

(3) *In rock ledges below surface.*—When the ledge is only slightly below the surface, a tablet set in the usual manner in the ledge will be sufficient, provided two reference marks are established. Where the ledge is so far below the surface that a surface mark is required, a tablet or copper bolt should be set in the ledge, the ledge carefully brushed or washed off for a space at least 18 inches in diameter, and a concrete surface mark placed above the subsurface mark. A tablet should be set in the surface mark directly over the subsurface tablet or bolt. If the rock ledge in which the subsurface mark is set is very smooth, it should be furrowed with a chisel to afford better anchorage for the concrete.

(4) *In concrete.*—(*a*) *Shape.*—The mark should be either a frustum of a cone or of a pyramid, or have the form of a post with an enlarged base. If of pyramidal or

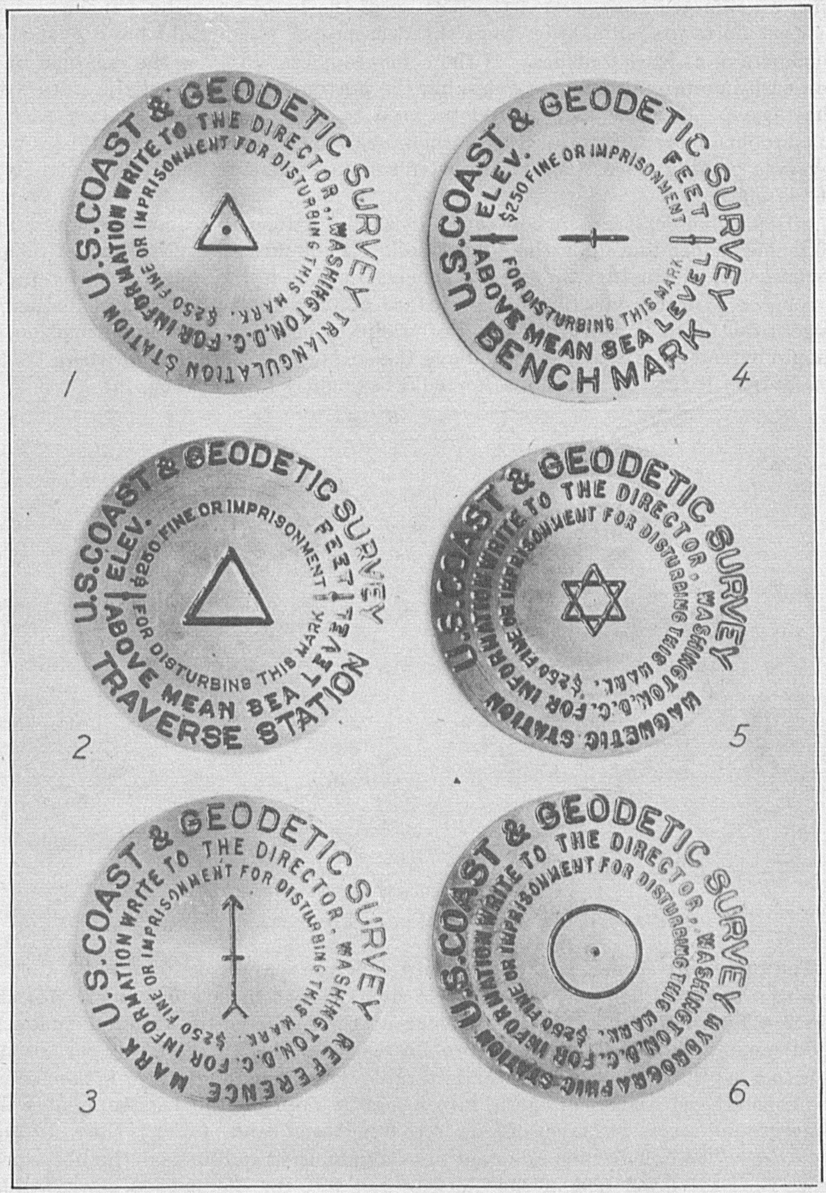

FIG. 15.—Standard marks of the United States Coast and Geodetic Survey

1. Triangulation station mark.
2. Traverse station mark.
3. Reference mark.
4. Bench mark.
5. Magnetic station mark.
6. Hydrographic station mark.

conical form, the sides should have a batter of at least 1 inch to 1 foot. When a post with an enlarged base is used the bottom of the base should be 4 inches larger in least horizontal dimension than the post proper and should have a vertical thickness of at least 6 inches. If the concrete is cast in place the enlarged base can easily be provided for by enlarging the bottom of the hole at the sides with the digger. Extreme care should be used to avoid making the mark with a mushroom top or with projecting corners near the surface, which would provide leverage points for frost action and would make easier the malicious destruction of the mark.

(*b*) *Size and depth.*—The concrete post should extend to a depth of from 30 to 36 inches, depending upon the kind of soil. It should be not less than 14 inches in diameter, except that the upper 12 inches may be in the shape of a frustum of a cone or pyramid with the upper surface not less than 12 inches in diameter. Where the mark is not in the path of traffic or in soil subject to cultivation, it should extend from 2 to 4 inches above the surface. When located where traffic passes over it the top of the mark should be slightly below the surface.

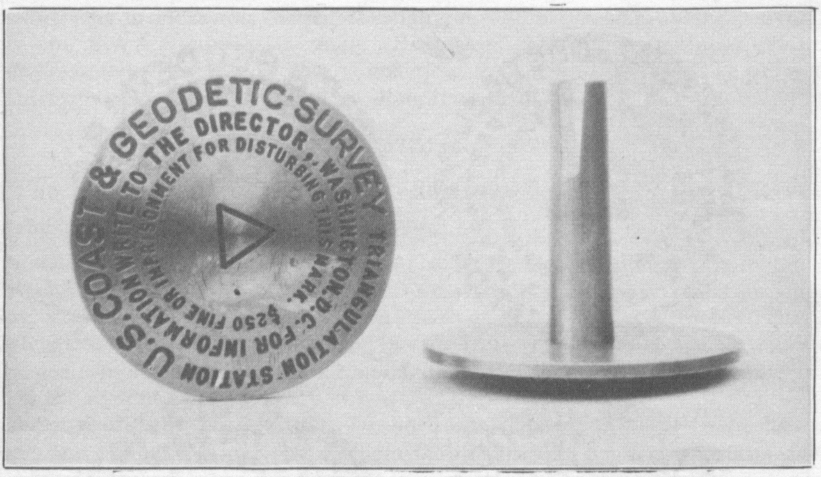

FIG. 16.—Triangulation station mark

The cleft shank is spread before the tablet is set in stone or concrete.

The procedure in making the standard concrete mark is as follows: A hole is dug to a depth of 3½ feet or more. It should be 16 inches in diameter for the top 2½ feet, and 10 inches in diameter at the lower end. Concrete made of good cement, sand, and gravel or broken rock is placed in the lower part of the hole to a depth of 6 inches. A standard tablet station mark (fig. 16) is then set in the concrete, with the top of the tablet slightly depressed. This completes the underground mark. A layer of from 4 to 6 inches of sand or dirt is then put into the hole. The hole is then enlarged about 2 inches in radius near the bottom in order that the lower end of the block of concrete for the surface mark will be mushroomed, and then the hole is filled with concrete to within 9 inches of the surface of the ground. Next a mold or frame 12 inches on a side at the top, 13 inches at the bottom, and 12 inches in depth is set in the hole on top of the concrete and filled in around the outside with dirt tamped firmly. The frame is then filled with concrete level with its top and a standard tablet station mark (fig. 16) is set in the center of the concrete, with the top of the tablet slightly depressed. The tablet must be centered exactly over the underground mark. The top of the concrete should be smoothed with a trowel and the frame should be left in place to protect the concrete until it becomes firmly set.

Care must be taken not to disturb the position of the tablet in the underground mark when placing the layer of sand or dirt and when pouring the concrete for the surface mark. A piece of thin board should be placed over the lower mark or other suitable means used to insure against any horizontal movement of the tablet due to the impact or pressure of the material above.

SPECIAL CONDITIONS

Under certain conditions special marks will often be required, and these should conform in size and durability to the marks described above.

(1) **Sand.**—In sand, which if used as a mold would spoil the concrete by absorbing the water from it, sewer tiles 8 inches in diameter and 30 inches long may be used, set with the bell end down, filled with concrete and with the base end set in concrete. A sheet-iron mold of the same dimensions filled with concrete may also be used. A metal tablet should be set in the center of the top.

(2) **Marsh.**—Where the surface of the ground is too soft to hold a mark of the usual type, a post of durable wood should be forced down vertically as far as it will go, its top cut off flush with the surface, and a sewer tile at least 6 inches in diameter set into the marsh around the top of the post. The tile should then be filled with concrete and a tablet set in the top. Where the marsh is very soft but dries out at certain seasons of the year, successive tiles can be forced down around the post, the post then can be withdrawn and the mud worked out from within the tiles, and the tiles then filled with a hydraulic cement mixture.

(3) **Land subject to cultivation.**—The subsurface or lower mark should be a tablet in a block of concrete 10 inches square or 10 inches in diameter and 6 inches thick, set with its top 3 feet below the surface. The upper mark should be a tablet set in a block of concrete 15 inches in least horizontal cross-section dimension and 20 inches high, with its top 12 inches below the surface of the ground. About 3 inches of dirt should be placed between the concrete blocks bearing the upper and lower marks.

All stations so marked should be referenced by two standard reference marks placed on property boundary lines, preferably along a well-established highway or quarter-section line, in a location where there is little likelihood of their being disturbed. The directions to the reference marks should be such as to give a good angle of intersection at the station. The reference marks may be as much as a half mile from the station, if necessary, provided they can be seen from the station. The distance to each reference mark should be carefully measured. Other distances, such as those to the center of a highway, the corner of a building, or the center of a well, should be measured if feasible. Two or more such measurements will intersect so near the station that the concrete block will be easily found with a small prodding rod. When measurements are made to buildings or other objects the directions must be also given. If measurements of this kind are made the station may usually be easily recovered though the reference marks may both be destroyed. The measurements to a road should always be to the center of the road and not to the fence line. All distances must be carefully measured and not estimated. Care should be taken in placing reference marks along highways, for nearly all States are widening the highways.

REFERENCE MARKS

Each reference mark should consist of a metal tablet similar in material and shape to the station mark, but bearing an arrow which points to the station. A reference mark should be stamped with the same designation as its station mark, and where there is more than one reference mark they should be numbered serially in a clockwise direction, the number to be stamped upon each one. Each should be set under the same conditions as specified for the station mark, except that the concrete post in which it is set may be 2 inches smaller in diameter and 6 inches shorter than for the station mark.

Each station mark must have at least one reference mark and should preferably have two. If the station mark, due to surface conditions, is entirely beneath the surface there should be two reference marks, unless there are permanent witness marks, such as road crossings, etc., which will serve to locate the station without an excessive amount of digging. If the station mark is on ground liable to be disturbed or washed away two reference marks should invariably be established. These should be so located as to avoid the probability of both being disturbed by the same cause. They should also preferably be so located as to give a good angle of intersection at the station, or else be placed in range with the station.

Material.—The main considerations in making concrete are to have clean materials, mix them well before adding water, have the mixture not too wet, and tamp well into the form. Each streak of dirt in concrete means a line of cleavage. Where rough aggregate is available the proportions may very well vary from 1–2–4 to 1–3–5, but the top 12 inches of the mark should be of considerably richer mixture. Where only cement and sand are available the lower part of the mark should be proportioned 1 part of cement to 3 parts of sand, and the upper part should be 1 part of cement to 2 parts of sand. With a mark of the proper size it will not be necessary to reinforce the concrete with metal rods or wire. To avoid cracking of the concrete, due to rapid drying, it should be covered with paper or cloth and then with earth or other material for a period of at least 48 hours.

INSTRUMENTS

THEODOLITES

Observations for both first and second order triangulation are usually made with direction theodolites of the highest class, although repeating theodolites may be used where the support for the instrument is so unstable that the proper accuracy can not be secured with a direction instrument.

The direction method of measuring angles consists of measuring the direction to each station from some one station taken as an initial. The directions are the angles measured from the initial station to each other station reckoned clockwise. The angle at a station between any two other stations is the difference of their directions.

In observing, a reading is taken upon the initial station and then upon each station around the horizon in a clockwise direction; the telescope is then reversed and the readings made back in a reverse direction. A direction theodolite does not usually have a slow-motion screw for the lower motion, though the direction method of observing may be used with a theodolite arranged for repetitions by keeping the lower motion clamped.

With the method of repetitions some multiple of the angle between any two stations is accumulated on the graduated circle between successive readings by recording the circle reading, pointing on the left-hand object with the lower slow-motion screw, which does not change the reading, then loosening the upper motion and pointing upon the right-hand object, perfecting the pointing with the upper slow-motion screw. The lower motion is then freed and the operation repeated until a number of measures of the angle, usually six, are accumulated on the circle, when the circle is read and recorded, the telescope reversed, and the same number of measures made on the explement of the angle.

Both classes of instruments should be read by micrometers for either first or second order work. Micrometer theodolites are preferred to vernier theodolites for the reason that the former may be read accurately to 1 or 2 seconds, and it is thus possible to secure with a single measure such an accurate value for an angle that the observer can detect changes in atmospheric refraction and also be better able to recognize and evaluate any instrumental errors. He is also able to locate intersection stations with sufficient accuracy by observing in only one or two positions of the circle. Moreover, with the direction instrument a prescribed accuracy can be obtained more quickly than with a repeating instrument of the same class. Finally, the program of observing used in the survey with a direction instrument avoids the necessity for making a station adjustment of the observations, which is usually required when a repeating instrument is used.

The repeating theodolite also has distinctive advantages, but is better adapted to triangulation of a lower grade than first or second order. Where the support is unstable, however, a repeating instrument can be used, for two observers can so place themselves as to make the pointings and read the circle of a repeating instrument without shifting their weight or disturbing the instrument in level or in azimuth.

Theoretically the method of repetitions is a very accurate one, since by accumulating several measures of an angle on the circle between successive readings the value of the single angle may be determined very accurately, even though the circle can be read to only the nearest 5 or 10 seconds. Experience has proved, however, that there are certain sources of error in the instrument which prevent the securing of extreme accuracy, due largely to the necessary play in the arrangement of the movable members around the vertical axis.

In the Coast and Geodetic Survey the micrometer direction theodolite, with a graduated circle 12 inches in diameter, has been associated many years with first-order triangulation. A theodolite of that size is not necessary, however, to secure the accuracy required on first-order work. Theodolites with circles only 8¼ inches in diameter have given first-order accuracy with the usual number of observations, while inferior instruments can be used if the observa-

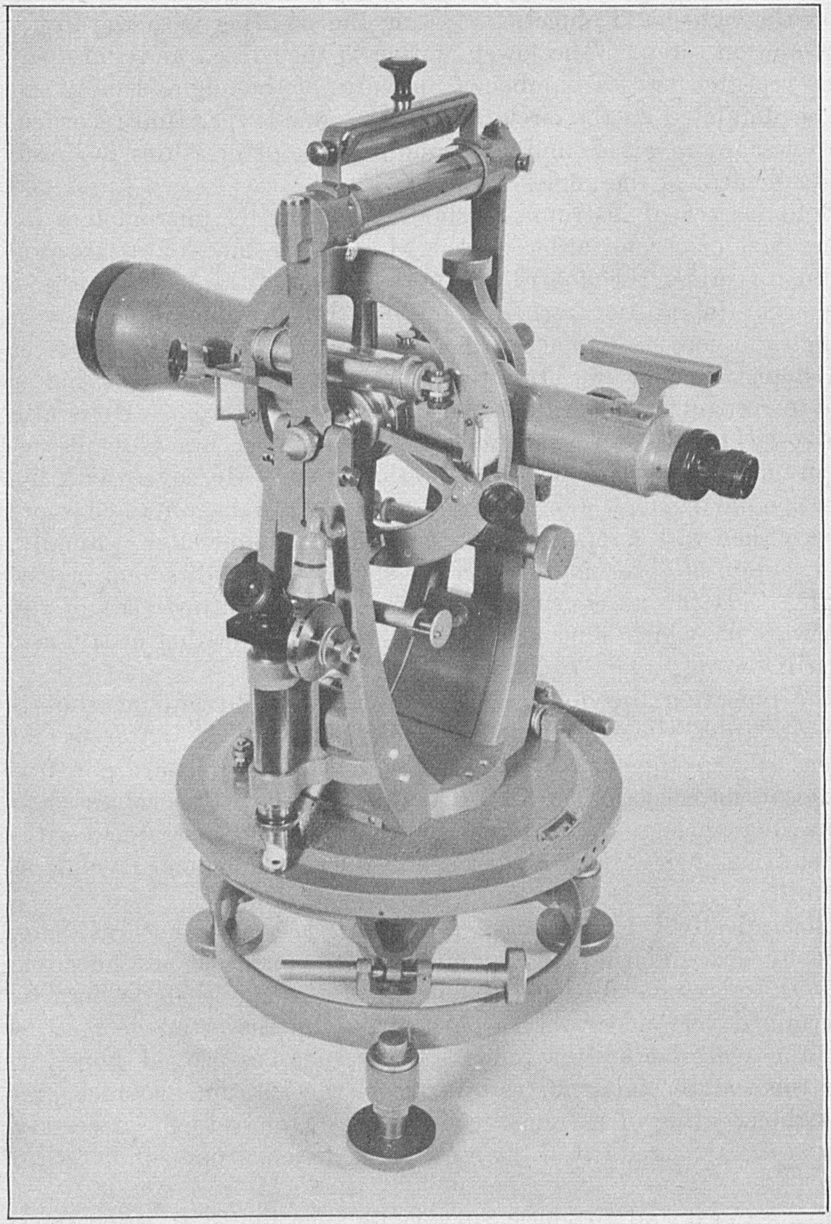

FIG. 17.—Micrometer theodolite

This instrument has a 9-inch circle, read by each of two micrometers to single seconds. When used as a direction instrument the spring opposite the lower tangent screw is replaced by a metal block in order that the azimuth center may be held firmly.

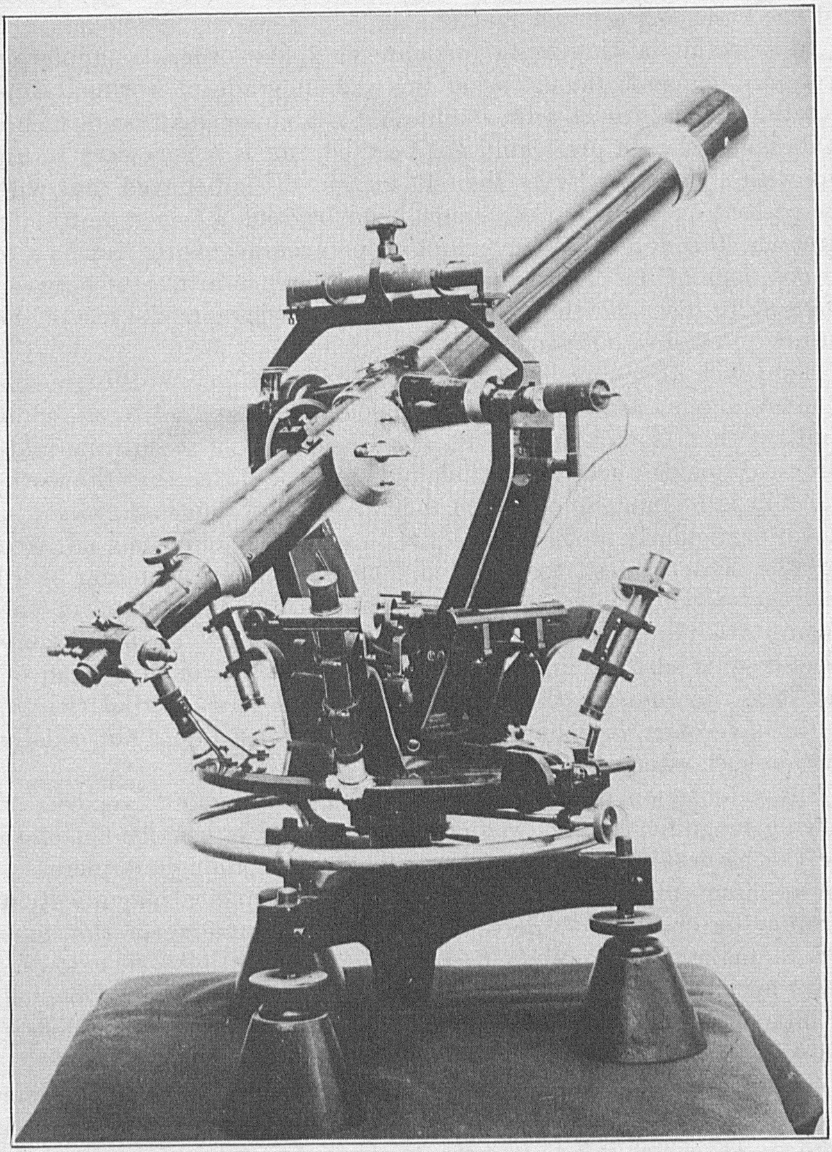

FIG. 18.—12-inch direction theodolite, Coast and Geodetic Survey model

For almost 30 years instruments like this model were used almost exclusively on first-order triangulation. Although somewhat bulky, these theodolites are very accurate.

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tions are increased in number. The highest class of instruments are ordinarily used on the more accurate grades of triangulation, as it is not economical to use inferior instruments, since the time required for the observing is much greater.

In selecting a theodolite for observing first-order triangulation economy demands that it be of the highest grade of workmanship. For the theodolites at present obtainable a circle less than 8 inches in diameter should preferably not be used, nor is it necessary to use one with a diameter larger than 12 inches. It is believed that with the present excellence in mechanical construction a 9-inch or 10-inch direction theodolite will give first-order accuracy with from 12 to 16 positions of the circle, and that the difference in weight between the 9 or 10 inch and the 12-inch theodolites warrants the use of the former on most of our work.

Eight-inch direction theodolites usually read to 2 seconds by means of micrometers, and many 10-inch theodolites also read to the same limit. Other 10-inch and all 12-inch theodolites of recent manufacture read to single seconds. It should again be stated that the workmanship is of more importance than the size of the instrument, as shown particularly in the accuracy with which the circle is graduated and the centers fitted together, and also in the construction of all micrometer and tangent screws. The proper adaptation of the magnification in the reading micrometers to the pitch of the micrometer screw is also an important factor. The micrometer head on any large micrometer theodolite may be read by interpolation to tenths of seconds, but the range of readings secured does not usually warrant such interpolation.

A theodolite is frequently supplied with two or more eyepieces of varying magnifications. On first-order work it is usually better to use the highest-power eyepiece available, even though it increases the apparent unsteadiness of the light. The higher magnification permits the observer to detect better any asymmetry of the light and to make an allowance therefor in the pointing. The lower-power eyepieces should be used in pointing upon objects to be located by intersections, which in a hazy atmosphere would not be clearly visible when a high-power eyepiece is used.

Whether or not a precise theodolite should have a vertical circle for the observing of zenith distances is a matter largely to be decided by the use to which it is to be put. A theodolite to be used on certain classes of triangulation might preferably be without a vertical circle, but for the general use of the Coast and Geodetic Survey it is a decided advantage to have in a single instrument the means of observing zenith distances as well as horizontal angles, for the problem of transportation of the instrumental outfit is thereby greatly simplified. The objection to having a vertical circle as a part of a precision theodolite is that it requires a higher mounting of the horizontal axis of the telescope above the graduated circle, especially if the telescope is to transit through the standards. Theoretically this is an objection, but in actual practice the observing of late years is made at such a rapid rate that there is little error due to the uncorrected differential change in the position of the wyes during the measurement of a single position.

On work in mountainous regions the weight of the theodolite is an important factor, yet if the instrument is made too light it is not sufficiently stable in azimuth. To increase the pressure upon the support a system of springs is sometimes used, together with an aluminum base which is screwed fast to the top of the stand supporting the instrument. Such an arrangement is shown in Figure 35, page 78.

Many other factors enter into the design of a theodolite to make it acceptable to the engineer using it, such as simplicity of construction, convenience in manipulation, compactness, and ease of adjustment; but the observer must usually direct his energies to making the best use of the instrument furnished him, instead of deciding upon the theoretically best instrument for his particular purpose. This should not deter him, however, from informing himself as fully as possible regarding the principles underlying the design and construction of theodolites, for such knowledge will enable him to better estimate the capabilities and weaknesses of any instrument supplied to him.

THEODOLITE ADJUSTMENTS

When measuring angles with the accuracy required on first and second order triangulation it is best to keep the theodolite in good adjustment, even though the program of observing tends to eliminate most of the errors due to lack of perfect adjustment. The mechanical devices provided for making the adjustments vary somewhat on different instruments, but an inspection will usually quickly disclose the method of operation. A general knowledge of the structural features of a theodolite is assumed in the description of adjustments which follows. If a new type of theodolite is to be used, and the mechanical means of adjusting it are not readily seen, proceed carefully, for a strained and weakened joint or a stripped screw thread may necessitate the return of the instrument to the shop.

Plate-level adjustment.—The purpose of this adjustment is to make vertical the line passing through the center of the spindle, which is designed to be the vertical axis of rotation, and incidentally to make the plane of the graduated circle horizontal, since it is necessary to assume that the circle is mounted perpendicular to the vertical axis. If this assumption is true, then leveling the theodolite with a properly adjusted plate bubble will achieve both results.

With the upper motion loose, bring a plate level parallel to the line joining two of the leveling screws, and bring the bubble to center with the leveling screws. Turn the alidade bearing the level 180°, checking the angle by sighting over the wyes or telescope or by reading the circle. Correct half the bubble error by the leveling screws and half by the adjusting screws on the bubble. If the theodolite is considerably out of level turn the alidade 90° and bring the bubble to center by the third leveling screw. Then place the bubble in its original position and repeat the operation until the bubble is in adjustment within less than one division, always checking the final movement of the adjusting screws by an additional testing of the bubble by reversal. After the instrument is properly leveled the bubble of the second plate level can at once be brought to the center by the adjusting screws.

Lack of verticality of the vertical axis introduces an error in the measured angle which can not be eliminated by the method of observing, and it is therefore advisable to test this adjustment and relevel the instrument at comparatively frequent intervals. Since on an inclined circle one diametral line can be drawn which is horizontal, it follows that the directions observed will be unequally affected. The error on any pointing is given by the formula

$$\text{error} = i \tan h,$$

where i is the angle of inclination in seconds as given by the bubble at right angles to the line of sight, and h is the angle above or below the horizon of the object sighted upon. The error of any angle is obtained by combining the mean errors of the two directions involved. The magnitude of the errors due to an inclined circle is indicated by the following table:

Errors in horizontal directions due to inclination of vertical axis of instrument

Inclination of the vertical axis (i)	Angle of elevation or depression of the line of collimation (h)	Correction to horizontal direction ($i \tan h$)
″	′	″
10	20	0.06
20	20	0.12
30	20	0.17
10	40	0.12
20	40	0.23
30	40	0.35
10	60	0.17
20	60	0.35
30	60	0.52

Striding-level adjustment.—In describing the adjustments of the level the term "bubble axis" denotes that horizontal line, tangent to the surface of the centered bubble, which lies in the vertical plane through the axis of the bubble tube.

The purpose of the striding-level adjustment is to make the bubble axis parallel to the horizontal axis of the telescope. Strictly speaking, the bubble axis is brought into parallelism with a line which approximates more closely the true horizontal axis the more nearly the telescope pivots assume the form of true cylinders having equal diameters and with their major axes in alignment.

To make the adjustment place the level in position on the pivots of the horizontal axis of the telescope. Bring the bubble to the center with the leveling screws. Test for wind by rocking the level slowly forward and back on its supports. If the bubble does not remain centered then the bubble axis and the horizontal axis of the telescope lie at an angle to each other when projected upon a horizontal plane, and the level is said to have wind. Correct the defect by the screws at one end of the tube, which permit a lateral adjustment.

After the adjustment for wind is perfected bring the bubble to the center again by the leveling screws and then reverse the level. If the bubble does not return to the center adjust half the discrepancy by the foot screws and half by the adjusting screws on the level. Repeat the test and adjustment until the lack of adjustment does not exceed one or two divisions of the level.

Adjustment of standard.—The purpose of this adjustment is to make the horizontal axis of the telescope perpendicular to the vertical axis of rotation of the alidade.

If the plate level is but little inferior in sensitiveness to the striding level, as is usually the case with the larger direction theodolites, the adjustment is most easily made by first adjusting the plate and striding levels, as previously described. Next make the alidade axis truly vertical by means of the plate level. Place the striding level in position; if the bubble of the striding level is not centered, adjust the whole discrepancy by the device provided for raising or lowering one of the standards. In a few instruments no mechanical arrangement is made for the standard adjustment, and the only way it can be made is by carefully filing down and polishing the higher standard. This should be done in the field only when the lack of adjustment is large, and will usually not be necessary unless one of the standards has been knocked out of true position. If the pivots of the telescope are appreciably unequal in diameter, allowance must be made for that inequality in adjusting the standard.

If the vertical axis of the alidade has not been made truly vertical by the plate level, then the amount by which the striding-level bubble is displaced by rotating the alidade 180° will have to be corrected, half by the standard adjustment and half by the leveling screws of the instrument, the testing and the adjusting being continued until the bubble shows no displacement upon rotation through 180°

When the axis of rotation of the alidade is vertical the error introduced in the measured horizontal angles by the horizontal axis of the telescope not being at right angles to the vertical axis of the alidade is completely corrected for by the reversing of the instrument in the middle of the observation.

Inequality of pivots.—If the pivots are unequal in diameter the defect can be detected and the amount of the inequality determined by placing a carefully adjusted striding level on the pivots, bringing the bubble to center, reversing the telescope in the wyes so that each pivot lies in a different wye than at first, and then reading the level. With the method of observing in use in the Coast and Geodetic Survey, whereby there is no change of the pivots in the wyes during even a season's work, no error in the angle measures results from the pivots being unequal in diameter.

Irregularity of pivots.—Ordinarily if the pivots are not truly cylindrical a striding level placed in position on the pivots will change its reading as the telescope is slowly rotated on its horizontal axis. This irregularity will cause errors in the measured horizontal angles if it is of any considerable size.

If either irregularity or inequality of pivots exists that fact should be called to the attention of the office when the instrument is returned at the end of the season. in order that the pivots may be reground.

Focusing adjustment.—The error due to change of focus is eliminated by the usual method of reversal of the telescope during the observations.

Adjustment for parallax.—Point the telescope toward a light surface, such as the sky. Screw the eyepiece of the telescope in or out until the wires show the sharpest and blackest. Next focus the telescope on a distant object, and then test the adjustment by moving the eye slowly across the front of the eyepiece. If the wires appear to move over the image of the object sighted upon, parallax is present. The focus of the object glass of the telescope should be changed until the objective is at the proper distance to cause the image of the object sighted upon to fall exactly on the plane of the cross wires; in this position no movement of the wires over the field of view will be apparent when the eye is moved across the eyepiece. The adjustment for parallax must be closely watched, for the error due to lack of proper adjustment is not eliminated by the method of observing. It is especially noticeable if the eye of the observer, because of a strained position, is not in front of the exact center of the eyepiece.

The adjustment of the eyepiece must also be tested frequently, for as the eyes of the observer tire the focal distance of the lenses of the eye changes, causing a blurring of the wires and an increased effort in the centering of the image of the light between the sighting wires.

Adjustment for verticality of sighting wires.—To test if this adjustment is necessary, point upon a well-defined object. Swing the telescope slowly in elevation while watching the position of the object in the field of view. If the object changes its position with relation to the vertical wires as the telescope changes in elevation, the diaphragm must be rotated around the longitudinal axis of the telescope. An examination of the telescope will usually quickly disclose the mechanical means for accomplishing this purpose.

Collimation adjustment.—The purpose of this adjustment is to make the line of collimation of the telescope perpendicular to its horizontal axis. To make the adjustment point the telescope upon some sharply defined object, and with the alidade clamped lift the telescope from the wyes, rotate it 180° around its longitudinal axis, and replace it in the wyes, the pivots being in different wyes than in the original position. If the object is not bisected after reversal correct half of discrepancy by shifting the reticule by means of the screws provided, and again bisect the object by using the tangent screw. Repeat the test to check the adjustment.

If the instrument can not be reversed in the wyes set a stake, *A*, several hundred feet distant and bisect it with the wires. Plunge the telescope with the alidade clamped and set a second stake, *B*, in the opposite direction at almost the same distance to avoid having to change the focus. Both points should be as nearly as possible in the plane of the horizon of the instrument to prevent errors due to imperfect leveling. Rotate the alidade about its vertical axis and then bisect the stake *A*. Plunge the telescope with the alidade clamped, and if the wires do not bisect stake *B* set another stake, *C*, in the line of sight close to *B*. Set a fourth stake, *D*, one-fourth the distance from *C* toward *B* and adjust the wires by means of the reticule screws to bisect *D*. Check the adjustment.

A procedure which will give an approximately correct adjustment for collimation is to point on some sharply defined distant object with both motions clamped, read both micrometers, plunge or reverse the telescope, loosen upper motion and set the micrometers so the mean reading will be exactly 180° from the mean of the first readings. If object is not bisected correct half the discrepancy by the reticule screws and half by the tangent screw. Repeat the test as a check.

If the collimation adjustment is perfect the line of sight defined by the center point of the sighting wires describes a plane perpendicular to the horizontal axis when the telescope is rotated on the horizontal axis. When the adjustment is not perfect the line of sight describes a cone. The correction for the error of collimation is equal to $c \sec h$, where c is the angle of error in the horizon and h is the altitude of the object sighted upon. The error of collimation is eliminated from the result by taking the mean of the direct and reversed readings.

Adjustment of centers.—In precision theodolites so little tolerance is permitted in the fit of the centers that an adjustment is often necessary to regulate the variations of friction caused by wear and by changes of temperature. This usually takes the form of a nut or screw at the lower end of the vertical axis which, by pressing upward upon the lower end of the spindle, lessens the weight of the alidade and telescope upon the conical bearing surfaces. This adjustment must be made with caution, for if too much of the weight is removed from the bearing surfaces there is play in the centers, with a resultant loss of accuracy. To make the adjustment, raise the vertical axis with the adjusting screw until the alidade appears to move just freely enough on its vertical axis. Test for play in the conical bearings by pointing on an object and noticing if a slight pressure on the alidade will move the telescope off its pointing. A better test is to point the telescope on some well-defined object, read the circle, swing the telescope around clockwise, and again point and read. Repeat the process, except that the alidade is swung in the opposite direction. If a series of three or four sets of such alternating pointings shows the effect of drag, the pressure of the screw upward on the vertical axis should be increased; if the readings are erratic and cover a considerable range, the pressure should be lessened.

Adjustment of level on vertical circle.—Such theodolites of this survey as are equipped with vertical circles have that circle rigidly attached to the horizontal axis of the telescope, with the level bubble mounted on the vernier frame, the verniers not being adjustable. Since the method of observing entails the reversal of the level during each measure of a vertical angle, thus eliminating the error of the level, it is customary to leave the level bubble on the vertical circle unadjusted.

Micrometer adjustment.—The micrometer microscope is a most satisfactory device for measuring accurately the angular value of any part of the interval between adjacent marks of a graduated circle. It consists essentially of a compound microscope with a micrometer box mounted between the objective and eyepiece, at such a distance from each that the movable wires in the micrometer box can be brought into the focal plane of each lens system. The principle of operation will be more readily understood after the mechanical details of the micrometer box have been described.

The mechanical arrangements of the box vary somewhat on different instruments, but a typical arrangement illustrated in Figures 19, 20, and 21, will be described. An outer case, c, into which are screwed from opposite sides the objective tube and the eyepiece, contains a slide, *d*, carrying the comb, *e*, the center notch of which, taken in conjunction with the zero of the micrometer head, *f*, furnishes a fiducial point for all readings. The slide for the comb is adjustable transversely by the screw, *g*, acting against the spring, *h*, movement in other directions being prevented by the machined surfaces of the slot in which the slide moves.

A moveable slide, *i*, bearing two wires, is fastened rigidly to a finely machined screw, *k*, on which the micrometer head, *f*, and the attached

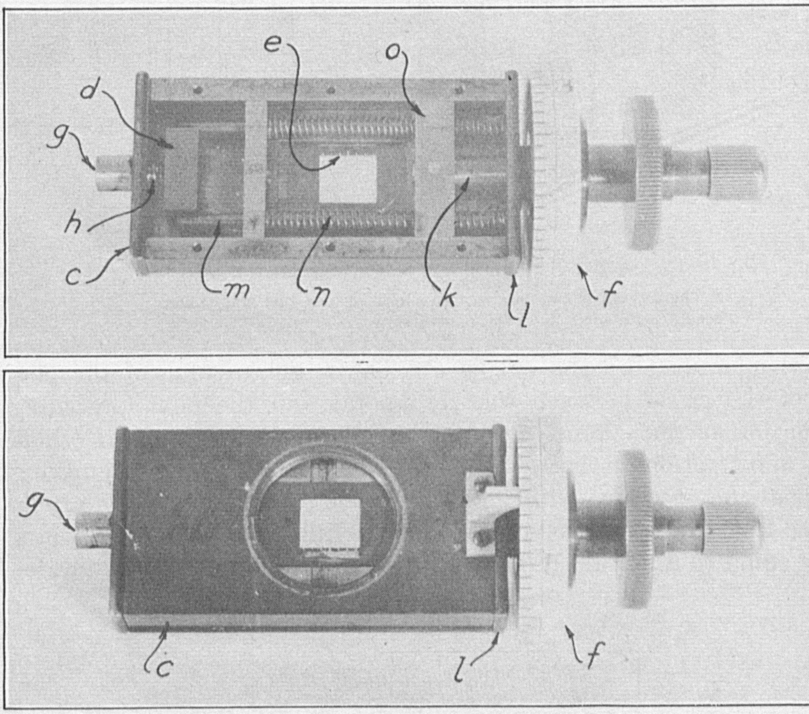

FIG. 19.—Micrometer box

Upper view: Lower side of box with plate removed.
Lower view: Upper side of box with eyepiece removed.

spindle works, a washer on the micrometer head bearing against the end, *l*, of the micrometer case. In reality there are usually two pairs of these parallel wires, but for simplicity the function and adjustment of one pair will be first explained and the use of the second pair will be explained later. Rigidly attached to the case end, *l*, on its inner side are two rods, *m*, around each of which is a helical spring, *n*. The rods and springs pass through holes in the upright partition, *o*, of the slide, *i*, and the rods fit into depressions in the other upright partition of the slide, while the springs bear against its inner surface, thus maintaining a pressure of the slide and its screws against the threaded bearing in the head, preventing slackness and play.

With this construction the theory of operation is readily seen. The objective forms a magnified image of a small portion of the graduated circle in the plane of the parallel wires. This image in

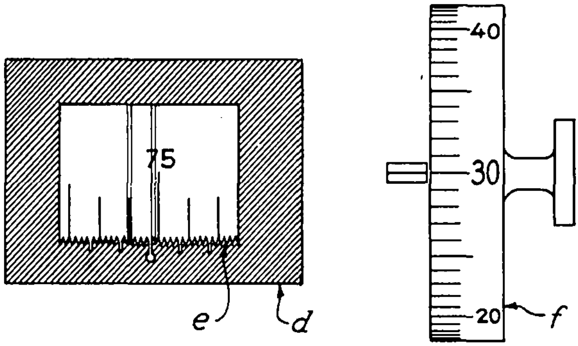

FIG. 20.—Field of view of micrometer box

This diagram shows the appearance in the field of view of the microscope, comb, wires, and circle graduations.

turn is magnified by the eyepiece. The angular value of the portion of the circle between the zero point and the next preceding graduation of the circle can then be measured in terms of whole turns and fractional turns of the micrometer screw. The magnifying power of the objective and the pitch of the micrometer screw are so related that the adjustment can make one full turn of the micrometer screw equal to a minute of arc, or on some instruments to 2 minutes

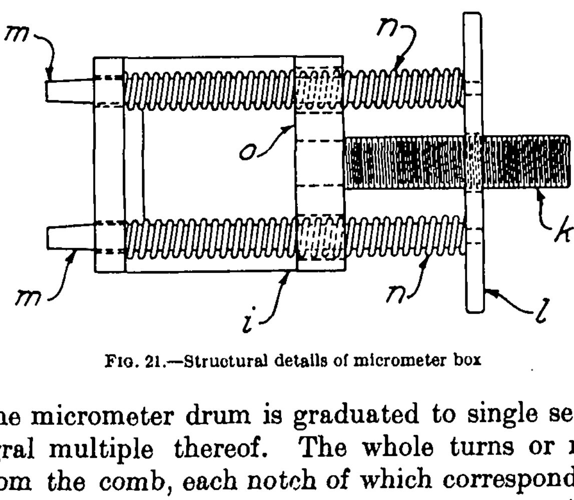

FIG. 21.—Structural details of micrometer box

of arc. The micrometer drum is graduated to single seconds, or to some integral multiple thereof. The whole turns or minutes are read off from the comb, each notch of which corresponds usually to 1 minute of arc, while the fractional part of a minute is read from the micrometer drum in seconds.

A single micrometer microscope is in correct adjustment when the following conditions are fulfilled: First, when the micrometer lines and image of the graduated circle are so closely in the same plane that no parallactic movement can be detected by shifting laterally the position of the eye; second, when five revolutions of the screw will exactly traverse a 5-minute space on the circle (the adjacent graduations of the circle being usually 5 minutes apart on theodolites of precision); and third, when the micrometer reads zero seconds with the pair of parallel wires coinciding with the zero point on the comb. *After* these three conditions are fulfilled the micrometers must be spaced around the circle at equal intervals.

Eyepiece adjustment.—Adjust the eyepiece by sliding or screwing it in or out until the parallel wires are in the most distinct focus. When this is accomplished the comb will be sufficiently visible.

Adjustment for focus.—Loosen the screws which hold the microscope tube in the bracket clamps and move the tube up or down until the fine scratches on the circle due to polishing are as distinct as possible, then tighten the screws.

Radial adjustment of microscope.—There is usually some sort of hinged joint provided for adjusting the objective of the microscope radially. This adjustment should be made so as to bring the outer edge of the graduations near the center of the field of view, yet still leave the degree numerals visible.

Adjustment for parallelism.—If the parallel wires of the diaphragm are not parallel to the graduation marks on the circle, either turn the micrometer box slightly on the tube or loosen the binding screws and turn the tube slightly in its supports.

Adjustment of comb.—If the zero notch of the comb, usually marked by a deeper cut than the other notches or by a hole beneath it, is not in the center of the field of view adjust it by the screw *g*.

Adjustment of graduated drum.—Center the pair of parallel wires on the zero notch of the comb, then hold the spindle of the micrometer firmly and turn the graduated drum on its friction mounting until the zero of the drum coincides with the index line, *p*, mounted on the micrometer case adjacent to the drum. Check to see that the parallel wires have not moved off the zero of the comb. If the drum is held in position by a screw as well as by a friction mounting, the screw must, of course, be loosened before making the adjustment.

If the micrometer screw is threaded to have one revolution equal to 2 minutes of arc, then the graduated drum will have two graduations on it marked zero seconds, the numerals of the second minute usually having some distinguishing characteristic. Care must be taken always to adjust and read to the proper zero mark.

Adjustment for run.—By "run" of the micrometer is meant the difference in seconds of arc between the intended value of one turn of the micrometer screw and its actual value as determined by measuring with the micrometer the space between two adjacent graduation marks of the circle. This quantity has sometimes been called the "error of run," but the former term seems preferable and is more generally used.

Assuming that adjacent graduations of the circle are 5 minutes apart and that one revolution of the micrometer screw will cause the parallel wires to traverse approximately one-fifth of the interval between adjacent graduations, the purpose of the adjustment for run is to bring about, as closely as may be, the condition that exactly five revolutions of the screw will move the parallel wires from one graduation to the next. The theory of the adjustment may be seen from the fact that the portion of the graduated circle and its magnified image are at conjugate foci of the objective, and that the magnification of the image is represented by f'/f, where f and f' are the distances of the circle and its image, respectively, from the center of the objective. The magnification of the image may be increased by decreasing the distance of the objective from the circle, and decreased by moving it in the opposite direction. Therefore, if fewer than five complete turns of the micrometer screw are necessary to move the parallel wires from one graduation to the next the image needs to have its magnification increased, and the objective should be moved nearer the circle by protruding it from the tube by its screw adjustment.

When the objective is protruded from the tube to correct for run the image is thrown farther up in the tube. Without changing the relation of the eyepiece to the micrometer wires, move the whole micrometer tube upward to bring the image of the circle into the plane of the micrometer wires. This process may have to be repeated several times to get the run down to the required limit.

If more than five complete turns of the screw are required to make the parallel wires traverse the space between adjacent graduations the adjustment should be made in the direction opposite to that described above.

In actual practice the preliminary tests for run should be made on five or six equidistant parts of the circle, since the error of run will vary somewhat due to eccentricity of the circle, and the adjustment should be made on a portion of the circle where the error is near the mean. Also, the final adjustment should be tested by taking the mean of at least 10 readings of the screw value of the space between graduations, and these readings should be made a part of the record of horizontal directions.

The mean run of a single micrometer should not exceed 2 seconds, and the algebraic sum of the runs of all micrometers should not exceed 1 second. If the magnification of the micrometer is small and the required correction is very slight, the adjustment may be made by raising or lowering the entire tube instead of changing the objective in the tube, since the change in distance affects the run at a much more rapid rate than it does the parallax or distinctness of vision.

The accuracy of micrometric readings depends largely upon the proper mounting of the pairs of wires used for reading. They should be at such distance apart that a narrow strip of the bright surface of the arc is visible on either side of the graduation when the wires are accurately centered astride it. They should be parallel as closely as the eye can judge and should be adjusted to be parallel to the graduations. If the wires are not parallel or are slack they should be remounted, as described on page 42. They should also be heavy, black, and smooth.

The formula for the correction for run may be found in various texts in different forms. That given below from Crandall's Geodesy is perhaps as convenient as any.

With a single pair of wires:

Let a = backward reading of micrometer.
b = forward reading of micrometer.
r = average run of micrometer, plus when $a > b$.

$$\text{Correction to } a = \frac{-r}{300''}\, a.$$

$$\text{Correction to } b = \frac{r}{300''}\,(300'' - b).$$

$$\text{The mean, } \quad m = \frac{a+b}{2}.$$

$$\text{Correction to } m = \frac{r}{300''}\,\frac{300'' - (a+b)}{2}$$

$$= \frac{r}{2} - m\frac{r}{300''}. \qquad (1)$$

For convenience and speed in making the readings it is usual in this survey to have two pairs of wires mounted approximately 4 minutes apart. The reading on the graduation next preceding the zero of the comb, called the forward reading, is made with the (apparent) left-hand pair of wires, and the reading on the mark next following the zero of the comb, called the backward reading, with the right-hand pair.

A consideration of formula (1) above shows the following:

(*a*) When reading a single pair of wires on the two graduation marks adjacent to the fiducial point of the micrometer, the total correction for run is the same whether applied separately to the two readings or to their mean.

(*b*) Where two pairs of wires are used mounted at an arbitrary distance apart (less than 5 minutes), the correction for run can be applied to the reading of each pair of wires separately, as when a single pair of wires is read on one graduation only at a setting.

(*c*) With either one or two pairs of wires the correction for run may be disregarded, provided (1) the run for any one micrometer is less than 2 seconds and the algebraic mean of all is less than 1 second, and (2) that the initial settings are distributed approximately uniformly throughout the space between adjacent divisions.

Adjustment for equidistance of microscopes.—After the individual adjustments have been made, set microscope A exactly on an even degree graduation mark with micrometer comb and drum reading zero. To set microscope B at 120° or 180° distance around the circle, depending on whether it is a three-micrometer or a two-micrometer theodolite, proceed as follows: Note if the zero of the comb on B, when near the center of the field of view, is very distant from the proper degree mark. If it is move microscope B in the proper direction by whatever mechanical means for such adjustment is provided until the proper degree mark is near the zero of the comb. (Microscope A is not usually adjustable circumferentially.) Then move the comb by the screw *g* until the zero mark coincides with the proper degree mark, and adjust the graduated drum to read zero when the parallel wires read zero on the comb, as already described. If the theodolite has a third micrometer microscope it should be adjusted to microscope A in the same way.

Because of eccentricity of the circle, the micrometers will not maintain a constant difference when read on different portions of the circle. A sufficient number of readings should be taken around the circle to determine the approximate amount of this eccentricity and the adjustment for equidistance should be made at a point where the eccentricity can be closely estimated. It facilitates the taking of means of micrometer readings if microscope B is set sufficiently ahead of microscope A that B will seldom read less than A.

Adjustment of reading microscope.—On three-micrometer theodolites it is usual to have a small reading microscope with a single wire on its diaphragm by which the degrees and the next preceding 5-minute graduation of the circle is read. Such a microscope should be adjusted to read minutes and seconds with micrometer microscope A by whatever means are provided for such adjustment.

Illumination of circle.—When the circle is illuminated by artificial light for making micrometer readings it is important that the circle be evenly illuminated from above, or else that the light be held normal to the circle directly opposite the graduation to be read; otherwise there will be an appreciable error due to phase.

CARE OF THE THEODOLITE

The most important rules to observe in caring for a delicate surveying instrument are to handle it carefully and keep it clean. When an instrument is received from the office unpack it carefully and slowly, noting the exact manner in which it is fitted into its packing case, and when replacing it in its case always avoid forcing any part into place. Avoid knocks and jars as far as possible, for many of the parts of the instrument are delicate and easily damaged. When handling a theodolite lift it entirely by the tribarch or by the lifting ring, and never by the standards or micrometer arms. Avoid setting up screws too tightly, especially capstan screws operated by adjusting pins, for the fine threads are easily stripped.

A necessary antecedent to caring for, adjusting, and repairing an instrument is for the observer to familiarize himself with the principles and details of its construction. This does not necessarily mean that before observing with a new instrument it should be dismantled and reassembled, but the observer should seize the first opportunity to see any part with which he is not familiar dismantled in the instrument shop of the survey, or by a more experienced observer. He should also study out any detail of construction which is not at once evident, in order to recognize more quickly the cause of any trouble which might develop with the instrument.

When moving by truck, a large sack containing excelsior or similar material, or a pad made of blankets placed under the theodolite box, will lessen the effect of vibration and jars. Always make sure that the theodolite is securely fastened in its case. When packing for a long shipment fill vacant spaces around the instrument with paper, as this will prevent any object which may become loose from injuring the instrument; but do not use loose excelsior for that purpose, for the dust from it is very penetrating and is injurious to the working surfaces of the instrument.

A good observer almost invariably keeps a clean instrument. The atmospheric conditions met with in field work are very severe on metal surfaces, which will rust if not oiled but will collect dust and grit if an excess of oil is used. The best thing to do is to go over all exposed surfaces each time the instrument is used, allowing time for this and for the adjustments before it is time to begin observing. When the instrument is first set up brush off the dust from the enameled or painted surfaces first, then go over the working surfaces, such as wyes, pivots, and exposed screws, with a soft rag very lightly oiled with a light oil. Next rub a soft dry rag over the surfaces which have been oiled to remove all oil except the film adhering to the metal. If the air is dusty during the observing clean the pivots and the wyes frequently to avoid error and wear. A clean finger will clean the wyes of dust and will usually leave the right amount of oil on the metal. Moving the telescope through a small arc on either side of its horizontal position at intervals during the observing may also improve the seating of the pivots in the wyes when the atmosphere is very dusty. When packing an instrument for a long shipment, especially by sea, all polished surfaces should be coated with a fairly heavy oil to prevent rusting.

The centers and the micrometer slides require special treatment. It frequently happens that a large amount of friction develops in them when low temperatures are encountered. This often indicates that there is an excess of oil on the bearing surfaces, which are fitted together with a very small tolerance. The parts affected should be taken down, all the old oil wiped off, porpoise oil applied and again wiped off with a dry, soft rag free from lint, and the parts again assembled. The film of oil left on the metal surfaces will afford sufficient lubrication. On some theodolites there is an adjustment provided for the centers for change of temperature, but even in such cases there should be no excess of oil in cold weather, for if it exists there is either excessive friction or undue play.

The outer surfaces of the lenses require frequent cleaning, but should be rubbed as little as possible. First brush off the dust with a camel's-hair brush, then take soft paper or an old linen rag and lightly flick the surface to remove what dirt may remain. If further cleaning is necessary soft paper free from silicious particles, which are found in most paper, may be moistened and rubbed very lightly over the surface. A greasy film may be removed with paper or rag moistened in alcohol, but if an excess of alcohol is used it may penetrate between the component parts of the lens and affect the balsam which is sometimes used to cement the lenses together. A lens which still remains cloudy after the above treatment can not be cleared by field methods.

The component parts of a compound lens should not be taken apart in the field except in an emergency. When it is necessary to take apart a compound lens the component lenses must be so marked that they can be reassembled exactly in their former relative positions

Emergency repairs.—Even though extreme care be observed, the conditions of transportation incident to field work frequently make necessary a certain amount of repairs to instruments during the field season. The delay to the party which would result from awaiting a relief theodolite makes it advisable for the observer to make emergency repairs where possible.

The repair job most frequently encountered in the field is replacing one or more sighting "wires," either in the telescope or the reading microscopes. This requires care and patience, but is not difficult if the proper materials and appliances are at hand. In anticipation of such a contingency each chief of party should obtain from the office a spider's cocoon, a solution of pure shellac in alcohol, and a small piece of beeswax. A watchmaker's magnifying glass is a convenience, though an ordinary magnifying glass may be used, or a binocular objective, or the reading glass for the verniers of a theodolite may be mounted in a position to answer the purpose very well. A description of the method of installing the wires follows:

Take the micrometer apart carefully, in order not to break any wires which do not need replacing. Clean off with alcohol all dirt and shellac from the slide where the wires are to be mounted. If only one wire of a pair is broken it is often impossible to properly clean the slide without removing the other wire; but if this is necessary it is of little consequence, for a pair of wires can be installed almost as easily as one.

After cleaning the slide place it in a stable position on a good reflecting surface so the wires will be easily seen. Attach a bit of beeswax to each of the points of a pair of dividers, or to each end of a piece of wire bent into the form of a V with the points turned down, and to one point attach one end of a thread of the cocoon. With the cocoon suspended from the point wrap the thread two or three times around the point and then catch the thread on the other point, wrapping it two or three times around that point before cutting it. Stretch the thread until the kinks disappear, then hold it for a few seconds in warm (not hot) water; stretch it a bit more, again immerse it in water, and repeat the operation until two or three threads are broken and you can judge when to stop the stretching just short of the breaking point. Then with a thread fully stretched between the points of the dividers, place the points astride the slide so the thread will be in approximately the correct position, as shown by the scratchers on the slide. Block up the points of the dividers so the thread will not be stretched too much.

If a pair of wires is to be mounted use another pair of dividers and place the second thread in position also, the points of one pair of dividers falling outside the points of the other pair. With the aid of a magnifying glass adjust the threads with a needle until they are exactly parallel and properly spaced, as shown by the scratches on the slide. Finally, with the eye end of a needle place a very small drop of shellac on each end of each wire to cement it to the slide, and after allowing it to dry for a few minutes the dividers may be cut loose. The shellac must be of such quality that it will spread immediately upon application into a thin film over the metal surface, otherwise the thread will not be held taut. A web so mounted will rarely slacken in wet weather. The observing wires in the telescope should be mounted from 25 to 35 seconds apart to secure the best results on lights of average size. This can be tested by computing a 30-second intercept at some convenient measured distance.

Instead of cocoon threads, fine tungsten wire may be used for diaphragm wires, but the tungsten must be thoroughly cleaned with alcohol, stretched in place under considerable tension, and fastened by several coats of a very thin solution of collodion. The cocoon threads are much easier to mount in the field than the tungsten wire.

The repair of broken parts is largely a matter of ingenuity, combined with a knowledge of what is essential to the proper working of an instrument. Stripped threads on screws may sometimes be made to hold temporarily with gum or sealing wax, provided they are not such as require moving in adjusting, or a pin of hardwood may be used in place of the broken screw. Broken plate-level mountings have been temporarily replaced with sealing wax, and even a broken micrometer microscope bracket has been made sufficiently rigid with properly shaped pieces of wood wrapped with cord or wire which was then stretched with wedges. Resourcefulness is a necessary quality for a triangulator in unfrequented regions.

If erratic results are being obtained in the observations which can not be otherwise accounted for, the entire structure of the theodolite should be scrutinized in detail. See that the lenses of the objective are tight in the case, and tighten the inner screw ring if they are not. Next examine the eyepiece end of the telescope to see that the eyepiece tube fits tightly into the telescope barrel and that no screws are loose. Examine the foot screws to see that the clamping screws hold them firmly. Inspect the horizontal axis, standards, tangent-screw assembly, and microscope brackets for fractures, and see that the graduated circle is screwed firmly to its seat. Also test the junction of the barrel of the telescope and its horizontal axis, and the junction of the seating of the object glass and the telescope.

If the agreement of the separate measures of a direction is satisfactory but the closing errors of triangles large, the cause is probably not in the instrument but in those atmospheric conditions which cause lateral refraction, or else is due to instability in the stand or in the mounting of the theodolite. The trouble may also be due to the eccentricity of the lamps or other objects sighted on. There is always a reason for poor results, and the observer should not rest satisfied until he has found it.

HOW TO ESTIMATE THE QUALITY OF A THEODOLITE

In deciding what observing program will give most economically the accuracy desired for any class of work it is necessary to know the quality of the instrument, which, however, is not measured by its size or by the minuteness of the least reading of the micrometers. The best measure of the excellence of a theodolite is its performance in actual field work, but if a new theodolite is being used it is necessary to apply other tests.

A preliminary examination will show a great deal about the workmanship and accuracy of the instrument. The four structural features which must be scrutinized to form an estimate of the accuracy of a theodolite are: The graduation of the circle; the design of the micrometers and the workmanship on them; the fit of the centers and tangent screws; and lastly, the optical properties of the telescope. Although the accuracy of the graduation of the circle is the most important one, each of these features must be entirely satisfactory in order to secure the best results.

Before testing an instrument place it in as good adjustment as possible by methods which are explained on pages 29 to 40. Test the centers for fit and play by pointing the micrometer on graduations on different parts of the circle, and with the micrometer wires centered on a graduation and the circle clamped test the movement of the wires resulting from gentle pressure upon different parts of the graduated circle and upon the alidade. The friction in the centers can be judged by unclamping the alidade and applying a gradually increasing tangential force to the alidade until motion about its vertical axis occurs.

The relation between the greatest magnification obtainable by the telescope and the pitch of the tangent screw should be such that a barely perceptible movement of the tangent screw should cause a barely perceptible movement of an object across the telescope wires. A similar relation should exist between the magnification of the micrometer telescopes and the pitch of the micrometer screws.

The tangent screw assembly should be tested for friction by noting in the telescope if any lag is apparent in the motion of the telescope when the tangent screw is moved slightly away from the spring. The micrometer screws should be tested in the same way. Theoretically the final motion of a tangent screw or a micrometer screw should always be against the spring. As a matter of practice it has been found by extensive tests that if micrometers and tangent screws are properly made and kept clean there are no appreciable errors resulting from making the final pointing by moving the screw indiscriminately against or away from the spring. If it is found, however, that with the screw properly cleaned and oiled there is a lag when the screw is moved away from the spring, then either the spring must be strengthened or the final movement of the screw must always be against the spring.

The design and construction of the micrometers can best be tested by clamping the alidade and taking a series of readings, about 20 in number, upon some one graduation. The total range of the readings should not exceed two divisions on the micrometer drum.

There is no method available to the engineer in the field to determine accurately the errors of graduation of the horizontal circle, but the following process will give a very good measure of the combined excellence of the graduation and the micrometers. Careful readings are taken on each of two micrometers at equal intervals around the circle, say 10° apart, and from those readings a curve similar to that shown in Figure 22 is drawn as a mean of the plotted differences of the readings.

Such a curve will show three things: First, the algebraic mean of the differences (A-micrometer reading minus B-micrometer reading) represents the failure of the B-micrometer to be exactly 180° from A, and also indicates the amount the horizontal reference line would have to be moved to make the sum of the plus ordinates equal to the sum of the minus ordinates; second, the amplitude of the mean curve drawn through the points represents the eccentricity of the graduated circle with reference to the axis of rotation of the micrometers, combined with the accidental and short-period errors at the maximum and minimum points, the eccentricity curve being a sine curve; and third, the variation of the plotted differences from the mean curve is a measure of the local and accidental errors of graduation, combined with those resulting from reading the micrometers. With a 10 or 12 inch circle this variation from the mean curve should seldom exceed 1 second and should never exceed 2 seconds. Errors due to eccentricity are entirely eliminated by reading two or more micrometers spaced at equidistant intervals around the circle.

With some theodolites a tendency is noticed for certain positions of the circle to give values for all directions either higher or lower than the mean, irrespective of the size of the angle between the initial station and the object sighted upon. Such a condition can be caused only by the graduation at the setting for the initial station being in error in its angular position with reference to the mean of the other graduations of the circle. After a number of stations have been occupied, inspection will often disclose the existence of this defect. It can be verified by tabulating for each setting the variations from the mean value of a direction, taking a sufficient number of directions to insure that the effect of the errors in position of the graduations read when the object other than the initial is sighted upon will approach zero in value. Figure 22a shows a curve constructed to show the results of such a tabulation. It indicates that position No. 7, corresponding to a setting of 67° 03′ 10″, is consistently too low, and that position No. 14, corresponding to a setting of 146° 01′ 50″, is too high. When such a condition is found to exist the initial settings affected should be changed an even degree, either higher or lower. If the error of graduation is accidental in character, or is one of short period, the tendency for that position to give directions higher or lower than the mean may be corrected by the change in the initial setting. Such a small deviation in the

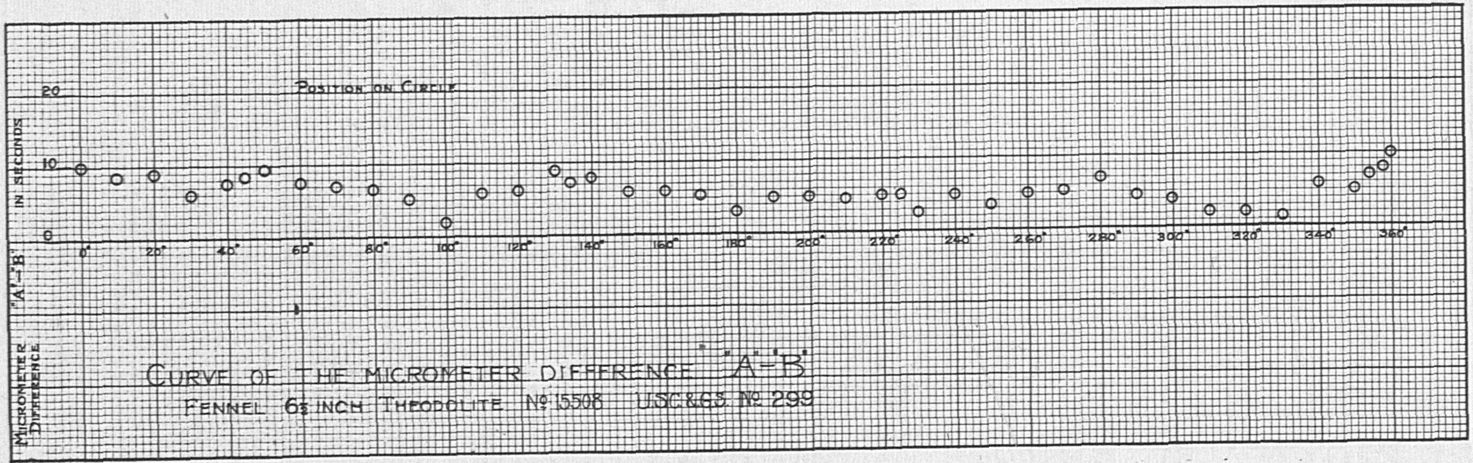

FIG. 22.—Differences of readings, A-micrometer minus B-micrometer, at uniform intervals around the theodolite circle

The construction and study of such a diagram gives valuable information regarding the quality of a theodolite.

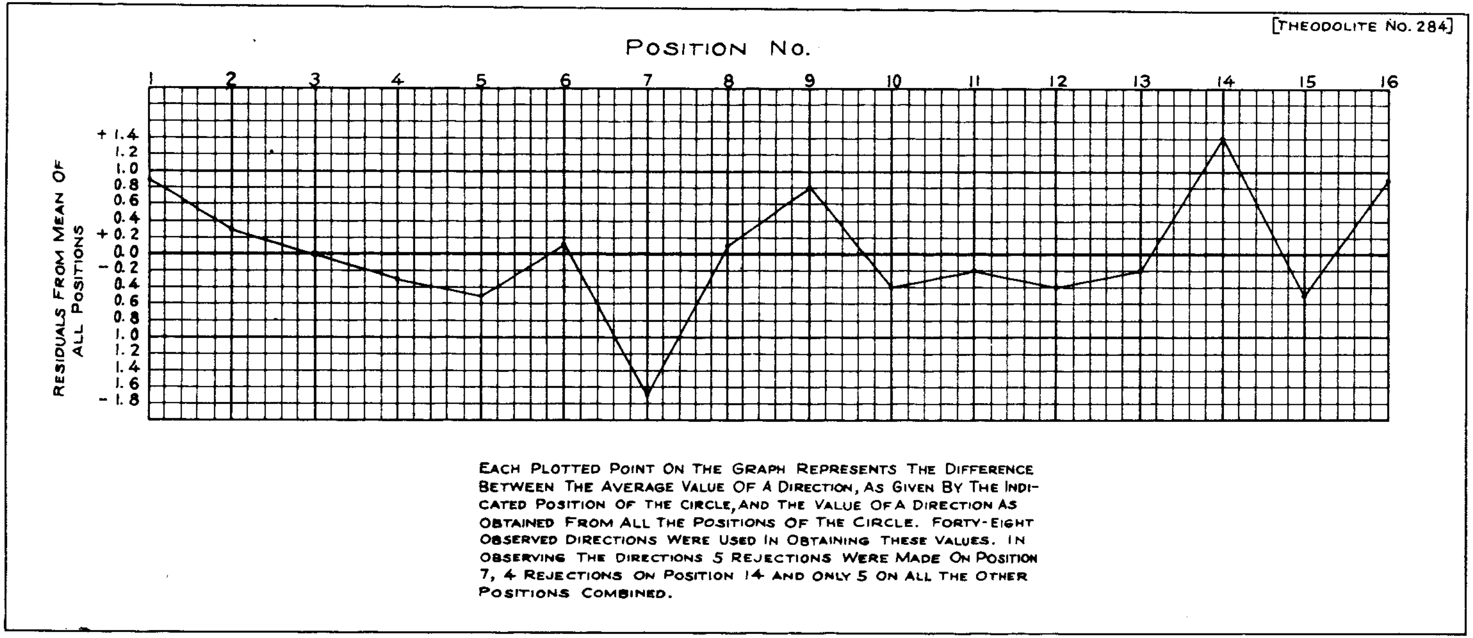

FIG. 22a.—Relative errors of graduations of theodolite circle at different position settings

settings from the theoretical best distribution for correcting errors of graduation will have a negligible effect upon the final accuracy.

A similar change in the setting of the circle may often be used to advantage when it is necessary to reobserve a position because of a rejected value for a direction, even though the initial setting is not suspected of being subject to unusual error, for a micrometer may read upon a faulty portion of the circle when the telescope is pointed upon a station other than the initial.

VERTICAL CIRCLE

Many direction instruments have no vertical circle attached, and it is then necessary to have a separate instrument with which to measure vertical angles. Such accessory instruments have become known in the Coast and Geodetic Survey as vertical circles. Some of them are universal instruments with the vertical circle the primary one. The horizontal circle is used only for laying off an angle roughly. Scarcely any two instruments in this class owned by the survey have the same mechanical arrangement, and therefore the adjustments can not be definitely described, but must be made in accordance with the general principles found to apply in the adjustments for the theodolite.

Some of these instruments have double centers for the vertical graduated circle in order that the instrument may, if desired, be operated by the method of repetitions. That method of observing vertical angles is seldom used in the survey, however, for single determinations of the zenith distance give better results than the same number of observations by the repetition method. On most of these instruments the graduated circle is either fixed or else movable for adjustment purposes only.

VERTICAL-CIRCLE ADJUSTMENTS

Adjustment of optical parts.—The telescopic adjustments for focus and parallax are the same as for the theodolite. (See p. 32.) The error of collimation is eliminated by the method of observing. The vernier microscopes must be adjusted to eliminate parallax.

Horizontal-axis adjustment.—The vertical circle is made truly vertical by leveling the horizontal axis, to which it is rigidly attached. This is accomplished by means of a striding level in the usual manner. (See p. 31.) Where no striding level is provided the vertical axis is made vertical by means of the level attached to the graduated circle or by means of whatever leveling arrangement is provided on the instrument, and the horizontal axis is assumed to be normal to the vertical axis.

Adjustment of graduated-circle level.—This level is sometimes attached to the back of the graduated circle, sometimes is mounted on the vernier frame, and in repeating circles is attached to the frame surrounding the horizontal axis. When attached to the vernier frame the bubble must either be brought to the center by a slow-motion screw after each pointing and before the circle is read, or else the ends of the bubble must be read on the tube graduations and corrections made later for lack of horizontality. If the level tube is attached to the frame or to the graduated circle it must be adjusted so the bubble will remain near the center after reversal

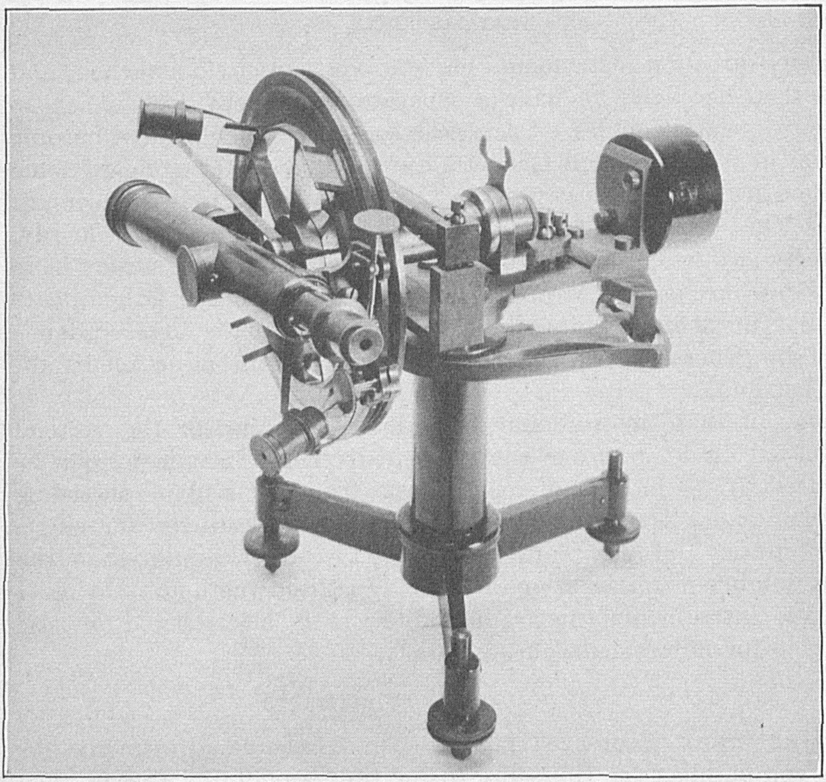

FIG. 23.—Vertical circle

This instrument has a repeating circle about 7 inches in diameter, read by each of 4 verniers to 10 seconds, and is used for measuring zenith distances.

of the instrument, though if the bubble is off the graduations after reversal it may be brought back with the foot screws before the reverse pointing is made. No error will result from this procedure. The bubble must *not* be brought back by the screw which on some instruments regulates the adjustment between the level bubble and the graduated circle. The relation between the bubble and the line joining the verniers and the relation between the horizontal wire of the telescope and the graduations of the circle must not be changed between the two pointings of a measurement.

METHODS OF OBSERVING VERTICAL ANGLES

The observing program is designed to give an angle which on all repeating vertical circles and on most others is twice the zenith distance of the object sighted upon. With certain instruments the angle obtained is twice the altitude of the object. The observations are made as follows:

(*a*) **Single measures of an angle, repeating circle.**—The outer clamp is the one which binds the vernier arc to the graduated circle, and the inner clamp is the one which clamps the graduated circle to the frame of the instrument. Each clamp is provided with a slow-motion tangent screw.

To observe, first level the instrument; then with the outer clamp tight and the inner clamp loose read and record the circle, no matter at what setting it may be. Next, point approximately upon the object with circle right, set up the inner clamp and perfect the pointing with the inner tangent screw. Read and record the level which is mounted parallel to the plane of the graduated vertical circle. Release the outer clamp, rotate the instrument 180° on its vertical axis, point upon the object with circle left, set up the outer clamp, and perfect the pointing with the outer tangent screw. Read the level and the circle. This completes a single measure of the double-zenith distance. The next measure should be begun with circle left and end with circle right, but before beginning it the reading of the circle should be changed at least a degree in order that the effect of an error in reading the circle may be confined to a single measure of an angle.

(*b*) **Fixed circle.**—With the instrument leveled and circle right, point upon the object; either read the level or bring the bubble to the center by the vernier tangent screw and read the circle. Next point upon the object with circle left and read the level and the verniers. The bubble may be brought to its center, if preferred to reading it, by the foot screws of the instrument before the final pointing or by the tangent screw after the pointing. For the second measure of the double-zenith distance begin with the circle left and finish with circle right.

With the type of instrument having a fixed circle and with a telescope which will not plunge through the standards but must be lifted out for reversing, time will be saved in measuring vertical angles on more than one object by making pointings and readings on all of them with circle right (or left) and then reversing the telescope and reading back on them in reverse order with circle left (or right). Theoretically this method is not so accurate as the usual one, since the elapsed time between the two pointings of a single measure of an angle gives an opportunity for instrumental changes, but if not more than 10 or 15 minutes are consumed in making a single measure of the zenith distances of a number of objects the errors due to such instrumental changes will usually be much smaller than those due to refraction.

The method of computing the double-zenith distance will vary with the system of graduation of the circle. A description of the methods to be used with different kinds of graduated circles is given on page 101, under Field Computations. In any event a statement describing the system of graduation on the circle should be made in the record book where the first observations with the instrument are entered.

When observing upon an object at a considerable elevation, using a straight eyepiece, care should be taken to eliminate the parallax as completely as possible, for otherwise a constant error may enter into the observations. When observing upon stars near the prime vertical for the determination of local time the error due to parallax is of opposite sign for east and west stars.

Formulas for the application of the level correction may be found on page 104.

VERTICAL COLLIMATOR

This instrument, shown in Figure 24, is used to center the theodolite, lamp, or heliotrope on an observing tower over the center of a station mark or to set a station mark directly under a definite point on the tower.

In principle the instrument consists of a telescope fitted with a diaphragm bearing cross wires, a tribrach with three leveling screws and a long vertical collar, into which the telescope is placed with the eye end uppermost. Near the eye end of the telescope and eccentrically placed is a level at right angles to the axis of the telescope.

To adjust the collimator place the cross wires in the focus of the eyepiece by pulling out or pushing in the eyepiece until the wires are as sharply defined as possible, then focus the telescope on the object beneath so that there is no shifting of the intersection of the wires over the object as the eye is moved across the eyepiece. Next the level is adjusted in the usual manner until there is no movement of the bubble when the telescope is rotated. Finally the cross wires are adjusted by means of the diaphragm screws until the intersection of the wires remains on a point in the field of view when the telescope is rotated. When so adjusted the point covered by the intersection of the cross hairs in the field of view is in the vertical line passing through the center of the telescope.

If the collimator is slightly out of adjustment a vertical line can be established by marking the four points in the field of view covered successively by the intersection of the wires when the telescope is rotated to four different positions approximately 90° apart. The intersection of the lines joining the diagonally opposite points will be the point sought.

After the instrument has been adjusted and it is desired to mark a point on the tower directly over the station mark, the axis of collimation of the telescope is brought directly over the center of the mark. If the center strip of the cap block has been removed to permit free vision with the collimator the point can be marked by two threads intersecting beneath the point of the plunger. It

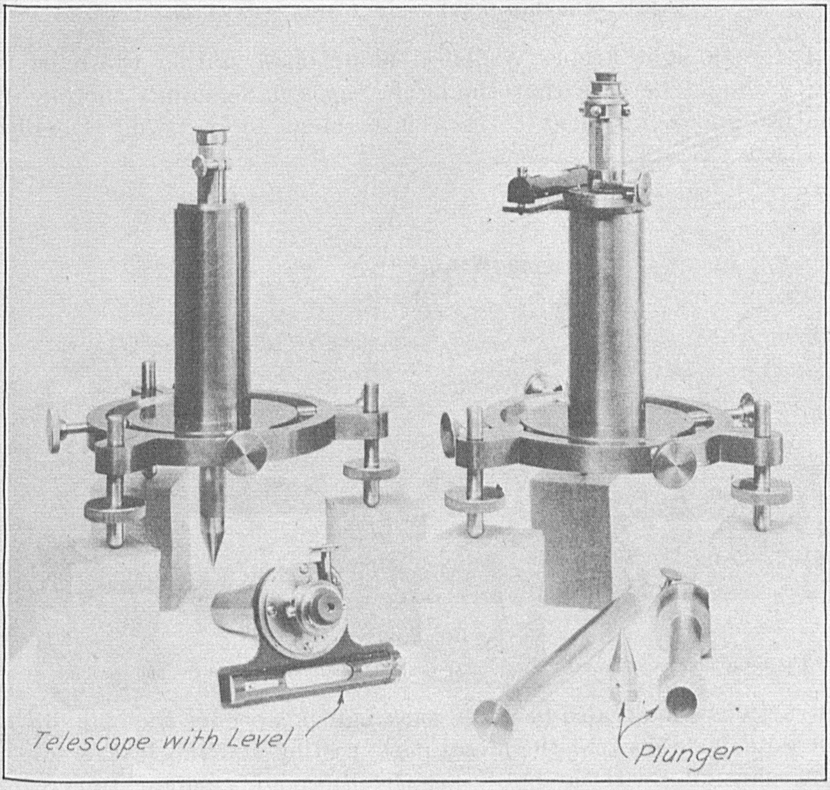

FIG. 24.—Vertical collimator

The collimator on the right has the telescope, with attached level, in place. The telescope has been removed from the collimator on the left and in its place is the plunger which serves to mark the point determined by the telescope.

is the usual practice, however, to bore a small hole through the cap block of such size that the vertical collimator telescope may be sighted through it and yet small enough to hold the screw which fastens the lamp or heliotrope to the tripod head. When the signal is being built it is best to center the instrument over the hole in the loose cap block and then slide the cap into a position where the hole will be directly over the station mark. The cap can then be nailed securely to the tower.

After a tower has been built for a few days or weeks the center of its cap block is likely to have been disturbed by the drying out and warping of the lumber. In such a case as this it is frequently best to find the point on the station mark directly beneath the center of the hole in the cap block, measure on the station mark the eccentric distance and direction of the projected point, and then transfer these dimensions to the cap, thus obtaining easily the point on the cap directly above the center of the station mark.

LIGHT-KEEPING INSTRUMENTS AND APPARATUS

Before a light keeper is placed alone on a station the chief of party should be sure that the light keeper understands thoroughly the use and adjustment of each instrument to be used. Detailed

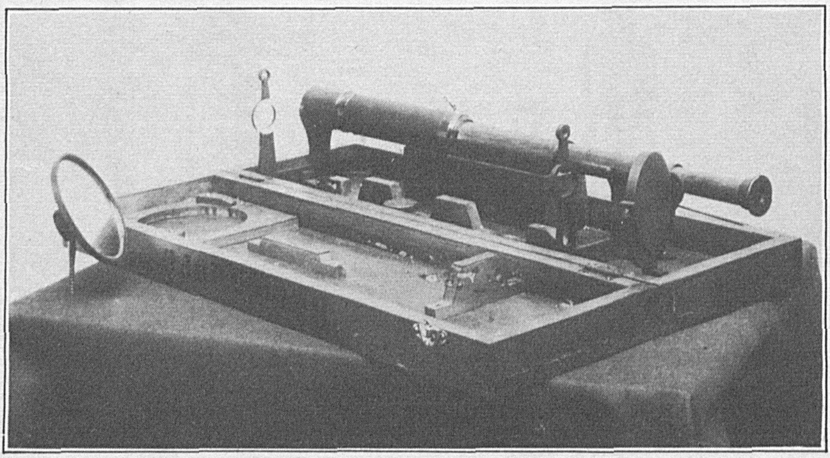

FIG. 25.—Heliotrope, box type

The illustration shows the arrangement of the sighting rings, auxiliary mirror, and telescope.

instructions should also be given each light keeper for the care of the instruments to prevent the metal parts rusting and the leather cases from shrinking. If a light keeper thoroughly understands his apparatus he will be able to make many of the emergency repairs which become necessary from time to time.

Compass.—The compass furnished each light keeper should either be compared with other compasses or else tested on a line whose azimuth is known. The effect of magnetic objects should be explained to him, and he should be cautioned against carrying the compass with the needle resting upon the pivot point. The principles underlying the orientation of maps and sketches and the application of the magnetic declination should be fully explained, with actual tests in them, before the light keeper takes his first station.

Heliotrope.—An undamaged heliotrope needs no adjustment. The only possible lack of adjustment occurs when one of the sighting devices is bent. Should this have occurred the line joining the sighting points is not parallel to the line passing through the centers of the alignment rings. To test, point the heliotrope at some nearby object, such as a rock or tent, and center the reflected light so the shadow of the ring next the mirror falls fair on the forward ring. The sights should point to a spot exactly above the center of the illuminated surface and just as far above as the line of the sights is above the line passing through the centers of the rings. If such is not the case the affected part should be removed from the box, straightened, and replaced.

Each light keeper should be shown how to make an emergency helio, shown in Figure 25*a*, by driving two nails vertically about 2 feet apart into a board, the heads of the nails to be used as sighting points for the beam of reflected sunlight. Place the board on the

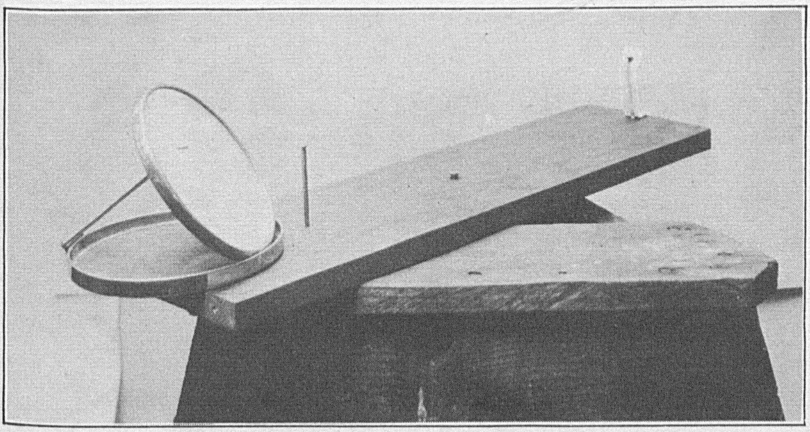

FIG. 25a.—Emergency heliotrope

A shaving mirror, board, and a couple of nails will make a serviceable heliotrope when no other is available.

stand and align the heads of the nails with the station of the observing party. Next fit a narrow strip of paper to the front side of the forward nail, the strip projecting slightly above the nailhead. With a common mirror a few inches in diameter throw the reflected rays of the sun along the line of the nailheads. This will be accomplished when the shadow of the head of the rear nail falls on and exactly covers the head of the forward nail. The paper strip mentioned above enables one to make this exact contact. An emergency heliotrope like the above has been satisfactorily observed upon from a station 40 miles away. The center of the mirror should be held approximately in line with the nailheads to avoid eccentricity of the light shown the observer. If the direction to the observer is nearly opposite the direction to the sun it may be necessary to use an auxiliary mirror to reflect the sun's rays onto the mirror which is in line with the nails.

Signal Lamp.—The electric signal lamp supplied with current by dry batteries has almost entirely superseded the acetylene lamp which was in use many years. The 1924 model of the electric lamp is shown in Figure 26. It was designed especially for regions where the lines of sight are not greatly in excess of 25 miles and where it is important to have the outfit as light as possible. A larger lamp, shown in Figure 28, is used where a more brilliant light is needed on account of long lines or hazy atmosphere.

Aside from the electric connections only two adjustments are needed for either lamp, one for focus and the other for the sighting devices, and these should be tested frequently

The focusing adjustment is made by the screw socket into which the bulb fits. Since it frequently differs for different bulbs of the

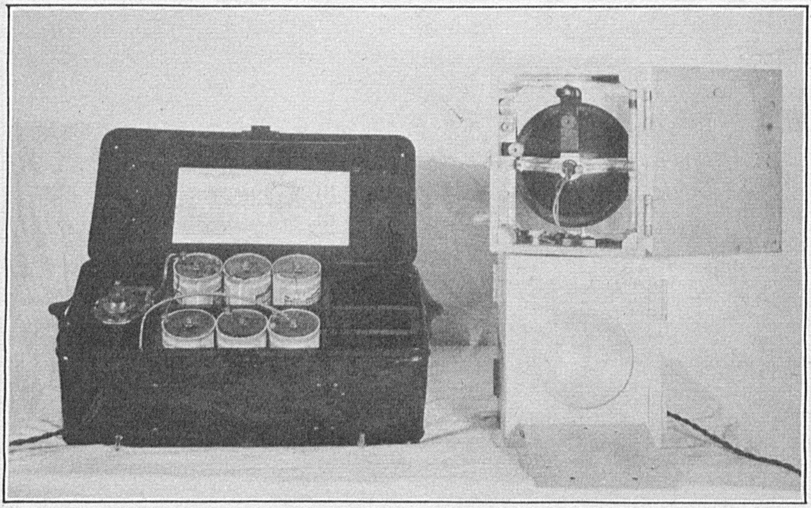

FIG. 26.—Small signal lamp, with automatic lighter

The eight-day clock shown in the battery box on the left can be set to turn the light on for a definite number of hours each night.

same apparent size, it should be made each time a new bulb is used. It is best done by directing the light upon a flat surface, such as a tarpaulin about 100 feet or more away, and varying the adjustment until the brightest part of the disk is but little larger than the lens of the lamp. After this has been done the sighting device should be adjusted to point exactly above the center of the brightest part of the illuminated surface and as far above it as the sighting tube is above the center of the lens of the lamp. As the transportation of the lamp from station to station is apt to disturb both of these adjustments, they should be tested before the lamp is posted at a new station. If the lamp is to be operated automatically and is to be posted during the day and immediately left, the adjustments should be tested the night preceding. It is just as satisfactory to make the adjustment in daylight by changing the focus until the reflector as viewed from a point two or three hundred feet away is evenly illuminated. A stake can also be set with its top at a point where the light is the brightest and the sighting tube adjusted to the proper distance above that point.

ORGANIZATION OF PARTY

SELECTION OF PLAN OF OPERATION

Before deciding upon a general plan of procedure for field work on a contemplated project the chief of party should become as fully informed as possible concerning the conditions to be encountered, especially with regard to railroads and highways, weather, accessibility of stations, availability of supplies, and the kind of transportation best adapted to that particular region. If detailed reconnaissance for the selection of triangulation stations has been made in advance, the reconnaissance report and the detailed descriptions of stations will in general give all the information needed when studied in conjunction with the best maps available. It is sometimes helpful to make for each station in the scheme a tabulation of the distances to the nearest point of supply, to the nearest point approachable by truck, to the nearest water, and also the distance it will be necessary to horse pack or back pack the instrumental outfit.

If a detailed reconnaissance report is not available, reliance must be placed upon maps, weather reports, and upon such information as can be obtained from persons familiar with the region. If time permits, valuable information regarding roads and trails in mountainous regions can often be obtained by writing to postmasters and forest rangers who live in the country through which the work is to be carried.

After the chief of party has obtained a general knowledge of the average conditions to be met it is usually easy to choose the means of transportation which will be most economical. The next step is to decide upon the number of transportation units and the number of employees needed. This step is a most important one, as it has a great effect upon the unit costs of the work and warrants a close study of all factors affecting it.

The observing party proper usually consists of from three to five men, including the chief of party. If trucks are to be used a good organization is to have the chief of party as chief observer, a junior officer as assistant observer, a recorder who can drive a truck as a part of his duties, and a machinist truck driver. Local conditions may make it necessary to modify somewhat this plan of organization. Under the conditions ordinarily encountered two trucks of the three-quarter-ton or 1-ton type will be sufficient to move the observing party and even to post occasional lights.

The light keepers on a first-order triangulation party will vary from two to eight in number. The most economical number can only be determined by carefully making out two or three schedules of moves, using different combinations of light keepers and methods of moving them. By estimating the time required for moving each element of the party, and assuming for purposes of estimation that one night is sufficient for the observer to finish a station if all light keepers have had time to reach their stations, the relative cost of each combination can be approximated. The plan selected should be the one which will give the lowest cost per station occupied rather than the lowest cost per month or the greatest number of stations occupied per month.

An example of a schedule of moves is shown in the diagram below. It is adapted to a region where it is advisable to have a light keeper on each station all the time and where the automatic feature of the modern electric signal lamp would be used only as an auxiliary, if at all.

Schedule of moves for observer and light keepers

Observer	Light keeper "B"	Light keeper "D"	Light keeper "H"	Light keeper "K"	Light keeper "P"	Light keeper "U"	Light keeper, extra
Haystack	Rawhide	Hobbs	Willow	Coleman	Notch	Chugwater	
Coleman	Haystack	do	do	do	do	do	
Notch	do	Whitaker	Ragged	do	do	do	
Chugwater	do	do	do	do	do	do	
Whitaker	Wadhill	do	do	Greentop	do	do	
Ragged	do	do	do	do	do	do	

Each light keeper will be assigned a letter so chosen as not to be identical with or similar to any of the code signals. The stations which each light keeper is to occupy in succession are shown in the vertical column under his letter, while the horizontal lines show the location of the observer and the various light keepers at any time.

Thus when the observing party was at Haystack, light keeper "B" was at Rawhide, "D" at Hobbs, etc., and were showing their lights to Haystack; then the observer moved to Coleman, "B" moved to Haystack, and the other light keepers kept their stations, changing the pointing of their lights so as to show them to the observer at Coleman.

The column in the schedule headed "light keeper, extra," is to cover cases where more lights must be observed on from a station than there are light keepers. In such cases the light is shown either by a truck driver or by a local man hired temporarily. If there are frequently more stations than can be taken care of by the regular light keepers and by truck drivers temporarily detached from the observing party, it may be more economical to have an additional regular light keeper to avoid delays to the party.

Different methods of planning the moves of light keepers may be used, and the advantages of each can best be shown by using typical cases.

Where the scheme of triangulation is small and the stations accessible, with few extra stations, five light keepers can be used to advantage, provided two of them are supplied with a truck apiece. Two light keepers, one with a truck and one without, keep ahead of the observing party all the time, and another pair with a truck take care of the rear stations. The fifth light keeper shows a light from the station in the scheme which is opposite the observing party. When the observing party moves ahead to a new line the forward light

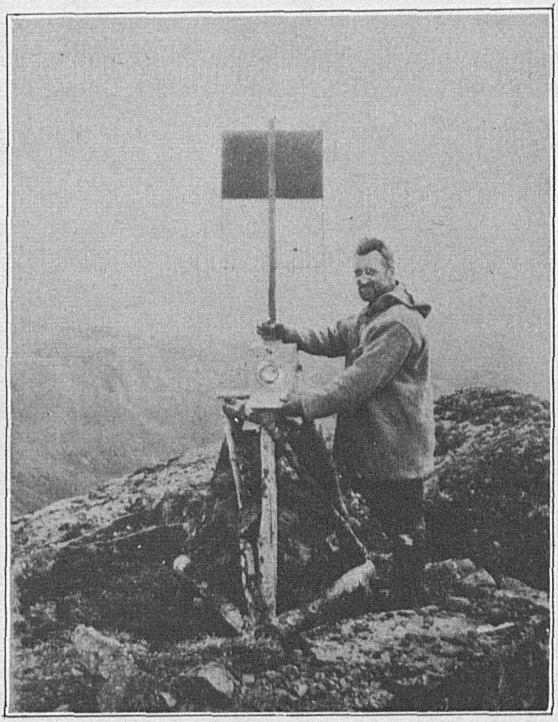

FIG. 27.—Small signal lamp, with target centered above

An aluminum cup-shaped plate is made for this type of lamp, which may be attached to the top of the lamp and which automatically centers the base of the target over the center of the reflector of the lamp.

keeper with the truck picks up the other forward light keeper, takes him to his next station, and then moves to his own new station. The two rear light keepers move ahead one line in the same manner, while a truck from the observing party usually picks up the light keeper working opposite and posts him at his new station ahead.

When the automatic lights are used under the same conditions one truck driver can post the lights in front and another in the rear, a truck from the observing party posting the light at the station across the belt of triangulation from where the observing is being done. If sufficient lamps are available, the amount of travel to post lamps may be reduced by posting two lamps at each forward and rear station not watched over by a truck driver or light keeper, the lamps pointing to different stations.

Where the distances between stations are long, or where many of the stations are on mountains, it may be more economical to have a larger number of light keepers and for the rear light keepers to move ahead of all the others in the direction of progress when directed to move by signals from the observer. Where there is only a single truck to move light keepers, this is the plan which will usually have to be followed. It may also be followed in principle when automatic lamps are used. A tabulation of the relative total mileage required

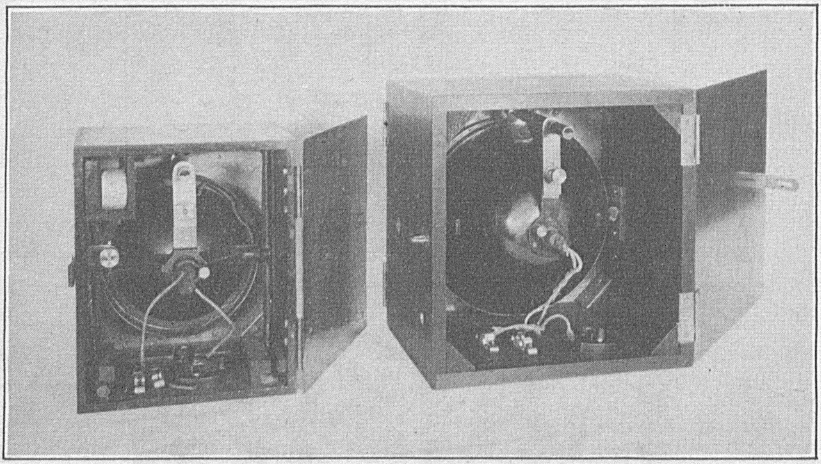

FIG. 28.—Signal lamps, back view

The comparative sizes of the large and small lamps are shown in this illustration. Both have the same general arrangement and are used with the battery box and clock for automatic control shown in Figure 26. A bulb with a less concentration of filament and greater amperage is used with the larger lamp.

by each of the systems under consideration, and the probable delays to the observing party under each plan, will often be of material assistance in deciding upon the best plan.

The schemes of organization suggested above will indicate the numerous combinations which can be effected. Often different systems will have to be used during a single season to suit the changing conditions, but a close study of available data will usually disclose what transportation should be used and the most economical number of employees. Too much transportation is almost as fatal to low unit costs as too little, for the extra mileage run, the repairs required by the extra trucks, and depreciation will cause the costs of the party to mount rapidly.

SELECTION OF PERSONNEL

The excessively long hours of work and the arduous labor involved on most triangulation parties make it necessary for the chief of party to use the utmost care in selecting the men on his party. They must be industrious, dependable, and self-reliant to a high degree, apart from their special qualifications for the work they are to do. An inexperienced man can be taught, but a lazy or indifferent worker is difficult to change. Moreover, unless a man has a distinct liking for the nomadic life required he is not apt to last through the season. As a rule only young men of sufficient age to have held down jobs involving hard manual labor and some responsibility will have acquired the ability to give satisfactory service on a triangulation party. The chief of party will have little enough time to teach a man his duties without having to teach him how to work also.

For the men on the observing party one other requirement must be borne in mind. Under the stress of observing at night, traveling between stations, or doing camp work and computing during the day, good humor becomes a great asset. A fault-finding person can ruin the spirit of an observing party, and no one, from the chief of party down, should be on the work unless he can smile and joke when he is dead tired. The pleasure of the party in the work and its progress can be entirely spoiled by one of its members being a grumbler.

Truck mechanics and drivers.—The position of mechanic or chief truck driver is the hardest to fill, because the men with adequate skill and experience to be satsifactory do not as a rule relish the exposure and long hours incident to the work. There may be three or four truck drivers on the party, but the head driver or mechanic must not only know how to make all adjustments in the mechanism of the truck, but also must be skilled at diagnosing quickly the source of any trouble and be able to make emergency repairs under field conditions. In addition to his work on the trucks he must do his share in making and breaking camp and in other routine work.

The other truck drivers should at least have sufficient skill to make simple adjustments to the truck mechanism and to be good and careful drivers. Since it is necessary for them to go frequently on detached duty, they must be so reliable that the chief of party may feel satisfied that they will do their utmost to carry out his orders. The rate of progress of the party depends so much upon the proper care and use of trucks that a careless or reckless driver should not be kept.

Recorder.—For selfish reasons, if for no other, the chief of party should select a competent recorder. The recorder should be very rapid and accurate in mental calculations, otherwise the observer must spend time, after the night's observing program is finished, in checking over the abstract of directions to see if additional observations are needed before signaling instructions to the light keepers. The recorder must be industrious and self-reliant, else the chief of party must take it upon himself to see that all record books and abstracts are properly made out and checked. He must be rugged physically, where there is much back packing of the instrumental outfit, if the observer desires to avoid carrying most of it himself. All these points should be considered in selecting an untried man, for too many student applicants for summer jobs have the idea that being a recorder in a surveying party is just a means of having their expenses paid while on an outing. Where the other qualifications are present the young man who has partly worked his way through college or who has successfully held a job involving hard manual labor has far the best chance of being satisfactory as a recorder.

Light keepers.—A light keeper's duties require him to be alone on a station for days or even weeks at a time, showing a helio to the observer in the afternoon for two or three hours and watching by the signal lamp at night. If he is on a mountain station there will be plenty of climbing and back packing. He must cook his own food, and often arrange for his supplies and for his transportation from station to station. If he camps on top of the mountain he must carry his food and often all the water he uses, while if he camps at the foot of the mountain near water and supplies he must go down in the darkness after each night's work.

Under such working conditions two qualities stand out as being essential in a light keeper. The first is faithfulness and a will to be always on the job and to show a light under all conditions, even when the chances are small that the observer can see the light. The second is a liking for the loneliness of his work, or at least the ability to ignore it, though usually the light keeper who does not learn to like the isolation will leave the job after a short time. It is no job for a young boy, especially one who has been raised in a city. Youngsters will often protest that it is just the work they adore, and then leave at their first station. If a new light keeper is not experienced in being alone and doing for himself, only an unusual amount of grit and determination will carry him through.

One thing which each chief of a triangulation party soon learns by experience is to discharge a man just as soon as he demonstrates that he is unfit for the work. After having done his utmost to select the right man, the chief of party will usually find it best to let a man go without recriminations if he wishes to leave and to discharge him just as quickly if he does not measure up to the standard. It is usually possible to have one or two prospects in readiness for a summons to fill vacancies occurring on the party during the season.

OUTFIT

In deciding upon the outfit to be carried by the observing party and by each light keeper two conflicting considerations must be reconciled. Since the work is usually in unfrequented places, the outfit must be adequate to meet the conditions of working and living. On the other hand, each article must be handled, packed, and unpacked many times during the season, each operation requiring time and labor. With the conditions of transportation constantly in mind the chief of party should consider carefully the need for each article of equipment, and discard or place in temporary storage any surplus outfit or supplies. The same rule must be applied to the clothing and personal effects of the members of the party. The most successful chief of party is almost invariably the one who has learned how to work comfortably with the least equipment.

Trucks.—These will, as a rule, have been overhauled at the end of the previous season, but at the first opportunity each one should be

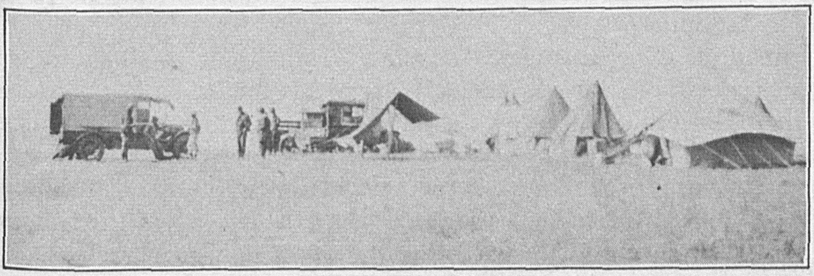

FIG. 29.—Camp of base-line party

The same types of tents and trucks are used by the triangulation parties.

thoroughly inspected and tested by the best mechanic on the party. A list of the tools and accessories which must be purchased should also be made as early as possible, in order that each truck may be fully equipped and its inventory made out and checked before the party takes the field. Besides the usual accessories, each truck should carry at all times a spade, axe, and towing rope. One truck on the observing party should also be supplied with extra appliances for making emergency repairs, such as a blowtorch, breast drill, soldering iron, small vise, emery wheel, and special wrenches.

Tents.—If tents are obtained from storage each one should be carefully inspected and needed repairs made. All weak ropes should be replaced. If the tents are old it is often advisable to furnish each light keeper with a piece of canvas, sail needle, thread, and sewing palm.

If new tents are to be purchased they should be of the standard type of center-pole tent, specifications for which can be obtained from the office. The standard tents are of two sizes, 9 by 9 feet and 7 by

7 feet. The smaller size is for the use of light keepers, while the larger size will comfortably house two men, or three in an emergency. The standard tents are more strongly reinforced than ordinary stock tents. A center-pole tent can be pitched by one man even in very windy weather, and will stand in a strong wind better than a ridge pole tent. The center pole itself is usually made in two or three sections, held together by ferrules of brass tubing, thus making it easy to pack.

Other equipment.—In order to secure an outfit sufficiently uniform to make it easily transportable, cots, bed tarps, and packing boxes are usually furnished to all temporary employees. In regions such as Alaska, where pack trains are used extensively and weight must be reduced to the utmost degree, sleeping bags and aluminum mess kits are also furnished, but ordinarily each temporary employee furnishes his own bedding, mess outfit, and food, being paid a flat rate for his services. This saves the chief of party a great deal of accounting, and besides it has been found impossible to provide satisfactory subsistence in kind for the men under the conditions encountered in triangulation.

Purchase of equipment.—If time permits, bids on all equipment needed should be secured sufficiently far in advance to have it delivered at the point of outfitting by the date on which the party is to be assembled. If part of the outfit is to be obtained from storage such articles should be examined as soon as possible and any supplemental outfit purchased at once. It is essential to the success of the training period which usually precedes the beginning of field work that the entire outfit be on hand when the party is assembled.

Packing boxes and apparatus.—The ease and speed with which the party can make and break camp and prepare the instrumental outfit for tranpsortation by truck, pack animals, or by man packing will depend upon the thoroughness with which the packing details are thought out and proper equipment provided. Packing boxes of convenient size, with rope handles, should be provided for the observing party and light keepers. In these boxes all small articles, including mess outfit and food supplies, should be kept packed. The boxes should be of such dimension that they can be loaded into the truck bodies without wasting space.

The instrumental outfit requires the greatest care and consideration. If trucks can be driven to each station the instruments may be transported in their regular packing cases, but if horse packing and man packing must be resorted to frequently a special crate, obtained by requisition from the office, should be used for the theodolite. If transportation is partly by truck and partly by packing, an outer box must be provided for the theodolite crate while it is being carried by truck, the theodolite being always carried in its crate. The instrumental outfit must be so packed that within a few minutes after the trucks stop the instruments can be made ready for the pack up the mountain. If there is to be considerable back packing by the members of the party adequate packing apparatus must be provided, such as pack boards, pack sacks, or special apparatus.

Either ropes of convenient length or webbing straps should be provided for fastening all bundles, such as bed rolls and tents. It is a waste of time and energy to put an elaborate lashing on a bed roll or tent when by using care in its arrangement a couple of straps will be adequate to make it compact and secure. Leather straps should not be used because they are very expensive, require cleaning and oiling frequently, and are much appreciated as food by all classes of rodents.

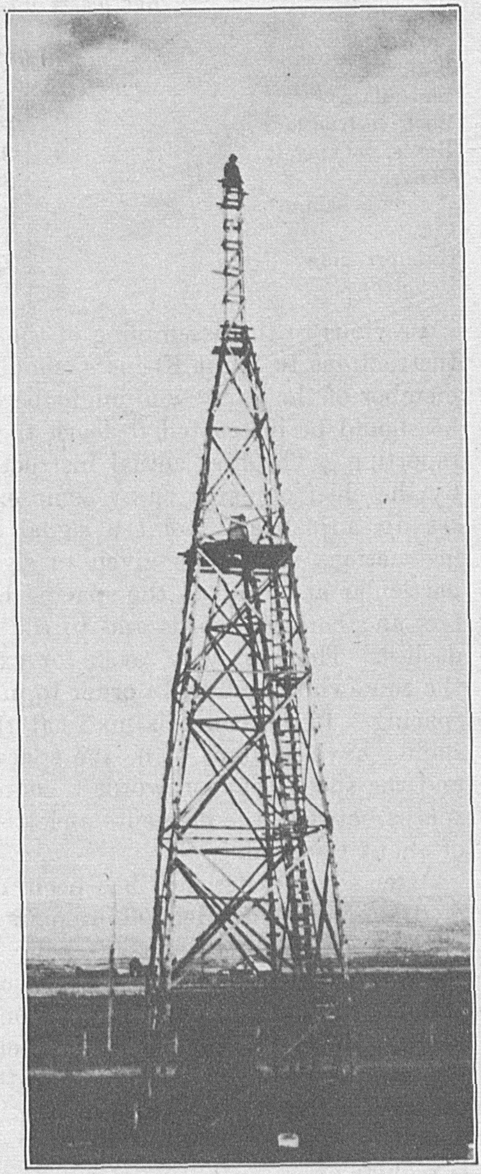

FIG. 30.—Triangulation tower, with superstructure

From his perch at the top of the superstructure the light keeper shows his light to the observer's station.

TRAINING OF LIGHT KEEPERS

If there are more than one or two inexperienced light keepers on the party a period of from four to six days can usually be spent to good advantage in training them in the work before beginning field operations. This can be done best by establishing camp, preferably at the edge of some town, where the outfit can be gone over in detail under approximately field conditions.

After all the tents are pitched and assigned, each light keeper should be given the necessary instruments and equipment. A receipt should be taken for the articles issued to each man on detached duty and he should be held responsible for their care and safety. Each light keeper should be furnished with the following articles as the minimum equipment:

Equipment of light keepers

Ax.	Lamp, signal.
Batteries.	Maps and descriptions of stations.
Binoculars.	Nails, screws, etc.
Book, instruction.	Plummet.
Boxes, packing, 2.	Screw driver.
Canvas.	Spade.
Compass, azimuth.	Stationery and accounts forms.
Cot.	Tarp, bed.
Hammer, claw.	Tarp, extra, small.
Heliotrope.	Tent, complete.

Previous to the assembling of the party a copy of the publication, Instructions to Light Keepers, should be sent to each inexperienced member of the party who might be called upon to show a light and he should be instructed to learn the Morse code thoroughly before reporting. The first actual instruction should be a demonstration by the chief of party or by some experienced employee on how to set up, adjust, and point a signal lamp and a heliotrope. Next, instructions should be given in signaling with the lights, paying particular attention to the spacing between the elements of the letters and between words and to the relative length of the dots and dashes. The best way, even for experienced men, is to count all the time while sending in order to maintain uniformity in speed and spacing. If the count is made at the rate of 2 per second a dash should have a count of 6, the space between letters a count of 6, and the space between words a count of 20, while the dots and the spaces between the elements of letters should each have a duration of about 1 second.

After some experience has been obtained in group signaling the men should be divided off in pairs and placed about a half mile apart, and at least three hours a day should be spent in sending and receiving messages. The same two men should not practice together all the time. One man alone can obtain good practice in both sending and receiving by directing his light on a wall or rock about 50 meters away. By concentrating his attention on the illuminated object while he is signaling he can judge as to how the light would appear if seen from a distance and whether or not the spacing is regular.

Each day some instruction should be given in the use of the compass in orienting a map, for that may be the only method the light keeper can use in pointing his light correctly toward the observer's station. The light keeper should be warned that the compass orientation should not be made by placing the compass on the light stand, since the nails in the head of the stand will often deflect the compass needle appreciably. The light keeper should also be taught to perfect his orientation after the first compass orientation is made by the use of peaks or towns which are marked on his map and which can be identified from his station. After a preliminary demonstration by some experienced member of the party the light keeper himself should perform all operations, both in orientation problems and in adjusting his lamps and helio, with such supervision and correction as may be necessary.

The most important part of the instructions to light keepers relates to the proper pointing of the lamp, and the most experienced officer or employee on the party should have that part of the training in charge. The reconnaissance sketch showing the scheme of triangulation is usually not accurate, and the direction to a station as shown on the sketch may be in error several degrees. Where an automatic lamp is to be set it may, on a short line or in clear weather, be thrown slightly out of focus in order that the beam may cover a wider angle. This expedient should be used with caution on any except short lines, for the brightness of the light is inversely proportional to the cross-sectional area of the beam.

When either the observer or a light keeper is trying to communicate with a station whose direction is not exactly known, the light should be shifted every minute or so by about half the width of the beam until the horizon has been swept for several degrees on either side of the most probable direction. To sweep the beam rapidly around the horizon does little good, for that procedure produces no more than a flicker of light as viewed from the distant station and is usually not noticed.

A couple of evenings during the training period can be spent profitably by assembling the entire party and talking over the prospective season's work. This will afford a good opportunity to explain in detail the accounting expected of light keepers and may save the chief of party a large amount of correspondence on that subject. It will also give each one of the party a chance to ask questions regarding signaling and the general plan of the work.

A by-product of these meetings, the value of which must not be overlooked, is the building up of a team spirit due to a greater interest in the plans for the season and in the other members of the party. It gives a chance for the chief of party and his experienced men to imbue in the new men a spirit which will make the work of all easier during the season, for with each one alert and doing his utmost the observing hours at night will be materially shortened. The failure of one light keeper to show his light on time or to answer a calling signal promptly will usually require all to remain later at their posts, and may delay by a day or even a week in the event of bad weather the completion of a station. The presence of one or two good, experienced light keepers on the party will do much to set a high standard of performance, for the talk always turns to past feats of endurance and hardship which enabled the observing party to complete a station earlier than it otherwise could have done. The unwritten story of the faithful vigils of the light keeper on his lonely mountain station, chilled by storms and often short of rations, constitutes as fine a record of devotion to duty as can be found in civil life, and the knowledge of such a background is an inspiration to the whole party.

Part of the time of the evening conferences should be devoted to talks on camp routine, for a neat, clean camp induces contentment. Camp sanitation should be discussed, and the necessity for "policing up" when leaving a camping place. It takes only a few minutes to dig a trench for garbage and tin cans and to fill it up when leaving, and the camp fire will dispose of papers. No matter how remote the camp site this procedure should be invariably followed, and the good repute of the entire party will be enhanced thereby.

Cleanliness and orderliness are just as important in the daily routine, for an experienced camper can be recognized at once by clean dishes and the neat arrangement of all camp appurtenances. Only a tenderfoot will leave dishes dirty from one meal to the next, except under the stress of a real emergency. Where rocks or trees are at hand the construction of ovens, benches, and tables will do much to add to the interest and convenience of a light keeper's existence.

A man not accustomed to cooking for himself will often rely too much on canned goods, to the detriment of his health and pocketbook. The chief of party should see to it that all who need it are given instructions in the simpler forms of camp cooking, including the baking of camp bread; otherwise he is apt to have vacancies on the party when he least desires it. Assistance should also be given the inexperienced men in making a list of food supplies for their first stations.

Another topic which should be emphasized is the care of instruments. Binoculars may be ruined by being left out in the rain and dew or by exposing the objective for long periods to the direct rays of the sun. Leather goods after being wet should be dried in the shade and oiled immediately. The silvered backing of heliotrope mirrors will be loosened by being wet. Instruments left at the station may be stolen or broken, and carelessness in handling them, especially on high towers, will often result in their destruction. The sum total of the instruments used by a party represents a large investment, and proper methods of handling them must be taught each member of the party.

No process of inductive reasoning is needed to show that the time of the chief of party is apt to be fully taken up during this training period. In addition to supervising the outfitting and training of the party, his position as disbursing officer makes heavy demands upon his time. This emphasizes the importance of purchasing outfit previous to the assembling of the party, for by personally looking after the outfitting and training of the men he secures a knowledge of them which will be of much benefit and will enable him to estimate their capabilities better when the party actually takes the field.

ROUTINE OF OBSERVING PARTY

The duties devolving upon the members of the observing party are so many and varied that it is necessary for the chief of party to establish a camp routine as quickly as possible, with certain duties and responsibilities assigned each man. Particularly the procedure of making and breaking camp must be so systematized that it can be done quickly and easily.

Camp routine.—Assuming, for example, that the observing party has to break camp, make a short move by truck to the next station and there prepare for observing, the usual program would be about as follows: If the observing the preceding night had not been finished until late, one man may have been sent to bed early in order that he might prepare breakfast before calling the others, in which case two of the party after the meal would wash dishes and pack mess outfits while the others struck the tents and loaded the trucks as rapidly as the outfit was packed. If all had lost the same amount of sleep all would turn out at the same time, but much of the packing could be done while breakfast was being made ready, and all could work on the mess outfit afterwards. In either case a trained party can be ready to leave camp with the outfit an hour after the general reveille is sounded. The man who requires several minutes to wake up or 15 minutes to dress does not belong on the observing party.

When the next camp site is reached the chief of party designates where the tents will be pitched and the outfit is unloaded at a place as convenient to the tent sites as possible. The tents are pitched, cots and bed rolls made ready for the night, mess outfit put in place, the observing outfit placed to one side ready to be taken to the station, and surplus outfit placed under a tarpaulin. This should not require more than 30 minutes, after which one man can start cooking, the truck driver work on the trucks, and the others work on records or accounts until time to start to the station. To keep the truck in good order and the record books and abstracts checked up to date requires constant forethought and application.

The ease with which all camp work is done will be greatly increased by training the members of the party to return each article to its proper place after being used. This applies especially to the mess outfit, for with each member of the party liable to be assigned to the rôle of cook much time will be lost in searching for articles if there is not a standardized method of packing. The labeling of bags and boxes containing food supplies is also helpful.

To maintain a clean, orderly, and sanitary camp is a matter of prime importance, for the good repute of the party with its neighbors will depend in a large measure upon it. Stray papers should be kept picked up, tin cans and garbage disposed of in a trench, and a final "policing up" made when leaving a camp site.

Observing routine.—The process of transporting the observing outfit from the camp to the station need not be dwelt upon, though often it is the most laborious work of all. If back packing for any considerable distance is necessary the most that can be done is to have the pack harness as well adapted to the loads as possible, and then let strong backs and stout hearts do the rest. Many observers

FIG. 31.—Horse-packing outfit of triangulation party in Alaska

Crossing the swift Alaska streams with pack trains is often a dangerous procedure.

prefer not to trust the theodolite to another, but carrying a 70-pound pack for several miles up a mountain probably does not materially assist the observer in securing small triangle closures.

Upon reaching the station the observer should first examine the small wooden stand used as a support for the signal lights and the theodolite as shown in Figure 27. The first test is to see if the small hole in the top of the stand, into which the signal lights are fastened and by which they are centered, is directly over the station mark. If it is not and the station has been observed upon from any other station the eccentric distance and angle must be measured with an accuracy such that the resultant correction to directions will never be in error by more than three or four hundredths of a second. The proper method of recording the eccentricity factors is shown on page 94.

On tall towers the vertical collimator, described on page 52 and shown in Figure 24, is used in determining the vertical line to the station mark. If the tower has a superstructure from which the signal lights are shown it is better first to establish a point on the tripod head directly over the station mark, next plumb down from the hole in the light stand at the top of the superstructure to a point which can be marked on the tripod head in order to measure the eccentricity, and then the hole in the light stand should be plumbed directly over the station mark to avoid eccentric reductions on the other lines.

On certain theodolites, such as the Coast and Geodetic Survey type which is shown in Figure 18, the vertical axis extends below the

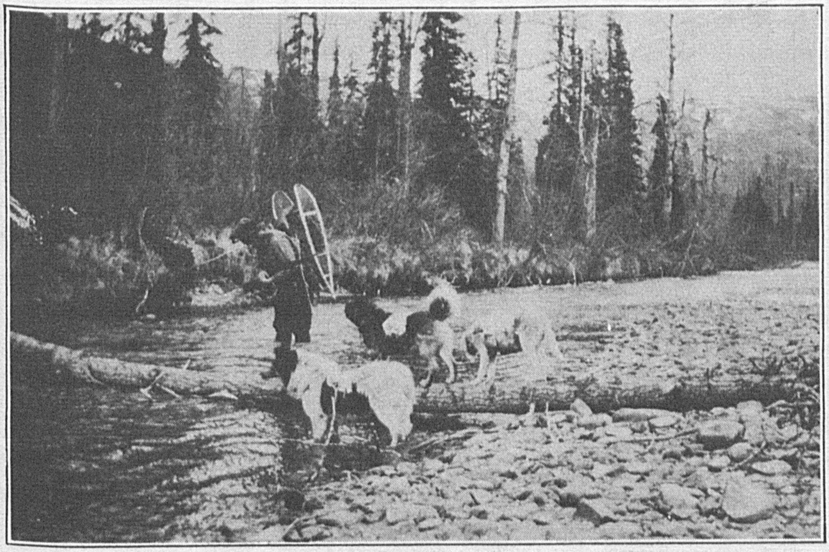

FIG. 32.—Packing with dogs in Alaska

On reconnaissance, where the outfit to be carried is light and trail conditions extremely difficult, the Alaskan sled dogs are often used to advantage. Each dog will pack about 30 pounds in the specially constructed pack bags.

plane of the lower ends of the leveling screws, requiring that a section of the top board of the tripod head be cut away. In this case a point in the vertical line through the station mark as determined by the collimator can be marked by threads intersecting at right angles, held in knife cuts in the edge of the tripod head.

TESTING STABILITY OF STAND

Before beginning the measurement of horizontal angles the observing stand should be closely examined to make sure that it is rigid and stable. If the observer is to stand upon the ground the telescope of the theodolite should be pointed upon some object and the pointing closely watched while some other person steps on the ground as near the tripod legs as the observer would stand, to make sure that there is no movement of the stand. Where the tripod legs are placed in broken rock it is very difficult to avoid having the weight of the observer transmit thrusts to the tripod legs which will prevent accurate measurements, and in tundra or in swampy soil the same trouble will occur.

The condition may be remedied by having three 1-inch planks, 8 or 10 inches wide, of such length as to fit around the tripod stand at a distance of about 6 inches from the legs, so supported upon pieces of 2 by 4 inch timbers that no pressure is put upon the surface of the ground near the tripod legs by the observer walking about the stand on these boards. If boards are not available the condition

FIG. 33.—Back-packing the outfit, triangulation party

The loads are made up of observing instruments, bedding, and food. To pack such loads for hours through underbrush and streams, over swamps, glaciers, and mountains, often in rain or snow, requires determination as well as strength and endurance.

may usually be remedied by digging the dirt or rocks away from the tripod legs for a depth of a foot or more, trusting to rocks piled tightly inside the stand upon boards resting upon the lower horizontal braces to give it the proper rigidity. Even where the ground is firm there should be a hole 3 or 4 inches deep around the tripod legs. Errors in angle measurements caused by pressure against the tripod legs are especially hard to detect, because the observer stands in practically the same spot when making the several observations upon any one object and each position will be in error by almost the same amount.

Whether or not the stand or tower is newly built it should be gone over and each nail set up, additional nails being driven if the old holes seem to be enlarged and if the wood will permit it without splitting. When tall towers are used upon which to mount the theodolite it is necessary that the tower as a whole be elastic, in order that it may immediately regain its normal position after each recurrent thrust by the wind.

OBSERVING TENT

When observing from towers it is well to have a tarpaulin about 5 feet high fitted with ties and of a proper length to fasten around the observing platform. This can be supplemented in rainy weather by a canvas canopy suspended from an overhead timber, with side walls which can either be tied up to permit observing or fastened down by ties to keep out the wind.

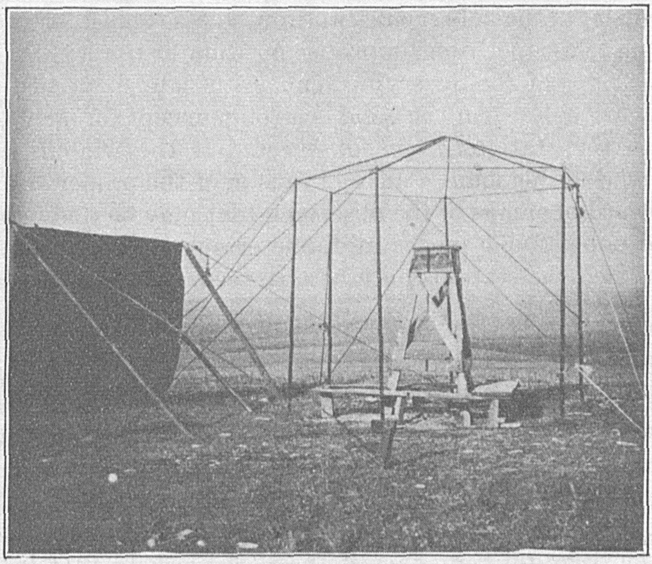

FIG. 34.—Steel frame for silk observing tent

The vertical poles are made in two sections, for convenience in packing. A ferrule on the top of each pole projects through an eye in the end of a radial arm. When the silk covering is taken down the frame is held in place by the guy ropes and by a rope passing around the frame and fastened to the top of each pole.

Where most of the observing is done on the ground a silk hexagonal tent of standard type is ordinarily used. This tent is supported by a frame of thin steel tubing, shown in Figure 34, and is very light. Whatever type of tent is used, it should be pitched before any observing is done. If heavy winds are anticipated the guy ropes should lead straight out from the stand and poles with two extra ropes leading from the top of one of the poles to the two adjacent tent pegs to prevent the tent from swinging in azimuth. In moderate or light winds the pegs may be driven on lines passing out from the station mark midway between each pair of poles and a single long rope used for the guys, passing alternately behind a peg and then in a hitch over the top of the next pole.

In windy regions it will be advisable to carry as a part of the observing outfit, usually as a wrapping for the poles of the tent, a piece of canvas about 6 by 16 feet, which can be used as a wind-break. The method of using it is shown in Figure 34. When the soil around the stand is soft and dry the dust is often bothersome, and a piece of light canvas used as a floor in the observing tent will prove helpful. Another light canvas laid outside the tent to windward, close to the bottom of the wall, will also decrease the annoyance from dust.

If wind of considerable strength is permitted to strike the theodolite, it delays the observing and may also cause errors by deflecting the telescope from its proper position in the wyes. For that reason the flaps of the observing tent are usually kept tied down on the windward side when the wind is strong, except for small openings on line to the stations to be observed. If the wind is especially constant and bothersome, a strip of muslin of the proper size to cover the windward openings of the observing tent may be tied or pinned in place and holes 4 or 5 inches in diameter cut on line to the stations through which the stations may be observed.

OBSERVING PROGRAM

Observations of vertical angles for trigonometric leveling should be made at the first opportunity. If the party does not arrive at the station until near the end of the period during which it is permissible to observe verticals (see Instructions, p. 20), the tent may be hastily put up and the verticals measured before the stand is tested. Verticals on both occupied and intersection stations should be made on two different days if the party is held that long at the station by failure to finish the horizontal measures the first night.

When verticals must be observed at night and there are available two observers and an instrument for observing vertical angles besides the one used for the horizontal directions, the instrument for observing vertical angles may be mounted on a tripod or other support outside the tent and the observations of horizontal directions and vertical angles carried on simultaneously, one man recording for both observers.

After the vertical angles have been observed the measurements necessary to reduce the elevations to the station mark should be made and recorded in the record book for vertical angles, commonly called the D. Z. D. (double-zenith distance) book. In order that no essential item may be forgotten, the recorder upon reaching a new station should write in his record book the following tabulation and later see that the measurements are recorded in the proper place:

		Meters
Height of tripod head above station mark		
Height of vertical circle above station mark		
Height of helio above station mark		
Height of lamp above station mark		
Reference mark No. 1	above station mark	
	below station mark	
Reference mark No. 2	above station mark	
	below station mark	

Horizontal directions to the intersection stations and the reference marks are usually observed immediately after the completion of the vertical-angle measurements. The distance to the reference marks should be determined at the same time and the description of the station written up. The description should invariably be written while at the station, to avoid the vagueness and errors which will otherwise be present.

Immediately upon reaching the station, if during the hours when light keepers are supposed to be on their stations, begin "calling" them with the signal light in order to give them a correct line to the observer's station. It is only after signals have been satisfactorily exchanged with each light keeper who is to show to the observer's station that there is any certainty of finishing the station that night. It is often of great assistance in calling the various stations to take a round of directions in the afternoon upon either the helios or upon the approximate positions of the stations as shown by the map, recording the directions on a flyleaf of the horizontal-direction record. Even an approximate direction will be of value in pointing a light after dark upon a station which is not showing a light.

A good recorder will do much to reduce the time spent in observing at a station. He will compute and tabulate the directions, and within five minutes after the observing program is finished he should be able to show the observer the mean direction to each station and also the means of each group of four positions on each station. The observer can then judge what positions must be rejected and reobserved. If the direction to any station shows a progressive change during the observing period, as revealed by the averages of the four-position groups, additional directions may have to be observed even though the observations already made are within the rejection limit.

Before the night's observing begins the recorder or observer will also have computed the angles which will exactly close such triangles as are to be closed at the station, so that an inspection of the recorder's abstract of directions will show at once if the closures are satisfactory.

Even before the observer finishes his program the recorder should be able to judge from the average directions given by the first 12 positions whether or not there will be many rejections and whether the values obtained will close the triangles satisfactorily. If everything is favorable the light keeper at the observer's station, about half an hour before the time when the observing will be entirely completed, can call all stations and give the light keepers "A A," signifying that they are to keep a constant watch for further signals. With all light keepers watching closely 10 or 15 minutes will often suffice for all signaling after the completion of the station.

In the preceding pages on the routine of party operation mention has been made of the need for systematic planning and for efficient speed. Only by keeping these two factors constantly in mind can the chief of party make rapid progress with the work and at the same time secure a proper amount of rest for his party. There is an exhilaration attendant upon swift and well-directed muscular movements which is lacking in leisurely motions. If there is an hour's work to do and two hours in which to do it, finish the work in an hour and then rest rather than prolong the work throughout the greater part of the two hours. In that way the entire party will keep keyed up and become trained to perform a complex operation like pitching camp in almost a constant period of time, even under very divergent conditions.

PRINCIPAL SOURCES OF ERRORS IN HORIZONTAL-ANGLE MEASUREMENTS

The experience of observers in the past has shown that practically all large errors in the measurement of horizontal angles, and many of the errors of smaller magnitude, are avoidable by the exercise of good judgment and care in the manipulation of the instrument and in the selection of observing conditions. The term "observing conditions" as here used does not mean atmospheric conditions alone but includes the requirements imposed by proper reconnaissance and the observance of adequate precautions against the various classes of systematic errors. A good observer is one who can consistently secure results commensurate with the possibilities of the instrument which he is using, or if the dictates of economy prescribe that rougher observations shall be made he must then be able to estimate the approximate magnitudes of the errors introduced by the less accurate methods employed. Such proficiency can be obtained only by careful study of the instrument and by constant exercise of good judgment. Experience is also a large factor, and it is to assist in the acquisition of this experience that the following suggestions are made.

It must be borne constantly in mind that it is futile to eliminate the factors which will cause minute errors and neglect at the same time gross errors which may be simply the result of haste and carelessness. Due regard must be had for the relative importance of the different classes of errors. Too often the distance to a reference mark is measured to millimeters but the record fails to state whether it is a horizontal distance or one measured along the slope; or a very precise estimate of the phase of a signal may be made and the observer may forget to tighten the screws which clamp the leveling screws firmly to the tribrach.

The actual pointing of an instrument on an object is a simple operation; it is a mistake to try to perfect the pointing after the perceptions are once satisfied that the object has been centered on the wires. The most satisfactory observations are usually those which are made quickly. Speed can be attained by the observer by training himself in deftness of movement in manipulating the instrument and in studying how he can perform the manipulations with the fewest movements.

There are four principal causes of error in the measurement of horizontal angles aside from blunders, such as reading a vernier incorrectly, and the allowable errors of pointing upon the object. These four causes, arranged in the order of the usual relative importance, are: Instrumental errors, phase, eccentricity, and horizontal refraction.

INSTRUMENTAL ERRORS

The adjustments of the theodolite and the effects of the errors resulting from lack of perfect adjustment have been described on pages 29 to 40. These errors may be summarized as belonging to two classes: First, those which may be practically eliminated by a proper observing program; and second, those which can not be so eliminated. To the first class belong: (1) The errors due to lack of proper collimation adjustment; (2) the unequal spacing of reading microscopes; (3) the eccentricity of the graduated circle with reference to the vertical axis of rotation of the reading micrometers; (4) the lack of horizontality of the horizontal axis of the telescope; and (5) the errors of graduation of the instrument, though the last error is only partly eradicated by the distribution of the readings about the circle. The methods by which these errors are eliminated are stated in the section referred to above.

Among the instrumental errors which can not be eliminated are those due to parallax and to lack of horizontality of the graduated circle. The effects of these errors are described in the section relating to the adjustments of the theodolite, page 29. There are other errors, however, which although real are much harder to evaluate.

Certain theodolites, because of excessive friction in the centers, or because of undue tolerance in the fit of the centers or instability in the support of the theodolite, exhibit a "drag" which may be manifested in either of two ways. If friction in the centers is causing errors they can usually be detected by reading upon the initial four times for each position, that is, at the beginning and end of each half-position, instead of only twice, as with the usual method. If the movement of the telescope alidade has dragged the graduated circle around with it slightly, a series of such readings will indicate it. If the drag is due to slight movements of the theodolite upon its supporting surface, the device shown in Figure 35 will correct the fault.

If there is a lack of fit in the centers or a yielding in the support of the theodolite it can often be detected by the same method of ob-

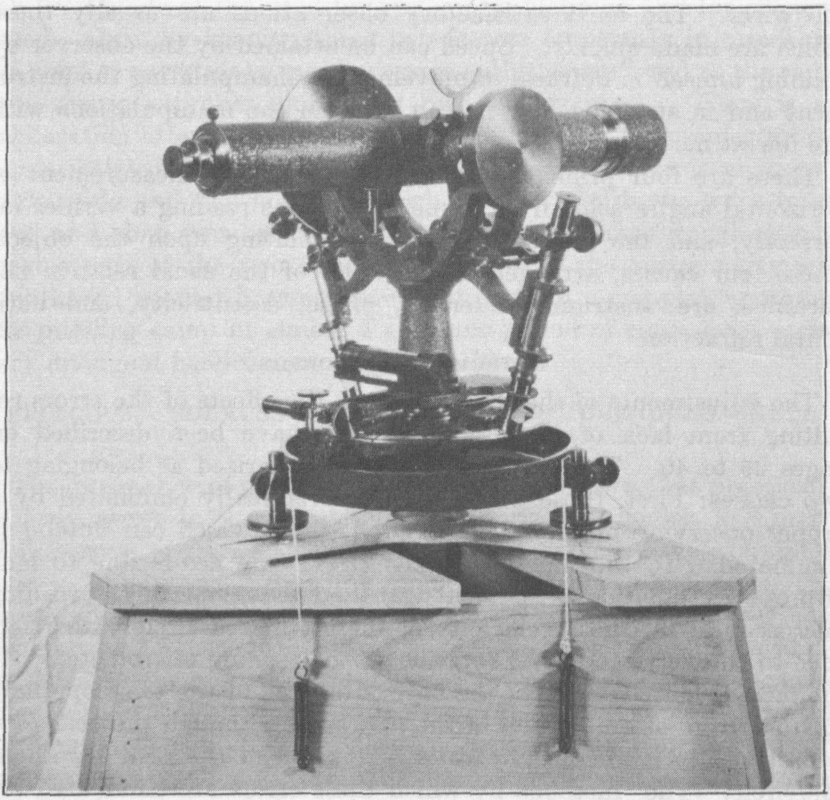

FIG. 35.—Device for holding light theodolite firmly to stand

The springs have the effect of making a light theodolite, which can be easily transported, rest as firmly upon its support as would a heavy theodolite.

serving. Its effect can be lessened by slowly swinging the telescope through rather a wide arc in a clockwise direction up to the initial to begin a position, in order to force all yielding parts to their extreme limit of motion in that direction. The same procedure is followed in pointing upon the other stations. After reversing, the telescope is brought up to each pointing by swinging in the opposite direction. If the telescope is swung past a station it is brought back and brought up to it again in the proper direction. It is a good plan to sight back upon the initial on two or three positions at each station in order to obtain information regarding the presence of drag. These verification readings to close the horizon should not be used in computing directions.

The changes in an instrument due to changes of temperature are such that even a small theodolite should be protected at all time from direct sunlight if good results are to be obtained. The effects of these temperature changes are manifested in various ways. A graduated circle will expand on the side next the sun and there will be a differential change in the eccentricity of the circle which will not be eliminated. Standards will change in their relative elevation, the collar of the diaphragm will change its adjustment, and since all of these are changing in amount from moment to moment no system of observing will eradicate them. This is one of the principal reasons why rapid pointings upon the object give more accurate results than slow pointings.

Another element which is often not appreciated is the manner of manipulation of the instrument. In an endeavor to make rapid pointings instrumental errors are sometimes introduced which are larger than those the observer is seeking to avoid. The hand should not rest heavily upon the instrument at any time. Slow-motion screws and the graduated heads on the screws of reading microscopes should be turned with a true rotary motion without lateral thrust. Slow-motion screws should always be tested to see if there is any dragging when the screw is turned in the direction which decreases the tension of the spring. If there is any appreciable dragging and the cause can not be found and corrected, then the final motion of the screw must always be made against the spring. In this connection it should be noted that many books on geodesy state that the observer should invariably make the last motion of the micrometer and slow-motion tangent screws against the spring. Extensive comparative tests and the experience of many observers on first-order triangulation over many years have shown, however, that with well machined, and clean, well-oiled screws acting against springs of the proper tension the errors introduced by making the final pointing in a direction away from the spring are, at least, no greater than those resulting in pointing or reading with the final movement always in one direction, since the latter method is liable to cause larger errors of estimation than the former.

The cause of large errors frequently lies in the support of the theodolite head. The leveling screws should always fit tightly into the arms of the theodolite base, and if a tripod is used the legs must be clamped firmly to the tripod head. The observer must school himself to step wide around the points of support of the tripod or stand upon which the instrument rests and to be careful that no lateral thrust from the weight of his body shall be transmitted through loose rocks or other means to the tripod legs. When observing from towers which have stood for some time, in which nail holes have become large and worn and the wood has lost its springiness, it is very difficult to secure good results.

PHASE

The errors in horizontal-angle measures due to phase, or to the unequal illumination of a target, are often of considerable magnitude. In effect it is an eccentricity which could be corrected for if its exact amount were known. The difficulty lies in the inability of the observer to estimate its amount accurately, for it depends upon factors which change rapidly. The angle of the sun with the line of sight, the opacity of the signal, the shape of the object sighted upon, the intensity of the sunlight, will each have its effect on the appearance of the signal.

Many textbooks give trigonometric formulas for the correction of phase which are based upon the direction of the sun. These are usually not practicable to apply because other factors enter in. The apparent penumbral zone lying between the surface having a full illumination and that having no direct sunlight upon it will vary in width with the intensity of the light. The formula would also apply only to cylindrical or spherical objects, whereas many of our observations are made upon squared timbers. A target made of signal cloth will show a different phase from one made of lumber of the same shape and dimensions. For these reasons the best rule, when the outlines of the signal can be seen, is for the observer to make a close examination of the signal through the best telescope available and decide upon what part of the illuminated surface it is necessary to observe in order to eliminate errors due to phase. If the outlines of the target can not be seen, a can or other object placed at some distance away in the direction of the signal will show what part of it is illuminated and will give a rough idea as to how to point upon the object, if it is remembered that most of our telescopes are inverting.

Squared timbers should be used for signal poles on short lines and one side should be exactly faced toward the observer. On first-order traverse a vertical pole with a T cross section has been used to advantage. The stem of the T is a 2 by 2 or 2 by 4 inch piece, while the bar of the T is a thin board, which is faced toward the observer. The thin edges of the board prevent any errors of phase, and the square timber, hidden from the observer's view, gives the necessary rigidity to the pole.

ECCENTRICITY

A target or instrument is said to be eccentric when its center is not in the vertical line passing through the point to which the observations are referred. The proper correction can always be made for eccentricity if the distance and direction to the true station are recorded. Often, however, the observer desires to make an estimate of the error which would be introduced by an approximate amount of eccentricity due to phase or to some other causes. This can easily be made by remembering that "a second is a foot at 40 miles," or that an inch represents about 3⅛ seconds at a distance of 1 mile. On short lines the improper centering of either the theodolite or the target will introduce large errors. Warped or crooked signals must be observed upon with care, and an equal amount of care used in testing them for eccentricity. The safest plan is to point always at some certain part of the signal, say at the bottom of the target, and to plumb that particular point over the station. Upon reaching a signal which has been observed upon it should be tested for eccentricity before its position is changed and the amount of its eccentricity recorded in such a manner that the computer can make no mistake when correcting the observations for it. (See p. 94.)

When using lamps on short lines care must be taken to point the lamps directly toward the observer, because the glass in front of the light is sufficiently illuminated to be visible to the observer and if not centered directly on the line will affect the accuracy of the observation.

HORIZONTAL REFRACTION

The horizontal component of the curved path of the rays of light which pass from the object observed upon to the theodolite of the observer constitutes an error which can not be corrected for. It can, however, be avoided in large measure by carefulness in the reconnaissance and in the selection of observing conditions. The existence of this horizontal refraction can not always be foreseen, but certain atmospheric and topographic conditions operate strongly to cause such errors, and these will be briefly pointed out.

Under ordinary conditions the air strata are of greater density near the ground and lie roughly parallel to it. Over a sloping terrain these strata of different densities are not horizontal, and a ray of light passing through them will be bent horizontally as well as vertically. The greater the difference in density in the air strata passed through, and the more they are inclined to the horizontal, the greater will be the horizontal bending of the light rays.

The most potent cause of this variation of density is the unequal heating of the air. The force and direction of the wind are also determining factors, for with a strong wind the differences in the temperatures and densities of adjacent air strata are less marked. A very usual condition met with on triangulation is that which is encountered when the line of sight passes alongside a bluff or mountain slope. Under these conditions it may be necessary to make observations in overcast weather or when the wind is blowing toward the bluff, if the first set of observations give indications of horizontal refraction. If the wind blows down a slope and across the line of sight the heated air from the slope will often cause trouble. A line passing near a building or stone wall will suffer a like change in direction. The nearer the cause of the disturbance to the observer the greater will be the angular distortion.

A line of sight passing over a level country may also be refracted horizontally by the influence of variations in the vegetation conditions under the line of sight. Its influence can be understood by considering an extreme case in which the line of sight passes over a region covered with forest, except at one place where there is a V-shaped plowed field, with its apex beneath the line of sight. With a gentle breeze blowing across the field toward the line of sight the light rays will be bent horizontally, because the heated air from the field will have a different density from the atmosphere adjacent to it on the sides and will form a vertical triangular prism.

Because of all of these factors night observations are more accurate than those made by daylight, although other errors due to similar causes may be introduced by night observations. Care must be taken in placing the lantern used by the recorder if of a type which gives out considerable heat, for if the observer is sighting through a column of heated air in the immediate vicinity of the instrument refraction errors must be expected. Trucks or tents under or near the line of sight may also cause appreciable errors, due to bending of the line of sight.

It must not be thought that errors caused by horizontal refraction are always of small magnitude. On first-order triangulation, where the probable error of a direction may average about one-half second, cases are not infrequent where horizontal refraction has caused an error of from 3 to 6 seconds. At times with such errors present a single night's observations will exhibit a very small range in the observed directions and there is nothing in the results of the observations to indicate that the line is in error. The triangle closures alone furnish a guide to the source of trouble.

One effect of the changes in density of the air is seen in the unsteadiness of signals and of signal lights when enlarged by the telescope. "Looming," "waving," "unsteady," etc., are the usual terms applied to targets to denote their appearance, while "flickering," "flaring," and "walking" are descriptive terms applied to lights. In general, it may be said that looming and unsteady targets and flickering lights need cause no hesitation or uneasiness on the part of the observer, but that waving targets and flaring lights should be examines closely to see if there is a semipermanent displacement of the center of the image. Set the cross wires of the telescope as precisely as possible on the mark and examine its behavior for a minute or two, when the extent and character of its motion will be evident. The term "walking" is applied to a signal or light when it suffers a semipermanent angular displacement. This displacement has been observed to be as much as 10 or 15 seconds with a period of 2 or 3 minutes; such a large angular displacement is very rare, but when it occurs there is little chance of obtaining satisfactory observations.

OTHER CONDITIONS AFFECTING THE ACCURACY OF OBSERVATIONS

Effect of brightness and size of light.—In laboratory experiments the size of an object sighted upon has a considerable effect upon the accuracy with which it can be bisected by a line. Likewise on triangulation the size of the light may affect somewhat the accuracy of the observed direction, but agencies for controlling the apparent size of the light are not always available on field work, and it has been found that the errors of observation due to pointing upon large lights are not large compared to those from other sources. Formulae are sometimes found in books on geodesy for determining the size of aperture of signal lights for different distances, but such formulae are not closely applicable to field conditions where the lines of sight are more than 10 miles long and where the apparent size of a distant light depends almost entirely upon the steadiness of the atmosphere. The light from an electric signal lamp may appear as a point of light one night and the next will subtend an angle of 40 seconds or more; in fact, an hour's time may cause that much change in the apparent size of a light. The use of diaphragm rings on a light when the atmosphere is unsteady will, on lines more than 5 or 10 miles long, reduce the intensity of the light and may even make it appear somewhat tenuous, but will not materially reduce its apparent size. On lines less than 2 or 3 miles long, however, it is usually advisable to use diaphragm rings, or else to use a lamp with a small reflector like a flashlight.

Another factor affecting the accuracy is the apparent brightness of the light pointed upon. Without doubt the greatest accuracy can be obtained when pointing upon lights of about equal brilliancy, for then the illumination of the diaphragm wires can be kept constant. On the other hand, many years of observing have shown it to be possible to obtain the accuracy required on first-order triangulation when the signal light is so faint that the proper settings must be made on both the horizontal and vertical circles before it can be picked up. When the light is very dim the pointing must be made by cutting down the illumination of the wires to the point when they are barely visible, then switching the illumination off until the signal light is picked up and brought toward the center of the wires. By successive approximations, since the signal light is not visible when the wires can be seen, a pointing is obtained where the position of the signal light as retained by the eye in the field of view after the light is obliterated by the illumination seems to be in the middle of the parallel diaphragm wires as they become visible. It is to secure pointings upon such faint lights that the switch controlling the illumination of the diaphragm wires is always made in the form of a pressure contact lever, placed where it can be operated by the hand which is on the tangent screw.

When the lights are large and flaring it seems almost as difficult to secure accurate pointings as when the lights are dim. With the parallel diaphragm wires including an angle of from 25 to 35 seconds, as is usually the case in large modern theodolites, and with a flaring light more than a minute of arc in diameter as a target, it seems hopeless to a novice to expect to measure the angle with the accuracy prescribed for first-order triangulation. Yet the experience of many field seasons shows that it can be done, and that the eye can bisect a wide target with a very small error, or at least with an error which is very consistent and which is largely eliminated by the method of observing.

When the light is very large and flaring there is a temptation to use a lower-power eyepiece in the theodolite, since this seems to make the outline of the light more regular by making less prominent the flaring fringes of the light. This is a mistake, for the greater magnification enables the observer to judge better what part of the light to sight upon. Usually the proper pointing is neither upon the most solid part of the light nor the center of the entire light area, but upon a point somewhere between the two. It is only by experience that the observer can learn to choose this point with any certainty.

Where the light is very intense there is a tendency for the diaphragm wires to "burn out," that is, to become less visible. Either one or both of two remedies may be used to correct this. A signal may be sent the light keeper to reduce the number of batteries on the light or to substitute old ones, or the observer may increase the illumination of the diaphragm wires. Another effective means of reducing the injurious effect of bright lights is to have the diaphragm wires heavy and black rather than fine and partly translucent. Heavy wires are also an advantage when the lights to be observed upon are very faint and the illumination is necessarily dimmed. Where two parallel wires are used as a sight it is only necessary that they be smooth and a uniform distance apart. Even on small instruments parallel vertical wires, if they are mounted sufficiently far apart, are better than the usual intersecting lines, since there is no chance for the parallel wires to blot out the target.

Partly obstructed light.—If part of a light, when large in diameter as viewed through the telescope, should be obstructed, the measured angle would be in error and that error would be revealed only by the triangle closures. In one instance an error of approximately 7 seconds was caused by the light from a station 23 miles away being partly obstructed by a tree about 8 miles from the observer. It sometimes happens on traverse that a line of sight passing under an obstruction is entirely clear by day when the reconnaissance is made, but at night the greater refraction will cause the light to be wholly or partly obstructed.

Rapidity of observing.—In the discussion of instrumental errors it was pointed out that speed in observing was an aid to accuracy as well as to economy. Since speed is a relative term an attempt will be made to give it a more exact meaning, for rapid observing does not require hurried movements nor does it imply careless work. Except in the rare cases where the pointing is being made upon a light which has a slow vibrating movement, not more than a second or two at the most should be spent in pointing the telescope upon the light *after the observer's perceptions are satisfied that the bisection is good.* It is better to quit the telescope the instant the perceptions are satisfied with the pointing. The micrometer should be read with even less hesitation. When all lights are visible to the naked eye and no time is lost in setting the circle for faint lights, a trained observer will average almost a pointing to the minute throughout the evening, reading all the micrometers of a three-micrometer instrument himself. This means that with a five-light station each position will take 10 minutes, including the settings for the initial. With a two-micrometer instrument, or when the observer has assistance in reading the micrometers, the time will be reduced.

ACCORDANCE OF OBSERVATIONS

The fact that the directions to a certain light given by the different positions or settings on the circle exhibit a considerable range does not necessarily mean that their average value will be far from the true one, nor does a small range of values indicate the absence of some constant error. The methods and instruments prescribed will usually give the results sought. If they do not the observer must locate the source of the trouble as quickly as possible and correct it. If the error is due to horizontal refraction it may suffice to elevate the instrument or some of the lights. If the refraction error is due to seasonal winds it may be advisable to proceed to other stations and reoccupy the station affected when the wind conditions have changed, though that is usually expensive. Above all, the observer must not try to force the observations by sighting on a different part of the light from that which his judgment says is the proper point. He must cultivate an impersonal attitude toward his results and read the angles as they are, for an angle forced to give a good triangle closure will often result in a large correction to a direction when the least-squares adjustment is made. The best results are secured by treating the readings as true observed quantities until the observations are completed.

DETERMINATION OF VALUE OF ONE DIVISION OF A LEVEL BUBBLE

It is frequently necessary to determine in the field, where a level trier is not available, the value of one division of a level bubble. This is readily done by the method described below. The principle of this method consists in measuring an intercept whose length is known, at a known distance, in terms of divisions of the bubble. The angular value of the intercept is then calculated and the value of one division thus obtained.

Fasten the level to be tested, by adhesive tape or otherwise, longitudinally along the top of the telescope of a theodolite having a clamp and slow-motion screw for moving the telescope in a vertical plane. Suspend vertically an accurately graduated tape at a known distance from the theodolite, say 50 or 100 feet, or else plumb an accurately graduated rod at that distance. If the bubble has a chambered vial adjust the length of the bubble so it will extend under about one-third to one-half the graduated portion of the vial.

Point upon some division of the tape or rod so selected that the bubble will be near the end of the vial toward the eye end of the telescope. Read and record both ends of the bubble, and repeat the pointing and readings until about 10 readings are obtained, bringing the cross wire of the telescope upon the mark half the time from above the mark and half the time from below, to neutralize the effect of any friction of the bubble against the vial. The mean of all the bubble readings will be the mean position of the center of the bubble.

Next point upon some mark on the tape or rod which will bring the bubble near the end of the vial toward the objective end of the telescope, take readings as before, and obtain the mean position of the center of the bubble. Measure carefully the distance from the rod to the horizontal axis of the instrument, compute the angular value of the intercept at that distance, and divide by the number of divisions of the bubble between the two mean positions. The value of one division of the bubble obtained by this method will be as accurate as that obtained on the average level trier.

DETERMINATION OF HEIGHT OF STATION BY OBSERVING SEA HORIZON

At times it may be difficult to connect a triangulation scheme to a bench mark. If some of the stations are within sight of the ocean the elevations of the stations, as determined by the vertical-angle measurements carried through the chain of triangles, can be checked and made more exact by observations upon the sea horizon. Elevations determined in this manner are not as accurate as when frequent connections can be made to bench marks, for the observations are nonreciprocal and an arbitrary value must be used for the coefficient of refraction, m, which may vary for daytime observations on the sea horizon from 0.078 to 0.130. The formula for computing the height of station from the observed angle of depression is:

$$h=\frac{1}{2}\,\rho\,\frac{\theta^2}{(1-m)^2}\sin^2 1'' + \frac{1}{8}\,\rho\,\frac{\theta^4}{(1-m)^4}\sin^4 1''$$

where h = elevation of station above sea level.

ρ = radius of curvature of the earth (in the same unit of length as h).

m = coefficient of refraction.

θ = observed angle of depression in seconds.

For ordinary purposes, and where θ does not exceed 120 seconds, the second term of the above formula may be disregarded, as well as powers of m above the first and the formula becomes,[1]

$$h=\frac{1}{2}\,\rho\,\frac{\theta^2}{(1-2m)}\sin^2 1''.$$

The time of the observations may be noted and a correction for height of tide applied if the accuracy of the work warrants it.

FIELD COMPUTATIONS

Only such computations need be made in the field as are required to inform the observer as to the accuracy which is being attained and to reduce the observations to such form as will make them readily available for office computation. If there is immediate need of the geographic positions of the stations on some portion of the line and it is necessary to compute the positions in the field, mention will be made of it in the instructions to the chief of party. All computations should be made in a neat and orderly manner, on the forms provided for the purpose, and should be kept up to date. The last requirement is the one which demands constant effort and watchfulness on the part of the chief of party.

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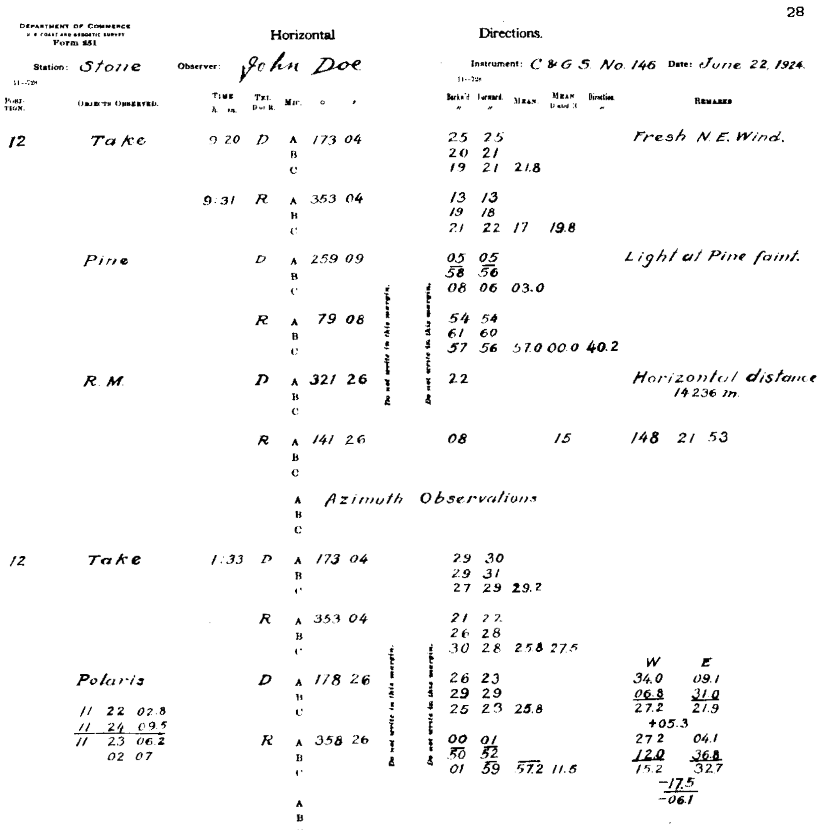
[1] Assuming a coefficient of refraction of 0.130, the error introduced by disregarding the second term is 0.1 meter (0.3 feet) when θ=60 seconds, 2.1 meters (6.9 feet) when θ=120 seconds, and 10.2 meters (33.5 feet) when θ=180 seconds.

The record and computation forms used in the field are listed below and a brief description of the proper preparation of each form will follow.

- Record, horizontal directions.
- Abstract of horizontal directions.
- List of horizontal directions.
- Reduction to center.
- Triangle-side computation.
- Position computation (if needed).
- List of geographic positions (if needed).
- Record, double-zenith distances.
- Abstract of zenith distances.
- Description of station (new stations).
- Description of station, recovery note (old stations).

Illustrations are given (figs. 36 to 47) for the purpose of showing how each form is filled out.



28

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 251

Horizontal Directions.

Station: *Stone* Observer: *John Doe* Instrument: *C & G S No. 146* Date: *June 22, 1924.*

11—728

Position	Objects observed	Time h. m.	Tel. D or R	Mic.	°	′	Back'd ″	Forward ″	Mean	Mean D and R	Direction ″	Remarks
12	Take	9 20	D	A	173	04	25	25				Fresh N. E. Wind.
				B			20	21				
				C			19	21	21.8			
		9:31	R	A	353	04	13	13				
				B			19	18				
				C			21	22	17	19.8		
	Pine		D	A	259	09	05	05				Light at Pine faint.
				B			58	56				
				C			08	06	03.0			
			R	A	79	08	54	54				
				B			61	60				
				C			57	56	57.0	00.0	40.2	
	R. M.		D	A	321	26	22					Horizontal distance 14236 m.
				B								
				C								
			R	A	141	26	08			15		148 21 53
				B								
				C								
				A	Azimuth Observations							
				B								
				C								
12	Take	1:33	D	A	173	04	29	30				
				B			29	31				
				C			27	29	29.2			
			R	A	353	04	21	22				
				B			26	28				
				C			30	28	25.8	27.5		W E
	Polaris		D	A	178	26	26	23				34.0 09.1
				B			29	29				06.8 31.0
	11 22 02.8			C			25	23	25.8			27.2 21.9
	11 24 09.5											+05.3
	11 23 06.2		R	A	358	26	00	01				27.2 04.1
	02 07			B			50	52				12.0 36.8
				C			01	59	57.2	11.6		15.2 32.7
												−17.5
												−06.1
				A								
				B								
				C								

Do not write in this margin.

FIG. 36.—Sample record of horizontal directions and azimuth observations

RECORD, HORIZONTAL DIRECTIONS

A double page of the record book is shown in Figure 36. The means should be checked before the abstract of directions is made out, preferably by some one other than the original computer. Each page checked should be so indicated by a check mark at the bottom of the page, with the initials of the person checking. There is also shown in Figure 36 one observation on Polaris for azimuth, showing how the time and level readings are recorded. The final seconds of the direction are not taken out for Polaris, since the mean angles for both Polaris and the mark are entered on the computation form.

ABSTRACT OF HORIZONTAL DIRECTIONS

It is important that this form be made out carefully, because the mean directions derived from the abstract of horizontal directions constitute the basis for all the later computations. Every position observed at a station, except observations on objects where only one or two positions are taken, should appear on the abstract, the rejected readings being indicated by the letter R. Sample forms are shown in Figures 37 and 37*a*.

Where more than one station is used as an initial there will frequently be different ways in which the observations can be combined to give the directions from some one initial station. Figures 37 and 37*a* will illustrate the proper way to form the combined direction in a number of typical cases.

At station Granite both South Base and Westedge were used as initials in the observations, but South Base was chosen as the initial for the list of directions. A supplemental abstract of directions, Figure 37*a*, was first made out for the observations in which Westedge was used as initial, and the abstract, shown in Figure 37, was then made out for such observations as had South Base for initial. It was then necessary to transfer the observations made with Westedge as initial to equivalent values with South Base as initial, marking such transferred directions with the letter T on the abstract to show their origin. For example, in Figure 37*a* the direction of Floyd from Westedge, position 1, is 271° 11′ 44.″0, while the direction of Westedge from South Base, position 1, is 17° 17′ 49.″5, and the sum of the two directions, 288° 29′ 33.″5, is the direction from South Base to Floyd, as shown for position 1, Figure 37. Similarly, the values for the other positions for Floyd and Williams are transferred from the supplemental abstract to the combined one, using for each position the corresponding value of the angle between South Base and Westedge.

It is not necessary to transfer Frisco from the supplemental abstract shown in Figure 37*a* to the combined abstract for the reason that a complete set was observed on that station from each initial. The mean of the directions on Frisco, with Westedge as initial, viz, 326° 55′ 07.″57, plus 17° 17′ 50.″76, the mean of the directions on Westedge with South Base as initial, gives 344° 12′ 58.″33. The mean of this value of the direction to Frisco and that obtained with South Base as initial, 344° 12′ 57.″69, is used in the list of directions shown in Figure 38. If more than 10 or 12 acceptable positions are obtained on any one night for a direction, that night should be given unit weight with any other night in determining the mean direction. In general, where 10 or more positions of a direction have been measured on each of two or more nights no one night's observations should be rejected unless it is more than one-half second from the mean of all the values for that direction. If the divergence from the mean is

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Form 470

ABSTRACT OF DIRECTIONS

State Arizona

Granite — Computed by O. P. S. — Date July 25-Aug 6

Observer C. V. H. — Checked by W. F. R. — Inst. No.

DO NOT WRITE IN THIS MARGIN

POSITION NO.	South Base (INITIAL) 0° 00' ″	Westedge 17° 17' ″	Union 89° 58' ″	Floyd 288° 29' ″	N. Base 318° 34' ″	Williams 325° 54' ″	Frisco 344° 12' ″	° ' ″
1	0.00	49.5	01.3	33.5T	47.8 (41.6)R	49.0T	57.0	
2	0.00	53.0	00.4	35.7T	46.5	53.0T	54.5	
3	0.00	49.6	59.6	32.4T	49.3	51.1T	56.7	
4	0.00	51.5	59.8	36.3T	47.3	53.3	59.6	
5	0.00	50.8	02.2	37.3T	46.2	51.3	58.3	
6	0.00	51.0	02.8	38.3T 36.6 }37.4	48.0 (42.5)R	51.8	60.7	
7	0.00	48.2	58.5	32.8	45.8	(57.2)R 52.8	56.5	
8	0.00	50.4	03.0	36.8	46.8	52.2	60.0	
9	0.00	52.2	05.0	35.5	45.8	53.0	54.5	
10	0.00	52.0	00.5	36.2	47.7	52.8	56.7	
11	0.00	50.5	00.6	35.7	46.4	52.2	55.7	
12	0.00	51.7	00.7	35.3	48.3	53.8	59.7	
13	0.00	50.5	02.5	32.3	48.0	51.0	56.5	
14	0.00	50.7	03.8	35.4	47.2	54.0	59.2	
15	0.00	48.8	59.2	34.2	47.0	52.8	59.6	
16	0.00	51.7	04.6	33.7	47.0	52.2	57.8	
Sum,		812.1	24.5	560.5	755.1	836.3	923.0	
Mean,		50.76	01.53	35.03	47.19	52.27	57.69	
Cor. for sec.,			−0.26					
Direction,		50.76	01.27	35.03	47.19	52.27	57.69	

FIG. 37.—Sample abstract of directions

greater than one-half second that night's observations should be selected which will best close the triangles, provided that at least 12 acceptable positions are available for the retained direction.

It will be noted that since the angles were measured from South Base to Westedge, from Westedge to Frisco, and from South Base to Frisco, the proper value of the direction from South Base to the other two points could be secured most accurately by a least-squares adjustment (see Station Adjustment, pp. 8–13, Special Publication No. 28). In most cases, however, the results obtained by this station adjustment do not justify the time required to make the com-

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U. S. COAST AND GEODETIC SURVEY
Form 470

ABSTRACT OF DIRECTIONS

State *Arizona*

Station *Granite* Computed by *O. P. S.* Date *July 25 – Aug. 6*

Observer *C. V. H.* Checked by *W. F. R.* Inst. No. *Wanschaff #2*

DO NOT WRITE IN THIS MARGIN

POSITION NO.	STATIONS OBSERVED: *Westedge*	*Floyd*	*William*	*Frisco*				
	(INITIAL) 0° 00′ ″	° ′ *271 11* ″	° ′ *301 17* ″	° ′ *326 55* ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	0.00	*44.0*	*59.5*	*07.2*				
2	0.00	*42.7*	*60.0*	*06.5*				
3	0.00	*42.8*	*61.5*	*07.1*				
4	0.00	*44.6* *45.0*		*09.0*				
5	0.00	*46.5*		*07.0*				
6	0.00	*47.3*		*10.0*				
7	0.00	*Transferred to sheet 1.*	*Transferred to sheet 1.*	*06.3*				
8	0.00			*08.7*				
9	0.00			*07.1*				
10	0.00			*05.9*				
11	0.00			*08.2*				
12	0.00			*08.0*				
13	0.00			*06.7*				
14	0.00			*07.0*				
15	0.00			*09.1*				
16	0.00			*07.3*				
Sum,				*121.1*				
Mean,				*07.57*				
Cor. for ecc.,								
Direction,				*07.57*				

FIG. 37a.—Sample abstract of directions for missing signals

putation, but a mean value for the sum angles can usually be obtained by arbitrary methods which will meet sufficiently well the final demands for accuracy. When a number of sum angles are measured, however, and especially when the means obtained by different combinations vary considerably, a station adjustment may be made.

The direction to triangulation station Floyd, Figure 37, has two acceptable values for position 6. In such cases the mean is taken of all values for a position and that mean given unit weight in the final mean, on the theory that a symmetrical distribution of the readings around the circle is essential to accuracy. With an accurately graduated circle it is probable that the variation due to the graduation is not quite so large as that due to errors in reading, but the rule of unit weight for each position is the safest to follow as a uniform procedure.

Rejection of observations.—The chief difficulty in making out the form lies in deciding what observations to reject. The usual formulæ for the rejection of observational quantities are too cumbersome to apply, and are not satisfactorily applicable to a short series of observations. It is, therefore, customary to apply an arbitrary limit of rejection, determined empirically from previous experience with the instrument used or with one of similar qualities. For observations with the type of theodolite usually used on first-order triangulation the rejection limit for the angular value of a direction on any one position of the circle may ordinarily be taken as 4 seconds from the mean. Specific ways in which the rejection limit is to be applied are indicated below. If it is found that with such a rejection limit the number of rejections is averaging much in excess of 10 per cent of the entire number of observations, application may be made to the director to increase the rejection limit, though before that is done every effort should be made to determine the cause of the large range.

The following rules will be a sufficient guide to the rejection of observed directions:

1. No reading should be rejected if it falls within the limit of retention (in the sample this limit is $\pm 4''.0$ from the mean) unless rejected at the time of taking the observation. The observer's reason for rejection should then appear in the original record. This rule will not apply to the case where one set of observations of a direction is rejected in favor of another set of 12 or more positions, as provided for on page 90.

2. If two or more readings have been taken for a single position the mean should be used if all readings come within the limit of retention.

3. If one reading falls without the limit and one within the limit, do not use a mean even though the mean be within the limit. Use instead the single reading within the limit.

4. If both readings fall without the limit, one being abnormally high and the other abnormally low, and the mean falls within the limit, reject both readings and try the position again. If a reading is not obtained within the required limit reject the position entirely, using the remaining positions to compute the mean direction.

5. In case the 16 readings seem to fall in two groups, the mean of one group differing from the other by two or more seconds, extreme care is necessary on the part of the observer. The mean of the 16 values is evidently an erroneous value for the direction in this case. The safest plan, and the one which should be followed if the average closing error is more than 1 second, is to observe the entire 16 positions a second time and use the new data to determine the value of the direction.

6. Before computing a trial mean any observations so far from the approximate mean as to be very evidently the result of blunders should be rejected. After a trial mean is obtained and the rejection limit applied, the observations so rejected should not be again included even though the new mean would bring them within the limit of rejection.

7. The results obtained by applying rigorously the limit of rejection, even though the quantities rejected are just outside the limit, will probably be but little different from those derived after long consideration, and much time can be saved the field party by a strict application of the rule.

LIST OF DIRECTIONS

On the list of directions, Figure 38, the mean directions of all unrejected observations are arranged in order of azimuth from some one selected initial. It includes not only the mean directions to the principal stations as listed and computed on the abstract of directions, but also the directions to intersection points and reference marks.

The data on this form constitute the material upon which the office computations are based, and these data should be so completely checked that there will be no need in the office to resort to the record book or the abstract of directions. The only exception to this rule is where there is not sufficient time in the field to make all the eccentric reductions without delay to the progress of the party, and this contingency is provided for in later paragraphs.

On the back of the form for the list of directions are instructions for its preparation. Only two points covered by those instructions need be mentioned here, viz, the number of decimal places to be shown in the mean angle and the treatment of eccentric directions. As regards the first, on first-order triangulation the directions to main-scheme stations should be carried to hundredths of a second, directions to second-order stations and to sharply defined permanently marked intersection stations to tenths of seconds, and directions to other points, such as mountain peaks, to seconds. Directions to near-by objects, such as witness or reference marks, need be taken to the nearest 10 seconds only. In general two uncertain figures should be given; that is, the third digit from the right in the number denoting the seconds should not be in error by more than one unit.

The second point to be emphasized in the preparation of the list of directions is the recording of the eccentricity and the reduction of the observed directions to center. If a direction has not been reduced to center the seconds pertaining to that direction should not be written on the form in ink, but in pencil. This rule should be

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FORM 24A

LIST OF DIRECTIONS

State: Arizona

Station Granite Computed by C. P. S. Station .... Computed by ....

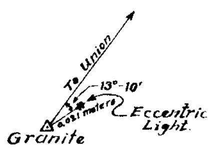
Observer C. V. H. Checked by W. F. R. Observer .... Checked by ....

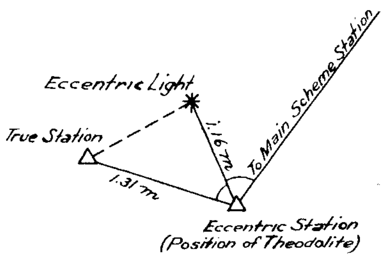
Do not write in this margin.

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″
South Base	00 00 00.00	
Westedge	17 17 50.76	
R. M. #1; dist. 6.738 m.	28 36 09.	
Union	89 58 01.27	
South Smoke stack sawmill	92 43 21.4	
Eccentric Light; dist. 0.021 m.	103 08	
R. M. #2; dist. 23.762 m.	200 25 08	
Floyd	288 29 35.00	
N. Base	318 34 47.19	
Williams	325 54 52.27	
Frisco	344 12 58.01	

Stations Observed	Directions After Local Adjustment ° ′ ″	Final Seconds ″

(Method of recording eccentricity when theodolite is eccentric and eccentric light is more than 0.1 m from the true station)





Light eccentric as given above when shown to Union and Williams.
No eccentricity of light to other stations.
No eccentricity of theodolite.

All directions reduced for eccentricity.

(Record must state to what stations the light was shown in above position.
Measuring the distance from the true station to the eccentric light will give a check on the values for the elements of the small triangle.)

FIG. 38.—Sample list of directions

invariably followed, for otherwise an unreduced direction may be used for a reduced one.

On the list of directions the main-scheme stations should be easily distinguishable from the other points listed. This can be achieved by printing in heavy letters the names of the main-scheme stations, while the names of the other points are written in with ordinary width of line. The sample list of directions shows the appearance produced by such a system. The distinction between the main-scheme points and others can be further accentuated by the use of asterisks if desired.

Whether the directions shown on the form have been reduced for eccentricity or not, any eccentricity of lights or instrument at a station should be recorded on the list of directions in a form that is entirely free from any possibility of misinterpretation. The form of record shown on the sample in Figure 38 should always be used when the light was in an eccentric position and the instrument was centered. The sketch should always be included.

Wherever it is possible to mount the theodolite directly over the mark it should be done, even though the light has been shown from an eccentric point. To have an instrument mounted eccentrically when it can be avoided simply causes unnecessary computation. Directions to eccentric lights can be easily corrected by the nomogram shown in Figure 39, or by tables computed decimally for lines of various lengths and for eccentric distances of different amounts.

When the theodolite must be mounted eccentrically to the station, for the purposes of field computation all directions may be reduced to the eccentric station and the station should be so named—as, "Roundtop Eccentric." In this case the angle and distance to the marked station should be recorded in the manner shown in Figure 38, making sure that the points on the sketch representing the eccentric station and the marked station are distinctively marked.

When at any station the theodolite is eccentric and the light at that station has occupied a different eccentric position the measurements to the eccentric light may be made and recorded in either of two ways. When the light is not farther from the marked station than a decimeter, one edge of a straightedge of some kind can be placed in the vertical line through the marked point and sighted successively at all stations to which the eccentric light was shown, the normal distance from the eccentric light to the true lines being measured and recorded as "Light at Granite when showing to Union 0.011 meters to the west of line to Union." When recorded in this manner there is no chance for ambiguity.

When the instrument is eccentric and the light is shown from a different eccentric point which is more than a decimeter from the vertical line through the marked station, it is better to measure the distance to the eccentric light from the point occupied by the theodolite and the angle at the theodolite point from the eccentric light to some main-scheme station. The distance and direction from the marked point to the eccentric light can then be found by solving the small triangle shown in Figure 38.

The importance of the proper recording of the eccentricity of lights and theodolites has been emphasized for the reason that more mis-

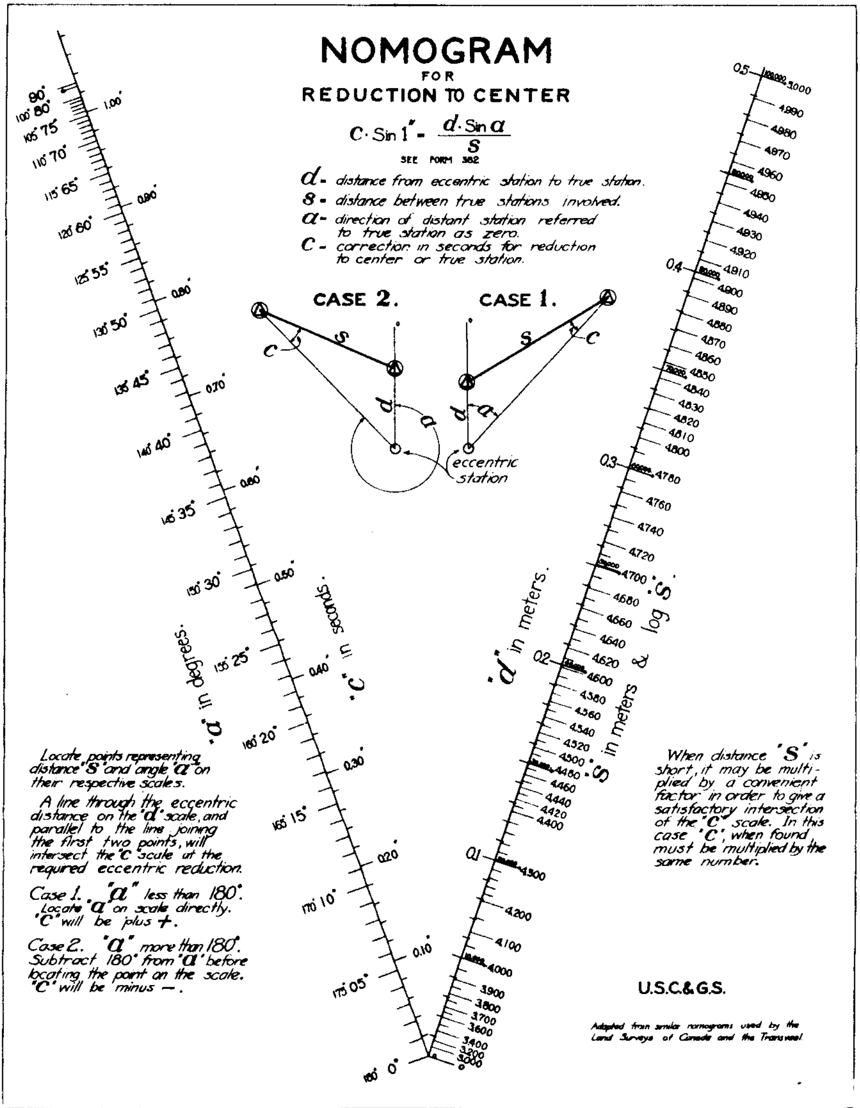


FIG. 39.—Nomogram for reduction to center

Copies of this nomogram on heavy paper can be obtained on requisition takes and ambiguities in triangulation records are traceable to that source than to any other.

REDUCTION TO CENTER

When observations are made upon an object which is not in the vertical line through the point to which the observations are to be referred, the object sighted upon is termed an eccentric object and

INSTRUCTIONS

The required reduction to center is, in seconds, $c=\frac{d \sin \alpha}{s \sin 1''}$, in which d is the distance from the eccentric station to the true station, and s is the length in meters of the line between the true stations involved, and, therefore, log s is taken directly from the computation of triangle sides. α is the direction of the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of α is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

Carry α to minutes only and all logarithms to five decimal places only. Do not in any case carry the derived reductions to more than two decimal places. There is no advantage in carrying them to more decimal places than the directions to which they are to be applied are carried on Form 24 A.

REDUCTIONS FOR AN ECCENTRIC INSTRUMENT

If the instrument is eccentric the first column of this form should contain the names of the stations observed from that eccentric position of the instrument.

The values in the fifth column are derived by subtracting those in the fourth column from those in the third. The values in the fourth column may need to be derived by successive approximations from the triangle side computations if the eccentric reductions are large. The values in the sixth column are obtained from those in the fifth by adding log $\frac{d}{\sin 1''}$, derived as indicated in the heading of the form, if d is expressed in meters. If d is expressed in feet, to the other two logarithms add also 9.48402 to convert to meters. To obtain a direction as shown on Form 24 A, subtract the reduction c for the station which is the initial on Form 24 A from the reduction c for the required direction and apply the difference to the observed direction. *Similarly, the correction to any angle is the difference of the reductions on this form to the two* directions involved in that angle.

REDUCTIONS FOR AN ECCENTRIC OBJECT OBSERVED

If the object observed is eccentric the heading "Eccentric Station ——" should be changed to "Eccentric Observed Object at Station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case α is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction *from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder* of the computation on this form is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions, at the stations named in the first column, to the eccentric object at the station named in the heading. The directions to which these reductions are to be applied are therefore found in various of the lists of directions on Form 24 A, not all in one list as is the case when the instrument is eccentric.

Compare the following example with that given on Form 24 A.

REDUCTION TO CENTER

Eccentric Station: Chase.

d = 10.987 *meters.*

Log d = 1.04088
Colog sin 1″ = 5.31443
Sum 6.35531

STATIONS	α °	′	Log sin α	Log s (s in meters)	Log $\frac{\sin \alpha}{s}$	Logarithms of Reduction in Seconds	Reduction = c ″
Center	0	00					
Bossing	179	18	8.08696	4.49198	3.59498	9.95029	+ 0.89
Central	224	27	9.84528	4.40254	5.44274	1.79805	− 62.81
Little River	242	47	9.94904	4.51928	5.42976	1.78507	60.96
Lyons, salt works	249	02	9.97025	4.30616	5.66409	2.01940	−104.57

FIG. 40.—Sample form for reduction to center

the computation necessary to correct the angles to make them refer to the true object is called the reduction to center. The same process is used to reduce to the true station the observations made at an eccentric station.

The computation for the reduction to center is most easily made on Form 382. The instructions given on that form, which include a numerical example of a reduction to center, are reproduced in Figure 40 and give ample information for making the computation.

TRIANGLE-SIDE COMPUTATION

The usual arrangement of this computation is shown in Figure 42. The sketch of the quadrilateral, Figure 41, shows the relative positions of the stations, the known line being Cosmos-Camp.

Every triangle should be included in this computation. The observed angles are taken from the list of directions previously compiled and the closure of the triangle obtained without regard to spherical excess. As a check on taking out the angles it may be noted that the sum of the closures of the two triangles formed by one diagonal is equal to the sum of the closures of the two triangles formed by the other diagonal. Thus, $-0.''08+1.''69=+0.''52+1.''09$.

Having checked the taking out of the observed angles in this manner, the angles are then used to compute preliminary lengths for use in the computation of spherical excess. Five places of decimals are sufficient for this computation in all cases, and four places will usually give the required accuracy.

The spherical excess is computed by the formula $e=a\,b\,\sin C\times m$, where e is the spherical excess, a, b, and C are two sides and the included angle, respectively, of the corresponding triangle, and m a factor depending upon the latitude of the triangle and the dimensions of the spheroid. Values of log m for the different latitudes, reproduced from page 16 of Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions, are given on page 165. The quadrilateral of the sample computation has a mean latitude of 57° 00′.

The computation of the spherical excess of triangle 1 is given below:

log distance Cosmos to Camp	$=3.92493$
log distance Cosmos to Creek	$=4.11975$
log sin 66° 08′ 32″.02	$=9.96121-10$
log m (57° 00′)	$=1.40280-10$
log e	$=9.40869-10$
e	$=0.26$

This computation may be made in small figures directly upon the triangle computation form. The same check prevails in the computation of spherical excess as was noted in the taking out of the observed angles. Thus, $0.26+0.13=0.14+0.25$.

The spherical excess is distributed among the three angles of the triangle as shown in the form, viz, one-third to each angle.

The closing errors of the triangles with the spherical excess considered are next obtained, being checked in the example by the equation, $+0.18+1.82=+0.66+1.34$.

This closing error is now distributed among the three angles of the triangle, one-third to each angle, and the spherical angles obtained. By subtracting one-third the spherical excess from each spherical

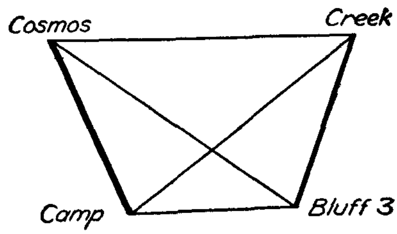


FIG. 41.—Quadrilateral

DEPARTMENT OF COMMERCE
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Form 25

COMPUTATION OF TRIANGLES

State: Alaska

NO.	STATIONS	OBSERVED ANGLE			CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLES AND DISTANCES	LOGARITHM
	2-3 Camp to Cosmos							8412.5	3.9249253
	1 Creek	38	12	45.52	+0.06	45.58	.08	45.50	0.2086029
	2 Camp	75	38	42.54	+0.06	42.60	.09	42.51	9.9862246
	3 Cosmos	66	08	32.02	+0.06	32.08	.09	31.99	9.9612085
	1-3 Creek to Cosmos				+0.18		0.26	13175.1	4.1197528
	1-2 Creek to Camp			00.08				12437.6	4.0947367
	2-3 Camp to Cosmos							8412.5	3.9249253
	1 Bluff 3	37	05	34.67	+0.22	34.89	.05	34.84	0.2196030
2	2 Camp	111	44	19.24	+0.22	19.46	.05	19.41	9.9679608
	3 Cosmos	31	10	05.57	+0.22	05.79	.04	05.75	9.7139549
	1-3 Bluff 3 to Cosmos				+0.66		0.14	12956.5	4.1124891
	1-2 Bluff 3 to Camp			59.48				7219.1	3.8584832
	2-3 Camp to Creek							12437.6	4.0947367
	1 Bluff 3	111	07	28.97	+0.61	29.58	.05	29.53	0.0302128
3	2 Camp	36	05	36.70	+0.61	37.31	.04	37.27	9.7701944
	3 Creek	32	46	52.64	+0.60	53.24	.04	53.20	9.7335471
	1-3 Bluff 3 to Creek				+1.82		0.13	7855.0	3.8951439
	1-2 Bluff 3 to Camp			58.31				7219.3	3.8584966
	2-3 Cosmos to Creek							13175.1	4.1197528
	1 Bluff 3	74	01	54.30	+0.45	54.75	.09	54.66	0.0170892
4	2 Cosmos	34	58	26.45	+0.44	26.89	.08	26.81	9.7583109
	3 Creek	70	59	38.16	+0.45	38.61	.08	38.53	9.9756545
	1-3 Bluff 3 to Creek				+1.34		0.25	7855.1	3.8951529
	1-2 Bluff 3 to Cosmos			58.91				12956.8	4.1124965

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FIG. 42.—Sample computation of triangles

angle the corresponding plane angles of the triangle are obtained. These plane angles are used only for the computation of the sides of the triangle.

The computation of the sides of the triangle is by the well-known law of sines. In triangle 1 of the sample 3.9249253 is the log of the distance in meters from Camp to Cosmos. This distance is known from previous computations. 0.2086029 is the log cosecant of 38° 12′ 45″.50. 9.9862246 and 9.9612085 are logs of the sines of 75° 38′ 42″.51 and 66° 08′ 31″.99, respectively. The sum of the first, second, and third of these logs gives 4.1197528, the log distance Creek to Cosmos, while the sum of the first, second, and fourth gives 4.0947367, the log distance Creek to Camp. Other triangles of the quadrilateral are computed in like manner, an approximate check being obtained upon the logarithmic work by the two values obtained for each of the three lines, Bluff 3-Camp, Bluff 3-Creek. and Bluff 3-Cosmos.

The line Bluff 3-Creek in tne example is used to carry the triangulation forward to other quadrilaterals. The length obtained from the strongest chain of triangles through the figure should be used, and not the mean of the two or more values obtainable by using various combinations of triangles. The strongest chain of triangles should likewise be used in computing positions. By the strongest chain is meant the chain with the smallest ΣR, and not necessarily the one with the smallest triangle closures.

In triangle 1, Figure 42, the lengths of the lines Creek-Camp and Creek-Cosmos are obtained. Either of those lines could have been used in triangle 3 to obtain the length Bluff 3 to Creek, but the line Camp-Creek was chosen because it gave a stronger determination of the line from which the lengths would be carried ahead through the scheme. Similarly, to go from the line Camp-Cosmos to the line Bluff 3-Cosmos and thence to the line Bluff 3-Creek is much stronger, as measured by the R's of the individual triangles, than to go by way of the line Bluff 3-Camp.

A word of caution.—Reference has been made repeatedly to the errors in closure of the triangles. This has been chosen as the criterion by which the observer must judge the accuracy of his work, because it is the test which can be most easily and quickly applied in the field.

It should be remembered, however, that this is but one of the tests that the season's work must pass. The lengths of the various lines of the triangulation when computed through the various chains of triangles must check to a degree of accuracy comparable to the triangle closure. When the triangles have been closed by applying one-third of the error of closure to each angle, and the triangle sides computed, the logarithms of the length of a side, as computed through the various chains, should differ by not more than one and one-half or two times the tabular difference for 1 second of the smallest angle entering into the computation of that logarithm side.

Finally, there must be satisfactory agreements between the azimuth as computed through the scheme and the corrected observed azimuth at each Laplace station, and also between the length of a base as computed through the triangulation from the last base and its measured length. If the observer bears in mind these additional tests which his work must meet he will not unduly force an angle in a certain direction in order to make a better triangle closure.

POSITION COMPUTATION

In Special Publication No. 8, Formulæ and Tables for the Computation of Geodetic Positions, is found all the information needed for computing the geodetic positions of points or of computing the distance and azimuth between the two points when their geodetic positions are known. Since these computations can not be made conveniently without the use of factors tabulated in Special Publication No. 8, the computation form is not reproduced here.

LIST OF GEOGRAPHIC POSITIONS

This form, infrequently used in the field on first-order triangulation, is little more than a tabulation of data from the position-computation sheet, arranged as shown in Figure 43. The equivalent in meters of the final seconds of latitude and longitude is computed in third-order triangulation from tables given in Special Publication No. 5, Polyconic Projection Tables, and tabulated for convenience in plotting the points on projections. This column need not be filled out for first and second order triangulation points.

To save time and space any one line of the triangulation is tabulated only once. For instance, the azimuth and length of the line San Juan-Handy is found only under San Juan.

RECORD, DOUBLE-ZENITH DISTANCES

The arrangement of the record book and a specimen computation of the zenith distance from observations with a vertical circle are shown in Figure 44. The mechanical arrangements of vertical circles are so varied that no one general rule will suffice for computing the zenith distance. Of the circles which are graduated from 0° to 360° some are repeating and others fixed, some are graduated clockwise and others counterclockwise, while the level bubble may be mounted on either the frame or the vernier circle.

When the circle is graduated clockwise from 0° to 360° twice the zenith distance is equal to the reading with the circle right minus the reading with the circle left. When the graduation is counter-

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Form 38 B

11—[illegible]

Field Computation

GEOGRAPHIC POSITIONS

Accession No. of Computation: ____________

Locality Rio Grande River *Datum* North American *State* Texas

STATION.	LATITUDE AND LONGITUDE. ° ′ ″	SECONDS IN METERS.	AZIMUTH ° ′ ″	BACK AZIMUTH. ° ′ ″	TO STATION.	DISTANCE. LOGARITHM (METERS)	DISTANCE. METERS.	DISTANCE. FEET.
Handy, 1917	26 05 36.89		215 16 06.1	35 17 30.2	Donna	3.962926	9181.8	30124
	d.m. 98 05 56.31		278 12 03.5	98 16 06.0	Rio	4.189998	15488.1	50814
San Juan, 1917	26 11 18.32		295 07 34.0	115 09 15.9	Donna	3.850585	7089.0	23258
	d.m. 98 06 36.51		353 55 49.4	173 56 07.2	Handy	4.023922	10566.3	34666
McAllen, 1917	26 14 01.95		291 10 58.2	111 14 24.7	San Juan	4.143554	13917.3	45660
	d.m. 98 14 23.87		317 45 45.4	137 49 29.2	Handy	4.321861	20982.7	68841
Hickley, 1917	26 08 48.05		197 43 13.2	17 44 02.3	McAllen	4.006096	10141.4	33272
	d.m. 98 16 15.08		288 51 10.0	108 55 42.4	Handy	4.259351	18169.8	59612

FIG. 43.—Sample list of geographic positions, field computation

DEPARTMENT OF COMMERCE
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Form 258

DOUBLE ZENITH DISTANCES.

Station: *Joaquin* State: *Calif.* Instrument: *Vertical Circle #75* Date: *Dec. 30, 1922.*
Observer: *John Doe* County: *Los Angeles*

Remarks: *Clear, Calm* — *Temp. 6.9 C.* — *Bar. 29.30*

Object Observed	Time	Level O.	Level E.	Circle Right or Left	Circle Reading °	′	Verniers A. ″	B. ″	C. ″	D. ″	Mean ″	Zenith Distance °	′	″	h	m	s
Star West	*1:30*	*10.2*	*05.7*	*R*	*141*	*15*	*40*	*45*	*60*	*30*	*43.8*				*12*	*39*	*03.0*
Aldebaran		*09.8*	*06.0*	*L*	*243*	*08*	*00*	*50*	*30*	*40*	*30.0*	*50*	*55*	*53.1*	*12*	*40*	*06.5*
		20.0	*11.7*											*-3.3*	*(12*	*39*	*34.8)*
		-8.3												*49.8*			
		09.7	*06.3*	*R*											*12*	*44*	*10.5*
(Value of 1 division of		*06.3*	*09.8*	*L*	*347*	*04*	*55*	*30*	*40*	*95*	*40.0*	*51*	*58*	*35.0*	*12*	*45*	*12.4*
level bubble = 1.596)		*16.0*	*16.1*											*0.0*	*(12*	*44*	*41.4)*
			+0.1											*35.0*			
		08.0	*08.0*	*R*											*12*	*48*	*26.0*
		08.6	*07.4*	*L*	*92*	*45*	*50*	*115*	*70*	*120*	*88.8*	*52*	*50*	*54.4*	*12*	*49*	*24.0*
		16.6	*15.4*											*-0.5*	*(12*	*48*	*55.0)*
		-1.2												*53.9*			
		12.0	*04.2*	*R*													
		07.6	*08.3*	*L*	*201*	*18*	*20*	*15*	*35*	*20*	*22.5*	*54*	*15*	*56.8*	*12*	*55*	*19.2*
		19.6	*12.5*											*-2.8*	*12*	*56*	*12.8*
		-7.1												*54.0*	*(12*	*55*	*46.0)*
	1:55	*07.8*	*08.3*	*R*													
		07.8	*08.3*	*L*	*311*	*24*	*10*	*25*	*50*	*20*	*41.2*	*55*	*02*	*39.4*	*12*	*59*	*01.0*
		15.6	*16.6*											*+0.4*	*1*	*00*	*04.4*
			+1.0											*39.8*	*(12*	*59*	*32.7)*

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FIG. 44.—Sample record, double zenith distance

clockwise the subtraction is made in the reverse order. If the circle reads altitudes or zenith distances direct the computation consists merely in taking means.

If the level bubble is mounted on the vernier circle the bubble is usually brought to the center of the tube before the circle is read and no level correction is made. If the bubble is read the level correction is computed by the following formulas:

With the level graduations increasing numerically from the middle toward both ends,

$$C = \tfrac{1}{4}\{(E+E_1)-(O+O_1\}d.$$

With the graduations numbered continuously from one end,

$$C = \tfrac{1}{4}\{(E_1-E)-(O-O_1)\}d,$$

in which O and E are the readings of the ends of the bubble toward the objective and eyepiece, respectively, when the larger numbers are toward the objective, and d is the value in seconds of one division of the level vial.

The value of one division of the level should be entered at the beginning of each volume of observations. If the value is not known it can be determined by the method described on page 86.

The observations of double-zenith distances, shown in Figure 44, were made upon a star, but the form would be the same for observations upon a triangulation station except that the time shown in the Remarks column would not be recorded, nor would the thermometer and barometer readings be given. In connection with each object observed upon, however, it should be noted whether the ground, tripod head, helio, or lamp was being pointed upon, in order that the observations may be properly referred to the station mark. In the Remarks column on the first page of observations at a station, a record should be made of the heights above the station mark of the tripod head of the tower or stand, the telescope of the vertical-angle instrument, the helio, and the lamp. (See page 75.)

In the record shown in Figure 44 some of the vernier readings have bars over them. One bar indicates that the seconds recorded refer to a minute on the graduated circle one less than the minute recorded for the A-vernier. For example, in the first measure of the zenith distance of Aldebaran the readings of the minutes and seconds of verniers A, B, C, and D with circle left (L) are, respectively, 8′ 00″, 6′ 50″, 7′ 30″, and 7′ 40″.

ABSTRACT OF ZENITH DISTANCES

A sample of this form completely made out is shown in Figure 45. Little difficulty should be encountered in the preparation of this abstract. Column 7, headed "Reduction to Line Joining Stations," is essentially a vertical eccentric reduction. The formula for its computation is

$$r = -\frac{t-o,}{s \sin 1''}$$

where r is the reduction in seconds, $t-o$ the difference in heights

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29

ABSTRACT OF ZENITH DISTANCES

Station Stump State Idaho

Observer John Doe Instrument Verticle Circle #46

DATE 1922	HOUR	OBJECT OBSERVED	OBJECT ABOVE STATION —o	TELESCOPE ABOVE STATION —t	DIFF. OF HEIGHTS t−o	REDUCTION TO LINE JOINING STATIONS	OBSERVED ZENITH DISTANCE	CORRECTED ZENITH DISTANCE
			Meters	Meters	Meters	″	° ′ ″	° ′ ″
8/21	2:30	Ball	1.055	1.370	+0.315	-0.7	90 39 37.5	
		(Helio)					90 39 37.3	
							90 39 34.2	
							90 39 32.7	
							90 39 33.3	
							90 39 35.5	
							90 39 35.9	
	3:05						90 39 38.3	
							35.6	90 39 34.9
8/22	3:22	Divide	0	1.370	+1.370	-70.3	93 42 30.4	
		(Ground)					93 42 30.7	
	3:28						93 42 27.7	
							29.6	93 41 19.3
8/22	3:30	Knoll	0	1.370	+1.370	−12.7	92 49 25.8	
		(Ground)					92 49 23.6	
	3:35						92 49 27.4	
							25.6	92 49 12.9
8/22	3:37	Holmer's, J.W. house, cross on windvane	0	1.370			90 30 43.	
8/22	3:40	Schoolhouse top of belfry	0	1.370			89 59 39.	

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FIG. 45.—Sample abstract of zenith distances

from the preceding column, and s the distance in meters between the stations involved. This reduction need be made only when the observations are reciprocal, and also may be omitted if it will delay the party. All other parts of the form should be completely made out and checked in the field.

DESCRIPTION OF STATIONS

Specifications for the marking of stations are given on page 20. It is very desirable that the mark for the station be permanent. Of equal importance to the future recovery of the station is the description of the station, which should be clear, concise, and complete. The first part of the description should enable one to go with certainty to the immediate vicinity of the station, while the latter part, the detailed description, by its measured distances to reference marks and its description of the station marks must inform the searcher of the exact location of the station and make its identification certain.

The original description should be written in the horizontal-direction record book or in a separate notebook carried by the recorder for that purpose. While at the station, or as soon as possible after leaving it, while the topography of the vicinity is fresh in mind, the written notes should be transferred to Form 525, on a typewriter if practicable. A single copy only is required to be sent to the office, but it is a good plan always to make one carbon copy to be retained in the field for reference until the end of the season, when the duplicates can be transmitted to the office for issue to field parties as needed. The form, after being completely filled out, should be read over carefully to see that there are no reversed directions and that no part of the description is vague, ambiguous, or erroneous.

The specimen description of triangulation station in Figure 46 illustrates the progressive localization referred to above, for, beginning with the naming of the State and county, the description gives the distance and direction of the station from the nearest town, its location in a particular quarter-section, and finally the distances from reference marks and near-by topographic features. As a guide the engineer should keep in his mind the thought, "What facts should be available to engineers in order that they may recover this station 25 years hence?" In particular, measurements should be made and recorded to section lines, road centers, wells, etc., and the location of reference marks with reference to such permanent features should also be noted.

Notes 1*a*, 7*a*, etc., on the specimen form refer to particular kinds of marks, as described below. The use of such a notation decreases the time required for writing the description, and also appreciably reduces the cost of printing the triangulation data. These notes describe the marks in general terms only, and any essential divergence from these types should be mentioned in the description.

If a tower was required at the station described, the description should state the height of the tower. Since it frequently happens that a tall tower is required to render one or two of the adjoining stations visible while others may be visible from the ground, the description should also state the approximate heights above the

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DEPARTMENT OF COMMERCE
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Form 525

DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: Gould
CHIEF OF PARTY: E. O. Heaton

STATE: Oklahoma COUNTY: Harmon
YEAR: 1923 LOCALITY: Gould

Surface-station mark,		Note,*	1a
Underground-station mark,		Note,*	7a
Reference mark,	No. 1	Note,*	12a
Reference mark,	No. 2	Note,*	11a
Witness mark,		Note,*	
Witness mark,		Note,*	
Height of signal above station mark			meters.
Height of telescope above station mark			meters.

DISTANCES AND DIRECTIONS TO REFERENCE MARKS AND PROMINENT OBJECTS

OBJECT	DISTANCE	DIRECTION	AZIMUTH
	meters	° ′ ″	° ′ ″
Eldorado		0 00 00	
Reference mark No. 1	35.15	36 48 31	
Reference mark No. 2	67.12	151 14 01	

Detailed description:

About 1/2 mile northeast of the town of Gould, in the NE 1/4 sec. 6, T. 2 N., R. 24 W., about 1/2 mile east of the residence of J.E. Willingham, which is the first house north of the railroad track and about 100 meters east of the highway. The station is on the highest point of the extreme southern end of a ridge, with bluff on south and east sides, and is 54 meters east of the north and south fence line, where this fence line turns southwest.

Reference mark No. 1 is south of the station and No. 2 is in the fence line northwest of the station.

Described by E. O. Heaton.   Marked by J. S. Bilby.

NOTE.—The initial direction must be to a main scheme station.   11—5761

*See p. 108.

FIG. 46.—Sample description of triangulation station.

ground at which the various stations observed upon become visible. Such information is of great value in deciding from what points new work should start.

Surface marks

Note 1.—A standard disk station mark set in the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 2.—A standard disk station mark wedged in a drill hole in outcropping bedrock (*a*) and surrounded by a triangle chiseled in the rock, (*b*) and surrounded by a circle chiseled in the rock, (*c*) at the intersection of two lines chiseled in the rock.

Note 3.—A standard disk station mark set in concrete in a depression in outcropping bedrock.

Note 4.—A standard disk station mark wedged in a drill hole in a bowlder.

Note 5.—A standard disk station mark set in concrete in a depression in a bowlder.

Note 6.—A standard disk station mark set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Underground marks

Note 7.—A block of concrete 3 feet below the ground containing at the center of its upper surface (*a*) a standard disk station mark, (*b*) a copper bolt projecting slightly above the concrete, (*c*) an iron nail with the point projecting above the concrete, (*d*) a glass bottle with the neck projecting a little above the concrete, (*e*) an earthenware jug with the mouth projecting a little above the concrete.

Note 8.—In bedrock (*a*) a standard disk station mark wedged in a drill hole, (*b*) a standard disk station mark set in concrete in a depression, (*c*) a copper bolt set in cement in a drill hole or depression, (*d*) an iron spike set point up in cement in a drill hole or depression.

Note 9.—In a bowlder 3 feet below the ground (*a*) a standard disk station mark wedged in a drill hole, (*b*) a standard disk station mark set in concrete in a depression, (*c*) a copper bolt set with cement in a drill hole or depression, (*d*) an iron spike set with cement in a drill hole or depression.

Note 10.—Embedded in earth 3 feet below the surface of the ground (*a*) a bottle in an upright position, (*b*) an earthenware jug in an upright position, (*c*) a brick in a horizontal position with a drill hole in its upper surface.

Reference marks

Note 11.—A standard disk reference mark with the arrow pointing toward the station set at the center of the top of (*a*) a square block or post of concrete, (*b*) a concrete cylinder, (*c*) an irregular mass of concrete.

Note 12.—A standard disk reference mark with the arrow pointing toward the station (*a*) wedged in a drill hole in outcropping bedrock, (*b*) set in concrete in a depression in outcropping bedrock, (*c*) wedged in a drill hole in a bowlder, (*d*) set in concrete in a depression in a bowlder.

Note 13.—A standard disk reference mark with the arrow pointing toward the station, set in concrete at the center of the top of a tile (*a*) which is embedded in the ground, (*b*) which is surrounded by a mass of concrete, (*c*) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (*d*) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Witness marks

Note 14.—A conical mound of earth surrounded by a circular trench.

Note 15.—A tree marked with (*a*) a triangular blaze with a nail at the center and each apex of the triangle, (*b*) a square blaze with a nail at the center and each corner of the square, (*c*) a blaze with a standard disk reference mark set at its center into the tree.

Recovery note, triangulation station.—Whenever a station established during some previous season is recovered or searched for, Form 526, Recovery Note, Figure 47, must be made out and transmitted to the office. Any deficiency or lack of accuracy in the original description, any change in the character of the marks or of the topography near the station, or any information which will make the station more readily recovered in the future, should be recorded on the recovery note. If the station is looked for and not found, the recovery note should describe in some detail the completeness of the search in order that the office may know whether or not to mark the station in the records as "lost." If the evidence seems conclusive that the station is lost a definite recommendation should be made on the recovery form by the officer in the field that the station be listed as such in the office records.

SEASON'S REPORT

Although not properly a part of the records, the season's report is such a valuable adjunct to them that it is worth while to indicate its general form and characteristics. Considered as an administrative and professional report on a completed project, it should cover in more or less detail the following points:

1. Purpose and scope of work, as shown by excerpts from instructions or by a condensed statement.
2. Locality, with description of such topographic and climatic and transportation conditions as affected organization or progress of party.
3. Organization of party—personnel, equipment, transportation, subparties, etc.
4. Chronology of progress of work.
5. Discussion of results, supported by progress sketch and statistics of field work.
6. Statement of costs.
7. Recommendations, or special features.

The report should be clearly written, logically arranged, and concise. It may vary in length from a 2-page report covering a routine piece of work to a 15 or 20 page report with photographs illustrating phases of the season's activities, when those activities are diversified. Even a short report, however, will necessarily embrace the features listed in the outline above, even though the text relating to some one or more of them might be condensed to a single paragraph or sentence.

Each season's report must be accompanied by statistics of field work on the form provided, and also by a progress sketch on which

DEPARTMENT OF COMMERCE,
U. S. COAST AND GEODETIC SURVEY.
Form 526.

RECOVERY NOTE, TRIANGULATION STATION.

R

NAME OF STATION: Laguna   STATE: California   COUNTY: Ventura

ESTABLISHED BY: W. E. Greenwell   YEAR: 1857   LOCALITY: Camarillo or Hueneme

RECOVERED BY: F. W. Hough   YEAR: 1923

Detailed statement as to the fitness of the original description: The station is on top of the high peak nearest the ocean of the first range south of Hueneme, and is easily identifiable as the westernmost peak. A two mile trail from the Upper Broome Ranch leads to the station. The station was found as described and in excellent condition. All the old reference marks were recovered. A standard triangulation disk was set in the drill hole in the center of the post, thus re-marking the center of the station. A new standard reference mark, note 12a, was established.

	Santa Clara	0° 00′ 00″
New reference mark 34.16 meters		179 58 30

GOVERNMENT PRINTING OFFICE   NOTE.—One of these forms must be used for every station recovered.   11—7902

FIG. 47.—Sample recovery note, triangulation station

is shown each point located. The discussion of results need include only a statement of triangle closures, discrepancies in length, etc., as given by the field computations, or it may be expanded to include the results of special investigations.

A statement of costs of a project is an essential part of any report upon it. In order to secure uniformity, the following tabulated forms are suggested:

Truck costs (from truck record books)

	Truck No. ---- Capacity ---- pounds	Truck No. ---- Capacity ---- pounds	Truck No. ---- Capacity ---- pounds
Total miles traveled ----------	----------	----------	----------
Total cost per mile ----------	----------	----------	----------

Triangulation costs

Total expenses (including total truck costs, salary of permanent employees plus annual leave earned and time spent in office preparing for field and working on reports. Cost of base measuring and reconnaissance should be stated separately) ----------
Linear miles of progress measured along the axis of the scheme ----------
Cost per mile of progress ----------
Number of square miles covered ----------
Cost per square mile ----------
Number of occupied stations of main scheme ----------
Number of occupied stations of subordinate schemes ----------
Cost per station occupied (two subordinate stations equal one main-scheme station) ----------
Total number of points whose geographic positions were determined ---
Cost per point determined ----------

Chapter 3.—BASE MEASUREMENT

GENERAL INSTRUCTIONS

On September 21, 1925, the Director of the Coast and Geodetic Survey approved the following condensed instructions for first-order base measurement:

These instructions for the measurement of first-order bases supersede all previous instructions for such work.

The instructions for reconnaissance, on pages 4 and 5, govern the distance between bases, but the engineer must bear in mind the close interrelation between reconnaissance, triangulation, and base measurement. With weak figures the ΣR_1 and ΣR_2 will increase rapidly, and bases must then be located closer together. Unusually large closing errors in the triangulation will sometimes warrant a reduction in the minimum prescribed ΣR between bases, or the interpolation of an additional base, even though there is a satisfactory agreement of the measured length of a base with the length as computed from the preceding base. In such cases the agreement may be due to compensating errors and intermediate lines may have inadequate accuracy.

The length of each first-order base will be determined by at least two complete measurements with standardized invar tapes, with a resultant accuracy represented by a probable error of about 1 part in 1,000,000 or better. Such precautions will be used in aligning the tape, in marking the tape lengths, and in determining the corrections for grade, tension, and temperature as will insure that the actual error in the length of the measured base due to any one of these causes will not exceed 1 part in 500,000. Methods of measurement should be adopted which will reduce to a proper degree the cumulative effect of systematic errors. Very little increase in the average accuracy of the lengths of the lines of triangulation between bases will result from an increase in the accuracy indicated above, and no additional time and expense should be expended in securing an accuracy beyond the limits stated.

At least three tapes will be used in measuring each base, the portions measured with each tape being approximately equal in total length. To secure this result the base will be measured in sections approximately 1 kilometer in length, except that one section may be longer or shorter than 1 kilometer, and the total length will be divided into three divisions of approximately equal length, each division beginning and ending with the end of some kilometer section. A different pairing of the tapes will be used on each of the three divisions, in order to secure a complete intercomparison of the lengths of the tapes. Each tape will be run forward on one division and backward on the other. The party should have a fourth tape for use in the measurement in the event that one of the other tapes is injured or shows an unwarranted discrepancy in length when compared with the others.

Only two measurements of any one section will be made unless the discrepancy in millimeters between the two measurements exceeds $10\sqrt{K}$ (where K is the length of the section in kilometers), in which case additional measurements of the section must be made, preferably with the tapes originally used, until two measurements are secured which agree within this limit.

Before the first base of a season is measured all tapes should be standardized at the Bureau of Standards. More than one base may be measured during a season without restandardization of the tapes, provided no perceptible injury is suffered by two or more of the tapes, and provided also that the intercomparison of the tapes on the base measurement when using the standardized lengths does not disclose abnormal changes in the lengths of two or more of them. All tapes should be returned to the office for restandardization immediately after the measurement of the last base of the season.

On each tape, near the terminal mark, is a small v near one edge of the tape, denoting which end of the fiducial marks was used in the standardization. The edge of the tape marked by the v should be used next the marking strip on all base measurements. If the v can not be distinguished, that end of the mark should be used which lies at the far edge of the tape when the observer with the tape extended, has the zero graduation mark at his left and the 50-meter mark at his right.

Where topographic conditions demand it a broken base may be used, provided the terminal stations are intervisible and the angle at each break and at each end is measured so as to form a closed polygon and with an accuracy necessary

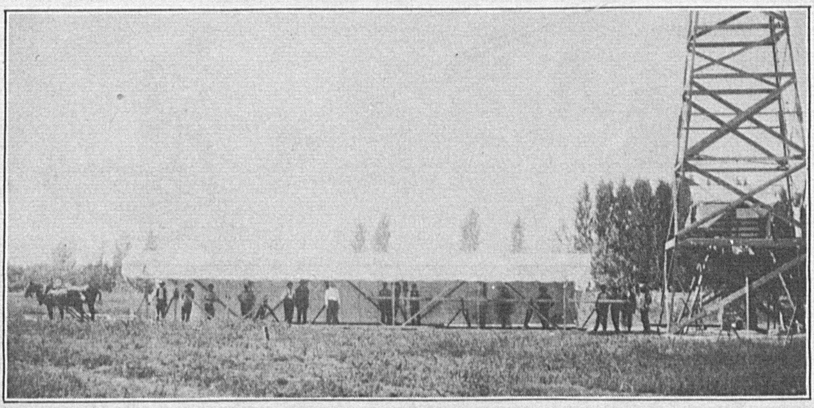

FIG. 48.—Measuring a base with the iced-bar apparatus

As the bars were moved forward the shelter, which was mounted on runners, was drawn forward by the team of horses. Invar tapes are now used exclusively by the Coast and Geodetic Survey for the measurement of first-order bases and the iced bar is used only for standardization comparisons.

to secure the precision in length indicated below. No considerable portion of the base should be inclined at an angle of more than 20° to the final projected line of the base, and this maximum should be kept down to 12° if possible. The total error due to projecting the elements of the base upon the straight line between the base ends should not exceed 1 part in 500,000 of the length of the base.

The slope of no tape length should, as a rule, exceed 10 per cent. Where this limit is approached or exceeded especial care should be used in determining the difference of elevation of the tape ends, and the points leveled upon should be the supports of the fiducial marks of the tape. The nomogram in Figure 51 shows the accuracy necessary to be attained in determining the difference of elevation between tape supports at different distances apart, in order that the proportionate error in the reduced length for that distance may not be greater than 1 part in 100,000.

Before leaving the base the computation of the lengths of the various sections should be carried far enough to insure that the accuracy prescribed above has been secured.

PREPARATION OF BASE

ALIGNMENT

Assuming that the sites of the terminal stations have been selected, the first step is to place poles or targets accurately on the line and close enough together to make it possible to place a theodolite readily and accurately on the line at a sufficient number of places to sight upon each stake to be used as a marking table. With a theodolite in proper adjustment it is easy to perfect the alignment so that no stake is more than 6 inches off the line between the terminal stations and no marking strip is more than 1 inch off the line joining the strips on the two adjacent stakes. When one or both of

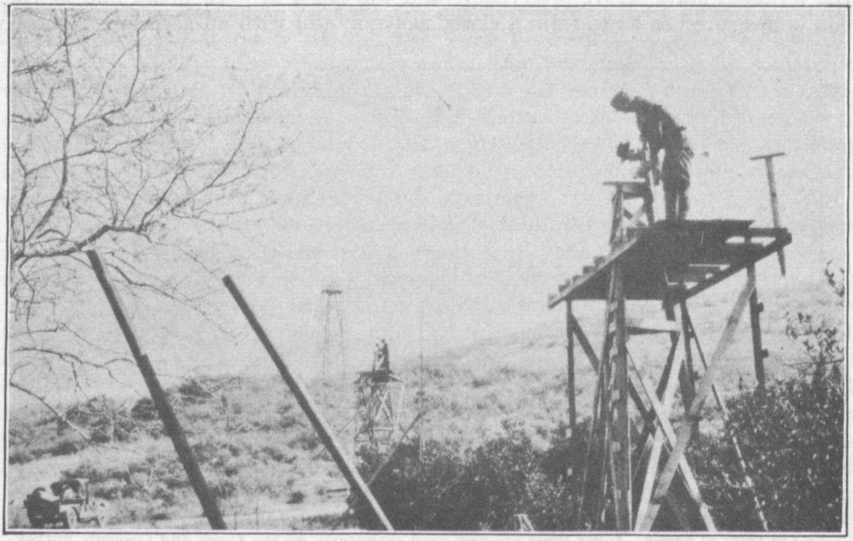

FIG. 49.—Platform and marking table used on the Pasadena base

At times these structures were used to reduce the slope of the tape on steep declivities; at other times they were needed to carry the measurement over the tops of the orange and lemon groves. The observer's platform was entirely detached from the marking table tripod.

the adjacent stakes are at a less distance than the usual 50 meters, the 1-inch tolerance in the alignment of the marking strips must be proportionately reduced. It is not necessary to have the alignment more exact than that.

CLEARING

The line must be cleared of obstructions in order that the tapes may hang free when under tension. Where only grass and weeds are on the line either a horse-drawn mowing machine or a scythe may be used to advantage, while trees and shrubs require axes and bush hooks. The cleared swath need not be more than 6 feet wide. Since those making the tape measurements work on only one side of the stakes, the cleared swath should extend farther on one side of the line of stakes than the other. Where the ground is such that a wagon or truck may be driven over it the cleared swath on one side of the line of stakes should be wide enough to serve as a roadway.

STAKING

The stake setting can be performed advantageously by from three to six men. The 4 by 4 inch stakes should be prepared before setting begins. The edges of the top end should be beveled off slightly to prevent splitting when the stakes are driven, and the lower end well tapered. Often arrangements can be made to have the stakes cut and sharpened at a planing mill in the town where the lumber is purchased, at a cost of about 3 cents per stake. The 4 by 4 inch stakes should have a 10-inch taper on each of the four sides, and the 2 by 4 inch stakes for the intermediate supports should have a 5-inch taper on the wider sides, with a 10-inch taper on the narrow sides.

Where the grade is not uniform the driven stakes should project about 2 to 2½ feet above the surface, which would require the stakes to

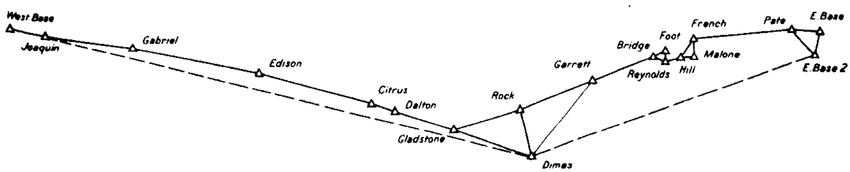

FIG. 50.—Pasadena base, California

An extreme example of a broken base. The solid lines were measured over, and the broken lines indicate projected lengths. It is believed that the projected lengths were determined with an actual error not greater than one part in a million. The probable error was less than one part in seven millions. The length of this base was 20.9 miles.

be about 3½ to 4 feet long for ordinary ground. Where the grade is uniform the stakes need not project more than 1½ to 2 feet above the surface, since the lower height makes them more rigid. A reconnaissance of the base line will show roughly how many of the tape ends will fall in depressions, where stakes of greater length must be used. Where deep depressions occur, or where it is necessary to reduce the slope of the tape by elevating the support, it may be necessary to build tripods with separate platforms for the observer. A type of structure used on the Pasadena base for this purpose is shown in Figure 49.

For first and second order bases each stake should be lined in with a theodolite as it is driven. On lower-order bases poles or targets can be placed on the line at very frequent intervals and the stakes lined in by eye and plumb bob. Where the latter method is employed extreme care must be taken in aligning the stakes. especially where the slopes are steep.

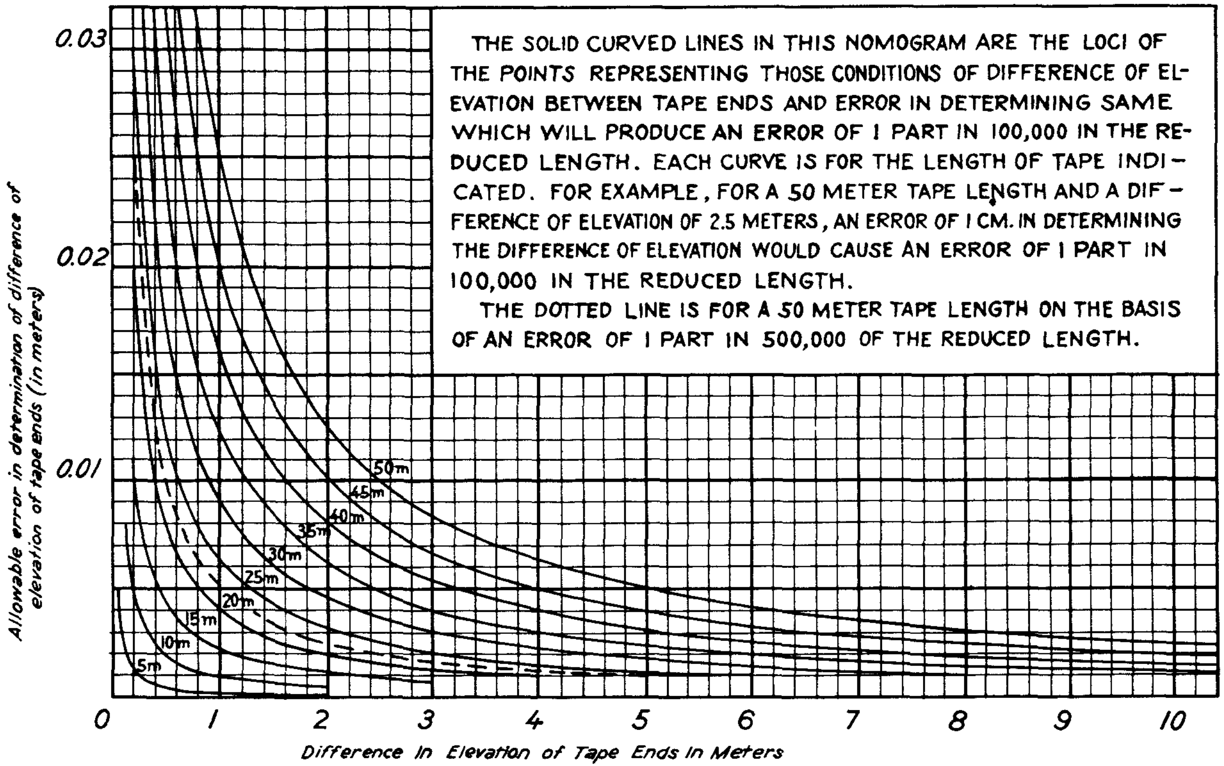

FIG. 51.—Nomogram for effect of errors in the relative elevations of the ends of a tape upon the reduced length

This nomogram was designed and constructed by Howard S. Rappleye, mathematician.

The actual staking can best be done by a party of five men. One man with the theodolite aligns the stakes, another, called the rear men, carries the rear end of the tape and holds the rear mark at the stake already set while the position of the forward stake is being located, the middle man at the center of the tape drives the center stake, the fourth, called the forward man, carries the forward end of the tape, locates the position for the forward stake and drives it, while the fifth man, or helper, drives the truck, distributes the stakes while the theodolite man is moving forward, and assists the forward tape man in driving the stakes. If only four men are available the stakes must be distributed beforehand, at intervals slightly less than that required, in order that there may be no need of going forward for a stake and carrying it back. Extra pieces of lumber should be dropped where they may be needed for bracing the stakes.

An old spliced invar tape, or a 50-meter steel tape, is usually used on the staking, since there is a possibility of injuring the tape. If a steel tape is used it should be roughly compared with a standard invar tape and an approximate allowance made for any difference in length and temperature in spacing the stakes, to avoid loss of time in making set-ups or setbacks while taping. The 25-meter point of the tape may be marked by a strip of adhesive while staking. Sixteen-pound iron sledges are best for driving the stakes. If the ground is hard or rocky a pointed iron bar should be used to make a hole for the stake before starting to drive it. The bar should be lined in with the theodolite, since the point of the stake will follow the hole made by the bar. The forward man and his helper carry sledges, a saw, hammer, nails, copper strips, and brads for fastening the strips to the stakes, while the middle man carries either a sledge or an axe, hammer, and nails.

To begin the staking the theodolite is set up accurately over the mark at the end of the base. After having been carefully adjusted the theodolite is sighted on the target at the other end of the base, or on some intermediate target which has been previously placed on line. The line of sight of the theodolite then defines the line of the base. If the station mark at the base station is low a bench should be built over it and a mark made on the bench directly above the station mark.

With the tape laid out along the base, the rear man holds the rear mark of the tape in contact with the mark over the base station, the forward man, approximately on line, applies to the tape with the spring balance attached to his tape stretcher the same tension which will later be used on the actual measurement, and the helper, holding the 4 by 4 inch stake opposite the forward mark on the tape, is waved into line by the theodolite man. While this is being done the middle man supports the middle of the tape approximately on the line between the ends and at the same time marks with the end of his stake the point on the ground under the middle mark of the tape.

With the forward stake in approximate alignment, the forward man lays aside the tape and drives the stake while the helper keeps it in position. While this is being done the theodolite man aligns the center stake, which is driven by the middle man. At the same time the instrument man watches the forward stake and signals if it is being driven out of line. To avoid confusion a black flag may be used for signalling to the middle man and a white flag for the forward man. When either the middle or forward man desires a check on the alignment of his stake he pats the top of the stake with his hand as a signal to the theodolite man.

As soon as the forward stake is driven and aligned the forward man lines the nail in vertically for the support of the tape at the middle stake, the theodolite man again checking the horizontal alignment to see that the tape properly clears the middle stake. The man at the theodolite then lines in a pencil or other object held vertically by the forward man on top of the forward stake, and this final alignment point is marked on the top of the stake. While the stakes are being driven and aligned the rear man stands aside with his stretcher so the line of sight may not be obstructed.

The tape is again brought into position with its rear mark in contact with the mark on the base station or on the bench above it and the proper tension is applied, with the edge of the tape touching the final aligning mark on the forward stake. A copper strip is then nailed in position flush alongside the tape on the aligning side, and at the same time a pencil mark is made on the top of the stake opposite the forward mark on the tape. This mark is used as a rear contact mark in setting the stakes for the next tape length. The tape is then moved forward and the process repeated, except that the theodolite need not be moved for several tape lengths. Some time can be saved by marking with a pencil on the forward stake the position of the copper strip and letting the rear tape man nail it in position while the next stake is being driven.

From 3 to 5 kilometers a day can be staked by a five-man party, the speed depending upon the nature of the soil and the proficiency of the party.

Lumber which does not split easily should be selected for the stakes. If a stake must be located on rocky ground into which it can not be driven, crosspieces may be nailed across its base with 1 by 4 inch braces from the outer ends to the top of the stake. The structure can then be weighted down with stones and made rigid. If a driven stake is inclined to be loose it should be braced with two or three 1 by 4 inch pieces; all stakes marking kilometer ends should be so braced.

If for any reason the intermediate support or supports between the tape ends can not be placed so they will be on grade with the terminal posts, such intermediate supports should be numbered with colored chalk and marked by a piece of cloth so they may be touched upon by the levelman. All stakes are numbered with colored crayon as they are driven, and intermediate supports not on grade should be given a fractional number.

At ravines or streams it is frequently necessary to place a stake on the edge of the bank and begin a tape length from that point. If it is less than half a tape length from the previous tape-end post, whose number for instance is 34, the fractional-length post should be numbered "34 set-up," the next terminal post being numbered 35. If the fractional-length post is more than half a tape length from the next preceding tape-end post, a 4 by 4 inch stake should be driven in at the half-tape mark and be given the number 34½, and the post on the edge of the obstruction should be numbered "34½ set-up." By following this system, which has been used in most cases for many years, there will be no uncertainty in the interpretation of records. Broken grades should be avoided whenever possible, because the tension through the tape and the shape of the catenary may be different on the two sides of the stake at the broken grade.

Other forms of middle support for the tape than that described above have been successfully used in the past. A hinged inverted V support may be used at the center of the tape, from the apex of which the tape is swung from a string or wire loop, the middle point of the tape being lined in vertically and horizontally by the front contact man. This method should be avoided where possible, for the taping party is delayed an appreciable amount of time by its use.

INSTRUMENTS AND APPLIANCES

The following instruments and appliances will usually be needed on the measurement of a first-order base:

2 awls, marking.
2 dividers, pairs.
1 level, wye, with rod and sunshade.
2 plummets.
2 scales, $\frac{1}{10}$ meter, boxwood, reading to millimeters.
1 stretcher, tape, complete, consisting of two staves with loops and tape attaching clips, two balances, and an apparatus for testing balances.
Strips, copper, for stake tops, of same thickness as tape, 20 per kilometer.
1 tape, steel, 30-meter, standardized.
1 tape, steel or invar, 50-meter, unstandardized, for marking out base.
4 tapes, invar, 50-meter, standardized.
1 theodolite, 7-inch.
3 thermometers, backed, for tapes.

Special conditions may necessitate the use of other instruments, such as a direction theodolite on a broken base, or movable iron tripods where a portion of the base is along a paved road in which stakes can not be driven.

HANDLING AND TESTING OF BASE APPARATUS

Invar tapes.—Of the base-measuring apparatus the invar tapes are the most important. When properly standardized and manipulated they are capable of giving a very high degree of accuracy, but they must be used with a full knowledge of their possibilities for error. To secure the best results two general conditions must be met: First, it is necessary to avoid accidents and methods of handling which may alter their lengths; second, they must be used, so far as possible, either under the same conditions as when standardized or under only such different conditions as can be corrected for. Among the ways in which the first condition may fail to be met may be mentioned: Kinking the tape; altering its mass by abrasion against the ground when measuring; or changing its length by stretching it beyond its yielding point. The second condition involves the determination of the corrections for tension, sag, grade, alignment, and temperature within the allowable limit of error. This will be discussed in detail later.

One thing to be borne in mind is that invar is a very unstable alloy. The four invar tapes used on the measurement of all our precise bases from about 1907 to 1916 exhibited a very satisfactory stability of length in their standardizations during that period, but later tapes have shown decided changes. Three 50-meter tapes from a particular lot of recent tapes, some of which had negative coefficients of expansion and others very low positive ones, showed changes of length while being standardized of from 0.3 millimeter to 1.0 millimeter, the largest change corresponding to 1 part in 50,000 of the length of the tape. Tapes subject to such erratic changes in length of course can not be used on first and second order bases.

The manufacturers of invar in Europe are experimenting on methods of stabilizing the material, and though the problem is a difficult one, no doubt improvements will result. The low coefficient of expansion of invar results from, first, a fairly definite proportion of about 35 per cent nickel to 65 per cent steel, and second, the metallurgical treatment after the alloy has been formed. Part of this treatment consists of a baking and gradual cooling extending over a period of several weeks. In obtaining the peculiar thermal qualities of the alloy a very unstable molecular arrangement results, as is strikingly shown by an experiment performed at the Bureau of Standards. An invar tape suspected of instability was first carefully standardized, then subjected to a rhythmic whipping for 100 times on a concrete floor, after which it was found on restandardization to have shortened by 1.5 millimeters.

From these considerations it is easy to infer, even when using tapes of fairly stable qualities, that care should be used in handling them to avoid possible extensive molecular change. They should not be reeled or unreeled rapidly or under a heavy tension, nor wound upon a reel having a small diameter, nor dragged over the ground, nor shaken violently, nor should they be subjected to sudden large changes in temperature.

In addition to the special precautions described above, which are designed to preserve the tape from changes in length, the usual care should be taken to keep the tape free from adhering substances and from corrosion. After the tape is used it should be cleaned and then oiled with a light oil before being reeled up. If reeled up in the rain a tape should be dried and oiled at the first opportunity.

Persons not accustomed to handling tapes will frequently kink them when unreeling or reeling them, as the tape channel on a reel is narrow and deep. Instructions on the proper way to handle the tapes should be given the members of the party when the tapes are first unreeled. Kinking a tape may render it useless as a precise measuring instrument, for whether the tape is allowed to remain kinked or is straightened, its effective length will be different from its previous standardized length. When a tape is kinked a note should be made in the record showing the exact time the kinking took place, in order that a different standardized length may be used for later measurements if necessary. If a tape becomes badly kinked a spare tape, if available, should be substituted for it.

As a result of the standardization of a tape at the Bureau of Standards, the following data are made available:

Weight of tape in grams per meter.
Coefficient of expansion per degree centigrade.
Length at specified temperature supported at 0, 25, and 50 meter points.
Length at specified temperature supported at 0, 12.5, 25, 37.5, and 50 meter points.
Length at specified temperature supported throughout.

The last value is usually computed from the values obtained by the standardizations when the tape is supported at three and at five points. It is to be noted that the standardized length for the tape supported throughout presupposes a frictionless surface as a support for the tape. In the field, when the tape is supported throughout, a railroad rail is ordinarily used as the support. A correction to lengths measured with the tape supported throughout can be obtained by comparing the length of a kilometer measured in that way with the length measured with the tape supported at three points. The difference in the lengths is the correction sought. On bases of the first order it is not desirable to have the tape supported throughout, because the error due to friction will vary with the surface conditions of the rail.

Before beginning the actual measurement of a base a test kilometer should be measured with each of the standardized tapes, in order to secure a check on the length of each tape derived from the standardization in Washington. This will indicate any change in length of a tape since its standardization or any typographic error in the standardization data. A test kilometer should also be measured with all the tapes at the completion of the base measures, in order to fix in point of time any suspected change in length of any tape prior to the next standardization.

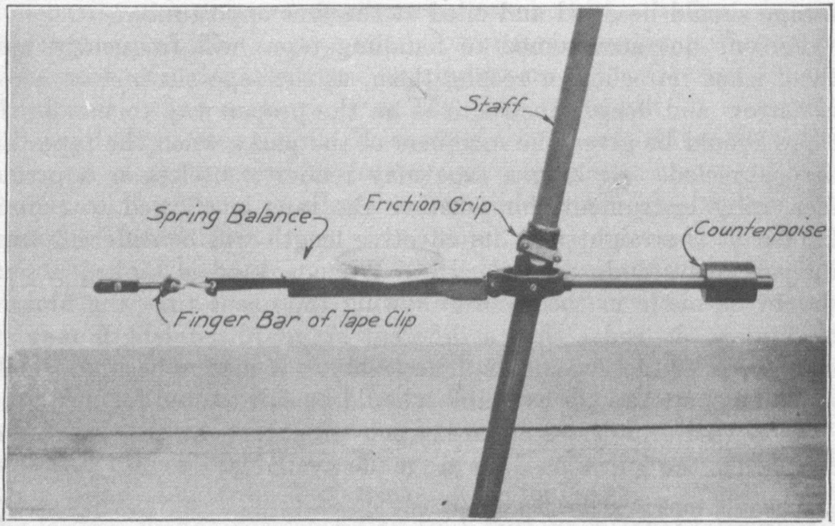

FIG. 52.—Tape stretcher and spring balance

This is the tension apparatus used at the front end of the tape. By means of the friction grip the balance can be adjusted in height on the staff.

Tape-stretching apparatus.—This is shown in Figure 52. It consists essentially of two staves of steel tubing, pointed at the bottom and with wooden tops. A loose-fitting leather loop with an attachment to receive the looped end of the tape slips over the staff used at the rear end of the tape. The leather loop can be easily slipped up and down on the staff according to the height of the rear stake. A frame for holding the spring balance is attached to the forward staff by means of a spring friction grip. The tape is fastened to the balance by the attachment shown in Figure 52. There is a finger bar by which the forward contact man carries the end of the tape when moving forward.

Spring balances.—The balance used, shown in Figure 52, is a commercial one, altered in the instrument division of the survey to reduce the internal friction as much as possible. The counterweight, shown in the figure, may be so adjusted as to prevent any drag of the frame of the balance on the drawbar when tension is applied.

A testing apparatus for the spring balances is sent with the balances. This should be used in testing before and after each day's work, also at midday if practicable, and oftener if it is suspected that the position of the dial pointer has changed. In one form of tester the weight is applied to the balance held vertically, after which the pointer is adjusted exactly to 15 kilograms. When this is done the balance will indicate true tensions when it is used in a horizontal

FIG. 53.—Testing spring balance with standard weight and frictionless pulley

position. In other words, the weight is of 15 kilograms mass, as weighed by the spring of the balance at Washington, minus the weight of the drawbar and other movable parts of balance below the spring. For the area of the United States no correction for change in gravity with elevation and latitude need be applied for a weight made standard for Washington, D. C.

Another method of testing consists of using a weight of exactly 15 kilograms mass, as weighed at Washington, suspended over a frictionless pulley, as shown in Figure 53. The spring balance is in this case held in a horizontal position. When tested by this method the weight is suspended on a wire passing over the pulley and leading to the balance. The system is permitted to come to rest, after which the balance is gently pulled away from the pulley until the pulley begins to move on its bearings, when a reading of the dial is taken. The system is again brought into equilibrium and the balance eased toward the pulley until the pulley again begins moving, when another reading of the dial of the balance is made. These readings should be repeated several times. The mean difference of what may be called the backward and forward readings will measure twice the effective friction of the pulley, and the mean should be free from this error. Care should be taken that the knife-edge axial support of the pulley remains near its middle position.

Where very accurate results are required the error resulting from the varying temperature of the balance spring should be corrected for. A number of typical balances tested at the Bureau of Standards showed a temperature correction of −4.15 grams for each degree

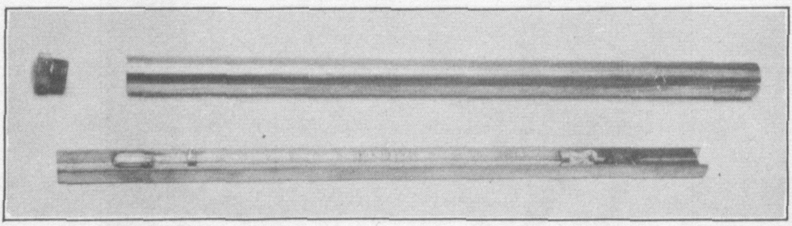

FIG. 54.—Base tape thermometer and tubular case

The channel-bar backing for the thermometer prevents the glass tube breaking when the tape is flexed.

centigrade increase of temperature. This correction is made in the office computation only, from the recorded temperature of the tapes and of the balance at the time of testing against the standard weight.

The most common injury to a spring balance used on base measurement results from the tension being suddenly released, allowing the drawbar to snap back. This may change the position of the dial pointer by several hundred grams, and even result in injury to the spring. For this reason the tension on the tape should always be released gradually. If through accident the drawbar of the balance is allowed to snap back, the balance should be tested before measuring is resumed.

Thermometers.—These are special and rather expensive thermometers, correct to within 0°.3 C. and usually to within 0°.1 C., within the ordinary range of temperature. They are tested at the Bureau of Standards before being sent to the field. Field computations need not take into account the graduation errors of the thermometers found by the standardization. The glass tube is supported in a channel-bar holder, to prevent the breaking of the tube when the tape is being handled or flexed. (See fig. 54.)

During standardization a thermometer of the type described above is fastened at each end of the tape, at a point 1 meter toward the center from the terminal mark, the distance being measured from the mark to the nearer end of the thermometer. On base measurement the thermometers should always be fastened in this same position by narrow bands of adhesive tape.

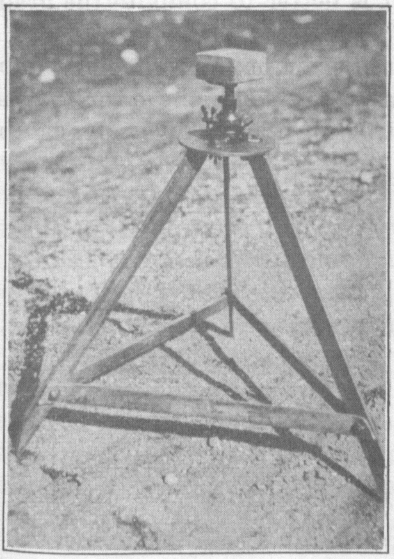

FIG. 55.—Movable iron tripod for base measurement

The wooden marking table, which carries a strip of coppeı on which the mark for the tape end is made, can be placed on grade by the ball-and-socket base and then clamped in position.

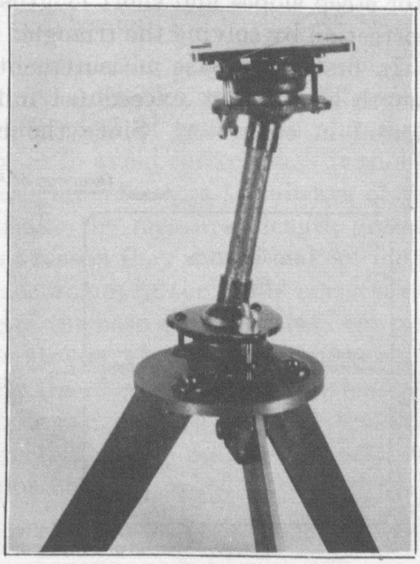

FIG. 56.—Movable iron tripod with double ball and socket

This tripod permits the marking table to be adjusted over a point as well as placed on grade.

CORRECTIONS TO MEASURED LENGTHS AND PRECAUTIONS AGAINST ERRORS

GRADE CORRECTION

The data for the correction for the slope of the tape are usually obtained by spirit leveling, by which the differences of elevation of the stakes supporting the two ends of the tape are obtained. If l is the inclined length, and h the difference of elevation of the two ends, the correction

$$C_G = -\left(l - \sqrt{l^2 - h^2}\right) = -\frac{h^2}{2l} - \frac{h^4}{8l^3} - \frac{h^6}{16l^5} - \cdots.$$

A table of the grade corrections for 50-meter tape lengths for various differences of elevation in both meters and feet is found on page 162.

Since for a 50-meter tape length the second term $\frac{h^4}{8l^3} < 0.1$ millimeter where $h < 3.1$ meters, on ordinary grades the correction will vary directly as the square of the difference of elevation. For this reason the leveling must be more accurately done on steep grades, and an inspection of the rate of change of the values for the correction in the tables should be the guide in deciding upon what accuracy is necessary in the leveling. The nomogram on page 116 also will indicate what accuracy is required in leveling for different lengths of tape and different slopes. For steep slopes and short lengths it is better to compute the grade correction by solving the triangle.

In first-order base measurement the error in C_G for a single tape length should not exceed 0.1 millimeter, though the error is accidental in character. Since the correction varies inversely as the

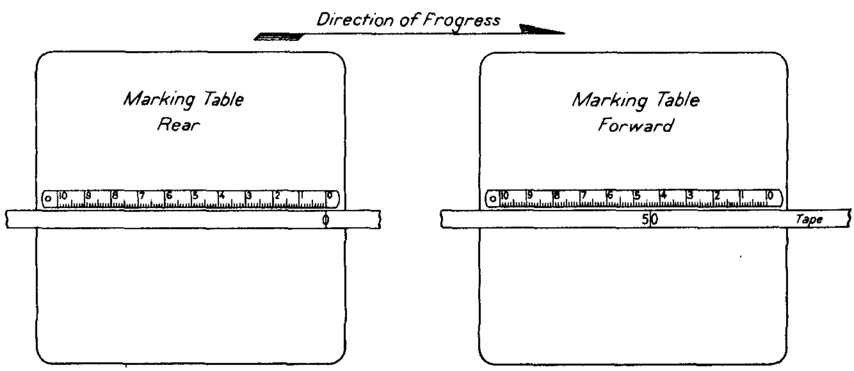

FIG. 57.—Method of using graduated invar strips on marking table

The marking tables of movable tripods for base measurement are frequently fitted with these strips. With the arrangement as shown, and the rear or zero mark of the tape held opposite the zero mark of the strip on the rear marking table, the reading of the forward mark on the tape would always be recorded as a set-up, or additive quantity.

distance, fractional tape lengths are liable to a larger error in the grade corection.

The error in the grade correction which must be most closely guarded against is due to the failure to note and correct for the break in the grade of the tape at the middle support or supports. For that reason such supports should always be flagged with cloth and given a number, such as 34½, the number signifying that the broken grade was at the half tape length between stakes 34 and 35. The tape record will then contain the note in the remarks column, "Broken grade at 34½." Before the grade corrections for any section are summed up a special check should be made to learn whether the levelman has touched upon the support at each broken grade and at the ends of fractional tape lengths, and that the leveling record has the same system of numbering of the tape supports as the tape record. By doing this confusion or doubt will be avoided when the final computation is made. In general broken grades should be avoided whenever possible and long set-ups used instead, to avoid the errors due to the friction of the tape on the middle support.

The leveling should either be run twice, once in each direction, or else a rod used which is graduated in feet on one side and in meters on the other. In the latter case the levels are run in only one direction, but both sides of the rod are read at each rod station and the differences carefully compared before the party leaves the base. If any material difference is found the discrepancy must be checked in the field.

ALIGNMENT CORRECTION

This should more properly be called the alignment error, for although the same correction formula and tables apply to differences in alignment of the tape as to differences of grade, the alignment can usually be made sufficiently exact to avoid correction. It should be borne in mind, however, that alignment errors are always of the same algebraic sign, tending to make the measured length greater than the actual length, and for that reason they should be kept much smaller in magnitude than the inaccuracies in the grade corrections. The section describing the staking of the base gives details of the precautions to be taken in aligning the stakes. In addition to that some member of the taping party, usually the rear contact man or the front stretcher man, should check each tape length to see that the tape does not change its horizontal direction at the middle support, and also that the forward stake has not been disturbed in alignment.

CORRECTIONS DUE TO SAG OF TAPE AND TO STRETCHING

The effective length of the tape when suspended between supports is affected by the shortening due to the sag and to the stretching due to the tension. The correction due to the sag is given by the formula

$$C_S = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3,$$

where n = number of sections into which the tape is divided by the equidistant supports.

l = length of section in meters.

w = weight of tape in grams per meter.

t = tension in grams.

To illustrate by an example: For tape No. 922, supported at three points under a tension of 15 kilograms,

$$n = 2$$
$$l = 25$$
$$w = 25.6$$

and

$$C_S = -\frac{1}{24} \times 2 \times 25.6^2 \times 25^3 \times \frac{1}{15000^2} = -0.00379 \text{ meter.}$$

If it is desired to determine the effect of small variations in the tension the formula for sag may be differentiated with respect to t and put in the following form:

$$\text{Change in length} = \Delta L = +\frac{1}{12} n w^2 l^3 \frac{\Delta t}{t^3}.$$

For tape No. 922 under the conditions of support given above a change of 100 grams in the tension would by this formula produce a change in length of 0.05 millimeter.

The change in the effective length of the tape due to the stretching under tension can best be determined by experiment, since the modulus of elasticity of invar varies greatly with the percentage of nickel in its composition and also with temperature. Five 50-meter invar tapes tested at the Bureau of Standards showed a mean change in length of 0.43 millimeter for a 500-gram change from a tension of 15 kilograms. It is thus seen that a change of 100 grams in the tension on tape No. 922 supported at three points under a total tension of 15 kilograms has more effect in stretching of the tape than in changing the catenary correction.

Precautions should be taken to insure that differences in tension of more than 100 grams from the standard tension are determined and corrected for. The spring balances must be tested both before and after each day's measurement by using the standard weights provided. The record books must show clearly the reading of the dial of the balance during the taping. The dial pointer on the spring balance should be adjusted to its proper reading whenever the tests show it to be appreciably in error. When the dial pointer can not be adjusted to the proper reading the even 15-kilogram dial reading may be used when measuring and the necessary corrections for erroneous tension applied later. This does not greatly increase the time spent in computing if corrections for the temperature of the spring of the balance must be applied. The effect of changes in temperature of the balance may make a difference even greater than 100 grams between the real and the indicated tensions. To enable this correction to be applied it is necessary that the temperature be recorded whenever the balance is tested with the standard weight. The temperature of the balance during the measuring is assumed to be the same as that given by the thermometers attached to the tapes.

Factors for computing catenary correction [1]

[Combined side and top arguments=weight of tape in grams per meter. Tabular values$=\frac{1}{24}\left(\frac{w}{t}\right)^2\times 10^{10}$ when $t=15{,}000$ grams. To obtain catenary correction between supports multiply by cube of length between supports and point off ten decimal places]

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	740. 74	748. 17	755. 63	763. 13	770. 67	778. 24	785. 85	793. 50	801. 19	808. 91
21	816. 67	824. 47	832. 30	840. 17	848. 08	856. 02	864. 00	872. 02	880. 08	888. 17
22	896. 30	904. 47	912. 67	920. 91	929. 19	937. 50	945. 85	954. 24	962. 67	971. 13
23	979. 63	988. 17	996. 74	1, 005. 35	1, 014. 00	1, 022. 69	1, 031. 41	1, 040. 17	1, 048. 97	1, 057. 80
24	1, 066. 67	1, 075. 58	1, 084. 52	1, 093. 50	1, 102. 52	1, 111. 58	1, 120. 67	1, 129. 80	1, 138. 97	1, 148. 17
25	1, 157. 41	1, 166. 69	1, 176. 00	1, 185. 35	1, 194. 74	1, 204. 17	1, 213. 63	1, 223. 13	1, 232. 67	1, 242. 24
26	1, 251. 85	1, 261. 50	1, 271. 18	1, 280. 91	1, 290. 67	1, 300. 46	1, 310. 30	1, 320. 17	1, 330. 07	1, 340. 02
27	1, 350. 00	1, 360. 02	1, 370. 07	1, 380. 17	1, 390. 29	1, 400. 46	1, 410. 66	1, 420. 91	1, 431. 18	1, 441. 50
28	1, 451. 85	1, 462. 24	1, 472. 67	1, 483. 13	1, 493. 63	1, 504. 17	1, 514. 74	1, 525. 35	1, 536. 00	1, 546. 69
29	1, 557. 41	1, 568. 17	1, 578. 97	1, 589. 80	1, 600. 67	1, 611. 58	1, 622. 52	1, 633. 50	1, 644. 52	1, 655. 58
30	1, 666. 67									

[1] Computed by Howard S. Rappleye, mathematician.

Amount of sag.—It is sometimes desired to compute the sag of the tape, that is, the vertical distance from the lowest point of the catenary to the line joining the adjacent points of support. For all practical purposes this is given by the formula

$$\text{Amount of sag}=y=\frac{wl^2}{8t}$$

where w, l, and t represent the same quantities as in the formula for the correction to the length due to sag. (See p. 127.)

For example, taking again tape No. 922 supported at three points under a tension of 15 kilograms:

$$y=\frac{25.6\times 25^2}{8\times 15000}=0.1333 \text{ meter.}$$

CORRECTIONS FOR METHOD OF SUPPORT OF TAPE AND FOR CHANGE IN WEIGHT

The change in the effective length of the tape due to a change in the number of supports can be determined by substituting in the formula for the correction due to sag different values for n and l, since a variation in the number of supports does not appreciably affect the stretching of the tape. Changes in the effective length of the tape due to a change in the weight of the tape per unit length, or in a lack of equidistance of the supports, can be determined by differentiating the same formula with respect to w and l, respectively. Moisture or grease on the tape will change its weight from 1 to 10 per cent. When thoroughly wet, an invar tap is 10 per cent heavier than when dry, and after being sharply shaken to remove the drops of water, it is still 1 or 2 per cent heavier than normal. An increase of 1 per cent in the weight of the tape changes its length about 1 part in 700,000 when supported at the 0, 25, and 50 meter points and an increase of 10 per cent changes its length about 1 part in 70,000. *A base should not be measured in rain or heavy fog.*

STANDARDIZATION DATA

The standardization values of the tapes furnished the field party are usually given in the following form:

United States Coast and Geodetic Survey 50-meter invar tape No. 922

Length when under a horizontal tension of 15 kilograms and when supported at the 0, 25, and 50 meter points:

(0 to 50 meters) = 49.99654 meters at 27°.4 C.

When supported at the 0, 12.5, 25.0, 37.5, and 50 meter points:

(0 to 50 meters) = 49.99945 meters at 27°.4 C.

When supported on a horizontal surface throughout its entire length (value computed from observations taken on the tape when supported at three and at five points):

(0 to 50 meters) = 50.00046 meters at 27°.4 C.

For the first and second of the above conditions, thermometers weighing 45 grams each were attached at points 1 meter inside the terminal marks during the tests.[1]

Temperature coefficient = 0.00000104 per degree centigrade.
Mass per meter = 25.6 grams.

Since the tapes are standardized when supported at three and at five points and throughout under a tension of 15 kilograms, the formulas given previously need be used only in the cases where an unusual number of supports are used, when it is desired to apply corrections for small changes in tension, or when it is necessary to investigate the effect of small changes in one of the many factors affecting the length of the tape.

FRICTION OVER SUPPORTS

With proper handling of tapes the error due to this cause is negligible. Before the tension is applied the devices attaching the tape to the stretcher staves should be so adjusted in height that the tape will be a millimeter or so above the tops of the marking posts when the tension is fully on. Frequently it is necessary to slacken off and adjust to the proper height after the tension has been partly applied. The tape must not at any time be permitted to drag over the rear post because of the danger of moving the post, which holds the mark from the previous tape length. With the tape in equilibrium under tension just above the posts a touch of the finger will depress it into position for marking.

If a nail driven into a piece of lumber is used as a middle support, the middle tape man while the tension is being applied keeps the

[1] Near one end of the terminal marks on each tape is a small x or v cut into the tape; that end of the marks should be used on all standardizations and measurements.

tape vibrating on the nail by tapping it rapidly on its under side with a light stick. This will suffice in most cases to prevent any large amount of friction between the tape and the nail, but a more accurate way is to suspend the tape in a wire stirrup several inches long suspended from the middle nail in such fashion that it will swing freely in the direction of the length of the tape.

TEMPERATURE CORRECTION

With most invar tapes the effect of errors due to incorrect temperatures are far from negligible. Experiments have shown that on sunshiny days the readings on the attached thermometers may differ as much as 3° or 4° from the actual temperature of the tape. With a temperature coefficient for the tape of 1 part in 1,000,000 per degree centigrade an error in temperature of 4° C. would affect the measured length 1 part in 250,000. On cloudy days the difference in temperature of the thermometer and tape is usually less than 1°. The coefficient of expansion of the tapes in use will determine largely the care which must be exercised in controlling the temperature error. The lag of the thermometer is partly due to the lack of intimate contact between the tape and the mercury bulb, but probably is due more to a difference in the absorption rates of the tape and the glass-inclosed mercury column of the thermometer when exposed to radiation from the ground and to the direct rays of the sun. The best method for minimizing the effect of lag in the thermometer is to make one half of the measurements of the base with a rising temperature and the other half with a falling temperature, on the theory that the lag will be approximately equal in amount under the two conditions named but of opposite sign. By this method the two measures will differ more in their computed length but the mean will be more nearly correct. Invar tapes can be secured with practically a zero coefficient of expansion, but tapes with a very low coefficient usually have great molecular instability and are subject to sudden changes in length.

Thermometers should be steadied in reading by grasping the tape a few inches on either side of the thermometer with the hands, care being taken not to allow the hands or the breath to touch them.

The thermometers used should be the same weight as the ones used during the standardization of the tapes, and they should be fastened in the same position on the tape. A formula has been developed by Walter D. Lambert, a mathematician of the Coast and Geodetic Survey, to compute the change in length caused by a change in the weight of the thermometers or a change in their position on the tape. This formula is:

$$s-s_0=\left(\frac{a^2-h^2}{2c^2}\right)\left(l+\frac{l^2}{2a}\right),$$

where s is the length of the catenary with the thermometer attached, s_0 is the length without the thermometer, a is one-half the distance between supports, h is the distance from the center of the catenary to the point at which the thermometer is attached, $c=\frac{T}{m}$, in which T is the tension and m is weight per linear unit of the catenary, and $l=\frac{p}{m}$, in which p is the weight of the thermometer and m is the same as above. For a 50-meter tape weighing 25 grams per meter, supported at the 0, 25, and 50 meter points, bearing two thermometers, each weighing 25 grams, at distances 1 meter from the ends and under a tension of 15 kilograms, this formula becomes

$$s-s_0=\left(\frac{12.5^2-11.5^2}{2\left(\frac{15000}{25}\right)^2}\right)\left(\frac{25}{25}+\frac{\left(\frac{25}{25}\right)^2}{25}\right)=0.0000347 \text{ meter.}$$

This is the value for half the tape and should be doubled for the whole tape. Taking $p=45$ grams, $s-s_0=0.0000643$ meter. The change in length, therefore, caused by changing the weight of the thermometers from 25 grams to 45 grams is 0.0000592 meter or about 1 part in 840,000 for the whole tape.

PARALLAX IN MARKING

Throughout the entire measurement of the base one man should make the mark for the forward contact, for the reason that this error in marking tends to be constant in size and direction for any one person. If then the person marking remains always on the same side of the tape and stakes, the marks which tend to make the measured length too long when measuring in one direction will make it too short when measuring in the opposite direction. It is probable that there is no parallax in making the rear contact. If there is any it is exceedingly small and need not be taken into account.

WIND EFFECT

The error caused by the bowing of the tape horizontally by the wind is always of the same sign and tends to make the measured length too long. A wind which merely flutters the tape need cause no concern, but where there is a perceptible bending of the tape out of horizontal alignment between supports the measuring should either be postponed or additional supports provided. When an increased number of supports are used they must be put more precisely on grade to avoid the greater liability of grade errors.

BLUNDERS

The numbering of the stakes and the inclusion of those numbers in the record of both the tape measures and the levels practically precludes the possibility of a dropped tape length. There is a chance for error, or at least for confusion, if the record of set-ups and half tape lengths is not clear and consistent throughout all the record books. There is a chance for a compensating error in recording a small set-up as a setback, or vice versa, when measuring in one direction, with a like error of approximately the same size in the opposite running, but with care and the usual system of having the recorder repeat clearly to the observer all data to be recorded, the probability of such an error remaining undetected is rather remote. There is one rule, however, which should invariably be observed: Any discrepancy in the records, whether of tape or level, *should be checked by field measurements*, even though the chief of party may think he has discovered the cause of the discrepancy through an examination of the record.

An added check against a dropped tape length in the record, or the recording of a half tape length as a full one, is to measure the base roughly with a 300-foot tape, checking on each kilometer section mark. Two men can do this in a short time, and the resulting certainty is worth the time spent.

Among other blunders which may occur is a tension on one tape length of 10 kilograms instead of the required 15 kilograms, since the dial pointer marks 5 kilograms at each revolution. The effect of this error is about 6 or 7 millimeters in a tape length. The possibility of making this blunder will be greatly reduced if both the forward stretcher man and the forward contact man each time the tension is applied will check the tension by a glance at the drawbar of the balance on which the 5, 10, and 15-kilogram lines are marked. The 15-kilogram line should be marked by white paint to distinguish it from the other lines.

If the stake holding the forward mark is jarred while the tape is being moved forward an error may be caused. In such a case the previous tape-length measurement should be repeated before the forward progress is resumed. Any movement of the stake holding the mark designating the end of a section is usually shown by the discrepancies between the forward and backward measures of the two sections involved having opposite signs and approximately the same size.

MEASUREMENTS ON RAILROAD TANGENTS

Bases have been measured along tangents of a railroad, using the rails for supporting the tape throughout its length, as on first-order traverse. This has not proved entirely satisfactory for the reason that there is friction between the tape and the rail which can not be adequately corrected for. The error caused by this friction has been found to be from 3 to 5 millimeters per kilometer, but it varies with the condition of the rail. If a rail support is used for the tape in measuring a first-order base, small wooden rollers should be placed between the tape and the rail to decrease friction. For example, with tape No. 922 under 15-kilogram tension and supported at the 8⅓, 25, and 41⅔ meter points, rollers 6.0 centimeters in diameter would elevate the tape sufficient to just clear the rail between supports with the tape just touching the rail at the terminal marks.

Since the rails are always somewhat uneven, the rollers would need to be slightly larger than the dimension computed above, but must not be so large that the tape at the terminal marks fails to lie close to the rail. If the tape is not standardized with such a system of supports the corrections due to the sag of the tape must be computed and applied.

MEASURING WITH TAPES

PERSONNEL AND DUTIES

Six men are ordinarily required for the operation of a 50-meter tape on base measurement, and their usual designations are as follows: Front contact man, rear contact man, front stretcher man, rear stretcher man, middle man, and recorder. Aside from the recorder, who must have the special qualifications usual to that position, the assignments to positions, in the order of experience and ability of the personnel, should be in the order given above. Usually the chief of party makes the forward contact, as in that position he can best supervise the manipulation of the tape and can set the pace of measurement. It is better to have experienced men for both contacts if they are available.

If any of the men are inexperienced it is better to measure 1 or 2 practice kilometers before the recorded measurements are begun. During the preliminary work the chief of party or some other experienced officer drills each man in turn in the minutiæ of his duties. A résumé of the precautions to be observed at each position follows, beginning with the least difficult.

The middle man carries the middle of the tape high off the ground when moving forward, places the tape on the middle support when the tension is to be applied, takes the necessary precautions to see that the friction over the middle support has a negligible influence on the effective length of the tape, sees that the tape is not in contact with weeds, brush, or other obstructions, notifies the recorder of all middle supports marked "broken grade," carries and places the tape so that there is no twist in it, and each time makes sure that the middle support is not more than a decimeter distant from the middle mark on the tape. If a nail is used as a middle support for the tape he must rapidly and lightly tap the under side of the tape near the support with a stick somewhat larger than a pencil until the front contact man calls "ready," in order to lessen the friction over the nail. The tapping must not be continued during the marking of the front contact. If a wire loop or other swinging device is used for a center support the middle man places the tape on it and sees that the support swings clear and is vertical.

The rear stretcher man with the rear tape stretcher holds the tape in position during the time the tension is on, so that the rear terminal mark on the tape is opposite to, or slightly forward of, the mark on the copper strip on the rear stake. As he comes up to the rear stake he must place the rear staff firmly in the ground at the proper distance back of the rear stake directly in line with the stakes, and at the same time slips the leather loop bearing the tape link to the proper height on the staff, so that when the full tension is applied the tape will be a few millimeters above the top of the stake. It must not drag over the top of the stake at any time. In order to maintain a steady position of the staff it is better to have the top of the staff back of one of his shoulders, the body being forward of the staff. One foot should be forward of and against the base of the staff to brace it. With practice all these movements can be so coordinated that they require only a few seconds, and the man operating the rear stretcher can keep the rear mark of the tape in the proper position as the tension is applied. As soon as the front contact man calls "mark" and the thermometers are read the tension is slackened off. The tape is then carried forward without being detached from the rear stretcher, the rear stretcher man maintaining just enough tension on the tape to keep it from touching the ground.

The front stretcher man applies the proper tension to the tape as measured by the spring balance attached to the front stretcher. In moving forward he carries the front stretcher and balance, detached from the tape. By hurrying he usually reaches the forward stake in time to check the vertical and horizontal alignment of the middle support, by sighting back over the tops of the stakes, before the front contact man reaches the stake with the front end of the tape. The checking is necessary to make sure that the stakes have not been moved since they were set or last aligned. As the tape is brought forward into position the front stretcher man with one hand holds the balance out horizontally with the hook in such position that the tape can be quickly attached. As the tape is attached he places the staff in line with the stakes at the proper distance from the front stake and applies the tension smoothly, rapidly at first but with increasing slowness as the 15-kilogram point is neared. Jerking motions must be avoided, as they may injure the balance or tape. With the staff held in the same manner as described for the rear stretcher man, and with one hand steadying the balance so the drawbar swings free, the front stretcher man can quickly bring the tape into equilibrium under the proper tension, at which time he informs the front contact man that the tension is correct. When under tension the tape must just clear the top of the forward stake and must not drag over it, otherwise the full tension will not be transmitted throughout the tape. Care must be taken that the stretcher staff is moved in the vertical plane through the stakes, for otherwise the balance will be twisted and friction exerted on the drawbar. The tension must be kept constant at 15 kilograms and watched closely, for if the dial pointer indicates more than 20 grams from 15 kilograms when the front contact man calls "mark," the front stretcher man should immediately tell the front contact man in order that the marking may be repeated. If the tension is satis-

FIG. 58.—Base measurement: Making rear contact

factory at the call of "mark" the tension is quickly but smoothly slackened off while the front contact man is reading the forward thermometer, the balance is held out for the detachment of the tape by the front contact man in the same manner as for its attachment, and the advance begun to the next position.

The rear contact man makes the rear contact and reads the rear thermometer. As the tape is brought up to a new position he steadies the tape as the rear staff is being placed in position and the tension applied, taking care that the tape does not drag over the rear stake. As the tension is applied he advises the rear stretcher man whether to ease off or take up on the tape. Standing directly opposite the mark on the copper strip nailed to the top of the stake, with one hand gloved he firmly grasps the tape between the staff and the mark and with the other hand he lightly touches the tape on the opposite side of the stake to steady it. He can then flex the tape with his gloved hand and bring exactly opposite each other the marks on the tape and copper strip. Before flexing the tape it is necessary, of course, that the mark on it shall be slightly forward of the mark on the strip. The marks are thus held in coincidence until the front contact man calls "ready" when the rear contact man calls "right" and the front contact man answers "mark," denoting the completion of the marking of the tape length. If more than a few seconds elapse after the calling of either "ready" or "right" before the next response can be made with accuracy the entire process should be repeated. Immediately following the call

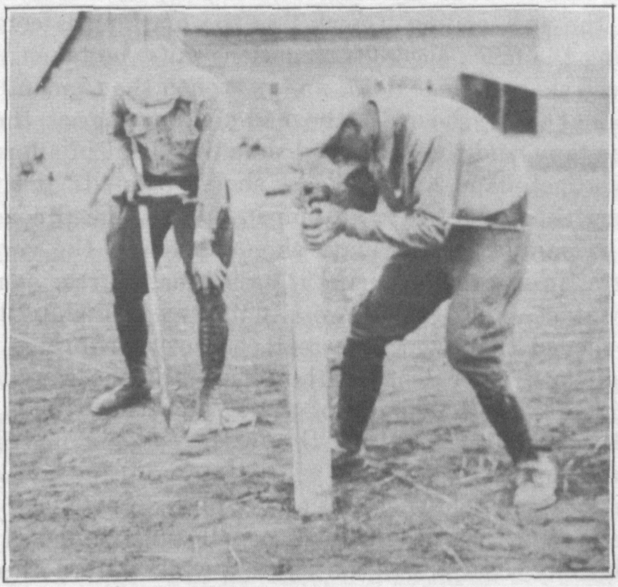

FIG. 59.—Base measurement: Making forward contact

"mark" the rear contact man reads the rear thermometer and the tape moves forward to the next position, unless some one at the forward end of the tape calls "hold" or "tension."

The front contact man has the hardest and the most important job of anyone handling the tape, for he must decide when all the conditions which affect the tape as a measuring unit are complied with, satisfying himself that the tape is in proper equilibrium under proper tension and support before he makes the forward mark. He also carries the forward end of the tape when moving forward, and thus to a large extent sets the pace for the entire operation. The sequence of his movements during the measurement of a single tape length is as follows: As the forward stake is reached he lowers the front end of the tape from its position above his shoulder and attaches the link to the hook of the balance, grasping and guiding the hook to make sure that the attachment is made with a single movement. He then steps quickly back to a position alongside the forward stake, where he steadies the tape into its proper position just clear of the top of the stake, alongside the copper strip and between him and the strip. As the tension is perfected and the tape approaches equilibrium he places the point of the sharp, symmetrically pointed awl on the edge of the copper strip next to the tape and keeps it opposite the terminal mark on the tape until he is satisfied that conditions are right. After glancing at the balance and down the tape he calls "ready," as above described. When he hears the response of "right" from the rear contact man he marks the copper strip with the awl, calling "mark" as the marking is completed.

In making the mark several precautions must be taken. The awl must be very sharp; it should *at no time* touch the tape in the region of the terminal mark; the eye of the man making contact, the terminal mark of the tape, and the entire awl should be kept in approximately the same vertical plane, and the mark should be made by the contact man moving the awl away from him in order to keep constant any error due to parallax. The mark should begin at the very edge of the copper strip in order to make it easier for the rear contact man to make contact. A straightedge is not used in making the mark, for it is believed that when the rear contact man brings the mark on the tape into coincidence with the end of the scratch on the strip and ignores the remainder of the scratch the error is less than that which would be caused by the less exact aligning of the awl point with the mark on the tape when a straightedge is used.

In order to avoid confusion the marks placed on the copper strips during the second measurement are usually distinguished from the first markings by a bar across the scratch, while a third measurement would have a second bar. On the first measurement of a kilometer section the end mark of the section is given a distinguishing mark on the copper strip. On the second measurement, as the stake marking the end of each section is reached a set-up or setback is taken to the original section mark on the strip, and the measurement of the next section is begun at the section mark. It is frequently better on the second measurement to make a set-up of a centimeter or so from the section mark in beginning a new kilometer, in order that the marks of the two runnings may not fall closely together.

Immediately following his call of "mark" the front contact man reads the forward thermometer as the tension is released, then detaches the front tape link from the balance as he starts to pull the tape forward to the next position. If the tape is always carried in the hand which is toward the rear when making the contact there is no danger of causing a half twist in the tape, for when detaching the tape from the balance the finger clip is caught in the rear hand, fingers pointing down, then the end of the tape is elevated above the shoulder without changing the grip, and at the next position is again attached with the hands in the same relative position.

When the tape is brought into position with its rear mark in contact with the mark on the rear stake the forward mark on the tape will sometimes fall short of or beyond the copper strip on the forward stake. If the staking was properly done and the stakes have not been moved a second mark can be made on the strip on the rear stake, in front of or behind the first mark, at such distance as to permit of a contact mark being made on the forward strip. If the front contact man calls "set-up, 3," meaning that the forward terminal mark on the tape lacks 3 centimeters of reaching the copper strip, the rear contact man makes a mark 3 centimeters or more ahead of the mark on the rear stake, measures its distance from the previous mark with a boxwood scale, calls out the distance to the recorder, making sure that the recorder repeats it properly, measures the distance again as a check, then signals that he is ready to make contact on the new mark. Small set-ups and setbacks should be measured with an error not greater than one or two tenths of a millimeter. As an additional safeguard the recorder may check the measurement of set-ups and setbacks, and have the contact man in turn check his entry in the record book.

Some base tapes have marks at approximately 5-meter intervals along the tapes, the intermediate intervals having had their lengths determined with secondary accuracy. Where such tapes are available long set-ups are usually so staked as to enable the invar tape to be used for the greater part of the set-up, that is, for some multiple of 5 meters, the remainder of the set-up being measured with a standardized steel tape. For instance, if a 23-meter set-up is necessary a stake would be set at the 20-meter point and another one at 3 meters beyond. The 20-meter interval would in such case be measured with the invar tape with the standard 15-kilogram tension. The method of supporting the tape and the tape temperature should be noted in the Remarks column. The 3-meter interval would be measured with the steel tape, using the standardization tension, which is usually 5 kilograms, and noting in the Remarks column the temperature and method of support. For each set-up the record should also show the part of the tape used in making the measurement. For instance, a set-up of 20 meters could be made either from the 50-meter to the 30-meter mark or from the zero to the 20-meter mark, while the standardized distance between the two pairs of marks would probably be different.

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The recorder should be an experienced man. If he is not, the chief of party must frequently inspect the record, especially where broken grades or set-ups and setbacks are to be recorded. It is the province of the recorder to be sure that no blunders are committed such as the dropping or adding of a tape length, or recording a half tape length as a full one, or a set-up as a setback. He should check the chalked numbers on each stake as it is reached and make all notes necessary to a definite and correct interpretation of the record. With an experienced front contact man it is often advisable for the chief of party to assume the recorder's duties, as he can then not only record but can watch in turn the work of each man on the tape and correct such practices as need it.

If a truck can be driven along the base and a truck driver is available he can save the party a great deal of time by keeping at hand extra tapes, tools, and stakes, preparing fences for the passage of the tape, and laying out tapes. A tape should be unreeled 10 or 15 minutes before being used on the base measurement in order that it may assume the temperature of the air. It is preferable to have it lie on weeds, brush, or stakes rather than in contact with the ground.

FORMS FOR RECORDS AND COMPUTATIONS

Form No. 590 should be used for recording the tape measurements, and Form No. 634 for recording the wye levels. The computations of the lengths of the various sections of the base should be on Form No. 589. Form No. 635 should be used for the abstract of wye levels and the computations of the grade corrections. Samples of all these forms are shown in Figures 60 and 61.

Explanation of Form No. 590 (tape measurements).—In recording the tape measures on Form No. 590 two thermometer readings indicate a full 50-meter tape length and one thermometer reading a half tape length or a set-up. Each half tape length or large set-up should be recorded on a separate line, and not on the same line with a full tape length. The numbering of the stakes should plainly indicate the full tape lengths and the partial lengths. (See page 119 for method of numbering.) Notes in the Remarks column should clearly explain any unusual conditions.

Form No. 634 (wye leveling).—The sample form shows the wye-level record when using a rod graduated in meters on one side and feet on the other. When such a rod is used the levels are run in only one direction, but both sides of the rod are read at each rod point, the reading in meters being recorded as the forward running and the reading in feet as the backward running. If the rod used is graduated on only one side a forward and backward running is necessary. The numbering of the stakes in this record should correspond to the numbering on Form No. 590. Extreme care should be taken to get readings on all broken grades and partial tape lengths, and these should be plainly indicated in the record. In the columns headed "meters or feet" the one not used should be crossed out. This is especially important where the rod used is graduated on only one side, because the mathematician making the office computation has no other way of knowing whether the rod used was graduated in meters or feet.

89 DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 634

WYE LEVELING

From △ E. Base To T. E. 20
FORWARD RUNNING

Date 7/7/'24 Instrument 95 Rods 118

Point	Backsight. Meters	Foresight. Meters	Difference. Meters	Remarks
△ E.B.	2.754			
△ E.B. set-up		2.438	+0.316	
1		2.004	+0.434	
2		1.937	+0.067	
3		1.540	+0.397	
4		1.309	+0.231	
5	2.511	1.110	+0.199	
6		2.438	+0.073	
6½ (B.G.)		2.440	−0.002	
7	0.471	2.779	−0.339	
8		0.799	−0.328	
9		0.976	−0.177	
10		1.273	−0.297	
10 set-up		1.300	−0.027	
11	1.647	1.738	−0.438	
12		1.962	−0.315	
13		1.437	+0.525	
14		0.537	+0.900	
15		1.734	−1.197	
16	1.247	1.469	+0.265	
17		1.837	−0.590	
18		2.004	−0.167	
19		2.540	−0.536	
20		2.486	+0.054	

90

From △ E. Base To T. E. 20
BACKWARD RUNNING

Date 7/7/'24 Instrument 95 Rods 118

Point	Backsight. Feet	Foresight. Feet	Difference. Feet	m.	Remarks Mean m.
△ E.B.	9.035				
△ E.B. set-up		7.999	+1.036	+0.316	+0.316
1		6.575	+1.424	+0.434	+0.434
2		6.354	+0.221	+0.067	+0.067
3		5.052	+1.302	+0.397	+0.397
4		4.290	+0.762	+0.232	+0.232
5	8.270	3.648	+0.642	+0.196	+0.198
6		8.005	+0.265	+0.081	+0.077
6½ (B.G.)		8.005	+0.000	+0.000	−0.001
7	1.550	9.121	−1.116	−0.340	−0.340
8		2.625	−1.075	−0.328	−0.328
9		3.202	−0.577	−0.176	−0.176
10		4.176	−0.974	−0.297	−0.297
10 set-up		4.265	−0.089	−0.027	−0.027
11	5.404	5.712	−1.447	−0.441	−0.440
12		6.437	−1.033	−0.315	−0.315
13		4.714	+1.723	+0.525	+0.525
14		1.765	+2.949	+0.899	+0.900
15		5.692	−3.927	−1.197	−1.197
16	4.100	4.820	+0.872	+0.266	+0.266
17		6.031	−1.931	−0.589	−0.590
18		6.575	−0.544	−0.166	−0.166
19		8.330	−1.755	−0.535	−0.536
20		8.149	+0.181	+0.055	+0.054

FIG. 60.—Sample record of wye leveling on base measurement

Form No. 635 (abstract of wye levels).—In the first column on this form are recorded the stake numbers, corresponding to the numbers on Forms No. 590 and No. 634. The second column gives the distances between stakes, each distance being that between the stake recorded on the same line as the distance and the stake on the line preceding. The third column gives the mean differences of elevation between the two stakes noted. This column is headed "Meters or feet," and here again it is important to cross out the word not applicable. The fourth column gives the grade or inclination corrections in millimeters. For 50-meter tape lengths these corrections can be obtained from the tables on pages 162 to 164. These tables are made out for differences of elevation in both feet and meters,

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 590

Base
~~TRAVERSE~~

From Station △ E. Base to Station T. E. 20

Forward measurement

Date: 7/7/'24 Time: 8:10 Tape No.: 922 Bal. No.: 302 Ther. Nos.: 1518 33260

11—6328

Section.		Temp.		Set up.	Set back.	Tape support.	Remarks.
From—	To—	Forward.	Rear.				
		° C.	° C.	Meters.	Meters.		
△ E. B.	△ E.B set-up	20.0		20.0000		2	0 — 20
△ E.B set-up	1	21.3	21.0			3	
1	2	21.2	21.0			3	
2	3	20.6	20.0	0.0714		3	
3	4	21.2	21.0			3	
4	5	21.0	20.5			3	
5	6	20.0	19.8			3	
6	7	20.0	20.0		0.0214	3	B.G. at 6½
7	8	20.2	20.0			3	
8	9	20.4	20.5			3	
9	10	20.6	20.6			3	
10	10 set-up		21.5	4.7000	0.0381	2	← Steel Tape 872
10 set-up	11	20.7	21.0			2	← Crossing Gully
11	12	20.8	21.0	0.0027		3	
12	13	20.8	21.0		0.0732	3	
13	14	21.0	21.0			3	
14	15	21.0	20.0			4	Supp. 0-12½-25-50
15	16	20.8	20.5			3	
16	17	20.5	20.3			3	
17	18	20.7	20.5			3	
18	19	20.8	20.4			3	
19	20	21.0	20.5			3	

FIG. 61.—Sample record of base measurement

(In first two columns above, "△ E.B.+20" should be changed to "△E. B. set-up" and "10'" to "10 set-up.")

so either feet or meters may be used in the third column. The corrections for lengths less than 50 meters must be computed as explained on pages 125 and 159. The sum of these corrections for the section of the base is entered on Form No. 589, Computation of Base Line, in the column headed "Inclination."

It is very important that all broken grades and partial tape lengths be indicated on this form and that the grade correction be computed for the corresponding distance. The most frequent mistake made in computing grade corrections arises from using a 50-meter length instead of the real length.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 635

ABSTRACT OF WYE LEVELS
AND
COMPUTATION OF INCLINATION CORRECTIONS

POINT	DISTANCE	MEAN DIFFERENCE OF ELEVATION	INCLINATION CORRECTION	ELEVATION	MEAN ELEVATION	REMARKS
	Meters	*Meters* ~~or feet~~	*mm.*	*Meters*	*Meters*	
△ E. B.				237.67		
△ E. B. set-up	20	+ 0.318	2.5			
1	50	+ 0.434	1.8			
2	50	+ 0.067	0.0			
3	50	+ 0 397	1.6			
4	50	+0.232	0.5			
5	50	+0.198	0.4			
6	50	+0.077	0.1	239.39		
6½	25	−0.001	0.0			
7	25	−0.340	2.3			
8	50	−0.328	1.1			
9	50	−0.176	0.3			
10	50	−0.297	0.9			
10 set-up	4.6619	−0.027	0.1			
11	50	−0.440	1.9			
12	50	−0.315	1.0	237.47		
13	50	+0.525	2.8			
14	50	+0.900	8.1			
15	50	−1.197	14.3			
16	50	+0.266	0.7			
17	50	−0.590	3.5			
18	50	−0.166	0.3			
19	50	−0.536	2.9			
20	50	+0.054	0.0	236.72	237.8	
			47.1			

FIG. 62.—Sample abstract of wye levels

Form No. 589 (computation of base line).—On Form No. 589 the first correction to be entered is the correction for temperature. This is computed as follows:

Temperature correction $= (T - T_s) \times$ temperature coefficient $\times 50 \times$ number of tape lengths, in which T is the mean temperature for the section and T_s is the temperature of the tape at standardization. The value of T is entered in the column headed "Temperature," and is the mean of the thermometer readings recorded on Form No.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 589

COMPUTATION OF *Exemplar* BASE LINE

11—[illegible]

SECTION	DATE 1924	DIR. OF MEAS.	TAPE NO.	UNCORRECTED LENGTH		TEMP.	CORRECTIONS					REDUCED LENGTH	ADOPTED LENGTH	(v)	(vv)
				Tape lengths	Meters		Temp.	Tape and Catenary	Set-up Set-back	Inclination	Sea level				
						°C	Meters	Meters	Meters	Meters	Meters	Meters	Meters	mm.	mm.
△ E. B. to T. E. 20	7/7	F	922	—	—	20.0	-0.0002	-0.0007	+20.0000	-0.0471	-0.0382	1024.4704			
	"	"	922	20	1000	20.6	-0.0071	-0.0790	- 0.0586						
	"	"	872	—	—	21.5	0.0000	+0.0013	+4.7000						

FIG. 63.—Sample computation of base line

590. The value of T_s is given in the standardization data for the tape. (See p. 130.) The temperature coefficient is the change in length per meter for each degree centigrade change in temperature, and is also given with the standardization data. The number of tape lengths is given in the column headed "Tape lengths," and is the number of full tape lengths recorded on Form No. 590. For tapes with a positive coefficient of expansion the temperature correction is, of course, + or −, according to whether the mean temperature is greater or less than the standard temperature. There are a few tapes which have negative coefficients of expansion, and for these the correction would have the opposite sign.

The correction in the column headed "Tape and catenary" is obtained from the standardization data for the tape or by computing the catenary correction when the tape is supported in an unusual manner. The tape correction is the difference between 50 meters and the length of the tape as given for the proper method of support. For instance, in the sample on Form No. 590 there are 18 tape lengths supported at three points, 1 at four points, and 1 at two points. The correction for a tape supported at three points is obtained directly from the standardization values, multiplied by 18 for the 18 tape lengths in this case. Referring again to the sample on Form No. 590, the correction for the one tape length supported at four points is obtained by combining the proper fractional parts of the corrections for the three and the five point supports. The correction for the tape supported at two points must be computed by the catenary formula. (See p. 127.) The algebraic sum of these corrections is entered in the column headed "Tape and catenary," the sign depending on whether the length of the tape is greater or less than 50 meters.

In the column headed "Set-up and setback" is entered the algebraic sum of the set-ups and setbacks recorded on Form No. 590, the set-ups being plus and the setbacks minus. In the sample shown in Figure 63 the two large set-ups and the corrections to them are recorded separately. All set-ups, however, could have been combined, and also the temperature, tape, and catenary corrections for the large set-ups could have been combined with the same corrections for the full tape lengths, and then the entire computation of a section would have been on one line. It simplifies the checking of the computation somewhat, however, to enter each large set-up on a separate line, as shown on the sample form.

The sum of the inclination corrections is obtained from Form No. 635. Then the algebraic sum of the uncorrected length and all corrections gives the reduced length for the section, and the mean of the reduced lengths from the forward and backward measurements gives the adopted length. The columns headed "(v)" and "(vv)" are used in computing the probable error of the measurement of the base.

PROBABLE ERROR OF MEASUREMENT

A method of computing the probable error of the measured length of the base, with separate values for the probable error due to standardization of the tapes, the determination of the coefficients of expansion, and the accidental errors of the measurement, is given in Appendix No. 4, Report for 1910, pages 160–161. The probable error is usually computed, however, by the method described in the following paragraph. This method is based on the theory that the errors of standardization and of the determination of the coefficients of expansion are either largely included in or are masked by the discrepancies in the measured lengths of the sections.

The probable error of each section is computed by the formula

$$\text{p. e.} = 0.6745 \sqrt{\frac{\Sigma v^2}{n\ (n-1)}},$$

where v is a residual and n the number of measures of the section. Where a section is measured only twice the probable error will, of course, be 0.6745 times one-half the difference between the two measured lengths. The probable error of the entire base is the square root of the sums of the squares of the probable errors of the component sections.

REDUCTION TO SEA LEVEL

Since the lines of a scheme of triangulation are reduced to their equivalent lengths at sea level, the length of any base must be likewise reduced to sea level before it can be used in adjusting the triangulation to which it is connected. It is frequently necessary to reduce the base to sea level in the field in order to compare the measured length with the length as computed through the triangulation from the previous base.

This requires the connection of the base line levels to a bench mark and the computation of the elevation above sea level of the tape supports in order to obtain a mean elevation for the base. Only enough elevations need be used in computing the mean elevation for a section to give a value correct to within 5 meters.

The formula used in reducing a base to sea level is

$$C = -S\frac{h}{r} + S\frac{h^2}{r^2} - S\frac{h^3}{r^3} + \ . \ . \ . \ .$$

in which C is the correction to reduce to sea level a section of length S, of a mean elevation h, with r the radius of curvature of the earth's surface for that section. Only the first term of the formula need be used for any field reduction.

The computation of the correction for sea level shown on sample Form 589, page 144, is given below, the mean latitude of the base being 40° 30′ and its azimuth 75°, giving a value for log r (see table p. 167) of 6.80521. The mean elevation as obtained from Form 635 is 237.8 meters.

$$\begin{aligned} \log 1{,}024.5 &= 3.01051 \\ \log 237.8 &= 2.37621 \\ \operatorname{colog} r &= 3.19479 \\ \hline \log C &= 8.58151 \\ C &= 0.0382 \text{ meters.} \end{aligned}$$

The error per kilometer of base line for each 1-meter error in the elevation above sea level as used in computing the reduction varies from 0.000158 meters to 0.000156 meters, depending upon the latitude and azimuth of the base. This corresponds to a proportionate error in length of from 1 part in 6,329,000 to 1 part in 6,410,000.

Chapter 4.—AZIMUTHS

LAPLACE AZIMUTHS

The accumulation of angular errors in the triangulation, and a tendency for a part of those errors to be systematic and to give a twist to the direction of a triangulation scheme, make it necessary to include in the adjustment of the triangulation certain azimuth control points, called Laplace stations. A Laplace station is a station of the triangulation at which both the astronomic azimuth and the astronomic longitude have been determined. A Laplace azimuth is an astronomic azimuth corrected for the deflection of the vertical. On pages 17–21, Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy, is an explanation of the method of using Laplace azimuths on triangulation.

FIRST-ORDER AZIMUTHS

On first-order triangulation the azimuths at Laplace stations are designated as first-order azimuths. They are located at main-scheme stations of the triangulation from six to eight figures apart, and are observed by the triangulation party as the work progresses. If possible they should be located at stations to which trucks can be driven, in order that they may be readily accessible to the longitude party with its heavy instruments. Where the stations of the main scheme are so inaccessible that it is necessary to locate a supplemental point for a Laplace station it need be connected to the main scheme by a single well-shaped triangle only, but the angle measures must be made with first-order accuracy. Formerly it was necessary to locate Laplace stations where a telegraphic connection could be secured, but by the radio method of receiving time signals on longitude work such a connection is no longer necessary.

The azimuth at a Laplace station is determined by observations upon Polaris at any hour angle, using some one station of the main scheme as the azimuth mark. The station last sighted upon before pointing on Polaris is the one to be used as the azimuth mark, in order that the instrumental changes shall be as small as possible between the times of pointing upon the mark and star. Other things being equal, it is advantageous to use for initial the first station of the scheme to the right of north. This will cause the main-scheme station just to the left of north to be the azimuth mark and Polaris to be at a convenient angle from the mark. A sample record for azimuth observations is shown on page 88. The azimuth observations are usually made during the measurement of the directions to the stations of the triangulation scheme, the time required to make the observations upon Polaris being approximately equal to that required to make the observations upon two of the triangulation stations.

The observations for a first-order azimuth at a Laplace station should be made upon at least two separate nights. The final result should have a probable error which seldom exceeds 0″.30. It should depend upon at least 22 acceptable positions with a direction instrument, not less than 8 of them being on any one night.

The following routine for observing upon Polaris will reduce the time required for the operation and also minimize the chances for mistakes and omissions.

After having completed the pointing and readings upon the mark, point upon Polaris, bringing the star within a half minute or so of the vertical wires in the middle of the field; clamp the horizontal motion; place the stride level in position on the standards, at the same time calling "stand by" to the recorder; perfect the pointing, calling "tip" sharply at the moment of bisection; and then make mental note of the stride-level readings, but do not call out the readings until the recorder who is marking down the time, calls "ready." After noting the stride-level readings reverse the stride level and move to the first micrometer, calling out the stride-level readings, west end always first, then the micrometer readings in order, and lastly the readings of the reversed level, again west end first. Remove the level, turning it end for end as it is removed, and place it on the nails provided for the purpose on the south side of the stand, in position to be placed later on the instrument with the same end to the west as on the first reading. This is not necessary, but will often enable the computer to detect mistakes in the recording.

Next loosen the upper horizontal clamp, turn the instrument 180 degrees, reverse the telescope and point again upon Polaris, going through the same procedure as described for the first pointing. This routine permits the stride-level bubble to come fully to rest without delay to the observing. With an extra man to read one micrometer 16 positions can be observed on Polaris and the mark in little more than an hour.

The principal precautions to take in observing upon Polaris are to train the recorder to note accurately the chronometer time corresponding to the call of "tip," and then to give him sufficient time to make record of it before he is confused by other readings. The recorder should be trained to carry mentally a staccato count of the seconds of the chronometer, as, "twenty′ half, twenty-one′ half, twenty-two′ half,"—the accented word or syllable and the word "half" synchronizing with the half-second beat. It is then easy for him to note within a quarter of a second the time of the observer's "tip."

If a sidereal watch is used instead of a chronometer its fifth-second beat can not readily be followed, but with practice the mental staccato half-second count can be regulated so that the count of the whole second can be made to coincide with the passage of the second hand over each successive second division of the dial as registered by the eye and the time of the "tip" noted with reference to the count. This is more accurate than noting the time by the eye alone.

To avoid errors due to the instrument being out of level the azimuth mark, whether a distant triangulation station or some near-by object, should be as nearly as possible in the horizon. If it is elevated or depressed more than 1 degree out of the horizon, stride-level readings should be taken when reading upon the mark in the same manner as when reading upon Polaris.

In northern latitudes, such as Alaska, particular care must be taken that the instrument stand is stable and not disturbed by the movements of the observer about it. Because of the high altitude of Polaris the stride level used should be very sensitive and care taken that the bubble has come to rest before the readings are made. A 2-millimeter division of the bubble should have a value of not to exceed 4 seconds of arc for azimuth work in high latitudes.

The record book in which are recorded the observations upon Polaris should always contain a record of the eccentricity of both the light and the theodolite. If there is no eccentricity of either it should be so stated. Often there is uncertainty as to whether the eccentricity recorded for the regular angle observations should be applied to the azimuth observations also, if the azimuth is measured separately from the other directions.

SECOND-ORDER AZIMUTHS

An azimuth at a first-order triangulation station which is not to be used as a Laplace station is called a second-order azimuth, and is observed to secure data regarding the deflection of the vertical. In general, a second-order azimuth should be observed at intervals of from 40 to 60 miles through the scheme, though it is desirable that an azimuth be observed at any station where the observing party has to observe on more than one night, if the weather permits azimuth work. In observing a second-order azimuth the same procedure should be followed as described for a first-order azimuth, but all observations may be made on a single night. Such an azimuth should not depend on less than 10 acceptable positions and should have a probable error of less than $0''.50$, though in Alaska this limit may be extended to $0''.65$.

CHRONOMETER CORRECTION

Since observations for azimuth are made upon Polaris at any hour angle, the correction to the chronometer time should be known to within one second. An error of one second in the time would cause an error of about $0''.3$ in the computed azimuth of Polaris near culmination in latitude 30°, and about $0''.6$ in latitude 60°.

Occasionally the chronometer correction may be obtained by comparison with the time signals from the Naval Observatory sent out by radio or over commercial telegraph lines, but these signals are apt to prove unsatisfactory for the following reasons: Because of temperature and winding errors the daily rate of the chronometer obtained by successive comparisons with the noon signals may be very different from the actual rate during the interval from the noon signal to the mean epoch of the time of observing. Transporting the chronometer from place to place also affects its rate. Chronometer corrections obtained by observations upon stars near the time of the azimuth observations will, in general, prove the more accurate and satisfactory, and of the many methods available observed altitudes of both east and west stars near the prime vertical will prove the most advantageous with the instruments usually at hand. With this method no star catalogue or observing list is needed, and any star near the prime vertical of greater altitude than 30° can be used, even though it shows for only a few minutes through a broken field of clouds.

The instrument used for the trigonometric leveling between triangulation stations is used for measuring the zenith distances of the time stars, whether it be a so-called vertical circle similar to the one shown in Figure 23, or the vertical circle of a theodolite of the universal type shown in Figure 17. In either case the graduated circle is usually about 6 inches in diameter, read by two or more verniers to 10 seconds, and has a fairly sensitive level bubble.

The method of observing depends upon whether the bubble is attached to the vernier circle or to the frame of the graduated circle. With either method the star is brought near the intersection of the middle vertical and horizontal wires, the horizontal wire brought just ahead of the star in the direction in which the star is moving, and the star allowed to make contact, thus eliminating the error due to the thrust upon the instrument when the horizontal wire is moved into contact with the star.

With an instrument having the bubble attached to the vernier circle the observing routine is as follows: With circle right bring the star near the intersection of the middle wires as described above, call "tip" to the recorder for the noting of the chronometer time as the star makes contact with the horizontal wire, bring the bubble to the center of the tube with the vernier screw, watching it three or four seconds to see that it remains centered, read the verniers, loosen the vernier clamp, reverse the telescope and with circle left bring the star into approximate position with the vernier screw, calling "tip" to the recorder as the star makes contact, bring the bubble to the center and read the verniers. This constitutes one measure of the zenith distance. The next measure should be made beginning circle left and ending circle right. Four measures of the zenith distance of an east star should be made and a similar number on a west star before Polaris is observed upon for azimuth. This constitutes what is commonly called a time set. Like observations for time should be made after the observations upon Polaris are completed for the second time set. The difference between the chronometer corrections given by the two sets is distributed throughout the intervening period as a chronometer rate.

With an instrument having the bubble attached to the frame or to the graduated circle the bubble must be read, and a different observing routine is followed. With the circle right read the verniers, bring the star near the horizontal wire with the tangent screw which does not change the vernier readings, call "tip" to the recorder as the star makes contact, then read the bubble, objective end first. Loosen the vernier clamp, reverse the instrument, and with circle left bring the star into position with the vernier slow-motion screw, call the contact, read the bubble and then the verniers.

If the bubble is out of adjustment and one end falls beyond the graduations after reversal, it can be brought back by moving one or more foot screws of the instrument before pointing upon the star. The relation between the line tangent to the axis of the bubble and the line passing through the zero of the graduated circle and the graduation mark 180° from the zero must not be changed between the times of the two star contacts in a single measure of the zenith distance.

If a repeating circle is being used the reading of the circle should be changed by at least a degree between each two measures; otherwise an error in a single reading would cause the rejection of two measures of the zenith distance.

ERRORS IN TIME OBSERVATIONS

It is more difficult to secure good time sets than to secure good observations upon Polaris. For that reason the observer should always compute his time sets, though for primary azimuths after the first azimuth station or so he need only abstract on the proper forms his observations upon Polaris. The computation of both time and azimuth is fully explained in Special Publication No. 14.

Some of the more common sources of error in time observations are mentioned below. The remedy for each is indicated.

1. **Incorrect noting of time.**—An inexperienced recorder should be trained in the way explained in the paragraphs relating to the observations on Polaris. Do not confuse him by calling out the readings of the verniers or levels before he has finished recording the time.

2. **Incorrect circle readings.**—The difficulty of securing an even illumination of the verniers by flashlight increases the chances of incorrect readings. Check carefully the minutes of each vernier reading, for the mistakes are more apt to occur in the minutes than in either the degrees or seconds.

3. **Wrong star.**—The effect of this error can be nullified by measuring roughly the horizontal direction to each time star from either Polaris or some station of the triangulation, noting the time of the measurement. This should invariably be done, and the angle and time recorded for each time star.

4. **Refraction errors.**—The zenith distances are corrected for refraction, the correction angles being taken from tables given in Special Publication No. 14. These tabulated values must themselves be corrected for temperature and barometric pressure, so thermometer and barometer readings must be recorded for each time set. The differential effect of an incorrect index of refraction being used for the tabulated values for the correction will be lessened by having the east and west stars as nearly as possible of the same zenith distance, though usually the error from this source is not serious if no star is used of an altitude less than 30°.

5. **Poor selection of stars.**—More serious errors will be introduced by selecting stars too far from the prime vertical. In the early evening there is always the temptation to use the first stars visible in order to begin the night's work. A delay of a quarter of an hour is usually not serious, and will often result in securing time stars which will give a much more accurate chronometer correction.

6. **Parallax.**—The effect of parallax is very apt to be evident in the computed times, for the effect of this error is almost invariably opposite in sign for east and west stars. It is very essential that both east and west stars be observed upon for each time set, for the mean will be measurably free from this error unless there is a great difference in their zenith distances.

Chapter 5.—SPECIAL SURVEYS

Frequently the geodetic engineer is called upon to execute a control survey for a special purpose, and must decide from investigation what accuracy is necessary and what methods must be used. A proper decision requires the exercise of experience and good judgment, for unnecessary accuracy is wasteful and a survey possessing too little accuracy is apt to prove useless.

Bearing in mind the purpose for which the horizontal control is being established, the first thing to decide upon is the limit of permissible error in position and in scale—or length—in the detailed control. If the area to be covered is at all considerable it will usually be more economical to span the area with control of either the first or second order, as may be required, while the detailed control would be of a lower order of accuracy. The process to use, whether traverse or triangulation, will depend largely upon topographic conditions, while the choice of instrumental methods will be influenced by the class of instruments available and the accuracy which must be secured. The amount of money available must also be considered as a major factor, but this should be permitted to affect the quantity rather than the quality of the control.

METROPOLITAN CONTROL SURVEYS

Several cities, both large and medium-sized, have either completed or begun detailed topographic surveys for use in connection with the various engineering operations of the city or to assist in what is generally known as city planning. The horizontal control for such surveys, especially those in which property boundaries are connected to the control surveys, requires an accuracy of the highest order obtainable at a reasonable expense. The rapidly increasing value of urban real estate makes it necessary to secure an accuracy greater even than that specified for the first-order control surveys which span the country and coordinate the surveys of the nation. The estimated future value of the real property rather than its present value should dictate the accuracy to be secured on the control surveys.

As an illustration of the combination of various grades and processes of control on a single project, there is given the following extracts from the specifications for the horizontal-control survey of an urban district where the Coast and Geodetic Survey cooperated with the city authorities in making the survey:

The work to be executed will *consist of both first-order triangulation* and first-order traverse, the two being closely connected into a strong scheme of horizontal control.

Connections shall be made to adjusted first-order triangulation stations of the national survey in that vicinity, in order to place the surveys upon the North American datum, but the lengths for the control should be obtained by measuring two bases located on opposite sides of the city and strongly connected with the scheme of triangulation.

The ΣR_1 and ΣR_2 between bases, as defined on pages 12 to 25 of Special Publication No. 26 of the Coast and Geodetic Survey, should be not greater than 60 and 90, respectively. If it is necessary to do so a broken base may be used, provided the terminal stations are intervisible and the loop closure of observed angles is made, and provided also that no considerable element of the measured base has an inclination of more than 15° to the final projected line.

Each base should be measured with an actual total error from all sources of not to exceed 1 part in 300,000, and with a probable error of not more than 1 part in 1,000,000. The measurement of each base should be made with at least three invar tapes, so used as to give an accurate and equal intercomparison of their effective lengths on the measurement. The tapes should be standardized at the Bureau of Standards both before and after the two bases are measured. In order to secure the accuracy stated above it is advisable that the uncertainties due to the measured temperatures and tensions, to the grade corrections derived from measured elevations of tape ends, and to the marking of the tape ends should be eliminated to such an extent that the error from no one source shall exceed, for the entire base, more than 1 part in 500,000. Whenever an error tends to be systematic and to be greater in magnitude than the amount stated above such a method of measuring should be adopted as will tend to eliminate it from the final mean of the measured length.

Triangulation.—Such methods should be used on the main-scheme triangulation as will give an agreement of better than 1 part in 100,000 between the measured length of the base line and its length as computed through the triangulation from the base on the opposite side of the city, after the side and angle equations have been satisfied by the least-squares adjustment. Since this agreement can not be known until after the triangulation is entirely completed, it is suggested that methods of observing be adopted which will secure a triangle closure of not to exceed 3″, with an average not greatly in excess of 1″.25, which should give a probable error for a direction of about 0″.75. Special attention must be paid in selecting times for observing when the adverse effect of horizontal refraction will be a minimum.

The principal lines of the triangulation scheme should generally be from 1 to 5 miles in length. It is desirable to have some overlapping of figures whenever the strength of the entire net would be materially strengthened thereby. A distribution of first-order stations over the area as uniformly as possible should be secured with an average of about one for each 5 square miles of area. The stations on the outer lines of the scheme should be so chosen as to permit of expansion of the scheme outward at a future date.

In addition to the occupied first-order stations a supplemental first-order station for each square mile of area should be determined by intersection from at least three stations of the main scheme, these supplemental stations to be distributed as uniformly over the area as possible. The observations upon these supplemental stations should be made with such methods as will give a probable error for a direction not greatly in excess of 1 second.

The instrument to be used on the triangulation is the regular 12-inch Coast and Geodetic Survey model direction theodolite. Observations upon supple-

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mental stations can be made either with this instrument or with the smaller direction instrument.

Traverse.—Since it is upon the first-order traverse stations that the city engineer must rely for connections to his subordinate control lines, particular care must be given to the location and measurement of the traverse.

Location of lines.—The traverse lines between connections to first-order triangulation stations should contain not more than 15 angle stations each. The ends of the traverse lines should be connected to the triangulation in both position and azimuth. Additional connections in position should be made between the traverse and supplemental triangulation stations whenever opportunity presents itself, but accurate connections in azimuth can usually be made only to triangulation stations which have been occupied.

The first-order traverse lines should be distributed over the area with as much uniformity as practicable, and in general should not be more than 1 mile apart. Traverse within the 1-mile squares will be of a lower order of accuracy. Long, narrow loops should be avoided, and when the distance between two traverse lines is less than 15 per cent of the distance around the loops which have those lines as component parts a connecting traverse line should be run across the loop. Where two lines of first-order traverse cross each other they should invariably have an angle station at the intersection.

When a connection in azimuth is to be made between a first-order traverse and a first-order triangulation station the angle observations for the connection should have an error of not to exceed 2''.0, and the connection in distance should have an error of not to exceed 0.05 foot, with a check determination of the distance. The chief function of the triangulation is to provide proper position and angle control for the first-order traverse, and for that reason the connections between the two systems should be as strong as can be made economically.

The first-order traverse stations should be spaced along the traverse lines at a distance apart of not to exceed 500 yards. Often two first-order traverse stations on the same line, not adjacent, are intervisible, in which case the azimuths should be carried through the line by the fewest stations, avoiding the shortest lines of sight. Care should be taken, however, that no considerable portion of a supplemental line makes an angle of more than 25° with the line of the main scheme upon which it is to be projected.

Accuracy.—The first-order traverse system will be composed of two sets of lines. The first set will consist of those to be included in the first adjustment, which will be intimately connected with the main-scheme triangulation and will divide the area up into blocks 3 or 4 miles across. The second set of first-order traverse lines will divide these large blocks into smaller blocks containing about a square mile, and will be adjusted into the first-order traverse lines forming the perimeter of the block. Traverse within the 1-mile squares will be of a lower order of accuracy.

The angle measures on the lines to be held in the first adjustment should be made with such accuracy that the correction per angle to close a loop of the precise traverse will not greatly exceed $2''.5\sqrt{a}/a$, where a is the number of main-angle stations on the line. On the lines to be held in the second adjustment the correction per angle to close the loop should not greatly exceed $3''.5\sqrt{a}/a$, where a has the same significance as above.

First-order traverse lines included in the first adjustment should be given two precise measures with invar tapes. In making rejections an estimate of the accuracy of the mean of two or more measures should be considered rather than their lack of agreement. Extreme care should be taken to eliminate systematic

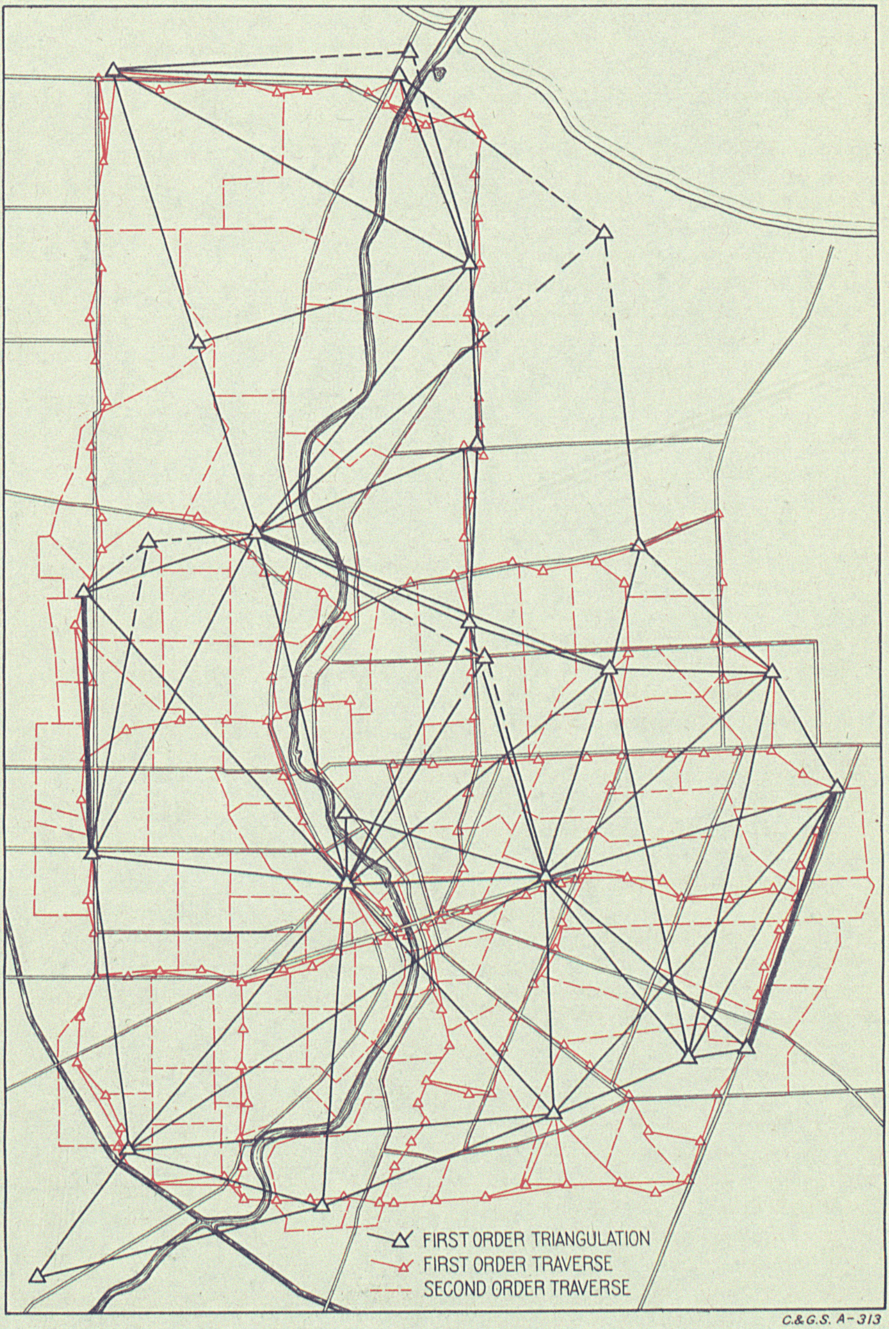

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Fig. 64-Scheme of first and second order control for a city survey

errors on the traverse measurements, or at least to eliminate their effect from the mean of the measures.

First-order traverse lines included in the second adjustment need be given but one precise measurement with an invar tape, but a rough check measurement should be made with a steel tape to detect any blunders before the computations are begun.

A first-order traverse line included in the first adjustment, starting from a point of the adjusted triangulation, should close in position upon another point of the adjusted triangulation within 1 part in 25,000 of the length of the traverse line after all tape corrections and angle corrections for loop closure have been applied. If the first closing error in position is greater than 1 part in 25,000 either the angle or tape measurement, or both, must be strengthened until the required agreement is secured. On lines of the second adjustment the closing error in position upon an adjusted point, either traverse or triangulation, should not exceed 1 part in 20,000 of the length of the line after the tape corrections and angle corrections for loop closure have been made.

For the 38 triangles of first-order triangulation executed under the above instructions the average triangle closure was less than 1″, and only two triangles had a closure greater than 1″.70.

In Figure 64 there is shown a somewhat idealized sketch of a scheme where first-order triangulation and both first and second order traverse are combined into a closely coordinated whole to conform to the specifications given above. The triangulation is first adjusted, then the traverse shown by heavy lines on the sketch is adjusted to the triangulation, then the remainder of the first-order traverse lines are computed in a series of adjustments. Finally, second-order traverse lines having an accuracy represented by a proportionate error in length and position of about 1 part in 20,000 are run about one-fourth mile apart. Traverses on the remaining streets are third order in character and would have an error in length represented by a closure of not to exceed 1 part in 10,000.

With such a coordinated scheme of control errors can not accumulate over long distances, and yet a comparatively large part of the traverse is second and third order in character, with a consequent lessening of the total cost.

On any horizontal-control survey made for a special purpose reconnaissance is an exceedingly important part of the operation. The preliminary reconnaissance should be made in sufficient detail to assure the engineer in charge that the methods chosen are the best possible ones to adequately and economically meet the conditions and the requirements.

When the general plan of operations has been decided upon the detailed reconnaissance should be kept far enough ahead of the observing to obviate the necessity for frequent reoccupation of stations because of new work being added. Before the triangulation stations are occupied the reconnaissance should be completed for all supplemental stations and traverse connections, with all stations marked and prepared which are to be observed upon. Reconnaissance for the first-order traverse lines should also include all connecting lines, since a slight shifting of a main-scheme station may often prove of great advantage in connecting a subsidiary line to it. On all of the operations accuracy is essential and speed desirable, but it is on the reconnaissance and in the proper planning of operations that the greatest opportunity for economy lies.

Chapter 6.—MISCELLANEOUS TABLES

Grade corrections for various horizontal lengths

[For differences of elevation up to 3 meters. Correction $=(l-\sqrt{l^2+h^2})=-\frac{1}{2}\frac{h^2}{l}+\frac{1}{8}\frac{h^4}{l^3}-\ \ldots$ (always negative)]

Length of section in meters	Difference of elevation in meters										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
10	0.0	0.5	2.0	--------	--------	--------	--------	--------	--------	--------	--------
20	--------	0.2	1.0	2.2	4.0	--------	--------	--------	--------	--------	--------
30	--------	0.2	0.7	1.5	2.7	4.2	6.0	8.2	10.7	--------	--------
40	--------	0.1	0.5	1.1	2.0	3.1	4.5	6.1	8.0	10.1	12.5
50	--------	0.1	0.4	0.9	1.6	2.5	3.6	4.9	6.4	8.1	10.0
60	--------	0.1	0.3	0.8	1.3	2.1	3.0	4.1	5.3	6.8	8.3
70	--------	0.1	0.3	0.6	1.1	1.8	2.6	3.5	4.6	5.8	7.1
80	--------	0.1	0.2	0.6	1.0	1.6	2.2	3.1	4.0	5.1	6.2
90	--------	0.1	0.2	0.5	0.9	1.4	2.0	2.7	3.6	4.5	5.6
100	--------	0.0	0.2	0.4	0.8	1.2	1.8	2.4	3.2	4.0	5.0
110	--------	--------	0.2	0.4	0.7	1.1	1.6	2.2	2.9	3.7	4.5
120	--------	--------	0.2	0.4	0.7	1.0	1.5	2.0	2.7	3.4	4.2
130	--------	--------	0.2	0.3	0.6	1.0	1.4	1.9	2.5	3.1	3.8
140	--------	--------	0.1	0.3	0.6	0.9	1.3	1.7	2.3	2.9	3.6
150	--------	--------	0.1	0.3	0.5	0.8	1.2	1.6	2.1	2.7	3.3
160	--------	--------	0.1	0.3	0.5	0.8	1.1	1.5	2.0	2.5	3.1
170	--------	--------	0.1	0.3	0.5	0.7	1.1	1.4	1.9	2.4	2.9
180	--------	--------	0.1	0.2	0.4	0.7	1.0	1.4	1.8	2.2	2.8
190	--------	--------	0.1	0.2	0.4	0.7	0.9	1.3	1.7	2.1	2.6
200	--------	--------	0.1	0.2	0.4	0.6	0.9	1.2	1.6	2.0	2.5
210	--------	--------	0.1	0.2	0.4	0.6	0.9	1.2	1.5	1.9	2.4
220	--------	--------	0.1	0.2	0.4	0.6	0.8	1.1	1.5	1.8	2.3
230	--------	--------	0.1	0.2	0.3	0.5	0.8	1.1	1.4	1.8	2.2
240	--------	--------	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.7	2.1
250	--------	--------	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.6	2.0
260	--------	--------	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.6	1.9
270	--------	--------	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.5	1.9
280	--------	--------	0.1	0.2	0.3	0.4	0.6	0.9	1.1	1.4	1.8
290	--------	--------	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.4	1.7
300	--------	--------	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.3	1.7

Grade corrections for various lengths of tape—Continued

Length of section in meters	Difference of elevation in meters									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
40	15.1	18.0								
50	12.1	14.4	16.9	19.6	22.5	25.6				
60	10.1	12.0	14.1	16.3	18.8	21.3	24.1	27.0	30.1	33.6
70	8.6	10.3	12.1	14.0	16.1	18.3	20.6	23.1	25.8	28.3
80	7.6	9.0	10.6	12.2	14.1	16.0	18.1	20.2	22.6	25.0
90	6.7	8.0	9.4	10.9	12.5	14.2	16.1	18.0	20.1	22.2
100	6.0	7.2	8.4	9.8	11.2	12.8	14.4	16.2	18.0	20.0
110	5.5	6.5	7.7	8.9	10.2	11.6	13.1	14.7	16.4	18.2
120	5.0	6.0	7.0	8.2	9.4	10.7	12.0	13.5	15.0	16.7
130	4.7	5.5	6.5	7.5	8.7	9.8	11.1	12.5	13.9	15.4
140	4.3	5.1	6.0	7.0	8.0	9.1	10.3	11.6	12.9	14.3
150	4.0	4.8	5.6	6.5	7.5	8.5	9.6	10.8	12.0	13.3
160	3.8	4.5	5.3	6.1	7.0	8.0	9.0	10.1	11.3	12.5
170	3.6	4.2	5.0	5.8	6.6	7.5	8.5	9.5	10.6	11.8
180	3.4	4.0	4.7	5.5	6.3	7.1	8.0	9.0	10.0	11.1
190	3.2	3.8	4.4	5.2	5.9	6.7	7.6	8.5	9.5	10.5
200	3.0	3.6	4.2	4.9	5.6	6.4	7.2	8.1	9.0	10.0
210	2.9	3.4	4.0	4.7	5.4	6.1	6.9	7.7	8.6	9.5
220	2.7	3.3	3.8	4.5	5.1	5.8	6.6	7.4	8.2	9.1
230	2.6	3.1	3.7	4.3	4.9	5.6	6.3	7.0	7.8	8.7
240	2.5	3.0	3.5	4.1	4.7	5.3	6.0	6.7	7.5	8.3
250	2.4	2.9	3.4	3.9	4.5	5.1	5.8	6.5	7.2	8.0
260	2.3	2.8	3.2	3.8	4.3	4.9	5.6	6.2	6.9	7.7
270	2.2	2.7	3.1	3.6	4.2	4.7	5.4	6.0	6.7	7.4
280	2.2	2.6	3.0	3.5	4.0	4.6	5.2	5.8	6.4	7.1
290	2.1	2.5	2.9	3.4	3.9	4.4	5.0	5.6	6.2	6.9
300	2.0	2.4	2.8	3.3	3.7	4.3	4.8	5.4	6.0	6.7

Length of section in meters	Difference of elevation in meters									
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
70	31.5	34.6	37.8	41.1						
80	27.6	30.2	33.1	36.0	39.1	42.2	45.6	49.0		
90	24.5	26.9	29.4	32.0	34.7	37.6	40.5	43.6	46.7	50.0
100	22.0	24.2	26.4	28.8	31.2	33.8	36.4	39.2	42.0	45.0
110	20.0	22.0	24.0	26.2	28.4	30.7	33.1	35.6	38.2	40.9
120	18.4	20.2	22.0	24.0	26.0	28.2	30.4	32.7	35.0	37.5
130	17.0	18.6	20.3	22.1	24.0	26.0	28.0	30.2	32.3	34.6
140	15.7	17.3	18.9	20.6	22.3	24.1	26.0	28.0	30.0	32.1
150	14.7	16.1	17.6	19.2	20.8	22.5	24.3	26.1	28.0	30.0
160	13.8	15.1	16.5	18.0	19.5	21.1	22.8	24.5	26.3	28.1
170	13.0	14.2	15.6	16.9	18.4	19.9	21.5	23.1	24.7	26.5
180	12.2	13.4	14.7	16.0	17.4	18.8	20.2	21.8	23.4	25.0
190	11.6	12.7	13.9	15.2	16.5	17.8	19.2	20.6	22.1	23.7
200	11.0	12.1	13.2	14.4	15.6	16.9	18.2	19.6	21.0	22.5
210	10.5	11.5	12.6	13.7	14.9	16.1	17.4	18.7	20.0	21.9
220	10.0	11.0	12.0	13.1	14.2	15.4	16.6	17.8	19.1	20.5
230	9.6	10.5	11.5	12.5	13.6	14.7	15.8	17.0	18.3	19.6
240	9.2	10.1	11.0	12.0	13.0	14.1	15.2	16.3	17.5	18.7
250	8.8	9.7	10.6	11.5	12.5	13.5	14.6	15.7	16.8	18.0
260	8.5	9.3	10.2	11.1	12.0	13.0	14.0	15.1	16.2	17.3
270	8.2	9.0	9.8	10.7	11.6	12.5	13.5	14.5	15.6	16.7
280	7.9	8.7	9.4	10.3	11.2	12.1	13.0	14.0	15.0	16.1
290	7.6	8.3	9.1	9.9	10.8	11.7	12.6	13.5	14.5	15.5
300	7.3	8.1	8.8	9.6	10.4	11.3	12.1	13.1	14.0	15.0

Differences of elevation and grade corrections for varying angles of inclination

[Length=50 meters. Argument is inclination angle]

α	Difference of elevation	Grade correction	α	Difference of elevation	Grade correction	α	Difference of elevation	Grade correction	α	Difference of elevation	Grade correction	α	Difference of elevation	Grade correction	α	Difference of elevation	Grade correction
° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.
0 00	0.00	0.0	0 30	0.44	1.9	1 00	0.87	7.6	1 30	1.31	17.1	2 00	1.74	30.5	2 30	2.18	47.6
01	.01	.0	31	.45	2.0	01	.89	7.9	31	.32	17.5	01	.76	31.0	31	.20	48.2
02	.03	.0	32	.47	2.2	02	.90	8.1	32	.34	17.9	02	.77	31.5	32	.21	48.9
03	.04	.0	33	.48	2.3	03	.92	8.4	33	.35	18.3	03	.79	32.0	33	.22	49.5
04	.06	.0	34	.49	2.4	04	.93	8.7	34	.37	18.7	04	.80	32.5	34	.24	50.2
0 05	0.07	0.1	0 35	0.51	2.6	1 05	0.95	8.9	1 35	1.38	19.1	2 05	1.82	33.0	2 35	2.25	50.8
06	.09	.1	36	.52	2.7	06	.96	9.2	36	.40	19.5	06	.83	33.6	36	.27	51.5
07	.10	.1	37	.54	2.9	07	.97	9.5	37	.41	19.9	07	.85	34.1	37	.28	52.1
08	.12	.1	38	.55	3.1	08	.99	9.8	38	.42	20.3	08	.86	34.7	38	.30	52.8
09	.13	.2	39	.57	3.2	09	1.00	10.1	39	.44	20.7	09	.88	35.2	39	.31	53.5
0 10	0.15	0.2	0 40	0.58	3.4	1 10	1.02	10.4	1 40	1.45	21.2	2 10	1.89	35.7	2 40	2.33	54.1
11	.16	.3	41	.60	3.6	11	.03	10.7	41	.47	21.6	11	.90	36.3	41	.34	54.8
12	.17	.3	42	.61	3.7	12	.05	11.0	42	.48	22.0	12	.92	36.9	42	.36	55.5
13	.19	.4	43	.63	3.9	13	.06	11.3	43	.50	22.4	13	.93	37.4	43	.37	56.2
14	.20	.4	44	.64	4.1	14	.08	11.6	44	.51	22.9	14	.95	38.0	44	.38	56.9
0 15	0.22	0.5	0 45	0.65	4.3	1 15	1.09	11.9	1 45	1.53	23.3	2 15	1.96	38.6	2 45	2.40	57.6
16	.23	.5	46	.67	4.5	16	.10	12.2	46	.54	23.8	16	.98	39.1	46	.41	58.3
17	.25	.6	47	.68	4.7	17	.12	12.5	47	.56	24.2	17	.99	39.7	47	.43	59.0
18	.26	.7	48	.70	4.9	18	.13	12.9	48	.57	24.7	18	2.01	40.3	48	.44	59.7
19	.28	.8	49	.71	5.1	19	.15	13.2	49	.59	25.1	19	.02	40.9	49	.46	60.4
0 20	0.29	0.8	0 50	0.73	5.3	1 20	1.16	13.5	1 50	1.60	25.6	2 20	2.04	41.5	2 50	2.47	61.1
21	.31	.9	51	.74	5.5	21	.18	13.9	51	.61	26.1	21	.05	42.1	51	.49	61.8
22	.32	1.0	52	.76	5.7	22	.19	14.2	52	.63	26.5	22	.06	42.7	52	.50	62.6
23	.33	1.1	53	.77	5.9	23	.21	14.6	53	.64	27.0	23	.08	43.3	53	.52	63.3
24	.35	1.2	54	.79	6.2	24	.22	14.9	54	.66	27.5	24	.09	43.9	54	.53	64.0
0 25	0.36	1.3	0 55	0.80	6.4	1 25	1.24	15.3	1 55	1.67	28.0	2 25	2.11	44.5	2 55	2.54	64.8
26	.38	1.4	56	.81	6.6	26	.25	15.6	56	.69	28.5	26	.12	45.1	56	.56	65.5
27	.39	1.5	57	.83	6.9	27	.27	16.0	57	.70	29.0	27	.14	45.7	57	.57	66.3
28	.41	1.7	58	.84	7.1	28	.28	16.4	58	.72	29.5	28	.15	46.3	58	.59	67.0
29	.42	1.8	59	.86	7.4	29	.29	16.8	59	.73	30.0	29	.17	47.0	59	.60	67.8
0 30	0.44	1.9	1 00	0.87	7.6	1 30	1.31	17.1	2 00	1.74	30.5	2 30	2.18	47.6	3 00	2.62	68.5

Grade corrections for 50-meter tape lengths

[For differences of elevation up to 7.50 meters. Cor. $= -0.01h^2 - 0.000001h^4$]

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
0.00	0.000	0.0	.65	2.133	4.2	1.30	4.265	16.9	1.95	6.398	38.0
.01	.033	.0	.66	2.165	4.4	1.31	4.298	17.2	1.96	6.430	38.4
.02	.066	.0	.67	2.198	4.5	1.32	4.331	17.4	1.97	6.463	38.8
.03	.098	.0	.68	2.231	4.6	1.33	4.364	17.7	1.98	6.496	39.2
.04	.131	.0	.69	2.264	4.8	1.34	4.396	18.0	1.99	6.529	39.6
.05	.164	.0	.70	2.297	4.9	1.35	4.429	18.2	2.00	6.562	40.0
.06	.197	.0	.71	2.329	5.0	1.36	4.462	18.5	2.01	6.594	40.4
.07	.230	.0	.72	2.362	5.2	1.37	4.495	18.8	2.02	6.627	40.8
.08	.262	.1	.73	2.395	5.3	1.38	4.528	19.0	2.03	6.660	41.2
.09	.295	.1	.74	2.428	5.5	1.39	4.560	19.3	2.04	6.693	41.6
.10	.328	.1	.75	2.461	5.6	1.40	4.593	19.6	2.05	6.726	42.0
.11	.361	.1	.76	2.493	5.8	1.41	4.626	19.9	2.06	6.759	42.5
.12	.394	.1	.77	2.526	5.9	1.42	4.659	20.2	2.07	6.791	42.9
.13	.427	.2	.78	2.559	6.1	1.43	4.692	20.4	2.08	6.824	43.3
.14	.459	.2	.79	2.592	6.2	1.44	4.724	20.7	2.09	6.857	43.7
.15	.492	.2	.80	2.625	6.4	1.45	4.757	21.0	2.10	6.890	44.1
.16	.525	.3	.81	2.657	6.6	1.46	4.790	21.3	2.11	6.923	44.5
.17	.558	.3	.82	2.690	6.7	1.47	4.823	21.6	2.12	6.955	45.0
.18	.591	.3	.83	2.723	6.9	1.48	4.856	21.9	2.13	6.988	45.4
.19	.623	.4	.84	2.756	7.1	1.49	4.888	22.2	2.14	7.021	45.8
.20	.656	.4	.85	2.789	7.2	1.50	4.921	22.5	2.15	7.054	46.2
.21	.689	.4	.86	2.822	7.4	1.51	4.954	22.8	2.16	7.087	46.7
.22	.722	.5	.87	2.854	7.6	1.52	4.987	23.1	2.17	7.119	47.1
.23	.755	.5	.88	2.887	7.7	1.53	5.020	23.4	2.18	7.152	47.5
.24	.787	.6	.89	2.920	7.9	1.54	5.052	23.7	2.19	7.185	48.0
.25	.820	.6	.90	2.953	8.1	1.55	5.085	24.0	2.20	7.218	48.4
.26	.853	.7	.91	2.986	8.3	1.56	5.118	24.3	2.21	7.251	48.9
.27	.886	.7	.92	3.018	8.5	1.57	5.151	24.6	2.22	7.283	49.3
.28	.919	.8	.93	3.051	8.6	1.58	5.184	25.0	2.23	7.316	49.8
.29	.951	.8	.94	3.084	8.8	1.59	5.217	25.3	2.24	7.349	50.2
.30	.984	.9	.95	3.117	9.0	1.60	5.249	25.6	2.25	7.382	50.7
.31	1.017	1.0	.96	3.150	9.2	1.61	5.282	25.9	2.26	7.415	51.1
.32	1.050	1.0	.97	3.182	9.4	1.62	5.315	26.2	2.27	7.447	51.6
.33	1.083	1.1	.98	3.215	9.6	1.63	5.348	26.6	2.28	7.480	52.0
.34	1.115	1.2	.99	3.248	9.8	1.64	5.381	26.9	2.29	7.513	52.5
.35	1.148	1.2	1.00	3.281	10.0	1.65	5.413	27.2	2.30	7.546	52.9
.36	1.181	1.3	1.01	3.314	10.2	1.66	5.446	27.6	2.31	7.579	53.4
.37	1.214	1.4	1.02	3.346	10.4	1.67	5.479	27.9	2.32	7.612	53.9
.38	1.247	1.4	1.03	3.379	10.6	1.68	5.512	28.2	2.33	7.644	54.3
.39	1.280	1.5	1.04	3.412	10.8	1.69	5.545	28.6	2.34	7.677	54.8
.40	1.312	1.6	1.05	3.445	11.0	1.70	5.577	28.9	2.35	7.710	55.3
.41	1.345	1.7	1.06	3.478	11.2	1.71	5.610	29.2	2.36	7.743	55.7
.42	1.378	1.8	1.07	3.510	11.4	1.72	5.643	29.6	2.37	7.776	56.2
.43	1.411	1.8	1.08	3.543	11.7	1.73	5.676	29.9	2.38	7.808	56.7
.44	1.444	1.9	1.09	3.576	11.9	1.74	5.709	30.3	2.39	7.841	57.2
.45	1.476	2.0	1.10	3.609	12.1	1.75	5.741	30.6	2.40	7.874	57.6
.46	1.509	2.1	1.11	3.642	12.3	1.76	5.774	31.0	2.41	7.907	58.1
.47	1.542	2.2	1.12	3.675	12.5	1.77	5.807	31.3	2.42	7.940	58.6
.48	1.575	2.3	1.13	3.707	12.8	1.78	5.840	31.7	2.43	7.972	59.1
.49	1.608	2.4	1.14	3.740	13.0	1.79	5.873	32.0	2.44	8.005	59.6
.50	1.640	2.5	1.15	3.773	13.2	1.80	5.906	32.4	2.45	8.038	60.1
.51	1.673	2.6	1.16	3.806	13.5	1.81	5.938	32.8	2.46	8.071	60.6
.52	1.706	2.7	1.17	3.839	13.7	1.82	5.971	33.1	2.47	8.104	61.0
.53	1.739	2.8	1.18	3.871	13.9	1.83	6.004	33.5	2.48	8.136	61.5
.54	1.772	2.9	1.19	3.904	14.2	1.84	6.037	33.9	2.49	8.169	62.0
.55	1.804	3.0	1.20	3.937	14.4	1.85	6.070	34.2	2.50	8.202	62.5
.56	1.837	3.1	1.21	3.970	14.6	1.86	6.102	34.6	2.51	8.235	63.0
.57	1.870	3.2	1.22	4.003	14.9	1.87	6.135	35.0	2.52	8.268	63.5
.58	1.903	3.4	1.23	4.035	15.1	1.88	6.168	35.3	2.53	8.301	64.0
.59	1.936	3.5	1.24	4.068	15.4	1.89	6.201	35.7	2.54	8.333	64.6
.60	1.968	3.6	1.25	4.101	15.6	1.90	6.234	36.1	2.55	8.366	65.1
.61	2.001	3.7	1.26	4.134	15.9	1.91	6.266	36.5	2.56	8.399	65.6
.62	2.034	3.8	1.27	4.167	16.1	1.92	6.299	36.9	2.57	8.432	66.1
.63	2.067	4.0	1.28	4.199	16.4	1.93	6.332	37.2	2.58	8.465	66.6
.64	2.100	4.1	1.29	4.232	16.6	1.94	6.365	37.6	2.59	8.497	67.1

Grade corrections for 50-meter tape lengths—Continued

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
2.60	8.530	67.6	3.25	10.663	105.7	3.90	12.795	152.3	4.55	14.928	207.5
2.61	8.563	68.2	3.26	10.696	106.4	3.91	12.828	153.1	4.56	14.961	208.4
2.62	8.596	68.7	3.27	10.728	107.0	3.92	12.861	153.9	4.57	14.993	209.3
2.63	8.629	69.2	3.28	10.761	107.7	3.93	12.894	154.7	4.58	15.026	210.2
2.64	8.661	69.7	3.29	10.794	108.4	3.94	12.926	155.5	4.59	15.059	211.1
2.65	8.694	70.3	3.30	10.827	109.0	3.95	12.959	156.3	4.60	15.092	212.0
2.66	8.727	70.8	3.31	10.860	109.7	3.96	12.992	157.1	4.61	15.125	213.0
2.67	8.760	71.3	3.32	10.892	110.3	3.97	13.025	157.9	4.62	15.157	213.9
2.68	8.793	71.9	3.33	10.925	111.0	3.98	13.058	158.7	4.63	15.190	214.8
2.69	8.825	72.4	3.34	10.958	111.7	3.99	13.091	159.5	4.64	15.223	215.8
2.70	8.858	73.0	3.35	10.991	112.4	4.00	13.123	160.3	4.65	15.256	216.7
2.71	8.891	73.5	3.36	11.024	113.0	4.01	13.156	161.1	4.66	15.289	217.6
2.72	8.924	74.0	3.37	11.056	113.7	4.02	13.189	161.9	4.67	15.321	218.6
2.73	8.957	74.6	3.38	11.089	114.4	4.03	13.222	162.7	4.68	15.354	219.5
2.74	8.989	75.1	3.39	11.122	115.1	4.04	13.255	163.5	4.69	15.387	220.4
2.75	9.022	75.7	3.40	11.155	115.7	4.05	13.287	164.3	4.70	15.420	221.4
2.76	9.055	76.2	3.41	11.188	116.4	4.06	13.320	165.1	4.71	15.453	222.3
2.77	9.088	76.8	3.42	11.220	117.1	4.07	13.353	165.9	4.72	15.486	223.3
2.78	9.121	77.3	3.43	11.253	117.8	4.08	13.386	166.7	4.73	15.518	224.2
2.79	9.154	77.9	3.44	11.286	118.5	4.09	13.419	167.6	4.74	15.551	225.2
2.80	9.186	78.5	3.45	11.319	119.2	4.10	13.451	168.4	4.75	15.584	226.1
2.81	9.219	79.0	3.46	11.352	119.9	4.11	13.484	169.2	4.76	15.617	227.1
2.82	9.252	79.6	3.47	11.384	120.6	4.12	13.517	170.0	4.77	15.650	228.0
2.83	9.285	80.2	3.48	11.417	121.3	4.13	13.550	170.9	4.78	15.682	229.0
2.84	9.318	80.7	3.49	11.450	122.0	4.14	13.583	171.7	4.79	15.715	230.0
2.85	9.350	81.3	3.50	11.483	122.7	4.15	13.615	172.5	4.80	15.748	230.9
2.86	9.383	81.9	3.51	11.516	123.4	4.16	13.648	173.4	4.81	15.781	231.9
2.87	9.416	82.4	3.52	11.549	124.1	4.17	13.681	174.2	4.82	15.814	232.9
2.88	9.449	83.0	3.53	11.581	124.8	4.18	13.714	175.0	4.83	15.846	233.8
2.89	9.482	83.6	3.54	11.614	125.5	4.19	13.747	175.9	4.84	15.879	234.8
2.90	9.514	84.2	3.55	11.647	126.2	4.20	13.780	176.7	4.85	15.912	235.8
2.91	9.547	84.8	3.56	11.680	126.9	4.21	13.812	177.6	4.86	15.945	236.8
2.92	9.580	85.3	3.57	11.713	127.6	4.22	13.845	178.4	4.87	15.978	237.7
2.93	9.613	85.9	3.58	11.745	128.3	4.23	13.878	179.2	4.88	16.010	238.7
2.94	9.646	86.5	3.59	11.778	129.0	4.24	13.911	180.1	4.89	16.043	239.7
2.95	9.678	87.1	3.60	11.811	129.8	4.25	13.944	181.0	4.90	16.076	240.7
2.96	9.711	87.7	3.61	11.844	130.5	4.26	13.976	181.8	4.91	16.109	241.7
2.97	9.744	88.3	3.62	11.877	131.2	4.27	14.009	182.7	4.92	16.142	242.7
2.98	9.777	88.9	3.63	11.909	131.9	4.28	14.042	183.5	4.93	16.175	243.6
2.99	9.810	89.5	3.64	11.942	132.7	4.29	14.075	184.4	4.94	16.207	244.6
3.00	9.842	90.1	3.65	11.975	133.4	4.30	14.108	185.2	4.95	16.240	245.6
3.01	9.875	90.7	3.66	12.008	134.1	4.31	14.140	186.1	4.96	16.273	246.6
3.02	9.908	91.3	3.67	12.041	134.9	4.32	14.173	187.0	4.97	16.306	247.6
3.03	9.941	91.9	3.68	12.073	135.6	4.33	14.206	187.8	4.98	16.339	248.6
3.04	9.974	92.5	3.69	12.106	136.3	4.34	14.239	188.7	4.99	16.371	249.6
3.05	10.007	93.1	3.70	12.139	137.1	4.35	14.272	189.6	5.00	16.404	250.6
3.06	10.039	93.7	3.71	12.172	137.8	4.36	14.304	190.5	5.01	16.437	251.6
3.07	10.072	94.3	3.72	12.205	138.6	4.37	14.337	191.3	5.02	16.470	252.6
3.08	10.105	95.0	3.73	12.238	139.3	4.38	14.370	192.2	5.03	16.503	253.6
3.09	10.138	95.6	3.74	12.270	140.1	4.39	14.403	193.1	5.04	16.535	254.7
3.10	10.171	96.2	3.75	12.303	140.8	4.40	14.436	194.0	5.05	16.568	255.7
3.11	10.203	96.8	3.76	12.336	141.6	4.41	14.468	194.9	5.06	16.601	256.7
3.12	10.236	97.4	3.77	12.369	142.3	4.42	14.501	195.7	5.07	16.634	257.7
3.13	10.269	98.1	3.78	12.402	143.1	4.43	14.534	196.6	5.08	16.667	258.7
3.14	10.302	98.7	3.79	12.434	143.8	4.44	14.567	197.5	5.09	16.699	259.8
3.15	10.335	99.3	3.80	12.467	144.6	4.45	14.600	198.4	5.10	16.732	260.8
3.16	10.367	100.0	3.81	12.500	145.4	4.46	14.633	199.3	5.11	16.765	261.8
3.17	10.400	100.6	3.82	12.533	146.1	4.47	14.665	200.2	5.12	16.798	262.8
3.18	10.433	101.2	3.83	12.566	146.9	4.48	14.698	201.1	5.13	16.831	263.9
3.19	10.466	101.9	3.84	12.598	147.7	4.49	14.731	202.0	5.14	16.863	264.9
3.20	10.499	102.5	3.85	12.631	148.4	4.50	14.764	202.9	5.15	16.896	265.9
3.21	10.531	103.1	3.86	12.664	149.2	4.51	14.797	203.8	5.16	16.929	267.0
3.22	10.564	103.8	3.87	12.697	150.0	4.52	14.829	204.7	5.17	16.962	268.0
3.23	10.597	104.4	3.88	12.730	150.8	4.53	14.862	205.6	5.18	16.995	269.0
3.24	10.630	105.1	3.89	12.762	151.6	4.54	14.895	206.5	5.19	17.028	270.1

Grade corrections for 50-meter tape lengths—Continued

Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction	Difference of elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
5.20	17.060	271.1	5.80	19.029	337.5	6.40	20.997	411.3	7.00	22.966	492.4
5.21	17.093	272.2	5.81	19.062	338.7	6.41	21.030	412.6	7.01	22.999	493.8
5.22	17.126	273.2	5.82	19.094	339.9	6.42	21.063	413.9	7.02	23.031	495.2
5.23	17.159	274.3	5.83	19.127	341.0	6.43	21.096	415.2	7.03	23.064	496.6
5.24	17.192	275.3	5.84	19.160	342.2	6.44	21.129	416.5	7.04	23.097	498.1
5.25	17.224	276.4	5.85	19.193	343.4	6.45	21.161	417.8	7.05	23.130	499.5
5.26	17.257	277.4	5.86	19.226	344.6	6.46	21.194	419.1	7.06	23.163	500.9
5.27	17.290	278.5	5.87	19.258	345.8	6.47	21.227	420.4	7.07	23.195	502.3
5.28	17.323	279.6	5.88	19.291	346.9	6.48	21.260	421.7	7.08	23.228	503.8
5.29	17.356	280.6	5.89	19.324	348.1	6.49	21.293	423.0	7.09	23.261	505.2
5.30	17.388	281.7	5.90	19.357	349.3	6.50	21.325	424.3	7.10	23.294	506.6
5.31	17.421	282.8	5.91	19.390	350.5	6.51	21.358	425.6	7.11	23.327	508.1
5.32	17.454	283.8	5.92	19.423	351.7	6.52	21.391	426.9	7.12	23.360	509.5
5.33	17.487	284.9	5.93	19.455	352.9	6.53	21.424	428.2	7.13	23.392	511.0
5.34	17.520	286.0	5.94	19.488	354.1	6.54	21.457	429.5	7.14	23.425	512.4
5.35	17.552	287.0	5.95	19.521	355.3	6.55	21.489	430.9	7.15	23.458	513.8
5.36	17.585	288.1	5.96	19.554	356.5	6.56	21.522	432.2	7.16	23.491	515.3
5.37	17.618	289.2	5.97	19.587	357.7	6.57	21.555	433.5	7.17	23.524	516.7
5.38	17.651	290.3	5.98	19.619	358.9	6.58	21.588	434.8	7.18	23.556	518.2
5.39	17.684	291.4	5.99	19.652	360.1	6.59	21.621	436.2	7.19	23.589	519.6
5.40	17.716	292.5	6.00	19.685	361.3	6.60	21.654	437.5	7.20	23.622	521.1
5.41	17.749	293.5	6.01	19.718	362.5	6.61	21.686	438.8	7.21	23.655	522.5
5.42	17.782	294.6	6.02	19.751	363.7	6.62	21.719	440.2	7.22	23.688	524.0
5.43	17.815	295.7	6.03	19.783	364.9	6.63	21.752	441.5	7.23	23.720	525.5
5.44	17.848	296.8	6.04	19.816	366.1	6.64	21.785	442.8	7.24	23.753	526.9
5.45	17.881	297.9	6.05	19.849	367.4	6.65	21.818	444.2	7.25	23.786	528.4
5.46	17.913	299.0	6.06	19.882	368.6	6.66	21.850	445.5	7.26	23.819	529.9
5.47	17.946	300.1	6.07	19.915	369.8	6.67	21.883	446.9	7.27	23.852	531.3
5.48	17.979	301.2	6.08	19.947	371.0	6.68	21.916	448.2	7.28	23.884	532.8
5.49	18.012	302.3	6.09	19.980	372.3	6.69	21.949	449.6	7.29	23.917	534.3
5.50	18.045	303.4	6.10	20.013	373.5	6.70	21.982	450.9	7.30	23.950	535.7
5.51	18.077	304.5	6.11	20.046	374.7	6.71	22.014	452.3	7.31	23.983	537.2
5.52	18.110	305.6	6.12	20.079	375.9	6.72	22.047	453.6	7.32	24.016	538.7
5.53	18.143	306.7	6.13	20.112	377.2	6.73	22.080	455.0	7.33	24.049	540.2
5.54	18.176	307.9	6.14	20.144	378.4	6.74	22.113	456.3	7.34	24.081	541.7
5.55	18.209	309.0	6.15	20.177	379.7	6.75	22.146	457.7	7.35	24.114	543.1
5.56	18.241	310.1	6.16	20.210	380.9	6.76	22.178	459.1	7.36	24.147	544.6
5.57	18.274	311.2	6.17	20.243	382.1	6.77	22.211	460.4	7.37	24.180	546.1
5.58	18.307	312.3	6.18	20.276	383.4	6.78	22.244	461.8	7.38	24.213	547.6
5.59	18.340	313.5	6.19	20.308	384.6	6.79	22.277	463.2	7.39	24.245	549.1
5.60	18.373	314.6	6.20	20.341	385.9	6.80	22.310	464.5	7.40	24.278	550.6
5.61	18.405	315.7	6.21	20.374	387.1	6.81	22.342	465.9	7.41	24.311	552.1
5.62	18.438	316.8	6.22	20.407	388.4	6.82	22.375	467.3	7.42	24.344	553.6
5.63	18.471	318.0	6.23	20.440	389.6	6.83	22.408	468.7	7.43	24.377	555.1
5.64	18.504	319.1	6.24	20.472	390.9	6.84	22.441	470.0	7.44	24.409	556.6
5.65	18.537	320.2	6.25	20.505	392.2	6.85	22.474	471.4	7.45	24.442	558.1
5.66	18.570	321.4	6.26	20.538	393.4	6.86	22.507	472.8	7.46	24.475	559.6
5.67	18.602	322.5	6.27	20.571	394.7	6.87	22.539	474.2	7.47	24.508	561.1
5.68	18.635	323.7	6.28	20.604	395.9	6.88	22.572	475.6	7.48	24.541	562.6
5.69	18.668	324.8	6.29	20.636	397.2	6.89	22.605	477.0	7.49	24.573	564.1
5.70	18.701	326.0	6.30	20.669	398.5	6.90	22.638	478.4	7.50	24.606	565.7
5.71	18.734	327.1	6.31	20.702	399.7	6.91	22.671	479.8			
5.72	18.766	328.3	6.32	20.735	401.0	6.92	22.703	481.2			
5.73	18.799	329.4	6.33	20.768	402.3	6.93	22.736	482.6			
5.74	18.832	330.6	6.34	20.800	403.6	6.94	22.769	484.0			
5.75	18.865	331.7	6.35	20.833	404.9	6.95	22.802	485.4			
5.76	18.898	332.9	6.36	20.866	406.1	6.96	22.835	486.8			
5.77	18.930	334.0	6.37	20.899	407.4	6.97	22.867	488.2			
5.78	18.963	335.2	6.38	20.932	408.7	6.98	22.900	489.6			
5.79	18.996	336.4	6.39	20.965	410.0	6.99	22.933	491.0			

Spherical excess.—The spherical excess is computed by the formula:

$$\epsilon = \frac{a_1 b_1 \sin C_1 (1 - e^2 \sin^2 \phi)^2}{2a^2 (1 - e^2) \sin 1''} = a_1 b_1 \sin C_1 \times m.$$

In this formula ϵ is the spherical excess; a_1 b_1 and C_1 are two sides and the included angle, respectively, of the corresponding triangle;

e^2 is the square of the eccentricity, and a the major semiaxis of the spheroid of reference; and ϕ is the mean latitude of the three vertices of the triangle. That part of the above expression which depends only on the latitude and the dimensions of the spheroid may be designated by a single letter, m, as shown. In the following table the logarithms of m are given with the latitude as an argument.

The above formula gives the spherical excess too small by one one-hundredth of a second for an equilateral triangle with 200-kilometer sides, or for a nonequilateral triangle of the same area. For an equilateral triangle of 100-kilometer sides, or an equivalent nonequilateral triangle, the excess as given by this formula is too small by less than one one-thousandth of a second.

In cases where a more accurate value of the spherical excess is required the formulæ given on page 51 of Special Publication No. 4, The Transcontinental Triangulation, may be used. These formulæ give a slightly unequal distribution of the spherical excess among the three angles of the triangle.

Table of log m

[Computed for the Clarke spheroid of 1866 as expressed in meters]

Latitude	log m	Latitude	log m	Latitude	log m	Latitude	log m
° ′		° ′		° ′		° ′	
0 00	1.40695 −10	20 00	1.40626 −10	40 00	1.40452 −10	60 00	1.40253 −10
0 30	695 −10	20 30	623 −10	40 30	446 −10	60 30	249 −10
1 00	695 −10	21 00	619 −10	41 00	441 −10	61 00	244 −10
1 30	694 −10	21 30	616 −10	41 30	436 −10	61 30	240 −10
2 00	694 −10	22 00	612 −10	42 00	431 −10	62 00	235 −10
2 30	694 −10	22 30	608 −10	42 30	426 −10	62 30	231 −10
3 00	693 −10	23 00	605 −10	43 00	421 −10	63 00	227 −10
3 30	693 −10	23 30	601 −10	43 30	416 −10	63 30	223 −10
4 00	692 −10	24 00	597 −10	44 00	411 −10	64 00	219 −10
4 30	691 −10	24 30	594 −10	44 30	406 −10	64 30	215 −10
5 00	690 −10	25 00	590 −10	45 00	400 −10	65 00	210 −10
5 30	689 −10	25 30	586 −10	45 30	395 −10	65 30	207 −10
6 00	688 −10	26 00	582 −10	46 00	390 −10	66 00	203 −10
6 30	687 −10	26 30	578 −10	46 30	385 −10	66 30	199 −10
7 00	686 −10	27 00	573 −10	47 00	380 −10	67 00	195 −10
7 30	685 −10	27 30	569 −10	47 30	375 −10	67 30	192 −10
8 00	683 −10	28 00	565 −10	48 00	369 −10	68 00	188 −10
8 30	682 −10	28 30	560 −10	48 30	364 −10	68 30	185 −10
9 00	680 −10	29 00	556 −10	49 00	359 −10	69 00	181 −10
9 30	679 −10	29 30	552 −10	49 30	354 −10	69 30	178 −10
10 00	677 −10	30 00	548 −10	50 00	349 −10	70 00	174 −10
10 30	675 −10	30 30	544 −10	50 30	344 −10	70 30	171 −10
11 00	673 −10	31 00	539 −10	51 00	339 −10	71 00	168 −10
11 30	671 −10	31 30	534 −10	51 30	334 −10	71 30	164 −10
12 00	669 −10	32 00	530 −10	52 00	329 −10	72 00	1.40161 −10
12 30	667 −10	32 30	525 −10	52 30	324 −10		
13 00	665 −10	33 00	520 −10	53 00	319 −10		
13 30	663 −10	33 30	516 −10	53 30	314 −10		
14 00	660 −10	34 00	511 −10	54 00	309 −10		
14 30	658 −10	34 30	506 −10	54 30	304 −10		
15 00	655 −10	35 00	501 −10	55 00	299 −10		
15 30	653 −10	35 30	496 −10	55 30	295 −10		
16 00	650 −10	36 00	491 −10	56 00	290 −10		
16 30	647 −10	36 30	486 −10	56 30	285 −10		
17 00	644 −10	37 00	482 −10	57 00	280 −10		
17 30	642 −10	37 30	477 −10	57 30	276 −10		
18 00	639 −10	38 00	472 −10	58 00	271 −10		
18 30	636 −10	38 30	467 −10	58 30	266 −10		
19 00	632 −10	39 00	462 −10	59 00	262 −10		
19 30	1.40629 −10	39 30	1.40457 −10	59 30	1.40257 −10		

Logarithms of radii of curvature of the earth's surface (in meters)

[Based upon Clarke's spheroid of 1866 as expressed in meters].

Azimuth (degrees)	Latitude								
	0°	1°	2°	3°	4°	5°	6°	7°	8°
0	6.80175	6.80175	6.80175	6.80176	6.80177	6.80178	6.80180	6.80181	6.80183
5	177	177	178	178	179	180	182	184	186
10	184	184	184	185	186	187	188	190	192
15	195	195	195	196	197	198	199	201	203
20	209	209	210	210	211	212	214	215	217
25	227	228	228	228	229	230	232	233	235
30	248	249	249	250	250	251	252	254	256
35	272	272	272	273	273	274	276	277	278
40	296	297	297	297	298	299	300	301	303
45	322	322	322	323	324	324	325	326	328
50	348	348	348	348	349	350	351	352	353
55	373	373	373	373	374	374	375	376	377
60	396	396	396	396	397	398	398	399	400
65	417	417	417	418	418	418	419	420	421
70	435	435	436	436	436	437	437	438	439
75	450	450	450	450	451	451	452	452	453
80	461	461	461	461	462	462	463	463	464
85	468	468	468	468	468	469	469	470	470
90	470	470	470	470	471	471	472	472	473

Azimuth (degrees)	Latitude								
	8°	9°	10°	11°	12°	13°	14°	15°	16°
0	6.80183	6.80186	6.80188	6.80191	6.80194	6.80197	6.80201	6.80204	6.80208
5	186	188	190	193	196	199	203	206	210
10	192	194	197	200	202	206	209	213	217
15	203	205	207	210	213	216	219	223	227
20	217	219	222	224	227	230	233	236	240
25	235	237	239	242	244	247	250	254	257
30	256	257	260	262	264	267	270	273	276
35	278	280	282	284	287	289	292	295	298
40	303	304	306	308	310	313	315	318	321
45	328	329	331	333	335	337	339	342	344
50	353	354	356	358	359	361	364	366	368
55	377	379	380	382	383	385	387	389	391
60	400	401	403	404	406	407	409	411	413
65	421	422	423	424	426	427	429	430	432
70	439	440	441	442	443	444	446	447	449
75	453	454	455	456	457	458	460	461	463
80	464	465	466	467	468	469	470	471	473
85	470	471	472	473	474	475	476	478	479
90	473	474	474	475	476	477	478	480	481

Azimuth (degrees)	Latitude								
	16°	17°	18°	19°	20°	21°	22°	23°	24°
0	6.80208	6.80213	6.80217	6.80222	6.80226	6.80232	6.80237	6.80242	6.80248
5	210	215	219	224	228	234	239	244	250
10	217	221	225	230	234	239	244	250	255
15	227	231	235	239	244	249	254	259	264
20	240	244	248	252	257	262	266	271	277
25	257	261	265	269	273	277	282	287	292
30	276	280	284	287	292	296	300	305	309
35	298	301	305	308	312	316	320	324	329
40	321	324	327	330	334	338	341	345	350
45	344	347	350	353	357	360	364	367	371
50	368	371	373	376	379	382	386	389	392
55	391	394	396	398	401	404	407	410	413
60	413	415	417	419	422	424	427	430	432
65	432	434	436	438	440	443	445	448	450
70	449	451	453	454	456	459	461	463	465
75	463	464	466	468	470	472	473	476	478
80	473	474	476	478	479	481	483	485	487
85	479	480	482	483	485	487	489	490	492
90	481	482	484	485	487	489	490	492	494

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	24°	25°	26°	27°	28°	29°	30°	31°	32°
0	6.80248	6.80254	6.80260	6.80266	6.80272	6.80279	6.80285	6.80292	6.80299
5	250	256	262	268	274	280	287	294	300
10	255	261	267	273	279	285	292	298	305
15	264	270	276	282	288	294	300	306	313
20	277	282	288	293	299	305	311	317	324
25	292	297	302	308	313	319	325	331	337
30	309	314	319	324	330	335	340	346	352
35	329	333	338	343	348	353	358	363	369
40	350	354	358	362	367	372	377	382	386
45	371	375	379	383	387	391	396	400	405
50	392	396	399	403	407	411	415	419	423
55	413	416	420	423	426	430	434	437	441
60	432	435	438	442	445	448	451	455	458
65	450	453	455	458	461	464	467	470	473
70	465	468	470	473	475	478	481	484	486
75	478	480	482	484	487	489	492	494	497
80	487	489	491	493	495	498	500	502	505
85	492	494	496	498	501	503	505	507	510
90	494	496	498	500	502	504	507	509	511

Azimuth (degrees)	Latitude								
	32°	33°	34°	35°	36°	37°	38°	39°	40°
0	6.80299	6.80306	6.80313	6.80320	6.80327	6.80335	6.80342	6.80350	6.80357
5	300	307	314	322	329	336	344	351	359
10	305	312	319	326	333	340	348	355	363
15	313	320	326	333	340	348	355	362	369
20	324	330	337	343	350	357	364	371	378
25	337	343	349	355	362	368	375	382	388
30	352	358	364	370	376	382	388	394	401
35	369	374	380	385	391	397	402	408	414
40	386	392	397	402	407	412	418	423	429
45	405	410	414	419	424	429	434	439	444
50	423	428	432	436	441	445	450	454	459
55	441	445	449	453	457	461	465	469	474
60	458	462	465	469	472	476	480	484	487
65	473	476	480	483	486	489	493	496	500
70	486	489	492	495	498	501	504	507	510
75	497	500	502	505	508	510	513	516	519
80	505	507	510	512	515	517	520	523	525
85	510	512	514	517	519	522	524	527	529
90	511	514	516	518	521	523	526	528	531

Azimuth (degrees)	Latitude								
	40°	41°	42°	43°	44°	45°	46°	47°	48°
0	6.80357	6.80365	6.80373	6.80380	6.80388	6.80396	6.80404	6.80411	6.80419
5	359	366	374	382	389	397	404	412	420
10	363	370	378	385	393	400	408	415	423
15	369	376	384	391	398	406	413	420	428
20	378	385	392	399	406	413	420	427	434
25	388	395	402	408	415	422	429	436	442
30	401	407	413	420	426	433	439	446	452
35	414	420	426	432	438	444	450	456	462
40	429	434	440	446	451	457	462	468	474
45	444	449	454	459	464	470	475	480	485
50	459	464	468	473	478	482	487	492	496
55	474	478	482	486	490	495	499	503	508
60	487	491	495	499	502	506	510	514	518
65	500	503	507	510	514	517	520	524	528
70	510	514	517	520	523	526	529	532	536
75	519	522	525	528	531	534	536	539	542
80	525	528	531	534	536	539	542	544	547
85	529	532	534	537	540	542	545	548	550
90	531	533	536	538	541	544	546	549	551

Logarithms of radii of curvature of the earth's surface (in meters)—Continued

Azimuth (degrees)	Latitude								
	48°	49°	50°	51°	52°	53°	54°	55°	56°
0	6.80419	6.80426	6.80434	6.80442	6.80449	6.80457	6.80464	6.80471	6.80479
5	420	428	435	443	450	458	465	472	479
10	423	430	438	445	453	460	467	474	481
15	428	435	442	450	457	464	471	478	485
20	434	441	448	455	462	469	476	483	489
25	442	449	456	463	469	476	482	489	495
30	452	458	465	471	477	484	490	496	502
35	462	468	474	480	486	492	498	503	509
40	474	479	485	490	496	501	506	512	517
45	485	490	495	500	505	510	515	520	525
50	496	501	506	510	515	520	524	528	533
55	508	512	516	520	524	528	533	537	541
60	518	522	526	530	533	537	541	544	548
65	528	531	534	538	541	545	548	551	555
70	536	539	542	545	548	551	554	557	560
75	542	545	548	551	554	557	559	562	565
80	547	550	553	555	558	561	563	566	568
85	550	553	555	558	560	563	566	568	570
90	551	554	556	559	561	564	566	569	571

Azimuth (degrees)	Latitude								
	56°	57°	58°	59°	60°	61°	62°	63°	64°
0	6.80479	6.80486	6.80493	6.80500	6.80506	6.80513	6.80520	6.80526	6.80532
5	479	486	493	500	507	514	520	526	532
10	481	488	495	502	509	515	522	528	534
15	485	492	498	505	511	518	524	530	536
20	489	496	502	509	515	521	527	533	539
25	495	501	508	514	520	526	531	537	542
30	502	508	514	519	525	530	536	541	546
35	509	515	520	525	531	536	541	546	551
40	517	522	527	532	537	542	546	551	556
45	525	530	534	539	543	548	552	556	560
50	533	537	542	546	550	554	558	562	565
55	541	545	548	552	556	560	563	567	570
60	548	552	555	558	562	565	568	572	575
65	555	558	561	564	567	570	573	576	579
70	560	563	566	569	572	574	577	580	582
75	565	568	570	573	575	578	580	583	585
80	568	571	573	576	578	580	583	585	587
85	570	573	575	578	580	582	584	586	588
90	571	574	576	578	580	583	585	587	589

Azimuth (degrees)	Latitude								
	64°	65°	66°	67°	68°	69°	70°	71°	72°
0	6.80532	6.80538	6.80544	6.80550	6.80555	6.80560	6.80565	6.80570	6.80575
5	532	538	514	550	555	561	566	570	575
10	534	540	545	551	556	562	566	571	576
15	536	542	547	553	558	563	568	572	577
20	539	544	550	555	560	565	570	574	578
25	542	548	553	558	562	567	572	576	580
30	546	551	556	561	565	570	574	578	582
35	551	556	560	564	569	573	577	581	584
40	556	560	564	568	572	576	580	583	587
45	560	564	568	572	576	579	583	586	589
50	565	569	573	576	579	583	586	589	592
55	570	574	577	580	583	586	589	591	594
60	575	578	581	584	586	589	591	594	596
65	579	582	584	587	589	592	594	596	6.80598
70	582	585	587	590	592	594	596	6.80598	6.80600
75	585	587	590	592	594	596	598	6.80600	601
80	587	589	591	593	595	597	6.80599	601	602
85	588	590	592	594	596	598	6.80600	601	603
90	589	591	593	595	597	598	600	602	603

Length of 1 degree of the meridian at different latitudes

Latitude (degrees)	Meters	Statute miles	Geographic miles (1′ of the equator)	Latitude (degrees)	Meters	Statute miles	Geographic miles (1′ of the equator)
0	110,568.5	68.703	59.594	45	111,132.1	69.054	59.898
1	110,568.8	68.704	59.594	46	111,151.9	69.067	59.908
2	110,569.8	68.705	59.595	47	111,171.6	69.079	59.919
3	110,571.5	68.706	59.596	48	111,191.3	69.091	59.929
4	110,573.9	68.707	59.597	49	111,210.9	69.103	59.940
5	110,577.0	68.709	59.598	50	111,230.5	69.115	59.951
6	110,580.7	68.711	59.600	51	111,249.9	69.127	59.961
7	110,585.1	68.714	59.603	52	111,269.2	69.139	59.972
8	110,590.2	68.717	59.606	53	111,288.3	69.151	59.982
9	110,595.9	68.721	59.609	54	111,307.3	69.163	59.992
10	110,602.3	68.725	59.612	55	111,326.0	69.175	60.002
11	110,609.3	68.729	59.616	56	111,344.5	69.186	60.012
12	110,617.0	68.734	59.620	57	111,362.7	69.198	60.022
13	110,625.3	68.739	59.625	58	111,380.7	69.209	60.032
14	110,634.2	68.745	59.629	59	111,398.4	69.220	60.041
15	110,643.7	68.751	59.634	60	111,415.7	69.230	60.051
16	110,653.8	68.757	59.640	61	111,432.7	69.241	60.060
17	110,664.5	68.763	59.646	62	111,449.4	69.251	60.069
18	110,675.7	68.770	59.652	63	111,465.7	69.261	60.077
19	110,687.5	68.778	59.658	64	111,481.5	69.271	60.086
20	110,699.9	68.786	59.665	65	111,497.0	69.281	60.094
21	110,712.8	68.794	59.672	66	111,512.0	69.290	60.102
22	110,726.2	68.802	59.679	67	111,526.5	69.299	60.110
23	110,740.1	68.810	59.686	68	111,540.5	69.308	60.118
24	110,754.4	68.819	59.694	69	111,554.1	69.316	60.125
25	110,769.2	68.829	59.702	70	111,567.1	69.324	60.132
26	110,784.5	68.838	59.710	71	111,579.7	69.332	60.139
27	110,800.2	68.848	59.719	72	111,591.6	69.340	60.145
28	110,816.3	68.858	59.727	73	111,603.0	69.347	60.151
29	110,832.8	68.868	59.736	74	111,613.9	69.354	60.157
30	110,849.7	68.879	59.745	75	111,624.1	69.360	60.163
31	110,866.9	68.889	59.755	76	111,633.8	69.366	60.168
32	110,884.4	68.900	59.764	77	111,642.8	69.372	60.173
33	110,902.3	68.911	59.774	78	111,651.2	69.377	60.177
34	110,920.4	68.923	59.784	79	111,659.0	69.382	60.182
35	110,938.8	68.934	59.794	80	111,666.2	69.386	60.186
36	110,957.4	68.946	59.804	81	111,672.6	69.390	60.189
37	110,976.3	68.957	59.814	82	111,678.5	69.394	60.192
38	110,995.3	68.969	59.824	83	111,683.6	69.397	60.195
39	111,014.5	68.981	59.834	84	111,688.1	69.400	60.197
40	111,033.9	68.993	59.845	85	111,691.9	69.402	60.199
41	111,053.4	69.005	59.855	86	111,695.0	69.404	60.201
42	111,073.0	69.017	59.866	87	111,697.4	69.405	60.202
43	111,092.6	69.029	59.876	88	111,699.2	69.407	60.203
44	111,112.4	69.042	59.887	89	111,700.2	69.407	60.204
45	111,132.1	69.054	59.898	90	111,700.6	69.407	60.204

Length of 1 degree of the parallel at different latitudes

Latitude (degrees)	Meters	Statute miles	Geographic miles (1′ of the Equator)	Latitude (degrees)	Meters	Statute miles	Geographic miles (1′ of the Equator)
0	111, 321. 9	69. 171	60. 000	45	78, 850. 0	48. 995	42. 498
1	111, 305. 2	69. 162	59. 991	46	77, 466. 5	48. 135	41. 753
2	111, 254. 6	69. 130	59. 964	47	76, 059. 2	47. 261	40. 994
3	111, 170. 4	69. 078	59. 918	48	74, 628. 5	46. 372	40. 223
4	111, 052. 6	69. 005	59. 855	49	73, 174. 9	45. 469	39. 440
5	110, 901. 2	68. 911	59. 773	50	71, 698. 9	44. 552	38. 644
6	110, 716. 2	68. 796	59. 673	51	70, 200. 8	43. 621	37. 837
7	110, 497. 7	68. 660	59. 556	52	68, 681. 1	42. 676	37. 018
8	110, 245. 8	68. 503	59. 420	53	67, 140. 3	41. 719	36. 187
9	109, 960. 5	68. 326	59. 266	54	65, 578. 8	40. 749	35. 346
10	109, 641. 9	68. 128	59. 095	55	63, 997. 1	39. 766	34. 493
11	109, 290. 1	67. 909	58. 905	56	62, 395. 7	38. 771	33. 630
12	108, 905. 2	67. 670	58. 697	57	60, 775. 1	37. 764	32. 757
13	108, 487. 3	67. 411	58. 472	58	59, 135. 7	36. 745	31. 873
14	108, 036. 6	67. 131	58. 229	59	57, 478. 1	35. 715	30. 979
15	107, 553. 1	66. 830	57. 969	60	55, 802. 8	34. 674	30. 076
16	107, 037. 0	66. 510	57. 690	61	54, 110. 2	33. 622	29. 164
17	106, 488. 5	66. 169	57. 395	62	52, 400. 9	32. 560	28. 243
18	105, 907. 7	65. 808	57. 082	63	50, 675. 4	31. 488	27. 313
19	105, 294. 7	65. 427	56. 751	64	48, 934. 3	30. 406	26. 374
20	104, 649. 8	65. 026	56. 404	65	47, 178. 0	29. 315	25. 428
21	103, 973. 2	64. 606	56. 039	66	45, 407. 1	28. 215	24. 473
22	103, 265. 0	64. 166	55. 657	67	43, 622. 2	27. 106	23. 511
23	102, 525. 4	63. 706	55. 259	68	41, 823. 8	25. 988	22. 542
24	101, 754. 6	63. 227	54. 843	69	40, 012. 4	24. 862	21. 566
25	100, 953. 0	62. 729	54. 411	70	38, 188. 6	23. 729	20. 583
26	100, 120. 6	62. 212	53. 963	71	36, 353. 0	22. 589	19. 593
27	99, 257. 8	61. 676	53. 498	72	34, 506. 2	21. 441	18. 598
28	98, 364. 8	61. 121	53. 016	73	32, 648. 6	20. 287	17. 597
29	97, 441. 9	60. 548	52. 519	74	30, 780. 9	19. 126	16. 590
30	96, 489. 3	59. 956	52. 006	75	28, 903. 6	17. 960	15. 578
31	95, 507. 3	59. 345	51. 476	76	27, 017. 4	16. 788	14. 562
32	94, 496. 2	58. 717	50. 931	77	25, 122. 8	15. 611	13. 541
33	93, 456. 3	58. 071	50. 371	78	23, 220. 4	14. 428	12. 515
34	92, 387. 9	57. 407	49. 795	79	21, 310. 8	13. 242	11. 486
35	91, 291. 3	56. 726	49. 204	80	19, 394. 6	12. 051	10. 453
36	90, 166. 8	56. 027	48. 598	81	17, 472. 4	10. 857	9. 417
37	89, 014. 8	55. 311	47. 977	82	15, 544. 7	9. 659	8. 378
38	87, 835. 6	54. 578	47. 341	83	13, 612. 2	8. 458	7. 337
39	86, 629. 6	53. 829	46. 691	84	11, 675. 5	7. 255	6. 293
40	85, 397. 0	53. 063	46. 027	85	9, 735. 1	6. 049	5. 247
41	84, 138. 4	52. 281	45. 349	86	7, 791. 7	4. 841	4. 200
42	82, 854. 0	51. 483	44. 656	87	5, 845. 9	3. 632	3. 151
43	81, 544. 2	50. 669	43. 950	88	3, 898. 3	2. 422	2. 101
44	80, 209. 4	49. 840	43. 231	89	1, 949. 4	1. 211	1. 051
45	78, 850. 0	48. 995	42. 498	90	0. 0	0. 000	0. 000

Fractional change in a number corresponding to a change in its logarithm

[Computed from the formula $\frac{\Delta N}{N} = \frac{\Delta \log N}{\mu}$, where μ = modulus of common logarithms = 0.43429448]

For $\Delta \log N$ = 1 unit in	$\frac{\Delta N}{N}$	For $\Delta \log N$ = 4 units in	$\frac{\Delta N}{N}$ (in round numbers)
4th place	$\frac{1}{4342}$	4th place	$\frac{1}{1000}$
5th place	$\frac{1}{43429}$	5th place	$\frac{1}{10000}$
6th place	$\frac{1}{434294}$	6th place	$\frac{1}{100000}$
7th place	$\frac{1}{4342945}$	7th place	$\frac{1}{1000000}$

CONVERSION TABLES

Lengths—Feet to meters (from 1 to 1000 units)

[Reduction factor: 1 foot=0.3048006096 meter]

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
0	0.0	50	15.24003	100	30.48006	150	45.72009	200	60.96012
1	0.30480	1	15.54483	1	30.78486	1	46.02489	1	61.26492
2	0.60960	2	15.84963	2	31.08966	2	46.32969	2	61.56972
3	0.91440	3	16.15443	3	31.39446	3	46.63449	3	61.87452
4	1.21920	4	16.45923	4	31.69926	4	46.93929	4	62.17932
5	1.52400	5	16.76403	5	32.00406	5	47.24409	5	62.48412
6	1.82880	6	17.06883	6	32.30886	6	47.54890	6	62.78893
7	2.13360	7	17.37363	7	32.61367	7	47.85370	7	63.09373
8	2.43840	8	17.67844	8	32.91847	8	48.15850	8	63.39853
9	2.74321	9	17.98324	9	33.22327	9	48.46330	9	63.70333
10	3.04801	60	18.28804	110	33.52807	160	48.76810	210	64.00813
1	3.35281	1	18.59284	1	33.83287	1	49.07290	1	64.31293
2	3.65761	2	18.89764	2	34.13767	2	49.37770	2	64.61773
3	3.96241	3	19.20244	3	34.44247	3	49.68250	3	64.92253
4	4.26721	4	19.50724	4	34.74727	4	49.98730	4	65.22733
5	4.57201	5	19.81204	5	35.05207	5	50.29210	5	65.53213
6	4.87681	6	20.11684	6	35.35687	6	50.59690	6	65.83693
7	5.18161	7	20.42164	7	35.66167	7	50.90170	7	66.14173
8	5.48641	8	20.72644	8	35.96647	8	51.20650	8	66.44653
9	5.79121	9	21.03124	9	36.27127	9	51.51130	9	66.75133
20	6.09601	70	21.33604	120	36.57607	170	51.81610	220	67.05613
1	6.40081	1	21.64084	1	36.88087	1	52.12090	1	67.36093
2	6.70561	2	21.94564	2	37.18567	2	52.42570	2	67.66574
3	7.01041	3	22.25044	3	37.49047	3	52.73051	3	67.97054
4	7.31521	4	22.55525	4	37.79528	4	53.03531	4	68.27534
5	7.62002	5	22.86005	5	38.10008	5	53.34011	5	68.58014
6	7.92482	6	23.16485	6	38.40488	6	53.64491	6	68.88494
7	8.22962	7	23.46965	7	38.70968	7	53.94971	7	69.18974
8	8.53442	8	23.77445	8	39.01448	8	54.25451	8	69.49454
9	8.83922	9	24.07925	9	39.31928	9	54.55931	9	69.79934
30	9.14402	80	24.38405	130	39.62408	180	54.86411	230	70.10414
1	9.44882	1	24.68885	1	39.92888	1	55.16891	1	70.40894
2	9.75362	2	24.99365	2	40.23368	2	55.47371	2	70.71374
3	10.05842	3	25.29845	3	40.53848	3	55.77851	3	71.01854
4	10.36322	4	25.60325	4	40.84328	4	56.08331	4	71.32334
5	10.66802	5	25.90805	5	41.14808	5	56.38811	5	71.62814
6	10.97282	6	26.21285	6	41.45288	6	56.69291	6	71.93294
7	11.27762	7	26.51765	7	41.75768	7	56.99771	7	72.23774
8	11.58242	8	26.82245	8	42.06248	8	57.30251	8	72.54255
9	11.88722	9	27.12725	9	42.36728	9	57.60732	9	72.84735
40	12.19202	90	27.43205	140	42.67209	190	57.91212	240	73.15215
1	12.49682	1	27.73686	1	42.97689	1	58.21692	1	73.45695
2	12.80163	2	28.04166	2	43.28169	2	58.52172	2	73.76175
3	13.10643	3	28.34646	3	43.58649	3	58.82652	3	74.06655
4	13.41123	4	28.65126	4	43.89129	4	59.13132	4	74.37135
5	13.71603	5	28.95606	5	44.19609	5	59.43612	5	74.67615
6	14.02083	6	29.26086	6	44.50089	6	59.74092	6	74.98095
7	14.32563	7	29.56566	7	44.80569	7	60.04572	7	75.28575
8	14.63043	8	29.87046	8	45.11049	8	60.35052	8	75.59055
9	14.93523	9	30.17526	9	45.41529	9	60.65532	9	75.89535

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
250	76.20015	300	91.44018	350	106.68021	400	121.92024	450	137.16027
1	76.50495	1	91.74498	1	106.98501	1	122.22504	1	137.46507
2	76.80975	2	92.04978	2	107.28981	2	122.52985	2	137.76988
3	77.11455	3	92.35458	3	107.59462	3	122.83465	3	138.07468
4	77.41935	4	92.65939	4	107.89942	4	123.13945	4	138.37948
5	77.72416	5	92.96419	5	108.20422	5	123.44425	5	138.68428
6	78.02896	6	93.26899	6	108.50902	6	123.74905	6	138.98908
7	78.33376	7	93.57379	7	108.81382	7	124.05385	7	139.29388
8	78.63856	8	93.87859	8	109.11862	8	124.35865	8	139.59868
9	78.94336	9	94.18339	9	109.42342	9	124.66345	9	139.90348
260	79.24816	310	94.48819	360	109.72822	410	124.96825	460	140.20828
1	79.55296	1	94.79299	1	110.03302	1	125.27305	1	140.51308
2	79.85776	2	95.09779	2	110.33782	2	125.57785	2	140.81788
3	80.16256	3	95.40259	3	110.64262	3	125.88265	3	141.12268
4	80.46736	4	95.70739	4	110.94742	4	126.18745	4	141.42748
5	80.77216	5	96.01219	5	111.25222	5	126.49225	5	141.73228
6	81.07696	6	96.31699	6	111.55702	6	126.79705	6	142.03708
7	81.38176	7	96.62179	7	111.86182	7	127.10185	7	142.34188
8	81.68656	8	96.92659	8	112.16662	8	127.40665	8	142.64669
9	81.99136	9	97.23139	9	112.47142	9	127.71146	9	142.95149
270	82.29616	320	97.53620	370	112.77623	420	128.01626	470	143.25629
1	82.60097	1	97.84100	1	113.08103	1	128.32106	1	143.56109
2	82.90577	2	98.14580	2	113.38583	2	128.62586	2	143.86589
3	83.21057	3	98.45060	3	113.69063	3	128.93066	3	144.17069
4	83.51537	4	98.75540	4	113.99543	4	129.23546	4	144.47549
5	83.82017	5	99.06020	5	114.30023	5	129.54026	5	144.78029
6	84.12497	6	99.36500	6	114.60503	6	129.84506	6	145.08509
7	84.42977	7	99.66980	7	114.90983	7	130.14986	7	145.38989
8	84.73457	8	99.97460	8	115.21463	8	130.45466	8	145.69469
9	85.03937	9	100.27940	9	115.51943	9	130.75946	9	145.99949
280	85.34417	330	100.58420	380	115.82423	430	131.06426	480	146.30429
1	85.64897	1	100.88900	1	116.12903	1	131.36906	1	146.60909
2	85.95377	2	101.19380	2	116.43383	2	131.67386	2	146.91389
3	86.25857	3	101.49860	3	116.73863	3	131.97866	3	147.21869
4	86.56337	4	101.80340	4	117.04343	4	132.28346	4	147.52350
5	86.86817	5	102.10820	5	117.34823	5	132.58827	5	147.82830
6	87.17297	6	102.41300	6	117.65304	6	132.89307	6	148.13310
7	87.47777	7	102.71781	7	117.95784	7	133.19787	7	148.43790
8	87.78258	8	103.02261	8	118.26264	8	133.50267	8	148.74270
9	88.08738	9	103.32741	9	118.56744	9	133.80747	9	149.04750
290	88.39218	340	103.63221	390	118.87224	440	134.11227	490	149.35230
1	88.69698	1	103.93701	1	119.17704	1	134.41707	1	149.65710
2	89.00178	2	104.24181	2	119.48184	2	134.72187	2	149.96190
3	89.30658	3	104.54661	3	119.78664	3	135.02667	3	150.26670
4	89.61138	4	104.85141	4	120.09144	4	135.33147	4	150.57150
5	89.91618	5	105.15621	5	120.39624	5	135.63627	5	150.87630
6	90.22098	6	105.46101	6	120.70104	6	135.94107	6	151.18110
7	90.52578	7	105.76581	7	121.00584	7	136.24587	7	151.48590
8	90.83058	8	106.07061	8	121.31064	8	136.55067	8	151.79070
9	91.13538	9	106.37541	9	121.61544	9	136.85547	9	152.09550

Lengths—Feet to meters (from 1 to 1000 units)—**Continued**

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
500	152.40030	550	167.64034	600	182.88037	650	198.12040	700	213.36043
1	152.70511	1	167.94514	1	183.18517	1	198.42520	1	213.66523
2	153.00991	2	168.24994	2	183.48997	2	198.73000	2	213.97003
3	153.31471	3	168.55474	3	183.79477	3	199.03480	3	214.27483
4	153.61951	4	168.85954	4	184.09957	4	199.33960	4	214.57963
5	153.92431	5	169.16434	5	184.40437	5	199.64440	5	214.88443
6	154.22911	6	169.46914	6	184.70917	6	199.94920	6	215.18923
7	154.53391	7	169.77394	7	185.01397	7	200.25400	7	215.49403
8	154.83871	8	170.07874	8	185.31877	8	200.55880	8	215.79883
9	155.14351	9	170.38354	9	185.62357	9	200.86360	9	216.10363
510	155.44831	560	170.68834	610	185.92837	660	201.16840	710	216.40843
1	155.75311	1	170.99314	1	186.23317	1	201.47320	1	216.71323
2	156.05791	2	171.29794	2	186.53797	2	201.77800	2	217.01803
3	156.36271	3	171.60274	3	186.84277	3	202.08280	3	217.32283
4	156.66751	4	171.90754	4	187.14757	4	202.38760	4	217.62764
5	156.97231	5	172.21234	5	187.45237	5	202.69241	5	217.93244
6	157.27711	6	172.51715	6	187.75718	6	202.99721	6	218.23724
7	157.58192	7	172.82195	7	188.06198	7	203.30201	7	218.54204
8	157.88672	8	173.12675	8	188.36678	8	203.60681	8	218.84684
9	158.19152	9	173.43155	9	188.67158	9	203.91161	9	219.15164
520	158.49632	570	173.73635	620	188.97638	670	204.21641	720	219.45644
1	158.80112	1	174.04115	1	189.28118	1	204.52121	1	219.76124
2	159.10592	2	174.34595	2	189.58598	2	204.82601	2	220.06604
3	159.41072	3	174.65075	3	189.89078	3	205.13081	3	220.37084
4	159.71552	4	174.95555	4	190.19558	4	205.43561	4	220.67564
5	160.02032	5	175.26035	5	190.50038	5	205.74041	5	220.98044
6	160.32512	6	175.56515	6	190.80518	6	206.04521	6	221.28524
7	160.62992	7	175.86995	7	191.10998	7	206.35001	7	221.59004
8	160.93472	8	176.17475	8	191.41478	8	206.65481	8	221.89484
9	161.23952	9	176.47955	9	191.71958	9	206.95961	9	222.19964
530	161.54432	580	176.78435	630	192.02438	680	207.26441	730	222.50445
1	161.84912	1	177.08915	1	192.32918	1	207.56922	1	222.80925
2	162.15392	2	177.39395	2	192.63399	2	207.87402	2	223.11405
3	162.45872	3	177.69873	3	192.93879	3	208.17882	3	223.41885
4	162.76353	4	178.00356	4	193.24359	4	208.48362	4	223.72365
5	163.06833	5	178.30836	5	193.54839	5	208.78842	5	224.02845
6	163.37313	6	178.61316	6	193.85319	6	209.09322	6	224.33325
7	163.67793	7	178.91796	7	194.15799	7	209.39802	7	224.63805
8	163.98273	8	179.22276	8	194.46279	8	209.70282	8	224.94285
9	164.28753	9	179.52756	9	194.76759	9	210.00762	9	225.24765
540	164.59233	590	179.83236	640	195.07239	690	210.31242	740	225.55245
1	164.89713	1	180.13716	1	195.37719	1	210.61722	1	225.85725
2	165.20193	2	180.44196	2	195.68199	2	210.92202	2	226.16205
3	165.50673	3	180.74676	3	195.98679	3	211.22682	3	226.46685
4	165.81153	4	181.05156	4	196.29159	4	211.53162	4	226.77165
5	166.11633	5	181.35636	5	196.59639	5	211.83642	5	227.07645
6	166.42113	6	181.66116	6	196.90119	6	212.14122	6	227.38125
7	166.72593	7	181.96596	7	197.20599	7	212.44602	7	227.68606
8	167.03073	8	182.27076	8	197.51080	8	212.75083	8	227.99086
9	167.33553	9	182.57557	9	197.81560	9	213.05563	9	228.29566

Lengths—Feet to meters (from 1 to 1000 units)—Continued

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
750	228.60046	800	243.84049	850	259.08052	900	274.32055	950	289.56058
1	228.90526	1	244.14529	1	259.38532	1	274.62535	1	289.86538
2	229.21006	2	244.45009	2	259.69012	2	274.93015	2	290.17018
3	229.51486	3	244.75489	3	259.99492	3	275.23495	3	290.47498
4	229.81966	4	245.05969	4	260.29972	4	275.53975	4	290.77978
5	230.12446	5	245.36449	5	260.60452	5	275.84455	5	291.08458
6	230.42926	6	245.66929	6	260.90932	6	276.14935	6	291.38938
7	230.73406	7	245.97409	7	261.21412	7	276.45415	7	291.69418
8	231.03886	8	246.27889	8	261.51892	8	276.75895	8	291.99898
9	231.34366	9	246.58369	9	261.82372	9	277.06375	9	292.30378
760	231.64846	810	246.88849	860	262.12852	910	277.36855	960	292.60859
1	231.95326	1	247.19329	1	262.43332	1	277.67336	1	292.91339
2	232.25806	2	247.49809	2	262.73813	2	277.97816	2	293.21819
3	232.56287	3	247.80290	3	263.04293	3	278.28296	3	293.52299
4	232.86767	4	248.10770	4	263.34773	4	278.58776	4	293.82779
5	233.17247	5	248.41250	5	263.65253	5	278.89256	5	294.13259
6	233.47727	6	248.71730	6	263.95733	6	279.19736	6	294.43739
7	233.78207	7	249.02210	7	264.26213	7	279.50216	7	294.74219
8	234.08687	8	249.32690	8	264.56693	8	279.80696	8	295.04699
9	234.39167	9	249.63170	9	264.87173	9	280.11176	9	295.35179
770	234.69647	820	249.93650	870	265.17653	920	280.41656	970	295.65659
1	235.00127	1	250.24130	1	265.48133	1	280.72136	1	295.96139
2	235.30607	2	250.54610	2	265.78613	2	281.02616	2	296.26619
3	235.61087	3	250.85090	3	266.09093	3	281.33096	3	296.57099
4	235.91567	4	251.15570	4	266.39573	4	281.63576	4	296.87579
5	236.22047	5	251.46050	5	266.70053	5	281.94056	5	297.18059
6	236.52527	6	251.76530	6	267.00533	6	282.24536	6	297.48539
7	236.83007	7	252.07010	7	267.31013	7	282.55017	7	297.79020
8	237.13487	8	252.37490	8	267.61494	8	282.85497	8	298.09500
9	237.43967	9	252.67971	9	267.91974	9	283.15977	9	298.39980
780	237.74448	830	252.98451	880	268.22454	930	283.46457	980	298.70460
1	238.04928	1	253.28931	1	268.52934	1	283.76937	1	299.00940
2	238.35408	2	253.59411	2	268.83414	2	284.07417	2	299.31420
3	238.65888	3	253.89891	3	269.13894	3	284.37897	3	299.61900
4	238.96368	4	254.20371	4	269.44374	4	284.68377	4	299.92380
5	239.26848	5	254.50851	5	269.74854	5	284.98857	5	300.22860
6	239.57328	6	254.81331	6	270.05334	6	285.29337	6	300.53340
7	239.87808	7	255.11811	7	270.35814	7	285.59817	7	300.83820
8	240.18288	8	255.42291	8	270.66294	8	285.90297	8	301.14300
9	240.48768	9	255.72771	9	270.96774	9	286.20777	9	301.44780
790	240.79248	840	256.03251	890	271.27254	940	286.51257	990	301.75260
1	241.09728	1	256.33731	1	271.57734	1	286.81737	1	302.05740
2	241.40208	2	256.64211	2	271.88214	2	287.12217	2	302.36220
3	241.70688	3	256.94691	3	272.18694	3	287.42697	3	302.66701
4	242.01168	4	257.25171	4	272.49174	4	287.73178	4	302.97181
5	242.31648	5	257.55652	5	272.79655	5	288.03658	5	303.27661
6	242.62129	6	257.86132	6	273.10135	6	288.34138	6	303.58141
7	242.92609	7	258.16612	7	273.40615	7	288.64618	7	303.88621
8	243.23089	8	258.47092	8	273.71095	8	288.95098	8	304.19101
9	243.53569	9	258.77572	9	274.01575	9	289.25578	9	304.49581

Lengths—Meters to feet (from 1 to 1000 units)

[Reduction factor: 1 meter=3.280833333 feet]

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
0		50	164.04167	100	328.08333	150	492.12500	200	656.16667
1	3.28083	1	167.32250	1	331.36417	1	495.40583	1	659.44750
2	6.56167	2	170.60333	2	334.64500	2	498.68667	2	662.72833
3	9.84250	3	173.88417	3	337.92583	3	501.96750	3	666.00917
4	13.12333	4	177.16500	4	341.20667	4	505.24833	4	669.29000
5	16.40417	5	180.44583	5	344.48750	5	508.52917	5	672.57083
6	19.68500	6	183.72667	6	347.76833	6	511.81000	6	675.85167
7	22.96583	7	187.00750	7	351.04917	7	515.09083	7	679.13250
8	26.24667	8	190.28833	8	354.33000	8	518.37167	8	682.41333
9	29.52750	9	193.56917	9	357.61083	9	521.65250	9	685.69417
10	32.80833	60	196.85000	110	360.89167	160	524.93333	210	688.97500
1	36.08917	1	200.13083	1	364.17250	1	528.21417	1	692.25583
2	39.37000	2	203.41167	2	367.45333	2	531.49500	2	695.53667
3	42.65083	3	206.69250	3	370.73417	3	534.77583	3	698.81750
4	45.93167	4	209.97333	4	374.01500	4	538.05667	4	702.09833
5	49.21250	5	213.25417	5	377.29583	5	541.33750	5	705.37917
6	52.49333	6	216.53500	6	380.57667	6	544.61833	6	708.66000
7	55.77417	7	219.81583	7	383.85750	7	547.89917	7	711.94083
8	59.05500	8	223.09667	8	387.13833	8	551.18000	8	715.22167
9	62.33583	9	226.37750	9	390.41917	9	554.46083	9	718.50250
20	65.61667	70	229.65833	120	393.70000	170	557.74167	220	721.78333
1	68.89750	1	232.93917	1	396.98083	1	561.02250	1	725.06417
2	72.17833	2	236.22000	2	400.26167	2	564.30333	2	728.34500
3	75.45917	3	239.50083	3	403.54250	3	567.58417	3	731.62583
4	78.74000	4	242.78167	4	406.82333	4	570.86500	4	734.90667
5	82.02083	5	246.06250	5	410.10417	5	574.14583	5	738.18750
6	85.30167	6	249.34333	6	413.38500	6	577.42667	6	741.46833
7	88.58250	7	252.62417	7	416.66583	7	580.70750	7	744.74917
8	91.86333	8	255.90500	8	419.94667	8	583.98833	8	748.03000
9	95.14417	9	259.18583	9	423.22750	9	587.26917	9	751.31083
30	98.42500	80	262.46667	130	426.50833	180	590.55000	230	754.59167
1	101.70583	1	265.74750	1	429.78917	1	593.83083	1	757.87250
2	104.98667	2	269.02833	2	433.07000	2	597.11167	2	761.15333
3	108.26750	3	272.30917	3	436.35083	3	600.39250	3	764.43417
4	111.54833	4	275.59000	4	439.63167	4	603.67333	4	767.71500
5	114.82917	5	278.87083	5	442.91250	5	606.95417	5	770.99583
6	118.11000	6	282.15167	6	446.19333	6	610.23500	6	774.27667
7	121.39083	7	285.43250	7	449.47417	7	613.51583	7	777.55750
8	124.67167	8	288.71333	8	452.75500	8	616.79667	8	780.83833
9	127.95250	9	291.99417	9	456.03583	9	620.07750	9	784.11917
40	131.23333	90	295.27500	140	459.31667	190	623.35833	240	787.40000
1	134.51417	1	298.55583	1	462.59750	1	626.63917	1	790.68083
2	137.79500	2	301.83667	2	465.87833	2	629.92000	2	793.96167
3	141.07583	3	305.11750	3	469.15917	3	633.20083	3	797.24250
4	144.35667	4	308.39833	4	472.44000	4	636.48167	4	800.52333
5	147.63750	5	311.67917	5	475.72083	5	639.76250	5	803.80417
6	150.91833	6	314.96000	6	479.00167	6	643.04333	6	807.08500
7	154.19917	7	318.24083	7	482.28250	7	646.32417	7	810.36583
8	157.48000	8	321.52167	8	485.56333	8	649.60500	8	813.64667
9	160.76083	9	324.80250	9	488.84417	9	652.88583	9	816.92750

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
250	820.20833	300	984.25000	350	1,148.29167	400	1,312.33333	450	1,476.37500
1	823.48917	1	987.53083	1	1,151.57250	1	1,315.61417	1	1,479.65583
2	826.77000	2	990.81167	2	1,154.85333	2	1,318.89500	2	1,482.93667
3	830.05083	3	994.09250	3	1,158.13417	3	1,322.17583	3	1,486.21750
4	833.33167	4	997.37333	4	1,161.41500	4	1,325.45667	4	1,489.49833
5	836.61250	5	1,000.65417	5	1,164.69583	5	1,328.73750	5	1,492.77917
6	839.89333	6	1,003.93500	6	1,167.97667	6	1,332.01833	6	1,496.06000
7	843.17417	7	1,007.21583	7	1,171.25750	7	1,335.29917	7	1,499.34083
8	846.45500	8	1,010.49667	8	1,174.53833	8	1,338.58000	8	1,502.62167
9	849.73583	9	1,013.77750	9	1,177.81917	9	1,341.86083	9	1,505.90250
260	853.01667	310	1,017.05833	360	1,181.10000	410	1,345.14167	460	1,509.18333
1	856.29750	1	1,020.33917	1	1,184.38083	1	1,348.42250	1	1,512.46417
2	859.57833	2	1,023.62000	2	1,187.66167	2	1,351.70333	2	1,515.74500
3	862.85917	3	1,026.90083	3	1,190.94250	3	1,354.98417	3	1,519.02583
4	866.14000	4	1,030.18167	4	1,194.22333	4	1,358.26500	4	1,522.30667
5	869.42083	5	1,033.46250	5	1,197.50417	5	1,361.54583	5	1,525.58750
6	872.70167	6	1,036.74333	6	1,200.78500	6	1,364.82667	6	1,528.86833
7	875.98250	7	1,040.02417	7	1,204.06583	7	1,368.10750	7	1,532.14917
8	879.26333	8	1,043.30500	8	1,207.34667	8	1,371.38833	8	1,535.43000
9	882.54417	9	1,046.58583	9	1,210.62750	9	1,374.66917	9	1,538.71083
270	885.82500	320	1,049.86667	370	1,213.90833	420	1,377.95000	470	1,541.99167
1	889.10583	1	1,053.14750	1	1,217.18917	1	1,381.23083	1	1,545.27250
2	892.38667	2	1,056.42833	2	1,220.47000	2	1,384.51167	2	1,548.55333
3	895.66750	3	1,059.70917	3	1,223.75083	3	1,387.79250	3	1,551.83417
4	898.94833	4	1,062.99000	4	1,227.03167	4	1,391.07333	4	1,555.11500
5	902.22917	5	1,066.27083	5	1,230.31250	5	1,394.35417	5	1,558.39583
6	905.51000	6	1,069.55167	6	1,233.59333	6	1,397.63500	6	1,561.67667
7	908.79083	7	1,072.83250	7	1,236.87417	7	1,400.91583	7	1,564.95750
8	912.07167	8	1,076.11333	8	1,240.15500	8	1,404.19667	8	1,568.23833
9	915.35250	9	1,079.39417	9	1,243.43583	9	1,407.47750	9	1,571.51917
280	918.63333	330	1,082.67500	380	1,246.71667	430	1,410.75833	480	1,574.80000
1	921.91417	1	1,085.95583	1	1,249.99750	1	1,414.03917	1	1,578.08083
2	925.19500	2	1,089.23667	2	1,253.27833	2	1,417.32000	2	1,581.36167
3	928.47583	3	1,092.51750	3	1,256.55917	3	1,420.60083	3	1,584.64250
4	931.75667	4	1,095.79833	4	1,259.84000	4	1,423.88167	4	1,587.92333
5	935.03750	5	1,099.07917	5	1,263.12083	5	1,427.16250	5	1,591.20417
6	938.31833	6	1,102.36000	6	1,266.40167	6	1,430.44333	6	1,594.48500
7	941.59917	7	1,105.64083	7	1,269.68250	7	1,433.72417	7	1,597.76583
8	944.88000	8	1,108.92167	8	1,272.96333	8	1,437.00500	8	1,601.04667
9	948.16083	9	1,112.20250	9	1,276.24417	9	1,440.28583	9	1,604.32750
290	951.44167	340	1,115.48333	390	1,279.52500	440	1,443.56667	490	1,607.60833
1	954.72250	1	1,118.76417	1	1,282.80583	1	1,446.84750	1	1,610.88917
2	958.00333	2	1,122.04500	2	1,286.08667	2	1,450.12833	2	1,614.17000
3	961.28417	3	1,125.32583	3	1,289.36750	3	1,453.40917	3	1,617.45083
4	964.56500	4	1,128.60667	4	1,292.64833	4	1,456.69000	4	1,620.73167
5	967.84583	5	1,131.88750	5	1,295.92917	5	1,459.97083	5	1,624.01250
6	971.12667	6	1,135.16833	6	1,299.21000	6	1,463.25167	6	1,627.29333
7	974.40750	7	1,138.44917	7	1,302.49083	7	1,466.53250	7	1,630.57417
8	977.68833	8	1,141.73000	8	1,305.77167	8	1,469.81333	8	1,633.85500
9	980.96917	9	1,145.01083	9	1,309.05250	9	1,473.09417	9	1,637.13583

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
500	1,640.41667	550	1,804.45833	600	1,968.50000	650	2,132.54167	700	2,296.58333
1	1,643.69750	1	1,807.73917	1	1,971.78083	1	2,135.82250	1	2,299.86417
2	1,646.97833	2	1,811.02000	2	1,975.06167	2	2,139.10333	2	2,303.14500
3	1,650.25917	3	1,814.30083	3	1,978.34250	3	2,142.38417	3	2,306.42583
4	1,653.54000	4	1,817.58167	4	1,981.62333	4	2,145.66500	4	2,309.70667
5	1,656.82083	5	1,820.86250	5	1,984.90417	5	2,148.94583	5	2,312.98750
6	1,660.10167	6	1,824.14333	6	1,988.18500	6	2,152.22667	6	2,316.26833
7	1,663.38250	7	1,827.42417	7	1,991.46583	7	2,155.50750	7	2,319.54917
8	1,666.66333	8	1,830.70500	8	1,994.74667	8	2,158.78833	8	2,322.83000
9	1,669.94417	9	1,833.98583	9	1,998.02750	9	2,162.06917	9	2,326.11083
510	1,673.22500	560	1,837.26667	610	2,001.30833	660	2,165.35000	710	2,329.39167
1	1,676.50583	1	1,840.54750	1	2,004.58917	1	2,168.63083	1	2,332.67250
2	1,679.78667	2	1,843.82833	2	2,007.87000	2	2,171.91167	2	2,335.95333
3	1,683.06750	3	1,847.10917	3	2,011.15083	3	2,175.19250	3	2,339.23417
4	1,686.34833	4	1,850.39000	4	2,014.43167	4	2,178.47333	4	2,342.51500
5	1,689.62917	5	1,853.67083	5	2,017.71250	5	2,181.75417	5	2,345.79583
6	1,692.91000	6	1,856.95167	6	2,020.99333	6	2,185.03500	6	2,349.07667
7	1,696.19083	7	1,860.23250	7	2,024.27417	7	2,188.31583	7	2,352.35750
8	1,699.47167	8	1,863.51333	8	2,027.55500	8	2,191.59667	8	2,355.63833
9	1,702.75250	9	1,866.79417	9	2,030.83583	9	2,194.87750	9	2,358.91917
520	1,706.03333	570	1,870.07500	620	2,034.11667	670	2,198.15833	720	2,362.20000
1	1,709.31417	1	1,873.35583	1	2,037.39750	1	2,201.43917	1	2,365.48083
2	1,712.59500	2	1,876.63667	2	2,040.67833	2	2,204.72000	2	2,368.76167
3	1,715.87583	3	1,879.91750	3	2,043.95917	3	2,208.00083	3	2,372.04250
4	1,719.15667	4	1,883.19833	4	2,047.24000	4	2,211.28167	4	2,375.32333
5	1,722.43750	5	1,886.47917	5	2,050.52083	5	2,214.56250	5	2,378.60417
6	1,725.71833	6	1,889.76000	6	2,053.80167	6	2,217.84333	6	2,381.88500
7	1,728.99917	7	1,893.04083	7	2,057.08250	7	2,221.12417	7	2,385.16583
8	1,732.28000	8	1,896.32167	8	2,060.36333	8	2,224.40500	8	2,388.44667
9	1,735.56083	9	1,899.60250	9	2,063.64417	9	2,227.68583	9	2,391.72750
530	1,738.84167	580	1,902.88333	630	2,066.92500	680	2,230.96667	730	2,395.00833
1	1,742.12250	1	1,906.16417	1	2,070.20583	1	2,234.24750	1	2,398.28917
2	1,745.40333	2	1,909.44500	2	2,073.48667	2	2,237.52833	2	2,401.57000
3	1,748.68417	3	1,912.72583	3	2,076.76750	3	2,240.80917	3	2,404.85083
4	1,751.96500	4	1,916.00667	4	2,080.04833	4	2,244.09000	4	2,408.13167
5	1,755.24583	5	1,919.28750	5	2,083.32917	5	2,247.37083	5	2,411.41250
6	1,758.52667	6	1,922.56833	6	2,086.61000	6	2,250.65167	6	2,414.69333
7	1,761.80750	7	1,925.84917	7	2,089.89083	7	2,253.93250	7	2,417.97417
8	1,765.08833	8	1,929.13000	8	2,093.17167	8	2,257.21333	8	2,421.25500
9	1,768.36917	9	1,932.41083	9	2,096.45250	9	2,260.49417	9	2,424.53583
540	1,771.65000	590	1,935.69167	640	2,099.73333	690	2,263.77500	740	2,427.81667
1	1,774.93083	1	1,938.97250	1	2,103 01417	1	2,267.05583	1	2,431.09750
2	1,778.21167	2	1,942.25333	2	2,106.29500	2	2,270.33667	2	2,434.37833
3	1,781.49250	3	1,945.53417	3	2,109.57583	3	2,273.61750	3	2,437.65917
4	1,784.77333	4	1,948.81500	4	2,112.85667	4	2,276.89833	4	2,440.94000
5	1,788.05417	5	1,952.09583	5	2,116.13750	5	2,280.17917	5	2,444.22083
6	1,791.33500	6	1,955.37667	6	2,119.41833	6	2,283.46000	6	2,447.50167
7	1,794.61583	7	1,958.65750	7	2,122.69917	7	2,286.74083	7	2,450.78250
8	1,797.89667	8	1,961.93833	8	2,125.98000	8	2,290.02167	8	2,454.06333
9	1,801.17750	9	1,965.21917	9	2,129.26083	9	2,293.30250	9	2,457.34417

Lengths—Meters to feet (from 1 to 1000 units)—Continued

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
750	2,460.62500	800	2,624.66667	850	2,788.70833	900	2,952.75000	950	3,116.79167
1	2,463.90583	1	2,627.94750	1	2,791.98917	1	2,956.03083	1	3,120.07250
2	2,467.18667	2	2,631.22833	2	2,795.27000	2	2,959.31167	2	3,123.35333
3	2,470.46750	3	2,634.50917	3	2,798.55083	3	2,962.59250	3	3,126.63417
4	2,473.74833	4	2,637.79000	4	2,801.83167	4	2,965.87333	4	3,129.91500
5	2,477.02917	5	2,641.07083	5	2,805.11250	5	2,969.15417	5	3,133.19583
6	2,480.31000	6	2,644.35167	6	2,808.39333	6	2,972.43500	6	3,136.47667
7	2,483.59083	7	2,647.63250	7	2,811.67417	7	2,975.71583	7	3,139.75750
8	2,486.87167	8	2,650.91333	8	2,814.95500	8	2,978.99667	8	3,143.03833
9	2,490.15250	9	2,654.19417	9	2,818.23583	9	2,982.27750	9	3,146.31917
760	2,493.43333	810	2,657.47500	860	2,821.51667	910	2,985.55833	960	3,149.60000
1	2,496.71417	1	2,660.75583	1	2,824.79750	1	2,988.83917	1	3,152.88083
2	2,499.99500	2	2,664.03667	2	2,828.07833	2	2,992.12000	2	3,156.16167
3	2,503.27583	3	2,667.31750	3	2,831.35917	3	2,995.40083	3	3,159.44250
4	2,506.55667	4	2,670.59833	4	2,834.64000	4	2,998.68167	4	3,162.72333
5	2,509.83750	5	2,673.87917	5	2,837.92083	5	3,001.96250	5	3,166.00417
6	2,513.11833	6	2,677.16000	6	2,841.20167	6	3,005.24333	6	3,169.28500
7	2,516.39917	7	2,680.44083	7	2,844.48250	7	3,008.52417	7	3,172.56583
8	2,519.68000	8	2,683.72167	8	2,847.76333	8	3,011.80500	8	3,175.84667
9	2,522.96083	9	2,687.00250	9	2,851.04417	9	3,015.08583	9	3,179.12750
770	2,526.24167	820	2,690.28333	870	2,854.32500	920	3,018.36667	970	3,182.40833
1	2,529.52250	1	2,693.56417	1	2,857.60583	1	3,021.64750	1	3,185.68917
2	2,532.80333	2	2,696.84500	2	2,860.88667	2	3,024.92833	2	3,188.97000
3	2,536.08417	3	2,700.12583	3	2,864.16750	3	3,028.20917	3	3,192.25083
4	2,539.36500	4	2,703.40667	4	2,867.44833	4	3,031.49000	4	3,195.53167
5	2,542.64583	5	2,706.68750	5	2,870.72917	5	3,034.77083	5	3,198.81250
6	2,545.92667	6	2,709.96833	6	2,874.01000	6	3,038.05167	6	3,202.09333
7	2,549.20750	7	2,713.24917	7	2,877.29083	7	3,041.33250	7	3,205.37417
8	2,552.48833	8	2,716.53000	8	2,880.57167	8	3,044.61333	8	3,208.65500
9	2,555.76917	9	2,719.81083	9	2,883.85250	9	3,047.89417	9	3,211.93583
780	2,559.05000	830	2,723.09167	880	2,887.13333	930	3,051.17500	980	3,215.21667
1	2,562.33083	1	2,726.37250	1	2,890.41417	1	3,054.45583	1	3,218.49750
2	2,565.61167	2	2,729.65333	2	2,893.69500	2	3,057.73667	2	3,221.77833
3	2,568.89250	3	2,732.93417	3	2,896.97583	3	3,061.01750	3	3,225.05917
4	2,572.17333	4	2,736.21500	4	2,900.25667	4	3,064.29833	4	3,228.34000
5	2,575.45417	5	2,739.49583	5	2,903.53750	5	3,067.57917	5	3,231.62083
6	2,578.73500	6	2,742.77667	6	2,906.81833	6	3,070.86000	6	3,234.90167
7	2,582.01583	7	2,746.05750	7	2,910.09917	7	3,074.14083	7	3,238.18250
8	2,585.29667	8	2,749.33833	8	2,913.38000	8	3,077.42167	8	3,241.46333
9	2,588.57750	9	2,752.61917	9	2,916.66083	9	3,080.70250	9	3,244.74417
790	2,591.85833	840	2,755.90000	890	2,919.94167	940	3,083.98333	990	3,248.02500
1	2,595.13917	1	2,759.18083	1	2,923.22250	1	3,087.26417	1	3,251.30583
2	2,598.42000	2	2,762.46167	2	2,926.50333	2	3,090.54500	2	3,254.58667
3	2,601.70083	3	2,765.74250	3	2,929.78417	3	3,093.82583	3	3,257.86750
4	2,604.98167	4	2,769.02333	4	2,933.06500	4	3,097.10667	4	3,261.14833
5	2,608.26250	5	2,772.30417	5	2,936.34583	5	3,100.38750	5	3,264.42917
6	2,611.54333	6	2,775.58500	6	2,939.62667	6	3,103.66833	6	3,267.71000
7	2,614.82417	7	2,778.86583	7	2,942.90750	7	3,106.94917	7	3,270.99083
8	2,618.10500	8	2,782.14667	8	2,946.18833	8	3,110.23000	8	3,274.27167
9	2,621.38583	9	2,785.42750	9	2,949.46917	9	3,113.51083	9	3,277.55250

PUBLICATIONS OF THE COAST AND GEODETIC SURVEY RELATING TO TRIANGULATION AND BASE MEASUREMENT

Special Publication No. 5.—Tables for a Polyconic Projection of Maps, 20 cents. This publication contains the necessary explanation of the method employed in constructing a polyconic projection, and also gives the values in meters of the degrees, minutes, and seconds of latitude and longitude for all latitudes.

Special Publication No. 8.—Formulæ and Tables for the Computation of Geodetic Positions, 25 cents. Contains the formulæ, instructions, and data for computing the spherical coordinates for triangulation when the distances and angles are known.

Special Publication No. 14.—Determination of Time, Longitude, Latitude, and Azimuth, 35 cents. A manual describing the instruments and methods used in the Coast and Geodetic Survey on its astronomic field and office work.

Special Publication No. 26.—General Instructions for the Field Work of the Coast and Geodetic Survey. Includes condensed instructions for third-order triangulation, topography, hydrography, and tidal and current observations.

Special Publication No. 28.—Application of the Theory of Least Squares to the Adjustment of Triangulation, 25 cents. Explains the principles of least-squares adjustments of triangulation, with illustrative examples.

Special Publication No. 58.—General Instructions for Precise and Secondary Traverse, 10 cents. (Now called first and second order traverse, respectively.)

Special Publication No. 65.—Instructions to Light Keepers on Triangulation, 10 cents. A small pamphlet issued to light keepers, containing the signaling code used and instructions for adjusting and sighting the lights.

Special Publication No. 71.—Relation between Plane Rectangular Coordinates and Geographic Positions, 10 cents. Contains formulæ and tables for changing from plane to spherical coordinates and vice versa.

Special Publication No. 79.—Precise Traverse and Triangulation in Indiana, 20 cents.

Special Publication No. 86.—Precise Traverse, Racine, Wis., to Vandalia, Ill. 15 cents. In addition to the usual tables of results this and the publication above contain an account of the methods employed in the field work on traverse and in the office computation of the field measurements.

Special Publication No. 91.—Use of Geodetic Control for City Surveys, 20 cents. Outlines the reasons why accurate vertical and horizontal control is desirable for city surveys and describes briefly the methods and instruments used on work of that character.

Special Publication No. 93.—Reconnaissance and Signal Building, 30 cents. A manual covering reconnaissance for triangulation, the marking of stations, and the construction of triangulation targets and towers.

Special Publication No. 109.—Wireless Longitude, 15 cents. A description of the apparatus and methods employed in the accurate measurement of differences in longitude when radio time signals are used in place of signals sent over telegraph lines.

Any of the publications given above can be purchased by those outside the bureau from the Superintendent of Documents, Washington, D. C., at the price listed. Special Publication No. 26 can be obtained from the Director, Coast and Geodetic Survey. Numerous publications of this bureau contain the results of geodetic operations in the form of geographic positions of triangulation and traverse stations and elevations of bench marks. An engineer interested in securing those data for any particular locality should address his inquiry to the Director, Coast and Geodetic Survey.

A complete list of the publications of the Coast and Geodetic Survey is found in the List of Publications of the Department of Commerce, a copy of which may be obtained free of charge upon application to the Department of Commerce.

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