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CHARLES SAWYER, *Secretary*

COAST AND GEODETIC SURVEY
LEO OTIS COLBERT, *Director*

Manual of Leveling Computation and Adjustment

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By

HOWARD S. RAPPLEYE
Chief, Section of Leveling

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PREFACE

United States Coast and Geodetic Survey Special Publication No. 140, "Manual of First-order Leveling" by Henry G. Avers, was first published in 1929 and was reprinted in 1935, 1939, and 1943, without revision except for a supplement which was added to show changes in the general instructions for first-order leveling and to provide for second-order leveling.

During the nearly two decades in which the Manual of First-order Leveling has been in use, changes have taken place in the total amount of leveling accomplished, the uses being made of the resulting data, and the spacing of bench marks along the lines of levels. The wide use of second-order leveling for minor networks within first-order loops and other changes in field and office practice, resulting from experience gained since the manual was first issued, have made a revision necessary.

Special Publication No. 140 was divided into two parts: Part I.—Field Methods, and Part II.—Office Computations. Part I has been revised, expanded, and issued as Coast and Geodetic Survey Special Publication No. 239, "Manual of Geodetic Leveling." Part II has been revised, greatly expanded, and now becomes Special Publication No. 240, "Manual of Leveling Computation and Adjustment."

In developing this manual from Part II of Special Publication No. 140, much additional material has been included, particularly as regards additional methods for the adjustment of leveling, the discussion of the proper treatment of details at junctions of new and old lines, and the combination of old leveling and releveling along the same route.

This publication is intended as a guide to the office personnel engaged in processing field observations, from the time the field records, field computations, and descriptions of bench marks are received in the Office until the computations and adjustments have been completed and the resulting data are ready to be prepared for publication.

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MANUAL OF LEVELING COMPUTATION AND ADJUSTMENT

INTRODUCTION

First-order leveling was first undertaken by the United States Coast and Geodetic Survey in 1878 when work was started on the transcontinental line of levels which was designed primarily to furnish accurate vertical control for the transcontinental arc of triangulation following approximately the 39th parallel from the Atlantic coast in the vicinity of Chesapeake Bay to the Pacific coast in the vicinity of San Francisco Bay. The leveling was extended from a tide station at Sandy Hook, N. J., and elevations were published for a considerable time as "above mean tide level at Sandy Hook." This reference surface became known popularly as the "Sandy Hook Datum," and references to that datum still persist in the publications of some organizations.

The network of lines of levels in continental United States has been gradually expanded until it now includes well over 350,000 miles of first- or second-order leveling. The net was first adjusted in 1899, was partially readjusted in 1903, 1907, and 1912, and completely readjusted in 1929. For a brief history of the development of the vertical control net in the United States, see Coast and Geodetic Survey Special Publication No. 226, "Control Leveling."

Precise leveling, precision leveling, leveling of high precision, first-order leveling, second-order leveling, etc., are terms applied by various organizations to spirit leveling of various grades of accuracy, defined in various ways.

Probably the first attempt to secure international agreement on matters concerning spirit leveling, as to standards of accuracy and methods of defining them, was the action taken in 1867 by the Association for the Measurement of Degrees in Central Europe. That organization adopted the term "precise leveling" and defined it somewhat as follows: Leveling was to be considered as being precise if the probable error of the difference of elevation of two bench marks 1 kilometer apart did not exceed 3 millimeters on an average and was not greater than 5 millimeters as a maximum.

In 1912 the International Geodetic Association, at its meeting in Hamburg, Germany, introduced another class of leveling called "leveling of high precision," for which the limits of errors are plus or minus 1 millimeter per kilometer for the probable accidental error and plus or minus 0.2 millimeter per kilometer for the probable systematic error. This narrowing of the limits was made possible by important advances in the art of leveling during the period between 1867 and 1912.

A translation of the resolution adopted by the International Geodetic Association, at its Seventh General Conference, held at Hamburg, Germany, in September 1912, will be found in appendix A of this publication.

At the Sixth General Assembly of the International Geodetic and Geophysical Union, the International Association of Geodesy adopted a resolution redefining leveling of high precision and precise leveling. According to this definition leveling comes within the international classification of leveling of high precision if its total probable error ϵ, calculated by means of the adopted formulas, does not exceed 2 millimeters per

kilometer. It is classed as precise leveling if this total probable error does not exceed 6 millimeters per kilometer. A translation of the resolution will be found in appendix B of this publication.

The Board of Surveys and Maps of the Federal Government, in its "Specifications for Horizontal and Vertical Control Surveys," approved by the Board on May 9, 1933, and issued in mimeographed form for the guidance of all mapping agencies of the United States Government, classified leveling in four different grades as to its accuracy and specified the criteria to be used in classifying the work. The following is quoted from these specifications.

First-order leveling.—First-order leveling should be used in developing the main level net of the United States. The lines should be so placed that eventually no point in the country will be more than about 50 miles from a bench mark established by leveling of this order. All the lines should be divided into sections 1 to 2 kilometers in length, and each section should be run forward and backward, the two runnings of a section not to differ more than 4 mm. $\sqrt{K}$ or 0.017 foot $\sqrt{M}$, where K is the length of the section in kilometers and M its length in miles.

Second-order leveling.—Second-order leveling should be used in subdividing loops of first-order leveling until no point within the area is much more than 12½ miles from a first- or second-order bench mark. Second-order leveling will include lines run by first-order methods, but in only one direction, between bench marks previously established by first-order leveling, and all double-run lines of leveling whose sections, run in a backward and forward direction, check within the limits of 8.4 mm. $\sqrt{K}$ or 0.035 foot $\sqrt{M}$, where K is the length of the section in kilometers and M its length in miles.

Third-order leveling.—Third-order leveling may be used in subdividing loops of first- or second-order leveling, where additional control may be required. Third-order lines should not be extended more than 30 miles from lines of the first- or second-order; they may be single-run lines but must always be loops or circuits closed upon lines of equal or higher order. Closing checks are not to exceed 12 mm. $\sqrt{\text{kilometers in circuit}}$ or 0.05 foot $\sqrt{\text{miles in circuit}}$.

Leveling of lower order.—Leveling that allows closures greater than the limit stated for third-order work, such as trigonometric leveling, barometric leveling, or "flying" levels, shall be considered as belonging to the lower order of work. No bench marks established by leveling that is less accurate than that of the third order, as described above, shall be marked by standard bench-mark tablets, except that in mountainous regions inaccessible to ordinary spirit-level lines, standard marks may be used on the mountain summits to mark elevations determined by trigonometric leveling; such marks should be stamped in a distinguishing manner. Elevations inferior to the third order in accuracy shall not be published in such a way as to be confused with standard work of the third or higher orders.

In accordance with the recommendations of the Board of Surveys and Maps of the Federal Government, the leveling run by the Coast and Geodetic Survey is classified as first-order, second-order, etc., but the first-order leveling run by this Bureau comes within the international specifications for leveling of high precision and the second-order leveling would be classified under the international specifications as precise leveling.

Geodetic leveling parties of the Coast and Geodetic Survey, operating under special instructions issued for each project, carry on the field work of running lines of first- or second-order levels. The field work is done in accordance with the provisions of the General Instructions for Geodetic Leveling as set forth in Special Publication No. 239, "Manual of Geodetic Leveling." That manual amplifies and explains the General

Instructions and is the guide for the geodetic leveling parties in the field, as this manual is the guide for the office personnel engaged in processing the data received from the field.

It is recommended that any person not already familiar with the field procedures of precision leveling read Special Publication No. 239 prior to any intensive study of this manual.

This manual has two separate purposes to serve:

(1) It is intended to be a manual or reference volume for the experienced computer, and

(2) A text for use in training new computers.

This dual purpose makes it necessary to include some material which the experienced computer will regard as altogether too elementary. It will also require the inclusion of much material, especially on the adjustment of observations, which will be of little or no use to the new computer at first. It should be fully realized that not all of this manual will be of interest to any individual computer at any single stage of his development.

NOTATION

In trying to describe in plain English the various steps in the processing of the geodetic leveling records, one soon comes to a realization of the fact that both the writer and the reader must resign themselves to many long complex sentences, which are both difficult to write and hard to understand. The only apparent alternative seems to be a system of notation which will serve as a sort of "shorthand" in the writing and enable the writer to reduce the descriptions of most of the operations to formulas of a mathematical character.

On the other hand, since the observations made and recorded in the record books involve readings on both the faces and the backs of the rods in both the backsight and foresight positions; mean readings on the faces of the rods in both the backsight and foresight positions; half-wire and total-wire intervals on the faces of the rods in both the backsight and foresight positions; tota s and cumulative totals of cross-wire readings, mean readings, half-wire and total-wire inte vals on the faces of the rods, and cumulative totals of middle-wire readings on the backs of the rods; and differences in elevation in both forward and backward directions, etc.; it soon becomes evident that even the system of notation will become quite complicated and cumbersome.

The system of notation set forth in the following pages seems to be the best way out of a complicated situation. While this system of notation and its use in describing the various operations required in computing and checking the leveling data is admittedly complex, it does have the saving features of exactness and brevity, which is more than may be rightfully claimed for the attempt to cover the multitudinous details by the use of English as the only medium of expression.

There will be certain fundamental notations that must be memorized in order to read this manual efficiently. After these are mastered, their occurrence in the various combinations in which they will have to be used should present no great difficulty.

T denotes a reading of the top one of a group of three horizontal cross-wires on the face of a leveling rod.

M denotes a reading of the middle one of a group of three horizontal cross-wires on the face of a leveling rod.

B denotes a reading of the bottom one of a group of three horizontal cross-wires on the face of a leveling rod.

m denotes the mean of a group of three horizontal cross-wire readings on the face of a leveling rod.

Then

$$m=1/3\ (T+M+B)$$

I denotes the length of rod subtended between T and B.
Iu denotes the length of rod subtended between T and M.
Il denotes the length of rod subtended between M and B.

Then

$$Iu=T-M,$$
$$Il=M-B, \text{ and}$$
$$I=Iu+Il=T-B.$$

Furthermore, a b relates the preceding symbols to backsights and an f relates them to foresights. Then mb denotes the mean of a group of three horizontal cross-wire readings on the face of a rod in the backsight position.

Also, let the subscripts 1, 2, 3, 4, . . . n indicate the set-ups in their chronological order from the beginning of any running over any section of a line of levels, regardless of the direction of the running. The subscript $n+1$ would then indicate the first set-up of the next succeeding running. The subscript x will denote an extra foresight.

Let S=the sum of any series of values,

S_1=the sum of any series of values for the first set-up only of a running,

S_2=the sum of any series of values for the first two set-ups of a running,

S_3=the sum of any series of values for the first three set-ups of a running, etc., and

S_n=the sum of any series of values throughout the whole running of the section, that is, to and including the last set-up of the running of the section.

D=a difference of elevation,

D_F=a difference of elevation determined by leveling in the forward direction,

D_B=a difference of elevation determined by leveling in the backward direction, and

D_m=the mean difference of elevation for a section.

Let t_s=the temperature (in degrees centigrade) at which a leveling rod was standardized,

t_1=the temperature recorded for the leveling rod in the backsight position at the first set-up of a running of a section of a line of levels, and

$$t_2, t_3, \ldots\ldots t_n,$$

will be the temperatures recorded at any intermediate set-ups along the running of a section of a line of levels, t_{n+1} being the temperature recorded for the first set-up of the next succeeding running.

Let t_m=the mean temperature for a running.

C_s=the stadia constant of a leveling instrument.

C_e=the coefficient of expansion of the invar strip in a geodetic leveling rod.

C=the level constant of a leveling instrument as determined by observations made in accordance with the specifications in Special Publication No. 239.

c_i=the index correction to be applied to an observed difference in elevation for one running of a section of a line of levels.

c_r=the rod correction to be applied to the observed difference in elevation for one running of a section of a line of levels.

c_t=the temperature correction to be applied to the observed difference in elevation for one running of a section of a line of levels.

c_l=the level correction to be applied to the observed difference in elevation for one running of a section of a line of levels.

l_s=the length of any sight in meters.

L_s=the length of any section in kilometers.

e=excess of rod length in mm./m. as computed on the rod standardization sheet.

When any of the above notations are marked ′, the notation then refers to readings or values from the backs of the rods and the unit automatically changes from the metric to the English system.

For purposes of illustration, the following typical combinations of the above notations will serve to indicate the manner in which it is proposed to combine them for use in the following pages of this manual. These miscellaneous examples should show how the system of notation may be used as "shorthand" to abbreviate the text and to indicate exactly how the required numerical operations should be performed. The numerical values given in all of the following examples come from the sample of record shown in figure 1, page 7.

Tb_1=the reading of the top one of the group of three horizontal cross-wires on the face of the rod in the backsight position at set-up 1. Numerically, $Tb_1=1651$ mm.

Bf_3=the reading of the bottom one of the group of three horizontal cross-wires on the face of the rod in the foresight position at set-up 3. Numerically, $Bf_3=1359$ mm.

$M'f_4$=the reading of the middle one of the group of three horizontal cross-wires on the back of the rod in the foresight position at set-up 4. Numerically, $M'f_4=3.82$ feet.

$$mb_1=\frac{Tb_1+Mb_1+Bb_1}{3}=1549.0 \text{ mm., and}$$

$$mf_2=\frac{Tf_2+Mf_2+Bf_2}{3}=1359.7 \text{ mm.}$$

$$Iub_3=Tb_3-Mb_3=1667-1569=98 \text{ mm., and}$$

$$Ilb_3=Mb_3-Bb_3=1569-1470=99 \text{ mm.}$$

$$Ib_3=Iub_3+Ilb_3=98+99=197 \text{ mm.}=Tb_3-Bb_3=1667-1470=197 \text{ mm.}$$

$$\begin{aligned}STMBb_3&=(Tb_1+Mb_1+Bb_1)+(Tb_2+Mb_2+Bb_2)+(Tb_3+Mb_3+Bb_3)\\&=(1651+1549+1447)+(1716+1615+1514)+(1667+1569+1470)\\&=14198 \text{ mm.}\end{aligned}$$

$$SIf_2=If_1+If_2=204+199=403 \text{ mm.}$$

$$SM'f_4=M'f_1+M'f_2+M'f_3+M'f_4=4.20+4.45+4.78+3.82=17.25 \text{ feet.}$$

$$STMBb_n=28104 \text{ mm., and}$$

$$STMBf_n=25581 \text{ mm.}$$

$$Smb_n=9368.1 \text{ mm., and}$$

$$Smf_n=8527.1 \text{ mm.}$$

$$D_F=Smb_n-Smf_n=9368.1-8527.1=+841.0 \text{ mm.}=+0.8410 \text{ m.}$$

$$D'_F=SM'b_n-SM'f_n=30.69-27.93=+2.76 \text{ feet.}$$

MATERIAL RECEIVED FROM THE FIELD

The material received in the Office, from field parties engaged in geodetic leveling operations, consists of the following items:

1. Original records—Form 257, "Spirit Level Observations,"

2. Field computations—Form 45, "Computation of _____-order Levels,"

3. Descriptions of bench marks—Form 638 (Revised 1946), "Description of Bench Mark,"

4. Recovery notes—Form 685 A, "Recovery Note, Bench Mark,"

5. Progress sketches and season reports.

The following paragraphs contain detailed descriptions of the various items received from the field.

ORIGINAL RECORDS

The observations made during the running of a line of levels are recorded in specially printed field books (Form 257) about 5 by 7½ inches in size and containing about 100 pages. The cover label and title page of each book are identical and provide blanks in which data concerning the contents of the volume are entered. These entries include the following information:

(a) The name of the Director,

(b) The names of the States through which the line of levels was run,

(c) The title of the line,

(d) The number of the instrument used for the leveling,

(e) The date on which the records were received at the Office,

(f) The accession number assigned to the book or group of books by the Library and Archives Section,

(g) The name of the chief of party,

(h) The year in which the leveling was run,

(i) The number of the particular volume in the group of which it is a part, and

(j) The total number of volumes in the group.

All of the above items of information are entered in the field, except items (e) and (f) which are entered by the Library and Archives Section after the records have been received at the Office.

The record book (Form 257) is arranged so that at each opening of the book both pages are in use by the recorder at the same time. The record of the backsights is kept on the left-hand page, and the record of the foresights is kept on the right-hand page. Each opening of the book provides blanks for entering in the page headings the following information:

(a) The date on which the observations were made,

(b) Data regarding the relative directions of the sun and the progress of the running along the line,

(c) The direction of the running with respect to the general direction of progress along the line,

(d) The designation of the bench mark from which the leveling recorded in that opening of the book was started,

(e) The bench mark on which the same leveling ended, or toward which the leveling was progressing if the record for the section between the two bench marks required more than the two pages of one opening of the book,

(f) The strength of the wind at the time the leveling was done, and

(g) The time at which the leveling on the section was started.

Figure 1 (p. 7) shows one opening of a record book and the record of the observations made during one running of a section of a line of levels. In the sample record shown, we find that the levels were run on August 14, 1946, in the forward direction with respect to the general progress of the leveling along the line, and that the running was started from bench mark Y 159 and ended on temporary bench mark 2. We also

find that the sun was behind and to the left of the party as it moved along the line (see diagram and explanation at the bottom of Form 45, fig. 2, p. 10), that a wind of strength 1 was blowing, and that the leveling of the section was started from bench mark Y 159 at 1330 (1:30 p. m.).

The left-hand page of Form 257 is divided into seven columns, and the right-hand page is divided into six columns. The horizontal lines ruled across the pages do not show in figure 1.

On the left-hand page, the page on which the record of the backsights is made, column 1 contains the numbers of the instrument stations at which the observations were made. The instrument stations are numbered consecutively throughout the leveling on any one day, beginning anew with 1 for the first set-up of each day. It should be noted at

SPIRIT LEVELING

Date: Aug. 14, 1946 Forward—~~Backward~~ (Strike out one word) From B. M.: Y159 To B. M.: Z

Sun: 5 Wind: 1 Time: 1330

No. of Station	Thread Reading Back Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Rod and Temp.		Thread Reading Fore Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Remarks
	1651					375		1384					
53	1549	15490	508	102	204	33°		1282	12820	420	102	204	
	1447			102				1180			102		
	4647							3846					
	1716							1459					
54	1615	16150	530	101	202			1360	13597	445	99	199	
	1514	31640	1038	101	406			1260	26417	865	100	403	
	9492							7925					
	1667							1559					
55	1569	15687	513	98	197			1459	14590	478	100	200	
	1470	47327	1551	99	603			1359	41007	1343	100	603	
	14198							12302					
	1690							1267					
56	1591	15907	521	99	199			1167	11667	382	100	201	
	1491	63234	2072	100	802			1066	52674	1725	101	804	
	18970							15802					
	1823							1885					
57	1723	17227	564	100	201			1784	17840	585	101	202	
	1622	80461	2636	101	1003			1683	70514	2310	101	1006	
	24138							21154					
	1422							1575					
58	1322	13220	433	100	200			1476	14757	483	99	199	
	1222	93681	3069	100	1203			1376	85271	2793	100	1205	
	28104	85271	2793					25581				1203	
	25581	+0.8410 m.	2.76	f = 0.84 m.								2408	= 0.80 km.
	3/2523												
	+ 0.8410 m.												

FIGURE 1.—Sample of geodetic leveling record—one opening of Form 257.

this point that in geodetic leveling the instrument stations are numbered consecutively, not the turning points.

Column 2 contains four entries for each individual backsight, the first three being the readings of the three horizontal cross-wires on the face of the leveling rod and the fourth being the sum of the first three or the sum of the three just entered and the next preceding total. At any time during the running of a section, the total next following the entry of any group of three wire readings is the cumulative total of all the wire readings already recorded for that running. The last of the totals entered in column 2, therefore, shows the sum of all the backsight wire readings for the running.

In column 3 of the left-hand page the recorder enters, on the line opposite the middle one of each group of three wire readings, the mean of those three readings for each backsight. After the second mean reading has been entered, the total of the first two mean readings is entered on the next line below the second of the mean readings, and this cumulative total of the mean readings is carried down the column as each new

mean reading is entered, until the end of the section is reached, where the last total in column 3 is the sum of all the mean backsight readings for the running.

The readings in column 2 are made and recorded in meters, decimeters, centimeters, and millimeters. The mean readings are carried out one place farther, to tenths of millimeters. The last figure in any correctly computed mean reading will always be "0," "3," or "7." The last figures of the cumulative totals of the mean readings in column 3 will follow no set rule.

Column 4 of the left-hand page of Form 257 shows the individual readings of the middle wire on the back of the leveling rod for each successive backsight. Since the backs of the leveling rods are graduated in feet and tenths of feet, and are read by estimation to the nearest hundredth of a foot, the readings recorded in column 4 are in feet, tenths, and hundredths. Cumulative totals are also carried down column 4 as the leveling progresses.

In column 5 the entries will be found to be in pairs. The first entry of each pair is the difference between the first and second of the three wire readings entered in column 2 for the backsight at that instrument station, and the second entry of each pair in column 5 is the difference between the second and third wire readings entered in column 2 for the same backsight. The two entries of each pair in column 5 should be alike or nearly so. Any sudden departure from this condition is indicative of a misreading of one or more of the three horizontal wires on the face of the rod.

In column 6 the recorder enters the sum of each pair of entries in column 5, and again a cumulative total is carried down column 6 throughout the running of the section.

Column 7 of the left-hand page of Form 257 usually contains but two entries at the top of the column, and the rest of the column is then left blank, except under certain conditions which will be explained in the next paragraph. The two entries at the top of column 7 of the left-hand page of the record shown in figure 1 are arrived at as follows: The first, "375," shows that rod No. 375 was in backsight at instrument station 53 when the running of the section was started from bench mark Y 159 at 1330. The second entry, "33°," is the reading of the thermometer (in degrees centigrade) imbedded in the back of rod No. 375, at the time it was standing on bench mark Y 159 when the backsight from set-up 53 was taken.

Sometimes double entries will be found at the top of column 7. This will be the case when the temperature readings of the two rod thermometers are not in close agreement, and, in order to secure a mean reading, both thermometers are read and recorded. In a case of this sort the entries, reading down the column, might be as follows: "375, 33°, 382, 30°." Translated this would mean that, when the leveling was started, rod No. 375 was in backsight at a temperature of 33° and rod No. 382 was in foresight on the first turning point at a temperature of 30°. In other words, if both rod numbers and their corresponding temperature readings are recorded, the first number and first temperature recorded are for the rod held on the bench mark for a backsight, and the second rod number and second temperature apply to the rod in foresight on the first turning point away from the bench mark. Furthermore, it will be found that sometimes temperature readings are recorded at intermediate points during the running of a section, either because the section is long and the temperature is varying erratically or because the leveling had to be interrupted for a period of time, in which case temperature readings should be made and recorded both at the beginning and at the end of the period of the interruption. (See p. 23.)

The first five columns of the right-hand page of Form 257 correspond to columns 2 to 6, inclusive, of the left-hand page, except that on the right-hand page the record is for the foresights made from each successive instrument station.

Column 6 on the right-hand page, the "Remarks" column, is used for making miscellaneous notations concerning the work and also for a brief description of the point on which the rod was held in case a regular turning point happens also to be a point on which an "extra elevation" is to be determined. (See p. 68 of Sp. Pub. No. 239.)

The record for the running of a section is continued by computing and checking the difference in elevation. If the numerically smaller totals are carried over and subtracted from the corresponding totals on the opposite page, plus differences will be worked out on the left-hand or backsight page and minus differences will appear on the right-hand or foresight page. The difference of elevation to be carried over to Form 45 (fig. 2) is the difference of the totals in the columns containing the mean readings. This value is checked by taking the difference of the sums of the thread readings and dividing that difference by three. This check should be exact except for a little looseness in the last place resulting from rounding the mean readings to the nearest tenth of a millimeter. (See p. 17.)

A further check, especially against blunders in reading the meters and decimeters, is obtained by evaluating the expression 0.3048 $(SM'b_n - SM'f_n)$ and comparing the result with the numerical value of $Smb_n - Smf_n$.

The last step is the computation of the length of the section, thus:

$$L_s = C_s(SIb_n + SIf_n).$$

FIELD COMPUTATIONS

The results of the leveling as recorded and computed in the record books (Form 257) are abstracted in the field on Form 45, "Computation of _____-order Levels" (see fig. 2), and these computation or abstract sheets are forwarded to the Office when the field work has been completed.

All runnings over a single section of the line are grouped together, and the levelings over the successive sections are abstracted in the order in which they occur along the line. All portions of the abstract which refer to spur lines of any description are either entered in red ink or "boxed" as shown in figure 2, if red ink cannot be used, to distinguish them from the sections of the main line.

The entries in columns 1 to 8, inclusive, of Form 45 are taken directly from the record books. The entries in column 9 are cumulative totals of the entries in column 8 and show the distance of the end of each section from the beginning of the line of levels.

Entries in columns 10, 11, and 12 of Form 45 are also taken directly from the record books. Those entries in columns 13 and 14 are computed in the field according to rules which will be given farther along in this publication when the office computations (Form 31) are discussed. However, for the present it may be stated that the entries in column 13 are the discrepancies between the forward and backward runnings over a single section and those in column 14 are the progressive totals of the entries in column 13.

The entries in column 15 of Form 45 represent the means of the different runnings, section by section, along the line, and the entries in column 16 are the designations of the bench marks at the forward ends of the sections for which the differences in elevation are given in column 15. The algebraic sum of the differences in elevation up to any

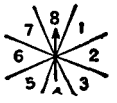
DEPARTMENT OF COMMERCE
U.S. COAST AND GEODETIC SURVEY
Form 45
Rev. Ed. Sept. 1934

COMPUTATION OF FIRST-ORDER LEVELS

Line Warsaw, Ind., to Goshen, Ind. Rods 370 & 375

Date (1)	Designation of Section (2)	Forward or Backward (3)	Time (4)	Sun (5)	Wind (6)	Temp. of Rods (7)	Length of Section (8) km.	Distance from B.M. (9) km.	ROD READINGS ΣB (10) meters	ROD READINGS ΣF (11) meters	Difference of Elevation Each Section (12) meters	DIVERGENCE (B—F) Partial (13) mm.	DIVERGENCE (B—F) Total Accumulated (14) mm.
	I8												
8-14	to J8	B	11^{25}	5	1	30	0.30		6.2987	6.7557	-0.4570		
8-14	J8 to	F	11^{50}	1	1	30	0.58		6.7290	9.8898	-3.1608		
8-14	TBM 1	B	11^{00}	7	1	30	0.57	0.57	8.2274	5.0664	+3.1610	-0.2	-0.2
8-14	TBM 1	F	13^{00}	4	1	32	0.77		6.2071	7.9444	-1.7373		
8-14	to Y159	B	10^{30}	7	1	30	0.77	1.34	8.1344	6.3948	+1.7396	-2.3	-2.5
8-14	Y159 to	F	13^{30}	5	1	34	0.80		9.3681	8.5271	+0.8410		
8-14	TBM 2	B	10^{10}	7	1	30	0.80	2.14	8.2094	9.0515	-0.8421	+1.1	-1.4
8-14	TBM 2	F	14^{10}						4.5884	4.6054	-0.0170		
8-14	to EFS	B	10^{00}						3.2447	3.2247	+0.0200	-3.0	
8-14	TBM 2	F	13^{15}	4	1	33	0.93		10.8355	12.6758	-1.8403		
8-14	to Z159	B	9^{45}	7	1	30	0.93	3.07	10.9845	9.1401	+1.8444	-2.6	-4.0
8-14		B	14^{20}	1	1	34	0.93		12.6672	10.8258	+1.8414		
8-14	Z159 to	F	15^{00}						2.9920	1.1610	+1.8310		
8-14	EFS	B	9^{40}						0.8350	2.6647	-1.8297	-1.3	
8-14	Z159 to	F	15^{00}	4	1	34	0.87		12.1111	7.4304	+4.6807		
8-14	TBM 3	B	9^{25}	7	1	28	0.87	3.94	7.1931	11.8748	-4.6817	+1.0	-3.0
8-14	TBM 3	F	16^{35}						1.5530	1.5560	-0.0030		
8-14	to EFS	B	9^{20}						2.8000	2.8010	-0.0010	+4.0	

Abbreviations for sun: O=cloudy. With sunshine, show direction of progress in relation to sun by numbers in diagram.
Abbreviations, strength of wind: O=calm; 1=moderate; 2=strong.



U.S. GOVERNMENT PRINTING OFFICE: 1934 11—10258

FIGURE 2.—Sample of field computation—one sheet of Form 45.

Level No. G 70 Observer G. Mueller Year 1946

Mean Difference of Elevation (15) meters	Designation of Bench Mark (16)	Elevation of Bench Mark (17) meters	CORRECTIONS Index Error mm. (18)	Length of Rod mm. (19)	Temp. of Rod mm. (20)	Level mm. (21)	REMARKS (22)
	I 8	251.8860					Check data: between I 8 & J 8
+ 0.4570	J 8	252.343					Old difference 0.4572
							New " 0.4570
							Divergence 0.0002
−3.1609	TBM 1	249.1821					
−1.7384	Y159	247.4437					
+0.8416	TBM 2	248.2853					
							0.5 mi N of Y159 1.2 mi. N of Warsaw Top of E rail of main N.Y.C.R.R. &
−0.0185	EFS	248.2668					N. end of easterly siding
−1.8416	Z159	246.4437					
							At Z159 1.9 mi N of Warsaw Top of E rail of N.Y.C.R.R.
+1.8304	EFS	248.2741					& E-W gravel road
+4.6812	TBM 3	251.1249					
							0.5 mi N of Z159 2.4 mi N of Warsaw Top of E rail N.Y.C.R.R. & E-W
−0.0010	EFS	251.1239					gravel road

given bench mark when added to the elevation of the starting bench mark results in the elevation of that mark shown in column 17. Each elevation in that column represents the field elevation for the bench mark whose designation appears directly to the left of the elevation.

Columns 18 to 21, inclusive, are not filled in by the field party but are left blank for the use of the computers in the Office. Column 22 is used for recording a variety of miscellaneous data: the record of the checks obtained at the beginnings and ends of lines of levels and at intermediate junctions; the descriptions of points for which extra elevations were determined; names of towns and cities along the route; and other miscellaneous notes concerning the leveling.

DESCRIPTION OF BENCH MARK

Designation	State	County
Nearest town	County	
Distance and direction from nearest town		
Character of mark		Stamping
Established by		
Detailed description		

Chief of party	Date
Described by	Checked by

U. S. Coast and Geodetic Survey—Form 638 (Rev. 1946) 16—49345-1 U. S. GOVERNMENT PRINTING OFFICE

FIGURE 3.—Form 638 (Rev. 1946) "Description of Bench Mark."

Further information concerning the making of field computations will be found on pages 68 to 71, inclusive, of Special Publication No. 239, "Manual of Geodetic Leveling." The computations made in the Office for filling in the entries in columns 18 to 21, inclusive, of Form 45 will be found on pages 27 to 30, inclusive, of this publication.

DESCRIPTIONS OF BENCH MARKS

For each permanent or semipermanent bench mark established or tied in during the course of the leveling, if that mark is brought into the vertical control net for the first time during the leveling, the field party is required to fill out a copy of Form 638 (Revised 1946). This form (fig. 3) constitutes the original description of the bench mark and is eventually filed in the Section of Leveling as the working copy of the description for use in the Section.

RECOVERY NOTES

In case any bench mark which has been brought previously into the vertical control net, as computed and adjusted by the Coast and Geodetic Survey, is searched for, a

copy of Form 685 A (fig. 4) must be filled out by the field party and forwarded to the Office along with the descriptions of the new bench marks established or tied in during the course of the leveling. This applies to any bench mark previously taken into the net which the party searches for, whether or not it is actually recovered. Form 685 A, when properly filled out, actually constitutes a report on the condition of the mark at the time it was searched for, a report on the adequacy of the description, and suggestions for the betterment of the description if it is not already adequate for the easy recovery and positive identification of the mark. The recovery notes, like the descriptions of the new bench marks, are filed in the Section of Leveling, and are used primarily for revising the descriptions.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 685 A

RECOVERY NOTE, BENCH MARK

R

Designation State County
Nearest town County
Distance and direction from nearest town
Character of mark Stamping
Established by
Present condition
Detailed report

Chief of Party Date
Recovered by Checked by

U. S. GOVERNMENT PRINTING OFFICE 16—40836-1

FIGURE 4.—Form 685 A "Recovery Note, Bench Mark."

PROGRESS SKETCHES AND SEASON REPORTS

When progress sketches and season reports are received from the field they are accessioned in the Library and Archives Section, routed through the Bureau so that all interested persons and units may have a chance to read them, and returned to the Library and Archives Section for filing. They are available for future use as needed. In case the progress sketch is needed for use in connection with the office work on the leveling records and computations, a blueprint or some similar reproduction of it is made, used by the computers handling the computations for the project, and then filed with the office computations of the leveling to which the sketch applies. Ordinarily the progress sketches are not sufficiently detailed to permit their use in computing orthometric corrections, but they are a valuable aid in identifying on the more detailed maps available in the Office the routes followed by the leveling parties. An exception to the above may be noted when the progress sketch is made up on a base map having a scale of 1: 500,000 or larger and the latitude and longitude lines as well as the locations of towns and cities along the routes of the various lines are shown clearly on the progress sketch.

PRELIMINARY PROCESSING BY THE SECTION OF FIELD RECORDS

All leveling records, field computations, descriptions of bench marks, and recovery notes forwarded by the geodetic leveling parties are routed to the Library and Archives Section, where they are listed, assigned accession numbers and shelf marks, and then turned over to the Section of Field Records of the Division of Geodesy.

In the Section of Field Records, the records and computations are given an initial inspection to make sure that, in general at least, the work has been done in accordance with the provisions of the General Instructions for Geodetic Leveling and the other provisions of Special Publication No. 239, "Manual of Geodetic Leveling."

The main points covered by the initial inspection of the records and computations may be outlined by the following list of questions, which are constantly in the mind of the person making the inspection.

(1) Are the records and computations legible, of sufficiently neat appearance, prepared in accordance with standard practice, and complete?

(2) Was the level constant, C, determined daily or oftener as required by the General Instructions?

(3) Were the computations of the level constant correctly made, and was the instrument adjusted and C redetermined if necessary?

(4) Were any readings "off the graduations"?

(5) Were changes made which have even the appearance of having been made after the instrument had been moved and, if so, were they adequately explained in the record and supported by the required releveling?

(6) Was the stadia constant of the instrument determined at or near the time when the instrument was first put in use and following any replacement of cross-wires in the field?

(7) Was proper use made of the "Remarks" column in recording any unusual features of the work or unusual circumstances?

(8) In the case of leveling in area developments, were the records properly indexed and cross-referenced?

(9) Were the checks obtained at the beginning and end of each line and at intermediate junctions along the line within the prescribed tolerances?

(10) Were any sections of double-run leveling left without sufficient releveling having been done to secure prescribed checks when the first two runnings failed to agree within proper limits?

(11) Were any unauthorized gaps or breaks left in the line which would require additional field work?

(12) Was the accumulation of $B-F$ predominantly of one sign and of such size as to indicate the presence of any appreciable systematic error?

While the above list of questions is by no means complete, it will serve to indicate the objectives to be gained by the initial inspection. If any departures from standard practice, violations of the provisions of the General Instructions, or evidences of carelessness are found, these matters are made the subject of a letter, through channels, to the observer concerned, and he is advised of his shortcomings and cautioned against continued repetition of items of bad practice.

Upon the completion of the initial inspection and the preparation of the letter to the field party, if such a letter is necessary, the records and computations are sent to the Section of Leveling for use in making the office computations and adjustments necessary to produce the standard elevations of the bench marks for publication.

The descriptions of bench marks and recovery notes are first inspected to make sure that they have been carefully prepared and typed in such a manner as to permit their reproduction by photolithography. If only minor corrections are required, these are often made during the inspection. However, if the cards—Form 638 (Revised 1946) and Form 685 A—have been carelessly prepared or poorly typed they are returned to the field party for correction or retyping.

If the cards are acceptable, or can be made so by a small amount of retyping in the Office, they are then photostated. The photostat copies are placed in the "HA" cahiers and sent to the Library and Archives Section for storage. The photostats of the original cards thus become the permanent record copy in the archives.

Next, the original cards are "stuck up" on 16- by 21-inch sheets and prepared for photolithographic reproduction. This preparation for reproduction also includes the preparation of title cards, standard paragraphs, and other material included in the heading and introductory paragraphs at the beginning of each list.

After the photographic negatives and printing plates have been prepared, the cards are removed from the large sheets and sent to the Section of Leveling, where they are used in a number of ways to be discussed at other places in this manual. They are eventually filed in the Section of Leveling as the "working copy" of the descriptions, for ready reference and any other uses to which it may be necessary or desirable to put them. It is often necessary to rephotostat the original cards in order to comply with requests for data concerning newly established bench marks, when such requests are received prior to the printing of the lithographed lists of descriptions.

PRELIMINARY COMPUTATIONS

When the records, field computations, descriptions of bench marks, and recovery notes reach the Section of Leveling, they are placed in temporary storage to await their turn for computation and adjustment. The data received from the field are not always processed in the order received, since there is usually a considerable backlog of this work on hand and special considerations often require the quick processing of certain items while in other cases the time element is not so important.

When the time comes for the processing of the material for a particular project, the preliminary computation is the first of a chain of operations which eventually results in the production of standard elevations for the bench marks established or tied in during the leveling.

First it is desirable to make certain comments on the general features of the work, as a whole, before embarking upon the extremely detailed explanations which will be necessary to cover all of the multitudinous details which must be considered.

To restate objectives, this manual aims to serve two separate and quite distinct purposes:

(1) To provide a standard procedure to be followed in making all preliminary computations and, as far as may be possible, to provide for uniform treatment of the resulting data during adjustment, and

(2) To aid in the training of new personnel.

The office computations for geodetic leveling are usually handled so that a team of computers will be involved in each step of the work. In actual practice the two computers of a team do not always work together, since computers A, B, and C may be assigned to the initial work and computer D may be assigned to check the work of the other three. One group may be assigned to carry forward certain operations, and a different group may pick up the work at the place where the first group finished and

proceed from that point. Regardless of the division of labor, however, there still remains the semblance of team work, since checking is always done by some person other than the one who makes the original computations.

The term "computer" as used in this manual should be interpreted to mean the first person who carries forward a new step in the chain of operations. The term "checker" should be understood to mean the person who checks the work of a computer. For purposes of illustration, in checking from books to sheets, the computer does the work, and the checker goes over the work done by the computer and verifies any changes which the computer may have made. In a sense the computer was really a checker if we consider the computer in the field. However, since this manual covers the office work only, the person who first makes the check from books to sheets is designated as a computer.

The terms "computer" and "checker" are used as defined above for reasons of convenience. Both are computers in the usually accepted sense, inasmuch as they both make numerical calculations.

It is desired to instill in each computer and checker engaged in the processing of geodetic leveling records a healthy worship of accuracy. All records and computations are to be carefully and thoroughly checked. This system of checking is necessary because of the fact that, no matter how honest, competent, and conscientious an employee may be, some mistakes are bound to be made. The checker should always approach his task of checking the work of a computer with the attitude of eliminating honest mistakes from the work of a conscientious fellow-worker, rather than with the attitude of obtaining data for use in criticizing or casting reflection on the work or abilities of another. The checker should also remember that in case of a disagreement the mistake *may* be his. All mistakes found by the checker should be taken up with the computer, and before they are finally disposed of both computer and checker should be satisfied that the correct figures are being used. However, to avoid interrupting the work of the computer too frequently, it is usually best for the checker to wait until the work of checking a particular piece of computing has been completed and then discuss at one time all the changes which have been made by the checker.

It is generally accepted practice in the Section of Leveling to make all office entries in the record books in red ink to distinguish them from entries made as a part of the original record, which is usually kept in blue or blue-black ink. In case the field computation is entirely in blue or blue-black ink, the same rule should apply. However, field computations may be encountered in which the spur lines are indicated by means of red ink rather than "boxes," and when this occurs all office entries should be made in ink of such a color as to distinguish the office entries from the original computation. The checker should use a pencil or ink of a color such as will make his check marks or changes distinct from both the entries and check marks of the original records and computations as well as from those added by the computer.

Throughout this manual, when a computer or checker is required to "check" an entry, it is required that the person doing the work must assure himself that the entry is correct (or within the specified tolerance when any leeway from an absolute check is permitted) *and* that he must also place a check mark against the entry which he has found to be acceptable.

Actual check marks placed on records, computations, or changes, should be small, neatly made, and so placed (usually to the right of, or above and to the right of, the entries to which they apply) that there will be no possibility of confusion as to which

entry has been checked. Obviously it is bad practice to place any check mark in such a position that it will tend to obscure or confuse any entry in a record or computation.

In many of the operations involved in the computation and adjustment of leveling more decimal places are used in the course of the computations than it is necessary or advisable to retain in the final results. In dropping the extra decimal places no question arises if the figures to be dropped represent either more or less than one-half a unit of the last place retained, for if they are less the last figure retained remains unchanged and if more it is increased by one. For example, if the two numbers 13.2821 and 15.3368 are to be rounded to three decimal places, the adopted numbers will be 13.282 and 15.337, respectively.

However, when the figures in the dropped decimal places represent exactly one-half a unit in the last place retained, the number adopted may have either of two values both of which are equally correct. In order to avoid confusion we have arbitrarily adopted the convention of using the nearest *even* figure for the last decimal place retained in such cases. For purposes of illustration the following examples are given:

1.2345 rounds to 1.234,
1.2355 rounds to 1.236,
1.2365 rounds to 1.236, and
1.2375 rounds to 1.238, etc.

The work on the preliminary computations made during the processing of the records and field computations of geodetic leveling is discussed in the following pages under four general headings as follows:

Checking from books to sheets,
Preparation of office abstracts,
Computation of orthometric corrections, and
Preparation of summary sheets.

CHECKING FROM BOOKS TO SHEETS

The work discussed under this heading involves the checking of the transfer of data from the record books (Form 257) to the field-computation sheets (Form 45) or to other record forms in the Section of Leveling. This is the most tiresome, tedious, and monotonous part of the work. However, it is basically the most important, since any uncorrected mistakes will carry through into the final publication of the results. In checking books to sheets, the computer should be sure to examine every page of every volume of the field record, including even the inside pages of the covers and the flyleaves. Records concerning the determination of stadia constants, records of C determinations, or notes pertinent to the leveling, are sometimes found at the back of a partially filled record book or even on the flyleaves or inside pages of the covers. It is not safe to assume that, because the continuous record in the volume extends only part way through the book, nothing will be found farther along in the volume.

The process of checking from books to sheets involves the following steps:

1. Checking determinations of stadia constants,
2. Checking determinations of the level constant, C,
3. Checking headings and columns 3, 7, 8, 12, and 22 of Form 45,
4. Computing entries for columns 18, 19, 20, and 21 of Form 45,
5. Checking the abstracting of extra-foresight runnings,
6. Consideration of check points, and
7. Computation of river crossings.

NOTE.—The column numbers as given above refer to the numbers shown at the tops of the columns in the sample field-computation sheet in figure 2, page 10. Also, it should be noted that the entries in columns 1, 2, and 4 are checked by inspection only, and no check marks are placed opposite them. Columns 5 and 6 are usually accepted without check. Columns 10 and 11 are utilized only as a check on the signs of the entries in column 12 and are checked only in case they show a discrepancy in sign. Columns 9, 13, 14, 15, 16, and 17 are given no consideration in the office computation because the entries in corresponding columns of Form 31 (fig. 10, p. 36) are made without reference to the sheets of the field computation.

The exact routine to be followed in carrying out the steps outlined above may be varied to suit the individual computer. The following routines are suggested for trial, and one or the other will probably be found to be acceptable. The exact routine is a minor matter except that, in the interest of efficiency, the above outlined steps should all be accomplished in two or three trips through the record books, depending on whether or not the extra foresights are handled separately.

Routine 1.—On the first trip through the books, check determinations of stadia constant, C_s, and determinations of the level constant, C. On the second trip through the books, check columns 3, 7, 8, 12, and 22 and compute the level corrections to be entered in column 21. The abstracting of the extra foresights may be checked at the same time column 12 is checked, or may be handled as a separate job.

Routine 2.—On the first trip through the books, check determinations of the stadia constants; determinations of the level constant, C; and also check columns 3, 7, and 12. These are the columns which involve the differences of elevation. On the second trip through the books, check column 8 and compute level corrections to be entered in column 21. Again the extra foresights may be handled in connection with the checking of column 12 or may be considered a separate job.

1. Checking determinations of stadia constants.—Determinations of stadia constants are rather infrequent since each observing unit usually determines the stadia constant for an instrument only at the beginning of the season, when a different instrument replaces one which has been in use, or when new cross-wires are installed in the field.

The determination of a stadia constant in the field is described on pages 20 and 21 of Special Publication No. 239, and a sample computation of a stadia constant is shown in figure 5, page 19, of this manual.

The duties of the computer are to check the thread intervals, the summation of the thread intervals, and the summation of the distances. He should also check the division of the summation of the distances by the summation of the thread intervals and compute the stadia constant. This constant should be rounded to three decimal places.

The checker will check only the changes made by the computer. He will also see that the new stadia constant is properly recorded on the appropriate card in the file showing the stadia constants of all geodetic levels. In case work of this sort is being carried on in an Office outside of Washington, the new stadia constant should be entered in the corresponding file at that Office, and a copy must be sent to the Washington Office, marked for the attention of the Section of Leveling, in order that the permanent file of stadia constants in that Section may be kept up to date.

2. Checking determinations of the level constant, *C*.—The field procedure for the determination of the level constant, C, is given on pages 6, 7, and 19 of Special Publication No. 239, and a sample of the record and computation for a C determination is

shown in figure 6, page 20, of this publication. The level constant is the amount by which the actual line of sight through a leveling instrument (when the bubble is centered in its vial) departs from the truly horizontal line through the optical center of the instrument, computed in millimeters per millimeter of stadia interval.

In the office, the computer will check the individual thread intervals, the summation of the thread intervals, the computation of the individual mean readings, and the summation of the mean readings for both the short sights and the long sights. He will also compute the curvature-and-refraction correction for each long sight—*not* for the sum of the two. This correction is computed by multiplying the total thread interval for the sight by the stadia constant for the level, to determine the length of

Stadia Constant SPIRIT LEVELING *Level No. G47*

Date: *Nov. 7, 1945* Forward—Backward (Strike out one word) From B. M.: To B. M.:

Sun: Wind: Time: *10:00*

No. of Station	Thread Reading Back Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Rod and Temp.		Thread Reading Fore Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Remarks
	1218												
75	1106			-112									
	0994			112	224				75 / 224 =	0.335			
	1174												
85	1047			127									
	0919			128	255				85 / 255 =	0.333			
					479								
	1111												
95	0969			142									
	0827			142	284				95 / 284 =	0.335			
					763								
	1059												
105	0902			157									
	0744			158	315				105 / 315 =	0.333			
					1078								
	1018												
115	0846			172									
	0674			172	344				115 / 344 =	0.334			
					1422								
	0972												
125	0786			186									
	0599			187	373				125 / 373 =	0.335			
					1795								
										6 / 2.005			
600										0.334			
		600 / 1795 =	0.334										

FIGURE 5.—Sample of record and computation of stadia-constant determination.

the sight in meters. That length is then used as the argument when entering table I (p. 142) to obtain the correction for curvature and refraction which must be applied to the rod reading. The two separate corrections may be added together and then applied to the summation of the mean readings for the long sights. The curvature-and-refraction correction is not applied to the short sights, simply because they are always so short that the correction will never amount to as much as 0.1 mm., or one in the last place. An inspection of a few records and the first few entries in table I will soon show that this is the case.

If the sum of the mean readings for the short sights is numerically smaller than the corrected sum of the mean readings for the long sights, it should be transferred to the right-hand page of the record book and subtracted from the corrected sum of the mean readings for the long sights. If the corrected sum of the mean readings for the long sights is numerically smaller than the sum of the mean readings for the short sights, it should be transferred to the left-hand page of the record book and subtracted from the sum of the mean readings for the short sights.

Determination of C SPIRIT LEVELING

Date: May 8, 1946 Forward—Backward (Strike out one word) From B. M.: To B. M.:

Sun: Wind: Time: 0840

No. of Station	Thread Reading Back Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Rod and Temp.		Thread Reading Fore Sight	Mean	Middle Thread Reading Back of Rod	Thread Interval	Sum of Intervals	Remarks
	1603							1784					
	1593	15930		10				1628	16280		156	312	
	1583			10	20			1472			156		
	4779							4884					
	1547							1666					
	1537	15367		10				1510	15100		156	312	
	1526	31297		11	21			1354	31380		156	624	
	9389				41			9414	-12			-41	
									31368			583	
									31297				
									583/7.1	-0.012			
									583				
		Instrument adjusted							1270				
	1547							1661					
	1536	15363		11				1505	15053		156	311	
	1526			10	21			1350			155		
	4609							4516					
	1587							1763					
	1576	15757		11	23			1609	16087		154	309	
	1564	31120		12	44			1454	31140		155	620	
	9336							9342	-11			-44	
									31129			576	
									31120				
									576/0.9	-0.002			
		Observer	T. C. Cowart										
		Recorder	J. C. Chambers										
		Instrument	G 71										
		Rods	368 & 387										
		Weather	Clear - Warm										

FIGURE 6.—Sample of record and computation of a determination of the level constant, C.

The level constant, C, is computed by means of the following formula:

$$C=\frac{\Sigma N_r-\Sigma F_r}{\Sigma F_t-\Sigma N_t}, \text{ where}$$

ΣN_r = the sum of the mean readings for the short sights,

ΣF_r = the sum of the mean readings for the long sights, after the correction for curvature and refraction has been applied,

ΣF_t = the sum of the total intervals for the long sights, and

ΣN_t = the sum of the total intervals for the short sights.

The computer will also prepare a C-sheet for the line or area and record the C values on it. Summary-sheet paper (9-column, 8- by 10½-inch) is usually used, though any standard-size sheet having at least four columns may be used. A sample of a C-sheet is shown in figure 7, page 21, and it will be noted that the entry for any one determination of C includes, in the order named, the date, the time, the numerical value of C, and notes as to whether the C was determined before, during, or after the adjustment of the level. In cases where the first determination of C on a particular day showed the level to be in such good adjustment that no adjustment was made, no entry is made in the fourth column of the C-sheet.

In any case in which the first determination of C for the day shows the need of adjustment of the level and the adjustment is made, the computer should compute and record both values of C and carry appropriate notes in the fourth column, as shown in figure 7.

If the instrument was found to be enough out of adjustment so that an adjustment of the level was made and two or more values of C were determined, the computer

should attempt to identify the set-ups from which the C observations were taken. This can usually be done by comparing the readings of the first long sight of the C determination with the foresight readings for the running which was being made at the time the C determination was made. If the set-up is identified, note the number of the set-up on the page on which the C was computed and put "C" in the remarks column of the leveling record, opposite the record for the set-up from which the C

C - Sheet

2 Mi. S.E. of Carson to Randle, Washington 944 HGZ 1946 R

I. R. Rubottom

1946

L-11949

Date	Time	"C"	
Instrument G 71 (x 0.291)			
1946			
Apr. 4	7:45	+0.006	
" 5	8:32	-0.004	
" 8	8:50	+0.001	
" 9	9:42	0.000	
" 12	9:50	-0.006	
" 15	8:58	+0.001	
" 17	8:55	-0.009	
" 18	8:33	+0.001	
" 19	8:33	-0.006	
Instrument G 81 (x 0.336)			
1946			
Oct. 15	10:15	+0.011	Before
		-0.001	After
" 16	10:45	+0.002	
" 18	9:10	-0.003	
" 23	9:40	-0.002	
" 24	9:45	-0.003	
" 25	9:25	-0.003	
" 26	9:28	+0.006	

Date	Time	"C"	
Instrument G 86 (x 0.293)			
1946			
Aug. 1	8:20	+0.009	
" 9	9:30	-0.004	
" 10	8:20	0.000	
" 12	8:10	-0.001	
" 13	8:00	-0.003	
" 14	8:00	+0.004	
" 15	8:00	-0.012	Before
		+0.008	After
" 16	8:20	+0.008	
" 17	7:50	-0.001	
" 19	8:20	+0.011	
" 20	8:10	+0.006	
" 21	8:10	+0.005	
" 22	8:00	+0.006	
" 23	7:50	-0.006	
" 24	8:00	-0.010	

FIGURE 7.—Sample of C-sheet.

observations were begun. If no adjustment of the instrument was made, this is not necessary since the same C will be used in applying the level corrections to all of the observations made on that day.

Occasionally an observer will encounter difficulty in bringing his instrument into proper adjustment, and more than two values of C will be determined. In most cases only the first C and the last C of the group will be computed and recorded. However, some observers, in spite of definite instructions to the contrary, sometimes continue

leveling along the line of levels during the process of adjusting the instrument. In such cases, *all* of the values of C are to be computed and recorded, inserting "During" in column 4 of the C-sheet opposite the intermediate values of C.

If, even in spite of definite instructions to the contrary, no C is determined in the course of a day on which leveling is done, the date should be entered on the C-sheet and no entries made on the same line in columns 2, 3, and 4.

The checker will check all changes made by the computer, all computations made by the computer, and the recording of the various values of C on the C-sheet.

3. Checking headings and columns 3, 7, 8, 12, and 22 of Form 45.—In checking the headings and the entries in the various columns of the field-computation sheets (Form 45) the computer will check the transfer of the data from books to sheets, and the checker will confine his attention to checking *changes* made by the computer, except as noted below.

The computer should make sure that the headings on each field-computation sheet are correct. Particular attention should be paid to the rod numbers, since they control the corrections, c_r and c_t, in columns 19 and 20 of Form 45. If more than one pair of rods has been used during the leveling abstracted on a single field-computation sheet, a red line should be drawn across the four correction columns (columns 18 to 21, inclusive) at the place where the change of rods took place, and the proper rod numbers should be entered both above and below this line. If a single pair of rods was used throughout the leveling abstracted on any one field-computation sheet, it is only necessary to check the numbers of the rods as given in the heading of the sheet.

The computer will inspect carefully the entries in columns 1, 2, and 4 (Form 45) and make necessary changes in any cases where the data have been incorrectly transcribed, but will not take the trouble to place actual check marks opposite correct entries in these three columns.

Careful attention should be given to the "F" and "B" entries in column 3 of Form 45, since these control the sign of the mean difference in elevation (D_m) for each section. In first-order leveling and in double-run second-order leveling the forward (F) running is abstracted normally preceding the backward (B) running, regardless of which running was the first chronologically. Should a case be found in which the backward running has been abstracted before the forward running, the computer should circle the entries for that section in column 12 and add the note "Reversed" so as to call the unusual procedure to the attention of the computers who will prepare the office abstracts (Form 31) at a later time.

In checking the mean temperatures entered in column 7 (Form 45), it should be remembered that the coefficients of expansion (C_e) of the invar strips in the Coast and Geodetic Survey geodetic leveling rods are so small that, in most cases, a variation of half a degree in the mean temperature of the rods for a running of a section will change the temperature correction (c_t) by an amount so small as to be negligible. Except in the most rugged country, a section having a difference of elevation (D) of as much as 50 meters is a rare occurrence. Assume a section $D=50$ m., and a pair of rods having $C_e=0.000001$ meter per degree centigrade. Then:

$$c_t=D\times C_e=50\times 0.000001=0.00005 \text{ m.},$$

per degree variation in the mean temperature. Since the corrections (columns 18 to 21, inclusive, on Form 45) are only carried out to the nearest 0.1 mm., the uncertainty in the above example due to a variation of a whole degree in the mean temperature is still only one-half of one in the last place and may be neglected. With this in mind, the

rules given later in this section regarding the checking of entries in column 7 of Form 45, will not impress the novice as being as loose as they might otherwise seem.

The temperature of one or both rods is entered in the record book at the beginning of each running of each section when the leveling is continuous. The General Instructions for Geodetic Leveling (see Special Publication No. 239) provide for reading and recording additional temperatures at any time the leveling is discontinued and again when the leveling is resumed after the interruption.

In most cases the mean temperature for a section is

$$t_m=\frac{t_1+t_{n+1}}{2}.$$

In case two temperatures are recorded at a single set-up, the mean of the two should be used in lieu of t_1 or t_{n+1} in the above formula.

An interruption during a running over a section may result in the recording of extra temperatures. The method of computing the mean temperature for the running of a section under these conditions can be illustrated best by means of a numerical example. Assume a running of 9 set-ups. Then $t_n=t_9$. Also, assume an interruption between set-ups 4 and 5, and that $t_1=25°$, $t_4=28°$, $t_5=30°$ and $t_{n+1}=31°$. Then,

$$t_m=\frac{4}{9}\left(\frac{t_1+t_4}{2}\right)+\frac{5}{9}\left(\frac{t_5+t_{n+1}}{2}\right)$$

$$=\frac{4}{9}\left(\frac{25+28}{2}\right)+\frac{5}{9}\left(\frac{30+31}{2}\right)$$

$$=11.8+16.9=28.7, \qquad \text{say } 29°.$$

In this connection it must be remembered that the sole purpose of computing the mean temperatures for the various runnings is to provide for the computation of the temperature corrections in column 20 of Form 45, and, as has been pointed out, the mean temperature to the nearest degree is sufficient. Furthermore except under unusual conditions, a variation of a whole degree in the mean temperature for a running is of no significance.

The following rule should be observed by computers in checking entries in column 7 of Form 45:

Make no change if the computed t_m is no more than 1° different from the value already entered on Form 45, but, if the computed t_m differs by more than 1°, make the change. If no change is made, place a check mark opposite the original entry.

The use of the above rule obviates the necessity for the checker to go over a large number of small changes which have no practical significance and also avoids many disagreements between computer and checker over minor differences which will have no appreciable effect on the final temperature corrections to be computed and entered in column 20 of Form 45. Unusual conditions may, at times, require slightly greater refinement in computing t_m and good judgment and experience will be required to know when it is necessary to depart from the above rule.

The entries in column 8 of Form 45 serve five useful purposes and should be checked carefully. These entries, the section lengths, are used as follows:

(a) In computing the probable error of double-run leveling,

(b) In computing the lengths of the links in making adjustments of the leveling,

(c) In distributing adjustment corrections through the various links or lines of levels,

(d) In checking distance references in the descriptions of the bench marks, and

(e) In determining whether or not the specified tolerances between forward and backward measures over a section have been exceeded and whether or not proper checks have been obtained between the new leveling and previous leveling with which it is connected.

The second decimal place in the length of a section has little significance for use in connection with items (a) to (d), inclusive, but has very real significance at times when item (e) is under consideration.

The stadia constant (C_s) of a leveling instrument is determined in the field and furnishes a means of computing the length of the section from the summation of the wire intervals. In actual practice the stadia constants of the Coast and Geodetic Survey geodetic leveling instruments range from about 0.290 to about 0.340 and, when used with the decimal point placed as shown above, are used to compute the length in meters of any single sight, thus:

$$l_s = I \times C_s.$$

Assuming a value of 0.326 for C_s and $I=204$,

$$l_s = 204 \times 0.326 = 66.5 \text{ m.}$$

For use in computing the length of a section (L_s) in kilometers, either use C_s as computed in the field but with the decimal point moved 3 places to the left or arrange to divide the result by 10^3 or 1,000. For example: assume $C_s=0.326$ as before, assume $SIb_n=1346$ and $SIf_n=1358$.

Then

$$L_s = \left(\frac{SIb_n + SIf_n}{1000}\right) C_s$$

$$= \frac{(1346+1358)\ 0.326}{1000}$$

$$= 0.88 \text{ km.}$$

In checking the entries in column 8 of Form 45, observe the following rules:

(1) Always check the additions, $SIb_n + SIf_n$.

(2) When there is only one running of a section, accept SIb_n and SIf_n as entered in the record book as correct, without actually checking these totals.

(3) When there are two or more runnings over a section and L_s for each running differs from the others by not more than 0.05 km., accept the values entered in the record book for SIb_n and SIf_n as correct without actually checking them.

(4) If there are two or more runnings over a section and the individual values of L_s differ by more than 0.05 km., and no logical explanation appears in the record book, check SIb_n and SIf_n. If no adding-and-subtracting machine is available for use, check all entries for Iu, Il, I, and SI_n as they were computed in the field for both the backsights and the foresights. However, if a machine is available for addition and direct subtraction, the values of SIb_n and SIf_n may be checked as follows:

$$SIb_n = Tb_1 - Bb_1 + Tb_2 - Bb_2 + \ldots\ldots\ldots\ldots + Tb_n - Bb_n$$

$$SIf_n = Tf_1 - Bf_1 + Tf_2 - Bf_2 + \ldots\ldots\ldots\ldots + Tf_n - Bf_n$$

Subtotals should be struck at SIb_3, SIb_6, SIb_9, etc., and SIf_3, SIf_6, SIf_9, etc., to enable one to localize the error quickly if one is found.

If no error is found, circle the smallest value of L_s for the section and mark it "to be used" so that, under these circumstances, the shortest length will be used in preparing Form 31.

If an error is found, make the correction in red ink as required at the place where the error occurred, correct at each subtotal provided for above, correct SIb_n and/or SIf_n as may be required and correct the sum SIb_n+SIf_n.

In computing or checking values of L_s on sections involved in obtaining checks on old leveling, at the beginning of a line, at the end of a line, at intermediate junctions along the line, or in releveling continuously along an old line, it may be necessary at times to be more particular about the second decimal place in certain values of L_s when there is doubt as to whether or not a check on a particular section of an old line was obtained.

Columns 10, 11, and 12 of the field-computation sheets are considered as a unit, since they serve one purpose—obtaining the correct difference of elevation for each running. It will be well for the novice to review the material on pages 62 to 71, inclusive, of Special Publication No. 239, "Manual of Geodetic Leveling." As will be noted, the entries in these columns are transcribed directly from the record books. However, the computer, in checking from books to sheets, is responsible for *more* than a routine check of the transcription. Before placing check marks on these entries on Form 45, the computer must make the following checks and comparison:

(1) Check the copying of $\frac{1}{3}$ $(STMBb_n)$ and Smb_n to the right-hand page or $\frac{1}{3}$ $(STMBf_n)$ and Smf_n to the left-hand page, as the case may require,

(2) Check $STMBb_n-STMBf_n$, or
$STMBf_n-STMBb_n$, as the case may require,

(3) Check $\frac{1}{3}$ $(STMBb_n-STMBf_n)$, or
$\frac{1}{3}$ $(STMBf_n-STMBb_n)$, as the case may require, and

(4) Compare $\frac{1}{3}$ $(STMBb_n-STMBf_n)$ with (Smb_n-Smf_n), or
$\frac{1}{3}$ $(STMBf_n-STMBb_n)$ with (Smf_n-Smb_n), as the case may require.

If this comparison shows a difference of 2 or less in the last place (0.2 mm.), assume that the discrepancy is due to an accumulation in rounding the individual means to four decimal places in meters (one decimal place in millimeters) and accept the value of (Smb_n-Smf_n) or (Smf_n-Smb_n), as the case may be, as entered in the record book as correct without further check.

However, if the comparison reveals a discrepancy of more than 2 in the last place, find out why.

Each time a value for m ends in a 3, $\frac{1}{3}$ of 1 in the last place has been dropped. Each time a value for m ends in a 7, $\frac{1}{3}$ of 1 in the last place has been added. If the 3's outnumber the 7's by 3, Sm_n is smaller than $\frac{1}{3}$ $(STMB_n)$ by 1 in the last place. If the 7's outnumber the 3's by 3, Sm_n is larger than $\frac{1}{3}$ $(STMB_n)$ by 1 in the last place. If the difference between $\frac{1}{3}$ $(STMB_n)$ and Sm_n is less than $n/3$, the discrepancy *may* be due to accumulation; if it is greater than $n/3$, there is an error in the record.

Whenever $\frac{1}{3}$ $(STMBb_n-STMBf_n)$ differs by more than 0.2 mm. from (Smb_n-Smf_n) or whenever $\frac{1}{3}$ $(STMBf_n-STMBb_n)$ differs by more than 0.2 mm. from (Smf_n-Smb_n), follow the procedure given below:

(1) Count the 3's and 7's occurring as end figures among the values of mb. If the net excess of 3's over 7's, or vice versa, accounts for the discrepancy between $\frac{1}{3}$ $(STMBb_n)$ and Smb_n, accept Smb_n as correct. Then count the 3's and 7's occurring

as end figures among the values of *mf*. If the net excess of 3's over 7's, or vice versa, accounts for the discrepancy between ⅓ ($STMBf_n$) and Smf_n, accept Smf_n as correct and disregard the following steps, since the discrepancy has been satisfactorily accounted for as an accumulation resulting from dropped or added thirds.

(2) When the procedures outlined in (1) fail to account satisfactorily for a specific discrepancy between ⅓ ($STMBb_n$) and Smb_n or ⅓ ($STMBf_n$) and Smf_n, the location of the mistake in the record will be localized to either the backsights or the foresights. Next, apply the necessary portion of the procedure outlined in (1) at the midpoint of the running. This will serve to localize the mistake in either the first or the second half of the running. Next, apply the necessary portion of the procedure outlined in (1) up to the midpoint of the half of the running to which the trouble has been traced. Repeat this routine as may be necessary to eventually locate the individual set-up in the record for which the mistake was made.

(3) Required changes should then be made in the record and on Form 45.

When the entry D_F or D_B in the record book is correct or has been corrected, check the entry in column 12 of Form 45 or make the necessary change, as may be appropriate. Care should be exercised to make sure that the algebraic sign as well as the numerical value of the entry is correct. No check marks need be placed opposite the entries for Smb_n and Smf_n in columns 10 and 11 of Form 45, but, if errors in these values have been found and corrected in the record book, the appropriate changes should be made in the corresponding entries on the field-computation sheet.

After all of the entries in column 12 of a sheet of field computation have been checked as outlined above, make an additional check on the sign of the entries in column 12 by comparing the entries in column 10 with the corresponding entries in column 11. If an entry in column 10 is numerically greater than the corresponding entry in column 11, the sign of the entry in column 12 should be plus (+). If the entry in column 10 is numerically smaller than the corresponding entry in column 11, the sign of the corresponding entry in column 12 should be minus (—). If this last check indicates that the sign of any entry in column 12 is incorrect, go back to the record books and make certain that the entries in columns 10 and 11 are correct before making the change in the sign of the entry in column 12.

Occasionally a running over a section is completed in the record books but is rejected by the observing party and is not abstracted on Form 45. Any such runnings should be transcribed to Form 45 and included in the computations even though the office computation supports the rejection made by the field party. The reason for this provision is that it sometimes happens that runnings rejected by the field party are found to be retainable after all corrections are applied and new means are taken. If all runnings are not shown on Form 45, it *may* happen that a good running will be left out of the final computation.

The entries made by the field party in column 22 of Form 45 concerning the location of extra-foresight points should be inspected to see that they have been correctly transcribed from the record books. If, in these entries, mileage references along the railroad or road to the nearest station or town are not included, compute them as instructed in item 5 (p. 30, et seq.). The entries concerning checks against old leveling need not be checked, since this feature will be taken care of in connection with the junction-detail studies which will be made after the corrected differences are available from the computations made on Form 31.

Column 22 of Form 45 should also be used for the following purposes:

(a) To record cross references at all known junctions with other lines.

(b) To record notes showing spur lines when these are to bench marks described on Form 638. Except for extra foresights to rails opposite stations and mileposts, the presence of a brief description in column 22 will show the section abstracted to be a spur to an extra-foresight point.

(c) To record any other notes or comments which are pertinent to the leveling.

4. Computing entries for columns 18, 19, 20, and 21 of Form 45.—At the present time the geodetic leveling rods of the Coast and Geodetic Survey are standardized at the National Bureau of Standards and are then paired before being sent to the field. For this reason and unless the rods are mismated in use, the index correction (c_i) is always zero. No consideration need be given to column 18 of Form 45 if the rods in use on the leveling formed a proper pair. We continue to carry column 18 for

ROD STANDARDIZATION SHEET

923
HGZ
1946
R

2 mi. North of Stephan to Rousseau, S. Dak.

F. A. Riddell
1946

L-11662

ROD	TEMP.	LENGTH	COEFF.	EXCESS
322	27°	2.9999	.000001	
325	24°	2.9999	.000001	
Mean	26°	2.9999	.000001	-0.033 mm/m
249	27°	3.0000	.0000025	
320	27°	3.0000	.0000010	
Mean	27°	3.0000	.0000018	0.000 mm/m

FIGURE 8.—Sample of rod-standardization sheet.

emergency use should it happen that for any reason a field party was forced to mismate rods temporarily in order to continue leveling.

Note.—If only one rod is used the index correction to an observed difference of elevation is zero. When two rods having the same index error are used the index correction will still be zero. When two rods having different index errors are used and each rod stays on its own point, for both the foresight from the preceding instrument station and the backsight from the succeeding instrument station, the index correction to the observed difference of elevation will be either zero or the difference between the two index errors, depending on whether there is an even or an odd number of instrument stations or set-ups in the running. Furthermore, if two rods having different index errors are used and both rods move forward when the instrument is moved forward along the line, the index correction to the observed difference of elevation will be n times the difference between the two index errors, when "n" is the number of instrument stations or set-ups in the running.

Preparatory to computing corrections (c_r and c_i) to be entered in columns 19 and 20 of Form 45, the computer will prepare a rod-standardization sheet. (See fig. 8.)

789223°—49——3

Unless some emergency or accident prevents, the rods are always used in pairs, and each pair of rods should be considered as a unit on the rod-standardization sheet. A file is maintained which includes the standardization data for each of our geodetic leveling rods. From that file the computer will obtain for each rod the temperature at which the rod was standardized, the coefficient of expansion of the invar strip, and the length of the rod (from the 0.2-meter to 3.2-meter graduations) at the temperature at which it was standardized. He will then compute:

(1) The mean of the temperatures at which the two rods of the pair were standardized. This mean temperature is the t_s of the notation used in this manual and is computed to the nearest degree.

(2) The mean of the coefficients of expansion of the two invar strips in the pair of rods. This mean coefficient is the C_e of the notation used in this manual and is computed to the same number of decimal places as are used in the individual coefficients in the file of standardization data.

(3) The mean of the two rod lengths to the nearest 0.1 mm.

(4) The excess (e) of the actual mean rod length, at the standardization temperature, over the nominal length of the rod. This is computed to three decimal places in millimeters per meter. The sign of the excess will be plus if the rods are actually longer than their nominal length, but minus if they are shorter than their nominal length.

In the sample rod-standardization sheet shown in figure 8, the complete series of computations outlined above has been carried out for two different pairs of rods and should be self-explanatory.

The corrections to be entered in column 19 of Form 45 for each running are computed by means of the following formula:

$$c_r = D \times e$$

and due care should be exercised to take into account the algebraic signs of both D and e. The following numerical example will serve as an illustration.

Assume

$$D = -7.3742 \text{ m., and}$$

$$e = +0.033 \text{ mm./m.}$$

Then

$$c_r = -7.3742 \times +0.033$$

$$= -0.2 \text{ mm.}$$

It should be noted that c_r is computed to the nearest 0.1 mm., and that the excess (e) is in *millimeters per meter*, which has the effect of making the above computation the same as

$$c_r = -7.3742 \times +0.000033$$

$$= -0.0002 \text{ m.}$$

$$= -0.2 \text{ mm.}$$

The corrections to be entered in column 20 of Form 45 for each running are computed by means of the following formula:

$$c_t = (t_m - t_s) \times D \times C_e$$

Again, due care should be exercised to take into account the algebraic signs of $(t_m - t_s)$, D, and C_e. The following numerical example will serve as an illustration.

Assume

$$t_m = 35°,$$
$$t_s = 26°,$$
$$D = -8.4735, \text{ and}$$
$$C_e = +0.000001$$

Then

$$\begin{aligned} c_t &= (35° - 26°) \times -8.4735 \times +0.000001 \\ &= +9 \times -8.4735 \times +0.000001 \\ &= -0.000076 \text{ m.} \\ &= -0.076 \text{ mm.,} \end{aligned}$$

which, when rounded to the nearest 0.1 mm., becomes 0.1 mm.

The level correction (c_l) to be entered in column 21 of Form 45 for each running is applied to take care of the lack of adjustment of the leveling instrument. Actually the instrument is adjusted in the field if the level constant, C, is found to be greater than a prescribed limit. This limit is such that at all times the instrument is in reasonably good adjustment. However, standard practice in all forms of precision surveying is to have the instrument in good adjustment and then to devise an observing program which will either eliminate entirely the last little bit of maladjustment or measure it and correct for it. In precision leveling we attempt to eliminate the effect of slight inaccuracy of adjustment by keeping backsights and foresights equal in length, but, as this cannot be done exactly, we measure the small amount that the level is out of adjustment and then correct the observations for the error thus introduced.

As explained in Special Publication No. 239 (p. 6) and on page 20 of this manual, the level constant, C, is to be determined daily, adjustment of the instrument is made if required, and, if adjustment is made, the value of C is again determined.

The level correction (c_l) is simply the product of the level constant (C) and the difference in the accumulated lengths of the backsights and foresights for the running, thus:

$$c_l = (SIb_n - SIf_n)\ C.$$

Due care must be exercised that the algebraic signs of the various items are strictly observed. The following numerical example will serve to illustrate the computation of level corrections.

Assume

$$SIb_n = 2737 \text{ mm.,}$$
$$SIf_n = 2760 \text{ mm., and}$$
$$C = -0.005$$

Then

$$\begin{aligned} c_l &= (2737 - 2760) \times -0.005 \\ &= -23 \times -0.005 \\ &= +0.115 \text{ mm.,} \end{aligned}$$

which, when rounded to the nearest 0.1 mm., becomes +0.1 mm.

It should be noted that the selection of the proper value of C to be used in each case requires reference to the C-sheet (fig. 7, p. 21) and the observance of the following rules.

(1) If only one C was observed during a day's leveling, use that C in computing all values of c_l for runnings made on that day.

(2) If the instrument was adjusted, use the first C before adjustment in computing values of c_l for all runnings made on that day before the running in which the adjustment was made. Use the C after adjustment for computing all values of c_l for runnings made on that day after the running in which the adjustment was made.

(3) The c_l for the running in which the adjustment was made should be handled as follows:

(a) Compute c_l to three decimal places for the set-ups preceding and including the C set-up, using the C before adjustment.

(b) Compute c_l to three decimal places for set-ups following the C set-up, using the C after adjustment.

(c) Combine the corrections computed in accordance with (a) and (b) and round the result to the nearest 0.1 mm., to obtain the c_l for the running in which the adjustment was made.

(4) If no C was determined on a particular day, use the mean of the last C for the previous day and the first C of the following day.

(5) If no C was determined on the first day of the leveling, use the first C to be determined on the line.

(6) If no C was determined on the last day of the leveling, use the last previously determined C.

(7) It is possible that, in case an instrument has met with an accident, an extra C determination or an extra group of C determinations may be made at some unusual time during the day. To decide just what value of C to use in such cases may involve good judgment and experience not yet in the possession of a new computer. In such cases he is advised to seek advice from his supervisor, at least until he has attained sufficient familiarity with the methods of handling such exceptional cases as will permit him to proceed by himself with some assurance that he will be using the proper methods.

Corrections in columns 18, 19, 20, and 21 of Form 45 are computed and entered on the form by the computer. The checker will check all values entered in those four columns, make sure that all C's have been entered on the C-sheet, and that all extra-foresight runnings have been abstracted, in addition to checking changes and entries in other columns as provided for in the preceding pages.

5. Checking the abstracting of extra-foresight runnings.—Paragraphs 12, 13, and 14 of the General Instructions for Geodetic Leveling (Special Publication No. 239, pp. 4 and 5) provide for taking extra foresights. When leveling along railroads these extra foresights are taken on the top of the rail on bridges, opposite railroad stations, opposite mileposts, and at important crossings, and at other points which may be useful in connection with topographic surveying either by ground methods or aerial methods. When the parties are leveling along roads or highways, extra foresights are taken on the surface of the roadway at bridges, at crossings, and at a variety of other useful points. The abstracting of these extra foresights has been a particularly troublesome feature of the training of new computers in the Section of Leveling and is covered in considerable detail in this manual in the hope that this feature of the work can be presented so that the novice, with only a limited knowledge of field methods, may be able to interpret properly the field records concerning runnings to extra-foresight points.

As a sort of background to the following discussion, it may be well to outline briefly the fundamentals of spirit leveling. In its simplest form, differential spirit leveling is carried on as follows:

A leveling rod, graduated from the bottom upwards, is held in a vertical position on a bench mark or point of known or assumed elevation. The leveling instrument is then set up a convenient distance forward along the route to be followed. After the instrument has been leveled and the telescope pointed and focused on the rod, the line of sight intersects the rod on the bench mark at a distance above the bottom of the rod or the bench mark which can be observed by noting the reading on the rod at the point where the line of sight intersects it. Then, since the line of sight is horizontal, the reading on the rod is a measure of the amount by which the elevation of the line of sight through the instrument exceeds the elevation of the bench mark on which the rod is held. The reading, the backsight, is added to the elevation of the starting bench mark to arrive at the elevation of the line of sight through the instrument—commonly called the "height of instrument" or "H. I."

Then if the same rod or another one like it is held in a vertical position on another point still farther along the route to be followed, the reading taken on the second rod is a measurement of the amount by which the elevation of the point on which the rod is held is lower than the elevation of the line of sight through the instrument or H. I. This reading subtracted from the H. I. gives the elevation of the turning point or other point on which the rod was held when the second reading was taken.

The turning point then becomes a point of known elevation, and, if the instrument is again moved to a point farther along the line, a new backsight can be taken on the turning point and a second foresight can be taken on a rod held on a second turning point still farther along the line. This second turning point will then in its turn become a point of known elevation and can be used as the starting point for a third set-up.

This operation is repeated over and over along the line until a second bench mark is reached and one running of the section between two bench marks has been completed.

Inasmuch as the set-ups and turning points are only temporary (simply a means of carrying the line forward), we have no interest in the H. I. of each set-up or in the elevations of the temporary turning points. Departing from standard practice in ordinary engineering leveling, we record the backsights on the left-hand page of each opening of the field book and the foresights on the opposite page. Then, by totaling all of the backsights, totaling all of the foresights, and subtracting the total of the foresights from the total of the backsights, we obtain the observed difference of elevation for the running between the bench marks at the ends of the section.

Now it will frequently happen that, in a running between two bench marks, a party will find a point for which an extra elevation should be determined, such as the top of the rail in front of a railroad station. The elevations of such points are not published to the same degree of refinement as are the elevations of the bench marks, and the most economical method of obtaining elevations for such points is by reading extra foresights on rods held on them.

An extra foresight is just what the name implies, an extra measurement downward from the H. I. to the extra point. A little reflection will show that, in order to obtain a difference in elevation from the starting bench mark in a running to the extra-foresight point, the leveling is the same up to and including the backsight for the set-up from which the extra foresight was taken. Then, if we substitute the extra foresight for the regular foresight at that same set-up, we have a complete running

from the starting bench mark of the running to the extra-foresight point. Also, if the extra-foresight point happens to be near the end of a running of a section of the main line, it may be easier to work backward from the end bench mark to the extra-foresight point. However, this phase of the subject will be discussed later. (See p. 33.)

The sketch in figure 9 shows in diagrammatic form (in plan, not in profile) the situation when a running has an extra foresight.

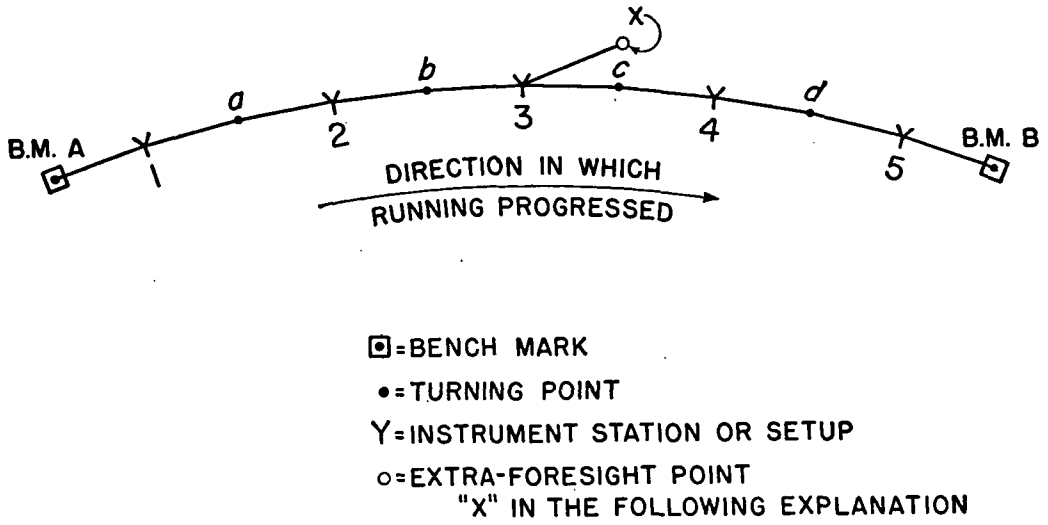


FIGURE 9.—Sketch showing the relationship of an extra foresight to the main line of levels.

According to the system of notation in use in this manual, the observed difference in elevation from B. M. A to B. M. B is:

$$D_F = Smb_5 - Smf_5,$$

and the running in the same direction from B. M. A to the extra-foresight point (X) is:

$$D_F = mb_1 + mb_2 + mb_3 - mf_1 - mf_2 - mf_{x3}$$
$$= Smb_3 - Smf_2 - mf_{x3}.$$

It is important to remember that any running to an extra-foresight point is considered as a spur line. When the difference in elevation is abstracted as a difference in elevation *from* the point in the main line *to* the extra-foresight point the running is a *forward* running. When abstracted as a difference in elevation *from* the extra-foresight point *to* a point in the main line the running is a *backward* running, regardless of the actual geographic direction of the running of the spur or its relation to the direction of the running along the main line.

It may be well for a new computer to diagram the first few cases of extra-foresight spurs which he has occasion to check or abstract, but this will not be necessary after he gets the knack of handling them.

To obtain the difference of elevation from X to B. M. A (backward),

$$D_B = mf_{x3} + mf_2 + mf_1 - mb_3 - mb_2 - mb_1$$
$$= Smf_2 + mf_{x3} - Smb_3.$$

It will be noted that we have now interchanged backsights and foresights. This is legitimate, since in computing D_B we are assuming that the point X is our starting point and that to get back to B. M. A we have measured up from X to H. I.$_3$, down from H. I.$_3$ to turning point b, up from turning point b to H. I.$_2$, etc.

To abstract the leveling diagramed in figure 9 as if it were run *from* B. M. B *to* X,

$$\begin{aligned} D_F &= mf_5+mf_4+mf_3-mb_5-mb_4-mf_{x3} \\ &= Smf_5-Smf_2-Smb_5+Smb_3-mf_{x3} \\ &= -(Smb_5-Smf_5)-Smf_2+Smb_3-mf_{x3}, \text{ and} \\ D_B &= mf_{x3}+mb_4+mb_5-mf_3-mf_4-mf_5 \\ &= mf_{x3}+Smb_5-Smb_3-Smf_5+Smf_2 \\ &= mf_{x3}+(Smb_5-Smf_5)-Smb_3+Smf_2. \end{aligned}$$

Extra foresights taken on the top of the rail opposite railroad stations and opposite mileposts are sufficiently identified by means of the names of the railroad stations or the milepost numbers and no additional geographic references need be carried in column 22 of Form 45 or in column 12 of Form 31. Extra foresights to points on bridges and at road crossings, highway intersections, etc., should include a distance-and-direction reference to the nearest town as a part of the description of each point.

If a distance-and-direction reference to the nearest town is given in the record book or on Form 45 for the point on which an extra foresight was taken, only an approximate check need be made to make certain that the reference is sufficiently accurate to correctly identify the point. In general the following inspection check will be sufficient. If the extra foresight was taken during a running from bench mark A to bench mark B, the distance-and-direction reference should place the extra-foresight point between B. M. A and B. M. B; i. e., if the extra foresight was taken about midway in the running from B. M. A to B. M. B, the distance-and-direction reference should show the extra-foresight point to be about midway between the two distance-and-direction references for these two bench marks as given in the descriptions of them on Form 638. Of course, if the route of the line is such that a number of closely spaced bridges, crossings, or intersections may be expected, a closer check should be made to make certain that the points on which the extra foresights were taken are correctly described. In such cases proceed as outlined below for cases in which the distance-and-direction references to extra-foresight points were not computed and entered by the field party, either in the record books or on Form 45.

If it is necessary to *compute* the distance from the nearest town in order to check or complete the description of an extra-foresight point, first compute the distance from the bench mark in the main line, from which the spur to the extra-foresight point was computed, to the extra-foresight point by means of the intervals. In the case shown in figure 9, the distance from B. M. A to the extra-foresight point is

$$\begin{aligned} L_s &= (Ib_1+If_1+Ib_2+If_2+Ib_3+If_{x3})C_s \\ &= (SIb_3+SIf_2+If_{x3})C_s. \end{aligned}$$

This gives L_s in kilometers and this value must be converted to miles before it is applied, with the appropriate sign, to the distance reference in the description (in this case, of B. M. A on Form 638), to arrive at the distance reference for the extra-foresight point.

Had it so happened (see fig. 9) that the extra-foresight point had been located between set-up 3 and turning point b and this fact had been made known by the presence of a B on the left-hand page of the record book opposite the record of the observation of the extra foresight, the sign of the If_{x3} member of the above equation would be minus (−) instead of plus (+) as shown above. However, if neither F nor B has been entered in the record book, so that the direction to the extra-foresight point is not known, neglect the term If_x in computing L_s for the spur to the extra-foresight point.

Normally, extra foresights are taken only to the top of a rail, the surface of a roadway, the floor of a bridge, etc., and elevations thus determined are published only to the nearest tenth of a foot. The corrections in columns 18 to 21, inclusive, of Form 45 are therefore insignificant and need not be computed for any runnings of spurs to extra-foresight points. However, it sometimes happens that a bench mark is touched upon by means of an extra foresight. In such cases, the corrections in columns 18 to 21, inclusive, of Form 45 will be computed. Also, runnings to permanent bench marks touched upon by means of extra foresights are always to be abstracted from the *nearest* bench mark in the main line.

If runnings to extra-foresight points have been abstracted by the field party, the abstracting will be checked by the computer, and the checker will confine his attention to checking changes made by the computer and to a general inspection to make sure that none has been overlooked. If extra-foresight runnings have not been abstracted by the field party, they will be abstracted by the computer and checked completely by the checker.

6. Consideration of check points.—Bench marks which are established to hold the elevation of the line overnight or longer are referenced by check points as specified in paragraph 17 of the General Instructions for Geodetic Leveling. (See p. 5 of Special Publication No. 239.) Whenever these check points are encountered in the record books, they are to be computed on scratch paper and a check made to make sure that the bench mark has remained stable in relation to the check point during the interval between the two observations of the difference in elevation between the bench mark and its check point, and that the elevation has been carried forward from the proper point.

7. Computation of river crossings.—River crossings occur relatively infrequently. The details of the computations involved are quite different from those encountered in the normal computation of a line of levels. The field methods used in making river-crossing observations and the details of the method of computing the difference of elevation resulting from the observations are covered on pages 51 to 61, inclusive, of Special Publication No. 239.

Because of the fact that the necessity for checking a set of computations for a river crossing occurs only now and then, it is not believed to be necessary or advisable to repeat all of the details of the computing in this manual. The reader is referred to Special Publication No. 239 for the discussion of these details, and only comments of special interest to office personnel are included in the following paragraphs.

The office computer who is checking a set of river-crossing computations will have at his disposal the original record of the observations (at least two volumes) and the field computation made by the observing party.

The first step will require checking all computed entries for near-rod readings shown in the record books. The items to be checked will be $STMBb_1$, $STMBb_1 \div 3$, mb_1, Iub_1, Ilb_1, and Ib_1. $STMBb_1 \div 3$ should be compared with mb_1 and any discrepancy greater than one in the last place (0.1 mm.) should be investigated.

The second step will require checking column totals and mean values for each of

the three columns of micrometer readings on each right-hand page throughout the record. If possible, this should be done by using an adding machine for summing the columns and a computing machine for obtaining the mean values. Any errors discovered in the field values should, of course, be corrected.

The third step includes the checking of the entries on the computation sheet which were transferred from the record books.

The fourth step includes the checking of all remaining entries on the computation sheet, to and including the final mean difference of elevation between the bench marks at the ends of the crossing.

All discovered errors should be corrected, and care should be exercised to make sure that any change affecting any later entries is taken into account in the recomputation of those entries. Special attention should be paid to such items as the position of the decimal point, reversed figures, algebraic signs, and to the correct positioning of data transferred from the record books.

On the computation sheet shown in figure 15, pages 58 and 59, Special Publication No. 239, the column subheadings under "Micrometer Interval" should be read "Top *to* Bottom" and "Level *to* Bottom" rather than "Top *minus* Bottom" and "Level *minus* Bottom," to avoid negative signs in these columns, since the micrometer readings *increase* as the middle horizontal wire travels *downward* on the rod. However, should this point be neglected and negative signs be entered in these columns, no damage will be done if strict adherence to signs is maintained throughout the computation, since in the sample computation on page 57 of Special Publication No. 239,

$$\frac{(24.025)(0.5200)}{49.940}=0.2502,$$

would become

$$\frac{(-24.025)(0.5200)}{-49.940}=0.2502$$

and the result would still be the same.

Since there is no stadia determination of the length of a river crossing, such as is determined for the length of a section of a line run in the orthodox manner, the length of the crossing is usually scaled from the best map available in the Office when a check on the length of the crossing is made.

NOTE.—In the process of checking from books to sheets, irregularities in field procedure may be noted. The computer should prepare a list of all such irregularities noted, giving the character of each irregularity and a volume-and-page reference showing the place in the record where each one occurs. This list should be attached to the title sheet of the computations for the line or area and should be called to the attention of the person in charge of the group of computers carrying on this portion of the work.

PREPARATION OF OFFICE ABSTRACTS

When the procedures discussed under the heading "Checking from Books to Sheets" have been completed, a computer undertakes the preparation of the first of two independent office abstracts, using Form 31. (See fig. 10, p. 36.) This form is 10½ inches high and 16 inches wide and is folded along a 10½-inch line midway of the 16-inch width to form a 4-page folder. This form is printed exactly alike on both sides of the sheet. All work on Form 31 is to be done in blue or blue-black ink, except when red ink is used to indicate spurs.

In beginning an office abstract, page one of the first 4-page folder is used as a title page. On it are shown the library shelf mark and the accession number of the computations. These should appear near the right-hand edge of the page and about halfway down that portion of the page which is ruled with horizontal lines. The other information shown on the title page includes:

1. The order of accuracy of the leveling,
2. The title of the line or area,
3. The name of the chief of party, and
4. The year or years in which the leveling was done.

A typical example of the entries on such a title page is shown below.

Second Order
Poway Area, Calif.
John Laskowski
1935

941
HGZ
1935
L
L–6852

In making the office abstract for the leveling in an area, only the area title should appear on the title page. The individual line titles and numbers should appear at the beginning of the abstract for each individual line and be separated by a skipped line from both preceding and following computations.

The abstract is begun on pages 2 and 3 of the first 4-page folder and is continued

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 31
Ed. Sept. 1939

ABSTRACT OF

STATE Indiana INSTRUMENT G 70 RODS 370 & 375

U. S. GOVERNMENT PRINTING OFFICE 16—11482

(1) Date 1946		From B. M. to B. M. (2)	Distance (3)	Difference of Elevation: Forward Line (4)	Backward Line (5)	Mean (6)	Discrepancy: Partial (7)	Total accumulated (8)
August			km.	m.	m.	m.	mm.	mm.
	14	I8 - J8	0.30		−0.4570	+0.4570		
14	14	J8 - TBM 1	0.57	−3.1607	+3.1608	−3.1608	−0.1	−0.1
14	14	TBM 1 - Y159	0.77	−1.7373	+1.7395	−1.7384	−2.2	−2.3
14	14	Y159-TBM 2	0.80	+0.8410	−0.8422	+0.8416	+1.2	−1.1
14	14	TBM 2 - E. Rail		−0.0170	+0.0200	−0.0185	−3.0	
14	14	TBM 2 - Z159	0.93	−1.8402	+1.8443	−1.8415	−2.6	−3.7
	14				+1.8413			
14	14	Z159 - E. rail Gravel Rd.		+1.8310	−1.8297	+1.8304	−1.3	
14	14	Z159 - TBM 3	0.87	+4.6806	−4.6816	+4.6811	+1.0	−2.7
14	14	TBM 3 - E. rail Gravel Rd		−0.0030	−0.0010	−0.0010	+4.0	

FIGURE 10.—Sample of office abstract—one sheet of Form 31.

by using page 4 of the first folder with page 1 of the second, and so on throughout the length of the abstract.

When abstracting check runnings along old lines, the entries should be made in red ink if no new marks were established between the old marks. If new marks were established during the new leveling, the check leveling should be abstracted in blue or blue-black ink. The main line is abstracted in blue or blue-black ink and spurs in red ink, except as provided for below. Long spurs to provide connections to tidal bench marks, leveling of other organizations, etc., may be abstracted in blue or blue-black ink. In such cases, leave one blank line across Form 31 after the last main-line section before a long spur. On the next line enter "Spur Line to ------," giving the name of the place to which the spur line leads. On the next line enter in column 9 the designation of the bench mark which forms the base of the spur line. The spur line is then abstracted on the following lines until it is completed. Then leave one blank line. On the next line enter "End of Spur Line" or "Main Line Continues." On the next line repeat in column 9 the designation of the bench mark which is both the base of the spur and the mark from which the main line continues. Continue abstracting the main-line sections as before.

The data for preparing the office abstract (Form 31) are obtained from the field-computation sheets (Form 45). After the headings have been filled in, obtain the entries for the various columns as outlined below. The column numbers used in referring to Form 31 are as shown in figure 10 (p. 36), and the column numbers used in referring to Form 45 are as shown in figure 2 (p. 10). These numbers do not appear on the printed forms but were added to figures 2 and 10 for convenience of reference.

NOTE.—For the reason that two colors could not be used readily in preparing figures 2 and 10, spur lines are "boxed" rather than shown in red as provided for in this manual.

FIRST -ORDER LEVELS

OBSERVER G. Mueller COMPUTER Compton Frye

Designation of B. M. (9)	Distance from B. M. J8 (10)	Observed Elevation Above Mean Sea Level (11)	(12)
	km.	m.	
I8		251.8858	No. 74017
J8	0.00	252.3428	See: Warsaw, Ind., to Jackson, Mich.
TBM 1.			
Y159	1.34	247.4436	
TBM 2.	2.14		
E. Rail		248.2667	1.2 mi. N. of Warsaw, 0.5 mi. N. of Y159. Top of E. rail main NYCRR & N end E. siding
Z159	3.07	246.4437	
E. Rail Gravel Rd.		248.2741	1.9 mi. N. of Warsaw, at Z159, Top of E. rail of NYCRR at & E-W gravel road.
TBM 3.	3.94		
E. Rail Gravel Rd.		251.1238	2.4 mi. N. of Warsaw, 0.5 mi. N. of Z159, Top of E. rail NYCRR & & E-W gravel road.

In column 1 of Form 31 it is customary to enter the year at the top of the column and in the blank space below the heading "DATE" and to enter the month of the year in the upper portion of the first blank space in the column. The day of the month on which a forward running was made goes in the left-hand part of the column and the day of the month on which a backward running was made goes in the right-hand part of the column, as shown in figure 10.

NOTE.—Figure 10 represents the office abstract of the leveling shown in the field computation in figure 2, except that the corrections entered in the Office in columns 18, 19, 20, and 21 of Form 45 are not shown in figure 2.

The entries in columns 2 and 3 of Form 31 are copied directly from columns 2 and 8 of Form 45.

The entries in columns 4 and 5 of Form 31 are taken from Form 45 by correcting the entries in column 12 of Form 45 by the algebraic sum of the corresponding corrections in columns 18 to 21, inclusive, of Form 45. Corrected differences in elevation for forward runnings are entered in column 4 of Form 31 and corrected differences in elevation for backward runnings are entered in column 5 of Form 31. If more than two runnings over a section were made, enter the corrected differences for the extra runnings on succeeding lines as may be necessary, but do not repeat the entries in columns 2 and 3. (See sixth section abstracted in fig. 10, TBM 2 to Z 159.)

The entries in column 6 of Form 31 are derived from those in columns 4 and 5 in accordance with the following rules.

Rule 1. If there is only a forward running of a section, the entry in column 6 is the same as that in column 4.

Rule 2. If there is only a backward running of a section, the entry in column 6 is the same numerically as the entry in column 5, but the sign is opposite to that of the corresponding entry in column 5.

Rule 3. If there are two runnings over a section, one forward and one backward, the entry in column 6 is the mean of the numerical values in columns 4 and 5, but the sign is that of the forward running or the same as that of the entry in column 4—except in special cases covered by rule 4.

Rule 4. If, as sometimes happens, the mean difference in elevation for a section is smaller than the difference between the forward and backward runnings, the sign of the entry in column 6 *may* be the same as that of the corresponding entry in column 5. Since the entry to be made in column 6 is the mean difference in elevation in the forward direction, it may be computed by changing the sign of the backward running (column 5) and taking the mean of the two values with due regard for signs. As an illustration of this type of special case, note the following examples.

I. Assume a forward running of +0.0024 and a backward running of +0.0010. Changing the sign of the backward running, we have +0.0024 and −0.0010, and the mean is +0.0007.

II. Assume a forward running of −0.0006 and a backward running of −0.0019. Changing the sign of the backward running, we have −0.0006 and +0.0019, and the mean is +0.00065 or, when rounded to four places, +0.0006.

Rule 5. If there are more than two runnings over a section, take a mean of all of the forward and backward runnings, changing the signs of the backward runnings. Then, in first-order leveling if any of the runnings diverge from that indiscriminate mean by amounts in excess of ± 6.0 mm. $\sqrt{K}$ (when K is the length of the section in kilometers), such runnings shall be rejected. After the necessary rejections have been made, take a mean of all retained forward runnings and a mean of all retained back-

ward runnings. The mean of these two means is the numerical value to be entered, with the sign of the forward running, in column 6 of Form 31. In dealing with sections having lengths of 0.25 km. or less, assume K to be 0.25 km. Then, on such sections, the rejection limit will be $\pm$ 6.0 mm. $\sqrt{0.25}=\pm 3.0$ mm. With double-run second-order leveling the same procedure is followed, except that the rejection limit is ± 9.0 mm. $\sqrt{K}$. For sections of double-run second-order leveling 0.25 km. or less in length, assume K to be 0.25 km. Then, for these short sections the rejection limit is the constant ± 9.0 mm. $\sqrt{0.25}=\pm 4.5$ mm.

CAUTION. Following rule 5 blindly may in some cases produce ridiculous results. For example, if among the runnings over a section there is one containing an obvious blunder, the inclusion of that running in the indiscriminate mean would result in the rejection of *all* the runnings. Furthermore, if a section had four runnings over it and if among the runnings there were two which agreed well, a third which had an obvious blunder of small size, and a fourth which contained a blunder of large size, the inclusion of all runnings in the indiscriminate mean might result in the rejection of all runnings except the one containing the obvious blunder of small size. Therefore, before taking the indiscriminate mean provided for in rule 5, all runnings containing obvious blunders of large size are to be arbitrarily rejected. If doubt arises as to whether a particular running is to be arbitrarily rejected, refer the matter to the supervisor. Also, marginal cases, which seem to require the exercise of good judgment rather than a strict application of rule 5, should also be referred to the supervisor, at least until the novice has become familiar with the line of reasoning followed in treating such cases.

Occasionally a section will be encountered over which there are only two runnings, and these two runnings do not agree within the allowable limits for field check (4.0 mm. $\sqrt{K}$ for first-order leveling, or 8.4 mm. $\sqrt{K}$ for second-order leveling). Such cases can be divided into three groups, as follows:

Group 1. Marginal cases in which the application of the index, rod, temperature, and level corrections has thrown the corrected differences just outside the allowable tolerance or limit of divergence.

Group 2. Cases, in single-run second-order leveling, where for some reason the field party has suspected a blunder; has made an extra running which does not constitute a proper check; and has noted in the record a logical reason for rejecting one value and retaining the other.

Group 3. Cases in which the observer has failed to obtain a check within allowable limits and has offered no explanation of his failure to do so.

NOTE.—*If* the field party follows strictly the provisions of paragraphs 24 to 33, inclusive, of the General Instructions for Geodetic Leveling (see pp. 8 and 9 of Special Publication No. 239, "Manual of Geodetic Leveling"), no cases will be encountered which will fall in group 3 as outlined above. Cases that fall in group 2 will be properly explained if the General Instructions are followed strictly.

The procedure in handling cases falling in group 1 is to use the mean of the two corrected differences in elevation and make no notation as to the divergence or partial being slightly in excess of the allowable limit. If a case is encountered which falls in group 2, accept the running retained by the field party and reject the one which they rejected, since the field party presumably had a good and legitimate reason for suspecting a blunder in the running or they would not have made the second running over the section. If a case is encountered which falls in group 3, use the mean of the two runnings but furnish a memorandum to the Chief, Section of Leveling, in which will be included the names of the chief of party, the observer, and the recorder; the name of

the line or area; a statement as to whether the leveling is first- or second-order; the volume and page references where the runnings may be found; and a statement showing the terminal bench marks of the section, the observed differences of elevation, the actual discrepancy between them, and the allowable divergence for the class of leveling involved. Such memoranda should be prepared by the computer discovering the trouble, checked by the supervisor, and passed on to the Chief, Section of Leveling.

The entries in column 7 of Form 31 are derived from the corresponding entries in columns 4 and 5. Except in the special cases noted below, the entry in column 7 is the difference between the numerical values in columns 4 and 5 and takes the sign of the smaller of the numerical values. On sections where more than two retained runnings occur, the entry in column 7 is the difference between the mean of the forward runnings and the mean of the backward runnings, with the sign of the smaller of the two means. This may be illustrated by reference to figure 10 (p. 36). In the case of the section from Y 159 to TBM 2, the forward running is +0.8410, the backward running is −0.8422, the difference of the numerical values is 0.0012 m., or 1.2 mm., and takes the plus sign since the 0.8410 of the forward running is the smaller of the two numerical values. In the case of the section from TBM 2 to Z 159, the forward running is −1.8402, the mean of the backward runnings is +1.8428, the numerical difference is 0.0026 m., or 2.6 mm., and the partial is −2.6 mm., since the smaller numerical value (that of the forward running) has the negative sign.

CAUTION. This rule or procedure fails in the relatively rare case in which the difference of elevation between the bench marks at the ends of the section is so small that both forward and backward runnings produce differences of elevation of the same sign. In such a case, the best way to *understand* the computation of the partial is to assume an elevation for the starting bench mark; apply the forward difference to the assumed elevation to get an elevation for the terminal mark; then apply the backward difference to the elevation last computed to arrive at a second elevation for the starting bench mark of the section. The correction necessary to make this second elevation of the initial bench mark agree with the assumed elevation used as a start will be the partial or entry in column 7 for that section. On the other hand, probably the best way to *compute* the partial in such a case is to evaluate the expression $(-D_B-D_F)$, which is equivalent to taking the algebraic sum of the differences of elevation for the forward and backward runnings of the section and then changing the sign of the sum.

This may be illustrated by referring to the section TBM 3 to E. Rail in figure 10 (p. 36). In this case, if we assume an elevation of one meter for TBM 3 the computation of the partial is as follows:

Assumed elevation of TBM 3	1.0000 m.
Forward running	−0.0030
Computed elevation E. Rail	0.9970
Backward running	−0.0010
Computed elevation of TBM 3	0.9960
Partial	+0.0040
Assumed elevation of TBM 3	1.0000

Thus we find that the partial is +4.0 mm. Evaluating the expression $(-D_B-D_F)$ we have

$$+0.0010+0.0030=+0.0040 \text{ (Check).}$$

That this is in accordance with the previously stated rule for use in all but these special cases may be verified by computing the partial for the section TBM 1 to Y 159 according to this method.

Assumed elevation of TBM 1	10.0000 m.
Forward running	−1.7373
Computed elevation for Y 159	8.2627
Backward running	+1.7395
Computed elevation of TBM 1	10.0022
Partial	−0.0022
Assumed elevation of TBM 1	10.0000

It is thus seen that the partial is −0.0022 m., or −2.2 mm., when computed in this manner and that this value is the same as shown in figure 10 (p. 36), and is the difference between the numerical values of the two runnings with the sign of the smaller. Furthermore, the evaluation of the expression $(-D_B - D_F)$ gives the same result, as shown below:

$$-1.7395 + 1.7373 = -0.0022 \text{ (Check).}$$

Column 9 of Form 31 is the same as the right-hand half of column 2 of the same form except that, since we compute elevations for only the permanent bench marks and the extra-foresight points, it is good practice to enter the designations of the marks for which elevations are to be computed at the right side of column 9 and the designations of the other marks at the left side of the column. Some temporary bench marks are of a semipermanent character and are described by the field party on Form 638 (Revised 1946). In such cases these marks should be treated in the same manner as the permanent bench marks, and the designations of them should be set over to the right side of column 9 of Form 31. This not only tends to simplify the preparation of the summary sheets, a later step in the computation and adjustment of the leveling, but greatly simplifies the later work of making up the master copies for published lists of elevations, especially when preliminary elevations are to be published in advance of the adjustment of the leveling.

When the first computer has completed his abstract through column 9 of Form 31, the field records and field computations are turned over to the second computer for the preparation of a second or duplicate abstract. The preparation of this second abstract should be a completely independent job, and, to make sure that it is, the first computer keeps his abstract in his possession until after the second abstract has been completed, in accordance with the procedure already outlined, up to and including column 9.

The two abstracts (Form 31) are then compared by copy reading columns 2, 3, 6, and 7, only. By "copy reading" we mean that one computer reads aloud from his abstract while the other watches his abstract. The watcher stops the reader whenever a discrepancy occurs and both refer back to the source material before reaching a conclusion as to which abstract is correct. When the correct entry has been decided upon, the erroneous abstract is corrected and the copy reading proceeds. If the two abstracts are not in exact agreement at any point, never assume that one is right and the other wrong, even though an experienced computer may be teamed with a novice. Both computers should always check back to the source material before reaching a decision as to the correctness of a questionable entry. In case they cannot agree after each one has checked his own work, the matter should then be referred to the supervisor

for decision. However, the need for referring a case to the supervisor will occur quite infrequently if the foregoing procedures have been followed carefully.

From this point on the computations are best made by having the two computers of the team work simultaneously, each on his own abstract.

The entries in column 8 of Form 31 represent the cumulative totals of the entries in column 7. Since these totals usually remain small, the necessary additions and subtractions are usually made mentally, and the two computers compare results at the end of each page. When they agree, they proceed with the computation. If they do not agree, the error is localized by copy reading down the column until the place of first disagreement is located. Then both check the discordant entries, and the necessary corrections are made on the faulty abstract.

Next, enter the designation of the bench mark which is selected as the starting bench mark in the indicated space in "Distance from B. M. _____" in the heading of column 10 and enter the distance "0.00" in column 10 opposite the same designation in column 9. For convenience, the starting bench mark should be one of the old marks proved by the check leveling to have remained stable (see pp. 52 to 65) and ordinarily should be the one nearest the beginning of the new leveling route. Entries in column 10 (disregarding all spur entries) are the cumulative totals of the entries in column 3 but are only filled in opposite the designations appearing at the right side of column 9, that is, for permanent and semipermanent bench marks and for temporary bench marks which form the bases for spur lines.

It is best for each computer to use an adding-and-subtracting machine for this work and to strike a subtotal at each permanent or semipermanent bench mark and at each temporary bench mark which forms the base of a spur. The two computers should compare their totals for the last entry to be placed in column 10 at the end of each page. If they agree, the computation is continued. If they do not agree, the two adding-machine tapes should be compared, item by item from the last check forward, until the discrepancy is located. When the necessary corrections have been made the computation is continued. The subtotals may then be copied from the tape to column 10. No comparison of these copied subtotals need be made, since the method of computing and checking the summary sheets will disclose any errors which may have crept into the entries in column 10 of Form 31 in copying from the tapes to the computation sheets. (See p. 51.)

The elevation of the starting bench mark (the one the designation of which appears in the heading of column 10) is then entered in column 11 on the same line on which its designation also appears in column 9. This will locate it just to the right of the "0.00" entry in column 10. Use for the elevation of the starting bench mark the *first* of the following types of elevations which may be available at the time the elevations are computed and entered in column 11:

1. The adjusted elevation from a published list or a completed summary sheet,
2. The orthometric elevation from an uncompleted summary sheet,
3. The observed elevation from an office abstract, or
4. The elevation used for the same mark by the field party in making up the field computation, Form 45.

The elevations are then computed simultaneously by the two computers, as follows:

1. After clearing the machine, place the starting elevation in it,
2. Disregarding spurs (or making special provision for them as noted below), add or subtract the successive differences from column 6, with due regard for sign,

3. Strike a subtotal at each permanent or semi-permanent bench mark and at each temporary bench mark forming the base of a spur,

4. Compare the elevations on the two tapes at the end of each sheet and correct any mistakes that may be discovered, and

5. Copy the subtotals from the tapes into column 11 of Form 31.

SPURS. No entries are made in column 10 for marks on spurs, except when long spurs are encountered which connect with tidal bench marks or to other leveling. Entries in column 11 for bench marks or extra-foresight points on spurs may be computed by either of the two routines outlined below.

1. At the time the tape is run for the main line, by adding algebraically the spur value from column 6 of Form 31, striking a subtotal and then subtracting algebraically the same spur value from column 6, to get back to the elevation of the bench mark at the base of the spur before proceeding along the main line. In case this procedure is followed, it is well to let the adding machine print the subtotal for the point on the spur twice, as a signal that, when copying from the tape to column 11 of Form 31, the elevation should be copied in red ink.

2. After the elevations of the bench marks in the main line have been computed and copied into column 11 of Form 31, the elevations of the marks on the spur lines may be computed separately by adding algebraically the spur-line differences in elevation to the elevations of the bench marks forming the bases of the spurs.

COMPUTATION OF ORTHOMETRIC CORRECTIONS

After the completion of the office abstracts (Form 31) through column 11, the observed elevation of each permanent bench mark or other point along the line is available. Ordinarily, the next step is the computation of the orthometric corrections, using the formula (see appendix C for theory and derivation) $c_o = -C_o\, h\, d\phi$, where c_o is the orthometric correction to be applied to a small portion of the line of levels, C_o is a constant depending on the latitude (tabulated in table II, p. 142), h is the average elevation of the portion being considered, and $d\phi$ is the difference in latitude between the ends of the portion.

Figure 11 (p. 44) shows the tabulation involved in an orthometric-correction computation. Present practice in the Section of Leveling is to make this tabulation in column 12 of Form 31 (in a manner to be described later), but here it is shown as a separate tabulation for the sake of clarity.

Referring to figure 11, it will be noted that the entries in column 1 are merely the designations of the permanent bench marks and that they are arranged down the column in the order in which they occur along the line of levels.

The entries in column 2 are the latitudes of the bench marks the designations of which are tabulated in column 1. These latitudes are scaled from reliable maps and are entered to the nearest minute of latitude.

In column 3, opposite the first instance of change in latitude, enter the average elevation of the line from the beginning to that point. Opposite the second instance of change in latitude, enter the average elevation of the line between the points of first and second change in latitude. Proceed similarly for each portion of the line between successive changes in latitude.

If two successive bench marks are listed as being at the same latitude, $d\phi$ (to the nearest minute) will be zero, and from the formula $c_o = -C_o\, h\, d\phi$, the correction between these two bench marks will be zero. Consequently, the average elevations are entered

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Computation of Orthometric Corrections
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B.M.	Latitude ° ′	Avg. Elev. (m.)	Ortho. Corr'n. (mm.)	Total Corr'n. (mm.)
A20	42 57			0.0
B20	58	540	-0.8	-0.8
H218	59	570	-0.9	-1.7
A36	43 00	580	-0.9	-2.6
Y315	00			-2.6
B36	01	580	-0.9	-3.5
Spur Line				
B36	43 01			-3.5
C36	01			-3.5
Mag. Sta. S	02	590	-0.9	-4.4
" " N	02			-4.4
D36	03	600	-0.9	-5.3
Z315	04	590	-0.9	-6.2
E36	04			-6.2
End of Spur Line				
B36	43 01			-3.5
F1 (USE)	01			-3.5
F2 "	01			-3.5
F3 "	01			-3.5
F4 "	01			-3.5
F5 "	01			-3.5
F6 "	01			-3.5
F7 "	00	580	+0.9	-2.6
F8 "	00			-2.6
F9 "	00			-2.6
F10 "	01	550	-0.8	-3.4
F11 "	01			-3.4
F12 "	02	550	-0.8	-4.2
F13 "	02			-4.2
F14 "	02			-4.2
F15 "	02			-4.2
F16 "	02			-4.2
F17 "	01	530	+0.8	-3.4

FIGURE 11.—Sample of computation of orthometric corrections.

in column 3 only opposite points of successive change in latitude, and for the purpose of orthometric-correction computation the line is broken up into portions bounded by these values listed in column 3. The average elevation is considered to be the average elevation of the portion (the average height of the profile of the portion of the line of levels) rather than a mean of the end points of the portion or an indiscriminate mean of all bench-mark elevations in the portion. This average elevation is shown only to the nearest 10 meters.

After the average-elevation entries have been made, the orthometric correction can be computed for each portion by the above formula and entered in column 4. For example, in computing the orthometric correction for the portion ending at bench mark D 36 in figure 11, C_o is found from table II, page 145, to be +0.000001534 for latitude 43°03′, and knowing that the average elevation of the line of levels from Mag. Sta. S to D 36 is 600 meters and that the difference of latitude between the ends of the portion is 1 minute,

$$\begin{aligned} c_o &= -C_o\, h\, d\phi \\ &= -0.000001534 \times 600 \times 1 \\ &= -0.0009 \text{ m.} \\ &= -0.9 \text{ mm.} \end{aligned}$$

Since the sign of $d\phi$ is positive when the latitude values are increasing and negative when the latitude values are decreasing, the negative sign before the C_o serves to make the orthometric correction negative when the line has a trend toward the pole and positive when the line has a trend toward the equator (except in the case of a negative h, such as might be encountered in Death Valley). Thus in the northern hemisphere with positive elevations, the sign of the correction is plus when the direction of progress over a portion is from north to south and minus when it is from south to north.

Since the orthometric corrections are to be applied to observed elevations (rather than to differences of elevation), the cumulative sum of the orthometric corrections for the portions are shown for each bench mark in column 5 of figure 11.

Figures 12 and 13 (pp. 46 and 47) show two nomograms designed by E. J. Parkin for the graphic solution of the formula $c_o = -C_o h d\phi$ in computing orthometric corrections. Figure 12 covers the area between $\phi = 20°$ and $\phi = 70°$, while figure 13 covers the belt between $\phi = 5°$ and $\phi = 20°$. These two nomograms were designed to cover all territory included in continental United States, the Philippines, the Territory of Hawaii, and all but the most northerly portion of Alaska.

In using these nomograms, a right line from the appropriate latitude (ϕ) to the appropriate average elevation (h) intersects the correction scale at the proper point so that the orthometric correction may be read in thousandths of the unit used for h. Thus, if h is in meters, yards, or feet, the orthometric correction will be in millimeters, milliyards, or millifeet, respectively.

The task of arriving at the average elevation of a particular portion of a line of levels is simplified by reason of the fact that the average elevation need only be computed to the nearest 10 meters. The average elevation may often be determined simply by an inspection of column 11 of Form 31. For example, in the case of the section (see fig. 11) from B 20 to H 218, the observed elevation of B 20 was 569.7722 m., and that of H 218 was 572.2468. Assuming a fairly uniform route between these marks, a condition which could be readily inferred from an inspection of the leveling record, the average elevation would obviously be 570 meters, when taken only to the nearest 10 meters. On the other hand, that portion of the line from F 12 (USE) to F 17 (USE) cannot be so readily averaged by simple mental calculation. In this case there is a distinct break in grade within the portion of the line between these two bench marks, and some more certain method of deciding upon the average elevation should be used. Various schemes have been tried with varying degrees of success. The method outlined below is general, gives the needful accuracy, and can be modified

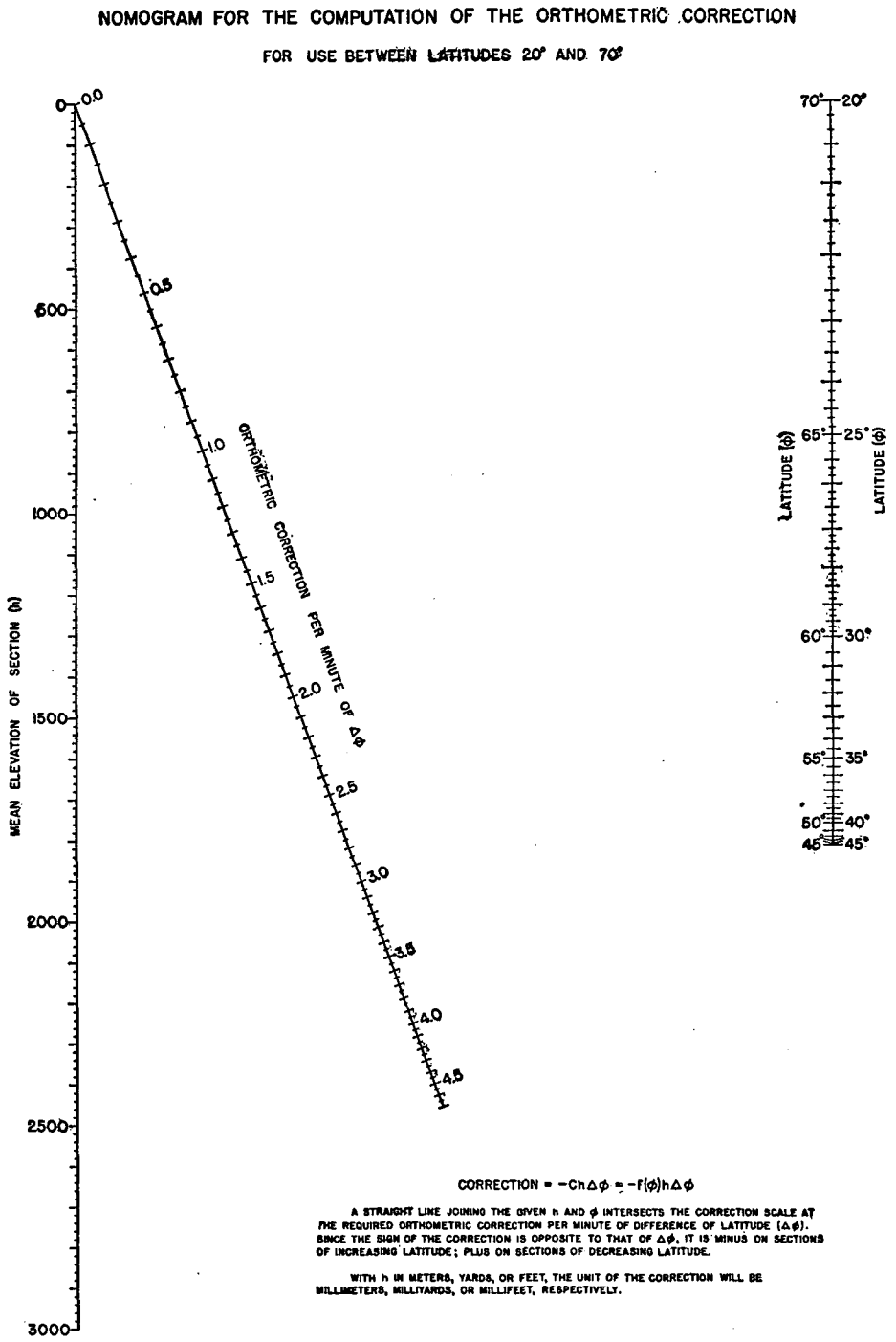


FIGURE 12.—Nomogram for the computation of the orthometric correction, for use between latitudes 20° and 70°.

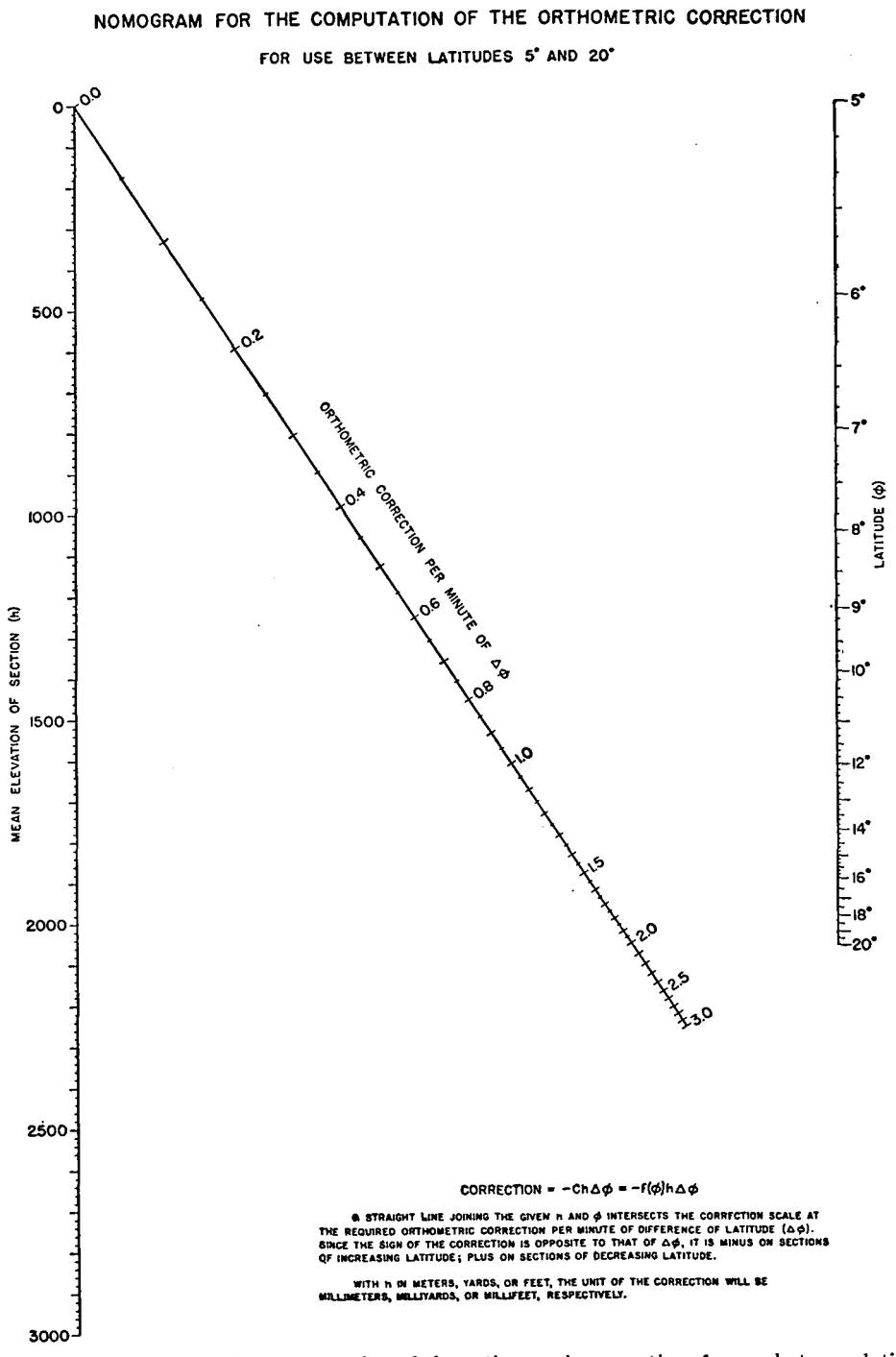


FIGURE 13.—Nomogram for the computation of the orthometric correction, for use between latitudes 5° and 20°.

to eliminate some of the work under special conditions and still maintain the required accuracy in the results.

An inspection of Form 31 shows that for the portion of the line from F 12 (USE) to F 17 (USE) the various marks have the following observed elevations and are located at distances apart as shown in the following table.

B. M.	*Distance (km.)*	*Elevation (m.)*
F 12 (USE)		547. 5
	1. 60	
F 13 (USE)		547. 2
	1. 73	
F 14 (USE)		546. 8
	1. 37	
F 15 (USE)		542. 9
	1. 53	
F 16 (USE)		545. 3
	0. 80	
TBM 1B		475. 1
	1. 07	
F 17 (USE)		440. 1

Then the average elevation (E_a)

$$
\begin{aligned}
&=[(1.60\times 547.5)\\
&\quad+(1.60+1.73)\ 547.2\\
&\quad+(1.73+1.37)\ 546.8\\
&\quad+(1.37+1.53)\ 542.9\\
&\quad+(1.53+0.80)\ 545.3\\
&\quad+(0.80+1.07)\ 475.1\\
&\quad+(1.07\times 440.1)]\div 2\ (1.60+1.73+1.37+1.53+0.80+1.07)\\
&=530.7 \text{ or } 530, \text{ to the nearest 10 meters,}
\end{aligned}
$$

all of which can be very easily and quickly computed on a calculating machine, as follows:

1. Multiply 1.60 by 547.5 and leave the product in the machine.
2. Add 1.60 and 1.73 mentally and multiply the result, 3.33, by 547.2, allowing the product to accumulate in the machine.
3. Continue the mental addition of the pairs of section lengths and multiply each of these sums by the corresponding elevations, allowing all products to accumulate in the machine.
4. After multiplying 1.07 by 440.1, still allowing the product to accumulate, add all the section lengths either mentally or on the machine, double the total, and use that as a divisor for the accumulation of the products.
5. The quotient is then the average elevation sought. It should be rounded to the nearest 10 meters for entry on the sheet on which the orthometric corrections are being computed.

All necessary data are obtainable from Form 31 and no scratch-pad work is necessary.

NOTE.—In lieu of adding the section lengths the total may be obtained by subtracting the distances located in column 10 of Form 31 opposite the terminal bench marks of the portion of the line being considered.

In case the section lengths are all approximately equal, the above may be shortened by assuming a uniform section length of unity and the computation then shortens to

$$E_a = \frac{E_1 + 2E_2 + 2E_3 + \ldots\ldots + E_n}{2n}$$

where

E_a = the average elevation sought, and

$E_1, E_2, \ldots\ldots E_n$ are the observed elevations of the successive bench marks along the portion of the line under consideration.

Furthermore, if the sections are all of approximately the same length and the successive differences of elevation are all approximately the same, as might be the case in leveling along a canal or a long uniform railroad grade, a mean of the two elevations of the end bench marks might give the average elevation with sufficient accuracy.

In the Section of Leveling it has become standard practice to compute the orthometric corrections in column 12 of Form 31. This eliminates the necessity for copying all of the bench-mark designations on another computation sheet. The latitudes of the permanent bench marks are entered at the left side of column 12. The average elevations are not actually written down. The orthometric corrections to the differences of elevation are written down the middle of column 12 and the cumulative totals are entered at the right-hand edge of column 12. The computer does this work in pencil and after the computations have been checked and any changes verified the cumulative totals only are inked. This method eliminates a certain amount of writing and reduces the number of separate computation sheets which must be referred to in making up the summary sheet. Also, since the average elevation is not actually written down by the computer, the checker must make an absolutely independent estimation or computation of the average elevation. If he agrees with the computer's estimate or computed value of the average elevation closely enough so that no change of as much as 0.1 mm. is made in the correction, the average elevation must, perforce, have been determined with sufficient accuracy by both the computer and the checker.

At the beginning of this discussion of orthometric-correction computation, it was stated that *ordinarily* such computation followed the completion of the office abstracts through column 11. There are two exceptions. If the line being computed is a releveling of an old line, the orthometric corrections may be taken directly from the computations for the old line for the old bench marks. For new marks established during the releveling, orthometric corrections can be determined by interpolating according to distance between the known values at the old marks. If the computation is being made for an area network consisting of approximately straight links of reasonably uniform slope or involving no large differences of elevation, the orthometric correction may be made link by link as a part of the work of adjusting the leveling in the area.

PREPARATION OF SUMMARY SHEETS

The final step in the preliminary computing is the preparation of the summary sheet. This sheet is a further abbreviation of the data concerning the line of levels. It contains the data necessary for use in preparation for an adjustment of the leveling and provides a means of computing the adjusted elevations after the adjustment has been completed.

The summary sheets are made up on standard 8- by 10½-inch 9-column data sheets. The shelf mark and accession number of the line or area are entered in the upper right-hand corner of the sheet. The words "Summary Sheet," the name of the line or area, the name of the chief of party, and the year in which the leveling was done are also entered at the top of the sheet. Furthermore, if the leveling to be summarized on the sheet is of second-order accuracy, the words "SECOND ORDER" are stamped in red by means of a rubber stamp in the heading of the sheet. See figure 14, below.

Summary Sheet 923 HGZ 1946 R

Fairfax to Lake Andes, S. Dak.

F. A. Riddell

Sheet 1 of 2 1946 **SECOND-ORDER** L-11706

B.M.	Dist. (km.)	Obs. Elev. (m.)	Ortho. Corr'n. (m)	Adj. Corr'n. (m)	Adjusted Elev. (m.)	Adjusted Elev. (feet)	Remarks
A20 (S No ✓)	0	514.0647	0.0000				
B20	0	569.7722	-0.0008				
H218	2	572.2468	- 17				
A36	3	580.8497	- 26				
Y315	4	577.8395	- 26				
B36	6	588.8219	-0.0035				
Spur Line							
B36	6	588.8219	-0.0035				
C36	7	594.4270	- 35				
Mag. Sta. S	7	590.3225	- 44				
" " N	7	589.8237	- 44				
D36	9	606.0916	- 53				
Z315	11	577.4864	- 62				
E36	13	578.5094	-0.0062				
Main Line Continues							
B36	6	588.8219	-0.0035				
F1 (USE)	8	589.4147	-0.0035				
F2 "	10	584.4422	- 35				
Rd. Int. (S No ✓)	10	587.5288	- 35				
F3 (USE)	12	588.4415	- 35				

FIGURE 14.—Sample of summary sheet—prior to adjustment.

For the reason that it is very frequently necessary to photostat summary sheets, all work done on summary sheets must be in *black* ink. In the Section of Leveling all summary-sheet work is done either in "Eternal" or waterproof India ink.

It will be noticed that the designations of the bench marks are entered in the order in which they are encountered along the line in listing them down column 1 of the summary sheet. Column 2 contains the distances of the various bench marks from the starting bench mark. These distances are here rounded to the nearest kilometer since that is sufficiently accurate in almost all cases for adjustment purposes. In some special cases of small, tight networks with short links, it might be well to round the distances to the nearest 0.1 kilometer.

Column 3 of the summary sheet contains the observed elevations from column 11 of Form 31, and column 4 contains the orthometric corrections from the orthometric computation sheets or from column 12 of Form 31, if they were computed in that column.

The entries $\left(\begin{matrix} S \\ No \\ \checkmark \end{matrix}\right)$, found now and then on the line between columns 1 and 2 of the summary sheet, indicate that the points opposite which they appear were touched upon by single-run spur lines and, not being in the main line, are no-check points. An entry $\left(\begin{matrix} S \\ \checkmark \end{matrix}\right)$ would be used to indicate a spur which has been leveled at least twice.

The distance entry for a no-check spur or other short spur to a single bench mark is the same as the distance entry of the bench mark from which the spur originated. Thus, since the no-check spur to "Rd. Int." in figure 14 originated from the bench mark F 2 (USE) the distance is the same as that for F 2 (USE). Also A 20 was touched upon by a single-run spur in obtaining the starting check and for this reason also carries the "0" distance of the bench mark B 20 from which the spur to A 20 was run. On the long spur from B 36 to E 36 the distances are entered so that the length of the spur is shown on the summary sheet.

The entries in the first three columns of the summary sheet are put on the sheet from the *first* of the duplicate office abstracts (Form 31), and the checking of the entries in these first three columns is done by using the *second* of the office abstracts. This serves to check the copying from the adding machine tapes to Form 31 and explains why it was stated on page 42 that no check of the copying from the tapes to the sheets had to be made at the time.

As a final check on the preliminary computations an adding-machine tape is run up on which the observed elevations from the summary sheet are put in the machine with the positive sign and the corresponding elevations from the field computation (Form 45) are put in with the negative sign. A full total is struck after each pair of elevations so that the printed totals represent the differences between the elevations computed in the Office and those computed in the field. If any discrepancies are found, which are larger than can be accounted for by the corrections in columns 19 to 22, inclusive, of Form 45 and the small changes due to accumulation of small errors in rounding means to four decimal places in meters, a detailed comparison of the field and office computations must be made to find the cause of the discrepancy. If the mistake is found to be in the field computation, only a note at that point in the field computation need be made. However, if the mistake is found in the office abstracts or summary sheet, make the necessary correction at the place where the mistake was made and carry the correction through all following values that are affected by the change.

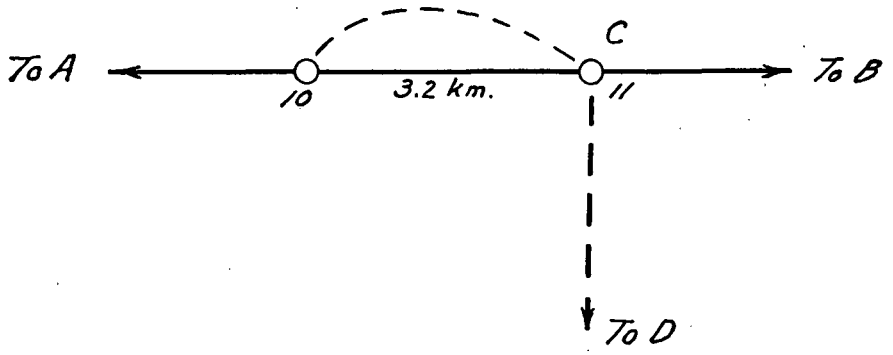
After the above procedures have been completed, prepare a title sheet for the line, fold and punch the field sheets, and bind the following items in the following order:

1. Title sheet.
2. One copy of office abstract.
3. Orthometric sheets.
4. "C" sheet.
5. Rod standardization sheet.
6. Field sheets.
7. Backing sheet.

The summary sheets are to be stapled at the upper left-hand corner and placed in the cahier—not bound with the computations. One copy of the office abstract is also to be stapled and placed in the cahier—not bound.

JUNCTION DETAILS

Paragraphs 1 to 5, inclusive, of the General Instructions for Geodetic Leveling (Special Publication No. 239, "Manual of Geodetic Leveling," p. 3) require that checks with the old leveling be obtained at the beginning and end of each new line of levels and at any intermediate junctions of the old and new leveling which may be encountered along the new line. To obtain such checks, new leveling is run between bench marks established during the previous work until a new difference of elevation is obtained which agrees with the difference of elevation determined by the previous leveling between the same marks within specified limits.



———	A to B	First-order	Doe	1920
— — —	C to D	"	Roe	1925

FIGURE 15.—The simplest case of a junction detail.

THE SIMPLEST CASE

The simplest case of a junction detail occurs when the leveling is as illustrated in figure 15. This is a simple junction or connection in which there was no difficulty in obtaining a check on the first two old marks leveled over and no complications were encountered.

NOTE.—In preparing junction-detail sketches the standard practice is to use place names as they occur in the titles of the lines involved, to show bench-mark designations as they actually appear in the computations and on the summary sheets, to show single-run differences by means of dashed lines and double-run differences by means of solid lines, and to indicate the various lines of levels involved in a single sketch by means of different colors. However, in this manual the use of more than one color is impractical. For this reason the double-run leveling is shown by means of heavy lines and single-run leveling by means of light lines. The various levelings included in a single sketch are differentiated by means of the varying character of the lines in the sketches—solid, dashed, dotted, etc. Furthermore, since all of the sketches in figures 15 to 18, inclusive, represent hypothetical and not actual situations, letters have been sub-

stituted for place names, and Arabic numerals have been substituted for the usual letter-and-number designations of the bench marks. Also, in the following discussion of junction details, distances between bench marks will be shown to the nearest tenth of a kilometer only, whereas office practice is to show and use these distances to hundredths of a kilometer as they are given in the office abstracts.

Referring to figure 15, we find the following to be the situation at the junction of the first-order lines A to B and C to D. In 1920, leveling by Doe was run from A to B and bench marks 10 and 11 were included in that line, bench mark 11 being at C. In 1925, Roe's party obtained its starting check by a single running from 11 to 10 and continued first-order leveling from C to D. Assuming that observed differences of elevation were determined as shown below, the circuit closure is obtained by summing differences in a clockwise direction around the circuit, as follows:

11 to 10 (1920)	−5.7643 m.
10 to 11 (1925)	+5.7634 m.
Closure	−0.0009 m.=−0.9 mm.

Since the distance from 10 to 11 is shown to be 3.2 km., this is a very good check. Paragraph 2 of the General Instructions would allow

$$\pm 4.0 \text{ mm. } \sqrt{3.2} = \pm 7.2 \text{ mm.},$$

as the maximum permissible disagreement in this case, without requiring additional check leveling.

Still referring to figure 15, we may discuss a modification of this simplest of all types of junction details. Assume the leveling from 10 to 11 in 1920 as used above. Next, assume that the single running from 11 to 10 in 1925 resulted in an observed difference of −5.7563 m. The preliminary or trial closure would then be

11 to 10 (1920)	−5. 7643 m.
10 to 11 (1925)	+5. 7563 m.
	−0. 0080 m.=−8.0 mm.,

which is outside the permissible tolerance. Further leveling is required. As the next step, Roe's party levels from 10 to 11 and arrives at a second observed difference in elevation of +5.7610 m., which agrees with *his* single run in the opposite direction within the permissible tolerance. The mean of the two runnings in 1925 is then (5.7563+5.7610)÷2=5.7586, and the light dashed line from 11 to 10 would be replaced by a heavy dashed line in the sketch. The closure is then computed, using the mean of the two 1925 runnings, as follows:

11 to 10 (1920)	−5. 7643 m.
10 to 11 (1925)	+5. 7586 m.
	−0. 0057 m.=−5.7 mm.,

which, being smaller than the permissible tolerance of 7.2 mm., shows that a check was obtained within allowable limits of divergence.

Under the assumed conditions regarding this particular junction, the check leveling has, according to our accepted standards, proven the relative stability of bench marks 11 and 10 during the period from 1920 to 1925, and we are at liberty to adopt bench

mark 11 as the junction bench mark in this case. On the summary sheet for the line A to B enter in the remarks column opposite the data concerning bench mark 11 the note "Junction line C to D." On the summary sheet for the line C to D the information concerning bench mark 10 should be crossed out neatly with a single line to show that the elevation of that mark will be computed on another summary sheet. In the remarks column on that same line, enter the note "See line A to B."

"From B. M. to B. M." vs. "B. M. minus B. M." It so happens that in computing circuit closures involving leveling between junction points, the information used is often taken from the summary sheets, on which *elevations* of the bench marks appear, rather than from the abstracts (Form 31), on which the observed *differences* of elevation appear. For this reason a modification of the above-outlined procedure is recommended for use in many cases.

In computing the circuit closure indicated in figure 15, the differences of elevation were summed in a clockwise direction around the circuit, thus: from 11 to 10 (1920) and from 10 to 11 (1925). If this procedure is followed when using data from the summary sheets, it is necessary to obtain each of the differences of elevation by subtraction, set them down as indicated in the discussion of figure 15, and then compute the circuit closure.

On the other hand, if we proceed around the circuit in the opposite or counter-clockwise direction and, using an adding-and-subtracting machine, put in the elevation of the starting bench mark as a positive quantity and the elevation of the second bench mark as a negative quantity; then, switching to the other summary sheet, put in the elevation of the second bench mark as a positive quantity; and follow that with the elevation of the starting bench mark as a negative quantity; the total struck on the machine will be the circuit closure sought, and the whole computation will be carried out on the machine without the necessity for doing any writing at all, other than to copy down the resulting closure.

To illustrate this method, a tabulation is given below in which the assumed differences of the first discussion of figure 15, an assumed elevation for 10 in line A to B, and an assumed elevation for 11 in the line C to D have been used.

A to B	
Obs. elev. 10	37.6742 m.
Diff. 10 to 11	+5.7643 m.
Obs. elev. 11	43.4385 m.
C to D	
Obs. elev. 11	43.4370 m.
Diff. 11 to 10	−5.7634 m.
Obs. elev. 10	37.6736 m.

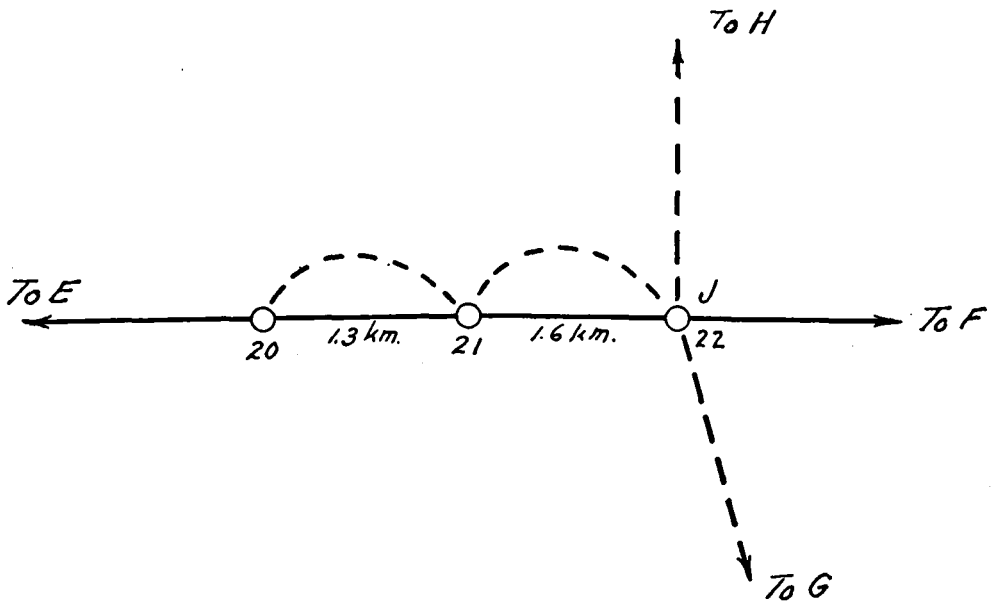
Proceeding from 11 and passing counter-clockwise around the circuit shown in figure 15, and using the method "B. M. minus B. M." we will place in the adding-and-subtracting machine the following:

+43.4370
−37.6736
+37.6742
−43.4385

and the total − 0.0009 m., or −0.9 mm., is exactly the same as when computed in the "from–to" sense in the clockwise direction around the circuit.

The value of this method of computing circuit closures becomes quite apparent when a large circuit, consisting of many lines or parts of lines, each coming from a separate summary sheet, is encountered.

In the following discussion of additional junction details, either method will be used as seems most convenient in each particular case. However, the fact should be kept in mind that, in using differences in the "from–to" sense, one should proceed in a clockwise direction around the circuit. When using elevations, in the sense of "B. M. minus B. M.," the counterclockwise route around the circuit should be followed. Carelessness in not observing this rule strictly will result in circuit closures of the wrong sign and may cause untold confusion in complicated junction details or in the adjustment of a large network of lines of levels.



———	E to F	First-order	Doe	1915
— — —	G to H	"	Roe	1935

FIGURE 16.—Junction detail with one old bench mark disturbed.

JUNCTION DETAIL WITH ONE OLD MARK DISTURBED

The state of affairs concerning the next type of junction detail to be considered is shown in figure 16. Doe, in 1915, ran a first-order line from E to F and during that leveling established bench marks 20, 21, and 22, bench mark 22 being at J. Roe, in 1935, ran another line of first-order levels from G to H, crossing the line E to F at J. Roe attempted to make a tie to the 1915 work by leveling from 22 to 21, failed to obtain a check and kept on to 20, where he obtained a check between 22 and 20. Roe leveled back from 20 to 21 and on to 22 for the reason that, in order to obtain a new first-order elevation for 21 which was evidently disturbed at some time between 1915 and 1935, he was required to have double-run leveling to the disturbed mark from the adjacent undisturbed marks.

The following tabulations represent in abbreviated form the essential data from Form 31 for the lines E to F and G to H.

Line E to F (1915)

Section	*Dist.*	*F*	*B*	*Mean*	*B. M.*	*Obs. Elev*
					20	96.8174
20 to 21	1.3	−3.7647	+3.7659	−3.7653	21	93.0521
21 to 22	1.6	−5.4761	+5.4731	−5.4746	22	87.5775

Line G to H (1935)

					22	87.5780
22 to 21	1.6	+5.4483	−5.4505	+5.4494	21	93.0274
21 to 20	1.3	+3.7918	−3.7938	+3.7928	20	96.8202

Using the "from-to" method in reconstructing the sequence of events in the field, a trial closure is computed, after the first check run reveals a forward difference of + 5.4483 meters from 22 to 21, as follows:

22 to 21 (1915)	+5.4746 m.
21 to 22 (1935)	−5.4483 m.
	+0.0263 m.=+26.3 mm.,

which is far beyond the permissible tolerance of

$$\pm 4.0 \text{ mm. } \sqrt{1.6} = \pm 5.1 \text{ mm.}$$

The 1935 leveling was continued from 21 to 20, and the trial closure between 21 and 20 is computed as follows:

21 to 20 (1915)	+3.7653 m.
20 to 21 (1935)	−3.7918 m.
	−0.0265 m.=−26.5 mm.,

which is again far outside the permissible tolerance of ± 4.0 mm. $\sqrt{1.3} = \pm 4.6$ mm.

However, using the same method of computing and assuming movement of 21 between 1915 and 1935, we find that the trial closure between 22 and 20 is

22 to 21 (1915)	+5.4746 m.
21 to 20 (1915)	+3.7653 m.
20 to 21 (1935)	−3.7918 m.
21 to 22 (1935)	−5.4483 m.
	−0.0002 m.=−0.2 mm.,

a most excellent check.

If all that was necessary at this junction was to obtain a check between lines E to F and G to H, the requirements would have been satisfied at this stage of the work. However, the disturbance of 21 having been discovered, it became necessary to double-run the leveling between 20 and 22 to provide a new first-order elevation for 21.

Accordingly the party leveled from 20 to 21 and from 21 to 22. The partials for these two sections of the 1935 leveling are +2.2 mm. and +2.0 mm., and are acceptable.

Using the means from the 1935 leveling against the means from the 1915 leveling the closures become

22 to 21 (1915)	+5.4746 m.
21 to 22 (1935)	−5.4494 m.
	+0.0252 m.=+25.2 mm.

and

21 to 20 (1915)	+3.7653 m.
20 to 21 (1935)	−3.7928 m.
	−0.0275 m.=−27.5 mm.

The closure between 20 and 22 is found to be

22 to 21 (1915)	+5.4746 m.
21 to 20 (1915)	+3.7653 m.
20 to 21 (1935)	−3.7928 m.
21 to 22 (1935)	−5.4494 m.
	−0.0023 m.=−2.3 mm.,

and, according to the other method of computing

Elev. of 20 from E to F	+96.8174 m.
Elev. of 22 from E to F	−87.5775 m.
Elev. of 22 from G to H	+87.5780 m.
Elev. of 20 from G to H	−96.8202 m.
	−0.0023 m.=
	−2.3 mm. (check).

It should be noted at this point that the closure between 22 and 20 can also be computed by adding algebraically the two closures (22 to 21 and 21 to 20), thus:

$$+25.2-27.5=-2.3 \text{ (check).}$$

This is true as long as the new leveling is drawn on the same side of the old leveling in the sketch and the established conventions are strictly adhered to in computing the closures. What happens when the new leveling is allowed to cross over the old leveling in drawing the junction-detail sketch will be illustrated in the discussion of the next type of junction detail.

This closure between 22 and 20, using the means from both levelings (1915 and 1935), is well within the permissible tolerance of

$$\pm 4.0 \text{ mm. } \sqrt{1.3+1.6}=\pm 6.8 \text{ mm.,}$$

and shows, according to our accepted standards, the stability of bench marks 20 and 22. Furthermore, we have the data necessary for computing a new first-order elevation for bench mark 21.

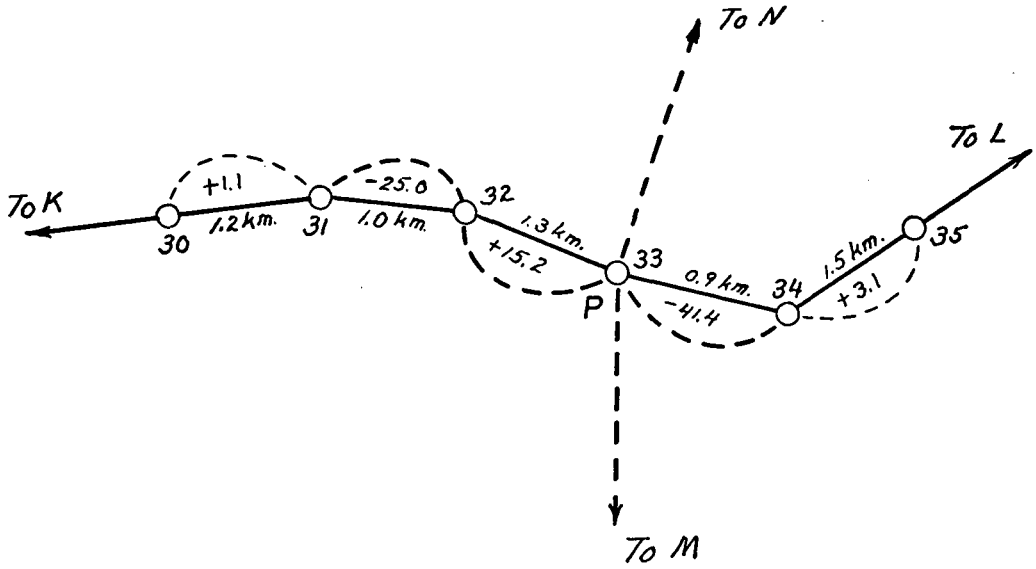
On the summary sheet for the leveling from E to F, the data for bench mark 21 should be crossed out neatly, by drawing a single line through the entries opposite the designation of that mark, and "See G to H" should be entered in the remarks column opposite the information that has been crossed out. Also, if a list of elevations or a list of descriptions and elevations has been published for line E to F, the computer

should cross out the published elevation of bench mark 21 on the file copy of the list, and insert the reference "For new elevation, see G to H." On the summary sheet for the line G to H, nothing is to be crossed out at this stage of the work. The treatment of the data on this summary sheet will depend on the adjustment procedure. If the 1915 leveling is held fixed and the 1935 leveling fitted to it, the treatment will be somewhat different from that which will be accorded these entries if both levelings are adjusted simultaneously.

JUNCTION DETAIL WITH MORE THAN ONE DISTURBED MARK AND AN "OFFSET" JUNCTION MARK

In figure 17 we find a situation which may have developed somewhat as follows:

Doe's party, in 1920, ran first-order levels from K to L and established bench marks 30 to 35, inclusive, during the course of the leveling, 33 being at P. In 1930,



————	K to L	First-order	Doe	1920
— — —	M to N	"	Roe	1930

FIGURE 17.—Junction detail with more than one disturbed mark and an "offset" junction mark

Roe's party, in leveling from M to N with first-order accuracy, passes through P and attempts to get a checked tie to the line K to L in the vicinity of P. In doing this Roe levels from 33 to 34 and fails to check the original leveling. Not knowing at this stage of the work whether the trouble is in his leveling or, if his leveling is accurate, which mark (33 or 34) has moved, he runs back from 34 to 33, verifies his own work, and continues to 32, again failing to check either with 33 or 34. He relevels from 32 to 33 and verifies his own leveling. He decides to continue toward K and levels from 32 to 31. He now finds that, while he has not checked any one of the three sections, 31 to 32, 32 to 33, or 33 to 34, his over-all check from 31 to 33 looks promising.

To settle the matter so that the leveling can be adjusted properly he continues on to 30. He finds that his first running from 31 to 30 checks the original leveling well within prescribed limits and that he has proven the relative stability of 30 and 31.

He then relevels from 31 to 32, verifying his first running, and now finds that the

over-all check between the two double-run lines (1920 and 1930) between 31 and 34 shows an acceptable closure.

Roe might have been justified in abandoning the tie at this stage, since he found two relatively stable bench marks, as indicated by his single running from 31 to 30, and has found 34 to be in agreement with the other two marks which check, and he has double-run first-order leveling enough to compute new first-order elevations for 32 and 33. However, he apparently reasoned that, if this locality is as unstable as his findings would seem to indicate, he had better take nothing for granted and would be justified in spending an additional hour or so on the problem. He then levels from 34 to 35 and obtains a check on his first attempt. He now has checks between 30 and 31, between 34 and 35, and the agreement from 31 to 34 is within limits for that distance. He has perfected an excellent tie to the old line from K to L and proceeds with his leveling from 33 toward N.

Skipping over the numerical details of all Roe's trial closures, etc., we now tabulate below the data which would be available (on Form 31) to the computer in making up the sketch as shown in figure 17, in checking the tie closures, and in deciding which mark to use as a junction mark.

Line K to L (Doe, 1920)

Section	*Dist.*	*F*	*B*	*Mean*	*B.M.*	*Obs. Elev.*
					30	100.3748
30 to 1a	0.5	+28.1746	−28.1740	+28.1743		
1a to 1b	0.3	+11.2691	−11.2678	+11.2684		
1b to 31	0.4	−37.0253	+37.0249	−37.0251	31	102.7924
31 to 1c	0.3	−18.9426	+18.9407	−18.9416		
1c to 1d	0.4	+33.7349	−33.7348	+33.7348		
1d to 32	0.3	−11.7858	+11.7836	−11.7847	32	105.8009
32 to 1e	0.5	−25.6925	+25.6917	−25.6921		
1e to 1f	0.4	+38.4760	−38.4765	+38.4762		
1f to 33	0.4	−17.5934	+17.5926	−17.5930	33	100.9920
33 to 1g	0.5	−25.6418	+25.6425	−25.6422		
1g to 34	0.4	+24.3253	−24.3241	+24.3247	34	99.6745
34 to 1h	0.6	+18.6828	−18.6810	+18.6819		
1h to 1j	0.4	+12.8436	−12.8439	+12.8438		
1j to 35	0.5	−29.1133	+29.1141	−29.1137	35	102.0865

Line M to N (Roe, 1930)

					33	100.8641
33 to 2n	0.4	−21.1775	+21.1790	−21.1782		
2n to 34	0.5	+19.9016	−19.9026	+19.9021	34	99.5880
34 to 2p	0.7	+23.4718		+23.4718		
2p to 35	0.8	−21.0629		−21.0629	35	101.9969
					33	100.8641
33 to 2q	0.6	+9.4810	−9.4824	+9.4817		
2q to 32	0.7	−4.6568	+4.6584	−4.6576	32	105.6882
32 to 2r	0.4	−14.3622	+14.3615	−14.3618		
2r to 31	0.6	+11.3783	−11.3783	+11.3783	31	102.7047
31 to 2s	0.7	+24.1890		+24.1890		
2s to 30	0.5	−26.6077		−26.6077	30	100.2860

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The computation of the closures may now be made, using the "B. M. minus B. M." method.

Elev. of—	
31 (1930)	+102. 7047 m.
30 (1930)	−100. 2860
30 (1920)	+100. 3748
31 (1920)	−102. 7924
	+ 0. 0011 m.=+1.1 mm.
32 (1930)	+105. 6882 m.
31 (1930)	−102. 7047
31 (1920)	+102. 7924
32 (1920)	−105. 8009
	− 0. 0250 m.=−25.0 mm.
33 (1920)	+100. 9920 m.
32 (1920)	−105. 8009
32 (1930)	+105. 6882
33 (1930)	−100. 8641
	+ 0. 0152 m.=+15.2 mm.
34 (1920)	+ 99. 6745 m.
33 (1920)	−100. 9920
33 (1930)	+100. 8641
34 (1930)	− 99. 5880
	− 0. 0414 m.=−41.4 mm.
35 (1920)	+102. 0865 m.
34 (1920)	− 99. 6745
34 (1930)	+ 99. 5880
35 (1930)	−101. 9969
	+ 0. 0031 m.=+3.1 mm.
34 (1920)	+ 99. 6745 m.
31 (1920)	−102. 7924
31 (1930)	+102. 7047
34 (1930)	− 99. 5880
	− 0. 0012 m.=−1.2 mm.

As mentioned in the discussion of the junction detail shown in figure 16, we have now allowed the dashed line representing the new leveling to cross over to the opposite side of the solid line representing the old leveling in this sketch (fig. 17) to illustrate the method of combining individual circuit closures to obtain an over-all closure for check purposes.

$$-41.4+15.2-(-25.0)=-1.2 \text{ mm. (check).}$$

It should be noticed that the signs of the individual closures must be changed, after a cross-over on the sketch, in summing the closures of the individual circuits.

Usually, in drawing junction details, the computer will attempt to keep the lines

representing older leveling below or to the right of lines representing newer leveling throughout the sketch. However, undesirable complexities may result occasionally if this practice is followed too rigidly.

The following tabulation of the computed closures and the permissible tolerance shows that in three cases the observed closures are well within allowable limits.

Closure between	*Closure mm.*	*Permissible tolerance*
30 & 31	+1.1	± 4.0 mm. $\sqrt{1.2} = \pm 4.4$ mm.
31 & 34	−1.2	± 4.0 mm. $\sqrt{3.2} = \pm 7.2$ mm.
34 & 35	+3.1	± 4.0 mm. $\sqrt{1.5} = \pm 4.9$ mm.

Having proved 30, 31, 34, and 35 to have remained relatively stable and 33 and 32 to have moved, we are at liberty to choose our junction bench mark. Either 31 or 34 may be used as an "offset" junction, but for practical reasons we will choose 34, it being the one most directly related to the actual crossing of the two lines of levels.

On the summary sheet for the 1920 leveling, cross out the data for bench marks 32 and 33 and enter "See M to N" in the remarks column. On the summary sheet for the 1930 leveling cross out the data opposite the designations for 30 and 35, entering "Check Run" in the remarks column in each case. The treatment to be accorded the data on the lines for bench marks 31 and 34 will be determined by the adjustment procedure. If elevations have been published for the 1920 leveling, cross out the old elevations for 32 and 33 on the file copy and enter the reference "For new elevations, see M to N."

JUNCTION DETAIL INVOLVING THREE LINES OF LEVELS AND CONSIDERABLE MOVEMENT OF BENCH MARKS

As it is obviously impossible to cover all of the possible types of junction details in this manual, we will now consider a fairly complicated junction detail involving three levelings and movement of three bench marks. The discussion of this detail will bring out one more method of studying the behavior of marks that have moved.

Referring to figure 18, the sequence of events there represented is about as follows:

1. Doe, in 1910, ran a line of first-order levels from P, through S, to Q. He established bench marks 40 to 45, inclusive, all in or near the city of S, 42 and 45 being on spur lines from 41 and 44, respectively.

2. In 1920, Roe, in tying out a line of first-order levels from R to S, levels directly from 45 to 43 and fails to check. He continues to 42, establishing a new mark (50) on the way, and finds that 42 also fails to check, either with 43 or 45. He continues his leveling to 40, fails to check between 40 and 42, but finds that 40 seems to check with 43. This gives him a checked tie-out in accordance with the provisions of the General Instructions, and he is finished with his line.

3. Poe, in 1930, in attempting to start a new line of first-order levels, which he is to run from S to T, encounters considerable trouble with movement of bench marks and continues leveling until he has removed all reasonable doubt as to his having secured a checked start for his line. He leveled from his first new mark (60) to 50, established in 1920 by Roe, and then to 43 of the 1910 line. His difference between 50 and 43 failed to check Roe's work. He returned to 60 and leveled to 42 and found he also had a bad closure in the circuit 60, 42, 50, and 60. He continued leveling to 40 and found that he checked Doe's 1910 difference between 40 and 43. He might have stopped there but, having run into so much trouble and being anxious to get a really sound start

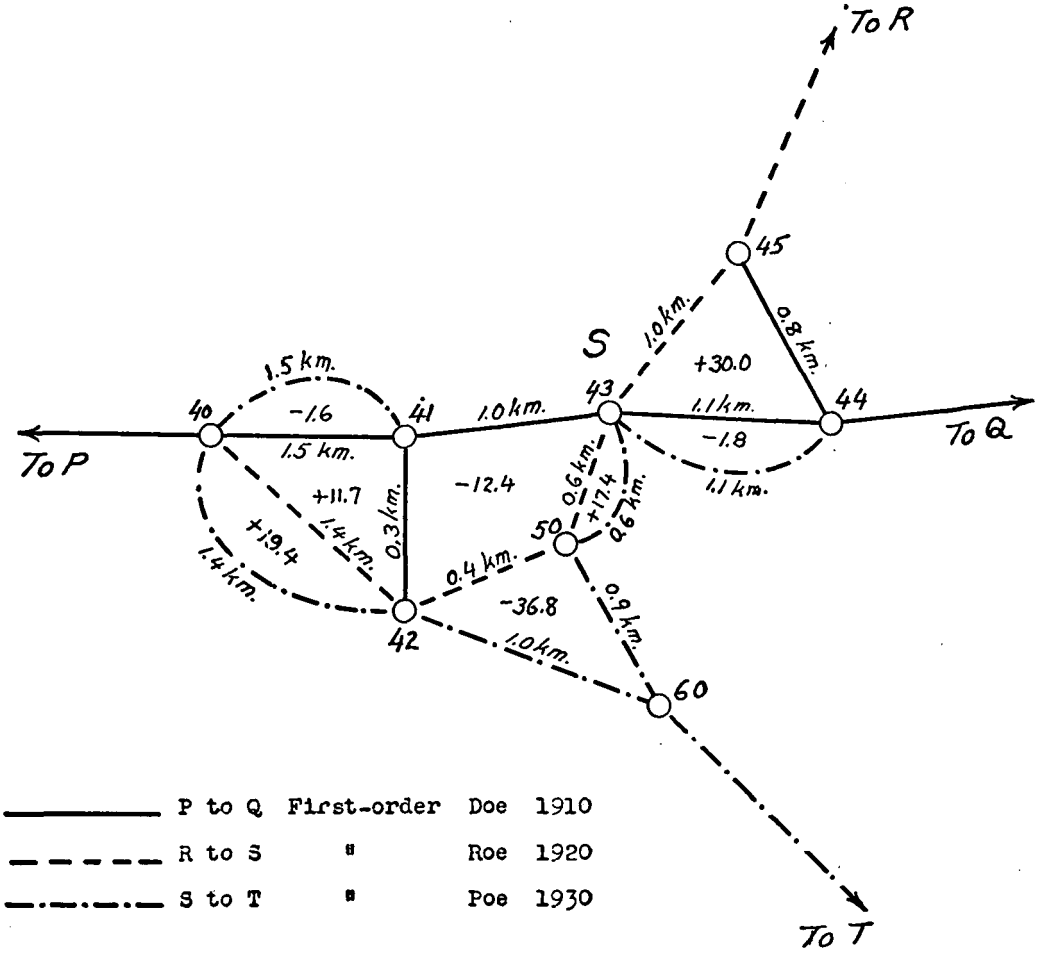

FIGURE 18.—Junction detail involving three lines of levels and considerable movement of bench marks.

for his line to T, he leveled from 40 to 41 and from 43 to 44, securing good checks in both cases. He then took up the work on his main line from 60 toward T.

Abbreviated tabulations of the pertinent data for these three levelings in the vicinity of S are shown below. Only mean differences are shown, since it is assumed in this case that all of the leveling is double-run first-order.

Line P to Q (Doe, 1910)

Section	*Dist.*	*Obs. Diff.*	*B. M.*	*Obs. Elev.*
			40	60. 3185
40 to 41	1. 5	+1. 7648	41	62. 0833
41 to 42	0. 3	+2. 0041	42	64. 0874
41 to 43	1. 0	−0. 8008	43	61. 2825
43 to 44	1. 1	−2. 4172	44	58. 8653
44 to 45	0. 8	+1. 9049	45	60. 7702

Line R to S (Roe, 1920)

Section	*Dist.*	*Obs. Diff.*	*B. M.*	*Obs. Elev.*
			45	60.7091
45 to 43	1.0	+0.4823	43	61.1914
43 to 50	0.6	−1.1841	50	60.0073
50 to 42	0.4	+3.9766	42	63.9839
42 to 40	1.4	−3.7572	40	60.2267

Line S to T (Poe, 1930)

			60	58.5608
60 to 50	0.9	+1.4848	50	60.0456
50 to 43	0.6	+1.1667	43	61.2123
43 to 44	1.1	−2.4154	44	58.7969
60 to 42	1.0	+5.4246	42	63.9854
42 to 40	1.4	−3.7378	40	60.2476
40 to 41	1.5	+1.7632	41	62.0108

Computing the various circuit closures by summing differences clockwise, we have:

43 to 45 (1920)	1.0 km.	−0.4823 m.
45 to 44 (1910)	0.8	−1.9049
44 to 43 (1910)	1.1	+2.4172
	2.9 km.	+0.0300 m.=+30.0 mm.
43 to 44 (1910)	1.1 km.	−2.4172 m.
44 to 43 (1930)	1.1	+2.4154
	2.2 km.	−0.0018 m.=−1.8 mm.
43 to 50 (1930)	0.6 km.	−1.1667 m.
50 to 43 (1920)	0.6	+1.1841
	1.2 km.	+0.0174 m.=+17.4 mm.
42 to 41 (1910)	0.3 km.	−2.0041 m.
41 to 43 (1910)	1.0	−0.8008
43 to 50 (1920)	0.6	−1.1841
50 to 42 (1920)	0.4	+3.9766
	2.3 km.	−0.0124 m.=−12.4 mm.
42 to 50 (1920)	0.4 km.	−3.9766
50 to 60 (1930)	0.9	−1.4848
60 to 42 (1930)	1.0	+5.4246
	2.3 km.	−0.0368 m.=−36.8 mm.
40 to 41 (1910)	1.5 km.	+1.7648 m.
41 to 42 (1910)	0.3	+2.0041
42 to 40 (1920)	1.4	−3.7572
	3.2 km.	+0.0117 m.=+11.7 mm.
40 to 42 (1920)	1.4 km.	+3.7572 m.
42 to 40 (1930)	1.4	−3.7378
	2.8 km.	+0.0194 m.=+19.4 mm.
40 to 41 (1930)	1.5 km.	+1.7632 m.
41 to 40 (1910)	1.5	−1.7648
	3.0 km.	−0.0016 m.=−1.6 mm.

As a check on the computation of the eight individual circuit closures shown above, we can compute an over-all circuit closure by adding and subtracting elevations counter-clockwise around the outside lines of the junction net, as follows:

41 (1930)	+62.0108 m.
44 (1930)	−58.7969
44 (1910)	+58.8653
45 (1910)	−60.7702
45 (1920)	+60.7091
43 (1920)	−61.1914
43 (1910)	+61.2825
41 (1910)	−62.0833
	+ 0.0259 m. = +25.9 mm.,

which, if all is in order, should equal the algebraic sum of all of the individual circuit closures in this net: +30.0−1.8+17.4−12.4−36.8+11.7+19.4−1.6=+25.9 (check).

In the analysis of the behavior of bench marks 42, 45, and 50, which appear to have been disturbed for one reason or another, we may proceed as follows:

The 1910 leveling *from* 43, via 44, *to* 45 resulted in an observed difference of elevation of −0.5123 m. The 1920 leveling *from* 43 directly *to* 45 resulted in an observed difference of −0.4823 m. Bench mark 43 having been shown to be relatively stable, the decrease in the size of this "downhill" difference indicates that 45 has moved upward during the period from 1910 to 1920.

Using the same procedure, we find that the observed differences in elevation *from* 43 *to* 42, as determined by the various levelings, are as tabulated below:

1910	+2.8049 m.
1920	+2.7925 m.
1930	+2.7731 m.

The progressive decrease in the size of this "uphill" difference indicates a progressive settlement of bench mark 42.

When we consider bench mark 50 in relation to 43, we find we have observed differences in elevation *from* 43 *to* 50 as given below:

1920	−1.1841 m.
1930	−1.1667 m.

Here, again, the decrease in the size of a "downhill" difference indicates that bench mark 50 has heaved during the period between the two levelings.

Another method of exhibiting this same information, and one which is perhaps more useful, especially when a complicated net is encountered, is discussed in the following paragraphs.

"NEW MINUS OLD" TAPES

In the analysis of complicated junction details, the device known as a "new minus old tape" often proves valuable. Such a tape is made up by starting at a convenient bench mark, usually the first one common to the two levelings under consideration, and placing the new elevation for that mark in the adding-and-subtracting machine as a positive quantity. Then the old elevation for the same mark is placed in the machine as a negative quantity and a full total is struck. Then the new elevation for

the next mark common to both levelings is put in as a positive quantity and the old elevation for the same mark is put in as a negative quantity, and again a full total is struck. Continuing this process throughout the series of marks which are common to both levelings gives a comparison of results.

Such a procedure for the three levelings discussed in connection with figure 18 would result in three new—old tapes:

1920—1910
1930—1920
1930—1910.

The following is a tabulation made up from the new—old tapes for the three levelings under consideration.

	New—Old		
B. M.	*1920—1910* *mm.*	*1930—1920* *mm.*	*1930—1910* *mm.*
40	−91.8	+20.9	−70.9
41			−72.5
42	−103.5	+1.5	−102.0
43	−91.1	+20.9	−70.2
44			−68.4
45	−61.1		
50		+38.3	

A study of the above tabulation will show that the 1920 leveling checked the 1910 leveling between bench marks 40 and 43, since the new—old entries in the 1920—1910 column are closely alike. It also shows that bench mark 42 has settled, since the new—old entry for that mark in the 1920—1910 column is smaller (a larger negative quantity) than the corresponding entries for the marks on which the checks were obtained. Furthermore, this column of the tabulation exhibits the heaving of bench mark 45, since the entry opposite B. M. 45 and in the 1920—1910 column is larger (a smaller negative quantity) than the corresponding entries for the marks upon which checks were obtained.

Passing to the 1930—1920 column of the tabulation, 40 and 43 again show relative stability, while 42 shows further settlement and 50 exhibits an upward movement during the period from 1920 to 1930. The last column exhibits the state of affairs for the period from 1910 to 1930.

When any mark is common to all three levelings the entry in column "1930—1910" is the algebraic sum of the corresponding entries in the two preceding columns.

This tabulation or, in practice, the simultaneous consideration of the three new—old adding-machine tapes, clearly exhibits the whole story concerning the behavior of the various bench marks in the net.

The foregoing is *not* to be considered as a complete presentation of this phase of the work of adjusting leveling. It should, however, acquaint the novice with the standard methods of attack on this type of problem and promote uniformity in the work of a group of computers, so that one may readily pick up where another has left off on any particular project with a minimum of delay and lost motion.

The sketches which are drawn to show the details of the various levelings at a junction do not have to be fine examples of drafting. They are usually freehand sketches made with pencils of various colors. The prime requisites are that they be reasonably neat and readily legible.

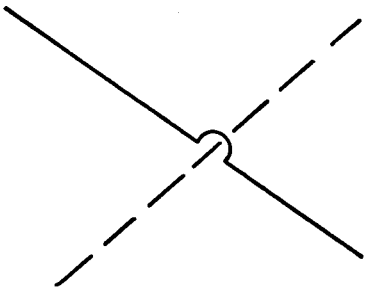

FIGURE 19.—Method of indicating crossing of two lines of levels at a point where no connection has been made.

When it becomes necessary, as it sometimes will, to let two lines of levels in a sketch cross each other at a point where no connection between the two levelings exists, this fact should be plainly exhibited as shown in figure 19.

Strict adherence to the rules set forth in the preceding discussions of the various junction details is necessary if confusion is to be avoided. The price the computer must pay for success in the analysis of a complicated junction detail is constant vigilance and care. He must avail himself of every possible chance for a check on his work and, when the work is not self-checking, have another computer check carefully all that he has done before it is accepted and put to use in the next stage of the work.

COMBINATION COMPUTATIONS

Another preliminary step which must sometimes be taken, prior to the adjustment of a network of lines of levels, is the consideration of the results of releveling along one or more lines leveled previously. As the level net is developed and especially in connection with leveling originally run as single-run second-order leveling, circuit closures may indicate that mistakes or blunders have been made in certain lines or parts of lines. It then becomes necessary to relevel such lines. If the releveling is done strictly in accordance with the specifications for first-order leveling, the releveling is usually substituted for the original leveling and the original leveling is discarded. However, if the levelings involved are single-run levelings, a combination abstract is prepared. The purpose of the combination is to determine the best observed elevations obtainable from the data at hand.

The instructions outlined in the following pages cover the procedures involved in making combinations of single-run second-order levelings, but the same general principles may be applied in preparing combinations for double-run leveling of either first- or second-order accuracy. *These procedures are designed to meet the peculiar problems presented by combination work and are not to be confused with the procedures previously discussed for the preliminary computations of first-order or double-run second-order leveling.*

The chief difference between combination computations and the preliminary computations previously discussed is in the manner in which mean differences in elevation are computed. In first-order or double-run second-order leveling, the difference in elevation for any section depends upon at least one running in each direction, a check being required between two runnings in opposite directions. In single-run second-order leveling the difference in elevation for any given section depends upon a single running which may have been made in either the forward or backward direction with respect to the general progress along the line. In releveling a single-run second-order line a check is required either between a single running of the releveling and the single running of the original leveling without regard to direction, or between two or more runnings of the releveling which check in accordance with the specifications for double-run leveling. As a result, the difference in elevation for a particular section in a combination abstract may depend on only two runnings, both in the same direction. Under these conditions the direction of a running loses some of its significance. In computing a mean difference in elevation in combination work, a mean of all runnings which fall

within the rejection limits is used, without regard to the directions of the individual runnings.

The office abstracts (Form 31) for the original leveling and the releveling are prepared in the usual manner, as covered previously in this manual, before undertaking the combination computations. The preparation of the combination abstract (Form 31, modified as shown in fig. 22) is carried out as the work of combination progresses.

For a hypothetical line "Smith to Brown, Tex." which has been assumed to have been leveled originally in 1935 and releveled in 1938, figure 20 shows the left-hand side of the office abstract for the original leveling, figure 21 shows the left-hand side of the office abstract for the releveling, and figure 22 shows the combination abstract.

The first step in combination work is the preparation of the strip detail. Figure 23 shows the strip detail for the hypothetical line mentioned above. These strip details are usually drawn in one continuous strip and are not broken into sections as in figure 23.

All of the original leveling and all of the releveling is shown on the strip detail. The circuit closures are computed and checked, as explained in the foregoing discussions of the various types of junction details. If the strip detail is drawn with the releveling above the original leveling, as in figure 23, the circuit closures are equivalent to the new-minus-old values required in the combination abstract.

The strip detail is then analyzed and unit details are prepared for those units of the combination which are too complicated to be handled directly on the combination abstract. (The term "unit" in this discussion refers to the smallest piece of the leveling for which the differences of elevation can be computed without reference to other pieces of leveling. A unit may be a single section or any group of sections which are so tied together that they cannot be considered independently.) The number of details required for the combination of the various levelings of a line depends upon (1) the complexity of the units and (2) the experience of the computers. The inexperienced computer will often require a detail in order to analyze some unit which an experienced computer might handle successfully directly on the combination abstract. While these unit details are ordinarily prepared before the combination abstract is started, it seems advisable to discuss our hypothetical line, unit by unit, and to defer discussion of Detail No. 1 (fig. 24, p. 75) until the preceding units have been discussed.

After the unit details have been prepared and checked, the combination abstract is made on Form 31 by either of two routines, (1) preparing two abstracts and checking one abstract against the other by copy-reading methods or (2) preparing a single abstract which another computer carefully checks by repeating all of the steps in the computation. The decision as to which routine is to be used in a given instance rests with the supervisor. In general, however, if both computers are experienced and thoroughly familiar with combination-computation procedures, one abstract with a thorough check should give accurate results with a minimum of effort. If even one of the computers in the team is relatively inexperienced, the duplicate abstracts are probably safer. In case duplicate abstracts are to be prepared, the first computer should make sufficient notes on the strip detail to enable the second computer (the checker) to proceed with his abstract without the necessity of referring to the first combination abstract.

The computer will enter on the first page of the combination abstract: The title of the combination; the title, chief of party, date, library shelf number, and accession number of the original leveling; and the title, chief of party, date, library shelf number,

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Form 31
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ABSTRACT OF

STATE Texas INSTRUMENT G 46 RODS 254, 255

U. S. GOVERNMENT PRINTING OFFICE 16—11482

Date (1)	From B. M. to B. M. (2)	Distance (3)	Difference of Elevation: Forward Line (4)	Difference of Elevation: Backward Line (5)	Difference of Elevation: Mean (6)	Discrepancy: Partial (7)	Discrepancy: Total accumulated (8)
1935 May		km.	m.	m.	m.	mm.	mm.
7	A – B	0.78	+2.1636		+2.1636		
7	B – A1	1.12	+1.6063		+1.6063		
7	A1 – B1	1.63	+0.5869		+0.5869		
7	B1 – C1	1.58	−1.4970		−1.4970		
7	C1 – 1	1.17	−2.7894		−2.7894		
7	1 – D1	0.89	−3.0167		−3.0167		
7	D1 – E1	1.78	+2.1043		+2.1043		
7	E1 – F1	1.69	+4.2031		+4.2031		
8	F1 – G1	1.82		+3.6109	−3.6109		
8	G1 – H1	1.72		+4.1106	−4.1106		
8	H1 – J1	2.46		−1.9637	+1.9637		
8	J1 – K1	1.69		+1.2617	−1.2617		
8	K1 – L1	1.58		+0.6753	−0.6753		
8	L1 – M1	1.72		−1.2146	+1.2146		
9	M1 – RV1	0.08	+0.4216		+0.4216		
9	M1 – N1	1.68	+2.6879		+2.6879		
9	N1 – P1	1.61	+3.6543		+3.6543		
9	P1 – USGS 538	0.32	−1.7889		−1.7889		
9	P1 – Q1	2.06	+1.1126		+1.1126		
9	Q1 – RV2	0.06	−0.2174		−0.2174		
9	Q1 – R1	2.12	−2.9055		−2.9055		
10	R1 – S1	1.04		+4.1892	−4.1892		
10	S1 – T1	0.17	−1.1304		−1.1304		
10	S1 – U1	0.97		+2.1602	−2.1602		
10	U1 – X	0.74		+1.9843	−1.9843		
10 10	X – Y	1.16	+0.7809	−0.7823	+0.7816	+1.4	
10 10	Y – Z	0.17	+0.0463	−0.0460	+0.0462	−0.3	

FIGURE 20.—Left-hand page of office abstract (Form 31) for original leveling of hypothetical line "Smith to Brown, Tex."

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ABSTRACT OF

STATE Texas INSTRUMENT G 72 RODS 344, 346

U. S. GOVERNMENT PRINTING OFFICE 16—11452

Date (1)	From B. M. to B. M. (2)	Distance (3)	Difference of Elevation: Forward Line (4)	Backward Line (5)	Mean (6)	Discrepancy: Partial (7)	Total accumulated (8)
1938 Nov.		km.	m.	m.	m.	mm.	mm.
12	A - B	0.77	+2.1647		+2.1647		
12	B - A1	1.13	+1.6043		+1.6043		
12 14	A1 - B1	1.63	+0.5291	-0.5307	+0.5299	+1.6	
12 14	B1 - C1	1.59	-1.4416	+1.4423	-1.4420	-0.7	
12	C1 - A2	1.00	-2.5948		-2.5948		
14	A2 - D1	1.04		+3.2122	-3.2122		
15 14	D1 - E1	1.78	+2.1852	-2.1821	+2.1836	-3.1	
15 14	E1 - F1	1.68	+4.1988	-4.1902	+4.1945	-8.6	
15 14	F1 - G1	1.81	-3.6233	+3.6709 (R)	-3.6204	+5.9	
15				+3.6174			
14	G1 - B2	1.67		+3.0061	-3.0061		
14	B2 - C2	1.36		-1.3624	+1.3624		
14	C2 - J1	1.14		+0.5044	-0.5044		
15 16	J1 - D2	1.49	+0.9824	-0.9871	+0.9848	+4.7	
15 16	D2 - L1	1.78	-2.8127	+2.8144	-2.8136	-1.7	
15	L1 - RV1	1.76	+1.6386		+1.6386		
15 15	RV1 - N1	1.66	+2.3345	-2.3338	+2.3342	-0.7	
15	N1 - USGS 538	1.84	+1.8728		+1.8728		
16	USGS 538 - Q1	2.16		-2.8957	+2.8957		
16	Q1 - RV2	0.06		+0.2180	-0.2180		
16	RV2 - R1	2.11		+2.6814	-2.6814		
17 17	R1 - T1	1.18	-5.2491	+5.2514	-5.2502	-2.3	
17 17	T1 - E2	1.03	+0.7846	-0.7864	+0.7855	+1.8	
17 17	E2 - U1	0.18	-1.8897	+1.8917	-1.8907	-2.0	
17 17	E2 - X	0.68	-3.9642	+3.9654	-3.9648	-1.2	
17	X - Y	1.18	+0.7817		+0.7817		
17	Y - Z	0.16	+0.0457		+0.0457		

FIGURE 21.—Left-hand page of office abstract (Form 31) for releveling of hypothetical line "Smith to Brown, Tex."

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Form 31
Ed. Sept. 1939

ABSTRACT OF

STATE Texas INSTRUMENT _____ RODS _____

U. S. GOVERNMENT PRINTING OFFICE 16—11482

Date (1)	From B. M. to B. M. (2)	Distance (3)	Difference of Elevation: 1935 Forward Line (4)	Difference of Elevation: 1938 Forward/~~Backward~~ Line (5)	Difference of Elevation: Accepted Diff. ~~Mean~~ (6)	Discrepancy: 1938-1935 ~~Partial~~ (7)	Discrepancy: Total accumulated (8)
		km.	m.	m.	m.	mm.	mm.
	A – B	0.78	+2.1636	+2.1647	+2.1642	+1.1	
	B – A1	1.12	+1.6063	+1.6043	+1.6053	−2.0	
	A1 – B1(1935)	(3.26) 1.63	+0.5869 (−0.7)		+0.5862		
	B1(1935) – C1	(3.16) 1.58	−1.4970 (−0.7)		−1.4977		
	A1 – B1(1938)	1.63		2] +0.5299 (+0.3)	+0.5302	−2.0	9.64
	B1(1938) – C1	1.59		2] −1.4420 (+0.3)	−1.4417		
	C1 – D1	2.06	−5.8061 (−0.3)		−5.8066		
	C1 – A2	1.00		−2.5948 (+0.2)	−2.5946	−0.9	4.10
	A2 – D1	1.04		−3.2122 (+0.2)	−3.2120		
	D1 – E1	1.78	+2.1043 (R)	2] +2.1836	+2.1836	+7.3	
	E1 – F1	1.68	+4.2031	2] +4.1945	+4.1974	−8.6	
	F1 – G1	1.81	−3.6109	2] −3.6204	−3.6172	−9.5	
	G1 – H1	1.72	−4.1106 (−0.2)		−4.1108		
	H1 – J1	2.46	+1.9637 (−0.4)		+1.9633		
	G1 – B2	1.67		−3.0061 (+0.2)	−3.0059	−1.2	8.35
	B2 – C2	1.36		+1.3624 (+0.2)	+1.3626		
	C2 – J1	1.14		−0.5044 (+0.2)	−0.5042		
	J1 – K1	1.69	−1.2617 (R)				
	K1 – L1	1.58	−0.6753 (R)				
	J1 – D2	1.49		2] +0.9848	+0.9848	+108.2	
	D2 – L1	1.78		2] −2.8136	−2.8136		
	L1 – M1	1.72	+1.2146 (+1.2)		+1.2158	+2.4	3.56
	M1 – RV1	0.08	+0.4216 (0.0)		+0.4216		
	L1 – RV1	1.76		+1.6386 (−1.2)	+1.6374		
	M1 – N1	1.68	+2.6879 (R)			+67.9	
	RV1 – N1	1.66		2] +2.3342	+2.3342		
	N1 – USGS 538	1.84		+1.8728 (−3.4)	+1.8694		
	N1 – P1	1.61	+3.6543 (+3.0)		+3.6573	−6.4	3.45
	P1 – USGS 538	0.32	−1.7889 (+1.0)		−1.7879		
	P1 – Q1	2.06	+1.1126 (−2.3)		+1.1103		
	USGS 538 – Q1	2.16		+2.8957 (+2.5)	+2.8982	−4.8	4.22

FIGURE 22.—Combination abstract (Form 31, modified) for the hypothetical line "Smith to Brown, Tex."

COMBINATION

SECOND -ORDER LEVELS

OBSERVER K.L. Dixon N.M. Moore COMPUTER C.E. McC.

Designation of B. M. (9)	Distance from B.M. B (10) km.	Observed Elevation Above Mean Sea Level (11) m.	Dates: Smith to Brown, Texas, May 7-10, 1935 Rel. Smith to Brown, Texas, Nov. 12-17, 1938 (12)
A		162.8188	See: Dale, via Smith, to West, Texas
B	0.00	164.9830	
A1	1.12	166.5883	
B1	2.75	167.1745	← 1935 Elev.
(C1)		(165.6768)	Check
B1	2.75	167.1185	← 1938 Elev.
C1	4.34	165.6768	
(D1)		(159.8702)	Check
A2	5.34	163.0822	
D1	6.38	159.8702	
E1	8.16	162.0538	
F1	9.84	166.2512	
G1	11.65	162.6340	
H1	13.37	158.5232	
(J1)		(160.4865)	Check
B2	13.32	159.6281	
C2	14.68	160.9907	
J1	15.82	160.4865	
K1	17.51	No accurate elev.	
D2	17.31	161.4713	
L1	19.09	158.6577	
M1	20.81	159.8735	
RV1	20.89	160.2951	
(RV1)		(160.2951)	Check
N1	22.55	162.6293	
(USGS 538)		(164.4987)	Weighted Mean from P1 to USGS 538
P1	24.16	166.2866	A = +7.4 L V
USGS 538	24.48	164.4987	B = −5.8 1 3.45 −6.4
(Q1)		(167.3969)	Check 2 0.32 +1.0
Q1	26.64	167.3969	3 4.22 −4.8

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 31
Ed. Sept. 1939

ABSTRACT OF

STATE Texas INSTRUMENT _____ RODS _____

U. S. GOVERNMENT PRINTING OFFICE 16—11482

Date (1)	From B. M. to B. M. (2)	Distance (3)	Difference of Elevation 1935-1938: Forward Line (4)	Forward ~~Backward~~ Line (5)	Accepted Diff. ~~Mean~~ (6)	Discrepancy 1935-1938 ~~Partial~~ (7)	Total accumulated (8)
		km.	m.	m.	m.	mm.	mm.
	Q1 - RV2	0.06	-0.2174	-0.2180	-0.2177	-0.6	
	RV2 - R1	2.11		-2.6814 (-3.2)	-2.6846	+6.4	4.23
	Q1 - R1	2.12	-2.9055 (+3.2)		-2.9023		
	R1 - T1(1938)	1.18			-5.2491		
	R1 - S1	1.04			-4.1911	-6.0	
	S1 - T1(1935)	0.17	See Detail No. 1		-1.1304		
	S1 - U1	0.97			-2.1620		
	U1 - E2	0.18			+1.8905		
	E2 - X	0.68			-3.9648	-89.8	
	X - Y	1.17	[2] +0.7816	+0.7817	+0.7816	+0.1	
	Y - Z	0.17	[2] +0.0462	+0.0457	+0.0460	-0.5	

FIGURE 22.—Combination abstract (Form 31, modified) for the hypothetical line "Smith to Brown, Tex."—Continued

and accession number of the releveling. (When completed the combination abstract is filed with the computations for the releveling.)

In preparing combination abstracts, the column headings for columns 4, 5, 6, and 7 are changed as shown in figure 22. The inclusive dates of the levelings are recorded in the blank at the top of column 12 as shown in figure 22. Red ink (indicated by boxed entries) is used not only to indicate the check runs and spurs which are shown in red on the regular abstracts but also to indicate other entries which are to be considered independently in computing the distances and elevations in columns 10 and 11, respectively.

On a combination abstract columns 1 and 8 are not used. The entries in columns 2, 3, 9, 10, and 11 are similar to corresponding entries on a regular abstract. The mean differences of elevation in the forward direction as obtained from the original leveling are entered in column 4. If a difference in elevation is derived from more than one running, the number of runnings is indicated by a small boxed or circled number at the left-hand side of the column. The mean differences of elevation (also in the forward direction) as obtained from the releveling are entered in column 5, and again small boxed or circled numbers at the left-hand side of the column indicate the number of runnings, if more than one.

The accepted differences of elevation in the forward direction, as obtained by combining the results of the original leveling and the releveling, are entered in column 6, and column 7 shows the difference, releveling minus the original leveling. This difference is computed and entered in column 7 for each successive unit, except when an adjustment has been involved.

The hypothetical line exhibited in figures 20 to 23, inclusive, does not represent normal expectancy in combination work, since the majority of the sections will be

COMBINATION

SECOND -ORDER LEVELS

OBSERVER K.L. Dixon N.M. Moore COMPUTER C.E. Mc C.

Designation of B. M. (9)	Distance from B. M. B (10)	Observed Elevation Above Mean Sea Level (11)	(12)
	km.	m.	
RV2	26.70	167.1792	
R1	28.81	164.4946	
(R1)		(164.4946)	Check
T1	29.99	159.2455	← 1938 Elev.
S1	29.85	160.3035	
T1		159.1731	← 1935 Elev.
U1	30.82	158.1415	
E2	31.00	160.0320	
X	31.68	156.0672	See: Black to Derby, Texas
Y	32.85	156.8488	New elev.
Z	33.02	156.8948	

similar to the sections from B to A 1 and from C 1 to D 1 of the hypothetical line. This line has been deliberately arranged to cover as many different problems as possible in a minimum of space.

The various procedures to be followed in preparing combination abstracts are discussed under the following headings:

Discussion of the hypothetical line;
Spurs and extra foresights;
Reset bench marks;
Starting distances and elevations;
Preparation of the summary sheet;
Complicated problems; and
Combinations involving three or more levelings.

As explained before, since only one color can be used in the preparation of this manual, boxed entries on Form 31 indicate the use of red ink in the actual computations and, in figures 23 and 24, light lines indicate single-run leveling, heavy lines indicate sections leveled two or more times, and the character of the lines distinguishes the original leveling from the releveling; while in actual practice the character of the line is used to distinguish between single-run and multiple-run sections, and colors are used to distinguish the levelings of different lines.

DISCUSSION OF THE HYPOTHETICAL LINE

A unit-by-unit discussion of the hypothetical line "Smith to Brown, Tex." is contained in the following paragraphs, and a careful study of the methods used in

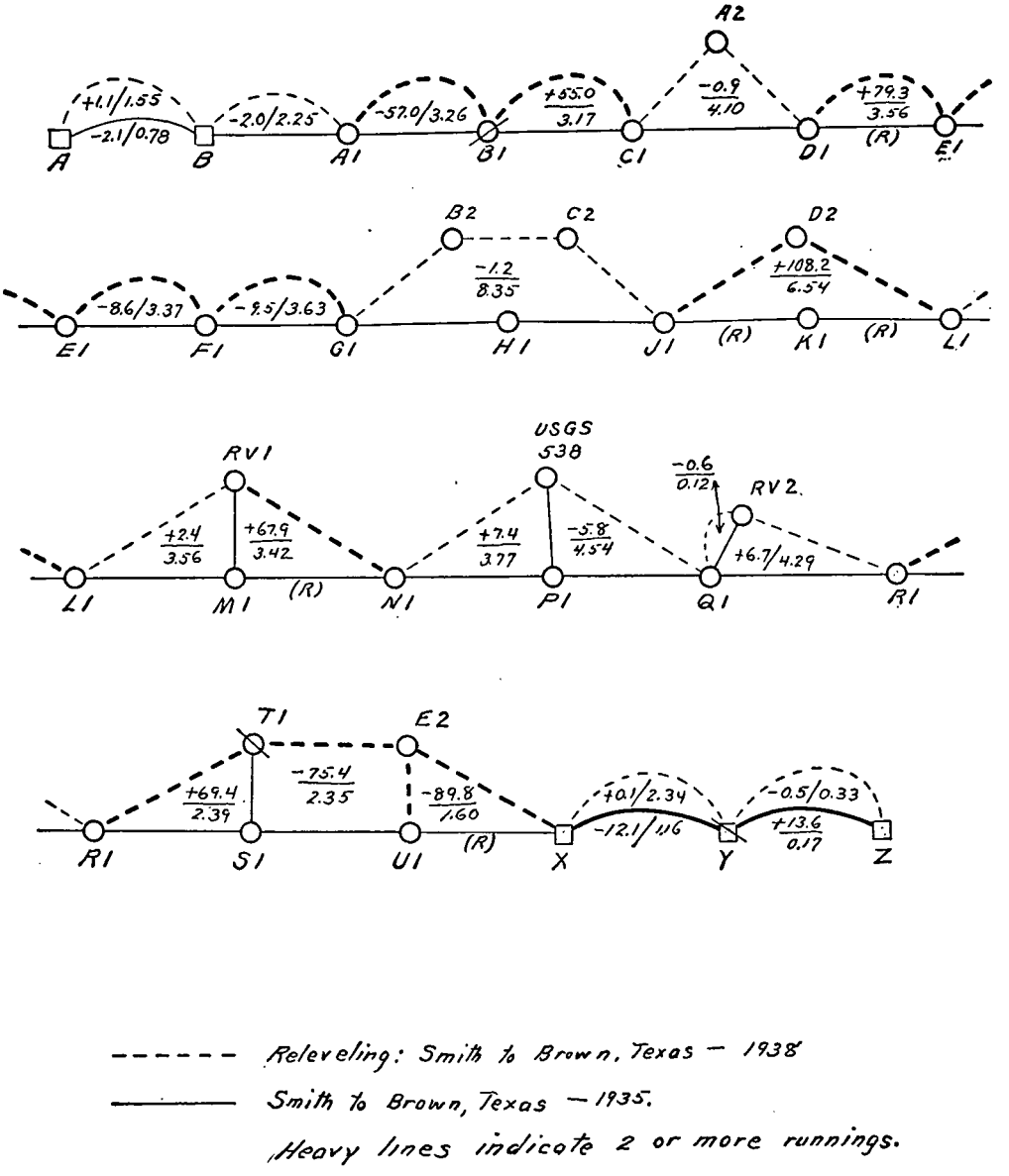
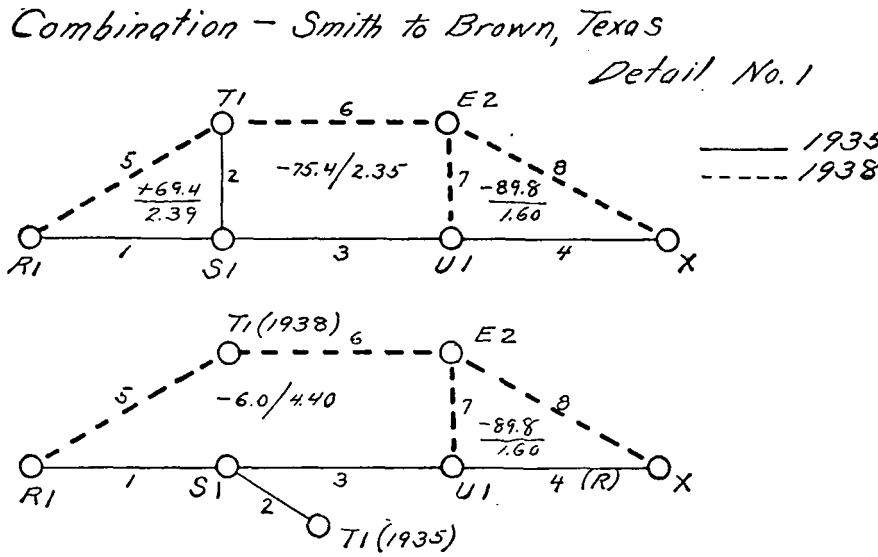


FIGURE 23.—Strip detail for hypothetical line "Smith to Brown, Tex."

combining the levelings assumed to have been run in 1935 and 1938 should serve to clarify combination-abstract problems which are likely to be encountered frequently. As in the study of junction details we could not hope to cover all possible cases, so in the study of the methods of making combinations we cannot possibly cover all of the various problems that may be encountered. However, we have attempted to cover the cases which will be encountered quite often and by means of these to outline the methods to be used in attacking the more complicated cases which will be encountered only occasionally.

The section from A to B may be analyzed as follows:

In figure 23, bench marks A and B are indicated by small squares to show that these



Link	BM-BM	Length	Obs. Diff.	Corr'n.	Adj. Diff.
1	S1 - R1	1.04	-4.1892	-1.9	-4.1911
2	T1(35)-S1	0.17	-1.1304	0.0	-1.1304
3	U1 - S1	0.97	-2.1602	-1.8	-2.1620
4	X - U1	0.74	-1.9843	Reject	
5	T1(38)-R1 ⊗	1.18	-5.2502	+1.1	-5.2491
6	E2-T1(38) ⊗	1.03	+0.7855	+1.0	+0.7865
7	U1 - E2 ⊗	0.18	-1.8907	+0.2	-1.8905
8	X - E2	0.68	-3.9648	0.0	-3.9648

⊗ Double-run
Use ½ length in distribution.

FIGURE 24.—Detail No. 1 of the hypothetical line "Smith to Brown, Tex."

are the bench marks established during leveling run prior to the "original" leveling of 1935 from Smith to Brown and are the bench marks on which the 1935 leveling obtained its check to start the line to Brown. The entry −2.1/0.78 shown below the circuit from A to B indicates that in 1935 a check of −2.1 mm. was obtained against the previous leveling and that the two marks are 0.78 km. apart along the line of levels. Referring to figure 20, we find that the difference of elevation from A to B is +2.1636 m. Figure 21 shows that in 1938 this difference was found to be +2.1647 m. The circuit closure is found to be +1.1 mm. and indicates a good check between the 1935 and 1938 levelings from A to B. Since no new bench marks or new elevations for old bench marks occur on this check run, it is abstracted (fig. 22) in red. A mean of the 1935 and 1938 distances is entered in column 3, the 1935 difference of elevation is entered in column 4, the 1938 difference of elevation is entered in column 5, the mean of the 1935 and 1938 runnings is entered in column 6, and the difference between the two runnings (column 5 minus column 4) is entered in column 7.

The levelings from B to A 1 show a check between the original leveling and the releveling. The 1935 and 1938 differences of elevation are entered, the mean is com-

puted and entered, and the difference (releveling minus original leveling) is also computed and entered on the combination abstract. The entries for this section should be in black ink.

Considering the section from A 1 to B 1, we find that in 1935 the observed difference of elevation was +0.5869 m. The first running in 1938 resulted in an observed difference of elevation of +0.5291 m. and a trial closure of −57.8 mm. on a 1.63-km. section, which was not a check, and the leveling was repeated. The second running in 1938 resulted in an observed difference of elevation of −0.5307 m. The partial for the 1938 work being +1.6 mm., the two 1938 runnings were in good agreement, and, since the check on the previous section indicated the stability of A 1, the evidence at this point indicates movement of B 1.

Passing on to the section from B 1 to C 1, the first running from the 1938 leveling failed to agree with the original leveling in 1935, so a backward run was made. The 1938 partial for the section from B 1 to C 1 is −0.7 mm., an excellent check.

Computing the two circuit closures involved in the unit from A 1 to C 1, we find that they are −57.0 mm. and +55.0 mm., which shows a check of −2.0 mm. from A 1 to C 1 and substantiates the suspected movement of B 1.

Since we wish to compute two elevations for B 1, one based on the 1935 work and the other based on the 1938 work, the 1935 leveling is abstracted in red and the differences of elevation are entered in column 4. The 1938 leveling is then abstracted in black with the differences of elevation entered in column 5. The small numerals in the boxes at the left-hand side of the column show there were two runnings over each of the two sections in this unit in 1938.

Since the levelings between A 1 and C 1 involve differences of elevation for more than one section, we distribute the closing error around the circuit rather than take a series of means. The closing error, using the from–to method, is obtained by adding the entries in column 5 and subtracting those in column 4, thus

$$+(+0.5299)+(-1.4420)-(-1.4970)-(+0.5869)=-2.0 \text{ mm.},$$

and is entered as the numerator of the fraction in column 7, with a brace to indicate that it includes all of the runnings from A 1 through to C 1.

Since the releveling was double-run and the original leveling was single-run, the original leveling is given one-half weight in distributing the closure by doubling the distances on the single-run leveling. These weighted or doubled section lengths are entered in pencil in column 3 and are used only in distributing the closure. (The pencil entries are indicated in fig. 22 by means of parentheses.) The weighted distances are summed around the circuit, thus:

$$3.26+3.16+1.63+1.59=9.64 \text{ km.},$$

and the total is entered as the denominator of the fraction in column 7. This fraction is the distribution rate for the circuit. The correction to be applied to each difference of elevation is obtained by multiplying the weighted distance by the distribution rate, thus:

A 1 to B 1 (1935)	3.26(−2.0/9.64)=−0.7
B 1 to C 1 (1935)	3.16(−2.0/9.64)=−0.7
A 1 to B 1 (1938)	1.63(−2.0/9.64)=−0.3
B 1 to C 1 (1938)	1.59(−2.0/9.64)=−0.3
	−2.0 (check).

These corrections are entered above and to the right of the appropriate differences of elevation in columns 4 and 5, as shown in figure 22. Since the closure was computed on a new-minus-old basis, the corrections as computed above are of the proper sign (same sign as the closure) for use on the sections of the original leveling, but the signs must be changed to obtain the proper corrections to be applied to the sections of the new leveling. The corrected differences of elevation are then entered in column 6.

The distance entries for B 1 and C 1 in column 10 of the combination abstract are computed using the unweighted distances in column 3 and using 1935 and 1938 distances when computing through the 1935 and 1938 sides of the circuit, respectively.

The 1935 elevation of B 1 is shown in black in column 11. The elevation of C 1 carried through the corrected 1935 differences of elevation is computed and entered in column 11, in pencil, for later comparison with the other elevation for the same mark, computed by means of the corrected differences of elevation for the 1938 leveling. The pencil entry for the elevation of B 1 is marked "check" if it is later found to agree with the other elevation brought along the 1938 side of the circuit. If these two do not agree, some mistake has been made, and the error should be found and corrected before proceeding further with the combination abstract.

In computing the corrections as outlined above, the sum of the individual corrections may fail to agree exactly with the closure because of the accumulation of dropped or added hundredths in rounding the computed values to one decimal place. If so, an arbitrary change of one in the last place is made on one of the computed values (ordinarily on the one nearest the half in the second decimal place) so that the values sum correctly and permit an exact check between the elevations computed through both sides of the circuit. For example, assume a series of corrections computed to two decimal places, as follows: +3.49, +3.47, +3.55, and +3.49, which total correctly to +14.00. Now, rounding these to only one place of decimals, we have, according to the rules previously set forth (p. 17), a set of four corrections, as follows: +3.5+3.5+3.6+3.5= +14.1, so we slip the third one the wrong way in rounding and then have +3.5+3.5+ 3.5+3.5=+14.0, which produces the correct total and permits a check.

Referring again to figure 22, the 1938 elevation of B 1 and the second elevation of C 1 are computed, using the corrected 1938 differences from column 6, and since this elevation for C 1 agrees with the one in pencil, two lines above, the penciled elevation is marked "check" to exhibit the fact that no mistake has been made in distributing the corrections around the circuit, A 1, B 1 (1938), C 1, B 1 (1935), and A 1.

A line is drawn through the circle indicating bench mark B 1 on the strip detail (fig. 23) to indicate that no check was obtained on that mark and that the adjoining circuits were combined to produce a single circuit.

The releveling between C 1 and D 1 checks the original leveling between the same marks, but a new mark was established during the releveling. The 1935 leveling is abstracted in red and the 1938 leveling in black. It is noted in referring to the fifth and sixth lines of figure 20 that TBM 1 was leveled over in 1935 but, since it was not described in 1935 and not leveled over in 1938, no elevation is computed for it. The differences of elevation for the 1935 sections from C 1 to TBM 1 and from TBM 1 to D 1 are added together to produce a single entry for a section from C 1 to D 1 in column 4 of the combination abstract. The entries in columns 6 and 7 are again computed by the same method used for the unit from A 1 to C 1, and the elevation derived from the 1935 leveling is again entered in pencil, as a check.

Between D 1 and E 1 the releveling failed to check the original leveling and was rerun. The two 1938 runnings show a partial of −3.1 mm., which constitutes a good

agreement between the two runnings. Since the units from C 1 to D 1 and from E 1 to F 1 both show checks between the original leveling and the releveling, it must be assumed that both D 1 and E 1 have remained stable and that a blunder occurred in the 1935 leveling from D 1 to E 1. The entries in columns 2 to 5, inclusive, are made in the same manner in which they would have been made if a check had been obtained *except* that the distance in column 3 is taken from the releveling and the entry in column 4 is marked "(R)" to show that it has been rejected. The 1935 leveling of that section on the strip detail is also marked "(R)" for the same reason. The differences of elevation from the 1938 leveling are entered in column 6. The entry for column 7 is computed and entered even though the difference in elevation in column 4 was rejected in computing the entry to go in column 6. The entries of large size in column 7 show at a glance where blunders were found in the original leveling.

Between E 1 and F 1 we find that the observed difference of elevation from the original leveling is +4.2031 m. The first running in 1938 resulted in a difference of elevation of +4.1902 m. in the forward direction and the trial closure was −12.9 mm., a failure to obtain a second-order check on this section having a length of 1.69 km. (1935) or 1.68 km. (1938). The second running in 1938 showed a resulting difference of elevation of +4.1988 m. The 1938 partial then proved to be −8.6 mm., a second-order check, and the divergence between the mean of the two 1938 runnings and the single-run difference of 1935 is also −8.6 mm., still a second-order check.

The general rules for computing the accepted difference of elevation for all sections on which more than two runnings have been made are:

(1) Reject arbitrarily any running which is so divergent from the group as to indicate a blunder,

(2) Take a preliminary mean of the remaining runnings, weighting all runnings equally regardless of the directions of the runnings,

(3) Reject all runnings included in the preliminary mean which differ by more than ± 9.0 mm. $\sqrt{K}$ from that mean, and

(4) Repeat steps (2) and (3) until no further rejections are made.

These runnings are then retained, and their mean becomes the accepted difference of elevation. The entry to go in column 6 is the mean of the retained runnings.

Applying these rules to the unit from E 1 to F 1, we have

$$\begin{array}{r} 4.2031 \text{ m.} \\ 4.1902 \text{ m.} \\ \underline{4.1988 \text{ m.}} \\ 12.5921 \div 3 = 4.1974 \text{ m.} \end{array}$$

We then reject any running differing from that mean by more than ± 9.0 mm. $\sqrt{1.68} = \pm 11.7$ mm., or, in other words, any runnings falling outside the range from 4.1857 to 4.2091, and find that no rejections need be made. The first mean then becomes the accepted difference and is entered in column 6 of the combination abstract.

NOTE.—Good judgment indicates that rejection limits cannot be applied as strictly on combination work as in preliminary computations. For example, if, on a section 1 km. in length, two runnings of the releveling differ by 1.0 mm., and the original running differs from the mean of the two runnings of the releveling by 13.0 mm., all three runnings would fall within the rejection limits, yet good judgment would indicate that the original running should be arbitrarily rejected and that the mean of the two runnings of the releveling be used as the accepted difference of elevation However, the divergence between the two runnings of the releveling need

not increase very much in relation to the divergence of their mean from the original running before good judgment would dictate the application of the general rules set forth above. All deviations from a strict application of the rejection limits should be carefully considered before being allowed.

An inspection of the data pertaining to the section from F 1 to G 1 shows that the party had some difficulty in obtaining a check. The four runnings, in the order in which they were made and with the sign of the forward running, are as follows:

Original (1935)	−3.6109 m.,
Releveling (1938)	−3.6709 m.,
Releveling (1938)	−3.6233 m.,
Releveling (1938)	−3.6174 m.

Obviously the first running of the releveling should be arbitrarily rejected. The remaining three runnings should then be handled in the manner outlined above for the treatment of the data pertaining to the section from E 1 to F 1.

In the unit from G 1 to J 1 the releveling did not touch upon H 1, but two new marks, B 2 and C 2, were established during the releveling, which checked the original difference of elevation from G 1 to J 1. Again the 1935 leveling is abstracted in red and the 1938 leveling in black. The elevations for H 1, B 2, and C 2 are then computed using the same methods used in computing the elevations of B 1 (1935) and B 1 (1938) and A 2.

The releveling indicates a blunder in the original leveling between J 1 and L 1. The old bench mark K 1 was not included in the releveling, but a new mark (D 2) was established. In this case the entries in columns 2 to 5, inclusive, are made in the same manner as for the unit between G 1 and J 1. Since there is no way of localizing the blunder in the original leveling, both sections are rejected and no entries appear in column 6 opposite the 1935 differences of elevation. Even though no elevation is computed for K 1, it is to be listed in column 9 and its distance is computed and entered in column 10. In column 11 the entry "No accurate elevation" is made. The designation and distance for K 1 should also be listed on the summary sheet with the same notation. The 1938 leveling is accepted, and the entries in column 6 are the same as those in column 5 for the releveling from J 1 through to L 1.

In the next unit the 1938 party ran a single-run line from L 1 to RV 1, obtaining a check within prescribed limits; ran a double-run line from RV 1 to N 1; and did not level over M 1. Since a check was obtained between L 1 and RV 1, the circuit closure is distributed around the circuit, L 1, M 1, RV 1, and L 1, in the same manner as for the unit A 1 to C 1. Since a check was not obtained between RV 1 and N 1 and the releveling between these two marks was double-run, the original leveling from M 1 to N 1 is rejected and the releveling accepted from RV 1 to N 1. The leveling along the most direct route which will include all of the bench marks is abstracted in black and the remaining runnings in red. To facilitate the distribution of the closing error, the runnings are arranged on the combination abstract as shown in figure 22. After the accepted differences of elevation have been computed, the elevations in column 11 are computed by using the runnings entered in black. The elevation of RV 1 computed from L 1 by using the running entered in red is entered in pencil and compared with the other elevation for the same mark, as a check on the distribution of the closure in the closed circuit in the first part of the unit. In the last part of the unit, the rejection of the running from M 1 to N 1 prevents the computation of a check elevation for N 1. The entries in column 3 are obtained directly from the abstract of the original

leveling and the abstract of the releveling, and the distances from the runnings abstracted in black are the ones used in computing the distances entered in column 10. No attempt is made to make the distances entirely consistent throughout the unit since the discrepancies are of such small size as to be insignificant in adjusting the line to the level net.

Considering next the unit from N 1 to Q 1, we note that it is quite similar to the next preceding unit *except* that checks within prescribed limits were obtained in both of the circuits involved in the unit. The entries in columns 2 to 5, inclusive, are made in the same manner as for the unit from L 1 to N 1. However, we cannot distribute the closing error around each circuit independently because such distribution would usually result in two different values for the difference of elevation for the section running directly from P 1 to 538 (USGS).

An analysis of this unit shows that the difference of elevation from P 1 to 538 (USGS) may be computed in three independent ways:

(1) From P 1 to N 1 through the original leveling and from N 1 to 538 (USGS) through the releveling;

(2) From P 1 to 538 (USGS) through the original leveling only; and

(3) From P 1 to Q 1 through the original leveling and from Q 1 to 538 (USGS) through the releveling.

Since these three ways include all of the leveling in the unit, it is evident that if the most probable value for the difference of elevation from P 1 to 538 (USGS) can be determined, the most probable values for the differences of elevation of all the runnings can be computed. The most probable value for the difference of elevation from P 1 to 538 (USGS) is obtainable from a weighted-mean adjustment of the three independent differences of elevation. The corrections resulting from a weighted-mean adjustment can be computed from the following formulas, developed by C. E. McCombs:

If:

A=the closure of the circuit from N 1 to P 1 to 538 (USGS) to N 1,

B=the closure of the circuit from P 1 to Q 1 to 538 (USGS) to P 1,

L_1=the distance from P 1 to N 1 to 538 (USGS),

L_2=the distance from P 1 to 538 (USGS),

L_3=the distance from P 1 to Q 1 to 538 (USGS),

v_1=the correction to be applied to the difference of elevation from P 1 to 538 (USGS) via N 1,

v_2=the correction to be applied to the difference of elevation from P 1 directly to 538 (USGS), and

v_3=the correction to be applied to the difference of elevation from P 1 to 538 (USGS) via Q 1;

then

$$v_2=\frac{L_3A-L_1B}{L_1+L_3+\frac{L_1L_3}{L_2}},$$

$$v_1=v_2-A, \text{ and}$$

$$v_3=v_2+B.$$

To carry out this computation a table is prepared in column 12 of the combination abstract, as shown in figure 22. The values of A and B are taken from the strip detail. The values of the L's are obtained from column 3 of the combination abstract. The values of the v's are computed by means of the formulas given above. Since no more

than three significant figures are normally required to insure necessary accuracy of the numerical values of the v's, a slide rule can almost always be used for such a computation.

NOTE.—The circuit reduction method which is discussed in detail farther along in this manual may be applied to the solution of the above problem by using computation Form A. For such use, only the first three links of the diagram at the top of Form A would be needed, the points of arrows 1, 2, and 3 would all be lettered P 1, and the point representing the butts of all three arrows would be lettered 538 (USGS). The balance of the work of adjustment would be carried out on Form A in accordance with the methods discussed later.

v_2 is applied to the difference of elevation on the direct running from P 1 to 538 (USGS). v_1 is entered in column 7 and is distributed in proportion to the distances involved between the runnings from N 1 to P 1 and from N 1 to 538 (USGS). v_3 is also entered in column 7 and distributed between the runnings from P 1 to Q 1 and from 538 (USGS) to Q 1.

Though the numerical values of v_1 and v_3 are entered in column 7, these are not new-minus-old values but are corrections to be applied to the differences of elevation in the direction from P 1 to 538 (USGS). Therefore, the corrections should be applied to the runnings from N 1 to 538 (USGS) and from P 1 to Q 1 with the same signs as their respective v's, but the corrections applied to the runnings from N 1 to P 1 and from 538 (USGS) to Q 1 will have signs opposite to those of their respective v's.

After the corrections have been applied to the differences of elevation in columns 4 and 5 and the accepted differences have been entered in column 6, the entries to go in columns 10 and 11 are computed, using the runnings entered in black. Check elevations for 538 (USGS) and Q 1 are computed along the route N 1 to 538 (USGS) to P 1 to Q 1 and entered on the combination abstract in pencil. This serves as a check on the proper application of the corrections to the various runnings.

The unit between Q 1 and R 1 presents another weighted-mean problem and could be handled in the same manner as we handled the unit from N 1 to Q 1. However, the closure from Q 1 to RV 2 to Q 1 is quite small, and the distance from Q 1 to RV 2 is also quite small with respect to the distances from RV 2 to R 1 and from Q 1 to R 1. In this case the same result is obtained by taking a mean of the two runnings from Q 1 to RV 2 and then distributing the remaining closure between the runnings from RV 2 to R 1 and from Q 1 to R 1. The leveling is then abstracted as shown in figure 22.

NOTE.—This treatment of the unit from Q 1 to R 1 is one of the "short cuts" which are reserved for the use of thoroughly experienced computers. Short cuts should never be used unless the computer is *certain* that the results obtained will be the same as those which would be obtained by the application of the weighted-mean formulas. *If in doubt, use the formulas.* The novice should use the formulas and then recompute the unit by the short-cut method until he has handled a sufficient number of units of this type in both ways to make sure that he will recognize cases which may be handled successfully by the short-cut procedure.

The unit between R 1 and X presents a problem in analysis. Detail No. 1 (fig. 24, p. 75) was prepared to clarify the treatment and to simplify making the proper entries on the combination abstract. The first of the two diagrams, including the circuit closures, was copied from the strip detail. Since these closures indicate that T 1 has moved and that a blunder occurred in the original leveling from U 1 to X, the second diagram was drawn to show T 1 (1935) and T 1 (1938) as two separate marks and the rejection of the original leveling from U 1 to X.

Each link is assigned an arbitrary number. In this case, the links of the original

leveling were assigned consecutive numbers in the order in which the runnings appear in the abstract of the original leveling, and the links of the releveling were assigned consecutive numbers in the order in which the runnings appear in the abstract of the releveling. In more complicated unit details it may be preferable to number the links around each successive circuit in the counterclockwise direction.

The table of links is then prepared. The links may be written in either direction: i. e., link 1 could be written S 1 minus R 1 or R 1 minus S 1. In this case, link 1 was written S 1 minus R 1 in order that the observed difference of elevation in column 4 of the table of links could be copied directly from column 6 of the abstract of the original leveling without a change of sign. The data in columns 3 and 4 of the table of links are obtained from the abstracts of the original leveling and releveling. Since link 2 is shown as a spur in the second diagram of Detail No. 1 (fig. 24), its correction is 0.0 mm. Since there is a blunder in link 4, it is rejected. Link 8 is accepted and given a correction of 0.0 mm.

The closure of the circuit from R 1 to S 1 to U 1 to E 2 to T 1 (1938) to R 1 is then distributed in the usual manner around the circuit in proportion to the distances involved, and the computed corrections are entered in column 5 of the table of links. The entries in column 6 of the table of links are the algebraic sums of the corresponding entries in columns 4 and 5 of the same table.

Since T 1 (1935) has been treated as being on the end of a spur line, no direct route can be chosen for use in abstracting through the unit which will include all of the bench marks. R 1 to T 1 (1938) is therefore abstracted independently and S 1 to T 1 (1935) is abstracted as a spur from the main route. The entries on the combination abstract are made as shown in figure 22. Columns 4 and 5 carry only a reference to the unit detail. The entries in column 6 are obtained from the adjusted-difference column (column 6) of the table of links on the detail. The new-minus-old values between R 1 and U 1 and between U 1 and X are entered in column 7.

NOTE.—Units of this type, which require no adjustment other than a mere distribution of a single closure around the circuit, can be handled by any one of three different methods, as outlined below.

(1) The method illustrated and discussed above.

(2) Space can often be found on the strip detail for sketching the second diagram and preparing the table of links. This would avoid the necessity for preparing the detail as shown in figure 24, but would be primarily the same procedure as (1). The entries on the combination abstract would be the same except that the note in columns 4 and 5 would read, "See strip detail."

(3) All runnings can be entered on the combination abstract—the runnings along the main route through the unit being entered in black and the others in red. The running of the original leveling from U 1 to X would then be rejected, and the closure of the circuit, R 1 to S 1 to U 1 to E 2 to T 1 (1938) to R 1, is distributed on the combination abstract. This procedure may be a bit difficult to follow on the combination abstract, but it avoids the preparation of the unit detail and the table of links.

Each computer will develop a personal preference for some adaptation of one of these three methods and should use the one which he finds most efficient, provided that the method he uses is clear and straightforward enough to be readily understood by other computers who may be called upon to check his work.

The check run from X to Y to Z should be abstracted in black since it involves a new elevation for Y. The entries in columns 4, 5, 6, and 7 of the combination abstract are obtained in the same manner as for any section involving three or more runnings.

After the entries in column 11 have been computed, a new-minus-old comparison is made between the combination abstract and the abstract of the original leveling and a check is made to insure that all "jumps" in the new-minus-old values are accounted for by blunders detected in the original line.

SPURS AND EXTRA FORESIGHTS

Any spur on the original leveling which was not releveled should be abstracted on the combination abstract from the base of the spur, the difference of elevation being shown in columns 4 and 6. If the spur was run more than once during the original leveling, the number of runnings should be indicated in column 4 by means of a small circled or boxed number near the left-hand side of the column. Short spurs should be abstracted in red and long spurs should be set off from the main line in the same manner as in the preliminary computations. (See p. 37.)

Any spur leading to a new mark or to new marks established or tied in for the first time by the releveling should be treated in the same way as spurs from the original line *except* that the differences of elevation are entered in columns 5 and 6.

A spur from the original leveling, which was rerun during the releveling, is abstracted in red or set off from the main line as in preliminary computations. The accepted differences of elevation are computed in the same manner as for similar units of the main line.

Extra foresights or other spurs to permanent bench marks usually result in units similar to one of the three units between L 1 and R 1 of the hypothetical line discussed in the preceding pages. The distances from the marks on the main line to marks touched upon by means of extra foresights can usually be estimated satisfactorily from the abstract of either the original leveling or the releveling, but good judgment may indicate in some cases that reference should be made to the original record books and that the distance be computed from the information contained in them.

Extra foresights to rails, highway surfaces, bridge floors, etc., may be classified and treated as outlined below.

(1) *Extra foresights to points touched upon only by the original leveling.*—These are to be shown on the combination abstract in the same manner as spurs which were not releveled. Extra foresights which were taken on rejected runnings are to be retained if the releveling checks the original leveling within ± 40.0 mm., and (except in unusual cases) rejected if the releveling indicates a blunder greater than ± 40.0 mm. in the original running. Extra foresights which are rejected are listed on the combination abstract and summary sheet in the same manner as bench mark K 1 of the hypothetical line previously discussed.

(2) *Extra foresights to points touched upon only by the releveling.*—These are to be shown on the combination abstract in the same way as spurs to new bench marks established or tied in for the first time during the releveling.

(3) *Extra foresights to points touched upon by both the original leveling and the releveling.*—Since the elevations of extra-foresight points are usually subject to frequent changes due to track and highway maintenance work and since the elevations are only published to the nearest 0.1 foot, the values from the original leveling are usually disregarded, and only the values from the releveling are shown on the combination abstract. Care should be exercised to see that no points touched upon by the original leveling are omitted from the combination abstract except such as are duplicated on the releveling.

RESET BENCH MARKS

It is the present policy of the Coast and Geodetic Survey to have bench marks reset whenever possible when the original marks must for any reason be destroyed. The work of establishing such "reset" marks is very frequently done by some railroad, highway, or construction engineer affiliated with some organization other than this Bureau. These engineers usually determine and report to the Office the difference of elevation between the original mark and the reset mark, and we publish the elevations for both. (See Special Publication No. 226, "Control Leveling," edition of 1948, p. 18.)

Should a bench mark be reset between the time of the original leveling and the time of the releveling, the reset mark would normally be included in the releveling. In such cases, we consider only the leveling done by the field parties of this Bureau in preparing the combination abstract (treating the reset bench mark as a new mark and the original bench mark as an unrecovered mark) unless the inclusion of the difference of elevation between the original mark and the reset mark serves to aid in the localization of a blunder in the original leveling. For example, if, on our hypothetical line, we substitute "K 1 RESET" for D 2, we would add the difference of elevation between K 1 and K 1 RESET to our strip detail and compute the closures for the two circuits which would be thus created from the one originally appearing on the strip detail. If these two closures indicated that the blunder was confined to either the section J 1 to K 1 or the section K 1 to L 1, we would then have a unit similar to the one between L 1 and N 1 and would treat it accordingly on the combination abstract. The difference of elevation between K 1 and K 1 RESET could be shown in either column 4 or column 5 of the combination abstract, but should be annotated to show the source of the data.

STARTING DISTANCES AND ELEVATIONS

All combinations fall roughly into one of the four groups discussed below. The choice of starting distances and elevations to be used in computing entries in columns 10 and 11 of the combination abstracts varies according to the group into which a particular combination may happen to fall.

Group 1 includes any combination which includes all of the original line. The starting distance of a combination falling in this group would be 0.00 km., and the best available elevation is used as the starting elevation, as outlined on page 42 in connection with the discussion of preliminary computations.

Group 2 includes any combination which includes the last link or links of the original line. In computing adjusted elevations it is desirable that the distances and elevations in columns 10 and 11 of the combination abstract for a combination falling in this group be on the same basis or datum as the distances and elevations on the accepted portion of the original line. The starting distance and elevation for the first mark on the combination abstract are therefore obtained from the original line.

Group 3 includes any combination which includes the first link or links of the original line. Again it is desirable that the distance and elevations on the combination abstract be on the same basis or datum as those on the accepted portion of the original line. The distance and elevation of the *last* mark in the combination are obtained from the original line, and black (unboxed) entries in columns 10 and 11 of the combination abstract are computed by *subtracting* the successive values in columns 3 and 6 from the *end* of the abstract to the *beginning* of the abstract.

Group 4 includes any combination which includes the middle link or links of the original line. The starting distance and elevation to be used will be those which will place the combination on the same basis or datum as the larger of the accepted portions

of the original line. In general, the procedure for group 2 should be followed when the accepted portion of the original line which precedes the combination is longer than the accepted portion which follows the combination. When the accepted portion of the original line which follows the combination is longer than the accepted portion which precedes the combination, the procedure outlined for group 3 is to be followed.

NOTE.—The primary purpose of the above instructions is to facilitate the publication of revised lists of preliminary elevations. If a line has been partially releveled, a new list of preliminary elevations is ordinarily published in which the new elevations from the combination are on a consistent basis with those from the original leveling. However, if adjustment of the network involving the combination can be undertaken so soon after completion of the combination that a list of adjusted elevations can be published without the necessity for a temporary list of preliminary elevations, consistency between the combination and the original leveling is not so important, and the abstracts may be prepared as in group 1, above. In all cases, the portion of the abstracts and summary sheets for the original leveling and the releveling involved in the combination should be marked "SUPERSEDED" and cross-references should be made to the title of the combination.

PREPARATION OF THE SUMMARY SHEET

Unless adjustment work is to follow promptly, the summary sheet for the original line should be superseded by a new summary sheet including the data from both the accepted portion or portions of the original line and from the combination. The headings for the new summary sheet should include the titles and accession numbers for the original and for the combination, and an explanatory note should be inserted, preferably just below the column headings on sheet 1 of the summary, specifying which portions were taken from the original line and which portions were taken from the combination. In many cases it will be possible to salvage most of the original summary sheets for the accepted portion or portions of the original line.

The data appearing on the new summary sheet should all be placed on the same basis or datum. If the suggestions under "Starting Distances and Elevations" have been followed, most of the data on the abstracts will be already on the same datum, and it will be necessary to apply constant corrections to the distances and elevations on part of the original line only when the combination includes the middle link or links of the original line.

The check elevations which appear in pencil on the combination abstract are *not* to be listed on the new summary sheet. Items on the combination abstract are to be rearranged on the summary sheet so that they will appear in order according to distance. For example: items R 1, T 1 (1938), S 1, T 1 (1935), and U 1 on the combination abstract shown in figure 22 should be listed on the new summary sheet in the order R 1, S 1, T 1 (1935), T 1 (1938), and U 1.

The orthometric corrections for the combinations are obtained by accepting the orthometric corrections for the old bench marks from the original summary sheet and interpolating according to distance to obtain the orthometric corrections for the new bench marks.

COMPLICATED PROBLEMS

Units of leveling will often be encountered in combination work which are much more complicated than any of those included in the hypothetical line discussed above. If these units present problems in analysis, they may be handled by means of a detail, as was done for the unit R 1 to X of the hypothetical line; by an analysis on the strip

detail; or by appropriate entries on the combination abstract, according to the preference of the computer. However, if any of these units present problems in adjustment which are too complicated to permit the application of the weighted-mean formulas, a detail should be prepared and the differences of elevation made entirely consistent by means of one of the methods of adjustment discussed later in this manual.

Complicated units may be encountered in which some sections have been run only once, some twice, and some three or more times. In general, those sections which have been run three or more times are given the same weight as those which were only run twice, since the problem of weighting strictly in accordance with the number of runnings becomes too involved to be worth what it costs. There may be some cases in which good judgment may dictate a strict weighting according to the number of runnings, but such cases will probably occur quite infrequently and in only the more unusual details.

COMBINATIONS INVOLVING THREE OR MORE LEVELINGS

Some lines or parts of lines may be encountered now and then which have been releveled more than once. In such cases the original leveling and each releveling should be included in the combination. The following procedure is suggested for a combination which includes the original leveling and two relevelings:

(1) Enter the differences of elevation from the original leveling and the first releveling in columns 4 and 5 as was provided for in the discussion of the hypothetical line from Smith to Brown, Tex.

(2) Enter the differences of elevation from the second releveling, following those of the original leveling and the first releveling, in either column 4 or column 5, using a distinguishing color of ink or an asterisk to refer to an explanatory note in the remarks column in which information concerning the source of the data is given.

(3) Compute the accepted difference of elevation to be entered in column 6 by taking means of all runnings which fall within rejection limits.

(4) Since, under the circumstances, the new-minus-old values lose much of their significance, column 7 of the combination abstract should be utilized only for the purpose of showing distribution rates for units involving the application of the weighted-mean formulas, etc.

(5) Entries in the other columns would be similar to the entries made on the combination abstracts involving only the original leveling and one releveling.

The procedures outlined above can be expanded to include as many levelings as may be necessary, but practical considerations, especially if a long portion of an old line has been releveled more than once, may dictate the abandonment of Form 31 and the use of the wide (20-column) data sheet bearing the number "B–797–2(1)." When these sheets are used the differences of elevation from the original leveling and from each releveling may be kept in separate columns and there will be less chance of confusion. The main draw-back to the use of these forms is the necessity for filling in all column headings as the sheets are used.

ADJUSTMENT OF LEVELING

After the preliminary computations, junction details, combinations, and any other necessary preliminaries have been disposed of, and the network of lines of leveling is ready for adjustment, the work of adjustment may be carried out in any one of a variety of ways, depending on the complexity of the network and, to some extent at least, the personal preferences of the computer who is to make the adjustment. In the matter

of the complexity of the network, this may vary from the simplest case, in which a single link is to be fitted in between two fixed elevations, one at each end of the link; up through the case of the weighted mean, in which the lines to be adjusted radiate from a single unadjusted junction bench mark to fixed elevations at the ends of the radiating lines; to the case in which a large network involving many circuit closures may all have to be adjusted at the same time. The 1929 General Adjustment of the combined first-order nets of the United States and Canada required the solution of a set of 276 simultaneous equations.

THE SIMPLEST CASE

The simplest case encountered in the adjustment of leveling is that in which a single line or link of leveling is to be fitted in between fixed elevations at each end of the line and in which there are no intermediate ties or junctions. The procedure in a case of this sort is shown by the example given below.

Assume:

(1) The fixed elevation of A 307 to be 100.3748 m.,

(2) The fixed elevation of Q 347 to be 158.4927 m.,

(3) The observed orthometric difference of elevation from A 307 to Q 347 to be +58.1523 m., and

(4) The distance from A 307 to Q 347 to be 126 km.

Then the fixed difference of elevation from A 307 to Q 347 is 158.4927−100.3748= +58.1179 m. The discrepancy (fixed difference minus observed orthometric difference) is found to be 58.1179−58.1523=−0.0344 m., or −34.4 mm. The rate of correction necessary to fit the line in between the fixed elevations is then found to be −34.4 mm.÷126 km.=−0.273016 mm./km. The mechanics of the distribution of the correction along the line of levels will be discussed under the heading "Distribution of Corrections." Also, the reason for carrying out the computation of the rate of correction to six or seven decimal places will become apparent at that time.

THE WEIGHTED MEAN

A special case of the weighted mean and its treatment by a short-cut method have already been discussed in the section on combination computations. In order to illustrate the more general case in which adjustment of leveling may be accomplished by means of the weighted mean, let us assume that lines of levels have been run from an interior junction to each of four bench marks the elevations of which have been fixed by previous adjustment. The fixed elevations, observed orthometric differences of elevation along the lines radiating from the single interior junction to the bench marks whose elevations have been fixed, and the lengths of the radiating lines of levels are assumed to be as tabulated below:

B. M.	*Fixed elevation*
X 475	200. 7416 m.
Y 480	215. 4949 m.
Z 500	186. 8910 m.
A 375	175. 7541 m.

B. M.—B. M.	*Obs. Ortho. Diff.*	*Distance*
Q 632—X 475	−10. 3556 m.	76 km.
Q 632—Y 480	−25. 1298 m.	85 km.
Q 632—Z 500	+3. 4895 m.	131 km.
Q 632—A 375	+14. 6317 m.	42 km.

The unadjusted observed orthometric elevations for Q 632 as determined by means of the unadjusted differences from each of the fixed points are computed as follows:

From	*Fixed Elev.*	*Obs. Ortho. Diff.*	*Unadj. Elev. Q 632*
X 475	200. 7416 m.	−10. 3556 m.	190. 3860 m.
Y 480	215. 4949 m.	−25. 1298 m.	190. 3651 m.
Z 500	186. 8910 m.	+3. 4895 m.	190. 3805 m.
A 375	175. 7541 m.	+14. 6317 m.	190. 3858 m.

Then, on the assumption that an elevation brought in from a long distance has less weight than one brought in from a short distance, we compute the reciprocals of the lengths of the links to use as the weights for the various unadjusted elevations for bench mark Q 632, thus:

$$1/76=0.013158$$
$$1/85=0.011765$$
$$1/131=0.007634$$
$$1/42=0.023810$$

Multiplying the various elevations for Q 632 by their respective weights and dividing by the sum of the weights, we have

$$\begin{array}{rl} 190.3860\times 0.013158= & 2.505099 \\ 190.3651\times 0.011765= & 2.239645 \\ 190.3805\times 0.007634= & 1.453365 \\ 190.3858\times \underline{0.023810}= & \underline{4.533086} \\ \quad\quad\quad\quad\quad 0.056367 & 10.731195 \end{array}$$

$$10.731195\div 0.056367=190.3808.$$

Some work may be saved by replacing the complete elevations in the above computation by the decimal portion only of each elevation, cutting down the number of significant figures in the weights and the products, and arriving at the same result, as illustrated below.

$$\begin{array}{rl} 0.3860\times 0.01316= & 0.005080 \\ 0.3651\times 0.01176= & 0.004294 \\ 0.3805\times 0.00763= & 0.002903 \\ 0.3858\times \underline{0.02381}= & \underline{0.009186} \\ \quad\quad\quad\quad\quad 0.05636 & 0.021463 \end{array}$$

$$0.021463\div 0.05636=0.3808 \text{ (check).}$$

A slight additional saving in labor may be secured by replacing the common numerator of 1 in the computation of weights by the numerical value of one of the link lengths. This small short-cut is one of a type frequently used by experienced computers and will not be discussed further.

The adjusted differences of elevation are then computed as follows:

B. M.—B. M.	*Adjusted—Fixed*	*Adj. Diff.*
Q 632—X 475	190. 3808—200. 7416	=−10. 3608
Q 632—Y 480	190. 3808—215. 4949	=−25. 1141
Q 632—Z 500	190. 3808—186. 8910	=+ 3. 4898
Q 632—A 375	190. 3808—175. 7541	=+14. 6267

The corrections to be distributed along the various lines are computed as follows:

B. M.—B. M.	*Adj. Diff.*	*Obs. Ortho. Diff.*	*Adj.—Obs.*
Q 632—X 475	−10. 3608	−10. 3556	−5. 2 mm.
Q 632—Y 480	−25. 1141	−25. 1298	+15. 7 mm.
Q 632—Z 500	+3. 4898	+3. 4895	+0. 3 mm.
Q 632—A 375	+14. 6267	+14. 6317	−5. 0 mm.

The rates of correction in mm. per km. are then computed as follows:

B. M.—B. M.	*Correction*	*Distance*	*Rate*
Q 632—X 475	−5. 2 mm.	76 km.	−0. 068421
Q 632—Y 480	+15. 7 mm.	85 km.	+0. 184706
Q 632—Z 500	+0. 3 mm.	131 km.	+0. 002290
Q 632—A 375	−5. 0 mm.	42 km.	−0. 119048
			−0. 000473

The rates along the lines radiating from a mark the elevation of which has been determined by a weighted-mean adjustment should sum to a value close to zero. If they do, as in the case above, it may be considered a partial check on preceding operations, but all steps in the computation are to be checked by a different computer.

THE METHOD OF CONDITION EQUATIONS

The adjustment of a relatively small net as shown in figure 25 will be considered next, using the method of condition equations. In this case there are five closed circuits of spirit leveling. This net is also tied to two tidal bench marks, the elevations of which are assumed to have been determined by tide observations, and to two other bench marks, the elevations of which are assumed to have been fixed by previous adjustment.

In figure 25, the junction bench marks for which fixed elevations must be held without change in the adjustment are indicated by means of circles with crosses in them. The junction bench marks which are to be assigned adjusted elevations as a result of the adjustment are shown by means of circles only. The designations of the bench marks are shown near the small circles representing the marks. The numbers assigned to the various links of leveling are shown by means of arabic numerals placed near the middle of each link. The numbers assigned to the various circuits are enclosed in circles on the sketch to avoid the possibility of their being confused with the link numbers. The circuit numbers are placed near the middle of each closed circuit of spirit leveling and in such a position with relation to the other circuits involving fixed differences as to be as free as possible from confusion. In any event, if there is any confusion concerning circuits 1, 5, 8, and 9, it is believed that it will clear up completely as this discussion is developed.

In the following discussion of methods of adjustment no attempt has been made to include the underlying mathematics of the method of least squares. For such material the reader is referred to any of several excellent textbooks. The emphasis here is on a clear, detailed, and practical presentation of the methods and processes involved in the actual adjustment of networks of lines of levels, with particular reference to present practice in the Section of Leveling of the Coast and Geodetic Survey. Furthermore, in the practical examples presented to illustrate the various methods no attempt has been made to present them as models of peak efficiency. However, suggestions for short cuts and labor-saving procedures are introduced at appropriate places.

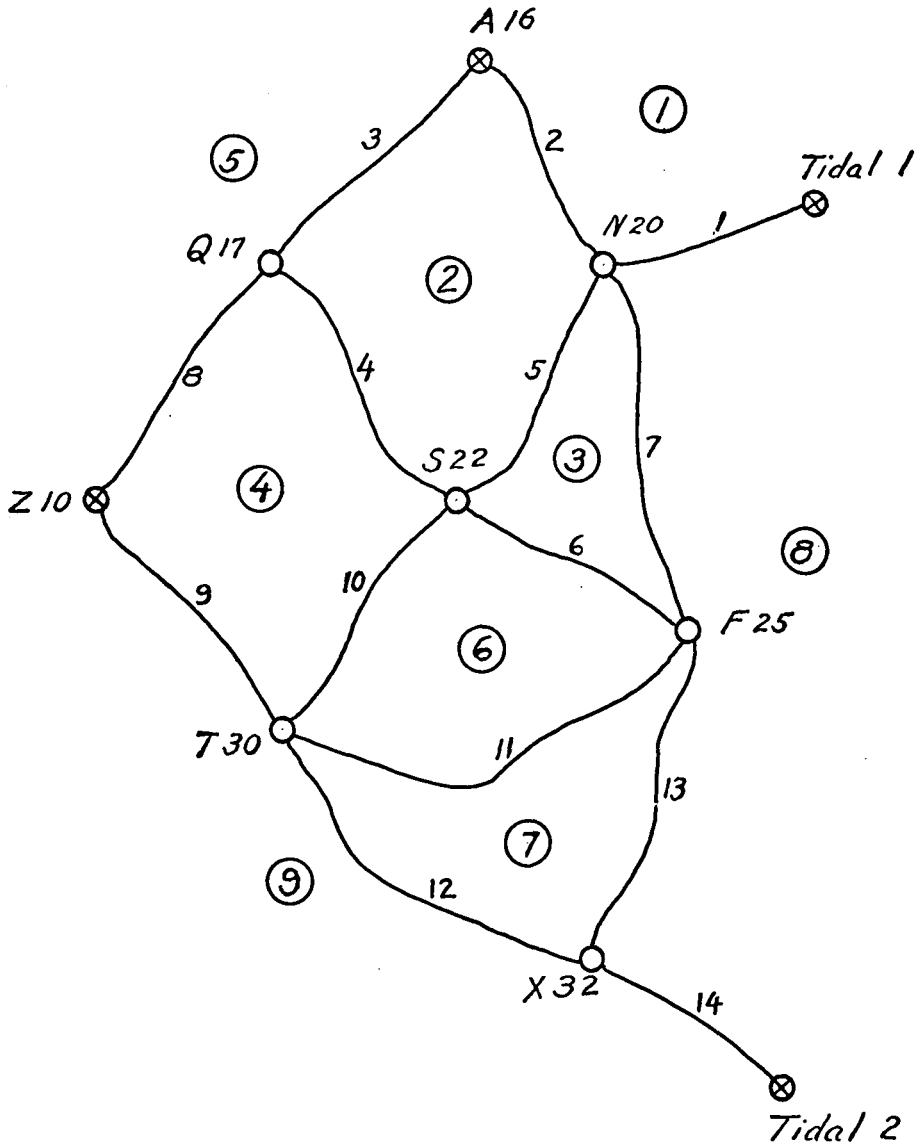


FIGURE 25.—Sketch of network of lines of levels to be adjusted by the method of condition equations and later by the method of observation equations.

The following tabulation shows the designations and elevations of the bench marks to be held at their predetermined elevations during the adjustment.

Fixed elevations

B. M.	*Elevation*
Tidal 1	1. 3752 m.
Tidal 2	2. 1654 m.
A 16	23. 7685 m.
Z 10	57. 1287 m.

The following tabulation contains in the first 5 columns the basic data for use in the adjustment, and columns 6, 7, and 8 exhibit results of the adjustment computations.

Table of links

Link	B. M.–B. M.	Dist. (km.)	Obs. Ortho. Diff. (m.)	"v"	Adj. Corr'n. (m.)	Adj. Diff. (m.)	Rate (mm./km.)
1	Tidal 1–N 20	20	−12. 3434	$+v_1$	−0. 0067	−12. 3501	−0. 335000
2	N 20–A 16	25	−10. 0410	$+v_2$	−0. 0022	−10. 0432	−0. 088000
3	A 16–Q 17	31	−15. 9121	$+v_3$	+0. 0040	−15. 9081	+0. 129032
4	Q 17–S 22	28	+3. 8128	$+v_4$	−0. 0014	+3. 8114	−0. 050000
5	S 22–N 20	37	+22. 1284	$+v_5$	+0. 0115	+22. 1399	+0. 310811
6	S 22–F 25	32	+10. 3317	$+v_6$	+0. 0007	+10. 3324	+0. 021875
7	F 25–N 20	41	+11. 8103	$+v_7$	−0. 0028	+11. 8075	−0. 068293
8	Q 17–Z 10	37	−17. 4588	$+v_8$	+0. 0067	−17. 4521	+0. 181081
9	Z 10–T 30	39	−2. 8147	$+v_9$	−0. 0029	−2. 8176	−0. 074359
10	T 30–S 22	41	+24. 0654	$+v_{10}$	+0. 0157	+24. 0811	+0. 382927
11	T 30–F 25	52	+34. 4186	$+v_{11}$	−0. 0051	+34. 4135	−0. 098077
12	T 30–X 32	48	+15. 4827	$+v_{12}$	−0. 0171	+15. 4656	−0. 356250
13	X 32–F 25	45	+18. 9476	$+v_{13}$	+0. 0003	+18. 9479	+0. 006667
14	X 32–Tidal 2	17	+42. 3215	$+v_{14}$	−0. 0062	+42. 3153	−0. 364706
		493					

In the above table, column 1 shows the number assigned to each link in figure 25. The links are arranged in numerical order for convenience later in using them in the computation of the circuit closures. Column 2 shows the designations of the bench marks at the termini of each link. The first in each case is the one the elevation of which takes the plus sign and the second is the one the elevation of which takes the minus sign in computing the numerical value of the observed orthometric difference shown in column 4. Column 3 gives the length of each link, and these lengths, except in very unusual cases, are rounded to the nearest kilometer. Column 5 shows the sign and number of each "v" or symbol representing the correction to be applied to the corresponding observed orthometric difference of elevation in column 4.

The entries in columns 6, 7, and 8 are filled in after the adjustment computations have been completed through the stage of the computation of the "v's" or corrections.

The next step is the computation of the circuit closures, and the following tabulation exhibits this computation for the net shown in figure 25.

Computation of circuit closures

Circuit	Link	B. M.—B. M.	Dist. (km.)	Obs. Ortho. Diff. (m.)	v	Adj. Corr'n. (m.)	Adj. Diff. (m.)
1		Tidal 1—A 16	----------	−22.3933	----------	----------	−22.3933
	2	A 16—N 20	25	+10.0410	$-v_2$	+0.0022	+10.0432
	1	N 20—Tidal 1	20	+12.3434	$-v_1$	+0.0067	+12.3501
			45	−0.0089		+0.0089	0.0000
2	2	N 20—A 16	25	−10.0410	$+v_2$	−0.0022	−10.0432
	3	A 16—Q 17	31	−15.9121	$+v_3$	+0.0040	−15.9081
	4	Q 17—S 22	28	+3.8128	$+v_4$	−0.0014	+3.8114
	5	S 22—N 20	37	+22.1284	$+v_5$	+0.0115	+22.1399
			121	−0.0119		+0.0119	0.0000
3	5	N 20—S 22	37	−22.1284	$-v_5$	−0.0115	−22.1399
	6	S 22—F 25	32	+10.3317	$+v_6$	+0.0007	+10.3324
	7	F 25—N 20	41	+11.8103	$+v_7$	−0.0028	+11.8075
			110	+0.0136		−0.0136	0.0000
4	4	S 22—Q 17	28	−3.8128	$-v_4$	+0.0014	−3.8114
	8	Q 17—Z 10	37	−17.4588	$+v_8$	+0.0067	−17.4521
	9	Z 10—T 30	39	−2.8147	$+v_9$	−0.0029	−2.8176
	10	T 30—S 22	41	+24.0654	$+v_{10}$	+0.0157	+24.0811
			145	−0.0209		+0.0209	0.0000
5		A 16—Z 10	----------	−33.3602	----------	----------	−33.3602
	8	Z 10—Q 17	37	+17.4588	$-v_8$	−0.0067	+17.4521
	3	Q 17—A 16	31	+15.9121	$-v_3$	−0.0040	+15.9081
			68	+ 0.0107		−0.0107	0.0000
6	6	F 25—S 22	32	−10.3317	$-v_6$	−0.0007	−10.3324
	10	S 22—T 30	41	−24.0654	$-v_{10}$	−0.0157	−24.0811
	11	T 30—F 25	52	+34.4186	$+v_{11}$	−0.0051	+34.4135
			125	+0.0215		−0.0215	0.0000
7	11	F 25—T 30	52	−34.4186	$-v_{11}$	+0.0051	−34.4135
	12	T 30—X 32	48	+15.4827	$+v_{12}$	−0.0171	+15.4656
	13	X 32—F 25	45	+18.9476	$+v_{13}$	+0.0003	+18.9479
			145	+0.0117		−0.0117	0.0000
8	1	Tidal 1—N 20	20	−12.3434	$+v_1$	−0.0067	−12.3501
	7	N 20—F 25	41	−11.8103	$-v_7$	+0.0028	−11.8075
	13	F 25—X 32	45	−18.9476	$-v_{13}$	−0.0003	−18.9479
	14	X 32—Tidal 2	17	+42.3215	$+v_{14}$	−0.0062	+42.3153
		Tidal 2—Tidal 1	----------	+0.7902	----------	----------	+0.7902
			123	+0.0104		−0.0104	0.0000
9	14	Tidal 2—X 32	17	−42.3215	$-v_{14}$		
	12	X 32—T 30	48	−15.4827	$-v_{12}$		
	9	T 30—Z 10	39	+2.8147	$-v_9$		
		Z 10—Tidal 2	----------	+54.9633	----------		
			104	−0.0262			

It should be noted at this point that whenever a fixed difference of elevation enters into the computation of a circuit closure there is no link number in column 2, no distance in column 4, no "v" in column 6, and no adjustment correction in column 7, and that the entries in columns 5 and 8 are identical.

It should also be noted that while circuit 9 is computed for check purposes it is not used in the adjustment since it is the negative of the sum of the other eight circuits, and

its inclusion in the solution would create an identity, cause possible errors, and, at the very least, require an arbitrary evaluation of the mathematically indeterminate expression 0/0.

In this particular case, circuit 9 shows a closure of −0.0262 m., which is exactly equal to the negative of the sum of the other eight circuit closures,

$$-8.9\ -11.9\ +13.6\ -20.9\ +10.7\ +21.5\ +11.7\ +10.4=+26.2 \text{ mm.},$$

and, except for the possibility of compensating errors, this shows that the eight circuit closures to be used in the adjustment are correctly computed. However, it is well to have both the table of links and the computation of circuit closures thoroughly checked before proceeding with the next step in the adjustment computations.

The following comments on the computation of the various circuit closures may serve to clarify the procedure. Referring to figure 25 and the above tabulation of the circuit-closure computations, we find that in circuit 1 we begin at bench mark Tidal 1 and, proceeding counterclockwise around the circuit using the B. M.—B. M. method, we take from the table of fixed elevations the elevation (1.3752) of bench mark Tidal 1 minus the elevation (23.7685) of bench mark A 16 to obtain the fixed difference of elevation (−22.3933) for the fixed link of the circuit. Passing to the next link we need the difference of elevation for bench mark A 16 minus N 20. We refer to the sketch, find this to be link 2, refer to the table of links, and find we have link 2 listed there as N 20—A 16 and with a $+v_2$ in column 5. We, therefore, change the sign of both the difference of elevation and the correction symbol and enter +10.0410 in column 5 and $-v_2$ in column 6. The closing link in circuit 1 is N 20—Tidal 1, and again we have to change the signs of both the difference of elevation and the correction symbol.

In other words, when the B. M.—B. M. column in the computation of circuit closures shows the designations of the bench marks in the order opposite to that in which they appear in the table of links, the algebraic signs of both the difference of elevation and the correction symbol must be changed. Otherwise the signs are as shown in the table of links.

The lengths of the circuits are the sums of the lengths of the links that are to take corrections. These sums are not used directly in the adjustment computations but are computed so that they may be compared with the corresponding circuit closures to see if any unusually large closures are in need of further investigation before proceeding with the actual adjustment.

If we divide the circuit closure (in mm.) by the perimeter of the adjustable portion of the circuit (in km.) we have the rate of correction around that circuit on the assumption that the closure is distributed uniformly around the adjustable portion of the circuit. This is at least an approximation to what may be expected in the way of adjustment corrections and will serve to show up any suspiciously large rate and permit further investigation. Performing this computation for each of the nine circuits tabulated above we have:

Circuit	*Average rate (mm./km.)*	*Circuit*	*Average rate (mm./km.)*
1	−0.20	6	+0.17
2	−0.10	7	+0.08
3	+0.12	8	+0.08
4	−0.14	9	−0.25
5	+0.16		

It will be noticed that circuit 9 is the only one which shows an average rate of correction of as much as 0.25 mm./km., and, since experience indicates that in first-order leveling any circuit which shows an average rate of 0.5 mm./km. or less will probably not require excessive corrections, we conclude that this particular net is ready for adjustment.

The next step is the preparation of the table of correlates. In this table there will be as many horizontal lines of entries as there are v's to be computed, in this case 14, and 3 more columns than the number of circuits to be used in the adjustment.

As shown in the following table the first column is headed "No. of v." The second is headed "$1/p$." The next eight are numbered from "1" to "8," inclusive, for the eight circuits in the adjustment, and the last is headed "Σ." In column 1 the 14 "v" numbers are entered in their proper numerical order.

Table of correlates

No. of v	$1/p$	1	2	3	4	5	6	7	8	Σ
1	0.80	−1							+1	0
2	1.00	−1	+1							0
3	1.24		+1			−1				0
4	1.12		+1		−1					0
5	1.48		+1	−1						0
6	1.28			+1			−1			0
7	1.64			+1					−1	0
8	1.48				+1	−1				0
9	1.56				+1					+1
10	1.64				+1		−1			0
11	2.08						+1	−1		0
12	1.92							+1		+1
13	1.80							+1	−1	0
14	0.68								+1	+1
	19.72									
	493/25=19.72 (check)									

Taking up the completion of the table, the entries in column 2 are the reciprocals of the weights assigned to the links. However, since, as we have shown before, the weight is the reciprocal of the length, the entries in column 2 of the table of correlates might be the lengths of the various links. However, to use the lengths of the links (in km.) for the reciprocals of the weights of the links would, as will be seen presently, result in inconveniently large numbers. Accordingly the reciprocal of the weight is computed by dividing each link length by some constant which will produce values of $1/p$ which will group as closely as may be reasonably convenient around unity. In this adjustment, each link length was divided by 25 (multiplied by 0.04) to secure the individual entries in column 2 of the table of correlates.

It will be noticed that in the table of links the total of column 3, the sum of the link lengths, has been shown and is 493 km. This is used in obtaining a check on the entries in column 2 of the table of correlates. The sum of the entries in column 2 is 19.72 which is equal to the sum of the link lengths (493) divided by 25 (or multiplied by 0.04) and is, except for the possibility of compensating errors, a check on the computed values of $1/p$.

The entries in the eight numbered columns of the table of correlates are simply a tabulation of the v's of the eight circuits. The numerical coefficient of each v is unity,

and, since only the coefficients of the v's are entered in the table of correlates, each entry in the eight numbered columns must be either $+1$ or -1.

Taking up circuit 1 we find in column 6 of the tabulation of the circuit-closure computations that we have entered there $-v_2$ and $-v_1$. Therefore, in the column headed "1" of the table of correlates we enter "-1" on line one and "-1" on line two.

The second circuit shows $+v_2$, $+v_3$, $+v_4$, and $+v_5$ in the tabulation of the circuit-closure computations, and we enter in the column headed "2" of the table of correlates a "$+1$" on line two, a "$+1$" on line three, a "$+1$" on line four, and a "$+1$" on line five.

This same procedure is followed until the coefficients of all of the v's in all of the eight circuit closures have been entered in the proper places in the table of correlates.

Each entry in the Σ column is the sum of the entries on the same horizontal line in the eight numbered columns. For example, the first entry in the Σ column is the sum of the -1 in the column headed "1" and the $+1$ in the column headed "8," and is zero.

It will be noticed that all of the entries in the Σ column are zero except on links 9, 12, and 14 and that these are the links of circuit 9 which is not used in the adjustment computation. Except in cases where overlapping circuits occur, this peculiarity of the Σ-column entries will be the regular thing and will serve as a partial check on the table of correlates. However, in any extensive adjustment the table of correlates should be checked thoroughly before proceeding with the next step, the formation of the normal equations.

The formation of the table of normal equations is a routine process which, while difficult to set forth in print, is found to be a simple one once it is clearly understood. Probably the best way to exhibit the details of the process will be to actually follow through the routine on enough of the practical example we are now considering to cover all of the necessary features.

Normal equations

1	2	3	4	5	6	7	8	N	Σ
+1.80	−1.00						−0.80	−0.89	−0.89
	+4.84	−1.48	−1.12	−1.24				−1.19	−1.19
		+4.40			−1.28		−1.64	+1.36	+1.36
			+5.80	−1.48	−1.64			−2.09	−0.53
				+2.72				+1.07	+1.07
					+5.00	−2.08		+2.15	+2.15
						+5.80	−1.80	+1.17	+3.09
							+4.92	+1.04	+1.72

In the column headed "1" in the table of correlates we find we have two entries, -1 on line 1 and -1 on line 2. Multiplying each of these entries by itself, we have $+1$ and $+1$, respectively. Then multiplying each of these by the corresponding entries in the $1/p$ column we have $+0.80$ and $+1.00$, respectively. The sum of these two values is the entry for line 1 in column 1 of the table of normal equations. In mathematical form this is

$$(-1\times-1\times0.80)+(-1\times-1\times1.00)=+1.80.$$

The entry for line 1 of column 2 of the table of normal equations is simply the product of the -1 on line 2 of column 1 of the table of correlates, multiplied by the $+1$ of line 2 of column 2 of the same table and then multiplied by the 1.00 of line 2 of the $1/p$ column, thus:

$$(-1\times+1\times1.00)=-1.00,$$

which is entered on line 1 of column 2 of the table of normal equations.

Now it will be seen that in columns 3 to 7, inclusive, of the table of correlates there are no entries on lines 1 and 2, so no entries will appear on line 1 of the table of normal equations in columns 3 to 7, inclusive.

The entry for line 1 of column 8 of the table of normal equations is

$$(-1\times+1\times0.80)=-0.80.$$

The entry on line 1 of the N column of the table of normal equations is merely the closure of circuit 1 and is copied directly from the tabulation of the circuit-closure computations. *For convenience, this entry is pointed off to represent the closure in centimeters.*

Multiplying entries in column 1 of the table of correlates by the corresponding entries in column Σ of the same table, and multiplying these products by the corresponding entries in the $1/p$ column, summing these products, and then correcting this total by the closure of circuit 1, expressed in centimeters, we have

$$\begin{aligned} -1\times0\times0.80&=0.00\\ -1\times0\times1.00&=\underline{0.00}\\ &\quad\;\; 0.00-0.89=-0.89, \end{aligned}$$

which is the entry for line 1 of the Σ column of the table of normal equations.

This completes the formation of the first of the normal equations and its formation may be checked by adding across line 1 of the table of normal equations to and including the N column and comparing the resulting total with the entry on line 1 of the Σ column, thus:

$$+1.80-1.00-0.80-0.89=-0.89 \text{ (check)}.$$

In forming the second normal equation we would begin by multiplying the entries in column 2 by those in column 1 and by those in column $1/p$, but this has already been done and the result entered on line 1 of column 2 of the table of normal equations.

We then proceed to multiply the entries in column 2 by themselves and by the corresponding entries in the $1/p$ column, thus:

$$\begin{aligned} +1\times+1\times+1.00&=+1.00\\ +1\times+1\times+1.24&=+1.24\\ +1\times+1\times+1.12&=+1.12\\ +1\times+1\times+1.48&=\underline{+1.48,}\\ &\quad\;\; +4.84, \end{aligned}$$

which is entered on line 2 of column 2 of the table of normal equations.

Treating columns 2 and 3 of the table of correlates we have

$$+1\times-1\times+1.48=-1.48,$$

which is the entry for line 2 of column 3 of the table of normal equations.

Next we operate on columns 2 and 4 of the table of correlates and have

$$+1\times-1\times+1.12=-1.12,$$

and this result is entered on line 2 of column 4 of the table of normal equations.

For columns 2 and 5 of the table of correlates we have

$$+1\times-1\times+1.24=-1.24,$$

and this result is entered on line 2 of column 5 of the table of normal equations.

Since no entries appear in the table of correlates in columns 6, 7, and 8 on lines 2 to 5, inclusive, no entries will appear on line 2 in columns 6, 7, and 8 of the table of normal equations.

The entry on line 2 of column N of the table of normal equations is the circuit closure of circuit 2, pointed off to show the closure in centimeters.

Operating on columns 2 and Σ of the table of correlates, we have

$$\begin{array}{l} +1\times 0\times +1.00=0.00 \\ +1\times 0\times +1.24=0.00 \\ +1\times 0\times +1.12=0.00 \\ +1\times 0\times +1.48=0.00 \\ \hline \qquad\qquad\qquad 0.00-1.19=-1.19, \end{array}$$

which is entered on line 2 of the Σ column of the table of normal equations.

The check on the formation of the second normal equation is obtained by adding down column 2 to and including line 2 and then along line 2 to the right, to and including the N column, thus:

$$-1.00+4.84-1.48-1.12-1.24-1.19=-1.19 \text{ (check)}.$$

This same routine is continued until the end of the table of normal equations is reached, but, in order to cover one of the cases in which the entries on the same line in the N and Σ columns are not identical, we will show below the actual computation of the seventh line of the normal equations.

$$\begin{array}{l} -1\times -1\times +2.08=+2.08 \\ +1\times +1\times +1.92=+1.92 \\ +1\times +1\times +1.80=+1.80 \\ \hline \qquad\qquad\qquad +5.80 \\ +1\times -1\times +1.80=-1.80 \\ \hline \qquad\qquad\qquad -1.80 \\ -1\times \quad 0\times +2.08= \quad 0.00 \\ +1\times +1\times +1.92=+1.92 \\ +1\times \quad 0\times +1.80= \quad 0.00 \\ \hline \qquad\qquad\qquad +1.92+1.17=+3.09 \end{array}$$

The +5.80 is entered on line 7 of column 7 of the table of normal equations; the −1.80 goes on line 7 of column 8; the entry for line 7, column N comes from the tabulation of the circuit-closure computations; and the +3.09 goes on line 7 of the Σ column.

The check on equation 7 is as follows:

$$-2.08+5.80-1.80+1.17=+3.09 \text{ (check)}.$$

In general, where i and k stand for any pair of integers, the entry in line i of column k of the normal equations is the sum of the products of the three elements of each line (one from the $1/p$ column, one from the i column, and one from the k column) of the table of correlates, considering all blanks as zero values. Since in an adjustment of a leveling network the coefficients of the v's are all either +1 or −1, it may be noticed that this routine reduces in practice to the algebraic summation of those $1/p$ values on lines having entries in both of the pair of numbered columns considered, with the sign of each $1/p$ value being indicated by the product of the entries in that pair of numbered columns. The formation of the normal equations may be carried out with no equipment other than an adding-and-subtracting machine, and many of the entries can easily be determined mentally.

We now have, in the case under consideration, a set of eight simultaneous linear equations in tabular form, and these may be solved in a mechanical fashion by what is known as the Doolittle Method.

Referring now to the tabulation showing the forward solution of this set of equations, we find that we have 8 columns numbered from 1 to 8, inclusive, for the 8 equations, an N column, and a Σ column. The forward solution is begun by simply copying the first of the normal equations on the first line of the forward solution.

Forward solution of normal equations

1	2	3	4	5	6	7	8	N	Σ
+1.8000	−1.0000						−0.8000	−0.8900	−0.8900
*C*1	+0.55556						+0.44444	+0.49444	+0.49444
	+4.8400	−1.4800	−1.1200	−1.2400				−1.1900	−1.1900
	−0.5556						−0.4444	−0.4944	−0.4944
	+4.2844	−1.4800	−1.1200	−1.2400			−0.4444	−1.6844	−1.6844
	*C*2	+0.34544	+0.26141	+0.28942			+0.10373	+0.39315	+0.39315
		+4.4000			−1.2800		−1.6400	+1.3600	+1.3600
		−0.5113	−0.3869	−0.4283			−0.1535	−0.5819	−0.5819
		+3.8887	−0.3869	−0.4283	−1.2800		−1.7935	+0.7781	+0.7781
		*C*3	+0.09949	+0.11014	+0.32916		+0.46121	−0.20009	−0.20009
			+5.8000	−1.4800	−1.6400			−2.0900	−0.5300
			−0.2928	−0.3241			−0.1162	−0.4403	−0.4403
			−0.0385	−0.0426	−0.1273		−0.1784	+0.0774	+0.0774
			+5.4687	−1.8467	−1.7673		−0.2946	−2.4529	−0.8929+1
			*C*4	+0.33769	+0.32317		+0.05387	+0.44853	+0.16327−1
				+2.7200				+1.0700	+1.0700
				−0.3589			−0.1286	−0.4875	−0.4875
				−0.0472	−0.1410		−0.1975	+0.0857	+0.0857
				−0.6236	−0.5968		−0.0995	−0.8283	−0.3015
				+1.6903	−0.7378		−0.4256	−0.1601	+0.3667+1
				*C*5	+0.43649		+0.25179	+0.09472	−0.21694−6
					+5.0000	−2.0800		+2.1500	+2.1500
					−0.4213		−0.5903	+0.2561	+0.2561
					−0.5711		−0.0952	−0.7927	−0.2886
					−0.3220		−0.1858	−0.0699	+0.1601
					+3.6856	−2.0800	−0.8713	+1.5435	+2.2776+2
					*C*6	+0.56436	+0.23641	−0.41879	−0.61797−5
						+5.8000	−1.8000	+1.1700	+3.0900
						−1.1739	−0.4917	+0.8711	+1.2854
						+4.6261	−2.2917	+2.0411	+4.3754+1
						*C*7	+0.49538	−0.44121	−0.94581−2
							+4.9200	+1.0400	+1.7200
							−0.3556	−0.3956	−0.3956
							−0.0461	−0.1747	−0.1747
							−0.8272	+0.3589	+0.3589
							−0.0159	−0.1321	−0.0481
							−0.1072	−0.0403	+0.0923
							−0.2060	+0.3649	+0.5384
							−1.1353	+1.0111	+2.1675
							+2.2267	+2.0322	+4.2587+2
							*C*8	−0.91265	−1.91256−9

This equation is then divided throughout by the first term with its sign changed. This makes the entry $C1$ equivalent to -1.00000, and all successive entries, $C2$, $C3$, ... $C8$, are to be understood to amount numerically to the same value, -1.00000.

The check at this point is obtained by adding the -1.00000 represented by the symbol $C1$ and all the entries to the right along the $C1$ line to and including the entry in the N column and comparing with the entry in the Σ column, thus:

$$-1.00000+0.55556+0.44444+0.49444=+0.49444 \text{ (check).}$$

Next, equation 2 from the table of normal equations is copied in as shown in the tabulation. The next line is obtained by multiplying the entry on the $C1$ line in column 2 by the entry next above it and by all of the entries to the right along the same line, thus:

$$\begin{aligned}
+0.55556\times -1.0000&=-0.5556\\
+0.55556\times -0.8000&=-0.4444\\
+0.55556\times -0.8900&=-0.4944\\
+0.55556\times -0.8900&=-0.4944.
\end{aligned}$$

These products are entered in the appropriate columns of the forward solution on the line next below the one on which line 2 of the normal equations was copied.

The entries copied from line 2 of the normal equations and the products computed in the preceding paragraph are added algebraically, column by column, as follows:

$$\begin{aligned}
+4.8400-0.5556&=+4.2844\\
-1.4800\qquad\quad&=-1.4800\\
-1.1200\qquad\quad&=-1.1200\\
-1.2400\qquad\quad&=-1.2400\\
-0.4444&=-0.4444\\
-1.1900-0.4944&=-1.6844\\
-1.1900-0.4944&=-1.6844.
\end{aligned}$$

These totals, when entered in the appropriate columns and on the next line above the $C2$ line, appear as shown in the tabulation of the forward solution.

The check at this point is as follows:

$$+4.2844-1.4800-1.1200-1.2400-0.4444-1.6844=-1.6844 \text{ (check).}$$

The $C2$ line is obtained by dividing all entries on the line next above by the first one with its sign changed, and the check at this stage becomes:

$$-1.00000+0.34544+0.26141+0.28942+0.10373+0.39315=+0.39315 \text{ (check).}$$

Next, we copy line 3 of the normal equations. The next line is obtained by multiplying the entry on the $C2$ line of column 3 by the entry next above it and by each one to the right of that. Totals are again taken, and the line of totals is divided through by the first one with its sign changed. The checks on the lines of totals and on the C lines are always obtained, as previously outlined, as soon as these lines are completed.

When we come to equation 4 we find that the process is slightly more complicated. Line 4 of the normal equations is copied; the next line is obtained by multiplying the entry on line $C2$ of column 4 by the entry next above it and by each one to the right of that; the next line is obtained by multiplying the entry on the $C3$ line of column 4 by

the entry next above it and by each one to the right of that; totals are taken and checked; the line of totals is divided by minus the first one; and the $C4$ line is checked by summing across to and including the entry in the N column and comparing with the entry on the same line in the Σ column.

These checks will not always be exact, and either of two methods of handling the small differences may be followed:

(1) The method followed in this solution was to indicate the check failure in terms of one in the fourth or fifth place as necessary and as shown in the tabulation of the forward solution. Consider the line just above the $C5$ line in the forward solution. The sum of the line of totals, to and including the N column, is

$$+1.6903-0.7378-0.4256-0.1601=+0.3668,$$

while the entry on the same line in the Σ column is $+0.3667$. The $+1$ entry shows that the Σ-column entry must be increased by 1 in the fourth place to produce an exact check. The check on the $C5$ line is

$$-1.00000+0.43469+0.25179+0.09472=-0.21700,$$

while the entry on this line in the Σ column is -0.21694, and the -6 entry shows that the entry in the Σ column must be decreased (algebraically) by 6 in the fifth place to produce an exact check. This appears to be legitimate since a discrepancy of one in the fourth place might easily amount to six in the fifth place because we divided the line of totals in this case by a number considerably smaller than two.

In continuing the solution as tabulated above, the uncorrected values were used in the succeeding steps in all cases, and the gradual increase in the size of the little corrections shown represents a sort of cumulative total of the looseness of the solution at each successive step.

(2) The other method is to either correct the entries in the Σ column to agreement with the line totals or to indicate the necessary correction and then use the corrected values when needed in the subsequent steps. This procedure keeps the corrections to a smaller size as the solution progresses, especially in a long solution, but it loses the advantage of showing the accumulated looseness throughout the solution.

The solution is continued, step by step as outlined above, until the last line is finished and checked. We are then ready to consider the backward solution. However, it is well to consider at this point an alternate method of making the forward solution. The following tabulation of this same forward solution was made by the same methods used for the solution previously discussed, except that, instead of computing and entering each one of the products of a group separately, the products were allowed to accumulate in the machine, the top entry in each group included by addition, and the entries for the lines of totals were each taken directly from the machine for entry on the forward-solution sheets. This system requires much less writing. The novice would do well to go through one or two solutions by setting down each product separately before attempting the short-cut method of solution.

Another advantage of this method of making the forward solution is that, since we can predict exactly the physical form of the tabulation of the forward solution, the normal equations may be entered directly on the sheets of the forward solution as they are formed and thus do away with the necessity for making up the separate table of normal equations shown on page 95.

Forward solution—alternate method

1	2	3	4	5	6	7	8	N	Σ
+1.8000 *C*1	−1.0000 +0.55556						−0.8000 +0.44444	−0.8900 +0.49444	−0.8900 +0.49444
	+4.8400	−1.4800	−1.1200	−1.2400				−1.1900	−1.1900
	+4.2844 *C*2	−1.4800 +0.34544	−1.1200 +0.26141	−1.2400 +0.28942			−0.4444 +0.10373	−1.6844 +0.39315	−1.6844 +0.39315
		+4.4000			−1.2800		−1.6400	+1.3600	+1.3600
		+3.8887 *C*3	−0.3869 +0.09949	−0.4283 +0.11014	−1.2800 +0.32916		−1.7935 +0.46121	+0.7781 −0.20009	+0.7781 −0.20009
			+5.8000	−1.4800	−1.6400			−2.0900	−0.5300
			+5.4687 *C*4	−1.8468 +0.33770	−1.7673 +0.32317		−0.2946 +0.05387	−2.4529 +0.44853	−0.8929 +0.16327
				+2.7200				+1.0700	+1.0700
				+1.6903 *C*5	−0.7378 +0.43649		−0.4256 +0.25179	−0.1601 +0.09472	+0.3667+1 −0.21694−6
					+5.0000	−2.0800		+2.1500	+2.1500
					+3.6855 *C*6	−2.0800 +0.56437	−0.8713 +0.23641	+1.5435 −0.41880	+2.2776+1 −0.61799−3
						+5.8000	−1.8000	+1.1700	+3.0900
						+4.6261 *C*7	−2.2917 +0.49538	+2.0411 −0.44121	+4.3754+1 −0.94581−2
							+4.9200	+1.0400	+1.7200
							+2.2269 *C*8	+2.0322 −0.91257	+4.2588+3 −1.91243−14

To illustrate the above-outlined routine, let us consider the computation of the entry "+3.6856" just above the symbol *C*6 in the first of the two forward solutions shown above. In this instance we have:

$$\begin{aligned} &\qquad\qquad\qquad\qquad +5.0000 \\ +0.32916\times-1.2800 &= -0.4213 \\ +0.32317\times-1.7673 &= -0.5711 \\ +0.43649\times-0.7378 &= -0.3220 \\ \hline &\qquad\qquad\qquad\quad +3.6856. \end{aligned}$$

In the second case we use the same figures but allow the products to accumulate positively or negatively in the machine and read off the final value at the end of the computation. In the following paragraph an attempt is made to show just what happens in this case.

We are multiplying 4-place numbers by 5-place numbers, so the products in the machine will run to 9 places of decimals. We start the recomputation of the above value by putting +5.0000 in the product dials in proper relation to the decimal-point indicator, so that these dials will read "5.000000000." We then set for negative multiplication and multiply +0.32916 by −1.2800 as before. At the end of this operation the product dials will register "4.578675200" but are not cleared. Leaving the machine set for negative multiplication, we multiply +0.32317 by −1.7673, and the

product dials will then read "4.007536859" but are not cleared. Then the multiplication of +0.43649 by −0.7378 is performed and the product dials then read "3.685494537" which is rounded to +3.6855 and entered in the spot next above the symbol *C*6 in the second or alternate version of the forward solution of this net. Then the machine is completely cleared, and the next series of accumulated products is started.

Often in making the forward solution of a set of normal equations some labor may be saved by rearranging the equations in other than a strict numerical order when setting up the table for the forward solution. Also the order in which the various condition equations are computed and entered in the table of correlates may often have a very considerable effect on the amount of computing necessary in the later stages of the adjustment. For example, in the adjustment of the net shown in figure 25 by the method of condition equations, had these equations been taken up in the order 9, 7, 6, 3, 4, 5, 2, 1, and circuit 8 omitted from the solution, the number of items to be written down in the first of the two forward solutions would have been reduced by about 25 percent and the number of multiplications to be performed would have been about one-third less.

On the other hand, it will often happen that more time will be wasted by an inexperienced computer in these attempts at labor-saving short cuts than can be saved by their use. This is especially true in short solutions involving a relatively small number of equations. In a large solution including 50, 100, or even several hundred equations, it is well to consider carefully the order in which the equations are introduced into the solution, since even a 10 percent saving in labor on a long forward solution is a real saving.

Two rules which it is well to follow are:

(1) In numbering circuits on the sketch make sure that at least one new link comes into the solution with each new circuit considered and, as much as possible, avoid skipping about on the sketch, and

(2) Use as the omitted circuit the one of the outside circuits having the largest number of links in it.

The backward solution or the computation of the various factors to be used in the computation of the *v*'s is the next step in the series of computations necessary in the adjustment of a network of lines of levels by this method.

Since it is a backward solution it seems logical to let it run backwards on the computation sheets. The backward solution eventually forms a triangular tabulation quite similar in general appearance to the table of normal equations. We, therefore, start the tabulation of the backward solution by numbering, in this numerical example, eight columns, thus: 8, 7, 6, . . . 1.

Backward solution of normal equations

8	7	6	5	4	3	2	1
−0.91265	−0.44121	−0.41879	+0.09472	+0.44853	−0.20009	+0.39315	+0.49444
−0.91265	−0.45211	−0.21576	−0.22980	−0.04916	−0.42092	−0.09467	−0.40562
	−0.89332	−0.50415					
		−1.13870	−0.49703	−0.36799	−0.37481		
			−0.63211	−0.21346	−0.06962	−0.18295	
				−0.18208	−0.01812	−0.04760	
					−1.08356	−0.37430	
						−0.30637	−0.17021
							−0.08139

Referring to the first of the two forward solutions shown above, we enter the value "-0.91265," from the $C8$ line of the N column of the forward solution, as the first entry in column 8 of the backward solution. The entry "-0.44121" from the $C7$ line of the N column of the forward solution then becomes the first entry in column 7 of the backward solution. We proceed onward across the first line of the backward solution by entering in succession the entries from the N column and C lines of the forward solution, in order, passing up the N column of the forward solution and to the right across the first line of the backward solution.

Next a total of the entries in column 8 of the backward solution is struck, which, of course, in the case of column 8 in this example, means merely copying the only entry in the column.

The total of column 8 of the backward solution is then used as a constant multiplier for all of the entries on the C lines in column 8 of the forward solution, and the individual products are entered to the right along line 2 of the backward solution in the columns corresponding to the numbers of the C lines from which the various factors in the series of products were obtained.

A total of column 7 of the backward solution is then struck and used as the constant multiplier for the C-line entries up through column 7 of the forward solution. The total for column 6 of the backward solution then becomes the constant multiplier for the C-line entries up through column 6 of the forward solution, and so on until the solution is completed by taking the total of column 1 of the backward solution.

It should be mentioned at this point, however, that it is not necessary to run the backward solution of a set of equations in the manner shown on page 102. The same result may be achieved by setting up the table with the columns numbered in the 1, 2, 3, ... order from left to right and progressing from right to left when filling in the entries in the table. Furthermore, if one wishes to do so, the backward solution may be carried out by letting the individual products accumulate in the machine in much the same way as was done in making the second of the forward solutions for the adjustment of the net shown in figure 25 by the method of condition equations. The novice will, however, do well to avoid some of these short cuts until he has become somewhat familiar with the basic methods.

We next provide for the computation of the v's by constructing a table quite similar in appearance to that of the table of correlates. The 14 lines of the table are numbered from 1 to 14, inclusive. The first column, the one in which the 14 v numbers are listed, is headed "No. of v." The next 8 columns are numbered 1, 2, 3, ... 8 to provide for the 8 equations used in this adjustment. The next four columns are headed, in order from left to right, "Σ," "$1/p$," "$\Sigma \times (1/p)$," and "Adopted v." Sometimes, for convenience, especially in a large adjustment, an extra column is used at the right side of the table and headed "No. of v" and the v numbers are repeated so as to be close to the adopted v's when filling in the last three columns of the tabulation of the computation of the circuit closures (p. 92).

This table is then filled in and completed as follows: The total of column 1 at the right of the backward solution is used as a constant multiplier for the entries in column 1 of the table of correlates, and the products are entered in the corresponding spots in column 1 of the computation of v's. The total for column 2 of the backward solution is then used as a constant multiplier for all entries in column 2 of the table of correlates, and these products are arranged in column 2 of the computation of v's. This operation is continued until all 8 numbered columns of the computation of v's have been filled in. At this point the 8 numbered columns of the computation of v's should show the same

Computation of v's

No. of v	1	2	3	4	5	6	7	8	Σ	$1/p$	$\Sigma\times(1/p)$	Adopted v (mm.)	No. of v
1	+0.0814							−0.9126	−0.8312	0.80	−0.6650	−6.7	1
2	+0.0814	−0.3064							−0.2250	1.00	−0.2250	−2.2	2
3		−0.3064			+0.6321				+0.3257	1.24	+0.4039	+4.0	3
4		−0.3064		+0.1821					−0.1243	1.12	−0.1392	−1.4	4
5		−0.3064	+1.0836						+0.7772	1.48	+1.1503	+11.5	5
6			−1.0836			+1.1387			+0.0551	1.28	+0.0705	+0.7	6
7			−1.0836					+0.9126	−0.1710	1.64	−0.2804	−2.8	7
8				−0.1821	+0.6321				+0.4500	1.48	+0.6660	+6.7	8
9				−0.1821					−0.1821	1.56	−0.2841	−2.9	9
10				−0.1821		+1.1387			+0.9566	1.64	+1.5688	+15.7	10
11						−1.1387	+0.8933		−0.2454	2.08	−0.5104	−5.1	11
12							−0.8933		−0.8933	1.92	−1.7151	−17.1	12
13							−0.8933	+0.9126	+0.0193	1.80	+0.0347	+0.3	13
14								−0.9126	−0.9126	0.68	−0.6206	−6.2	14

number of entries as are shown in the 8 numbered columns of the table of correlates, and the arrangement of these entries in the various columns and lines should be the same as for the corresponding entries in the table of correlates. It is well to make sure that this is actually so, since in an extensive table it is quite easy to get an entry in the wrong box now and then. If that happens the computed v's are incorrect, the circuits will not close, and lost motion results from going back to correct any mistakes that may have been made.

Each entry in the Σ column of the computation of v's is the sum of the entries on the same line in the eight numbered columns of the table. The column headed "$1/p$" is filled in by copying directly from the corresponding column of the table of correlates. Each entry in the column headed "$\Sigma\times(1/p)$" is the product of the corresponding entries in the two columns next to the left.

The column he ded "Adopted v" is then filled in simply by rounding the entries in the next column to the left and moving the decimal point so that the adopted v's are listed in millimeters. It should be remembered that in the computation of the circuit closures the closures were computed in meters. Then when the closures were entered in the N column of the normal equations and the forward solution of the normal equations, the decimal point was moved two places to the right so that the closures were shown in centimeters to two places of decimals. Now the computed v's in the column headed "$\Sigma\times(1/p)$" are in centimeters, and we round each to two places of decimals and then move the point one place to the right so that the adopted v's are shown in millimeters to one decimal place.

The adopted v's should be entered in pencil only at this stage of the work. The reason for this provision will become quite apparent as this discussion develops.

The adopted v's are next reconverted to meters by moving the decimal point three places to the left and are entered in the "Adj. Corr'n" column of the tabulation of the circuit-closure computations. This should also be done in pencil at this stage of the work.

It will be found that the adopted v's for links 1, 9, and 12 are shown in the tabulation of the computation of v's as −6.7, −2.9, and −17.1 while a strict rounding according to the rules set forth earlier in this manual (see p. 17) would make them −6.6, −2.8, and −17.2, respectively.

These changes are accounted for as follows: When the adopted v's, rounded strictly in accordance with the rules, were entered in pencil in the tabulation of the

circuit-closure computations, it was found that the sum of the v's for circuits 1, 4, 7, and 8 failed to total correctly by 0.1 mm., in each case, as shown below:

1		4		7		8	
$-v_2$	+0.0022	$-v_4$	+0.0014	$-v_{11}$	+0.0051	$+v_1$	−0.0066
$-v_1$	+0.0066	$+v_8$	+0.0067	$+v_{12}$	−0.0172	$-v_7$	+0.0028
	+0.0088	$+v_9$	−0.0028	$+v_{13}$	+0.0003	$-v_{13}$	−0.0003
		$+v_{10}$	+0.0157		−0.0118	$+v_{14}$	−0.0062
			+0.0210				−0.0103

when they should have closed as follows:

1	4	7	8
+0.0089	+0.0209	−0.0117	−0.0104

The other circuits closed properly. This condition made the use of "arbitrary corrections" necessary, and the routine of deciding on how to apply them was about as given below:

In looking over the situation we notice that link 1 is common to circuits 1 and 8 and that the computed v is 0.6650 cm., or 0.006650 m. We only have to change this computed value by 0.000001 m., in order to make it round to 0.0067 instead of 0.0066 and by so doing we bring circuits 1 and 8 into perfect agreement. Next we inspect circuits 4 and 7 and find that no single link is common to both circuits but that circuit 4 includes v_9 and circuit 7 includes v_{12}, both of which are included in circuit 9. We find these computed v's to be 0.002841 m., and 0.017151 m., respectively, and also that, by slipping the first by the amount of 10 in the sixth place and the second by 2 in the sixth place, they become 0.002851 and 0.017149, respectively, and then round to 0.0029 and 0.0171, respectively. By doing this (applying arbitrary corrections of a *maximum* size of 0.01 mm.) we make all circuits close exactly. Making these very slight changes, we ink in the adopted v's and the adjustment corrections which were previously entered in pencil.

As a final verification of the rigidity of the adjustment, the adjustment corrections arrived at as a result of the procedures just discussed are then entered in the table of links; the adjusted differences of elevation are computed; the rates of correction in mm./km. are computed; and the elevations of the junction bench marks are computed *using all links* in the manner shown below.

	Meters	
Fixed elevation of Tidal 1	1.3752	
Tidal 1 to N 20	+12.3501	
Adjusted elevation of N 20	13.7253	
N 20 to A 16	+10.0432	
Fixed elevation of A 16	23.7685	(check)
A 16 to Q 17	+15.9081	
Adjusted elevation of Q 17	39.6766	
Q 17 to S 22	−3.8114	
Adjusted elevation of S 22	35.8652	
S 22 to N 20	−22.1399	
Adjusted elevation of N 20	13.7253	(check)
Adjusted elevation of S 22	35.8652	
S 22 to F 25	−10.3324	
Adjusted elevation of F 25	25.5328	
F 25 to N 20	−11.8075	
Adjusted elevation of N 20	13.7253	(check)
Adjusted elevation of Q 17	39.6766	
Q 17 to Z 10	+17.4521	

	Meters	
Fixed elevation of Z 10	57.1287	(check)
Z 10 to T 30	+2.8176	
Adjusted elevation of T 30	59.9463	
T 30 to S 22	−24.0811	
Adjusted elevation of S 22	35.8652	(check)
Adjusted elevation of T 30	59.9463	
T 30 to F 25	−34.4135	
Adjusted elevation of F 25	25.5328	(check)
Adjusted elevation of T 30	59.9463	
T 30 to X 32	−15.4656	
Adjusted elevation of X 32	44.4807	
X 32 to F 25	−18.9479	
Adjusted elevation of F 25	25.5328	(check)
Adjusted elevation of X 32	44.4807	
X 32 to Tidal 2	−42.3153	
Fixed elevation of Tidal 2	2.1654	(check)

Since the adjusted elevations of the junction bench marks have now been completely verified, the adjustment is complete, and the adjusted elevations of the junction marks may be entered on the summary sheets in preparation for the distribution of the corrections along the various links or lines.

The completion of the summary sheets will be discussed later, after taking up the consideration of some alternate methods of adjustment.

THE METHOD OF OBSERVATION EQUATIONS

The application of the method of observation equations to the adjustment of a network of lines of levels will be illustrated by again adjusting the network shown in figure 25. This method of adjustment should produce the same results as were obtained by the condition-equation method, but, since the details of the work are in many respects quite different, it is believed wise to exhibit and discuss the complete adjustment of this same network by this other method for purposes of comparison. At times the observation-equation method will be the more economical of the two methods of adjustment. This point will be discussed later, after the method has been explained.

The first step, as was the case with the condition-equation method, is the preparation of a table of fixed elevations. This table is the same as the one given on page 91, but is repeated here for convenience.

Fixed elevations

B. M.	*Elevation*
Tidal 1	1. 3752 m.
Tidal 2	2. 1654 m.
A 16	23. 7685 m.
Z 10	57. 1287 m.

Next, and this is a departure from the previous routine, a table of "assumed elevations" is prepared. While the assumed elevations are being tabulated it is well to remember that they should be assumed at numerical values which will be reasonably close to the final values. This is desirable in order to keep the corrections to be computed and applied to the assumed elevations from becoming unduly large and thereby possibly introducing a slight looseness into the final results. Errors of "one in the last place" are encountered at times in any extensive calculations of this type, but the frequency of their occurrence can be kept to a minimum by proper methods of computation and by the use of good judgment in such matters as deciding upon weights, assuming elevations, etc. It may be well to take up in detail the method of arriving at a few of these assumed elevations before setting down the complete table.

We find from the office abstracts or the summary sheets that the difference of elevation from Tidal 1 to N 20 is +12.3434 m. Adding this to the fixed elevation of Tidal 1 we obtain an observed elevation of 13.7186 m. for N 20. Working in from A 16, we have an observed difference of elevation from A 16 to N 20 of −10.0410 m., which, when added to the fixed elevation of 23.7685 for A 16, gives an observed elevation of 13.7275 m. for N 20. It is proper at this point to assume an elevation of 13.7200 m. for N 20.

Then, with the assumed elevation of N 20 as 13.7200 m. and an observed difference of elevation of +22.1284 m. from N 20 to S 22, we arrive at an observed elevation of 35.8484 m. for S 22 and adopt 35.8500 m. as our assumed elevation for S 22.

Passing on to Q 17 we find that the observed elevation brought in from the fixed elevation of A 16 is 39.6806 m., and from the assumed elevation of S 22 is 39.6628 m., so we assume an elevation of 39.6700 m. for Q 17.

Proceeding to the other junction bench marks and following this same method of attack, we build up the following table of assumed elevations.

Assumed elevations

B. M.	*Elevation (meters)*
N 20	$13.7200+x_1$
S 22	$35.8500+x_2$
Q 17	$39.6700+x_3$

Assumed elevations—Continued

B. M.	*Elevation (meters)*
X 32	$44.4900+x_4$
F 25	$25.5400+x_5$
T 30	$59.9500+x_6$

The x's added to the assumed elevations in the above table are the symbols representing the corrections to be computed and applied to these assumed elevations in order to arrive at the adjusted elevations of these marks. It should also be understood that another computer, in working out assumed elevations for these marks, might assume elevations slightly different from those given above. However, that would make no difference in the final results as long as the assumed elevations are reasonably close to their final values and, once assumed, are adhered to strictly throughout the rest of the computations.

The next step in the adjustment by this method is to construct the table to be used in the formation of the observation equations. This in a rough way at least is about the same sort of tabulation as that known as the table of links when making an adjustment by the method of condition equations. The details, as will be noticed, are quite different. This table is shown below, and the following comments should make its construction clear.

Table for formation of observation equations

Link	B. M.'s		Difference of elevation			Weight (p)	Symbol	Adj. diff. of elevation h_1-h_2 (meters)	v (mm.)	pv
	(1)	(2)	Assumed (meters)	Observed (meters)	Assumed minus observed (meters)					
1	Tidal 1	N 20	−12.3448	−12.3434	−0.0014	1.25	$-x_1$	−12.3501	−6.7	−8.4
2	A 16	N 20	+10.0485	+10.0410	+0.0075	1.00	$-x_1$	+10.0432	+2.2	+2.2
3	A 16	Q 17	−15.9015	−15.9121	+0.0106	0.81	$-x_3$	−15.9080	+4.1	+3.3
4	S 22	Q 17	−3.8200	−3.8128	−0.0072	0.89	$+x_2-x_3$	−3.8114	+1.4	+1.2
5	N 20	S 22	−22.1300	−22.1284	−0.0016	0.68	$+x_1-x_2$	−22.1398	−11.4	−7.8
6	F 25	S 22	−10.3100	−10.3317	+0.0217	0.78	$+x_5-x_2$	−10.3324	−0.7	−0.5
7	N 20	F 25	−11.8200	−11.8103	−0.0097	0.61	$+x_1-x_5$	−11.8074	+2.9	+1.8
8	Q 17	Z 10	−17.4587	−17.4588	+0.0001	0.68	$+x_3$	−17.4522	+6.6	+4.5
9	Z 10	T 30	−2.8213	−2.8147	−0.0066	0.64	$-x_6$	−2.8175	−2.8	−1.8
10	S 22	T 30	−24.1000	−24.0654	−0.0346	0.61	$+x_2-x_6$	−24.0811	−15.7	−9.6
11	F 25	T 30	−34.4100	−34.4186	+0.0086	0.48	$+x_5-x_6$	−34.4135	+5.1	+2.4
12	T 30	X 32	+15.4600	+15.4827	−0.0227	0.52	$+x_6-x_4$	+15.4655	−17.2	−8.9
13	F 25	X 32	−18.9500	−18.9476	−0.0024	0.56	$+x_5-x_4$	−18.9480	−0.4	−0.2
14	Tidal 2	X 32	−42.3246	−42.3215	−0.0031	1.47	$-x_4$	−42.3153	+6.2	+9.1

In this table for the formation of the observation equations, the link numbers as shown in figure 25 are listed in the first column in numerical order. The designations of the bench marks at the junction points or fixed points are shown in columns 2 and 3.

Using the tables of fixed and assumed elevations, the assumed difference of elevation for each link is computed in the sense "elevation of (1) minus the elevation of (2)," and are entered in column 4. The corresponding observed orthometric differences of elevation from the summary sheets are entered in column 5. The entries in column 6 are the differences "Assumed minus observed" and are obtained by subtracting each entry in column 5 from the corresponding entry in column 4.

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The weights shown in column 7 were computed, in this case, by taking 25 times the reciprocal of the link length for each link. To have used the reciprocals of the link lengths would have resulted in inconveniently small weights. For this reason $25/L$ was used as the weight in each case. The choice of the numerator in this fraction representing the weights assigned to the links may be varied to suit any particular adjustment. For example, should an adjustment be encountered in which the links are of an order of size of from 80 to 125 km., it would probably be well to use $100/L$ as the weight of each link. However, once the numerator has been decided upon for any one adjustment it must be adhered to strictly throughout that particular adjustment.

The column headed "Symbol" shows the x's which are assigned as corrections to the assumed elevations and which, when computed and properly applied, result in the adjusted elevations of the bench marks not held fixed in the adjustment. The elevations held fixed, of course, remain undisturbed throughout. Each x is assigned a plus sign in the table of assumed elevations, but when it appears in column 8 of the above table the sign remains plus if the bench mark designation to which the x is related appears in the second or "B. M. (1)" column but becomes minus if the designation of the bench mark to which the x is related appears in the third or "B. M. (2)" column. Since there are no x's assigned to fixed elevations, any link involving a fixed elevation will have only one x in column 8, while for links between bench marks both of which have been assigned assumed elevations there will be two x's in column 8.

The remaining columns of the above table are filled in later after the adjustment has been completed and will be discussed at the proper place later in this manual.

The table for the formation of the normal equations is in some respects quite similar to the table of correlates used in connection with the adjustment by the method of condition equations. It will be seen from an examination of the table that the columns are headed, in order from left to right, "Link," "Weight (p)," "x_1," "x_2," ------"x_6," "N," "pN," "Σ," and "$p\Sigma$." The links are numbered down column 1 in numerical order, and the weights of the links are entered in column 2. The columns headed "x_1" to "x_6," inclusive, are filled in with the $+1$ or -1 coefficients of the x's, spotting them in the columns indicated by the subscripts of the x's and on the lines as indicated by the link numbers to which they are related.

The entries in the N column are the values from the "Assumed minus observed" column of the table for the formation of the observation equations, but the decimal point is moved three places to the right in each case to express these values in millimeters instead of meters as originally computed and listed in the table for the formation of the observation equations.

The entries in the Σ column are the sums of all entries in the columns x_1 to N, inclusive. The entries in the pN column are, as the heading indicates, the product of the entries in p and N columns. Similarly the entries in the pΣ column are the products of the entries in the p and Σ columns.

The table of normal equations is made up from the above table in the same manner as the table of normal equations was made up from the table of correlates in the adjust-

Table for formation of normal equations

Link	Weight (p)	x_1	x_2	x_3	x_4	x_5	x_6	N	pN	Σ	pΣ
1	1.25	−1						−1.4	−1.750	−2.4	−3.000
2	1.00	−1						+7.5	+7.500	+6.5	+6.500
3	0.81			−1				+10.6	+8.586	+9.6	+7.776
4	0.89		+1	−1				−7.2	−6.408	−7.2	−6.408
5	0.68	+1	−1					−1.6	−1.088	−1.6	−1.088
6	0.78		−1			+1		+21.7	+16.926	+21.7	+16.926
7	0.61	+1				−1		−9.7	−5.917	−9.7	−5.917
8	0.68			+1				+0.1	+0.068	+1.1	+0.748
9	0.64						−1	−6.6	−4.224	−7.6	−4.864
10	0.61		+1				−1	−34.6	−21.106	−34.6	−21.106
11	0.48					+1	−1	+8.6	+4.128	+8.6	+4.128
12	0.52				−1		+1	−22.7	−11.804	−22.7	−11.804
13	0.56				−1	+1		−2.4	−1.344	−2.4	−1.344
14	1.47				−1			−3.1	−4.557	−4.1	−6.027

ment using the method of condition equations, *except that* in this case the N column is treated in the same manner as the columns x_1 to x_6, inclusive.

The entry for the N column of the first normal equation could be thought of as the sum of products of corresponding entries in columns p, x_1, and N, thus:

$$(1.25)\ (-1)\ (-1.4) + (1.00)\ (-1)\ (+7.5) + (0.68)\ (+1)\ (-1.6) + (0.61)\ (+1)\ (-9.7) = -12.755,$$

but since the pN column shows the products of weights and N terms for each link, the first N entry may be more easily computed as the sum of the products of corresponding entries in the x_1 and pN columns, thus:

$$(-1)\ (-1.750) + (-1)\ (+7.500) + (+1)\ (-1.088) + (+1)\ (-5.917) = -12.755.$$

Similarly, the N term for the second equation is the sum of the products of corresponding entries in the x_2 and pN columns, and so on. Likewise, the first entry for the Σ column is the sum of the products of corresponding entries in the x_1 and pΣ columns, the second entry is the sum of the products of corresponding entries in the x_2 and pΣ columns, and so on. After all the terms in each normal equation have been computed, they may be checked by adding down and across to the N term as in the case of normal equations formed from condition equations.

In this case the forward solution of the normal equations was carried out by the method of allowing the products to accumulate in the machine, and for this reason the table of normal equations and the forward solution of the normal equations have been combined in one table. The forward solution of the normal equations and the backward solution are handled in the same way as outlined for the adjustment of this net when the method of condition equations was used, and no further comment on these two parts of this adjustment should be needed.

Combined table of normal equations and forward solution of normal equations

1	2	3	4	5	6	N	Σ
+3.5400 *C1*	−0.6800 +0.19209			−0.6100 +0.17232		−12.7550 +3.60311	−10.5050 +2.96751+1
	+2.9600 +2.8294 *C2*	−0.8900 −0.8900 +0.31455		−0.7800 −0.8972 +0.31710	−0.6100 −0.6100 +0.21559	−43.3520 −45.8021 +16.18792	−43.3520 −45.3699 +16.03517−1
		+2.3800 +2.1001 *C3*		−0.2822 +0.13437	−0.1919 +0.09138	−2.1100 −16.5171 +7.86491	−0.6200 −14.8911 +7.09066
			+2.5500 +2.5500 *C4*	−0.5600 −0.5600 +0.21961	−0.5200 −0.5200 +0.20392	+17.7050 +17.7050 −6.94314	+19.1750 +19.1750 −7.51961
				+2.4300 +1.8795 *C5*	−0.4800 −0.8134 +0.43277	+25.6270 +10.5740 −5.62596	+25.6270 +11.6401 −6.19319
					+2.2500 +1.6429 *C6*	+9.3980 +6.2007 −3.77424	+10.0380 +7.8436 −4.77424

Backward solution

6	5	4	3	2	1
−3.77424 −3.77424	−5.62596 −1.63338 −7.25934	−6.94314 −0.76964 −1.59422 −9.30700	+7.86491 −0.34489 −0.97544 +6.54458	+16.18792 −0.81369 −2.30194 +2.05860 +15.13089	+3.60311 −1.25093 +2.90649 +5.25867

The column totals of the backward solution represent the corrections to be applied to the assumed elevations of the junction bench marks. When these are rounded to one decimal place they are entered in the following table of the computation of the adjusted elevations, each assumed elevation is changed by the amount of the appropriate correction, and the adjusted elevations result.

Adjusted elevations of junction marks

B. M.	Assumed elevation (meters)	Correction (mm.)	Adjusted elevation (meters)	B. M.	Assumed elevation (meters)	Correction (mm.)	Adjusted elevation (meters)
N 20	13.7200	+5.3	13.7253	X 32	44.4900	−9.3	44.4807
S 22	35.8500	+15.1	35.8651	F 25	25.5400	−7.3	25.5327
Q 17	39.6700	+6.5	39.6765	T 30	59.9500	−3.8	59.9462

Having arrived at the adjusted elevations of the newly adjusted junction bench marks, the adjusted elevations would be entered on the summary sheets, and the distribution of the corrections through the various links would be carried out. However, this feature of the work will be discussed later under the heading "Distribution of corrections."

For purposes of comparison the following table exhibits the adjusted elevations of the six junction bench marks in this net as arrived at by the method of observation equations and as they resulted from the adjustment of the same net by the method of condition equations.

Comparison of results of adjustment by the two different methods

B. M.	Adjusted elevations by the method of—		B. M.	Adjusted elevations by the method of—	
	Observation equations	Condition equations		Observation equations	Condition equations
N 20	13.7253	13.7253	X 32	44.4807	44.4807
S 22	35.8651	35.8652	F 25	25.5327	25.5328
Q 17	39.6765	39.6766	T 30	59.9462	59.9463

It may be seen that the adjusted elevations of two of the six marks agree exactly, as determined by the two different methods of adjustment. The other four fail to agree by one in the last place or 0.1 mm. in each case. However, it is interesting to note that it would require changes of only from 6 to 25 microns in the computed values of the x's to cause them to round so as to produce perfect agreement between the adjusted elevations resulting from both adjustments.

It should be noted at this point that, in the adjustment of this network by the method of observation equations, only six normal equations were developed while seven were required for the adjustment by the method of condition equations. This, in itself, is some saving in labor, and cases will be found now and then when a much greater saving in labor may be accomplished by the use of the observation-equation method. This may be illustrated by the following example:

Assume a closed circuit of leveling and two lines of levels radiating from each of three points on the circuit to six bench marks which are located outside the circuit and which have had their elevations fixed by previous adjustment. In a case of this sort the condition-equation method would require six normal equations while only three would be developed if the observation-equation method was used. In general, when the number of new points to be adjusted is small in comparison with the number of fixed elevations to be considered in the adjustment, the observation-equation method is likely to prove to be the more economical of the two. The observation-equation method is perhaps not quite so completely self-checking as the other method, but the results will be the same for all practical purposes by the use of either method.

It should also be noted, when comparing the two methods of adjustment, that the observation-equation method results in the direct computation of the adjusted elevations of the junction marks rather than the adjusted differences of elevation for the various links from which the adjusted elevations must be computed if the condition-equation method is used. This would at first appear to be a saving, but, in order to obtain a check on the x's computed in the adjustment by the observation-equation method, it is necessary to complete the last three columns of the table for the formation of the observation equations.

Using the table of fixed elevations, together with the table of adjusted elevations, the adjusted differences of elevation are computed and entered in the table for formation of observation equations. (See p. 107.) The v's are then computed by subtracting each observed difference of elevation from the corresponding adjusted difference of elevation. Each v is then multiplied by the weight for the link to which it applies to obtain the entries in the last or pv column.

The check is obtained by summing the pv's for the links radiating from each of the junction points, as follows:

Approximate check on solution

x_1	x_2	x_3	x_4	x_5	x_6
+8.4	+1.2	−3.3	+8.9	−0.5	+1.8
−2.2	+7.8	−1.2	+0.2	−1.8	+9.6
−7.8	+0.5	+4.5	−9.1	+2.4	−2.4
+1.8	−9.6	0.0	0.0	−0.2	−8.9
+0.2	−0.1			−0.1	+0.1

This table for the summation of the pv's about each point adjusted is built up by multiplying the +1 and −1 coefficients in columns x_1 to x_6, inclusive, of the table for the formation of the normal equations by their appropriate pv's and summing these products column by column. These checks are not exact, as will be noticed in the above tabulation, but since the v's were computed only to the nearest 0.1 mm., and were later multiplied in some cases by weights greater than unity, some little looseness may be expected. This check, while not absolutely rigid, will, however, serve to detect any *blunders* that may have been made in the computations. The tabulations and computations up to and including the table for the formation of the normal equations must be thoroughly checked, but from there on the method is at least approximately self-checking as shown above.

A comparison of the v's resulting from the two adjustments shows 7 of them to be in exact agreement, and no discrepancy is greater than 0.1 mm., or one in the last place.

THE CIRCUIT-REDUCTION METHOD

When the fundamental vertical control net of the United States had been developed to such an extent that within most of the loops of first-order leveling, second-order lines were run with an approximate 50- or 25-mile spacing and later, when the 25-mile loops began to be broken down by the so-called 7½-mile lines, it became necessary to adjust a great many subordinate networks. Mr. Norman F. Braaten, mathematician, of the Section of Leveling of this Bureau, set about the development of a method of adjustment which was to attain, if possible, the following objectives:

(1) The method should produce the same results as would be arrived at if the adjustment were made by either of the two methods of adjustment already discussed, the method of condition equations or the method of observation equations,

(2) The corrections to the observed orthometric differences of elevation should be arrived at directly and by a process involving a minimum of computation, and

(3) The method should be applicable to a large percentage of the subordinate networks to be handled in this Office.

Mr. Braaten has developed, and gradually expanded the field of usefulness of, a method of adjustment which he calls "the circuit-reduction method." This method of adjustment has been used a great many times in the Section of Leveling of this

Bureau and also by some of the computers of the United States Geological Survey. The results have been uniformly satisfactory, and the method is gradually becoming known as "the Braaten method." The method has attained completely the goals set for it and, in addition, the computations are of such character that in most cases a slide rule may be used in lieu of computing machinery, thus making the method particularly useful for the small engineering office with a minimum of available computing machines. The method is becoming well known not only in this country but in other countries as well, particularly in Central and South America.

The use of this method is illustrated by a series of practical applications in the following pages. The development of the method and the derivation of the formulas are given in appendix D. Other information as to possible ways of expanding the field of usefulness of this method and also information concerning the practical limits to which this expansion may be carried will be included in appendix E.

As our first illustration of the application of this method, we will adjust a subordinate net such as is shown in figure 26.

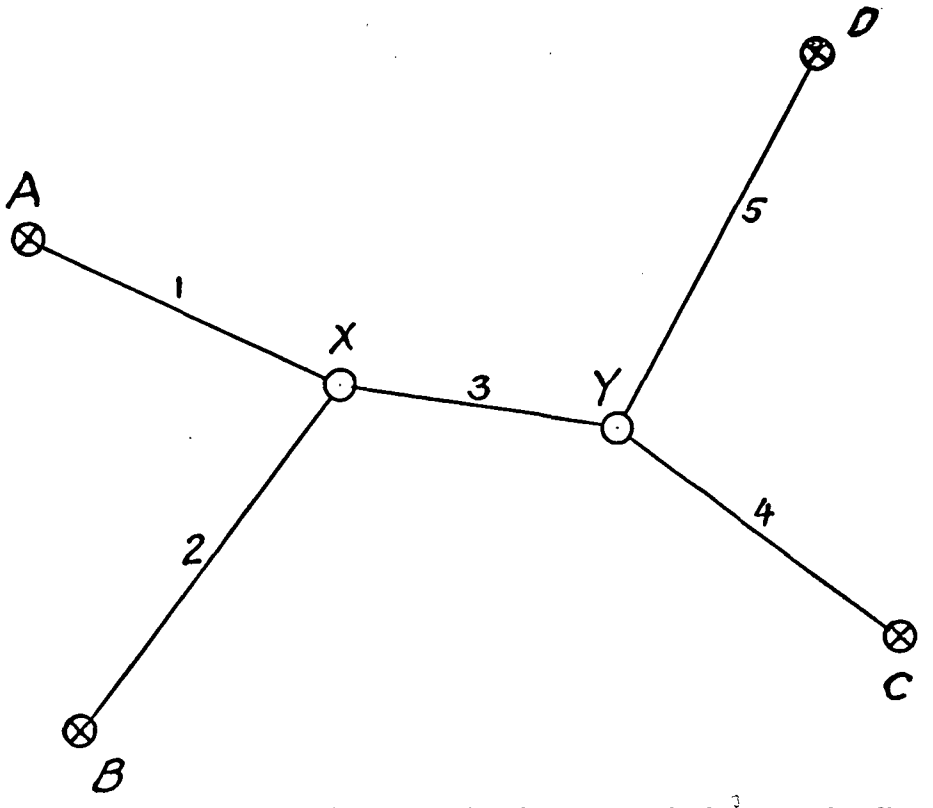


FIGURE 26.—Subordinate net, containing two new junctions and four fixed points, to be adjusted by the circuit-reduction method.

The basic data needed for this adjustment, which would normally come from the office abstracts (Form 31) or the summary sheets, have been tabulated below:

Table of fixed elevations

B. M.	*Elevation (m.)*	*B. M.*	*Elevation (m.)*
A	10.3748	C	24.3141
B	15.8916	D	8.1416

Table of links

Link No.	B. M. minus B. M.	Length (km.)	Obs. Ortho. Diff. of Elevation (m.)	v (mm.)	Adj. Diff. of Elevation (m.)
1	X−A	15.1	+21.7648	+1.7	+21.7665
2	X−B	21.7	+16.2532	−3.5	+16.2497
3	X−Y	19.6	+2.7514	+0.9	+2.7523
4	Y−C	30.2	+5.0680	+6.9	+5.0749
5	Y−D	24.0	+26.2517	−4.3	+26.2474

Obviously, columns 5 and 6 of the table of links are the results of the adjustment and are filled in *after* the adjustment computations are completed. It should also be noted at this point that as a computer becomes familiar with this method of adjustment it will not be necessary to set up the table of fixed elevations nor the table of links, as the material can go directly from the office computations or summary sheets, through the computing machinery when necessary, directly to the computation form shown in figure 27.

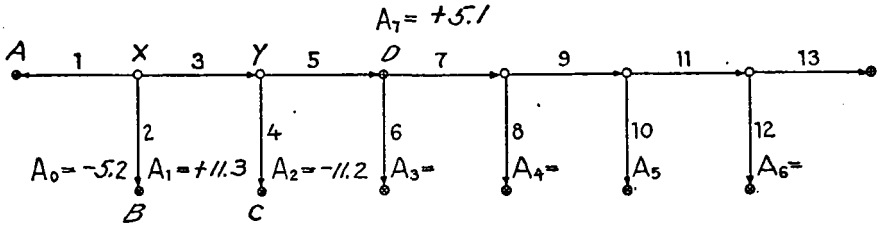
Referring now to figure 27, the diagram at the top of the form appears to be quite different from that shown in figure 26. However, if one visualizes the bench marks and lines of leveling shown in figure 26 as rings and lengths of light chain, respectively, and picks the network up by rings A and D, stretching them as far apart as possible in a horizontal direction, links 1, 3, and 5 would then make a straight horizontal line, links 2 and 4 would hang down vertically from rings X and Y, and the network would then very closely resemble the left part of the diagram at the top of the computation form shown in figure 27.

In the diagram at the top of figure 27 the bench marks whose elevations are to be held fixed are represented by small circles with crosses in them. These are already printed on the form for the marks designated A, B, and C, but the cross must be added to the diagram for the mark designated D in this case. The small circles designated X and Y are left open to indicate that these marks are the ones for which adjusted elevations will be derived as a result of the adjustment. It should be noted that Form A (fig. 27) provides for 13 links, but in this case we are only concerned with the first 5 and can disregard the remainder of the diagram. This amounts to assuming that the remaining links, 6 to 13, inclusive, are all zero, both as regards length and difference of elevation.

Using the "B. M. minus B. M." method and proceeding counterclockwise around each circuit, we compute the following circuit closures.

B–X	−16.2532 m.
X–A	+21.7648 m.
A–B	−5.5168 m.
	−0.0052 m., or −5.2 mm.
C–Y	−5.0680 m.
Y–X	−2.7514 m.
X–B	+16.2532 m.
B–C	−8.4225 m.
	+0.0113 m., or +11.3 mm.
D–Y	−26.2517 m.
Y–C	+5.0680 m.
C–D	+21.1725 m.
	− 0.0112 m., or −11.2 mm.

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



					V/l	Check
$l_1 = 15.1$ $l_2 = 21.7$		$A_0 =$	-5.2			
$E_1 = \frac{l_2 l_1}{l_1+l_2} =$	8.9	$K_1 = \frac{l_2 A_0}{l_1+l_2} =$	-3.1	$V_1 = -A_0 + V_2 =$	$+1.7$	$+0.113$	
$l_3 = 19.6$ $l_4 = 30.2$		$A_1 =$	$+11.3$	$V_2 = -A_1 + V_3 + V_4 =$	-3.5	-0.161	
$E_2 = \frac{l_4(E_1+l_3)}{(E_1+l_3+l_4)} =$	14.7	$K_2 = \frac{l_4(K_1+A_1)}{E_1+l_3+l_4} =$	$+4.2$	$V_3 = \frac{l_3(K_1+A_1-V_4)}{E_1+l_3} =$	$+0.9$	$+0.046$	-0.002
$l_5 = 24.0$ $l_6 =$		$A_2 =$	-11.2	$V_4 = -A_2 + V_5 + V_6 =$	$+6.9$	$+0.228$	
$E_3 = \frac{l_6(E_2+l_5)}{(E_2+l_5+l_6)} =$		$K_3 = \frac{l_6(K_2+A_2)}{E_2+l_5+l_6} =$		$V_5 = \frac{l_5(K_2+A_2-V_6)}{E_2+l_5} =$	-4.3	-0.179	$+0.003$
$l_7 =$ $l_8 =$		$A_3 =$		$V_6 = -A_3 + V_7 + V_8 =$			
$E_4 = \frac{l_8(E_3+l_7)}{(E_3+l_7+l_8)} =$		$K_4 = \frac{l_8(K_3+A_3)}{E_3+l_7+l_8} =$		$V_7 = \frac{l_7(K_3+A_3-V_8)}{E_3+l_7} =$			
$l_9 =$ $l_{10} =$		$A_4 =$		$V_8 = -A_4 + V_9 + V_{10} =$			
$E_5 = \frac{l_{10}(E_4+l_9)}{(E_4+l_9+l_{10})} =$		$K_5 = \frac{l_{10}(K_4+A_4)}{E_4+l_9+l_{10}} =$		$V_9 = \frac{l_9(K_4+A_4-V_{10})}{E_4+l_9} =$			
$l_{11} =$ $l_{12} =$		$A_5 =$		$V_{10} = -A_5 + V_{11} + V_{12} =$			
$E_6 = \frac{l_{12}(E_5+l_{11})}{(E_5+l_{11}+l_{12})} =$		$K_6 = \frac{l_{12}(K_5+A_5)}{E_5+l_{11}+l_{12}} =$		$V_{11} = \frac{l_{11}(K_5+A_5-V_{12})}{E_5+l_{11}} =$			
$l_{13} =$		$A_6 =$		$V_{12} = -A_6 + V_{13} =$			
				$V_{13} = \frac{l_{13}(K_6+A_6)}{E_6+l_{13}} =$			

FIGURE 27.—Form A, showing computations involved in the adjustment of the net shown in figure 26.

These three closures, in the order in which they have been computed are the numerical values for A_0, A_1, and A_2, respectively, and may be entered on the form as shown in figure 27.

The check on the computation of these closures is obtained as follows:

D–A	−7.2332 m.
A–X	−21.7648 m.
X–Y	+2.7514 m.
Y–D	+26.2517 m.
	+0.0051 m., or +5.1 mm.,

which is equal to the negative of the sum of the first three closures and constitutes a check on the computation of all four closures, except for the possibility of compensating errors.

This fourth or check closure is to be entered on the computation form (fig. 27) at the very top of the diagram and just to the right of "$A_7=$," which may be changed in this case to "$A_3=$" or left unchanged and merely be assumed to be "$A_3=$."

The part of the diagram below and to the right of the circle and cross representing bench mark D may be crossed out if desired though, as one becomes familiar with the application of this method, this will not be at all necessary.

The lengths of the links are entered on the computation form in the appropriate places just to the right of "$l_1=$," "$l_2=$," etc., in the left-hand column of the left half of the form.

We are then ready to compute E_1, K_1, E_2, and K_2 in the order given and as indicated by means of the formulas printed directly on the computation form. These computations are given in detail below, for this one illustration, but will not be given in such detail for the later adjustments which will be shown to further illustrate this method.

$$E_1=\frac{l_2l_1}{l_1+l_2}=\frac{21.7\times15.1}{15.1+21.7}=8.9$$

$$K_1=\frac{l_2A_o}{l_1+l_2}=\frac{21.7\times-5.2}{15.1+21.7}=-3.1$$

$$E_2=\frac{l_4(E_1+l_3)}{(E_1+l_3+l_4)}=\frac{30.2(8.9+19.6)}{8.9+19.6+30.2}=14.7$$

$$K_2=\frac{l_4(K_1+A_1)}{(E_1+l_3+l_4)}=\frac{30.2(-3.1+11.3)}{8.9+19.6+30.2}=+4.2$$

Since l_6 to l_{13}, inclusive, are all zero the rest of the E's and K's would all be zero.

We then proceed to compute v_5, v_4, ... v_1, in that order and in accordance with the formulas shown on the right side of the computation form, as follows:

$$v_5=\frac{l_5(K_2+A_2-v_6)}{E_2+l_5}=\frac{24.0(4.2-11.2-0.0)}{14.7+24.0}=-4.3$$

$$v_4=-A_2+v_5+v_6=+11.2-4.3+0.0=+6.9$$

$$v_3=\frac{l_3(K_1+A_1-v_4)}{E_1+l_3}=\frac{19.6(-3.1+11.3-6.9)}{8.9+19.6}=+0.9$$

$$v_2 = -A_1 + v_3 + v_4 = -11.3 + 0.9 + 6\ 9 = -3.5$$

$$v_1 = -A_0 + v_2 = +5.2 - 3\ 5 = +1.7$$

and these v's are entered on the computation form as shown in figure 27.

Next we compute the values of $\frac{v_1}{l_1}, \frac{v_2}{l_2}$, etc., entering them in the column headed "v/l."

The check on the computation then comes from summing the numerical values for v/l about each point which has been adjusted, as follows:

$$+0.113 - 0.161 + 0.046 = -0.002$$

$$-0.046 + 0.228 - 0.179 = +0.003.$$

The algebraic signs of the various values for v/l are used as shown as long as the arrows in the diagram point away from the junction about which the values of v/l are being summed. If, as in the case of link 3 in the summation of the v/l's about Y, the arrow points toward the point about which the sum of the v/l's is being taken, the signs of such values of v/l must be changed.

The corrections (v's) thus computed, when pointed off so as to be expressed in meters and properly applied to the observed orthometric differences of elevation, result in the adjusted differences of elevation from which the adjusted elevations of the junctions (in this case X and Y) may be computed according to the methods used on page 105.

The corrections (v's) are applied to the B. M.-minus-B. M. differences of elevation with the sign computed in figure 27 whenever the link sheet lists the difference for the link in the direction indicated by the arrow in the sketch at the top of the computation form. If the link sheet shows the difference in the opposite direction, the sign of the correction is changed. For example, had the B. M.-minus-B. M. difference for link 1 been listed on the link sheet as "A—X" (necessitating the observed orthometric difference of elevation being shown as "—21.7648") the v would have been entered with the opposite sign. The adjusted difference would then have been of the same numerical value but of opposite sign to that shown.

In order to show that these new adjusted elevations for bench marks X and Y are consistent, no matter from which fixed elevations they are computed, the computation of the adjusted elevations, using the adjusted differences shown on page 114, is set forth below.

A	10.3748		B	15.8916	
	+21.7665			+16.2497	
X	32.1413		X	32.1413	(check)
	−2.7523				
Y	29.3890		C	24.3141	
	−26.2474			+5.0749	
D	3.1416	(check)	Y	29.3890	(check)

By this time it has become quite obvious that none of the computations which have been made in the process of adjusting the subordinate network shown in figure 26 are beyond the capacity of a slide rule. Further the amount of tabular work is much less than with the two methods of adjustment discussed previously. One may, if we except perhaps a few preliminary trials, work directly from the office computations and summary sheets to Form A. The adjustment is set up, the computations are made, and the resulting corrections applied to the observed orthometric differences of elevation with a minimum of effort and in a very short time.

However, at this point this method has only been applied to a network which may be exhibited diagrammatically as a part of the figure shown in conventionalized form at the top of Form A (fig. 27). To show how this method may be applied to a somewhat more complicated network, let us consider a net as shown in figure 28.

Here we have a network which, if we assume it to be made up of rings and chains as before, pick it up at A and F, and stretch it out, would look quite like the network shown in figure 29.

It is admitted that the net shown in figure 28 could have been "picked up" by C and F. Then A and B would hang down from X, and D and E would hang down from Y. However, this particular illustration is being handled according to figure 29 in order to exhibit the fact that a link may be thrown over to the other side of the "backbone" line for convenience and without harm to the final results *if* the circuit closures are properly computed.

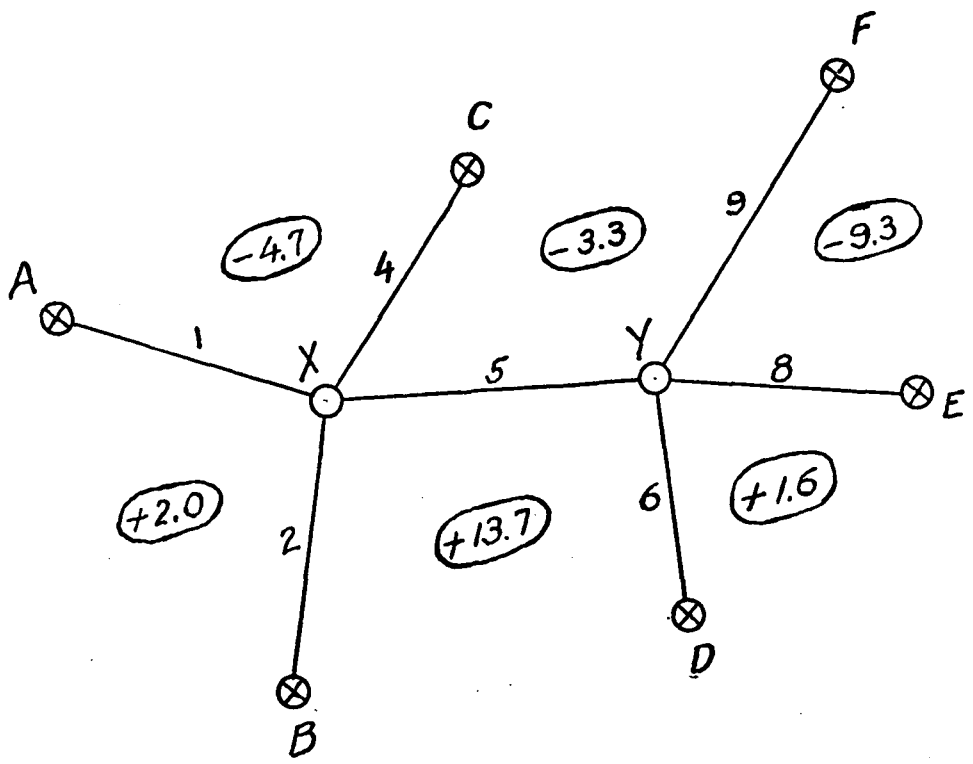


FIGURE 28.—Subordinate net, containing two new junctions and six fixed points, to be adjusted by the circuit-reduction method.

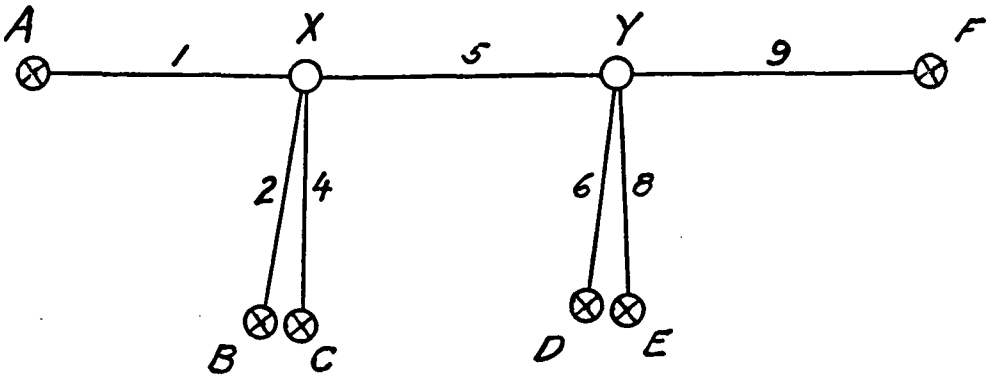


FIGURE 29.—The net of figure 28 "picked up" by A and F.

We will now assume the following tables of fixed elevations and links:

Fixed elevations

B. M.	*Elevation*	*B. M.*	*Elevation*
A	10.3724 m.	D	9.7648 m.
B	15.2615 m.	E	31.4709 m.
C	13.2141 m.	F	19.0976 m.

Table of links

Link No.	B. M. minus B. M.	Length (km.)	Obs. Ortho. Diff. of elevation (m.)	Link No.	B. M. minus B. M.	Length (km.)	Obs. Ortho. Diff. of elevation (m.)
(2) 1	X−A	20.8	+2.0090	6	Y−D	10.3	+8.5002
(4) 2	X−B	15.9	−2.8821	8	Y−E	15.0	−13.2075
(1) 4	X−C	17.4	−0.8374	9	Y−F	21.5	−0.8249
5	X−Y	30.5	−5.8993				

Then, using the B. M.-minus-B. M. method and proceeding counterclockwise around each circuit as shown in figure 28, the closures shown in the circles in that figure are computed and expressed in millimeters. That these have been correctly computed is shown by the fact that the sum of all six of the closures is 0.0 mm.

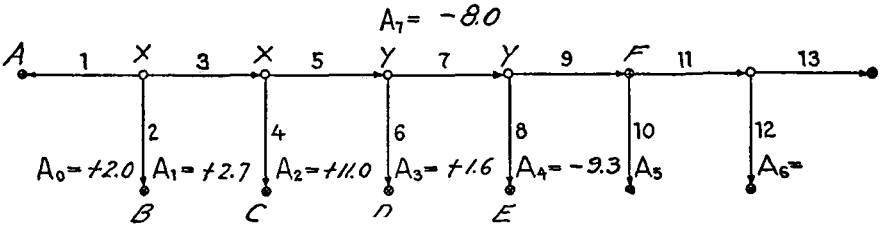
It will be noticed that certain numbers have been skipped in numbering the links in both figure 28 and figure 29. For the present, disregard the link numbers in parentheses in the preceding table of links.

If we assume link 3 to extend from X to X, both the difference of elevation and the length of the link will be zero. The same applies to link 7, from Y to Y.

Redrawing the network on this basis we have a diagram such as appears at the top of Form A as shown in figure 30. Links 3 and 7 thus have zero differences of elevation and zero lengths.

The closures are computed in accordance with the diagram at the top of Form A in figure 30 and are entered on the computation form in their proper places as they are computed. The closures to be computed are A_0, A_1, A_2, A_3, A_4, and $A_5=A_7$. They are computed, as before, using the B. M.-minus-B. M. method and proceeding counterclockwise around each circuit. It must be remembered that the differences

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



						$\frac{V}{l}$	Check
$l_1 = 20.8$ $l_2 = 15.9$		$A_0 =$	$+2.0$				
$E_1 = \frac{l_2 l_1}{l_1 + l_2} =$	9.0	$K_1 = \frac{l_2 A_0}{l_1 + l_2} =$	$+0.9$	$V_1 = -A_0 + V_2 =$	-4.1	-0.197	
$l_3 = 0.0$ $l_4 = 17.4$		$A_1 =$	$+2.7$	$V_2 = -A_1 + V_3 + V_4 =$	-2.1	-0.132	
$E_2 = \frac{l_4(E_1 + l_3)}{(E_1 + l_3 + l_4)} =$	5.9	$K_2 = \frac{l_4(K_1 + A_1)}{E_1 + l_3 + l_4} =$	$+2.4$	$V_3 = \frac{l_3(K_1 + A_1 - V_4)}{E_1 + l_3} =$	0.0	0.000	
$l_5 = 30.5$ $l_6 = 10.3$		$A_2 =$	$+11.0$	$V_4 = -A_2 + V_5 + V_6 =$	$+0.6$	$+0.034$	
$E_3 = \frac{l_6(E_2 + l_5)}{(E_2 + l_5 + l_6)} =$	8.0	$K_3 = \frac{l_6(K_2 + A_2)}{E_2 + l_5 + l_6} =$	$+3.0$	$V_5 = \frac{l_5(K_2 + A_2 - V_6)}{E_2 + l_5} =$	$+9.0$	$+0.295$	0.000
$l_7 = 0.0$ $l_8 = 15.0$		$A_3 =$	$+1.6$	$V_6 = -A_3 + V_7 + V_8 =$	$+2.6$	$+0.252$	
$E_4 = \frac{l_8(E_3 + l_7)}{(E_3 + l_7 + l_8)} =$	5.2	$K_4 = \frac{l_8(K_3 + A_3)}{E_3 + l_7 + l_8} =$	$+3.0$	$V_7 = \frac{l_7(K_3 + A_3 - V_8)}{E_3 + l_7} =$	0.0	0.000	
$l_9 = 21.5$ $l_{10} =$		$A_4 =$	-9.3	$V_8 = -A_4 + V_9 + V_{10} =$	$+4.2$	$+0.280$	
$E_5 = \frac{l_{10}(E_4 + l_9)}{(E_4 + l_9 + l_{10})} =$		$K_5 = \frac{l_{10}(K_4 + A_4)}{E_4 + l_9 + l_{10}} =$		$V_9 = \frac{l_9(K_4 + A_4 - V_{10})}{E_4 + l_9} =$	-5.1	-0.237	0.000
$l_{11} =$ $l_{12} =$		$A_5 =$		$V_{10} = -A_5 + V_{11} + V_{12} =$			
$E_6 = \frac{l_{12}(E_5 + l_{11})}{(E_5 + l_{11} + l_{12})} =$		$K_6 = \frac{l_{12}(K_5 + A_5)}{E_5 + l_{11} + l_{12}} =$		$V_{11} = \frac{l_{11}(K_5 + A_5 - V_{12})}{E_5 + l_{11}} =$			
$l_{13} =$		$A_6 =$		$V_{12} = -A_6 + V_{13} =$			
				$V_{13} = \frac{l_{13}(K_6 + A_6)}{E_6 + l_{13}} =$			

FIGURE 30.—Form A, showing the computations involved in the adjustment of the net shown in figures 28 and 29.

in elevation for links 3 and 7 are each 0.0000 meter. The sum of the six closures is again 0.0 mm., indicating that the closures have been correctly computed.

Let us now examine the changes which have been made in the circuit closures as a result of throwing link 4 over to the other side of the "backbone line" (A, through X and Y, to F). In doing this we have removed the boundary between circuits AXCA and CXYFC, and it will be noticed that the sum of the closures of these two circuits as shown in figure 28 is $(-4.7)+(-3.3)=-8.0$ and is the same as $A_5=A_7=-8.0$ in figure 30. This is as might be expected, for if we remove a boundary line between two circuits to combine the two into a single circuit the closure of the single combined circuit is the same as the algebraic sum of the two circuits before combining them.

The circuits ABXA, DEYD, and EFYE are the same in figure 30 as in figure 28, since throwing link 4 over into a position between links 2 and 5 does not affect any of these three circuits. However, the circuit BDYXB has now been split into two circuits, BCXB and CDYXC, and we find, as might be expected, that circuits A_1 (BCXB) and A_2 (CDYXC) in figure 30 sum to +13.7 mm., which was the closure for circuit BDYXB as shown in figure 28 before it was cut up by throwing link 4 over into it.

It is usually convenient to make the first computation of the circuit closure on the basis of a sketch which shows the various junction marks and links in approximately their correct geographical relationships. However, some nets will require that certain links be thrown over to the other side of the "backbone line" in order to make the sketch fit the diagram on the computation form. This need cause no concern and has no effect on the final results provided the closures are recomputed in accordance with the lay-out at the top of Form A and are used in that way in the adjustment computations.

The adjustment of this net is shown completely in figure 30, including the check on the computations by summing the numerical values for v/l around each new junction being adjusted. The check at X involves the sum $(v_1/l_1+v_2/l_2+v_4/l_4+v_5/l_5)$ and that at Y the sum $(-v_5/l_5+v_6/l_6+v_8/l_8+v_9/l_9)$.

As this computation is made in the same way as the one shown in figure 27 and since that has been discussed in detail, no further comment on the details of the computations shown in figure 30 will be made *except* to remind the reader that links 3 and 7 have zero lengths, zero differences of elevation, and zero for the numerical values of their v's.

Applying the resulting v's to the corresponding observed orthometric differences of elevation, we determine adjusted differences of elevation as follows:

B. M. minus B. M.	*Adj. Diff. of elevation*	*B. M. minus B. M.*	*Adj. Diff. of elevation*
X−A	+2.0049 m.	Y−D	+8.5028 m.
X−B	−2.8842 m.	Y−E	−13.2033 m.
X−C	−0.8368 m.	Y−F	−0.8300 m.
X−Y	−5.8903 m.		

We then compute and check the adjusted elevations of X and Y, as follows:

A	10.3724		Y	18.2676	
	+2.0049			+13.2033	
X	12.3773		E	31.4709	(check)
	+2.8842				
B	15.2615	(check)	D	9.7648	
				+8.5028	
C	13.2141		Y	18.2676	(check)
	−0.8368				
X	12.3773	(check)	Y	18.2676	
F	19.0976			−5.8903	
	−0.8300		X	12.3773	(check)
Y	18.2676				

In order to exhibit the fact that this last illustration produces the same results as would have been obtained had link 4 not been thrown over to the other side of the "backbone line," we now repeat this same adjustment on the assumption that the network was "picked up" by the points C and F, so that A and B hang down from X, and D and E hang down from Y.

The details of this adjustment are shown in figure 31. The table of fixed elevations on page 119 still stands as shown, but in the table of links on page 119, link 4 becomes link 1, link 1 becomes link 2, and link 2 becomes link 4. The rest of the links remain as shown in the table. The changed link numbers are shown in parentheses in the table.

It will be seen from an inspection of the circuit closures shown in figure 31 that the closures are now identical with those shown in figure 28.

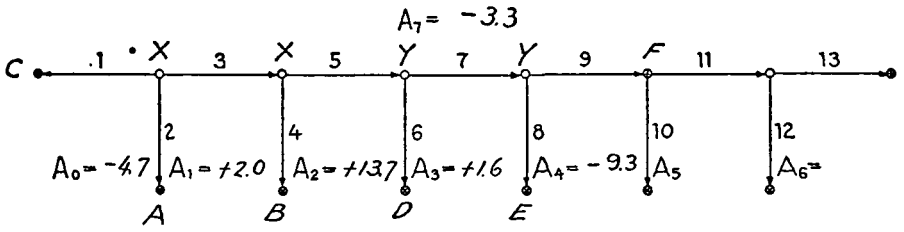
Applying the v's from figure 31 to the observed orthometric differences of elevation we determine adjusted differences of elevation, as follows:

B. M. minus B. M.	*Adj. Diff. of elevation*	*B. M. minus B. M.*	*Adj. Diff. of elevation*
X−C	−0.8369 m.	Y−D	+8.5027 m.
X−A	+2.0048 m.	Y−E	−13.2034 m.
X−B	−2.8843 m.	Y−F	−0.8301 m.
X−Y	−5.8903 m.		

The adjusted elevations of X and Y are again computed and checked, as follows:

C	13. 2141		F	19. 0976	
	−0. 8369			−0. 8301	
X	12. 3772		Y	18. 2675	
	−2. 0048			+13. 2034	
A	10. 3724	(check)	E	31. 4709	(check)
B	15. 2615		D	9. 7648	
	−2. 8843			+8. 5027	
X	12. 3772	(check)	Y	18. 2675	(check)
				−5. 8903	
			X	12. 3772	(check)

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



						$\frac{V}{l}$	Check
$l_1 = 17.4$ $l_2 = 20.8$		$A_0 =$	-4.7				
$E_1 = \frac{l_2 l_1}{l_1+l_2} =$	9.5	$K_1 = \frac{l_2 A_0}{l_1+l_2} =$	-2.6	$V_1 = -A_0 + V_2 =$	$+0.5$	$+0.029$	
$l_3 = 0.0$ $l_4 = 15.9$		$A_1 =$	$+2.0$	$V_2 = -A_1 + V_3 + V_4 =$	-4.2	-0.202	
$E_2 = \frac{l_4(E_1+l_3)}{(E_1+l_3+l_4)} =$	5.9	$K_2 = \frac{l_4(K_1+A_1)}{E_1+l_3+l_4} =$	-0.4	$V_3 = \frac{l_3(K_1+A_1-V_4)}{E_1+l_3} =$	0.0	0.000	
$l_5 = 30.5$ $l_6 = 10.3$		$A_2 =$	$+13.7$	$V_4 = -A_2 + V_5 + V_6 =$	-2.2	-0.138	
$E_3 = \frac{l_6(E_2+l_5)}{(E_2+l_5+l_6)} =$	8.0	$K_3 = \frac{l_6(K_2+A_2)}{E_2+l_5+l_6} =$	$+2.9$	$V_5 = \frac{l_5(K_2+A_2-V_6)}{E_2+l_5} =$	$+9.0$	$+0.295$	-0.016
$l_7 = 0.0$ $l_8 = 15.0$		$A_3 =$	$+1.6$	$V_6 = -A_3 + V_7 + V_8 =$	$+2.5$	$+0.243$	
$E_4 = \frac{l_8(E_3+l_7)}{(E_3+l_7+l_8)} =$	5.2	$K_4 = \frac{l_8(K_3+A_3)}{E_3+l_7+l_8} =$	$+2.9$	$V_7 = \frac{l_7(K_3+A_3-V_8)}{E_3+l_7} =$	0.0	0.000	
$l_9 = 21.5$ $l_{10} =$		$A_4 =$	-9.3	$V_8 = -A_4 + V_9 + V_{10} =$	$+4.1$	$+0.273$	
$E_5 = \frac{l_{10}(E_4+l_9)}{(E_4+l_9+l_{10})} =$		$K_5 = \frac{l_{10}(K_4+A_4)}{E_4+l_9+l_{10}} =$		$V_9 = \frac{l_9(K_4+A_4-V_{10})}{E_4+l_9} =$	-5.2	-0.242	-0.021
$l_{11} =$ $l_{12} =$		$A_5 =$		$V_{10} = -A_5 + V_{11} + V_{12} =$			
$E_6 = \frac{l_{12}(E_5+l_{11})}{(E_5+l_{11}+l_{12})} =$		$K_6 = \frac{l_{12}(K_5+A_5)}{E_5+l_{11}+l_{12}} =$		$V_{11} = \frac{l_{11}(K_5+A_5-V_{12})}{E_5+l_{11}} =$			
$l_{13} =$		$A_6 =$		$V_{12} = -A_6 + V_{13} =$			
				$V_{13} = \frac{l_{13}(K_6+A_6)}{E_6+l_{13}} =$			

FIGURE 31.—Form A, showing the computations involved in the readjustment of the net shown in figure 28 when "picked up" at C and F instead of at A and F.

It will now be seen that although 6 of the 7 v's computed as shown in figure 31 differ from the corresponding v's as shown in figure 30, the resulting adjusted elevations are still in agreement except for one in the last place (0.1 mm.) which, since the adjusted elevations of all bench marks are rounded to the nearest millimeter prior to publication, is of little practical importance. However, if greater refinement should be needed in some special cases, the carrying of more decimal places in the computations would serve to tighten the solution, but it should be noted that this would require the abandonment of the slide rule as a computing device.

It should also be noted that while the checks obtained by summing the numerical values of v/l around X and Y in figure 30 were both 0.000, this was undoubtedly due simply to the fact that the inaccuracies due to rounding in the computation tended to compensate completely. The corresponding checks shown in figure 31 are -0.016 and -0.021, respectively, and are larger than may ordinarily be expected. However, this happened to be an aggravated case in which several of the computed values were rounded to one decimal place from values very close to 5 in the second place. A repetition of the solution shown in figure 31 but with the E's, K's, and v's carried out to *three* decimal places, resulted in v's which, when rounded to one decimal place after the computation had been completed, agreed exactly in all 7 cases with those shown in figure 30.

The allowable divergence from 0.000 in these check sums will depend upon the number of significant figures carried throughout the computations and the relative size of the numerical values involved. Inasmuch as a rigorous determination of the allowable divergence is quite involved, it is suggested that in questionable cases, a change of one in the last place (and of opposite sign to that of the check sum) be arbitrarily made in the v of the shortest link involved in the particular check being investigated. If this arbitrary change in the v produces a new v/l such that the sign of the check sum changes when the new v/l is introduced, it may be accepted as proof that the original set of v's is correct. Actually the allowable divergence may be greater than that indicated by the above procedure, as in the case discussed just previously, but until the computer acquires sufficient experience to gauge the allowable divergence quickly and accurately, it is suggested that, if the indicated procedure fails to produce a change of sign in the check sum, all steps in the solution be carefully checked.

Another point which it will be well to discuss at this stage has to do with the mechanics of this type of computation. The arrangement of the computation form was designed to facilitate the computations. It is suggested that, during the computation, the numerical values be written over or under all terms given in parentheses in the first two columns, as these values are used again in the solution. Not only does the denominator of an E term reappear in the denominator of the corresponding K term, but also the second parts of the numerators of the E and K terms reappear in the v term on the same line. (In the examples shown the indicated sums are not written above or below the parenthetical items, as they would be in practice, to avoid possible difficulties in reducing the 8- by 10½-inch form to page size.) Note that E_2, E_3, etc., are each composed of fractions involving the three terms immediately above and that the corresponding K term is composed of the same fraction with the exception of the second term in the numerator which involves the two terms next above it.

In making a slide-rule computation of E_2 and K_2 in figure 31, a single setting suffices for the determination of both values (after the indicated summations have been made); i. e., divide l_4 by $(E_1+l_3+l_4)$ by setting 15.9 on the A scale opposite 25.4 on the B

scale, whereupon the value on the A scale which is opposite 9.5 on the B scale will be E_2 and the value on the A scale which is opposite 0.6 on the B scale will be K_2, numerically. Similarly the other E's and K's may be computed in pairs. (The C and D scales of the slide rule may be used instead of the A and B scales to obtain greater accuracy, but their use may require more manipulation of the slide.)

Up to this point in our discussion of the circuit-reduction method of adjustment we have only taken up cases in which a sketch on a geographical basis may be transformed quite simply to the conventionalized form as shown at the top of computation Form A. On the other hand, this system of adjustment is applicable to *any network of lines of levels in which each circuit contains one or two fixed points.* No circuit can have more than two fixed points, since the introduction of a third fixed point immediately breaks the one circuit up into two separate circuits. It often happens, however, that an inner circuit is encountered in which there are *no* fixed points and *to a limited extent only* can these be handled by the circuit-reduction method.

The method of handling some of the more complicated networks requires the use of certain auxiliary computation forms and slight modification of the use of Form A.

Let us now consider a network such as is illustrated in figure 32. For the sake of brevity and to eliminate a considerable amount of tabular work in the text, we will attempt in this and following problems to exhibit all data necessary for use in the adjustment on the sketch itself. In order to do this certain conventions must be adopted for the sake of clarity, and these are set forth below.

(1) Each circuit is given a Roman-numeral designation which is written on the sketch and within the circuit to which it applies.

(2) Each link is given either an Arabic-numeral designation or a designation consisting of an Arabic numeral followed by a lower-case letter, and these link designations are written on the sketch "below" and to the "left" along the line representing the link. (The words "below," "above," "left," and "right" as here used indicate relative positions of data and link when the sketch is oriented so that the data may be most conveniently read.)

(3) The length of each link is written along the line representing the link "below" and to the "right" and is expressed in kilometers.

(4) The difference of elevation (*from* bench mark *to* bench mark), in meters and with the proper sign, is written "above" each link.

(5) The arrow on each link is an indication of the "from—to" direction.

(6) Fixed points are indicated, as before, by means of circles with crosses in them, and adjustable junctions by means of open circles.

(7) The designation of each junction bench mark is written near the symbol representing it, and, at each fixed point, the fixed elevation, in meters, is also entered on the sketch along with the designation of the mark. (See p. 55.)

(8) The numerator of a fraction appearing within a circuit and near the Roman-numeral designation of the circuit is the closure of that circuit, expressed in millimeters and showing the proper sign.

(9) The denominator of each of these fractions is the sum of the lengths of the links involved in the circuit or, in other words, the length of the adjustable portion of the perimeter of the circuit.

To make sure that these conventions are clearly understood, refer to the sketch in figure 32. Link 4 lies between bench mark J 101 (fixed elevation=139.8872 m.) and bench mark N 150 (a point to be adjusted) and has a length of 20 kilometers. Its

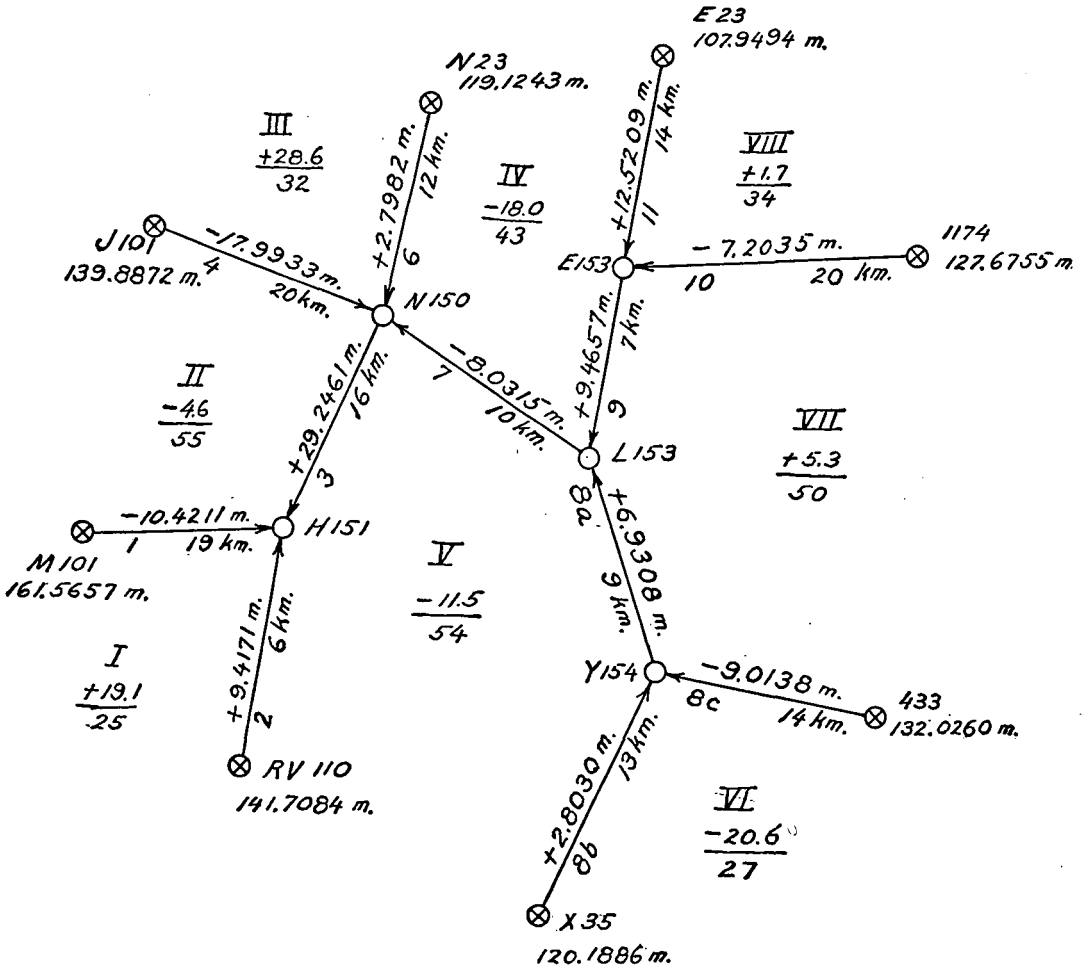


FIGURE 32.—Subordinate network, containing five new junctions and eight fixed points, to be adjusted by the circuit-reduction method.

difference of elevation is —17.9933 m., and the arrow indicates that this difference has been computed *from* J 101 *to* N 150. The combination of the minus sign and the direction in which the arrow points shows that N 150 is lower than J 101.

The system used in numbering the links will become evident later as this discussion develops.

Since we have already discussed the matter of "picking up" a network by such points as may be most convenient and letting certain links "fall over" to the opposite side of the "backbone line" we may now proceed to redraw our sketch on the assumption that it has been picked up by the points designated M 101 and E 23. The sketch now is as shown in figure 33. The circuit closures have been renumbered C_0, C_1, C_2, ... C_7, and recomputed when necessary owing to the changes brought about by letting certain links fall over to the opposite side of the "backbone line." It becomes evident now that we have already discussed the method of handling this situation, *except* insofar as it has been complicated by the introduction of the extra lines leading from Y 154 to X 35 and 433. If it were not for this new complication we could introduce link 5 as a "zero" link and proceed with the adjustment on Form A.

The proper treatment of the leveling along the lines radiating three ways from Y 154 calls for the use of Form C (p. 128), by means of which we can determine the length and "observed" difference of elevation for a single link (8) which, insofar as its effect on the whole adjustment is concerned, is the equivalent of the three links 8a, 8b, and 8c in figures 32 and 33.

Close examination of the sketches in the computation forms in figures 34 and 35 and the network sketches in figures 32 and 33 will reveal that the arrows on the computation forms are shown for all links in opposite directions to those for the corresponding links in the network sketches in figures 32 and 33. In the network sketches the use of arrows to point the B. M.-*to*-B. M. differences seems most reasonable and follows convention established in engineering literature. On the computation form the formulas were set up to derive the v's with correct sign for the B. M.-*minus*-B. M. differences

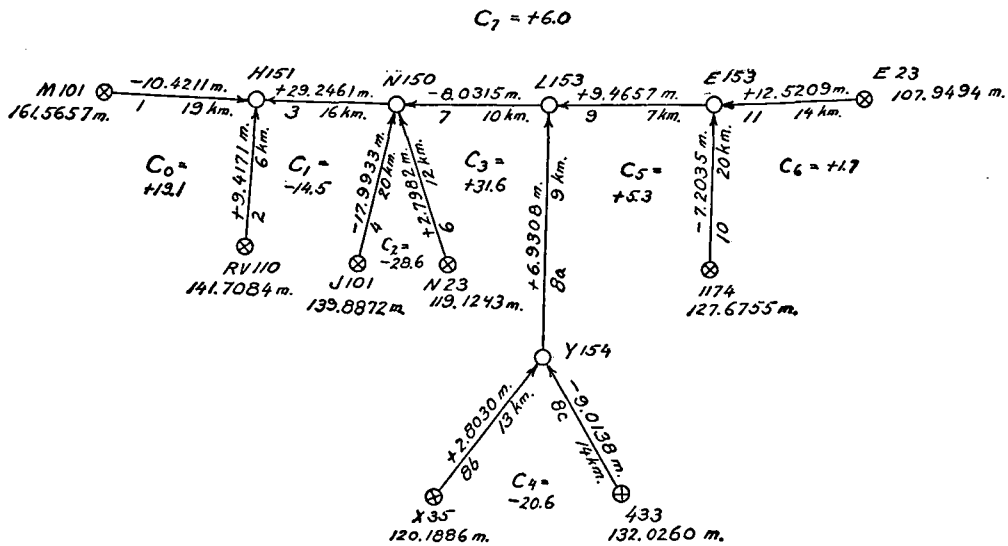


FIGURE 33.—The same net as shown in figure 32 after transformation by "picking it up" at M 101 and E 23.

to conform to office practice. Changing the sign of the derived v's on the computation form *once* to give correct values to apply to B. M.-*to*-B. M. differences, then changing them *again* to correct for the opposing directions of the arrows, results in allowing the set of v's as derived on the computation forms to be applied without change of sign to the B. M.-*to*-B. M. differences shown in figures 32 and 33.

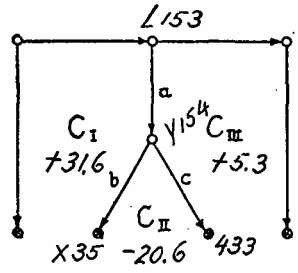
As shown in figure 34, we transfer the closures $C_3=+31.6$ mm., $C_4=-20.6$ mm., and $C_5=+5.3$ mm., from the sketch in figure 33 to the sketch in figure 34, renumbering them C_I, C_{II}, and C_{III}, and also enter them in the appropriate places to the right of the sketch. The lengths of the links 8a, 8b, and 8c are also entered as l_a, l_b, and l_c, respectively, at the appropriate locations on Form C.

Then, by means of the formulas printed on Form C, the values of l_x, A_I, and A_{II} are computed and entered on Form C. The l_x of Form C then becomes the length of link 8 in figure 35, and A_I and A_{II} become A_3 and A_4, respectively, of the same figure. We now have a single link (8) to replace 8a, 8b, and 8c, and may proceed with the adjustment by means of Form A as shown in figure 35.

This adjustment on Form A is shown completely in figure 35. The v_8 of figure 35 then becomes the v_x of figure 34, and v_a, v_b, and v_c are then computed and checked by

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT

FORM C.

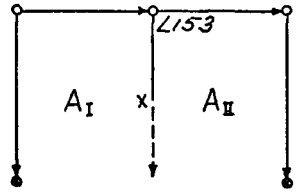


$l_a = 9 \qquad l_b = 13 \qquad l_c = 14$

$C_I = +31.6 \qquad C_{II} = -20.6 \qquad C_{III} = +5.3$

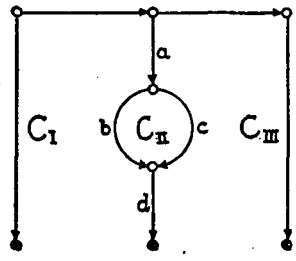
$l_x = l_a + \frac{l_b l_c}{l_b + l_c} = 15.7 \qquad K = \frac{l_b C_{II}}{l_b + l_c} = -9.9$

$A_I = C_I + K = +21.7 \qquad A_{II} = C_{II} + C_{III} - K = -5.4$



		$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} =$	+1.5	+0.167	
$V_b = -K + V_x - V_a =$	+11.1	+0.854	
$V_c = +C_{II} + V_b =$	-9.5	-0.679	+0.008

FORM D.

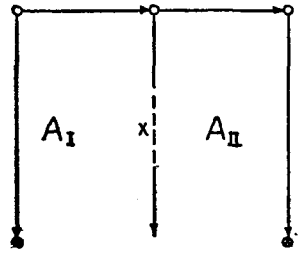


$l_a = \qquad l_b = \qquad l_c = \qquad l_d =$

$C_I = \qquad C_{II} = \qquad C_{III} =$

$l_x = l_a + l_d + \frac{l_b l_c}{l_b + l_c} = \qquad K = \frac{l_b C_{II}}{l_b + l_c} =$

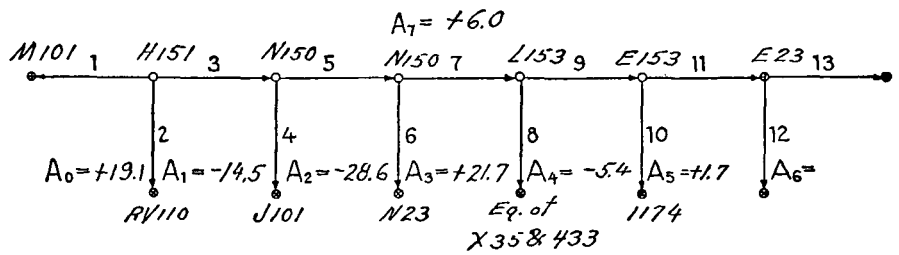
$A_I = C_I + K = \qquad A_{II} = C_{II} + C_{III} - K =$



	$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} =$		
$V_d = \frac{l_d V_x}{l_x} =$		
$V_b = -K + V_x - V_a - V_d =$		
$V_c = +C_{II} + V_b =$		

FIGURE 34.—Form C, showing the computations involved in obtaining a single-link equivalent for the three lines radiating from Y 154 in figures 32 and 33.

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



				V/l	Check
$l_1 = 19$ $l_2 = 6$	$A_0 = +19.1$				
$E_1 = \frac{l_2 l_1}{l_1+l_2} = 4.56$	$K_1 = \frac{l_2 A_0}{l_1+l_2} = +4.58$	$V_1 = -A_0+V_2 =$	-10.1	-0.532	
$l_3 = 16$ $l_4 = 20$	$A_1 = -14.5$	$V_2 = -A_1+V_3+V_4 =$	$+9.0$	$+1.500$	
$E_2 = \frac{l_4(E_1+l_3)}{(E_1+l_3+l_4)} = 10.14$	$K_2 = \frac{l_4(K_1+A_1)}{E_1+l_3+l_4} = -4.89$	$V_3 = \frac{l_3(K_1+A_1-V_4)}{E_1+l_3} =$	-15.6	-0.975	-0.007
$l_5 = 0$ $l_6 = 12$	$A_2 = -28.6$	$V_4 = -A_2+V_5+V_6 =$	$+10.1$	$+0.505$	
$E_3 = \frac{l_6(E_2+l_5)}{(E_2+l_5+l_6)} = 5.50$	$K_3 = \frac{l_6(K_2+A_2)}{E_2+l_5+l_6} = -18.15$	$V_5 = \frac{l_5(K_2+A_2-V_6)}{E_2+l_5} =$	0.0	0.000	
$l_7 = 10$ $l_8 = 15.7$	$A_3 = +21.7$	$V_6 = -A_3+V_7+V_8 =$	-18.5	-1.542	
$E_4 = \frac{l_8(E_3+l_7)}{(E_3+l_7+l_8)} = 7.80$	$K_4 = \frac{l_8(K_3+A_3)}{E_3+l_7+l_8} = +1.79$	$V_7 = \frac{l_7(K_3+A_3-V_8)}{E_3+l_7} =$	$+0.5$	$+0.050$	-0.012
$l_9 = 7$ $l_{10} = 20$	$A_4 = -5.4$	$V_8 = -A_4+V_9+V_{10} =$	$+2.7$	$+0.172$	
$E_5 = \frac{l_{10}(E_4+l_9)}{(E_4+l_9+l_{10})} = 8.51$	$K_5 = \frac{l_{10}(K_4+A_4)}{E_4+l_9+l_{10}} = -2.07$	$V_9 = \frac{l_9(K_4+A_4-V_{10})}{E_4+l_9} =$	-0.8	-0.114	$+0.008$
$l_{11} = 14$ $l_{12} =$	$A_5 = +1.7$	$V_{10} = -A_5+V_{11}+V_{12} =$	-1.9	-0.095	
$E_6 = \frac{l_{12}(E_5+l_{11})}{(E_5+l_{11}+l_{12})} =$	$K_6 = \frac{l_{12}(K_5+A_5)}{E_5+l_{11}+l_{12}} =$	$V_{11} = \frac{l_{11}(K_5+A_5-V_{12})}{E_5+l_{11}} =$	-0.2	-0.014	$+0.005$
$l_{13} =$	$A_6 =$	$V_{12} = -A_6+V_{13} =$			
		$V_{13} = \frac{l_{13}(K_6+A_6)}{E_6+l_{13}} =$			

FIGURE 35.—Form A, showing the computations involved in the adjustment of the net, shown in figure 32, after Form C has been used to produce the single link (8) which is equivalent to the three links radiating from Y 154 in figures 32 and 33.

summing the values of v/l on Form C. These then are the v's for links 8a, 8b, and 8c of figure 32, and the adjustment is complete except for the fact that it remains necessary to compute the adjusted elevations of the newly adjusted junction marks.

As a computer becomes thoroughly familiar with this method of adjustment, circuit closures such as those shown in the sketch in figure 33, when they differ from those in figure 32, may be quickly computed by making the proper combinations of the closures in the sketch drawn on a geographical basis. For example: $C_1=-I-II$; $C_2=-III$; and $C_3=+I+II+III+V$. Careful watch on the algebraic signs is necessary, and this short cut should not be used by the novice without checking the results by recomputation of the circuit closures until a thorough understanding of the possible pitfalls of this method of transforming circuit closures has been attained. Even then, in complicated cases, it is best and safest to completely recompute any circuit closures which change during a transformation of a sketch from one form to another.

The use of Form D is illustrated by the following adjustment of a network of lines of levels as shown on a geographical basis in figure 36.

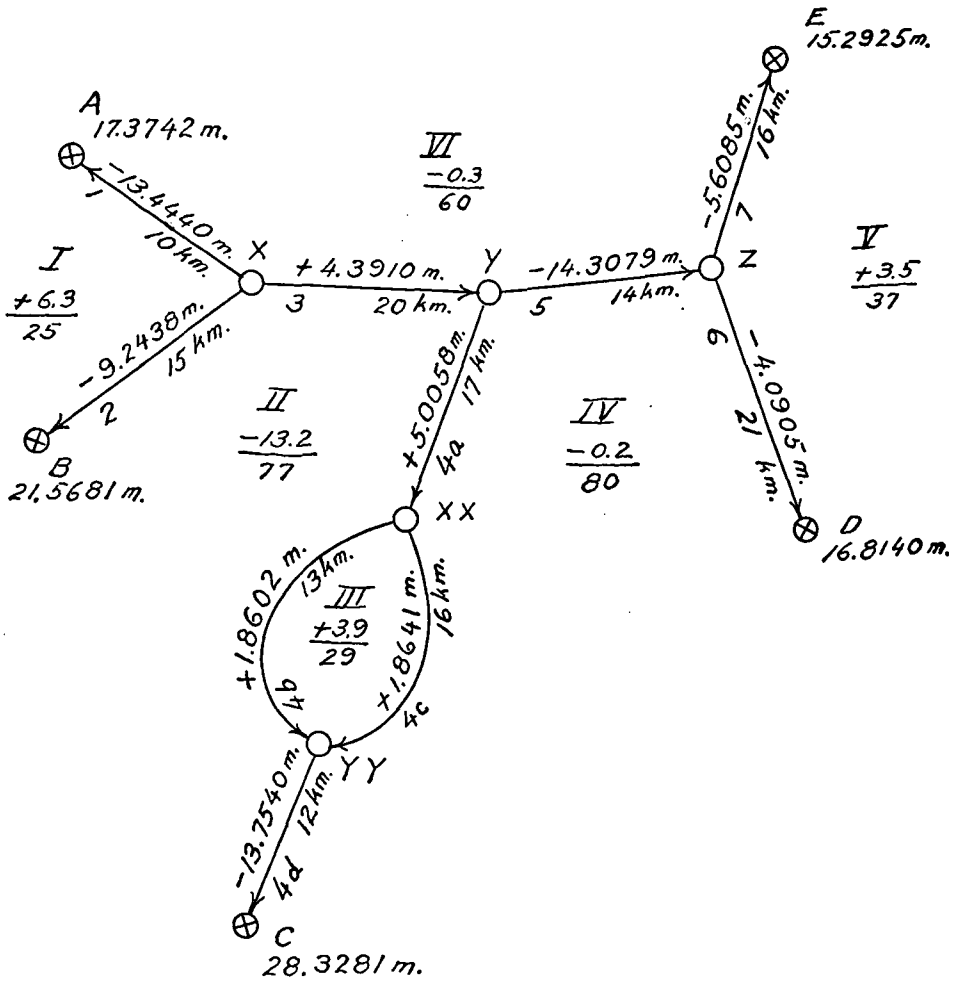
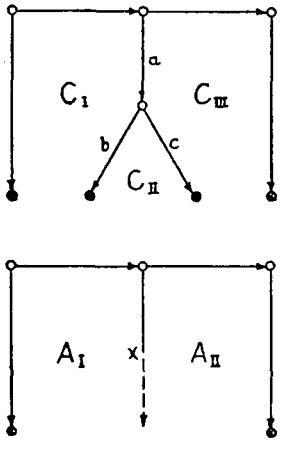


FIGURE 36.—Subordinate net; containing five new junctions, five fixed points, and one inner circuit having no fixed points; to be adjusted by the circuit-reduction method to illustrate the use of Form D.

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT

FORM C.



$l_a =$ $\quad l_b =$ $\quad l_c =$

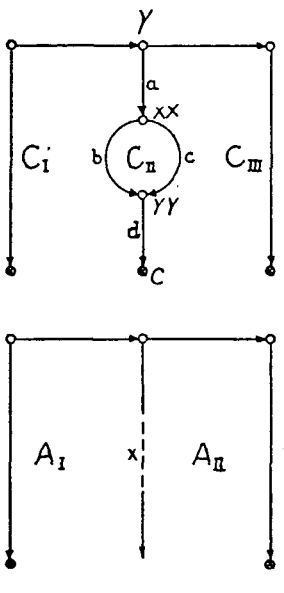
$C_I =$ $\quad C_{II} =$ $\quad C_{III} =$

$l_x = l_a + \frac{l_b l_c}{l_b + l_c} =$ $\quad K = \frac{l_b C_{II}}{l_b + l_c} =$

$A_I = C_I + K =$ $\quad A_{II} = C_{II} + C_{III} - K =$

	$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} =$		
$V_b = -K + V_x - V_a =$		
$V_c = +C_{II} + V_b =$		

FORM D.



$l_a = 17$ $\quad l_b = 13$ $\quad l_c = 16$ $\quad l_d = 12$

$C_I = -13.2$ $\quad C_{II} = +3.9$ $\quad C_{III} = -0.2$

$l_x = l_a + l_d + \frac{l_b l_c}{l_b + l_c} = 36.2$ $\quad K = \frac{l_b C_{II}}{l_b + l_c} = +1.7$

$A_I = C_I + K = -11.5$ $\quad A_{II} = C_{II} + C_{III} - K = +2.0$

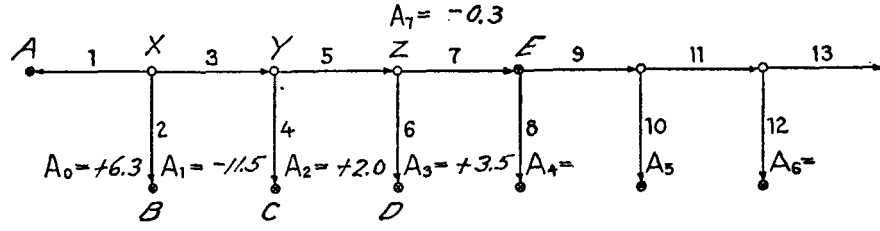
		$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} =$	-2.0	-0.118	
$V_d = \frac{l_d V_x}{l_x} =$	-1.4	-0.117	-0.001
$V_b = -K + V_x - V_a - V_d =$	-2.6	-0.200	
$V_c = +C_{II} + V_b =$	+1.3	+0.081	+0.002

FIGURE 37.—Form D, showing the computations involved in obtaining a single-link equivalent for the combination of links 4a, 4b, 4c, and 4d of figure 36.

We may now go directly to Form D to develop for use on Form A a single link which will be the equivalent of the combination of links 4a, 4b, 4c, and 4d as shown in figure 36.

In transferring data from figure 36 to Form D (fig. 37), closures II, III, and IV

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



						V/l	Check
$l_1 = 10$		$A_0 =$	$+6.3$				
$l_2 = 15$							
$E_1 = \frac{l_2 l_1}{l_1+l_2} =$	6.00	$K_1 = \frac{l_2 A_0}{l_1+l_2} =$	$+3.78$	$V_1 = -A_0 + V_2 =$	-1.7	-0.170	
$l_3 = 20$		$A_1 =$	-11.5	$V_2 = -A_1 + V_3 + V_4 =$	$+4.6$	$+0.307$	
$l_4 = 36.2$							
$E_2 = \frac{l_4(E_1+l_3)}{(E_1+l_3+l_4)} =$	15.13	$K_2 = \frac{l_4(K_1+A_1)}{E_1+l_3+l_4} =$	-4.49	$V_3 = \frac{l_3(K_1+A_1-V_4)}{E_1+l_3} =$	-2.6	-0.130	$+0.007$
$l_5 = 14$		$A_2 =$	$+2.0$	$V_4 = -A_2 + V_5 + V_6 =$	-4.3	-0.119	
$l_6 = 21$							
$E_3 = \frac{l_6(E_2+l_5)}{(E_2+l_5+l_6)} =$	12.20	$K_3 = \frac{l_6(K_2+A_2)}{E_2+l_5+l_6} =$	-1.04	$V_5 = \frac{l_5(K_2+A_2-V_6)}{E_2+l_5} =$	-0.2	-0.014	-0.003
$l_7 = 16$		$A_3 =$	$+3.5$	$V_6 = -A_3 + V_7 + V_8 =$	-2.1	-0.100	
$l_8 =$							
$E_4 = \frac{l_8(E_3+l_7)}{(E_3+l_7+l_8)} =$		$K_4 = \frac{l_8(K_3+A_3)}{E_3+l_7+l_8} =$		$V_7 = \frac{l_7(K_3+A_3-V_8)}{E_3+l_7} =$	$+1.4$	$+0.088$	$+0.002$
$l_9 =$		$A_4 =$		$V_8 = -A_4 + V_9 + V_{10} =$			
$l_{10} =$							
$E_5 = \frac{l_{10}(E_4+l_9)}{(E_4+l_9+l_{10})} =$		$K_5 = \frac{l_{10}(K_4+A_4)}{E_4+l_9+l_{10}} =$		$V_9 = \frac{l_9(K_4+A_4-V_{10})}{E_4+l_9} =$			
$l_{11} =$		$A_5 =$		$V_{10} = -A_5 + V_{11} + V_{12} =$			
$l_{12} =$							
$E_6 = \frac{l_{12}(E_5+l_{11})}{(E_5+l_{11}+l_{12})} =$		$K_6 = \frac{l_{12}(K_5+A_5)}{E_5+l_{11}+l_{12}} =$		$V_{11} = \frac{l_{11}(K_5+A_5-V_{12})}{E_5+l_{11}} =$			
$l_{13} =$		$A_6 =$		$V_{12} = -A_6 + V_{13} =$			
				$V_{13} = \frac{l_{13}(K_6+A_6)}{E_6+l_{13}} =$			

FIGURE 38.—Form A, showing the computations involved in the adjustment of the net shown in figure 36, after Form D has been used to produce a single-link equivalent for links 4a, 4b, 4c, and 4d of figure 36.

become C_I, C_{II}, and C_{III}, respectively, and the lengths of links 4a, 4b, 4c, and 4d become the lengths of links a, b, c, and d, respectively.

Solving the formulas on the upper portion of the right half of Form D results in numerical values for l_x, A_I, and A_{II} which carry over to Form A (fig. 38) as l_4, A_1, and A_2, respectively.

The remainder of the data to be entered on Form A before the computations are started comes directly from the sketch in figure 36.

The computations on Form A are next completed and checked. Then the v_4 of Form A becomes the v_x of Form D, and v_a, v_b, v_c, and v_d are then computed and checked by means of the formulas on the lower part of the right half of Form D.

All that remains to be done is to compute and check the adjusted elevations of the junction bench marks X, Y, Z, XX, and YY.

In the case just discussed, since the arrows in the sketch in figure 36 point the directions of the B. M.-*to*-B. M. differences and for each corresponding link coincide with the directions of the arrows on the computation forms, which point B. M.-*minus*-B. M. directions, the v's as computed on Forms A and D (figs. 38 and 37) must be changed to the opposite sign in each case before they are applied to the B. M.-*to*-B. M. observed orthometric differences of figure 36 to obtain the adjusted differences in the B. M.-*to*-B. M. direction.

To illustrate the use of Form B, consider a network such as is shown in figure 39. Proceeding directly from the sketch in figure 39 to Form B (fig. 40), circuit closures I, II, and III become A_0, A_1, and A_2, respectively, the link lengths are entered on Form B in the appropriate places, and the computations are completed as indicated by means of the formulas printed on the computation form.

As with Form A, only that portion of the form which is needed is used. In this case links 8, 9, and 10 are assumed to have zero lengths or zero differences of elevation.

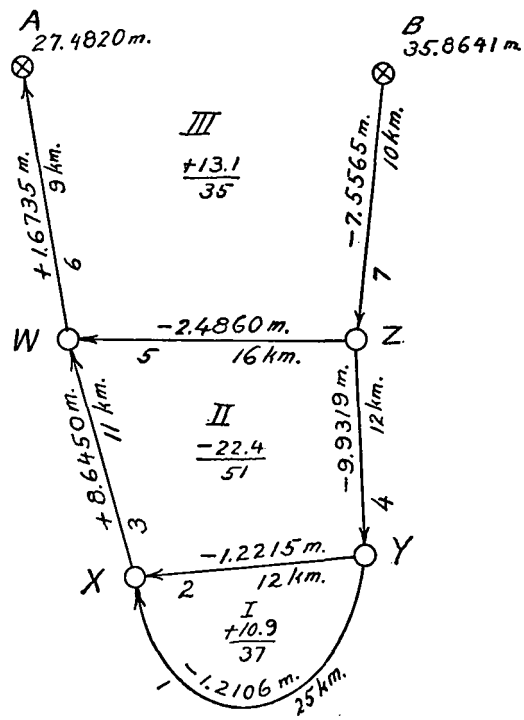
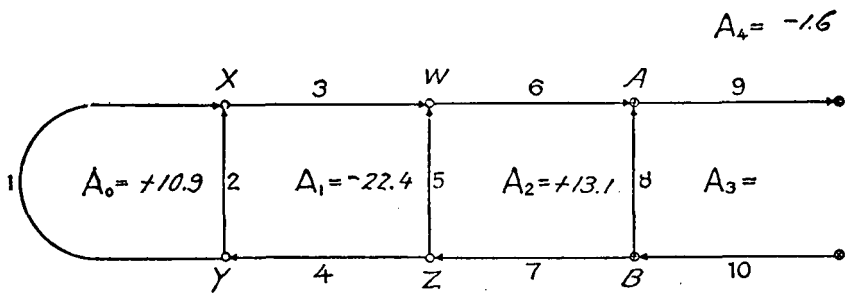


FIGURE 39.—Subordinate net; containing four new junctions, two fixed points, and two circuits having no fixed points; to illustrate the use of Form B.

The complete adjustment of the network shown in figure 39 appears in figure 40 and, as before, all that remains after the work on Form B is finished is to compute and check the adjusted elevations of the newly adjusted junction bench marks, using the adjusted differences determined by applying with reversed signs (see above) the corrections (v's) computed on Form B to the observed orthometric differences of elevation shown in figure 39.

So far, in our discussion of the application of the circuit-reduction method to the adjustment of leveling, we have handled no cases of circuits having no fixed points in them, except the relatively simple ones used to illustrate the use of Form B and Form

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM B.



				V/l	Check
$l_1 = 25$	$A_0 = +10.9$		$V_1 = +A_0 + V_2 =$ +4.8	+0.192	
$l_2 = 12$					
$E_1 = \frac{l_2 l_1}{l_1 + l_2} =$ 8.11	$K_1 = \frac{l_2 A_0}{l_1 + l_2} =$ +3.54		$V_2 = +A_1 - V_3 - V_4 + V_5 =$ -6.1	-0.508	
$l_3 = 11$			$V_3 = \frac{l_3(K_1 + A_1 + V_5)}{E_1 + l_3 + l_4} =$ -3.6	-0.327	-0.011
$l_4 = 12$	$A_1 = -22.4$		$V_4 = \frac{l_4(K_1 + A_1 + V_5)}{E_1 + l_3 + l_4} =$ -3.9	-0.325	+0.009
$l_5 = 16$					
$E_2 = \frac{l_5(E_1 + l_3 + l_4)}{(E_1 + l_3 + l_4 + l_5)} =$ 10.57	$K_2 = \frac{l_5(K_1 + A_1)}{E_1 + l_3 + l_4 + l_5} =$ -6.41		$V_5 = +A_2 - V_6 - V_7 + V_8 =$ +8.8	+0.550	
$l_6 = 9$			$V_6 = \frac{l_6(K_2 + A_2 + V_8)}{E_2 + l_6 + l_7} =$ +2.0	+0.222	-0.001
$l_7 = 10$	$A_2 = +13.1$		$V_7 = \frac{l_7(K_2 + A_2 + V_8)}{E_2 + l_6 + l_7} =$ +2.3	+0.230	-0.005
$l_8 =$			$V_8 = +A_3 - V_9 - V_{10} =$		
$E_3 = \frac{l_8(E_2 + l_6 + l_7)}{(E_2 + l_6 + l_7 + l_8)} =$	$K_3 = \frac{l_8(K_2 + A_2)}{E_2 + l_6 + l_7 + l_8} =$				
$l_9 =$			$V_9 = \frac{l_9(K_3 + A_3)}{E_3 + l_9 + l_{10}} =$		
$l_{10} =$	$A_3 =$		$V_{10} = \frac{l_{10}(K_3 + A_3)}{E_3 + l_9 + l_{10}} =$		

FIGURE 40.—Form B, showing the computations involved in the adjustment of the network shown in figure 39.

D. However, this method *can* be applied to even more complicated networks, though the practical limit seems to have been reached when a network requires the reduction of about 15 circuits or includes an inner circuit without fixed points and having from 5 to 7 links. This limit would vary according to the equipment available in a given office. Where computing machinery is available it would probably be most economical to use either the method of condition equations or the method of observation equations on any network having more than about 15 circuits or in which a multilink inner circuit is encountered. On the other hand, in a small office where it is a case of using either a slide rule or logarithms, it might be economical to push the use of the circuit-reduction method farther in order to be able to make use of the slide rule as a computing device.

In order to make the material a matter of permanent record, as well as to furnish the necessary information to anyone wishing to explore the application of this method to more complicated networks, we have included in appendix E a general discussion of the method of attack upon multilink inner circuits without fixed points and an example of the application of the method.

OTHER METHODS OF ADJUSTMENT

The author wishes it to be clearly understood that the methods for the adjustment of networks of lines of leveling discussed in detail in the preceding pages are by no means the only methods available for such use. The three methods already discussed seem to be the ones best adapted for use in the Section of Leveling of the United States Coast and Geodetic Survey as that section is staffed and equipped at the present time. Changes in our methods have taken place in the past two decades, and there is no logical reason to expect that no changes will be made in the future.

There is given below a partial list of papers pertaining to the subject of the adjustment of leveling. This is included for the use of any persons who may wish to explore this subject further. It is entirely possible that such further study may bring to light knowledge of methods which are better adapted to the needs of other organizations when conditions, especially as regards available staff and equipment, are such that the methods emphasized in this publication are found to be unsuited to their use.

The Adjustment of a Level Net by George H. Dell, Paper No. 1940 (including discussion) Transactions, American Society of Civil Engineers [vol. 101 (1936), pp. 834–856].

An Improved Method for Adjusting Level and Transit Surveys by Clarence Norris and Julius L. Speert, Paper No. 2087 (including discussion) Transactions, American Society of Civil Engineers [vol. 105 (1940), pp. 1376–1411].

Survey-net Adjustment by Electrical Analogue by Julius L. Speert, Transactions, American Geophysical Union [vol. 28, No. 6 (December 1947), pp. 831–837].

Additional references will be found in these papers, either in footnotes or, in the case of the paper by Speert in the Transactions of the American Geophysical Union, in the appended bibliography.

DISTRIBUTION OF CORRECTIONS

In the last part of the section of this manual which deals with preliminary computations (see pp. 49 to 52, inclusive) we discussed the preparation of the summary sheets, up to the point where they are laid aside to await the adjustment of the leveling. Now that we have covered the matter of adjustment, we are ready to discuss the completion of the summary sheets. This work consists of distributing the adjustment

corrections throughout the various lines, the computation of the adjusted elevations in meters, and the conversion of the metric elevations to feet.

There is shown, in figure 41, a completed summary sheet for the releveling of a short line of second-order levels "Murphy to 7 miles north of Murphy, Idaho."

The summary sheets for a line of levels are numbered as shown near the upper left-hand corner of figure 41. For a three-sheet job the sheets would be numbered 1 of 3, 2 of 3, and 3 of 3.

It will be noticed that the "no-check spur" from X 72 to W 72 has been crossed out. This shows that a satisfactory check at the beginning of the line was obtained as

SUMMARY SHEET 932 HGZ. 1945 S

Releveling - Murphy to 7 mi. N. of Murphy, Idaho

Stewart - 1945

SECOND ORDER L-11362

Sheet 1 of 1

	B.M.	Distance (km.)	Obs. Elev. (m.)	Ortho. Corr'n.	Adj. Corr'n.	Adjusted Elev. (m)	(ft.)	
	~~W 72 (S/No/V)~~	~~0~~	~~870.1310~~	~~0.0000~~	(See: Nampa, Idaho, to Jordan Valley, Oregon)			
Jct.	X 72	0	848.3430	0.0000	-0.0691	848.273[9]*	2783.046	* Jordan Valley, Ore, Supp. Adj. 1947
	Rd x-ing (S/No/V)	0	842.6923	0	- 691	842.623	2764.506	1 mi. S. of F 300
	F 300	2	817.2076	-0.0013	- 675	817.139	2680.897	
	℄ track (S/No/V)	2	815.1773	- 13	- 675	815.108	2674.233	At F 300
	G 300	3	799.1911	- 25	- 667	799.122	2621.786	
	Y 72 (S/No/V)	3	797.5179	- 25	- 667	797.449	2616.297	
	℄ track (S/No/V)	3	783.3739	- 37	- 667	783.304	2569.890	0.4 mi. S. of H 300
	H 300	5	774.1656	- 37	- 651	774.097	2539.683	
	J 300	6	756.7020	- 37	- 643	756.634	2482.390	
	Bridge (S/No/V)	6	760.9537	- 37	- 643	760.886	2496.340	At J 300
	K 300	9	727.6600	- 49	- 618	727.593	2387.111	
	℄ track	9	721.1456	- 49	- 618	721.079	2365.740	0.5 mi. N. of K 300
	F 295	10	696.6322	- 60	- 610	696.565	2285.314	
Jct.	P 73	12	697.1438	-0.0071	-0.0594	697.077[3]*	2286.993	
			10837.9060	-0.0453	-0.9108	10836.950	35554.226	
						.9499	.2264	

$\frac{+.97}{12} = \pm 0.08333$

FIGURE 41.—Completed summary sheet for the short line of levels, "Murphy to 7 miles north of Murphy, Idaho."

a result of the first running between bench marks X 72 and W 72. The junction detail was presumably quite similar to that shown in figure 15 (p. 52). Both bench marks (X 72 and W 72) are in the line "Nampa, Idaho, to Jordan Valley, Oregon," and, a check having been obtained between these two marks, only the one (X 72) from which the line of levels extends northward from Murphy is retained in this summary.

The adjusted elevation of W 72 would have to be obtained from the summary sheets for the line "Nampa, Idaho, to Jordan Valley, Oreg."

The asterisks above the adjusted metric elevations of X 72 and P 73 both refer to the note in the remarks column and show that these two adjusted elevations were derived from the "Jordan Valley, Oreg., Supplementary Adjustment of 1947."

The adjusted elevation of X 72 is shown as 848.273[9] for the reason that unless the adjusted metric elevations of the junction points are shown on the summary sheets to

the fourth decimal place in meters no *accurate* check on the distribution of the correction through the line would be obtainable. The "9" being set up as a sort of exponent is a signal to the person charged with preparing the material for publication that the elevation to be published is, according to the rules for rounding, 848.274 meters.

The preliminary elevation of X 72 is shown to be 848.3430; the orthometric correction is 0.0000; and this would make the orthometric elevation also 848.3430 meters. This is 0.0691 meter too high to agree with the adjusted elevation of X 72, so the adjustment correction for that mark is −0.0691 meter.

Referring now to the bottom of figure 41, we find that the observed orthometric elevation of P 73 is 697.1438−0.0071=697.1367 meters and that the adjustment correction necessary to make that elevation agree with the adjusted elevation of P 73 is −0.0594 meter.

Since, at any junction bench mark,

Obs. Elev.+Ortho. Corr'n.+Adj. Corr'n.=Adj. Elev.,

it follows that the unknown in this equation (the adjustment correction) may be computed as follows:

Adj. Corr'n.=Adj. Elev.−Obs. Elev.−Ortho. Corr'n.

In computing the required adjustment correction at any particular junction bench mark, strict attention to the algebraic signs of all items must be the rule. Negative elevations of considerable size are encountered in Death Valley, Calif., and negative elevations of small size *may* be encountered at times when a line of levels passes along or close to the shore of the ocean or a tidal estuary.

Since the observed orthometric difference of elevation *from* X 72 *to* P 73 (B. M. P 73 *minus* X 72) is 697.1367−848.3430=−151.2063 and the adjusted difference is 697.0773−848.2739=−151.1966, the adjustment correction for the link from X 72 to P 73 must have been +0.0097 meter in order to change the observed orthometric difference to the adjusted difference, thus: −151.2063+0.0097=−151.1966. Upon checking back to the adjustment computations, it was found that the v for this link was determined to be +9.7 mm. This checks the findings outlined above, and we are ready to proceed with the distribution of the adjustment correction throughout the line in proportion to the distances of the various intermediate bench marks from junction bench mark X 72.

The rate of correction throughout this link is seen to be +9.7/12=+0.8083333 mm./km. as shown in the left-hand margin of figure 41. The number of decimal places to which the rate should be computed is dependent to some extent on the size of the discarded portion of the rate when rounding to a given number of decimal places and on the length of the line or link to which the computed rate applies. Obviously the discarded portion of the computed rate can amount to as much as 5 in the first of the dropped places. On a line 100 km. in length this would accumulate during the computation of the adjustment corrections for the intermediate bench marks to as much as 5 in the next to the last place retained. As will be seen from the following discussion, it is just as easy to carry plenty of places as it is to carry a barely sufficient number of places, and, since this extra tightness in corrections is secured at little or no extra cost in time and labor, it is well to compute the rates to at least six decimal places.

It is the practice in the Section of Leveling to compute the rates to a uniform number of places except perhaps when special conditions require extra places on some particularly long line. The computer who works out the rate originally should record

that rate in the left-hand margin of the summary sheet as shown in figure 41. The checker should then check the rate to the same number of places, and both should use all places recorded in the margin in computing and checking the adjustment corrections for the intermediate bench marks along the line. If this is not done, disagreements between computer and checker over one in the last place will occur fairly frequently and cause considerable lost motion and confusion over numerous small changes which are of no practical importance.

The following detailed discussion of the computation of the adjustment corrections for the intermediate bench marks along the line from Murphy to 7 miles north of Murphy, Idaho, has been deliberately written around the practice necessary when one of the least adaptable of the calculating machines is used for the work. The reason for doing this is that if the method can be explained in such a way that a novice is able to follow through the routine correctly with the least adaptable equipment he certainly will be able to adapt the routine to machines having features which make the work simpler rather than more complicated.

The machine used as the basis for the following discussion is a desk-model calculating machine having a 10-bank keyboard, 10 multiplier dials, 20 product dials, complete carry-over in the product dials, no carry-over in the multiplier dials, and red figures showing in the multiplier dials as a result of the use of the minus bar after a complete clearing and new start with the machine.

NOTE.—It should be remembered that throughout the following discussion the banks of the keyboard and the windows showing the figures for both the multiplier and product dials are always assumed to be numbered from right to left across the machine.

With the carriage of the calculating machine set over to the left as far as it will go and all dials and the keyboard cleared, the following is a step-by-step description of the routine for distributing the adjustment correction through the line shown in figure 41.

Set the decimal-point indicator on the keyboard between banks 7 and 8. This places the decimal point so that all items punched on the keyboard must be expressed in millimeters. Next, punch 69.1 into the keyboard, this being the adjustment correction for X 72 expressed in millimeters. Press the plus bar down once and the numbers 691 will show in the 9, 8, and 7 windows of the product dials. It is assumed that the machine is set for "repeat," so we will now have to clear the keyboard manually. We will also have to clear the multiplier dials of the "1" which appeared in the No. 1 window as a result of our striking the plus bar once.

Now it will be convenient to set *two* decimal-point indicators in the row of product dials so that one of them will be between windows 10 and 11 and the other between windows 6 and 7.

This leaves windows 7 to 10, inclusive, of the product dials between the two decimal indicators, and these columns now represent the first four decimal places in meters. If all has been carried out correctly up to this point, the first 11 windows of the product dials will read as follows with the decimal-point indicators in the positions shown by the decimal points:

0.0691.000000

and it will be readily seen that this represents the adjustment correction, expressed in meters, for bench mark X 72, the starting bench mark for the line.

Next, punch the rate into the keyboard in the first seven columns so that the whole keyboard setting will be as follows: 000.8083333.

Since a negative product shows up in the product-dial windows as the arithmetic complement of the negative product and we are dealing with a negative correction, we put the −0.0691 into the machine as a positive quantity for convenience and must remember that all successive adjustment corrections which we determine must be entered on the summary sheet as negative corrections even though they show as positive on the calculating machine.

Since the distance column on the summary sheet shown in figure 41 shows the distance to "Rd. X-ing" also to be "0," the adjustment correction "−0.0691" is repeated for that point.

The distance to bench mark F 300 is 2 km. By striking the minus bar twice we get a red 2 showing in the first window of the row of multiplier dials and cause the product dials to read "0.0674.833334." The adjustment correction for F 300 is −0.0675 meter, and this same correction is repeated on the next line since the distance to that point is also 2 km.

Pressing the minus bar once more causes a red 3 to show in the first window of the multiplier dials, and the product dials then read "0.0666.750001." The adjustment correction for G 300 is −0.0667 meter, and this correction is repeated on the two following lines since the two following distances are also 3 km.

This process is continued until we arrive at K 300, when a red 9 shows in the first window of the multiplier dials, and the product dials read "0.0618.250003." The adjustment correction for K 300 is −0.0618 meter.

Since the machine we are using does not have complete carry-over in the multiplier dials, another depression of the minus bar would cause a *black* 9 to show in the first window of the multiplier dials when we should have a red 1 in the second window and a zero in the first window of that row. Accordingly we use the plus bar to back the red 9 in the first window down to a zero, move the carriage one place to the right, and strike the minus bar once. We then have a red 1 in the second window of the multiplier dials and a zero in the first of those windows, while the product dials now read "0.0610.-166670" and the adjustment correction for F 295 is −0.0610, as shown in figure 41.

Next, we shift the carriage back to its original position, strike the minus bar twice, and find the multiplier dials showing a red 12. The product dials then read "0.0594.-000004," and the adjustment correction for P 73 is −0.0594 meter. This agrees exactly with the entry already on the summary sheet except for the "4" in the first window of the product dials. This checks the distribution of the adjustment correction through the line, and the distribution is completed. The check on this work is the complete recomputation of all items by another computer who uses, if possible, another calculating machine for his work.

The procedure outlined above, with suitable modifications in the use of the plus and minus bars and the mental processes necessary to keep track of the signs of the corrections, will do for all four of the possible combinations of positive or negative starting corrections with positive or negative rates, when there is no change in the sign of the adjustment corrections before the end of the line is reached.

The following is an outline of the procedure to be used when the adjustment correction changes sign during the distribution. This procedure is based on the assumption that the same machine is in use. If a more adaptable machine is available there will be certain possible short-cuts which will be obvious to an experienced machine operator.

789223°—49——10

In order to make the following discussion as brief as possible, let us assume a starting correction of −0.0026 meter for a bench mark at 0 km. along the line; intermediate bench marks at 5, 7, 9, 13, and 19 km. along the line; and an adjustment correction of +0.0039 meter at the end of the line where the distance is 21 km. from the starting bench mark. The rate is then computed as follows: 6.5/21=+0.3095238 mm./km.

We punch the starting correction (2.6 mm.) on the keyboard and transfer it to the product dials by means of a stroke of the plus bar. The product dials then read "0.0026.000000." Clear the keyboard and the multiplier dials. The rate is then punched on the keyboard.

Since we have a negative starting correction and a positive rate, the starting negative correction (positive in the machine) must become smaller as we pass along the line. Therefore, we use the minus bar to build up the proper distances in the multiplier dials.

When we build up to a red 5 on the multiplier dials, the product dials read "0.0010.-523810" and the adjustment correction for the mark which is 5 km. along the line is −0.0011 meter. Building up to a red 7 in the multiplier dials, the product dials read "0.0004.333334" and the adjustment correction is −0.0004 meter for the bench mark 7 km. along the line. We strike the minus bar once more to bring a red 8 in the multiplier dials and find we have not yet reached the change of sign in the adjustment correction.

Striking the minus bar once more brings a red 9 in the multiplier dials and we find that the product dials have developed a string of 9's all the way out to the left-hand end of the carriage. Reading the product dials in the same columns as before we find that they read "9.9998.142858." Clear the keyboard and punch the arithmetic complement of the reading of the product dials on the keyboard. The keyboard will then read "000.1857142" and a single stroke of the plus bar will cause the multiplier dials to all read zero. This is a check on the taking of the arithmetic complement and its placement on the keyboard. Striking the plus bar once more causes the product dials to read "0.0001.857142," and the adjustment correction for the bench mark 9 km. along the line is +0.0002 meter.

We have upset the distance showing in the multiplier dials by striking the plus bar twice since the red 9 showed in the multiplier dials, and they now show a red 7 when we have actually progressed 9 km. along the line. Further, since we are now going to use the plus bar to build up our distances in the multiplier dials, we should have a black 9 in the multiplier dials at this point. This is accomplished as follows:

Clear the keyboard; use the plus bar to back the red 7 out of the multiplier dials and to build up a black 9 in the multiplier dials; punch the rate back into the keyboard; and then proceed to transform the black 9 of the multiplier dials to a black 13.

The product dials will then read "0.0014.238094," and the adjustment correction is +0.0014 for the mark 13 km. along the line.

Building the multiplier dials up to read a black 19 causes the product dials to read "0.0032.809522," and the adjustment correction for the mark 19 km. along the line is +0.0033 meter. Then, for a check, build up a black 21 in the multiplier dials and the product dials should read "0.0038.999998," which rounds to +0.0039 and checks the adjustment correction assumed for the end of the line.

This same routine, with only slight modifications, would be used to pass from a positive starting correction through a change of sign to a final correction of negative sign.

Summary sheets will be encountered frequently on which the starting distance will not be zero as in the two examples discussed in the preceding paragraphs. In such cases it is only necessary to build up (with the keyboard cleared, of course) the proper starting distance in the multiplier dials, instead of starting the distribution with those dials reading zero. Also, in such cases the denominator of the "rate fraction" is length of line from starting to ending bench marks of the portion through which the correction is being distributed, not the distance entry opposite the end bench mark as might be assumed from the two examples already discussed.

COMPUTATION OF ADJUSTED ELEVATIONS

When the adjustment corrections have been computed and entered on the summary sheet, the adjusted elevations of the intermediate marks are then computed by summing for each mark, with due regard for algebraic signs, the observed elevation, the orthometric correction, and the adjustment correction, thus, for G 300 (fig. 41):

$$\begin{array}{r} +799.1911 \text{ meters} \\ -0.0025 \text{ meter} \\ -0.0667 \text{ meter} \\ \hline +799.1219 \text{ meters} \end{array}$$

which rounds to 799.122 meters and is so entered on the summary sheet.

Next, each metric elevation, to three decimal places in meters, is multiplied by the conversion factor (3.2808333) to convert the metric elevations to feet. The resulting elevations in feet are rounded to three decimal places and entered on the summary sheet, thus, for G 300:

$$799.122 \times 3.2808333 = 2{,}621.786 \text{ feet.}$$

FINAL CHECK ON SUMMARY SHEETS

All operations on the summary sheets as so far outlined are to be checked independently by another computer after the original work has been completed. Then, as a final check of these sheets, the following operations are performed.

Run up totals on the adding-and-subtracting machine for columns 3 to 7, inclusive, including in the totals only the unscratched entries and making the additions sheet-by-sheet throughout the line.

The sum of the totals for columns 3, 4, and 5 should then be closely equal to the total for column 6. The total for column 6, when multiplied by 3.2808333 should be closely equal to the total for column 7. Ordinarily these checks will not be absolutely exact, due to the accumulation of small errors in rounding, but they are exact enough to be of great help in picking up blunders that may have been made in the calculations or in copying resulting data from the machine to the sheets.

The whole series of processes has now been covered and the adjusted elevations are ready for the use of the persons who will combine them with the descriptions of the marks and prepare the data for publication. Since editing and publication of results do not properly come within the scope of computation and adjustment, these features are not covered in this manual.

TABLES

TABLE I.—*Total correction for curvature and refraction*

[This table of total correction for curvature and refraction is for use in computing "*C*" and in general wherever the total correction is required. In computing this table, the refraction was assumed to be equal to one-eighth the curvature.]

Distance (meters)	Correction to rod reading (mm.)	Distance (meters)	Correction to rod reading (mm.)	Distance (meters)	Correction to rod reading (mm.)	Distance (meters)	Correction to rod reading (mm.)
0 to 27	0. 0	106 to 112	−0. 8	160	−1. 8	240	−4. 0
28 to 47	−0. 1	113 to 118	−0. 9	170	−2. 1	250	−4. 3
48 to 60	−0. 2	119 to 124	−1. 0	180	−2. 3	260	−4. 7
61 to 72	−0. 3	125 to 130	−1. 1	190	−2. 6	270	−5. 0
73 to 81	−0. 4	131 to 136	−1. 2	200	−2. 8	280	−5. 4
82 to 90	−0. 5	137 to 141	−1. 3	210	−3. 0	290	−5. 8
91 to 98	−0. 6	142 to 146	−1. 4	220	−3. 3	300	−6. 2
99 to 105	−0. 7	147 to 150	−1. 5	230	−3. 7		

TABLE II.—*Orthometric and dynamic corrections.*

[For explanation of the use of the tables, see pp. 43 and 159]

[C_ϕ is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C_ϕ and log D_1. In south latitudes, if ϕ is taken negative, C_ϕ is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_ϕ	Log C_ϕ	D_1	Log D_1	Latitude ϕ	C_ϕ	Log C_ϕ	D_1	Log D_1
° ′					° ′				
0 00	0	−∞	+2637	7. 4211	4 00	+214	3. 3294	+2611	7. 4169
10	+ 9	1. 9506	+2637	7. 4211	10	+222	3. 3470	+2609	7. 4165
20	+ 18	2. 2516	+2637	7. 4211	20	+231	3. 3640	+2607	7. 4161
30	+ 27	2. 4277	+2637	7. 4210	30	+240	3. 3802	+2605	7. 4157
40	+ 36	2. 5526	+2636	7. 4210	40	+249	3. 3959	+2602	7. 4153
50	+ 45	2. 6495	+2636	7. 4209	50	+258	3. 4110	+2600	7. 4149
1 00	+ 54	2. 7287	+2635	7. 4208	5 00	+266	3. 4256	+2597	7. 4145
10	+ 62	2. 7956	+2635	7. 4208	10	+275	3. 4396	+2594	7. 4140
20	+ 71	2. 8536	+2634	7. 4206	20	+284	3. 4533	+2592	7. 4136
30	+ 80	2. 9047	+2633	7. 4205	30	+293	3. 4665	+2589	7. 4131
40	+ 89	2. 9504	+2633	7. 4204	40	+302	3. 4793	+2586	7. 4126
50	+ 98	2. 9917	+2632	7. 4202	50	+310	3. 4917	+2583	7. 4121
2 00	+107	3. 0295	+2631	7. 4201	6 00	+319	3. 5038	+2580	7. 4115
10	+116	3. 0642	+2629	7. 4199	10	+328	3. 5155	+2576	7. 4110
20	+125	3. 0963	+2628	7. 4197	20	+336	3. 5269	+2573	7. 4104
30	+134	3. 1262	+2627	7. 4195	30	+345	3. 5380	+2570	7. 4099
40	+143	3. 1541	+2626	7. 4192	40	+354	3. 5488	+2566	7. 4093
50	+151	3. 1804	+2624	7. 4190	50	+362	3. 5593	+2563	7. 4087
3 00	+160	3. 2051	+2623	7. 4187	7 00	+371	3. 5696	+2559	7. 4080
10	+169	3. 2285	+2621	7. 4185	10	+380	3. 5796	+2555	7. 4074
20	+178	3. 2507	+2619	7. 4182	20	+388	3. 5894	+2551	7. 4068
30	+187	3. 2718	+2617	7. 4179	30	+397	3. 5989	+2547	7. 4061
40	+196	3. 2919	+2615	7. 4176	40	+406	3. 6082	+2543	7. 4054
50	+205	3. 3111	+2614	7. 4172	50	+414	3. 6173	+2539	7. 4047

TABLE II.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 43 and 159]

[C_e is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C_e and log D_1. In south latitudes, if ϕ is taken negative, C_e is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_e	Log C_e	D_1	Log D_1	Latitude ϕ	C_e	Log C_e	D_1	Log D_1
° ′					° ′				
8 00	+423	3. 6262	+2535	7. 4040	15 00	+ 767	3. 8850	+2285	7. 3588
10	+431	3. 6350	+2531	7. 4033	10	+ 775	3. 8893	+2277	7. 3573
20	+440	3. 6435	+2526	7. 4025	20	+ 783	3. 8936	+2269	7. 3558
30	+449	3. 6519	+2522	7. 4018	30	+ 790	3. 8979	+2261	7. 3543
40	+457	3. 6600	+2518	7. 4010	40	+ 798	3. 9020	+2253	7. 3528
50	+466	3. 6680	+2513	7. 4002	50	+ 806	3. 9062	+2245	7. 3513
9 00	+474	3. 6759	+2508	7. 3994	16 00	+ 813	3. 9102	+2237	7. 3497
10	+483	3. 6836	+2503	7. 3985	10	+ 821	3. 9143	+2229	7. 3481
20	+491	3. 6912	+2499	7. 3977	20	+ 828	3. 9182	+2221	7. 3465
30	+500	3. 6986	+2494	7. 3968	30	+ 836	3. 9222	+2213	7. 3449
40	+508	3. 7058	+2489	7. 3960	40	+ 843	3. 9260	+2204	7. 3432
50	+516	3. 7130	+2484	7. 3951	50	+ 851	3. 9299	+2196	7. 3416
10 00	+525	3. 7200	+2478	7. 3942	17 00	+ 858	3. 9336	+2187	7. 3399
10	+533	3. 7269	+2473	7. 3932	10	+ 866	3. 9374	+2179	7. 3382
20	+542	3. 7336	+2468	7. 3923	20	+ 873	3. 9410	+2170	7. 3364
30	+550	3. 7403	+2462	7. 3913	30	+ 880	3. 9447	+2161	7. 3347
40	+558	3. 7468	+2457	7. 3904	40	+ 888	3. 9483	+2152	7. 3329
50	+567	3. 7532	+2451	7. 3894	50	+ 895	3. 9518	+2143	7. 3311
11 00	+575	3. 7595	+2445	7. 3884	18 00	+ 902	3. 9553	+2134	7. 3293
10	+583	3. 7657	+2440	7. 3873	10	+ 909	3. 9588	+2125	7. 3274
20	+591	3. 7718	+2434	7. 3863	20	+ 917	3. 9622	+2116	7. 3256
30	+600	3. 7778	+2428	7. 3852	30	+ 924	3. 9656	+2107	7. 3237
40	+608	3. 7837	+2422	7. 3841	40	+ 931	3. 9689	+2098	7. 3218
50	+616	3. 7896	+2416	7. 3831	50	+ 938	3. 9722	+2089	7. 3198
12 00	+624	3. 7953	+2410	7. 3819	19 00	+ 945	3. 9755	+2079	7. 3179
10	+632	3. 8009	+2403	7. 3808	10	+ 952	3. 9787	+2070	7. 3159
20	+640	3. 8065	+2397	7. 3797	20	+ 959	3. 9819	+2060	7. 3139
30	+649	3. 8119	+2391	7. 3785	30	+ 966	3. 9850	+2051	7. 3119
40	+657	3. 8173	+2384	7. 3773	40	+ 973	3. 9881	+2041	7. 3098
50	+665	3. 8226	+2377	7. 3761	50	+ 980	3. 9912	+2031	7. 3077
13 00	+673	3. 8278	+2371	7. 3749	20 00	+ 987	3. 9942	+2021	7. 3056
10	+681	3. 8330	+2364	7. 3736	10	+ 994	3. 9972	+2011	7. 3035
20	+689	3. 8380	+2357	7. 3724	20	+1000	4. 0002	+2001	7. 3014
30	+697	3. 8430	+2350	7. 3711	30	+1007	4. 0031	+1991	7. 2992
40	+705	3. 8480	+2343	7. 3698	40	+1014	4. 0060	+1981	7. 2970
50	+713	3. 8528	+2336	7. 3685	50	+1021	4. 0088	+1971	7. 2947
14 00	+720	3. 8576	+2329	7. 3672	21 00	+1027	4. 0117	+1961	7. 2925
10	+728	3. 8623	+2322	7. 3658	10	+1034	4. 0145	+1951	7. 2902
20	+736	3. 8670	+2315	7. 3645	20	+1040	4. 0172	+1940	7. 2879
30	+744	3. 8716	+2307	7. 3631	30	+1047	4. 0200	+1930	7. 2855
40	+752	3. 8761	+2300	7. 3617	40	+1054	4. 0227	+1919	7. 2832
50	+760	3. 8806	+2292	7. 3602	50	+1060	4. 0253	+1909	7. 2808

TABLE II.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 43 and 159]

C_e is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C_e and log D_1. In south latitudes, if ϕ is taken negative, C_e is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_e	Log C_e	D_1	Log D_1	Latitude ϕ	C_e	Log C_e	D_1	Log D_1
° ′					° ′				
22 00	+1066	4. 0280	+1898	7. 2784	29 00	+1303	4. 1148	+1399	7. 1459
10	+1073	4. 0306	+1888	7. 2759	10	+1307	4. 1164	+1386	7. 1418
20	+1079	4. 0331	+1877	7. 2734	20	+1312	4. 1180	+1373	7. 1377
30	+1086	4. 0357	+1866	7. 2709	30	+1317	4. 1195	+1360	7. 1335
40	+1092	4. 0382	+1855	7. 2684	40	+1321	4. 1210	+1347	7. 1293
50	+1098	4. 0407	+1844	7. 2658	50	+1326	4. 1225	+1334	7. 1250
23 00	+1104	4. 0432	+1833	7. 2632	30 00	+1330	4. 1240	+1320	7. 1207
10	+1111	4. 0456	+1822	7. 2606	10	+1335	4. 1254	+1307	7. 1163
20	+1117	4. 0480	+1811	7. 2579	20	+1339	4. 1269	+1294	7. 1118
30	+1123	4. 0504	+1800	7. 2553	30	+1344	4. 1283	+1280	7. 1073
40	+1129	4. 0527	+1789	7. 2525	40	+1348	4. 1297	+1267	7. 1027
50	+1135	4. 0550	+1777	7. 2498	50	+1352	4. 1311	+1253	7. 0980
24 00	+1141	4. 0573	+1766	7. 2470	31 00	+1356	4. 1324	+1240	7. 0933
10	+1147	4. 0596	+1755	7. 2442	10	+1361	4. 1338	+1226	7. 0886
20	+1153	4. 0618	+1743	7. 2413	20	+1365	4. 1351	+1213	7. 0837
30	+1159	4. 0640	+1732	7. 2384	30	+1369	4. 1364	+1199	7. 0788
40	+1165	4. 0662	+1720	7. 2355	40	+1373	4. 1377	+1185	7. 0738
50	+1171	4. 0684	+1708	7. 2326	50	+1377	4. 1389	+1171	7. 0687
25 00	+1176	4. 0705	+1697	7. 2296	32 00	+1381	4. 1402	+1158	7. 0636
10	+1182	4. 0726	+1685	7. 2266	10	+1385	4. 1414	+1144	7. 0584
20	+1188	4. 0747	+1673	7. 2235	20	+1389	4. 1426	+1130	7. 0531
30	+1193	4. 0768	+1661	7. 2204	30	+1393	4. 1438	+1116	7. 0477
40	+1199	4. 0788	+1649	7. 2173	40	+1396	4. 1450	+1102	7. 0423
50	+1205	4. 0808	+1637	7. 2141	50	+1400	4. 1461	+1088	7. 0367
26 00	+1210	4. 0828	+1625	7. 2109	33 00	+1404	4. 1473	+1074	7. 0311
10	+1216	4. 0848	+1613	7. 2076	10	+1407	4. 1484	+1060	7. 0254
20	+1221	4. 0868	+1601	7. 2044	20	+1411	4. 1495	+1046	7. 0196
30	+1227	4. 0887	+1589	7. 2010	30	+1414	4. 1506	+1032	7. 0137
40	+1232	4. 0906	+1576	7. 1977	40	+1418	4. 1517	+1018	7. 0077
50	+1237	4. 0924	+1564	7. 1943	50	+1421	4. 1527	+1004	7. 0016
27 00	+1243	4. 0943	+1552	7. 1908	34 00	+1425	4. 1538	+ 989	6. 9954
10	+1248	4. 0961	+1539	7. 1873	10	+1428	4. 1548	+ 975	6. 9891
20	+1253	4. 0979	+1527	7. 1838	20	+1431	4. 1558	+ 961	6. 9827
30	+1258	4. 0997	+1514	7. 1802	30	+1435	4. 1568	+ 947	6. 9762
40	+1263	4. 1015	+1502	7. 1766	40	+1438	4. 1577	+ 932	6. 9695
50	+1268	4. 1032	+1489	7. 1729	50	+1441	4. 1587	+ 918	6. 9628
28 00	+1273	4. 1049	+1476	7. 1692	35 00	+1444	4. 1596	+ 903	6. 9559
10	+1278	4. 1066	+1464	7. 1654	10	+1447	4. 1605	+ 889	6. 9489
20	+1283	4. 1083	+1451	7. 1616	20	+1450	4. 1614	+ 875	6. 9418
30	+1288	4. 1100	+1438	7. 1577	30	+1453	4. 1623	+ 860	6. 9345
40	+1293	4. 1116	+1425	7. 1538	40	+1456	4. 1632	+ 846	6. 9271
50	+1298	4. 1132	+1412	7. 1499	50	+1459	4. 1640	+ 831	6. 9196

TABLE II.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 43 and 159]

[C_e is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C_e and log D_1. In south latitudes, if ϕ is taken negative, C_e is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_e	Log C_e	D_1	Log D_1	Latitude ϕ	C_e	Log C_e	D_1	Log D_1
° ′					° ′				
36 00	+1462	4. 1649	+816	6. 9119	43 00	+1534	4. 1859	+184	6. 2658
10	+1464	4. 1657	+802	6. 9040	10	+1535	4. 1861	+169	6. 2280
20	+1467	4. 1665	+787	6. 8960	20	+1535	4. 1862	+154	6. 1867
30	+1470	4. 1673	+772	6. 8879	30	+1536	4. 1864	+138	6. 1410
40	+1472	4. 1680	+758	6. 8795	40	+1536	4. 1865	+123	6. 0899
50	+1475	4. 1688	+743	6. 8710	50	+1537	4. 1866	+108	6. 0319
37 00	+1478	4. 1695	+728	6. 8623	44 00	+1537	4. 1867	+ 92	5. 9650
10	+1480	4. 1703	+713	6. 8534	10	+1537	4. 1868	+ 77	5. 8859
20	+1482	4. 1710	+699	6. 8443	20	+1538	4. 1869	+ 62	5. 7890
30	+1485	4. 1717	+684	6. 8350	30	+1538	4. 1869	+ 46	5. 6641
40	+1487	4. 1723	+669	6. 8254	40	+1538	4. 1870	+ 31	5. 4880
50	+1489	4. 1730	+654	6. 8157	50	+1538	4. 1870	+ 15	5. 1870
38 00	+1492	4. 1736	+639	6. 8057	45 00	+1538	4. 1870	00	—∞
10	+1494	4. 1743	+624	6. 7954	10	+1538	4. 1870	− 15	5.1870_n
20	+1496	4. 1749	+609	6. 7849	20	+1538	4. 1870	− 31	5.4880_n
30	+1498	4. 1755	+594	6. 7741	30	+1538	4. 1870	− 46	5.6641_n
40	+1500	4. 1761	+579	6. 7630	40	+1538	4. 1869	− 62	5.7891_n
50	+1502	4. 1766	+564	6. 7516	50	+1538	4. 1869	− 77	5.8860_n
39 00	+1504	4. 1772	+549	6. 7399	46 00	+1537	4. 1868	− 92	5.9651_n
10	+1506	4. 1777	+534	6. 7278	10	+1537	4. 1867	−108	6.0320_n
20	+1507	4. 1782	+519	6. 7154	20	+1537	4. 1866	−123	6.0900_n
30	+1509	4. 1787	+504	6. 7026	30	+1536	4. 1865	−138	6.1411_n
40	+1511	4. 1792	+489	6. 6894	40	+1536	4. 1864	−154	6.1868_n
50	+1513	4. 1797	+474	6. 6758	50	+1535	4. 1862	−169	6.2282_n
40 00	+1514	4. 1802	+459	6. 6617	47 00	+1535	4. 1860	−184	6.2659_n
10	+1516	4. 1806	+444	6. 6472	10	+1534	4. 1859	−200	6.3006_n
20	+1517	4. 1810	+429	6. 6321	20	+1533	4. 1857	−215	6.3328_n
30	+1519	4. 1815	+413	6. 6164	30	+1533	4. 1855	−230	6.3627_n
40	+1520	4. 1819	+398	6. 6002	40	+1532	4. 1852	−246	6.3906_n
50	+1521	4. 1822	+383	6. 5833	50	+1531	4. 1850	−261	6.4169_n
41 00	+1523	4. 1826	+368	6. 5657	48 00	+1530	4. 1848	−276	6.4416_n
10	+1524	4. 1830	+353	6. 5473	10	+1529	4. 1845	−292	6.4650_n
20	+1525	4. 1833	+337	6. 5281	20	+1528	4. 1842	−307	6.4872_n
30	+1526	4. 1836	+322	6. 5080	30	+1527	4. 1839	−322	6.5083_n
40	+1527	4. 1839	+307	6. 4869	40	+1526	4. 1836	−338	6.5284_n
50	+1528	4. 1842	+292	6. 4648	50	+1525	4. 1833	−353	6.5476_n
42 00	+1529	4. 1845	+276	6. 4414	49 00	+1524	4. 1829	−368	6.5660_n
10	+1530	4. 1848	+261	6. 4166	10	+1523	4. 1826	−383	6.5836_n
20	+1531	4. 1850	+246	6. 3904	20	+1521	4. 1822	−399	6.6005_n
30	+1532	4. 1853	+230	6. 3625	30	+1520	4. 1818	−414	6.6168_n
40	+1533	4. 1855	+215	6. 3326	40	+1519	4. 1814	−429	6.6324_n
50	+1534	4. 1857	+200	6. 3005	50	+1517	4. 1810	−444	6.6475_n

TABLE II.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 43 and 159]

[C_e is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. -10 is to be understood with log C_e and log D_1. In south latitudes, if ϕ is taken negative, C_e is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_e	Log C_e	D_1	Log D_1	Latitude ϕ	C_e	Log C_e	D_1	Log D_1
° ′					° ′				
50 00	+1516	4. 1806	− 459	6. 6621$_n$	57 00	+1407	4. 1482	−1077	7. 0320$_n$
10	+1514	4. 1801	− 474	6. 6762$_n$	10	+1403	4. 1471	−1091	7. 0377$_n$
20	+1512	4. 1797	− 490	6. 6899$_n$	20	+1399	4. 1459	−1105	7. 0432$_n$
30	+1511	4. 1792	− 505	6. 7031$_n$	30	+1396	4. 1448	−1119	7. 0487$_n$
40	+1509	4. 1787	− 520	6. 7159$_n$	40	+1392	4. 1436	−1133	7. 0541$_n$
50	+1507	4. 1782	− 535	6. 7283$_n$	50	+1388	4. 1424	−1147	7. 0594$_n$
51 00	+1505	4. 1777	− 550	6. 7404$_n$	58 00	+1384	4. 1412	−1160	7. 0646$_n$
10	+1504	4. 1771	− 565	6. 7521$_n$	10	+1380	4. 1399	−1174	7. 0698$_n$
20	+1502	4. 1766	− 580	6. 7635$_n$	20	+1376	4. 1387	−1188	7. 0748$_n$
30	+1500	4. 1760	− 595	6. 7746$_n$	30	+1372	4. 1374	−1202	7. 0798$_n$
40	+1498	4. 1754	− 610	6. 7854$_n$	40	+1368	4. 1361	−1216	7. 0848$_n$
50	+1496	4. 1748	− 625	6. 7959$_n$	50	+1364	4. 1348	−1229	7. 0896$_n$
52 00	+1493	4. 1742	− 640	6. 8062$_n$	59 00	+1360	4. 1335	−1243	7. 0944$_n$
10	+1491	4. 1736	− 655	6. 8162$_n$	10	+1356	4. 1321	−1256	7. 0991$_n$
20	+1489	4. 1729	− 670	6. 8260$_n$	20	+1351	4. 1308	−1270	7. 1038$_n$
30	+1487	4. 1723	− 685	6. 8356$_n$	30	+1347	4. 1294	−1283	7. 1084$_n$
40	+1485	4. 1716	− 700	6. 8449$_n$	40	+1343	4. 1280	−1297	7. 1129$_n$
50	+1482	4. 1709	− 715	6. 8540$_n$	50	+1338	4. 1266	−1310	7. 1174$_n$
53 00	+1480	4. 1702	− 729	6. 8629$_n$	60 00	+1334	4. 1251	−1324	7. 1218$_n$
10	+1477	4. 1695	− 744	6. 8716$_n$	10	+1329	4. 1237	−1337	7. 1262$_n$
20	+1475	4. 1687	− 759	6. 8802$_n$	20	+1325	4. 1222	−1350	7. 1305$_n$
30	+1472	4. 1680	− 774	6. 8885$_n$	30	+1320	4. 1207	−1364	7. 1347$_n$
40	+1470	4. 1672	− 788	6. 8967$_n$	40	+1316	4. 1192	−1377	7. 1389$_n$
50	+1467	4. 1664	− 803	6. 9047$_n$	50	+1311	4. 1176	−1390	7. 1430$_n$
54 00	+1464	4. 1656	− 818	6. 9126$_n$	61 00	+1306	4. 1160	−1403	7. 1471$_n$
10	+1461	4. 1648	− 832	6. 9203$_n$	10	+1302	4. 1145	−1416	7. 1511$_n$
20	+1459	4. 1639	− 847	6. 9279$_n$	20	+1297	4. 1129	−1429	7. 1551$_n$
30	+1456	4. 1631	− 862	6. 9353$_n$	30	+1292	4. 1112	−1442	7. 1590$_n$
40	+1453	4. 1622	− 876	6. 9426$_n$	40	+1287	4. 1096	−1455	7. 1629$_n$
50	+1450	4. 1613	− 891	6. 9497$_n$	50	+1282	4. 1079	−1468	7. 1667$_n$
55 00	+1447	4. 1604	− 905	6. 9567$_n$	62 00	+1277	4. 1062	−1481	7. 1705$_n$
10	+1444	4. 1595	− 920	6. 9636$_n$	10	+1272	4. 1045	−1493	7. 1742$_n$
20	+1441	4. 1585	− 934	6. 9704$_n$	20	+1267	4. 1028	−1506	7. 1779$_n$
30	+1437	4. 1576	− 948	6. 9770$_n$	30	+1262	4. 1010	−1519	7. 1815$_n$
40	+1434	4. 1566	− 963	6. 9835$_n$	40	+1257	4. 0993	−1531	7. 1851$_n$
50	+1431	4. 1556	− 977	6. 9900$_n$	50	+1252	4. 0975	−1544	7. 1887$_n$
56 00	+1428	4. 1546	− 991	6. 9963$_n$	63 00	+1246	4. 0957	−1557	7. 1922$_n$
10	+1424	4. 1536	−1006	7. 0025$_n$	10	+1241	4. 0938	−1569	7. 1956$_n$
20	+1421	4. 1526	−1020	7. 0086$_n$	20	+1236	4. 0919	−1581	7. 1990$_n$
30	+1417	4. 1515	−1034	7. 0146$_n$	30	+1230	4. 0901	−1594	7. 2024$_n$
40	+1414	4. 1504	−1048	7. 0205$_n$	40	+1225	4. 0881	−1606	7. 2058$_n$
50	+1410	4. 1493	−1062	7. 0263$_n$	50	+1220	4. 0862	−1618	7. 2091$_n$

TABLE II.—*Orthometric and dynamic corrections*—Continued

[For explanation of the use of the tables, see pp. 43 and 159]

[C_e is expressed in units of the ninth decimal place. D_1 is expressed in units of the sixth decimal place. −10 is to be understood with log C_e and log D_1. In south latitudes, if ϕ is taken negative, C_e is negative. The sign of D_1 is independent of the sign of the latitude.]

Latitude ϕ	C_e	Log C_e	D_1	Log D_1	Latitude ϕ	C_e	Log C_e	D_1	Log D_1
° ′					° ′				
64 00	+1214	4. 0843	−1630	7. 2123$_n$	68 00	+1071	4. 0296	−1906	7. 2800$_n$
10	+1209	4. 0823	−1643	7. 2155$_n$	10	+1064	4. 0270	−1916	7. 2825$_n$
20	+1203	4. 0803	−1655	7. 2187$_n$	20	+1058	4. 0243	−1927	7. 2849$_n$
30	+1197	4. 0782	−1667	7. 2219$_n$	30	+1051	4. 0216	−1937	7. 2872$_n$
40	+1192	4. 0762	−1679	7. 2250$_n$	40	+1045	4. 0189	−1948	7. 2896$_n$
50	+1186	4. 0741	−1691	7. 2280$_n$	50	+1038	4. 0162	−1958	7. 2919$_n$
65 00	+1180	4. 0720	−1702	7. 2311$_n$	69 00	+1031	4. 0134	−1969	7. 2942$_n$
10	+1175	4. 0699	−1714	7. 2341$_n$	10	+1025	4. 0106	−1979	7. 2965$_n$
20	+1169	4. 0677	−1726	7. 2370$_n$	20	+1018	4. 0077	−1989	7. 2987$_n$
30	+1163	4. 0656	−1738	7. 2400$_n$	30	+1011	4. 0048	−1999	7. 3009$_n$
40	+1157	4. 0633	−1749	7. 2429$_n$	40	+1004	4. 0019	−2010	7. 3031$_n$
50	+1151	4. 0611	−1761	7. 2457$_n$	50	+0998	3. 9990	−2020	7. 3053$_n$
66 00	+1145	4. 0589	−1772	7. 2485$_n$	70 00	+0991	3. 9960	−2030	7. 3074$_n$
10	+1139	4. 0566	−1784	7. 2513$_n$	10	+0984	3. 9929	−2039	7. 3095$_n$
20	+1133	4. 0543	−1795	7. 2541$_n$	20	+0977	3. 9899	−2049	7. 3116$_n$
30	+1127	4. 0519	−1806	7. 2568$_n$	30	+0970	3. 9868	−2059	7. 3137$_n$
40	+1121	4. 0496	−1818	7. 2595$_n$	40	+0963	3. 9836	−2069	7. 3157$_n$
50	+1115	4. 0472	−1829	7. 2622$_n$	50	+0956	3. 9805	−2078	7. 3177$_n$
67 00	+1109	4. 0448	−1840	7. 2648$_n$	71 00	+0949	3. 9773	−2088	7. 3197$_n$
10	+1102	4. 0423	−1851	7. 2674$_n$	10	+0942	3. 9740	−2097	7. 3217$_n$
20	+1096	4. 0398	−1862	7. 2700$_n$	20	+0935	3. 9707	−2107	7. 3236$_n$
30	+1090	4. 0373	−1873	7. 2726$_n$	30	+0928	3. 9674	−2116	7. 3255$_n$
40	+1083	4. 0348	−1884	7. 2751$_n$	40	+0921	3. 9640	−2125	7. 3274$_n$
50	+1077	4. 0322	−1895	7. 2776$_n$	50	+0913	3. 9606	−2134	7. 3293$_n$
					72 00	+0906	3. 9572	−2144	7. 3311$_n$

D_2 in units of the tenth decimal place

[See p. 159. D_2 is always positive]

ϕ \ h	0 meter	1,000 meters	2,000 meters	3,000 meters	4,000 meters	5,000 meters
°						
0	1575	1574	1574	1574	1574	1573
10	1575	1574	1574	1574	1574	1573
20	1574	1574	1574	1574	1573	1573
30	1574	1574	1574	1573	1573	1573
40	1574	1573	1573	1573	1573	1572
50	1573	1573	1573	1573	1572	1572
60	1573	1573	1572	1572	1572	1572
70	1573	1572	1572	1572	1572	1572
80	1572	1572	1572	1572	1572	1571
90	1572	1572	1572	1572	1571	1571

Log D_2

[See p. 159. −10 is to be supplied as part of the characteristic. D_2 is always positive]

ϕ \ h	0 meter	1,000 meters	2,000 meters	3,000 meters	4,000 meters	5,000 meters
°						
0	3. 1972	3. 1971	3. 1970	3. 1970	3. 1969	3. 1968
10	. 1971	. 1971	. 1970	. 1970	. 1969	. 1968
20	. 1971	. 1970	. 1970	. 1969	. 1968	. 1968
30	. 1970	. 1970	. 1969	. 1968	. 1968	. 1967
40	. 1969	. 1969	. 1968	. 1967	. 1966	. 1966
50	. 1968	. 1968	. 1967	. 1966	. 1965	. 1965
60	. 1967	. 1966	. 1966	. 1965	. 1965	. 1964
70	. 1966	. 1966	. 1965	. 1964	. 1964	. 1963
80	. 1966	. 1965	. 1965	. 1964	. 1963	. 1962
90	3. 1966	3. 1965	3. 1964	3. 1964	3. 1963	3. 1962

TABLE III.—*Allowable divergence between forward and backward runnings of a section of first-order levels.* (*4.0 mm.* $\sqrt{K}$)

Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)
0. 00 to 0. 26	2. 0	0. 75 to 0. 78	3. 5	1. 54 to 1. 59	5. 0
0. 27 to 0. 28	2. 1	0. 79 to 0. 83	3. 6	1. 60 to 1. 65	5. 1
0. 29 to 0. 31	2. 2	0. 84 to 0. 87	3. 7	1. 66 to 1. 72	5. 2
0. 32 to 0. 34	2. 3	0. 88 to 0. 92	3. 8	1. 73 to 1. 78	5. 3
0. 35 to 0. 37	2. 4	0. 93 to 0. 97	3. 9	1. 79 to 1. 85	5. 4
0. 38 to 0. 40	2. 5	0. 98 to 1. 02	4. 0	1. 86 to 1. 92	5. 5
0. 41 to 0. 43	2. 6	1. 03 to 1. 07	4. 1	1. 93 to 1. 99	5. 6
0. 44 to 0. 47	2. 7	1. 08 to 1. 12	4. 2	2. 00 to 2. 06	5. 7
0. 48 to 0. 50	2. 8	1. 13 to 1. 18	4. 3		
0. 51 to 0. 54	2. 9	1. 19 to 1. 23	4. 4		
0. 55 to 0. 58	3. 0	1. 24 to 1. 29	4. 5		
0. 59 to 0. 62	3. 1	1. 30 to 1. 35	4. 6		
0. 63 to 0. 66	3. 2	1. 36 to 1. 41	4. 7		
0. 67 to 0. 70	3. 3	1. 42 to 1. 47	4. 8		
0. 71 to 0. 74	3. 4	1. 48 to 1. 53	4. 9		

TABLE IV.—*Allowable divergence between forward and backward runnings of double-run second-order levels.* (*8.4 mm.* $\sqrt{K}$)

Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)
0. 00 to 0. 25	4. 2	0. 73 to 0. 74	7. 2	1. 47 to 1. 48	10. 2
0. 26	4. 3	0. 75 to 0. 76	7. 3	1. 49 to 1. 51	10. 3
0. 27 to 0. 28	4. 4	0. 77 to 0. 78	7. 4	1. 52 to 1. 54	10. 4
0. 29	4. 5	0. 79 to 0. 80	7. 5	1. 55 to 1. 57	10. 5
0. 30	4. 6	0. 81 to 0. 82	7. 6	1. 58 to 1. 60	10. 6
0. 31	4. 7	0. 83 to 0. 85	7. 7	1. 61 to 1. 63	10. 7
0. 32 to 0. 33	4. 8	0. 86 to 0. 87	7. 8	1. 64 to 1. 66	10. 8
0. 34	4. 9	0. 88 to 0. 89	7. 9	1. 67 to 1. 69	10. 9
0. 35 to 0. 36	5. 0	0. 90 to 0. 91	8. 0	1. 70 to 1. 73	11. 0
0. 37	5. 1	0. 92 to 0. 94	8. 1	1. 74 to 1. 76	11. 1
0. 38 to 0. 39	5. 2	0. 95 to 0. 96	8. 2	1. 77 to 1. 79	11. 2
0. 40	5. 3	0. 97 to 0. 98	8. 3	1. 80 to 1. 82	11. 3
0. 41 to 0. 42	5. 4	0. 99 to 1. 01	8. 4	1. 83 to 1. 85	11. 4
0. 43	5. 5	1. 02 to 1. 03	8. 5	1. 86 to 1. 89	11. 5
0. 44 to 0. 45	5. 6	1. 04 to 1. 06	8. 6	1. 90 to 1. 92	11. 6
0. 46	5. 7	1. 07 to 1. 08	8. 7	1. 93 to 1. 95	11. 7
0. 47 to 0. 48	5. 8	1. 09 to 1. 11	8. 8	1. 96 to 1. 99	11. 8
0. 49 to 0. 50	5. 9	1. 12 to 1. 13	8. 9	2. 00 to 2. 02	11. 9
0. 51	6. 0	1. 14 to 1. 16	9. 0		
0. 52 to 0. 53	6. 1	1. 17 to 1. 18	9. 1		
0. 54 to 0. 55	6. 2	1. 19 to 1. 21	9. 2		
0. 56 to 0. 57	6. 3	1. 22 to 1. 23	9. 3		
0. 58	6. 4	1. 24 to 1. 26	9. 4		
0. 59 to 0. 60	6. 5	1. 27 to 1. 29	9. 5		
0. 61 to 0. 62	6. 6	1. 30 to 1. 31	9. 6		
0. 63 to 0. 64	6. 7	1. 32 to 1. 34	9. 7		
0. 65 to 0. 66	6. 8	1. 35 to 1. 37	9. 8		
0. 67 to 0. 68	6. 9	1. 38 to 1. 40	9. 9		
0. 69 to 0. 70	7. 0	1. 41 to 1. 43	10. 0		
0. 71 to 0. 72	7. 1	1. 44 to 1. 46	10. 1		

TABLE V.—*Allowable divergence between forward and backward runnings of a section of first-order levels when special instructions call for special accuracy.* (*3.0 mm.* $\sqrt{K}$)

Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)	Length of section (kilometers)	Allowable divergence (mm.)
0. 00 to 0. 26	1. 5	0. 67 to 0. 72	2. 5	1. 33 to 1. 40	3. 5
0. 27 to 0. 30	1. 6	0. 73 to 0. 78	2. 6	1. 41 to 1. 48	3. 6
0. 31 to 0. 34	1. 7	0. 79 to 0. 84	2. 7	1. 49 to 1. 56	3. 7
0. 35 to 0. 38	1. 8	0. 85 to 0. 90	2. 8	1. 57 to 1. 64	3. 8
0. 39 to 0. 42	1. 9	0. 91 to 0. 96	2. 9	1. 65 to 1. 73	3. 9
0. 43 to 0. 46	2. 0	0. 97 to 1. 03	3. 0	1. 74 to 1. 82	4. 0
0. 47 to 0. 51	2. 1	1. 04 to 1. 10	3. 1	1. 83 to 1. 91	4. 1
0. 52 to 0. 56	2. 2	1. 11 to 1. 17	3. 2	1. 92 to 2. 00	4. 2
0. 57 to 0. 61	2. 3	1. 18 to 1. 24	3. 3		
0. 62 to 0. 66	2. 4	1. 25 to 1. 32	3. 4		

APPENDIX A—RESOLUTION OF THE INTERNATIONAL GEODETIC ASSOCIATION

The following is a translation of the resolution adopted by the International Geodetic Association, at its Seventh General Conference, held at Hamburg, Germany, September 1912:

Because of the important advances made in the art of leveling since the year 1867, when limits were first fixed for errors permissible in precise leveling, and because of the benefit accruing to the greater needs of geodesy in creating a new class of more precise leveling; that is, one with narrower limits of error and with its errors, both accidental and systematic, computed according to uniform rules:

Therefore, the Seventh General Conference of the International Geodetic Association, still preserving unchanged the limits of error of 1867 for precise leveling, decides to place hereafter in a new class of leveling, to be termed "leveling of high precision," every line, set of lines, or net which is run twice in opposite directions on different dates as far as possible, and whose errors, accidental and systematic, computed by the formulas hereinafter given, do not exceed—

± 1 mm. per km. for the probable accidental error,

or

± 1.5 mm. per km. for the mean accidental error;

± 0.2 mm. per km. for the probable systematic error,

or

± 0.3 mm. per km. for the mean systematic error.

If L denote the length of an unconnected line, or the length of the side of a polygonal circuit in the case of a net;

ΣL, the aggregate length of the set of lines, or of the net under consideration;

Δ, the discrepancy between the results of the two runnings between consecutive bench marks;

r, the distance between these two bench marks;

s, the entire systematic discrepancy between the results of the two runnings, either for a whole line or for the side of a circuit;

f, the error of closure of a circuit of the net after the orthometric correction has been applied;

Σf^2, the sum of the squares of the errors of closure of the circuits, including the error, Σf, of the outside circuit;

Then the probable or mean errors are to be computed by the following formulas:

1. For the probable accidental error, η_r, in the case of a set of lines, whether or not they form circuits:

$$\text{I.} \quad \eta_r^2 = \frac{1}{9}\left[\frac{\Sigma\Delta^2}{\Sigma L} - \frac{\Sigma r^2}{(\Sigma L)^2}\,\Sigma\,\frac{s^2}{L}\right]$$

2. For the probable systematic error, σ_r or σ_R:

(a) In the case of a set of lines not forming a net:

$$\text{II.} \quad \sigma_r^2 = \frac{1}{9\Sigma L}\,\Sigma\,\frac{s^2}{L}$$

(b) In the case of a net allowing at least 10 circuits:

$$\text{III.} \quad \sigma_R^2 = \frac{1}{\Sigma L^2}\left[\frac{2}{9}\,\Sigma f^2 - \eta_r^2 \Sigma L\right]$$

3. For the mean accidental error, or for the mean systematic error, the same formulas as above, after multiplying the second member by $\frac{9}{4}$.

APPENDIX B—RESOLUTION OF THE INTERNATIONAL ASSOCIATION OF GEODESY

The following is a translation of the resolution adopted by the International Association of Geodesy of the International Geodetic and Geophysical Union during the Sixth General Assembly, held at Edinburgh, Scotland, September 1936.

A. Evaluation of the Precision of a Method of Leveling

CLASSIFICATION AND DEFINITION OF THE ERRORS AFFECTING A METHOD OF LEVELING

Leveling is affected by two classes of errors called "η-errors" and "ζ-errors" which are independent of each other.

The "η-errors" are due to causes that act independently on each successive piece of leveling. They obey the laws of Gauss. They are characterized by a probable (accidental) "η-error" per kilometer η, such that the probable error due to errors of this class, in the difference in elevation between two bench marks D kilometers apart, is equal to $\eta\sqrt{D}$, whatever the distance D may be.

The "ζ-errors" are due to causes that act in a similar way on successive or nearby pieces of leveling. They do not obey the laws of Gauss. They act like accidental errors on two bench marks D kilometers apart only if the distance D exceeds a certain limit Z, of the order of magnitude of some tens of kilometers, which is subject to variation with the method of leveling. They are characterized by a probable (accidental) "ζ-error" per kilometer ζ, such that, if the distance D between two bench marks is greater than the limit Z, the probable error of their difference in elevation due to these sources of error is $\zeta\sqrt{D}$. If the distance D is less than the limit Z, the coefficient of proportionality of this error affecting the square root of D decreases from the value ζ to zero as D decreases from the limit Z to zero.

Accordingly, the combined effect of the "η-errors" and the "ζ-errors "is characterized by a single number, namely the total probable (accidental) error per kilometer ϵ, such that, for two bench marks separated by a distance D greater than the limit Z, the total probable error of their difference in elevation is equal to $\sqrt{D}$. The coefficient ϵ is connected with the coefficients η and ζ by the relation:

$$\epsilon^2=\eta^2+\zeta^2.$$

Notation:

Let:

r be the interval between two consecutive bench marks;

r_m, the mean value of this interval;

n_r, the number of intervals between consecutive bench marks;

Σ_r, the total length of these intervals, equal to the total length of the net ($\Sigma_r=\Sigma L=n_r\times r_m$);

ρ, the discrepancy between the results of the two leveling operations considered (if the leveling considered is the mean of two such operations) for two consecutive bench marks;

L, the length of one of the portions into which the net may be divided arbitrarily; the portions may be identical with entire links, or may be merely fractions of links, or may sometimes be composed of links in sequence on the ground (or even parts separated on the ground and joined together for the purposes of calculation), the name link being applied to every side common to two circuits, or to every isolated line taken as a whole;

L_m, the mean length of the portions;

n_L, their number;

ΣL, their total length, equal to the total length of the net ($\Sigma L=n_L\times L_m$);

Λ, the discrepancy between the results of two leveling operations (if the leveling in question is the mean of two such operations) between the ends of a portion;

P, the perimeter of a closed circuit;

P_m, its mean value;

n_P, the number of these circuits;

ΣP, the sum of their perimeters ($\Sigma P=n_P\times P_m$);

f, the closing residual of a closed circuit after all instrumental and theoretical corrections have been applied, particularly the orthometric or dynamic correction arising from the nonparallelism of the level surfaces of the earth.

The distances (r, L, P) are stated in kilometers, and the closing residuals (ρ, Λ, f) in millimeters

Formulas for the evaluation of the precision of a method of leveling:

Let:

$$\text{(a)}\quad e^2=\frac{1}{9}\text{ mean }\frac{\rho^2}{r},$$

$$\text{(b)}\quad e_L^2=\frac{1}{9}\text{ mean }\frac{\Lambda^2}{L},$$

$$\text{(c)}\quad e_P^2=\frac{4}{9}\text{ mean }\frac{f^2}{P}.$$

To obtain a satisfactory approximation these means should be evaluated for at least ten elements. In the calculation either equal weights or weights equal to r, L and P, respectively, are adopted; the first method yields a rather better approximation, especially when the number of elements is small; the second involves somewhat simpler calculations. The following formulas are thus obtained:

Equal weights	Weights equal to—
(a′) $e_r'^2=\frac{1}{9n_r}\sum\frac{\rho^2}{r}$,	(a″) $e_r''^2=\frac{1}{9\Sigma r}\sum\rho^2$,
(b′) $e_L'^2=\frac{1}{9n_L}\sum\frac{\Lambda^2}{L}$,	(b″) $e_L''^2=\frac{1}{9\Sigma L}\sum\Lambda^2$,
(c′) $e_P'^2=\frac{4}{9n_P}\sum\frac{f^2}{P}$,	(c″) $e_P''^2=\frac{4}{9\Sigma P}\sum f^2$.

Finally, let us denote E_L and E_P the limits toward which the respective quantities $e_L(e_L',e_L'')$ and $e_P(e_P',e_P'')$ tend as the mean length L_m of the portions, or the perimeter P_m of the circuits, increases beyond the limit Z defined above. To make sure that these limits are really reached, increase the mean lengths L_m and P_m (the latter, if necessary, by a grouping of circuits); an approximate determination of the limiting distance Z is thus obtained.

When the limit E_P is reached, it is preferable to take into account the outside circuit in the expressions (c′) and (c″) and write:

$$(c_1')\quad e_P'^2=\frac{4}{9}\frac{1}{n_P+1}\Sigma\frac{f^2}{P}\ \begin{pmatrix}\text{with}\\ \text{outside}\\ \text{circuit}\end{pmatrix}\qquad (c_1'')\quad e_P''^2=\frac{4}{9}\frac{1}{n_P+1}\left[n_P\frac{\Sigma f^2}{\Sigma P}+\frac{f^2}{P}\right].\ \begin{pmatrix}\text{without}\\ \text{outside}\\ \text{circuit}\end{pmatrix}\begin{pmatrix}\text{outside}\\ \text{circuit}\end{pmatrix}$$

In calculating E_P, if the number of circuits is small, the approximation is slightly improved by replacing E_P by the value obtained from the corrections C applied to the various portions having a mean length in excess of the limit Z, the corrections C to be derived from a rational adjustment of the net. The quantity E_P is then the limit, when L_m exceeds the limiting distance Z, of the expression g_P such that:

$$g_P=\frac{4}{9}\text{ mean }\frac{C^2}{L},$$

where g_P may be computed by either of the following formulas:

$$g_P'^2=\frac{4}{9}\Sigma\frac{C^2}{L}\qquad g_P''^2=\frac{4}{9\Sigma L}\Sigma C^2$$

This being so, the formulas to be used for computing the probable "η-error," the "ζ-error" and total error expressed in millimeters per kilometer, may be chosen from the following:

Probable "η-error" per kilometer η:

(1) $$(1')\ \eta^2=e_r'^2-\zeta^2\frac{r_m}{Z};\qquad (1'')\ \eta^2=e_r''^2-\zeta^2\frac{1}{Z}\frac{\Sigma r^2}{\Sigma r}.$$

Probable "ζ-error" per kilometer ζ:

(2) $$\zeta^2=E_L^2-\eta^2;\qquad (3)\ \zeta^2=E_P^2-\eta^2.$$

Total probable error per kilometer ϵ:

(1–2) $$\epsilon=E_L;\qquad (1\text{–}3)\ \epsilon=E_P.$$

These formulas are grouped together in the table on page 153 where the principal sources of inaccuracy tending to alter them are summarized.

FORMULAS FOR THE EVALUATION OF THE PRECISION OF A METHOD OF LEVELING

TYPES OF ERRORS		FORMULAS — Use of Weights: equal	FORMULAS — Use of Weights: equal to r, L, P	Influence of the different causes of inaccuracy (1): Arbitrary character of the choice of the portions	Non-independence of the two component levelings	Errors connected with the direction of running of the component levelings: In opposite directions	In the same direction
Probable error "η" per kilometer η deduced from the discrepancies ρ between consecutive bench marks	(1)	(1') $\eta^2 = e_r'^2 - \zeta^2 \frac{r_m}{Z}$ with (a') $e_r'^2 = \frac{1}{9n_r} \Sigma \frac{\rho^2}{r}$	(1'') $\eta^2 = e_r''^2 - \zeta^2 \frac{1}{Z} \frac{\Sigma r^2}{\Sigma r}$ with (a'') $e_r''^2 = \frac{1}{9\Sigma r} \Sigma \rho^2$	0	−	0	0
Probable error "ζ" per kilometer ζ — deduced from the discrepancies Λ between the ends of the portions	(2)	(2) $\zeta^2 = E_L^2 - \eta^2$ where E_L is the limit of e_L' and of e_L'' for $L_m > Z$, with: (b') $e_L'^2 = \frac{1}{9n_L} \Sigma \frac{\Lambda^2}{L}$	(b'') $e_L''^2 = \frac{1}{9\Sigma L} \Sigma \Lambda^2$	±	−	+	−
Probable error "ζ" per kilometer ζ — deduced from the errors of closure f of the circuits	(3)	(3) $\zeta^2 = E_P^2 - \eta^2$ where E_P is the limit of e_P' and of e_P'' for $P_m > Z$, with: (c') $e_P'^2 = \frac{4}{9n_P} \Sigma \frac{f^2}{P}$ When the limit E_P is reached, it is preferable to take the outside circuit into account, and to write: (c'') $e_P'^2 = \frac{4}{9} \frac{1}{n_P+1} \Sigma \frac{f^2}{P}$ (with outside circuit) If n_P is small, the corrections C for the portions L, deduced from the rational adjustment, furnish a little better value of E_P, the limit of g_P' and of g_P'' for $L_m > Z$, with: $g_P'^2 = \frac{4}{9n_L} \Sigma \frac{c^2}{L}$	(c'') $e_P''^2 = \frac{4}{9\Sigma P} \Sigma f^2$ (c''') $e_P''^2 = \frac{4}{9} \frac{1}{n_P+1} \left[n_P \frac{\Sigma f^2}{\Sigma P} + \frac{f^2}{P} \right]$ (without outside circuit) (outside circuit) $g_P''^2 = \frac{4}{9\Sigma L} \Sigma c^2$	0	+	0	0
Total probable error per kilometer ε ($\varepsilon^2 = \zeta^2 + \eta^2$) — deduced from the discrepancies Λ between the ends of the portions	(1-2)	(1-2) $\varepsilon = E_L$ E_L having the definition given above for formula (2)		±	−	+	−
Total probable error per kilometer ε ($\varepsilon^2 = \zeta^2 + \eta^2$) — deduced from the errors of closure f of the circuits	(1-3)	(1-3) $\varepsilon = E_P$ E_P having the definition given above for formula (3)		0	0	0	0

(1) The signs +, — and 0 indicate that the influence of the cause of the inaccuracy considered upon the value of η, ζ, or ε given by the formula tends to increase (+) or to decrease (—) it with respect to its theoretically exact value or is nothing or negligible (0).

DEFINITION OF LEVELING OF HIGH PRECISION AND OF PRECISE LEVELING

By definition a method of leveling comes within the international classification of Leveling of High Precision if its total probable error ϵ, calculated by means of the preceding formulas, does not exceed 2 millimeters per kilometer.

It is classed as Precise Leveling if this total probable error does not exceed 6 millimeters per kilometer.

B. Advice Concerning the Execution of Leveling of High Precision

Instruments

Level.—The magnification of the telescope should be at least 25 diameters, and if possible should reach 30 or 40 diameters. On the other hand, in order to insure proper illumination and definition, the aperture of the eyepiece should have a diameter of at least 1.5 millimeters.

The radius of curvature of the vial should be kept between 40 and 100 meters, the latter limit not to be exceeded for it is already extreme for observations like leveling made in the open air.

For the highest temperatures at which work is possible the length of the bubble should never become less than 25 millimeters, since shorter bubbles do not have sufficient mobility.

Rods.—In the case where the level is not equipped with a device eliminating the necessity of estimating fractions of the smallest graduations, the smallest division of the rod should be in proportion to the usual length of sight and therefore smaller when the sights are usually shorter. Thus, for operations in rough country, where the sights average but little more than 20 to 25 meters, the smallest graduation of the rod should be reduced to 2 millimeters It should not, on the other hand, exceed 1 centimeter.

The division shall be painted, as far as possible, on invar strips.

If this is not done, then in order that it may be possible to determine the variation of the lengths of the wooden rods daily, even in the field, and to take account of it in the computations, compensating rods should be used, in which a single strip of invar might replace the bimetallic strip (iron and brass) set into the body of the rod (Col. Goulier's model).

To insure their verticality at the moment of reading, the rods should be furnished with spherical levels of 0.2 to 0.5 meter radius of curvature. In addition, it may be convenient, the more easily to hold the rods steady in the vertical position, to provide them with handles and to use one, or better two, props.

Methods of Operation and Computation

Operations in the field.—

(1) Each line shall be run twice in opposite directions and, as far as possible, on different dates or at different times of the day.

(2) Two rods shall be used simultaneously.

(3) The sights should not be too long. The error, due to refraction and the unsteadiness of the graduations, increases more rapidly than the length of the sights. They should be regulated, according to atmospheric conditions, so that this error remains consistent with the definition of Leveling of High Precision.

(4) At each set-up the level shall be placed approximately at equal distances from the rods.

(5) The readings used in the computation of the difference in level should not be made on the first lower decimeters of the rod.

(6) The operations should be conducted in such a manner as to reduce as much as possible the influence of causes that might involve systematic errors.

Computations.—

(1) The computations should be made in duplicate and, as far as possible, by two different methods.

(2) The difference in elevation obtained, forward and backward, should be compared, at least from bench mark to bench mark. The interval of comparison should be chosen small enough to permit, in each case, a good check on the precision.

It is necessary, as a verification, to relevel the parts of the line for which the comparison brings out discrepancies greater than the following tolerances:

Let η, ζ and ϵ be the probable "η", "ζ" and total error in millimeters per kilometer as computed by the formulas given above for the leveling operations performed by the same method as the leveling operations in question:

"η discrepancy": the discrepancy for a comparison interval of r kilometers should not exceed, when expressed in millimeters, the value $\rho = \pm 6$ to $8\sqrt{r}$;

"ζ discrepancy": the discrepancy found for a length of L kilometers including at least some ten comparison intervals should not exceed, when expressed in millimeters, the following values:

For a length L less than several tens of kilometers:

$$\Delta = \pm 2 \text{ to } 2.5\ \epsilon\ L^{3/4};$$

For a length L greater than several tens of kilometers:

$$\Delta = \pm 6 \text{ to } 8\ \epsilon\ \sqrt{L}.$$

Finally, it is further necessary to be certain that the probable error ϵ, computed for the whole net. does not exceed the limit allowing the method to be classed as one of high precision.

Adjustment.—

(1) For the final calculation of the altitudes of bench marks, the orthometric or dynamic corrections should be applied to allow for the lack of parallelism of level surfaces.

(2) In the adjustment of the errors of a net, the total probable error to be expected on each link, due to errors of every sort, should be taken into account.

(3) In addition to the elevations of bench marks supplied to the public for practical use, it would be interesting from a scientific point of view if the States belonging to the Union would publish the elevations of at least their principal bench marks, particularly those to which tidal observations are referred, according to a system in which the adjustment is made on rational principles, and by assuming a priori the elevation of a single fixed point.

As regards the determination of standard elevations for practical purposes, each State remains free to adjust its level net, in accordance with local conditions, by following the method that seems to it most convenient, and especially by assuming as zero (or approximately zero) the mean levels of the sea as determined at various points.

APPENDIX C—ORTHOMETRIC AND DYNAMIC CORRECTIONS [1]

It has been found necessary to apply a correction to the observed elevations on account of the nonparallelism of level surfaces.

A level surface is one which is everywhere perpendicular to the direction of gravity as defined by the plumb line. The surface of still water at a uniform temperature is level. A level surface is also called an equipotential surface, meaning thereby that every point on it has the same gravity potential. The difference in potential between two equipotential surfaces is equal to the work done by or against gravity [2] in moving a unit mass from a point on one surface to a point on the other surface. The work done or difference of potential is independent of the particular points that may be chosen or of the path between them. The work in passing from one point to another on the same level surface is zero. Potential and equipotential surfaces and their properties are discussed in many books, such as Helmert's Die Mathematischen und Physikalischen Theorieen der Höheren Geodäsie: II Theil. In ordinary spirit leveling it is assumed that, if the field work could be made sufficiently exact, the same difference of elevation between two points would be obtained, irrespective of the route followed. It is implied that level surfaces are parallel to each other.

As stated above, this is not true, but the error introduced by this assumption in the results of leveling which is not of a high grade is small compared with the errors of leveling and may be neglected.

In geodetic leveling a correction must be applied to the observed differences in elevation on account of the nonparallelism of level surfaces, and this is most important on north-and-south lines, especially when the average elevation of the line is great. This correction would be small and probably negligible on a line run along a coastal plain.

[1] Lallemand discusses at length the orthometric and dynamic corrections on pp. 358-387 of his "Nivellement de Haute Précision" in the Encyclopédie des Travaux Publics, Paris et Liège, 1912.

[2] Gravity is considered to be the resultant of gravitation (the attraction of the earth's mass) and the centrifugal force due to the earth's rotation.

The surface of the sea and other level surfaces above and below it are approximately spheroidal in shape, but each of the surfaces above sea level has a greater proportional flattening than the sea surface, and consequently such a surface will be at a shorter distance from sea level at the poles than at the Equator. A level surface 1,000 meters above the sea at the Equator would be only 995 meters above the sea at the poles. The polar convergence of other level surfaces toward sea level is approximately in the same proportion to their elevation; for instance, a level surface 500 meters above the sea at the Equator would be 497½ meters above it at the poles, etc.

If we consider a line whose tangent is everywhere coincident with the direction of gravity—that is, a line which is perpendicular at every point to the level surface through that point—this line would not be straight but slightly curved and concave toward the earth's axis. Strictly speaking, there is some ambiguity in the expression "elevation" of a point above sea level. The distance between a point and the sea level might be measured (1) along a curved line of the kind just mentioned passing through the point; (2) along a straight line perpendicular to the level surface at the point in question and extending down to sea level; (3) along a straight line perpendicular to the sea-level surface and passing through the point in question. This third kind of line is the one commonly used in geodesy for reducing to sea level to obtain a point on the sea-level surface corresponding to a point above it. But as long as only the lengths of these lines are considered it makes no difference which length is measured, since if one of these lengths be 10,000 meters—greater than the elevation of the highest mountain on earth—the others will differ from it by less than a thousandth of a millimeter. This source of ambiguity may, therefore, be neglected, but the ambiguity arising from the route followed remains to be considered.

The manner in which discordant differences of level between two points are obtained by following different routes between them may be illustrated by a simple ideal case. Suppose a still lake, with axis lying north and south, is situated on the edge of a level plateau near the sea in the Northern Hemisphere. Let B be a point at the lake's surface near the middle and A a point at sea level in the same latitude as B. Let one line of levels be carried by water leveling north from A to C, a point at sea level and in the same latitude as the north end of the lake, then directly to D, a point on the surface at the north end of the lake, then by water leveling along the lake southward to B. The difference of level between A and B, or between A and any point on the lake's surface will come out equal to the elevation of D above C. Let another line of levels be run southward from A by water levels to E, a point at sea level and in the same latitude as the south end of the lake, then directly from E to F, a point on the surface at the south end of the lake, and northward by water levels on the lake to B. The difference of level between A and B, or between A and any point on the lake's surface, will now come out equal to the elevation of F above E and will be greater than the apparent elevation by the first route, since the two level surfaces approach each other as they near the north pole; and neither result will agree with the result of measuring directly from A to B.

There are two methods of correcting this ambiguity, so that except for errors of observation one may always arrive at the same result for the same point. One method is to correct the difference of level in such a manner that one may obtain the actual vertical distance between B and the sea level or, in the general case, between a point and the geoid. This elevation is called the orthometric elevation of B, and the correc-

tion to the measured difference of elevation to obtain the orthometric elevation from the observed results of leveling is called the orthometric correction. It is to be noted (1) that one may speak, not of the orthometric correction to an elevation, but of the orthometric correction to a difference of elevation for a given route; (2) that points on the same level surface have different orthometric elevations if they lie in different latitudes, and that therefore, conversely, points in different latitudes, having the same orthometric elevation, lie on different level surfaces. This inconvenience has led to the second method, which discards the simple conception of measured vertical distances altogether and gives to each surface a number of its own. Instead of the elevation of a point above sea level, a serial number is given to the level surface on which it lies. The points F, B, and D on the lake-level surface would bear the same number. For convenience, the system of numbering these surfaces is such that the number of a level surface is not very different from the height (in the unit chosen) of any point in the surface. The serial number of a level surface is called its dynamic number [3] and is defined as follows: In the metric system the dynamic number of a point is the work required to raise a mass of 1 kilogram against the force of gravity from sea level to the level surface passing through the point, the work being measured in standard kilogram-meters at sea level in latitude 45°. If the English system be used, the kilogram in the preceding statement is replaced by the pound and the kilogram-meter by the corresponding standard foot-pound. More generally, to get the dynamic number in any system of units, the work which is necessary to raise a unit mass from sea level to the level surface in question is expressed in absolute units and the result divided by g_{45}, where g_{45} is the normal acceleration of gravity at sea level in latitude 45°.

The quantity which must be added to the orthometric elevation of a point to obtain the dynamic number is called the dynamic correction.

The exact determination of orthometric and dynamic corrections would require a knowledge of the force of gravity at every point of the line of levels. To get a sufficient approximation to this knowledge by direct observation at numerous points on the line is, at present, impracticable. If it is assumed that the force of gravity is given by one of the usual mathematical expressions, a fair approximation to the truth will, in general, be obtained. The following expression [4] is taken for the force of gravity g at any point in latitude ϕ and at an orthometric elevation h:

$$g=g_{45}(1-\alpha \cos 2\phi+\beta \cos^2 2\phi-kh); \tag{1}$$

g_{45} has the meaning previously given; α, β, and k are constants, α and β being independent of the unit used and equal, respectively, to 0.002644 and 0.000007; $g_{45}=980.624$ dynes and $k=0.0000003147$, if h is in meters; or, more strictly,

$$k=\frac{3147}{10^{10}}\left(1+\frac{71}{10^5}\cos 2\phi-\frac{23}{10^8}h\right),$$

or for brevity, $k=k'\ (1+\gamma \cos 2\phi-ch)$.*

Let W be the work of raising a unit mass from sea level to a point at orthometric elevation h and H the dynamic number of the level surface on which the point lies. The

[3] The phrase "dynamic number" is an attempt to render into English the French expression "cote dynamique," or its equivalent, "dynamische Koto," as used by German writers.

[4] Equivalent to the expression given by Helmert.

*Sitzungsberichte der Königliche Preussischen Akademie der Wissenschaften, 1903, first Halbband, p. 651.

curved vertical of length h is chosen as the most convenient path for passing from sea level to the given point.

Then

$$W=\int_0^h g dh=g_{45}\int_0^h (1-\alpha \cos 2\phi+\beta \cos^2 2\phi-kh)dh, \text{ or}$$

$$W=g_{45}(1-\alpha \cos 2\phi+\beta \cos^2 2\phi)h-k'\frac{h^2}{2}\left(1+\gamma \cos 2\phi-\frac{2}{3}ch\right).$$

By definition $H=\frac{W}{g_{45}}$, or

$$H=h-(\alpha \cos 2\phi-\beta \cos^2 2\phi)h-k'\frac{h^2}{2}\left(1+\gamma \cos 2\phi-\frac{2}{3}ch\right). \tag{2}$$

This gives the connection between the orthometric elevation and dynamic number. Since H is by definition constant for a level surface, equation (2) may be regarded as the equation of the level surface in coordinates ϕ and h. Let there be a neighboring point on the surface with coordinates $\phi+d\phi$ and $h+dh$. The relation that $d\phi$ and dh must satisfy will be found by differentiating (2), noting that H is constant. It is

$$dh=\frac{-2h(\alpha \sin 2\phi-\beta \sin 4\phi+\frac{1}{2}k'h\gamma \sin 2\phi)d\phi}{1-\alpha \cos 2\phi+\beta \cos^2 2\phi-k'h(1+\gamma \cos 2\phi-ch)} \tag{3}$$

or

$$dh=-2h\alpha \sin 2\phi\left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi+k'h\left(1+\frac{\gamma}{2\alpha}\right)+\ldots\right]d\phi. \tag{4}$$

The term in $k'h$ may be neglected as insignificant in comparison with uncertainties due to errors of observation. On the line of levels from San Diego to Seattle, with an average elevation of about 1,000 meters, and stretching through a difference of latitude of about 15°, the orthometric correction amounted to about one and one-quarter meters, while the term in question had an effect of less than half a millimeter. The neglected terms in (4), involving higher powers in α, β, and $k'h$, have even less effect on the result, so for all practical purposes the equation may be written—

$$dh=-2h\alpha \sin 2\phi\left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi\right]d\phi. \tag{5}$$

By this equation one may compute the orthometric correction to an observed difference in elevation. After corrections have been applied for curvature and refraction, the two points sighted on the level rods are on the same level surface. The rods measure the vertical distances from this level surface to the points on which they rest, but since the point sighted on the more northerly rod (speaking for the Northern Hemisphere) is nearer sea level than the corresponding point of the level surface on the other rod by an amount which may be calculated from equation (5), the observed difference in rod readings must be corrected by equation (5) to obtain the required difference in elevation above sea level. Equation (5) may be extended to far greater distances than those of a single sight by using for h the average height of the instrument between the points whose difference in elevation is to be found.

Tables on pages 142 to 148 contain quantities useful in computing from equations (2) and (5) the orthometric and dynamic corrections.

In Table II the argument is the latitude; the quantities tabulated in the second and third columns are for computing by equation (5) and consist, respectively, of the factor $2\alpha \sin 2\phi \left[1+\left(\alpha-\frac{2\beta}{\alpha}\right)\cos 2\phi\right] \sin 1'$, called C_o, and its logarithm; $\sin 1'$ is included so that $d\phi$ may be taken in minutes of arc. The orthometric correction to the difference of elevation between two points (elevation of second point minus elevation of starting point) is then $-C_o h d\phi$, where h is the average elevation of the instrument between the points, and $d\phi$ is the difference of latitude in minutes, positive when the second point is north of the first. Any unit of length may be used for h; the orthometric correction computed as above is then expressed in the same unit.

The fourth and fifth columns of Table II contain, respectively, the quantities $\alpha \cos 2\phi - \beta \cos^2 2\phi$ (called D_1) and its logarithm.

The expression $-(\alpha \cos 2\phi - \beta \cos^2 2\phi)h = -D_1 h$ constitutes the dynamic correction for latitude and is expressed in the same unit as h. The remaining term of equation (2), $-k' \frac{h^2}{2}\left(1+\gamma \cos 2\phi - \frac{2}{3}ch\right)$, constitutes the dynamic correction for elevation. In the lower table on page 147, $\frac{k'}{2}\left(1+\gamma \cos 2\phi - \frac{2}{3}ch\right)$, abbreviated as D_2, is tabulated under the two arguments ϕ and h (expressed in meters), and in the following table $\log D_2$ is given for the same arguments. The total dynamic correction is then $-D_1 h - D_2 h^2$.

Example 1: If the elevation above sea level of Lake Michigan is 177 meters at Chicago, latitude 41° 53′, what is it at Milwaukee, in latitude 43° 03′? In making the computation the mean elevation of the lake, h, may be assumed to be 177 meters. $d\phi = +70'$. The tables give C_o for the middle latitude, 42° 28′, as 0.000001532; whence the elevation at Milwaukee $= 177 - C_o h d\phi = 177 - 0.0190 = 176.9810$ meters.

Example 2: Suppose levels are to be carried from A, in south latitude 30° 35′, to a point B, in south latitude 30° 17′, and that the average elevation of the instrument between these points is 600 meters. For the middle latitude 30° 26′ the table gives $C_o = -0.000001342$, the negative sign following from the rule at the head of the table, and $d\phi = -30° 17' - (-30° 35') = +18'$. The correction to the quantity (observed elevation at B − observed elevation at A) is then $-600 \times (-0.000001342) \times 18 = +0.0145$ meters.

An idea of how closely the actual gravity at a point conforms to the value derived from formula (1), which gives the value in "free air," may be obtained by consulting pages 59 and 60 of Special Publication No. 99 of the United States Coast and Geodetic Survey. This table shows the free-air anomalies for a total of 85 gravity stations at which observations were made during the period 1916 to 1923, inclusive, and only in one case (Buford, Wyo.) does the anomaly exceed 0.1 dyne. The average of the 85 computed anomalies with regard to sign was 0.000 dyne and without regard to sign the mean was 0.027 dyne.

It may be noted that, if numerous observations of the relative intensity of gravity have been made along the line of leveling, the dynamic numbers may be obtained, free from any hypothesis as to the change in the intensity of gravity, much more readily than the orthometric elevations. Suppose it is assumed that the dynamic number of a certain point is known and that it is desired to determine by leveling the dynamic number of a second point, not far distant. For this purpose, besides the observed difference of level, there is needed the average intensity of gravity along the line, which may be taken as the value of gravity at a point whose projection on the geoid is the middle point of the line joining the projections of the two points and whose elevation is the mean of the elevations, which need not be known with the greatest

exactness. This value could be obtained by direct observation or by interpolation from adjacent gravity stations. Let this value be called g and the observed difference of elevation $\Delta h'$. Then the difference in dynamic numbers is $\frac{\Delta h' g}{g_{45}}$.

A similar process, using theoretical instead of observed gravity, could be used to obtain the dynamic number of one point from another without first computing the orthometric elevations. For this purpose, if g now represents theoretical gravity, the formula can be written $\frac{g}{g_{45}}\Delta h' = \Delta h' - (D_1 + kh)\Delta h'$. The last term serves as a correction to the observed $\Delta h'$ to obtain the difference in the dynamic numbers. The value (approximate) of k is $\frac{3147}{10^{10}}$. (See p. 157.)

But should the law of variation of gravity with elevation and latitude not be known, and the orthometric elevation of a point be desired with only observed values of gravity available, then its dynamic number must first be found, and next both the distance and gravity must be measured along the vertical into the earth to the surface of the geoid, the dynamic number being derived at each step. The dynamic number zero marks the surface of the geoid, and the distance between the given point and the point on the surface whose dynamic number is zero is the orthometric elevation of the former.

APPENDIX D—DEVELOPMENT OF CIRCUIT-REDUCTION METHOD

Fundamentally this method consists of the reduction of particular networks of lines of levels into successively simpler but equivalent forms until the value of one of the unknowns is *readily* determined; whereupon the computations already made are utilized in determining the remainder of the unknowns. Inasmuch as this method gives results identical with those determined by the method of least squares, both in specific and general examples, no rigid mathematical derivation or proof will be given for all statements, but an attempt will be made to show how the method was developed from the principle of the weighted mean.

According to the principle of the weighted mean, if the elevation of a particular bench mark is determined to be X_1, by a line or link of levels of length l_1, and is determined to be X_2 by another link of levels of length l_2, the most probable resulting elevation of the bench mark (considering these two links alone) is

$$\frac{p_1 X_1 + p_2 X_2}{p_1 + p_2},$$

where p_1 (weight)$=\frac{1}{l_1}$ and p_2 (weight)$=\frac{1}{l_2}$. It is believed that the principle of the weighted mean is sufficiently well-known as to require no further discussion here.

Consider the network of lines of levels which is shown in figure 42. (Here the elevations and differences of elevation, as well as the circuit closures, are given in feet to three decimal places and the lengths of the links are given in miles.) It seemed to the originator of this method theoretically permissible to reduce the network shown in figure 42 to that shown in figure 43 if the following conditions were fulfilled: (1) That links 1 and 2 be replaced by a link E_1 of such a length (to be computed) as to be equivalent for adjustment purposes to the two links l_1 and l_2 combined. (2) That the observed orthometric differences of elevation from links 1 and 2 be replaced by a single difference (also to be computed) from S in such a manner as to derive the same elevation for

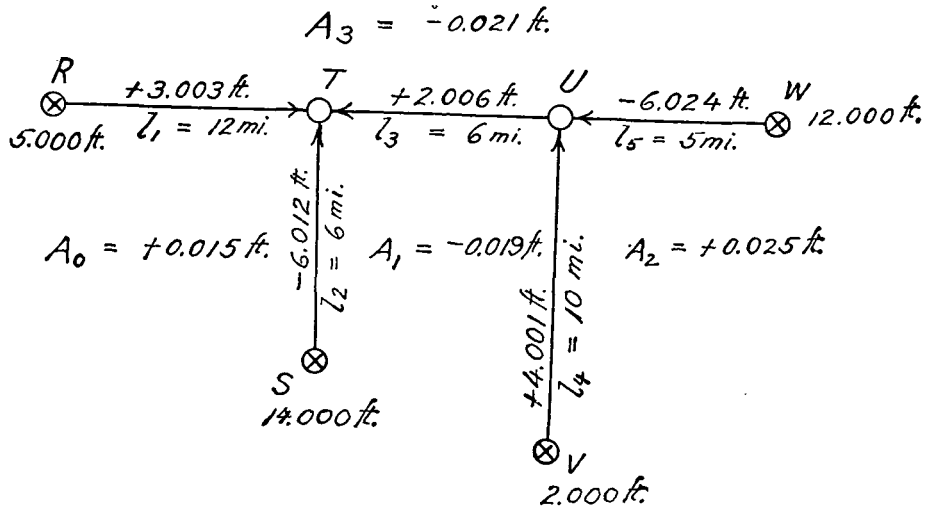


FIGURE 42.—Schematic diagram of a simple level net.

bench mark T from link E_1 as would be derived from a weighted-mean adjustment of links 1 and 2.

In determining such an equivalent length it is necessary that the combined weight of the two links equal the sum of the individual weights of links 1 and 2 or $\frac{1}{l_1}+\frac{1}{l_2}=\frac{l_1+l_2}{l_1l_2}$. It is desired to determine an equivalent length E_1 having a weight of $\frac{l_1+l_2}{l_1l_2}$, and since the length is inversely proportional to the weight, it follows that E_1 equals $\frac{l_1l_2}{l_1+l_2}$. In determining the equivalent difference of elevation, the principle of the weighted mean will be used.

Letting X equal the observed orthometric elevation of T from link 2, $X+A_0$ will equal the observed orthometric elevation of T from link 1; and the weighted-mean elevation of T, considering only links 1 and 2, will be

$$\frac{\frac{X+A_0}{l_1}+\frac{X}{l_2}}{\frac{1}{l_1}+\frac{1}{l_2}}=\frac{(l_1+l_2)X+l_2A_0}{l_1+l_2}=X+\frac{l_2A_0}{l_1+l_2}.$$

Thus it is seen that the equivalent difference varies from the observed orthometric difference from S to T by $+\frac{l_2A_0}{l_1+l_2}$, which will be called K_1.

However, it is desirable to consider the result in terms of changes in circuit closures rather than in terms of changes in differences of elevation. Referring to figure 43, where a single link of length E_1 and a difference of elevation equal to K_1 plus the original difference from S to T have been substituted for links 1 and 2 as shown in figure 42, it may be seen that the closure from S to T to U to V to S now becomes A_1+K_1. In terms of closures, then, it may be said that $K_1=\frac{l_2A_0}{l_1+l_2}$ is the proportional part of circuit closure A_0 that is absorbed by closure A_1 when such a substitution is made.

Similarly the network shown in figure 43 may be reduced to the equivalent form shown in figure 44. In figure 43 the leveling from S to T to U can obviously be con-

sidered as a single link of length l_3+E_1. By using the same reasoning as in the above paragraphs, it may be said that

$$K_2=\frac{l_4(A_1+K_1)}{l_4+l_3+E_1}$$

is the proportional part of circuit closure A_1+K_1 that is absorbed by closure A_2 when an equivalent link E_2 of length $\frac{l_4(l_3+E_1)}{l_4+l_3+E_1}$ is substituted for both link 4 and the link having the length l_3+E_1. This is equivalent to saying that if the difference of elevation determined along link 4 is replaced by that difference plus K_2, the same elevation for U would be derived as that resulting from a weighted-mean adjustment of links 1, 2, 3, and 4; and that a single link of length E_2 has an influence upon the point U equal to the combined influence of links 1, 2, 3, and 4.

With the network simplified to the form shown in figure 44, v_5 may be determined by simple weighted-mean procedure. If Y is considered to be the observed orthometric

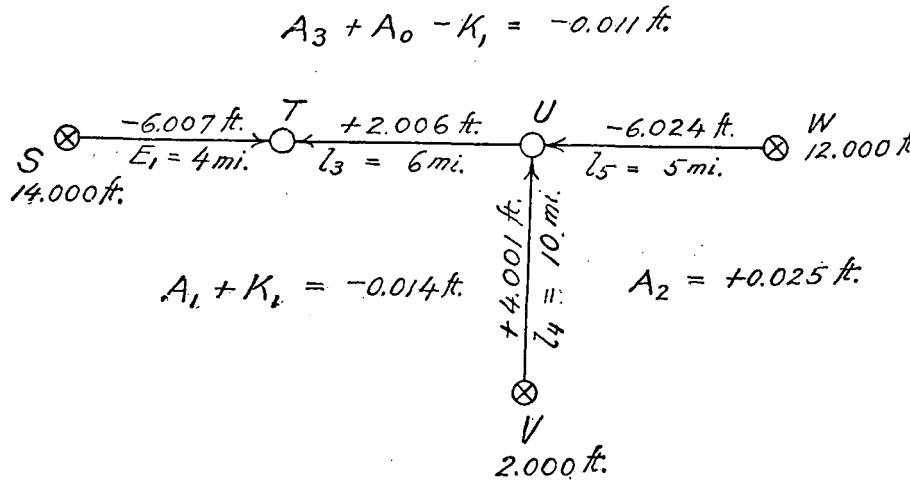


FIGURE 43.—First reduction of the net shown in figure 42.

elevation of U determined from link 5, $Y+A_2+K_2$ will be the elevation of U determined from the link E_2, and the adjusted elevation of U (using the same procedure as that used to determine X on p. 161) will be

$$\frac{\frac{Y+A_2+K_2}{E_2}+\frac{Y}{l_5}}{\frac{1}{E_2}+\frac{1}{l_5}}=\frac{(l_5+E_2)Y+l_5(A_2+K_2)}{l_5+E_2}=Y+\frac{l_5(A_2+K_2)}{l_5+E_2}.$$

Since by definition v_5 is the adjustment correction which must be applied to the observed orthometric difference from link 5 to obtain the adjusted elevation of U, it will be seen that

$$v_5=\frac{l_5(A_2+K_2)}{l_5+E_2}.$$

With the value of v_5 known, that of v_4 may be determined from the consideration that the sum of the corrections to be applied to the various links of a circuit must be equal and opposite in sign to the circuit closure; that is, in circuit A_2 in figure 43, the sum

of the adjustment corrections applied to the observed orthometric differences of elevation from V to U and from U to W should equal $-A_2$. Since v_5 was determined as the adjustment correction for the difference of elevation from W to U, it will be seen that the correction for the difference of elevation from U to V is $-v_5$. Thus, since $+v_4-v_5=-A_2$, $v_4=-A_2+v_5$.

Referring to the circuit A_1+K_1 in figure 43, it will be seen that similarly $+v_{E_1}-v_3-v_4=-A_1-K_1$ or that $v_3-v_{E_1}=A_1+K_1-v_4$. The adjustment correction for link 3 alone is a proportional part of the correction $(v_3-v_{E_1})$ from S to T to U; that is

$$v_3=\frac{l_3(A_1+K_1-v_4)}{l_3+E_1}.$$

Similarly, from circuit A_1 in figure 42, it may be seen that $+v_2-v_3-v_4=-A_1$, so $v_2=-A_1+v_3+v_4$. Also, since from circuit A_0 in figure 42, $+v_1-v_2=-A_0$, $v_1=-A_0+v_2$.

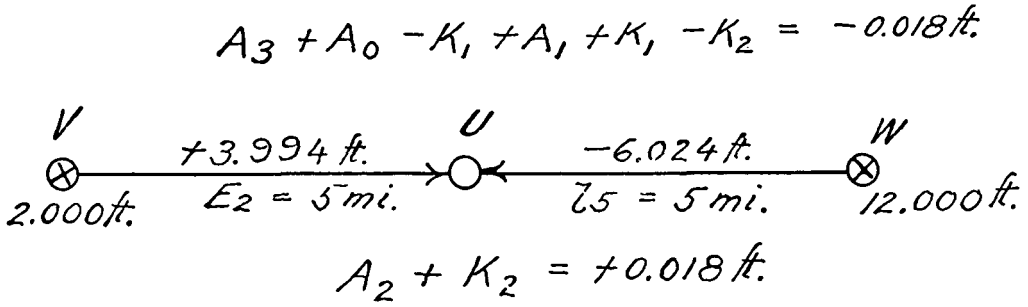


FIGURE 44.—Second reduction of the net shown in figure 42.

On the computation forms the directions of the arrows on each link have been changed from those shown in these figures illustrating the theory of the method, so as to show the directions in which the v's are to be applied to B. M.-*minus*-B. M. differences.

Referring now to computation Form A, as shown in figure 27 and several later figures, it will be seen that formulas are given for gradually reducing the entire network to successively simpler forms, where each K_n is the proportional part of the circuit closure $A_{n-1}+K_{n-1}$ that is absorbed by circuit A_n when a link of length E_n is substituted for two links of length l_{2n} and $l_{2n-1}+E_{n-1}$; and where v_n is the adjustment correction to be applied to the observed difference of elevation (*from* bench mark *to* bench mark as indicated by the arrows shown in the sketch) for link n.

Although the E's, K's, and v's are shown on computation Form A only as far as E_6, K_6, and v_{13}, the number of formulas can be increased as may be necessary by using the general expressions:

$$E_n=\frac{l_{2n}(E_{n-1}+l_{2n-1})}{E_{n-1}+l_{2n-1}+l_{2n}},$$

$$K_n=\frac{l_{2n}(K_{n-1}+A_{n-1})}{E_{n-1}+l_{2n-1}+l_{2n}},$$

$$V_{2n+1}=\frac{l_{2n+1}(K_n+A_n-v_{2n+2})}{E_n+l_{2n+1}}, \text{ and}$$

$$V_{2n}=-A_n+v_{2n+1}+v_{2n+2}.$$

APPENDIX E—APPLICATION OF THE CIRCUIT-REDUCTION METHOD TO NETWORKS INCLUDING INNER CIRCUITS HAVING NO FIXED POINTS

The following discussion of the application of the principles of the circuit-reduction method in operating on circuits containing more than two links will show that a proportionally greater time is required for the elimination of each circuit than is required for two-link circuits. The reader will be left to judge for himself as to the relative efficiency of the circuit-reduction method as compared with the methods using either condition or observation equations when dealing with the type of network to be discussed later on in this appendix.

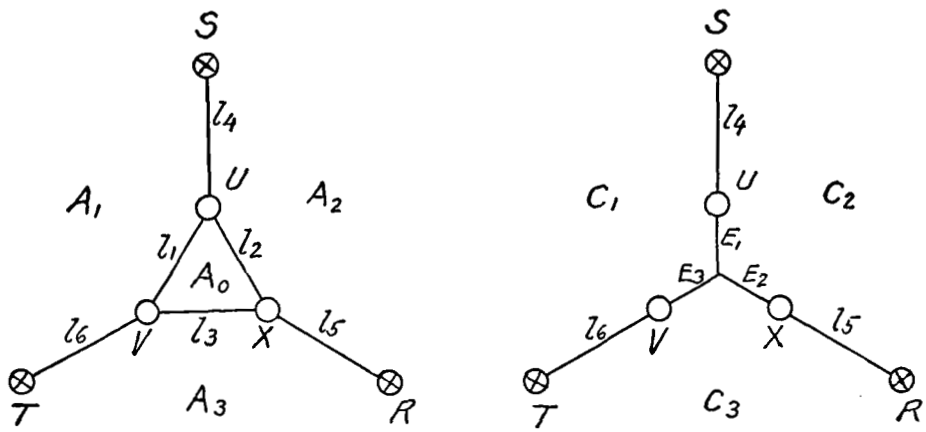


FIGURE 45.—Reduction of interior three-sided circuit.

Consider the network shown at the left in figure 45. For adjustment purposes this network is equivalent to that shown at the right in figure 45, if

$$E_1=\frac{l_1l_2}{l_1+l_2+l_3};$$

$$E_2=\frac{l_2l_3}{l_1+l_2+l_3};$$

$$E_3=\frac{l_1l_3}{l_1+l_2+l_3};$$

$$C_1=A_1+\frac{l_1A_0}{l_1+l_2+l_3};$$

$$C_2=A_2+\frac{l_2A_0}{l_1+l_2+l_3}; \text{ and}$$

$$C_3=A_3+\frac{l_3A_0}{l_1+l_2+l_3}.$$

In other words, the inner circuit A_0 can be eliminated if an equivalent set of three links, E_1, E_2, and E_3, are substituted for links l_1, l_2, and l_3, in the manner shown, with the outer circuit closures changed to include proper proportional parts of the closure

of the inner circuit. It may be proven that this is permissible by making least-square adjustments of both the original network and the reduced network, whereupon comparison of results will show that identical results are obtained.

After the network shown at the left in figure 45 has been reduced to the network shown at the right in the same figure, the adjustment corrections or v's for the composite links, l_4+E_1, l_5+E_2, and l_6+E_3, can be determined by the methods previously discussed and the use of Form A. The corrections for l_4, l_5, and l_6 can then be determined by taking proportional parts of the corrections for the composite links. Since the sum of the corrections (v's) in any circuit must be equal to, but of opposite sign to, the closure of that circuit and since A_1, A_2, A_3, v_4, v_5, and v_6 are already known, the remaining v's (v_1, v_2, and v_3) may be computed.

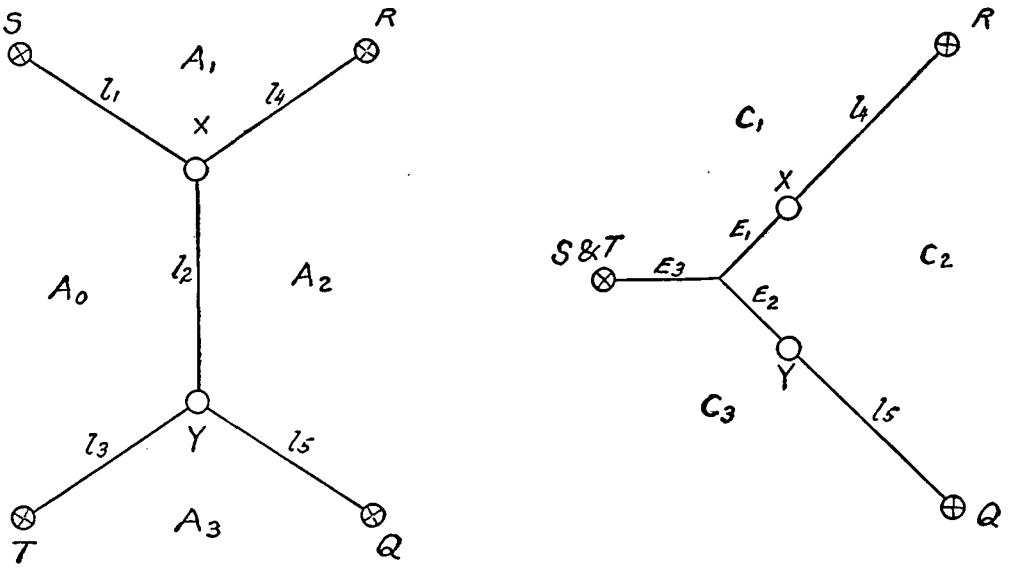


FIGURE 46.—Reduction of exterior three-sided circuit.

Similarly, it can be proven that the network at the right in figure 46 is equivalent for adjustment purposes to the network at the left in the same figure *if* the E's and C's have the values indicated by the above set of formulas (p. 164). If fixed points S and T are considered to be separated by a zero distance, circuit A_0 can be considered as a triangular circuit, so that this is but another example of the reduction of a triangular circuit as explained previously in this appendix.

By operating on circuit A_0 of the network shown at the left in figure 47 as explained in the preceding paragraph, a reduced equivalent network with an inner triangular circuit is obtained as shown at the right in figure 47. Once this network is obtained, it can be further reduced by following the method outlined at the beginning of this appendix.

In order to show the impracticability of operating directly on a four-link circuit, it may be stated that if the inner circuit A_2 of the left half of figure 48 is operated on, the reduced network takes the form shown at the right in figure 48. Here, not only does the elimination of a single circuit require the determination of six equivalent lengths and four equivalent circuits, but the placement of the equivalent links is not

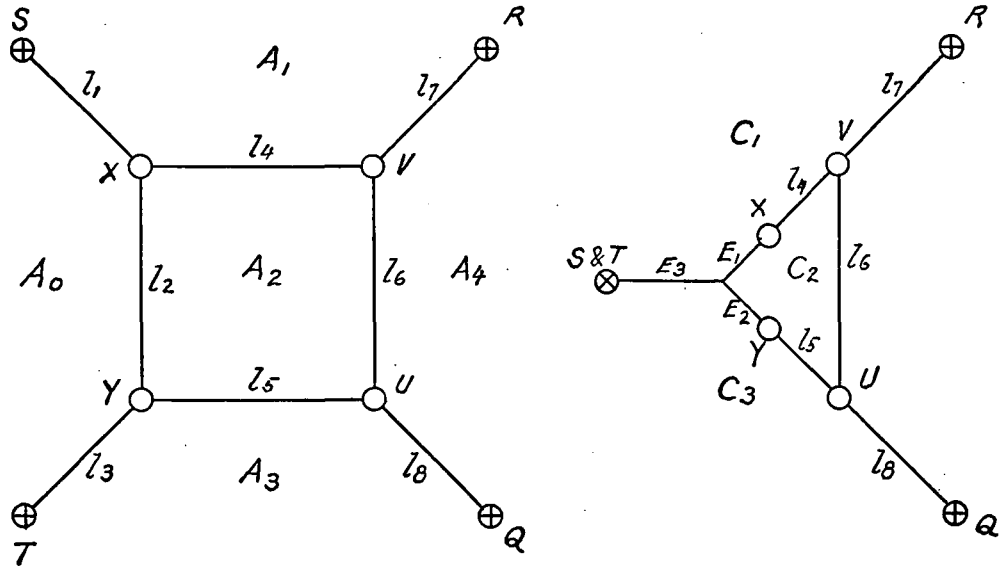


FIGURE 47.—Reduction of interior four-sided circuit (first method).

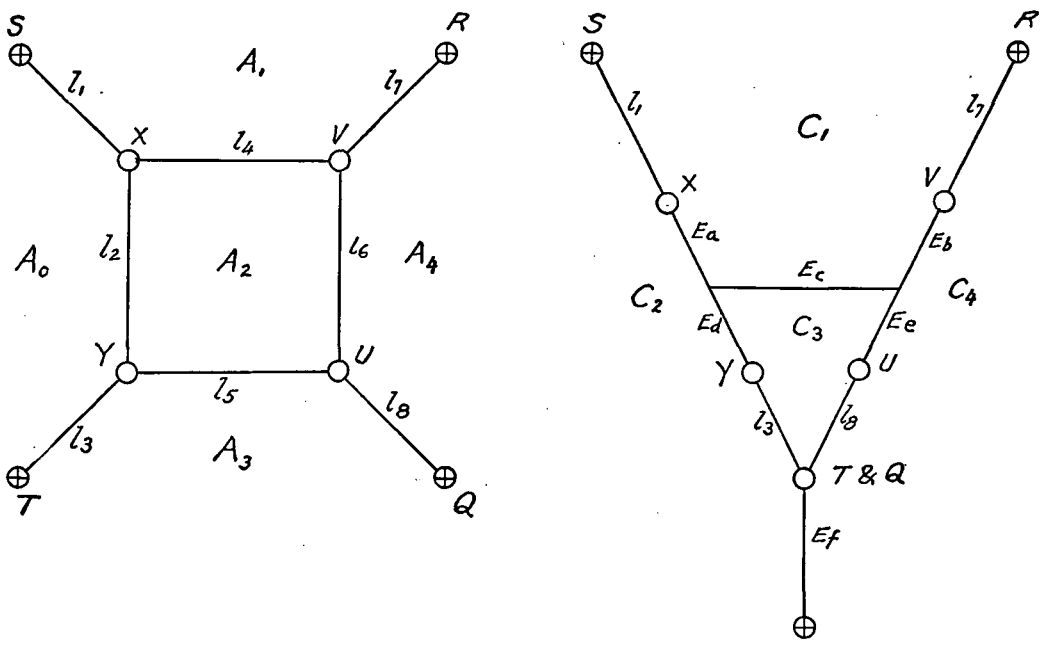


FIGURE 48.—Reduction of interior four-sided circuit (second method).

obvious. The formulas necessary to reduce the network in the left half of figure 48 to that shown in the right half of the same figure are as follows:

$$E_a=\frac{l_2l_4}{l_2+l_4+l_5+l_6},$$

$$E_b=\frac{l_4l_6}{l_2+l_4+l_5+l_6},$$

$$E_c=\frac{l_4l_5}{l_2+l_4+l_5+l_6},$$

$$E_d=\frac{l_2l_5}{l_2+l_4+l_5+l_6},$$

$$E_e=\frac{l_5l_6}{l_2+l_4+l_5+l_6},$$

$$E_f=\frac{l_2l_6}{l_2+l_4+l_5+l_6},$$

$$C_1=A_1+\frac{l_4A_2}{l_2+l_4+l_5+l_6},$$

$$C_2=A_0+\frac{l_2A_2}{l_2+l_4+l_5+l_6},$$

$$C_3=A_3+\frac{l_5A_2}{l_2+l_4+l_5+l_6}, \text{ and}$$

$$C_4=A_4+\frac{l_6A_2}{l_2+l_4+l_5+l_6}.$$

A study of the Forms D 1 and D 2 (see fig. 52) will show that the seven-link inner circuit is reduced one side at a time by operating on an outer circuit in the manner explained in connection with the discussion of figure 47 until an inner circuit composed of a triangle is derived, whereupon it is then reduced by the method explained in connection with the discussion of figure 45. This computation form (the combination of Forms D 1 and D 2) may be used in adjusting networks conforming to any of the sketches shown at the top of either of these forms. It may also be used in combination with the other forms previously discussed, as exemplified in the problem considered below.

For adjustment purposes the network shown on a geographical basis in figure 49 may be represented by the sketch in figure 50 if the fixed point common to links 1d and 24 in figure 49 is considered to be two separate fixed points having the same elevation, an obviously permissible procedure. Furthermore, if links 1d, 2d, 3d, 4d, 5d, 6d, and 7d are expressed in terms of a composite link 17, and links 7a, 7b, and 7c in terms of a composite link 7, with proper modification of the circuit closures, a simpli-

fied network will be derived which conforms to the second network from the left at the top of computation Form D 2 as shown in figure 52.

To determine the length of the composite link 17 and the resulting circuit closures, computation Form A (see fig. 53) may be used. The lengths of links 1d to 7d, inclusive, and closures C_0 to C_3, inclusive, are entered on the form, and the computation of the E's and K's through E_3 and K_3 is made. (The computation of the v's must, of course,

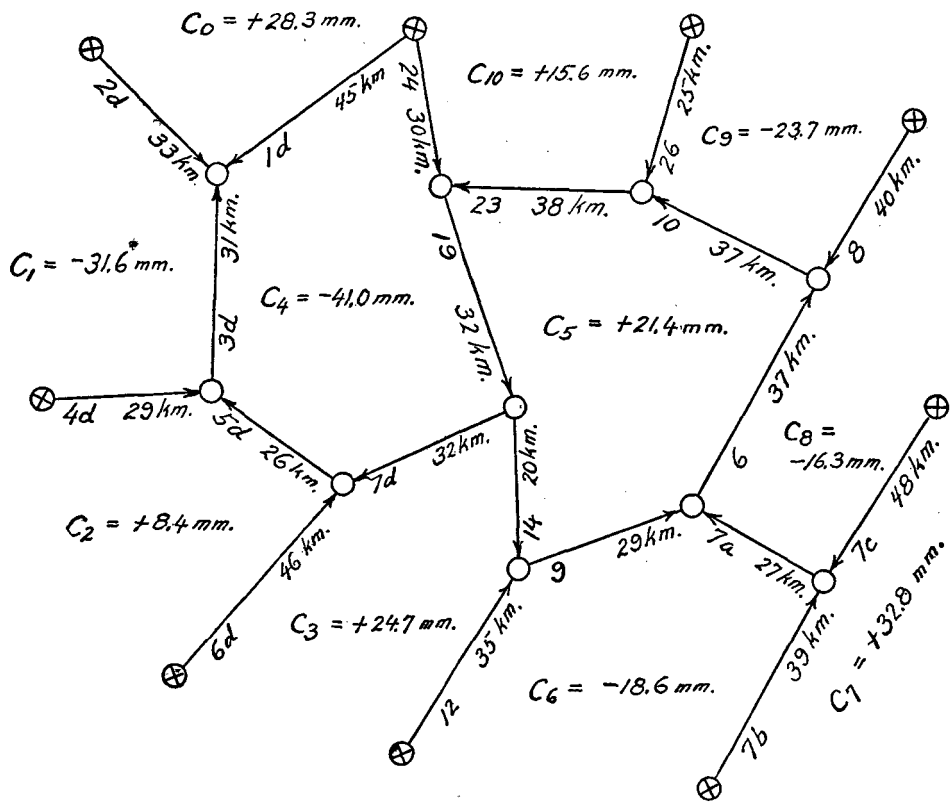


FIGURE 49.—Subordinate net; containing 10 new junctions, 9 fixed points, and 1 six-sided interior circuit with no fixed points; to be adjusted by the circuit-reduction method to illustrate the use of Forms D 1 and D 2.

wait until later.) The length of the composite link 17 is then taken to be E_3+l_{7d} and the closure below link 17 is K_3+A_3. The closure above link 17 can be determined by consideration of the fact that the sum of the closures to the left of the lines formed by links 12, 14, 19, and 24 should be the same as before, so that it will be equal to $C_0+C_1+C_2+C_3+C_4-K_3-A_3$.

Determination of the length of the composite link 7 and the resultant circuit closures can be made by using computation Form C in the manner explained in the previous discussion of the use of Form C. (See p. 127.) The computation in this case is shown in figure 51.

It will now be seen that our network has been reduced to a form similar to that shown in the second figure from the left at the top of Form D 2 (see fig. 52) and that at the start the numbering of the links in figures 49 and 50 was carried out in such a manner that when the network had been reduced to this form the numbering would correspond with that of the sketch at the top of the computation form. To carry the computation through this form, all known values of the l's and A's are entered, disregarding the formulas given for such known values. No values are known or need be computed for l_1 to l_5, inclusive, or for A_1, A_2, A_3, and D_1. The computations for the unknown values among the l's and A's are then made in order down the left-hand

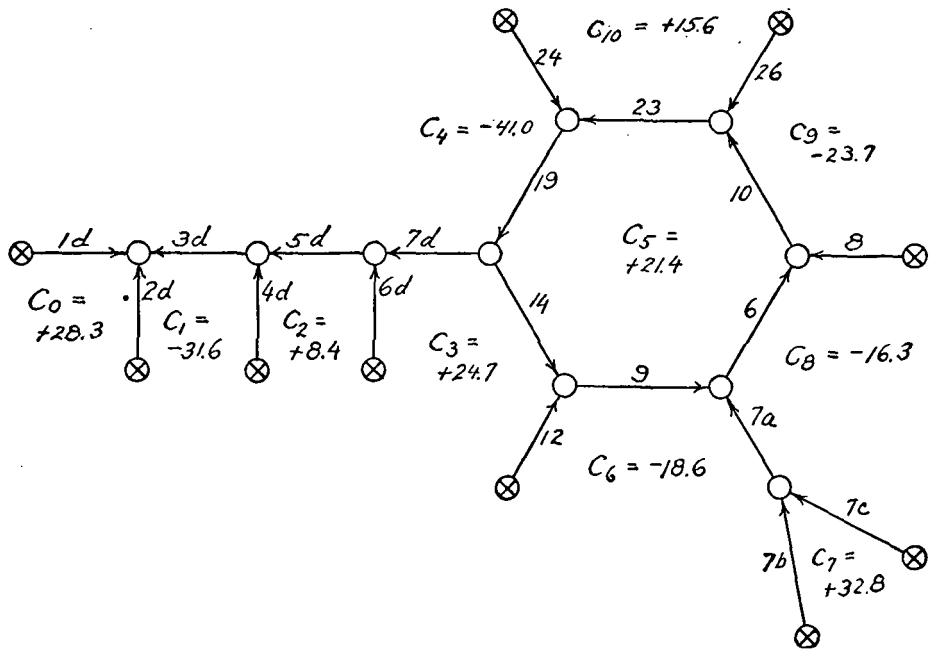


FIGURE 50.—Schematic diagram of the net shown in figure 49.

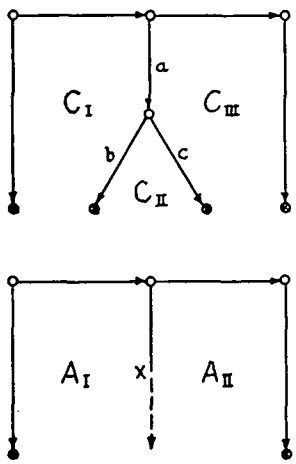
columns of Form D 2 and Form D 1, after which the v's are computed in reverse order up the right-hand columns of these forms. After v_{29}, v_{28}, and v_{27} are computed, the v/l's may be computed and summed as an intermediate check, but this check alone will not reveal all types of errors that could be made in the preceding work. Remaining v's are then computed, the v/l's are computed for all links shown in the second figure from the left at the top of Form D 2, and checks are made at each junction point of the figure.

The values of the v's in figures 51 and 53 may now be computed. v_7 from figure 53 is entered as v_x in figure 51, and v_{7a}, v_{7b}, and v_{7c} are computed as indicated by the formulas. Summation of the v/l values (with the sign of the v/l for link 1a changed) completes the check on the computed values of these v's. Considering the theory involved, it is apparent that v_{7d} in figure 50 (v_7 in fig. 53) should be a proportional part of v_{17} in figure

52, or that $v_{7d}=-\frac{l_{7d}\ v_{17}}{l_{17}}$. Note the negative sign, which is due to the fact that the arrows on the link are in opposite directions in figures 52 and 53. Now, from circuit closure C_3 in figure 50 it can be seen that $v_{6d}-v_{7d}+v_{14}-v_{12}=-C_3$, or $v_{6d}=-C_3+v_{7d}+v_{12}-v_{14}$, assuming that the arrows in this figure indicate B. M. to B. M. differences. The remaining v's can then be computed directly from the formulas shown on Form A (fig. 53) and the computation of the values of v/l and the summations for check are carried out as in previous examples of the use of Form A.

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT

FORM C.



$l_a = 27$ $\quad l_b = 39$ $\quad l_c = 48$

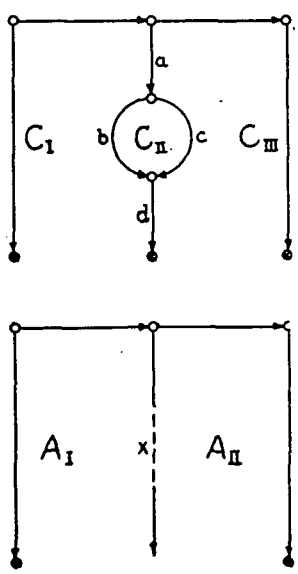
$C_I = -18.6$ $\quad C_{II} = +32.8$ $\quad C_{III} = -16.3$

$l_x = l_a + \frac{l_b l_c}{l_b + l_c} = 48.52$ $\quad K = \frac{l_b C_{II}}{l_b + l_c} = +14.70$

$A_I = C_I + K = -3.9$ $\quad A_{II} = C_{II} + C_{III} - K = +1.8$

	$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} = +1.09$ (+1.95)	+0.040	
$V_b = -K + V_x - V_a = -13.84$	−0.355	0.000
$V_c = +C_{II} + V_b = +18.96$	+0.395	

FORM D.



$l_a =$ $\quad l_b =$ $\quad l_c =$ $\quad l_d =$

$C_I =$ $\quad C_{II} =$ $\quad C_{III} =$

$l_x = l_a + l_d + \frac{l_b l_c}{l_b + l_c} =$ $\quad K = \frac{l_b C_{II}}{l_b + l_c} =$

$A_I = C_I + K =$ $\quad A_{II} = C_{II} + C_{III} - K =$

	$\frac{V}{l}$	check
$V_a = \frac{l_a V_x}{l_x} =$		
$V_d = \frac{l_d V_x}{l_x} =$		
$V_b = -K + V_x - V_a - V_d =$		
$V_c = +C_{II} + V_b =$		

FIGURE 51.—Form C, showing the computations involved in obtaining a single-link equivalent for links 7a, 7b, and 7c as shown in figure 50.

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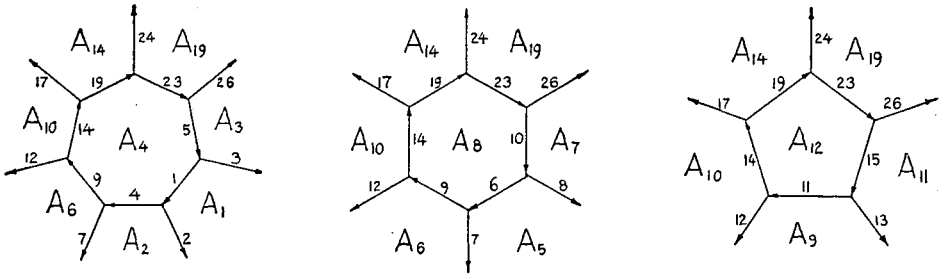
"CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT

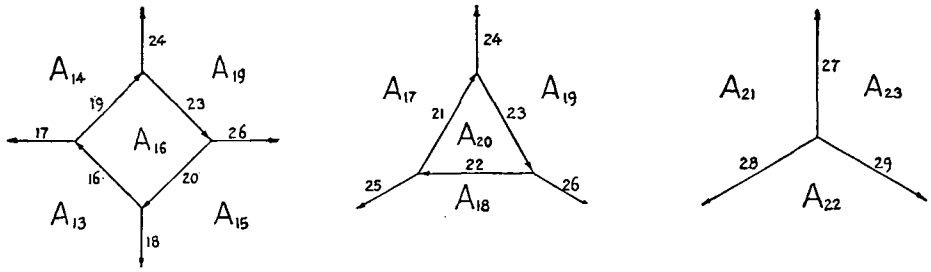
FORM D-2



			V/l	CHECK
$l_1=$	$A_1=$	$V_1=-A_1-V_2+V_3=$		
$l_2=$	$A_2=$	$V_2=+A_2+V_4+V_7=$		
$l_3=$	$A_3=$	$V_3=-A_3-V_5+V_{26}=$		
$l_4=$		$V_4=\frac{l_4V_6}{l_6}=$		
$l_5=$	$D_1=l_1+l_2+l_3=$	$V_5=\frac{l_5V_{10}}{l_{10}}=$		
$l_6=\left[\frac{l_1l_2}{D_1}+l_4\right]=37$	$A_5=\left[\frac{l_2A_1}{D_1}+A_2\right]=+1.8$	$V_6=-A_5-V_7+V_8=+3.29$	+0.089	$\frac{6,8,10}{0.000}$
$l_7=48.52$	$A_6=-3.9$	$V_7=+A_6+V_9+V_{12}=+1.95$	+0.040	
$l_8=\left[\frac{l_2l_3}{D_1}\right]=40$	$A_7=\left[\frac{l_3A_1}{D_1}+A_3\right]=-23.7$	$V_8=-A_7-V_{10}+V_{26}=+7.04$	+0.176	$\frac{6,7,9}{-0.001}$
$l_9=29$		$V_9=\frac{l_9V_{11}}{l_{11}}=+1.40$	+0.048	
$l_{10}=\left[\frac{l_1l_3}{D_1}+l_5\right]=37$	$D_5=l_6+l_7+l_8=125.52$	$V_{10}=\frac{l_{10}V_{15}}{l_{15}}=+9.82$	+0.265	$\frac{9,12,14}{0.000}$
$l_{11}=\left[\frac{l_6l_7}{D_5}+l_9\right]=43.30$	$A_9=\left[\frac{l_7A_5}{D_5}+A_6\right]=-3.20$	$V_{11}=-A_9-V_{12}+V_{13}=+2.09$		
$l_{12}=35$	$A_{10}=+25.31$	$V_{12}=+A_{10}+V_{14}+V_{17}=+4.45$	+0.127	
$l_{13}=\left[\frac{l_7l_8}{D_5}\right]=15.46$	$A_{11}=\left[\frac{l_8A_5}{D_5}+A_7\right]=-23.13$	$V_{13}=-A_{11}-V_{15}+V_{26}=+3.34$		
$l_{14}=20$		$V_{14}=\frac{l_{14}V_{16}}{l_{16}}=-1.58$	-0.079	
$l_{15}=\left[\frac{l_6l_8}{D_5}+l_{10}\right]=48.79$	$D_9=l_{11}+l_{12}+l_{13}=93.76$	$V_{15}=\frac{l_{15}V_{20}}{l_{20}}=+12.95$		

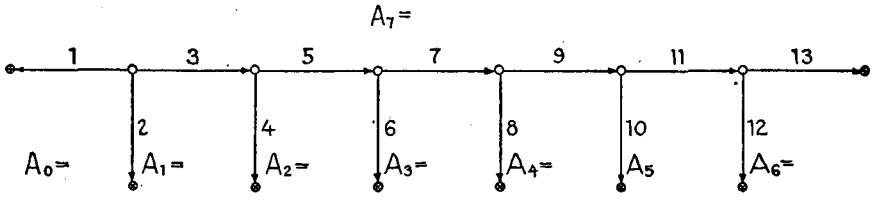
FIGURE 52.—Forms D 2 and D 1, showing the computations involved in the adjustment of the net shown in figures 49 and 50, after simplification by the use of Forms C (fig. 51) and A (fig. 53).

"CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM D-1



			V/l	CHECK
$l_{16}=\left[\frac{l_{11}l_{12}}{D_9}+l_{14}\right]=36.16$	$A_{13}=\left[\frac{l_{12}A_9}{D_9}+A_{10}\right]=+24.12$	$V_{16}=-A_{13}-V_{17}+V_{18}=-2.86$		
$l_{17}=54.58$	$A_{14}=-36.51$	$V_{17}=+A_{14}+V_{19}+V_{24}=-19.28$	-0.353	$\frac{14,17,19}{0.000}$
$l_{18}=\left[\frac{l_{12}l_{13}}{D_9}\right]=5.77$	$A_{15}=\left[\frac{l_{13}A_9}{D_9}+A_{11}\right]=-23.66$	$V_{18}=-A_{15}-V_{20}+V_{26}=+1.98$		
$l_{19}=32$		$V_{19}=\frac{l_{19}V_{21}}{l_{21}}=+8.77$	$+0.274$	$\frac{19,23,24}{0.000}$
$l_{20}=\left[\frac{l_{11}l_{13}}{D_9}+l_{15}\right]=55.93$	$D_{13}=l_{16}+l_{17}+l_{18}=96.51$	$V_{20}=\frac{l_{20}V_{22}}{l_{22}}=+14.84$		
$l_{21}=\left[\frac{l_{16}l_{17}}{D_{13}}+l_{19}\right]=52.45$	$A_{17}=\left[\frac{l_{17}A_{13}}{D_{13}}+A_{14}\right]=-22.87$	$V_{21}=-A_{17}-V_{24}+V_{25}=+14.38$		$\frac{10,23,26}{-0.001}$
$l_{22}=\left[\frac{l_{16}l_{18}}{D_{13}}+l_{20}\right]=58.09$	$A_{18}=\left[\frac{l_{18}A_{13}}{D_{13}}+A_{15}\right]=-22.22$	$V_{22}=-A_{18}-V_{25}+V_{26}=+15.41$		
$l_{23}=38$	$A_{19}=+15.6$	$V_{23}=-A_{19}+V_{24}-V_{26}=-0.30$	-0.008	
$l_{24}=30$	$A_{20}=-A_{17}-A_{18}-A_{19}=+29.49$	$V_{24}=\frac{l_{24}V_{27}}{l_{27}}=+8.46$	$+0.282$	
$l_{25}=\left[\frac{l_{17}l_{18}}{D_{13}}\right]=3.26$		$V_{25}=\frac{l_{25}V_{28}}{l_{28}}=-0.03$		
$l_{26}=25$	$D_{20}=l_{21}+l_{22}+l_{23}=148.54$	$V_{26}=\frac{l_{26}V_{29}}{l_{29}}=-6.84$	-0.274	
$l_{27}=\left[\frac{l_{21}l_{23}}{D_{20}}+l_{24}\right]=43.42$	$A_{21}=\left[\frac{l_{21}A_{20}}{D_{20}}+A_{17}\right]=-12.46$	$V_{27}=-A_{21}+V_{28}=+12.25$	$+0.282$	
$l_{28}=\left[\frac{l_{21}l_{22}}{D_{20}}+l_{25}\right]=23.77$	$A_{22}=\left[\frac{l_{22}A_{20}}{D_{20}}+A_{18}\right]=-10.69$	$V_{28}=-A_{22}+V_{29}=-0.21$	-0.009	$\frac{27,28,29}{0.000}$
$l_{29}=\left[\frac{l_{22}l_{23}}{D_{20}}+l_{26}\right]=39.86$		$V_{29}=\frac{l_{29}(A_{22}+K_1)}{l_{29}+E_1}=-10.90$	-0.273	
$E_1=\left[\frac{l_{27}l_{28}}{l_{27}+l_{28}}\right]=15.36$	$K_1=\left[\frac{l_{28}A_{21}}{l_{27}+l_{28}}\right]=-4.41$			

FORMULAS FOR "CIRCUIT-REDUCTION" METHOD OF ADJUSTMENT
FORM A.



				$\frac{V}{l}$	Check
$l_1 = 45$ $l_2 = 33$		$A_0 = +28.3$			
$E_1 = \frac{l_2 l_1}{l_1 + l_2} = 19.04$	$K_1 = \frac{l_2 A_0}{l_1 + l_2} = +11.97$		$V_1 = -A_0 + V_2 = -12.95$	-0.288	
$l_3 = 31$ $l_4 = 29$		$A_1 = -31.6$	$V_2 = -A_1 + V_3 + V_4 = +15.35$	$+0.465$	0.000
$E_2 = \frac{l_4(E_1 + l_3)}{(E_1 + l_3 + l_4)} = 18.36$	$K_2 = \frac{l_4(K_1 + A_1)}{E_1 + l_3 + l_4} = -7.20$		$V_3 = \frac{l_3(K_1 + A_1 - V_4)}{E_1 + l_3} = -5.50$	-0.177	
$l_5 = 26$ $l_6 = 46$		$A_2 = +8.4$	$V_4 = -A_2 + V_5 + V_6 = -10.75$	-0.371	-0.001
$E_3 = \frac{l_6(E_2 + l_5)}{(E_2 + l_5 + l_6)} = 22.58$	$K_3 = \frac{l_6(K_2 + A_2)}{E_2 + l_5 + l_6} = +0.61$		$V_5 = \frac{l_5(K_2 + A_2 - V_6)}{E_2 + l_5} = +5.02$	$+0.193$	
$l_7 = 32$ $l_8 =$		$A_3 = +24.7$	$V_6 = -A_3 + V_7 + V_{12} - V_{14} = -7.37$	-0.160	0.000
$E_4 = \frac{l_8(E_3 + l_7)}{(E_3 + l_7 + l_8)} =$	$K_4 = \frac{l_8(K_3 + A_3)}{E_3 + l_7 + l_8} =$		$V_7 = \frac{l_7(V_{17})}{E_3 + l_7} = +11.30$	$+0.353$	
$l_9 =$ $l_{10} =$		$A_4 =$	$V_8 = -A_4 + V_9 + V_{10} =$		
$E_5 = \frac{l_{10}(E_4 + l_9)}{(E_4 + l_9 + l_{10})} =$	$K_5 = \frac{l_{10}(K_4 + A_4)}{E_4 + l_9 + l_{10}} =$		$V_9 = \frac{l_9(K_4 + A_4 - V_{10})}{E_4 + l_9} =$		
$l_{11} =$ $l_{12} =$		$A_5 =$	$V_{10} = -A_5 + V_{11} + V_{12} =$		
$E_6 = \frac{l_{12}(E_5 + l_{11})}{(E_5 + l_{11} + l_{12})} =$	$K_6 = \frac{l_{12}(K_5 + A_5)}{E_5 + l_{11} + l_{12}} =$		$V_{11} = \frac{l_{11}(K_5 + A_5 - V_{12})}{E_5 + l_{11}} =$		
$l_{13} =$		$A_6 =$	$V_{12} = -A_6 + V_{13} =$		
			$V_{13} = \frac{l_{13}(K_6 + A_6)}{E_6 + l_{13}} =$		

FIGURE 53.—Form A, showing the computations involved in obtaining a single-link equivalent for links 1d to 7d, inclusive, as shown in figure 50.

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