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REPORT ON EARTH TIDES

[Report to the International Association of Geodesy of the International Union of Geodesy and Geophysics, International Council of Scientific Unions]

BY

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FOREWORD

This report on earth tides is one of a series of reports to the International Association of Geodesy presented at its triennial meetings. The first of the series was presented to the second assembly, held at Madrid in 1924. Previous reports have been printed by the International Association of Geodesy under the title: "(Number of Assembly) Assemblée Générale. (place of meeting) Travaux de la Section (or de l'Association) de Géodésie. Rapports Généraux. Paris (date)."

This report was presented to the Seventh General Assembly held at Washington, D. C., in September 1939. Because of disturbed conditions in Europe, it seemed advisable to print this report here.

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REPORT ON EARTH TIDES

By Walter D. Lambert, *Senior Mathematician, United States Coast and Geodetic Survey*

INTRODUCTION AND PLAN OF THE REPORT

A questionnaire on earth tides, similar in form to those distributed in 1927, 1930, 1933, and 1936 for the Prague, Stockholm, Lisbon, and Edinburgh meetings,[1] was sent out early in 1939. A preliminary version of this report was mimeographed and distributed at Washington. The reporter hopes that anyone noticing errors or omissions will promptly inform the reporter of them. His address is at the end of this report. (See p. 22.)

Tides of the so-called solid earth have been measured in three principal ways:

(1) By means of the tidal deflection of the vertical referred to the adjacent ground. Measured with horizontal pendulums or with long water levels.

(2) By means of the tidal deflection of the vertical referred to the axis of the earth. Measured by analysis of astronomical observations for latitude and for difference of longitude.

(3) By means of the tidal variation in gravity. Measured by sensitive gravimeters, which are of recent development.

The tide-producing forces can readily be calculated, also the effects that these forces would produce on an ideal unyielding earth. Since no substance is unyielding and the earth therefore yields, the ratio of the observed effect to the theoretical effect for an ideal unyielding earth is of interest.

Mathematical analysis has shown that this ratio is expressible in terms of three pure numbers, k, h, and l. These numbers are applicable only in connection with those second-degree spherical harmonics that represent the tidal forces and the deformations of like kind produced by these forces. They do not apply to the actual tidal deformation of the earth, for that is affected by the gravitational action of the ocean tides and by the surface tractions produced by the shifting load of tidal water. Every investigator knows how irregular the actual oceanic tide is as compared with the regularity of a second-degree spherical harmonic. The problem of correcting adequately for these irregularities and reducing the observed results to what they would be if only the oceanic tides were absent is one of the main problems of earth tides and it is still far from solution.

If, however, we assume that the observations are so remote from the sea that the influence of the ocean tides is negligible[2] and that other local effects are absent, then the factors for reducing *from* the predicted effect for an absolutely unyielding earth *to* the effect for the yielding earth are:

(1) Observations of the deflections of the vertical referred to the adjacent ground:

$$\text{Factor} = 1 + k - h$$

(2) Astronomical observations of the deflection of the vertical referred to the earth's axis:

$$\text{Factor} = 1 + k - l$$

(3) Observations for changes in gravity:

$$\text{Factor} = 1 + h - \frac{3}{2}k$$

The yielding referred to may be plastic as well as elastic. As a matter of fact, despite apparent evidence to the contrary, the yielding is probably on the whole elastic, with very little lag in phase between the force and the effect produced. If it were otherwise, tidal friction would be so great that it would be apparent in the observed secular acceleration of the moon.

[1] The text of the 1927 questionnaire is included in the report of the Prague meeting: Travaux de la Section de Géodésie, tome 6, Rapports généraux, Paris, 1928.

[2] This assumption is probably in general contrary to fact, for the ocean covers three-fourths of the earth and the gravitational and other effects are perceptible to distances greater than one might be inclined to guess.

The small secular acceleration that is not accounted for by gravitational theory seems to be about taken care of by the tidal friction in the waters of the shallow seas of the earth,[3] leaving little to be accounted for by friction in the body of the so-called solid earth.

The analytical expressions for h, k and l on the rather drastic simplifying assumptions that the earth is homogeneous and incompressible were given in the Lisbon (1933) report but for completeness are repeated here. They are based on the work of investigators of the nineteenth century, such as Kelvin and Darwin.

They are:

$$k=\frac{3f}{2f+1}$$

$$h=\frac{5f}{2f+1}$$

$$l=\frac{\frac{3}{2}f}{2f+1}$$

f being equal to $\frac{g\rho a}{19\mu}$,

where g is the acceleration of gravity=981 gals.
ρ is the mean density of the earth=5.52 gm./cm.^{3}
a is the mean radius of the earth=6.37×10^8 cm.
μ is the mean effective modulus of elasticity in dynes/cm.^{2}

The value of μ probably lies between that of steel, which is 8×10^{11} dynes/cm^2, and twice this value, probably nearer the latter value.

For these two values of μ we find as follows:

μ	k	h	l	$1+k-h$	$1+k-l$	$1+h-\frac{3}{2}k$
8×10^{11}	0.47	0.78	0.23	0.69	1.23	1.08
16×10^{11}	.28	.46	.14	.82	1.14	1.03

The assumptions are rather drastic but they do simplify the problem greatly. The assumption of incompressibility is particularly effective.[4] Several authors have treated the problem of the values of k and h under assumptions intended to represent the actual earth more nearly than do the assumptions just discussed. A list of their articles will be found on page 20 of this report.

The quantity l which is needed only in connection with astronomical observations, has not received much attention, since it is only recently that these observations have been discussed in connection with earth tides.

As in previous reports, the observations will be discussed under the three headings already mentioned in accordance with the proper factor to reduce from an ideal unyielding earth to a yielding one. This has some disadvantages, for in a few instances observations of more than one kind were made at the same place. This discussion will be followed by one on tides in wells remote from the sea and by a few remarks on the problem of earth tides.

[3] Harold Jeffreys. The Earth, its Origin, History, and Physical Constitution. Cambridge University Press, 2d Ed. 1929, p. 261. 1st Ed. 1924, p. 220.

[4] See A. E. H. Love. Some Problems in Geodynamics. Cambridge University Press, 1911. Chapters II and VII. Other chapters of the book are also of interest.

CHAPTER I, DEFLECTIONS OF THE VERTICAL REFERRED TO THE ADJACENT GROUND

ADDITIONS AND CORRECTIONS TO PREVIOUS REPORTS

The work of Dr. Schaffernicht with horizontal pendulums at Marburg/Lahn [1] was not published until after the Edinburgh meeting, but since the time fell within the period covered by the report an account of the work was incorporated in the printed and revised text.[2] For details, see Schaffernicht's article already referred to. It will be sufficient to mention the fact that the phase lags were negative and numerically less than 10° and that the observed values of $1+k-h$ were 0.87 and 0.65 respectively for the east-west and the north-south components of the deflection.

Dr. J. Egedal of the Danish National Committee for Geodesy and Geophysics has kindly communicated some remarks and corrections to those portions of the Edinburgh report dealing with the work done by him and by Dr. Fjeldstad in a disused railway tunnel at Bergen, Norway.[3]

It appears that the comparison of the device for reading the water level with the microscope originally employed for reading their long water levels at Lake Geneva, Wis., was inappropriate. Dr. Egedal says that the water level was determined by means of a variometer ("Niveau Variometer") for reading a scale directly. The scale was determined by means of a hydrostatic leveling instrument constructed by Dr. La Cour.

Dr. Egedal notes that the departures from the expected values are mainly due to tides in the sea at places remote from Bergen. This is a good point to emphasize. It is not generally realized to how great a distance oceanic tides affect the observed results, especially where there is a large tract of ocean having tides nearly in the same phase.

Additional values of the ratio $1+k-h$ have been computed by Dr. Egedal for N_2 and S_2, the assumption being that $1+k-h$ for these components is affected by the ocean tide in the same way as it is for M_2. For S_2 the computation is not very certain, as S_2 from direct analysis came out small but the agreement with the other components is satisfactory. The new values of $1+k-h$ from S_2 and N_2 are given below along with the values of this ratio as already published for the other components.

$1+k-h$ for Bergen, Norway

Component	$1+k-h$	Component	$1+k-h$
M_2	0.578	O_1	0.75
N_2	.66	K_1	.60
S_2	.63		

In addition to the joint report of Egedal and Fjeldstad already referred to, Dr. Egedal has published an article: On Observations of Vertical Movements in the Earth's Crust by Means of Improved Niveauvariometers. Report No. 19, Scandinavian Naturalist Congress, Helsinki, 1936, pages 294–5.

LETTAU'S WORK

The simple horizontal pendulum has long been a standard instrument for measuring very small changes in the direction of the vertical. Lettau [4] describes a double horizontal pendulum, which is two simple pendulums mechanically coupled together. By this process a higher sensitiveness was attained. What Lettau calls the "static angular magnification" (statische Winkelvergrösserung) in the apparatus actually used was from 400,000 to 600,000 with a magnification of about one million apparently attainable, whereas with the simple pendulum the magnification is of the order of from 1,000 to 10,000. It is not practicable to couple *any* two horizontal pendulums together; the results might be poor or unobtainable because of instability. The natural period of vibration of the two pendulums coupled together is different from the period of either of the component pendulums.

[1] W. Schaffernicht. Horizontalpendelbeobachtungen über Lotschwankungen in Marburg/Lahn. Annalen der Physik. 5 Folge, Bd. 29, p. 349.

[2] Published as part of vol. 14 of the Travaux de l'Association de Géodésie. Rapports Généraux. Paris, au Secrétariat de l'Association. 1938.

[3] See Edinburgh Report, p. 9, where reference is made to the work of J. Egedal and J. E. Fjeldstad, Observations of Tidal Motions of the Earth's Crust made at the Geophysical Institute, Bergen. Published by the Norwegian Academy of Sciences at Oslo as Geophysical Publication, vol. XI, No. 14. Also issued as a publication of the Danish Meteorological Institute at Copenhagen.

[4] Heinz Lettau. Das Horizontaldoppelpendel. Zeitschrift für Geophysik. Jahrgang 13, 1937, p. 25.

A pendulum actually used at Collm (Geophysical Observatory of the University of Leipzig) may be cited as an illustration of how different the component pendulums may be. The masses are 1393 and 4.5 grams, and the distances of the centers of gravity of these masses from the axes of rotation are 27.5 and 0.67 centimeters. For a discussion of the considerations involved in the design of the instrument see Lettau's article. Deflections of the vertical were observed with this instrument at Collm ($\phi=51°$ 19′ N., $\lambda=13°$ 00′ E., elevation=230 meters). Those for a few days in January 1936 are reproduced in the article. The curves shown are almost as regular as the predicted curves until the freezing of the ground disturbed the regularity, for unlike many horizontal pendulums, this one was not installed at such a depth as to be beyond the reach of frost.

The value of $1+k-h$ from these few days was about 0.6, but the emphasis of the article is on the design and performance of the instrument, rather than on the numerical results. Several paragraphs are devoted to the effects of barometric pressure. This subject, which cannot be dissociated from that of earth tides, is further discussed by Lettau in another article.[5]

Egedal suggests that it would be interesting to compare his apparatus with Lettau's by using both types of instrument at the same place, as in the disused tunnel at Bergen, where the observations of Egedal and Fjeldstad (see reference p. 3) were made.

WORK REPORTED BY TOMASCHEK

A very full report has been received from Professor R. Tomaschek of Munich regarding observations on earth tides made in Germany. Doubtless the matter covered by the report and allied topics will be published by Professor Tomaschek and his colleagues. Both horizontal pendulums and gravimeters were used, so that the method of treatment adopted in this report would separate the report into two parts, one for the horizontal pendulums and one for the gravimeters. However, gravimeters have been in operation at only two places, Marburg and Berchtesgaden. The results for Marburg were discussed in the Edinburgh report and those for Berchtesgaden are not yet available. We need therefore report only on the work with the horizontal pendulums. Assisting Dr. Tomaschek for part of the time were Dr. Schaffernicht and Engineers Heck, Scheffers and Sorber. When the report was submitted Drs. Voit and Gnass were collaborating with Professor Tomaschek.

The figures given below differ considerably from those given in the preliminary version of the report. The report of the changes was dated August 10, 1939, and was received too late to be embodied in the preliminary report.

The results for the horizontal pendulums are given under the names of the places of observation. The pendulums were of the type designated by Tomaschek and Schaffernicht.[6]

Pillnitz ($\phi=51°$ 01′ N.; $\lambda=13°$ 53′ E.; elevation=151 meters and 40 meters below ground). In operation since January 1, 1938. The values were obtained by special kind of harmonic analysis. (See p. 20.) Pillnitz is the main station for this earth-tide campaign. Two gravimeters and three horizontal pendulums were in operation there and it was there that instruments used elsewhere were tested. The results for deflection were:

	N—S	E—W
Amplitude of M_2	0″.00492±23	0″.00821±36
$1+k-h$	0.638	0.829
Phase lag=κ	+9°24′	+3°10′

The errors are mean-square errors in units of the last place.

Beuthen, Upper Silesia ($\phi=50°$ 21′ N.; $\lambda=18°$ 55′ E.; elevation = 300 meters and 540 below ground in a coal mine). In operation since July 1, 1938; the curves are subject to perturbations; the temperature in the mine is high.

	N—S	E—W
Amplitude of M_2	0″.00562±42	0″.00747±60
$1+k-h$	0. 726	0. 749
Phase lag=κ	+20°28′	+13°52′

Berchtesgaden ($\phi=47°$ 38′ N; $\lambda=12°$ 59′ E.; elevation=567 meters and 124 meters underground in a salt mine, lying about 1,000 meters within the mountain). In operation since February 9, 1939.

	N—S	E—W
Amplitude of M_2	0″.00425±55	0″.00815±50
$1+k-h$	0.481	0.715
Phase lag=κ	+16°57′	−2°28′

[5] Heinz Lettau. Über die unmittelbare Einwirkung atmosphärischer Kräfte auf die Erdkruste. Meteorologische Zeitschrift, 1937, p. 53.
[6] Annalen der Physik, vol. 29, 1937, p. 349.

A report on the Marburg observations was given in the Edinburgh report. No further work there is planned.

WORK OF LIVERPOOL TIDAL OBSERVATORY

The Tidal Observatory of the University of Liverpool, located at Bidston, near Birkenhead, England, reports as follows: [7]

"A Milne Shaw seismograph was maintained continuously in use as a Tilt Meter at Bidston (Lat. 53°24′ N., Long. 3°04′ W.), between March 5, 1935, and March 12, 1936. Observations covering a complete year have been reduced and discussed.

"The seismograph was specially purchased for use as a tilt meter. It is of the ordinary type but with a modified recording unit. The conditions of installation are similar to those described in an earlier paper (Load Tilt and Body Tilt at Bidston, Monthly Notices Royal Astronomical Society, Geophysical Supplement, vol. 3, 1932–36, p. 203).

"For the semidiurnal species, the Load Tilt has been separated from the Body Tilt, and in the method of treatment *the secondary effects due to tidal phenomena not in phase with the body tilt* have been partly eliminated.

"The assumptions made are:

"(1) That for the loading tide the ratios of the amplitudes of M_2, S_2 and N_2 and their phase differences are assumed to be the same as the corresponding quantities for the local oceanic tides, taking a mean of the results for Liverpool and Hilbre Island.[8]

"(The variation in these ratios in the loading tide is small over a large region near Bidston, and they may be considered to be well determined. Further, these ratios hold approximately over a large part of the world, and it has been shown by trial that small changes in the assumed ratios have a relatively small effect on the final results. Small changes in the phase lags of the load tilt are of considerable importance but these are *not assumed*, and are in fact deduced from the results).

"(2) That ratios of amplitudes of M_2, S_2 and N_2 in the body tilt are in the same ratios as amplitudes of M_2, S_2 and N_2 in the equilibrium tilt.

"(3) That the phase lag of the body tilt is small. (The method does not determine this phase lag but provides, for assumed values of the phase lag, the corresponding values of $1+k-h$.)"

"The results are as tabulated below:

Phase lag of body tilt	$1+k-h$	Phase lag of body tilt	$1+k-h$
°		°	
15	0.48	−5	.86
10	.63	−10	.94
5	.69	−15	1.12
0	0.77		

"For a value of $1+k-h$ near to the accepted value the phase lag of the body tilt must thus be nearly zero. If we assume that it is zero, then $1+k-h=0.77$, a value intermediate between that of Michelson and Gale (0.70) and that of Schweydar (0.84).

"The phase lags of both the semidiurnal and quarter diurnal load tilts correspond to loading tides intermediate between those at Liverpool and Hilbre Island and there is no indication of an appreciable lag of the load tilt on the loading tide. It requires careful analysis to separate the load tilt from the equilibrium tilt because the tide is large at Liverpool and the tidal observatory at Bidston is not far from the shore. For the moon the observed tilt is some seven times the theoretical tilt for a rigid earth.

"The diurnal results have proved disappointing. The probable errors of the diurnal constants from monthly determinations are much larger than would be expected, and no explanation for their inconsistency has been found.

"The instrument is badly situated for their determination for two reasons:

"(1) In the case of O_1 the load and body tilts at Bidston are in opposite phase, and so lead to a very small observed tilt.

"(2) The latitude of Bidston is such that the south-north diurnal body tilts are relatively small.

"When allowance is made for the probable errors of the constants, the diurnal load and body tilts are not found to be in positive disagreement with the semidiurnal and quarter diurnal tilts.

[7] The report has been somewhat condensed and revised in the light of Mr. Corkan's more extended discussion. R. H. Corkan, the Analysis of Tilt Records at Bidston, Geophysical Supplement to the Monthly Notices of the Royal Astronomical Society. Vol. 4, No. 7, June 1939, p. 481.

[8] Hilbre Island is at the very mouth of the River Dee, not far from Bidston, which lies near the open coast between the Mersey and the Dee. It is natural to suppose that the Hilbre Island tides would represent the loading effects due to the tide off the coasts somewhat better than would the Liverpool tides. In fact it is found that the loading effect is nearer to what would be found by computing with Hilbre Island alone than by computing with Liverpool alone. Reporter.

"Long-period tilting was given special attention during the observations. There is no evidence of appreciable permanent tilting of the ground at Bidston, but the existence is shown of large oscillations of tilt (maximum displacement about ± 2 secs. arc) which appear to be partly related to long-period oscillations of pressure.

"No relation was found between sudden tilts and the occurrence of world-shaking earthquakes. In fact, all sudden tilts have been traced to local origin in the pillar supporting the instrument."

WORK IN BOHEMIA

A horizontal pendulum was installed some time since in a deep mine at Przibram, Bohemia, but the results were vitiated by movements of the ground. Under date of February 27, 1937, Professor Cechura reported the intention of the Czechoslovakian National Committee on Geophysics to continue observations in a more favorable part of the mine.

JAPANESE WORK

Dr. Eiiti Nisimura of the Geophysical Institute, Kyoto Imperial University, has kindly furnished the reporter with summaries of various papers not yet accessible in English nor published in final form. From these abstracts those that concern the deflection of the vertical referred to the ground were selected for reproduction below:

Kenzo Sassa and Sadao Tani. "On tiltmetric observations (preliminary). Ten pairs of *tiltmeter of Zöllner suspension, which are of fused-silica-make and made in our observatory,* were put in operation at various points in Japan—Kyoto, Beppu, Aso and others.

"At Kamigamo Observatory in Kyoto ($\phi=35°$ N., $\lambda=135.7°$ E.), the comparison of the silica tiltmeter with that of Rebeur's type, which were once used for observation of earth tide by late Professor T. Shida, were made during more than one year at the same position. From this observation, it is ascertained that two tiltmeters of different type (Zöllner and point suspensions) show same results on earth tide but on secular change of inclination they do not always show similar results. On these points, detailed investigations were made."

Eiiti Nisimura. *"On earth tide observed at Beppu and Aso."*

"Eight pairs of silica tiltmeter of same type which were above described were set up at Beppu ($\phi=33.2°$ N., $\lambda=131.5°$ E.) (city of hot springs facing on the Bay of Beppu in Kyushu Province) and at Aso ($\phi=32.9°$ N., $\lambda=131.0°$ E.) (district of the active volcano, middle of Kyushu Province), and this time the semidiurnal lunar tides of these stations were investigated.

"At Beppu, the active fault line with 20 km. length runs along the southern boundary of the city of Beppu and its easterly end deeply penetrates into the Bay of Beppu. On the recent activity of this tectonic line, were recorded the great catastrophe which happened in 1596 A. D., making two islets of the Beppu Bay in a moment to sink into sea bottom. The observational results of tiltmeter show that at each point of Beppu, the bending of the earth surface by the tidal loading of sea-water of Beppu-Bay differs from each other as shown in figure 2. Especially at two points [9] (E and F) which are oppositely situated against the tectonic line, the vectorial direction of bending are just in opposition, that is, the earth surface of different side bordered by the tectonic line cave in opposite sense in time of high water of Beppu Bay. Thus, by tiltmetric observation, it is able to investigate the micro-structure of the superficial layer of the earth crust with aid of tidal loading of neighboring sea.

"On the other hand, at Aso (more than 50 km. distant from sea shore) earth tide observed at four points few kilometers apart from each other show similar results, and the diminishing factor calculated from primary component which were obtained by subtracting the effect of ocean tide from the observed value, was about 0.8. From this, we can say that even at the volcanic district there is no layer having special physical property with depth of 50 km. or more."

[9] Shown in a sketch not here reproduced. Reporter.

CHAPTER II, DEFLECTIONS REFERRED TO THE EARTH'S AXIS

LUNAR EFFECTS ON CLOCK CORRECTIONS

Earth-tide effects in astronomical observations have hitherto been studied by means of observations for the variation of latitude and for the differences in longitude between two or more observatories. Some rather large apparent effects have developed in the differences of longitude, but not in the latitude. In the longitude there is always the uncertainty of the transmission time of the signals.

Sollenberger and Clemence [1] of the U. S. Naval Observatory at Washington, D. C., ($\phi=38°$ 55′ N.; $\lambda=77°04'$ W.) have studied the change in clock corrections between the observation of two different star groups, the interval between the groups being long enough for tidal effects, if any, to manifest themselves. The following account is condensed from the article:

The instrument used was the Ross zenith tube, originally designed to measure the variation of latitude but refitted for the present work.

The regular observing list consists of 72 stars, all culminating within 10 minutes of arc of the Washington zenith. These are divided into 8 groups of 9 stars each, ranging over 1.5 to 3.3 hours of right ascension. Normally, 2 groups are observed on a night. The relative positions of the stars are based on observations with this instrument from December 1932 to June 1937. The probable error of an adopted star place is about $\pm0^s\!.0014$. For each night on which at least 3 stars were observed in each of 2 groups, the mean clock corrections were formed for the 2 groups and the differences tabulated, allowing for the rate of the clock in the interval. This rate was in general based on the mean of 2 or 3 Shortt clocks and on observations extending over about a week. There were 624 group differences obtained for the period March 1934 to February 1939. The mean value for the 5 years for each pair of groups were taken off to eliminate errors of star place and the residuals were tabulated for every 30° of the *Moon's* hour angle in 3 zones of declination, +27° to +15°, +15° to −15°, and −15° to −27°. The *Moon's* average declination for the 3 zones was +21°, 0°, and −21°. The 12 equations of condition for each zone are of the form

$$\Delta=a(\sin h_E-\sin h_L)+b\ (\cos h_E-\cos h_L)+c\ (\sin 2h_E-\sin 2h_L)+d\ (\cos 2h_E-\cos 2h_L)$$

where Δ is the group difference in the sense clock correction for the earlier group *minus* clock correction for the latter group and h is the *Moon's* hour angle reckoned west from the Washington meridian, the subscripts E and L denoting the earlier and later groups, respectively.

These equations were solved by least squares and the results compared with the theoretical results for an unyielding earth.

The results are:

Declination	Observation minus theory	
	Sin h	Sin $2h$
°		
+21	+0^s^.0005	+0^s^.0001
0	+.0004	−.0003
−21	+.0004	−.0001

The agreement is well within the probable error. It is evident that there are no appreciable tidal effects in our astronomical observations.

The reporter would prefer to say that these figures go to show that there *is* an approximately normal earth tide at Washington, since the theoretical tidal effects (for astronomical observations) are about the same for an ideal unyielding earth and the actual earth. The evidence would seem to show that unexplainably large, apparent variations in trans-Atlantic longitudes determined by wireless time signals are not due to tidal effects at Washington.

"Unless the effects discussed lag behind the *Moon* (as might be the case with a tidal effect), there are no terms in cos h and cos $2h$ to be expected from gravitational theory. However, the velocity of the *Earth* in its orbit around the center of gravity of the *Earth* and *Moon*

[1] Some Lunar Effects on Clock Corrections, Astronomical Journal, vol. 48, No. 1107, July 17, 1939, p. 78.

produces an aberrational effect, which for an observation at the Washington zenith, is nearly $T=+0^s0007 \cos h$. The observed effect is $-0^s0004 \cos h$. The discordance, as well as the existence of observed terms in $\cos 2h$, may well be accidental, but it is greater than in the case of the sine terms, and the possibility of an extension of theory should not be neglected. A longer series of observations will throw more light on this question."

In the paper by Sollenberger and Clemence reference is made to periodic terms in the earth's rotation, due to the attraction of the sun and moon on the nonspherical earth and the possible noncoincidence of the pole of figure with the pole of rotation, which is the cause of the variation of latitude. The terms are taken from Helmert [2] and were computed on the assumption of an unyielding earth. The effect of the tidal deformation of the earth has been computed by F. Andersson,[3] on the simplifying assumptions of Schweydar and Herglotz. The terms found have periods of a fortnight, a half year and 18 years. Their amplitudes are 0^s001, 0^s008 and 0^s278, respectively. As Sollenberger and Clemence remark of the terms taken from Helmert: "These terms should be present in the ΔT (difference of the clock corrections) for any given night * * * but they will disappear in the mean of any series of observations taken of a year or more." This would not strictly apply to the 18-year term but its presence should not materially affect the conclusions reached by Sollenberger and Clemence for their 5-year series.

TIDAL EFFECTS ON THE LATITUDE OF GREENWICH

Dr. H. Spencer Jones [4] has discussed 25 years of latitude observation made with the Cookson floating zenith telescope (1911–36). The result deduced from harmonic analysis of the observations arranged in groups for the lunar hours 1 to 24 is:

$$(0''.0050 \pm 0.13) \cos (2t - 2° \pm 15°),$$

where t is the hour angle of the true moon. If the mean moon had been used, a very slightly smaller coefficient should be expected. The observations were corrected for "(*a*) the latitude variations, read off from a smooth curve representing the final values from which the wind effects had been eliminated; (*b*) the group corrections using the final values from the 25 years discussion; (*c*) the wind effect, as derived in the definitive discussion of the 1911–36 observations; (*d*) the diurnal variation of latitude as derived from the same discussion." Previous discussions of the wind effect on the observed latitude will be remembered by those who have followed such matters.

Greenwich Observatory is near the Thames, where the spring tides rise nearly 21 feet and the neap tides 14. This means a gravitational effect on the plumb line of this mass of water, an effect having the same period as the M_2 component of the earth tide proper. A numerical estimate of the attraction of this load of tidal water was made by mechanical quadrature. The result was

$$+0''.0021 \cos (2t - 5°.5),$$

to be applied as a correction to the observed tide to obtain the earth tide proper. The result is

$$(0''.0071 \pm 0''.0013) \cos (2t - 3°).$$

The direction of the plumb line is referred to the earth's axis, not to the adjacent ground, as is the case with observations with the horizontal pendulum. No correction, therefore, is needed for the deformation of the ground by the tidal load. The factor is that for astronomical observations, $1+k-l$.

The theoretical effect is

$$0''.0077 (1+k-l) \cos 2t.$$

$1+k-l$ should presumably be greater than unity, but the observed result shows it to be less. Perhaps if a greater tidal area including the surrounding seas were considered, the result would be different. The fairly large probable error, which is found in spite of the uncommonly long series, should also be considered.

[2] Höhere Geodäsie, vol. II, p. 434, formula (23).

[3] Berechnung der Variation der Tageslänge infolge der Deformation der Erde durch fluterzeugende Kräfte. Arkiv for Matematik, Astronomi och Fysik. Vol. 26, 2d half, 1938, p. 1.

[4] The Tidal Effects on the Variation of Latitude at Greenwich. Monthly Notices Royal Astronomical Society. Vol. 99, Jan. 1939, p. 196. Also: Observations made with the Floating Zenith Telescope in the years 1927–36 at the Royal Observatory, Greenwich and the Determination of the Variation of Latitude and the Constant of Nutation from Observation in the years 1911–36 under the Direction of H. Spencer Jones. London, His Majesty's stationery office, 1939.

INTERNATIONAL LATITUDE STATIONS

In the Edinburgh report values were given for the deflections of the plumb line at the various International Latitude Stations by various authors and for various periods of time. At the time of the Edinburgh report the figures quoted from Dr. Eiiti Nisimura were based on a personal communication, but the article has since been published under the title Change of Plumb Line referred to the Axis of the Earth as found from the results of the International Latitude Observations (Memoirs of the College of Science, Kyoto Imperial University, series A, vol. XX, 1937, p. 192). The results themselves are the same as given in the Edinburgh Report but are here repeated because the now published paper contains the probable or mean-square errors.[5] The period covered is 1912.0 to 1922.7 and the harmonic analysis gives for the M_2-component of the variation of latitude:

Station	M_2-term in the latitude
Mizusawa	$0''.0032 \cos (2t-260°) \pm 0''.0014$
Carloforte	$0''.0118 \cos (2t-334°) \pm 0''.0008$
Ukiah	$0''.0105 \cos (2t-11°) \pm 0''.0013$

Considering the character of the data, the only peculiar station is Mizusawa and it may be affected by the Pacific tides. It may also be remarked that other authors have analyzed the data for other periods of time and have found for Mizusawa values rather closer to the theoretical value

$$0''.0077\ (1+k-l) \cos 2t.$$

Compare the international latitude stations with Greenwich, previously given in this report.

The equality of the coefficient $0''.0077$ for Greenwich and for the international stations on the parallel of 39° 08′ is due to the fact that, although the latitudes themselves are quite different, the latitude factor $\sin 2\phi$, is practically the same for both.

EARTH TIDES AND THE INTERNATIONAL TIME SERVICE

The International Time Service (Bureau international de l'Heure) studies the apparent differences of longitude as revealed by wireless time signal sent out from and received by various observatories in all parts of the globe. Mention has been made in previous reports of apparent tidal effects on the various differences of longitude. The variation of latitude, irregularities in the rotation of the earth and possible continental drift are also involved.

The subject is a little aside from the subject of this report, so the interested reader is referred to the publications of the Bureau international de l'Heure, also to an article by N. Stoyko.[6] In this article references will be found to other papers by Stoyko on the same subject.

[5] Which kind of error is not clear. Reporter.

[6] Variation apparente des longitudes. Acta Astronomica, vol. 3, July 1938, p. 98.

CHAPTER III, VARIATION IN THE VALUE OF GRAVITY AT ONE PLACE

MR. TRUMAN'S WORK

Mr. O. H. Truman [1] of Salt Lake City, Utah, has devised a portable gravimeter. Mr. Truman says in the article referred to:

"The gravity meter used by the writer operates on the spring and suspended-weight principle, with a device by which the effective length of the spring is greatly increased, so as to bring the deflections due to small changes of gravity up to the point where they can be successfully magnified by the optical-lever principle to such a degree that they can be read. The machine was invariably read by a null method, a line in the field of the eyepiece, which moves with variations of gravity, being moved by a micrometer screw acting upon a very weak auxiliary spring until it was upon a fixed line of a scale; and then the changes of gravity were obtained from the screw reading. Because of the almost incessant motion of the movable line to and fro, owing to microseisms, this method is much better than trying to estimate the position of this line at varying points along the scale. In fact, in this way the microseisms, which would otherwise be an evil, actually help the accuracy of reading, provided they are no larger than is common at the Gulf coast.

"Temperature is, of course, one of the great enemies in designing such an instrument. To prevent errors due to it, the instrument proper is inclosed in a heavy nonconducting case, in which the temperature is maintained very closely by a thermostat, at a point a few degrees above the highest expected air temperature. The temperature of the instrument, as shown on a Beckman thermometer, fluctuates a few hundredths of a degree, partly in a daily cycle, depending upon changing outside temperature; but these changes have been ascertained, by separate tests, to be wholly without effect upon the readings.

"In the instrument as used in the field, no attempt was made to keep the case airtight, since to make it so and to be sure it stayed that way would involve a great deal of trouble. Instead, air-pressure changes were merely corrected for. But in the accurate work to be done here, it was an essential that the uncertainty of these corrections should be entirely removed. At the same time, it was not desired to completely rebuild the machine with a new airtight inner case and new outer cases, etc., which this would have necessitated; instead, the whole apparatus was put inside an airtight case.

"It will be seen that, if this alone were done, the pressure inside the instrument would change with varying room temperature, since a rise of temperature would expand the air outside the insulating case and force some of it into the constant-temperature space. Also, small leaks are always to be apprehended. It was easiest to overcome all the troubles, therefore, by adding an electrically operated pressure-regulator, which would constantly hold the pressure about 50 mm. of mercury below the average outside pressure, and therefore always somewhat below the lowest outside pressures which might occur.

"This pressure-regulator had a mercury column which balanced the inside pressure against a Torricellian vacuum. Whenever the pressure rose, it broke an electrical contact in the mercury, which, through relays, started a small pump and pumped it down again. By providing a small leak, which constantly admitted a little air, together with what might get in through accidental leaks, the absolute pressure inside was kept the same, day after day. It might seem that there would be a pressure cycle, owing to intermittent operation of the pump; but by proper proportioning, this was kept down to a point where it could not be detected with certainty by a microscope on the mercury, and was certainly no more than 0.02 or 0.03 mm.—far too little to matter. Also, by properly choosing the long and short legs of the pressure-regulator, it can be made immune to varying temperature of the mercury and glass. This was done by calculation; and it was later verified, by special tests, that room temperature had no effect upon the gravity readings.

"The objection can be raised to this procedure that the moisture content of the incoming air will vary from time to time, thus gradually changing the moisture content of the air in the case; and it can be calculated that the resulting change in indicated gravity might become very important, although it would only occur slowly over a period of several days. To test this, ingoing air was alternately fed for several days through a bubbling water bottle, which must have caused

[1] Variations in Gravity at One Place. Astrophysical Journal, Vol. 89, 1939, p. 445.

it to be saturated with moisture, and through a calcium chloride bottle, which, with the slow rate of passage of air through it, must have dried the air entirely. Repeating this test several times showed, very gratifyingly, that there was no visible effect upon the gravity readings; and it can only be believed that the calculated effect spoken of above was in some way compensated.

"The instrument was set up in one of the rooms of an ordinary brick-veneer dwelling-house. By bracing the floor underneath securely with small concrete piers, vibration and changes of level due to people moving about were entirely done away with. The instrument was releveled before each reading, by means of sensitive levels; and, needless to say, proper tests had shown that these levels would check the position with the required accuracy. Disturbances due to traffic were wholly unimportant.

"Such a gravity meter is, however, a pretty sensitive seismometer, even though its sensitivity as such is purposely minimized as much as possible by the design. In this case a ground movement up and down of 0.0005 mm. would cause a motion of a whole division in the eyepiece, a very perceptible amount. However, most of the time in the Gulf coast the motion is only two or three of these divisions and, as was stated before, aids, rather than lessens, the accuracy. Earthquakes were very noticeable, when they occurred; and several times they made it necessary to suspend the observations.

"One fact was observed, however, which ought to be mentioned, as it may be an important contribution to the question of the origin of microseisms. During the winter time the Gulf coast is often visited by 'northers.' A norther is a period of very cold weather, with a high wind from the north, frequently initiated by the passage of a 'squall line' or 'wind-shift line,' and may or may not be accompanied by rain. Now during these northers the ground motion may increase to as much as 40 or more of the eyepiece divisions, often compelling the suspension of work. There is never any ground motion of importance excepting with a norther; it is never caused by a wind from any direction, no matter how high, unless coming with a norther. And a norther always brings ground motion.

"It is thus evident that microseisms at the Gulf coast are in some way caused by some peculiar conditions accompanying northers. The writer understands that in Europe they have been thought to be due to the pounding of the waves upon the rocky coasts of Norway at times of storms at sea; but in view of the above-mentioned observation, it seems much more probable that they are caused by something else accompanying the storm, and that the pounding of the waves is merely incidental. On the Gulf coast the shores are mostly low and gently sloping, and the high winds which accompany a norther would be offshore winds.

"In taking the reading, allowance was made for drift, both secular and irregular."

Observations were made at Houston, Tex. ($\phi=29°49'$ N; $\lambda=95°20'$ W. approx.), for a period from December 18, 1936, to April 21, 1937. The solar diurnal variation appears rather strongly in the results. Variations having this period appear in other observations to determine earth tides and have been attributed to meteorological cause, chiefly temperature and pressure, but Mr. Truman believes that the construction of his instrument eliminates effects of this sort. The origin is a mystery to him as it is to the reporter.

The results for the lunar semidiurnal effect were not very accordant when the observations were split up into series of 29 days each, but the average of all [2] gave an observed result 13 percent greater than for a rigid earth and a phase-lag of 4°.4. This agrees excellently with theory, quite as well as could be expected. Houston is rather near the coast, but the semidiurnal tide in the Gulf of Mexico is small.

The observed solar semidiurnal tides were found to be about twice the theoretical tide for a rigid earth with a negative lag of 9°. The first harmonic of the mysterious 24-hour effect may perhaps explain this rather anomalous result.

The O_1-component (period 25.819 hours) was a disappointment. Five 28-day series (the last two overlapping) gave rather discordant results. Usually results from this component are considered rather reliable, as it is a diurnal component and the shifting load of tidal water is usually relatively small, so that the effects of the ocean tide are more nearly negligible than for other components.

The K_1-component (period one sidereal day) was so entangled with the 24-hour component that no satisfactory conclusions could be drawn.

WORK OF THE GULF RESEARCH AND DEVELOPMENT COMPANY

The following are extracts, with interpolated explanations, from a report on a simultaneous test of a number of gravimeters of the Gulf Research and Development Co., of Pittsburgh, Pa.

[2] Mr. Truman notes, what is true of tides in general and of other periodic phenomena, that the amplitudes and phase-lags should be averaged as vectors and not as scalars.

The material was kindly furnished by Dr. E. A. Eckhardt of that organization. Dr. Eckhardt states:

"The work done on this project includes substantial contributions by a large number of men and it would be only fair to refer to the results as being a contribution of the geophysical staff of the Gulf Research and Development Company.

"This preliminary report is prepared to give some of the first results of our analysis of gravimeter observations of tide-producing forces, as it may be some time yet before all of the results are reduced to final form.

"All of our gravimeter field parties were requested to make special continuous observations of diurnal variations in gravity over the week-end of April 1, 1939. All of the 14 field parties responded splendidly and made the observations requested by reading a gravimeter at a fixed location at half hourly periods, most of which covered the interval [3] from 6 p. m., March 31, to 7 a. m., April 3, local standard time. Twelve of the fourteen sets of data so observed were quite usable. Two sets were discarded on account of erratic performance of the instruments used.

"In addition to the short period field observations, continuous recordings of the bifilar gravimeter have been made at the laboratory [4] for the periods January 5 to March 22, 1938, August 26 to November 28, 1938, and March 22, 1939, to the present. It is planned to continue this recording for an indefinite period. However, the value of these records is doubtful, as recent study of them suggests a periodic disturbance which probably is extraneous to any gravity effects. We may or may not be able to evaluate and remove this disturbance so as to use the records.

"Through the cooperation of the United States Coast and Geodetic Survey we have the theoretical tide-producing forces for the Harmarville [5] location, as calculated on the tide-predicting-machine for all of the periods for which continuous observations were recorded. Also, we have carried out approximate calculations for all of the times and positions where short period field observations were made, computed from the six largest diurnal and semidiurnal tide-producing components (M_2, N_2, S_2, O_1, P_1, K_1). From comparison of our approximate theoretical curve computed from these components only, with the complete calculation by the tide-predicting machine, we estimate that the errors of the approximate calculation are less than 10 percent of the amplitude of the theoretical tide-producing forces.

"On the whole, the results of the field observations were surprisingly good. In most cases the gravimeter was considerably more steady and regular in performance than would be expected from normal field operations, due undoubtedly to its undisturbed condition throughout the observing period. The ultimate reading accuracy of our gravimeters is about 0.01 milligal or slightly better. This precision is not really adequate for a reliable study of the details of the diurnal gravity variations but is sufficient to indicate clearly the general nature and amplitude of these changes.[6]

"The greatest source of uncertainty in the analysis of the results is the determination of the drift curve. The curve is based primarily on a running average of the observations taken over a 24-hour period. The values for these averages are shown by crosses along the drift curve. This average should not be sensitive to real diurnal or semidiurnal variations in the observations. It will, however, be sensitive to any curvature of the drift curve which is appreciable within a 24-hour period, and in some cases there is some departure between the drift curve actually used and the running averages, for this reason.

"The values shown on the chart are all in milligal units. There is no uncertainty about the scale, as all of the instruments used have been calibrated over a standard gravity difference at the laboratory which has been established by careful pendulum observations within a probable error of 0.2 percent. From frequent checks of calibrations in the field we are confident that the present scale values of the instruments are correct to better than 1 percent.

"The principal results of the observations are summarized in the table following. This table shows the position, latitude, and longitude of the 12 short period observations. The 'Quality' column is a relative indication of the general regularity and dependability of the observations. The two columns headed 'Range of departure from theoretical curve' show approximately the amount by which the observed diurnal variation exceeded the theoretical tide-producing force in amplitude. Thus a value, for instance, of +0.04 and −0.06 in these two columns means that on the average the peaks of the observed curve were 0.04 milligal higher than the peaks of the theoretical curve and that the valleys of the observed curve were 0.06 milligal lower than the valleys of the theoretical curve. The 'Theoretical range' gives the

[3] This period covers a time when the moon crosses the equator from north to south. The tide was, therefore, nearly semidiurnal. The moon was full on April 4 and in perigee April 1, 1939. The tidal effects are, therefore, relatively large. Reporter.

[4] The laboratory is at Harmarville, Pa. See next note. Reporter.

[5] The research laboratories of the Gulf Research and Development Co. are at Harmarville, Pa. (ϕ=40° 32′.7 N; λ=79° 49′.6 W). It is a few miles northeast of Pittsburgh, Pa. Reporter.

[6] Some account of a diagram showing the results at Carlsbad, N. Mex, is omitted here. The references that occur later on to the chart or curves are to this omitted part. Reporter.

amplitude (from maximum to minimum) of the theoretical curve. The 'Ratio of observed to theoretical' column gives the ratio of the average total amplitudes of the observed and theoretical curves as computed from the departure figures. Thus, for instance, at the Bakersfield, Calif., station the range of departure was +0.04 to −0.06, so that the total amplitude of the observed effect was 0.10 milligal greater than the theoretical. Therefore, the ratio of observed to theoretical is 0.26/0.16=1.6.

Tidal Variation in Gravity

[Chiefly from Mar. 31 to Apr. 1, 1930. Gulf Research and Development Co.]

Location	Latitude N.	Longitude W.	Quality	Range of departure from theoretical curve	Theoretical range	Ratio observation to theory	Remarks
	° ′	° ′		*Milligals*	*Milligals*		
Ardmore, Okla	34 10	97 07	Excellent	+0.03 to −0.03	0.16	1.4	
Alva, Okla	36 49	98 40	Good	+.03 to −.03	.15	1.4	
Bakersfield, Calif	35 23	119 01	Fair	+.04 to −.06	.16	1.6	
Bunkie, La	30 58	92 11	Good	+.06 to −.06	.17	1.7	
Brownfield, Tex	33 11	102 17	Good	+.10 to −.10	.16	2.2	2 instruments.
Carlsbad, N. Mex	32 27	104 14	Good	+.03 to −.04	.16	1.4	
El Dorado, Ark	33 13	92 40	Fair	+.05 to −.05	.16	1.6	
El Campo, Tex	29 12	96 16	Poor	(?) (?)	.18		Incomplete.
Perry, Okla	36 18	97 18	Fair	+.03 to −.05	.15	1.5	
Santa Barbara, Venezuela	9 37	63 37	Good	+.04 to −.04	.22	1.4	
Varde, Denmark	55 38	351 30	Good	+.01 to −.02	.07	1.4	
Wink, Tex	31 46	103 09	Fair	+.05 to −.04	.17	1.5	
Harmarville, Pa	40 33	79 50	(?)		.13		Incomplete.
Average				+.043 to −.047		1.57	

"It will be observed that in all cases the observed variations are greater than the theoretical, the average ratio being 1.57. The range of the amplitude ratio is from 1.36 to 2.25. We are not yet prepared to say whether or not there is any very definite correlation between the amplitude ratio and the geological or geographical situation of the point of observation. Of course, since the observations were made incidental to oil prospecting, they are all in areas of thick sedimentary deposits.

"We are not yet prepared to say that there is any distinct phase shift between the observed and theoretical gravity variations. In all cases the observed maxima and minima are fairly close to those of the theoretical curve.

"In some cases there are indications of breaks or irregularities in the observed variations which seem to be consistently repeated at 24-hour intervals. There are not enough of these and the duration of the observations is too short to establish their existence with certainty, but we feel that the indication is definite enough to suggest an interesting possibility that the earth's surface may not respond uniformly to the tide-producing forces. Perhaps with better instruments and longer observations it would be possible to establish a connection between such discontinuities and microseisms."

The ratio, 1.6, of the observed tidal change in gravity to the theoretical change on a rigid earth, that is, the value of $1+h-\frac{3k}{2}$, comes out unexpectedly large. Probably a harmonic analysis would reduce it somewhat.

For the only available information relative to the German earth-tide campaign with gravimeters see page 4.

CHAPTER IV, TIDES IN WELLS AS A MANIFESTATION OF EARTH TIDES

INTRODUCTION

Tides in wells have been known from early times. Pliny the Elder (23-79 A. D.) says :[1] "At Gades (Cadiz), which is very near the temple of Hercules, there is a spring enclosed like a well, which sometimes rises and falls with the ocean, and, at other times, in both respects contrary to it. In the same place there is another well, which always agrees with the ocean. On the shores of the Baetis (the Guadalaquivir), there is a town where the wells become lower when the tides rise, and fill again when it ebbs; while at other times they remain stationary. The same thing occurs in the town of Hispalis (Seville) while there is nothing perculiar in the other wells."

The late A. C. Veatch published a discussion of tides in wells, with special reference to wells in Long Island, N. Y.[2] It is of course possible that in the case of wells near the coast there is hydrostatic connection between the ocean and the well on shore, either direct and unobstructed, or more or less obstructed by porous rock or soil acting as a filter.

In the case of the Long Island wells, however, Mr. Veatch considers that the tidal effect is generally due to the deformation of the ground under the load of tidal water which acts on the reservoir of water or water-bearing stratum that supplies the well.

Mr. Veatch's monograph is not entirely confined to wells on Long Island. There is a two-page bibliography in fine print (pp. 67-69 of monograph), from which it appears that fluctuations in wells have often been attributed to the tide; all of the wells appear to be fairly near the sea coast.

Schureman [3] has published an article on tides in a well at Longport, N. J., near Atlantic City. His conclusion is that the tidal effect of the Longport well was due mainly to the varying load of tidal water; that is, by what has been called in these reports the secondary effects of the tidal forces. The tidal forces, themselves, appeared to have little effect but the level of the water in the well was affected by the actual level of the ocean water, whether that level was due to the normal effect of the tidal forces or to a storm.

However, there are wells remote from the coast that show tidal effects, and since the pressure of the load of tidal water must be unimportant at such places, we are driven to the conclusion that we are dealing with the direct effects of the tide-producing forces. Four rather clear-cut cases have come to the reporter's attention and are here mentioned for the sake of completeness. Probably these four cases are not the only ones. Two antedate the period normally covered by this report and are included for the sake of completeness, especially as the subject is now mentioned for the first time in this series of reports.

DUX (DUCHOV), BOHEMIA

In 1879 a whole connected series of lignite mines near Dux (Duchov) in Bohemia ($\phi=50°37'$ N; $\lambda=13°44'$ E.) was inundated. It was observed that the water did not rise regularly in the main shaft. In fact, a lunar effect was suggested. Suitable apparatus for measuring the level was installed and measurement was made over a period of about five months (April 8-September 15) and analyzed. The results showed a definite dependence on the barometer and clearly marked lunar, solar, and declinational effects. The observations were discussed before the days of the Darwinian tidal harmonic analysis. The reporter hopes sometime to put them into the more modern and more easily interpreted form. Duchov is about 300 miles from the Mediterranean and its arm, the Adriatic, also about the same distance from the North Sea. The tides in all these seas are inconsiderable. Moreover, Duchov is so far above sea level that any hydrostatic connection with the ocean is unthinkable.

The sources of information are:

(1) F. W. Klönne. Die periodische Schwankungen in den inundierten Kohlenschächten von Dux in der Periode vom 8. April bis 15 September 1879, Sitzungsberichte der Kaiserlchen Akademie der Wissenschaften in Wien. Vol. 81, p. 101.

[1] *Historia Naturalis*, book II (Bostock and Riley's translation) quoted by R. A. Harris, Manual of Tides, pt. I, p. 393. Appendix No. 8 to Report of Superintendent of U. S. Coast and Geodetic Survey for 1897.

[2] A. C. Veatch. Fluctuations of the Water Level in Wells, with Special Reference to Long Island, N. Y. Water Supply and Irrigation Paper No. 155 of the U. S. Geological Survey. Washington. 1906.

[3] Paul Schureman. Tides in Wells. The Geographical Review. Vol. 16, July 1926, p. 479.

This article sketches briefly the geology of the region, describes the apparatus used for measuring the height of the water and gives tables of the observed heights.

(2) Guilio Grablovitz. Sul fenomeno di marea osservato nelle minieri di Dux in Boemia. Bolletino della Società Adriatica di Scienze Naturali in Trieste, Vol. VI, 1880, p. 34. This publication describes the solar, lunar, and barometric effects.

The solar effect on the level of the water is represented by Grablovitz as the sum of two trigonometric terms

$$1.07 \cos (t-157°)+0.73 \cos (2t-168°)$$

where t is the hour angle of the sun[4] and the coefficients are in centimeters. There is, of course, no appreciable solar diurnal term in the tide-producing forces. The term $1.07 \cos (t-157°)$ represents partly the uneliminated effects of solar K_1 and P_1, for during the period discussed the sun was north of the equator, partly the effects of temperature and pressure and other phenomena having an approximately diurnal period. The phenomena may have first harmonics that affect the term in $2t$.

There is little profit in guessing why the term in $2t$ comes out just as it does, although it corresponds in period to the well-known tide-producing force of the sun, until we have some mental picture of the mechanism whereby this force acts on an underground reservoir of water high above the sea. The possible presence of a harmonic of t has been noted. The other thing to be noted is that the phase is nearly inverted; that is, the phase lag is nearly 180° instead of nearly zero, as it is for the normal tide in the earth itself. This characteristic will be found in other cases of tides in underground waters.

The lunar effect on the water level is represented by:

$$0.16 \cos (t-84°)+1.29 \cos (2t-174°)$$

where the amplitudes are in centimeters, as before, but t is now the hour angle of the moon. The semidiurnal term is the significant one. The phase is again approximately inverted from that of the earth tide.

The size of the solar effect as compared with the lunar suggest that the effect is due to the tidal forces in the crust rather than to any pressure of tidal water, if we can imagine such a pressure to be particularly effective as far inland as Duchov. We have for the ratio of solar semidiurnal to lunar semidiurnal effects (S_2/M_2) (amplitudes),

Theoretical ratio of forces	0.465
Ratio at Duchov	.57

Prevailing values in the ocean are from 20 to 33 percent.[5] This argument seems in fact to prove too much, but some allowances must be made for our ignorance of how much of the solar semidiurnal effect is really tidal.

Another criterion of whether the observed effect is due mainly to the tidal forces in the earth or to the variable load of tidal water is found by comparing the diurnal components with the semidiurnal. The tides in the ocean are prevailingly of marked semidiurnal type with little diurnal inequality, whereas the pure earth tide in middle latitude shows a marked diurnal inequality in the low waters and is sometimes of the mixed type, particularly at quadratures and in years when the moon attains a high declination.

Grablovitz's analysis does not bring out the diurnal tides as well as modern methods of analysis do, though he does discuss the subject and though his curves show a marked inequality in the low waters at quadratures, as compared with the semidiurnal effect. His figures show in a general way that the declinational effect is about what might be expected from the diurnal tidal forces in the body of the earth. For this reason particularly, the reporter hopes in the not too distant future to subject Klönne's fundamental data to harmonic analysis.

To obtain some idea of the reliability of his numerical results Grablovitz divided the observations into five periods. The results for each period separately agree in a general way with the results for the other periods and with the results obtained by discussing all available observations.

It was noted that the barometer affected the level of the water in the flooded mine. In general the higher the barometer the lower the water. The actual level was influenced by many things; the inflow of underground spring water into the flooded mine, rain, and change in level due to the change in the area flooded. Grablovitz's conclusion is that if we arbitrarily attribute everything to the barometer and assume a steady rise due to inflow of underground water, then the change in the level of the water is about 5.4 that of the barometer, the two

[4] Probably local mean time was used, though it is not so stated explicitly. (Central European time, now used in this region, was then unknown.)
[5] Of course, but very exceptionally, larger ratios are found.

effects being in opposite directions, as was to be expected. If the effect were purely static and if the water could betake itself to an infinite distance, the factor should be 13.6 instead of 5.4, because mercury is about 13.6 times as heavy as water.

TARKA BRIDGE, CRADOCK DISTRICT, SOUTH AFRICA

The work at Tarka Bridge was done mainly for irrigation purposes. The discovery that the water level in some of the boreholes shows a tide was incidental but when the discovery had been made the problems were deemed worthy of scientific investigation. The following account is made up mostly of verbatim quotations from Young's[6] account; omissions are not indicated.

"The position of the boreholes that I have studied in most detail is on a farm formerly called Driefontein, but now known as Tarka Bridge. It lies about 15 miles to the southeast of the town of Cradock,[7] and on it the junction of the Tarka River and Fish River occurs. The arable land forming the greater portion of the farm lies about 2,700 and 2,800 feet above sea level, while the mountainous portions in the southeast and east rise much higher, probably exceeding 4,000 feet in parts.

"Since the present owners, Messrs. Rayner and Roberts, came into possession in 1903, eight boreholes have been made, six of which are in the vicinity of natural springs. Five of these six have tapped supplies of sulphurous water which flows at the surface of the boreholes.

"The boreholes are all of 6-inch diameters, and the following boreholes have been named as follows:

"No. II is 204 feet deep, No. III is 167 feet deep, No. IV is 65 feet deep, No. V is 65 feet deep, and No. VI is 225 feet deep.

"As Messrs. Rayner and Roberts use the water of the boreholes for irrigation, it is only at certain limited periods that opportunities of using the recording apparatus occur.

"My personal observations began in January 1905. About the end of 1904 I heard that Messrs. Roberts and Rayner had noticed a serious periodic variation in the apparent yield of borehole No. V. They said that the variation was like a 'tide' and although I was very skeptical about the propriety of this term as applied to the fluctuation, I thought that this report was worthy of some inquiry.

"In January 1905, I spent a fortnight on the farm and made the following series of measurements. The measurements were continued at intervals until the present year." (Apparently 1912. An instrument was set up and curves obtained.)

"The most striking general feature of the curves obtained is their wonderful regularity. Each day's record shows two maxima and two minima, and these turning points occur at very regular and approximately equal intervals, as estimated by general inspection. By determining the first and last maxima in the May week and dividing the time by the number of complete wave lengths on the record, I quickly obtained by approximate estimate the average wave period as 12½ hours. By a similar process the records for the June fortnight gave the same result.

"The amplitudes (ranges) of the great 12½-hour period waves were obviously subject to considerable variation. A general inspection brought to light the fact that these amplitudes (ranges) attained a maximum of about 1½ inches on June 17, which happened to be the date of the full moon, and attained a minimum of less than ¾ inch when the moon reached its first quarter. Another minimum was attained about May 26 when the moon reached its last quarter."[8]

Mr. Young did not at first find any immediate connection between the height of the barometer and the height of the tide, but later by discussing the observations by Chrystal's method of residuation, he did. He comes to the conclusion: "A comparison of the curve H (the observed curve with the tidal fluctuations removed) makes it apparent that the former is substantially the vertical inversion of the latter, the maxima of one curve corresponding to the time of minima of the other." He repeats in another place his statement about maximum range and new and full moon and minimum at quadratures. He also states that comparison of heights with a thermograph record showed no apparent connection between temperature and height. He makes a more careful study that substantiates his earlier conclusions about the time between maxima and minima and compares the tides in the boreholes with tides in the ocean. He notes the obvious dissimilarity in range, 1–2 inches for the borehole as against several feet on the coast.

[6] Andrew Young. Tidal Phenomena in Inland Boreholes near Cradock. Transactions of the Royal Society of South Africa. Vol. 3, 1913, p. 61.

[7] Cradock: $\phi=32°10'$ S; $\lambda=25°37'$ E. The place is about 100 miles from the coast.

[8] This is pretty good evidence that these effects are mainly due to the tidal forces in the body of the earth and not to the loading effect of the water at a distance. There is no "age" for the observed springs and neaps and their ranges are approximately in the ratio of the forces. In the case of ocean tides the solar effect is usually smaller than would be inferred from the forces. Reporter.

He then notes another dissimilarity:

"The amplitude of the diurnal inequality seems to be relatively greater in the Tarka Bridge curve than in the coastal curves." This is as it should be if the tide in the boreholes is due to tidal forces in the body of the earth. He also finds evidence of a solar semidiurnal tide, also a K_1 component but apparently not an O_1 component.

The curves in Mr. Young's article are on such a small time-scale that numerical evaluation is difficult, but it is evident that low water and not high water occurs at about the time of the moon's transit, as was the case at Duchov. The low appears to lag a little behind the transit.

CARLSBAD, N. MEX., AND IOWA CITY, IOWA

A fairly complete account for the tidal well at Carlsbad, N. Mex., transmitted by the U. S. Geological Survey, is available. The account of the work at Iowa City is less full.

The details about the Carlsbad well are from a manuscript report by C. V. Theis, Geologist in Charge of Ground Water Investigation in New Mexico.

The well is in latitude 32°18′ N., longitude 104°00′ W. Its altitude is 2,955 feet and its distance from the sea about 500 miles. The water-stage recorder is apparently accurate to 1/100 inch; the time scale is 20 hours = 1 inch. The mercury barometer is read hourly, day and night, at the refinery of the United States Potash Co., 1¾ miles away.

The well is an artesian well 265 feet deep, tapping a water-bearing stratum of gypsum. The water is of density 1.20 because of the dissolved sodium chloride. No water is drawn from the stratum and the nearest natural outlet seems to be 5 miles away. The water level responds to barometric fluctuations, falling with barometric pressure without appreciable lag. The fluctuations amount to about 70 percent of those of a theoretical barometer using instead of mercury a fluid having density of 1.20.

All other water-bearing strata encountered in drilling the well, except the gypsum stratum in question, were cemented and cased off. The well is unusual in that an accurate log is available, that it was drilled under close technical supervision and that it taps a water-bearing stratum not subject to artificial withdrawals of water.

The automatically recorded graph of the water level was corrected for the barometer on the 70 percent basis just indicated. The residual curve shows practically no correlation with the barometer; it does show a slow fluctuation probably connected with the rate at which the stratum is supplied with water, and on this are superimposed the semidiurnal fluctuations. The residual curve (after correction for barometer) reaches a minimum at the times of the transit of the moon[9] and the fluctuations are most pronounced at times of new and full moon. At the quadratures the fluctuations are barely recognizable, being close to the limit of instrumental error. The maximum tidal fluctuation was apparently 0.06 or 0.07 foot.

It is planned to continue indefinitely the record of water level and of the barometer. A microbarograph will probably be installed at the well in an attempt to obtain a better barometric correction. When time permits, an average fluctuation over the lunar month will be derived.

Further discussion of this tide at Carlsbad is given at the end of this section.

The Iowa City well in Iowa is located in about latitude 41°30′ N., longitude 91°34′ W. Iowa City is even farther from the ocean than Carlsbad, N. Mex. The well there is drilled in limestone to a depth of 755 feet and the water is fresh instead of briny. The effect of the barometer was of the same general nature as at Carlsbad, the effect being 75 percent of that of a barometer composed of water (density 1.00). The tidal fluctuations were about twice those observed at Carlsbad and, as at Duchov, Tarka Bridge, and Carlsbad, low water occurs at about the time of the moon's transit. The solar inequality in amplitude was recognized. Nothing is said about the diurnal inequality. Further observations are planned.

It is remarkable that in all four cases cited, Duchov, Tarka Bridge, Carlsbad, and Iowa City, the *lowest* level of the water occurs at the time of the moon's transit. If we assume that we are dealing with vast underground lakes—a most improbable assumption—the time of tide, high or low may be estimated by the equilibrium theory, which is applicable to large, but not too large, inland lakes and seas.

According to this theory in simplest form the level of the water surface may be considered a plane with its normal tilted to coincide with the direction of gravity at the center of gravity of the surface as affected by the tidal forces. The time of high and low water would therefore depend on the position of the point of observation on the edge of the lake or sea and the range on the size of the lake or sea.

The amount of rise and fall would depend on the dimensions of the lake or sea. Apart from other difficulties, we can hardly imagine underground lakes of the necessary size. There is also the improbability that all four places here discussed would have the same situation with respect to their respective underground bodies of water.

[9] About as at Duchov and Tarka Bridge. Reporter.

The most probable explanation of tides in wells seems to be that they are due to a tidal decrease in the volume of the reservoir of underground water. This decrease forces the water into the mineshaft, well, or borehole, thus concentrating the entire apparent effect of the compression there. This serves to magnify the tidal effect. Compression corresponds to high water. Low water corresponds to an increase in the volume of the reservoir.

Why low water should take place at about the time of the moon's transit remains to be explained. This is true in all the four cases just discussed. Of course, this does not prove that the rule is general, but the conditions in the four cases were so different that some general explanation seems called for. (See appendix to this report, p. 23.)

Dr. C. V. Theis in a paper presented to the Washington meeting [10] reports tidal fluctuations in a well at Conchas Dam, N. Mex. ($\phi=35°24'$N., $\lambda=104°11'$W.). The range is given as perhaps 0.035 foot; the relation of high or low water is not stated. When Dr. Theis' paper was presented the record was too short to yield accurate results.

DISCUSSION OF TIDES IN THE WELL AT CARLSBAD, N. MEX.

The reporter has made a few preliminary computations based on the figures available in Dr. Theis' paper, namely, the depth of the water in the Carlsbad well, corrected for barometric pressure, for every even solar hour up to and including 12 hours before and after the moon's upper transit. These figures were given separately for new and full moon and for the first and last quarter. The figures were interpolated for lunar hours and analyzed harmonically in the usual way. The expression below is for the tidal oscillations in the depth of the water surface below the zero of the scale, that is, for the tide inverted.

Spring tides (new and full moon): $0.0249 \cos (2t+1°.8)$.

Neap tides (first and last quarters): $0.0058 \cos (2t+9°.3)$.

Mean of springs and neaps: $0.0153 \cos (2t+3°.2)$.

The unit of amplitude is the foot and $2t$ is twice the true hour angle of the moon.

The tides are inverted with respect to the forces, low water occurring when the moon crosses the meridian, except for a small and uncertain negative lag. The analysis gave lunar diurnal terms, that is, terms of the form $a \cos (t-\kappa)$, but these were small and uncertain and seemed to have no real relation to the small term of this kind that, from theoretical considerations, might be expected to occur.

The amplitude 0.0153 foot is approximately the amplitude of the M_2 component for 1938–39. The mean value would be a trifle smaller, 0.0150 foot. The difference $0.0249-0.0058$ is approximately 2 S_2 (or more accurately 2 $(S_2+\mu_2)$, where μ_2 is a small tidal component caused by the lunar perturbation known to astronomers as the variation. Neglecting μ_2 and the reduction of M_2 to mean amplitude we find for S_2/M_2

$$\frac{S_2}{M_2}=\frac{\frac{1}{2}(249-58)}{153}=0.62$$

The two neglected reductions have opposite effects on this ratio.

This value is even larger than was found at Duchov, Bohemia. The results seem accurate enough to show that the ratio at both places exceeds 0.46, the ratio of the corresponding tidal forces.

This value of the ratio tends to prove, if any proof be needed, that these tides in wells are not primarily due to pressure effects of the tides in distant oceans. If they were so due, then the ratio S_2/M_2 might be expected to be about the same as it is in those oceans; almost everywhere in the oceans this ratio is less, and markedly less than the theoretical value 0.46 and not greater, as here. Why the ratio S_2/M_2 for tides in wells should exceed this theoretical amount is not clear.

In this connection analysis for the diurnal components K_1, O_1, and P_1 would be of considerable interest. The reporter would have made it if data had been available. If tides in wells are like other earth tides the ratios of the amplitudes of these components to one another and to the amplitude of M_2 would be approximately the same as the ratios of the corresponding tidal forces; moreover the phase-lags should be nearly zero. These conditions are almost never all fulfilled in the ocean tides.

[10] Earth Tides expressed in Fluctuations of Water Level in Artesian Wells in New Mexico. Duplicated, not printed.

CHAPTER V, MISCELLANEOUS

BAROMETRIC EFFECTS

Barometric effects have been mentioned in connection with tides in wells. The reason for such an effect is immediately obvious. The most obvious part of it is the effect of changing pressure on the level of the water itself, an effect that would exist even if the earth's crust around the well were perfectly unyielding. But we know, of course, that the crust is not unyielding, also that some of the effect is due to this yielding of the ground.

The yielding of the ground operates also in observations with the horizontal pendulum, as was noted by Lettau [1] in two articles.

It was early realized that barometric highs and lows could cause considerable variations from the theoretical slope but Lettau's articles constitute the beginning of an observational study.

INSTRUMENTS

Among the instrumental developments not elsewhere noted in this report should be mentioned the two-spring gravimeter of Sorber.[2]

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[1] Heinz Lettau. (1) Lotschwankungen unter dem Einflusz von Gezeitenkräften und atmosphärischen Kräften. Gerlands Beiträge zur Geophysik. Vol. 51, 1937, p. 250. (2) Über die unmittelbare Einwirkung atmosphärischer Kräfte auf die Erdkruste. *Meteorologische Zeitschrift.* 1937, p. 453.

[2] Heinz Sorber. Über ein Zweifeder-Gravimeter. Physikalische Zeitschrift, 37 Jahrgang, 1936, p. 599.

CHAPTER VI, THEORETICAL DEVELOPMENTS

Two theoretical developments have come to the reporter's attention. They are of very different kinds.

The first is the use of selected days or short periods for harmonic analysis instead of the whole period available. This method is explained by Voit.[3] There are certain advantages in choosing short but especially undisturbed periods for analysis, to the exclusion of the remainder of the observations. Furthermore the ordinary methods of harmonic analysis require a *complete* series of observations, so that gaps must be filled by some process of extrapolation or estimate, a procedure that may falsify the results somewhat. It may be remarked, however, that *all* the observations must certainly be better than part of them and that all the observations or all the good ones may be utilized, if sufficient labor be expended in computation, though this would require the developments of methods to fit special cases not covered by the ordinary methods of harmonic analysis.

The method of evaluating the harmonic constants from selected periods only was used for the observations of the deflection of the vertical reported by Professor Tomaschek.

In evaluating the effects of the pressure of the shifting load of tidal water Boussinesq's well-known method has commonly been used. This neglects the curvature of the earth and treats it as a semi-infinite solid of uniform physical properties throughout. It is possible to dispense with the assumption of uniformity both as to elasticity and density. The following abstract from the Zentralblatt für Geophysik, Meteorologie und Geodäsie, volume I, January 28, 1938, page 392 will serve to indicate what has been done along this line.

"Sezawa, Katsutada, and Kiyoshi Kanai: On the elastic deformation of a stratified body subjected to vertical surface loads. Bull. Earthquake Res. Inst. Tokyo 15, 359–369 (1937).

"Terazawa gave a criterion that, in the case of a semi-infinite body, the vertical component of surface displacement caused by a vertical surface load should be directed downward (J. Phys.-Math. Soc. Jap. 1, 141–143 (1927)). The present paper deals with the same problem for the case of a stratified body. The authors restricted the problem to a two-dimensional case so as to apply Airy's function. By examining the positive or negative sign of the final solution of the form of Fourier's double integral, it was possible to ascertain that Terazawa's condition is valid even in the case of a stratified body. With a view to confirming the theory, some cases of assigned numerical relations were calculated. Terazawa."

From the paper itself it appears that both density and elastic constants are assumed to vary with the depth.

BIBLIOGRAPHY

This brief bibliography is not in the least intended to cover the whole subject of earth tides, nor even the entire mathematical theory of the subject. Its main purpose is to list the articles that treat mathematically of the effect of the earth's physical properties on earth tides, especially on the quantities h, k, l, for which formulas are given on page 2, but which are there subject to very drastic simplifying assumptions. The purpose of most of the papers here listed is to evaluate these quantities, or equivalent ones, for an earth presumed to approximate more to the actual earth than does the incompressible sphere for which h, k, and l are given in this report.

The designations h, k, and l are not always used. German practice interchanges the meanings of h and k as compared with English and American practice. As was noted, there is almost nothing explicitly written about the quantity l. This is the quantity that occurs in astronomical observations only.

1. L. M. Hoskins. A Simple Method of Determining the Free Nutation of a Yielding Spheroid. Bulletin of the American Mathematical Society, volume 9, 1903, page 299. Abstract only. Apparently anticipates at least some of the results of Nos. 4 and 5.

2. G. Herglotz. Über die Elastizität der Erde bei Berücksichtigung ihrer variablen Dichte. Zeitschrift für Mathematik und Physik. Vol. 52, 1905, p. 275.

3. Alfred Brill. Über die Elastizität der Erde. (Inaugural-Dissertation.) Göttingen, 1908.

4. A. E. H. Love. The Yielding of the Earth to Disturbing Forces. Proceedings of the Royal Society, vol. 82 A, 1909, p. 73.

5. J. Larmor. The Relation of the Earth's Free Precessional Nutation to its Resistance against Tidal Deformation. Proceedings of the Royal Society, vol. 82 A, 1909, p. 89.

[3] H. Voit. Über eine einfache Methode zur Ermittelung der wichtigsten Tiden bei Schwere- und Lotschwankung. Zeitschrift für die gesamte Naturwissenschaft, Heft. 11, 1939, p. 443.

6. A. E. H. Love. Some Problems in Geodynamics. (Adams Prize Essay.) Cambridge (England), 1911. Especially Chapters IV–VIII.

7. W. Schweydar. Untersuchungen über die Gezeiten der festen Erde und die hypothetische Magmaschicht. Publication No. 54 of the Prussian Geodetic Institute. 1912.

8. W. Schweydar. Harmonische Analyse der Lotstörungen durch Sonne und Mond. Publication No. 59 of the Prussian Geodetic Institute. 1914.

9. W. Schweydar. Die Polbewegung in Beziehung zur Zähigkeit und zu einer hypothetischen Magmaschicht der Erde. Publication No. 79 of the Prussian Geodetic Institute. 1919.

10. L. M. Hoskins. The Strain of a Gravitating Sphere of Variable Density and Elasticity. Transactions of the American Mathematical Society. Vol. 21, 1920, p. 1.

In regard to the quantity l, about which so little has been written, the reporter has a reprint of this article sent him by the late Professor Hoskins, in which the little table on page 40 of the memoir is completed in pencil in Professor Hoskin's handwriting to show the values of l in relation to other quantities according to the assumptions made by him about the earth.

The three additional columns, together with two columns in print, are as follows, the added columns being on the right:

$\frac{\mu_0}{\mu_1}$	$\frac{1+k-h}{q}$	$l-\frac{2\alpha_1}{c}$	$k-\frac{34f}{35c}$	$1+k-l$
2	0.217	0.013	0.428	1.415
5	.576	.115	.326	1.211
$6\frac{2}{3}$	.695	.128	.261	1.133
10	.779	.114	.201	1.087

k, h, and l have the same meaning as has been given them in this series of reports. q, α_1, c, and f are part of Hoskins' notation. μ_0 and μ_1 are the moduli of rigidity at the center and surface. We now suppose the central part to be fluid or nearly so. See No. 14.

11. R. Stoneley. The Elastic Yielding of the Earth. Monthly Notices, Royal Astronomical Society, Geophysical Supplement, vol. I, No. 7. June 1926, p. 356. A summary of results rather than a detailed exposition.

12. L. Rosenhead. Tides in a Two-layer Earth. Monthly Notices Royal Astronomical Society, Geophysical Supplement, vol. II, No. 4, Oct. 1929, p. 171.

13. A. Prey. Über die Elastizitätskonstante der Erde. Gerlands Beiträge zur Geophysik. Vol. 23, 1929, p. 379, and vol. 44, 1935, p. 77.

14. Harold Jeffreys. The Earth, its Origin, History and Physical Constitution. 2d ed. Cambridge University Press, 1929, p. 238.

CHAPTER VII, CONCLUSION

The conclusion of the Edinburgh report still holds good; it reads:

"The essential difficulty of the problem of the earth tides remains what it has been in the past. It is not the difficulty of getting good records of phenomena that are clearly of tidal origin, although considerable technical skill is needed to devise and operate the instruments. The real difficulty is one of interpretation, interpretation in terms of the earth as a whole and interpretation in terms of the local and regional peculiarities of crustal structure."

In previous reports the reporter was inclined to minimize the effect of local geological structure on observed earth tides. It now seems to him that local structure may be important and that observations of earth tides may be made to yield information about such structure. Before this happens, however, our ability to interpret must grow with study and experience.

The difficulties in the way of a direct calculation of the effects of the shifting load of tidal water remain substantially as before. The Liverpool Tidal Observatory has, however, found an ingenious way to dodge the difficulty to a certain extent. The assumptions on which the method is based are plausible and probably valid, or nearly enough so, for many parts of the earth. The method requires an analysis of the record for several tidal components.

This is the first report of this series of reports to say anything about tides in wells remote from the sea. It is a fascinating subject, perhaps because of the very difficulties in the way of a quantitative explanation. It is to be hoped that the subject will be studied further.

Please send any corrections or additions to the reporter, Walter D. Lambert, U. S. Coast and Geodetic Survey, Washington, D. C., U. S. A.

CHAPTER VIII, APPENDIX, NOTE ON TIDES IN WELLS

By C. L. Pekeris, *Department of Geology, Massachusetts Institute of Technology*

In his report on earth tides read at the 1939 Congress of the International Union of Geodesy and Geophysics, Mr. Lambert described four cases of observed tides in wells which are situated far enough inland to be unaffected by the tidal load of the coastal waters. Mr. Lambert suggests that these tides are the direct effect of the tide-producing forces and are a manifestation of earth tides. Evidence for this conclusion he sees in some of the features which are suggested by the available observations, namely the relatively large ratio of the solar semidiurnal to the lunar semidiurnal components, the marked diurnal inequality, etc.

In all the wells one finds the remarkable phenomenon that for semidiurnal tides the lowest level of the water occurs at the time of the sun's and moon's transit.

It seems that such a phase relationship would be expected if, as suggested by Lambert, the tides in the wells result from the compression and dilatation in the water-bearing strata due to earth tides. Since the apparent tidal forces of the moon pull outwards the portions of the earth that are facing the moon and those which are at its antipodes and tend to push inwards the intermediate regions, it is clear that at the time of the moon's transit, when the earth tide is high, the region underneath the station is under tension and is dilated, while 6 hours later, when the earth-tidal displacement is downwards, it is under compression.

Compression in the water-bearing region would squeeze the water into the well and would thus bring about a rise of the water level at a time when the displacement due to the earth tide is downwards. In order to verify the above inference and also to determine the order of magnitude of the dilatation which is associated with the earth tides I have used the solutions of the tidal problem given by Hoskins [1] and Stoneley,[2] in which compressibility was allowed for.

Stoneley computes the bodily tide in a gravitating sphere in which, according to Wiechert's hypothesis, the density has the constant value of 8.21 in a core extending from the center to 0.78 of the radius, and the constant value of 3.12 in the remaining outer shell. In the core, the values $\mu=38.2\times10^{11}$, $\lambda+2\mu=134.7\times10^{11}$ are adopted while for the shell these constants are assumed to be given by

$$\mu=(50.78-45.71\xi)\times10^{11}$$

$$\lambda+2\mu=(164.0-147.5\xi)\times10^{11}\ \text{dynes/cm}^2,$$

where ξ denotes the distance from the center in units of the earth's radius.

The perturbing potential is of the form $\epsilon r^2 P_2(\cos\theta)$ and is denoted by W_2. One finds for the radial displacement u and the dilatation Δ the expressions

$$u=\frac{1}{r}(2F+r^2G)W_2$$

$$\Delta=\left(\frac{2}{r}\frac{dF}{dr}+r\frac{dG}{dr}+5G\right)W_2,$$

where F and G are functions of r for which Stoneley assumes the following form:

$$F(r)=A_0\frac{r^2}{a^2}+A_1\frac{r^2}{a^2}\left(\frac{a-r}{a}\right)+A_2\frac{r^2}{a^2}\left(\frac{a-r}{a}\right)^2+\ \ldots\ ,$$

$$G(r)=\frac{B_0}{a^2}+\frac{B_1}{a^2}\left(\frac{a-r}{a}\right)+\frac{B_2}{a^2}\left(\frac{a-r}{a}\right)^2+\ \ldots\ .$$

From these expressions it follows that at the surface, where $r=a$,

$$u(a)=\frac{W_2(a)}{a}(2A_0+B_0),$$

$$\Delta(a)=\frac{W_2(a)}{a^2}(4A_0-2A_1-B_1+5B_0).$$

[1] L. M. Hoskins, Trans. Am. Math. Soc., vol. 21, p. 31, 1920.
[2] R. Stoneley, M. N. R. A. S., Geophysical Supplement, vol. 1, p. 356, 1926.

Stoneley gives the following values for the coefficients:

$$A_0=-0.02149,\ A_1=-0.4693,\ B_0=-0.3834,\ B_1=-0.6190$$

in which the unit is 1/1000. Love's constants h and k for this model are found to be 0.37 and 0.19, comparing favorably with the observed values of 0.38 and 0.22.

It follows that

$$u(a)=-0.4264\ \frac{W_2(a)}{a}$$

$$\Delta(a)=-0.4453\ \frac{W_2(a)}{a^2}$$

or that

$$\Delta(a)=1.04\frac{u(a)}{a}. \tag{1}$$

Now, since in the assumed model the mean density is constant near the surface, the change in density $\delta\rho$ associated with the tidal motion is given by

$$-\frac{\delta\rho}{\rho}=\Delta=\frac{\delta V}{V},$$

where V is the specific volume.

Hoskins solves the tidal problem for a sphere of constant density and elasticity as well as for models in which the density and elastic constants increase with depth. For our purpose it will be sufficient to compute the dilatation for the case of constant density and elasticity. The physical constants enter this problem only through the combinations (λ/μ) and $b=g\rho a/\mu$. As usual, (λ/μ) is assumed to be unity and b was taken to be equal to 2.

With this value of b, h comes out 0.47, as against the observed value of 0.38, but this discrepancy is not serious for our application. Hoskins denotes the disturbing potential by $\frac{cgr^2}{2a}S_2$, where $S_2=\left(\cos^2\theta-\frac{1}{3}\right)$, and $\frac{u(a)}{a}$ and $\Delta(a)$ by e_1S_2 and yS_2. In terms of the coefficients A_n, C_n, D_n that enter his solution

$$e_1=\sum_0^\infty A_n,\qquad y=\sum_2^\infty C_n.$$

To obtain these coefficients, one first determines the C_n and D_n in terms of C_2 and D_2 by Hoskins' equation (121), where n is taken to be zero. From the equations (120) and (122) one then finds that $D_2=-1.783\ C_2$, $A_0=1.689\ C_2$, $C_2=0.2090c$.

It follows that $y=0.0758c$ and, by equation (139), that $e_1=0.2345c$. The latter value checks the value of $0.235c$ given in table II of page 38 of the memoir. Hence

$$\Delta(a)=0.323\frac{u(a)}{a} \tag{2}$$

It is seen from (1) and (2) that the *dilatation has the same sign as the radial displacement* and that it is of the order of magnitude of the ratio of the tidal displacement to the radius of the earth. For the lunar semidiurnal tide this amounts to about 1.5×10^{-8}. A change of level of 1 cm. in a well of 1 meter radius could be brought about by the above dilatation in a water-bearing hemisphere of only 100 meters in radius.

The preceding discussion suggests the possibility of observing earth tides with a dilatometer. A cavity of 10^6 c. c. filled with a liquid and sealed by a capillary would undergo volume changes of the order of 10 cubic millimeters, which could be observed in the capillary. Disturbing effects of temperature fluctuations could be minimized by observing at some depth underground, as is usual in earth-tide work. Plans are under consideration to construct a dilatometer on the above principle or some variation of it with a view to using it for tidal observations.

O