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MANUAL OF GEODETIC TRIANGULATION

By
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PREFACE

This manual is a compilation of approved methods for triangulation surveys of the U. S. Coast and Geodetic Survey, and its primary purpose is to furnish specifications and instructions to personnel engaged on those field operations.

The progress in development of instruments, equipment, and methods during the past twenty years virtually necessitated the revision of Special Publication No. 120, "Manual of First-Order Triangulation," which was first published in 1926, and Special Publication No. 145, "Manual of Second- and Third-Order Triangulation and Traverse," which was first published in 1929. The changes introduced by the use of portable steel towers, multiple-unit observing parties, improved instruments, and schemes of area triangulation have caused modifications and improvements in field operations which are not considered in the older manuals.

The material in this manual is the product of accumulated experience in the field and office of many members of the U. S. Coast and Geodetic Survey for many years. Applicable material is taken freely from previously published manuals of this Bureau, and in particular from Special Publications Nos. 120 and 145, which were so ably written by the late C. V. Hodgson, formerly Assistant Chief, Division of Geodesy.

This publication has been prepared under the direction of Captain H. W. Hemple, Chief, Division of Geodesy, whose advice and criticism are greatly appreciated. Assistance in the preparation of this manual has been given by a large number of members of the Division of Geodesy. In particular, Mr. Lansing G. Simmons, Chief Mathematician, Division of Geodesy, and Commander C. I. Aslakson reviewed the entire manuscript and contributed many valuable suggestions. Captain A. J. Hoskinson, Commander J. H. Brittain, Mr. C. A. Whitten, Mr. W. D. Sutcliffe, Mr. Donald A. Rice, Mr. R. O. Williamson, and Mr. S. P. Hand gave valuable advice and criticism on various sections of this manual. Mr. C. N. Claire and Mr. N. F. Braaten edited the manuscript.

PREFACE TO 1959 EDITION

Since the original publication of this Manual in 1950 it has seemed desirable, if not necessary, to tighten the specifications for first- and second-order triangulation under certain prescribed conditions. Accordingly, first-order work has been separated into three classes and second-order work into two classes. Specifications under these classes appear in the revised table 1 (pp. xiv–xv). Other revisions, reflecting this concept, appear on pages 2–5, 9, 10, 13, 17, 114, 162, 165, 197, 251, and 255.

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MANUAL OF GEODETIC TRIANGULATION

INTRODUCTION

Geodetic triangulation is a very efficient method of controlling surveys over extensive areas of the earth's surface and is utilized in the basic horizontal control networks of the major countries of the world. Triangulation is a method of surveying in which the stations are points on the ground at vertices of triangles forming chains or networks. In these triangles, the angles are observed instrumentally, and the sides are determined by successive computations through the chains of triangles from selected triangle sides called base lines, the lengths of which are obtained from direct measurements on the ground. Triangulation in which the figure and size of the earth are taken into account is called geodetic triangulation. The highest form of survey engineering is involved in geodetic triangulation, necessitating extremely precise instrumental equipment and observational techniques, capable and conscientious personnel, and detailed computations.

The data resulting from geodetic triangulation are primarily horizontal control data which are expressed in the form of geodetic latitudes and longitudes (or equivalent plane coordinate values) of definite established points and include the distances and azimuths for all lines observed. These points are normally marked by bronze disks (most frequently set in concrete monuments) but also include many prominent objects, such as church spires, water tanks, and radio towers. Descriptions and geographic positions of these established points are published for public distribution.

The basic horizontal control net of this country has many very important uses. It provides a rigid framework for all types of accurate charting and mapping projects, including the nautical and aeronautical charts of this Bureau; it serves in locating national, State, and county boundaries, and will perpetuate the surveys of any private boundaries which are connected to it; it enables local surveys and mapping of all kinds to be fitted together and into the national mapping plan; it serves to increase the accuracy of and to provide checks for local and city surveys, and for surveys of lower order; and it assists in the perpetuation of marks established by such surveys. It is of over-all importance to the nation for military defense both in furnishing a framework for correlated mapping and for all activities concerned with long-range position control and accurate directions and distances. Horizontal control also has a number of scientific uses including studies of earth movement, figure of the earth, and other geophysical research. Because of its widespread ramifications, provision of an adequate horizontal control survey network is a function of the Federal Government.

Whereas horizontal control data may be obtained by either triangulation or traverse, this manual does not cover the field methods of traverse but, in successive chapters, gives discussions of the coordinated operations of reconnaissance, triangulation observations, base measurement, azimuth determination, and special surveys.

Basic requirements for horizontal control were set forth in "Specifications for Horizontal and Vertical Control" as approved by the Board of Surveys and Maps of the Federal Government May 9, 1933. These specifications are printed on pages 260 to 267 in the appendix of this manual. The Board of Surveys and Maps no longer exists, having been abolished in 1942. Instructions and specifications for triangulation of the U. S. Coast and Geodetic Survey are printed in chapter 2 of this manual.

CLASSIFICATION OF HORIZONTAL CONTROL SURVEYS

The basis of classification of horizontal control surveys is the accuracy with which the length and azimuth of a line of the triangulation or traverse are determined. Since it is impossible to ascertain the absolute error in the determination of the length or azimuth of each line of triangulation or traverse, indirect criteria must be used. On triangulation, the principal criterion is that the length check between measured bases, or adjusted lines, and the position closure (when applicable) shall not exceed the limits given in table 1. These minimum requirements apply to all triangulation length closures including loops and area triangulation. In general, most closures are much better than the prescribed requirements. Since this criterion of length agreement between bases cannot be applied until the necessary observations have been completed, other criteria, as listed in table 1 and defined in the specifications of chapter 2, are applied as observations progress in order to maintain uniform accuracy along the chain of triangles.

Schemes that are supplemental to the main triangulation are observed with the same instruments and methods as are used for first-order observations. The allowable closures are modified for this supplemental triangulation as listed in table 1 under the heading of second-order. The resulting triangulation, which often approaches first-order accuracy and which is usually of much higher quality than that executed with second-order methods and instruments, is called modified second-order triangulation. It is used principally to extend area coverage of triangulation.

Table 1 shows the limits for the principal items of the specifications for first-, second-, and third-order triangulation and traverse.

It will be noticed that the standards of accuracy prescribed in table 1 apply only to the field observations. Other standards are used for the adjusted work. The process of adjusting observations by the method of least squares makes the results consistent throughout but does not remove all errors. If the observational errors are small and indiscriminately plus and minus, then the adjustment will probably distribute them so that there will be but a slight accumulation of errors; or, if the accumulation of error in length between bases, or in azimuth between Laplace stations, is fairly constant in amount and direction, then the adjustment will probably distribute the errors approximately where they belong. Blunders and systematic errors of varying signs are not distributed correctly by the adjustment process.

Under certain conditions the specified allowable error in the length of a line may be found to be exceeded even when the triangulation meets the other specifications for that particular grade of control. Where two points are close together, as compared with the size of the triangulation figure of which they are a part, the distance between those points may be in error in excess of that indicated by the class of triangulation of the scheme. The accuracy of the computed length of any line can be estimated by computing the ΣR_1 from the base to that line in accordance with the formula for the strength of figure as given on page 267.

Triangle closure and agreement in length are not the only standards for triangulation which should be applied. It is possible by a lucky balancing of errors to secure small triangle closures in a short scheme of triangulation even when the observations are below standard. It is also possible to reduce triangle closures by omitting from the computations observations which differ greatly from the mean. It may also happen that a balancing of errors in computing a chain of triangles will result in a very small discrepancy in length on the next fixed line. The accuracy of triangulation is perhaps best indicated by the probable error of a direction, but since this gage of the work is not available until after the adjustment has been made, the triangle closure and the agreement in length, as given by the preliminary computations, are the best available field criteria. To insure that the requisite accuracy is maintained throughout the triangulation, it is essential to give careful consideration to the instrumental equipment and the methods of observing in order that the systematic and accidental errors may be kept within the prescribed limits and that no part of the triangulation will exhibit undue weakness.

The accompanying table (table 1) groups control surveys into orders and classes, in accordance with certain standards of accuracy. The recommended spacing or distance between survey stations is also indicated. These standards are primarily intended for the guidance of Federal agencies in performing and classifying their control survey operations. They were prepared by the Coast and Geodetic Survey in cooperation with other Federal agencies concerned in making control surveys or in utilizing their results.

HORIZONTAL CONTROL

Generally, the density of permanently marked control points should be in direct ratio to land values. In metropolitan areas and along interstate highway systems a spacing at 1 or 2 mile intervals may be required and in rural areas of high land value a spacing of 3 to 4 miles may be desirable. Although wider spacing may suffice for Federal topographic mapping, closer spacing may be needed for property surveys, highway programs, transmission lines, reclamation projects, and numerous other engineering activities. The more closely spaced stations should be so situated that they are readily available to local engineers.

TRIANGULATION

Economic, engineering, and scientific progress has brought an increasing number of requests for higher accuracies in basic first-order triangulation. The range of accuracies is so great that it is necessary to divide first order into three classes so that satisfactory standards of accuracy can be established.

First order, class I: The high value of land in urban areas, the study of small systematic movements in the earth's crust in areas subject to earthquakes, and the testing of military equipment for the national defense require that the triangulation used by engineers and scientists in these varied activities should have an accuracy of at least 1 part in 100,000. Extensive surveys of this nature should make adequate connections with the arcs that make up the national triangulation network. Surveys of such accuracy are designated as class I of first order.

First order, class II: The basic national horizontal control network consists of arcs of triangulation spaced about 60 miles apart in each direction, forming areas between the arcs which are approximately square. The arcs are planned as chains of quadrilaterals or central point figures, so that the lengths of the sides may be computed through two different chains of triangles. The program for the completion of the network in the United States includes establishing area networks of triangulation

TABLE 1.—*Requirements for Horizontal Control*

TRIANGULATION

Criterion for—	First order			Second order		Third order
	Class I (Special)	Class II (Optimum)	Class III (Standard)	Class I	Class II	
Principal uses	Urban surveys, scientific studies.	Basic network	All other	Area networks and supplemental cross arcs in national net.	Coastal areas, inland waterways and engineering surveys.	Topographic mapping.
Spacing of arcs or principal stations.[1]	Stations: 1–5 miles or greater as required.	Arcs: 60 miles. Stations: 10–15 miles.	Stations: 10–15 miles.	Stations: 4–10 miles.	As required	As required.
Strength of figure:						
ΣR_1 between bases:						
Desirable limit	25	60	80	80	100	125.
Maximum limit	30	80	110	120	130	175.
Single figure:						
Desirable limit:						
R_1	5	10	15	15	25	25.
R_2	10	30	50	70	80	120.
Maximum limit:						
R_1	10	25	25	25	40	50.
R_2	15	60	80	100	120	170.
Base measurement:						
Actual error not to exceed	1 part in 300,000.	1 part in 300,000.	1 part in 300,000.	1 part in 300,000.	1 part in 150,000.	1 part in 75,000.
Probable error not to exceed	1 part in 1,000,000.	1 part in 1,000,000.	1 part in 1,000,000.	1 part in 1,000,000.	1 part in 500,000.	1 part in 250,000.
Triangle closure:						
Average not to exceed	1″	1″	1″	1″.5	3″	5″.
Maximum seldom to exceed	3″	3″	3″	5″	5″	10″.

TABLE 1.—*Requirements for Horizontal Control*—Continued

Criterion for	First order: Class I (Special)	First order: Class II (Optimum)	First order: Class III (Standard)	Second order: Class I	Second order: Class II	Third order
Side checks:						
Ratio of maximum difference of logs of sides to tab. diff. for 1″ of log sine of smallest angle.	1.5	1.5–2	2	2–4	4	10–12.
Or in side equation test, average corr. to direction not to exceed.	0.″3	0.″4	0.″4	0.″6	0.″8	2″.
Astro. azimuths:						
Spacing-figures	6–8	6–10	8–10	8–10	10–12	12–15.
Probable error	0.″3	0.″3	0.″3	0.″3	0.″5	2.″0.
Closure in length (also position when applicable) after side and angle conditions have been satisfied, should not exceed.	1 part in 100,000.	1 part in 50,000.	1 part in 25,000.	1 part in 20,000.	1 part in 10,000.	1 part in 5,000.

TRAVERSE

	First order	Second order	Third order
Number of azimuth courses between azimuth checks not to exceed.	15	25	50.
Astronomical azimuth: Probable error of result.	0.″5	2.″0	5.″0.
Azimuth closure at azimuth check points not to exceed.[2]	2 sec. $\sqrt{N}$ or 1.0 sec. per station.	10 sec. $\sqrt{N}$ or 3.0 sec. per station.	30 sec. $\sqrt{N}$ or 8.0 sec. per station.
Distance measurements accurate within	1 in 35,000	1 in 15,000	1 in 7,500.
After azimuth adjustment, closing error in position not to exceed.[2]	0.66 ft. $\sqrt{M}$ or 1 in 25,000	1.67 ft. $\sqrt{M}$ or 1 in 10,000	3.34 ft. $\sqrt{M}$ or 1 in 5,000.

N is the number of stations for carrying azimuth.
M is the distance in miles.
[1] Additional stations of same accuracy may be interspersed among principal stations.
[2] The expressions for closing errors in traverse surveys are given in 2 forms. The expression containing the square root is designed for longer lines where higher proportional accuracy is required. The formula which gives the smaller permissible closure should be used.

within these squares or loops formed by the arcs. To maintain satisfactory mathematical consistency within the area networks, these basic arcs should be measured with an accuracy of at least 1 part in 50,000. Most of these primary arcs have closures in length and position which are of the order of 1 part in 75,000 or 1 part in 100,000. Triangulation of this standard of accuracy is designated as class II of first order.

First order, class III: There are many additional demands for first-order triangulation within this national framework, and in some cases even independent of the national net. State, county, and private engineering organizations as well as branches of the Federal Government have need for horizontal control that would have a minimum accuracy of 1 part in 25,000. Surveys of this accuracy have long been recognized both nationally and internationally as first order and have attained the status of a widely accepted standard.

In the adjustment of the first-order national network, the surveys of class I will have precedence and should not be distorted to adjust them to surveys executed under the specifications of class II. When the surveys of class III are rigidly adjusted to the basic network, their accuracy should be improved.

The placing of first- or second-order control points within the loops of the basic network requires the extension of area networks, cross arcs, or traverses. These specifications list two classes of second-order triangulation.

Second order, class I: This class includes the networks covering the areas within the arcs of the basic network and, if area nets are not feasible, it includes the cross arcs which would be used to subdivide the area. The internal closures of this class of survey should indicate an average accuracy of 1 part in 25,000, with no portion less than 1 in 20,000. This class approximates very closely what heretofore has been considered modified second order.

Second order, class II: This class of triangulation is used to establish control for hydrographic surveys along the coastline and inland waterways. It may also be used for further breakdown of control within any of the higher classes of triangulation. This class of survey or any of the higher classes may be used by engineers for controlling extensive property surveys. The minimum accuracy to be allowable in class II of second order is 1 part in 10,000.

Third-order triangulation: Triangulation of this order should be supplemental to triangulation of a higher order for the control of topographic or hydrographic surveys, or for such other purposes for which it may be suitable. Although it will usually be established as needed for a specific project, third-order triangulation should be permanently marked, and azimuths should be observed to visible prominent objects, so that the work may be available for future projects and miscellaneous uses in the area. Points located by third-order triangulation may be expected to have an absolute position determination within 10 feet or less in relation to the adopted datum defined by higher order positions in the area. The work should be performed with sufficient accuracy to satisfy the standards listed in Table 1.

Standards for surveys below third order are not included in these classifications.

BASES

Bases for the control of the lengths of lines in the triangulation should be measured by appropriate methods and instruments, so that the standards in table 1 are satisfied. Recent developments in electronics indicate that accuracies comparable to those obtained with invar tapes may be expected from the Bergstrand geodimeter or similar instruments. The intervals between bases should be such that the standards regarding strength of figure (ΣR_1) also are satisfied.

TRAVERSE

Traverses are used to supplement all orders and classes of triangulation, and to provide closer and more adequate spacing of horizontal control points. A triangulation net in an urban area provides a framework for a complete traverse network of first- and second-order accuracies. It is neither economical nor feasible to use triangulation for this closer spacing. There are some sections of the United States in addition to these urban areas where traverse can be used efficiently to subdivide the basic network and provide the fundamental spacing of control specified in the national program.

First-order traverses should preferably be connected to first-order triangulation stations of class I or class II. If they are connected to class III of first order they might be used and given some weight in the adjustment of this class of triangulation. The minimum requirement of accuracy for a first-order traverse is 1 part in 25,000, yet first-order traverse networks, properly executed, will average about 1 part in 40,000. This value is expected and desired. Detailed standards are listed in table 1.

Traverses of second- and third-order accuracy are tied to triangulation or traverse of the same or higher order. They are used extensively for cadastral or property surveys and mapping. For property surveys, the value of the property should, in general, determine the accuracy to be used. For map control, the scale of the map and the positional accuracy required usually govern. Details of these orders of traverse are also listed in table 1.

TRILATERATION

Electronic techniques are increasingly used for the measurement of distances and, through the geometric combination of these distances, networks of trilateration or traverse are developed. In general, the same standards in regard to position closure may be applied as are used in triangulation and traverse.

Chapter 1.—RECONNAISSANCE

GENERAL STATEMENT

All triangulation is necessarily preceded by some form of reconnaissance to select the station sites.

The general term "triangulation" properly includes, in addition to the observation of horizontal angles, the operations of reconnaissance, base measurement, and Laplace azimuth observations, since the specifications for each must be decided upon with due regard for the others. Starting with a base of specified accuracy, the computed lengths of the successive triangle sides will gradually become less accurate until finally a new base is necessary if the required accuracy is to be maintained. The strength of the geometrical figures through which the lengths are carried by the triangulation process and the accuracy of the observations are factors in determining the number of figures between measured bases.

RECONNAISSANCE PARTY

Reconnaissance for triangulation is normally carried on by a separate field party, usually operating well in advance of the observing party.

The requirements of reconnaissance are briefly discussed in this manual for the information of triangulation parties. Special Publication No. 225, "Manual of Reconnaissance for Triangulation," discusses reconnaissance in detail.

A reconnaissance party usually consists of a chief of party with two assistants, equipped with trucks, binoculars, altimeters, sextants, transits, compasses, protractors, packboards, and tree climbers.

The reconnaissance party is furnished project instructions defining the arc or area to be surveyed, specifying the class of triangulation required, stating the desirable spacing of stations, outlining necessary connections to previously established surveys of the Coast and Geodetic Survey and other organizations, and indicating the desired location of base lines. Any unusual use to be made of the survey is also described. Sketches, descriptions and geographic position data of existing triangulation stations, and available maps and charts of the area are furnished.

GENERAL INSTRUCTIONS FOR RECONNAISSANCE

SPECIFICATIONS FOR FIRST-ORDER RECONNAISSANCE

All reconnaissance for first-order triangulation must conform to the following specifications.

1. Character of figures.—The chain of triangulation between base nets shall be made up of completed quadrilaterals or of central-point figures, with all stations occupied. Single triangles should not be used on main-scheme arcs except by specific permission of the Director.

It is desirable to have two ways of computing the lengths through each figure. On the other hand, there should be no overlapping of figures and no excess of observed lines beyond those necessary to secure a double determination of every length, except as follows: In a four-sided central-point figure one of the diagonals may be observed; in a figure used in expanding from a base the observation of additional lines is often desirable;

and in a network of triangulation over a city or other wide area, a few overlapping figures to meet special conditions may be used.

2. Strength of figures.—Strength of figure is an expression of the comparative precision of computed lengths in a triangulation net as determined by the size of the angles, the number of conditions to be satisfied, and the distribution of base lines and points of fixed position. Strength of figure in triangulation is not based on an absolute scale but rather is an expression of relative strength. The number expressing the strength of a triangulation figure is really a measure of its weakness, since the number determined by formula increases in size as the strength decreases.

The strength of figure is derived from that portion of the formula for probable error of a triangle side which is independent of the accuracy of the observations. An explanation of the terms and the method of computing the strength of figure are given on pages 267 to 270.

In the chain of first-order triangulation of class II between base nets the value of the quantity $R=\left(\frac{D-C}{D}\right)\Sigma[\delta_A^2+\delta_A\delta_B+\delta_B^2]$ must not exceed 25 for any one figure in the selected best chain (call it R_1) nor exceed 60 in the second best (call it R_2) in units of the sixth place of logarithms. These are extreme limits never to be exceeded. The quantities R_1 and R_2 are kept down to the limits 10 and 30 for the best and second-best chains, respectively, whenever the estimated total cost does not exceed that for a chain barely within the extreme limits by more than 25 percent. See table 1, p. xv, for the limits for triangulation of classes I and III. The values of R may readily be obtained by the use of table 3 on page 268.

3. Length of lines.—A line of the first-order triangulation outside of the base nets should in general not be less than 3 miles in length, if it is to be used directly in carrying the length forward through the scheme, that is, if it is opposite a distance angle used in computing R_1. Lengths of lines of from 4 to 10 miles are most desirable in order to obtain better distribution of stations for mapping control and local use. Lengths of lines in various classes of triangulation are usually governed by the requirements of the project instructions, by topographic conditions, and by strength-of-figure considerations.

4. Frequency of bases.—If the character of the country is such that a base site can be found near any desired location, ΣR_1 between base lines in triangulation of class II should be made about 60. This will be found to correspond to a chain of from 10 to 20 triangles, according to the strength of the figures secured. With strong figures, relatively few base lines will be needed, and a corresponding saving will be made on this phase of the work. If topographic conditions make it difficult to secure a base site at the desired location, ΣR_1 may be allowed to approach but not exceed 80. See table 1, p. xv, for the limits for triangulation of classes I and III.

5. Base sites and base nets.—In selecting base sites it should be kept in mind that a base can be measured with the required degree of accuracy on any site where the grade on any 50-meter tape length does not exceed 10 percent, and that narrow valleys or ravines less than 50 meters wide in the direction of the base are not insuperable obstacles to measurement. The length of each base should in general not be less than 4 kilometers. In each base net great care should be taken to secure as good geometric conditions as possible. There should be no hesitancy in placing the base on rough ground, provided the roughness is not greater than that indicated above, if by doing so the geometric conditions in the base net are improved. Each base net should not be longer than two ordinary figures of the main chain between bases.

6. Stations near cities.—There should be established in or near the city limits of all cities and towns of more than about three thousand population at least one triangulation station which may be located from two or more stations of the main scheme. Base lines should be measured near cities with a population of about 100,000 or more.

7. Connections to existing triangulation.—In starting from or connecting with either first- or second-order triangulation, it is important to consider the strength of the previously determined connecting line. Observations shall always be made from the two ends of that line upon a third point of the existing triangulation as a check on the recovery of the old stations. (See p. 14.) Even when connecting to third-order triangulation it is better if possible to connect with a line rather than a point, for the comparison of the lengths of the line common to the two systems of triangulation may give information of great value in adjusting the weaker scheme. Connection in position alone, namely, to a single point, or in position and azimuth, namely, to a single point, but with a direction observed from that point to another old station, may sometimes advantageously be made at intervals between the connections in length.

8. Connection to surveys of other organizations.—Connections will be made to monuments established by other surveying and mapping agencies, including the U. S. Geological Survey, the U. S. Bureau of Reclamation, the U. S. Corps of Engineers, the Mississippi River Commission, the Missouri River Commission, the Bureau of Land Management (General Land Office), the U. S. Forest Service, and State geodetic organizations; and to boundary monuments of States, counties, and reservations.

9. Project instructions.—Standard specifications for reconnaissance may be modified or strengthened still further by the project instructions. The maximum and desirable limits of R_1 and R_2 of a single figure may be smaller than those listed in table 1, p. xv; also, length of main-scheme lines may be limited and additional supplemental figures required. Instructions may also include directions for connections to bench marks for use in trigonometric leveling, and state the desired spacing of these ties.

10. Supplemental figures.—For area triangulation and figures that are supplemental to first-order triangulation, the following modifications should be made to paragraph 1:

(a) The network should consist of simple figures, preferably single triangles, to avoid unnecessary observations and excessive computation.

(b) Chains or networks of single triangles shall have as many connections as practicable to lines of equal or higher order. Normally, if a point can be determined from only one known line, a chain of triangles should not be continued beyond that point unless the chain is to be connected to another known line.

(c) Whenever any two stations are relatively close together, either the line between them should be the observed side of a triangle, or the stations should be connected by a traverse measurement.

(d) It is desirable that stations be established near crossroads, or at other points easily identifiable on aerial photographs, whenever practicable.

11. Stations at airports.—Station sites should be selected at all airports within the project area. The airport manager should be contacted relative to the most desirable site. The signal-building party will be prepared to install such aircraft warning lights as may be required.

12. Stations at colleges.—Points observable from two or more main-scheme stations should be selected for occupation on or near the campus of all recognized technical colleges and universities. The college authorities should be contacted relative to the location.

13. Intersection stations.—As many prominent objects as possible, such as church spires, water tanks, standpipes, towers, beacons, tall chimneys, and prominent mountain peaks, should be selected as intersection stations. The three most suitable occupied stations from which the intersection stations are to be observed should be indicated by short rays from the intersection station's symbol on the reconnaissance sketch.

14. Connections to bench marks.—Connections should be made to bench marks wherever practicable. Bench marks may be used as reference marks at main-scheme stations, or as station marks at supplemental stations. Where vertical angles are to be observed, such connections are desirable about every third quadrilateral.

SPECIFICATIONS FOR SECOND-ORDER RECONNAISSANCE

The preceding specifications for reconnaissance for first-order triangulation also apply to second-order triangulation, except as modified in the following paragraphs.

15. Character of figures.—The main scheme of the triangulation shall be made up of figures of from four to seven points each, in which certain stations may be left unoccupied as indicated under paragraph 16, "Strength of figures." It may be reduced in exceptional cases to a chain of single triangles with all angles observed where otherwise the cost and time would be excessive. On the other hand, there should be no overlapping of figures, except that in a four-sided, central-point figure one of the diagonals of the figure may be observed, and with the above exception there should be no excess of observed lines beyond those necessary to secure a double determination of every length. It is permissible, however, to observe between stations which are not in the same figure in order to avoid a back computation in locating supplementary or intersection stations. Observations over lines which will make the main scheme any more complicated than that defined above will practically be wasted. If it is necessary to occupy other stations than those in the main scheme in order to locate by intersection certain stations which must be fixed to control hydrographic or topographic operations, these additional occupied stations (which will be called supplementary stations) should be connected with the main scheme by the simplest figures possible in which there is a check. Single triangles with all the angles measured will, in general, be sufficient for the purpose. (See par. 10 for area and supplemental triangulation.)

16. Strength of figures.—In second-order triangulation of class I the value of the quantity $R=\frac{D-C}{D}\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ for any one figure should not, in the selected best chain of triangles (call it R_1), exceed 25, nor in the second best (call it R_2) exceed 100 in units of the sixth place of logarithms. In second-order triangulation of class II the limits shall be 40 and 120 for R_1 and R_2, respectively. These are outside limits never to be exceeded except when it is extremely difficult under existing conditions to keep within them. The quantities R_1 and R_2 should be kept down to the limits 15 and 70 for class I (25 and 80 for class II) for the best and second-best chains, respectively, whenever the estimated total cost does not exceed that for the chain barely within the extreme limits by more than 25 percent. One station in each quadrilateral or central-point figure may be left unoccupied or certain lines in a figure may be observed over in one direction only, if the values of R_1 and R_2 do not exceed the specified limits and if a considerable saving of time can be effected thereby. When a supplementary station is connected to the main scheme by a single triangle, the angle at the supplementary station should be greater than 30° if possible.

17. Frequency of bases.—If the character of the country is such that a base site can be found near any desired location, ΣR_1 between base lines, whether these are actually measured base lines or lines of first-order triangulation used as bases, should be made about 80 for class I, 100 for class II. This value will be found to correspond to a chain of from 10 to 25 triangles, according to the strength of the figures involved. With strong figures, relatively few measured base lines will be needed, and a corresponding saving will be made on this part of the work. If topographic conditions make it difficult to secure a base site at the desired location, ΣR_1 may be allowed to approach but not to exceed 120 for class I, 130 for class II. There will be danger, when this is done, that an intervening base will be necessary to meet the requirements stated in the next sentence. If in any case the discrepancy between adjacent bases is found to exceed 1 part in 20,000 for class I, 1 part in 10,000 for class II, after the side and angle equations have been satisfied, an intermediate base must be measured or the angle observations made more accurate.

SPECIFICATIONS FOR THIRD-ORDER RECONNAISSANCE

The same principles and methods are applied in reconnaissance for third-order triangulation as for that of a higher order, though the allowable limits of some of the controlling factors are different, as noted below.

18. Strength of figures.—In the main scheme the value of R_1 (see p. 269) for any one figure must not exceed 50, and that of R_2 for the figure must not exceed 170, except when conditions are such that it is very difficult to keep within these limits. The quantities R_1 and R_2 for a single figure should be kept down to 25 and 120 for the best and second-best chains, respectively, whenever the total estimated cost for the reconnaissance and observing does not exceed that for the chain barely within the extreme limits by more than 25 percent.

19. Frequency of bases.—When a base site can be found near any desired location, ΣR_1 between base lines, whether these are actual measured bases or lines of first- or second-order triangulation used as bases, should be made about 125. Where topographic conditions make it difficult to measure a base near the desired location, ΣR_1 between bases may be increased but should never greatly exceed 175. When this upper limit is approached an intervening base may be necessary, for if the discrepancy in length between bases exceeds 1 part in 5,000, after the side and angle equations have been satisfied, either an intermediate base must be measured or the angle observations made more accurate.

RECONNAISSANCE FIELD PROCEDURE

The field party plots existing triangulation on available maps and then lays out on the map a desirable preliminary scheme that fulfills the requirements of the Specifications and of the project instructions. The points selected in this preliminary scheme are then visited in the field, and the scheme is revised as necessary to obtain suitable points for the necessary intervisible signals. Doubtful lines are given detailed investigation. The formula for determining heights of signal necessary to clear an intervening obstruction and the table of corrections for curvature and refraction are included on page 271. (See Special Publication No. 225 for complete discussion of reconnaissance methods and procedure.) Special effort should be made to avoid having any obstructed lines. With a steel-tower party, an obstructed line usually costs the loss of a day or more to one or two observing units, a tear-down party, and a building party and adds to the unit cost of operation. It is also economically important that as many stations as practical be in locations to which trucks may be driven.

The final selection of a station site is a compromise of several requirements which are principally: Intervisibility of stations, permanence of marks, strength of figures, wishes of property owners, accessibility to the triangulation party, and availability of the station for future use of surveyors, both governmental and private.

DATA FURNISHED TRIANGULATION PARTIES

The final product of the work of a reconnaissance party is the information furnished the triangulation party. This is submitted to the Washington Office in the form of a neat sketch of the scheme and reconnaissance descriptions for each station site selected. Multiple copies of these are made and furnished the triangulation party.

RECONNAISSANCE SKETCH

To be of maximum value to the triangulation party, the sketch should show:

1. Main scheme by double-weight lines.
2. Supplemental scheme by single-weight lines.
3. Intersection stations with short rays toward observing stations.
4. New stations with a triangle △.
5. Recovered stations with a triangle inscribed in a circle◬.
6. Intersection stations with a small circle ⊙.
7. Parallels of latitude and meridians of longitude.
8. Statute mile scale (graphic); and inches to mile scale (equivalent).
9. True north and magnetic north.
10. Title label including State, date, project number, and personnel, similar to:

UNITED STATES COAST AND GEODETIC SURVEY

LEO O. COLBERT, DIRECTOR

RECONNAISSANCE SKETCH

FIRST-ORDER TRIANGULATION

PROJECT G-555

MICHIGAN

HILLSDALE TO SAGINAW

.... CHIEF OF PARTY

.... ASSISTANT

JULY, 1930

11. Station names in capital letters near each station symbol. (A special effort should be made to obtain the correct spelling of names.)

12. Recovered station names followed by date of establishment.
13. Height in feet of signal required near each station name.
14. Approximate time of climbing at all pack stations.
15. A "D" after the name of each drive station, unless all stations on a sketch are stations to which a truck can be driven.
16. Roads with dashed lines (main highways and roads to all drive stations).
17. Highway numbers on roads, followed by a small letter to show condition, such as p. — paved; g. — gravel; d. — dirt.
18. Rivers sketched in and named, and road crossings indicated.
19. Towns with concentric circles; i.e., one circle for towns of less than 100 population, two circles for towns of between 100 and 1,000 population, three circles for towns of between 1,000 and 10,000 population, etc. Large cities may be indicated by cross hatching.
20. Names of towns, using a different style of lettering from that used for station names.
21. Initials after names of stations of other organizations, such as (USE), (USBR), etc.
22. Bench marks for trigonometric connections in mountainous regions.
23. Traverse ties in parentheses below station names, such as (USE tie), (GLO tie), (BLM tie), (USGS tie), etc.

The final sketch should be carefully checked against the rough field sketch and notes before being forwarded to the Washington Office.

RECONNAISSANCE DESCRIPTIONS

Reconnaissance descriptions of stations should include the following data:

1. General location with respect to State and county, and distances and directions from larger towns or other well-known and easily found features.
2. Location with respect to local features, namely settlements, highways, topographic features, township, range, section, etc.
3. Land ownership and whether consent of owner has been obtained to establish a station and special agreements, if any. (The reconnaissance party should obtain permission to enter upon private property to establish a station in all cases including those which may require correspondence. The observing party should be furnished all details.)
4. Directions for reaching the station starting from sizeable towns, adequately describing roads and junctions, stating mileages, and including descriptions of alternate routes, if advisable.
5. Detailed location with respect to roads, fence lines, buildings, trees, etc., including directions and paced or measured distances, and notes of special markings.
6. Special information such as location of supplies, state of cultivation, arrangements about crop damages, amount of clearing, special types of marks required, seasonal or wet-weather restrictions, etc.
7. Height and type of signal required.

A brief summary of additional instructions to reconnaissance parties regarding descriptions is as follows:

1. Check the spelling of all proper names and in particular the names to be given to stations and names of property owners.

2. For all except drive stations, state amount of back packing or horse packing required, or whether boat, plane, or other means of transportation is advisable. Indicate where proposed means of transportation can be hired.

3. Use the nearest towns or crossings of main numbered highways shown on standard highway maps as initial points for highway mileages shown in giving distances to station sites.

4. Show both direction of turn and new course, such as—Turn left (west).

5. Do not use the expression "Follow main travelled road" unsupported by additional information.

6. For recovered marks, include information on any work necessary to repair the monuments.

7. In describing marks of other organizations include complete data on the type and condition of existing marks. Include any old descriptions that are available.

8. Check sketch and descriptions against each other before forwarding them to the Washington Office.

Chapter 2.—TRIANGULATION

GENERAL STATEMENT

Triangulation is a method of surveying in which the location of a new point is determined from the mathematical solution of the triangle whose vertices are the new point and two other points of known position. The length and azimuth of the known side of the triangle are determined by accurate field measurements or from the computation of a previous triangle. The angles of the triangle are determined by accurate field measurements. Using these angle measurements, the lengths and azimuths of the two unknown sides of the triangle and the geographic position of the new vertex point are computed. The vertices (or triangulation stations) are permanently marked or monumented points. Successive points are computed through a continuous chain of triangles. These triangles may be in quadrilaterals, central-point polygons, or complex networks, designed to comply with the standards of a class of triangulation and the requirements of a project. Triangulation is required in executing surveys of high accuracy over large areas, particularly in areas of rugged terrain, and for precise surveys in small areas where traverse cannot be used economically.

First-order triangulation is the term applied to the highest grade of triangulation. It is attained by the use of precision instruments and methods, and by adherence to certain accepted high standards of accuracy.

This chapter will define and discuss the specifications and field operation methods now used by the U. S. Coast and Geodetic Survey to obtain triangulation surveys of first-order, modified second-order, second-order, and third-order accuracy.

GENERAL INSTRUCTIONS FOR TRIANGULATION

The general instructions for first-order and modified second-order triangulation are contained in the following specifications. Specifications for second- and third-order triangulation are contained in the sections which follow the first-order specifications. Specifications for the collateral operations of reconnaissance, base measurement, and azimuth observations are listed in this manual under their respective subjects.

SPECIFICATIONS FOR FIRST-ORDER TRIANGULATION

1. **Figures.**—The scheme and figures for first-order triangulation shall be designed in accordance with specifications for first-order reconnaissance as listed in chapter 1 in paragraphs 1 to 5 inclusive.

2. **Length requirement.**—For first-order triangulation, the discrepancy between a computed length and the measured length of a base or the adjusted length of a check line shall not exceed 1 part in 50,000 for class II (1 in 100,000 for class I and 1 in 25,000 for class III), after the angle and side conditions have been satisfied.

3. **Angle requirement.**—For first-order triangulation, the closure of each triangle shall not exceed 3 seconds and the average closure of the triangles shall seldom exceed 1 second. The closure of a triangle is the difference between the sum of the observed angles and 180° plus the spherical excess, the amount by which the sum of the angles of a spherical triangle exceeds 180°. (See p. 272 for formula.)

4. **Side checks.**—For first-order triangulation, the following limits shall be used in obtaining satisfactory side checks. For a regular quadrilateral in which the triangles

have been closed by applying one-third of the error of closure to each angle, the logarithms of the length of a side, as computed through the R_1 and R_2 chains, should not differ by more than one and a half to two times the tabular difference for one second of the log sine of the smallest angle entering into the computations of that length. (See table 1, p. xv.) For figures other than regular quadrilaterals, a comparable limiting value for side checks can be obtained by multiplying the tabular difference for one second of the smallest angle used in the computation by one-half of the total number of triangles involved.

Side equation tests may also be applied. The average correction to a direction obtained by dividing the constant term of the equation by the sum of the coefficients of the other terms without regard to sign should not be greater than 0.4 second for classes II and III (0.3 second for class I). Side equation tests should be applied to all figures with excessive side closures in order to help isolate the error and to indicate the station requiring a reoccupation. Using the short form of the side equation test which is described on page 165, the mean correction to an angle should not exceed an average of 0".7 for a first-order arc or area; should seldom exceed 0".7 for any figure; and, should not exceed 2".0 even in the exceptional case of an equation using only one small angle.

5. Instruments. In general, direction instruments of the highest grade should be used on first-order triangulation. Repeating theodolites are used only when the station to be occupied is in such a position as to be difficult of occupation with a direction instrument or when there is doubt that the instrument support is of such a character as to insure that the movement of the observer about the instrument does not disturb it in azimuth. Such conditions may occasionally occur on lighthouses, buildings, or in rough terrain. Also under these conditions, instead of a repeating theodolite, an optical reading theodolite, such as the Wild, may be used, measuring each angle separately and closing the horizon.

6. Circle settings.—At each station occupied with a direction theodolite, the angles between lines of sight to stations observed upon will eventually be determined by taking differences between their relative directions expressed in angular measure. One measure, with the telescope both direct and reversed, of the horizontal direction from the arbitrarily selected initial station to each of the other stations is called a position. For greater precision, the means of a number of positions determined with various initial circle settings are used for the final observed values of the directions, each position being given a number corresponding to the circle setting of the initial pointing. In order that the readings of the micrometer microscopes may be uniformly distributed around the graduated circle during the measurement of any set of directions, the graduated circle should be oriented to give successive readings on the initial station corresponding approximately to the settings listed in table 2.

Settings given in table 2 are for the "A" micrometer and with the telescope reversed on alternate positions require changing the circle almost 180° in orientation for each new position. When observations are made at night the differential temperature effects are negligible, therefore at even-numbered positions where observations are to be begun with telescope reversed it is permissible to add 180° to the settings listed, which in effect changes the orientation of the plate only about 11 degrees for each of the 16 positions.

The pattern of the minutes and seconds in the settings tabulated provides for a compensating distribution of the small errors due to the run of the micrometers. (See p. 277.)

Table 2.—*Circle settings*

Circle initial settings for direction theodolite
2-micrometer theodolite

TWO POSITIONS OF CIRCLE

Position No.	1 division of circle = 4′	1 division of circle = 5′	1 division of circle = 10′
	° ′ ″	° ′ ″	° ′ ″
1	0 01 00	0 01 00	0 02 30
2	90 03 00	90 03 40	90 07 30

FOUR POSITIONS OF CIRCLE

1	0 00 30	0 00 40	0 01 20
2	45 01 30	45 01 50	45 03 50
3	90 02 30	90 03 10	90 06 20
4	135 03 30	135 04 20	135 08 50

EIGHT POSITIONS OF CIRCLE

1	0 00 30	0 00 40	0 01 20
2	22 01 30	22 01 50	22 03 50
3	45 02 30	45 03 10	45 06 20
4	67 03 30	67 04 20	67 08 50
5	90 00 30	90 00 40	90 01 20
6	112 01 30	112 01 50	112 03 50
7	135 02 30	135 03 10	135 06 20
8	157 03 30	157 04 20	157 08 50

TWELVE POSITIONS OF CIRCLE
[One division of circle = 5 minutes]

2-micrometer theodolite		3-micrometer theodolite	
Position No.	Setting	Position No.	Setting
	° ′ ″		° ′ ″
1	0 00 40	1	0 00 40
2	15 01 50	2	15 01 50
3	30 03 10	3	30 03 10
4	45 04 20	4	45 04 20
5	60 00 40	5	65 00 40
6	75 01 50	6	80 01 50
7	90 03 10	7	95 03 10
8	105 04 20	8	110 04 20
9	120 00 40	9	130 00 40
10	135 01 50	10	145 01 50
11	150 03 10	11	160 03 10
12	165 04 20	12	175 04 20

SIXTEEN POSITIONS OF CIRCLE

1	0 00 40	1	0 00 40
2	11 01 50	2	15 01 50
3	22 03 10	3	30 03 10
4	33 04 20	4	45 04 20
5	45 00 40	5	64 00 40
6	56 01 50	6	79 01 50
7	67 03 10	7	94 03 10
8	78 04 20	8	109 04 20
9	90 00 40	9	128 00 40
10	101 01 50	10	143 01 50
11	112 03 10	11	158 03 10
12	123 04 20	12	173 04 20
13	135 00 40	13	192 00 40
14	146 01 50	14	207 01 50
15	157 03 10	15	222 03 10
16	168 04 20	16	237 04 20

Circle settings for a repeating theodolite with a 10-minute circle and two 10-second verniers

TWO SETS

Set No.	Setting		
	°	′	″
1	0	00	00
2	90	05	30

THREE SETS

1	0	00	00
2	60	03	30
3	120	07	00

FOUR SETS

1	0	00	00
2	45	02	30
3	90	05	00
4	135	07	30

SIX SETS

1	0	00	00
2	30	01	40
3	60	03	20
4	90	05	00
5	120	06	40
6	150	08	20

Settings for Wild T-3 theodolite

Position No.	Circle		Micrometer readings
	°	′	
1	0	00	10 units
2	11	00	25 units
3	22	00	35 units
4	33	00	50 units
5	45	00	10 units
6	56	00	25 units
7	67	00	35 units
8	78	00	50 units
9	90	00	10 units
10	101	00	25 units
11	112	00	35 units
12	123	00	50 units
13	135	00	10 units
14	146	00	25 units
15	157	00	35 units
16	168	00	50 units

Sixteen positions shall be observed on first-order and modified second-order triangulation. In exceptional cases, the number of acceptable positions may be reduced to 12 or even 10 on one or more directions at a station if the triangle closures have been satisfactory and if observations necessary to obtain the missing measurements would require an extra day or more at the station. However, every effort should be made to obtain 16 positions.

At each station occupied with a repeating theodolite, the angles between adjacent stations are usually measured by sets of six direct and six reversed repetitions of each angle. The initial circle settings for successive sets are also listed in table 2. Six sets are observed on first-order and modified second-order triangulation, although the occasional omission (or rejection without reobservation) of one of the sets is permitted when satisfactory triangle closures are obtained.

7. **Rejections.**—For first-order triangulation, any measurement of a direction deviating by more than 4 seconds from the mean shall be rejected and reobserved, using the same position setting.

8. **Incomplete sets.**—In the event that one or more lights are not visible during all or a part of the time that observations can be made upon other stations, observations are continued on as many lights as are visible. Little time should be spent in waiting for a station light to show. When a broken set is observed, the missing stations are to be observed upon later in connection with the chosen initial or some other one, and only *one, of the stations* already observed in that set. With this system of observing, no local adjustment is necessary. When a new initial is used for observations upon one or more stations in all positions of the circle, the regular tabulated position settings may be used on the new initial; but, when a new initial is used to complete a set on a station, that position angle between the old and the new initial should be added to the setting for each position given in the table in order to distribute the readings on each station properly about the circle.

The values of a direction determined from two or more sets of 12 or more positions should be given equal weight, regardless of whether observed on the same night or on different nights. When two sets of 12 or more positions differ by more than 1 second, the set which best closes the triangles is retained and the other set rejected. In case of three sets, sometimes two sets which agree with each other must be rejected, and the third retained. Usually the set or sets retained are those which are observed under the least harmful conditions of horizontal refraction. When there are more than two sets, any set which falls within one-half second of the individual or mean value which best satisfies the triangles should also be meaned with that value.

9. **Obstructed lines.**—Obstructed lines on first-order triangulation which prevent double determination of lengths should be corrected by extending the height of signals, or by revising the figure. Revision of the figure is usually most readily accomplished by adding a central-point station near the obstruction. In exceptional cases, the omission of an occasional R_2 diagonal on short arcs will be permitted. The requirement for double determination of lengths should be strictly adhered to for work in large unsurveyed areas.

In second-order area triangulation that is supplemental to first-order triangulation, connected networks of single triangles may be used. These figures can usually be rearranged to effect a substitution should there be obstructed lines. In rearrangement, unsupported spurs of single triangles consisting of more than one triangle should be avoided. It is also desirable to avoid placing two stations relatively close together unless they are connected.

10. Supplemental triangulation.—For triangulation that is supplemental to first-order triangulation, specifications for second-order triangulation (see p. 16) will be modified and improved as follows. Sixteen positions will be observed with a first-order theodolite, following the criteria: (1) Any measurement of a direction deviating by more than 5 seconds from the mean shall be rejected and reobserved, using the same position setting; and (2) a station in the supplemental scheme will be reoccupied only when the closure of a triangle involving that station is more than 5 seconds. The mean of the triangle closures shall not exceed 1.5 seconds for class I of second order or 3 seconds for class II of second-order triangulation.

In addition to improved average accuracy, modified second-order triangulation provides economy of operations. If only eight positions were taken it might often be necessary to reobserve; with 16 positions observed it is seldom necessary to reoccupy stations to improve closures. The most time-consuming work in occupation of a station is transportation and station preparation. Actual observing of 16 positions at a normal station can usually be completed in less than 2 hours.

11. Side checks, supplemental figures.—For triangulation that is supplemental to first-order triangulation, and is not to be used for further extension of control, the following limits shall be used. For regular quadrilaterals, in which the triangles have been closed by applying one-third of the error of the closure to each angle, the limit of the side check shall be two to four times the tabular difference for one second of the log sine of the smallest angle entering into the computation. (See table 1, p. xv.) For figures other than regular quadrilaterals, a comparable limiting value for side checks can be obtained by multiplying the tabular difference for one second of the log sine of the smallest angle used in the computation by the total number of triangles involved.

For all figures where side equation tests are used, a limit of 0.6 second for an average correction to a direction is specified for second-order triangulation of class I. The limit is 0.8 second for class II. Using the short form of the side equation test which is described on page 165, the mean correction to an angle for modified second-order triangulation should not exceed an average of 1″.2 for an arc or area, should seldom exceed 1″.2 for any figure, and should not exceed an upper limit of 3″.0 even in the exceptional case of an equation using only one small angle.

12. Intersection stations.—At each occupied first-order or modified second-order station, directions will be taken (with four positions, using first-order theodolites) on objects such as church spires, water tanks, towers, standpipes, lighthouses, prominent mountain peaks, and tall chimneys. A line of the occupied scheme should be used as an initial direction. (See par. 13 of ch. 1 on p. 4.)

The descriptions and the lists of directions should leave no doubt as to the exact points of various objects sighted upon. If an object is too large to bisect readily and accurately it is best to observe on its tangents.

It is important to have three lines to each intersection station in order to secure a check upon its position, but an intersection station which can be positively identified, should not be ignored because only two lines to it can be secured.

Prominent objects which are visible from the ground should be observed for additional azimuth points even when it is not practical to secure observations from other stations. Also, where there is a scarcity of intersection stations, observations should be made on prominent objects (visible from tower or ground) in order to furnish additional azimuths for photogrammetric use.

13. Connections to established triangulation.—In making line connections to established triangulation, check angles to a third previously established station shall be observed from each end of the line, except where the work connected to has been accomplished within the past two years. Any unusually large discrepancy that cannot be reduced by field investigation should be called to the immediate attention of the Office. (See par. 7 of ch. 1 on p. 3.)

Observed angles should normally check the previously observed angles within less than 3 seconds. When the value exceeds 1.5 seconds, a second set of 16 positions should be observed if this would not delay the progress of the work. If the value exceeds 3.0 seconds, additional investigation and report should be made. In case the values of previously observed angles are not furnished with the data accompanying the project instructions, they will be furnished by the Washington Office on request. If the discrepancy is in excess of 2 seconds the Washington Office should be notified.

In cases where the old station cannot be recovered, a new station should be established nearby and after the field position has been determined, an inverse computation between the two stations should be made. A further attempt at recovery should be made using the computed azimuth and distance. Should the old station be found, a precise connection to the new station should be made.

14. Marks.—Detailed specifications for marking stations are given on pages 84 to 95.

Each station will be permanently marked. Surface and underground marks are to be used where local conditions permit. There should be at least two reference marks at each triangulation station. These should be so located as to avoid the probability of both being disturbed by the same cause, and they should be set so that the angle at the station between these marks will approach 90°, if possible.

An azimuth mark is to be established at each station in a location that will be visible from the ground at the station and not less than ¼ mile distant. The primary purpose of azimuth marks is to furnish accessible azimuths to engineers and surveyors using geodetic data. Azimuth marks will not be required where another triangulation station, intervisible from the ground at a distance of not more than two miles, is available.

Since triangulation stations are sometimes used as magnetic stations they should not contain iron or steel reinforcing material.

Observations made on azimuth marks should consist of at least 4 positions with a first-order instrument. Observations on reference marks should consist of at least 3 positions with the direction theodolite or be made by closing the horizon with a transit. Extreme care must be taken in centering the instrument over the station mark and in leveling the theodolite so that the angles to nearby objects may be correctly measured. Linear distances from the station to the reference marks shall be measured with the steel tape in meters to thousandths and checked with a measurement in feet to hundredths. Where it is at all practicable to do so, the distance between the reference marks should be measured.

Standard witness posts should be established at stations as specified on page 89.

15. Connections to surveys of other organizations.—Connections by methods that insure second-order accuracy will be made to monuments established by other agencies. These monuments should be included in the scheme as occupied stations or connected by short traverses. See pages 3, 89 and 115.

16. Descriptions of stations.—A description on Form 525 shall be submitted for each new triangulation station established. The descriptions of the intersection stations shall be submitted on Form 526b, and whenever possible a personal visit to the site of the intersection station shall be made to obtain information for the description.

Form 525 will also be submitted for each station of another organization for which no position has been previously determined by the Coast and Geodetic Survey.

A thorough search should be made for all previously established stations within the project area. A recovery note on Form 526 shall be submitted for each previously established station recovered or for which a search has been made.

17. Landmarks for charts.—A report on Form 567, "Landmarks for Charts," should be submitted for those prominent objects and landmarks which should appear on nautical and aeronautical charts. The positions given in latitude and longitude may be those obtained from the field computations. Positions are given in seconds for points in the interior of the country and in seconds and meters for points to be listed on nautical charts (i.e. coastal points).

18. Vertical angle measurements.—Reciprocal observations of double zenith distances shall be made at all occupied stations in areas specified by project instructions. Three determinations of the double zenith distance of each object will be made. Each determination shall consist of one direct and one reversed pointing. If the separate determinations of the zenith distance fall within a range of 10 seconds, no further observations are required. If they do not, then additional observations will be made until a satisfactory set is obtained. The telescope must be reversed between consecutive pointings on the same object in order to insure a separate and independent value for each pointing. If the instrument is of unusual construction a sketch of the vertical circle and verniers will be placed on the first page of each volume for each instrument, with a notation whether diagram shows circle left or circle right (see p. 104).

The index correction of the vertical circle should be kept low, generally within two or three minutes in order to facilitate checking and computation, and should not exceed five minutes.

Unless called for in the project instructions, computations of elevations determined by vertical angles are not made in the field, but a check on the consistency should be made so that errors and omissions may be detected and corrected before the records are transmitted to the Washington Office.

An approximate check may be made in the following manner: After the observations have been reduced for $t-o$ (see p. 184), multiply the length of the line (over which the observations have been made) in kilometers by 0.46. The result should equal, very nearly, the number of minutes by which the sum of the two zenith distances exceeds 180°. A difference exceeding about one minute will normally indicate an error in the observations or computation, or unusual refraction conditions.

All computations and all transfers from the records to the abstracts and other entries on the abstracts will be carefully checked in the field.

Observing procedure is described in detail on pages 103 to 106.

It is desirable that these observations be made between 12:00 noon and 4:00 P.M. since refraction is smaller and more constant during that part of the day. When observations cannot be obtained to all stations during this period, then the vertical angles to the stations omitted may be measured on lights at night. If this is done, it is desirable that observations should also be made on the lights of one or more of the stations on which vertical-angle observations were obtained during the afternoon period. When frequent connections are made to bench marks, vertical-angle observations may be made later than the time of least refraction, but then reciprocal observations should be made at all stations at nearly the same time of day. Whenever practicable, connections should be made to bench marks established by spirit levels and, in general, such connections should be made about every third quadrilateral.

19. Base measurement and azimuth observations.—Base measurement and azimuth observations are an integral part of the work of geodetic triangulation. Specifications for and discussion of these important operations are treated separately in chapters 3 and 4 of this manual.

20. Field computations.—Unless otherwise directed by the project instructions, the field computations shall be carried through the determination of geographic positions for all stations, including supplemental and intersection stations. Original record books should be forwarded to the Washington Office as soon as the record books, abstracts, and lists of directions are checked. Records, descriptions, and computations are to be forwarded monthly by registered mail to the Washington Office. No records are to be kept on hand in the field more than one month. To prevent total loss in mailing, the abstracts, lists of directions, and computations should not be sent on the same day as the original record books involved. Final field computations and progress sketches are to be submitted within one month following the completion of an arc or project.

Permission to depart from the procedure outlined in this section must be obtained from the Director.

21. Sketches.—A large sketch to scale showing the scheme in detail shall be submitted on tracing cloth for each project or season's work. (See p. 191.)

A small (preferably 8- by 10½-inch) generalized sketch showing types of work accomplished and locality covered are required for each project at the end of the fiscal year. (See p. 192.)

22. Reports.—An informal mid-month report of progress in letter form is required. (See p. 189.)

A formal report shall be made each month on Form 20, Monthly Report and Journal of Field Party. (See p. 189.)

A detailed season's report is required on completion of each season or project. (See p. 190.)

An annual statistical report by projects is required at the end of each fiscal year. (See p. 191.)

23. Recommendation of changes.—If at any time the local conditions are such that in the opinion of the chief of party it would be advisable to change the triangulation scheme or the program of carrying on the work, the Director should be informed and any recommendations considered pertinent should be submitted.

SPECIFICATIONS FOR SECOND-ORDER TRIANGULATION

The preceding specifications for first-order triangulation also apply to second-order triangulation, except as modified in the following paragraphs. Second-order triangulation is frequently observed on pole targets by daylight. It is used principally in coastal areas by the Coast and Geodetic Survey.

24. Figures.—Figures used shall be as described in paragraph 15 of chapter 1 (p. 4).

25. Length requirement.—For second-order triangulation, the discrepancy between a computed length and the measured length of a base or the adjusted length of a check line shall not exceed 1 part in 20,000 for class I or 1 part in 10,000 for class II, after the angle and side conditions have been satisfied.

26. Angle requirement.—For second-order triangulation, the closures of each triangle shall seldom exceed 5 seconds for both classes and the average closure of the triangles shall not exceed 1.5 seconds for class I or 3 seconds for class II.

27. Side checks.—For second-order triangulation, the limits described in paragraph 11 shall be used as criteria for satisfactory side checks.

28. Instruments.—Either a direction or a repeating instrument may be used in triangulation of this class, though the required results can usually be obtained more quickly and economically with a direction theodolite with micrometers.

29. Circle settings.—The initial settings for successive positions observed with a direction theodolite or successive sets observed with a repeating theodolite should differ by amounts depending upon the number of positions or sets to be observed and the number of micrometers or verniers on the theodolite. The interval in degrees between successive settings is given approximately by the formula $I = \frac{360}{mn}$, where I is the interval, m the number of micrometers or verniers, and n the number of positions or sets. In addition, the pattern of the minutes and seconds in the settings used should provide a compensating distribution of the small errors due to the run of the micrometers or the error of graduation of the verniers. The most commonly used circle settings are tabulated in table 2 on page 11.

30. Observations with a repeating theodolite.—A set of observations should consist of six repetitions of the angle with the telescope in the direct (or reversed) position, followed immediately by six repetitions of the explement of the angle with the telescope in the reversed (or direct) position. With the common type of 7-inch repeating theodolite equipped with verniers reading to 10 seconds, the accuracy specified for second-order triangulation will usually be obtained by making from two to three sets of observations of each angle.

With a repeating theodolite, measure only the single angles between adjacent lines of the main scheme, including the angle necessary to close the horizon. In the cases in which the failure of adjacent signals to show at the same time prevents carrying out this program, make as near an approach to it as possible and then take the remaining signals in another series together with some one, and only one, of the signals observed in the first series. Measurement of an angle which is the sum of two or more observed angles should be avoided. With this scheme of observing, the only local adjustment made is to distribute each horizon closure uniformly among the angles measured in that series. Sets which contain only angles measured between the same stations may be meaned directly. Other sets may be combined on summary or analysis sheets in the field in the same manner as described on page 148 for combining abstracts of directions.

31. Observations with a direction theodolite.—Depending on the type of direction theodolite available, from 6 to 12 positions are usually observed to secure second-order accuracy. Any measurement of a direction deviating by more than 5 seconds from the mean shall be rejected and reobserved, using the same position setting.

32. Intersection stations observed from second-order triangulation stations.—At each occupied second-order station, observations will be taken on intersection stations as described in paragraph 12 on page 13 with the following exceptions. Observations may be made with two positions of a direction theodolite provided they agree within a range of 6 seconds (on well-defined objects). If more than two positions are observed the regular 5-second rejection limit should be used. If observations are made with a repeating theodolite, a set of three direct and reversed repetitions should be made of each angle and the horizon should be closed. Objects which are not sharp and well-defined should be included in separate sets using one and only one previously observed well-defined signal in subsequent sets. It is usually undesirable to observe more than 6 to 10 objects in a set (without closing the horizon). A line of the occupied scheme should be included in each set of intersection station observations.

SPECIFICATIONS FOR THIRD-ORDER TRIANGULATION

The preceding specifications for higher orders of triangulation also apply to third-order, except that the requirements are less rigid, thus permitting fewer observations and smaller instruments. These requirements are summarized in the tabulation of table 1 on page xv. Most third-order observations are made on pole targets by daylight. Third-order triangulation has been largely replaced in the Coast and Geodetic Survey by higher orders of triangulation.

PREPARATIONS

The purpose of this section is to summarize the principal steps which are preliminary to beginning field work by a Coast and Geodetic Survey triangulation party. For further details, refer to Serial No. 685, "Regulations of the Coast and Geodetic Survey" and to serial C. & G. S. circulars.

Field work in the Coast and Geodetic Survey is initiated by the Director who issues instructions to a chief of party for a specific project. On receipt of the project instructions, the chief of party prepares estimates for all anticipated expenditures; makes requisitions on the Washington Office for instruments, books, and stationery; makes arrangements for transfer or purchase of equipment and consumable supplies; and arranges for assembly of personnel. Normally, project instructions for triangulation are issued to parties which are already organized and at least partly equipped for the project. Although the chief of party should be qualified to organize and train a new party, it is seldom necessary for him to organize an entirely new party. A skeleton force of experienced personnel is usually transferred to a new party.

PROJECT INSTRUCTIONS

Project instructions are the detailed instructions issued by the Director for each project. Geodetic projects are numbered serially preceded by a "G," as G-840. Project instructions designate the chief of party, name of project (usually by type of survey and general locality), project number, date, and other preliminary conditions for beginning project. Frequently a specific purpose of the project is mentioned. The specific project is then defined, including limits, reference to reconnaissance sketches included, classes of work, order or priority of operations, beginning and connecting lines, junctions, base lines, and azimuths required, and appropriations to which expenses of operation are chargeable. Project instructions refer to and include specifications of the General Instructions (see pp. 9 to 18 of this manual) and appropriate instructions published in other manuals of this Bureau, with any necessary additions or modifications required for the project.

Data which are furnished the field party along with project instructions usually include the following:

Reconnaissance sketches (about 25 copies for a standard party).
Reconnaissance descriptions (about 25 copies for a standard party).
Triangulation diagram of State.
Geographic positions of old stations.
Descriptions of old stations.
Data consisting of observed directions at old stations where check angles are required.
Standardization data for tapes.
Copy of reconnaissance party's instructions.
Leveling index map of State (where vertical angles are required).
Elevations of bench marks (where needed for vertical angles or base lines).
Copies of existing maps of other organizations, such as Geological Survey quadrangles, Forest Service maps, and county maps.

Supplemental instructions may be issued, as work on the project progresses, to limit the work or to define additional work.

ESTIMATES

Estimates are prepared by the chief of party on Form 1 (Estimates) on receipt of instructions for the project; or, for continuing projects and parties, by six-month periods beginning 1 July and 1 January. The principal expenses are usually grouped under: pay of personnel; consumable supplies; tools and equipment and other inventorial items; rentals; repairs to trucks and equipment; hire of animals, tractors, and emergency vehicles; liquidated damages to property; and miscellaneous expenses, such as tolls, shipping, communications, and utilities. These items are arranged by monthly expenses where practicable and otherwise as seasonal items. Where unusual expenditures are necessary for putting the party in the field or taking the party from the field, these are shown as separate items.

No expenditures can be made until the estimates are approved and an allotment is made. The allotment may not necessarily be as much as the amount of the estimate. Expenditures must not exceed the amount allotted.

Supplemental estimates (Form 493) shall be submitted when necessity arises for an expenditure not covered by any previously approved estimate.

PROCUREMENT OF PERSONNEL

The total complement of a field party is limited by a personnel ceiling which is fixed by the Washington Office. Personnel employed are further limited to the number and positions approved in the estimates. Most of the personnel have Civil-Service ratings. Party complements are filled mostly by transfers on official orders and from Civil-Service registers. Usually a few temporary positions may be filled by the chief of party from any qualified personnel available in the locality.

In moving personnel between projects, travel orders must be requested from the Director for each individual, with the method of travel specified.

PROCUREMENT OF EQUIPMENT AND SUPPLIES

Typical lists of items used by triangulation parties appear on pages 278 to 282.

All instruments are requisitioned from the Washington Office on Form 12.

Stationery and forms are requisitioned from the Washington Office on Form 11a.

Technical books and logarithmic tables are requested from the Washington Office by letter.

A few items of equipment such as tents, field desks, observer's bags, and special items are requisitioned from the Washington Office on Form 12.

Principal items of equipment such as trucks, towers, and tools may be obtained from storage, by transfer from other parties, or by purchase. In general, trucks and steel towers are purchased by the Washington Office. (See par. 899 of Regulations.)

Consumable supplies are purchased for delivery at the field base camp.

Items such as gasoline, oil, tires, batteries, and spark plugs are purchased on annual Treasury or Navy contracts. Most mobile parties use credit cards for gasoline purchases on these contracts. Items such as brushes, brooms, and canvas usually have to be purchased from the Federal Prison Industries.

Facilities of the Federal Supply Service should be used for purchases of required classes of items, and for other classes wherever practicable. (Refer to pars. 881–4 of Regulations.)

For certain purchases direct from dealers, the chief of party may issue invitations for bids and award the contract.

Purchases may be made in open market (without contract) up to a limited amount (usually $100). (See par. 892a of Regulations.)

Petty purchases not exceeding $5.00 may be made on sub-receipts. However, whenever possible, it is desirable to use the purchase voucher (Standard Form 1034) and to make payment by Government check.

Except for sub-receipt and emergency purchases, serially numbered purchase orders (Form 98) issued by the chief of party should be used for all purchases.

TRANSFER OF PARTY

When a triangulation party is transferred from one chief of party to another the following steps are taken:

1. Separate lists of instruments, general property, books, and accountable forms such as Government bills of lading, tax exemption books, and identification cards are itemized on Form 573. Note: Blank checks and transportation requests cannot be transferred in the field.
2. Pay cards showing dates and amounts earned and amounts paid by each assistant disbursing officer are initialed.
3. The new chief of party is furnished an itemized list of uncompleted field and office work.
4. A season's report to date on all completed or partially completed projects is submitted.
5. A statement of allotment balances (Form 474) to date of transfer with careful listing of outstanding bills is furnished. The new chief of party takes over party's allotment and assumes all proper outstanding bills.
6. Relieving chief of party telegraphs the Director, U. S. Coast and Geodetic Survey, on effective date of transfer: "Relieved ------------ as chief of party this date."

MOVES BETWEEN PROJECTS

When a party shifts base between projects from a working area in one part of the country to a working area in another part of the country, steel towers are usually shipped by freight as carload shipments. Steel-tower shipments should be classified as structural steel to obtain minimum rates.

In order to provide for continuous efficient operation of all un ts of a triangulation party, the work in the area being completed should be so planned that a carload of towers and an advance building unit can be sent ahead to begin signal building on a new project from one to three weeks in advance of the main party.

ORGANIZATION

First-order triangulation, as now carried on by the Coast and Geodetic Survey, is executed by mobile field parties usually operating from a central camp conveniently located in each project working area. Transportation is generally by truck (except for pack horses in some mountain areas, and boat and airplane in remote sections, such as Alaska). The basic building unit consists of from 2 to 5 men, depending on the type of signal to be built. The building parties drill or dig holes for marks and tower footings, mix the concrete and set the marks, build and collimate wooden or steel signal towers, and clear any needed lines. On a standard steel-tower party, a special unit of 4 builders acts as a tear-down unit. The basic observing unit consists of an observer, recorder, and lightkeeper. The lightkeeper also reads the "B" micrometer. Triangulation parties usually operate with from 2 to 4 observing units, with the necessary building units, lightkeepers, and computers.

TABLES OF ORGANIZATION

Typical organization charts of first-order triangulation field parties are shown in figures 1 and 2.

O-party is the designation of the first observing unit, OO-party is the designation of the second observing unit, etc.

PERSONNEL

Personnel generally have professional or sub-professional Civil-Service ratings. Chiefs of parties can usually hire a limited number of temporary non-Civil-Service personnel in the working area. The duties of most of the field party personnel are quite flexible depending on the current requirements of the project. Many of the older and more experienced personnel can do any job on the field program. Most of the personnel are carried under the title "Cartographic Survey Aid," and may be shifted by the chief of party to any necessary duty for which the man is capable. In addition to technical qualifications for the job, a common requirement of all field party personnel is good physical condition with ability to climb mountains and to work on high survey towers, to drive trucks, and to be adaptable to mobile camp life. All personnel should learn as much of all field party duties as opportunity and time permit. In particular, each employee should learn the duties of the next higher job to which he might be promoted.

Duties may change to meet various project requirements, and also to fit the capabilities of the personnel available. Any employee on a triangulation party may be assigned by a chief of party to any duty in which he is needed and which he is considered capable of performing.

CHIEF OF PARTY

The chief of party is responsible for the efficiency and economy of operation of his party in carrying out instructions issued by the Director. He sees that the party is governed in accordance with the Regulations of the Coast and Geodetic Survey; that the technical specifications for the surveys are rigidly adhered to; that economy in expenditures is exercised; and that proper measures are taken to protect Government property in his custody from loss or damage.

When bonded by the Secretary of Commerce as certifying officer and designated as an assistant disbursing officer, he acts as a disbursing officer for Government funds and complies strictly with the regulations of the Treasury Department and General Accounting Office in matters relating to the custody and expenditure of funds.

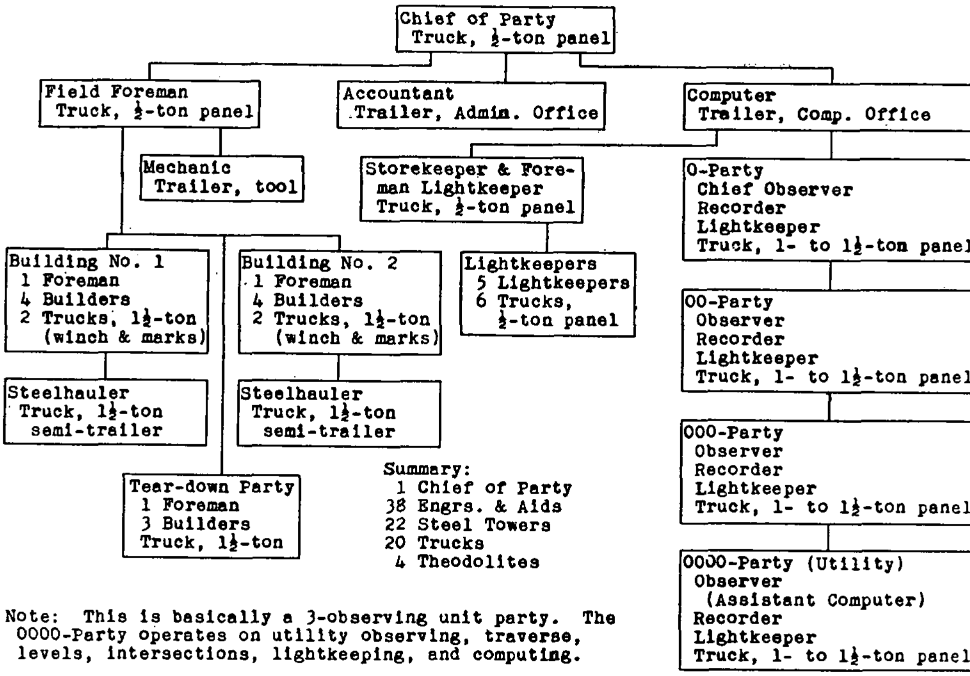


FIGURE 1.—Organization diagram, typical steel tower triangulation party (1949).

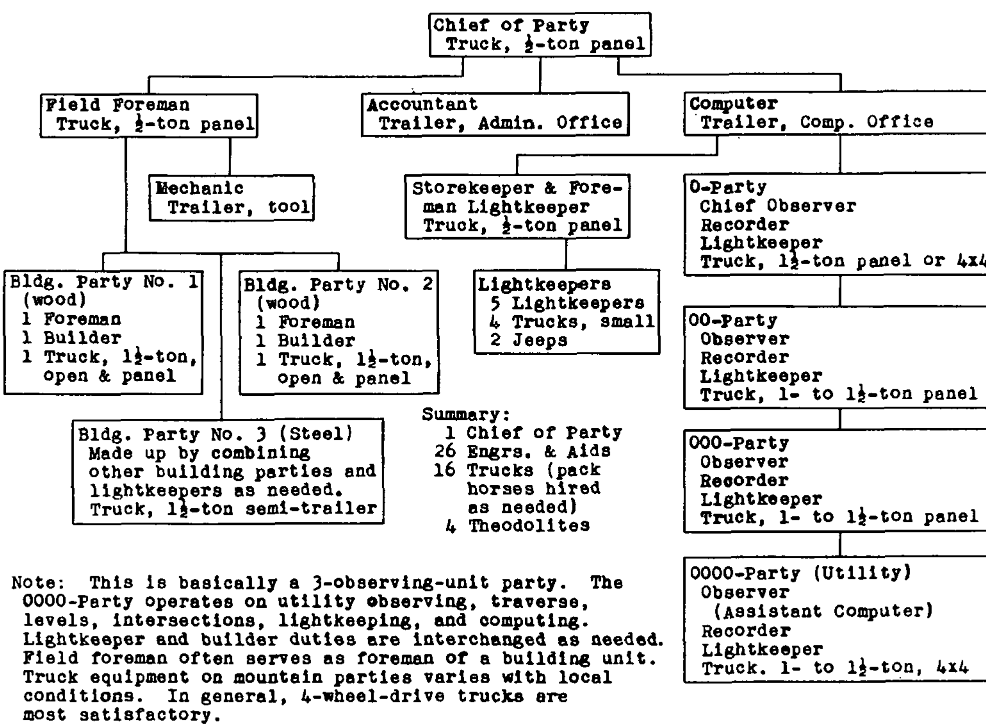


FIGURE 2.—Organization diagram, typical mountain triangulation party (1949).

He is responsible for the disposition of personnel and for the scheduling of operations so as to most effectively accomplish the field surveys and for seeing that the field records and computations are complete and kept up to date.

CAMP ADMINISTRATION

Multiple-unit first-order triangulation parties normally operate from a base camp with sub-camps where needed. This base camp is usually moved at about monthly intervals so as to maintain a central location in the working area.

Base-camp sites are selected to fulfill as many of the following considerations as practical: Proximity and accessibility to triangulation stations; availability of supplies, including building supplies, food, drinking water, and electricity; suitable accomodations for personnel; and economy of expenditures.

A contact man, usually the field foreman, makes the contract for the camp site in advance of each move. Suitable temporary camp sites can occasionally be obtained on property controlled by Federal, State, or local governments by obtaining a permit from the proper authorities. Recreation parks and fairgrounds can usually be obtained at very nominal rentals in off-seasons. It is sometimes necessary to rent space in a commercial camp or in private fields. Whenever possible, it is desirable in the interests of camp sanitation, morale, and efficiency of personnel to obtain camp sites with toilet facilities, piped drinking water, shade, and a minimum of dust and mud. When toilet facilities are not available, tents should be maintained over sanitary dug latrines. Camp sites in or near small modern towns are most satisfactory. This also permits those personnel who so desire to obtain room and board in town.

The camp layout is usually made by the field foreman with approval of the chief of party. Living quarters should be placed farthest from the entrance where they will be least disturbed by truck traffic. Usually, house trailers which are occupied mostly by families are placed in one group, tents of bachelor personnel in another group, and supply tents and office trailers in another group. Groups should have an orderly alinement or arrangement. Trucks not in use should be kept parked in orderly lines.

At each move, definite details of personnel should be assigned for the following duties: Digging and fill-in of latrines, piping water supply, making electrical connections, loading and unloading supplies, setting up supply tents and office trailers, disposing of rubbish, and policing of camp site. All personnel living in camp are required to keep the camp, and in particular their immediate surroundings, clean and sanitary. In addition, a detail, usually of two men daily, is assigned in rotation for general policing, sanitation, and disposal of garbage and rubbish. Personnel living in house trailers pay their own utility bills. An elected committee from these personnel usually handles these utility funds and reports to the chief of party that all bills are paid before camp moves.

The use of privately owned house trailers should be encouraged on a triangulation party, since they provide a family home on a necessarily nomadic life, and therefore are conducive to a happier and more permanent personnel.

Privately owned automobiles are personally convenient, but a part of each man's job on a mobile field party is to drive a truck. Trucks have priority on a camp move, and if the number of trucks exceeds personnel without private cars, some personnel will inevitably have to make private arrangements to move their private cars between camps.

An outdoor glass-covered bulletin board should be maintained near the office trailers for posting operation schedules and other announcements. A mail box should also be maintained in the same vicinity.

A board for truck keys should be maintained in the office trailer and keys for all trucks should be kept thereon when not in official use. Arrangements should be made for turning in keys whenever trucks return to camp.

The storekeeper should keep a record in which individuals sign for instruments and major items of equipment. The computer should keep a list on which observers receipt for pre-numbered record books.

INSTRUMENTS

A typical list of instruments used by a triangulation party is given on page 278.

Instruments in daily use are normally issued to the custody of personnel in charge of various units and to individual personnel. The storekeeper keeps a record of all instruments and the individuals to whom assigned. He should keep all instruments not in use in locked storage. The chief of party should make frequent checks to insure that instruments are being cared for properly.

THEODOLITES

A theodolite is a precision surveying instrument used for the accurate measurement of angles. It consists of an alidade with telescope, mounted on a base carrying an accurately graduated horizontal circle, and equipped with necessary levels and reading devices. The alidade usually carries a graduated vertical circle. There are two general types of theodolites, namely, repeating theodolites and direction theodolites. A repeating theodolite is so designed that successive measures of an angle may be accumulated on the graduated circle. The reading of the accumulated sum is divided by the number of repetitions to obtain the observed angle. With a direction theodolite the circle remains fixed while the telescope is pointed on a number of signals in succession with the circle being read for each direction. There are two principal types of direction theodolites: In one the circle, usually graduated on silver, is read by equally spaced micrometer microscopes (usually two which are diametrically spaced); in the other type, the circle graduations are usually on glass and are read through a single auxiliary microscope by means of prisms which bring diametrically opposite portions of the graduated circle into view, along with a micrometer scale on which is read the proportional part of the movement needed to bring opposite circle marks into optical coincidence.

The theodolite is the principal instrument used in triangulation. The quality of the theodolite has a direct relation to the quality of the results obtained. Direction theodolites of the best workmanship are the preferred instrument for first-order triangulation. Although a repeating instrument gives good results, its mechanical operation is not readily adaptable to an efficient observing program. Use of a repeating theodolite on first-order triangulation is confined to stations where there is not room for a direction instrument or where the support is unstable.

The Coast and Geodetic Survey has a number of instruments of various types and designs, both of domestic and foreign make. The Parkhurst and Wild theodolites, which are representative of their general types of direction theodolites, and a common type of a repeating theodolite are described in the following sections of this manual.

Parkhurst theodolites.—The standard first-order theodolite of the Coast and Geodetic Survey is the Parkhurst theodolite with a 9-inch graduated circle (fig. 3). This instrument was designed by D. L. Parkhurst, Chief of the Instrument Division of the Bureau. It is generally favored by observers because of its comparatively rugged and simple construction and its ease of operation and adjustment.

The distinctive design features of this theodolite are its non-binding centers, ball-bearing clamp ring, electrical connection through the central axis for illumination, discontinuous conical bearings for the horizontal circle, illuminated glass micrometer drums, and improved designs for tangent-screw assembly, clamp-block assembly, and micrometer mountings.

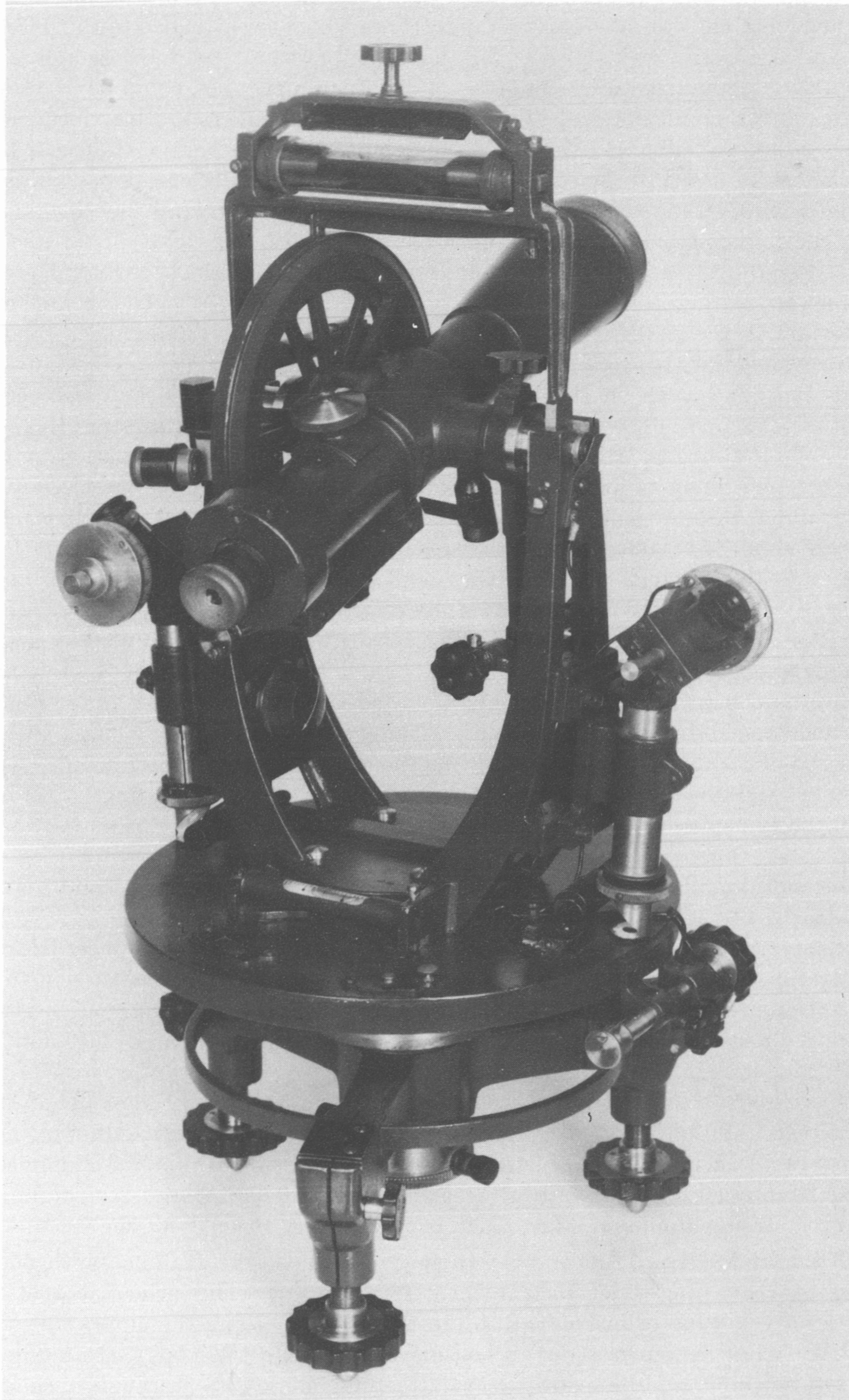


FIGURE 3.—Parkhurst first-order theodolite (9-inch circle).

This theodolite has two micrometer microscopes which are attached to the telescope standards and are easily adjustable. The micrometer drums read to seconds. The nine-inch circle is graduated to five minutes. The micrometer box contains two sets of parallel wires and a comb notched to minutes. Degree numbers are inscribed on the circle. In reading the circle it is helpful to remember that the part of the circle being read is at the center notch of the comb. The degrees and minutes can be read directly, checking the minutes when necessary by the notches on the comb, and then reading the seconds on the micrometer drum twice, once backward with the right pair of micrometer wires centered over the first circle graduation mark to the right of the comb center, and then forward with the left pair of micrometer wires centered over the first circle graduation mark to the left of the comb center. These readings should check to about 3 seconds provided the two sets of hairs are spaced properly. There are clamps and slow-motion tangent screws on the horizontal motion of the alidade and the vertical motion of the telescope. The circle has no clamps or slow-motion screw. It is held fixed during observations by a friction disk and is easily set for a new position by horizontal pressure and finger tapping on small knobs on the lower side of the circle plate. The instrument has three leveling screws with a clamp screw on each. The plate level has a sensitivity of about 15 to 20 seconds per division and is easily adjustable. Striding levels are usually of 5 to 6 seconds in sensitivity, and are freely adjustable. One of the standards is adjustable. The reticle ring is fully adjustable. The telescope has internal focusing. Illumination of the telescope is controlled by a rheostat and also by a small reflecting mirror at the intersection of the optical and horizontal axes of the telescope. A small lamp bulb lighting the telescope is mounted at the right end of the horizontal axis of the telescope and is easily accessible. There are separate spring contact switches for each micrometer and separate bulbs lighting the circle and micrometer drum. There is also a lighted sighting tube along the telescope which facilitates pointings. All lamp bulbs are ordinary commercial 2.8- or 3.5-volt screw-base flashlight bulbs and are easily replaceable.

The vertical circle is attached to the telescope. It is read by 10-second verniers which are attached to an adjustable level.

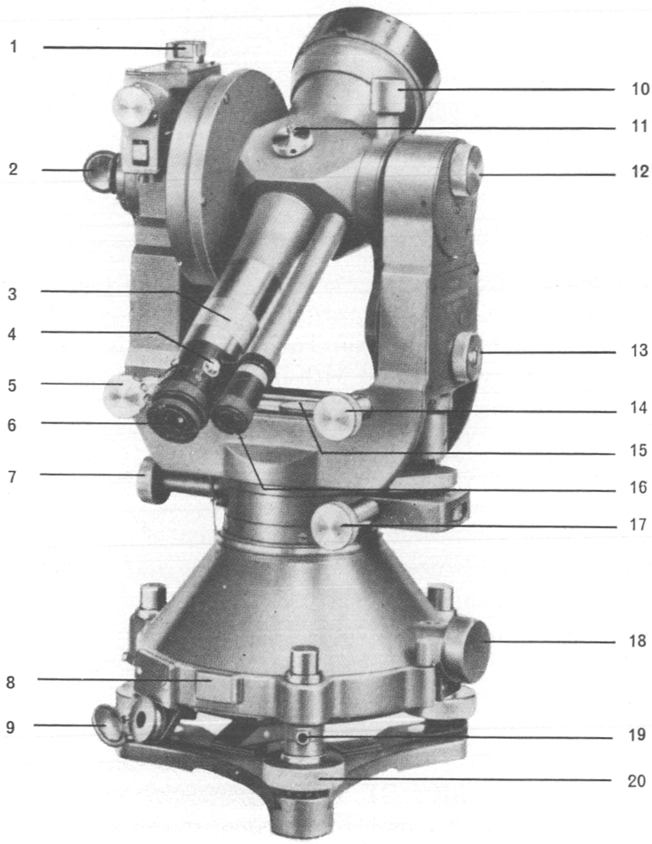
Additional details of the Parkhurst first-order theodolite are given on pages 32 to 37.

The Parkhurst second-order theodolite is similar in design and construction to the first-order theodolite, except for smaller dimensions and larger graduation intervals. The horizontal circles are 6½ inches in diameter. The circle is graduated to 10 minutes, and the micrometer drum is graduated to 2 seconds.

The latest design of the Parkhurst second-order theodolite, called the model of 1948, has been further improved with a circle which is strengthened to prevent warping, a tangent screw with an improved adjustment for fit of the screw, improved illumination of the circle, improved micrometer bearings, and lighter weight because of aluminum telescope and a strong aluminum alloy which is used largely throughout the instrument.

Wild theodolites.—The Wild T–3 is a prism-microscope type of direction theodolite and is a satisfactory first-order instrument. (See fig. 4.) The principal operating characteristic of this type of instrument is the method of reading the circles by means of an auxiliary telescope (microscope) alongside the sighting telescope. Both sides of the circle are reflected simultaneously in this reading microscope through a chain of prisms. A micrometer in this optical system is arranged so that movement of the micrometer screw will bring the opposite sides of the circle into optical coincidence with the amount of the movement being read on the micrometer. Shifting of a change-over knob on the side of the instrument allows either the vertical or horizontal circle to be read from the single microscope eyepiece. The vertical circle has a coincidence-type level.

Detailed operation of this instrument is described on page 141.



1 Prism reader for altitude level
2 Illuminating mirror for vertical circle
3 Focussing ring for telescope
4 Adjusting screw for cross-lines
5 Slow-motion screw for altitude level
6 Telescope eyepiece
7 Horizontal clamp
8 Fixing lugs for packing
9 Illuminating mirror for horizontal circle
10 Vertical clamp
11 Knob for cross-line illumination
12 Micrometer milled head
13 Change-over knob
14 Vertical slow-motion screw
15 Plate spirite level
16 Reading eyepiece
17 Horizontal slow-motion screw
18 Milled head for circle setting
19 Adjusting screw for foot-screw
20 Foot-screw

FIGURE 4.—Prism microscope theodolite, Wild T-3.

The following technical data of the Wild T-3 theodolite are listed in the manufacturer's publication.

Glass circles .... 360°
Diameter of horizontal circle .... 5.5 in.
Graduation-interval of horizontal circle .... 4′

Diameter of vertical circle	3.8 in.
Graduation-interval of vertical circle	8′
Graduation-interval of micrometer drum	0″.2
Clear aperture of object glass	2.4 in.
Length of telescope	10.2 in.
Magnifications of telescope	24, 30, 40×
Sensitivity of plate level	7″ per 2 mm.
Sensitivity of vertical-circle level	12″ per 2 mm.
Coincidence-adjustment of vertical-circle level to	0″.2
Weight of instrument	24.2 lbs.
of steel case	8.3 lbs.
of packboard carrier	3.9 lbs.
of tripod	16.5 lbs.

Repeating theodolites.—Commonly used types of repeating theodolites have 7-inch horizontal circles which are read by two verniers to 10 seconds. (See fig. 5.) Repeating instruments have an upper motion (with clamp and tangent screw) which carries the alidade and verniers, and a lower motion with its clamp and tangent screw which control the movement of the plate circle.

DAMAGE TO INSTRUMENTS

All personnel should be impressed with the necessity of taking proper care of delicate instruments. Many instruments used are very valuable and in some cases irreplaceable.

In the event of loss or damage of any instrument, the chief of party should make a thorough investigation, require all personnel involved to make written reports to him, then make a report to the Washington Office for action and disposition.

Damaged, useless, and unused instruments should be returned to the Washington Office promptly.

CARE OF THEODOLITES

One of the characteristics of a good observer is that he takes proper care of his assigned instruments. Instruments that are kept clean and oiled and in the best of repair and adjustment are a good indication that an observer is making every effort to obtain the best possible results.

It should be kept in mind that a theodolite is a delicate precision instrument. Jolts and rough handling should be carefully avoided. When an instrument is received, it should be unpacked carefully and slowly, noting the exact manner in which it is fitted in its case. When replacing the instrument in its case, avoid forcing any part into place. When not in use, the instrument should be properly secured in its case and kept in an upright position. Only objects that can be properly secured should be placed in the case with the instrument. Observing-party trucks should have specially prepared boxes, padded with sponge rubber and secured well forward in the truck, in which the cased theodolite is carried. The theodolite should always be lifted by the lifting ring on the tribrach, and never by the standards or micrometer arms. Adjusting and clamping screws should be set securely but not tight enough to cause undue strains or strip the fine threads. In use in the field, the theodolite should never be left unattended. It

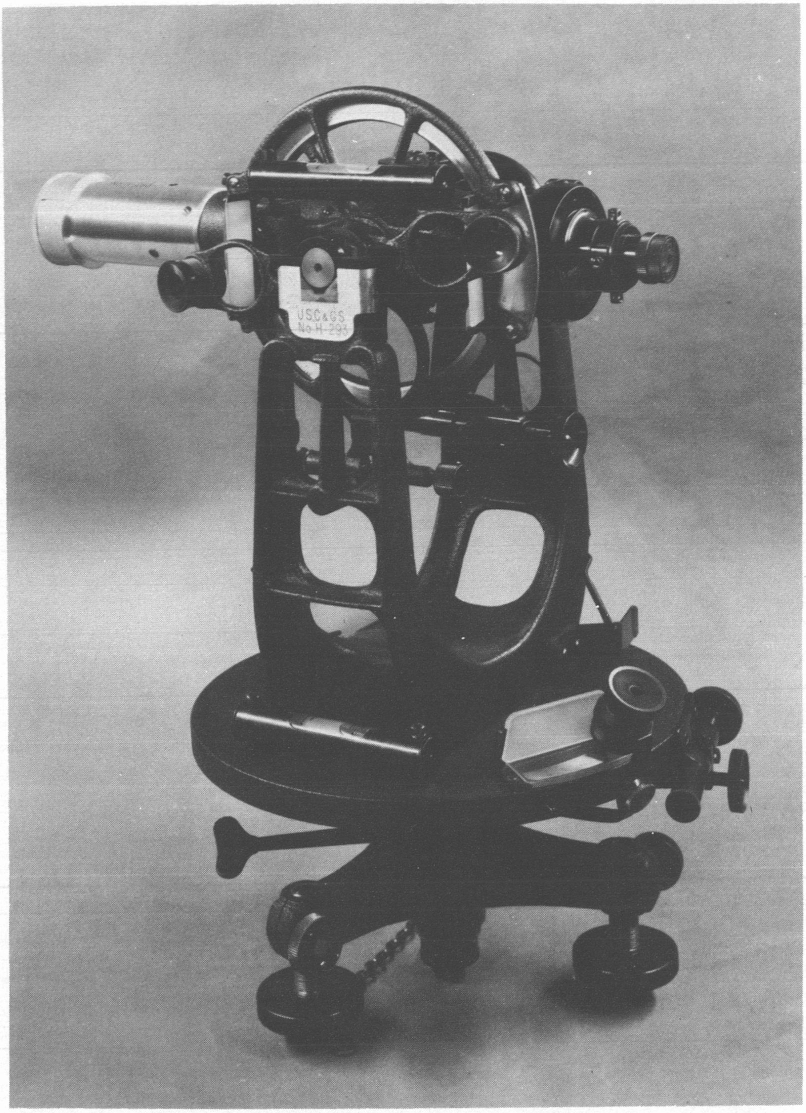


FIGURE 5.—Repeating vernier theodolite (7-inch circle).

should be protected from the sun and weather. If it should become wet or dusty, it should be cleaned as soon as possible.

Detailed instructions for cleaning and oiling of Parkhurst theodolites are given in the following section. Frequent cleaning is desirable. Parts to be cleaned in the field are lenses, axis centers, tangent screws, micrometer screws, telescope pivots, standard bearings, clamp collars, and foot screws. All moving parts should move freely and not bind. Binding may cause not only erratic results but also permanent damage to the instrument. When cleaning and oiling the center and micrometer bearings and other parts which require partial dismantling of the instrument, the work should be done in a clean work space in a room as dust-free as possible, and parts should be laid out in order

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so as not to be misplaced or damaged. Particular care is necessary never to allow the hand, oil, or anything harmful to touch the fine circle graduations.

The outer surfaces of the lenses require frequent cleaning but should be rubbed as little as possible. First brush off the dust with a soft camel's-hair brush, then take lens tissues or clean soft lint-free linen and lightly flick the surface with a circular brushing motion to remove any dirt which may remain. If necessary, cloth may be moistened with water or alcohol, but no alcohol should be allowed to remain on lens or get between the lenses as it affects the balsam cement.

Cleaning and oiling Parkhurst-type theodolite in the field.—There are certain parts of the theodolite that require cleaning and lubrication in the field if satisfactory performance is to be attained.

Any disassembling of the instrument should, as far as possible, be done in a place free from dust and dirt. Use only the oil provided with the instrument, except in cases of definite emergency, and then only clock or watch oil should be used.

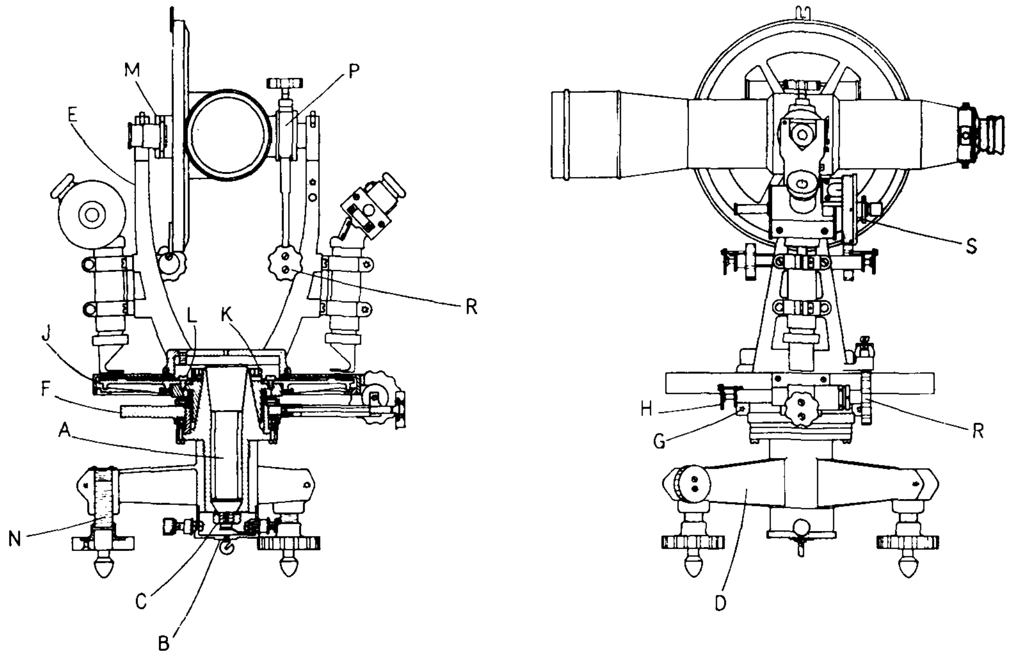

FIGURE 6.—Sectional view (front and side) of Parkhurst theodolite.

Frequency of cleaning and lubrication will depend upon circumstances of use. Hot, dry, or dusty conditions will cause the instrument to need more frequent attention. Never use the instrument if it drags or binds. Locate the cause and correct it, otherwise serious damage can be caused quickly.

The following description and illustrations may prove helpful to the user in caring for this instrument.

Normally, operations requiring field attention are:

(1) The vertical axis "A" should be cleaned and oiled frequently. To remove, turn the contact cup "B" to the left until it stops, then pull down (battery wires need not be removed). Remove nut "C" and pull back and lock the tangent-screw plunger

"H." Hold the leveling head "D" with one hand and lift up on standards "E" giving it a twisting motion. The oil sometimes causes considerable suction and if this occurs, press upward on the lower end of the axis. When the axis has been loosened, lift straight up. Promptly plug the socket with a lint-free cloth. (See figs. 6 to 12.)

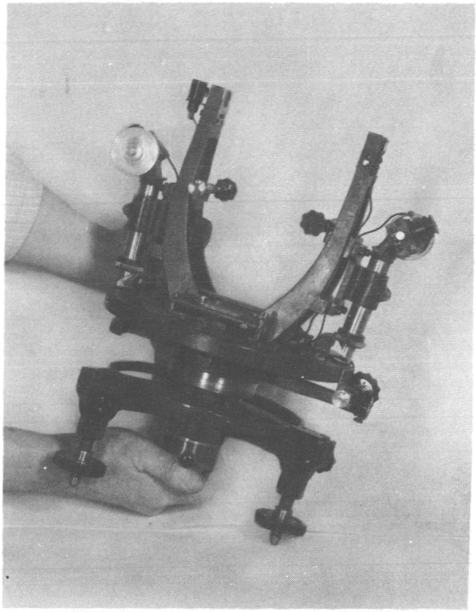

FIGURE 7.—Removal of vertical axis, step 1. Turn contact cup to left and pull down.

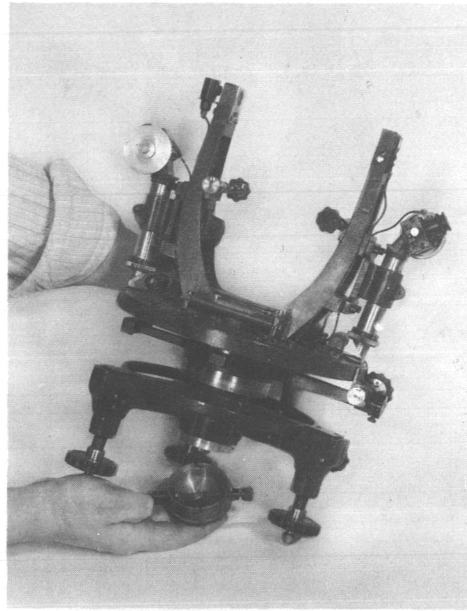

FIGURE 8.—Removal of vertical axis, step 2. Remove contact cup thereby exposing axis holding nut.

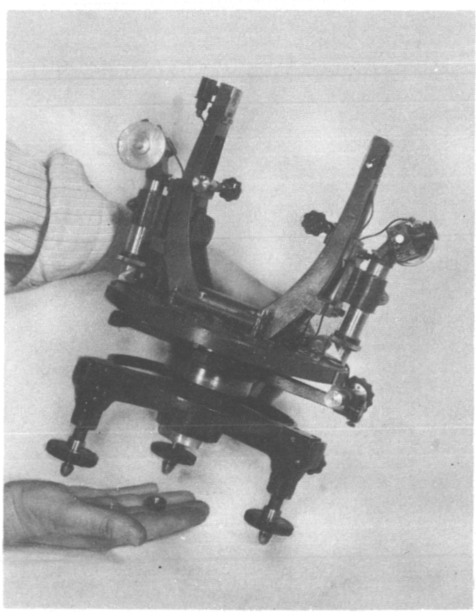

FIGURE 9.—Removal of vertical axis, step 3. Remove axis holding nut.

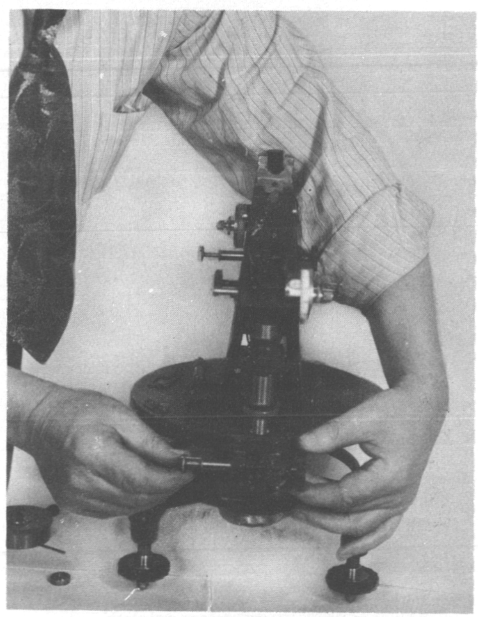

FIGURE 10.—Removal of vertical axis, step 4. Pull back tangent screw plunger and lock by turning until pin will catch.

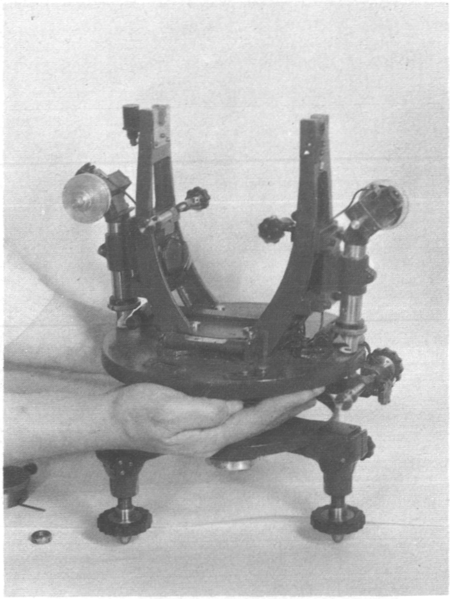

FIGURE 11.—Removal of vertial axis, step 5. Lift up on alidade, turning back and forth. Lift out straight up, vertically. If the axis does not come out easily, even though it turns freely, oil is probably causing suction. To relieve, press up on end of axis.

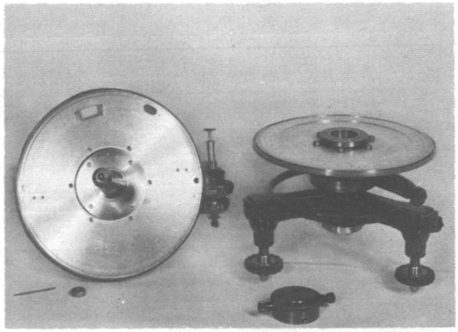

FIGURE 12.—Removal of vertical axis, step 6. When disassembled as shown, plug socket with lint-free cloth.

FIGURE 13.—Disassembly of circle bearing, step 1. Remove the circle nut. Be careful, in every move in this operation, not to touch the polished ring carrying the graduations. Do not loosen or tighten the six holding-down screws.

FIGURE 14.—Disassembly of circle bearing, step 2. Place thumbs on top of socket and press up on under side of circle, keeping palms of hands off the graduated surface.

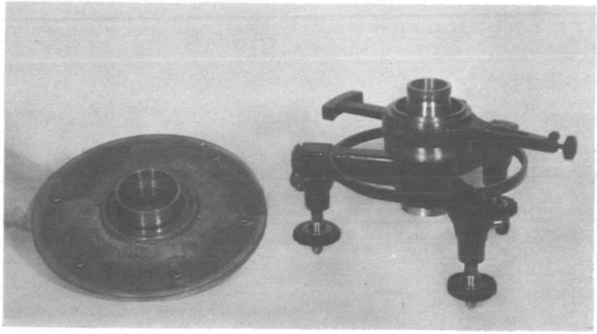

FIGURE 15.—Disassembly of circle bearing, step 3. Hold so as to protect the graduated surface when cleaning and oiling the bearing surfaces.

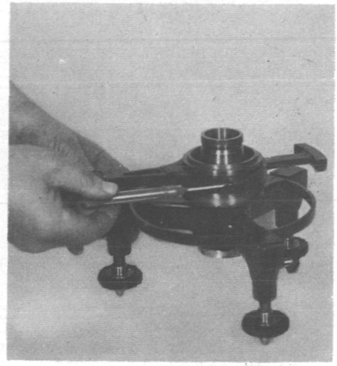

FIGURE 16.—Disassembly of horizontal clamp, step 1. Loosen the two screws by turning backward about half a dozen turns. Screws are made so that they will not fall out. It is not necessary to remove the circle or alidade, as was done in this illustration.

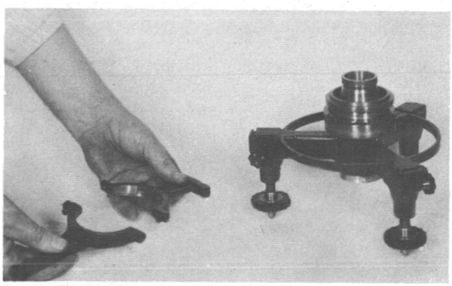

FIGURE 17.—Disassembly of horizontal clamp, step 2. Pull clamp apart.

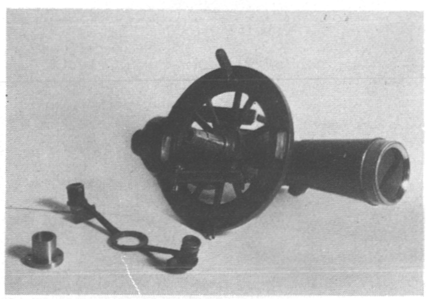

FIGURE 18.—Disassembly of vertical circle, step 1. Remove nut and lift off reading glass arm.

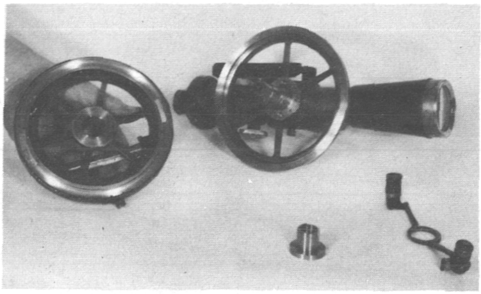

FIGURE 19.—Disassembly of vertical circle, step 2. Twist vernier circle slightly and lift off.

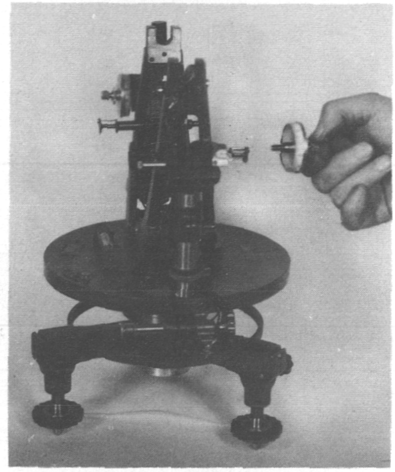

FIGURE 20.—Removal of micrometer screw. Turn drum counterclockwise. Quite a number of turns may be required before anything seems to happen. The micrometer slide eventually comes against a stop after which the micrometer screw will back out. When reassembling, be careful not to cross the threads.

Clean both axis and socket with alcohol or benzine, then oil using only enough to lightly smear the bearing surfaces. A few drops are sufficient, and too much oil will cause inaccurate results.

(2) If the bearing for the horizontal circle "J" requires cleaning, back off nut "K" and lift out the circle and its bearing (do not touch screws "L"). Lift the circle out very carefully being careful not to touch the polished and graduated surface. Do not allow anything harmful to touch this surface as it is easily damaged. If dusty, clean with the camel's-hair brush provided with the instrument. Do not wipe. Do not attempt to clean the graduated surface if it is tarnished. Unless the tarnish is very bad, it will not appear bothersome in the microscope. (See figs. 6, 13, 14, and 15.)

(3) Clamp "F" may require frequent cleaning. (See figs. 6, 16, and 17.) First pull plunger "H" back as far as it will go, then turn until its pin catches and holds in the "out" position. Loosen the two large screws "G" at the sides. In order to prevent their loss, these are designed so that they will not drop out. Half a dozen turns should allow the two halves of the clamp to be pulled apart. (Note: When reassembling, lower plunger "H" against the clamp. Do not let it snap into position.)

The ball bearings for the clamp cannot be readily removed, nor is removal necessary. The bearing may easily be cleaned and oiled in the same manner as the axis, using a more liberal amount of oil.

(4) The leveling screws "N" should not ordinarily give any trouble. If after considerable use they commence to run dry, clean and oil, or use a good cup grease (do not use fiber grease).

(5) The vertical circle assembly may be easily taken apart after the telescope is lifted from the wyes, by removing nut "M." (See figs. 6, 18, and 19.) Its bearing is frequently neglected, whereas it should be oiled with reasonable frequency. This also applies

to the telescope clamp "P," which can be disassembled in a similar manner. Be careful not to lose the small clamp block when taking apart.

(6) Tangent screws "R" should be oiled frequently, and a drop of oil should occasionally be put on the micrometer screw "S." This screw is removed by turning the micrometer drum backward until it comes off. On replacing, take care that the thread is not crossed. (See figs. 6 and 20.)

These are the principal parts of the theodolite that may require field lubrication. The telescope trunnion bearings should be cleaned frequently but not oiled.

Emergency repairs.—Except for a few minor repairs, it is usually desirable to return an instrument to the Washington Office for repairs.

When extra reticle rings and micrometer slides mounted with wires are kept on hand in the field it becomes a minor repair job to replace either set of wires whenever they become slack or broken. These replacements for a Parkhurst theodolite are furnished mounted and packed in a protective package suitable for keeping on hand until needed.

The reticle ring of a Parkhurst theodolite is removed by unscrewing the eyepiece assembly and balance cover ring, then removing the four capstan-headed adjusting screws which hold the reticle ring in the telescope tube. The new reticle ring is installed by the reverse process. A match or adjusting pin temporarily in one of the screw holes of the reticle ring permits it to be handled without damage to the wires. After installing the new reticle ring, it is necessary to center and adjust it for verticality and collimation of the wires. These adjustments are described in the section beginning on page 56. The micrometer wires can be replaced by removing the top cover from the micrometer box (4 screws), then removing the four small screws which fasten the parallel wire plate to the slide block, and lifting the plate off the slide. Replace with a new plate mounting having new wires, and reassemble. Cross wires can be replaced in the field from a spider's cocoon. However, that method requires considerable skill, time, and patience and is not always satisfactory.

Other minor repairs, such as replacing level bubbles, tangent screws and springs, electrical fittings, and an occasional lost screw or setting knob, can be done easily in the field after requisitioning the necessary parts from the Washington Office. There have also been occasions in remote localities where emergency repairs were largely a matter of ingenuity, with such items as sealing wax and adhesive tape taking a prominent part.

QUALITY OF A THEODOLITE

The quality of an instrument is not measured by its size nor by the minuteness of the least reading of the micrometers. The best measure of the excellence of a theodolite is its performance in actual field work, but if a new theodolite is being used it is necessary to apply other tests.

A preliminary examination will show a great deal about the workmanship and accuracy of the instrument. The four structural features which must be scrutinized to form an estimate of the accuracy of a theodolite are: The graduation of the circle; the design of the micrometers and the workmanship on them; the fit of the centers and tangent screws; and lastly the optical properties of the telescope. Although the accuracy of the graduation of the circle is the most important, each of these features must be entirely satisfactory in order to secure the best results.

Before testing an instrument, place it in as good adjustment as possible by the adjustments described in the section beginning on page 51. Test the centers for fit and play by setting the micrometer microscope on graduations on different parts of the circle and, with the micrometer wires centered on a graduation and the alidade clamped, test the movement of the wires resulting from gentle pressure upon different parts of the graduated circle and upon the alidade. The friction in the centers can be judged by unclamping the alidade and applying a gradually increasing tangential force to the alidade until motion about its vertical axis occurs.

The relation between the greatest magnification obtainable by the telescope and the pitch of the tangent screw should be such that a barely perceptible movement of the tangent screw causes a barely perceptible movement of an object across the telescope wires. A similar relation should exist between the magnification of the micrometer microscope and the pitch of the micrometer screw.

The tangent-screw assembly should be tested for friction by sighting through the telescope and noting if any lag is apparent in the motion of the telescope across an object when the tangent screw is moved slightly away from the spring. The micrometer screws should be tested in the same way. Theoretically the final motion of a tangent screw or a micrometer screw should always be against the spring. As a matter of practice it has been found by extensive tests that if micrometer and tangent screws are properly made and kept clean there are no appreciable errors resulting from making the final pointing by moving the screw indiscriminately against or away from the spring. If it is found, however, that with the screw properly cleaned and oiled there is a lag when the screw is moved away from the spring, then either the spring must be strengthened or the final movement of the screw must always be against the spring.

The design and construction of the micrometers can best be tested by clamping the alidade and taking a series of readings, about 20 in number backward and forward with one pair of wires upon some one graduation. The total range of the readings should not exceed two divisions on the micrometer drum.

With some theodolites a tendency is noticed for certain positions of the circle to give values for all directions either higher or lower than the mean, irrespective of the size of the angle between the initial station and the object sighted upon. Such a condition can be caused only by the graduation at the setting for the initial station being in error in its angular position with reference to the mean of the other graduations of the circle. The relative position of the graduations may be fairly well determined from a set of observations made in the field.

Curve to indicate the quality of a theodolite.—A curve which will indicate the quality of a theodolite circle may be made from field observations in the following manner. An Abstract of Directions should be selected from any station where observing conditions were good and at which at least five lines, well distributed around half the horizon, were observed. Figure 21 is an abstract of actual field observations. The means of all directions are computed to the nearest tenth of a second for this purpose. The upper half of figure 22 lists the residuals, i.e., the amounts necessary to correct each observation to the mean. In column A these are zero, as station C(GLO) was used as the initial. In column G are listed the average residuals for each position, with the sign reversed. For the first position this is +1.7, the mean of all residuals, with opposite sign, including the zero for the initial. The lower half of figure 22 lists the corrected residuals for all directions including the initial. The values are derived by applying algebraically for each position the values of column G to those in the other columns. The figures in

State Arizona

Station BN 33, 1947 *Computed by* L. G. B. *Date* 5-26-47

Observer L. G. B. *Checked by* M. G. *Inst. No.* G-301

POSITION NO.	STATIONS OBSERVED						
	C(GLO)	Rocky	Pole 842	White Tank	White	Litchfield	
	(INITIAL) 0° 00′	25° 23′	60° 18′	106° 29′	122° 16′	160° 15′	° ′
	″	″	″	″	″	″	″
1	0.00	56.2	22.2	03.8	05.2	06.8	
2	0.00	54.4	19.3	01.0	03.5	02.9	
3	0.00	55.6	20.4	04.6	04.8	06.8	
4	0.00	52.6	19.6	$\overline{59.3}$	03.4	03.8	
5	0.00	51.9	21.2	$\overline{59.7}$	03.2	05.5	
6	0.00	52.6	21.4	00.3	02.9	05.2	
7	0.00	50.8	18.2	01.6	03.8	01.2	
8	0.00	54.6	21.8	04.3	06.2	07.4	
9	0.00	52.7	22.1	02.0	07.9	06.5	
10	0.00	49.5	19.1	02.6	06.7	04.4	
11	0.00	53.0	23.8	02.5	01.5	01.5	
12	0.00	49.7	21.1	$\overline{59.0}$	02.1	01.3	
13	0.00	55.3	21.1	$\overline{58.3}$	03.2	04.8	
14	0.00	55.4	23.2	$\overline{59.2}$	03.5	05.1	
15	0.00	53.1	24.9	00.7	02.9	03.2	
16	0.00	53.0	19.7	01.1	04.5	04.6	
Sum,							
Mean,		53.2	21.2	01.2	04.1	04.4	

FIGURE 21.—Abstract of directions selected for circle test.

	A	B	C	D	E	F	G
POSITION NO.	C(GLO)	ROCKY	POLE 842	WHITE TANK	WHITE	LITCHFIELD	$-\frac{\text{SUM}}{6}$
1	0.0	-03.0	-1.0	-2.6	-1.1	-2.4	+1.7
2	0.0	- 1.2	+1.9	+0.2	+0.6	+1.5	-0.5
3	0.0	- 2.4	+0.8	-3.4	-0.7	-2.4	+1.4
4	0.0	+ 0.6	+1.6	+1.9	+0.7	+0.6	-0.9
5	0.0	+ 1.3	0.0	+1.5	+0.9	-1.1	-0.4
6	0.0	+ 0.6	-0.2	+0.9	+1.2	-0.8	-0.3
7	0.0	+ 2.4	+3.0	-0.4	+0.3	+3.2	-1.4
8	0.0	- 1.4	-0.6	-3.1	-2.1	-3.0	+1.7
9	0.0	+ 0.5	-0.9	-0.8	-3.8	-2.1	+1.2
10	0.0	+ 3.7	+2.1	-1.4	-2.6	0.0	-0.3
11	0.0	+ 0.2	-2.6	-1.3	+2.6	+2.9	-0.3
12	0.0	+ 3.5	+0.1	+2.2	+2.0	+3.1	-1.8
13	0.0	- 2.1	+0.1	+2.9	+0.9	-0.4	-0.2
14	0.0	- 2.2	-2.0	+2.0	+0.6	-0.7	+0.4
15	0.0	+ 0.1	-3.7	+0.5	+1.2	+1.2	+0.1
16	0.0	+ 0.2	+1.5	+0.1	-0.4	-0.2	-0.2

1	+1.7(0)	-1.3(25)	+0.7(60)	-0.9(106)	+0.6(122)	-0.7(160)
2	-0.5(11)	-1.7(36)	+1.4(71)	-0.3(117)	+0.1(133)	+1.0(171)
3	+1.4(22)	-1.0(47)	+2.2(82)	-2.0(128)	+0.7(144)	-1.0(2)
4	-0.9(33)	-0.3(58)	+0.7(93)	+1.0(139)	-0.2(155)	-0.3(13)
5	-0.4(45)	+0.9(70)	-0.4(105)	+1.1(151)	+0.5(167)	-1.5(25)
6	-0.3(56)	+0.3(81)	-0.5(116)	+0.6(162)	+0.9(178)	-1.1(36)
7	-1.4(67)	+1.0(92)	+1.6(127)	-1.8(173)	-1.1(9)	+1.8(47)
8	+1.7(78)	+0.3(103)	+1.1(138)	-1.4(4)	-0.4(20)	-1.3(58)
9	+1.2(90)	+1.7(115)	+0.3(150)	+0.4(16)	-2.6(32)	-0.9(70)
10	-0.3(101)	+3.4(126)	+1.8(161)	-1.7(27)	-2.9(43)	-0.3(81)
11	-0.3(112)	-0.1(137)	-2.9(172)	-1.6(38)	+2.3(54)	+2.6(92)
12	-1.8(123)	+1.7(148)	-1.7(3)	+0.4(49)	+0.2(65)	+1.3(103)
13	-0.2(135)	-2.3(160)	-0.1(15)	+2.7(61)	+0.7(77)	-0.6(115)
14	+0.4(146)	-1.8(171)	-1.6(26)	+2.4(72)	+1.0(88)	-0.3(126)
15	+0.1(157)	+0.2(2)	-3.6(37)	+0.6(83)	+1.3(99)	+1.3(137)
16	-0.2(168)	0.0(13)	+1.3(48)	-0.1(94)	-0.6(110)	-0.4(148)

FIGURE 22.—Computation of residuals, theodolite circle test.

parentheses represent the portion of the circle, to the nearest degree, at which each residual was developed. It will be noted that these figures are for the portion of the circle between 0° and 180°. The residuals are the result of the combined portions of the circle 180° apart, as they are derived from readings taken by two opposing micrometers. The numerical work may be checked by adding horizontally and vertically the rows and columns of corrected residuals. These sums should approach zero.

Figure 23 shows the plotting of the residuals at the corresponding portions of the circle. A series is plotted for each column or direction and a curve drawn for each series (actually the points are connected by straight lines). A series of mean values is then computed at intervals of 10° and a mean curve plotted. The example in figure 23 indicates that readings taken at the 30° and 210° portions of the circle should be corrected by $-1''.5$ to bring them into accordance with the average position of all other portions of the circle.

The observed value of the direction to station WHITE, ninth position, is $7''.9$ which is rather high in comparison with the mean value of $4''.1$. The two portions of the circle used in this measurement were near 90° and 212° or 32°. As taken from the mean curve the correction at 90° is $+1''.0$ and 32° is $-1''.5$. The total correction to

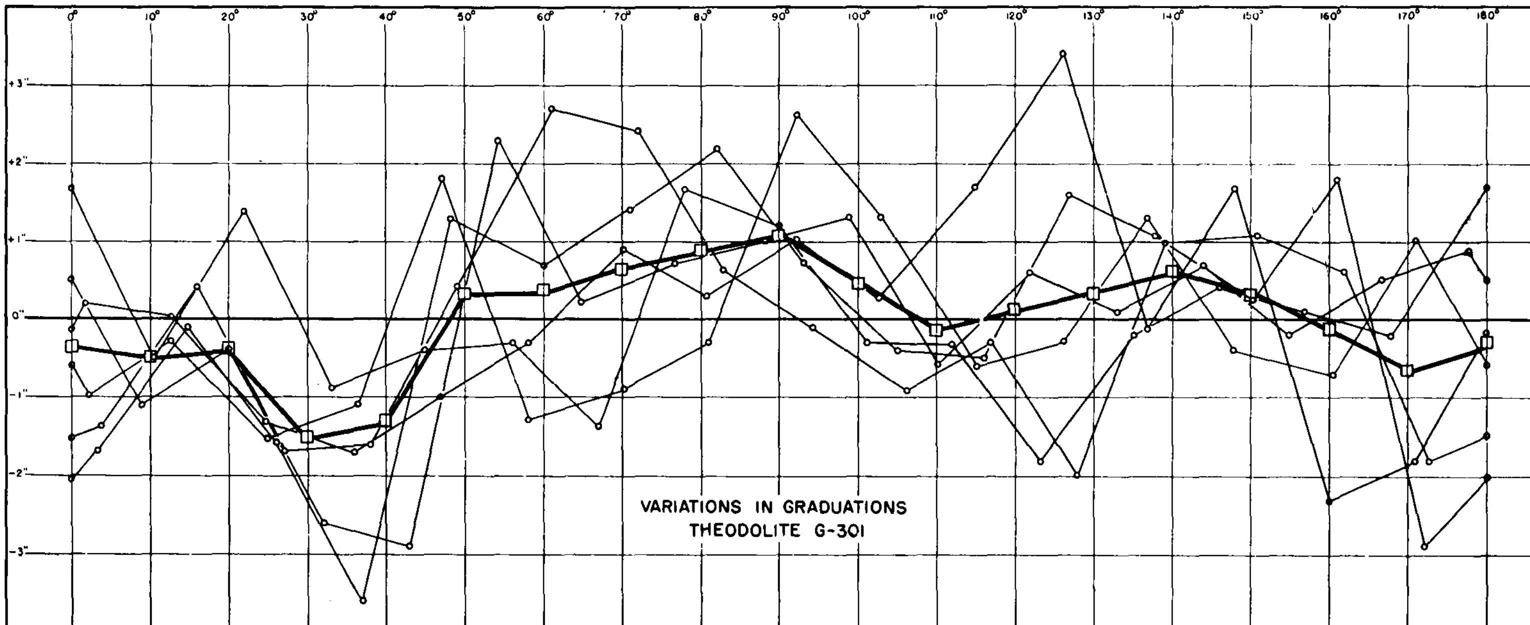

FIGURE 23.—Variations in graduations shown by circle test.

this direction would then be $-(+1''.0)+(-1''.5)$ or $-2''.5$ which applied to the observed value of $7''.9$ will give, as a corrected direction, $5''.4$. Thus it is shown that when the corrections are taken from the mean curve in figure 23 and applied to the observations, the reduced observations will be more closely in agreement. However, the effects of periodic errors in a well-graduated circle are eliminated from the results if the position settings are properly made.

If the micrometers have been carefully adjusted for run, the mean curve for the variations of graduations for a first-order circle, as determined from field observations, should not have a range greater than 3.5 seconds.

This test can be made with better results in a laboratory, employing collimators as sighting targets. Under these conditions the range of the variations should not exceed $2''.4$ for the best circles.

AUXILIARY INSTRUMENTS

Auxiliary instruments are listed on pages 278 to 280. A few of the principal types in use will be described in the following sections.

Vertical collimator.—A vertical collimator is a telescope so mounted that its collimation axis may be made to coincide with the vertical. This instrument serves as an optical plumb line designed to place a mark directly under the center of a signal tower, or more generally to project the center point of the station mark vertically to the top of a high tower for centering of the light plate and the instrument, and also for use in checking and determining eccentricities of the signal light or instrument.

A vertical collimator is illustrated in figure 24. It consists of a broken telescope, mounted with the objective tube vertical in supporting collar bearings. A mirror set at a 45° angle in the tube reflects the vertical line of sight to the horizontal eyepiece section of the telescope. The telescope can be rotated about the vertical axis of the collar bearings for about 300° of arc for purposes of adjustment. The frame carrying the center bearing collars is supported by three leveling foot screws. The instrument may be used on a tripod (fig. 25), or may be set directly on the station monument (fig. 26). An adjustable level bubble is mounted on the vertical part of the telescope at right angles to the collimating axis. The reticle is mounted in the eyepiece section in the focal plane of the objective and eyepiece. There is a hook for a plumb line at the bottom of the vertical telescope axis in the plane of the foot screws.

To adjust the instrument, set it up under a signal tower. (1) Adjust the level bubble in the same manner as the plate level of a theodolite (p. 53) as follows: Bring the bubble axis parallel to two foot screws; center the bubble with foot screws; rotate the telescope 180° about the vertical axis; adjust half of the displacement of the bubble with the foot screws and half with the bubble adjusting screws, rotate 90° and level with the third screw; repeat above adjustment until the bubble remains centered to within less than one division for all directions of rotation about the vertical axis of the instrument. (2) Focus the eyepiece on the cross wires and the objective on the target in the usual manner to obtain clear and distinct images, and at the same time to eliminate parallax. (3) Place temporarily a fixed horizontal target near the top of the tower (such as a board clamped to the top of the tripod, or a piece of adhesive tape stuck to the under side of the light plate). Line-in a pencil point with the cross wires of the collimator and make a dot on this target. Rotate the telescope 180° and make a second dot. Join the two dots with a straight line. Rotate the telescope about 90°, and make a third dot. Rotate the telescope 180° from

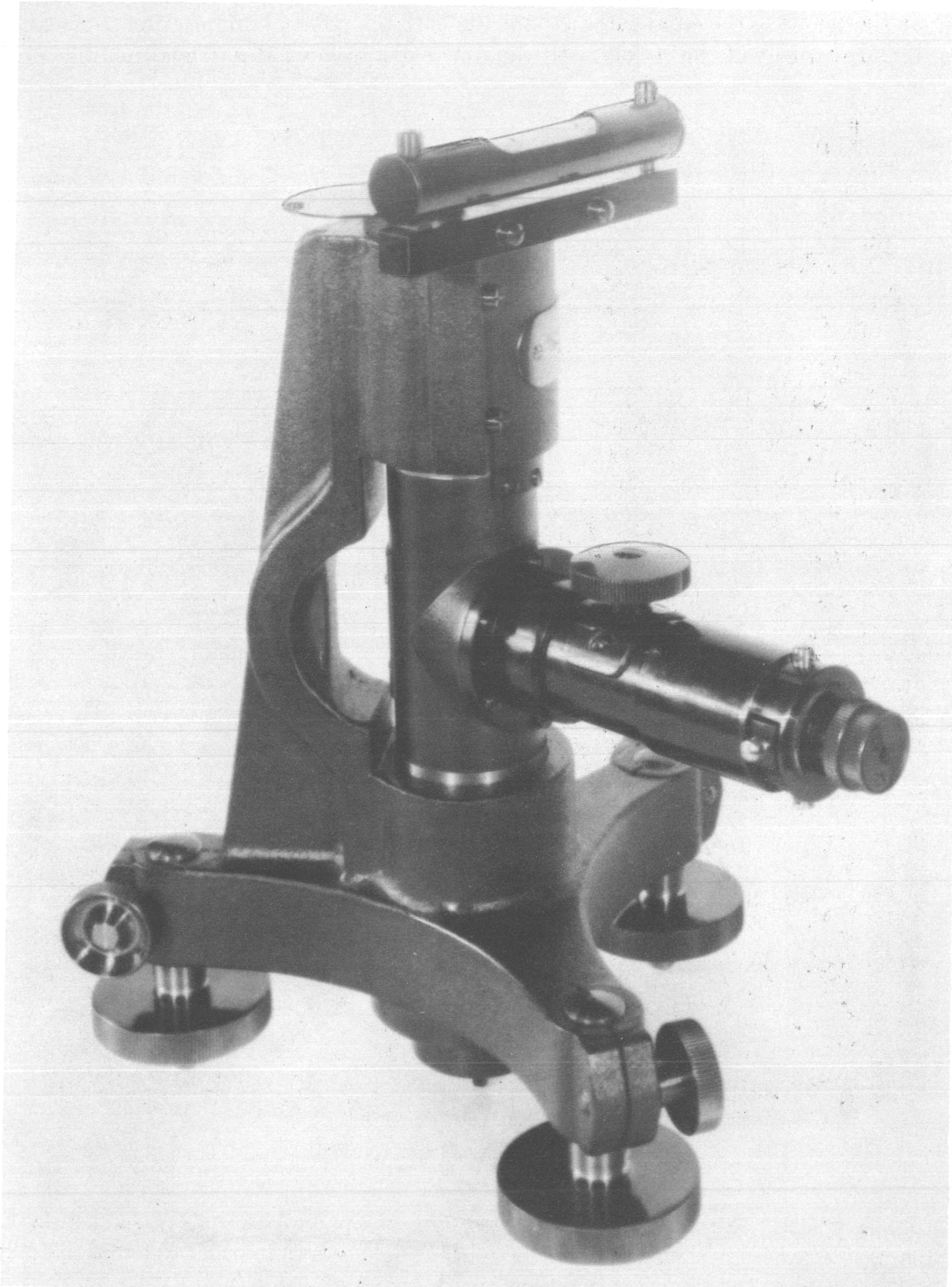

FIGURE 24.—Vertical collimator. This instrument is centered and leveled on the ground over the station mark. The telescope projects a vertical line of sight by which the light plate and theodolite on the tower are centered.

the third position and make a fourth dot. Join the third and fourth dots with a straight line. The intersection of the two lines marks the true center of prolongation of the vertical axis of the instrument. Adjust for the collimation error of the telescope by bringing the center of the cross wires to coincide with the point of intersection of the above lines, by adjustment of the reticle with the four capstan-headed reticle adjusting screws

FIGURE 25.—Vertical collimator on tripod.

near the eyepiece. Adjust these screws by opposite pairs, alternately slackening and tightening. Care is necessary not to overtighten these screws in order to avoid distorting the reticle ring and thus causing the wires to slacken, and also to avoid stripping the fine threads. When the adjustment is properly made, the intersection of the cross hairs will remain on a point during rotation of the telescope. It should be noted that the adjust-

FIGURE 26.—Vertical collimator on mark.

ment of the level and the adjustment of the collimation axis are two separate and independent adjustments. Although both these adjustments must be completed before the instrument can be used properly, it is not essential that the instrument be level while making the adjustment of the cross-wire collimation. It is important not to disturb the position of the vertical axis in any way during the collimation adjustment.

Signal lamp.—Signal lamps are almost invariably used as the targets on first-order triangulation and on some second-order triangulation. Figures 27 through 30 illustrate some typical designs of signal lamps now in use. Signal lamps consist of a frame or box

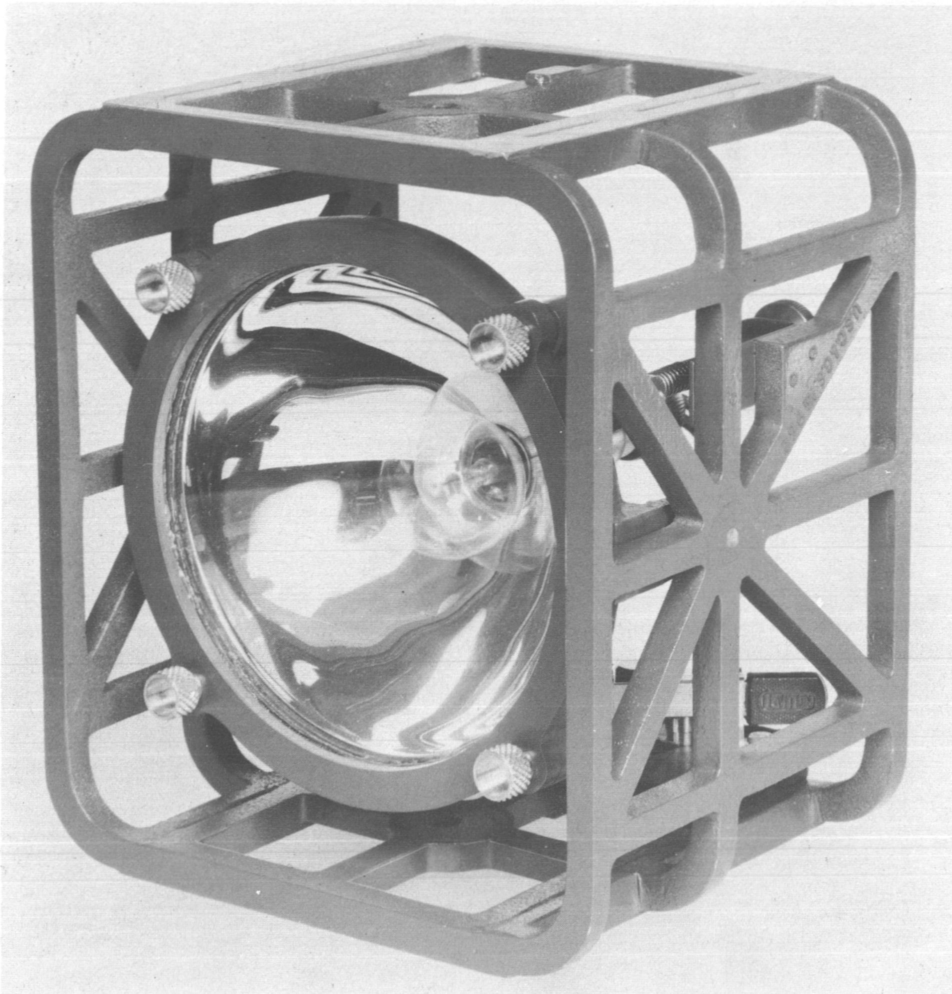

FIGURE 27.—Signal lamp (front).

with the top and bottom faces parallel to the horizontal axis of the lamp and with the vertical alinement screw holes in the plane of the focal point. The frame carries a lamp case pivoted about the horizontal axis. The case has a plate-glass cover lens and contains a parabolic reflector and an electric light bulb. Usually a 3.7-volt 0.6-ampere bulb is used. Higher amperage and also 6-volt bulbs are sometimes used if necessary. Lamps are operated with combinations of 1½-volt dry cell batteries. There is usually an adjustable focusing screw on the back of the case. Vertical tilt of the case in the frame is controlled by an adjusting screw and counter spring. Bulbs and lenses are readily replaceable. Lamps are equipped with switches and binding posts for electrical connections.

Signal lamps are adjusted by pointing on a wall or some other vertical flat surface. Adjust the focus by moving the lamp socket in or out in relation to the reflector until the brightest part of the disk of light on the wall is but little larger than the lens of the lamp.

Focusing should be checked each time a light is used. In addition to the above method, focusing can be tested by walking several hundred feet in front of the light and looking into the lens. Even illumination indicates a good focus. At ground stations setting a stake at the point where the best illumination is seen will give a lone lightkeeper a reference point to check the pointing. At all stations, as soon as it is dark enough to

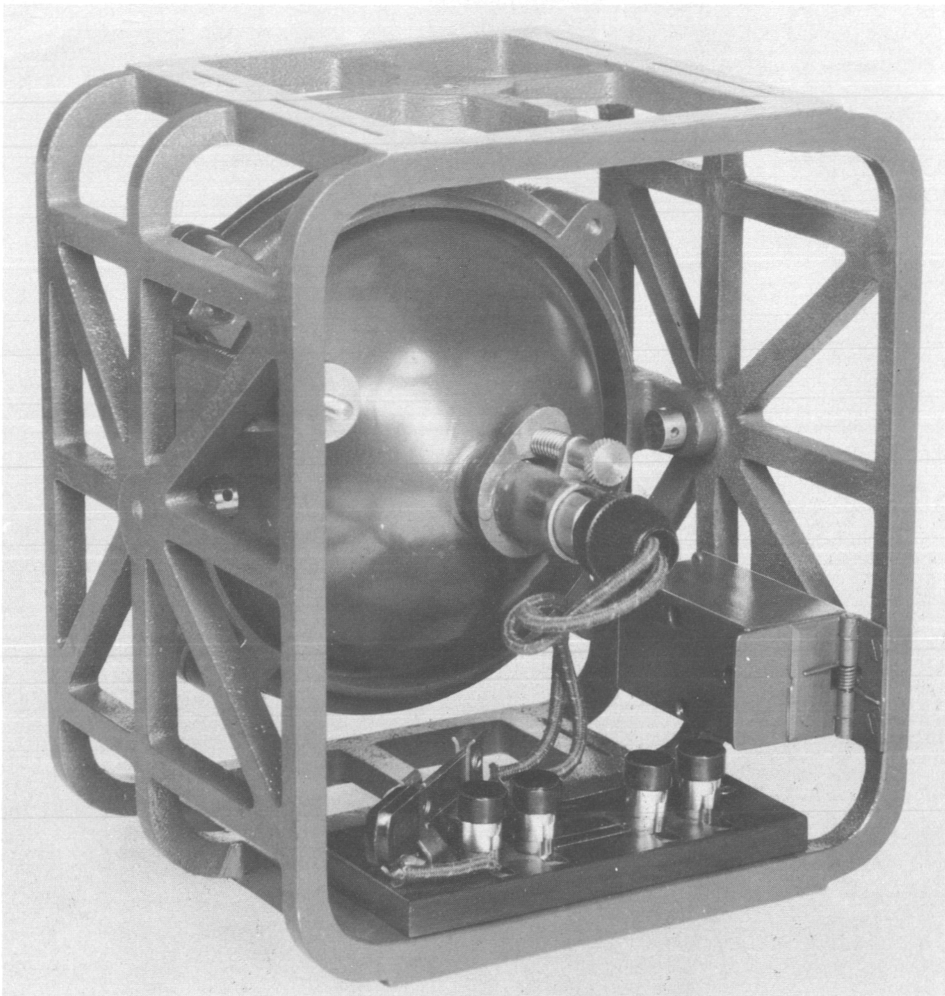

FIGURE 28.—Signal lamp (back).

see the beam and to see the observer's call light, the pointing of all lights should be checked both vertically and horizontally, since pointing on the side of a reflector will cause an error due to an unknown eccentricity. A full discussion of pointing procedures is given in Special Publication No. 65, "Instructions to Lightkeepers on First-Order Triangulation."

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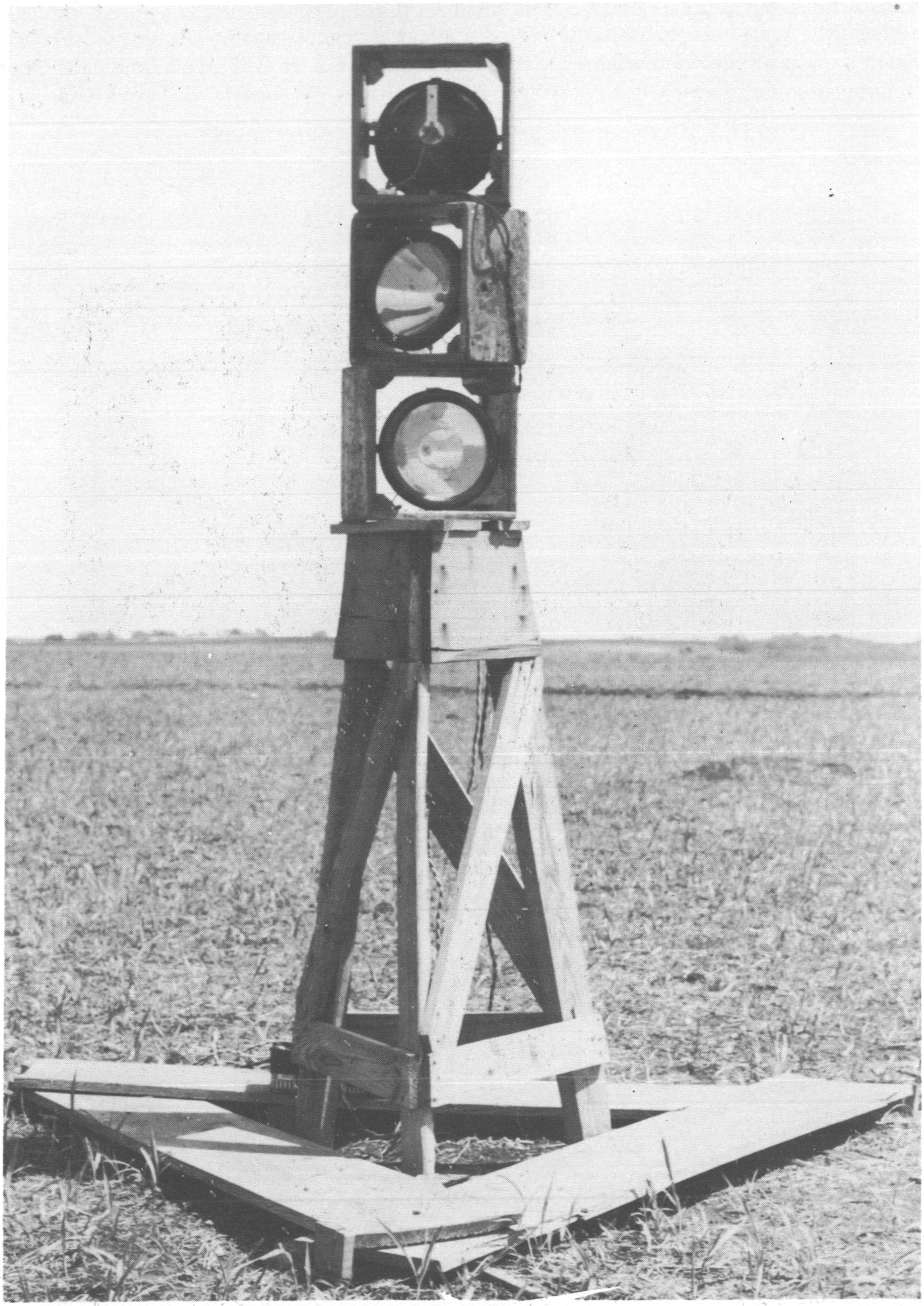

FIGURE 29.—Signal lamps on four-foot stand.

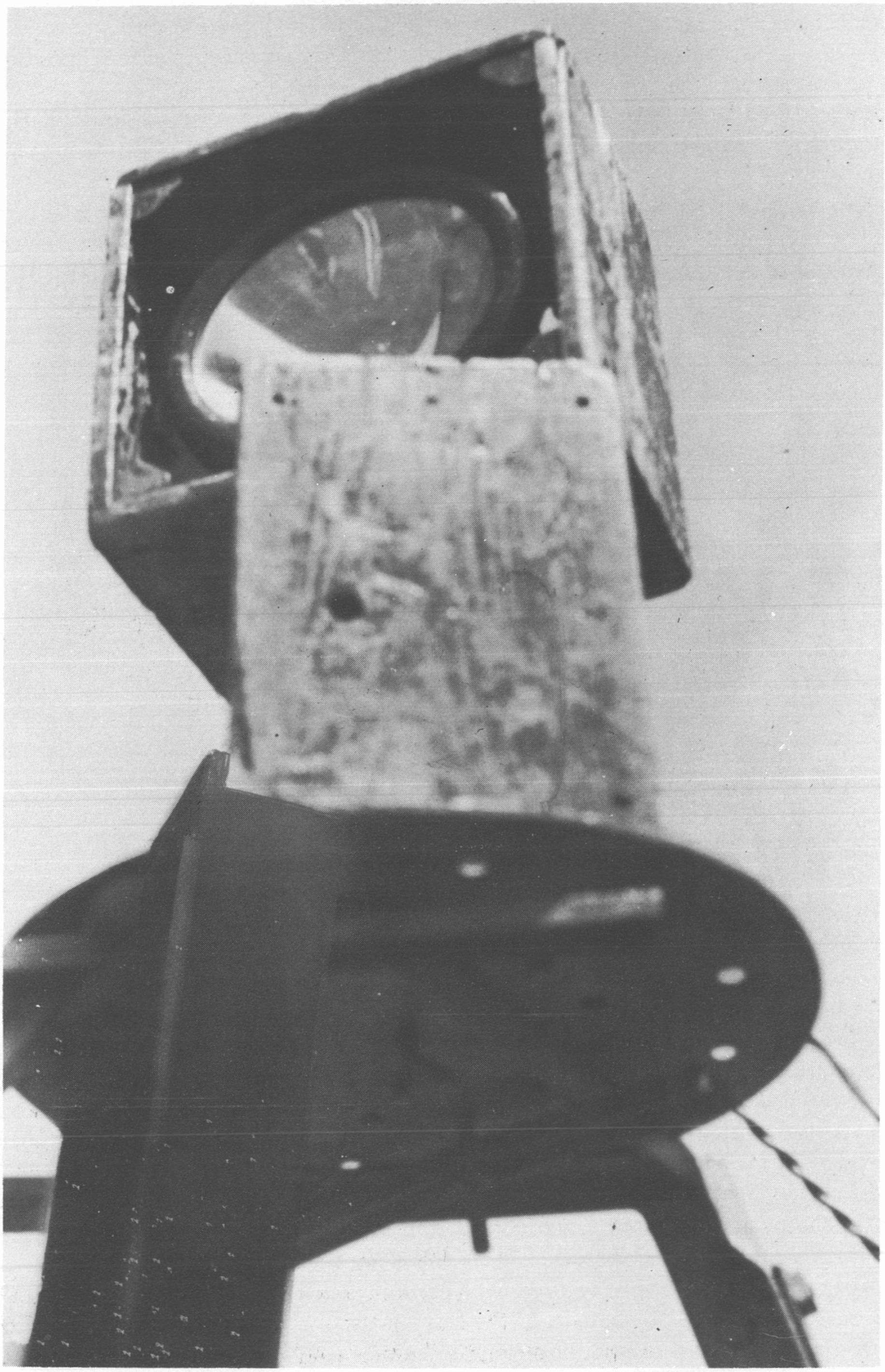

FIGURE 30.—Signal lamps on steel tower.

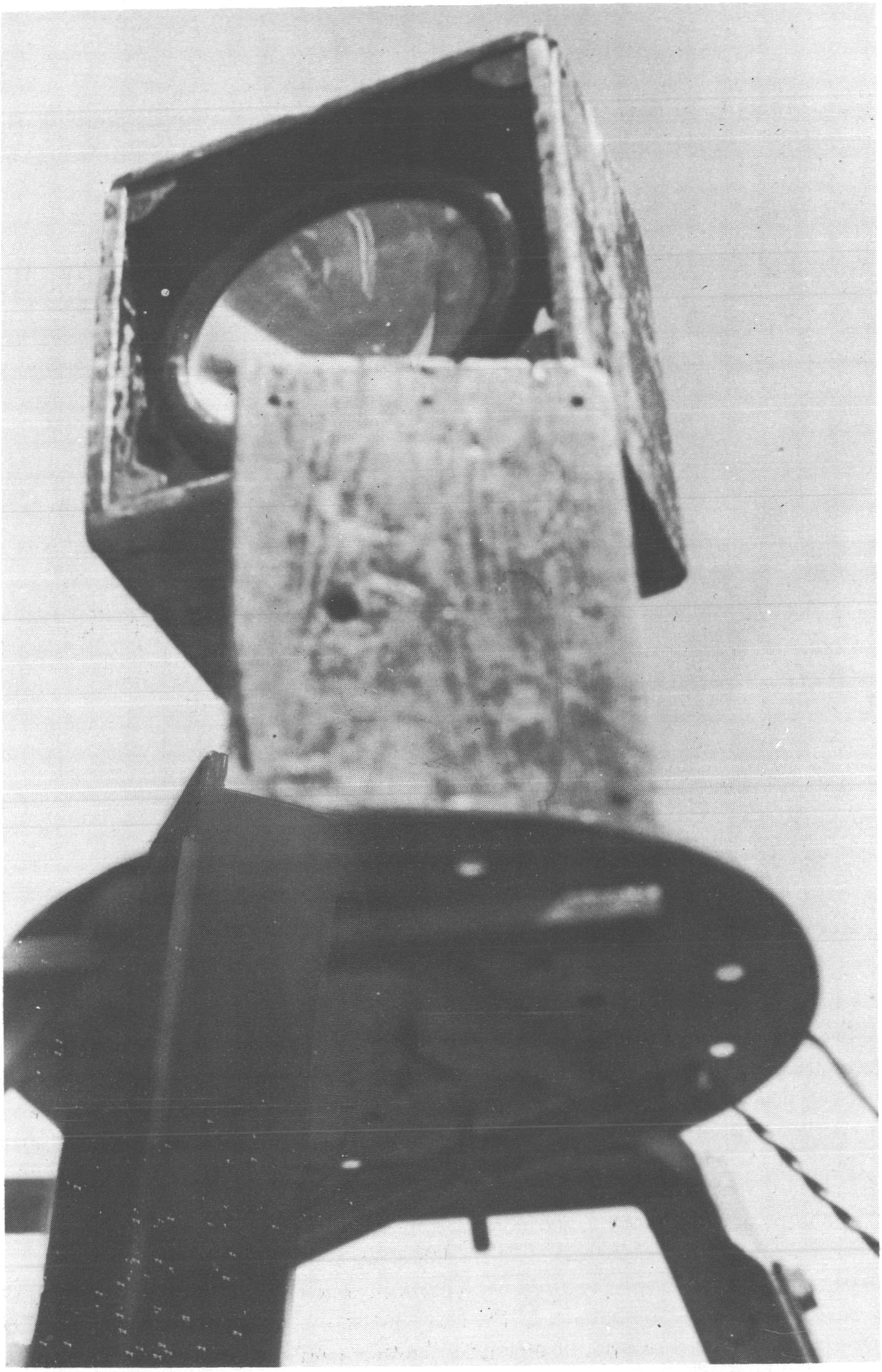

FIGURE 30.—Signal lamps on steel tower.

daytime initial for intersection-station cuts; or there may be adverse night conditions over certain lines which make desirable daylight observations on a heliotrope. Heliotropes are also frequently used on second- and third-order triangulation when the length of line is too great to see the targets clearly.

Tribrach plate.—A tribrach plate is a three-armed cast aluminum plate with radial grooves and is used as a foot plate for the leveling screws of theodolites and other instruments. There is a hole through the center for plumbing the instrument and several holes through the arms for securing with screws or bolts. On wooden stands and signal towers, tribrach plates are used singly and screwed down to the top of the stand. On steel towers, tribrach plates are used in pairs clamped together by bolts and wing nuts, with one above and one below the top horizontal angle piece of the inner tower. (See fig. 32.) Usually three fine wires are stretched diametrically across the center hole of the upper tribrach plate to facilitate centering with the collimator, or a plastic templet with concentric circles may be used to advantage instead of wires as a sighting target for centering the tribrach plate.

Tape.—The principal tape used in measurements at marks and eccentric stations is a steel tape graduated on one side for 30 meters and on the other side for 100 feet, and reeling into a pocket leather case. The dual graduations provide an independent check of all measurements. Standardized base-line and traverse tapes are discussed on page 198.

Compass.—Pocket azimuth compasses are carried by all units of a field party and are used to assist in finding and describing stations and in orienting reconnaissance sketches of the scheme. The needle should always be lifted off the pivot when the compass is carried. Usually this is done automatically when the sight vanes are closed down. Compasses should be tested for sluggishness of the needle and compared with other compasses at least seasonally.

The magnetic variation of the locality should be known to all compass users. This is usually shown on the reconnaissance sketch.

In orienting sketches with a compass from a steel tower it must be borne in mind that the steel will deflect the compass needle. The pocket azimuth compass may be used on the lightkeeper's stand if it is held well above the steel. If a distant point or object can be seen from the ground, the magnetic bearing to it may be determined from a point at least thirty feet from the tower. This bearing can then be plotted on the sketch and used for orientation from the tower.

ADJUSTMENTS OF THEODOLITES

A good observer will keep a theodolite in good adjustment, even though the program of observing tends to eliminate many of the errors due to lack of perfect adjustment. The mechanical devices provided for making the adjustments vary somewhat on different instruments, but usually an inspection will disclose the method of operation. Most of the adjustment descriptions which follow apply especially to the Parkhurst first-order theodolite. If another type of theodolite is used, and the mechanical means for adjusting it are not readily apparent, proceed carefully, for a strained and weakened joint or a stripped screw thread may necessitate the return of the instrument to the shop.

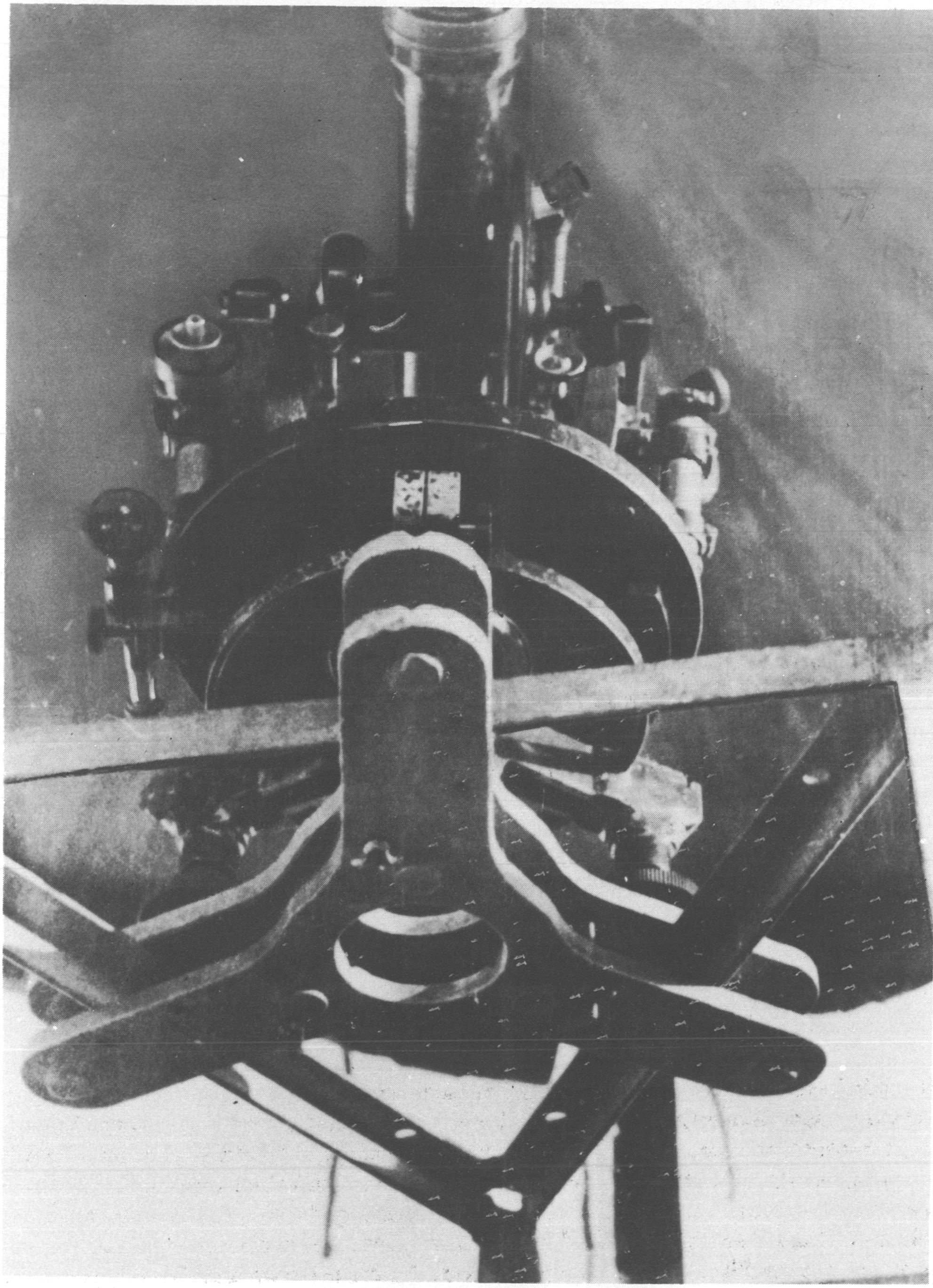

FIGURE 32.—Tribrach plates in use on steel tower.

In order to avoid any ambiguity or misunderstanding in the following discussion of theodolite adjustments, the principal terms used are defined:

The *alidade* is the upper motion part of a theodolite. It includes the standards, telescope, micrometer microscopes, and levels.

The *bubble axis* is the horizontal line tangent to the upper surface of the centered bubble, which lies in the vertical plane through the longitudinal axis of the bubble tube.

The *horizontal axis* is the center line through the center of the pivots of the telescope. It is the axis about which the telescope is tilted or plunged.

The *vertical axis* is the vertical line through the center of the spindle. It is the central vertical axis of rotation of the alidade and circle, and is perpendicular to the horizontal circle.

ADJUSTMENTS INVOLVING LEVEL VIALS

Plate-level adjustment.—The purpose of this adjustment is to make vertical the line passing through the center of the spindle which is designed to be the vertical axis of rotation, and incidently to make the plane of the graduated circle horizontal. Assuming that the circle is mounted perpendicular to the vertical axis, leveling of the theodolite with properly adjusted plate levels will achieve both results. Mounting of the circle perpendicular to the vertical axis is a shop function. However, any abnormal amount of either warping or tilting from the perpendicular position would be discernible in the variation of the run of a micrometer for different sectors of the circle and in variations in focus around the circle.

To test and adjust the plate level, the alidade is rotated until the level is parallel to a line joining two of the leveling foot screws, then the bubble is approximately centered with these two foot screws. The alidade is then rotated about 90° and the bubble is again centered with the third foot screw. The bubble is rechecked over the first two foot screws, then the alidade is rotated 180°. The number of divisions by which the bubble fails to return to center is the bubble error. Half the bubble error is corrected with the foot screws, and half with the adjusting screws at the end of the bubble tube. The leveling is again checked at 90° rotation, and the 180° rotation and adjustments are repeated until the bubble remains centered to within less than one division for any and all pointings of the alidade. The observer makes these tests (and adjustments when necessary) each time the theodolite is set up.

Lack of verticality of the vertical axis introduces an error in the measured angle which cannot be eliminated by the method of observing, and it is therefore advisable to test this adjustment and relevel the instrument at comparatively frequent intervals. Since on an inclined circle one diametrical line can be drawn which is horizontal, it follows that the directions observed will be unequally affected. The error on any pointing is given by the formula

$$\text{error} = i \tan h$$

where i is the angle of inclination in seconds as given by the bubble at right angles to the line of sight and h is the angle above or below the horizon of the object sighted upon. The error of any angle is obtained by combining the mean errors of the two directions involved. The magnitude of the errors due to an inclined circle is indicated in the following table:

Errors in horizontal directions due to inclination of vertical axis of instrument

Inclination of the vertical axis (i)	Angle of elevation or depression of the line of collimation (h)	Correction to horizontal direction (i tan h)
″	′	″
10	20	0.06
20	20	0.12
30	20	0.17
10	40	0.12
20	40	0.23
30	40	0.35
10	60	0.17
20	60	0.35
30	60	0.52

Adjustments of the striding level and standard.—The adjustment of the striding level and the standard are interrelated to such an extent that it is best to perform these adjustments at the same time.

The purpose of the striding-level adjustment is to make the bubble axis parallel to the horizontal axis of the telescope. Strictly speaking, the bubble axis is brought into parallelism with the line which approximates more closely the true horizontal axis the more nearly the telescope pivots assume the form of true cylinders having equal diameters and with their major axes in alinement.

The purpose of the standard adjustment is to make the horizontal axis of the telescope perpendicular to the vertical axis of rotation of the alidade.

First, the striding level should be tested and adjusted for wind. With the striding level in its normal position on the pivots and with the bubble centered, rock the level slowly forward and backward on its supports. If the bubble does not remain centered, then the bubble axis and the horizontal axis of the telescope lie at an angle to each other when projected upon a horizontal plane and the level is said to have wind. Make lateral adjustment with the side adjusting screws at one end of the level tube until the bubble remains centered when rocked forward and backward.

Next level the alidade by means of the plate bubble. In most of the present-day instruments, however, a greater refinement in making sure that the vertical axis is truly vertical is required. Place the striding level in position and rotate the alidade until the bubble axis is parallel to the line of two foot screws. Make sure that the bubble is free, that is, not against one end of the vial and note its position by reading the lower end of the bubble (the end where the figures are lower). It is not necessary to read both ends as long as the length of the bubble remains constant. Rotate the alidade 180° and read the lower end of the bubble again. The mean of these two readings is the "reversing position" of the bubble. Adjust the foot screws until the bubble assumes this reversing position within a half division in all directions of the alidade. The vertical axis of the theodolite is now truly vertical, but neither the striding level nor the standards are necessarily in adjustment.

Reverse the striding level (for the first time) end for end on the pivots and read the bubble again at the lower end. Adjust half the discrepancy by the device provided for lowering or raising one of the standards. This should be checked and the adjustment touched up until the bubble is not displaced more than one division of the striding level after reversing it end for end.

The standards are now so adjusted that the horizontal axis of the telescope is truly horizontal.

It should be noted that up to this time the striding-level bubble has not been adjusted except, perhaps, for wind. All that remains to be done is to center the bubble with its adjusting screws. A note of caution should be added here. During these operations it will be necessary to check from time to time the level of the theodolite, as the progression of the adjustments assumes first that the vertical axis has been made and kept truly vertical.

Inequality of pivots.—If the pivots are unequal in diameter the defect can be detected and the amount of the inequality determined by placing a carefully adjusted striding level on the pivots, bringing the bubble to center, reversing the telescope in the wyes so that each pivot lies in a different wye than at first, and then reading the level. No adjustment is practical in the field. The inequality should be noted when the instrument is sent to the Washington Office for shop repairs. Tests for inequality should be made seasonally, or whenever the standards are adjusted, as allowance must be made for the amount of inequality in adjustment of the standards. It will be a very rare occasion when any appreciable inequality is found in the field. With the method of observing in use in the Coast and Geodetic Survey whereby there is no change of the pivots in the wyes during even a season's work, no error in the angle measures will result from the pivots being unequal in diameter.

Irregularity of pivots.—Ordinarily if the pivots are not truly cylindrical, a striding level placed in position on the pivots will change its reading as the telescope is slowly rotated on its horizontal axis. This irregularity of pivots is not adjustable in the field. It requires regrinding in the instrument shop of the Washington Office.

If the irregularity is of any considerable size, it will cause errors in the measured horizontal angles. Tests should be made at least annually. It will be a very rare occasion when any appreciable irregularity of pivots is found in the field.

Adjustment of the vertical-circle level.—The purpose of this adjustment is to reduce the index error of the vertical-angle observations. The adjustment is not essential to the accuracy of the results, but facilitates the computation of zenith distances and the detection of blunders.

To make the adjustment, first observe the vertical angle or zenith distance of some well-defined object by pointings with the circle left and the circle right (see p. 104). The vernier bubble should be centered before each reading. Compute the zenith distance for these observations and repoint on the object with the circle left. Set the verniers by means of the vernier tangent screw so that their mean reading is the computed zenith distance, then center the bubble with its adjusting screw.

ADJUSTMENTS INVOLVING OPTICAL EQUIPMENT

Focusing adjustment.—The large adjusting screw on top of the telescope is used to regulate the focus for the maximum clearness of image. It is not desirable to change the focus except between positions or half positions (just before new circle setting or at reversals of the telescope). However, if the reference marks, eccentric stations, or other nearby points are taken in the same set with distant objects, pointings should be made first on the initial and all distant objects using sidereal focus, then pointings should be made on nearby objects requiring change of focus. After the telescope is plunged, pointings should be made on the nearby objects, then using the sidereal focus, pointings should be made on the distant objects, and finally on the initial.

Adjustment for parallax.—The object of this adjustment is to keep the image of the object in the plane of the cross wires.

Point the telescope toward a light surface, such as the sky. Slide the eyepiece of the telescope in or out until the wires show the sharpest and blackest. Next focus the objective of the telescope on a distant object, and then test the adjustment by moving the eye laterally across the front of the eyepiece. If the wires appear to move with respect to the object sighted upon, parallax is present. The focus of the object glass of the telescope should be changed until the objective is at the proper distance to cause the image of the object sighted upon to fall exactly within the plane of the cross wires. In this position no movement of the wires over the field of view will be apparent when the eye is moved across the eyepiece. The adjustment for parallax must be closely watched, for the error due to lack of proper adjustment is not eliminated by the method of observing. It is especially noticeable if the eye of the observer, because of a strained position, is not in front of the exact center of the eyepiece.

The adjustment of the eyepiece must also be tested frequently, for as the eye of the observer tires, the focal distance of the lens of the eye changes, causing a blurring of the wires and an increased effort in the centering of the image of the light between the sighting wires.

A theodolite is sometimes supplied with two or more eyepieces of varying magnifications. On first-order work it is usually better to use the highest-power eyepiece available, even though it increases the apparent unsteadiness of the light. The higher magnification permits the observer to detect better any asymmetry of the light and to make an allowance therefor in the pointing. The lower-power eyepieces should be used in pointing upon objects to be located by intersections, which in a hazy atmosphere would not be clearly visible if a high-power eyepiece were used.

Adjustment of verticality of sighting wires.—The object of this adjustment is to make the vertical wire of the telescope vertical. Point on a well-defined object with the horizontal axis of the telescope level, swing the telescope slowly in elevation, watching to see if the object changes its position with relation to the vertical wire. If a change of relative position is observed, two adjacent capstan screws holding the reticle ring are temporarily loosened sufficiently to rotate the reticle ring to a position in which the image of the object remains in the same relative position along the wire as the telescope is tilted.

Collimation adjustment.—The purpose of this adjustment is to make the line of collimation (line of sight) of the telescope perpendicular to the horizontal axis of rotation of the telescope.

There are several methods of making this adjustment. An approximate method frequently used in the field which gives results with a first-order theodolite which are within satisfactory limits is as follows: On a stable set-up with the instrument properly leveled, point on a distant signal lamp or well-defined object, read both micrometers, plunge the telescope, rotate the alidade 180°, point on the same object, and read micrometers. The difference between the mean micrometer readings with the telescope direct and reversed is twice the collimation error. Advance or turn back, according to the direction of the error, one of the micrometers by an amount equal to the collimation error. The alidade may then be moved this amount with the tangent screw by noting when the circle division is again straddled by the pair of micrometer cross wires. Then bring the center of the cross wires to the object with the horizontally opposed capstan-headed reticle adjusting screws. Repeat the test to check the adjustment. If a previous set of observations with the same instrument is available, the mean of the differences of D's and R's for each position can be used in determining the mean collimation error, then that portion of the circle can be used for test and adjustment where the D and R differences are closest to the mean. This refinement is seldom necessary since collimation need not be reduced below 5 seconds for any practical purpose when the observing program includes both direct and reversed pointings on each object.

Another variation of the above method is to set the micrometers after plunging the telescope to read exactly 180° from the first reading. Then if the object is not bisected, correct half of the discrepancy by the reticle screws and half by the tangent screw.

A third method involves reversal of the telescope in the wyes. The telescope is pointed on some sharply defined object, and with the alidade clamped, the telescope is lifted and reversed in the wyes so that each pivot lies in a different wye than at first. If the object is not centered between the wires after reversal, correct half of the discrepancy with the reticle adjusting screws and half with the tangent screw. Repeat the test to check the adjustment.

The program of observing by taking the mean of direct and reversed readings eliminates the error of collimation from the results. However, it is desirable to check the collimation frequently and to adjust as necessary in order to facilitate taking of means and discovery of blunders. Erratic collimation errors are usually an indication that cross wires are slack, or abnormally affected, as by temperature changes, or that something about the instrument or its support is loose or unstable.

In adjusting the collimation special care must be taken not to exert excessive pressure on the capstan adjusting screws with the adjusting pins. These screws are to be secured only with sufficient firmness to prevent the reticle ring from being jarred loose, but not tightened hard enough to distort the reticle ring and slacken the wires, or to injure the screw threads.

ADJUSTMENT OF MICROMETER MICROSCOPES

The micrometer microscope is a most satisfactory device for measuring accurately the angular value of any part of the interval between adjacent marks of a graduated circle. It consists essentially of a compound microscope with a micrometer box mounted between the objective and eyepiece, at such a distance from each that the movable wires in the micrometer box can be brought into the focal plane of each lens system. The principle of operation will be more readily understood after the mechanical details of the micrometer box have been described.

The mechanical arrangements of the box vary somewhat on different models of theodolites. The typical arrangement illustrated and described is that now in use on

Parkhurst first-order theodolites (fig. 33). An outer case, *c*, into which are screwed from opposite sides the objective tube and the eyepiece, contains a frame, *d*, carrying the comb, *e*, the center notch of which, taken in conjunction with the zero of the micrometer head, *f*, furnishes a fiducial point for all readings. The frame for the comb is adjustable transversely by screw, *g*, movement in other directions being prevented by the machined surfaces of the slot in which the frame moves. A separate movable slide, *i*, which carries two pairs of parallel cross wires, *j*, has the right end of the slide threaded through to fit

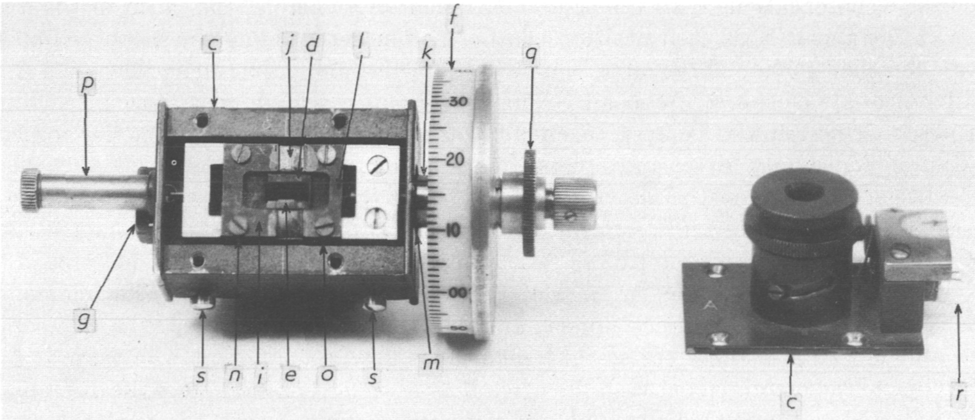

FIGURE 33.—Parkhurst first-order theodolite micrometer.

the finely machined screw, *k*, on which the micrometer head, *f*, and the attached spindle works. The point of the screw, *k*, bears against a hard plate, *l*, rigidly attached to the box. The length of the micrometer shaft (or screw), *k*, is sufficient so that the only point of bearing is at *l*. The bearing around the circumference of the shaft at *m* is merely a guide bearing and takes no thrust. The slide, *i*, moves on cylindrical bearings, *n*, located in tracks, *o*. A single tension spring, *p*, maintains pressure of the point of screw, *k*, against thrust bearing plate, *l*. The nut, *q*, is a friction clamp which permits setting of micrometer head or drum, *f*, on screw, *k*, to readings, as desired. A small lamp bulb, *r*, illuminates the drum, *f*. Screws, *s*, adjust the cylindrical bearing track.

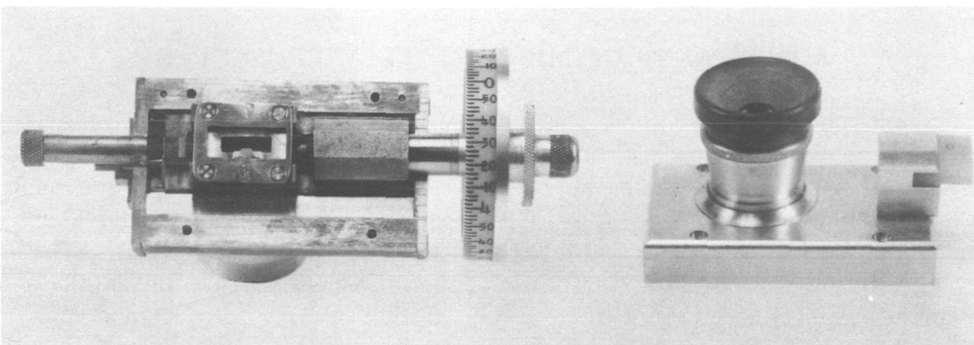

FIGURE 34.—Parkhurst second-order theodolite micrometer (Model of 1947).

A redesigned model of micrometer microscope, called the model of 1947, has been made in the Coast and Geodetic Survey instrument shop and is now being installed on later model theodolites. This is the model shown in figure 34. The principal design changes are: (1) The slide which carries the parallel wires has been lengthened and a sleeve bearing added to the slide to reduce wear on the thread. The slide, including the threaded portion and the sleeve bearing, is made from an oil-retaining sponge bronze for better lubrication; (2) Fixed closed illumination of the circle has been added to eliminate phase; (3) An improvement in design has been made to facilitate adjustment for focus and run. Adjusting screws now allow positive and accurate adjustments for focus and run instead of the old method of trial and error.

The method of operation is as follows: The objective forms a magnified image of a small portion of the graduated circle in the plane of the parallel wires. This image and the parallel wires are in turn magnified by the eyepiece. The angular value of the portion of the circle between the zero point of the comb and the graduation mark to the left can then be measured in terms of whole turns (minutes or multiples) and fractional turns of the micrometer screw (seconds). (See fig. 35.) The magnifying power of the objective and the pitch of the micrometer screw are so related that the adjustment can make one full turn of the micrometer screw move the parallel wires 2 minutes in arc (for Parkhurst first-order theodolite). The micrometer drum is graduated to single seconds. The comb is notched to minutes of arc.

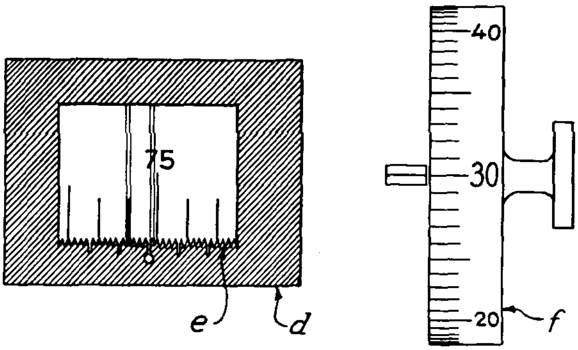

FIGURE 35.—Diagram of drum and field of view of micrometer.

The following conditions are essential for correct adjustment of a single micrometer microscope:

1. The micrometer wires and image of the graduated circle should be so closely in the same plane that no parallactic movement can be detected by shifting laterally the position of the eye.

2. To be in perfect adjustment, two and one-half revolutions of the screw (300 seconds on the drum) would traverse exactly a 5-minute space on the circle (adjacent graduations of circle 5 minutes apart).

3. The micrometer should read zero seconds with either pair of parallel wires coinciding with the zero point of the comb. (Micrometers must also be spaced at equal intervals around the circle.)

Eyepiece adjustment.—Adjust the eyepiece by sliding or screwing it in or out until the parallel wires are in the most distinct focus. When this is accomplished the comb will be sufficiently visible.

Adjustment for focus on graduations.—Loosen the screws which hold the micrometer microscope in the bracket clamps and move it up or down until the fine scratches on the circle due to polishing are as distinct as possible, then tighten screws. On 1947 models, there are two opposing screws on the side of the case for raising and lowering the micrometer microscope.

Radial adjustment of microscope.—Radial adjustment of Parkhurst theodolite microscopes is ordinarily unnecessary and cannot be made in the field. On some makes of theodolites, the micrometer microscope can be adjusted radially by moving the objective end toward or away from the center of the circle so as to bring the outer edge of the graduations near the center of the field of view, yet still leave the degree numbers visible.

Adjustment for parallelism.—If the parallel wires of the diaphragm are not parallel to the graduation marks on the circle, either turn the micrometer box slightly on the tube or loosen the binding screws and turn the tube slightly in its supports.

The accuracy of micrometer readings depends largely upon the proper mounting of the pairs of wires used for reading. Each set should be at such distance apart that a narrow strip of the bright surface of the arc is visible on either side of the graduation when the wires are accurately centered astride it. The two wires should be parallel to each other as closely as the eye can judge, and should also be adjusted to be parallel to the graduations. If the wires are not parallel or are slack, they should be replaced as described on page 37. They should also be heavy, black, and smooth.

Adjustment of comb.—If the zero notch of comb, usually marked by a deeper cut than the other notches or by a hole beneath it, is not in the center of the field of view, adjust it by the screw, *g*. (Also see p. 58.)

Adjustment of graduated drum.—Center a pair of wires on the zero notch of the comb, then, holding the spindle knob of the micrometer firmly (and loosening the friction clamp if necessary), turn the graduated drum on its friction mounting until the zero of the drum coincides with the index line mounted on the micrometer case adjacent to the drum. Check to see that the parallel wires have not moved off the zero of the comb.

Adjustment for run.—By "run" of the micrometer is meant the difference in seconds of arc between the circle graduation value (such as 5 minutes or 300 seconds) and its value as determined by measuring with the micrometer the space between two adjacent graduation marks of the circle.

The purpose of the adjustment for run is to bring about, as closely as practicable, the condition that the actual reading of the micrometer will equal the scale value between adjacent graduations of the circle. The theory of the adjustment may be seen from the fact that the portion of the graduated circle and its magnified image are at conjugate foci of the objective, and that the magnification of the image is represented by f'/f where f and f' are the distances of the circle and its image, respectively, from the center of the objective. The magnification of the image may be increased by decreasing the distance of the objective from the circle and decreased by moving it in the opposite direction. Therefore, if the micrometer reads lower than the angular distance between graduations, the image needs to have its magnification increased, and the objective should be moved nearer the circle by moving it along the axis of the tubular body of the microscope.

On the 1947 model microscopes, the lens holder has been threaded so that if it is turned the lens will be advanced or withdrawn, as desired. Turning of the lens holder is accomplished by having the teeth of a worm gear cut upon it, and a small brass worm, which has a capstan head for turning with an adjusting pin, is mounted on the microscope tube and engages the gear teeth cut in the lens holder. Turning the worm gear in a clockwise direction moves the lens away from the circle.

When the objective is protruded from the tube to correct for run, the image is thrown farther up in the tube. Without changing the relation of the eyepiece to the micrometer wires, move the whole micrometer microscope upward to bring the image of the circle into the plane of the micrometer wires. This process may have to be repeated several times to get the run down to the required limit.

If the micrometer reads higher than the angular distance between adjacent graduations, the adjustment should be made in the opposite direction to that described above.

If the required correction is very slight, the adjustment may be made by raising or lowering the entire micrometer microscope instead of moving the objective in the tube, since the change in distance affects the run at a much more rapid rate than it does the parallax or distinctness of vision.

The mean run of a single micrometer should not exceed 2 seconds and the algebraic sum of the runs of all micrometers should not exceed 1 second.

The preliminary tests for run should be made on five or six equidistant parts of the circle. Since the error of run may vary slightly due to eccentricity of the circle, the adjustment should be made on a portion of the circle where the error is near the mean. The final adjustment should be tested by taking the mean of at least 10 readings of the micrometer value of the space between graduations.

The error of observations due to run of micrometers may be very considerable unless the run adjustment is kept small and the setting on the circle for each position is kept close to the proper reading in order to give a symmetrical distribution of each group of readings throughout a single graduation space. When the run is kept adjusted to minimum value and when initial settings are approximately evenly distributed, correction for run is disregarded in making computations. (See pp. 277 to 278 for formula for correction for run.)

For convenience and speed in making the readings, Parkhurst first-order theodolites have two pairs of wires mounted approximately 4 minutes apart. The reading on the graduation next preceding the zero of the comb, called the forward reading, is made with the (apparent) left-hand pair of wires, and the reading on the mark next following the zero of the comb, called the backward reading, with the right-hand pair.

Tests and adjustments for run are made with either one of the two pairs of wires. It should be noted that the spread between backward and forward readings using two pairs of wires may be due to the wires not being set exactly 4 minutes apart. Run should first be checked and adjusted with one pair of wires only; then the average spread between backward and forward readings using two pairs of wires should be determined. If run is properly adjusted with one pair of wires, the spread between the readings of two pairs of wires, if consistent, will not affect the accuracy of observations. No correction is applied to readings when making observations using a micrometer in which the two pairs of wires are not exactly 4 minutes apart, the value of the average spread being taken into consideration in rejecting readings.

Adjustment for equidistance of microscopes.—The microscopes of Parkhurst theodolites cannot be moved circumferentially in the field. Adjustment for equidistance is made by setting the zero notches of the combs on the A and B micrometers close to the center of the field of view of the microscope and centered on circle graduations which are 180° apart. (See p. 60 for adjustment of comb and drum.) In some makes of theodolites, the entire B microscope may also be adjusted circumferentially.

Because of the slight eccentricities of the circle, the micrometers may not maintain a constant difference when read on different portions of the circle. A sufficient number of readings should be taken around the circle to determine the approximate amount of the eccentricity and the adjustment for equidistance should be made at a point where the eccentricity can be closely estimated. Alternatively and preferably, the following short method may be used: With the zero notch of the comb of the A micrometer set close to any selected graduation mark, read and record the backward readings only for the two micrometers. Then rotate the alidade until the comb of the A micrometer is close to the diametrically opposite graduation and read and record the backward readings of the two micrometers. Now if the first A reading minus the first B reading equals the second B reading minus the second A reading, then the micrometers are 180° apart around their axis of rotation regardless of the eccentricity of the circle. If not equal, slip the drum of the B micrometer (without moving parallel wires) to the correct reading which is the last B-micrometer reading plus the mean of the A reading minus the mean of the B readings. Then the zero notch of the comb should be adjusted into accordance with the drum reading.

Miscellaneous micrometer adjustments and tests.—When imperfections are found in the operation of a micrometer, it is usually necessary to return it to the instrument shop for repairs. It is a waste of time and money to attempt to continue to use any instrument which cannot be readily made to operate satisfactorily in the field by normal adjustments.

(a) Binding.—Binding in the micrometer slide is indicated if the parallel wires appear to lag or jump as the micrometer drum is slowly rotated forward and backward. If binding is present, remove the micrometer box cover and clean the slide bearings and guides without dismantling. If this does not remove binding, slack off slightly on the small screws, *s*, on the bottom edge of the box (see fig. 33). Never tighten these screws in the field as the hard bearings will roughen the slide guides. Note: These screws do not take side play out of the micrometer screw. The 1947 micrometers have cylindrical rods in place of ball bearings and a sleeve over the main screw which reduces side play.

(b) Dents in ball race.—Dents in the ball race caused by previous binding of the guide on the balls can be detected by feel while pulling the spindle in and out against the tension spring. If dents are present, shop repairs are necessary.

(c) Bearing imperfections.—Imperfections in the screw bearings of the micrometer screw may be detected by making readings and drawing a graph whose ordinates are the differences of backward and forward micrometer readings, and whose abscissas are the small increments of movement of the comb index (such as, 10-second increments of drum), extending over a total range of about 6 minutes. If the plotted results show the periodicity of a sine curve, bearing defect is probably present and of consequence in proportion to the amplitude of curve. If bearing imperfections are present, shop repairs are necessary. This test is seldom necessary with more modern micrometers in which the only thrust is at the point of the micrometer screw.

The above method is for a Parkhurst first-order theodolite in which the micrometer drum is graduated for 2 minutes per revolution. The same system would apply to any micrometer which makes ½ revolution or an odd multiple thereof between backward and forward readings.

For one-minute drums, or those which make single complete or multiple revolutions between backward and forward readings, set targets (two pencil marks on a card or a smooth wall), whose distance apart equals about ½ of value of drum (30 seconds for one-minute drum). In this case, the target remains fixed, but the plate is set ahead about 10 to 15 seconds each reading in order to give the small increment of comb index for the abscissa of the curve. The curve is plotted in the same manner as above.

(d) Illumination of circle.—For making micrometer readings it is important that the circle be evenly illuminated from above, or else that the light be admitted normal to the circle and directly opposite the graduation to be read, as otherwise there will be an appreciable error in the readings due to phase.

ALIDADE ADJUSTMENTS

Adjustment of centers.—The purpose of this adjustment is to prevent drag and irregular movements of the alidade or circle, the causes of which are the variation in fit of bearings around the vertical axis. In the field the principal remedies are correct cleaning and oiling of bearings. Dirt or grit will cause the bearing to bind or wear unevenly. Excess oil will gum or may allow wobble and cause variable eccentricity and irregular readings. There is no mechanical adjustment of centers to be made in the field on Parkhurst theodolites.

Some theodolites (for example, Kerns, Fennels, and Hildebrands) have an adjustment for fit of centers to regulate the variations of friction caused by wear and by changes of temperature. This usually takes the form of a nut or screw at the lower end of the vertical axis which, by pressing upon the lower end of the alidade axis, lessens the weight of the alidade and telescope upon the conical bearing surfaces. This adjustment must be made with caution, for if too much of the weight is removed from the bearing surface, there is play in the centers, with a resultant loss of accuracy. To make the adjustment, raise the vertical axis with the adjusting nut or screw until the alidade appears to move just freely enough on its vertical axis. Test for play in the conical bearings by pointing on an object and noticing if a slight pressure on the alidade will move the telescope off its pointing. A better test is to point the telescope on some well-defined object, read the circle, swing the telescope around clockwise, and again point and read. The process is repeated except that the alidade is swung in the opposite direction. If a series of three or four sets of such alternating pointings shows the effect of drag, the pressure of the screw upward on the vertical axis should be increased; if the readings are erratic and cover a considerable range, the pressure should be lessened.

Eccentricity of centers.—Adjustment for eccentricity of centers is a shop adjustment requiring special equipment. Neither the screws which hold the circle plate to the axis bearings nor the plate-centering adjusting screws are to be touched in the field.

Tangent-screw adjustment.—Occasionally lag in the counterspring of a tangent screw will cause inaccurate pointings. This can be tested by pointing on some distant well-defined object and while backing the tangent screw slowly away from the spring, noting if there is any lag or irregular movement. Lag is usually corrected by thoroughly

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cleaning and oiling the screw, the spring, and the plunger. Then, if lag persists, the affected parts of a standard instrument such as the Parkhurst theodolite can be requested from the Washington Office and replaced in the field.

ADJUSTMENTS PROHIBITED TO FIELD PARTIES

There are two adjustments which are not to be attempted in the field. There have been previous cases of serious and permanent damage by field parties.

1. The screws on the side of the micrometer box which adjust the tracks on which the movable slide of the micrometer runs should never be tightened in the field, since tightening may cause the tracks to become dented.

2. The six screws which hold the silver circle to its bearing and the four plate-centering adjusting screws should never be touched in any way in the field, as this adjustment requires special instruments for centering, and manipulation of the adjusting screws may cause permanent distortion of the circle.

If any theodolite is not giving accurate observations, and the reason is not apparent, a full report concerning the difficulty, accompanied by an Abstract of Directions, should be forwarded to the Washington Office.

CONDITIONS AFFECTING OBSERVED ECCENTRICITIES IN THE THEODOLITE

This section is included for general information only. None of the adjustments indicated are to be attempted in the field.

There are three main parts of the theodolite whose relative positions govern the observed eccentricity in the instrument itself, namely:

a. The alidade spindle.

b. The circle bearing, which rides on the exterior surface of the alidade spindle bearing.

c. The circle itself.

There are four principal conditions of relative arrangement of these parts, and these conditions determine the amount of eccentricity present. These are noted below:

1. The first, an ideal condition, is when the alidade spindle, circle bearing, and the rings of graduations on the circle are all concentric. Under this condition there is no eccentricity and the instrument is in perfect adjustment.

2. Under a second condition, the ring of graduations may be set to one side of the center of the alidade spindle. This condition can be remedied by adjustment of the four screws until the ring of the graduations and the alidade spindle are concentric.

3. The third condition may be that in which the circle bearing is eccentric with respect to the alidade spindle. Under this condition, it is possible in one position for the center of the circle to be coincident with that of the alidade spindle. Consequently, it is always necessary to test for more than one position of the circle. It is not possible to adjust for this eccentricity. It can be corrected only by remachining the circle bearing with respect to the alidade spindle.

4. A fourth condition may arise in which the circle bearing is eccentric and the ring of graduations on the circle is also eccentric to a degree such that in no position will the center of the circle coincide with that of the alidade spindle.

In an instrument which has been giving erratic readings, it is possible that condition No. 4 exists. This condition may be analyzed by making a series of readings and plotting the results. (See p. 38.) The amount and location of the eccentricity should be noted and in the Washington Office shop an effort should be made to adjust for it as though condition No. 2 existed. If it is found, by repeated shop adjustments and plotting of the results, that the eccentricity is reduced but not eliminated, this is a clear indication that condition No. 3 exists and that the bearing must be refitted.

In all of the above cases, assumption is made that the ring of graduation is a true circle. Unfortunately, this is not always the case. There have been a number of definite instances observed in which either the circle by some mischance was not graduated with a true ring of lines, or in which some change occurred in the material of the circle after graduation, causing it to be distorted radially. There is no remedy for a situation of this sort except regraduation of the circle. Detection of this condition requires a rather extensive set of observations and careful plotting of the results to insure proof that the circle is in the above condition. Circles with this defect are sometimes usable if the bearing elements are in satisfactory condition. The A minus B micrometer observations will not show a true sine curve, but if the error in the circle is very small the deviation may not make the circle unusable.

Another condition in the adjustment for eccentricity which must occasionally be considered is present when the plane of the ring of graduations is not normal to the axis of rotation. This may cause the graduation lines to appear less sharp in one position of the micrometer microscope than in another. It also may cause errors in reading. This is checked in the shop by use of special measuring instruments and, if a 9-inch circle tilts more than one thousandth of an inch, the location of the cause of the tilt is sought and the matter corrected. The most likely source of trouble will be that the seat of the circle on the circle bearing is not normal to the axis of rotation. This must be corrected by machining in position on the instrument. The same difficulty, however, may be due to a circle whose graduated surface has warped out of the true plane. In this case, it must be remachined and regraduated.

DETERMINATION OF VALUE OF ONE DIVISION OF LEVEL BUBBLE

This determination can be made most thoroughly and accurately in the instrument shop on a special instrument called a level trier. However, it occasionally happens that the field party has a striding level for which the value is unknown or for which a check value is desired. This is determined in the field by measuring an intercept of known length at a known distance, in terms of divisions of the level. The angular value of the intercept is then calculated and the value of one division is thus obtained.

Fasten the level to be tested, by adhesive tape or otherwise, longitudinally along the top of the telescope of a theodolite having a clamp and slow-motion screw for moving the telescope in a vertical plane. Suspend vertically, in a fixed position, an accurately graduated tape at a known distance from the theodolite, say 50 or 100 feet; or else, plumb an accurately graduated rod at that distance. If the bubble has a chambered vial, adjust the length of the bubble so it will extend under about one-third to one-half of the graduated portion of the vial.

Point upon some division mark of the tape or rod, so selected that the bubble will be near the end of the vial toward the eyepiece end of the telescope. Read and record both ends of the bubble (estimate tenths of divisions) and repeat the pointings and readings until about 10 readings are obtained, bringing the cross wire of the telescope upon the mark half the time from above the mark and half the time from below, to neutralize the effect of any friction of the bubble against the vial. The mean of all the bubble readings will be the mean position of the center of the bubble. Next point upon some mark on the tape or rod which will bring the bubble near the end of the vial toward the objective end of the telescope, take readings as before, and obtain the mean position of the center of the bubble. Measure carefully the distance from the rod to the horizontal axis of the telescope; and, using the known intercept of the rod or tape, compute the angular value of the intercept at that distance. Divide this angular value by the number of divisions of the level between the two mean positions in order to obtain the value of one division of the level. The original data should be forwarded to the Washington Office with the season's records. A copy of these data should be retained in the field-party files and the accepted value should be lettered on a piece of adhesive tape and fastened to the level frame.

ADJUSTMENT OF VERTICAL-CIRCLE VERNIERS

Vertical-circle verniers should be kept adjusted as closely as possible to the same minute base to avoid excessive use of a bar (vinculum) in the recording of readings. This adjustment requires disassembling of the vertical circle.

Verniers are adjusted by first removing the cover-ring clamp, then removing the three screws from the side of the cover guard and lifting the cover guard off the circle. The vernier ring and reading glasses can then be temporarily assembled on the circle so that the two screws at the ends of each vernier are exposed. Loosening these screws allows each vernier to be moved over a short arc because of the elongated holes in the verniers. These verniers can be adjusted for equidistance in the same manner as was described as a short method for micrometers on page 62. Usually the A vernier is set on zero and held fixed and the B vernier tapped into place at its proper reading. The screws are tightened and the readings checked, and the vertical circle reassembled.

EQUIPMENT

A typical list of equipment for a standard triangulation party is listed on pages 278 to 280. Some of the major items and a few items having special uses will be described in the following sections.

BILBY STEEL TOWERS

The Bilby steel tower (fig. 36) is described in detail, including detailed drawings and list of parts with dimensions, weights, and stock numbers, in Special Publication No. 158, "Bilby Steel Tower for Triangulation."

Bilby steel towers are specially designed for triangulation. The steel survey tower (or signal) consists of two demountable skeleton steel tripods, one within the other. The inner tripod which supports the theodolite does not have any contact with the outer tripod which supports the personnel, signal lamps, and observing tent. Each of the structural members of which the towers are built can be handled by one man. The towers are composed of angle structural steel pieces except for the upper diagonal steel tie rods, steel light plate, and wooden platform and anchor plate boards. The towers are built piece by piece from the ground up, the various structural members being bolted

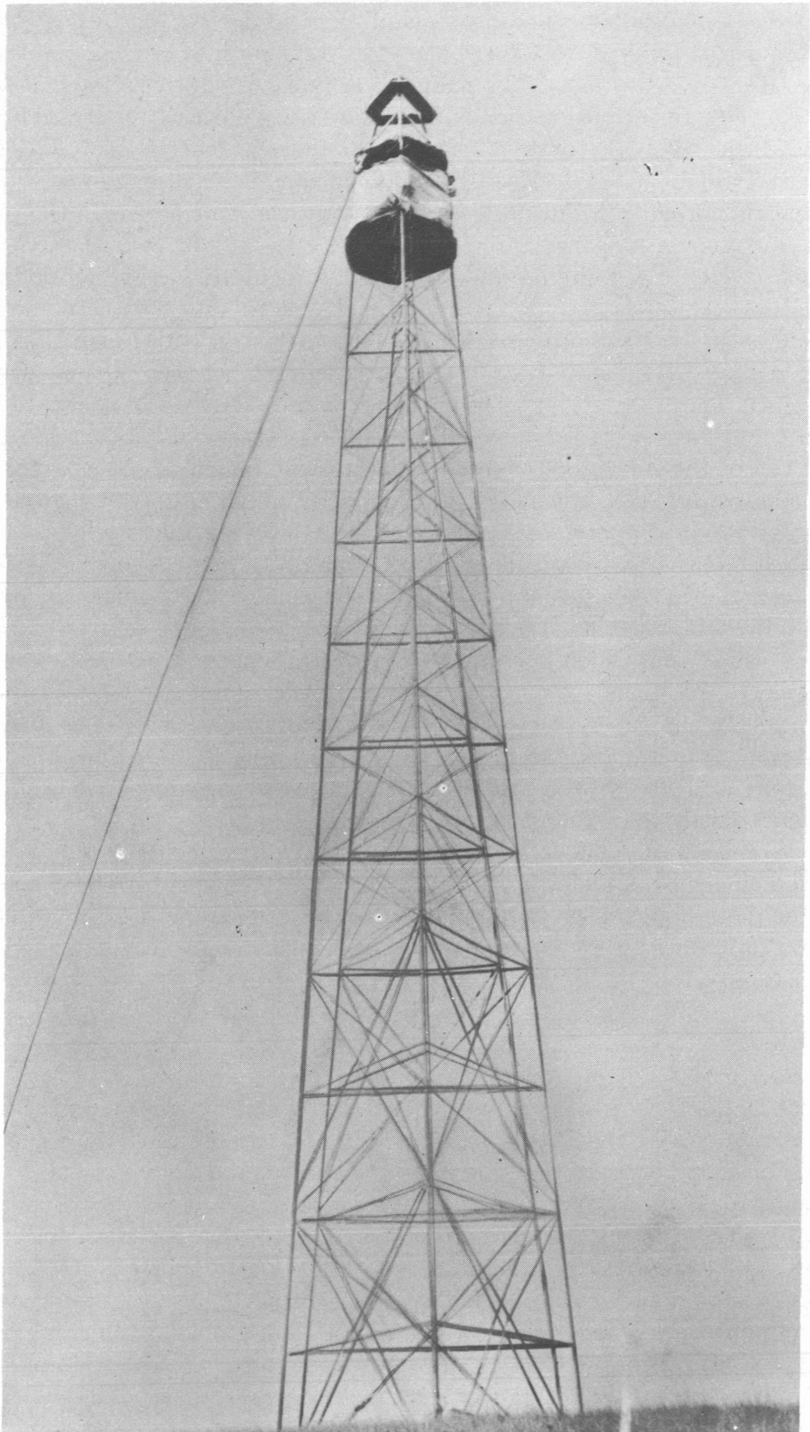

FIGURE 36.—Bilby steel tower, observing tent in place.

together with ⅜-inch bolts. The bottoms of the legs are bolted to heavy oak boards which are wedged down in 4- to 5-foot holes, and serve both as bearing plates and anchor plates to prevent overturning. The main leg sections are 13 feet 8½ inches long, and there are two bents to each leg section. Steel towers are normally classified by the height of the inner (or instrument) tower. There are 37-foot, 50-foot, 64-foot, 77-foot, 90-foot, 103-foot, and 116-foot towers. The light plate on top of the outer tower (or total height of the signal) is normally about 10 feet higher than the inner tower. One or more 10-foot vertical extension sections can be inserted near the top of both the inner and outer towers when needed. A building unit of five men regularly erects any of the standard towers in a day or less, including digging of anchor holes and setting of the five bronze station, reference, and azimuth marks at each station. A four-man unit dismantles two towers a day. Dismantled towers are transported between station sites on semi-trailer trucks. The weight of a 103-foot tower is only slightly in excess of three tons.

These towers are designed to facilitate the operations of the observer. The cap section on top of the inner tower is adjustable for the height of the observer with convenient U-clamps. The sections at the line of sight are free of obstructions except for the three legs which support the light plate, and these legs have adjustable swivel U-sections which can be easily turned to clear a line of sight. There is sufficient space on the hexagonal 6-foot observing platform for personnel and equipment. An observing tent, with adjustable openings for lines of sight converts the observer's platform of the tower into a work room which is well protected for all normal weather. The light plate and lightkeeper's platform for showing lights to other observing units working at the same time are on the superstructure above the observer, with light centered over the instrument and station mark and in no way interfering with operations of the observer.

Towers are a necessity for extending triangulation over many sections of the country which are covered by trees and other obstructions.

The use of the portable steel tower for triangulation has contributed toward marked economy and efficiency of operation of the field units. Wooden towers which were formerly used for these high signals required several times the amount of time to erect, were usually used only once, were consequently more expensive for any extended work, and were not as satisfactory an observing support as the steel towers.

TRUCKS

In most sections of the continental United States, motor trucks are the basic means of transportation for triangulation parties. Typical trucks used by standard parties are outlined on party organization diagrams of figures 1 and 2 on page 23.

Types of trucks used vary with available commercial and military models.

A semi-trailer type (fig. 37) is used for transporting the structural members of the dismantled steel towers. The tractor part of this steel-hauling truck is a standard commercial 1½-ton model with cab, dual rear wheels, extra low gears (4 or more forward speeds), and power-brake equipment. The trailer should be of 3-ton capacity and have an open type body about 2½ feet deep by 16 feet long by 5 feet wide and have dual wheels with brakes controlled from the cab.

Each steel-building unit needs one open-body 1½-ton dual-wheel truck to transport materials (cement, sand, gravel, and water) and mortar box for constructing station monuments, and in addition, a 1½-ton panel-body truck for transporting instruments, tools, and personnel. Either or both of these trucks usually has the right rear wheel equipped with a special hub flange for attaching a winch head for hoisting the steel tower parts. Units of 2 or 3 men who build low wooden signals usually use a single truck.

Observing units require a 1- to 1½-ton panel-body truck for transporting and safekeeping of their instruments and equipment. A special sponge-rubber-padded box should be installed in the truck for the cased theodolite. Cabinets or chests for smaller instruments and a reel for the hauling line are also desirable. Four-wheel-drive trucks are usually most satisfactory for observing units which operate in rugged country.

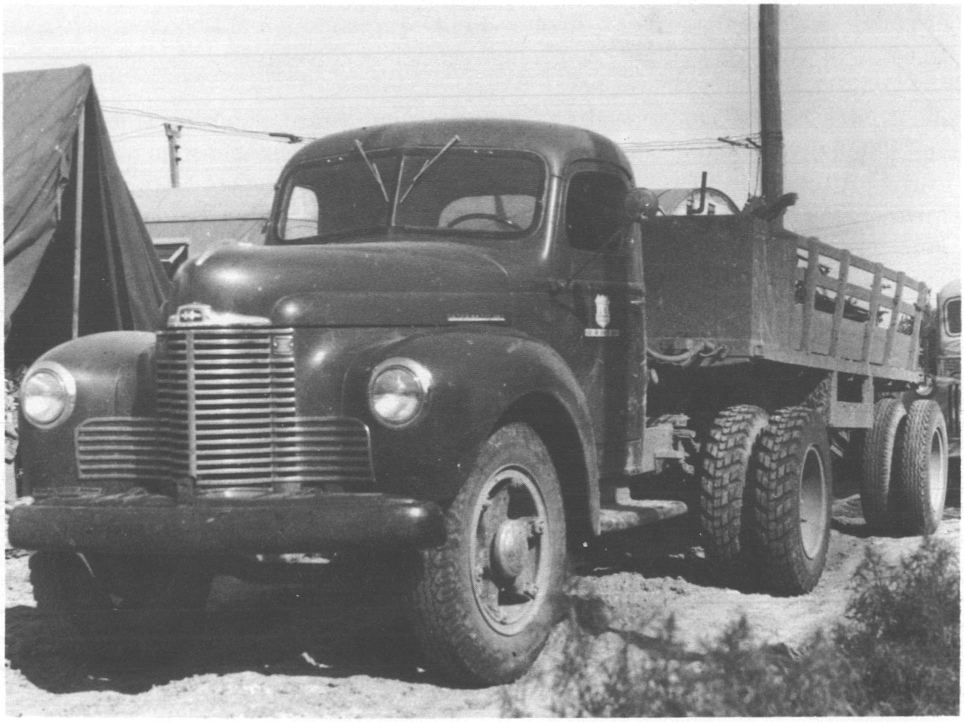

FIGURE 37.—Steel-hauling semi-trailer truck.

Lightkeepers require small light trucks. Small ¼- to ¾-ton panel and pick-up models are most satisfactory, provided the truck also has an enclosed cab and high road clearance. A locked and weatherproof chest for instruments and equipment should be installed in open-body trucks. A party operating in rugged country should have 4-wheel-drive jeep-type trucks for use of lightkeepers when needed.

The following general characteristics are desirable for all triangulation party trucks: Since stations are often in out-of-the-way localities and trucks frequently have to be operated away from improved roads, high road clearance and extra low gears (4 or more forward speeds) are most desirable qualities. In some types of terrain, oversized tires, dual wheels, or four-wheel drives are helpful. On these mobile parties, valuable instruments and equipment are transported and kept stored in trucks, therefore, it is important to have good locked panel bodies or locked waterproof chests on most of the trucks of the party.

Trucks carry assigned government license plates throughout the life of the truck. Decalcomania official-use shields and the department and bureau names should be maintained in good condition on door panels of truck cabs.

Installed equipment on all trucks should include all lights and equipment required by State laws, clearance lights, windshield wipers, rear-view mirrors, horn, heater, and adequate seating for all personnel who are assigned to use the truck (usually three or four men).

Each truck should be provided with a spare tire, a set of tire chains, a set of three highway flares, a fire extinguisher, a jack, a tire tool, a lug wrench, a crescent wrench, a pair of pliers, and a screw driver. Each item of equipment should be stamped, painted, or tagged with the truck number. Items should be inventoried in the truck record book and charged against the driver.

All drivers should be licensed operators in some State requiring a driver's examination. Each driver who has a truck assigned to him will be responsible for its condition and care. All drivers should carry public-liability and property-damage insurance. Excessive speeds should not be allowed. All driving precautions should be observed with due regard to the condition of the road and visibility. It is often better to pack the gear to the station instead of taking chances of getting stuck or of having an accident on poor, unimproved roads, or on cross-country driving.

Keys to all trucks not in use should be turned in to the field office.

OFFICE TRAILERS

Office trailers greatly increase the efficiency of field party administration. In recent years, with parties more permanently organized and with most of the personnel under Civil Service regulations, it has become necessary for mobile field parties to carry extensive files. When files, desks, and other office equipment are installed in the trailer, frequent movement of base camp, as work progresses, presents no unusual interruption of office work.

A small office trailer with two or more desks, files, and storage cabinet space is desirable for use as field administrative headquarters (with desk for chief of party), and for personnel records, accounts, and typing equipment (with desk for accountant).

A second and larger office trailer for use as a computing trailer should provide desk space for the computer, assistant computer, and three observers.

Office trailers (see fig. 38) should be sturdily constructed to carry the load of desks, files, and other office equipment and office supplies. The trailer should have either dual or tandem wheels, an efficient hitch and brakes, and clearance lights which comply with all State regulations. For efficient operation, they should have adequate interior heating, lighting, and cooling systems. In most places in the continental United States, where base camps will be established, electric lighting can be obtained by connection to the local 110-volt supply. In other localities, battery-operated lights can be installed.

Commercial house trailers without built-in cooking stoves, refrigeration, plumbing, or furnishings are satisfactory for conversion to office trailers when constructed heavily enough to carry the load.

Office equipment.—Built-in desks are more satisfactory and take up less room in office trailers than commercial desks. Standard steel drawer files are satisfactory. The heaviest files should be installed over the wheels and the weights should be well balanced.

Storage cabinets should be built-in. Stove and cooling and lighting systems should be installed. Portable equipment should include typewriters, adding and calculating machines, hectograph, technical manuals, books and numerical tables, drafting instruments, complete supply of forms, and stationery items. See lists on pages 278 to 282.

FIGURE 38.—Office trailers of a triangulation party.

TENTS

Observing tents.—Observing tents are specially designed as ground observing tents and as tower observing tents. They are obtained on requisition from the Washington Office.

The ground tent has a frame composed of six metal poles which knock down into two half-length sections each, and a six-arm metal spider top section. The fabric side covering extends almost to the height of eye and the top covering has flaps which can be lashed down to the sides so that the instrument can be protected from the wind and weather except for openings along the line of sight. Where back-packing is required to the station, aluminum frames and lightweight fabrics have been made available. Observing tents should be white in mountain areas so as to be easily visible as a target for lightkeepers and for vertical-angle observations. (See fig. 39.)

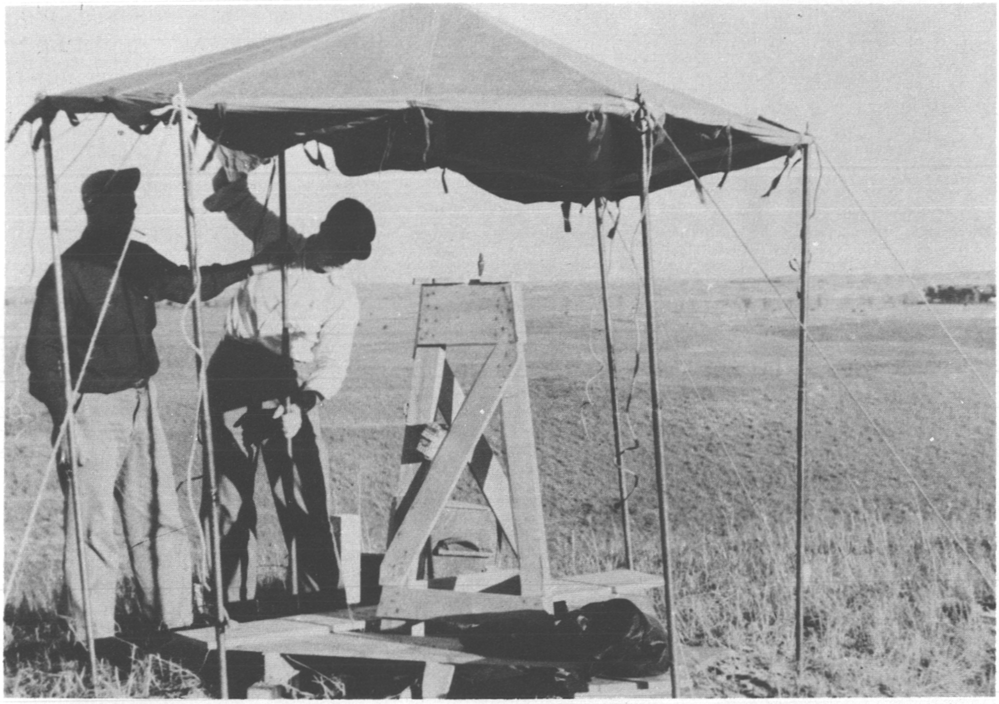

FIGURE 39.—Observing tent, ground type (side walls not yet in place).

The tower observing tent is similar in design to the ground tent, except that it has no frame and is made to fit around the observer's platform of a Bilby steel tower. (See fig. 36.)

Camp tents.—Large 14- by 14-foot ridge pole tents are usually used as storage tents for cement, building tools and supplies, observer's gear, lightkeeper's gear, and storage chests, and as garage repair tents.

To house personnel, 9- by 9-foot center pole tents with floor tarps are used. These tents can be set up by one man.

Many of the personnel, especially married members of the party, usually choose to live in privately-owned house trailers.

BUILDING EQUIPMENT

Typical building equipment is included in the lists on pages 278 to 280. Most of these items are well-known standard commercial articles.

HOISTING EQUIPMENT

Hauling lines are made up of manila rope ½ to ¾ inch in diameter and usually in 230-foot lengths. The line is roved through a 3-inch single fixed block which is hooked to a convenient part of the tower. The running end of the line is attached to a ring or a small block which runs on the hauling part. Rope straps about 6 feet long are hitched around parts to be hoisted and attached to rings on the running end of the line with heavy harness hooks. Building parties have a winch drum head which is bolted to a special flange on the hub of a right rear wheel of a truck. Steel is hoisted by taking turns of the hauling end of line about this revolving drum. A light haul-back line is attached to a ring on the running end of line. Observing parties use similar hauling lines with special slings and bags for instruments and equipment. Observing parties hoist all their instruments and equipment by hand to avoid risk of damage.

PACKING EQUIPMENT

Packboards (fig. 40) are the principal item of satisfactory backpacking equipment. These consist of canvas stretched tightly over a wooden frame with attached shoulder straps. The shoulder straps and stretched canvas form the principal bearing surfaces

FIGURE 40.—Observing unit back-packing to station.

against the body of packer. The frame allows the load to be fastened relatively high in a balanced position which provides more comfort and endurance to the packer. Pack-sacks for equipment and straps for the instrument case are easily attached to the pack-boards. There are several very satisfactory commercial models of packboard equipment.

SIGNAL BUILDING

BUILDING SCHEDULES

On a multiple-unit triangulation party, building schedules are usually posted by the field foreman after consultation with the chief of party. In most cases, the field foreman has previously visited the station site and checked the reconnaissance description and type of signal required. He has also contacted the property owner for permission to enter upon the land in order to build the tower and establish the station. Steel-tower parties usually have regularly established building units of five men each, and tearing-down units of four men each, with the building routine almost the same from day to day. The work schedule for each unit is listed under the unit foreman's name.

There follows a typical schedule which is posted daily on the bulletin board of steel-tower triangulation parties:

Building schedule
Tuesday, 2 Nov.

V. Brindley
Build ODELL 64′

N. Banks
Build CORWIN 90′

L. Blem
Tear down HEDDRICK and OWASCO

Steelhaulers:

J. Reynolds
Haul to: ODELL
Pick up: HEDDRICK

J. Early
Haul to: CORWIN
Pick up: OWASCO

/s/ B. Kelley
Field Foreman

Mountain triangulation building units include from two to five men, depending on the type of signal required and accessibility of station. The names of all personnel on a unit are usually listed on the building schedule for mountain parties. Schedules are sometimes posted for a week or more at a time.

Courtesy to property owners.—Stations must ordinarily be established on property owned by individuals or private corporations. Although many States have laws which give the right of entry upon private property to Government surveyors engaged on official work, it has not been the policy of this Bureau to insist upon this right. In practically all cases the owner of the property is willing to have stations established on his land if the object of the survey is explained to him. The chief of party should always arrange to secure permission to enter upon the premises and to establish the necessary marks. If it is necessary to damage crops, shrubs, or trees, the regulations must be closely followed in securing beforehand a written agreement which shall state the amount of damages to be paid. The good will and cooperation of property owners and their agents are very essential to the units which follow the reconnaissance and building parties, and any source of future controversy should be avoided. The signal-building party should exercise courtesy and tact as it is the advance agent of the other units of the triangulation party.

STEEL-TOWER SIGNALS

Steel-tower signals are described in Special Publication No. 158, "Bilby Steel Tower for Triangulation."

Building procedure.—A standard steel-tower-building unit consists of five men including the foreman. This provides for one man to work on each of three legs; one man to assist on the tower by passing parts to the three leg men, tightening bolts, and tending the hoist line; and the fifth man (usually the foreman) to tend to the hoisting of parts in correct order and to direct operations from the ground.

The holes for the legs are usually laid out using a builder's level or small transit. The signal builders dig the holes for the legs and marks.

A Bilby steel signal tower can be erected in a few hours by an experienced building unit. (See figs. 41, 42, and 43.)

Building accessories.—Construction platforms are three triangular platforms, one for each of the inner corners of a steel tower. They are 24 inches on each side with 2- by 2-inch cleats nailed on the under side to fit closely along the outside of the horizontal ties on which the platform rests.

Bolt bags worn by each builder consist of three canvas pouches on a broad leather belt. An S-type end wrench has been found most satisfactory. Each builder uses two.

Hoisting equipment is described on page 72. Building equipment is included in the lists given on pages 278 to 280.

Dismantling steel towers.—The unit which dismantles steel towers after observing is completed is known as a tearing-down party. The unit usually consists of four men, one of whom acts as foreman.

The tearing-down party usually dismantles two towers per day.

Steelhauling.—It saves extra handling and is to the advantage of both the tearing-down party and the steelhauler if the steelhauler is present during tearing-down operations and loads the parts as they are dismantled.

To avoid pilfering, it is desirable that dismantled steel tower parts be removed promptly from the station site.

The steelhauler should check over and straighten or have repairs made to damaged parts whenever necessary.

BUILDERS' REPORTS

For each station which is marked, or where a signal is built, the building foreman or man in charge of the building unit shall submit daily a report on Form 749, "Daily Report of Building Foreman on Establishing of Station." This report is to be turned in to the chief of party or to the field computing office, as may be directed. It is used principally for the information of the observing party.

This report shall always include approximate distances and directions from the station mark to reference and azimuth marks, a brief description of location of the azimuth mark, information on the kind of truck (4-wheel drive, for instance) and time required by the building party for both driving and backpacking. New directions to reach the station should be written on the back of the form, if necessary.

Figure 44 is a sample copy of a completed form.

FIGURE 41.—Bilby steel tower under construction.

FIGURE 42.—Bilby steel tower under construction.

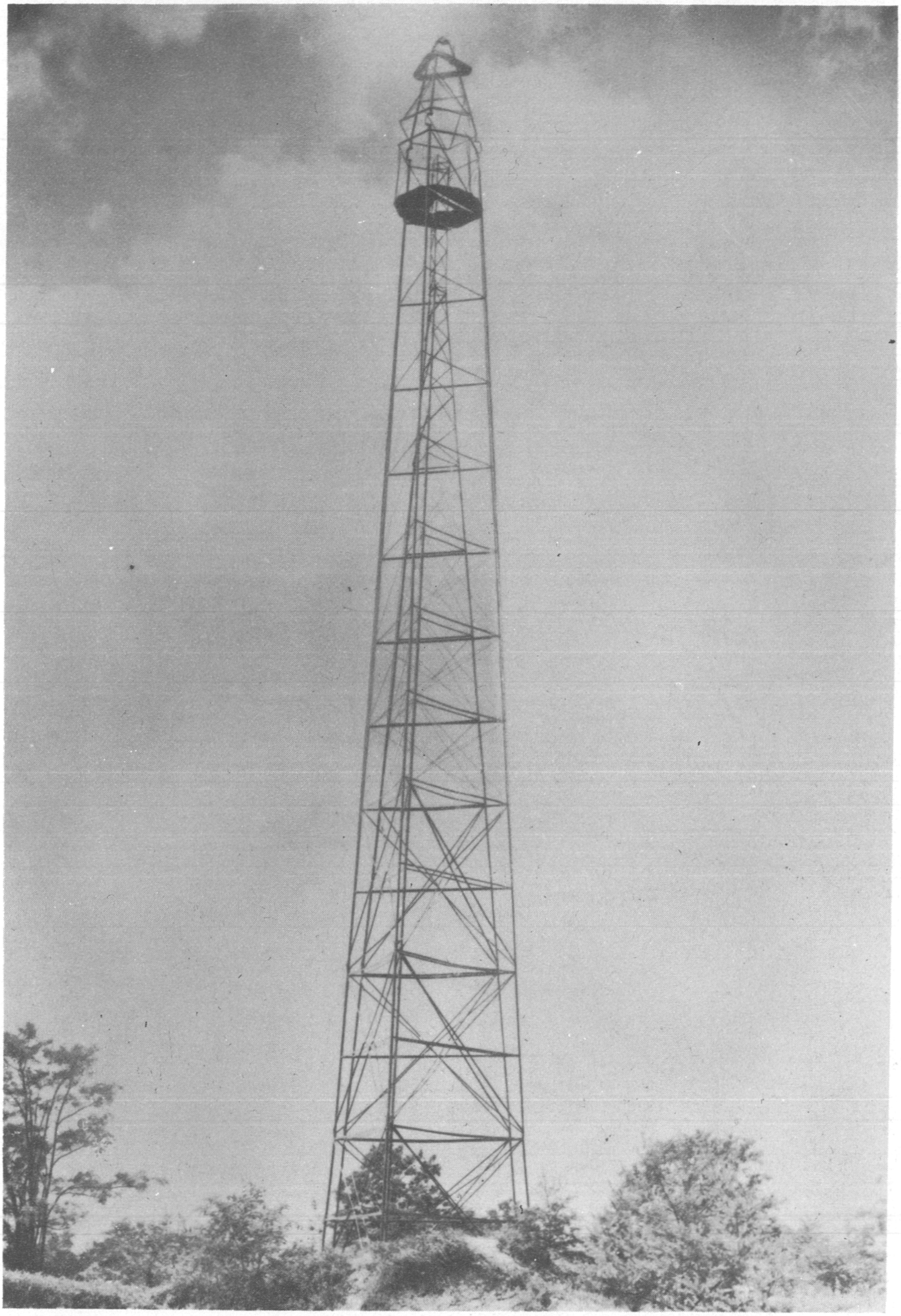

FIGURE 43.—Completed Bilby steel tower.

Form No. 749
DEPARTMENT OF COMMERCE
U. S. Coast and Geodetic Survey
Ed. May 1940

DAILY REPORT OF BUILDING FOREMAN

ON

ESTABLISHING OF STATION

Station BROWN Date built 22 June 1948

MARKS

	Surface	Underground	R. M. No. 1	R. M. No. 2	R. M. No. 3 (or azimuth mark)
Standard disk	x	x	x	x	x
Concrete mark:					
Square top	x		x	x	
Round top		x			
Diameter	12"	10"	10"	10"	
Boulder or bedrock					x
Height of mark above or below ground (+ or −)	+3 in.	-3 ft.	+4 in.	+4 in.	0
Distance from station	x	x	90 ft.	70 ft.	0.4 mile
Direction from station	x	x	NW	NE	S

Stamping on marks (print) BROWN 1948

TOWER

Steel 90 ft. Wood ------

Light stand 100 ft.

Actual height built as above

Height specified on reconnaissance blueprint 90 feet.

Was owner of land interviewed? Yes. Mr. John B. Brown

REMARKS: Enter lane on west side of Mr. Brown's barnyard, instead of passing thru barnyard. Obtain key to inner gate from tenant 1/4 mile east of barnyard. Azimuth mark is in fence corner at NE side of intersection of county roads 1/2 mile south of barnyard lane.

(Note any errors found in reconnaissance description. Give any information which may be useful to observers, such as location of marks, etc.)

/s/ J.A. White

Building Foreman, Party No. ....

U. S. GOVERNMENT PRINTING OFFICE 16—[illegible]-1

FIGURE 44.—Daily report of building foreman.

SAFETY PRECAUTIONS IN SIGNAL BUILDING

All extra workers, children, and onlookers should be kept from underneath the tower and at a safe distance during erection or tearing down of towers. People who have no business thereon should be warned and discouraged from climbing the towers at any time. No one should be allowed underneath the tower while hoisting steel. When any material is thrown from the tower (as by tearing-down crew), workers should look to see that the area underneath is clear and also shout a warning. Warning should be shouted when bolts, wrenches, or other materials are accidently dropped.

The foreman must be very careful in handling the hauling line to keep from getting it fouled in the rotating truck wheel or around himself, and thus injuring himself or men on the tower, or possibly pulling the tower over. An extra ignition cut-off switch near the winch is a good safety feature to stop the engine in case of trouble. Also, rotating the jacked-up winch wheel with truck in third gear with throttle set equivalent to speed of about 20 miles per hour will probably kill the engine if the rope jams. An iron pinch bar stuck upright in the ground several feet away from the winch wheel is useful in keeping the free end of the rope away from the rotating wheel.

All bolts and rivets through mudsills and anchor posts should be checked before each tower is built, and replaced where necessary. Welding repairs should be made if needed.

The boards of the lightkeeper's triangular platform, the observer's main platform, and the anchor mudsills should be renewed immediately when they become split or otherwise damaged or deteriorated in any manner which will endanger personnel.

The lightkeeper's platform should be securely bolted in place on all three legs. In windy areas, the main platform boards should be lashed on.

The vertical stays in the bottom sections of the Bilby towers should not be omitted. Failure of several towers under strong winds has been attributed to the signal builder's omission of these vertical stays.

The chief of party should inspect the signal builder's performance on stations periodically.

WOODEN SIGNALS

Building of wooden signals is described in detail in Special Publication No. 234, "Signal Building."

Wooden signals are used for 4-foot stands and where low towers are required. Most signals used in the western mountain areas are made of wood. Four-foot stands are used wherever elevation and non-obstructed lines of sight permit. They are economical in time of construction, are easily transported to stations difficult of access, and are stable for observations.

Four-foot stands.—Four-foot stands (fig. 45) are tripods which are usually built of standard commercial sizes of lumber. The two-by-fours for legs of low stands can often be purchased from the mill with a 60° bevel lengthwise along one edge, or two-by-sixes may be ripped or cut lengthwise at a 60° angle into two boards of equal cross section. This helps to make a better nailing surface and therefore a more stable stand. It is often advantageous to make up a pattern and cut the lumber to standard sizes in the base camp with a power saw. Four-foot stands can often be made up in advance in camp as available men and weather permit.

Holes should be dug for the legs of the tripod stand whenever possible. The tops of these holes should be left open to avoid transmitting surface disturbances to the stand. The stand should be securely staked or cribbed in place.

Footboards or a platform for the observer should be placed around the stand. This should be supported as far as practicable from the legs of the instrument stand. Testing of a stand for stability is described later under the discussion of observing procedure.

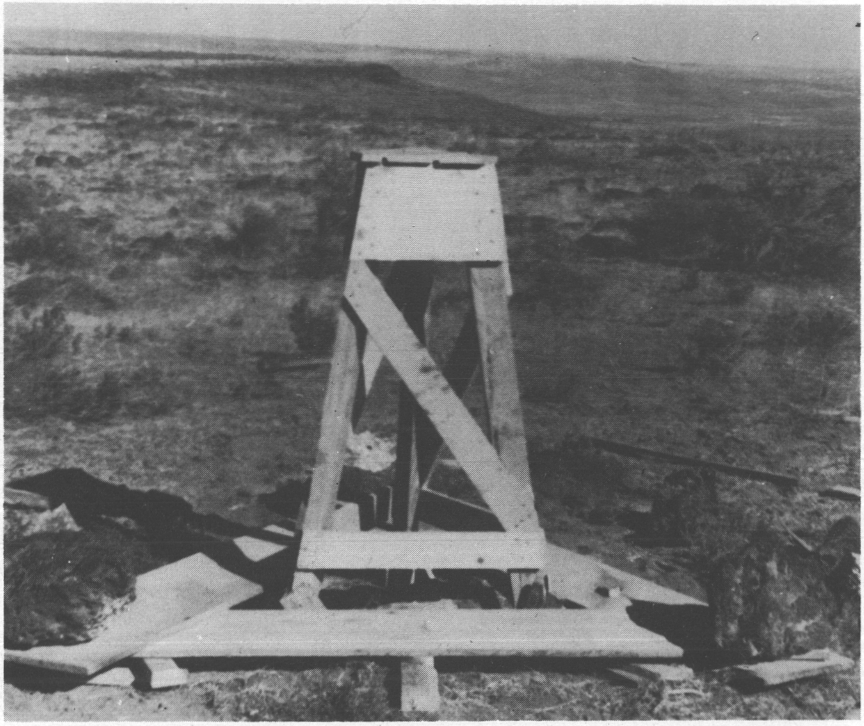

FIGURE 45.—Four-foot stand.

Wooden towers—6 to 25 feet.—Wooden tripods and scaffolds (see figs. 46 and 47) are used on most stations requiring from 6- to 25-foot signals, since it is not very satisfactory to build a standard Bilby steel tower of less than 37 feet. However, a 25-foot Bilby steel tower can be used by adapting special anchors for the inner tower.

The party should have standard designs for 6-, 10-, 15-, 20-, and 25-foot wooden stands. Usually, the materials at hand and easiest handled are 2″ x 4″ x 16′, 1″ x 4″ x 16′, and 1″ x 12″ x 16′ boards. For signals over 15 feet, two-by-fours are doubled on the lower two-thirds or so of the legs of the signal. The inner or instrument tower is built as a tripod, usually completed on the ground and set up in leg holes as one piece. The scaffold supporting the observer's platform is usually four-legged with two opposite sides being put together on the ground and then stood up around the tripod and the other two sides nailed on. All legs are laid out and marked on the ground. One side each of the tripod and scaffold is used as a pattern to cut the horizontal ties and diagonals the same for all sides.

On the wooden towers, the observer's platform is usually made 8 feet square, so that the ground observing tent can be set up on the platform.

Precautions to protect stability.—To increase the stability of wooden tripods: The diagonals should be cut and nailed to butt firmly against the horizontal pieces; all joints should be well nailed and should be firmly set up by rehammering before using; bottom horizontal ties should be within a few inches of the ground to reduce vibration of unbraced leg sections; legs should not rest on loose rock or loose surface material.

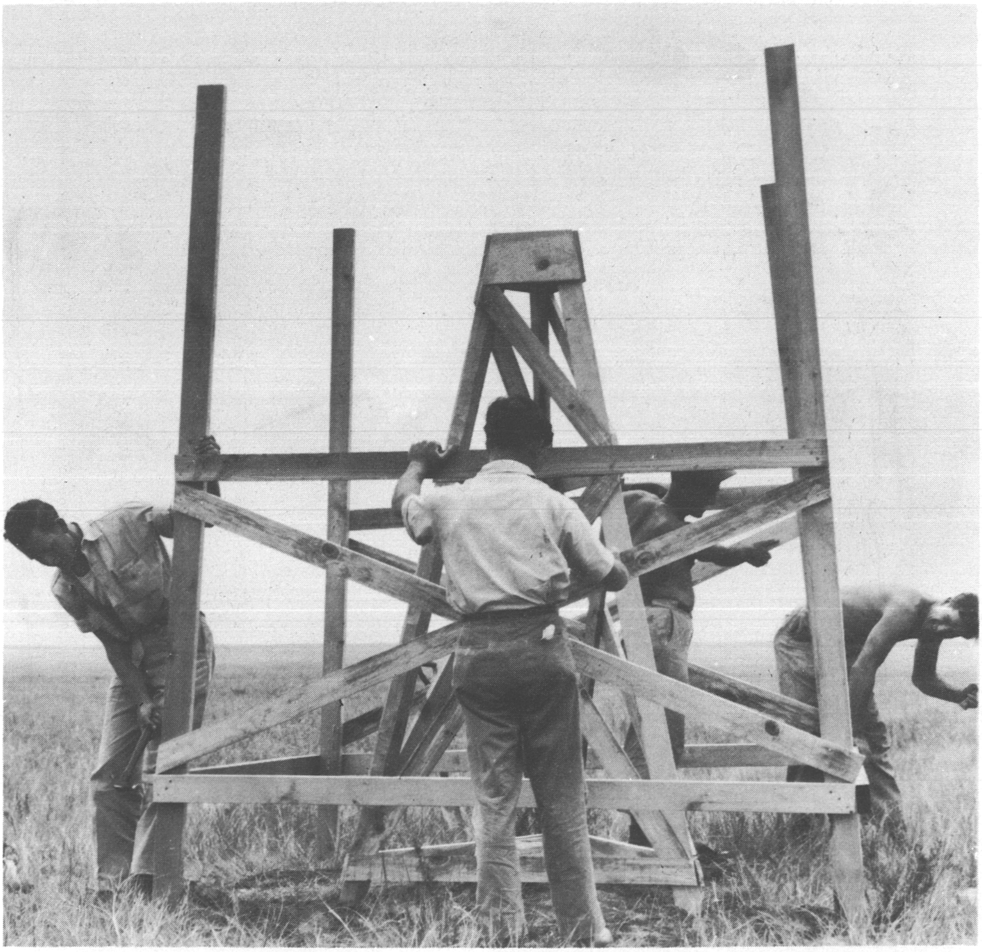

FIGURE 46.—Ten-foot tower under construction.

SIGNAL BUILDING FOR SECOND- AND THIRD-ORDER TRIANGULATION

The same types of signals as described in the preceding sections are frequently used for second- and third-order triangulation, except that, since these observations are usually made in the daytime, banded pole targets are often used instead of signal lamps. The point of the pole sighted upon is the section next to the tower. Targets and flags of various shapes are sometimes fastened to the top of the pole to assist in identifying the

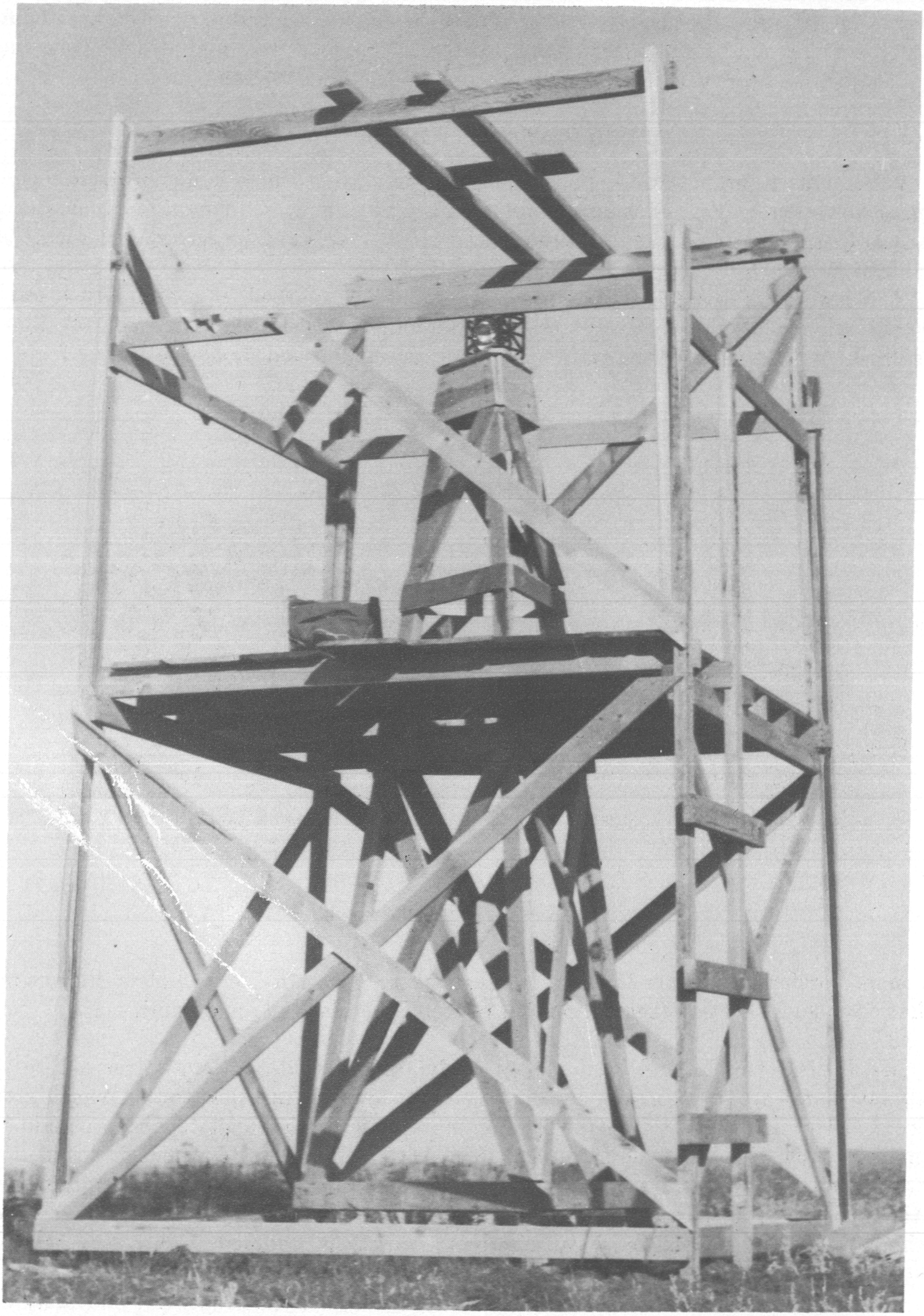

FIGURE 47.—Fifteen-foot tower.

signal. At ground stations the theodolite tripod is frequently used instead of four-foot stands. In this case, it is usually desirable to dig shallow holes and to drive stakes about flush with the ground for support of the points of the tripod legs, and also to place the usual footboards around the instrument to support the weight of the observer.

SPECIFICATIONS FOR MARKS

1. Stations to be marked.—Each triangulation station which is not in itself a permanent mark (such as a lighthouse, a spire, or a tank) but which is in a location where it can be permanently marked and referenced shall be marked in accordance with the specifications which follow.

2. Metal disks.—A triangulation station should be marked by a standard bronze disk so fastened as to effectively resist extraction, change of elevation, or rotation. (See fig. 48.) The name of the station and the year established should be stamped upon

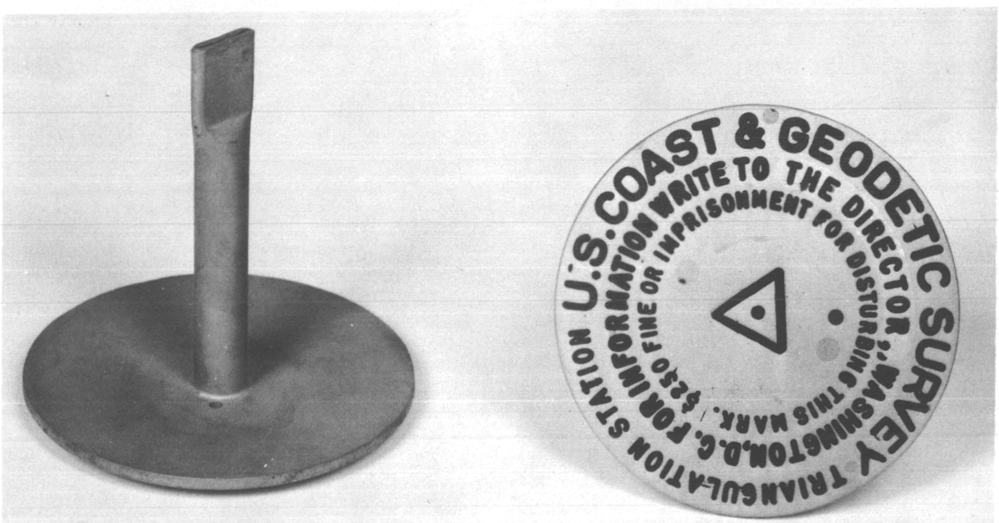

FIGURE 48.—Triangulation station mark.

the mark, preferably before it is set in the rock or concrete. A complete station installation includes station (center) marks, reference marks, and an azimuth mark. (See fig. 49.)

Metal disks which have been moved or defaced so that they no longer can serve as survey marks are to be broken out of the monuments and returned to the Washington Office. Lettering on stone, concrete, or any material other than metal markers should be hammered or chiseled off if the monument has been moved from its proper location.

3. Naming of stations.—The triangulation party normally uses the name assigned by the reconnaissance party unless there is some reason for changing it. Correct spelling of the name should be checked locally before the mark is stamped. The correct name as stamped on the mark will be used throughout the records.

The name of the locality is preferable but the name of the property owner may be used for the designation of the station. To avoid ill feeling, it should be made certain

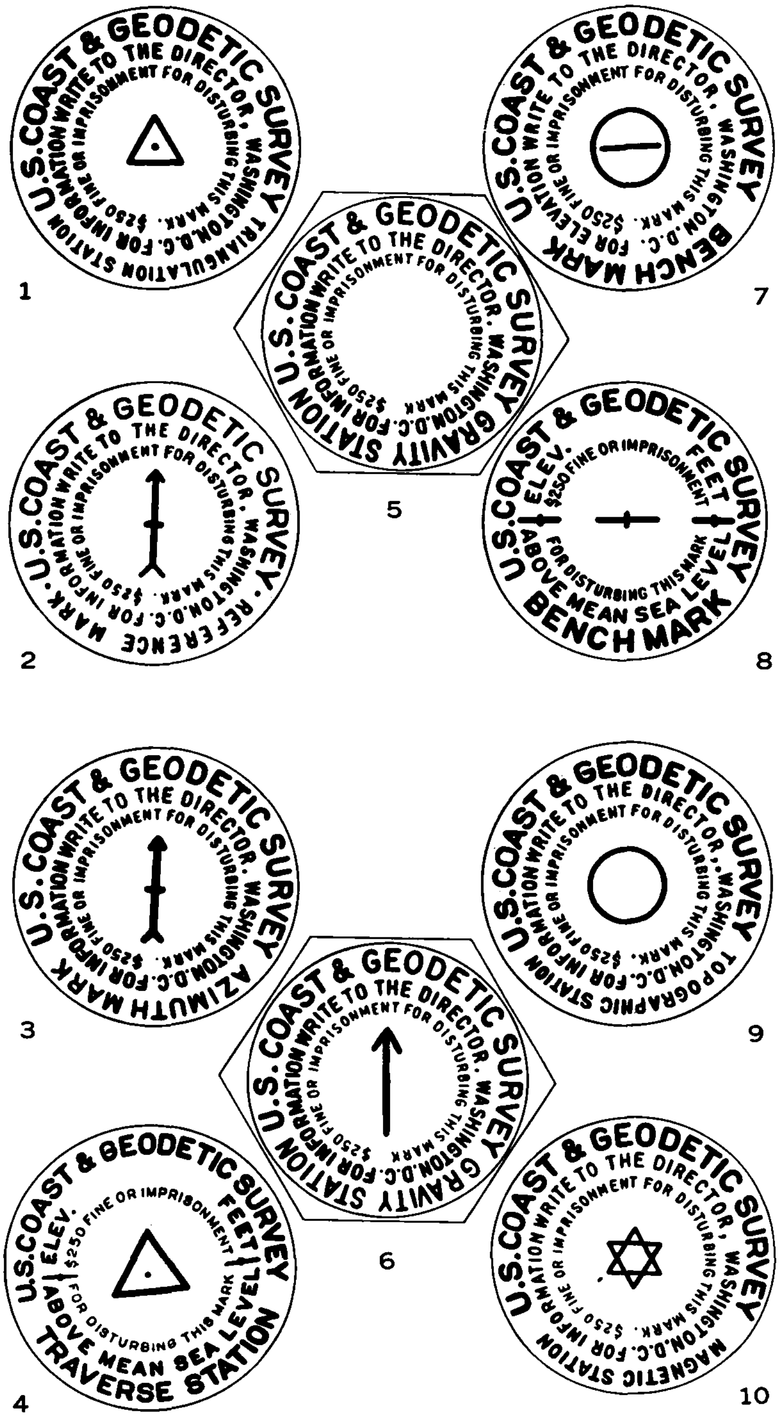

FIGURE 49.—Standard marks of the U. S. Coast and Geodetic Survey.

1. Triangulation station mark.
2. Reference mark.
3. Azimuth mark.
4. Traverse station mark.
5. Gravity station mark.
6. Gravity station reference mark.
7. Tidal bench mark.
8. Geodetic bench mark.
9. Topographic station mark.
10. Magnetic station mark.

that the station is actually on the named property, and that the owner's name is correctly spelled.

Double names should be avoided if practical as they cause extra work throughout the recording and computing. Also the double name including the word "peak," or "mountain," or "point," is not usually necessary, since the description should state that the station is on a peak or mountain or point of that name.

Names of stations of other organizations should be retained exactly as stamped on that organization's station mark. If additional azimuth or reference marks are established by a party of this Bureau, the stamping should include the original station name, the initials of the other organization, and the year in which the additional marks are established.

If the name of a recovered intersection station (not marked with a bronze disk) is incorrect, the correct name should be typed in the heading of Form 525b. The first statement in the body of the note should then list any previous triangulation names of the same object, as well as map names and any other names in current usage.

4. Rules and examples for marking stations.—All triangulation stations shall be marked or re-marked, and the disks stamped, in accordance with the following rules:

(a) Each newly established triangulation station shall be marked with a standard station-mark disk which shall be stamped with the name of the station and the year of establishment.

(b) Each reference-mark disk shall be stamped with the name of the station, the number of the reference mark, and the year.

(c) Each recovered station which is re-marked shall be stamped with the original name of the station, the original date of establishment, and the year in which it is re-marked.

(d) Additional reference marks may be established when a station is recovered and reoccupied. The name and date, or dates, on the reference-mark disks shall be the same as those on the station-mark disk at the time the reference mark is established.

(e) Do not renew an old reference mark. If it is in poor condition, either reinforce it or destroy it and set a new reference mark which will be numbered with the next consecutive unused number, regardless of the existence or absence of any of the reference marks established previously.

(f) The abbreviation "Ecc." (for eccentric) should never be stamped on a disk. Its use in the records should be solely to indicate that the observations made at that point must be reduced to the station center to close the triangles before being used in the subsequent computations.

(g) All new stamping on disks for station and reference marks shall be done with $\frac{3}{16}$-inch dies.

The following examples relative to stamping triangulation-station and reference-mark disks shall be strictly followed:

CASE I.—A new station is established.

In the center of the station-mark disk is a small triangle. The year of establishment is stamped under one side and the name above the opposite apex of the triangle. Two reference marks shall be established and the disks shall be stamped with the station name, number, and year. The reference marks shall be numbered consecutively in a clockwise direction from true north. They must be set so that the arrows on the disks point toward the station mark. Azimuth-mark disks shall be stamped with the name of the station and the year of establishment.

Example.—Station EAGLE is established in 1940. The disks shall be stamped as follows:

Station mark: "EAGLE 1940"
Reference marks: "EAGLE NO. 1 (or 2) 1940"
Azimuth mark: "EAGLE 1940."

CASE II.—The station mark, reference mark, or azimuth mark is reinforced but not re-marked or otherwise disturbed in any way.

If any or all of the above marks are reinforced only, the original stamping on the disks shall be retained without change, alteration, or addition.

CASE III.—The station is re-marked in the precise original position, and a new reference mark is established.

The original name and date of the station shall be retained and, in addition, the year the station is re-marked shall be stamped under the date of original establishment. In re-marking the station, a new station-mark disk shall be set if none was used formerly, or if the original disk cannot be re-used.

If the two original reference marks are in good condition and in no danger of being destroyed in the near future, they should not be altered.

Example.—Station TABLE 1925 is re-marked in 1940 and one new reference mark established.

The station-mark disk shall be stamped "TABLE 1925-1940."

The new reference-mark disk shall be stamped "TABLE NO. 3 (or next unused consecutive number) 1925-1940."

CASE IV.—The station mark only is moved.

If a station is to be moved, it is generally more practicable to establish a new mark in the new location, and when this is done the old mark must be completely destroyed. The station name shall be preserved but the number "2" shall be stamped after the name. The year the station is moved shall be stamped on the station-mark disk. The date of establishment of the original station shall not be stamped on a new disk, and if the old mark and disk are re-used, the original date shall be effaced by light tapping with the rounded end of a ball-peen hammer and the disk restamped with the new year.

At least one new reference mark shall be established, stamped with the new station name and year of moving the station and given the next unused consecutive reference-mark number. The stamping on the previous reference-mark disks shall not be changed.

Example.—The station mark for LUTKE 1925 is moved in 1940 but the reference marks are not moved.

The disk in the moved mark shall be stamped "LUTKE 2 1940." If there were two previous reference marks the new reference mark shall be stamped "LUTKE 2 NO. 3 1940."

CASE V.—The station mark and one or more of the reference marks or the azimuth mark are moved.

The station mark shall be treated as in CASE IV. The newly established reference-mark disks shall be stamped with the name of the station, the following consecutive numbers, and the year the station was moved. The old date of establishment should not appear on the disks of any of the moved marks. Should it be more practicable to reset one of the moved reference marks rather than establish a new one, the stamping on the disk which is no longer in order shall be effaced as in CASE IV and the correct notation restamped. The former reference marks which have been moved or destroyed shall be reported as nonexistent.

Example.—Station SITKA 1925 and its reference mark No. 2 are moved in 1940.

The station-mark disk shall be stamped "SITKA 2 1940." Reference mark No. 1 shall not be restamped. Reference mark No. 2, having been reset in a new location, shall be restamped "SITKA 2 NO. 3 (or the next unused consecutive number) 1940." Should reference mark No. 2 be destroyed and a new reference mark set, the stamping on the new disk shall be exactly the same as in the preceding sentence. If the azimuth mark is moved it is stamped "SITKA 2 1940." No old disk is restamped unless it is moved, or reset.

CASE VI.—The station mark is not re-marked or moved but one or more of the reference marks are moved or one or more new reference marks are added.

The original station name shall be preserved. The number of the reference mark moved shall be canceled and the next consecutive unused number stamped on the disk.

Example.—Reference mark No. 2 for RAVEN 1925 is moved in 1940 but the station mark is not re-marked or moved.

The reference mark shall be stamped "RAVEN NO. 3 (or next consecutive unused number) 1925." If both reference marks are moved, the new reference marks shall be designated Nos. 3 and 4 (or the next consecutive unused numbers) and the year 1925. The same shall apply if one or two new reference marks are established even though the two old reference marks are in good condition and their positions are not disturbed.

CASE VII.—The station mark is not re-marked or moved but the azimuth mark is moved.

In this case the azimuth mark shall be stamped with the name of the station, the year of the original establishment, and the year it is moved.

Example.—The disk in the azimuth mark for station MIAMI 1925 shall be stamped "MIAMI 1925 RESET 1940."

5. **Station mark.**—Each station center should be marked with a standard triangulation mark of the type illustrated in figure 48. The mark should be set in the manner described in section 10. An underground station mark should also be set under the surface mark wherever conditions permit. Marks of other organizations may be used as described in section 9. The upper station mark may also be set underground when necessary, as when the station is in a cultivated field.

6. **Reference marks.**—Each station should have at least two reference marks. A reference-mark disk is illustrated in figure 49. The disk bears an arrow which is set to point toward the station mark. Reference marks are stamped with the name and date of station and are numbered serially clockwise from north (for new marks). When needed, monuments are constructed similar to the surface station monument but may be two inches smaller in diameter. They should be 30 inches or more in length as may be necessary to extend below the active frost line. No underground marks are used with reference marks. The directions to the two reference marks required at each new station should intersect in a good angle, preferably near a 90° angle, or the marks should be on range with the station. Reference marks should be located where they are least liable to be disturbed, such as in or near fence lines. It is also necessary that reference marks be

placed where direct unobstructed measurements can be made to them from the station mark, and where the line of sight from the instrument to reference marks is clear both from the top of the tower and from the ground. Distances to reference marks from the station should preferably be kept less than a 30-meter tape length to facilitate taping, and far enough from the tower so that the line of sight will not be obstructed by the tower platform boards. It is the responsibility of the building foreman to see that lines of sight and measurement to reference marks do not hit tower legs or other obstructions, and that necessary clearing is done or plumb benches constructed.

In certain cases (for example, when a tower can no longer be built over a station because a power line has been constructed over it), a new reference mark may be established nearby, occupied as a station, and connected by a short traverse to the original station mark.

Additional standard reference marks should be established at recovered stations where needed to insure two or more good reference marks at each station.

7. Azimuth mark.—An azimuth-mark disk is illustrated in figure 49. The disk is labeled "AZIMUTH MARK" and bears an arrow which is set to point toward the station mark. The monument is constructed in the same manner as the reference-mark monument. Each station should have an azimuth mark established not less than ¼ mile distant therefrom and in such a location that it will be visible from the ground and from the top of the tower at the station.

The principal purpose of an azimuth mark is to furnish an azimuth at each station which will be available to local surveyors or engineers from an ordinary ground instrument set-up and without the necessity of building any high towers.

Azimuth marks are most frequently placed in or near a fence line along a road which leads to the triangulation station.

8. Witness posts.—In order to aid in the preservation and to serve as a means of easy recovery of the monuments being established, a wooden post will be set adjacent to the concrete station monument or near one of the reference marks at each station, preferably at the station mark. This post shall be 4″ x 4″ in cross section, about 3½ feet in length, and shall be set to project 15 to 18 inches above the ground surface. The top should have a ¼- to ½-inch bevel. It shall be painted white and shall have lettered thereon in black the legend "U.S. Δ" or U.S. Δ R.M." These posts will be set for monuments established along public highways, in rural districts, along the rights-of-way of railroads, and along the shore lines of rivers and lakes. They need not be set for monuments established along business streets, in residential sections of cities, on the grounds of schools and churches, in cemeteries, in cultivated farm lands, or on bare mountain tops. For survey stations established in cultivated fields and marked with an underground mark, the post shall be set at a reference mark.

9. Marks of other organizations.—If a satisfactory station mark of another organization is found at the station site in good condition, it should be used without alteration as the station mark of a new Coast and Geodetic Survey station. Reference marks and azimuth mark should be placed, as necessary, to bring the station installation up to the requirements of sections 6, 7, and 8.

In case the existing mark of the other organization is not in good condition for a station mark, a new Coast and Geodetic Survey station mark should be established in the vicinity, and the mark of the other organization should be used as an extra reference mark. The stamping of additional marks should be done as indicated in paragraph 3.

Care should be taken not to displace a mark of another organization in horizontal position, or even in vertical position if there is a possibility that it could have been used as a bench mark. The mark should not be altered without permission from the organization by which it was established.

10. Setting of disks.—The location of the station, composition of the ground or presence of rock, and the availability of materials will usually control the choice of the most suitable type of setting for the metal disks. The principal settings used for metal disks are concrete monuments and drill holes in bedrock (see fig. 50) and in partially buried boulders.

It should be emphasized that the continued value of triangulation is dependent on permanence of the station marks. Special care and effort should be exerted to make each mark as permanent as possible.

The settings for reference and azimuth marks conform in general to those described below for various types of surface station marks, except that when concrete monuments are used, slightly smaller dimensions are acceptable for these marks.

A discussion of several typical kinds of marks follows:

(a) In concrete monument.—The concrete monument is normally poured in place in a hole dug in the ground, using a top form only. The hole is dug to a depth of 3½ to 5 feet (sufficient to extend below the frost line) with either a square or circular cross section (depending on shape of top form used), and about 14 inches or more in diameter, except that the lower six-inch section is made about 10 inches in diameter for the underground station mark. (See fig. 51.) The concrete is poured and tamped in the lower six inches of the hole for an underground station mark and the disk is set. A point is plumbed directly over the center of the underground mark, on a plumb bench, signal stand, or collimator (see fig. 52). This point is maintained during the pouring of the surface monument, so that the surface mark disk may be plumbed over the underground station mark. The underground mark is covered by a thin board to prevent disturbing, and then by several inches of soil. The bottom of the hole for the surface monument is enlarged about 2 inches in radius, tapering upward for about ½ foot in order to make the bottom of the monument bell-shaped. Concrete is poured and tamped in the hole until a level is reached where the top form when set on the concrete will protrude from 2 to 6 inches from the ground. The top form may be in the shape of a frustum of a cone or a pyramid, or a cylinder. It is usually made of 1″ x 12″ boards with a 1-inch batter, a 12-inch square inside cross section at top of the form and a 14-inch square at the bottom. The form should be tried for fitting into the hole before concrete is poured in order to avoid any shoulders or mushrooming effect near the top of the monument which might afford purchase for frost action. The pouring, tamping, and back-filling are completed, and the top of the monument smoothed off and beveled with a trowel. The surface disk is then plumbed into position and set in the concrete monument.

A paper cement bag may be used as a top form for a concrete monument. Use of the paper cement bag as a form has the advantage of greater economy in materials, and the smooth rounded surface is less susceptible to damage by frost or vehicles than a square top. When a cement bag is used as a top form, a cylindrical hole is dug about 14 inches in diameter and belled out as before to about 4 inches greater diameter at the bottom. The ends of the bag are trimmed, leaving about an 18-inch cylindrical section about 12 inches in diameter. After the hole is filled with concrete to within about 1 foot of the surface, the bag is set on the poured concrete and then carefully filled

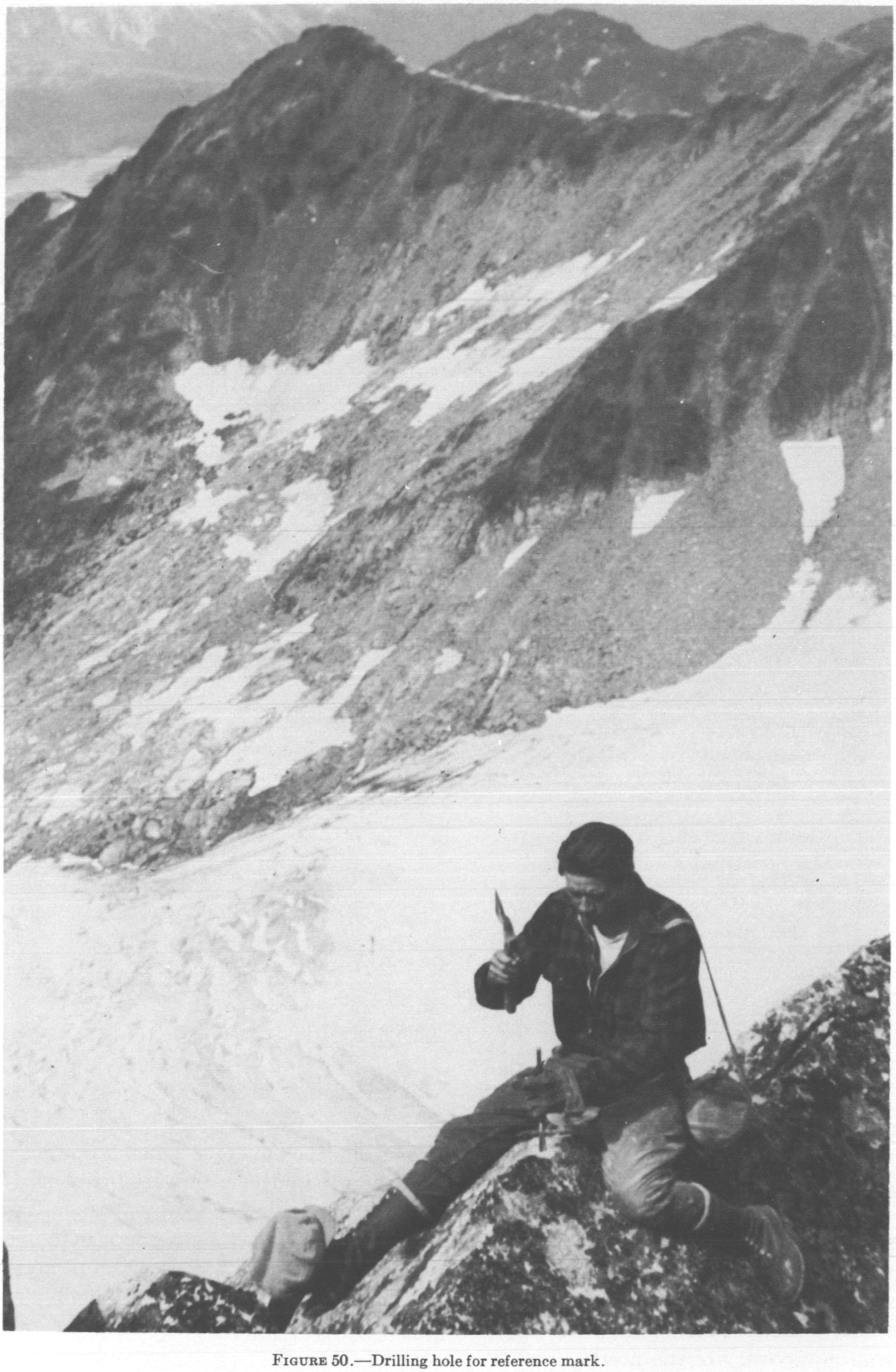

FIGURE 50.—Drilling hole for reference mark.

with concrete, working it around the edges with a trowel to prevent honeycombing. Care is necessary to keep the cross section of the bag circular and the bag vertical. A pair of cylindrical metal forms may be used for this purpose. The outer form is about 18 inches long and about 12 inches in diameter, and the inner metal form is 9 inches long and 11 inches in diameter. Both forms have a 1-inch flange around their top rims. The

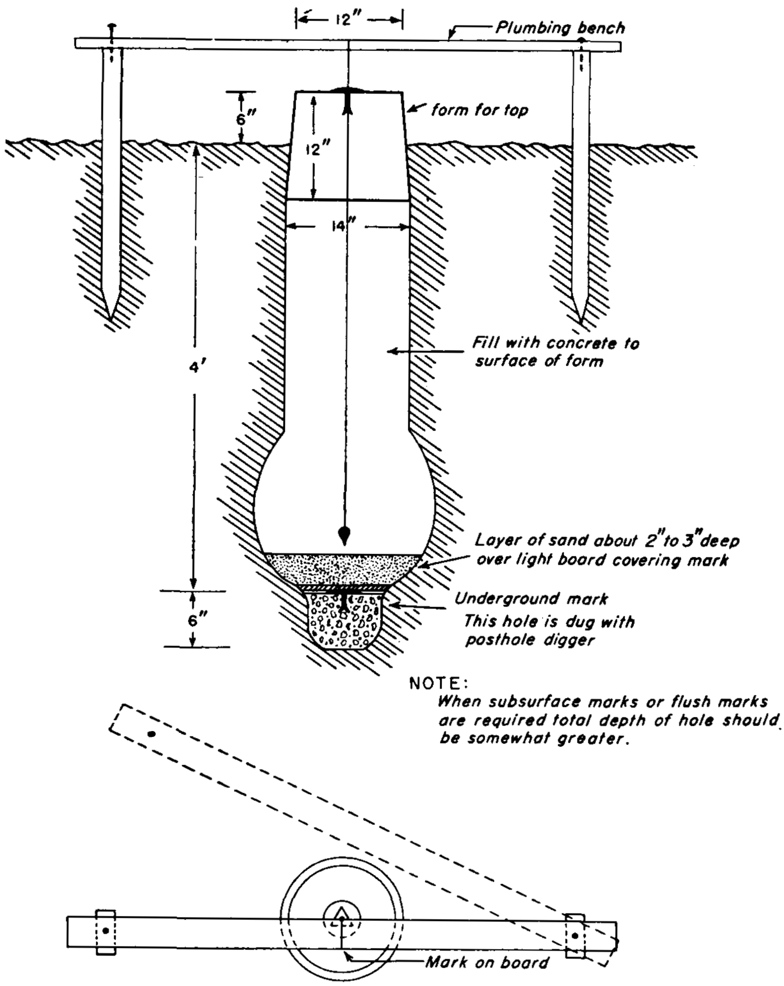

FIGURE 51.—Diagram of installation of typical triangulation station monuments.

bag is held in position between these two forms while the concrete is being poured. Immediately after the pouring, first the inner and then the outer forms are lifted off.

The same type of concrete station monument is used in land subject to cultivation, except that the hole is dug deeper; the upper monument may be made a little shorter; no top form is used; and top of upper monument is 12 to 15 inches or more below the ground surface. When the station mark is below the ground surface, small pieces of broken glass, crockery, tile, etc., should be mixed with the dirt covering the mark to assist in recovery of the mark.

FIGURE 52.—Setting plumbing bench over underground mark.

(b) In rock outcrop.—The rock in which a mark is set should be hard and a part of the main ledge and not a detached fragment. The disk should be countersunk and well cemented in a drill hole.

(c) In boulders.—The boulder should be of durable rock and as large or larger than a standard concrete monument. The boulder should be firmly imbedded in the ground. Unless the boulder is very large, a hole should be dug, and the boulder buried so as to protrude from the ground about 2 to 4 inches in the same manner as a concrete monument. In areas where boulders are prevalent, a truck and log chain can frequently be used to advantage in dragging a boulder to a point where a hole has been dug in a suitable location for a mark. The disk should be set in a drill hole in the same manner as in rock outcrop.

(d) In rock ledges below surface.—When the ledge is only slightly below the surface, a disk set in the usual manner in the ledge will be sufficient, provided two surface reference marks are established. Where the ledge is so far below the surface that a surface mark is required, a disk or copper bolt should be set in the ledge, the ledge carefully brushed or washed off for a space at least 18 inches in diameter, and a concrete surface monument placed above the underground mark. A disk should be set in the surface monument directly over the underground disk or bolt. If the rock ledge in which the underground mark is set is very smooth, it should be furrowed with a chisel to afford better anchorage for the concrete.

(e) In areas of permafrost and other places where monuments or drill holes are unsuitable.—The disk is brazed to the end of a pipe. Pipes 2 to 4 inches in diameter and 4 feet or more in length are desirable. Wrought-iron water pipes and cast-iron soil pipes are most durable. If one end of the pipe is belled or flanged, that end should be placed down. The lower end of the pipe should preferably be set in a mass of concrete. Pipe marks are used in sand areas, especially where there are drifting dunes. They are also used occasionally at pack stations where there is no suitable rock for setting the disks in drill holes.

If a mark is placed in a permafrost area, the pipe should extend through the active (freeze and thaw) layer into a good bond with the permafrost layer. It is desirable that the length of the pipe which is in the permafrost be twice the thickness of the active layer. Holes should be drilled in the lower end of the pipe and the lower part should be filled with water for an ice bond with the permafrost. Above the ice most of the upper half of the pipe should be filled with sand with a layer of peat near the surface for insulation. Sand with a peat cover should also be placed around the part of the pipe extending through the active layer.

(f) In marsh.—Tile forms for marks may be used in marsh areas. Preference should be given to the largest diameter that is practical. A wooden post which has been previously water soaked (to prevent later swelling and bursting of the tile) is forced vertically into the marsh as far as it will go, then a tile pipe or pipes are forced down to encircle the post. The tile is cleaned out and filled with concrete and a disk set in the top.

(g) In buildings.—Marks in buildings should preferably be set in drill holes in concrete or stone. The method of setting will frequently depend on the wishes of the owner and ingenuity of the mark setter. On thin roof slabs the disk can sometimes be set in a small rounded concrete monument which is poured over slanting lag screws set into the roof slab.

In cases where it is not practicable or not permissible to place a station mark in the roof, two reference marks may be placed in the parapet as close as practicable to the unmarked station and in such positions that their directions from the station intersect at about 90°. In this case, a third reference mark should be placed for a check distance.

MATERIAL FOR CONCRETE MONUMENTS

The main considerations in making concrete are: Have clean materials, mix them well before adding water, have the mixture not too wet, and tamp well into the form. No dirt should be allowed in the mixture as each streak of dirt in concrete means a line of cleavage. Where rough aggregate is available, the proportions should be 1–2–3, with the top 12 inches of the mark of slightly richer mixture. Where only cement and sand are available, the lower part of the mark should be proportioned 1 part of cement to 3 parts of sand, and the upper part should be 1 part of cement to 2 parts of sand. No reinforcement should be used because triangulation stations are frequently occupied as magnetic stations. To avoid cracking of the concrete due to rapid drying, it should be covered with paper or cloth and then with earth or other material for a period of at least 48 hours.

FIELD PROCEDURE FOR MARKING STATIONS

Marking of stations is an integral part of the work of a signal-building unit. An open-body truck is most convenient for transporting cement, sand, gravel, and water to the station. These materials are usually stocked and loaded at the base camp. A mortar box about 3 by 6 feet with a metal bottom and sloping ends (as part of regular equipment) is most satisfactory for mixing concrete although concrete is occasionally mixed on canvas by shovel or tossing. Water is usually carried in 5- or 10-gallon cans. Drums of 55-gallon capacity, fastened in trucks and with attached faucet and hose, have been found convenient for use at drive stations. A bag of cement per mark or an average of five bags for marking a complete station are used with good sand and gravel material. The metal disks should be stamped before being set in concrete. The materials of the concrete used in marks are first thoroughly mixed dry, then water is mixed and worked in until the mass has a stiff consistency. As concrete is poured, it should be constantly tamped in the holes and forms. Types of marks used are described on page 90. While pouring concrete into a hole dug in the ground, great care should be taken to avoid a cleavage plane due to dirt falling in the hole. If the ground is loose, this can be avoided by using a removable galvanized funnel with a tube approximately the size of the hole and carefully removing the funnel after pouring.

RESETTING AND RELOCATING STATION MARKS

Serial No. 632, "The Preservation of Triangulation Station Marks," describes in detail methods suitable for use when it becomes necessary to move triangulation station marks.

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OBSERVING

The methods and step-by-step operating routine of observing units of first-order triangulation parties are discussed in this section.

OBSERVING SCHEDULES

Observing schedules are posted on the party bulletin board. These schedules are prepared by the chief of party or by the computer after consultation with the chief of party. On steel-tower triangulation parties, where tower releases are an important factor in the progress of the party, schedules are usually posted daily. On mountain triangulation parties, schedules are often posted for a week or more in advance. With multiple observing units on area triangulation, daily schedules (where practical) are most advantageous, because, in cases where one or more units have obstructed lines or incompleted stations, the next day's schedule can be arranged for the most economical use of lightkeepers. A typed carbon copy of the observing schedule is furnished each observing unit and each lightkeeper.

Several different arrangements of observing schedules have been used by triangulation parties. The method shown in figure 53 is one of the briefest and least susceptible to misunderstanding.

OBSERVING SCHEDULE

Monday, 1 July 1948

O-PARTY	STATION OCCUPIED	NAMES OF LIGHT STATIONS								
		ROCKY	SUMMER	NORRIS	DALE	KORN	ZINC	HOM		
				Names of lightkeepers						
0	OREVILLE	Wilson	Black		Johns	Doe	Doe (post)			
00	BALDY	Wilson		Jarrel	Johns					
000	IRONTON	Wilson	Black	Jarrel				Richey		
0 & 00 show 000 show 0										

FIGURE 53.—Typical observing schedule for a triangulation party of three observing units. Lightkeepers named above are at stations shown at the heads of the columns and show lights to stations in second column on same line with their names. A note is also added listing personnel of O-parties whenever any changes are made in personnel.

A copy of the reconnaissance sketch mounted on a piece of soft wallboard material is helpful as an operations board in working up schedules on a multiple-unit triangulation party. The status of the work can be shown by a system of colored tacks which are kept up to date daily. For instance, blue tacks are inserted in the station symbol when the towers are built and stations are marked. A yellow tack is inserted beside the blue tack when the station is scheduled for observation. The blue tack is removed when the observations are completed. A red tack is substituted for the yellow tack when angle and side checks are completed and the tower is released for dismantling. This gives essential information at a glance. If desired, additional colored tacks can be used for other pertinent information, such as a green tack when permission to establish a station is obtained from the property owner, a white tack when a tower is delivered to the station site, and a purple tack when the tower has been torn down. Also different colors or small head and large head tacks can be used to denote wood stands and steel towers respectively.

OBSERVING PARTY PREPARATION

Observing parties should leave camp in time to arrive at the station in ample time to perform the following operations before dark: Write the description; measure the distances between the station and reference marks; measure the eccentricity; check the stability of the signal tower; collimate; set up the observing tent; set up the instrument; take vertical angles (if required); and take cuts to intersection stations, reference marks, and azimuth mark. Arrival at the station about two hours before sunset will usually allow sufficient time unless there is an unusually large number of intersection stations. If vertical angles are required, the observing party should arrive at the station earlier in the afternoon. It is the responsibility of the observer in charge of an observing unit to plan his departure from camp so as to arrive at the station in ample time to complete all his required daylight work. Before departure from camp he should have the truck fueled and checked, and should check to see that all necessary personnel, instruments, and equipment are in the truck. The observer's bag should contain the schedule, reconnaissance sketch and descriptions, a copy of the builder's report, blank record books and abstract forms, computation data for any triangles for which closures are to be obtained that night and for which his unit is responsible, and the triangulation manual. Typical observing party instruments and equipment are listed on page 278.

On most parties, observers are allowed to study the station requirements for observing and set their own time for leaving camp. If the chief of party finds that any of his observers cannot be relied upon to get on the station in time to execute all required work properly, he may find it necessary to set a definite hour for the party to be on the station. Before leaving camp the observers inform the lightkeepers, and in particular the one who is to show light from the proposed initial, of the time that the signal lights will be required.

STATION ROUTINE

NOTES FOR DESCRIPTION OF STATION

In the record books, the first set of horizontal-angle data for each station should be preceded by complete original notes for the description of that station. Each item of the description notes should be independently checked by someone other than the person writing the notes, and appropriate check marks and checker's initials should be entered in the record book to indicate that this has been done.

Enroute to the station the route distances and directions should be checked and notes made of speedometer distances for the description of station. On reaching the vicinity of the station (unless it is necessary to start observing vertical angles immediately), go to the azimuth mark, make local measurements (to roads, fences, buildings, etc.), write description notes in the record book, then post at the azimuth mark a small-type signal lamp pointed at the station. Next, drive or pack the gear to the station, check speedometer distance from the station to the azimuth mark, or pace or estimate approximate distance if necessary, make measurements to local objects such as roads, fences, houses, ditches, blazes, witness posts, etc., and record notes in the record book for description of station and reference marks. A note should be made of the exact stamping on all marks, the type of mark, the size of the monuments, and the distances they project above the ground. The writing of the final description is discussed on page 115.

Measurements to reference marks.—Measurements from the station mark to each reference mark should be carefully made independently in feet to hundredths and in meters to thousandths. A 30-meter (100-foot) steel tape marked in meters on one side and in feet on the other side is usually used. Measurements should be made horizontally if possible, using a plumb line or plumb benches if necessary (see fig. 54). When each measurement is made, it should be recorded directly into the record book. Also the check conversion computation of feet to meters should be shown. Measurements in feet and meters should be repeated until a check is obtained to 0.003 meter. A good method for obtaining completely independent measurements in feet and meters, is to hold the zero foot mark of the tape on the station mark, and read the feet, tenths, and hundredths at the reference mark. Then using the other side of the tape, hold the nearest greater decimeter mark on the reference mark, and read the set-back in centimeters and millimeters at the station mark. All original tape readings, including the set-back reading, and arithmetic computation of the final distance should be shown in the record book. Additional measurements directly between reference marks should also be made as a check, when practicable.

The horizontal distance to the reference mark is needed to compute the geographic position of the reference mark in case the station mark is lost. The horizontal distance only should be shown on the list of directions and on the upper part of the description card. If slope distance and vertical distance are also available, they should be listed on the description of station card in the paragraph on the detailed description of the reference mark.

If inclined measurements are necessary, they should be recorded in the record book as slope measurements. In order to compute the horizontal distance, the difference in elevation of the tape ends should be determined directly, or the zenith-distance angle should be measured and recorded in the record book along with the slope distance. The computation for reduction to the horizontal should be made in the record book on the pages with the measurement notes. To simplify the reduction of a slope distance to a horizontal distance, slope measurements preferably should be made mark to mark, or from benches at the same vertical distances above both marks (with no broken grades between tape ends). It also simplifies the computation to measure the zenith-distance angle to a temporary target which is at the same vertical distance above the reference mark as the telescope is above the station mark. However, if the zenith-distance angle is measured direct to the reference mark, the horizontal distance can be determined provided the height of instrument is also measured. If the tape measurement is made from the telescope center to the reference mark and the same zenith-distance angle measured, the computation of the horizontal distance is also simple; but the inclined measurement from the telescope to the reference mark should be clearly noted as such and not as a ground slope distance as is normally used in the description of station to assist in recovery of stations. Notes and sketches in the original record should clearly indicate the definite points between which measurements of slope distances and their accompanying zenith-distance angles were made.

For other short measurements usually made at the station site, see page 100 for measurement of eccentricity and page 115 for connections to marks of other organizations.

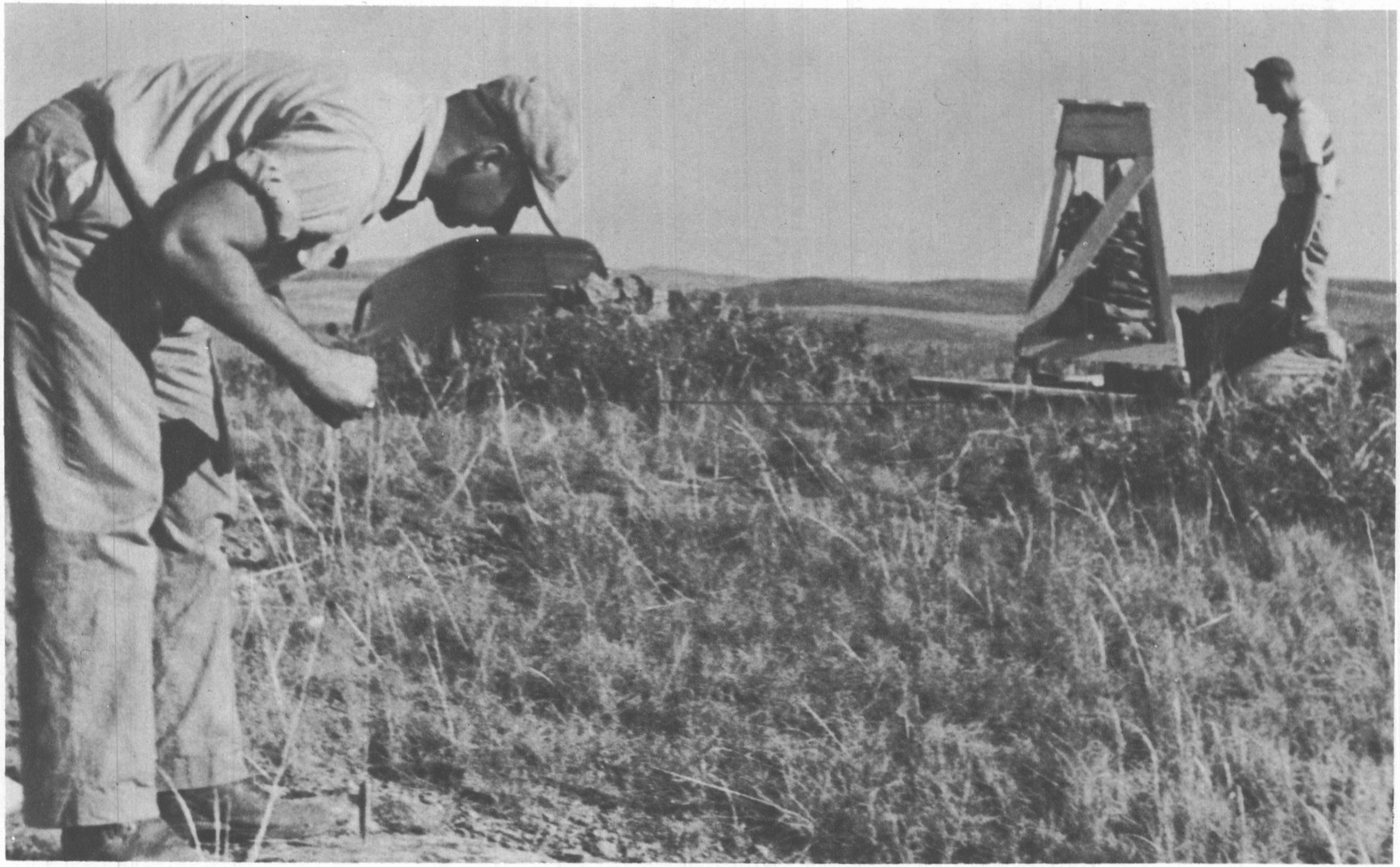

FIGURE 54.—Making horizontal measurement to reference mark.

SET-UP PROCEDURE

Measurement of eccentricity.—If a station has been previously occupied by a light-keeper, the light plate of a steel tower or the center hole of the top of a wooden stand or tripod should be checked and measured for eccentricity before anything is disturbed at the station.

A device consisting of concentric circles which are inscribed with one-half centimeter spacing on a small sheet of clear plastic material should be attached over the center hole of the tribrach plate when an observing party collimates a tower. This device facilitates: (1) The accurate centering of the tribrach plate, (2) the determination of the eccentricity of the light plate for lights previously shown, and (3) the centering of the light plate. The plastic sheet is readily centered over the center hole of the tribrach plate by eye, using the concentric circles, and may be fastened in place temporarily with scotch tape. The collimator is set up and carefully leveled over the station mark. The tribrach plate is centered by sighting on the attached concentric circles and signaling for movement of the tribrach plate until the circles are centered on the collimator cross wires. The tribrach plate is clamped in place. Next, the observer sights on the center hole of the light plate. He then signals the man on top of the tower to move a pen point just above the plastic until the point appears to coincide with the center of the hole in the light plate, at which point the tower man makes an ink dot. The tower man then climbs up to the light plate and is given signals so that he can adjust the light plate into centered position for future observations. When the observer reaches the top of the tower, he determines the eccentricity of the old light position by measuring the distance from ink dot to center and by lining in an object so that the direction of the previously eccentric light plate can be measured. The exact value for the metric distance of light from the vertical collimation line is $s + as/h$, where s is metric distance off center measured from plastic sheet, h is height of inner tower in feet, and a is distance in feet from top of inner tower to light plate (usually 10 feet on steel towers). The as/h term may often be small enough to be disregarded.

On four-foot stands and other signals which are too low to use a collimator, eccentricity is measured by using a plumb bob which is shielded from wind by a tarpaulin or the observing tent. In this case, the center hole in the top of the tripod board is plumbed down and a temporary mark made on the station-mark disk. Measurement is then made from the center of the station mark to this plumbed mark and a stake or distant point is sighted on line which is used for measurement of the angle of eccentricity after the theodolite is set up.

If any eccentricity is found, record its amount and direction in the horizontal direction record book, together with the names of all stations from which observations have been made on the signal while in this position. A sketch should always be included. (See fig. 57 on p. 109.)

When it is possible to mount the theodolite vertically over the mark it should be done, even though the light has been previously shown from an eccentric point. To have an instrument mounted eccentrically when it can be avoided causes unnecessary computation.

Checking signal for stability.—After the eccentricity has been measured, the instrument stand or tower should be checked for stability. Drive nails in tightly on wooden signals and add additional ones if needed. Investigate to see that footings are solid and that stand is not resting on loose rock or spongy earth, and that holes are cut away around legs of four-foot stands to prevent surface movements of the turf from disturbing the stand. Be sure that the outer and inner towers are not in contact. Check and tighten bolts on the inner of the steel towers, particularly at the bottoms of the two upper sections. After adjusting the upper adjustable part of the inner steel tower for height of the observer, be sure that the U-clamps securing this part are set up firmly on all three legs.

An additional check on stability, after the instrument is set up, is to point on some definite distant object and see if the pointing shows any movement while a man is moving around the instrument. It is important that the foot screws of the instrument are well seated on the tribrach plate, and that the clamps on these foot screws are tight.

Check of vertical collimation.—After checking the signal for stability, the light plate is centered with the vertical collimator which has been adjusted and leveled as described on page 42. Next, after adjusting the movable top part of the inner tower for the height of the observer, the double tribrach plates described on page 51 are centered over the station mark with the collimator, and secured in place at the top of the inner tower.

On wooden stands and towers, tribrach plates are used singly and are attached to the top of the stand with wood screws. On low towers and stands, tribrach plates are centered using shielded plumb bobs to assure verticality over the station mark.

All collimating is rechecked after adjustable parts of the signal tower are clamped securely. If a swivel ("window") section of the leg of the outer tower is moved during observations, the light plate should be recollimated before any further observations are made on that light.

During the collimation process, the observer operates the collimator, and the light-keeper centers and adjusts the light plate and tribrach plates as directed by observer. In the meantime, the recorder assembles and prepares the instruments and gear for hoisting.

Observing tent and gear on tower.—Observing tents are used to protect the instrument from the weather, principally the wind, rain, and sun. Observing tents are described on page 71.

At tower stations, after the collimation is completed, the two men on the ground hoist the tent, theodolite, signal lamps, batteries, and observing bags on the hauling line by hand. (It usually takes three loads.) The gear should be pulled inside of the tower onto the platform before unfastening from the hauling line, and then stowed securely where it will not be kicked or blown off. After the gear is hauled up, the free end of the hauling line should be hauled taut and secured to the bottom of an outer leg on the lee side of the tower, and the surplus line coiled nearby. This is done to prevent the wind from slapping the rope against the tower during observations, and also to prevent passing vehicles from becoming entangled in the free end of the rope and endangering the tower.

Signal lamps are passed up to the light plate and bolted thereon, and connecting wires are led to the dry batteries. In the meantime, the side wall of the observing tent is lashed around the rail of the tower. Then the roof section of the tent is lashed in place.

Instrument set-up and check.—After the tent and other gear are secured in place the theodolite base section is carefully removed from its box and set with its foot screws in the grooves of the tribrach plate. The telescope section is removed from its box. Wyes and pivots are cleaned and then the telescope is placed in the wyes with the vertical circle towards the A micrometer. The Parkhurst theodolite is held down on the tribrach plate by its own weight. Connect wires from two or three dry cells (depending on bulb voltage) to the binding posts on the bottom of the theodolite. An extension cord down through the upper part of the inner tower and under the platform to a battery box on the edge of the platform is convenient. Added conveniences are extension cords (with battery clips on ends of wires) from the battery box on the platform: (1) To signal lamps on the light plate; (2) to the call light used for signaling to lightkeepers; and (3) to the recorder's light. The recorder uses the instrument boxes for a seat and for a table.

The theodolite is checked to see that all moving parts work freely, that lights and electrical connections are satisfactory, and that the focuses and other adjustments are satisfactory.

The theodolite is leveled with the plate level and checked with the striding level. Both levels should be adjusted as described on pages 53 and 54. Optical adjustments of focusing, parallax, verticality of wires, and collimation should be made as described on pages 55 to 57. The micrometers should be tested and adjusted as described on page 57. All other adjustments should be made when necessary as described in section on adjustments of theodolites beginning on page 51.

Pointing signal lamps.—The observer orients the reconnaissance sketch and identifies as many stations as he can see through the telescope, and searches for other stations by turning off approximate angles which were computed from previous observations or scaled from the sketch. He has a temporary note made of directions from some zero initial to each station as soon as it is identified. (It always helps to identify all lights before complete darkness.) He then aids his lightkeeper in pointing signal lamps to all other observing parties which are to point on his station.

At four-foot stands, signal lamps shown from one observing party to other observers

FIGURE 55.—Signal lamp shown on range to a distant station.

are lined-in on range to other stations with the theodolite. These are usually shown from an extra collimator tripod or a light bench at the most favorable spot from about 15 to 100 feet from the station. (See fig. 55.) It is good practice to use a C-clamp to secure the signal lamps to the tripod or bench.

Confirming and perfecting signal lamp pointings.—As soon as it is dark enough to signal, observing parties should call all lightkeepers and send the code letter "Z" or "ZZ," etc. This is repeated in reverse to the observing parties by the lightkeepers. This serves the purposes of: (1) Notifying the lightkeeper that his light is seen and observations are to begin; (2) giving the lightkeeper a definite point on which he can perfect his pointing (perfecting pointing of signal lamps is very important because, otherwise, unknown eccentricities may be introduced by observing on the side of the reflector); (3) definitely identifying the distant station to observer (this is often a necessity in thickly settled areas where there are many other lights).

OBSERVING OF VERTICAL ANGLES

Vertical angles are measured and recorded as double zenith distances and are used to determine elevations of points by trigonometric leveling.

Vertical-angle observations are seldom required of steel-tower triangulation parties. When vertical-angle observations are required (usually in mountainous country), it is so specified in the project instructions.

The general instructions for vertical-angle observations are discussed on pages 15 and 16.

Daylight observations are usually made on tops of observing tents, tops of towers, heliotropes, and signal lamps at occupied stations; and to the tops of mountains, tops of tanks, stacks, towers, cupolas, and spires of intersection stations. Observations to the ground at a station are usually indefinite and unsatisfactory. The necessary measurements are made at all occupied stations to reduce observations to ground elevations.

Reciprocal vertical-angle observations are taken between occupied main- and supplemental-scheme stations. Nonreciprocal observations are taken to unoccupied intersection stations.

The zenith distances of both the tops and bottoms of intersection stations, such as elevated tanks and radio towers, should be measured in order to compute the heights of the objects, unless such heights are not needed or are determined by other methods.

Observing procedure, vertical angles.—The vertical-circle level is adjusted and the index error is determined as described on page 55. Verniers are adjusted as described on page 66.

Measure the heights of stand, instrument, top of observing tent, top of tower, and each light shown. All height measurements are referenced to the height of observer's station mark as zero, with "+" meaning "above" and "−" meaning "below." Measure vertical distances to the nearest millimeter. Check measurements shall be made to the nearest hundredth of a foot.

The height of an eccentric light posted on line to another station may be determined by setting the horizontal wire of the telescope on the distant station to which the light is posted, then measuring the vertical distance from the line of sight of the telescope to the center of the light, and applying this measurement algebraically to the height of the instrument. If this is not feasible, vertical-angle observations may be made on the posted light and a slope distance measured from the center of the telescope to the light, from which data both horizontal and vertical distances to the light may be computed. If the height of the posted light is determined by the measurement of the vertical distance below the line of sight, the horizontal distance from the station to the posted light should be recorded to the nearest decimeter. Inasmuch as the line of sight from the distant instrument to the posted light is not parallel to the line of sight between the two instruments, a discrepancy is introduced which may be computed. Under ordinary conditions, wherein the posted light is not in excess of 8 feet below the line of sight, the discrepancy is negligible, provided the horizontal distance to the light in meters is no more than 7 multiplied by the length of line between the two stations, as expressed in miles which usually can be scaled from the progress sketch. For example, if the line sighted over is 5 miles long, the light should be kept within 35 meters of the station. Ordinarily, this condition will be easily satisfied. In very rare cases where the line joining two stations is inclined more than 10° or 12° from the horizontal, the allowable distance to the posted light should be appreciably shortened.

When making pointings, the observer calls out to the recorder the name of the object and the part of the object sighted upon. The pointing is made with the middle of the horizontal cross wire sighted upon the object. In order to obtain separate and distinct pointings for each observation, the rule has been made that consecutive observations on a given object should never be made without intermediate reversing of instrument. Each observed determination consists of one direct (circle left) and one reversed (circle right) pointing. Three determinations of double zenith distance should be made, both of observations on occupied and intersection stations. If the range of zenith distances exceeds 10 seconds, additional observations should be made until a set of three determinations within a 10-second range is obtained. Observations may be completed on one object at a time, or may be made with the telescope in one position on several objects in succession.

After each pointing and before reading the verniers, the vernier level bubble is carefully centered with its tangent screw. Then both A and B verniers are read. The degrees, minutes, and seconds of the A vernier and the seconds of the B vernier are recorded. Care is necessary to check the minutes of the B vernier. If the minutes are greater than the A reading, add 60 to the B seconds for each minute that B exceeds A. If the minutes are less on B than A, place a bar (vinculum) over the B seconds for each minute that the B vernier is less than the A vernier. (Verniers should be adjusted for equidistance to eliminate differences of the minute base as much as practical.)

It is important to remember that the vertical pointing is made using the telescope clamp and the slow-motion screw and that the level bubble on the vernier is centered independently with the tangent screw of the vernier system before each reading.

If a type of instrument is used where the vernier level bubble is not centered with a tangent screw, the readings of both ends of the bubble should be recorded for each pointing, and the value of one division of the level should be entered at the beginning of each volume of observations.

Recording routine for vertical angles.—A sample page of record book, Form 252, "Observations of Double Zenith Distances," is shown in figure 56.

If any instrument other than a Parkhurst or Wild T-3 is used, a sketch of the vertical circle showing the zero point, the system of the graduations, and locations of A

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 252
Rev. Feb. 1936

DOUBLE ZENITH DISTANCES 45

Station *Brushy PK* State *California* Instrument *G 368* Date *4/3/47*

Observer *E.A.M.* County *Alameda*

Object Observed	Time	Level O.	Level E.	Circle Right or Left	Circle Reading °	′		Verniers A ″	B ″	Mean ″	Zenith Distance °	′	″	Remarks
Livermore East	1604			L	91	38		00	10	05				Light shown to TESLA
Base (Lite)				R	268	20		20	$\overline{30}$	$\overline{55}$				1.730 m. below line of
+26.35				DZD	183	18				10	91	39	05.0	sight
														Ht. stand 1.415m
				L	91	38		00	10	05				above sta. mk.
				R	268	20		20	$\overline{30}$	$\overline{55}$				Ht. Telescope 1.82m.
				DZD	183	18				10	91	39	05.0	Ht. Lite to TESLA = 0.09m.
				L	91	38		00	10	05				
				R	268	20	Do not write in this margin	30	$\overline{30}$	00				
				DZD	183	18	Do not write in this margin			05	91	39	02.5	
													04.2	
														r EAM

Figure 56.—Example, double zenith distances, Form 252.

and B verniers will be placed on the first page of the record book of each volume, noting whether the sketch shows circle left or circle right.

At the beginning of each day's observation at each station, record: (1) Height of the stand, (2) height of the instrument, (3) height of the top of the observing tent, (4) height of the top of the tower, and (5) height of each light shown to another observer; all referenced to the elevation of station mark being occupied as zero. Also, list the names of the observing party and weather data.

The recorder should repeat back all data to the observer when recording observations. Do not abbreviate the names of objects the first time they are used at a station. The observer should designate the part of an object sighted upon, and the recorder should record these details. Record the standard time of the first pointing on each object as a four-figure group (0945 is 9:45 a.m. and 2311 is 11:11 p.m.). If time of observations at a station continues through midnight, keep the same date in the heading and add 2400 to the time (1:30 a.m. for observations continued after midnight would be recorded as 2530 of the pre-midnight date).

When the observer calls out circle left or circle right for each pointing, place an L or R in column 5. Next the degrees, minutes, and seconds of the A-vernier reading are recorded and called back to the observer; then the B-vernier reading is recorded to the same minute base as the A vernier, using a bar (vinculum) over the B seconds if the minute reading is lower than for the A vernier. A- and B-vernier readings are meaned immediately in column 9. The telescope is then plunged and the alidade rotated 180° and a pointing again made on the same object and the vernier readings repeated, with the A vernier first in the same manner as before. On the next line is entered the DZD (double zenith distance) which is equal to the mean L reading minus the mean R reading when the vertical circle is graduated from 0° to 360° counter-clockwise (or R minus L when the circle is graduated clockwise). Divide the DZD value by two to obtain the value of the zenith distance which is then written in column ten under "Zenith Distance." This completes one determination. The same procedure is repeated until three determinations of zenith distance fall within a range of 10 seconds. Usually the observations on one object are completed before pointing on the next object, in which case the telescope is plunged between each pointing. If single pointings are taken on a round of objects in succession similar to the method of horizontal directions, the telescope, of course, is not plunged until the completion of a single round of pointings.

OBSERVING MARKS AND INTERSECTION STATIONS

Making observations to the reference and azimuth marks and intersection stations is a part of the daylight observing program at a station. If vertical angles are not required, observations to the marks and intersection points are started as soon as the observing tent and the instrument are set up and the instrument stability and adjustments checked.

A small signal lamp is usually used as a target on the azimuth mark. Various types of target can be centered on the reference marks. A vertical black line on a light-colored board or on a white tape attached to a small lead weight, or a black dot on the white head of a round match make satisfactory targets when properly centered and weighted down in place. It is frequently possible to see the mark clearly through the telescope and no target is necessary in that case. A plumb bench is frequently necessary on the azimuth mark and occasionally necessary on reference marks. Reference marks are easier to observe during daylight, but can readily be observed at night with a flash-light shining on the target.

Three positions are taken on the reference marks with those positions which are in excess of 20 seconds from the mean rejected and reobserved. Four positions are taken on the azimuth mark and intersection stations with a rejection limit of 5 seconds from the mean.

Intersection stations are usually natural and structural objects such as mountain peaks, tanks, standpipes, lookout towers, beacons, chimneys, stacks, spires, and cupolas. Most of these objects are unlighted and necessarily have to be observed during daylight hours. (See p. 13.)

It is important that all triangulation parties determine the positions of as many prominent visible objects as practicable. Location of these objects is particularly useful for photogrammetric map compilation, as landmarks for the nautical and aeronautical charts, and as azimuth marks for local surveys. Where there is a scarcity of intersection stations, observations should also be made on prominent objects which cannot be intersected, in order to furnish azimuths for photogrammetric control.

Observations on intersection points are desired from three stations if possible in order to obtain a check, but observations from two stations will be acceptable if the pointings are carefully identified. Wherever there is any choice, the nearest occupied stations which will give the best intersection should be used. Observations from the nearest stations are desirable in order that distances from them to the intersection stations may be based on direct observations. When an intersection station is within a short distance of an occupied station, the distance should be taped. Observations on an intersection station from more than three of the nearest and most suitable occupied stations are not necessary unless the object is visible from the ground. All such objects should be observed for azimuth purposes at all stations at which they are visible from the ground, regardless of whether they can be intersected from other stations or not. Observers should be constantly on the alert for all possible additional intersection stations, and should not limit their observations to objects that are shown on the reconnaissance sketch.

The same name for an object is used by all units, although only one person need visit the intersection station to write the description. A daily informal conference in camp between the observers and computers is helpful. On some parties in areas where intersection stations are numerous, one man (usually the assistant computer) is designated to canvass the area in advance of the observing, and select and write descriptions of the intersection stations. These are discussed with the observers, and scaled angles with correct names assigned are given to each observing unit prior to observing. After observations are completed the assistant computer draws a graphic plot of the observed cuts to intersection stations, which serves as a check on identity of the object and the number of suitable cuts. The basis of this temporary plot can be a sketch to any convenient scale on blank drawing or chart paper with the occupied stations plotted by computed distances taken from the preliminary triangle computations.

Observing procedure for reference marks and intersection stations.—Point on the same initial station that is to be used on the main-scheme observations, if practicable. (See p. 111.) If the principal initial station cannot be seen, use any main- or supplemental-scheme station which is convenient. While waiting for an initial to show, the azimuth mark may be used as an initial for the reference marks and nearby objects only.

With the telescope direct and pointed on the initial, the observer sets the circle for the first position (see circle settings on p. 11) and then perfects the pointing with the tangent screw. He then moves around to the A micrometer and calls out the degrees and minutes by reading the comb. To determine the backward reading in seconds, he turns the micrometer drum until the apparent right pair of wires is centered over the first graduation to the right of the center of the comb and mentally notes the drum reading. To obtain the forward reading, he turns the micrometer drum until the apparent left pair of wires is centered over the first graduation to the left of the center of the comb. He then calls out both drum readings.

In a similar manner, the lightkeeper of the observing party reads the B micrometer, calling out the backward and forward readings in seconds. Before doing so, he checks to see whether his minute reading is the same as that for the A micrometer. (Though the micrometers should have been set to read as closely together as practical when the instrument adjust ments were checked, the normal small variations between readings will sometimes throw the B reading to either the next higher or the next lower minute base.) Whenever the minute base of the B micrometer is 1 more than that of the A micrometer, 60 seconds should be added to the B reading in seconds; whenever it is 1 less, the lightkeeper should call out "barred" after the B reading in seconds so that the recorder will place a vinculum (short bar) over this reading in the record book. This system places all readings on the same recorded minute base.

The B-micrometer reader must be very careful not to touch or kick the tripod and should contact the reading light switch with a very light vertical touch. He should study the observer's movements and keep out of his way. He should be standing in position ready to read the B micrometer as soon as the pointing is completed.

Backward and forward readings are not to be meaned mentally and values called out as if they read the same or one second different. The actual reading for each pair of wires must be recorded with the (apparent) right pair of wires always recorded first (in backward column). A mental meaning of the backward and forward readings prevents the original record from indicating the true condition of the micrometers, and has a tendency to induce sloppy readings.

When the spread of the backward and forward readings exceeds the mean spread of the adjusted micrometer by over 3 seconds, the readings should be repeated. If the spread still remains, it may be due to a poor graduation mark on the circle, and readings can be tried on adjacent graduation marks. If the spread between the forward and backward readings appears to vary for different settings, the test for faulty bearings described on page 62 should be made. If a micrometer which has been adjusted for run has a consistently large spread of readings, it means that the two pairs of wires are not spaced exactly an even number of minutes apart. This will not affect the accuracy of results, but must be allowed for by the observer in determining the allowable spread. (Run adjustment is made with one pair of wires as described on p. 60.)

After completion of the above readings on the initial station, pointings and readings are made clockwise in turn on each additional object with the telescope direct; then the telescope is plunged and the alidade is rotated 180 degrees, and the pointings and readings are made on the objects in the reverse order to and including the initial object. This completes a round of direct and reversed pointings with the plate circle remaining in the same position, and is called a position.

Observations are usually started on the first position with the telescope direct. Then, successive circle positions are observed without plunging the telescope between positions.

Pointings on nearby objects, such as reference marks which require a change of focus, should be taken in a group immediately before and after plunging the telescope and not intermingled with other pointings. For instance, start with the initial and make all distant pointings, then the nearby pointings requiring a change of focus, then plunge the telescope and observe the nearby pointings, then the distant pointings, then the initial.

It is permissible to omit the B-micrometer readings on nearby reference marks when they are within 100 feet of the station.

Observations are continued in the same manner using four positions or circle settings for intersection stations and azimuth marks. Only three positions are used for reference marks if results check within the rejection limit.

Recording routine for reference marks and intersection stations.—A sample page of record book, Form 251a, "Observations of Horizontal Directions, 2-Micrometer Theodolite," is shown in figure 57. All recording is done in ink. Nothing is ever erased or obliterated. A slant mark is drawn through erroneous entries without obscuring the original entries, and a corrected entry is legibly made above the original. An explanatory note should be made for all corrections to the original recorded figures. It is essential that all recorded figures be neat and legible.

Fill in all headings on each page used. List the names and duties of each member of the observing party and note the weather conditions on the first page of each night's record, and when any changes occur during the observations. Also, make notes of the height and type of signal and other pertinent remarks which will clarify the record of

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 251a

Horizontal Directions 40

Station: MARBLE 2 Observer: J.J.D. Instrument: 320 Date: 12 July 1949

Position	Objects Observed	Time h. m.	Tel. D or R	Mic.	°	′		Backw'd ″	Forward ″	Mean	Mean D and R	Direction ″	Remarks
1.	BALDY 1939	1637	D	A	0	00		40	42				Obs. J.J.Doe
				B				51	52	46.2			Rec. W.W.Smith
													B-mike B.K. Blank
		1642	R	A	180	00		32	33				Weather: Clear and hot
				B				37	39	35.2	40.7		Wind - SE, 10 miles
													Visibility - 20 miles
	Greenville, Municipal		D	A	40	35		10	10				V.G. Ht of instr. 5.5 ft.
	Tank, (finial)			B				11	13	11.0			(4 ft wooden stand)
	(about 3 mi. N.)												
	(aluminum with		R	A	220	35		01	$\overline{59}$				
	black Greenville)			B				03	05	02.0	06.5	25.8	40° 34′
	R.M. 1 (NE)		D	A	82	13		17	17				
				B				19	19	18.0			
			R	A	262	13		02	05				
				B				03	03	03.2	10.6	29.9	82° 12′
	Az. Mk. (0.3 mi. E)		D	A	128	00		35	37				
				B				38	40	37.5			
			R	A	308	00		33	33				
				B				33	33	33.0	35.2	54.5	127° 59′
	Ecc. light (at Marble 2)		D	A	175	51							Shown to RUSH only
	(0.025 meters SE)			B									on 7/11/49 only.
			R	A	355	51							
				B									
													✓ JJD

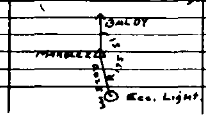

FIGURE 57.—Example, horizontal directions, Form 251a.

conditions affecting observations. The remarks column should be used freely throughout the observations.

Record the position number in the first column. Position numbers used with a first-order instrument are numbered 1 to 16 and refer to the circle settings as listed on page 11.

Names of the objects observed are listed in the second column. Complete names of intersection and occupied stations should be used on the first position of each set of observations and on the first pages of each record book. Names may be abbreviated on subsequent positions, but the abbrevation should still identify the object, such as the first syllable of the name appearing in full on the first position. Do not substitute letters or numbers for names in the later positions. The name of a station should be followed by the date of establishment if other than the current year. In general, the name of an intersection station should contain: First, the name of the locality, then the name of the owner, then the common name of the object. (A standard list of common names of objects is given on p. 283.) This should be followed by the name of the part of the object sighted upon, and a note of the approximate distance and bearing from the station to assist in identification for the triangle computations. A brief note or a sketch of any distinguishing feature will assist the observers in making identification from other stations and when the intersection station is visited for the purpose of writing a description. Compass bearings (to the nearest intercardinal point) should also be noted after the names of the azimuth and reference marks. If a magnetic bearing is shown in degrees, it should be followed by "(mag.)."

The standard time of each position should be noted in the third column in hours and minutes, recording in four figures from 0000 to 2400 beginning at midnight.

The recorder enters D's and R's (or R's and D's if position is begun with telescope reversed) alternately in the fourth column on every third line down the page.

The reading of degrees and minutes and the backward and forward readings of seconds called out by the observer at the A micrometer are written on the first line and repeated back by the recorder. Then the backward and forward readings of seconds (to the same minute base as the A micrometer) called out by the lightkeeper at the B micrometer are written on the second line and repeated back by the recorder, who immediately writes in the mean of these four readings to tenths of a second.

As successive pointings to other objects are made by the observer without reversing the telescope, the recorder writes the designations of objects and the A readings on every sixth line down the page, with B readings immediately below the A readings. Then as readings are called out in reverse order after reversing the telescope, the first of these A readings is written on the third line below the last previous A reading, with following readings being written on every sixth line up the page. This skipping of lines reserved for pointings with telescope reversed places both sets of readings on each object adjacent to each other and facilitates the taking of means.

As soon as both the mean D and R second values are obtained in column 10, their mean value is written in column 11, to tenths of seconds. Next, in order to reduce all directions to a zero initial, the mean value of the initial reading is subtracted from itself and from the mean values for each of the other objects. The seconds for the direction to each object with zero initial are written in column 12. An experienced recorder will complete this column while the observer is making the circle setting for the next circle position. Degrees and minutes for each direction with zero initial are written in the remarks column (column 13). On the first position, V. G. is entered after all observed objects which are visible from the ground (height of eye) at the station. All computations in columns 10, 11, 12, and 13 should be checked and so marked by the observer. The checker should place his initials in the lower right-hand corner of the page. A rough abstract is made at the station on Form 470, "Abstract of Directions," to facilitate obtaining means of directions to positions and determining those to be rejected and for which reobservations may be required.

OBSERVING MAIN AND SUPPLEMENTAL SCHEMES

A first-order triangulation party makes no distinction between the observing procedure of main-scheme (first-order) and the supplemental-scheme (modified second-order) triangulation. The differences in classes are introduced in the rejection limits and the criteria for angle and side checks. The observation of the full 16 positions for modified second-order triangulation provides triangulation of a higher accuracy than second-order standards with very little additional cost, since the additional eight positions require a small amount of observing time at the station in comparison to the time required for transportation, setting up the instrument, and other station routine. Observing 16 positions also usually eliminates reoccupations because of excessive triangle closures. These observations are nearly always made on signal lamps which are centered and posted or tended at other marked stations. These observations are usually started as soon after completion of the observations to reference marks and intersection stations (see p. 107) as the lights are visible. This is usually soon after sunset, and on cloudy days may be earlier in the afternoon. Each lightkeeper should be called as soon as it is dark enough, and signals exchanged for identification and perfection of pointing of lights. Lightkeepers are also directed when necessary to moderate or to brighten lights.

Sixteen positions are observed on both the main and supplemental schemes using the plate circle settings listed in table 2 on page 11. A rejection limit of 4 seconds is used for first-order and 5 seconds for second-order observations.

Observing procedure for main and supplemental schemes.—Any suitable main-scheme line may be used as an initial. The direction normally selected for the initial pointing is the left-hand line when facing directly across the scheme, or the line forming the right clockwise limb of the largest angle. Steadiness of the target and reliability for uninterrupted use of the light are prime considerations in selecting an initial. Lines liable to abnormal horizontal refraction should be avoided. Very short lines are undesirable. If a short distance line is used as an initial, small variations in the target position such as wind effect on the tower, sighting on the side of the light reflector, and local horizontal refraction are apt to cause more rejections and have a multiple effect on the accuracy of the longer lines. Observations on very long lines are more apt to cause interruptions because of the faintness of the lights, fog, and haze. A desirable initial would be at a medium distance.

Although the theodolite was completely checked and adjusted (see p. 102) prior to the observation of intersection stations, the levels and focuses are again checked before observation of the main scheme.

While pointing on the initial with the telescope direct, set the circle to the first position setting. (See position settings in table 2 on p. 11.) After setting the circle, repoint on the initial light using the tangent screw. Readings are then made using the same routine as described on page 107.

After completion of the direct readings on the initial light, direct pointings and readings are made in turn clockwise on each additional signal lamp of the schedule, then the telescope is plunged and pointings and readings made on the signal lamps in reverse order, to and including the initial light. This completes position number one. With the telescope remaining reversed, the circle is set for position two, and pointings on the initial and other lights again made in the same manner as for position one. This procedure is repeated for 16 positions for first-order triangulation. The rejected positions are reobserved.

At stations from which directions are to be observed that have an inclination of two degrees or more from the horizontal, the leveling of the theodolite should be checked with the striding level. The striding level is placed on the telescope pivots with the lower figures on the vial toward the vertical circle (left when telescope is direct) and kept in this position. Level the theodolite as closely as the plate level will permit, and complete the refinement by touching up the foot screws until the striding level reads the same in all directions of the alidade. This condition should be maintained within one division of the striding level throughout the observing period. For this purpose, the same end of the striding level should always be over the same telescope pivot.

Do not change the leveling of the theodolite during the observing of any one position. A note should be made in the remarks column whenever the instrument is releveled. The note, "levels checked," or "instrument releveled," should appear at least once near the middle of every set of 16 main-scheme observations.

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For lines which are inclined over 5° from the horizontal, striding-level readings should be recorded in the record book in addition to the precautions of maintaining the instrument level as explained in the previous paragraph. It will be necessary to read and record both ends of the bubble of the striding level only once for each pointing of the telescope over the inclined line. For these readings, the striding level shall at all times remain placed with the lower figures on the vial toward the vertical circle end of the telescope axis.

Observing should be rapid but not hurried. Do not deliberate on a pointing unless lights are vibrating unsteadily. Experienced observers under good observing conditions average about 30 to 50 seconds per direction per position.

Although every effort should be made to complete all scheduled observations at a station, failure of one or more lights should not interrupt completion of the rest of the observations at a station. Call any missing light at intervals. Messages can sometimes be sent to the missing station through other stations, or sometimes a man from the observer's station or a nearby station can be sent to investigate. Little time should be spent in waiting for a light to show. Instructions for incomplete sets are contained on page 12.

Recording routine for main and supplemental schemes.—A sample page of record book, Form 251a, "Observations of Horizontal Directions, 2-Micrometer Theodolite," is shown in figure 58.

Recording is done in the same manner as described on page 108. A sample recording of a striding-level reading on an observation over an inclined line is also shown in figure 58.

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Form 251a

Horizontal **Directions** 20

Station: MARBLE 2 Observer: J.J.D. Instrument: 320 Date: 12 July 1949

Position	Objects Observed	Time A. M.	Tel. D or R	Mic.	°	′	Do not write in this margin	Backw'd ″	Forward ″	Mean	Mean D and R	Direction ″	Remarks
5.	BALDY 1939	1845	D	A	45	00		35	37				Re-leveled, before position.
				B				36	36	36.0			Wind increased to 20mi E.
													Low haze beginning
		1851	R	A	225	00		29	31				1 div. stride level = 6.2″
				B				29	31	30.0	33.0		$\frac{6.2}{4} = 1.55$
													Level reads:
	VALE		D	A	124	47		59	61				L R
				B				61	60	60.2			6.2 21.4
			R	A	304	47		46	46				20.8 5.7
				B				48	48	47.0	53.6	20.6	14.6 −1.1 15.7
												−0.2	Vert Angle = +6°37′; tan = +0.113
	RUSH		D	A	149	10		12	13			20.4	Corr'n to direction =
				B				14	16	13.8			(1.55)×(−1.1)×(+0.113) = −0.19″
			R	A	329	10		01	02				
				B				59	59	00.2	07.0	34.0	
				A									
				B									
				A									
				B									
				A									
				B									
				A									
				B									
													√ J.J.D

FIGURE 58.—Example, horizontal directions, Form 251a.

Recording of the time of observations is a part of the data on observing conditions which is sometimes useful in comparing and evaluating different sets of observations.

Abstracts.—As a position is observed and the directions to each of the objects of that circle position are computed in the record book, these values are copied onto Form 470 "Abstract of Directions." Every direction observed at a station should appear on the abstract with rejected readings indicated by "R."

Before computing a trial mean, any direction differing from the approximate mean enough to be very evidently the result of a blunder should first be recomputed in the record book to ascertain if it is in error. If the blunder is not discovered, that position should be rejected. A trial mean of each column is then taken on Form 470. Observations which deviate more than 4 seconds from the mean for first-order lines and over 5 seconds from the mean for second-order lines are rejected and immediately reobserved. A new mean is taken and rejections and additional reobservations made, if necessary. (See below for further discussion of reobservations, and p. 125 for preparation of abstracts.)

REOBSERVATIONS

Reobservations indicated by the abstract.—If the observation of any direction shows a progressive change in value as indicated by changes in successive means of eight positions, the set should be reobserved, as changes are probably caused by variable horizontal refraction conditions.

If any observed direction is rejected and the reobservation also falls outside of the rejection limit, the error may be caused by a defect in a graduation mark of the circle. The circle can be shifted one or two graduation marks and observations again repeated.

Reobservations indicated by triangle closures.—A preliminary check of triangle closures is made on the station each night, usually by the chief observer. Any observer measuring the last observed angle of an isolated triangle usually determines the closure of that triangle.

With multiple observing units, when an observer completes all angular observations (including reobservations of rejected directions) and the abstract of directions is meaned, the lightkeeper attached to his unit will signal to the main observing unit the values in seconds of the directions necessary for the computations of triangles for which all angular measurements were completed.

If there are several angles to be sent and the initial used is not the normal initial or the initial previously agreed on for messages, then the message should begin with LF followed by the name of the initial. Normally, only seconds and tenths of seconds of directions are transmitted by signal at night. If directions from an abstract were as follows:

$$\begin{array}{rrr} 30^\circ & 45' & 25''.22 \\ 90 & 26 & 12.98 \\ 135 & 38 & 11.23 \end{array}$$

the message would be sent: NF 25 R 2 BT 13 R 0 BT 11 R 2.

The message is written down by the recorder as received and is repeated back exactly by the O-party lightkeeper.

The same procedure is repeated by the other observing units.

Computation of Triangles (Form No. 25) is partially completed before leaving camp, by inserting thereon the angles previously observed and the approximate spherical excess. The angles to complete the computations are determined from the directions observed on that night, entered on the form, and the closures are determined.

Normally, immediate reobservations of second sets of 16 positions are made by observing units for the directions involved in triangles for which the closures are in excess of the allowable average closure of 1 second for first-order or 1.5 seconds for class I of second order and 3 seconds for class II of second order. (See pp. 131 through 137, particularly p. 135, for a discussion of conditions affecting observations.)

Where reobservations are indicated, the chief observer has a message signaled to the observing unit concerned in the following form, which is repeated back to the sender: REO (followed by the name of the station or stations).

If triangles fail to close within 3 seconds for first-order or 5 seconds for second-order after reobservations and if observing conditions have not improved, stations are reoccupied on another night, usually by a different observer and instrument, and after the camp computing office has analyzed the triangle closures and side checks to determine which stations to reoccupy.

Completion of observations.—The chief observer makes the on-station decision as to when observations are completed and results are acceptable or when conditions are unfavorable for further improvement of observations.

On steel-tower parties and other parties where all units return to a base camp after each night's observations, the signal DG which means "finished for the night" is sent to all observing units, and in turn from all observing units to all lightkeepers.

On receipt of DG, each unit packs up its gear and returns to camp.

ECCENTRIC STATIONS

Small eccentricities of the light plate are measured as described on page 100.

In cases where it will not be practicable to occupy the true station in the future (see pp. 89 and 130), an eccentric station which is far enough from the center to allow room for a separate monument should be marked with a reference mark.

Where the signal is built over an eccentric station marked by a reference mark, all field computations are made for this occupied mark as though it were a new station. A traverse connection is then made to the old station center mark. The horizontal distance from the eccentric to the true station should be carefully measured in feet and meters. Except for very short distances, standardized tapes, spring balances, and tape thermometers should be used and inclination corrections determined. Horizontal direction observations of an eccentric direction (at either eccentric or true station) should be made and recorded in the same manner as for observations to reference marks. (See p. 117 for description of station of an occupied reference mark and p. 184 for list of geographic positions of an occupied reference mark.)

Directions to the azimuth mark should be observed at both the eccentric and true station provided another station can be seen from both set-ups. If observations at the true station are not possible, then, in addition to the observations on the azimuth mark from the eccentric station, four positions should be measured of the directions at the azimuth mark between the true and eccentric stations. If the true station and azimuth mark cannot be made intervisible (because of construction, for example) an additional azimuth mark should be established that will see both true and eccentric stations. If it is no longer possible to use the true station (for example, if it is under a structure but not physically destroyed) observations to the azimuth mark may be made only from the eccentric station if marked with a reference mark, and provided an appropriate explanation is made in the record book and on the description of station card.

A sketch of each eccentricity is required in the record book and on the list of directions.

TRAVERSE CONNECTIONS TO MARKS OF OTHER ORGANIZATIONS

Connections to marks of other organizations should be made with second-order accuracy. Standardized tapes should be used for traverse measurements. Short distances may be measured horizontally between plumb benches built over the marks. If plumb benches cannot be built at the same elevation, spirit-level observations should be made over the tape supports. A spring balance for applying a tape tension of 5 kilograms and a tape thermometer should be used on all distance measurements exceeding 5 meters. For distances exceeding 30 meters, stakes should be used for support of the tape ends. For distances of 100 meters or more, invar tapes and second-order traverse methods are used as described in chapter 3 for base measurements. All linear measurements should be checked by independent measurements in meters and feet.

Where distance measurements necessitate invar-tape traverse methods, the taping is done by a special traverse party which is organized by combining two or more units. Several short traverses are usually measured per day under this procedure. (See p. 235.)

OBSERVING PARTY OFFICE DUTIES

Observing units are required to turn in record books, abstracts, lists of directions, and typed descriptions of stations, all of which have been checked. For efficient completion of the observing party records, desk or table space should be provided at the base camp for the observing party's office duties. This may be in a separate tent with adequate heat and light, or preferably, in an office trailer. A portable typewriter should be a part of each observing unit's equipment.

Efficient use of stand-by time at stations in checking books and abstracts and in writing rough descriptions will materially reduce the time required in camp for office duties.

On steel-tower parties, it is frequently required that observing parties turn in their checked abstracts at night on their return to camp in order that the computers may make the triangle computations to ascertain which towers may be dismantled by the party on the following day.

Field party volume numbers for record books (Form 251a) usually start with 1 for each project. Blank volumes are numbered in advance and each observing unit receipts for each volume which is taken from the field computing office. Pre-numbering is done to prevent both duplicating of numbers and misplacing of the original records. Reference is made to the field numbers of record books on each list of directions as the source of the original data appearing thereon. When observations are continued from one record book to another, a cross reference is placed in each volume stating "continued to Vol. —." and "continued from Vol.—."

DESCRIPTION OF STATION

The description of station is one of the end products of triangulation. In order for triangulation to be of maximum use, complete and accurate descriptions of stations are of equal importance to permanently marked stations and accurate position determinations. The descriptions forwarded to the Office by field parties should be completely edited and neatly typed suitable for immediate lithographing and distribution.

"Description of Triangulation Station" card, Form 525, is used for all new marked stations and for marks of other organizations when used for the first time by parties of this Bureau. Form 525b is used for descriptions of intersection stations.

Recovery Note, Form 526, is the description form used whenever a station has been previously located and described by a party of this Bureau.

Descriptions are written and typed by one member of the observing party (usually the recorder) and completely checked by another member (usually the observer). Under normal conditions, this work should be completed the day following a night's observations and prior to departure for the next station. The description is later thoroughly edited by the computer (see p. 147) or some other designated member of the party having at least the rating of observer, and is examined by the chief of party before being forwarded to the Washington Office.

Better descriptions will result if a rough draft is written before typing. Descriptions are written from the original data contained in notes in the record book and the builder's report. (See pp. 97 and 79.) The description should not be copied from the reconnaissance notes. Any material repeated from the reconnaissance descriptions should be checked by the observing party personnel. Temporary features, such as signboards, haystacks, ruts, etc., have no place in a permanent description.

The description should be clear, concise, and complete. It should enable one to go with certainty to the immediate vicinity of the mark, and by the measured distances to reference points and the description of the character of the mark it should inform the searcher of the exact location of the mark and make its identification certain. It should include only essential details of a permanent character.

Typing of descriptions should be with black record typewriter ribbon. The typewriter type should be cleaned frequently, and the ribbon changed often enough to insure clear clean copy. Typing should not extend into the margins of the card, nor into the space scored for folding the 1948 revision of Form 525.

Names must be spelled correctly. The names of new stations as assigned by the reconnaissance party should be thoroughly checked for correct spelling. A triangulation party may change a new reconnaissance name completely if desirable. The name of a recovered station must be used as stamped on the mark even though misspelled. The correct name of a station of another organization which is used, whether re-marked or not, should be ascertained and retained, usually with the initials of that organization placed in parentheses after the name. Some typical examples of the names of marks of other organizations follow:

PT 22 (USGS)
TT 23 (USGS)
V.A. 5963 (USGS)
BALDY (USGS)
T 2 N, R 4 E, SEC. 29, NE CORNER
ROCKY (USE)
DEER (MRC)
PINEY (MORC)
BLUFF (USBR)

NO. 632 (IND. GEOD. S.);—*not* USC&GS AND SS.
CHICAGO-NEW YORK AIRWAY BEACON NO. 28.

Examples of names of marks of this Bureau which were previously established for purposes other than triangulation are: BENCH MARK P 231, GREENVILLE MAGNETIC, UNIVERSITY GRAVITY, etc.

When occupying a station previously established by this Bureau, or by another organization, it should be clearly stated what old marks are found, if and how the station was re-marked, and what new marks were established, if any. The description of the marks and monuments should include the letters and numbers used to identify the station.

Descriptions of intersection stations should, wherever practicable, be compiled from notes taken during a personal visit to the site of the object sighted upon.

If a season's work on a project must be abruptly halted because of weather conditions or priority assignments, description cards should also be furnished for all stations which are marked but not yet geodetically located. For each such station, a note should be placed on the description card stating that the station has not yet been located and listing details of any partial locations, such as directions from one or more named stations. This will also apply to any mark whose position, for any reason, is not determined.

If a reference mark is occupied and its geographic position is computed in the field, a new station description card on Form 525 should be furnished for this mark, and a recovery note on Form 526 should be furnished for the station mark to which the occupied mark refers. The word "eccentric" should not be used in the heading of the description card or on a list of geographic positions in connection with the name of the station or reference mark. (See p. 86.)

Description card Form 525.—Examples of descriptions of stations are shown in figures 59 and 60.

The principal parts of an original description are: (1) Station name and heading data, (2) numbered notes, (3) box data, (4) detailed description of locality, (5) directions to reach station, and (6) details of individual marks.

(1) Station name and heading data.—The name should be exactly the same as it appears in the stamping on the mark, the record books, lists of directions, and the computations. The correct name of a station of another organization, whether re-marked or not, should be ascertained and retained, with the initials of that organization placed in parentheses after the name. The blank spaces for the name of the State, county, and chief of party are self-explanatory. The year to be entered on the card is that in which the position is determined. If a previous date is stamped on the mark, that fact should be noted in the body of the description. Heights of light and telescope above station mark are entered to tenths of meters when vertical angles are observed, otherwise to even meters.

(2) Numbered notes.—The numbers used under the word "NOTE" refer to standard descriptions of marks on pages 121 and 122. In order to make all new descriptions independent of reference to numbered notes, the types of marks should also be briefly described in the body of the description, and where no standard note applies to the type of mark used, the description should be written in detail.

(3) Box data.—This part of the description card as indicated in the box heading is for distances and directions to the azimuth mark, reference marks, and prominent

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Form 525
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DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: BROWN STATE: Missouri COUNTY: Blank

CHIEF OF PARTY: W.X.A. YEAR: 1948 Described by: F.F.E.

NOTE.*	HEIGHT OF TELESCOPE ABOVE STATION MARK 27.7 METERS.†				HEIGHT OF LIGHT ABOVE STATION MARK 30.5 METERS.		
1a 7a	Surface-station mark, Underground-station mark	DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION					
	OBJECT	BEARING	DISTANCE feet	DISTANCE meters	DIRECTION‡		
	BALDY				0	00	00.0
	Greenwich, Mun. Water Tank	SW	(10 miles)		40	34	26
desc.	RM 1	SW	98.43	30.001	42	03	29
12c	RM 2	NW	13.01	3.965	137	51	17
16a	Az. Mk.	SE	(0.2 mile)		317	51	17.2

Detailed description:

10 miles northeast of village of Greenwich, in the NE 1/4 sec. 2, T6N R24W, in Walnut Ridge district, 1½ miles north of Walnut Ridge Mission Church, 1½ miles north of State Highway 201, 0.2 mile west of County Road D, about ¼ mile south of Walnut Creek, 0.2 mile northwest of house and barnyard of Samuel T. Brown, on summit of middle and highest of three rocky knobs.

To reach from post office at Greenwich, proceed north on US Highway 435 for 5.6 miles, turn right, east, on State Highway 201 for 7.1 miles, turn left north around Walnut Ridge Mission Church, onto county road D for 1.4 miles to S.T. Brown's red brick house, twin silos, and large barn on left, west, side of road. Follow winding lane 0.2 mile through barnyard and northwest across pasture to top of highest rocky knob and station.

Station mark, a standard disk in a concrete monument, stamped BROWN 1948 is 98 feet east of electric power line, 10 feet south of wire fence and 3 feet south of white 4" x 4" witness post. Monument projects 3 inches from ground.

Reference mark 1, a standard disk stamped BROWN No. 1 1948, is in northeast concrete footing of skeleton, steel, power-line tower No.E-19B2.

Reference mark 2, a standard disk set in a drill hole in a boulder and stamped BROWN NO. 2 1948, is in fence line 90 feet east of power line.

Azimuth mark, a standard disk set in a square concrete monument and stamped BROWN 1948, is opposite entrance to Mr. Brown's lane 35 feet east of center line of County Road D, and 20 feet south of a 6-foot drainage ditch.

*Refers to notes in manuals of triangulation and state publications of triangulation. ‡Direction-angle measured clockwise, referred to initial station.
†To nearest meter only, when no trigonometric leveling is being done. 16—50392-1 U. S. GOVERNMENT PRINTING OFFICE

FIGURE 59.—Example, description of triangulation station, Form 525.

objects which can be seen from the ground. A monumented station should be listed as an object for the zero initial. If a station is available which is V.G. (i.e., visible from the ground), it should be given preference as an initial for the directions listed in the box. It should then be followed by "V.G." (No other names in the box need be followed by "V.G.," since other objects would not be listed there unless they were visible from the ground.) The directions from the zero initial should be entered in the last column for all objects listed. The full name of one or two most prominent intersection stations visible from the ground and also nearby marks which are connected by traverse should be listed. The distance to the azimuth mark should be entered to the nearest tenth of a mile and may be estimated if necessary, but should then be labeled "approx." Horizontal distances to reference marks and nearby marks of other organizations are listed as measured to thousandths of meters and to hundredths of feet. The bearing to the nearest of the eight points of the compass is entered by the letter abbre-

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Form 525
Rev. Aug. 1948

DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: STANDARD STATE: ALASKA COUNTY:

CHIEF OF PARTY: A.N.S. YEAR: 1942 Described by: J.I.K.

NOTE.*	HEIGHT OF TELESCOPE ABOVE STATION MARK 1.3 METERS.†				HEIGHT OF LIGHT ABOVE STATION MARK 1.4 METERS.
2 / none	Surface-station mark, / Underground-station mark	DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION			
	OBJECT	BEARING	DISTANCE feet	DISTANCE meters	DIRECTION‡ ° ′ ″
	LINCOLN				0 00 00.0
12a	RM 1	E	13.24	4.035	37 25 52
17a	Az. Mk.	SE	(0.3 mile)		100 21 37.2
12a	RM 2	SW	29.00	8.839	150 16 27

Detailed description

Station is about 27 miles west and 2 miles south of Fairbanks, 3½ miles north and 3½ miles west of Standard section house which is on the Alaska Railroad at mile post 439.5. The station may be seen from a point about 0.1 mile east of the section house, on the highest peak on the skyline at magnetic bearing 281°, and it appears as the middle and most distant of three peaks. The station is on a small peak on the extreme south end of a prominent ridge lying west of Standard Creek. The top of the ridge is bare except for a few small bushes about 4 feet high.

To reach from Fairbanks, go by rail about 30 miles southwest to Standard section house at milepost 439.5. From a point about 100 yards west of section house, follow a tractor road, northwest for about 2 miles to its end and about ½ mile from the top of a heavily wooded peak. Continue northwest through the timber, to top of peak. Bear right and go north along a ridge, through a very thick growth of small spruce trees, for about 1½ miles to the base of a second peak. Bear left, northwest, skirt the base of the peak, passing through a heavy growth of alder and birch, for about a mile to a small saddle between second peak and another to the north. Go directly to top of peak to the north and the station. This is a 4-hour pack. To reach azimuth mark from station go south-southeast to first knoll.

All marks are standard disks set in drill holes in outcropping bedrock.

Station mark is stamped STANDARD 1941. The outcrop of quartz rock is about 1 foot square and projects about 4 inches. No underground or witness marks were established.

Reference mark 1 is about 1 meter lower than station on southeast slope of peak. It is stamped STANDARD NO 1 1941. The outcrop of quartz rock is about 2 feet square and projects 6 inches.

Reference mark 2 is about 1 meter lower than station on southwest slope of peak. It is stamped STANDARD NO 2 1941. The outcropping bedrock is about 1 foot square and projects 3 inches.

Azimuth mark is about 0.3 mile south-southeast of station, on the highest part of a slight knoll, about 25 feet lower than station. Knoll is brush-covered with a few small birch trees growing on southeast slope. Mark is stamped STANDARD 1941. Rock is about 1 foot square and projects 2 inches.

* Refers to notes in manuals of triangulation and state publications of triangulation. ‡ Direction-angle measured clockwise, referred to initial station.
† To nearest meter only, when no trigonometric leveling is being done. 16—56392-1 U. S. GOVERNMENT PRINTING OFFICE

FIGURE 60.—Example, description of triangulation station, Form 525.

viation (as N, NE, E, SE, etc.). Ordinarily all bearings should be referred to the true meridian, but magnetic bearings may be shown if labeled "(mag.)."

(4) Detailed description of locality.—This part of description begins by referring to the distance and direction from the nearest well-known mapped geographic feature, usually the *nearest* town or post office, but sometimes a local district, bay, mountain,

or reservation. From this sometimes distant point the definite objects and features to which the location of the station is referred are progressively localized down to the immediate vicinity of the station. In typical cases, this part of a description will take the form of the distance in miles and the nearest cardinal or inter-cardinal point of the compass of the station site from the nearest town, then from a school or church or crossroads store, then from nearest highways, railroads, bridges, shorelines, rivers, or other prominent features, then from fences, ditches, power or telephone lines, culverts, farm roads, houses, barns, wells, then any particular feature of immediate location, as cultivated field, rocky hill, bare knoll, or wooded hill. If station is on a hill or ridge that has a local name, that should be given. If a ridge is a long one with a number of knobs, the description should identify the particular knob by a designation such as north, highest, middle, etc. In general, the rule should be followed of working from the general to the specific. Distances in this part of the description should be in miles or in feet. Detailed measurements which appear elsewhere in the description should not be repeated in this paragraph. Points of the compass should be spelled out. The name of the owner of the property should be stated.

(5) Directions to reach the station.—This section is one of the most useful parts of a description. It usually enables a stranger to go directly to a station without the delay due to a detailed study of maps or of making local inquiries. It is a route description which should start from a definite point, such as (a) the post office of the *nearest* town, or (b) the *nearest* intersection of named main highways (which are shown on commonly used road maps), or (c) some definite and well-known geographical feature. (A description of station is no place for detailed directions of how to get from one town to another.) Where motor vehicle transportation can be used, distances should be given in speedometer readings to the nearest tenths of miles. The general directions of travel should be given. Turns from one road to another road should be indicated by the kind of turn (right or left) followed by a point of the compass (spelled out). The place of the end of truck travel should be mentioned. The approximate time required for packing should be noted. If pack horses are needed, a place where they can be obtained should be mentioned. If travel to the station is by boat, the place of landing should be indicated. This section should also describe the route from the station to the azimuth mark. If a station is placed on a prominent or unique point which is well known in the vicinity, and to which there are mapped roads, directions on how to reach it need not be given.

(6) Details of individual marks.—This section should give a short description of the station mark, each reference mark, and the azimuth mark. The exact stamping on each mark, the amount it projects above the surface or is covered by the ground, and the type of mark should be stated. The detailed location of each mark should include measurements from definite objects in the vicinity such as witness posts, center lines of roads, fences, ditches, power poles, prominent trees, wells, houses, barns, cairns, and shorelines. If slope distances were measured, they should be stated in this paragraph. Horizontal measurements listed in the box section should not be repeated here.

In certain cases, such as in isolated sections of Alaska, the recovery of station marks may be greatly facilitated by sketches indicating station sites in relation to topographic details. Whenever the occasion warrants it, a sketch should be made in black ink on Form 525, but this should be supplemental to (not a substitute for) the description in words.

Standard numbered notes for description of marks.—The following notes have been used for many years in published descriptions and other publications of the Coast and Geodetic Survey.

Surface marks

Note 1.—A standard triangulation-station disk set in the top of (a) a square block or post of concrete, (b) a concrete cylinder, (c) an irregular mass of concrete.

Note 2.—A standard triangulation-station disk cemented in a drill hole in outcropping bedrock, (a) and surrounded by a triangle chiseled in the rock, (b) and surrounded by a circle chiseled in the rock, (c) at the intersection of two lines chiseled in the rock.

Note 3.—A standard triangulation-station disk set in concrete in a depression in outcropping bedrock.

Note 4.—A standard triangulation-station disk cemented in a drill hole in a boulder.

Note 5.—A standard triangulation-station disk set in concrete in a depression in a boulder.

Note 6.—A standard triangulation-station disk set in concrete at the center of the top of a tile (a) which is embedded in the ground, (b) which is surrounded by a mass of concrete, (c) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (d) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Underground marks

Note 7.—A block of concrete about 3 feet below the ground containing at the center of its upper surface (a) a standard triangulation-station disk, (b) a copper bolt projecting slightly above the concrete, (c) an iron nail with the point projecting above the concrete, (d) a glass bottle with the neck projecting a little above the concrete, (e) an earthenware jug with the mouth projecting a little above the concrete.

Note 8.—In bedrock, (a) a standard triangulation-station disk cemented in a drill hole, (b) a standard triangulation-station disk set in concrete in a depression, (c) a copper bolt set in cement in a drill hole or depression, (d) an iron spike set point up in cement in a drill hole or depression.

Note 9.—In a boulder about 3 feet below the ground (a) a standard triangulation-station disk cemented in a drill hole, (b) a standard triangulation-station disk set in concrete in a depression, (c) a copper bolt set with cement in a drill hole or depression, (d) an iron spike set with cement in a drill hole or depression.

Note 10.—Embedded in earth about 3 feet below the surface of the ground (a) a bottle in an upright position, (b) an earthenware jug in an upright position, (c) a brick in a horizontal position with a drill hole in its upper surface.

Reference marks

Note 11.—A standard reference-mark disk, with the arrow pointing toward the station, set at the center of the top of (a) a square block or post of concrete, (b) a concrete cylinder, (c) an irregular mass of concrete.

Note 12.—A standard reference-mark disk, with the arrow pointing toward the station, (a) cemented in a drill hole in outcropping bedrock, (b) set in concrete in a depression in outcropping bedrock, (c) cemented in a drill hole in a boulder, (d) set in concrete in a depression in a boulder.

Note 13.—A standard reference-mark disk, with the arrow pointing toward the station, set in concrete at the center of the top of a tile (a) which is embedded in the ground, (b) which is surrounded by a mass of concrete, (c) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (d) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Previously used notes 14 and 15 referred to seldom-used types of witness marks and are purposely omitted.

Azimuth marks

Azimuth-mark notes are almost identical to reference-mark notes 11 through 13, which have been previously used for azimuth marks. The following numbers 16, 17, and 18, are to refer specifically to azimuth-mark disks.

Note 16.—A standard azimuth-mark disk, with the arrow pointing toward the station, set at the center of the top of (a) a square block or post of concrete, (b) a concrete cylinder, (c) an irregular mass of concrete.

Note 17.—A standard azimuth-mark disk, with the arrow pointing toward the station, (a) cemented in a drill hole in outcropping bedrock, (b) set in concrete in a depression in outcropping bedrock, (c) cemented in a drill hole in a boulder, (d) set in concrete in a depression in a boulder.

Note 18.—A standard azimuth-mark disk, with the arrow pointing toward the station, set in concrete at the center of the top of a tile (a) which is embedded in the ground, (b) which is surrounded by a mass of concrete, (c) which is fastened by means of concrete to the upper end of a long wooden pile driven into the marsh, (d) which is set in a block of concrete and projects from 12 to 20 inches above the block.

Description of station tied in by traverse.—Stations which are located by short traverse ties from nearby triangulation stations may be described on Form 525b (fig. 61). The words "Triangulation Intersection" are X-ed out in the heading of the form and the word "Traverse" is typed above. The body of the description should include a statemen of the distance and direction to the nearby triangulation station.

DEPARTMENT OF COMMERCE
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Form 525 b

TRAVERSE
DESCRIPTION OF [illegible] STATION

NAME OF STATION: TT 57 DWP (USGS)

CHIEF OF PARTY: C.A.E. YEAR: 1946 STATE: Indiana COUNTY: Washington

Description, including sketch of object:

2.0 miles south of the junction of State Highway 66 and U.S. Highway 150 (which junction is 0.2 mile east of Hardinsburg), on top of the timbered divide east of Honey Creek, 24 feet west of the center line of Highway 66, and 5.5 feet east of a power line pole with P.B.M. painted on it.

A traverse connection was made to triangulation station HONEY, distance being 81.70 feet (24.902 meters) east of station HONEY. The mark is a standard U.S.Geological Survey disk stamped TT 57 DWP 1946, and is set in a concrete post which projects 4 inches.

Described by C.B.A.

U. S. GOVERNMENT PRINTING OFFICE 16—[illegible]

FIGURE 61.—Example, description of station (traverse connection), Form 525b.

Description of intersection station, Form 525b.—The station names of intersection stations should contain first the name of the locality, then the name of the owner, then the common name of the object. (See pp. 283 to 285 for lists and definitions of standardized common names of objects.) The description of an intersection station should include its location with relation to nearby roads, towns, and other geographic features, and the ownership of the object observed. (See fig. 62.) If the definite point observed

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Form 525b

DESCRIPTION OF TRIANGULATION INTERSECTION STATION

NAME OF STATION: SALEM, WASHINGTON COUNTY COURTHOUSE, CUPOLA

CHIEF OF PARTY: C.A.E. YEAR: 1946 STATE: Indiana COUNTY: Washington

Description, including sketch of object:

The apex of the Washington County Courthouse Cupola, located in the town square in the central part of Salem. The structure is built of stone and is about 100 feet high.

Described by J.D.

U. S. GOVERNMENT PRINTING OFFICE

FIGURE 62.—Example, description of triangulation intersection station, Form 525b.

cannot be briefly and accurately described, a sketch should be made of the feature or structure with the point observed on labeled. Distinguishing features which identify the object should be included in the description and sketch. The height above the ground of prominent intersection stations, such as radio masts, in excess of 250 feet in height, should be determined to within the nearest five feet. This can sometimes be done from an adjacent occupied station by observing the zenith distances of the base and the highest point of the object. Special trips should be made to intersection stations by one of the observing parties designated by the chief of party for the purpose of obtaining the above information whenever it cannot be obtained in the regular trips to and from camp.

Recovery note, Form 526.—An example of a recovery note of a triangulation station is shown in figure 63.

Recovery notes should be written on Form 526 for stations previously established by this Bureau and for stations established by other organizations if these stations have been used and described previously by the Coast and Geodetic Survey. This will include all occupied and unoccupied stations recovered or searched for, giving whatever information is available.

The contents of a recovery note depend on whether conditions in the immediate area of a station, or along the route traveled to reach it, have changed, as well as on the completeness and accuracy of the previous description. Information for reaching and identifying a station and reference marks should be correct, specific, and adequate. Always state which marks are recovered and the stampings thereon. Also, if any changes are made by your party, make a clear statement of what these changes are. Differentiate between changes of marks or markings found and those made by your party. A statement such as "the station mark was set in concrete" (when originally described as in a boulder) may indicate an error in the original description, a change in condition found by your party, or a change in condition made by your party.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 526
(Rev. Feb. 1948)

RECOVERY NOTE, TRIANGULATION STATION

R

NAME OF STATION: MILLER
ESTABLISHED BY: G.A.F. YEAR: 1889 STATE: Indiana
RECOVERED BY:* C.A.E. YEAR: 1946 COUNTY: Jackson

Detailed statement as to the fitness of the original description; including marks found, stampings, changes made, and other pertinent facts:
The station marks and one stone reference post were recovered in good condition. A new surface mark, two new reference marks, and a new azimuth mark were set. New marks are standard bronze disks set in concrete as described by notes 1a and 11a.
A complete description follows:

Station is about 2½ miles south of Brownstown, on the highest hill in vicinity, about 200 feet west of an apple orchard, 50.88 feet west of center of Skyline Lookout Tower, and 33 feet north of the lane which leads to the lookout tower.

To reach from the junction of U.S. Highway 50 and State Highway 250 in south Brownstown, go west on Highway 50 for one block, turn left, south, on a gravel side road and go south 3.5 miles to Skyline Lookout Tower and station on left.

Underground station mark is an earthenware pyramid about 6 inches square, about 3 feet underground, and bears the raised letters U.S.C.S. Surface station mark projects 3 inches and the disk is stamped MILLER 1889 1946.

The one remaining stone reference post is 38.48 feet north of the station. It is 4 inches square at the top and about 30 inches long. The

* Name of chief of party should be inserted here. The officer who actually visited the station should sign his name at the end of the recovery note.

U. S. GOVERNMENT PRINTING OFFICE NOTE.—One of these forms must be used for every station recovered. 16—26488-2

Card 2 of 2.

Station: MILLER 1889-1946 State: Indiana County: Jackson

post projects 2 inches and the top of the mark bears a chiseled arrow which points toward the station.

Reference mark 4 is 84 feet south-southeast of the lookout tower. The mark projects 4 inches and the disk is stamped MILLER NO 4 1946.

Reference mark 5 is 41 feet northeast of the lookout tower. The mark projects 6 inches and the disk is stamped MILLER NO 5 1946.

Azimuth mark is 19 feet southwest of the road which leads to the station and 3 feet southeast of a white witness post. The mark projects 6 inches and the disk is stamped MILLER 1946.

Object	1946 Distances		1946 Directions
SMART	meters	feet	0 00 00.0
Reference mark 4 S	29.292	96.10	39 57 27
Azimuth mark W	0.4 mile		143 35 08.8
Stone reference post N	11.73	38.48	226 05 54
Reference mark 5 E	30.068	98.65	307 13 39
Skyline Lookout Tower E	15.51	50.89	327 02 11.6

Described D.L.W.

FIGURE 63.—Example, recovery note, Form 526.

When a new measurement differs from an old one, the correct value should be verified before leaving the station. If the old value was not correct, a statement to this effect should be made on the recovery note. The distances should be measured twice, once in meters and once in feet. The two measurements should then be compared, and if not in agreement remeasurements should be made.

If a station is recovered exactly as previously described and all marks are in good condition, the distances and directions are verified, and the description is entirely adequate, a statement to that effect will be sufficient on the recovery note. If the station is removed, re-marked, reinforced, reset, replaced, or new reference marks are established, this information, with a brief description of the new marks, reference measurements and directions, and stampings on the marks must be submitted. If the original description is inadequate or changes have taken place since the station was established, such as new roads, real estate development, erection of new structures, changes in property ownership, changes in natural topography due to storms, erosion, excavation, improvements, etc., the recovery note shall contain a complete new description of the station, such as would be written for a new station.

If a complete new description (on Form 526) is warranted, this should follow the notation of important changes. Do not copy old descriptions or recovery notes, especially distances and directions to reference marks, etc. In addition to being misleading, any typographical errors appear to be corrections.

The finished recovery note, as furnished the Office should be correct as to data, grammatically correct, clear, concise, and in sufficient detail to insure that no factual material is left out which would aid future recovery.

Checking of descriptions by observing party.—The typed descriptions are checked by a member of the observing party other than the man who wrote the description. Descriptions should be checked from original notes and record books for factual data.

The following should be checked: The completeness and arrangement of the data; the grammar and spelling; and all data for consistency, including feet with meters, directions with compass bearings, and spelling of proper names. The checker should place a small check mark by each figure and direction checked. This mark should not obscure or otherwise interfere with any of the typing. The check mark should be made with blue pencil or blue ink which is unlikely to be conspicuous on the lithographed copies. After completely checking the typed description, the checker should also place a check mark followed by his initials at the end of the description.

Descriptions are further edited in the field computing office before being forwarded to the Washington Office. (See p. 147.)

ABSTRACTS

Abstract of directions, Form 470.—An example of an abstract of directions is shown in figure 64.

Final abstracts are made in ink after the record books are completely checked. Directions from the record book for all completed positions are copied onto the abstract. Only such observations as those on a wrong object or those resulting from a kicked tripod are rejected in the record books and not entered on the abstract. All other rejections of observations are made on the abstract and indicated by the letter R. Rules for rejecting observations are listed on pages 149 to 151.

Normally, the observing party turns in to the camp office a separate checked abstract for each set of original observations. The combining of sets is usually a duty of the field party computers.

Form 470, Abstract of Directions, is filled in as follows: Make entries of all headings in the blanks as indicated. Write the name of the initial station over the initial column. Write names of successive observed objects at the heads of each of the next columns. Below the name of each observed object write the degrees and minutes of the direction to that object referred to zero initial. These degrees and minutes should be checked from several positions in the record book, usually from the first, fifth, ninth, and thirteenth

DEPARTMENT OF COMMERCE
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Form 470
Ed. Oct., 1932

ABSTRACT OF DIRECTIONS

State Colorado

Station MARBLE 2 Computed by W.S. Date 12 July 1949

Observer J.J.D. Checked by J.J.D. Inst. No. 320

DO NOT WRITE IN THIS MARGIN

POSITION NO.	STATIONS OBSERVED							
	BALDY	VALE	RUSH					
	(INITIAL) 0° 00′	° ′ 79 47	° ′ 104 09	° ′	° ′	° ′	° ′	° ′
	″	″	″	″	″	″	″	″
1	0.00	19.3	31.0 (30.1)R					
2	0.00	22.4	35.4					
3	0.00	19.1	31.0					
4	0.00	21.6	31.4					
5	0.00	20.4	34.0					
6	0.00	19.8	34.3					
7	0.00	22.2	32.9					
8	0.00	21.2	34.9					
9	0.00	19.1	34.1					
10	0.00	23.0	35.1					
11	0.00	18.3	37.1					
12	0.00	19.9	35.4					
13	0.00	20.3	36.0					
14	0.00	24.5 (28.3)R	36.0					
15	0.00	20.6	$\left.\begin{matrix}37.6\\38.0\end{matrix}\right\}37.8$					
16	0.00	21.0	36.8					
Sum,		16) 12.7	16) 73.2					
Mean,		20.79	34.58					
Cor. for ecc.,								
Direction,								

FIGURE 64.—Example, abstract of directions, Form 470.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 (Ed. Dec. 1945)

ABSTRACT OF ZENITH DISTANCES

Vol. 11, p. 34

Station BRUSHY PK State California

Observer E.A.M. Instr. G-368 Height of Stand 1.41 m.

DO NOT WRITE IN THIS MARGIN.

Date 1947	Hour	Object Observed	Object above station =o (Meters)	Telescope above station =t (Meters)	Diff. of heights t−o (Meters)	Reduction to line joining stations (″)	Observed Zenith Distance (° ′ ″)	Corrected Zenith Distance (° ′ ″)
1/17	1600	TESLA	1.35	1.81	0.46	-8.7	91 24 27.5	
		(light)					25.0	
							30.0	
							91 24 27.5	91 24 18.8
1/17	1615	DOOLAN	0.00	1.81	1.81	-32.4	90 46 57.5	
		(light)					57.5	
							57.5	
							90 46 57.5	90 46 25.1
4/3	1604	LIVERMORE	26.34	1.81	-24.53	+404.7	91 39 05.0	
		EAST BASE					05.0	
		(light)					02.5	
							91 39 04.2	91 45 48.9

Date	Light Shown to Station	Height of Light* Above Station (Meters)	Date	Light Shown to Station	Height of Light* Above Station (Meters)
1947					
1/17	TESLA	0.09			
1/17	DOOLAN	0.25			
4/1	LIVERMORE EAST BASE	1.78			

*Height of Light (or object above station) should also be entered on Abstract of Zenith Distances of station to which light was shown.

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FIGURE 65.—Example, abstract of zenith distances, Form 29.

positions. The seconds and tenths of seconds for each position are then entered from the record book (Form 251a) into the column headed by the name of the observed station and on the line of the same number as that of the position (circle setting). If any observations are so far from the approximate mean as to be very evidently the result of blunders, these should be rejected before taking the trial mean. Two or more acceptable observations on one position number are meaned. The seconds for 16 positions in each column are then summed using the mean value or the single observation value on each position. The sum is divided by the number of positions used (usually 16) for a trial mean. (See p. 149 for summary of rejection rules.) Using a 4-second rejection limit for first-order triangulation, any observations (including the individual ones forming a position mean) which differ by more than 4 seconds from the trial mean are rejected and R is placed by them. The rejected positions are reobserved. Using a new mean, the 4-second rejection limit is again applied to the observations. If any of these observations fall outside the rejection limit, these are also rejected and new reobservations and new means taken until all accepted positions are within 4 seconds of the final mean. The above rules should be rigidly applied. Care should be taken not to force the observations. Normally under good observing conditions with experienced observers and well-adjusted first-order instruments, there is a very small percentage of rejections.

The following is a summary of rejection limits:

First-order—4 seconds,

Second-order—5 seconds,

Azimuth marks—5 seconds,

Reference marks—20 seconds, and

Intersection stations—5 to 10 seconds, depending on sharpness and nearness of object.

Abstract of zenith distances, Form 29.—An example of an abstract of zenith distances is shown in figure 65.

Neat abstracts are made in ink or typed by each observing unit after the record books are completely checked.

The headings, and the data in columns 1, 2, 3, 5, and 8 are entered by the observing unit from the original data abstracted from the record book (Form 252). The date and hour are the date and hour of observations. The hour entry is a four-figure group in hours and minutes. The full station name plus the part of the object on which pointings were made (as light, top of O-tent, finial, etc.) should be listed in column 3, for all objects observed from the station named in the heading of the form. The observing party enters the height of their own instrument on this form. A space is provided on the lower end of the sheet for abstracting the dates and heights of all objects and lights shown to other stations from the station named in the heading of the form. The data in column 8 are the zenith distance angles as computed in the next to last column of the record book. Three values which fall within a range of 10 seconds must be observed for each object. These values are meaned in column 8 of Form 29. Data in columns 4, 6, 7, and 9 are entered or computed later in the field computing office.

LIST OF DIRECTIONS, FORM 24A

An example of a list of directions is shown in figure 66.

A list of directions is typed (usually in duplicate) by each observing unit as soon as practicable after the record books and abstracts are checked, and (where parties return to

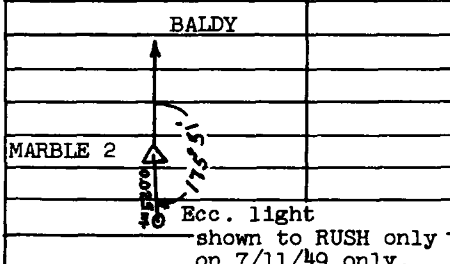
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 24A
Rev. Oct., 1953

LIST OF DIRECTIONS

Sheet 1 of 3.

Station MARBLE 2 | State Colorado | Recorded in Vol. 25.
Chief of party C.B.A. | Date 7/12/49 | Computed by J.J.D.
Observer J.J.D. | Instrument 320 | Checked by W.W.S.

OBSERVED STATION	Observed direction	Eccentric reduction	Sea level reduction*	Corrected direction with zero initial	Adjusted direction*
	° ′ ″	′ ″	″	° ′ ″	′ ″
BALDY 1939	0 00 00.00			0 00 00.00	
VALE	79 47 20.79				
RUSH (ecc.)	104 09 34.58				
MINOR	171 16 37.93				
MARBLE 2 (ecc. light)	175 51				
(0.025m.SE) 7/11/49					
#Greenville, Municipal					
Tank, (3 mi. N.)	40 34 27.2				
R.M. 1, (NE) 98.42 ft.	82 12 30				
30.001 m.					
Azimuth mark,	127 59 55.1				
(0.3 mi. E.)					
R.M. 2, (SE) 13.01 ft.	175 51 08				
3.965 m.					



* These columns are for office use and should be left blank in the field.
# Objects visible from the ground.

FIGURE 66.—Example, list of directions, Form 24A.

the base camp) before departure for the next station. The original copy is forwarded to the Washington Office after completion and editing by the party computer. It is usually advantageous to retain a duplicate in the party files until the end of the project.

Separate lists of directions may be submitted for each night's observation for the use of the field computer. This is an administrative matter for the decision of the chief of party. The field computer prepares for the Washington Office a combined list where necessary with rejected directions marked "R." He will also attach to the abstracts any analysis sheets showing how observations were combined in the field, in particular for those cases involving more than one initial or a shift of the initial.

The list of directions on Form 24A is prepared as follows, with most of the data obtained directly from the checked abstract of directions on Form 470.

The data in the heading are copied from Form 470. If the instrument is eccentric, the name of the station should be followed by "(ecc.)" in the heading of this form. If a reference mark is occupied instead of the center mark, and if it is planned to carry all field computations through the occupied reference mark, with a traverse tie to the center station mark, the notation "(ecc.)" should be omitted, and the name of the occupied station in the top of the form will be that of the reference mark, such as "BALDY RM 2 1935." The field volume number of the record book is recorded in the heading.

In column 1 the name of the initial is listed on the first line. On successive lines, the names of all objects observed from the station are listed in clockwise order. Names of the first-order main-scheme stations are listed in capital letters. Names of supplemental stations are listed in lower-case letters. Names of intersection stations, azimuth marks, reference marks, traverse ties, and eccentric points are listed in lower-case type. Names of these are followed by the approximate distances of azimuth marks in tenths of miles, and double-measured distances of reference marks, eccentric points, and traverse ties in feet to hundredths and in meters to thousandths. Each other object which is visible from the ground at the station is labeled "V.G." or marked in accordance with the explanatory note to this effect placed on the form. Names of eccentric objects should be followed by the notation "(ecc.)."

In column 2 under "Observed direction" is entered the value of each direction obtained from the mean of the positions on Form 470. If the instrument is eccentric or if any of the objects observed are eccentric, it is a good plan to enter the seconds for these observed directions in pencil until after the eccentric reductions have been applied. Directions to first-order main-scheme stations are listed to hundredths of seconds. Directions to supplemental (modified second-order) stations which have been observed with from 12 to 16 positions with a first-order theodolite are also listed to hundredths of seconds. Directions to intersection stations and azimuth marks are listed to tenths of seconds, and reference marks and other nearby objects are listed to seconds. Eccentric points if very close may be listed to minutes. Directions on lists of directions for main-scheme stations on second- and third-order triangulation are listed to tenths of seconds.

Entries in columns 3 and 5 are usually made by the field party computer. (See pp. 154 and 156.)

A sketch showing lines to the initial and eccentric points with names of the objects, angles, and distances labeled, is drawn on the lower part of Form 24A for each station at which the instrument is eccentric or at which the light or target shown to another observing unit is eccentric. The sketch should be followed by notes with a full explanation of dates and stations for which the eccentricity was effective.

Since the list of directions is the basis of all computations, it should be very carefully checked. A small check mark should be made for each entry checked, and the man checking the list should place his initials in ink in the blank indicated. If the list is copied, the copy-checker's initials should also be placed on the copy.

If an identifiable point is located especially for photogrammetric use, it should be called a substitute station. For example, in column 1, enter "Substitute Station," the compass bearing, and the taped distance, and enter the observed direction in column 2 in degrees and minutes. Near the bottom of the list of directions, give a brief description, such as "Substitute Station is the intersection of the center lines of State Highway 54 and drainage ditch and is 15 feet lower than the station."

OBSERVING PRECAUTIONS

Because of the high standards of accuracy required for first-order triangulation, constant precaution is necessary to counteract as much as possible all the sources of error.

The chief of party should inspect periodically the performance on station of all observing units.

To summarize operational methods previously discussed, refer to the following: (1) Checking signal and instrument for stability (see p. 100). Lack of stability can make all other refinements useless. (2) Instrument adjustment tests (see pp. 51 through 64). Particular attention should be given to errors of parallax and inclination of the plate circle, as these errors are not eliminated by the observing program. Since the effect of an inclined plate circle increases with the inclination of line of sight, striding-level readings are required as described on pages 111 and 112. (3) Make careful measurements of the eccentricity and apply corrections (see pp. 100 to 114). Careful pointing of the signal lamps eliminates eccentricities introduced by observing on the side of a lens or reflector. (See p. 103.) (4) When all other known precautions have been taken, one of the principal causes of reobservations described on pages 113 through 114 is horizontal refraction. Sometimes higher signals will reduce effects of horizontal refraction, but frequently the only solution without altering the scheme is reobserving under different atmospheric conditions. (5) Other operational precautions for accurate observations are: Repoint on the initial after each circle setting; check the plate level frequently; and protect the instrument from the wind, sun, and weather with an observing tent. Do not disturb the instrument during observation of a position by releveling, or by lateral thrust on a clamp, tangent screw, electric switches, or by striking the instrument or its support. (See pp. 111 and 134.)

PRINCIPAL SOURCES OF ERROR IN HORIZONTAL DIRECTION MEASUREMENTS

A good observer is one who can consistently secure results commensurate with the possibilities of the instrument which he is using. Proficiency can be attained only by a careful study of the instrument, by constant exercise of good judgment, and by making a careful study of all the factors affecting the accuracy of theodolite observations. Effort should be made to eliminate all known sources of error even those very minute. Due regard must be taken of the relative importance of the different classes of errors. It is possible to spend considerable time centering the theodolite exactly over the mark but fail to tighten the leveling screws. It is also possible to be very careful in perfecting all the instrumental adjustments, but omit a careful testing of the stability of the support for the theodolite, though the errors from the latter source are potentially very serious.

It must be borne constantly in mind that it is futile to eliminate the factors which cause minute errors and neglect at the same time gross errors which may be simply the result of haste and carelessness.

The actual pointing of an instrument on an object is a simple operation. It is a mistake to try to perfect a pointing after the perceptions are once satisfied that the object has been centered between the wires. The most satisfactory observations are usually those which are made rapidly and methodically but not carelessly. Speed can be attained by the observer by training himself in deftness of movement in manipulating the instrument and in studying how he can perform the manipulations with the fewest movements.

It is very difficult to secure the required accuracy unless the theodolite is sheltered from the direct rays of the sun and also protected from wind of any considerable strength. Temperature changes in the instrument are particularly prejudicial to accurate results. If the temperature inside the theodolite case differs greatly from that of the outside air, it is well to give the theodolite time to assume the temperature of the air before beginning the main-scheme observations. The intervening time can be used to advantage in measuring to reference marks or in making the observations upon intersection stations.

Aside from blunders, such as incorrect readings and inaccuracies of pointing upon an object, there are four principal causes of errors in the measurement of horizontal angles after properly sheltering the instrument from sun and wind. These are instability of instrument support, instrumental errors, phase and eccentricity, and horizontal refraction. The relative importance of these factors will vary with the field conditions encountered.

Instability of support.—It is essential that the theodolite be supported in a manner which will maintain the orientation of the bearing surfaces of the graduated circle and the alidade axis while the observations are being made; a first-order Parkhurst theodolite is set with foot screws in the grooves of an aluminum tribrach plate which is rigidly screwed or bolted to the top of the observing stand or inner tower. The bottoms of the leveling screws should be checked to see that they are well seated in the grooves.

Clamp screws for tightening the foot screws in the instrument base are always provided, and they must be tightened after the leveling of the instrument is perfected.

The cause of large errors frequently lies in the support of the theodolite. The leveling screws should always fit tightly into the arms of the theodolite base, and if a portable instrument tripod is used, the legs must be clamped firmly to the tripod head. The observer must school himself to step wide around the tripod or stand upon which the instrument rests and to be careful that no lateral thrust from the weight of his body shall be transmitted through loose rocks or by other means to the tripod legs. When observing from stands or towers which have been erected for some time, in which nail holes have become large and worn and the wood has lost its springiness, or bolts of steel towers loosened, it is very difficult to secure good results. Before observing, particular care should be taken to tighten the fastening of both steel and wooden towers which have stood for some time.

A proper footing or ground support for a stand is frequently hard to secure. It must be sufficiently firm so that the theodolite may be kept level and maintained in azimuth. Any effects of the observer's movements must not be transmitted to the instrument. Before beginning observations and after the instrument has been adjusted and leveled, the telescope should be pointed upon some well-defined object and the pointing watched closely while some other person steps around the instrument as near to the feet of the tripod legs as the observer would usually stand. If there is a movement of the wires with respect to the object, or if the levels of the theodolite show any change, the footings of the tripod must be made more firm. Separately supported platforms or footboards are used around a first-order theodolite for support of the observer. If the instrument must be mounted upon the same structure which supports the observer, such as a tank or building, two observers and an optical-reading direction theodolite or a repeating instrument may be necessary to obviate the effects of an unstable support. Each observer stands so as to face one of the stations to be observed upon, and each makes the pointing upon his assigned station with as little shifting of his weight as possible.

Instrumental errors.—The adjustments of the theodolite and the effects of the errors resulting from lack of perfect adjustment have been described on pages 51 through 66. These errors may be summarized as belonging to two classes: First, those which may be practically eliminated by a proper observing program, and second, those which cannot be so eliminated. To the first class belong: (1) The errors due to lack of proper collimation adjustment, (2) the unequal spacing of reading microscopes, (3) small eccentricities of the graduated circle with reference to the vertical axis of rotation of the reading microscopes, (4) the lack of horizontality of the horizontal axis of the telescope (standards out of adjustment), and (5) the errors of graduation of the instrument, though the last error is only partly eradicated by the distribution of the readings about the circle. The methods by which these errors are eliminated are stated in the section on adjustments beginning on page 51.

Among the instrumental errors which cannot be eliminated are those due to parallax and to lack of horizontality of the graduated circle. The effects of these errors are described in the section relating to the adjustments of the theodolite. There are other errors, however, which although real are much harder to evaluate.

The changes in an instrument due to changes of temperature are such that even a small theodolite should be protected at all times from direct sunlight and wind if good results are to be obtained. The effects of these temperature changes are manifested in various ways. A graduated circle will expand on the side next the sun, and there will be a differential change in the eccentricity of the circle which will not be eliminated. One side of the telescope will expand faster than the other, with a consequent change in collimation adjustment. Standards will change in their relative elevation, the collar of the diaphragm will change its adjustment, and since all of these are changing in amount from moment to moment, no system of observing will eradicate them. This is one of the principal reasons why rapid pointings upon the object give more accurate results than slow pointings.

If friction in the centers is causing errors, they can usually be detected by reading upon the initial four times for each position, that is, at the beginning and end of each half-position, instead of only twice, as with the usual method. If the movement of the telescope alidade has dragged the graduated circle around with it slightly, a series of such readings will indicate it. In Parkhurst theodolites, there is no contact between the moving alidade and the horizontal circle.

If there is a lack of fit in the centers or a yielding in the support of the theodolite, it can often be detected by the same method of observing. Its effect can be lessened by slowly swinging the telescope through rather a wide arc in a clockwise direction up to the initial to begin a position, in order to force all yielding parts to their extreme limit of motion in that direction. The same procedure is followed in pointing upon the other stations. After reversing, the telescope is brought up to each pointing by swinging in the opposite direction. If the telescope is swung past a station, it is brought back and brought up to it again in the proper direction. It is a good plan to sight back upon the initial on two or three positions at each station in order to obtain information regarding the presence of drag. These verification readings to close the horizon should not be used in computing directions.

Another element which is often not appreciated is the manner of manipulation of the instrument. In an endeavor to make rapid pointings, instrumental errors are sometimes introduced which are larger than those the observer is seeking to avoid. The hand should not rest heavily upon the instrument at any time. Slow-motion screws and the graduated heads on the screws of reading microscopes should be turned with a true rotary motion without lateral thrust. Slow-motion screws should always be tested to see if there is any dragging when the screw is turned in the direction which decreases the tension of the spring. If there is any sensible dragging and the cause cannot be found and corrected, then the final motion of the screw must always be made against the spring. Tests for drag and for looseness of the centers have already been described.

Phase and eccentricity.—A target or instrument is said to be eccentric when its center is not in the vertical line passing through the point to which the observations are referred. The proper correction can always be made for eccentricity if the distance and direction to the true station are recorded. Often, however, the observer desires to make an estimate of the error which would be introduced by an approximate amount of eccentricity due to phase or some other causes. This can easily be made by remembering the approximation "a second is a foot at 40 miles" (actually at 39.065 miles), or that an inch represents about 3¼ seconds at a distance of 1 mile. On short lines, the improper centering of either the theodolite or the target will introduce large errors. Warped or crooked signal poles must be observed upon with care, and an equal amount of care used in testing them for eccentricity. The safest plan is to point always at some certain part of the signal, such as at the bottom of the target, and to plumb that particular point over the station. Upon reaching a signal which has been observed upon, it should be tested for eccentricity. Any eccentricity found should be recorded in such a manner that the computer can be certain of the facts when correcting the observations.

When using lamps on short lines, care must be taken to point the lamps directly toward the observer, because the glass in front of the light is sufficiently illuminated to be visible to the observer, and failure to point directly on the line will affect the accuracy of the observation. Occasionally, a heliotrope or signal lamp must be shown from a point which is a short distance from the station and on range with the distant station at which an observer is working. Errors due to eccentricity of the light are very apt to occur when this is done unless extreme care is taken to line-in the light and to maintain it in position. A theodolite, if available, should be used to line-in the light; otherwise a plumb line should be used. The relative accuracy with which the light must be placed on line depends upon the distance from the observer's station, as indicated in the preceding paragraph.

When observations are made on targets or other objects in daylight, the errors in horizontal-angle measures due to phase, or to the unequal illumination of a target, are often of considerable magnitude. In effect, it is an eccentricity which could be corrected for if its exact amount were known. The difficulty lies in the inability of the observer to estimate its amount accurately, for it depends upon factors which change rapidly.

The angle of the sun with the line of sight, the opacity of the signal, the shape of the object sighted upon, and the intensity of the sunlight, will each have its effect on the appearance of the signal.

Trigonometric formulas for the correction of phase have appeared in some text-books. These are based upon the direction of the sun and are not usually practicable to apply because other factors enter in. The apparent penumbral zone lying between the surface having a full illumination and that having no direct sunlight upon it will vary in width with the intensity of the light. The formulas would also apply only to cylindrical or spherical objects, whereas many observations are made upon squared timbers. A target made of signal cloth will show a different phase from one made of lumber of the same shape and dimensions. For these reasons, the best rule, when the outlines of the signal can be seen, is for the observer to make a close examination of the signal through the best telescope available and decide upon what part of the illuminated surface it is necessary to observe in order to eliminate errors due to phase. If the outlines of the target cannot be seen, a can or other object placed at some distance away in the direction of the signal will show what part of it is illuminated and will give a rough idea as to how to point upon the object. It should be remembered that most of our telescopes are inverting.

Horizontal refraction.—The rays of light which pass from the object observed upon to the theodolite of the observer may curve horizontally as well as vertically. Horizontal refraction causes an error which is hard to detect and for which a correction cannot be applied. It can, however, be avoided in large measure by carefulness in the reconnaissance and in the selection of observing conditions. The existence of this horizontal refraction cannot always be foreseen, but certain atmospheric and topographic conditions operate strongly to cause such errors, and some of these will be briefly pointed out.

Air strata are generally of greater density near the ground and lie roughly parallel to it. Over a sloping terrain these strata of different densities are not horizontal, and a ray of light passing through them will be bent horizontally as well as vertically. The greater the difference in density in the air strata passed through, and the more they are inclined to the horizontal, the greater will be the horizontal bending of the light rays.

The most potent cause of this variation of density is the unequal temperature of different strata of the air. The force and direction of the wind are also determining factors, for with a strong wind the differences in the temperatures and densities of adjacent air strata are less marked, and horizontal bending of the rays of light is less apt to occur. A condition frequently met with on triangulation is that which is encountered when the line of sight passes along a bluff or mountain slope. Under these conditions, it may be necessary to make observations in overcast weather or when the wind is blowing toward the bluff, if the first set of observations gives indications of horizontal refraction. If the wind blows down a slope and across the line of sight, the hotter or colder air from the slope will often cause trouble. A line passing near a building, a stone wall, or even the brace of a signal tower may suffer a like change in direction. The nearer the cause of the disturbance to the observer the greater will be the angular distortion.

A line of sight passing over level terrain may also be refracted horizontally by the influence of variations in the vegetation conditions under the line of sight. Its influence can be understood by considering an extreme case in which the line of sight passes over a region covered with forest, except at one place where there is a V-shaped plowed field, with its apex beneath the line of sight. With a gentle breeze blowing across the field toward the line of sight the light rays will be bent horizontally, because the heated air from the field will have a different density from the atmosphere adjacent to it on the sides and will form a vertical triangular prism. When the line of sight passes partly over a body of water, such as the edge of a lake or a river, there is apt to be horizontal refraction.

Because of all these factors, night observations are more accurate than those made by daylight, although some errors due to similar causes may be introduced into night observations. Care must be taken in placing the light used by the recorder if it is of a type which gives out considerable heat. If the observer is sighting through a column of heated air in the immediate vicinity of the instrument, refraction errors must be expected. Trucks or tents under or near the line of sight may also cause appreciable errors, due to bending of the line of sight.

At times with horizontal-refraction errors present, a single night's observations may exhibit a very small range in the observed directions and there may be nothing in the immediate results of the observations to indicate that the line is in error. At such times, the triangle closures alone furnish a guide to the source of trouble.

The errors caused by horizontal refraction may be of considerable size. Night observations have occasionally been found to be in error by five or six seconds because of horizontal refraction and daylight observations by two or three times that amount. Undoubtedly, smaller errors due to this cause are frequently present but are more or less masked by errors due to other causes. If a line is suspected of having horizontal refraction, it should be reobserved, if practicable, under very different atmospheric conditions, especially with the wind in the opposite direction from that prevailing during the first observations.

Other conditions affecting observations.—As long as the objects are visible there should be no hesitation in observing upon them, even though the observing conditions are apparently unfavorable. An unsteady object or a flickering or flaring light does not usually produce inaccurate observations. It is only when there is a semi-permanent displacement or persistent creeping of the image of the object sighted upon that observations must be made more slowly or stopped altogether. This movement may be readily detected by centering the image upon the wires of the diaphragm and watching it closely for a minute or two. Occasionally, the amplitude of this movement may reach 10 or more seconds within a period of two or three minutes of time. Under such conditions, it is not possible to secure satisfactory observations. When the period of vibration is a very short time interval, the mean location of the image can be estimated.

The fact that the different observations upon a station exhibit a considerable range does not necessarily mean that their average value will be far from the true one, nor does a small range conclusively indicate the absence of some constant error. If the methods and instruments used do not give the results sought, the observer must systematically investigate the possible sources of error and locate the trouble. If the error is due to horizontal refraction it may suffice to elevate the instrument or some of the lights. If the refraction error is due to seasonal winds it may be advisable to proceed to other stations and to reoccupy the station affected when the wind conditions have changed, though that is usually expensive. Above all, the observer must not try to force the observations by sighting upon a different part of the object from that which his judgment indicates is the proper point. A poor triangle closure may be due to an error at any one of the three stations involved. He must cultivate an impersonal attitude toward his results and always make pointings and read the micrometers without bias.

When daylight observations are being made in an unsteady atmosphere, the observer may point upon the wrong object. This kind of blunder is particularly easy to make when sighting through a vista cut through timber which frequently renders observing conditions very unfavorable. The error caused by a mistake of this kind is usually easy to detect, but it may make a reoccupation necessary. A light or signal partly obstructed from view by intervening objects may cause an error of several seconds. Occasionally changes in vertical refraction will cause a light or target to be partly obstructed which at other times is entirely clear.

LIGHTKEEPER PROCEDURES

A lightkeeper's duties usually require that he work alone. Sometimes on mountain triangulation, he must be prepared to take care of himself for days or weeks at a time. A lightkeeper's duties, instruments, and equipment are described in detail in "Instructions for Lightkeepers," Special Publication No. 65, which lists the three fundamental rules for a lightkeeper as follows:

(1) Know the code.
(2) Know your instruments and take care of them.
(3) Get your light through.

In most areas of continental United States, where the principal method of triangulation party transportation is by truck, each lightkeeper may be assigned a truck and may return to the base camp on completion of the night's work.

It is essential that each lightkeeper have a good working knowledge of all his instruments and be able to use the lightkeeper signal code before being sent out alone. He should learn and practice the International Morse Code at every opportunity until he becomes proficient. When first employed, he should be sent out with an experienced lightkeeper to learn the routine of his job. It is desirable to assign less experienced lightkeepers single-light stations at first and gradually work them into caring for multiple lights and tending to posted lights. It is desirable to hold occasional practice sessions for lightkeepers in camp. The foreman lightkeeper and the chief of party should inspect all lightkeepers' performance on station periodically.

LIGHTKEEPERS' PREPARATION FOR DAY'S WORK

Lightkeepers obtain a copy of the observing schedule from the camp office. They should have a correct reconnaissance sketch and descriptions for the stations assigned. The chief observer should be consulted for information as to the time when lights or heliotropes will be required. Each lightkeeper should have sufficient lights and batteries for the schedule and see that all his equipment is in good operating condition, that spare bulbs are on hand, and that the truck is serviced.

STATION PROCEDURE

A tarpaulin is used at low-stand stations while checking the plumbing of the center hole for lights over the station mark center. On high towers, the lightkeeper ascertains that the light plate is fastened securely to the top of the outer tower but does not shift its position or attempt to plumb it.

The sketch is oriented with a pocket azimuth compass as described on page 51.

Signal lamps are tested and set up as described on page 45.

The pointing of lights to the observer's station is checked by sighting a card in the beam at arm's length, or by setting a stake on line at ground stations. It is important to check both direction and tilt of the beam. Lights are turned on well before dark, sometimes in the middle of the afternoon, at times previously instructed by the chief of party or requested by the observers. (See fig. 67.) Usually as many batteries as the bulb will stand are used in daylight. Batteries are gradually reduced in number as darkness approaches.

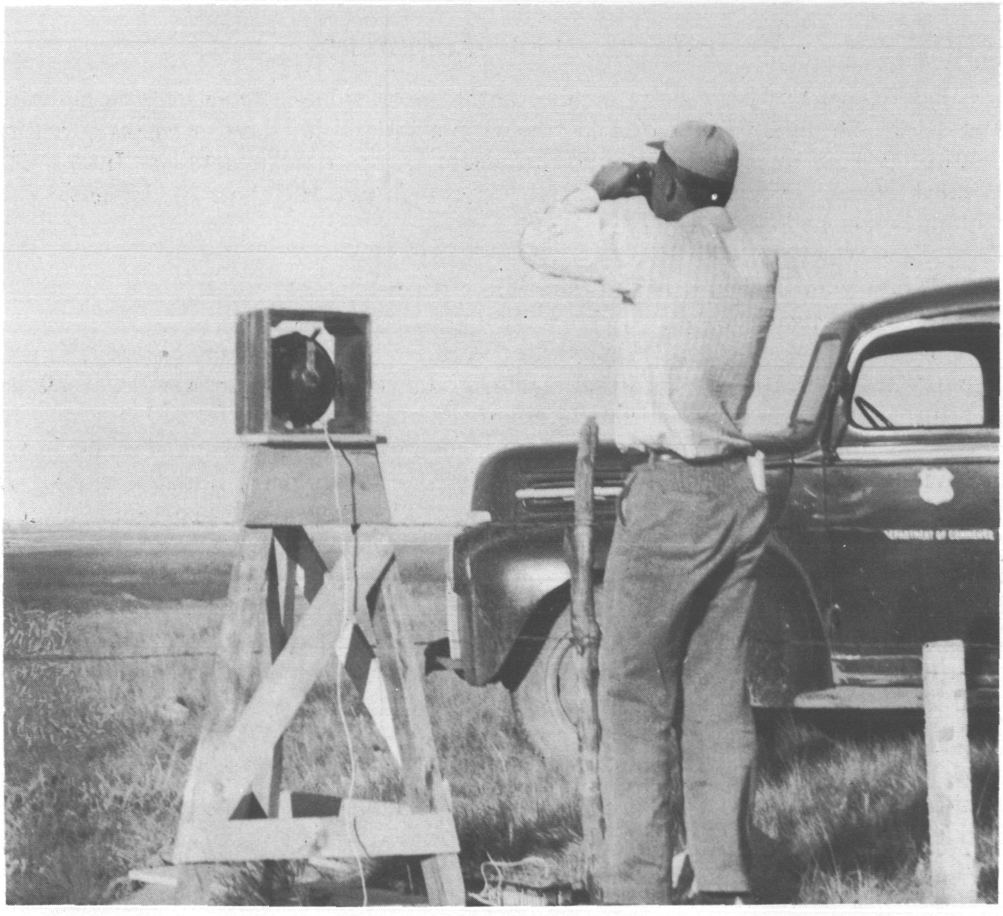


FIGURE 67.—Lightkeeper on station.

As soon as it is sufficiently dark, observing parties will send call letters to all lightkeepers. The lightkeepers check their pointings and repeat the call letters backwards to each observer. It is important to remember that a light pointing off in azimuth may cause an unknown eccentricity by reason of the observer pointing on the side of the reflector. The observer may send code letters to moderate or to brighten the light. Lights should be dimmed by removing battery cells and not by tilting the beam.

Eccentric signal lamps.—Except for four-foot stands, lightkeepers should show lights from the signal as they find it, as long as the light plate is secured firmly in place. The observer should be informed if the light is known to be eccentric.

It sometimes happens that the light from the station center is obstructed to the observer, but if the light is moved a few feet to one side or the other, it can then be seen. In this case, the light should be set up where it can be seen and the point marked temporarily so as to remain identifiable. The observer is informed that the light is eccentric and of its approximate distance and direction. Then when the station is occupied the eccentric point is accurately measured.

POSTED LIGHTS

Posted lights are used when the number of lines to be observed exceeds the number of lightkeepers available. Posted lights are frequently necessary in area triangulation, where a lightkeeper may have one or more posted light stations to attend in addition to his occupied station. Posted lights are used most successfully on short to medium length lines where the observer's station is visible by daylight for accurate pointing. It is desirable that stations for posted lights be selected along the same road or in the general vicinity of the lightkeeper's other lights, so that he can readily return to the posted light if he is sent a message that it requires additional attention. Only the most experienced lightkeepers should be directed to post lights, since the light must be exactly pointed to avoid eccentricities.

HELIOTROPES

Heliotropes are sometimes required for the daylight observations, particularly on initial lines. See page 50 for description of heliotropes.

HEIGHTS OF LIGHTS FOR VERTICAL ANGLES

In areas where vertical angles are being observed, data are required concerning the height of each light. Each lightkeeper turns in daily, to the field computing office, a notation showing the name of the station, date, and vertical number and type of light shown each observing unit. The light attached to the light plate is called No. 1, and the next light above No. 1 is called No. 2, etc. The height of each type of light is measured and kept on file in the field computing office. Since the observing parties measure and record the height of the stand and the light plate, the data of vertical number and type of light shown at each station obtained by the lightkeeper complete the necessary information to determine the height of each light above the station mark. A typical field form for the lightkeeper data is shown in figure 68. Normally, it is not necessary for the lightkeeper to make any measurements of heights, and only the heading and the first two columns need to be filled in if measurements of each type of light are kept on file in the field office. Heights of signals are entered later from the records of the observing unit.

If necessary to show the light from some point other than the top of a wooden stand or a steel-tower light plate, detailed measurements and a complete description of the point used should be furnished by the lightkeeper. A lightkeeper should leave a temporarily marked point at all eccentric stations from which lights are shown for later check measurements by the observing party.

Daily Height of Light Report

Station BLUFF

Date 16 April 1948

No. of Light from bottom up	Type of Light	Pointed to Station	Height above station mark
1.	large wood	WHITE	4.6 ft.
2.	large wood	JONES	5.6 ft.
3.	alum. box	PENN	6.4 ft.
4.	plastic	RUST	7.0 ft.
Ecc.	large wood	MOSS	(-)2.3 ft.

Height of stand 4.1 ft.

Any eccentric lights (x) Yes to MOSS only
( ) No

Distance ecc. light from mark 5.2 ft.

Offset distance right ( ) left (x) from line of sight to Station MOSS is 1.51 ft.

(Note: Leave a marked point at all eccentrics for later check measurements by O-party.

Signature John Doe

FIGURE 68.—Daily height of light report.

PROCEDURE FOR DISCONTINUING SHOWING OF LIGHT

Detailed signaling codes are given on pages 11 to 17 of Special Publication No. 65. For parties where all units return to base camp nightly, the most frequently used signal for ending of day's work is "DG." To avoid any misunderstanding, it is usually desirable for a lightkeeper to continue showing light to each of the multiple observing units until he receives a separate "DG" from each unit shown.

OBSERVATIONS WITH OPTICAL-PRISM-READING THEODOLITES

Observations are made with optical-prism-reading theodolites using the same general operating procedure as described in preceding sections, except for the method of making and recording readings which will be described in the following paragraphs. The following description refers to the Wild T–3 type of theodolite.

Settings for the Wild theodolite are shown in table 2 on page 11. These settings need only be true within five micrometer units, and can readily be made without loss of time. If only eight positions are to be taken, use alternate or odd-numbered degree-and-minute settings (adding 180° for those positions in which telescope is reversed) and successive micrometer scale increments; for example, use the following successive settings:

D	0° 00′	10 units
R	202° 00′	25 "
D	45° 00′	35 "
R	247° 00′	50 "
D	90° 00′	10 " etc.

Horizontal circle readings.—The horizontal circle is graduated to four-minute intervals and is read by an index to two minutes. The remaining reading is made on the micrometer scale. (See fig. 69.) To effect a reading, proceed as follows:

a. Point the telescope as usual.

b. See that the circle selector, which is a milled-head screw mounted on the right-hand standard (telescope direct), near the vertical-motion tangent screw, is turned to the right as far as it will go.

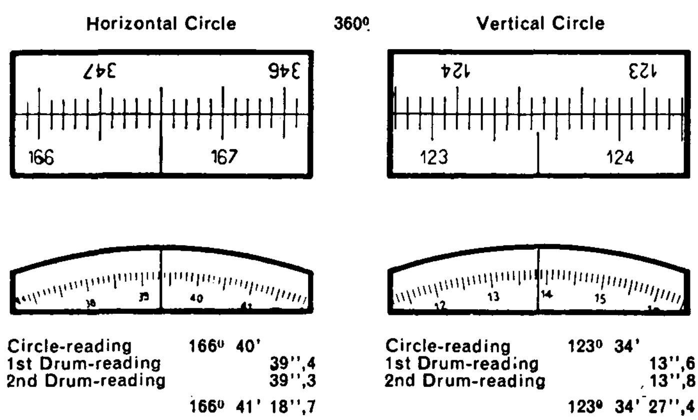


FIGURE 69.—Example, readings of Wild T–3 theodolite.

c. Look into the micrometer eyepiece. A view will be seen of two images of the limb, the graduations of the lower being erect, and the upper inverted. The graduations are on the adjacent edges and have the appearance of a vernier, except that they are uniformly spaced on both limbs. An image of the micrometer scale will also be seen beneath that of the limbs.

d. Turn the micrometer screw, which is located near the top, and on the outside face of the right-hand standard until the graduations of the two circle images are exactly alined (in coincidence). It will be observed that turning the screw apparently causes the two limbs to travel in opposite directions, and also the micrometer scale to move under its index. While the circle scales are uniform, only one point of coincidence can be found (except on readings of zero), on account of stops on the motion of the screw.

e. The index of the circle reading is a fine vertical line near the center of the images. If the instrument is in adjustment, this index will either be directly on one of the coinciding graduations or halfway between two of them.

f. If directly on a mark, read the degrees from the lower of the two circle images, and add four minutes for each subdivision between the degree mark read and the index. If the index is between two graduations, add four minutes for each full subdivision plus two more minutes. It will be noted the degrees of the scale read increase from left to right and are upright. Read the degree graduation to the left of the index and count subgraduations (four minutes each) toward the right, to the index. Should the index be out of adjustment, and not point to or halfway between the circle marks, it may cause confusion. In this case, the reading can always be found as follows: Read proper degree as above, then count the four-minute subdivisions from this degree toward the right to the diametrically opposite degree mark on the upper, inverted, circle image. Multiply the number of these subdivisions by one half of their value, in this case two minutes, to obtain the proper number of minutes read by the index.

The remainder of the reading is taken from the micrometer scale. This appears as a small arc with uniform graduations, six hundred in number. Every tenth graduation is numbered from zero to sixty, increasing from left to right, the small divisions being 0.1 unit. Now, note that when the micrometer screw is turned through two minutes of the circle, the micrometer scale moves from 0 to 60. Hence, one large unit of this scale is 120″/60 or 2″, and each subdivision is 120″/600 or 0.2″. This is known as half numbering, and requires that the micrometer scale reading be multiplied by two before it can be added to the reading of the circle scale. However, in precise observations, two readings of the micrometer scale are always taken as a check, and, instead of doubling a single reading, these two readings are added together to obtain the final value. The advantages of the half-numbering system are apparent, in that the graduations are spaced twice as far apart, therefore more easily read, and division by two in order to extract a mean is eliminated.

EXAMPLE: The circle reads 166°, index points to tenth subgraduation to right of this degree mark, and micrometer scale reads first 39.4 and second 39.3. What is the complete reading? (See fig. 69.)

ANSWER: Circle reading 166° plus 10 × 4 minutes equals 166°40′. Micrometer scale reading is 39.4 plus 39.3 equals 78″.7 or 1′18″.7. Complete reading is 166°40′ plus 1′18″.7 equals 166°41′18″.7.

g. Since only two micrometer readings are made instead of four as with a conventional two-micrometer instrument, and each of these readings is a mean and any error is not to be reduced by subsequent division, greater care and refinement in the coincidences and readings are required than is common practice with filar-micrometer theodolites. The coincidence must be made very carefully, and the scale read to the nearest small graduation (0.1 double second). With proper care and skill, the two successive readings will seldom differ more than 0.3 double-second unit (0″.6), and usually will agree within 0.1 double-second unit. If they vary more than 0.3 double-second unit, repeat readings until two properly agreeing values are obtained.

h. In order to eliminate the effect of lost motion, the two coincidences should be made by turning the screw in the same direction. This should be, by preference, clockwise. After the first coincidence is made, turn the screw back counterclockwise until the circle images are definitely out of coincidence. Then turn back clockwise until coincidence is re-established. In case the coincidence is overrun, turn backward, then establish coincidence by a clockwise motion. This does not eliminate lost motion from the reading, but makes it constant, and therefore compensating, for all readings.

Vertical circle readings.—Turn circle selector screw to the left as far as it will go, and read the circle and micrometer in the same eyepiece as for the horizontal circle. The vertical circle as well as the micrometer is half numbered, so that neither the zenith distance nor the altitude can be read directly from a single pointing, but altitude (elevation or depression) is obtained by subtracting a reading made in the reverse position from one made in the direct position. To obtain the altitude from a single reading, it is necessary to double the difference between the reading and 90°. In precise work, it will be observed as the difference between the readings taken in the reversed and direct positions of the telescope.

a. The prisms which are attached to a control level of the vertical circle must always be manipulated to center the level before reading the micrometer. This level is of the coincidence type. Images of diametrically opposite quarters of the bubble are placed side by side by means of a prismatic system, and centering is indicated by coincidence of the two quarters, apparently forming one end of a bubble in a vertical position. The leveling is done by means of a tangent screw on the left-hand standard.

b. In geodetic records, when speaking of vertical-angle observations, the direct position is usually referred to as "circle left" and reversed position as "circle right." This nomenclature will be used henceforth.

c. To measure vertical angles proceed as follows:

(1) Perfect the pointing in the usual manner, and record as circle left or circle right, as the case may be.

(2) Center the control-level bubble by its slow-motion screw. The two quarter bubbles must exactly coincide to form an image of one half bubble, with no displacement.

(3) With the selector turned for vertical-circle readings, look into the micrometer eyepiece, and bring the images of the opposite limbs into coincidence by the micrometer screw. Read the circle scale and micrometer exactly as directed for horizontal readings.

(4) Reverse the telescope, record circle right or circle left, point, center the bubble, and read the circle and micrometer as before.

(5) In the record, complete the angle readings by adding the sums of the two micrometer readings to the circle scale readings.

(6) To obtain the altitude, subtract the circle-right reading from the circle-left reading. A plus sign indicates an angle of elevation; a minus sign indicates an angle of depression. (See fig. 69.)

EXAMPLE: *Circle left:* circle scale reads 123° and there are eight and one-half subgraduations to the index. Micrometer scale reads 13.6 and 13.8. *Circle right:* circle scale reads 56° plus 6 subdivisions, and micrometer reads 43.3 and 43.5. What is the vertical angle?

ANSWER: Circle-left reading 123° plus $8\frac{1}{2} \times 4$ minutes plus 13".6 plus 13".8 equals 123°34'27".4. Circle-right reading is 56° plus 6×4 minutes plus 43".3 plus 43".5, equals 56°25′26″.8. Circle left minus circle right equals + 67°09′00″.6. The plus sign shows it to be an angle of elevation.

EXAMPLE: Circle right 93°32′ $\begin{Bmatrix} 45.9 \\ 46.2 \end{Bmatrix}$

Circle left 86°26′ $\begin{Bmatrix} 13.3 \\ 13.3 \end{Bmatrix}$

ANSWER: −7°07′05″.5 (depression angle).

Illumination.—There are separate sources of illumination for the horizontal circle and for the vertical circle and telescope reticle. For daylight illumination, reflecting mirrors may be used. These require adjustment for the direction of the light, and are subject to shadows, so that it is difficult to secure uniform lighting. For precise observations, electrical illumination should be used, even in daylight, since it can be adjusted for intensity and is uniform at any azimuth. This illumination is provided by bulbs installed in the light path and supplied by dry cells. Either 2.5- or 3.8-volt bulbs are satisfactory, the latter being preferable with two number 6 dry cells in series. Since the instruments are of European design, American bulbs should be tested for fit before taking to the field.

a. There are means for adjusting the intensity of the cross-hair illumination by a rheostat placed either in the vertical-circle circuit, or in the main circuit, and by revolving the reflecting mirror on the telescope. All lights remain on as long as the battery circuit is closed. Some experienced observers prefer to pick up the signal lights in a dark telescope, and to switch on the cross-hair illumination by a finger switch mounted near the horizontal clamp and tangent screws. It is possible to interrupt the vertical-circle circuit and carry its lead wire through such a switch, in which case it would be necessary to close the switch while reading the vertical circle as well as while pointing the telescope.

b. A soft diffused light is essential for properly reading the horizontal circle. If this is not provided by the frosted bulbs furnished with the theodolite, or such bulbs are not available, the tops of the bulbs may be painted white. Provision should be made for this before leaving headquarters.

Packing and transporting.—The instrument should always be carried in a vertical position to avoid strain on its centers. The metal case affords good protection while being back-packed, or carried by horse on good trails. When hauled by truck, cart, or by animal pack on dangerous trails, the metal case should be placed in an outer box or crate with adequate shock-absorbing packing. The frustum shipping case will suffice if not too bulky. The Wild T–3 theodolite is a fairly rugged instrument. However, if any of the numerous prisms is deranged even slightly, shop repairs will be necessary.

OBSERVATIONS WITH A REPEATING THEODOLITE

The procedure used in making a set of observations is as follows: Set the circle approximately at one of the readings given in the appropriate table of circle settings in table 2 on page 11 and record the exact reading. Point on the left-hand object by means of the lower clamp and tangent screw, which does not change the reading. Then unclamp the upper motion and point upon the right-hand object, perfecting the pointing with the upper clamp and tangent screw. Record the approximate reading of the circle. This completes one measure of the angle. The lower clamp is then released and the operation repeated, except that the circle is not read. The circle reading, if made, would equal the original setting plus twice the angle measured. The process of adding the angle to itself is continued until several measures, usually six, are accumulated on the circle. A reading of the circle is made and recorded after the third repetition as a check on the value of a single angle, and a reading is also made at the completion of the sixth repetition.

Next revolve the telescope about its horizontal axis, keeping the upper clamp tight and point upon the right-hand object by means of the lower clamp and screw. Then loosen the upper clamp, move the telescope clockwise, and point upon the left-hand object. This completes one measure of the explement of the angle. Make the same number of measures of the explement as was made of the angle, after which the circle reading should be nearly the same as on the original setting. The circle should then be read. (For sample of the record, see fig. 70.) Before beginning another set, the circle reading should be changed in order that an error in reading may not affect two angles. (See table 2 on p. 11.)

Department of Commerce
U. S. Coast and Geodetic Survey
Form 250

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HORIZONTAL ANGLES

Station: Dab State: P. I. Island or County: Luzon Date: Feb. 7, 1906
Observer: E.H.P. Instrument: B.&B. 7-in. theodolite No. 134

Objects observed	Time h. m.	Tel. D or R	Rep's	Angle ° '			A "	B "	Mean of verniers	Angle Mean D and R ° ' "	Remarks
Pet - Dog	a.m. 8:00		0	0 00			00	00	00		
			1	88 59			50				
			3	266 59			20	20			
		D	6	173 58			40	40	40	88 59 46.7	
(Dog - Pet)		R	6	0 00			10	20	15	44.2	45.5 - 0.7 = 44.8
Dog - Bat			0	0 00			15	25	20		
			1	42 30			15				
			3	127 30	Do not write in this margin	Do not write in this margin	35	45			
		R	6	255 01			15	25	20	42 30 10.0	
		D	6	0 00			25	25	25	09.2	09.6 - 0.7 = 08.9
Bat - Kow			0	0 00			10	10	10		
			1	27 34			10				
			3	82 43			10	20			
		D	6	165 26			20	30	25	27 34 22.5	
		R	6	0 00			50	00	55	25.0	23.7 - 0.8 = 22.9
Kow - Bol			0	0 00			00	10	05		
			1	37 40			40				
			3	113 02			10	20			
		R	6	226 04			20	30	25	37 40 43.3	
		D	6	0 00			10	20	15	41.7	42.5 - 0.8 = 41.7
Bol - Pet			0	0 00			20	30	25		
			1	163 15			10				
			3	129 45			30	30			
		D	6	259 30			40	40	40	163 15 02.5	
		R	6	0 00			20	30	25	02.5	02.5 - 0.8 = 01.7
										360 00	03.8 00.0

Figure 70.—Example, horizontal angles (repetition method), Form 250.

FIELD COMPUTATIONS

The methods and operating routine of field computers of triangulation parties are discussed in this section.

Triangulation parties are normally required to complete unadjusted field computations through the determination of geographic positions for main-scheme stations, and trigonometric leveling computations (when required by the project instructions) through the abstracting of zenith distances. There is usually one computer in the complement of each large field party whose primary duty is to make these computations.

The principal purposes of the field computations are: To insure the completeness of the data; to make a thorough field check of the observations; to inform the chief of party as to the accuracy which is being attained; to reduce the observations to such form as will make them readily available for the office computation and adjustment; and frequently, to furnish field values for immediate mapping needs.

All computations should be made in a neat and orderly manner, on the forms provided for the purpose, and should be kept up to date.

Field party computation duties include: Editing of the descriptions of stations; reviewing record books, abstracts, and lists of directions turned in by observing units; computing and checking the combined lists of directions, the triangles, eccentricities, and side-equation tests; completing and checking the abstracts of zenith distances; computing and listing the geographic positions; and preparing the progress sketch. Forms used are listed on pages 280 to 282.

The initials of persons making and checking all computations should be placed on each sheet as the work progresses. In rounding off extra decimal places, the last place to be retained should be its nearest value. When the decimal to be dropped is exactly a half unit, the last retained decimal should be rounded to the even figure. For instance, 1.05 would be rounded to 1.0, 1.15 to 1.2, etc. Every computation, unless self-checking, should be checked, preferably by someone other than the one who made the original computation. The checker should make his changes in pencil, then he and the original computer should agree on the proper values before the final corrections are made in ink. Do not write one figure so as to obscure another. Never erase or obscure an original recorded figure. A computed figure may be erased when superseded, or a line may be drawn through it and the correct figure written above. In using the Shortrede logarithmic tables, the proportional parts at the right of each page and tabular difference for one second at the bottom of each minute column should be used except for very small angles. For small angles, where the tabular difference for one second is changing rapidly, the difference should be taken out for the particular second used. When dividing by three to distribute the triangle closure, it is desirable to place any odd correction value on the largest angle.

Records and computations should not be allowed to accumulate at the risk of loss or damage in the field. They should be forwarded by registered mail as rapidly as completed and at intervals not exceeding 30 days. Computations and the original record books to which they refer are forwarded in separate mailings. Packages should be serially numbered for each project and Form 413, "Letter Transmitting Field Records," should be forwarded in duplicate under a separate cover and a copy should be enclosed in each package of the shipment.

EDITING OF RECORDS AND COMPUTATIONS RECEIVED FROM OBSERVING UNITS

Observing units are required to turn in to the field computing office checked and neatly typed descriptions of stations, checked record books, checked abstracts, and checked typed lists of directions. These are edited and rechecked as necessary by the computer and examined by the chief of party before being forwarded to the Washington Office.

DESCRIPTIONS OF STATIONS

Descriptions are edited for arrangement, factual data, grammar, spelling, consistency, and appearance. The arrangement should conform to the standard forms described on page 116 and shown in figure 59. The data furnished should be complete and correct. The phrasing should be grammatically correct, clear, and concise. All spelling should be rechecked and in particular, the spelling of geographic names and the names of owners of property. All names should be spelled exactly the same when appearing more than once in the description. The name of the station should be spelled exactly the same on all other field party records and computations as it appears on the mark and in the description. In general, abbreviations should be avoided in the body of the description. Feet and meters should be rechecked for consistency. Directions and compass bearings should be checked for consistency. The detailed descriptions and the measurements to marks must be consistent with each other. Directions on the description card should be rechecked from the final list of directions. The finished description card should present a neat appearance. It should be typed with black-record typewriter ribbon using clean type and a dark ribbon. Typing should not extend into the margins of the card. Check marks should be in blue and should not obscure or interfere with letters or figures.

RECORD BOOKS

Each record book of horizontal-direction observations is examined: For completeness of the record, headings, and notes; for thorough checking; for neatness; and for continuation notes. All headings should be complete. Notes for the description of and measurements to marks should be completed in the record book at the first occupancy (on the project) of each station. Personnel, weather, eccentricity, height, measurement, and plate-leveling notes should be complete for each occupancy of a station. There should be a sketch for each eccentricity of the light or instrument with a full explanation of the dates and stations involved. Names of all objects observed upon should appear in full on the first position and may be abbreviated on subsequent positions. In the case of intersection stations, the exact part of the object sighted upon should be recorded. Every computation on each page of the record book should be checked by the observer, and the checker's initials should appear at the bottom of each page. All figures should be neat and legible. All recording should be in ink. There should be no erasures nor obscuring of the original figures. Original numbers may be crossed out, but not obliterated, and corrected numbers are written above. The circumstances of the correction on all original figures should be explained in the remarks column at the time of correction. Directions in degrees and minutes should be noted in the remarks column for all intersection stations, reference marks, and the azimuth mark; and for four or more scattered positions of all other stations. The front cover or flyleaf of each record book should contain a list of the stations from which observations are made and recorded in that volume. Continuation notes from and to each volume should be checked. The record-book volumes are prenumbered for the project by the computer. Observing units receipt for each volume.

Record books of double-zenith-distance observations are checked in the same manner as those of horizontal directions. A sketch of the vertical circle of the theodolite should appear in the front of each record book used with an instrument other than a Parkhurst or Wild T-3. All headings should be filled in completely. Data for heights of stand, instrument, tent, and each light shown should be complete for each station. Computations should be checked and initialed. The time should be recorded for each set of observations. It is essential that the specific part of the object sighted upon should appear after the name of the object.

ABSTRACTS

The abstracts of directions turned in by observing units are the checked abstracts for one occupancy of the station. The computer examines these abstracts to see that they are complete and thoroughly checked and that all positions observed have been abstracted. Where there are more than one set and more than one date of occupancy of a station, combining and meaning of abstracts are usually done by the computer.

SUMMARY OF ABSTRACTS

State: Texas

Station: DOG Computed by: C.B.A. Date: 4/6/48,4/7/48

Observer: J.J.D. Checked by: F.E.D. Inst. No.: 320

	Date	ABLE	BAKER	CHARLIE
		[00°00′]	[48°45′]	[92°18′]
I	4/6/48	00″00	38″18	17″84
II	4/7/48	00″00	42″10	20″88
Mean		00″00	40″14	19″36
Shifting initial to Baker:				
I	4/6/48	(03″92)^R	42″10^F	21″76
II	4/7/48	00″00	42″10	20″88
Mean		00″00	42″10	21″32

Figure 71.—Summary of abstracts.

Figure 64 on page 126 is a sample abstract of observed directions for first-order triangulation, as received in the field office, and from which the list of directions is computed.

It is important that this form be made out carefully, because the mean directions derived from the abstract of horizontal directions constitute the basis for all the later computations. Every position observed at a station, except observations on objects where only one or two positions are taken, should appear on the abstract, the rejected readings being indicated by the letter R.

See page 150 for a summary of the principal requirements for abstracts and the rejection of observations.

Several occupations of a station are sometimes necessary in order to close triangles or obtain side checks within the required limits. These reoccupations may be due to any of the sources of error discussed on page 131 but are probably most generally the results of variable and unusual horizontal refraction conditions. When there are several occupations of a station a separate summary or analysis should be compiled to mean the abstracts. This summary sheet should be attached to the individual occupation abstracts and forwarded to the Office.

For sets of observations from two or more occupations of a station, it sometimes is desirable in computing means to shift the initial in order to use the maximum number of acceptable sets of observations of directions.

Summary at BANKS

Date	BLACK	MIDDLE	WILLOW	MEDICK	SAGE
1949 8/25 I	[0° 00′] 00″00	[52° 42′] 56″36	[125° 38′] 07″18	[231° 02′] 55″84	[260° 17′] 23″35
8/25 II	00″00	56″06	05″51	55″61	22″05
8/29		[0° 00′] 00″00	[72° 55′] 09″35	[178° 19′] 58″40	[207° 34′] 26″62

Reducing all direction to one initial:

Date	BLACK	MIDDLE	WILLOW	MEDICK	SAGE
8/25 I	[0° 00′] 00″00	[52° 42′] (56″36)R	[125° 38′] 07″18	[231° 02′] 55″84	[260° 17′] (23″35)R
8/25 II	(01″67)R	57″73	07″18F	(57″28)R	(23″72)R
8/29		57″83	07″18F	56″23	24″45*
Mean	00″00	57″78	07″18	56″04	24″45

*Closes triangles
F=directions held fixed in reducing through Willow

FIGURE 72.—Reductions of different lists of directions to one initial.

The relatively simple example shown in figure 71 is almost self-explanatory. The degrees and minutes of the directions from station DOG to stations ABLE, BAKER, and CHARLIE are shown in brackets under the station names. The mean values of the seconds for each direction as determined on Form 470 from the two sets of 16 positions are shown on the next two lines, with means of these shown on the following line. Presumably, however, triangle computations made as described on pages 159 to 162 disclosed the fact that all angles in the second set and the angle between BAKER and CHARLIE in the first set gave satisfactory angle closures and side checks, but that all others failed to give acceptable results. This would indicate an error in the initial in the first set and would justify shifting the initial to BAKER on this set. This is done by simply adding 3.92 seconds (the difference between the two directions to BAKER) to each of the directions first listed for Line I. The direction to BAKER is marked "F" to indicate that it is held fixed from the other determination, and the direction to ABLE is marked "R" to indicate that it should be rejected. Directions from the second set are then copied on the line below, and the means listed in the last line show the final adopted values. The form and method illustrated were arbitrarily selected from several now in use.

When this method or any other form of summary of abstracts of several occupations is used in the field, it should be performed step by step with all rejections indicated, and the sheet should be attached to the original abstract sheets so that the field selection of values can be checked in the Washington Office.

A field example involving several initials and combinations is shown in figure 72.

Abstracts of directions should be page numbered for each station, and the summary of abstracts should be the last page number for the station.

Summary of principal requirements for abstracts and rejections of observations.—

1. Record all positions observed on Form 470, "Abstract of Horizontal Directions." If two or more observations are made on the same position of a direction, list all these observations in the same box of Form 470 and take a mean for that position.

2. Examine list on Form 470, and, for all positions which appear to vary greatly from the mean, check the computations in the record book, giving particular attention to differences of the minute base. Then reject any positions which vary widely from the mean and reobserve these positions.

3. Determine a mean of the number of positions required for the class of triangulation being executed (see table 1 on p. xv). Reject all observations which differ from the mean by more than 4 seconds for first-order, or more than 5 seconds for second-order, observations. On Form 470 enclose rejected observations in parentheses followed by the letter R. In applying the rejection limit, the mean value of a direction should be rounded off to the nearest tenth of a second, and the criterion applied to each observation to a tenth of a second.

4. Reobserve rejections and then determine new means as before.

5. No reading should be rejected if it falls within the limit of retention unless rejected at the time of observation in which case the observer's reason for rejection should appear in the original record.

6. If one reading of two or more on a position falls without and one within the limit of retention, do not use the mean even though it comes within the limit. Use instead only the reading within the limit.

7. If two readings fall without the limit, one being abnormally high and the other abnormally low, and the mean falls within the limit, both readings should be rejected.

8. If there is a progressive change in the values of the positions of a direction, or, if the mean of the first half of the positions differs appreciably from the mean of the last half of the positions, it is desirable to observe another complete set of positions while at the station.

9. If during the same occupation of a station repeat positions are observed at any circle setting, the results of each additional observation should be combined with that of the original circle position. If, on first-order or modified second-order triangulation, 12 or more additional positions are observed, they should be meaned separately and combined by sets.

10. On first-order and modified second-order triangulation, two or more sets of 12 or more positions should be given equal weight, regardless of whether observed on the same or on different nights. When two sets of 12 or more positions differ by more than 1 second, the set which best closes the triangles is retained and the other set rejected. In case of three sets, sometimes two sets which agree with each other must be rejected and the third retained. Usually the set or sets retained are those which are observed under the least harmful conditions of horizontal refraction. When there are more than two sets, any set which falls within one-half second of the individual or mean value which best satisfies the triangles should also be meaned with that value.

11. If it appears that the initial is causing a large number of rejections, the observations can be listed by positions and directions on abstract Form 470 with some other station as the zero initial. In this case the angle between the new initial and the original initial should be added to the listed circle settings to obtain the correct values to use when reobserving rejections, unless a complete new set of observations is to be made with the new initial.

In cases where at the same occupation of a station a partial set of observations on one initial is continued on another initial, the angle between initials should be added to all remaining circle settings in table 2 in order to distribute the readings around the circle properly. If, for any reason, the provisions of the preceding sentence are not carried out at the time of the observations and it is desired to transfer a few positions using one initial to an abstract using another initial, the angle between the new and the old initials should be added to the original circle settings for all positions which are transferred in order to determine the nearest-numbered position settings on which the transferred positions are inserted on the new abstract. The values which are entered on the abstract using the new initial are obtained by adding the angle between the new and the old initials to the values of the positions from the old initial. When partial sets are transferred to another initial, the letter "T" is placed beside the transferred positions on the abstract which uses the new initial.

12. In all cases where observations are combined or initials are transferred, explanatory notes should be made. If the field procedure is so involved as not to be sufficiently clear on the abstract, a separate analysis or summary sheet should be attached showing the step-by-step procedure. An analysis sheet is particularly desirable when sets are combined or rejected.

LISTS OF DIRECTIONS

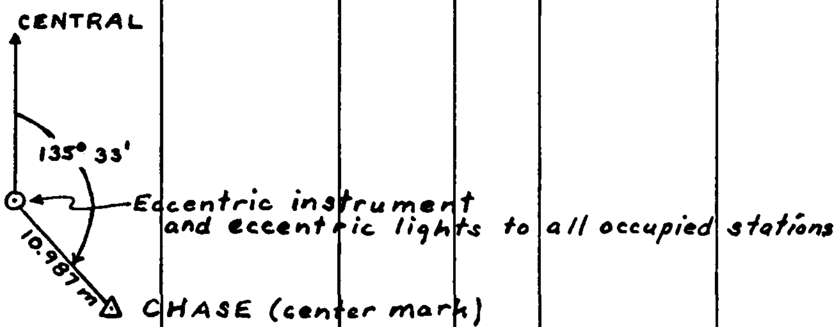
The typed list of directions which is turned in to the field computing office by each observing unit for each occupation of a station (see p. 128) is examined to determine if it is complete and in correct form, and thoroughly checked.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 24A
Rev. Oct., [illegible]

LIST OF DIRECTIONS

Station CHASE (ecc.) *State* Kansas
Chief of party J.J.D. *Date* 4/15/27 *Computed by* C.B.A.
Observer J.J.D. *Instrument* 120 *Checked by* J.J.D.

OBSERVED STATION	Observed direction	Eccentric reduction	Sea level reduction*	Corrected direction with zero initial	Adjusted direction*
	° ′ ″	′ ″	″	° ′ ″	′ ″
CENTRAL	0 00 00.00	-01 02.81		0 00 00.00	
LITTLE RIVER (ecc)	18 20 10.78	+00.39 -01 00.96		18 20 13.02	
Lyons, salt work, centerhoist	24 34 53.0	-01 44.57		24 34 11.2	
CHASE (center mark) NE 36.05 ft., 10.987 m.	135 33 02				
BOSSING	314 51 23.61	+00.89		314 52 27.31	



* These columns are for office use and should be left blank in the field.

FIGURE 73.—List of directions, illustrating eccentricity.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 24A
Rev. Oct., 1932

LIST OF DIRECTIONS

Station LITTLE RIVER *State* Kansas

Chief of party J.J.D. *Date* 4-16-27 *Computed by* C.B.A.

Observer J.J.D. *Instrument* 120 *Checked by* J.J.D.

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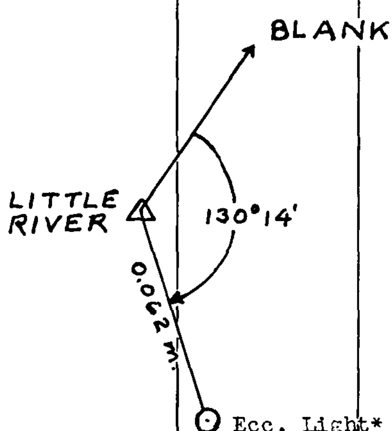
OBSERVED STATION	Observed direction	Eccentric reduction	Sea level reduction*	Corrected direction with zero initial	Adjusted direction*
	° ′ ″	′ ″	″	° ′ ″	′ ″
BLANK	0 00 00.00			0 00 00.00	
CHASE (ecc.)	44 55 10.57	-01 00.96		44 54 09.61	
*Ecc. Light, S, 0.062m. (0.20ft.)	130 14				
WHITE	137 03 23.16				



* Eccentric Light shown to CHASE only 4/15/27.
Observations recorded in Vol. 50.
No eccentricity of instrument.
No eccentricity of lights here this date.
Eccentric light shown from CHASE.

* These columns are for office use and should be left blank in the field.

FIGURE 74.—List of directions, illustrating eccentricity.

Particular care is necessary to ascertain that all eccentricities are noted. There should be a sketch with explanatory notes at all stations where the instrument is eccentric (see fig. 73) or where the signal lamp to any other station was shown eccentrically either at the time of the instrument occupation or by a lightkeeper (see fig. 74). If the instrument was eccentric, "(ecc.)" should follow the name of the station at the top of this form and the seconds on the list of directions of all stations observed with the eccentric instrument should be left in pencil by the observer. If part of the observations were made with the instrument eccentric and part with the instrument centered, the observing party should furnish a separate list of directions for each occupation. If a station pointed on was eccentric, its name on the list of directions should be followed by "(ecc.)" and the seconds on the list of directions should be left in pencil by the observing party as a reminder to the field computer. The field computer types in the observed seconds after computing and entering all eccentric values from the eccentric-reduction computations. (See p. 156.)

The field computer makes a combined list of directions for all observations at any one station, reduced to one of the main-scheme lines as the initial. Combined directions for the main- and supplemental-scheme stations are obtained from the summary of abstracts described on page 148. Other directions are obtained from the observing unit's checked lists of directions. Each of the observing unit's lists of directions should be given a page number beginning with 1 for each station, and the combined list should be given the last page number for the station and labeled "Combined List" at the top of the sheet.

The names of all stations shown on a list of directions including intersection stations are checked to verify that the name is exactly the same on all descriptions, records, computations, and sketches.

All measured distances shown on the list of directions are rechecked from the original record.

ECCENTRICITY

Reduction-to-center computations are made on Form 382. Directions for use of this form are printed on the back of the form. Computations are made horizontally across the form. Examples used are based on the illustration shown on the back of Form 382. (See fig. 75.)

Sketches with distances and directions for each eccentric instrument or light are required in the record book and on the list of directions. The original data for an eccentric instrument should always appear with the observations and list of the same date. Data for eccentric objects are recorded in the record book and appear on the lists at the time when the station at the eccentric object is occupied. However, when pointings are made on an object known to be eccentric, the station name should be followed by "(ecc.)" in both the record book and the list of directions. Form 382 is computed from the data shown on the list of directions. The computed eccentric reductions are then applied to complete columns three and five of Form 24A.

The computation on Form 382 is made in the same manner regardless of whether the instrument is eccentric (see fig. 75) or the object is eccentric (see fig. 76). The key to the use of the form is the proper determination of the angle a. As noted on the back of the form, angle a is always entered on the form as though measured at the eccentric point with the direction toward the true station as the initial. Angle a can be

INSTRUCTIONS

The required reduction to center is, in seconds, $c=\frac{d \sin a}{s \sin 1''}$, in which d is the distance from the eccentric station to the true station, and s is the length in meters of the line between the true stations involved, and, therefore, log s is taken directly from the computation of triangle sides. a is the direction of the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of a is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

Carry a to minutes only and all logarithms to five decimal places only. Do not in any case carry the derived reduction to more than two decimal places. There is no advantage in carrying them to more decimal places than the directions to which they are to be applied are carried on Form 24 A.

REDUCTIONS FOR AN ECCENTRIC INSTRUMENT

If the instrument is eccentric the first column of this form should contain the names of the stations observed from that eccentric position of the instrument.

The values in the fifth column are derived by subtracting those in the fourth column from those in the third. The values in the fourth column may need to be derived by successive approximations from the triangle-side computations if the eccentric reductions are large. The values in the sixth column are obtained from those in the fifth by adding $\log\frac{d}{\sin 1''}$ derived as indicated in the heading of the form, if d is expressed in meters. If d is expressed in feet, to the other two logarithms add also 9.48402 to convert to meters. To obtain a direction as shown on Form 24 A, subtract the reduction c for the station which is the initial on Form 24 A from the reduction c for the required direction and apply the difference to the observed direction. Similarly, the correction to any angle is the difference of the reductions on this form to the two directions involved in that angle.

REDUCTIONS FOR AN ECCENTRIC OBJECT OBSERVED

If the object observed is eccentric the heading "Eccentric Station ——" should be changed to "Eccentric Observed Object at Station ——," the first column should contain the names of the stations from which this eccentric object was observed, and in each case a is the direction from the eccentric object to the distant station involved, reckoned in a clockwise direction as usual, but referred to the direction from the eccentric object to the true station, or center, taken as zero. (No distinction need be made between the direction from the eccentric object to the distant station and the direction from the true station to the distant station except when the eccentric reduction is more than one minute.) The remainder of the computation on this form is made in the manner indicated above with reference to an eccentric instrument. The reductions to directions are, however, to be applied to observed directions, at the stations named in the first column, to the eccentric object at the station named in the heading. The directions to which these reductions are to be applied are therefore found in various of the lists of directions on Form 24 A, not all in one list as is the case when the instrument is eccentric.

REDUCTION TO CENTER

Eccentric Station: Chase.

Log d = 1. 04088
Colog sin 1″ = 5. 31443
Sum = 6. 35531

d = 10. 987 *meters*

STATION	a	LOG SIN a	LOG s (s in meters)	LOG $\left(\frac{\sin a}{s}\right)$	LOGARITHM OF REDUCTION IN SECONDS	REDUCTION $=c$
	° ′					″
Center	0 00					
Bossing	179 18	8. 08696	4. 49198	3. 59498	9. 95029	+ 0. 89
Central	224 27	9. 84528	4. 40254	5. 44274	1. 79805	− 62. 81
Little River	242 47	9. 94904	4. 51928	5. 42976	1. 78507	− 60. 96
Lyons, salt works	249 02	9. 97025	4. 30616	5. 66409	2. 01940	−104. 57

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FIGURE 75.—Example, reduction-to-center computation, reverse side of Form 382.

obtained: (1) In cases where the instrument was at the eccentric point, by shifting the zero initial (using the regular list of directions) to the direction of station center; (2) in cases where the directions were measured with the instrument at the station center, by adding 180° to the direction of the eccentric object as listed on Form 24A and subtracting this sum from the other directions.

Log sin a is entered in the third column of Form 382. The sign of this term according to the quadrant of the angle a determines the sign of the correction.

The s of column 4 is the distance in meters from the true station to the distant observed object. The first value of log s is usually obtained from the preliminary triangle computation. The first preliminary values of the eccentric reductions are applied to the angles, and the sides are again computed. This process is repeated until the changes in the value of s (after applying the latest reduction to angles) no longer cause changes in hundredths of the reduction (reduction to tenths is sufficient for second- and third-order triangulation). When eccentric distances are large, sometimes as many as three repetitions of computations of triangles and eccentric reductions are required. The number of repetitions of the computation can usually be reduced by concluding the angle at the eccentric station for the first preliminary computation. If the dis-

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Ed. June 1929

REDUCTION TO CENTER

Eccentric Station: Observed Object at station LITTLE RIVER — Log d = 8.79239

Colog sin 1″ = 5.31443

d = 0.062 *meters* — Sum = 4.10682

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STATION	a	Log sin a	Log s (s in meters)	Log $\left(\frac{\sin a}{s}\right)$	Logarithm of Reduction in Seconds	Reduction $= c$
	°					″
Center	0 00					
BLANK	49 46					
CHASE	94 41	+9.99855	4.51928	5.47927	9.58609	+0.39

FIGURE 76.—Example, reduction-to-center computation, Form 382.

tance between the true stations is known, that value should be used for s to reduce the number of repetitions of the computation. In cases where the distance between the eccentric station and the distant station is known it is most advantageous to use the method described on page 111 of the January 1948 issue of "The Journal, Coast and Geodetic Survey," as follows: "Compute the preliminary reduction, using the length between the eccentric station and the distant station involved. Add the resulting reductions algebraically to the direction in the a column of the computation form. Using the new value for log sin a and holding fixed the preliminary length and the eccentric distance d, the correct value for the reduction to center is obtained."

The value in column 5 is obtained by subtracting column 4 from column 3. The value in column 6 is obtained by adding the sum at the top of the form to column 5. Column 7, which is the reduction in seconds, is the number corresponding to the logarithm in column 6. The sign of the reduction is the same as the sign of the sine of the angle a. Reduction is plus for values of a from 0° to 180° and minus for values of a from 180° to 360°.

This reduction (always with the same sign as in column 7) is copied into column 3 of the list of directions, Form 24A, in minutes and seconds to hundredths. In cases where the instrument is eccentric, reductions will be entered on the lists of directions of stations named in the headings of Forms 382 and 24A and on the horizontal lines for these stations. In cases where the observed object is eccentric, reductions will be entered on the lists of directions headed by names appearing in column 1 of Form 382 and on the horizontal line of the station named in the heading of Form 382. If both the instrument and observed object are eccentric, two corrections will be entered on the same line of column 3 of the list of directions. Whenever an eccentric correction is applied to the initial in column 3 of Form 24A, the corrected directions in column 5 are found by summing values in columns 2 and 3 and applying the eccentric reduction on the initial with reversed sign to all these values. The fact that eccentric reductions have been applied will indicate that the values shown in column 5 apply to the true stations rather than to the eccentric stations listed in the heading and in the first column.

There are other methods of measuring eccentricity such as measuring the perpendicular offset distance for each line, and by measuring the sides of a triangle formed by the center, eccentric point, and a reference mark or a perpendicular to some known direction. There are also other methods of determining eccentric corrections, such as by use of nomograms and tables. However, use of the above outlined method of making the computation on Form 382 produces more consistent results and is less likely to be misinterpreted.

TRIANGLES

Triangulation parties compute the triangles for all observations made in the field. Computations are made by logarithms on Form 25, "Computation of Triangles," using Shortrede's seven-place tables of "Logarithms of Sines and Tangents for Every Second." Logarithms used for the sides are logarithms of the lengths in meters. The computation of the lengths of the sides of each triangle from one known (previously determined) length and three determined angles is by the well-known law of sines. Form 25 is conveniently arranged so that by masking the 3 line of logarithms the upper three logarithms can be added for the line 1–3, and by masking the 2 and 1–3 lines, the three remaining logarithms can be added for the 1–2 line. Convenient templates can be cut out of heavy paper to facilitate this operation. A similar template can also be made for the spherical excess computation (see p. 162).

Examples of triangle computation are shown in figures 78 and 79 on pages 161 and 164. Figure 78 shows an example of a complete computation made in the field. Angles may be carried to tenths of a second and logarithms to six places for second- and third-order triangulation. The C.D. entries in column 1, side-check data in column 7, and average triangle closure at bottom of sheet are shown here for the information of the field party, and are not usually included in the copy sent to the Washington Office.

LAYOUT OF TRIANGLES

Triangles to be computed are usually laid out by the chief of party or the computer in advance of the observations. Starting from a base line or previously computed main-scheme line, triangles are laid out by geometric figures, usually quadrilaterals and three-, four-, or five-sided central-point figures, the objective for main-scheme figures being to have a double determination of the length of the line from which the computations are continued. Supplemental figures are often chains of single triangles which connect to other lines of higher order. Intersection-station triangles are laid out to have two triangles with a line common to both serving as a check line. Usually separate series of computation of triangles, in individual ring binders, are laid out for the main scheme, supplemental scheme, and intersection stations. For convenience and for cross-referencing, it is desirable to serially number each series of computations of triangles for a project, with triangles for the main scheme starting with 1, triangles for supplemental determinations with 100, and triangles for intersection stations with 500 (or any other convenient numbering system). All possible triangles should be laid out and computed by the field party, regardless of the size or importance of the triangles, with the exception that two triangles (affording a check on a common side) will be sufficient for intersection stations.

The source of the 2–3 known line for each triangle should be written on the first line, immediately after the 2–3 printed number. The source designation will usually consist of the number of the previously computed triangle from which the logarithm of the known length is obtainable.

In laying out a triangle on Form 25, the newly used station is always number 1 and the others are numbered 2 and 3 in a clockwise order, with the line 2–3 always the known line. Names of stations at the vertices are written on the 1, 2, and 3 lines of column two of Form 25.

The chain of triangles which is geometrically strongest for carrying the length computation through a figure is called the R_1 chain of triangles. (See p. 269.) The smaller the R_1, the greater the strength of figure. The second-strongest chain of triangles is called the R_2 chain.

In laying out quadrilaterals and other figures in the field, it is advantageous to lay out the R_1 triangles first (starting from the known line), then the R_2 and all remaining triangles of the figure. This system makes it less confusing to follow through the strength of figures for field computations and facilitates the checking of closures across diagonals. For a central-point figure in which the strength of figure does not go through the central point, all the R_2 triangles are usually laid out with the central point as the number one vertex.

In selecting the strongest triangles for a preliminary layout, the distance angles (that is, the angle opposite the known side and the angle opposite the side desired) may be scaled to the nearest degree from the reconnaissance sketch. The distance angles are used to obtain values of $[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ from "Table for determining relative strength of figures in triangulation" on page 268. These values can be written lightly in the left margin of Form 25, and for purposes of selecting R_1 and R_2 triangles of any single figure, there is no necessity of multiplying by the factor $\frac{D-C}{D}$. In doubtful cases of relative strength of figure, values can later be checked using the observed angles instead of the scaled angles.

In area triangulation and in other supplemental schemes, the field lengths of the triangle sides are computed through the strongest possible chain of triangles from the nearest main-scheme line. The resulting adjacent chains have frequent common check lines. It is usually advantageous to lay out the triangles so as to make the common check lines fall on the shortest lines, and to avoid a rapid expansion from a short to a long line. Also, it is frequently advisable to make side equation tests (see pp. 165 to 171) using as many as possible of the new stations as poles (usually of central-point figures).

For intersection stations, triangles are laid out with the concluded angle at the intersection station entered at the number 1 vertex of the triangle. Since the principal object of observations on an intersection station is usually to determine its position for mapping use, the strongest triangles are those in which the concluded angles are nearest to 90 degrees. It is desirable that there be at least two triangles with a common side (for check purposes) for each intersection station. In case there is not an observed line between the number 2 and 3 vertices of an intersection triangle, an inverse solution can be made to determine the necessary values. (See p. 179.) However, this situation causes considerable extra computing and should be avoided if it is possible to lay out a check triangle that does not require an inverse solution.

COMPUTATION OF TRIANGLES

Computation of triangles is made on Form 25. The observational data used on this form are taken from Form 24A, as illustrated in figure 77, for the four vertices of a quadrilateral of first-order triangulation. Figure 78 is an example of the field computation made on Form 25, for the four triangles of the quadrilateral. Second- and third-order computations are made in a similar manner, but usually with fewer significant figures.

In column one the first figure for each triangle is the serial number of the triangle. (See p. 157.)

In column two (headed "STATION") the number in parentheses on the 2–3 line is the number of the triangle from which the logarithm of the length of the 2–3 side is obtainable. If this value is obtained from a previous season's triangulation, the date of the previous triangulation is entered on this line. The names on the 1, 2, and 3 lines are names of stations at the vertices of the triangle.

For column three (headed "OBSERVED ANGLE") the data are obtained from Form 24A (see fig. 77). From the lists of directions for the stations named in the preceding column, each angle is obtained as the difference in the directions of the other two stations of the triangle. Or, in taking out directions from Form 24A, the direction of the line forming the left side of an angle of a triangle is always considered negative and the direction of the line forming the right side of the angle is considered positive. These two directions are combined algebraically to form the angle, adding 360° to the positive direction if necessary. These angles are entered in column 3 to hundredths of seconds on the same-named horizontal line as the heading of the list of directions. The three angles of the triangle are then added, and the sum is placed on the bottom line of that triangle computation. The same procedure is then repeated, and observed angles are entered on Form 25 for the other three triangles of the quadrilateral. If there are no mistakes, the observed angles should sum to within a few seconds of 180°. As a check on taking out the angles, the sum of these preliminary closures (temporarily disregarding spherical excess) of the two triangles formed by one diagonal should be equal to the sum of the closures of the two triangles formed by the other diagonal. Thus, in figure 78, $+1.''65-1.''37=-0.''53+0.''81$. In general, for a given set of observations, the algebraic sum of the closures of a set of figures covering a given area without overlaps will equal the sum of the closures of any other set of figures covering the identical area without overlaps. This rule also holds for spherical excess.

Columns 4, 5, and 6 are passed over temporarily.

In column 7, the seconds of the plane angles are obtained by applying corrections equally to the three observed angles to make them sum up to exactly 180°.

In column 8, the entry on the 2–3 line is the logarithm to seven places of the known side of the triangle which is obtained from previous computations. The entry on the 1 line is the log of the reciprocal of the sine (colog sine or log 1 − log sine) of the first plane angle. The entry on the 2 line is the log sine of the second plane angle of the triangle. The entry on the 3 line is the log sine of the third plane angle of the triangle. The value to be entered on line 1–3 is equal to the sum of the values on lines 2–3, 1, and 2. The value to be entered on line 1–2 is equal to the sum of the values on lines 2–3, 1, and 3.

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COMBINED
LIST OF DIRECTIONS sheet 4 of 4

Station BRUMLEY *State* Missouri
Chief of party M.E.W. *Date* 7/2/47 and 7/8/47 *Computed by* M.E.T.
Observer Various *Instrument* Various *Checked by* R.C.J.

OBSERVED STATION	Observed direction ° ′ ″	Eccentric reduction ′ ″	Sea level reduction* ″	Corrected direction with zero initial ° ′ ″	Adjusted direction* ′ ″
IBERIA	0 00 00.00			0 00 00.00	
WOLF	87 46 21.01				
AUGLAIZE	133 10 32.69				
KAISER	202 36 32.22				

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COMBINED
LIST OF DIRECTIONS sheet 3 of 3

Station WOLF *State* Missouri
Chief of party M.E.W. *Date* 7-2-47 & 7-10-47 *Computed by* M.E.Z.
Observer Various *Instrument* Various *Checked by* L.D.F.

OBSERVED STATION	Observed direction ° ′ ″	Eccentric reduction ′ ″	Sea level reduction* ″	Corrected direction with zero initial ° ′ ″	Adjusted direction* ′ ″
KAISER	0 00 00.00			0 00 00.00	
BRUMLEY	29 39 55.20				
AUGLAIZE	289 20 07.52				

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COMBINED
LIST OF DIRECTIONS sheet 3 of 3

Station KAISER *State* Missouri
Chief of party M.E.W. *Date* 7-3-47, 7-24-47 *Computed by* M.E.Z.
Observer Various *Instrument* Various *Checked by* G.L.S.

OBSERVED STATION	Observed direction ° ′ ″	Eccentric reduction ′ ″	Sea level reduction* ″	Corrected direction with zero initial ° ′ ″	Adjusted direction* ′ ″
BRUMLEY	0 00 00.00			0 00 00.00	
WOLF	35 29 51.94				
AUGLAIZE	81 42 13.82				

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COMBINED
LIST OF DIRECTIONS sheet 3 of 3

Station AUGLAIZE *State* Missouri
Chief of party M.E.W. *Date* 7-9-47, 7-10-47 *Computed by* M.E.Z.
Observer Various (See individual lists) *Instrument* *Checked by* R.C.J.

OBSERVED STATION	Observed direction ° ′ ″	Eccentric reduction ′ ″	Sea level reduction* ″	Corrected direction with zero initial ° ′ ″	Adjusted direction* ′ ″
KAISER	0 00 00.00			0 00 00.00	
BRUMLEY	28 51 45.84				
WOLF	63 07 47.01				

FIGURE 77.—Combined lists of directions.

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Form 25
Ed. Nov 1946

COMPUTATION OF TRIANGLES

State: Missouri log m = 1.40472

(1)	STATION (2)	OBSERVED ANGLE (3)	CORR'N (4)	SPHER'L ANGLE (5)	SPHER'L EXCESS (6)	PLANE ANGLE AND DISTANCE (7)	LOGARITHM (8)
21	2-3 (18)						4.182 7101
(C.D.) +2.95	1 KAISER	35 29 51.94	+0.70	52.64	−0.15	52.49	0.236 0681
−0.97	2 BRUMLEY	114 50 11.21	0.70	11.91	0.15	11.76	9.957 8511
+3.70	3 WOLF	29 39 55.20	0.70	55.90	0.15	55.75	9.694 5485
	1-3						4.376 6293
	1-2						4.113 3267
		179 59 58.35	+2.10		0.45		9.658 61
22	2-3 (21)						4.376 6293
+1.07	1 AUGLAIZE	63 07 47.01	−0.09	46.92	−0.37	46.55	0.049 6200
+2.02	2 KAISER	46 12 21.88	0.09	21.79	0.37	21.42	9.858 4361
+0.74	3 WOLF	70 39 52.48	0.09	52.39	0.36	52.03	9.974 7859
	1-3						4.284 6854
	1-2						4.401 0352
		180 00 01.37	−0.27		10 ~~1.099~~		0.040 83
23	2-3 (18)						4.182 7101
+3.09	1 AUGLAIZE	34 16 01.17	+0.07	01.24	−0.24	1.00 ~~00.99~~	0.249 4535
+2.08	2 BRUMLEY	45 24 11.68	0.07	11.75	0.24	11.5~~0~~1	9.852 5197
−0.38	3 WOLF	100 19 47.68	0.06	47.74	0.25	47.~~54~~49	9.992 9032
	1-3					+2.1/6.2	4.284 6833
	1-2						4.425 0668
		180 00 00.53	+0.20		0.73		9.865 01
24	2-3 (23)						4.425 0668
+0.31	1 KAISER	81 42 13.82	+0.55	14.37	−0.28	14.09	0.004 5686
+0.79	2 BRUMLEY	69 25 59.53	0.54	60.07	0.27	59.80	9.971 3982
+3.82	3 AUGLAIZE	28 51 45.84	0.54	46.38	0.27	46.11	9.683 6899
	1-3					+1.6/62	4.401 0336
	1-2					+1.4/76	4.113 3253
		179 59 59.19	+1.63		0.82		9.914 51

Total = 4.20 = Av. 1.05
Prev. total = 18.12
24)22.32 = 0.93 Av. c/as.

Comp. J.J.D.
Ck. C.B.A

FIGURE 78.—Example of working copy of complete field computation of triangles, Form 25.

Taking of these sums is facilitated by covering or masking the logarithms on line 3 in the former case and lines 2 and 1–3 in the latter. Lines 1–2 and 1–3 are logarithms of lengths of these two sides of the triangle. Sides are usually expressed only in logarithms on this form.

The computations on Form 25 may be made by electric calculating machine if desired, using distances in meters and natural functions of the plane angles. In doing so, at least seven significant figures should be shown for the lengths.

Spherical excess.—The spherical excess is the amount that the sum of the observed angles of a triangle should exceed 180° due to being measured on the spheroidal surface of the earth. The formula for spherical excess and a table of values of log *m* for different latitudes are given on pages 272 and 273.

Referring to figure 78, by masking line 1, line 2, and line 1–2, and adding (to no more than five places of logarithms) line 2–3, line 3, line 1–3, and log *m*, the logarithm of the spherical excess is obtained to five places of logarithms. (Three places will normally furnish sufficient accuracy for field computations.) This logarithm is entered on the last line of column 8 for each triangle, and the antilog is entered on the same line in column 6.

The total spherical excess of the two triangles formed by one diagonal is equal to the spherical excess of the two triangles formed by the other diagonal of a quadrilateral. In case the sums fail to check by 0.″01 (0.″1 for second and third order) due to rounding of the last decimal place, they should always be made to check by arbitrarily changing the spherical excess of one of the triangles by that amount.

Angle corrections.—The triangle closure is equal to 180° plus the spherical excess, minus the sum of the observed angles of the triangle. If the sum of observed angles is the greater, the sign of the closure is minus. To be acceptable in first-order triangulation, this closure should not exceed a maximum of 3 seconds, and the average closure should not exceed 1 second. (See p. 9.) Cumulative averages for the scheme are kept by the field parties, usually on the bottom of working copies of Form 25. The allowable closures for second- and third-order triangulation are shown in table 1 on page xv.

The sum of the closures of the two triangles formed by one diagonal of a quadrilateral figure should be equal to the sum of the closures of the two triangles formed by the other diagonal. The closure is divided by three and entered in column 4 with the correct sign and to hundredths of seconds. Applying the correction in column 4 algebraically to the observed angle in column 3 gives the seconds of the spherical angle in column 5. (This is the angle used in computing geographic positions.) In column 6, the spherical excess is distributed equally among the three angles. Subtracting the spherical excess in column 6 from the spherical angle in column 5 gives the plane angle in column 7. This will ordinarily be the same as the plane angle previously computed. If any changes have been introduced in the final plane angle due to the distribution of odd fractions of the corrections, the log sine should be corrected and the side recomputed.

Side checks.—At the same time that the logarithms of the plane angles are taken from the Shortrede tables, it is convenient to note the difference of the log sine value of one second for that angle. This may be pencilled lightly in column one (of Form 25) on the line opposite the number of the angle to which it applies. This tabular difference, in units of the sixth decimal place, is usually taken from the line marked "C.D." (common difference) at the bottom of the minute column of the angle in the Shortrede tables where it is given in units of the seventh decimal place. In the computation of a quadrilateral, there are always three lines which have two determinations of their length. One of the requirements of first-order triangulation is that the difference between the two values for the logarithm of a line be no greater than one and a half to two times (see table 1, p. xv) the log sine difference for one second of the smallest distance angle involved in the computation of either value. (See p. 10.) Requirements for second- and third-order triangulation are given in table 1 on page xv. A simple method of applying this check is to pencil in lightly in column 7 of Form 25, on the same line as the side in question, the difference between the two logs of the same side. This value should be entered opposite the second place the side appears, as the numerator of a fraction. The denominator of the fraction should be two times the C.D. value entered in column one which is opposite the smallest (distance) angle involved in the determination of that side. Then whenever the numerator is greater than the denominator, that side check is not satisfactory. An additional check by side-equation tests is discussed on pages 165 to 171.

MAIN-SCHEME TRIANGLES

The computations described in the preceding sections were for first-order triangulation main-scheme triangles. Special features of these computations are that the angles are carried to hundredths of seconds and logarithms to seven places. See pages 9 and 10 for allowable closures and side checks. See table 1 and pages 17 and 18 for the requirements of second- and third-order triangulation.

SUPPLEMENTAL TRIANGLES

For modified second-order triangulation which is observed with 16 positions of a first-order theodolite and allowed a rejection limit of 5 seconds, computations of triangles are made in the same manner as for first-order triangles, listing seconds to hundredths and using seven-place logarithms. See page 13 for allowable closures and side checks.

In ordinary second-order triangulation, observed with second-order instruments and methods, seconds are usually carried to tenths and logarithms to six places.

INTERSECTION TRIANGLES

First-order triangulation parties compute triangles for intersection points using seven places of logarithms and carrying seconds to tenths. Spherical excess is computed to tenths of seconds. (See fig. 79.) The concluded angle at the intersection station is obtained by adding the spherical excess to 180° and subtracting the sum of the two observed angles. The spherical excess in column 6 is then subtracted directly from the angles in column 3 to obtain the plane angles in column 7. Log sines are then obtained from tables and the sides computed in the same manner as described on page 159. Side checks are obtained on the sides common to two or more triangles. See page 107 for preliminary graphic checks of observations to intersection stations. Computations of the two best triangles for a side check are sufficient in the field. Intersection triangles on second- and third-order work may be computed with the angles in seconds, using six-place logarithms.

TOWER RELEASES

It is very desirable to obtain both angle and side checks before dismantling towers. An important factor in the rate of progress of a steel-tower triangulation party is a quick and steady stream of releases of the signal towers, in order to keep the dismantling and building units on an efficient schedule. The fact that the angles have been closed

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26
Ed. Jan., [illegible]

11—9121

COMPUTATION OF TRIANGLES

State: Missouri

Do not write in this margin

NO.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
537	2-3	(173)			-		3.964 7122
	1 St. Elizabeth, St. Lawrence R.C. Church, Spire	(43 49 03.1)			0.1	03.0	0.159 6658
	2 Lurton	83 06 28.9			0.1	28.8	9.996 8504
	3 Eugene	53 04 28.3			0.1	28.2	9.902 7736
	1-3						4.121 2284
	1-2						4.027 1516
					0.3		9.39~~3~~4 ~~43~~
538	2-3	(173)					4.122 7193
	1 St.E.St.L.Ch., Spire	(80 39 21.5)			0.1	21.4	0.005 8011
	2 Lurton	46 59 20.2			0.1	20.1	9.864 0491
	3 Henley	52 21 18.6			0.1	18.5	9.898 6219
	1-3						3.992 5695
	1-2						4.027 1423
					0.3		9.4~~18~~20 ~~63~~
539	2-3	(173)					3.900 9837
	1 St.E,St.L,Ch,Spire	(36 50 17.7)			0.1	17.6	0.222 1690
	2 Eugene	47 45 25.2			0.1	25.1	9.869 4077
	3 Henley	95 24 17.3			0.0	17.3	9.998 0648
	1-3						3.992 5604
	1-2						4.121 2175
					0.2		9.29~~8~~7 ~~32~~
	* Source triangle repeated here for illustration.						
173*	2-3	(172)			-		4.122 7193
	1 Eugene	100 49 53.50	0.25	53.25	0.06	53.19	0.007 8070
	2 Henley	43 02 58.74	0.25	58.49	0.06	58.43	9.834 1859
	3 Lurton	36 07 08.69	0.25	08.44	0.06	08.38	9.770 4574
	1-3						3.964 7122
	1-2						3.900 9837
		180 00 00.93	-0.75		0.18		9.262 61

FIGURE 79.—Computation of intersection triangles.

to within allowable limits on the station by the observing party does not always insure that the side checks will be satisfactory. It is, therefore, undesirable to release towers before satisfactory side checks have been computed. Where there is an immediate need for towers, observing parties are required to turn in checked abstracts of directions to the field computing office on completion of the night's work. The computer then computes the triangle sides in time to post releases for the morning's tearing-down parties.

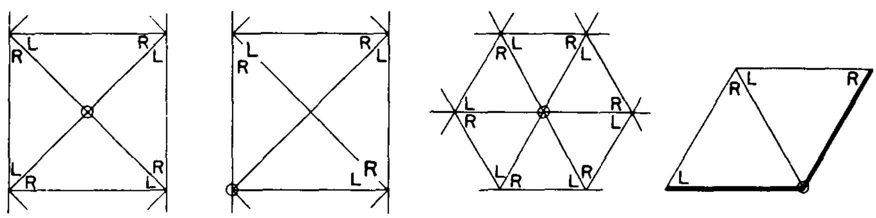
SIDE TESTS

In addition to meeting the specifications for average triangle closures, the lengths of the common sides of the triangles in the figures of a scheme must agree within fixed limits as computed through various chains. As previously stated, in a quadrilateral of first-order triangulation the computed logarithms of the sides should differ by not more than one and a half to two times the tabular difference for 1 second of the log sine of the smallest angle entering into the computation of the logarithm of that side. In area triangulation the discrepancy may be greater since the lengths must be carried through six to eight triangles before a check is obtained. A reasonable check for figures other than quadrilaterals would be half the number of triangles involved times the tabular difference for 1 second of the smallest angle used in the computation.

The above checks apply to first-order triangulation. For second-order triangulation the limits should be double those of first-order and for modified second-order the limits should be one and one-half times those of first-order triangulation. (See table 1, p. xv.)

These checks will not always indicate the directions which should be reobserved in order to reduce large side closures. Side-equation tests should be made when the closures are large in order to isolate the direction that is poor.

A side equation is based on the fundamental law of sines. The derivation of the equation is given in manuals for the adjustment of triangulation. A side equation



1. Pole at intersection of diagonals
2. Pole at vertex station
3. Pole at central station
4. Between fixed lengths

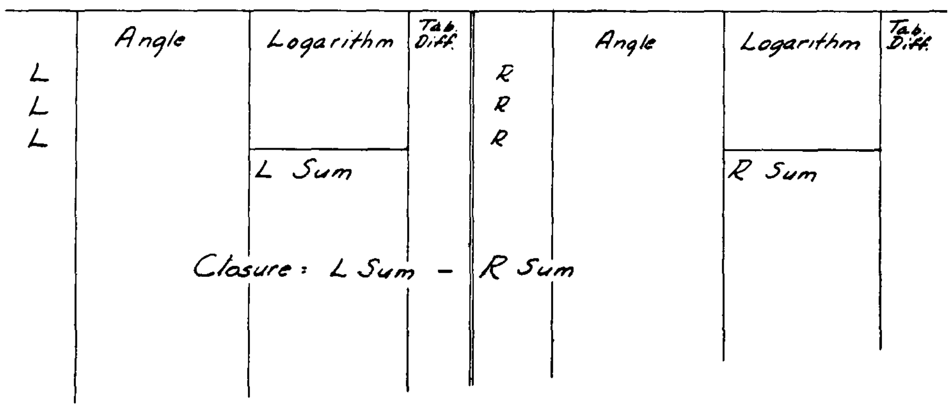


FIGURE 80.—Field form for side equation tests.

Department of Commerce
U. S. Coast and Geodetic Survey
Form 25
Ed. Nov. 1946

FIRST SET
COMPUTATION OF TRIANGLES

State: ....

TAB. DIFF.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3						4.260 5754
4	1 Linville	79 47 16.23				15.92	0.006 9353
23	2 Holton	42 26 33.31				33.01	9.829 2072
13	3 Ellis	57 46 11.37				11.07	9.927 3250
	1-3		-0.42		0.49		4.096 7179
	1-2						4.194 8357
		0.91					
	2-3						4.194 8357
20	1 Kelat	46 24 51.78				51.80	0.140 0545
3	2 Holton	81 01 41.15				41.18	9.994 6537
16	3 Linville	52 33 26.99				27.02	9.899 8007
	1-3		+0.75		0.67		4.329 5439
	1-2						4.234 6909
		59.92					
	2-3						4.260 5754
38	1 Kelat	29 10 58.03				57.75	0.311 9397
-14	2 Holton	123 28 14.46				14.18	9.921 2539
41	3 Ellis	27 20 48.35				48.07	9.662 1659
	1-3		-0.18		0.66		4.493 7690
	1-2						4.234 6810
		0.84					
	2-3						4.493 7690
-19	1 Linville	132 20 43.22				43.23	0.131 2979
68	2 Kelat	17 13 53.75				53.75	9.471 6361
36	3 Ellis	30 25 23.02				23.02	9.704 4773
	1-3		+0.51		0.50		4.096 7030
	1-2						4.329 5442
		59.99					

FIGURE 81.—Computation of triangles (first set for side equation test).

sufficiently complete for field tests may be written from the plane angles listed in the triangles. The form used in tabulating the respective angles is shown in figure 80.

The angles marked "L" are considered as left-hand angles and those marked "R" as right-hand angles. An equation may be formed by tabulating the logarithms of the sines of the angles in two columns, "L" and "R." It will be noted that the left-hand angle is opposite the unknown side and the right-hand angle opposite the known side when the computations are carried through a figure. The tabular difference of 1 second should also be recorded with each value. The difference of the sums of the two

SIDE TESTS USING PLANE ANGLES

FIRST SET

Pole at intersection of diagonals

Hol	42 26 33.01	9.829 2072	23	Ell	27 20 48.07	9.662 1659	41
Ell	30 25 23.02	9.704 4773	36	Lin	79 47 15.92	9.993 0647	4
Lin	52 33 27.02	9.899 8007	16	Kel	17 13 53.75	9.471 6361	68
Kel	29 10 57.75	9.688 0603	38	Hol	81 01 41.18	9.994 6537	3
		9.121 5455				9.121 5204	
			+251	÷ 229	= +1.″1		

Pole at Holton

Ell	57 46 11.07	9.927 3250	13	Lin	79 47 15.92	9.993 0647	4
Lin	52 33 27.02	9.899 8007	16	Kel	46 24 51.80	9.859 9455	20
Kel	29 10 57.75	9.688 0603	38	Ell	27 20 48.07	9.662 1659	41
		9.515 1860				9.515 1761	
			+99	÷ 132 =	+0.″8		

Pole at Ellis

Lin	132 20 43.23	9.868 7021	−19	Kel	17 13 53.75	9.471 6361	68
Kel	29 10 57.75	9.688 0603	38	Hol	123 28 14.18	9.921 2539	−14
Hol	42 26 33.01	9.829 2072	23	Lin	79 47 15.92	9.993 0647	4
		9.385 9696				9.385 9547	
			+149	÷ 166	= +0.″9		

Pole at Kelat

Hol	123 28 14.18	9.921 2539	−14	Ell	27 20 48.07	9.662 1659	41
Ell	30 25 23.02	9.704 4773	36	Lin	132 20 43.23	9.868 7021	−19
Lin	52 33 27.02	9.899 8007	16	Hol	81 01 41.18	9.994 6537	3
		9.525 5319				9.525 5217	
			+102	÷ 129	= +0.″8		

Pole at Linville

Kel	46 24 51.80	9.859 9455	20	Hol	81 01 41.18	9.994 6537	3
Hol	42 26 33.01	9.829 2072	23	Ell	57 46 11.07	9.927 3250	13
Ell	30 25 23.02	9.704 4773	36	Kel	17 13 53.75	9.471 6361	68
		9.393 6300				9.393 6148	
			+152	÷ 163	= +0.″9		

FIGURE 82.—Side tests using plane angles (first set).

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Ed. Nov. 1946

SECOND SET
COMPUTATION OF TRIANGLES
REOBSERVATION - LINVILLE

State: ....

TAB. DIFF.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3						
	1 Linville	No CHANGE					
	2 Holton	SEE					
	3 Ellis	FIRST SET					
	1-3						
	1-2						
	2-3						4.194 8357
20	1 Kelat	46 24 51.78				52.74	0.140 0526
3	2 Holton	81 01 41.15				42.11	9.994 6539
16	3 Linville	52 33 24.19				25.15	9.899 7977
	1-3		+3.55		0.67		4.329 5422
	1-2						4.234 6860
		57.12					
	2-3						
	1 Kelat	No CHANGE					
	2 Holton	SEE					
	3 Ellis	FIRST SET					
	1-3						
	1-2						
	2-3						4.493 7690
-19	1 Linville	132 20 40.42				41.36	0.131 2942
68	2 Kelat	17 13 53.75				54.69	9.471 6425
36	3 Ellis	30 25 23.02				23.95	9.704 4806
	1-3		+3.31		0.50		4.096 7057
	1-2						4.329 5438
		57.19					

FIGURE 83.—Computation of triangles after reobservation (second set).

columns represents the closure. The average correction to an angle may be computed by dividing the closure by the sum of the tabular differences in both columns (without regard to sign). The mean of these average corrections for first-order triangulation should not exceed 0".7. In the exceptional case of an equation using only one small angle, corrections as large as 2".0 per angle may be tolerated. For second-order triangulation these limits should be 2".0 and 4".0. For modified second-order triangulation the average should be 1".2 with an upper limit of 3".0.

In a quadrilateral a test around the intersection of diagonals will check all angles if the side closure is small but will not assist in locating the source of error if the closure is large.

When a large side closure exists, the faulty direction may be found by forming side equations using successive stations as poles. The pole of a side equation is the point about which the equation is written. It is an end point of both the starting and the ending lines of the equation. It forms a vertex of every triangle appearing in the equation, but the angles at a pole are not used directly in that equation. If one of these equations has a small closure there is evidence that the directions from the station being used as a pole are poor.

The example given illustrates how side tests may be used in the field. When the triangles are computed and the logarithms of the plane angles listed, the tabular difference should also be listed. The column in the left-hand margin could be used. All the quantities needed for the tests are on the triangle computation form.

SIDE TESTS USING PLANE ANGLES

SECOND SET

Triangle closures indicate tests should be made at Kelat and Linville

Pole	at Kelat						
Hol	123 28 14.18	9.921 2539	-14	Ell	27 20 48.07	9.662 1659	41
Ell	30 25 23.95	9.704 4806	36	Lin	132 20 41.36	9.868 7058	-19
Lin	52 33 25.15	9.899 7977	16	Hol	81 01 42.11	9.994 6539	3
		9.525 5322				9.525 5256	
			+66	÷ 129	= +0.″5		
Pole	at Linville						
Kel	46 24 52.74	9.859 9474	20	Hol	81 01 42.11	9.994 6539	3
Hol	42 26 33.01	9.829 2072	23	Ell	57 46 11.07	9.927 3250	13
Ell	30 25 23.95	9.704 4806	36	Kel	17 13 54.69	9.471 6425	68
		9.393 6352				9.393 6214	
			+138	÷ 163	= +0.″8		

Tests indicate poor direction at Kelat

FIGURE 84.—Side tests using plane angles (second set).

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 25
Ed. Nov. 1916

THIRD SET
COMPUTATION OF TRIANGLES
REOBSERVATION - KELAT

State: ....

TAB DIFF.	STATION	OBSERVED ANGLE	CORR'N	SPHER'L ANGLE	SPHER'L EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3						
	1 Linville	No CHANGE					
	2 Holton	SEE					
	3 Ellis	FIRST SET					
	1-3						
	1-2						
	2-3						4.194 8357
20	1 Kelat	46 24 54.88				54.80	0.140 0485
3	2 Holton	81 01 41.15				41.08	9.994 6536
16	3 Linville	52 33 24.19				24.12	9.899 7961
	1-3		+0.45		0.67		4.329 5378
	1-2	0.22					4.234 6803
	2-3						
	1 Kelat	No CHANGE					
	2 Holton	SEE					
	3 Ellis	FIRST SET					
	1-3						
	1-2						
	2-3						4.493 7690
-19	1 Linville	132 20 40.42				40.32	0.131 2922
68	2 Kelat	17 13 56.85				56.75	9.471 6565
36	3 Ellis	30 25 23.02				22.93	9.704 4770
	1-3		+0.21		0.50		4.096 7177
	1-2	0.29					4.329 5382

FIGURE 85.—Computation of triangles (third set).

The observations used in the first set of triangles (fig. 81) give good triangle closures. The length that is to be used to carry the scheme forward also checks. However, the lengths of the side lines do not agree within the required limits.

The equat ons shown in figure 82 consist of one using the intersection of the diagonals as a pole and four using the stations as poles. All of these equations indicate excessive average corrections. When such a condition exists, two or more directions are in error.

Reobservation at Linville gives new angles for a second set of triangles (fig. 83) and equations (fig. 84). The angle closures indicate that there is a poor direction at one end of the Kelat-Linville line. The pole equation at Kelat indicates that the trouble is at that station.

Reobservation at Kelat (fig. 85) gives satisfactory results. A test equation (fig. 86) using the intersection of the diagonals as a pole shows that the side condition checks.

These side closures may be computed directly from the triangles without making separate tabulations. The proper log sines may be selected by following a sketch and adding or subtracting in a calculating machine. The sum of the tabular differences may be obtained in the same manner.

SIDE TESTS USING PLANE ANGLES

					THIRD	SET	
Pole	at intersection of diagonals for final side check						
Hol	42 26 33.01	9.829 2072	23	Ell	27 20 48.07	9662 1659	41
Ell	30 25 22.93	9.704 4770	36	Lin	79 47 15.92	9.993 0647	4
Lin	52 33 24.12	9.899 7961	16	Kel	17 13 56.75	9.471 6565	68
Kel	29 10 57.75	9.688 0603	38	Hol	81 01 41.08	9.994 6536	3
		9.121 5406				9.121 5407	
			-1	÷ 229 = -0.004″			

FIGURE 86.—Side tests using plane angles (third set).

There may be occasions when conclusive results cannot be obtained using the logarithms of the plane angles. In such cases, tests may be made using the observed angles. This method has the disadvantage of requiring that additional logarithms be computed and tabulated but an advantage that only the logarithms of the reobserved angles need to be changed when several reobservations are made.

The example given in figures 87, 88, and 89 shows the results that are obtained using observed angles. It should be noted that the equations after the second set of observations show more conclusively that the faulty direction is at Kelat than did the set in which plane angles were used.

TWO-SIDES-AND-INCLUDED-ANGLE COMPUTATION

Triangle computations using two sides and included angle are made on Form 665. (See fig. 90 for example.) A sketch should be made of the triangle to be computed. The included angle is labeled C and the opposite side c. The longer side of the included angle is labeled a and angle opposite is A, which leaves the shorter side of included angle as b and angle opposite as B. The form is printed so that its computation follows step by step and is practically self-explanatory. $\frac{1}{2}(A+B)$ is obtained from angle C, and $\frac{1}{2}(A-B)$ is obtained from sides a and b by computation down the middle section of the form using the two formulas shown at the top of the form. ϕ is an auxiliary angle. $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$ are added and subtracted to determine angles A and B. The

SIDE TESTS USING OBSERVED ANGLES

Pole at intersection of diagonals FIRST SET

Hol	42 26 33.31	9.829 2079	23	Ell	27 20 48.35	9.662 1670	41
Ell	30 25 23.02	9.704 4773	36	Lin	79 47 16.23	9.993 0648	4
Lin	52 33 26.99	9.899 8007	16	Kel	17 13 53.75	9.471 6361	68
Kel	29 10 58.03	9.688 0613	38	Hol	81 01 41.15	9.994 6536	3
		9.121 5472				9.121 5215	
			+257	÷229	= +1.″1		
Pole	at Holton						
Ell	57 46 11.37	9.927 3254	13	Lin	79 47 16.23	9.993 0648	4
Lin	52 33 26.99	9.899 8007	16	Kel	46 24 51.78	9.859 9455	20
Kel	29 10 58.03	9.688 0613	38	Ell	27 20 48.35	9.662 1670	41
		9.515 1874				9.515 1773	
			+101	÷ 132	= +0.″8		
Pole	at Ellis						
Lin	132 20 43.22	9.868 7022	-19	Kel	17 13 53.75	9.471 6361	68
Kel	29 10 58.03	9.688 0613	38	Hol	123 28 14.46	9.921 2535	-14
Hol	42 26 33.31	9.829 2079	23	Lin	79 47 16.23	9.993 0648	4
		9.385 9714				9.385 9544	
			+170	÷ 166	= +1.″0		
Pole	at Kelat						
Hol	123 28 14.46	9.921 2535	-14	Ell	27 20 48.35	9.662 1670	41
Ell	30 25 23.02	9.704 4773	36	Lin	132 20 43.22	9.868 7022	-19
Lin	52 33 26.99	9.899 8007	16	Hol	81 01 41.15	9.994 6536	3
		9.525 5315				9.525 5228	
			+87	÷ 129	= +0.″7		
Pole	at Linville						
Kel	46 24 51.78	9.859 9455	20	Hol	81 01 41.15	9.994 6536	3
Hol	42 26 33.31	9.829 2079	23	Ell	57 46 11.37	9.927 3254	13
Ell	30 25 23.02	9.704 4773	36	Kel	17 13 53.75	9.471 6361	68
		9.393 6307				9.393 6151	
			+156	÷ 163	= +1.″0		

FIGURE 87.—Side tests using observed angles (first set).

SIDE TESTS USING OBSERVED ANGLES

SECOND SET

Pole at intersection of diagonals						
Hol	SEE FIRST SET			Ell	SEE FIRST SET	
Ell	" "	"		Lin	" "	"
Lin	52 33 24.19	9.899 7962		Kel	" "	"
Kel	SEE FIRST SET			Hol	" "	"
		9.121 5427				9.121 5215
			+212	÷ 229	= +0."9	
Pole at Holton						
Ell	SEE FIRST SET			Lin	SEE FIRST SET	
Lin	52 33 24.19	9.899 7962		Kel	" "	"
Kel	SEE FIRST SET			Ell	" "	"
		9.515 1829				9.515 1773
			+56	÷ 132	= +0."4	
Pole at Ellis						
Lin	132 20 40.42	9.868 7076		Kel	SEE FIRST SET	
Kel	SEE FIRST SET			Hol	" "	"
Hol	" "	"		Lin	" "	"
		9.385 9768				9.385 9544
			+224	÷ 166	= +1."3	
Pole at Kelat						
Hol	SEE FIRST SET			Ell	SEE FIRST SET	
Ell	" "	"		Lin	132 20 40.42	9.868 7076
Lin	52 33 24.19	9.899 7962		Hol	SEE FIRST SET	
		9.525 5270				9.525 5282
			−12	÷ 129	= −0."1	
Pole at Linville						
Kel	SEE FIRST SET			Hol	SEE FIRST SET	
Hol	" "	"		Ell	" "	"
Ell	" "	"		Kel	" "	"
		9.393 6307				9.393 6151
			+156	÷ 163	= + 1."0	

Tests indicate poor direction at Kelat

FIGURE 88.—Side tests using observed angles (second set).

side c is then computed by sines as indicated. Then the entire triangle is computed by angles and sines as a check triangle. In triangulation networks, the side c will often form the third side of a triangle other than the one from which it is computed. Combination (by addition or subtraction) of the computed angles A and B with the observed angles at the common vertices of these two triangles may then allow an independent check by a regular computation of the second triangle.

Two-sides-and-included-angle computations are sometimes preferable to inverse computations when it is desired to form an intersection triangle using two stations of the

SIDE TESTS USING OBSERVED ANGLES

					THIRD	SET
Pole	at intersection of diagonals for final side check					
Hol	SEE FIRST	SET		Ell	SEE FIRST	SET
Ell	" "	"		Lin	" "	"
Lin	" SECOND	"		Kel	17 13 56.85	9.471 6572
Kel	" FIRST	"		Hol	SEE FIRST	SET
		9.121 5427				9.121 5426
			+1	÷229	= 0.0"	

FIGURE 89.—Side tests using observed angles (third set).

triangulation scheme which are not in the same figure. The former computation is quicker and more readily checked than an inverse computation, and should give more consistent values because logs of lengths are used directly, whereas in an inverse the geographic positions used have been rounded off in the computations. In these cases, the angle C is best obtained by the differences in the computed azimuths of the lines a and b, and log a and log b are obtained directly from the scheme's triangle computations.

Two-sides-and-included-angle computations can also be used to recompute the R_2 diagonals of quadrilaterals in order to obtain more consistent field values, when it is desired to use these lines in computing positions of supplemental or intersection stations.

SPECIAL ANGLE COMPUTATIONS

Form 655a is used for certain special problems. The various cases in which the form is applicable are shown by sketches in the heading of the form (see fig. 91). A,B,C,D,E, and F are measured or concluded angles. The lines a and b, and the angle G are previously determined values. The points circled are the points to be determined.

The three-point problem is shown in Case 1 and illustrated in the sample computation. The two angles A and B are measured at an undetermined point between three known points shown here at the vertices of a previously determined triangle. The form and the example are self-explanatory.

If, in Case 1, the unknown point is situated on the circumference of the circle passing through the vertices of the known triangle, the problem is indeterminate.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 665
Ed. Dec. 1929

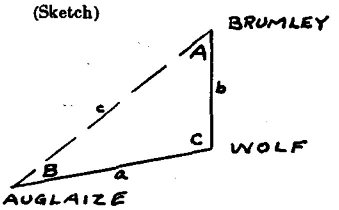
TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE

$\left[\frac{a}{b}=\tan(45°+\phi)\right.$ (Call longer side a): $\tan\frac{1}{2}(A_p-B_p)=\tan\phi\tan\frac{1}{2}(A_p+B_p)$: $\left.c=\frac{a\sin C_p}{\sin A_p}\right]$*

	°	′	″				
C_s	100	19	48.29	Log a	4.284 6854	Log m	1.40472
$\frac{\text{Sph. excess}}{3}$			.25	Log b	4.182 7101	Log sin C_s	9.99290
C_p	100	19	48.04	Log tan $(45°+\phi)$	0.101 9753	Log a	4.28469
$\frac{1}{2}C_p$	50	09	54.02	$(45°+\phi)$	51° 39′ 56.68″	Log b	4.18271
$90°-\frac{1}{2}C_p=\frac{1}{2}(A_p+B_p)$	39	50	05.98	ϕ	6 39 56.68	Log sph. ex	9.86502
$\frac{1}{2}(A_p-B_p)$	5	34	05.58	Log tan ϕ	9.067 6916	Sph. excess	0.733
Sum $=A_p$	45	24	11.56	Log tan $\frac{1}{2}(A_p+B_p)$	9.921 2722		
Diff $=B_p$	34	16	00.40	Log tan $\frac{1}{2}(A_p-B_p)$	8.988 9638		
C_p	100	19	48.04				

Log a	4.284 6854
Log sin C_p	9.992 9030
Colog sin A_p	0.147 4801
Log c	4.425 0685

(Sketch)



CHECK COMPUTATION

No.	STATION	SPHERICAL ANGLE	SPHERICAL EXCESS	PLANE ANGLE AND DISTANCE	LOGARITHM
	2-3				4.425 0685
	1 WOLF	100 19 48.29	0.25	48.04	0.007 0970
	2 AUGLAIZE	34 16 00.64	0.24	00.40	9.750 5446
	3 BRUMLEY	45 24 11.80	0.24	11.56	9.852 5199
	1-3				4.182 7101
	1-2				4.284 6854
			0.73		
	2-3				
	1				
	2				
	3				
	1-3				
	1-2				

*The subscripts s and p on this form refer to spherical and plane angles respectively.

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FIGURE 90.—Example, triangle computation using two sides and included angle, Form 665.

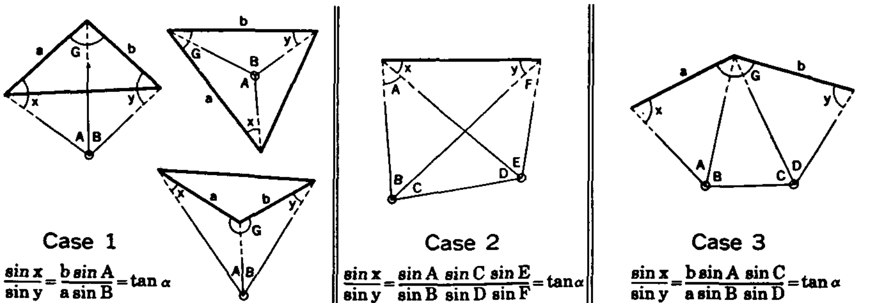
COMPUTATION OF GEOGRAPHIC POSITIONS

Geographic positions may be computed either by natural functions or by logarithms. When natural functions are used an electric desk calculator is required. The machine method is slightly faster and causes considerably less strain on the computer. In either case, computations are made from two known points over computed distances and direc-

494470 O -59 -13

Form 655a
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

SPECIAL ANGLE COMPUTATION



Case 1: $\frac{\sin x}{\sin y} = \frac{b \sin A}{a \sin B} = \tan \alpha$

Case 2: $\frac{\sin x}{\sin y} = \frac{\sin A \sin C \sin E}{\sin B \sin D \sin F} = \tan \alpha$

Case 3: $\frac{\sin x}{\sin y} = \frac{b \sin A \sin C}{a \sin B \sin D} = \tan \alpha$

$$\frac{1}{2}(x+y) = \begin{cases} \text{Case 1: } 180° - \frac{1}{2}(A+B+G) = 83° \ 49' \ 36.''5 \\ \text{Case 2: } \frac{1}{2}(C+D) = \\ \text{Case 3: } 270° - \frac{1}{2}(A+B+C+D+G) = \end{cases}$$

A = 84° 12′ 57.9
B = 21 38 06.8
G = 86 29 42.3

Leave blanks below here for values not involved in the CASE used.

log b =	3.644409	log a =	4.109221
log sin A =	9.997783 -10	log sin B =	9.566668 -10
log sin C =		log sin D =	
log sin E =		log sin F =	
* ① Sum =	3.642192	* ② Sum =	3.675889
-②		-①	3.642192
log tan α =		log tan α =	0.033697
α =	° ′ ″	α =	47° 13′ 14.″0
α - 45° =		α - 45° =	2 13 14.0

log tan ½(x+y) =		log tan ½(x+y) =	0.965928
log tan (α - 45°) =		log tan (α - 45°) =	8.588556 -10
Sum = log tan ½(x-y) =		Sum = log tan ½(y-x) =	9.554484 -10
½(x-y) =	° ′ ″	½(y-x) =	19° 43′ 21.″1
½(x+y) =		½(y+x)	83 49 36.5
x		y	103 32 57.6
y		x	64 06 15.4

α is an auxiliary angle needed only for the computation: it is always between 45° and 90°

* Where ① is greater than ② use only the left side of the form below here, and vice-versa.

FIGURE 91.—Example, special angle computation, Form 655a

tions to a common third point. This makes the computation self-checking. In the following examples the data labeled "(A)" are from previously determined positions, the data labeled "(B)" are from triangle computation Form 25, and the data labeled "(C)" are from published tables.

The type of form used is governed by the length of line and the class of triangulation.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 26—Rev. Apr. 11, 1939

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

				°	′	″					°	′	″
α	2	to 3	(A)	359	32	57.22	α	3	to 2	(A)	179	33	00.23
2d ∠		&	(B)	+414	50	11.91	3d ∠		&	(B)	−29	39	55.90
α	2	to 1		114	23	09.13	α	3	to 1		149	53	04.33
Δα				−	04	59.58	Δα				−	05	02.15
				180	00	00.00					180	00	00.00
α′	1	to 2		294	18	09.55	α′	1	to 3		329	48	02.18 +1
		First Angle of Triangle (B)		35 / 34	29 / 48	52.64 / 02.19							

	°	′	″			°	′	″		°	′	″			°	′	″
(A) φ	38	04	53.539	2 BRUMLEY	(A) λ	92	25	20.354	(A) φ	37	56	39.572	3 WOLF	(A) λ	92	25	15.447
Δφ	+	02	53.560		Δλ	+	08	05.452	Δφ	+	11	07.527		Δλ	+	08	10.360
φ′	38	07	47.099	1 KAISER	λ′	92	33	25.806	φ′	38	07	47.099	1 KAISER	λ′	92	33	25.807 −1

	Logarithms		″		Logs				Logarithms		″		Logs		
(B) s	4.113 3267	(1)	−173.8389	½	9̄.699	½(φ+φ′)	38 06 20.32	(B) s	4.376 6293	(1)	−667.8214	½	9̄.699	½(φ+φ′)	38 02 13.34
cos α	9.615 8237	(2)	+ 0.2781	s²			Logarithms	cos α	9.937 0242	(2)	+ 0.2824	s²			Logarithms
(C) B	8.510 9965	Sum	−173.5608	K		(B) s	4.113 3267	(C) B	8.511 0068	Sum	−667.5390	K		(B) s	4.376 6293
(1)=h	2.240 1469	(3)	+ 0.0007	E		sin α	9.959 4161	(1)=h	2.824 6603	(3)	+ 0.0107	E		sin α	9.700 4824
s²	8.22665	(4)	+ 0.0003	(5)		(C) A′	8.509 1655	s²	8.75326	(4)	+ 0.0011	(5)		(C) A′	8.509 1655
sin² α	9.91883	(5)	−	3	0.477	sec φ′	0.104 2381	sin² α	9.40096	(5)	−	3	0.477	sec φ′	0.104 2381
(C) C	1.29879	(6)	+	cos² α		Sum	2.686 1469	(C) C	1.29666	(6)	+	cos² α		Sum	2.690 5153
(2)=K	9.44427	(7)	+	(6)		Arc−sin corr.	+ 1	(2)=K	9.45088	(7)	+	(6)		Arc−sin corr.	− 6
(hφ)²	4.4803	−Δφ	−173.5598	(colog) E		Δλ	2.686 1465	(hφ)²	5.6493	−Δφ	−667.5272	(colog) E		Δλ	2.690 5147
(C) D	2.3795	Δφ/2		A⁴sec²1″/3	5̄.912	sin ½(φ+φ′)	9.790 3649	(C) D	2.3790	Δφ/2		A⁴sec²1″/3	5̄.912	sin ½(φ+φ′)	9.789 7011
(3)	6.8598			sec² φ		sec Δφ/2		(3)	8.0283			sec² φ		sec Δφ/2	6
−h	2.2401			(7)		(approx.) −Δα	2.476 5114	−h	2.8247			(7)		(approx.) −Δα	2.480 2164
s² sin² α	8.1455					do	″	s² sin² α	8.1542					do	″
(C) E	6.0650	Arc−sin corr.				(8)		(C) E	6.0622	Arc−sin corr.				(8)	
(4)	6.4506	for s	− 3.0	(Δλ)²		−Δα	299.579	(4)	7.0411	for s	− 10.0	(Δλ)²		−Δα	302.146
		for Δλ	+ 4.0	(C) F						for Δλ	+ 4.1	(C) F			
		Total	+ 1.0	(8)		Δλ	485.4522			Total	− 5.9	(8)		Δλ	490.3597

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FIGURE 92.—Example, position computation, first-order triangulation, Form 26.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 27
Ed. April, 1929

St. Elizabeth, St. Lawrence R.C. Church, Spire

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

			°	′	″				°	′	″
α	2	to 3	216	27	56.7	α	3	to 2	36	31	17.9
2d ∠		&	+ 46	59	20.2	3d ∠		&	− 52	21	18.6
α	2	to 1	263	27	16.9	α	3	to 1	344	09	59.3
Δα			+	04	29.3	Δα			+	01	08.4
			180	00	00.0				180	00	00.0
α′	1	to 2	83	31	46.2	α′	1	to 3	164	11	07.7
		First Angle of Triangle	80	39	21.5						

	°	′	″			°	′	″		°	′	″			°	′	″
φ	38	14	43.188	2 Lurton	λ	92	23	17.698	φ	38	20	29.059	3 Henley	λ	92	17	53.046
Δφ	+		39.130		Δλ	−	07	14.968	Δφ	−	05	06.741		Δλ	−	01	50.316
φ′	38	15	22.318	1 Spire	λ′	92	16	02.730	φ′	38	15	22.318	1 Spire	λ′	92	16	02.730

	Logarithms	Values in seconds			Logarithms	Values in seconds		Logarithms	Values in seconds			Logarithms	Values in seconds
s	4.027 1423			½(φ+φ′)	38 15 02.8		s	3.992 5695			½(φ+φ′)	38 17 55.7	
(−) Cos α	9.056 8622						(+) Cos α	9.983 2015					
B	8.510 9843			s	4.027 1423		B	8.510 9771			s	3.992 5695	
h	1.599 9888	1st term	−39.3540	(−) Sin α	9.997 1600		h	2.486 7481	1st term	+306.7242	(−) Sin α	9.435 9133	
s²	8.05428			A′	8.509 1624		s²	7.98514			A′	8.509 1624	
Sin² α	9.99432			Sec φ′	0.104 9921		Sin² α	8.87183			Sec φ′	0.104 9921	
C	1.30133			Δλ	2.638 4568	−434.9675	C	1.30282			Δλ	2.042 6373	−110.3157
	9.34993	2d term	+ 0.2238	Sin ½(φ+φ′)	9.791 7641			8.15979	2d term	+ 0.0144	Sin ½(φ+φ′)	9.792 2254	
h²	3.1899			−Δα	2.430 2209	−269.29	h²	4.9735			−Δα	1.834 8627	−68.37
D	2.3801						D	2.3804					
	5.5700	3d term	+ 0					7.3539	3d term	+ 0.0023			
		−Δφ	−39.1302						−Δφ	+306.7409			

FIGURE 93.—Example, position computation, third-order triangulation, Form 27.

LOGARITHMIC COMPUTATION

Computation of geographic positions by logarithms is made on Form 26. An example is shown in figure 92. A detailed description of the method is given in Special Publication No. 8, "Formulae and Tables for Computation of Geodetic Positions." The latitude functions listed therein are necessary for use of Form 26.

Positions are computed through the R_1 chain of triangles, using the spherical angles and logarithms of the sides from Form 25.

An example of position computation of an intersection station on Form 27 is shown in figure 93.

MACHINE COMPUTATION

Machine computations of geographic positions are made on Form 26a. An example is shown in figure 94. Tables, formulas, and details of the method are given in Special Publication No. 241, "Natural Tables for the Computation of Geodetic Positions." Natural sines and cosines to eight decimal places are listed in Special Publication No. 231, "Natural Sines and Cosines to Eight Decimal Places."

SHORT METHOD OF MACHINE COMPUTATION

This short method is strongly recommended for the computation of positions in the field or for any work which is unadjusted. (See fig. 95.) The method will, in general, permit position checks within $0\overset{''}{.}01$ for lines not exceeding 35,000 meters in length. All computations may be performed with an eight-bank calculating machine. Tables of the necessary functions are listed in "Natural Tables for the Computation of Geodetic Positions," Special Publication No. 241, and natural sines and cosines are listed in Special Publication No. 231.

1. Round off azimuths to nearest 0.1 second.
2. Take out sines and cosines to nearest 7th place. Interpolation can be made by inspection as the last decimal place need not be exact.
3. Round off x and y to nearest centimeter.
4. Compute a to 3 decimal places using x to nearest meter only.
5. Enter y_0 in thousands of meters, taking it out to nearest 0.1 thousand. This can be made by inspection as the last place need not be exact. y_1 is needed to compute V to 3 decimals. $\frac{1}{2}(y_0+y_1)$ is needed to enter table for differences per second of meridian arc.
6. Determine y_1 by adding y to y_0. This need be done to nearest 0.1 thousand only.
7. Interpolate, to fourth decimal place by inspection, the difference per second of the meridian arc for the mean of y_0 and y_1.
8. Take out V to 3 decimal places by inspection. This V can be used on both sides of the computation.
9. Determine Va to nearest centimeter.
10. Combine y and Va algebraically and divide the result by difference per second to obtain $\Delta\phi$ seconds.
11. Take out H for ϕ'. This H is used on both sides of the computation.
12. The product Hx is equal to $\Delta\lambda$ seconds.
13. Take out sin ϕ and sin ϕ' to 5 decimal places by inspection.

14. Multiply the mean of these two sines by $\Delta\lambda$ seconds to obtain $-\Delta\alpha$ seconds.
15. Round off positions to nearest 0.01 second.

INVERSE POSITION COMPUTATION

This consists of the computation of the distance and azimuths between two points whose geographic positions are known. Inverse position computations are not self-checking. They are best checked by an independent forward computation between the two points. Frequently a check triangle can be formed when a third point with previously computed distances and azimuths from the two points is available. Then, check angles can be determined from the azimuths, and a triangle on the third point computed with the inverse side as a base. Two-sides-and-included-angle computation should be used in place of the inverse position computation wherever possible. (See p. 174.)

Logarithmic computation of an inverse.—An example of logarithmic computation on Form 662 is shown in figure 96. The form is labeled in a self-explanatory manner. The first two lines contain the latitude and longitude of known points. The remaining quantities are determined by following through the form as labeled, using Special Publication No. 8 for A and B factors. The angle $\alpha+\frac{\Delta\alpha}{2}$ is obtained from the log tangent, which was obtained by subtracting log (s × cosine of the angle) from log (s × sine of the angle).

Form 26a
U. S. COAST AND GEODETIC SURVEY
DEPARTMENT OF COMMERCE
(Ed. Sept. 1945)

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION
(For calculating machine computation)

				°	′	″					°	′	″
α	2	to 3	(A)	359	32	57.22	α	3	to 2	(A)	179	33	00.23
2d ∠		&	(B)	+ 114	50	11.91	3d ∠		&	(B)	− 29	39	55.90
α	2	to 1		114	23	09.13	α	3	to 1		149	53	04.33
Δα				−	04	59.58	Δα				−	05	02.15
				180	00	00.00					180	00	00.00
α′	1	to 2		294	18	09.55	α′	1	to 3		329	48	02.18 +1
		First Angle of Triangle	(B)	35	29	52.64							

	°	′	″			°	′	″		°	′	″			°	′	″
φ (A)	9	04	53.539	2 Brumley	λ (A)	92	25	20.354	φ (A)	37	56	39.572	3 Wolf	λ (A)	92	25	15.447
				s = 12,981.555	Δλ	+	08	05.452					s = 23,802.868	Δλ	+	08	10.359
φ′	38	07	47.099	1 Kaiser	λ′	92	33	25.806	φ′	38	07	47.099	1 Kaiser	λ′	92	33	25.806

Δφ	2 53.6	(log s = (B) 4.113 3267) b = (y/10,000)²		Δφ	11 07.5*	(log s = (B) 4.376 6293) b = (y/10,000)²	
sin α	+ 0.91078552	x cor. = −½ fb	−0.001*	sin α	+ 0.501 74422	x cor. = −½ fb	−0.021*
cos α	− 0.41287982	x′	+11,823.411	cos α	− 0.865 01603	x′	+11,942.930
x = s sin α	+11,823.412	H	0.0410 58542	x = s sin α	+11,942.951	H	0.0410 58542
y = −s cos α	+ 5,359.822	Hx′ = (approx. Δλ″)	+485.4520	y = −s cos α	+20,589.862	Hx′ = (approx. Δλ″)	+490.3593
a = (x′/10,000)²	1.3979	Arc − sin / cor = + V (Va) / 15	+ 2*	a = (x′/10,000)²	1.4263	Arc − sin / cor = + V (Va) / 15	+ 2*
y cor. = + fa	+ 0.006*	Δλ″	+485.4522	y cor. = + fa	+ 0.024*	Δλ″	+490.3595
y_0	4,216,342.628	sin φ	0.616 78228	y_0	4,201,112.757	sin φ	0.614 89548
y′	+ 5,359.828	sin φ′	0.617 44440	y′	+ 20,589.886	sin φ′	0.617 44440
y_1	4,221,702.456	1 + cos Δφ	1.999 99964	y_1	4,221,702.643	1 + cos Δφ	1.999 99476
Va	− 8.591	(sin φ + sin φ′) / (1 + cos Δφ)	0.617 11345	Va	− 8.765	(sin φ + sin φ′) / (1 + cos Δφ)	0.616 17155
y_2	4,221,693.865	−Δα″ (approx.)	+ 299.579	y_2	4,221,693.878	−Δα″ (approx.)	+ 302.146
V	6.14536	+ F (Δλ″)³	0.000	V	6.14537	+ F (Δλ″)³	0.000
K (Va/1,000)² +	0.000	− Δα″	+ 299.579	K (Va/1,000)² +	0.000	− Δα″	+ 302.146

FIGURE 94.—Example, position computation (calculating machine), first-order triangulation, Form 26a. Entries marked with an asterisk are determined from nomograms shown in figures 134, 135, and 136 on pages 331 to 333. It should be noted that these are algebraic terms and not correction factors as described in Special Publication No 241.

Form 27a
U. S. COAST AND GEODETIC SURVEY
DEPARTMENT OF COMMERCE

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION
(For calculating machine computation)

			°	′	″				°	′	″
α	2	to 3				α	3	to 2			
2d ∠		&	+			3d ∠		&	−		
α	2	to 1	241	02	45.5	α	3	to 1	322	38	35.6
$\Delta\alpha$			+	11	14.4	$\Delta\alpha$			+	5	46.8
			180	00	00.0				180	00	00.0
α'	1	to 2	61	13	59.9	α'	1	to 3	142	44	22.4

First Angle of Triangle

	°	′	″			°	′	″		°	′	″			°	′	″
ϕ	33	44	23.21	2 Lincoln	λ	82	31	39.01	ϕ	34	04	58.84	3 Parsons	λ	82	21	47.61
	+	9	17.46	s = 35,583.75	$\Delta\lambda$	−	20	11.77		−	11	18.17	s = 26,270.23	$\Delta\lambda$	−	10	20.37
ϕ'	33	53	40.67	1 Williams	λ'	82	11	27.24	ϕ'	33	53	40.67	1 Williams	λ'	82	11	27.24

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			(Thousands)				(Thousands)
sin α	−0.875 0084	y_0	3,734.6	sin α	−0.606 7764	y_0	3,772.7
cos α	−0.484 1077	y	+ 17.2	cos α	+0.794 8726	y	−20.9
$x = s \sin \alpha$	−31,136.08	y_1	3,751.8	$x = s \sin \alpha$	−15,940.16	y_1	3,751.8
$y = -s \cos \alpha$	+17,226.37	$\frac{1}{2}(y_0+y_1)$	3,743.2	$y = -s \cos \alpha$	−20,881.49	$\frac{1}{2}(y_0+y_1)$	3,762.2
Va −	51.01			Va −	13.37		
Δy	+17,175.36	H	0.0389 18594	Δy	−20,894.86	H	(Same as left side)
diff. per sec.	30.8100	$Hx = \Delta\lambda''$	−1,211.772	diff. per sec.	30.8108	$Hx = \Delta\lambda''$	−620.369
V	5.261	sin ϕ	0.55542	V	(Same as left side)	sin ϕ	0.56039
$a = (x/10{,}000)^2$	9.695	sin ϕ'	0.55767	$a = (x/10{,}000)^2$	2.541	sin ϕ'	(Same as left side)
$\Delta\phi'' = \Delta y$ / diff. per sec.	+557.″461	$-\Delta\alpha'' = \frac{1}{2}(\sin\phi + \sin\phi')\Delta\lambda''$	−674.″40	$\Delta\phi'' = \Delta y$ / diff. per sec.	−678.″167	$-\Delta\alpha'' = \frac{1}{2}(\sin\phi + \sin\phi')\Delta\lambda''$	−346.″80

FIGURE 95.—Example, position computation (calculating machine), third-order triangulation, Form 27a.

An inverse computation for a short line can be conveniently made on one side of Form 27. The computation is made in reverse order until log (s sin α) and log (s cos α) are obtained. Then the second of these log terms is subtracted from the first to obtain log tan α, and α is then taken out of log tangent tables. After α is obtained, s is computed from either term, preferably the larger one.

Machine computation of an inverse.—Machine computation is most conveniently made by a backwards computation on Form 26a. (See fig. 97.)

The terms labeled "(E)" are the known latitudes and longitudes of the two stations.

The terms labeled "(G)" are determined directly from (E) or by using (E) as arguments in the tables. The arc-sine correction $V(Va)/15$ is equal to $(39174 \times 10^{-9})(\Delta\lambda'')^2 \sin^2 \phi'$. Multiplying this factor by H gives a correction which is applied directly to H. This corrected H is divided into $\Delta\lambda''$ to obtain x'.

An approximate V is determined using y_2 for argument. a is determined from x', then Va is determined, then y_1 is determined. A corrected value of V is determined after obtaining y_1. $y_1 - y_0 = y'$. The y correction is computed as fa. This is applied as a factor to y' to determine y, using the sufficient approximation $y = y'(1 - fa)$ and remembering that fa is expressed in units of the seventh decimal place. b is then computed and from it the x correction, which is applied as a factor to x' to determine x, using the sufficient approximation $x = x'(1 + \frac{1}{2}fb)$ and remembering that $\frac{1}{2}fb$ is expressed in units of the seventh decimal place. Then s is equal to the square root of the sum of the squares of x and y, and dividing s into x and y gives values for sin α and cos α. $\Delta\alpha$ is computed as usual at the bottom of the form. The computation is then checked.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 662
Rev. Sept. 1942

INVERSE POSITION COMPUTATION

$$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$$

$$s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$$

$$-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$$

in which $\log \Delta\lambda_1=\log(\lambda'-\lambda)$ − correction for arc to sin*; $\log \Delta\phi_1=\log(\phi'-\phi)$ − correction for arc to sin*; and $\log s=\log s_1+$ correction for arc to sin*.

NAME OF STATION

1. ϕ	37°	56′	39.572″	WOLF	λ	92°	25′	15.447″
2. ϕ'	38	07	47.099	KAISER	λ'	92	33	25.806
$\Delta\phi\ (=\phi'-\phi)$	+ 0	11	07.527	$\Delta\lambda\ (=\lambda'-\lambda)$		+ 0	08	10.359
$\frac{\Delta\phi}{2}$		05	33.7635	$\frac{\Delta\lambda}{2}$			04	05.1795
$\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$	38	02	13.3355					
$\Delta\phi$ (secs.)		+ 667.527″		$\Delta\lambda$ (secs.)			+ 490.359″	

log $\Delta\phi$	2.8244688			**log $\Delta\lambda$**	2.6905141
cor. arc−sin	− 2			cor. arc−sin	− 1
log $\Delta\phi_1$	2.8244686			log $\Delta\lambda_1$	2.6905140
log cos $\frac{\Delta\lambda}{2}$	9.9999997			**log cos ϕ_m**	9.8963126
colog B_m	1.4890001			**colog A_m**	1.4908322
log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	4.3134684 n	(opposite in sign to $\Delta\phi$)		**log$\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$**	4.0776588 p
				log$\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$	4.3134684 n
log $\Delta\lambda$	2.6905141	3 log $\Delta\lambda$	8.072	**log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	9.7641904 n
log sin ϕ_m	9.7897011	log F	7.874	$\alpha+\frac{\Delta\alpha}{2}$	149° 50′ 33.34″
log sec $\frac{\Delta\phi}{2}$	0.0000006	log b	5.946	**log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	9.7010300 p
log a	2.4802158 p			**log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$**	9.9368396 n
a	+ 302.145			**log s_1**	4.3766288
b	0.000			cor. arc−sin	+ 3
$-\Delta\alpha$ **(secs.)**	+ 302.145″			log s	4.3766291
$-\frac{\Delta\alpha}{2}$	+ 151.07″				
	02′ 31.07″				
$\alpha+\frac{\Delta\alpha}{2}$	149 50 33.34				
α (1 to 2)	149 53 04.41				
$\Delta\alpha$	− 05 02.14				
	180				
α' (2 to 1)	329 48 02.27				

* Use the table on the back of this form for correction of arc to sin.

NOTE.—For log s up to 4.0 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 3′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 7 decimal places.

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FIGURE 96.—Example, inverse position computation, Form 662.

Form 26a
U. S. COAST AND GEODETIC SURVEY
DEPARTMENT OF COMMERCE
(Ed. Sept. 1945)

INVERSE POSITION COMPUTATION, FIRST-ORDER TRIANGULATION
(For calculating machine computation)

		°	′	″		
α	2 to 3				α	3 to 2
2d ∠	&	+			3d ∠	&
α	2 to 1	149	53	04.39	α	3 to 1
Δα		−	05	02.14	Δα	
		180	00	00.00		
α′	1 to 2	329	48	02.25	α′	1 to 3

First Angle of Triangle

	°	′	″			°	′	″		°	′	″	
(E) φ	37	56	39.572	2 Wolf	(E) λ	92	25	15.447	φ				3
				s= 23,802.858	Δλ	+	08	10.359					s=
(E) φ′	38	07	47.099	1 Kaiser	λ′ (E)	92	33	25.806	φ′				1

(G) Δφ	0 11 07.5	(log s = 4.376 6291) b = (y/10,000)²	4.239	Δφ	
sin α	+0.501 74395	x cor. = −½ fb	17.4	sin α	
cos α	−0.865 01621	x′	11,942.919	cos α	
x = s sin α	+11,942.940	H (G)	0.0410 58542 +15	x = s sin α	
y = −s cos α	+20,589.858	Hx′ = (approx. Δλ″)		y = −s cos α	
a = (x′/10,000)²	1.4263	Arc−sin cor = + V (Va)/15	3.6	a = (x′/10,000)²	
y cor. = +fa	11.7	Δλ″ (G)	+490.359	y cor. = +fa	
(G) y₀	4,201,112.757	sin φ	0.614 89548	y₀	
y′	+20,589.882	sin φ′	0.617 44440	y′	
y₁	4,221,702.639	1 + cos Δφ	1.999 99476	y₁	
Va	− 8.765	(sin φ + sin φ′)/(1 + cos Δφ)	0.616 17155	Va	
(G) y₂	4,221,693.874	−Δα″ (approx.)	302.145	y₂	
V	6.1453~~5~~7	+ F (Δλ″)³		V	
K (Va/1,000)² +		− Δα″	+302.145	K (Va/1,000)² +	

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FIGURE 97.—Example, inverse-position computation (calculating machine), Form 26a.

LIST OF GEOGRAPHIC POSITIONS

Lists of geographic positions are made on Form 28B for all positions computed by the field party. (See fig. 98.) This is a tabulation of essential data from the position computation forms (Forms 26, 26a, 27, or 27a). This form should be very carefully made out and thoroughly checked. The field party's list of geographic positions should always be labeled across the top "Unadjusted Field Computations."

To save time and space, length and azimuths of any one line of the triangulation are tabulated only once. Only the latitude, longitude, lengths, azimuth, and back azimuth determined in each position computation need be listed. Stations are listed in the order that they were computed through the scheme. Separate sheets should be used for first-order, second-order, third-order, and intersection positions with the appropriate designation labeled in the heading of Form 28B. The data for the previously determined stations, used as the starting line in computing the new triangulation, should be given as shown in the first two entries of figure 98.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 28 B
Ed. April 1940

GEOGRAPHIC POSITIONS

Unadjusted Field Computations

Accession No. of Computation: ______

Locality Trinity River *North American 1927 Datum* First *-order Triangulation. State* Texas

STATION		LATITUDE AND LONGITUDE ° ′ ″	SECONDS IN METERS	AZIMUTH ° ′ ″	BACK AZIMUTH ° ′ ″	TO STATION	DISTANCE LOGARITHM (METERS)	DISTANCE METERS	DISTANCE FEET
COZBY, 1935		32 44 05.483							
r. '47	d.m.	97 25 01.921							
BOWMAN, 1935		32 42 28.109		102 31 51.45	282 27 10.40	COZBY	4.141 9956	13,867.4	
r. '47	d.m.	97 16 22.007		142 13 55.5		Azimuth Mark			
DICKESON, 1946		32 34 50.244		175 09 24.42	355 08 54.38	COZBY	4.234 6443	17,165.0	
r. '47	d.m.	97 24 06.261		220 35 31.11	40 39 41.54	BOWMAN	4.269 1213	18,583.2	
				267 19 46.0		Azimuth Mark			
SELMAN, 1947		32 34 09.432		94 49 14.36	274 44 03.87	DICKESON	4.178 8144	15,094.4	
r. '47	d.m.	97 14 29.579		169 12 28.63	349 11 27.99	BOWMAN	4.194 1885	15,638.3	
				183 18 02.9		Azimuth Mark			
THOMPSON, 1947		32 38 06.493		313 44 21.94	133 46 59.50	SELMAN	4.023 5761	10,557.9	
	d.m.	97 19 22.004		50 49 09.92	230 46 36.74	DICKESON	3.980 6593	9,564.4	
				343 00 19.4		Azimuth Mark			
BURLESON, 1947		32 34 19.645		176 17 19.49	356 17 10.12	THOMPSON	3.845 2528	7,002.5	
	d.m.	97 19 04.615		272 29 25.37	92 31 53.43	SELMAN	3.856 1945	7,181.2	
				163 51 19.9		Azimuth Mark			
CADDO, 1946		32 28 54.165		183 18 48.04	3 19 01.13	DICKESON	4.040 8738	10,986.9	
r. '47	d.m.	97 24 30.607		238 11 32.83	58 16 55.99	SELMAN	4.265 9602	18,448.5	
				209 36 45.1		Azimuth Mark			
EGAN, 1947		32 27 36.519		99 57 39.26	279 52 58.15	CADDO	4.142 4149	13,880.8	
	d.m.	97 15 47.003		189 28 24.23	9 29 05.85	SELMAN	4.088 8656	12,270.6	
				332 18 17.3		Azimuth Mark			

[1] No check on this position. Abbreviations used: d.=described; m.=marked; n.=not; r.=recovered; l.=lost; p.=probably. (Examples: n. d.=not described; p. l.=probably lost.)

FIGURE 98.—Example, list of geographic positions, Form 28B.

In cases where the short method of machine computation of geographic positions has been employed (see p. 178), it will be permissible to list the positions and azimuths to the number of decimal places computed; i.e., two for position and one for azimuth.

All stations located by triangulation should be entered on the list of geographic positions, and the names should correspond to those given in the descriptions, records, and computations. This includes geographic positions of occupied reference marks. Each name, including new stations, should always be followed by the date (year) that it was established, and by an abbreviated note to show whether or not the station was marked and described. The following abbreviations are employed for this purpose: d. = described, m. = marked, n.d. = not described, l. = lost, and r. = recovered. For example, d.m. after the name of a station means that it is described and marked; d. means that it is described but not marked. The azimuth to the azimuth mark should always be listed on Form 28B as shown in figure 98. It is not necessary for the field party to enter seconds in meters in column three or distance in feet in the last column of Form 28B.

The list of geographic positions is one of the most important parts of the computation of triangulation and should always be carefully made out in black ink, or typewritten, and should be completely checked before being sent to the Washington Office. Check marks should be small blue marks which in no way interfere with or obscure any of the listed data.

LANDMARKS FOR CHARTS

A report on Form 567, "Landmarks for Charts," should be submitted for those prominent objects and landmarks which should appear on nautical or aeronautical charts. Latitudes and longitudes of objects listed are obtained from the field geographic position computations. The seconds values of the latitudes and longitudes should be listed both in seconds and in meters in the "D.M. meters" and "D.P. meters" columns, except in areas which are not covered by our nautical charts. In the latter cases the values in meters may be omitted. The landmark name should be the identifying name for charting, such as SPIRE, TANK, DOME, TOWER, etc. (see pp. 283 to 285), followed by the full triangulation-station name, such as Greenville, St. Marys Church, spire. A detailed discussion of landmarks for charts may be found in section 8534 of Special Publication No. 143, "Hydrographic Manual."

VERTICAL ANGLES

COMPUTATION OF DATA

An Abstract of Zenith Distances, Form 29, is shown in figure 65 on page 127. The data which the observing units are required to complete on this form were discussed on page 128. These data are edited for completeness by the computer who then compiles all lightkeeper and observing-party data on the heights of objects observed upon (see p. 139) and enters the heights for each object in column 4 headed "*o*." The lower part of abstract Form 29 is filled in as a summary from lightkeeper and observer reports showing dates and heights of lights shown at the station named in the heading of Form 29. He then computes values for $t-o$ for column 6. Column 7, headed "Reduction to line joining stations," is essentially a vertical eccentric reduction. The formula for its computation is

$$r=-\frac{t-o}{s \sin 1''},$$

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 A

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.

Station 1, occ.	Tesla	Livermore East Base	Doolan			
Station 2, obs.	Brushy Pk	Brushy Pk.	Brushy Pk			
ζ_1	88 40 10.9	88 20 22.8	89 19 02.6			
ζ_2	91 24 18.8	91 45 48.9	90 46 25.1			
$\zeta_2-\zeta_1$	+2 44 07.9	+3 25 26.1	+1 27 22.5			
$\frac{1}{2}(\zeta_2-\zeta_1)$	+1 22 04.0	+1 42 43.0	+0 43 41.2			
$\frac{1}{2}(\zeta_2-\zeta_1)$ in secs.	4924.0	6163.0	2621.2			
log ditto	3.69232	3.78979	3.41850			
T	4.68566	4.68570	4.68560			
log s	4.03868	4.09824	4.06148			
log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$	2.41666	2.57373	2.16558			
log A	+ 2	+ 1	+ 3			
log B	+ 1	+ 1	0			
log C	0	0	0			
log (h_2-h_1)	2.41669	2.57375	2.16561			
h_2-h_1	+261.03	+374.76	+146.42			
h_1	255.31	140.14	369.29			
h_2	516.34	514.90	515.71			
2 log s	8.07736	8.19648	8.12296			
log $p=9-2$ log s	0.92264	0.80352	0.87704			
p of (h_2-h_1)	8.37	6.36	7.53			
α and mean ϕ	2° 38°	38° 38°	75° 38°			
$\zeta_1+\zeta_2-180°$	04' 29.7	06' 11.7	05 27.7			
$\zeta_1+\zeta_2-180°$ in sec.	269.7	371.7	327.7			
log ditto	2.43088	2.57019	2.51548			
log ρ	6.80343	6.80412	6.80513			
colog s	5.96132	5.90176	5.93852			
log $\frac{\sin 1''}{2}=\bar{4}.38454$	4.38454	4.38454	4.38454			
log $(0.5-m)$	9.58017	9.66061	9.64367			
$(0.5-m)$	0.3803	0.4577	0.4402			
p of $(0.5-m)$ *	1.19	1.57	1.33			

* Since $(0.5-m)$ varies as s^2, the weight $p=\frac{s^2}{N}$, where N is constant for a set and is preferably a power of 10.

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FIGURE 99.—Example, computation of elevations and refractions from reciprocal observations, Form 29A.

where r is the reduction in seconds, $t-o$ the difference in heights from the preceding column, and s the distance in meters between the stations involved. This reduction is made only when observations are reciprocal. Values in column 9 are obtained by applying the reduction from column 7 to the mean of observations in column 8.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 B

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS

Station 1, occ.	Livermore East Base	Tesla	Doolan			
Station 2, obs.	Positas	Positas	Positas			
Object sighted	light	light	light			
ζ_1	89 42 32.5	90 30 26.7	90 45 58.3			
α and mean ϕ	68° 38°	13° 38°	107° 38°			
log (0.5−m)	9.66061	9.58017	9.64367			
log s	3.95602	3.66009	4.09154			
colog ρ	3.19500	3.19648	3.19491			
colog sin 1″	5.31443	5.31443	5.31443	5.31443	5.31443	5.31443
log (k in secs.)	2.12606	1.75117	2.24455			
k in secs.	133.7	56.4	175.6			
(90°−ζ_1+k) in secs.	+1181.2	−1770.3	−2582.7			
log ditto	3.07232	3.24805″	3.41207″			
T	4.68558	4.68559	4.68560			
log s	3.95602	3.66009	4.09154			
log [s tan (90°−ζ_1+k)]	1.71392	1.59373″	2.18921″			
log A	+ 1	+ 2	+ 3			
log B	0	0	− 1			
log C	0	0	0			
log (h_2-h_1)	1.71393	1.59375″	2.18923″			
h_2-h_1	+51.75	−39.24	−154.61			
h_1	140.14	255.31	369.29			
$t-o$	+23.04	−0.34	+0.62			
Corrected elevation	214.93	215.73	215.30			
log p=9−2 log s	1.08796	1.67982	0.81692			
p	12.25	47.84	6.56			
Weighted mean elevation of sta. obs.		215.54 m.				

FIGURE 100.—Example, computation of elevations from nonreciprocal observations, Form 29B.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 C

COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS.
(By calculating machine)

Station 1, occ.	Tesla	Livermore East Base	Doolan			
Station 2, obs.	Brushy Pk.	Brushy Pk.	Brushy Pk.			
ζ_1	88 40 10.9	88 20 22.8	89 19 02.6			
ζ_2	91 24 18.8	91 45 48.9	90 46 25.1			
$\zeta_2-\zeta_1$	+2 44 07.9	+3 25 26.1	+1 27 22.5			
$\frac{1}{2}(\zeta_2-\zeta_1)$	+1 22 04.0	+1 42 43.0	+0 43 41.2			
$\tan\frac{1}{2}(\zeta_2-\zeta_1)$	0.023876	0.029888	0.012709			
s	10931.4	12538.3	11520.6			
A	1.000040	1.000022	1.000058			
B	+ 21	+ 29	+ 12			
C	+ 2	+ 2	+ 2			
h_2-h_1	+261.01	+374.76	+146.43			
h_1	255.31	140.14	369.29			
h_2	516.32	514.90	515.72			
$\frac{1}{s^2}=p$ of (h_2-h_1)	8.37	6.36	7.53			
α and mean ϕ	2° 38°	38° 38°	75° 38°			
$\zeta_1+\zeta_2-180°$	04′ 29.″7	06′ 11.″7	05′ 27.″7			
$\zeta_1+\zeta_2-180°$ in sec.	269.7	371.7	327.7			
$\rho\,\frac{\sin 1''}{2}$	15.416	15.440	15.476			
s	10931.4	12538.3	11520.6			
$(0.5-m)$	0.3803	0.4577	0.4402			
p of $(0.5-m)$ *	1.19	1.57	1.33			

Weighted mean elev. 515.71 m.

M-2085-2 (2)

$$h_2-h_1 = s\tan\tfrac{1}{2}(\zeta_2-\zeta_1)\,A\,B\,C$$

$$(0.5-m) = \frac{\zeta_1+\zeta_2-180° \text{ in sec. } \rho\,\frac{\sin 1''}{2}}{s}$$

* Since $(0.5-m)$ varies as s^2, the weight $p=\frac{s^2}{N}$, where N is constant for a set and is preferably a power of 10.

FIGURE 101.—Example, computation of elevations and refractions from reciprocal observations (by calculating machine), Form 29C.

COMPUTATION OF ELEVATIONS

Trigonometric elevations are computed by methods discussed in publications referred to in the following sections. Whenever practicable, connections should be made to bench marks established by spirit levels and, in general, connections should be made about

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 29 D

COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS.

(By calculating machine)

Station 1, occ.	Livermore East Base	Tesla	Doolan			
Station 2, obs.	Positas	Positas	Positas			
Object sighted	light	light	light			
ζ_1	89 42 32.5	90 30 26.7	90 45 58.3			
α and mean ϕ	68° 38°	13° 38°	107° 38°			
$(0.5-m)$	0.4577	0.3803	0.4402			
s	9036.9	4571.9	12346.5			
$\rho \sin 1''$	30.943	30.838	30.950			
k in secs.	133.7	56.4	175.6			
$(90°-\zeta_1+k)$	+0 19 44.2	-0 29 30.3	-0 43 02.7			
$\tan(90°-\zeta_1+k)$	0.005727	0.008583	0.012522			
A	1.000022	1.000040	1.000058			
B	+ 4	- 3	- 12			
C	+ 1	+ 1	+ 2			
h_2-h_1	+51.76	-39.24	-154.61			
h_1	140.14	255.31	369.29			
$t-o$	+23.04	-0.34	+0.62			
Corrected elevation	214.94	215.73	215.30			
$\frac{1}{s^2} = p$ of (h_2-h_1)	12.25	47.84	6.56			
Weighted mean elevation of sta. obs.		215.54				

M-2085-2

$$k \text{ in secs.} = \frac{(0.5-m)\,s}{\rho \sin 1''}$$

$$h_2-h_1 = s \tan(90°-\zeta_1+k)\,A\,B\,C$$

FIGURE 102.—Example, computation of elevations from nonreciprocal observations (by calculating machine), Form 29D.

every third quadrilateral. (See p. 16.) Levels are run to bench marks within about a mile of a triangulation station. Where connections are necessary for distances over about a mile, an extra-figure triangulation connection is made to a bench mark and reciprocal vertical angles are observed. On projects requiring vertical-angle observations and on which the elevations are not computed in the field, a check on the observations should be made as described on page 15.

Logarithmic computation of elevations from reciprocal observations.—These computations are made on Form 29A (see fig. 99). The values of ζ_1 and ζ_2 are the corrected zenith distances obtained from column 9 of Form 29 (fig. 65 on p. 127) for each of the two stations concerned. Special Publication No. 138, "Manual of Triangulation Computation and Adjustment," gives a detailed discussion of this computation.

Logarithmic computation of elevations from nonreciprocal observations.—Form 29B is used for computing differences of elevation from nonreciprocal observations. (See fig. 100.) The quantities ζ_1 and $(t-o)$ are obtained from Form 29. (See fig. 65.) The quantities log s, α, and "mean ϕ" are all obtained from the list of geographic positions, or from the triangle and position computations. Colog ρ (ρ being the earth's radius of curvature) is determined by subtracting from 1 the value of log ρ as taken from the tables on pages 304 to 308. Log A, log B, and log C are obtained from the tables on page 276. The value of $(0.5-m)$ is obtained from the computation of the reciprocal observations involving the same station. In case the elevation was fixed and there were no reciprocal observations from this station, then there are no previous determinations of $(0.5-m)$, and the value of $m=0.071$ may be used.

Computation of elevations and refractions from reciprocal and nonreciprocal observations by calculating machine.—These computations are made on Forms 29C and 29D (see figs. 101 and 102). Tables and detailed discussion of these computations are available at present in lithographed form (G-56) and will be published in the contemplated revision of Special Publication No. 138, "Manual of Triangulation Computation and Adjustment."

REPORTS

MONTHLY REPORTS

In accordance with paragraph 259 of Serial No. 685, "Regulations of the Coast and Geodetic Survey," the Monthly Report and Journal of Field Party should be submitted to the Director monthly on Form 20.

Care should be taken to avoid counting the same stations or area twice in successive months for statistics on the Monthly Report and Journal (Form 20). A colored pencil line may be drawn on the operations sketch described on page 96 to define the limits of area counted for each month's statistics. It is helpful to maintain a statistics notebook by projects with the necessary entries for data required on Forms 20, 21, and 615 and for the season's and annual reports.

Progress letter.—An informal report of progress to the Washington Office in letter form should be made monthly about the fifteenth of each month. This report should include a general statement on the limits of work accomplished for the month preceding the date of the report, a brief discussion of operations or items of special interest, a discussion of weather or other conditions affecting the operation of the party (both favorable and unfavorable), status of the computations, brief general plans for the next period, the probable location of the next camp and probable time of the move, changes in key personnel or duties, inspections of field units by the chief of party, and public contacts of special interest.

Miscellaneous reports to the Chief, Division of Geodesy.—The following reports on forms indicated should be mailed directly to the Chief, Division of Geodesy, at the end of each calendar month:

Summary of Monthly Truck Reports, Form 702 (original)

Storage Report, Form 46 (original)

Payrolls, Standard Form 1128a (the triplicate copy)

Monthly Statement of Allotment Balances and Obligations Incurred, Form 474 (original and duplicate).

Personnel and accounts forms.—Accounts are submitted monthly in accordance with the "Regulations of the Coast and Geodetic Survey," Serial No. 685, and current circulars.

SEASON'S REPORT

The season's report of a triangulation party is an administrative and professional report on a project. Season's reports should be submitted within 30 days after the completion of a season's work on a project, and on completion of an entire project. Season's reports are submitted under the name of the chief of party. In general, when there is a transfer of a party while work is in progress, the original chief of party should submit a season's report up to the date of his detachment.

The report should be clearly written, logically arranged, and concise. The following items should be covered by the season's report of a triangulation party:

1. Authority, purpose, scope, and time of beginning and ending field work.
2. Locality, area covered, limits, and junction with other work (generalized).
3. Conditions affecting organization or progress (favorable and unfavorable), including those of topography, climate, and transportation.
4. Organization of party personnel, equipment, transportation, and sub-parties.
5. Field work, methods, and chronology (brief). Particular mention should be made of any non-routine processes or developments.
6. Discussion of results, changes from reconnaissance due to obstructed lines, status of records and computations, and discrepancies.
7. Statistics.—Number of triangles, and average and maximum closures of each class of work. Field closures in latitude, longitude, azimuth, and length. Attach Form 21 with all statistics filled in. There must be no overlapping of fiscal years for statistics or cost data.
8. Statement of costs.—Attachment of completed Forms 615 and 21 furnishes a summary of cost data. Cost should include expenditures (both disbursed and incurred) from all appropriations. An item should be included for truck depreciation as explained in the preface of the motor truck record book, Form 625. Cost of leave should be proportioned to projects. Cost of travel to, or at the beginning of, a project should be charged to that project. Cost per station is the average for occupied stations of all orders. Cost per mile is based on statute miles along the major axis of main-scheme figures. Cost per square statute mile is based on total completed area covered by all classes of triangulation for each project. Cost of base lines is listed separately on the line labeled "traverse," with the cost proportional to the man-days involved.

9. Recommendations and special features not covered by above.
10. Photographs of operations, if available.

ANNUAL REPORT

An Annual Statistical Report on a special form (plate No. M–1133–5) is required in duplicate immediately after 30 June of each year. Brief résumés are required (by projects) of field work statistics, expenditures, and personnel for the fiscal year. This information is used in preparing the Director's annual report. This report is made by the chief of party who is in charge of the party on 30 June for all projects on which the party has been engaged for the entire fiscal year of the party. The data are most readily compiled from field copies of season's reports made during the fiscal year. The total expenditures of all projects must be equal to total expenditures shown on the statement of balances for the fiscal year, plus truck depreciation.

MISCELLANEOUS REPORTS

Semi-annual "Summary of Motortruck Record" is submitted on Form 625a on June 30 and December 31.

Annual inventories are submitted on Form 14 at the end of each calendar year. These inventories include those: (1) For instruments; (2) for general property; and (3) for books received from the Coast and Geodetic Survey Library.

SKETCHES

PROGRESS SKETCH

It is advantageous to construct a projection on cloth-backed drawing paper early in the project. The projection should cover the area of the project in one or more sheets. Scales of from 1:100,000 to 1:250,000 are usually convenient to work on, but the scale should be large enough so that the sketch will show all details satisfactorily. Main-scheme stations are plotted by their geographic positions, supplemental stations can be plotted by computed sides, and intersection stations by angles with a protractor. This affords a check on the completeness and identity of cuts to intersection stations. On completion of the project, a neat tracing on tracing cloth is made of the working progress sketch. (See fig. 103.)

It is very desirable that a copy of the progress sketch should reach the Washington Office along with records, computations, and descriptions shortly after the completion of each project or season's work. A tracing made over a revised reconnaissance sketch of proper scale will be acceptable.

On the tracing, a border is placed around the area covered, and the ends of the projection lines are drawn at the borders and labeled. Principal intersections of the projection are also shown with light-weight short lines when they do not interfere with the principal drawing. It is suggested that 30-minute intersections be shown on sketches of 1:250,000 scale, 10-minute intersections on sketches of 1:125,000 scale, and 5-minute intersections on sketches of 1:62,500 scale. The following will indicate the proper form for the title of the drawing:

494470 O - 59 - 14

U. S. COAST AND GEODETIC SURVEY
L. O. COLBERT, DIRECTOR

PROGRESS SKETCH

FIRST- AND SECOND-ORDER TRIANGULATION

MISSOURI VALLEY, MISSOURI

PROJECT G-840 APRIL-JUNE 1947

JOHN DOE—CHIEF OF PARTY
GRAPHIC SCALE IN MILES

New stations are indicated by small equilateral triangle symbols. For recovered stations a circle is circumscribed around the triangle. Intersection stations are shown with a small circle. Full station names in bold block capital letters are lettered by each station symbol, except that a number may be placed at intersection and short traverse stations, and their full names tabulated in some convenient blank area of the drawing. The main scheme should be shown with heavy-weight lines, base lines with double-weight lines, and the supplemental scheme with lighter-weight lines. Lines which are observed at only one end are shown dashed at the unobserved end. Reconnaissance lines should be dotted if shown on the sketch with observed triangulation, although occasion for showing reconnaissance lines will be very rare. If it is desirable for clarity to show any lines of a previously observed scheme over which no observations were made during the current project, they may be shown with long dashed lines. Only short segments of ends of lines are shown at intersection stations. The height of the signal in feet should be shown under the name of each station.

ANNUAL SKETCH

Sketches on the size of letter-paper sheets (8 by 10½ inches) are required for all work executed during the fiscal year to accompany the annual statistical report. These sketches may be photographically reduced copies of the progress sketches, or the sketch may be generalized without showing all actual geometric figures or any names of stations. Its principal object is to show the areas covered by the various classes of work. Each sketch should include the same general style of title information as shown in the preceding section.

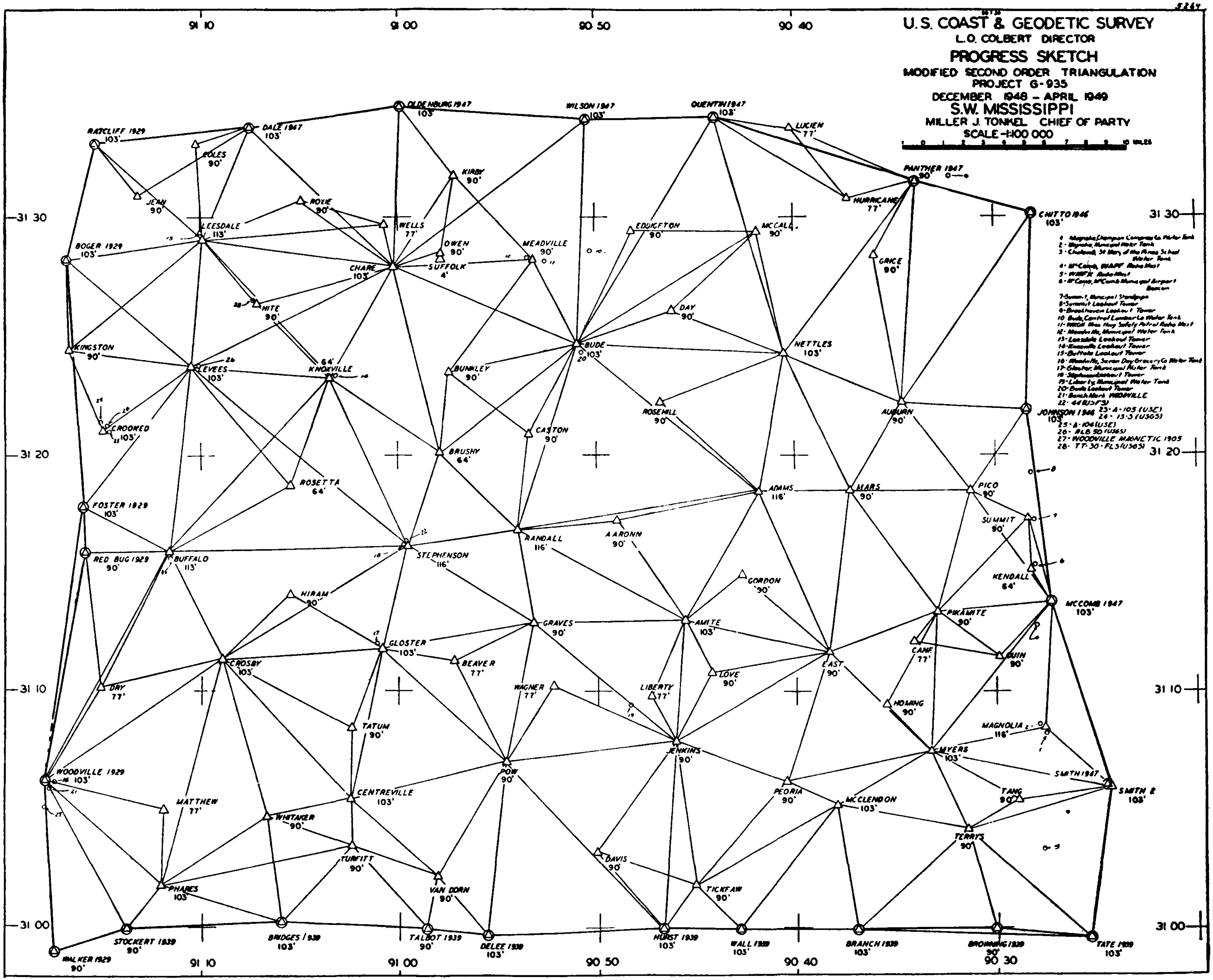

FIGURE 103.—Example, progress sketch, modified second-order (area) triangulation.

494470 O - 59 (Face p. 192)

Chapter 3.—BASE MEASUREMENT

GENERAL STATEMENT

A base line as used in triangulation constitutes the measured side of one of a series of connected triangles. Base lines are used as length control for triangulation, both as starting lines of known length and as check lines to insure the required degree of accuracy of the triangulation. Base lines are incorporated in the triangulation scheme to fulfill the requirements of the class of triangulation of the project.

The operation of base-line measurement is conducted with refinements to obtain a very high accuracy. These refinements include: The use of standardized tapes of small coefficient of thermal expansion; measurement with standard tape supports and tension; limiting conditions of alinement and grade; and corrections for grade, temperature, and any variations of alinement, tension, and support.

Common types of base lines which require slightly different adaptations of equipment and measuring techniques are those measured on stakes, those measured on a railroad track, and those measured on pavements. Where conditions are suitable, a base measured on a rail is usually more economical and satisfactory, principally because time and expense of staking are eliminated, effect of wind on the supported tape is less, and rails form a continuous and uniform marking table with fewer set-ups and set-backs.

The classes of bases are designated in table 1 on page xv. The specifications and field operation methods of base measurement are described in the following sections of this manual.

GENERAL INSTRUCTIONS FOR BASE MEASUREMENT

SPECIFICATIONS FOR FIRST-ORDER BASE MEASUREMENT

1. **Reconnaissance.**—The specifications for reconnaissance for geodetic triangulation (see p. 1) govern the character and number of figures between bases. There is a close interrelation between the specifications for reconnaissance, triangulation, and base measurement. The frequency of bases, the geometrical strength of the intervening figures, and the accuracy of measurement of angles and bases are factors which determine the final accuracy of the adjusted lengths, azimuths, and geographic positions. If one factor is weakened, one or more of the others must be correspondingly strengthened to maintain the required accuracy. For instance, unsatisfactory strength-of-figure accumulation or unsatisfactory length checks may necessitate additional bases.

2. **Site of base.**—A base may be measured with tapes with the required accuracy over fairly rough ground spanning small ravines and streams less than 50 meters wide, and over fairly steep slopes up to a grade of 10 percent. It is sometimes necessary to use occasional built-up stands in somewhat rugged terrain in order to span ravines or to reduce grades. The best grades and conditions requiring the least preparation of the line are usually found on long tangents of railroads or highways. Measurements along a rail can be made very satisfactorily.

3. **Alinement.**—The alinement of the stakes (or tape ends) should be done with such care that the base will contain no errors in length from this source in excess of 1 part in 500,000. This requirement can usually be met by the following limitations. No marking strip or point marking a 50-meter tape end should be more than 1 inch off the line joining the two adjacent marked tape-end points. The 1-inch tolerance in alinement

should be decreased proportionally for distances of less than 50 meters. No point between terminal stations should be more than 6 inches off the line.

A broken base (one composed of more than one tangent) may be used, where topographic conditions demand it, provided the terminal stations are, or can be made, intervisible and the angles at each break and at each end are measured so as to form a closed polygon with an accuracy necessary to secure the precision in length indicated below. No considerable portion of the base should be inclined at an angle of more than 20° to the final projected line of the base, and this maximum should be kept down to 12° if possible. The total error due to projecting the elements of the base upon the straight line between the base ends should not exceed 1 part in 500,000 of the length of the base.

4. Instruments, standardization, and tests.—The principal base-measuring tapes are standardized 50-meter ribbon tapes of a low coefficient of thermal expansion, usually composed of the nickel-steel alloy called invar. These tapes are used in sets of four, one of which is used as a standard for field comparison and as an emergency substitute tape.

Tapes should be standardized at the National Bureau of Standards before the first base of a season is measured and immediately after completion of the last base of a project. If only one base is scheduled for a project, the tapes should be returned to the Washington Office for restandardization on completion of this base. The 50-meter tapes should be standardized for support at the 0-, 25-, and 50-meter marks as usually used on stakes, and at the 0-, 12.5-, 37.5-, and 50-meter points as usually supported on rails and pavements, and also for support throughout. The 30-meter steel tape used for measuring short distances is standardized for each meter supported throughout and for two-point support at distances in 5-meter increments from the zero mark.

An intercomparison of the four 50-meter tapes should be made in the field over a test section of at least four tape lengths immediately before and after the measurement of each base. The terminal points of the test section should be carefully maintained while the base is being measured. If stakes are used for this test section, they should be especially well braced in four directions, and in a place where they will be free from disturbance.

The spring balance should be tested and set to read correctly by using the 15-kilogram test weight. This test is recorded before and after each day's work.

The special tape thermometers are tested at the National Bureau of Standards before being sent to the field.

5. Tape measurements.—At least three tapes will be used in measuring each base, the portions measured with each tape being approximately equal in total length. To secure this result the base will be measured in sections approximately 1 kilometer in length, except that one section may be longer or shorter than 1 kilometer, and the total length will be divided into three divisions of approximately equal length, each division beginning and ending with the end of some kilometer section. A different pairing of the tapes will be used on each of the three divisions, in order to secure a complete intercomparison of the lengths of the tapes. Each tape will be run forward on one division and backward on another. The party should have a fourth tape for use in the measurement in the event that one of the other tapes is injured or shows an unwarranted discrepancy in length when compared with the others.

Only one forward and one backward measurement of any one section will be made unless the discrepancy in millimeters between the two measurements exceeds $10\sqrt{K}$ (where K is the length of the section in kilometers), in which case additional measurements of the section must be made, preferably with the tapes originally used, until forward and backward measurements are secured which agree within this limit. A rough check measurement of the total length should be made with a 300-foot steel tape.

6. Slope measurements.—The slope of no tape length should, as a rule, exceed 10 percent. Where this limit is approached or exceeded especial care should be used in determining the difference of elevation of the tape ends, and the points leveled upon should be the supports of the fiducial marks of the tape. The nomogram in figure 104 shows the accuracy necessary to be attained in determining the difference of elevation between tape supports at different distances apart, in order that the proportionate error in the reduced length for that distance may not be greater than 1 part in 100,000.

Rod readings are to be obtained at all tape-end supports and broken-grade intermediate supports. The front and back sides of a rod which is graduated in feet on one side and in meters on the other should be read on each point. If the only rods available are graduated in only one unit, the spirit levels should be run both forward and backward over the base line.

7. Criteria.—A first-order base shall be measured with such accuracy that the computed probable error will not be greater than 1 part in 1,000,000. Such precautions should be used in alinement of the tape, in marking the tape lengths, and in determining the corrections for grade, tension, and temperature as will insure that the actual error (see p. 267) in length of the measured base due to any one of these causes will not exceed 1 part in 500,000.

8. Reduction to sea level.—In order to reduce the measured length of a base line to sea level, observations should be made to connect the base-line levels to a bench mark or other point of known elevation. Where Coast & Geodetic Survey bench marks are not available, elevations of marks of other organizations (if referred to mean sea level) may be used, if available. Under ordinary conditions in areas where trigonometric elevations are determined by the triangulation party, the trigonometric leveling will supply adequate elevations for sea-level reductions; provided, of course, a sufficient number of ties to bench marks are made in the trigonometric network and the observations indicate that sufficient accuracy of observations was obtained. The sea-level reduction will be within the requirements for precision of the base if the mean elevation above sea level of a base line is determined to within 6 meters.

9. Computation of base lines.—A field computation of the length of the base line should be made and checked. The original records and field computation should be forwarded to the Washington Office by registered mail in separate shipments.

SPECIFICATIONS FOR SECOND-ORDER BASE MEASUREMENT

The specifications for first-order base measurement listed in the preceding paragraphs also apply to second-order base measurement except as modified in the following paragraphs.

10. Alinement.—Where topographic conditions demand it, a broken base may be used, provided the terminal stations are intervisible and the angles at each break and at each end are measured so as to form a closed polygon with an accuracy necessary to secure the precision in length indicated below. No considerable portion of the base

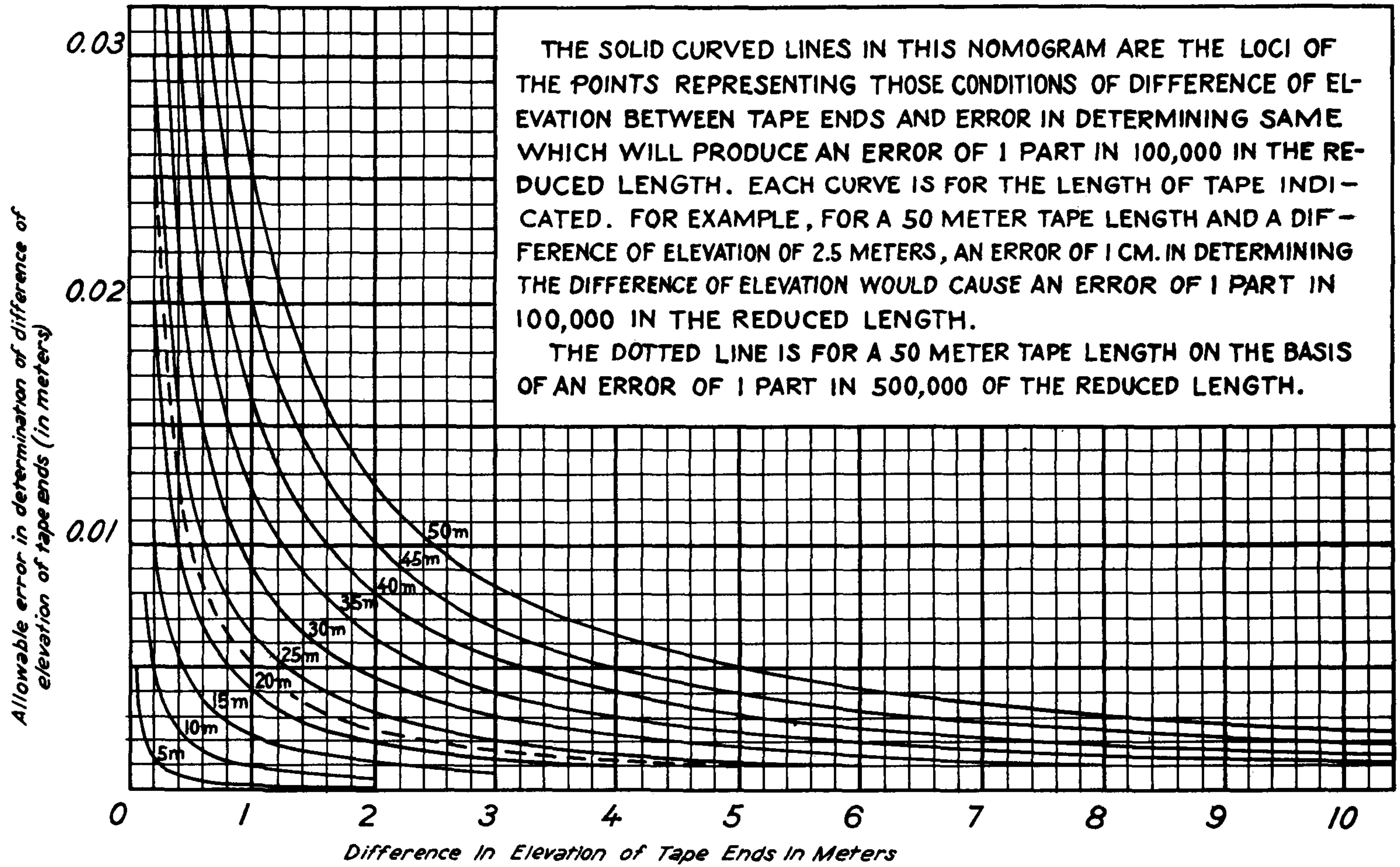

FIGURE 104.—Nomogram for effect of errors in the relative elevations of the ends of a tape upon the reduced length.

should be inclined at an angle of more than 30° to the final projected line of the base, and this maximum should be kept down to 20° if possible. The total error due to projecting the elements of the base upon the straight line between the base ends should not exceed 1 part in 300,000 of the length of the base.

11. Criteria.—A second-order base shall be measured with such accuracy that the total actual error due to known causes shall not exceed 1 part in 300,000 of the length of the base for Class I, or 1 part in 150,000 for Class II. The computed probable errors shall not be greater than 1 part in 1,000,000 and 1 part in 500,000 for Classes I and II, respectively. Very little increase in the average accuracy of the lengths of the lines of the triangulation will result from an increase in the accuracy indicated above, and no additional time and expense should be expended in securing an accuracy beyond the limits stated.

12. Checks and standardization.—Two measurements shall be made of the base with tapes which have been properly standardized. If the discrepancy in millimeters between the two measurements of a section exceeds $20\sqrt{K}$ (where K is the length of the section in kilometers), additional measurements of that section should be made until two measures made in opposite directions are secured which agree within this limit. At the end of the season, or sooner if practicable, the tapes should be returned to the Washington Office for restandardization.

13. Instruments.—In the Coast and Geodetic Survey, invar tapes, because of the greater economy of operation, have so completely replaced steel tapes for all accurate measurements that in these specifications the use of invar tapes is assumed unless steel tapes are specifically mentioned. If steel tapes are used, the measurement should be made either at night or when the sky is heavily overcast.

14. Tape measurements.—Two tapes shall be used on the measurement of a base, and a third tape should be available as a field standard. The third tape should be used in case the measurements made with the other two tapes indicate injury to one or both of them. The base should be measured in sections about 1 kilometer in length with the exception of one section which may be longer or shorter than this. Each section should be measured at least twice, one measurement being made in the forward direction and the other in the backward. At the beginning and at the ending of the measurement of a base, a one-kilometer section should be measured with all three tapes, in order to secure a field comparison to detect any changes from the standardized lengths.

15. Reduction to sea level.—If the mean elevation of the base is more than 25 meters above sea level, a connection should be made between the base-line levels and some point of known elevation which will give the elevation of the base with an error not exceeding 25 meters. An error of 25 meters in elevation causes an error of about 1 part in 250,000 in the reduced length of the base; in most cases the mean elevation of the base can be easily determined with much greater accuracy than that specified.

SPECIFICATIONS FOR THIRD-ORDER BASE MEASUREMENT

The same methods are used in measuring a third-order base as those prescribed for second order, and the precautions to be used in guarding against error are the same. The only differences are in the allowable discrepancies, as noted in the following paragraphs.

16. Criteria.—On a third-order base the total actual error from all known sources shall not exceed 1 part in 75,000 of the length of the base and the computed probable error shall not be greater than 1 part in 250,000. Two measurements shall be made of the base with two different standardized tapes, the base being divided into kilometer sections and the two measurements of each section being made in opposite directions as on second-order base measurement. If the discrepancy in millimeters between the two measurements of a section exceeds $25\sqrt{K}$ (where K is the length of the section in kilometers), additional measurements should be made until forward and backward measurements are secured which agree within the specified limit.

17. Reduction to sea level.—If the mean elevation of the base is more than 50 meters above sea level, the mean elevation shall be determined with an error of not to exceed 50 meters in order that the measured length may be reduced to its sea-level equivalent.

18. Tape measurements.—Such precautions shall be taken in staking and measuring the base that the errors due to lack of alinement, wind effect, or support of the tape shall not for any one of these causes be more than 1 part in 150,000 of the length of the tape. The tension should not be in error by more than 150 grams. The error in the grade corrections and the error in projecting a broken base upon the line between the terminal stations should not exceed that specified for a second-order base.

INSTRUMENTS

Instruments used in base-line measurement are listed on page 279. The principal characteristics of some of the more important items are described in the following paragraphs.

TAPES

The 50-meter base tapes are manufactured of a nickel-steel alloy with a low temperature-expansion coefficient. This alloy is ordinarily called invar, though other trade names, such as nilvar and lovar, are also used.

A standardization certificate is furnished by the National Bureau of Standards as shown in figure 105. A photostat of the premeasurement standardization certificate is furnished the field party for use in the field computations. The principal data shown on the standardization certificate are as follows:

Weight of tape in grams per meter.

Coefficient of expansion per degree centigrade.

Length at specified temperature when supported at 0-, 25-, and 50-meter points.

Length at specified temperature when supported at 0-, 12.5-, 37.5-, and 50-meter points.

Length at specified temperature when supported throughout.

For base tapes which are marked for each 5 meters of length, standardization values are also furnished for these marks with the tape supported on a horizontal flat surface. Lengths between intermediate marks under other conditions of support can be computed from these standardization data.

The restandardization of intermediate marks on the tape is not normally made each time the full tape length is standardized, unless there is evidence of an appreciable change.

Since the graduation lines are not always perfectly straight and parallel, the standardization data apply to distances between points defined by the ends of the graduation lines and at one particular edge of the tape. Unless otherwise noted in the standardization data, this edge is the one farther from the observer when the zero of the tape is to his left. Usually this edge is marked by small dots near the defining ends of the graduation lines.

NBS 584 (Rev. 8/12/48)

UNITED STATES DEPARTMENT OF COMMERCE
WASHINGTON

National Bureau of Standards

Certificate

FOR

50-METER IRON-NICKEL ALLOY TAPE

(Low Expansion Coefficient)

NBS No. 3242

U.S.C. & G.S. No. 917

SUBMITTED BY

U. S. Coast and Geodetic Survey,
Washington 25, D. C.

This tape has been compared with the standards of the United States under a horizontal tension of 15 kilograms. The interval (0 to 50 meters) has the following lengths at 25 °C under the conditions given below:

Supported at the 0-, 25-, and 50-meter points: 49.99941 meters.

Supported at the 0-, 12.5-, 37.5-, and 50-meter points, with the 12.5- and 37.5-meter points 6 inches above the plane of the 0- and 50-meter supports: 49.99904 meters.

The weight per meter of this tape, previously determined, is 25.8 grams.

Thermometers weighing 45 grams were attached at points 1 meter inside the terminal marks.

These comparisons were made on the section of the lines near the end on the edge of the tape marked with a small "x" or "v" or dots near the graduation.

The values for the lengths are not in error by more than 1 part in 500,000 ; the probable error does not exceed 1 part in 1,500,000.

The values for the lengths were obtained from measurements made at 24.4 °C, and in reducing to 25 °C, the thermal expansion of +0.055 millimeter per 50 meters per degree centigrade was used.

For the Director *by*

Lewis V. Judson
Lewis V. Judson
Chief, Length Section,
Metrology Division.

Test No. 2.1/ 119092
Test completed: February 10, 1949.

U. S. GOVERNMENT PRINTING OFFICE 16—68418-1

-2- NBS No. 3242
U.S.C. & G.S. No. 917

Length at 25° C under a horizontal tension of 15 kilograms, supported on a horizontal flat surface (value for the total length computed from observations taken on the tape when supported at three points):

Interval	Length	Corrected to 1949 Standard
(0 to 5 meters)	5.00094 meters	5.00094 m.
(0 to 10 ")	10.00150 "	10.00151 "
(0 to 15 ")	15.00169 "	15.00170 "
(0 to 20 ")	20.00213 "	20.00215 "
(0 to 25 ")	25.00176 "	25.00178 "
(0 to 30 ")	30.00231 "	30.00234 "
(0 to 35 ")	35.00241 "	35.00244 "
(0 to 40 ")	40.00274 "	40.00278 "
(0 to 45 ")	45.00304 "	45.00308 "
(0 to 50 ")	50.00322 "	50.00327 "

E C Crittenden
Acting Director
Lyman J. Briggs, Director
Hvis

II-1/Tw 98477

Test completed: June 10, 1943

FIGURE 105.—Certificate of tape standardization (50-meter invar)

Base tapes must be handled very carefully. These tapes are delicate precision instruments. The nickel-steel alloy used to obtain a low coefficient of expansion has been known to be very unstable. When properly standardized and manipulated, the tapes are capable of giving a very high degree of accuracy, but they must be used with a full knowledge of their possibilities for error. To secure the best results three general conditions are necessary: First, all possible precautions should be taken to avoid accidents to the tapes; second, methods of handling the tapes must be such as will not alter their lengths; and third, tapes must be used, so far as possible, either under the same condition as when standardized or only under such different conditions as can be corrected for. Among the ways in which the first two conditions may occur may be mentioned: Kinking the tape; altering its mass by abrasion against the ground, such as in carrying it between stakes when measuring; or by changing its length by stretching it beyond its yielding point. The third condition involves the determination of the corrections for tension, sag, grade, alinement, and temperature within the allowable limit of error. Kinks or rough handling will change the standardization values. When unreeled, the tape should be carried high and extended to its full length while moving forward. It should never be allowed to rub or strike against the ground or any other object. It should never be bent around a support or in any sharp turn. When not in use, it should always be rolled up on the special reel provided with the tape. Tapes should be kept clean and dry.

Standardized 30-meter steel tapes are used for short partial tape lengths. A standardization certificate is furnished by the National Bureau of Standards as shown in figure 106. The 30-meter steel tapes are standardized for each meter of length while supported on a horizontal flat surface under a tension of 5 kilograms, and for two-point support for each 5 meters of length from the zero. Also included are the weight per meter of tape and the coefficient of expansion, so that the standard length can be computed under other support and temperature conditions.

An unstandardized 50-meter tape is used for staking the base line. See page 207 for comparison procedure for making corrections to staking tapes.

A 300-foot steel tape is used for a rough check measurement of the total length of the base.

THERMOMETERS

Special thermometers are used on base tapes. (See fig. 107.) These thermometers are standardized by the National Bureau of Standards before being sent to the field to insure that they are accurate to within 0°.3 centigrade (most are within 0°.1 to 0°.2). No thermometer standardization correction is used in field computations. Tape thermometers are mounted in light metal channel-bar holders suitable for attaching to the base tape. During standardization and base measurements a thermometer in its channel bar should be attached to each end of the base tape one meter toward the center from the 0- and 50-meter contact points, the distances being measured to the near ends of the thermometers. The weights (usually 45 grams) of the thermometers used during standardization are shown on the standardization certificate. If thermometers of different weights are used, corrections should be computed.

Where standardized steel tapes are used for the measurement of short distances, a standard base-tape thermometer should be exposed and read in the immediate vicinity of the measurement, since the 30-meter steel tapes are standardized without a thermometer being attached to them.

Form [illegible]
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U. S. DEPARTMENT OF COMMERCE
WASHINGTON

National Bureau of Standards

Certificate

FOR

30-Meter Steel Tape

NBS No. 7815

Maker's Identification Mark
The Lufkin Rule Co.,
U.S.C.&G.S. Reel No. H-4130

SUBMITTED BY

United States Coast & Geodetic Survey,
Washington, D.C.

This tape has been compared with the standards of the United States. It complies with the specifications for a standard tape, and the intervals indicated have the following lengths at 68° Fahrenheit (20° centigrade under the conditions given below:

Supported on a horizontal flat surface:

Tension	Interval	Length
4 1/2 kilograms	(0 to 30 meters)	30.0018 meters

Supported on a horizontal flat surface:
Tension: 5 kilograms

Interval	Length	Interval	Length
(0 to 1 meter)	1.0001 meters	(0 to 16 meters)	16.0011 meters
(0 to 2 meters)	2.0001 meters	(0 to 17 meters)	17.0011 meters
(0 to 3 meters)	3.0002 meters	(0 to 18 meters)	18.0013 meters
(0 to 4 meters)	4.0003 meters	(0 to 19 meters)	19.0012 meters
(0 to 5 meters)	5.0004 meters	(0 to 20 meters)	20.0014 meters
(0 to 6 meters)	6.0002 meters	(0 to 21 meters)	21.0014 meters
(0 to 7 meters)	7.0006 meters	(0 to 22 meters)	22.0016 meters
(0 to 8 meters)	8.0007 meters	(0 to 23 meters)	23.0015 meters
(0 to 9 meters)	9.0005 meters	(0 to 24 meters)	24.0016 meters
(0 to 10 meters)	10.0008 meters	(0 to 25 meters)	25.0017 meters
(0 to 11 meters)	11.0007 meters	(0 to 26 meters)	26.0019 meters
(0 to 12 meters)	12.0009 meters	(0 to 27 meters)	27.0018 meters
(0 to 13 meters)	13.0007 meters	(0 to 28 meters)	28.0020 meters
(0 to 14 meters)	14.0011 meters	(0 to 29 meters)	29.0020 meters
(0 to 15 meters)	15.0012 meters	(0 to 30 meters)	30.0022 meters

Test No. II-1/Tw 104855

The comparisons of this tape with the United States Bench Standard were made at a temperature of 22° Centigrade and in reducing to 68° Fahrenheit (20° centigrade), the coefficient of expansion of the tape is assumed to be 0.00000645 per degree Fahrenheit (0.0000116 per degree centigrade).

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Tape Certificate (continued) Page 2 NBS No. 7815

Tension: 5 kilograms

Points of Support	Interval	Length
0 and 5 meters	(0 to 5 meters)	5.0003 meters
0 and 10 meters	(0 to 10 meters)	10.0005 meters
0 and 15 meters	(0 to 15 meters)	15.0003 meters
0 and 20 meters	(0 to 20 meters)	19.9997 meters
0 and 25 meters	(0 to 25 meters)	24.9984 meters
0 and 30 meters	(0 to 30 meters)	29.9965 meters

The above values for the lengths are not in error by more than 0.0002 meter.

The weight per meter of this tape was found to be 11.2 grams.

The comparisons of this tape with the Bench Standard have been made at the centers of the lines on the edge of the tape farthest from the observer when the zero of the tape is at his left hand.

L J Briggs

Lyman J. Briggs, Director
HWB

Test No. II-1/Tw 104855
Test completed: April 3, 1945

FIGURE 106.—Certificate of standardization of a 30-meter steel tape

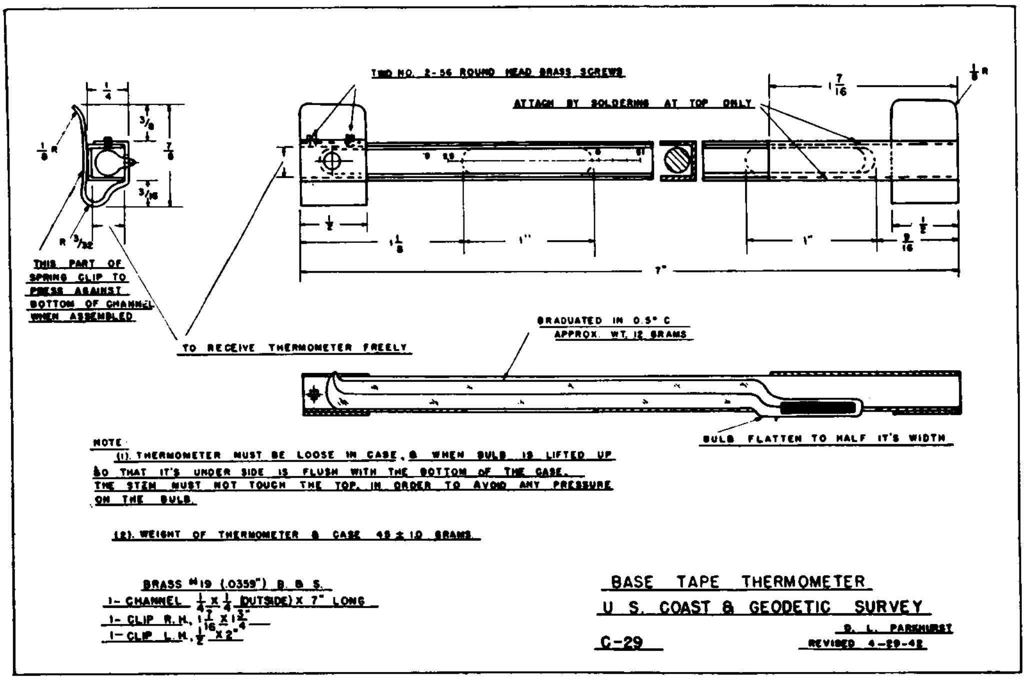

FIGURE 107.—Base tape thermometer and holder.

TAPE STRETCHERS

There are three types of tape stretchers in use. The type of stretcher used for measurements over stakes consists of a round staff, pointed at the lower end, and with an attachment for fastening the tape which can be moved up and down on the staff to adjust for various heights of the stakes. The attachment on the front stretcher has a pivoted fitting to which the spring balance is fastened on one side, and an arm with an adjustable counterpoise weight extends from the opposite side.

Special equipment using a four-point method of support has been developed for base measurement along railroads because it was found that friction prevented the securing of satisfactory results with the tape supported throughout on a rail.

The type of stretcher used for measurements along the rails of a railroad track consists of a channel-shaped shoe which fits over the top of the rail. A staff is pivoted 6 inches above the shoe forming a short lever arm to which the tape is attached at the bottom of the staff. The hooks for connection to the tape end and spring balance are about 1 inch above the rail when a 15-kilogram tension is applied. The front and rear stretchers for rail measurements are identical. The spring balance is fastened with hooks between the front stretcher and the base tape. The shoes are usually lined with brake lining. An important part of the outfit for both rail and pavement stretchers is the pair of stirrups carrying the nearly frictionless rollers which support the base tape 6 inches above the rail or pavement at the 12.5- and 37.5-meter points. These supports are so designed that with the tape stretched under 15-kilogram tension, the only direct contacts between the rail and the tape occur at the 0- and 50-meter points while marking.

The type of stretcher used on pavements is the same as the rail type except that the channel of the shoe is filled with a block of wood, and a friction surface of brake lining or corrugated rubber is attached to the bottom of the block.

SPRING BALANCES

A high-grade type of commercial balance graduated in kilograms is used for measuring and maintaining tension on the base tapes (see fig. 108). The circular dials of these balances read 5 kilograms per revolution. It is necessary to read the drawbar which is graduated by 5-kilogram intervals in order to check the number of revolutions on a 15-

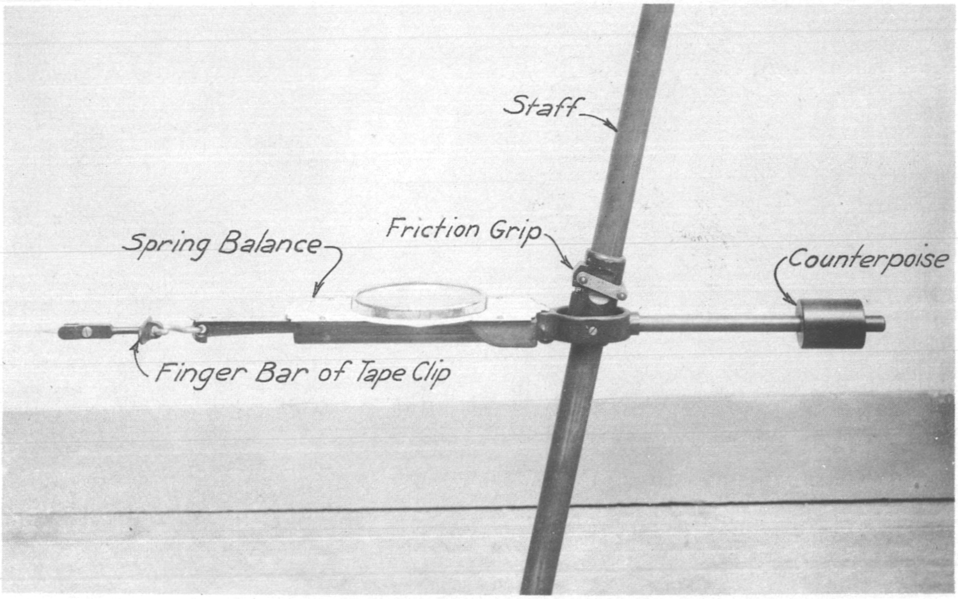

FIGURE 108.—Tape stretcher and spring balance.

kilogram reading. Spring balances should be tested with a standard weight each day before and after use, also at midday if practicable, and oftener if the temperature range is large or if it is suspected that the position of the dial pointer has changed. All tests should be recorded and the dial reset when there is any change. There is an adjusting screw alongside the drawbar by which the dial can be reset with a screw driver.

The method of testing consists of using a weight of exactly 15 kilograms mass, as weighed at Washington, suspended on a nearly frictionless pulley, as shown in figure 109. The weight is suspended on a short section of fine wire passing over the pulley and leading to the balance which is held in a horizontal position, and tension is applied using a stake-type stretcher. The balance is permitted to come to rest, after which it is gently pulled away from the pulley until the latter begins to move on its bearing, when a reading of the dial is taken. The balance is again brought to rest and then eased toward the pulley until the latter again begins moving, when another reading of the dial is made. These readings should be repeated several times. If the mean reading is not 15 kilograms, the balance should be adjusted until a mean reading of 15 kilograms is obtained. (See fig. 110.)

FIGURE 109.—Testing spring balance.

FIGURE 110.—Adjusting spring balance.

The most common injury to a spring balance used on base measurement results from the tension being suddenly released, allowing the drawbar to snap back. This may change the position of the dial pointer by several hundred grams, and even result in injury to the spring. For this reason the tension on the tape should always be released gradually. If through accident the drawbar of the balance is allowed to snap back, the balance should be tested before measuring is resumed. The balances should always operate smoothly and there should be no perceptible friction between the drawbar or spring and the sides of the balance.

A small cylindrical hand type of spring balance is used to apply 5-kilogram tension to 30-meter steel tapes.

PREPARATION

PRELIMINARY ARRANGEMENTS

The base-line site is selected by the reconnaissance party in accordance with chapter 1. The base-measurement party checks this selection. The permission of all property owners to enter on the site is verified. The preliminary alinement of the entire length is determined usually with a small transit or theodolite to insure against obstructions; to check maximum grades; to determine clearing, staking, and signal-building requirements; and to plan the measurement operations.

ALINEMENT

When the final alinement of the base has been decided, the stations at the end of the base are permanently marked and the necessary signals are built. Temporary intermediate points are then set so that a small theodolite can be set up and oriented at various points along the line as may be necessary to aline all stakes, or check the rail alinement, or line-in points on pavements.

CLEARING AND BUILDING

Trees, brush, weeds, high grass, and other obstructions should be cleared from the base line sufficiently to allow the base-measurement party to move freely along the line while making measurements. The width of the clearing need be only as required for the tape to swing freely (during tension) between the stakes without coming in contact with other objects and for the free passage of the party members. For base lines that are located on railroads or highways, there is very little clearing except perhaps for short distances near the ends of the base where the lines are projected beyond the roadways.

The stations at the ends of the base must be made intervisible. This usually necessitates towers at one or both ends. Intermediate towers are sometimes required to cross ravines, or at angle stations of a broken base. Ordinarily a suitable site can be located without intermediate towers. Signal building and marking are described on pages 74 to 95.

Benches to facilitate taping are usually constructed over the base monuments. A copper marking strip is plumbed over the center mark and attached to this bench.

STAKING

Most bases require some staking. Bases measured along railroad rails and along pavements usually require short staked sections at the ends. Cross-country bases and those along the sides of highways are usually measured over stakes throughout their length. Movable iron tripods (see figs. 111 and 112) have been used, but their operation

FIGURE 111.—Portable iron tripod for tape support, single joint. The wooden marking table, which carries a strip of copper on which the mark for the tape end is made, can be placed on correct slope by the ball-and-socket base and then clamped in position.

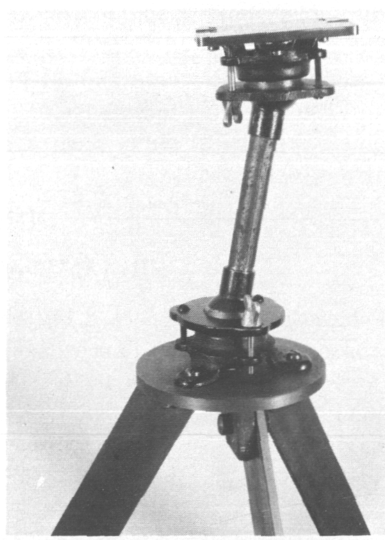

FIGURE 112.—Portable iron tripod for tape support, double joint. This tripod permits the marking table to be adjusted over a point as well as placed on correct slope.

is considerably slower and less satisfactory. Their use is usually confined to localities where it is impractical to drive stakes or to use pavement-type stretchers. On sections of a base line which are measured over stakes, the tape is supported (as standardized) on stakes at the 0-, 25-, and 50-meter points, with heavy rigid stakes at the 0- and 50-meter points and a lighter stake at the 25-meter point which furnishes support (usually on grade) for the middle of the tape.

STAKING EQUIPMENT

Stakes for the tape ends are usually 4 by 4 inches or 3 by 4 inches in cross section. The narrower stakes are easier to drive. The length of the stakes, the taper at the bottom, and the method of sharpening are matters to be determined by local conditions. The average length used is 3½ to 4 feet, with stakes projecting about 2 feet on uniform grades. Stakes should be of dressed lumber. A stake point with a 10-inch mill-made taper on all four sides is usually most satisfactory. A slight bevel around the top edge or the use of a driving cap will reduce splitting in hard ground. Stakes whose tops are battered in driving should be sawed off smooth. The end stakes of kilometer sections, test-section stakes, and also any stakes whose rigidity is questionable are cross-braced with 1- by 4-inch stakes.

The middle stakes may be 1- by 4-inch or 2- by 4-inch stakes, whichever are most suitable to local conditions, and are several inches longer than the end stakes.

The standardized base tapes should not be used for setting stakes. An obsolete base tape or a 50-meter steel tape may be used. It is important to make a comparison between the staking tape and the standardized tapes before staking so that a proper correction can be applied to the staking tape. In case of a large change of temperature during the staking a second comparison should be made. This is to avoid numerous set-ups and set-backs during the base measurements.

A small theodolite or transit mounted on a surveyor's tripod is used to aline the base-line stakes.

A heavy maul with a large flat-surfaced head is used to drive stakes in order to avoid splitting them. A 16-pound cast-iron maul or a heavy iron-bound wooden maul has been found very satisfactory.

STAKING PARTY

Six men can be used to advantage on a staking party. By combining duties, it is possible, but not as satisfactory an operation, to stake a base with as few as three men. The following are the principal necessary duties: Alinement of stakes with theodolite or transit; front contact and copper strip setting; rear contact and rear stretcher; front stretcher and stake driving; middle stake support and middle stake setting; and truck driving, stake distributing, and general utility duties.

STAKING PROCEDURE

A theodolite or transit is set up over one base station and sighted on the other base station or on an intermediate target which has been previously placed on line. The 50-meter staking tape is stretched out along the line under a 15-kilogram tension, manned by a rear stretcher man, a front stretcher man, and a middle support man. The rear stretcher man, who also usually acts as rear contact man while staking, holds the zero

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of the staking tape, or the corrected point determined by comparison with the standard tapes, on the mark on the bench which is plumbed over the station mark. The front contact man holds a stake or an iron digging bar vertical at the 50-meter point of the staking tape. The instrument man signals to aline the stake. Tension is then relaxed on the tape. The front stretcher man lays the tape aside, picks up the maul, and drives the stake. The instrument man checks the alinement while the stake is being driven and also at any time that a man at the stake pats the top of the stake with his hand as a signal for an alinement check. While the tape was under tension for setting the front stake, the middle support man marked the distance for the 25-meter stake. The middle stake is then alined to just clear the line and is driven at the same time as the front stake. The instrument man sometimes uses a white flag to refer to the front stake and a black or colored flag to refer to the middle stake in signaling for alinement. If the front stake is split or burred in driving, the top section is sawed off parallel to the slope of the line. As soon as the top of the front stake is in its final position, the rear contact man sights along the top of the rear and forward stakes and lines-in a nail on grade on the side of the middle stake. Or, if a 6-inch wire loop or stirrup is to be used at the middle support during taping, the bottom of a 6-inch measuring stick is lined-in to space the nail 6 inches above grade on the middle support stake. Meanwhile, a pencil is held vertically on the top of the front stake as a target for the instrument man, and a point is marked on the stake for alinement. Tension of 15 kilograms is again applied to the tape, the front contact man makes a pencil line along the side of the tape through the alinement point, and a short line perpendicular to the tape at its 50-meter mark. The front contact man tacks a copper strip along the far side of the pencil line, and at the same time the tape is moved forward. The operation is repeated at successive stakes with the rear contact being held on the perpendicular pencil mark on the last previous stake set. The middle support man and the stretcher men should be provided with machetes for any necessary additional clearing of grass or brush. The instrument is moved forward as necessary to maintain a clear view of the tops of the stakes (usually every 3 to 5 tape lengths).

Tape-end stakes are numbered with colored keel, successively as driven, by the front contact man, and numbering is checked by the rear contact man. The numbers start with the first tape-end stake after the beginning base station, as number one. Mid-tape or other intermediate supports are not numbered if they are on grade between the tape ends. If grade is broken at the middle support, it should be given a fractional number, as stake number 24½. Unless the break in grade at the midpoint is very slight, it is usually desirable to set a marking stake at the midpoint, number the stake, and make the measurements as a 25-meter set-up. When a partial tape length is measured for any reason, such as rough terrain, small streams, ravines, or end of an odd-length section, a number or designation should be given to each stake to which measurements are made.

Examples of numbering of stakes for tape ends of partial tape lengths are: For invar tape, 14+25, 38+45, etc.; for steel tape, 10 set-up, 17 set-up, 38+25 set-up, etc. It is sometimes necessary to have a series of measurements of less than a tape length. For instance, if, after stake 62, successive partial invar-tape lengths of 10, 20, and 25 meters are required, the stakes following stake 62 would be numbered 62+10, 62+10+20, and 62+10+20+25, respectively. Every stake marked solely by an integer must be a full 50 meters distant from the preceding marked stake regardless of the marking of the preceding stake. Thus if a full tape length can be used in proceeding from stake 62+10+20+25, the stake 50 meters ahead is numbered 63.

The utility man drives a truck, distributes stakes, supplies tools and materials as needed, carries food and water, assists in driving stakes, and performs other duties as needed.

The stakes at the ends of the kilometer sections should be firmly braced in four directions. The copper strip on the section end stakes should have a distinctive well-marked scratch across the middle of the strip to mark the end of the section. Additional diagonal lines near the ends of this scratch help to maintain its identity. The kilometer stakes are normally numbered with a multiple of 20.

If a stake must be located on rocky ground into which it cannot be driven, cross-pieces may be nailed across its base with 1- by 4-inch braces from the outer ends to the top of the stake. The structure can then be weighted down with stones and made rigid. If a driven stake is inclined to be loose, it should be braced with two or more 1- by 4-inch braces. Where a base line leaves a rail tangent at a curve, the base is continued on stakes, and the usual staked-base methods are used. Sometimes some unusually short or unusually long stakes are required when leaving the roadbed. It is sometimes convenient to drive stake supports to grade at the 12.5- and 37.5-meter points for support of rail stirrups instead of using a middle support when one tape end is still on the rail.

OFFSETS

Although it is preferable that the monumented stations of a base which is measured along a railroad or highway be in prolongation of the tangent over which the principal measurements are made, it frequently occurs that the monumented stations must be offset from this line. Where an offset measurement is necessary, checks on the offset distance and angle are obtained by measuring the third side of a triangle formed by vertices at the offset point, the monumented base station, and a tape-end point several tape lengths back along the base line. This type of triangle is known in the field as a shunt triangle, and the tape-end point along the base line is called the shunt take-off station. The number of tape lengths between the offset point and the shunt take-off depends on local conditions and the length of the offset distance, and should preferably be so selected as to make a small angle of about 6 degrees or less at the shunt take-off. For average offset distances of 10 to 20 meters, this will usually mean about 2 to 4 tape lengths. The sides of a shunt triangle which lead to the monumented base station are staked in the same manner as described on page 207, and since they are measured as separate lines, each line is given its own series of stake numbers. Measurements are discussed on page 221.

MEASUREMENT PROCEDURE

Prior to measurement of the first section of the base line, an intercomparison of the four standardized base tapes is made over a test section which is at least 4 tape lengths long. (See p. 194.)

Spring balances are tested as described on page 203.

A diagram of the base line should be prepared showing the number of the tape with which it is planned to measure each of the three divisions of the base in each direction as described on page 194.

PERSONNEL

The operation of measuring with a 50-meter base tape over stake supports requires a minimum of 6 men, and 7 men when measurements are made on railroads or pavements. The principal duties required are those for front contact, rear contact, recording, front stretcher, rear stretcher, middle support, and an additional support for four-point supports. The man in charge of the measurement operation usually fills the front contact or recorder job. Extra men, as flagmen, truck drivers, and utility men, may be required because of local conditions. If any of the men operating the tape are inexperienced, practice measurements with detailed instructions should be made over a kilometer section until all personnel are qualified in their jobs.

MEASUREMENT PROCEDURE OVER STAKES

The tape is carefully unreeled along the line, kept from contact with the ground or obstructions by at least three of the taping personnel. The tape thermometers are attached with adhesive tape as described on page 200. A test and adjustment of the spring balance is also made at this time and recorded in the record book. The tape is moved into position over the stakes, it is checked against twist, and stretchers are

FIGURE 113.—Middle tape support on nail.

attached. Prior to beginning taping, the tape should be wiped with a dry cloth throughout its length.

The middle man carries the middle of the tape high off the ground when moving forward, places the tape on the middle support when the tension is to be applied, takes the necessary precautions to see that the friction over the middle support has a negligible influence on the effective length of the tape, sees that the tape is not in contact with weeds, brush, or other obstructions, notifies the recorder of all middle supports marked "broken grade," carries and places the tape so that there is no twist in it, and each time makes sure that the middle support is not more than a decimeter distant from the middle mark on the tape. (See fig. 113.) If a nail is used as a middle support for the tape he must rapidly and lightly tap the under side of the tape near the support with a stick somewhat larger than a pencil until the front contact man calls "ready," in order to lessen the friction over the nail. The tapping must not be continued during the marking of the front contact. If a wire loop or other swinging device is used for a center support, the middle man places the tape on it and sees that the support swings clear and is vertical. (See fig. 114.)

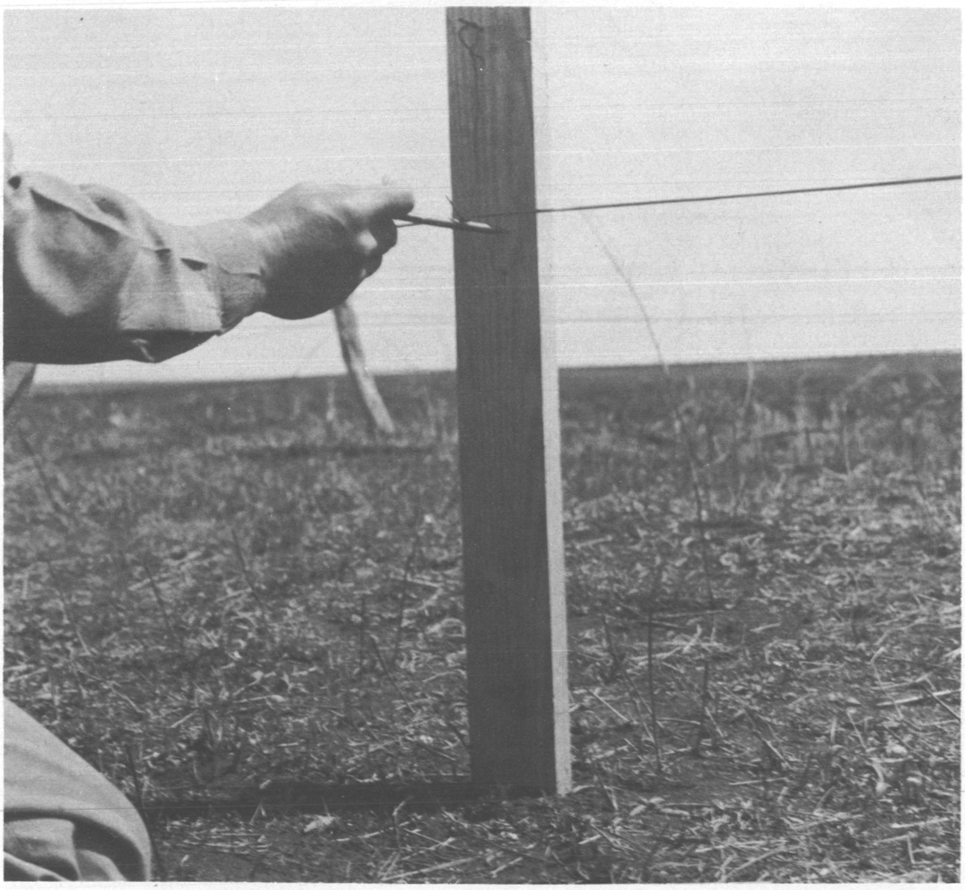

FIGURE 114.—Middle tape support on stirrup.

The rear stretcher man with the rear tape stretcher holds the tape in position during the time the tension is on, so that the rear contact man can by flexing the tape hold the rear terminal mark of the tape on the mark on the copper strip on the rear stake. (See fig. 115.) As he arrives at the rear stake, he places the rear staff firmly in the ground at the proper distance back of the stake directly in line with the forward stake. At the same time he slips the leather loop bearing the tape link to the proper height on the staff,

FIGURE 115.—Rear contact, base measurement over stakes.

so that when the full tension is applied the tape will be a few millimeters above the top of the stake. It must not drag over the top of the stake at any time. In order to maintain a steady position of the staff it is better to have the top of the staff back of one of his shoulders, the body being forward of the staff. One foot should be forward of and against the base of the staff to brace it. With practice all these movements can be coordinated so that they require only a few seconds, and the man operating the rear stretcher can keep the rear mark of the tape in the proper position as the tension is applied. As soon as the front contact man calls "mark" and the thermometers are read, the tension is slackened off. The tape is then carried forward without being detached from the rear stretcher, the rear stretcher man maintaining just enough tension on the tape to keep it from touching the ground.

The front stretcher man applies the proper tension to the tape as measured by the spring balance attached to the front stretcher. (See fig. 116.) In moving forward he carries the front stretcher and balance, detached from the tape. By hurrying he usually reaches the forward stake in time to check the vertical and horizontal alinement of the middle support, by sighting back over the tops of the stakes, before the front contact man reaches the stake with the front end of the tape. The checking is necessary to make sure that the stakes have not been moved since they were set or last alined. As the

FIGURE 116.—Front contact, base measurement over stakes.

tape is brought forward into position the front stretcher man with one hand holds the balance out horizontally with the hook in such position that the tape can be quickly attached. As the tape is attached he places the staff in line with the stakes at the proper distance from the front stake and applies the tension smoothly, rapidly at first but with increasing slowness as the 15-kilogram point is neared. Jerking motions must be avoided, as they may injure the balance or tape. With the staff held in the same manner as described for the rear stretcher man, and with one hand steadying the balance so the drawbar swings free, the front stretcher man can quickly bring the tape into equilibrium under the proper tension, at which time he informs the front contact man that the tension is correct. When under tension the tape should just clear the top of the forward stake and must not drag over it, otherwise the full tension will not be transmitted throughout the tape. Care must be taken that the stretcher staff is moved in the vertical plane through the stakes, for otherwise the balance will be twisted and friction exerted on the drawbar. The tension must be kept constant at 15 kilograms and watched closely. If the dial pointer indicates more than 20 grams different from 15 kilograms when the front contact man calls "mark," the front stretcher man should immediately tell the front contact man in order that the marking may be repeated. If the tension is satisfactory at the call of "mark" the tension is quickly but smoothly slackened off while the front contact man is reading the forward thermometer, the balance is held out for the detachment of the tape by the front contact man in the same manner as for its attachment, and the advance begun to the next position. Some parties find it desirable to steady the stretcher arm with a light wooden brace which rests on the ground in line with the tape, and which has a groove cut in its top fitting the top of the stretcher staff and moving up and down on it.

The rear contact man makes the rear contact and reads the rear thermometer. (See fig. 115.) As the tape is brought up to a new position he steadies the tape as the rear staff is being placed in position and the tension applied, taking care that the tape does not drag over the rear stake. As the tension is applied he advises the rear stretcher man whether to ease off or take up on the tape. Standing directly opposite the mark on the copper strip nailed to the top of the stake, with one hand he applies a tape flexor to the tape between the staff and the mark and with the other hand he lightly touches the tape on the opposite side of the stake to steady it. (Neither hand should exert any lateral pressure against the sides of the stake.) He can then flex the tape and bring the mark on the tape exactly opposite the mark on the copper strip. Tape flexors are hand grips with small metal plates with beveled grooved notches to slip over the tape to facilitate flexing without hand strain or injury to the tape. Some rear contact men prefer to flex the tape with a gloved hand. Before flexing the tape it is necessary, of course, that the mark on it shall be slightly forward of the mark on the strip. The marks are thus held in coincidence as the front contact man calls "ready" when the rear contact man calls "right" and until the front contact man answers "mark," denoting the completion of the marking of the tape length. If more than a few seconds elapse after the calling of either "ready" or "right" before the next response can be made with accuracy, the entire process should be repeated. Immediately following the call "mark" the rear contact man reads the rear thermometer and the tape is moved forward to the next position, unless someone at the forward end of the tape calls "hold" or "tension." An experienced base-measuring party frequently prefers the use of whistle signals, employing a whistle similar to a traffic whistle.

The whistle signals are:

2 short blasts by front contact man – "Ready"
1 short blast by rear contact man – "Right"
1 short blast by front contact man – "Mark"
A series of short blasts are a signal to repeat the contact.

The front contact man has the most difficult and the most important job of anyone handling the tape, for he must decide when all the conditions which affect the tape as a measuring unit are complied with, satisfying himself that the tape is in proper equilibrium under proper tension and support before he makes the forward mark. He also carries the forward end of the tape when moving forward, and thus to a large extent sets the pace for the entire operation. The sequence of his movements during the measurement of a single tape length is as follows: As the forward stake is reached he lowers the front end of the tape from its position above his shoulder and attaches the link to the hook of the balance, grasping and guiding the hook to make sure that the attachment is made with a single movement. He then steps quickly back to a position abeam of the forward stake, where he steadies the tape into its proper position just clear of the top of the stake, alongside the copper strip and between him and the strip. As the tension is perfected and the tape approaches equilibrium he places the point of the sharp, symmetrically pointed awl on the edge of the copper strip next to the tape and keeps it opposite the terminal mark on the tape until he is satisfied that conditions are right. After glancing at the balance and down the tape he calls "ready," as above described. When he hears the response of "right" from the rear contact man he marks the copper strip with the awl, calling "mark" as the marking is completed. (See fig. 116.)

In making the mark several precautions must be taken. The awl must be very sharp; it should at no time touch the tape in the region of the graduation mark; the eye of the man making contact, the graduation mark of the tape, and the entire awl should be kept in approximately the same vertical plane, in order to keep constant any error due to parallax. The mark should be made by the contact man moving the awl away from him so as not to scratch the tape. The mark should begin at the very edge of the copper strip in order to make it easier for the rear contact man to make contact. A straightedge is not used in making the mark, for when the rear contact man brings the mark on the tape into coincidence with the end of the scratch on the edge of the strip and ignores the remainder of the scratch, the error is less than that which would be caused by the less exact alinement of the awl point with the mark on the tape when a straightedge is used.

In order to avoid confusion the marks placed on the copper strips during the second measurement are usually distinguished from the first markings by a bar across the scratch, while a third measurement would have a second bar. The end mark of the section is given a distinguishing mark on the copper strip prior to making contact. A short set-up or set-back is then made to this mark from the 50-meter terminal mark. The measurement of the next section is begun at the section mark. On the second measurement, as the stake marking the end of each section is reached, a set-up or set-back is taken to the same section mark on the strip. It is sometimes desirable on the second measurement to make a set-up of a centimeter or so from the section mark in beginning a new kilometer, in order that the marks of the two runnings may not fall closely together. There will nearly always be a small set-up or set-back at the end of a section, since the same section terminal marks are used for all measurements.

Immediately following his call of "mark" the front contact man reads the forward thermometer as the tension is released, then detaches the front tape link from the balance as he starts to walk with the tape to the next stake. If the tape is always carried in the hand which is toward the rear when making the contact there is no danger of causing a half twist in the tape, for when detaching the tape from the balance the finger grip is held in the rear hand, fingers pointing down, then the end of the tape is elevated above the shoulder without changing the grip, and at the next position is again attached with the hands in the same relative position.

When the tape is brought into position with its rear mark in contact with the mark on the rear stake, the forward mark on the tape will sometimes fall short of or beyond the copper strip on the forward stake. If the staking was properly done and the stakes have not been moved, a second mark can be made on the strip on the rear stake, in front of or behind the first mark, at such distance as to permit making a contact mark on the forward strip. If the front contact man calls "set-up, 3," meaning that the forward terminal mark on the tape lacks 3 centimeters of reaching the copper strip, the rear contact man makes a mark 3 centimeters ahead of the mark on the rear stake, measures its distance from the previous mark with a boxwood scale, calls out the distance to the recorder, making sure that the recorder repeats it properly, measures the distance again as a check, then signals that he is ready to make contact on the new mark. Small set-ups and set-backs should be measured with an error not greater than one or two tenths of a millimeter. As an additional safeguard, the recorder should check the measurement of set-ups and set-backs, and have the contact man in turn check his entry in the record book.

The recorder should be an experienced man. The chief of party should frequently inspect the record, especially where broken grades or set-ups and set-backs are recorded. It is the direct responsibility of the recorder to be sure that no blunders are committed such as the dropping or adding of a tape length, or recording a half tape length as a full one, or a set-up as a set-back. He should check the chalked numbers on each stake as it is reached and make all notes necessary to a definite and correct interpretation of the record. With an experienced front contact man it is often advisable for the man in charge of the taping party to assume the recorder's duties. He can then watch the work of each man as well as record, and correct such practices as may be needed.

If a truck can be driven along the base and a truck driver is available he can save the party a great deal of time by keeping at hand extra tapes, tools, and stakes, preparing fences for the passage of the tape, and laying out tapes. A tape should be unreeled 10 or 15 minutes before being used on the base measurement in order that it may assume the temperature of the air. It is preferable to have it lie on weeds, brush, or stakes rather than in contact with the ground. The tape should be wiped off after every contact with any foreign object and after partial tape length measurements.

A taping party works on the same side of the base line for both backward and forward measurements. For instance, if the forward measurements are made on the south side of the base, the backward measurements are also made on the south side of the base. Either side whichever is more convenient for measurement may be chosen. The standardized side of the tape should always be the side away from the observer. The standardized side is usually marked with two dots, and is normally the far side of the tape when the zero is on the left and the 50-meter mark on the right. Either the 0- or the 50-meter end of the tape may be fastened to the front stretcher. The front stretcher with spring balance is always on the forward end of the tape in the direction the tape is being moved at the time.

MEASUREMENTS OF LESS THAN 50 METERS

Some base tapes also have marks standardized at approximately 5-meter intervals along the tapes. Where such tapes are available long set-ups are usually so staked as to enable the invar tape to be used for the greater part of the set-up, that is, for some multiple of 5 meters, the remainder of the set-up being measured with a standardized steel tape. For instance, if a 23-meter set-up is necessary a stake would be set at the 20-meter point and another one at 3 meters beyond. The 20-meter interval would in such case be measured with the invar tape with the standard 15-kilogram tension on the full tape. The method of supporting the tape and the tape temperature should be noted on a separate line. Special care is necessary when measuring a partial tape length with a 50-meter tape to avoid friction over supports, or bends in the tape at support points. The 3-meter interval would be measured with the steel tape, using its standardization tension, which is usually 5 kilograms, and noting on a separate line the temperature and method of support. For each set-up a note in the remarks column should show the part of the tape used in making the measurement. The standardized value for the distance between different pairs of marks will probably be different. For instance, a set-up of 20 meters might be made either from the 50-meter to the 30-meter mark or from the 0- to the 20-meter mark. The zero end should be held whenever practicable.

A rule to be followed is to avoid large set-backs. Except in very unusual circumstances no set-back should be greater than 0.1 meter. If a full tape end falls several decimeters beyond a mark a set-up of + 45 should be made and a second set-up should be made with a steel tape.

MEASUREMENT ON RAILROAD RAIL

The same general procedure is used for measurements on rails as over stakes with the following modifications: The tape is supported at the 0-, 12.5-, 37.5-, and 50-meter points. (See fig. 117.) The two intermediate points are supported six inches above the rail on small rollers set in stirrup frames which are held in a vertical position. The special-type stretchers which hold the ends of the tape under tension and slightly above the rail are described on page 202.

FIGURE 117.—Base measurement along a railroad rail.

Tape ends are marked on a small strip of Bristol board which is set on the edge of a piece of friction tape (both sides sticky) so that the edge of the Bristol board will be flush with the standardized side of the base tape. The contact point is marked with an awl or pricker in the same manner as on a copper strip. The strips are placed as the tension is applied. Because of the passage of trains, it is usually necessary to place separate strips for forward and backward measurements. An alternate method may be used with care by making a scratch directly on the rail and circling the point with yellow keel. This requires a very hard sharp point. It has the disadvantage of being very difficult to see, and of being subject to parallax because of the thickness of the tape on the rail. A glass cutter should not be used to mark tape ends because of even greater danger of parallax and danger of injury to the tape. Also, at tape ends, lines or arrows are placed on the rail flange with yellow keel or lumber crayon and the usual tape end numbers are written on the ties, or flange, or both. Terminal points of kilometer sections are marked by the intersection of a horizontal line normal to the direction of the base and a diagonal cross line cut on the rail with two file strokes, or by a small round hole made with a center punch. Such section points are usually located so as to require a set-up of several centimeters from tape end marks. Set-ups are read as usual using a small boxwood scale graduated to millimeters (estimated to tenth of a millimeter). In order to determine a correction for rail movement of the section point marked on the rail (due to passing trains between measurements), twenty-penny nails are driven into ties flush with the flange of the rail, and marked by a straight line which is cut with a glass cutter, normal to the rail and across the nail and flange. The nail heads should be very close but not in actual contact with the flange. About ten nails should be placed on alternate sides of the rail, with each nail preferably in a different cross-tie. The nails should be along the same

FIGURE 118.—Front contact, base measurement on rail.

section of the rail as that on which the tape end has been marked. The rail displacement (parallel to the base line) is read with a small boxwood scale to tenths of a millimeter for the mark on each nail head with respect to the mark on the rail flange. Wild readings that may be caused by displacement of the nail due to train movement, loose nail, or loose tie are rejected. All readings are recorded in the remarks column and meaned, then the mean is entered as a set-up or set-back. Remembering that the nail head is fixed, a note on the direction of rail movement should also be entered, as "rail moved east."

When a train approaches during the measurement operations, a tape-end point is marked in the same manner as at a section point so that after passage of the train, rail movement can be determined from the marks on the flange and the nail heads in the ties and applied as a set-up or set-back. Measurements are then resumed from the point which was marked on the rail with a center punch. Several members of the taping party (usually the front contact man, recorder, and utility man) carry hammers, nails, center punches, and glass cutters to be prepared for an approaching train. A constant lookout should be maintained for approaching trains, and the first member of a party who sees or hears a train approaching should call out a warning. The warning should be repeated by all members of the party.

FIGURE 119.—Rear contact, base measurement on rail.

During the operation of measuring on the rail the forward movement of the tape is halted by the rear stretcher man calling out "tape" or blowing a blast on his whistle when he approaches the tape end point. The men at the 12.5- and 37.5-meter marks (who carry the tape high when the tension is off), insert the tape (without twist) in the stirrups, then hold the tape up clear of the rail until the front stretcher man calls "tension." As tension is applied, they set the stirrup down to the rail, carefully keeping the low point of the catenary from touching the rail. The front stretcher man holds a light tension on the tape as he sets his stretcher on the rail. The remaining operations are carried on in the same manner as described on pages 210 to 216 for measurement over stakes, except that the rear contact man, instead of flexing the tape, coaches the rear stretcher man to position the tape for rear contact. (See figs. 118, 119, and 120.)

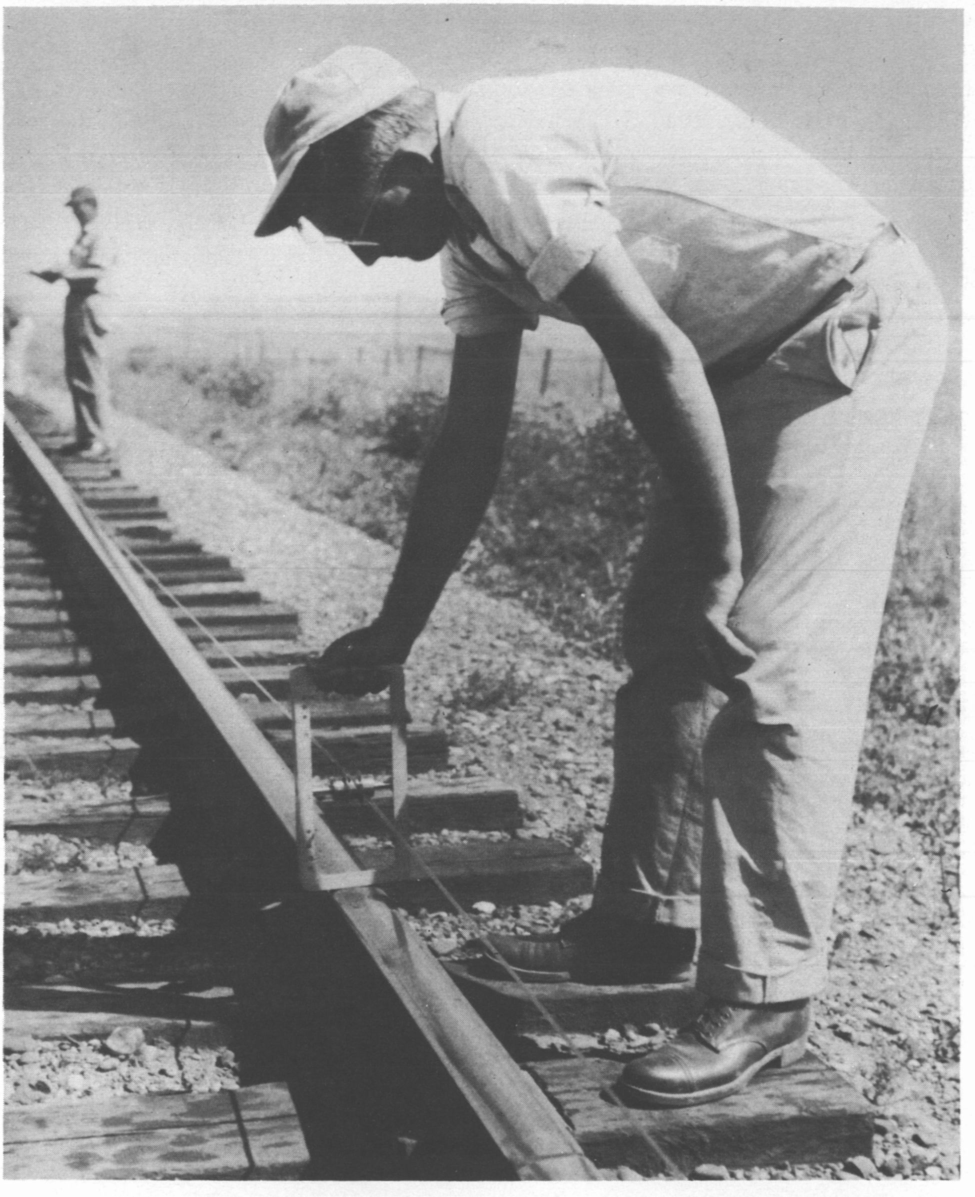

FIGURE 120.—Intermediate tape support, base measurement on rail.

MEASUREMENT OVER PAVEMENTS

Measurements over pavements are usually made with methods and equipment similar to rail measurements. The pavement-type stretcher equipment is described on page 203. Only comparatively smooth pavements are suitable for use of this equipment. It is usually necessary to mark the alinement and positions for tape-end points on the pavement with paint prior to base measurements. The principal problem is attachment of marking strips for tape ends and the marking of kilometer section points. The solution varies with local conditions; however, a clean dry surface is essential for even temporary attachment of marking strips. Bristol board set on friction tape similar to the rail method has been used with the surface cleaned and dried with a blow torch prior to setting the marking strip. Section points are usually marked with a rivet, plug, or small disk set flush and firmly in a drill hole in the pavement.

It is sometimes necessary to use movable iron tripods similar to those illustrated in figures 111 and 112, and stake-type stretcher equipment on pavements. Since the tripods are moved ahead as the measurements progress, it is necessary to determine the height of each tripod set-up above a ground mark (established for later use by a leveling party) or to operate a leveling party along with the measurement party because of variable heights of the top of the table above the pavement surface.

MEASUREMENT OF OFFSETS

If an offset is necessary, a shunt triangle is usually staked out as described on page 209. The side of the shunt triangle, from the shunt take-off point to the offset point, is measured as a part of the main line of the base. The sides from the offset point to the monumented base station and from the shunt take-off point to the base station are measured over stakes and benches with the same methods as described for the rest of the base line. When rail movement occurs, corrections are applied to the shunt take-off point and the offset point, and temporary corrected points are marked on the rail as vertices for any measurements of distances or angles of the shunt triangle. All three angles of a shunt triangle are measured with four positions of the horizontal circle as an additional check. The closure of a shunt triangle (in seconds) multiplied by the offset distance (in meters) should not exceed 400.

Owing to grades, stone walls, streams, and other natural obstructions, it is sometimes impossible on offset or shunt lines to use either the 0- or 50-meter marks on the tape. In such cases contact may be made on any of the 5-meter marks on the tape but the record should clearly show the marks used as follows: "10–45, 10 near rear stretcher." It is very undesirable to make a measurement in this manner since the tape cannot be flexed by the rear stretcher man because of danger of injury to the tape. However, such measurements are occasionally unavoidable.

LEVELING

The leveling for determining grade corrections may be accomplished with an ordinary wye or dumpy level. If a precision level is available the leveling can usually be done more accurately and with greater speed. Rods graduated on the face in meters and on the back in feet with readings made on both sides at each rod point should be used on a base. If rods graduated on both sides are not available, the leveling should be run in both directions. The leveling party usually consists of two or three men, an observer-recorder (or an observer and a recorder) and a rodman. Rod readings are observed on each tape-end support (both for full or partial tape lengths), on each broken grade, and on each station mark and each bench over a station mark. The same designation is used for all points as that used in the measurement with tapes.

Connections for elevation above sea level should also be made for reduction of the measured length to sea level. (See p. 195.)

RECORDS

When entering notes in the record books, both the recorder and the chief of party in the field should always have in mind that someone in the Office, unfamiliar with the actual job, will have to work with the books and interpret the records. Therefore, the record should be clear and well written; all steps should be included in detail, and no pertinent facts which will improve interpretation should be omitted.

RECORDING OF TAPE MEASUREMENTS

Tape measurements are recorded in "Traverse Measurement" record book, Form 590. A sample record is shown in figure 121.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 590

Base
~~TRAVERSE~~

From Station E. Base to Station 20

Forward measurement 1518

Date 7/7/50 Time 8:10 Tape No. 917 Bal. No. 302 Ther. Nos. 33260

Section		Temp.		Set up	Set back	Tape support	Remarks
From—	To—	Forward	Rear				
		°C.	°C.	Meters	Meters		
E. Base	E Base+20	20.0		20.0000		2	0 - 20
E. Base +20	1	21.3	21.0			3	
1	2	21.2	21.0			3	
2	3	20.6	20.0	0.0714		3	
3	4	21.2	21.0			3	
4	5	21.0	20.5			3	
5	6	20.0	19.8			3	
6	7	20.0	20.0		0.0214	3	B.G. at 6½
7	8	20.2	20.0			3	
8	9	20.4	20.5			3	
9	10	20.6	20.6			3	
10	10 set-up		21.5	4.7000	0.0381	2	Steel tape 4130
10 set-up	11	20.7	21.0			2	Crossing gully
11	12	20.8	21.0	0.0027		3	
12	13	20.8	21.0		0.0732	3	
13	14	21.0	21.0			3	
14	15	21.0	20.0			4	Supp. 0-12½-25-50
15	16	20.8	20.5			3	
16	17	20.5	20.3			3	
17	18	20.7	20.5			3	
18	19	20.8	20.4			3	
19	20	21.0	20.5			3	

FIGURE 121.—Example, recording of base measurements on Form 590.

The headings of the form should be completed in detail. The organization of the taping party and weather notes should be entered on the first page of each day's work. Notes on the testing of the spring balance should also be included. Data are recorded and summarized by sections of the base.

The entries in columns 1 and 2 are the names of the stations and numbers of tape ends. A partial tape length is given a separate line and the number of the preceding tape end followed by the meters or the word "set-up." (See p. 208.) The figure ½ should be used only for broken grades. If a stake with a contact strip is set at the half-tape point and contact is made to it, it is designated as a + 25 stake.

In columns 3 and 4, the entry of two temperatures indicates that the measurement was made with a full 50-meter tape length, and one thermometer reading indicates a half-tape length or a set-up. In cases of partial tape lengths, explanatory notes should be included in the remarks column. Those notes should include the part of the tape used in the measurement, such as 0 to 25, 25 to 50, 0 to 10, etc.

In columns 5 and 6, set-up and set-back measurements are entered in meters to 4 decimal places. These entries in addition to various partial tape-length measurements, include rail-movement measurements (usually the mean of 10 per rail tape-end point), and set-up or set-back measurements at the ends of sections and also at intermediate stakes used to make tape ends fall on the copper strips. When set-up measurements are made with a 30-meter tape, "hold and cut" measurements (made by holding a decimeter point at one contact and noting the difference between the other contact and the zero of the tape) may be entered as a set-up and a set-back.

The number of points of tape support are entered in column 7. A "T" is entered when a tape is supported throughout. Measurements are seldom made with a tape supported throughout because of friction. If these measurements are made the tape should be vibrated.

Entries in the remarks column are very important for the proper interpretation of the record. Any unusual method of support should have an accompanying explanation in the remarks column. All broken grades should be noted. The part of the tape used in a partial tape-length measurement should be indicated. The number of the steel tape used should be entered opposite all measurements made with a steel tape. (Unless otherwise noted, all measurements are made with the base tape whose number is shown at the top of the page.) Any accident to a tape (noting exact point of injury), or to equipment, or any change in weather should be entered. When any measurements are made to a bench over a mark, it should be indicated by a specific note in the remarks column. This applies both to the main base-line measurements and to offsets and shunt-triangle measurements. Since it sometimes clarifies interpretation of the record to know in which direction the zero of the tape was used, the top of the remarks column of each page (or wherever there is a change) should carry the note "Zero at front stretcher," or "Zero at rear stretcher."

The ten or more rail-movement measurements which are made at each section point and for each passing train are recorded and meaned in the remarks column. The mean is then entered as a set-up or set-back on the line number of the proper tape end, and with the direction of rail movement noted.

The intercomparison of the four base tapes prior to and after each base measurement should be recorded in the record book.

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RECORDING OF LEVELING

The spirit levels which are run over the base line to determine the differences of elevation between adjacent tape supports are recorded in "Wye Leveling" record book, Form 634. A sample record is shown in figure 122. The sample form shows the leveling record when using a rod which is graduated in meters on one side and in feet on the other. When such a rod is used, levels are run in only one direction, but both sides of the rod are read at each rod point, the reading in meters being recorded as the forward running and the reading in feet as the backward running. If the rod used is

WYE LEVELING

From E. Base To 20

FORWARD RUNNING

Date 7/7/'50 Instrument 95 Rods 118

Point	Backsight	H. I.	Foresight	Elevation	Difference	Remarks
	Meters ~~or feet~~	Meters ~~or feet~~	Meters ~~or feet~~	Meters ~~or feet~~	Meters ~~or feet~~	
E. Base	2.754					
E.Base +20			2.438		+0.316	
1			2.004		+0.434	
2			1.937		+0.067	
3			1.540		+0.397	
4			1.309		+0.231	
5	2.511		1.110		+0.199	
6			2.438		+0.073	
6½(BG)			2.440		-0.002	
7	0.471		2.779		-0.339	
8			0.799		-0.328	
9			0.976		-0.177	
10			1.273		-0.297	
10 set-up			1.300		-0.027	
11	1.647		1.738		-0.438	
12			1.962		-0.315	
13			1.437		+0.525	
14			0.537		+0.900	
15			1.734		-1.197	
16	1.247		1.469		+0.265	
17			1.837		-0.590	
18			2.004		-0.167	
19			2.540		-0.536	
20			2.486		+0.054	

From E. Base To 20

BACKWARD RUNNING

Date 7/7/'50 Instrument 95 Rods 118

Point	Backsight	H. I.	Foresight	Elevation	Difference	Remarks	
	~~Meters or~~ feet	~~Meters or~~ feet	~~Meters or~~ feet	~~Meters or~~ feet	~~Meters or~~ feet	m.	Mean m.
E. Base	9.035						
E.Base +20			7.999		+1.036	+0.316	+0.316
1			6.575		+1.424	+0.434	+0.434
2			6.354		+0.221	+0.067	+0.067
3			5.052		+1.302	+0.397	+0.397
4			4.290		+0.762	+0.232	+0.232
5	8.270		3.648		+0.642	+0.196	+0.198
6			8.005		+0.265	+0.081	+0.077
6½(BG)			8.005		0.000	0.000	-0.001
7	1.550		9.121		-1.116	-0.340	-0.340
8			2.625		-1.075	-0.328	-0.328
9			3.202		-0.577	-0.176	-0.176
10			4.176		-0.974	-0.297	-0.297
10 set-up			4.265		-0.089	-0.027	-0.027
11	5.404		5.712		-1.447	-0.441	-0.440
12			6.437		-1.033	-0.315	-0.315
13			4.714		+1.723	+0.525	+0.525
14			1.765		+2.949	+0.899	+0.900
15			5.692		-3.927	-1.197	-1.197
16	4.100		4.820		+0.872	+0.266	+0.266
17			6.031		-1.931	-0.589	-0.590
18			6.575		-0.544	-0.166	-0.166
19			8.330		-1.755	-0.535	-0.536
20			8.149		+0.181	+0.055	+0.054

FIGURE 122.—Example, leveling record for base measurements, Form 634.

graduated on only one side, both forward and backward runnings are necessary. The leveling record should agree with the notations in the measurement record (Form 590) on the identification of all tape ends. Extreme care should be taken to record leveling readings on all broken grades and partial tape lengths, and these should be plainly indicated in the record. In the columns headed "meters or feet" the one not used should be crossed out.

Rod readings should be recorded both for the bench and for the mark and should be explicitly labeled.

CORRECTIONS TO MEASURED LENGTHS

GRADE CORRECTION

The data for the correction for the slope of the tape are usually obtained by spirit leveling, by which the difference of elevation of the stakes supporting the two ends of the tape are obtained. If l is the inclined length, and h the difference of elevation of the two ends, the correction

$$C_G = -(l - \sqrt{l^2 - h^2}) = -\frac{h^2}{2l} - \frac{h^4}{8l^3} - \frac{h^6}{16l^5} \cdots\cdots$$

Tables of grade corrections for 50-meter lengths and others in multiples of 5 meters as required for various differences of elevation in both meters and feet are found on pages 289 to 300. Since for a 50-meter tape length the second term $\frac{h^4}{8l^3}$ is less than 0.1 millimeter where h is less than 3.1 meters, on ordinary grades the correction will vary directly as the square of the difference of elevation. For this reason the leveling must be more accurately done on steep grades, and an inspection of the rate of change of the values for the correction in the tables should be the guide in deciding upon what accuracy is necessary in the leveling. For steep slopes and short lengths it is better to compute the grade correction by solving the triangle.

In first-order base measurement the error in C_G for a single tape length should not exceed 0.1 millimeter, though the error is accidental in character. Since the correction varies inversely as the distance, fractional tape lengths are liable to a larger error in the grade correction.

The error in the grade correction which must be most closely guarded against is due to the failure to note and correct for the break in the grade of the tape at the middle support or supports. For that reason such supports should always be flagged with cloth and given a number, such as 34½, the number signifying that the broken grade was at the half-tape point between stakes 34 and 35. The tape record will then contain the note in the remarks column "Broken grade at 34½." Before the grade corrections for any section are summed up a special check should be made to learn whether the levelman has touched upon the support at each broken grade and at the ends of fractional tape lengths, and that the leveling record has the same system of numbering of the tape supports as the tape record. By doing this, confusion or doubt will be avoided when the final computation is made. In general, broken grades should be avoided, and whenever possible long set-ups used instead, to avoid the errors due to the friction of the tape on the middle support.

If any material difference is found between forward and backward levels, the discrepancy must be checked in the field.

ALINEMENT CORRECTION

This should more properly be called the alinement error, for although the same correction formula and tables apply to differences in alinement of the tape as to differences of grade, the alinement of all except rail bases can usually be made sufficiently exact to avoid correction. It should be borne in mind, however, that alinement errors are always of the same algebraic sign, tending to make the measured length greater than the actual length, and for that reason they should be kept much smaller in magnitude than the inaccuracies in the grade corrections. The section describing the staking of the base gives details of the precautions to be taken in the alinement of the stakes. In addition to that, some member of the taping party, usually the rear contact man or the front stretcher man, should check each tape length to see that the tape does not change its horizontal direction at the middle support, and also that the forward stake has not been disturbed in alinement.

If an alinement check of a rail base indicates that the rail tangent deviates more than

6 inches from a straight line, sufficient observations should be made and recorded to compute alinement corrections. Angle observations should form a closed figure or start with orientation on an azimuth mark at one base end, and end with orientation on the azimuth mark at the other base station, as a check. An alternative method that is seldom justified for small alinement corrections is to run a straight line between ends of the rail tangent and measure the offset distances to the rail at each tape end, then apply alinement corrections to each tape length from differences in the offset distances in the same manner as for grade corrections.

STANDARDIZATION CORRECTIONS

The standardization values furnished on the National Bureau of Standards certificate as illustrated in figure 105 on page 199 in addition to furnishing calibration correction, also include catenary correction under the usual conditions of measurement (using standard supports and tension).

Since the tapes are standardized when supported at three and at four points under a tension of 15 kilograms, the formulas given previously need be used only in the cases where an unusual number of supports are used, when it is desired to apply corrections for small changes in tension, or when it is necessary to investigate the effect of small changes in one of the many factors affecting the length of the tape.

CORRECTIONS DUE TO SAG OF TAPE AND TO STRETCHING

The effective length of the tape when suspended between supports is affected by the shortening due to the sag and to the stretching due to the tension. The correction due to the sag is given by the formula

$$C_S = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3,$$

where n = number of sections into which the tape is divided by the equidistant supports,
l = length of section in meters,
w = weight of tape in grams per meter, and
t = tension in grams.

To illustrate by an example: For tape No. 917, supported at three points under a tension of 15 kilograms,

$n=2$
$l=25$
$w=25.8$

and

$$C_S = -\frac{1}{24}\times 2\times 25.8^2\times 25^3\times\frac{1}{15{,}000^2} = -0.00385 \text{ meter.}$$

If it is desired to determine the effect of small variations in the tension the formula for sag may be differentiated with respect to t and put in the following form:

$$\text{Change in length} = \Delta L = +\frac{1}{12} n w^2 l^3 \frac{\Delta t}{t^3}.$$

For tape No. 917 under the conditions of support given above, a change of 100 grams in the tension would by this formula produce a change in length of 0.05 millimeter.

The change in the effective length of the tape due to the stretching under tension can best be determined by experiment, since the modulus of elasticity of invar varies greatly with the percentage of nickel in its composition and also with temperature. Five 50-meter invar tapes tested at the National Bureau of Standards showed a mean change in length of 0.43 millimeter for a 500-gram change from a tension of 15 kilograms. It is thus seen that a change of 100 grams in the tension on tape No. 917 supported at three points under a total tension of 15 kilograms has more effect in stretching of the tape than in changing the catenary correction.

Precautions should be taken to insure that differences in tension of more than 100 grams from the standard tension are determined and corrected for. The spring balances must be tested both before and after each day's measurement by using the standard weights provided. The record books must show clearly the reading of the dial of the balance during the taping. The dial pointer on the spring balance should be adjusted to its proper reading whenever the tests show it to be appreciably in error. When the dial pointer cannot be adjusted to the proper reading, the even 15-kilogram dial reading may be used when measuring and the necessary corrections for erroneous tension applied later. The effect of changes in temperature of the balance may make a difference even greater than 100 grams between the real and the indicated tensions. Making additional tests of the spring balance during the day is the best method of determining whether the balance is greatly affected by changes in temperature. Normally no corrections are applied. To enable a correction to be applied it will be necessary that the temperature be recorded whenever the balance is tested with the standard weight. The temperature of the balance during the measuring is then assumed to be the same as that given by the thermometers attached to the tapes.

A table of factors for computing catenary correction and a table of catenary corrections for various lengths and weights of tape (both tables assuming two-point support and 15-kilogram tension) are given on pages 300 to 302. The corrections tabulated are values to be applied to the standardized lengths when supported throughout. These tables will allow easy computations of corrections to partial tape lengths. Full tape lengths under unusual support conditions may be corrected by considering sections between supports separately.

Amount of sag.—It is sometimes desired to compute the sag of the tape, that is, the vertical distance from the lowest point of the catenary to the line joining the adjacent points of support. For all practical purposes this is given by the formula

$$\text{Amount of sag} = y = \frac{wl^2}{8t},$$

where w, l, and t represent the same quantities as in the formula for the correction to the length due to sag.

For example, taking again tape No. 917 supported at three points under a tension of 15 kilograms:

$$y = \frac{25.8 \times 25^2}{8 \times 15{,}000} = 0.1344 \text{ meter.}$$

CORRECTIONS FOR METHOD OF SUPPORT OF TAPE AND FOR CHANGE IN WEIGHT

The change in the effective length of the tape due to a change in the number of supports can be determined by substituting in the formula for the correction due to sag different values for n and l, since a variation in the number of supports does not appreciably affect the stretching of the tape. Changes in the effective length of the tape due to a change in the weight of the tape per unit length can be determined by differentiating the same formula with respect to w. Moisture or grease on the tape will change its weight from 1 to 10 percent. When thoroughly wet, an invar tape is 10 percent heavier than when dry, and after being sharply shaken to remove the drops of water, it is still 1 or 2 percent heavier than normal. An increase of 1 percent in the weight of the tape changes its length about 1 part in 70,000. A base should not be measured in rain or heavy fog.

TEMPERATURE CORRECTION

With most invar tapes the effects of errors due to incorrect temperatures are far from negligible. Experiments have shown that on sunny days the readings on the attached thermometers may differ as much as 3° or 4° from the actual temperature of the tape. With a temperature coefficient for the tape of 1 part in 1,000,000 per degree centigrade an error in temperature of 4° C. would affect the measured length 1 part in 250,000. On cloudy days the difference in temperature of the thermometer and tape is usually less than 1°. The coefficient of expansion of the tapes in use will govern the care which must be exercised in controlling the temperature error. The lag of the thermometer is partly due to the lack of intimate contact between the tape and the mercury bulb, but probably is due more to a difference in the absorption rates of the tape and the glass-enclosed mercury column of the thermometer when exposed to radiation from the ground and to the direct rays of the sun. The best method for minimizing the effect of lag in the thermometer is to make one-half of the measurements of the base with a rising temperature and the other half with a falling temperature, on the theory that the lag will be approximately equal in amount under the two conditions named but of opposite sign. By this method the measures will differ more in their computed length but the mean will be more nearly correct. Invar tapes can be secured with practically a zero coefficient of expansion, but tapes with a very low coefficient usually have great molecular instability and are subject to sudden changes in length.

A thermometer should be steadied in reading by grasping the tape a few inches on either side of it, care being taken not to touch or breathe upon the thermometer.

The thermometers used should be the same weight as the ones used during the standardization of the tapes, and they should be fastened in the same position on the tape. A formula which can be used to compute the change in length caused by a change in the weight of the thermometers or a change in their position on the tape is:

$$s-s_0=\left(\frac{a^2-h^2}{2c^2}\right)\left(l+\frac{l^2}{2a}\right),$$

where s is the length of the catenary with the thermometer attached, s_0 is the length without the thermometer, a is one-half the distance between supports, h is the distance from the center of the catenary to the point at which the thermometer is attached, $c=T/m$, in which T is the tension and m is the weight per linear unit of the catenary, and $l=p/m$, in which p is the weight of the thermometer and m is the same as above. For a 50-meter tape weighing 25 grams per meter, supported at the 0-, 25-, and 50-meter points, bearing two thermometers, each weighing 25 grams, at distances 1 meter from the ends and under a tension of 15 kilograms, this formula becomes

$$s-s_0=\left(\frac{12.5^2-11.5^2}{2\left(\frac{15{,}000}{25}\right)^2}\right)\left(\frac{25}{25}+\frac{\left(\frac{25}{25}\right)^2}{25}\right)=0.0000347 \text{ meter.}$$

This is the value for half the tape and should be doubled for the whole tape. Taking $p = 45$ grams, $s - s_0 = 0.0000643$ meter. The change in length, therefore, caused by changing the weight of the thermometers from 25 grams to 45 grams is 0.0000592 meter for the whole tape, or about 1 part in 840,000.

PRECAUTIONS AGAINST ERROR

There are certain errors which must be guarded against in measurements with a tape because no definite corrections are practicable. Some of the principal errors and methods of avoidance are discussed in the following paragraphs.

FRICTION OVER SUPPORTS

With proper handling of tapes the error due to this cause is negligible. Before the tension is applied the devices attaching the tape to the stretcher staves should be so adjusted in height that the tape will be a millimeter or so above the tops of the marking stakes when the tension is fully on. Frequently it is necessary to slacken off and adjust to the proper height after the tension has been partly applied. The tape must not at any time be permitted to drag over the rear stake because of the danger of moving the stake, which holds the mark from the previous tape length. With the tape in equilibrium under tension just above the stakes a touch of the finger will depress it into position for marking.

If a nail driven into a piece of lumber is used as a middle support, the middle tape man keeps the tape vibrating on the nail by tapping it rapidly on its under side with a light stick while the tension is being applied. This will suffice in most cases to prevent any large amount of friction between the tape and the nail, but a more accurate way is to suspend the tape in a wire stirrup several inches long suspended from the middle nail in such fashion that it will swing freely in the direction of the length of the tape. In the latter case the nail should be driven in at the proper height so that the tape point suspended in the stirrup will be on grade.

PARALLAX IN MARKING

Throughout the entire measurement of the base one man should make the mark for the forward contact, for the reason that this error in marking tends to be constant in size and direction for any one person. If then the person marking remains always on the same side of the tape and stakes, the marks which tend to make the measured length too long when measuring in one direction will make it too short when measuring in the opposite direction. It is probable that there is no parallax in making the rear contact. If there is any it is exceedingly small and need not be taken into account.

WIND EFFECT

The error caused by the bowing of the tape horizontally by the wind is always of the same sign and tends to make the measured length too long. A wind which merely flutters the tape need cause no concern but where there is a perceptible bending of the tape out of the horizontal alinement between supports the measuring should either be postponed or additional supports provided. When an increased number of supports are used they must be put more precisely on grade to avoid the greater liability of grade errors.

BLUNDERS

The numbering of the stakes and the inclusion of those numbers in the record of both the tape measures and the levels practically precludes the possibility of a dropped tape length. There is a chance for error, or at least for confusion, if the record of set-ups and half-tape lengths is not clear and consistent throughout all the record books. There is a chance for a compensating error in recording a small set-up as a set-back, or vice versa, when measuring in one direction, with a like error of approximately the same size in the opposite running, but with care and the usual system of having the recorder repeat clearly to the observer all data to be recorded, the probability of such an error remaining undetected is rather remote. There is one rule, however, which should invariably be observed: Any discrepancy in the records, whether of tape or level, should be checked by *field measurements, even though the chief of party may think he has discovered the cause* of the discrepancy through an examination of the record.

An added check against a dropped tape length in the record, or the recording of a half-tape length as a full one, is to measure the base roughly with a 300-foot tape, checking on each kilometer section mark. Two men can do this in a short time, and the resulting certainty is worth the time spent.

Among other blunders which may occur is a tension on one tape length of 10 kilograms instead of the required 15 kilograms, since the dial pointer marks 5 kilograms at each revolution. The effect of this error is about 6 or 7 millimeters in a tape length. The possibility of making this blunder will be greatly reduced if both the forward stretcher man and the forward contact man each time the tension is applied will check the tension by a glance at the drawbar of the balance on which the 5-, 10-, and 15-kilogram lines are marked. The 15-kilogram line should be marked by white paint to distinguish it from the other lines.

If the stake holding the forward mark is jarred while the tape is being moved forward an error may be caused. In such a case the previous tape-length measurement should be repeated before the forward progress is resumed. Any movement of the stake holding the mark designating the end of a section is usually shown by the discrepancies between the forward and backward measures of the two sections involved being of opposite sign and approximately the same size.

EXTRA-WEIGHT TAPE

A base should not be measured in rain or heavy fog because of effect of unknown increased weight of the tape on the catenary. (See p. 228.) The tape should be kept clean, dry, and free from dust, to avoid changing its standardization value.

FIELD COMPUTATION OF MEASURED BASES

Bases should be computed in the field immediately after measurement and before the party leaves the vicinity in order to insure that all data are complete and that the measurements are in accordance with the specifications for the project.

In order to make a rapid and accurate preliminary computation of the two measures of a section while at the base site it is convenient to prepare correction tables for each tape from the standardization data. These tables can be arranged for a kilometer and for a tape length to show temperature corrections for each tenth of a degree centigrade for the expected range of temperature and catenary correction for each tape cumulative for from 1 to 20 tape lengths. These tables are particularly useful if more than one base is to be measured with a set of tapes.

COMPUTATIONS IN FIELD RECORD BOOKS

The only computations made directly on Form 590, "Traverse Measurements," are the mean temperatures of the full tape lengths for each section.

The computations made in Form 634, "Wye Leveling," are the mean differences in elevation between adjacent tape ends or broken grades. If forward and backward levelings are recorded in different units, the differences in feet are converted to meters and then both differences meaned in the last column of the right page as shown in figure 122 on page 224. It is not necessary to compute elevations in this volume.

ABSTRACT OF LEVELS

Levels are abstracted from record book Form 634 onto Form 635, "Abstract of Wye Levels and Computation of Inclination Corrections." A sample computation is shown in figure 123. In the first column on this form are recorded the stake numbers, corresponding to the numbers on Forms 590 and 634, the record books for traverse measurements and wye leveling respectively. The distances between stakes are listed in the second column, each distance being that between the stake recorded on the same line as the distance and the stake on the line preceding. The mean differences of elevation between successive stakes are listed in the third column. This column is headed "Meters or feet," and here again it is important to cross out the word not applicable. The grade or inclination corrections in millimeters are listed in the fourth column. For 50-meter lengths and others in multiples of 5 meters, these corrections can be obtained from the tables on pages 289 to 300. These tables are made out for differences of elevation in both meters and feet, so either meters or feet may be used in the third column. The corrections for partial lengths not in multiples of 5 meters may be computed by solving the triangles, or if the grades are small by using the approximation $C_G = -h^2/2l$. The sum of these corrections for the section of the base is entered on Form 589, "Computation of Base Line," in the column headed "Inclination." (See fig. 124.)

It is very important that all broken grades and partial tape lengths be indicated on Form 635 and that each grade correction be computed for the corresponding distance. The most frequent mistake made in computing grade corrections arises from using a 50-meter length instead of the real length.

COMPUTATION OF BASE LINE

Computations of lengths are made on Form 589, "Computation of Base Line." A sample computation is shown in figure 124. On Form 589 the first correction to be entered is the correction for temperature. This is computed as follows: Temperature correction $=(T-T_s)\times$temperature coefficient$\times 50\times$number of tape lengths, in which T is the mean temperature for the section and T_s is the temperature of the tape at standardization. The value of T is entered in the column headed "Temperature" and is the mean of the thermometer readings recorded on Form 590. The value of T_s is given in the standardization data for the tape. The temperature coefficient is the change in length

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 685

ABSTRACT OF WYE LEVELS AND COMPUTATION OF INCLINATION CORRECTIONS

POINT	DISTANCE	MEAN DIFFERENCE OF ELEVATION	INCLINATION CORRECTION	ELEVATION	MEAN ELEVATION	REMARKS
	Meters	Meters ~~or feet~~	mm.	Meters	Meters	
E. Base				237.67		
E. Base +20	20	+ 0.316	2.5			
1	50	+0.434	1.8			
2	50	+0.067	0.0			
3	50	+0.397	1.6			
4	50	+0.232	0.5			
5	50	+ 0.198	0.4			
6	50	+0.077	0.1	239.39		
6½	25	-0.001	0.0			
7	25	-0.340	2.3			
8	50	-0.328	1.1			
9	50	-0.176	0.3			
10	50	-0.297	0.9			
10 set-up	4.6619	-0.027	0.1			
11	50	-0.440	1.9			
12	50	-0.315	1.0	237.47		
13	50	+0.525	2.8			
14	50	+0.900	8.1			
15	50	-1.197	14.3			
16	50	+0.266	0.7			
17	50	-0.590	3.5			
18	50	-0.166	0.3			
19	50	-0.536	2.9			
20	50	+0.054	0.0	236.72	237.8	
			47.1			

FIGURE 123.—Example, abstract of wye levels, Form 685.

per meter for each degree centigrade change in temperature and is also given with the standardization data, expressed as thermal expansion in millimeters per 50 meters per degree centigrade. The number of tape lengths is given in the column headed "Tape lengths," and is the number of *full* tape lengths recorded on Form 590. For tapes with a positive coefficient of expansion the temperature correction is, of course, + or −, according to whether the mean temperature is greater or less than the standard temperature. There are a few tapes which have negative coefficients of expansion, and for these the correction would have the opposite sign. Temperature corrections for partial tape lengths are computed separately, and usually are entered on a separate line.

The correction in the column headed "Tape and catenary" is obtained in one of two ways: (1) When the tape is used under conditions of support and tension shown in the standardization data, simply by subtracting the graduated length from the standardized length, or (2) when the tape is used under other conditions of support or tension, by applying to the standardized length (when supported throughout) the computed correction for variation in the manner of support and for variation in tension before subtracting the graduated length. Correction for tension is included only if the tension used varies appreciably from the standard tension. For example, in the sample computation on Form 589 in figure 124, the tape and catenary correction is computed as follows: For line 1, 18 $(49.99941-50)=-0.0106$. For line 2, $50.00327-0.01541-50=-0.0121$. The value of 15.41 millimeters is taken from table 14 on page 302 for 25.8 grams and a length of 50 meters between supports. If the value of the tape supported

COMPUTATION OF Sample BASE LINE

SECTION	DATE 1950	NO. OF MEAS.	TAPE NO.	TAPE SUPPORT	UNCORRECTED LENGTH: Tape lengths	UNCORRECTED LENGTH: Meters	TEMP. °C	CORRECTIONS: Temp. (Meters)	Tape and Catenary (Meters)	Set-up Set-back * (Meters)	Inclination (Meters)	Sea level (Meters)	REDUCED LENGTH (Meters)	ADOPTED LENGTH (Meters)
E Base to 20	7/7	F	917	3	18	900	206		-0.0106	-0.0205				
10 set-up to 11	7/7	F	917	2	1	50		-0.0048	-0.0121					
14 to 15	7/7	F	917	4	1	50			-0.0004 +0.0013		-0.0471	-0.0382	1,024.5310	
E Base to E Base+20	7/7	F	917	2	–	*	20.0	-0.0001	+0.0012	+20.0000 -0.0381				
10 to 10 set-up	7/7	F	4130	2	–	*	21.5	0.0000	+0.0004	+4.7000				

FIGURE 124.—Example, computation of base line, Form 589.

throughout is not included in the most recent standardization data, it can be computed as follows: $49.99941+0.00193+0.00193=50.00327$. The 49.99941 is the standardized value with the tape supported at 0, 25, and 50 meters. The catenary correction from table 14 for 25.8 grams and 25 meters between supports is 1.93 mm. Since this computed length is only 0.05 mm. greater than the 1943 value of figure 105 and entries in Form 589 are made to 0.1 mm., it may be either disregarded in field computations or proportionate corrections may be made to the 1943 values of intermediate graduations, as shown in the handwritten column in figure 105. For line 3, the computation is broken down by sections of the tape between supports. For the 25- to 50-meter section, $(50.00327-25.00178)-0.00193-25=-0.0004$. For the section supported at 0, 12.5, and 25 meters, the factor 1,233 is taken from table 13 on page 300 for 25.8 grams and multiplied by 12.5^3 and $10^{-10}=0.00024$. Then, $25.00178-2(0.00024)-25=+0.0013$. For line 4, $20.00215-0.00099-20=+0.0012$. The 20.00215 is taken from the corrected values of figure 105, and 0.99 mm. is taken from table 14 for 25.8 grams and 20 meters. For line 5, from the catenary formula, $C_S=-\frac{1}{24}\times 11.2^2\times 4.7^3\times\frac{1}{5{,}000^2}=-0.00002$. The terms in the C_S formula are described on page 226. Standardization values for tape 4130 are taken from figure 106. Then $4.7004-0.00002-4.7=+0.0004$.

In the column headed "Set-up and set-back" is entered the algebraic sum of the set-ups and set-backs recorded on Form 590, the set-ups being plus and the set-backs minus. In the sample shown in figure 124 the two large set-ups and the corrections to them are recorded separately. All set-ups, however, could have been combined, and also the temperature, tape, and catenary corrections for the large set-ups (partial tape lengths) could have been combined with the same corrections for the full tape lengths, and then the entire computation of a section would have been on one line. It simplifies the checking of the computation somewhat, however, to enter each large set-up on a separate line, as shown on the sample form.

The sum of the inclination corrections is obtained from Form 635. Then the algebraic sum of the uncorrected length and all corrections gives the reduced length for the section, and the mean of the reduced lengths from the forward and backward measurements gives the adopted length. The columns headed "(v)" and "(vv)" are used in computing the probable error of the measurement of the base.

PROBABLE ERROR OF MEASUREMENT

The probable error is usually computed by the method described in the following paragraph. This method is based on the theory that the errors of standardization and of the determination of the coefficients of expansion are either largely included in or are masked by the discrepancies in the measured lenghts of the sections.

The probable error of each section is computed by the formula

$$\text{p.e.} = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}},$$

where v is a residual and n the number of measures of the section. Where a section is measured only twice the probable error will, of course, be 0.6745 times one-half the difference between the two measured lengths. The probable error of the entire base is the square root of the sums of the squares of the probable errors of the component sections.

REDUCTION TO SEA LEVEL

Since the lines of a scheme of triangulation are reduced to their equivalent length at sea level, the length of any base must be likewise reduced to sea level before it can be used in adjusting the triangulation to which it is connected. It is necessary to reduce the base to sea level if a comparison is made in the field between the measured length and the length as computed through the triangulation from the previous base.

This requires the connection of the base-line levels to a bench mark and the computation of the elevation above sea level of the tape supports in order to obtain a mean elevation for the base.

The formula used in reducing a base to sea level is

$$C = -S\frac{h}{r} + S\frac{h^2}{r^2} - S\frac{h^3}{r^3} + \cdot \cdot \cdot \cdot \cdot \cdot \cdot ,$$

in which C is the correction to reduce to sea level a section of length S, of a mean elevation h, with r the radius of curvature of the earth's surface for that section. Only the first term of the formula need be used for any field reduction.

The computation of the correction for reduction to sea level as shown on Form 589 in figure 124 is given below, the mean latitude of the base being 40°40′ and its azimuth 75°, giving a value for log r (see table 16 on p. 306) of 6.80521. The mean elevation as obtained from Form 635 is 237.8 meters.

$$\begin{aligned} \log 1{,}024.5 &= 3.01051 \\ \log 237.8 &= 2.37621 \\ \operatorname{colog} r &= 3.19479 - 10 \\ \hline \log Sh/r &= 8.58151 - 10 \\ C &= -0.0382 \text{ meter} \end{aligned}$$

The error per kilometer of base line for each 1-meter error in the elevation above sea level as used in computing the reduction varies from 0.000158 meter to 0.000156 meter, depending upon latitude and azimuth of the base. This corresponds to a proportionate error in length of from 1 part in 6,329,000 to 1 part in 6,410,000.

DIAGRAM

If a base is a broken base, or has offsets and shunt triangles, a sketch or diagram should be included with the computations showing all reduced angles and lengths for projecting the measurements to the straight line between the two monumented base stations. Field computations of the projected length should also be attached.

SUPPLEMENTAL BASE LINES

In triangulation, it sometimes occurs that a main-scheme station is on a building, a tank, or other structure, and a more accessible station is desired on the ground; or, a station may be on a rugged mountain and a more accessible supplemental station is desired along a road. In such cases if only one established station is visible from the desired station site, a short base line may be measured so as to form a triangle which includes the established station at one vertex and two supplemental stations at the ends of the supplemental base line. Angles of the supplemental triangle are measured at all three vertices. Azimuth is carried to the supplemental figure through occupation of the established station. Measurement of the supplemental base line furnishes distances for solution of the supplemental triangle and for computation of the geodetic positions of the supplemental stations. Such supplemental base lines are normally measured with methods to obtain second-order accuracy of measurement.

TRAVERSE TIES

Triangulation parties frequently make short spur traverse connections from occupied stations to other nearby marked stations, particularly to those of other surveying and mapping organizations where the location or condition of these other stations make it impractical to include them in an observed triangle. Such traverse ties which are usually for distances of from a few meters to about a kilometer should be made with methods to insure second-order accuracy. Methods of measurement described on pages 115 and 209 to 217 may be used for making traverse ties as modified by the following specifications:

1. Tapes used for making traverse ties should be standardized 30-meter steel tapes, and standardized second-order 50-meter invar tapes. The invar tapes may be used in sets of two with one tape for forward measurements and the other tape for backward measurements. They need not be returned to the Office for restandardization unless the tape is injured or field results indicate that standardization has changed. They should

be compared in the field with first-order base tapes with sufficient frequency to assure the chief of party that standardization values are being maintained.

2. The backward and forward measurements of a traverse tie should check to within 20 mm.$\sqrt{K}$ where K is in kilometers; or in the case of short distances to within 3 millimeters.

3. All measurements of traverse ties should be made with the tapes under standardized tension. Temperature readings should be recorded for each tape measurement. Grade corrections should be determined. Corrections should be made for method of support. Methods should be used to insure that the corrections due to each of these factors are determined to an accuracy within 10 mm.$\sqrt{K}$; or, in the case of short distances to within 1.5 millimeters.

Chapter 4.—AZIMUTHS

GENERAL STATEMENT

The accumulation of angular errors in triangulation, and a tendency for a part of those errors to be systematic, and to give a twist to the direction of a triangulation scheme, make it necessary to include azimuth-control points, called Laplace stations, in the adjustment of the triangulation. A Laplace azimuth is an astronomic azimuth corrected for deflection of the vertical. The Laplace correction and several methods of determining azimuth are discussed in Special Publication No. 237, "Manual of Geodetic Astronomy." The description of azimuth observations which immediately follows in this manual is confined to the method used by first-order triangulation parties, who make the azimuth observations along with the angle measurements of triangles, using the same personnel, first-order theodolites, survey towers, observing tents, and other equipment as was described in chapter 2, plus additional equipment described on page 238. First-order Laplace azimuths are the only azimuths now used in the adjustment of triangulation of the Coast and Geodetic Survey. Second- and third-order azimuths are described in the last section of this chapter.

GENERAL INSTRUCTIONS FOR FIRST-ORDER AZIMUTHS

1. Site of azimuth station.—The locality of the station at which astronomic azimuth observations are required is usually specified in the project instructions. On a first-order triangulation arc, Laplace azimuth stations are usually 6 to 10 quadrangles apart. Since at each Laplace station it will be necessary for an astronomic field party to make longitude observations, it is very desirable that a station be selected to which trucks with comparatively heavy astronomic instruments and equipment may be driven directly. Where the stations of the main scheme are inaccessible, it may be necessary to locate an accessible supplemental point for a Laplace station. This station need be connected to the main scheme by a single well-shaped triangle only, but the angle measures must be made with first-order accuracy.

2. Star.—In the northern hemisphere, except near the equator, Polaris preferably should be used, though any other circumpolar will do, especially near elongation. With time determinations of an accuracy of better than one second, Polaris may be used at any hour angle. See Special Publication No. 237 for special methods of observations in high latitudes.

3. Mark.—The directions to two or more main-scheme triangulation stations should be included along with those to the star in each set of azimuth observations. For each circle position the pointing on the star should be the last pointing before reversal of the telescope and the first pointing after reversal.

4. Criteria.—(a) No observation (circle position) which gives a residual of 5″ or greater from the mean should be accepted.

(b) The azimuth should depend upon at least 24 acceptable observations, not less than 12 of them being on any one night.

(c) Observations should be made on at least two nights.

(d) The probable error of the azimuth should not exceed 0″.30.

5. Time.—Chronometer corrections should be determined before and after each observing period of azimuth observations and at about two-hour intervals by either a comparison with U. S. Navy or National Bureau of Standards radio time signals, or by four zenith-distance observations on an east star and four on a west star near the prime vertical.

INSTRUMENTS AND EQUIPMENT

THEODOLITES

The theodolites used by first-order triangulation parties in observing a first-order azimuth are the same instruments as used for the measurement of the horizontal angles of the triangulation scheme. These theodolites are described on pages 26 to 30. The principal instrument requirement is a direction-type theodolite of first-order quality. The theodolite should have a striding or plate level of about 6 seconds or better sensitivity per division and should be equipped with electric illuminating devices for reading the circles. The theodolite should also have a vertical circle reading to 10″ per division or better if time stars are to be observed. Tests and adjustments of theodolites are described on pages 51 to 66.

CHRONOMETER

Any standard chronometer with a good rate is satisfactory for use in azimuth observations. Readings of the chronometer are used in determining the correct sidereal time of all pointings on the stars. Chronometers are delicate instruments and must be handled and packed with care. The maintenance of a uniform rate between time comparisons is essential for accurate azimuth determinations. A chronometer should not be jarred or rotated rapidly during the observing period between time comparisons. When a chronometer is transported, the balance wheel should first be stopped by allowing it to strike gently against a small piece of paper until motion ceases. Then, two small soft cork wedges should be inserted simultaneously under opposite sides of the rim of the balance wheel near the spokes. These wedges should not be forced and should not contact any of the adjusting screws of the balance wheel. (See p. 19 of Special Publication No. 237.)

RADIO RECEIVER

The radio receiver should be an easily portable short wave battery-operated set with a frequency range of from about 1 to 30 megacycles per second. When radio time comparisons are used, they should be made at the station, so that the chronometer will not be transported between comparisons.

MISCELLANEOUS EQUIPMENT

Signal lamps, survey towers, observing tents, and other equipment used on azimuth observations are the same as were described for triangulation parties on pages 42 and 66.

BOOKS AND TABLES

In addition to the books and numerical tables listed on pages 280 and 281, a copy of the "American Ephemeris and Nautical Almanac" for the current year is needed for astronomic azimuth observations and computations.

ORGANIZATION OF PARTY

Five men are usually needed for azimuth observations. The organization of a party for observation of azimuths is the same as for a standard observing unit for triangulation (see p. 21). There are an observer, a recorder, and a combined lightkeeper and "B"-micrometer reader on the observing unit at the occupied station, and lightkeepers to show lights at two or more adjacent stations. The observer has the additional duties of making pointings on the star and readings of the striding level. The recorder has the additional duty of making chronometer readings for all star pointings.

TIME DETERMINATION

Generally, the timepiece used in the azimuth observations is a sidereal chronometer. It is usually set approximately at local sidereal time, though this is not essential. Chronometer corrections and rate must be obtained so that the recorded time of observations can be corrected to local sidereal time. The two principal methods used are discussed in the following paragraphs.

TIME BY RADIO

The simpler and more accurate method of obtaining the chronometer corrections and rate is by means of radio time signals. A radio-chronometer comparison should be made just before the azimuth observations begin and at every two hours thereafter, including a comparison immediately after the conclusion of the observations.

Any scientific time signals, such as the U. S. Navy time signals, NSS or NPG, or the National Bureau of Standards signal, WWV, will be satisfactory. (See Special Publication No. 237, p. 40.) The latter is particularly convenient because it is transmitted continuously throughout the 24 hours of the day. It is sufficient to obtain the chronometer times of several identifiable radio breaks by ear to the nearest quarter of a second. This method requires that the astronomical longitude of the observer's station be accurately known for final computation, but the geodetic longitude may be used in the field computation. See page 244 for discussion of the computation of the chronometer correction.

TIME BY ZENITH DISTANCES OF STARS NEAR THE PRIME VERTICAL

This method may be used when a radio is not available. It consists in the observation of the zenith distances of an east and a west star, each of which is within 30° of the prime vertical and not less than 15° above the visible horizon. Four observations (L and R) should be made on each star. This combination of star observations is commonly known as a time set.

The chronometer time is recorded for each pointing on a star. The difference between the local sidereal time computed from the zenith-distance observation and the recorded chronometer time gives a chronometer correction. See pages 245 to 247 for computation of chronometer correction. For the reduction of observed chronometer time of the azimuth observations to their local sidereal time, two time sets must be observed, one immediately preceding the azimuth observations and the other immediately following them. Using the rate of change of the difference between chronometer time and local sidereal time of the two sets, the local sidereal time of each azimuth observation is computed.

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OBSERVING PROCEDURE FOR TIME STARS

East and west stars are selected whose positions at the time of the observation fulfill the requirements of the preceding paragraph. Stars selected should preferably be among those listed in the Ephemeris. Stars may be identified by recognizing the constellations, by use of a Rude star finder, by use of Olcott's "Field Book of the Stars," by use of Hydrographic Office Publication No. 214, "Tables of Computed Altitude and Azimuth," or by the use of numerous other tables and charts of the skies. Observers should be careful not to use a planet which will appear as a definite round disk in the telescope. When star catalogs are available, any bright star used can be later identified if the horizontal angle from the north or from some main-scheme line is recorded to the nearest minute at the time of the zenith-distance observation.

Observations on time stars are recorded in Form 252 "Observations of Double-Zenith Distances," a sample page of which is shown in figure 125. The name of the star is entered in column one when identified. In order to check the definite identification of each star, the following data should be entered: (1) The horizontal angle between

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 252
Rev. Feb. 1935

DOUBLE ZENITH DISTANCES

Station Doyle State Ohio Instrument 368 Date Dec 3, 1943

Observer R.L. Tucker County Wayne

Object Observed	Time	Level O.	Level E.	Circle Right or Left	Circle Reading		Verniers A	Verniers B	Verniers Mean	Zenith Distance	Remarks
Star East	h m	s			° ′		″	″	″	° ′ ″	Δ from pole 96°05′
α Arietis	18 55	22.5		R	314 47		40	50	45.0		
										45 00 20.0	Barometer = 28"7
	18 58	34		L	44 48		20	30	25.0		Temperature = 3°5 C
mean	18 56	58.2									
	19 00	27.5		L	44 27		40	25	32.5		"B" Micro: Ed. Hartley
										44 10 28.8	Recorder: A.L. Powell
	19 02	26.5		R	316 06		30	40	35.0		Checker: M.Y. Poling
mean	19 01	27.0				Do not write in this margin Do not write in this margin					
	19 04	19.5		R	316 28		00	55	57.5		
										43 27 08.8	
	19 06	17.5		L	43 22		25	05	15.0		
mean	19 05	18.5									
	19 08	07.0		L	43 01		30	30	30.0		
										42 46 20.0	
	19 09	47.5		R	317 28		40	60	50.0		
mean	19 08	57.2									

FIGURE 125.—Example, observations of double zenith distances of star for time determination, Form 252.

Polaris or an adjacent triangulation station and the star in order that its azimuth may be determined; (2) the altitude of the star at the same recorded time; (3) an approximate comparison of the recorded time of observation and standard time. Additional notes should be entered regarding the organization of the party and weather data. The identification numbers of chronometer, barometer, and thermometer should be entered. The barometer and the temperature readings should be recorded for each time set, since these data are necessary to make refraction corrections to the zenith-distance observation.

The following description applies to Parkhurst first-order theodolites and similar instruments. The procedure is the same as was described on page 103 for observation of vertical angles for trigonometric leveling except that chronometer readings are recorded for each pointing on the star. Observations with optical prism-reading theodolites are made in the same manner. (See p. 143.) Observations with other types of vertical circles are described in Special Publication No. 237, page 96.

The image of the star is brought near the intersection of the wires of the telescope with the horizontal wire just ahead of the star in the direction in which the star appears to be moving. The star is allowed to make the contact, thus eliminating both the error due to thrust upon the instrument and that due to movement of the telescope when the horizontal wire is moved into contact with the star. A stand-by signal is given by the observer and at the instant of contact the observer calls out "tip," and the recorder reads and records the chronometer to the half second. The observer then centers the vertical-circle vernier level with the tangent screw and reads both verniers. Each determination of a zenith-distance angle consists of a circle-left and a circle-right pointing on the star. The recorded times for the two pointings are also meaned. As previously stated four determinations are made for an east star and four for a west star for each time set. Computation of time sights are described on page 245.

ERRORS IN TIME OBSERVATIONS

Some of the more common sources of error in time observations with the vertical circle are mentioned below, with the remedy for each indicated:

1. Incorrect noting of time.—An inexperienced recorder should be trained in the way explained in the paragraphs relating to the observations on Polaris. Do not confuse him by calling out the readings of the verniers or levels before he has finished recording the time.

2. Incorrect circle readings.—The difficulty of securing an adequate illumination of the verniers by flashlight increases the chances of incorrect readings. Check carefully the minutes of each vernier reading, for the mistakes are more apt to occur in the minutes than in either the degrees or seconds.

3. Wrong star.—The approximate azimuth from Polaris and the time of its measurement will serve to correct the identification of the star.

4. Refraction errors.—The zenith distances must be corrected for refraction by means of tables on pages 318 to 320 in accordance with the barometric pressure and temperature at the time of observation.

The error introduced by using one temperature and barometer reading for the whole time set instead of separate temperature and barometer readings for each observation will be diminished by having the zenith distances of the east and west stars as nearly equal as possible. Usually, however, the error from this source is negligible if no star is used whose altitude is less than 15°.

5. Poor selection of stars.—Serious errors may be introduced by selecting stars too far from the prime vertical. For instance, at 30° from the prime vertical the effect of an error in zenith distance on time is 15 percent more than it would be exactly in the prime vertical; at 20° from the prime vertical the effect on time is 6 percent more than when exactly in the prime vertical. In the early evening there is always the temptation to use the first stars visible in order to begin the night's work. A delay of a quarter of an hour

is usually not serious and will often result in securing time stars that will give a much more accurate chronometer correction.

6. Parallax.—The effect of parallax is almost invariably opposite in sign for east and west stars. Hence, to eliminate this effect, an east star and a west star should always be observed, as the mean of the computed times for the two stars will then be free from any measurable effect of parallax, unless there is a great difference in their zenith distances.

AZIMUTH OBSERVATIONS

PREPARATION

Before making azimuth observations, the theodolite should be tested and adjusted as described on pages 51 to 66. The striding level and the standards should be in especially close adjustment for azimuth observations. Any inclination of the horizontal circle will affect the accuracy of the star observations, and unequal standards will cause trouble in centering the striding level.

RECORDING OF OBSERVATIONS

Observations are recorded in Form 251a, "Observations of Horizontal Directions" in the same manner as was described on pages 108 and 112. A sample record is shown in figure 126. In addition to notes regarding the organization of the party and weather data, the identification numbers of the chronometer, barometer, thermometer, and strid-

DEPARTMENT OF COMMERCE
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Form 251a

Horizontal Directions

Station: Doyle Observer: R.L. Tucker Instrument: 368 Date: Dec. 3, 1943

Position	Objects Observed	Time A. m.	Tel. D or R	Mic.	°	′		Back'd "	Forw'd "	Mean	Mean D and R	Reduction	Remarks
1	Barberton		D	A	00	00		41	41				1 division of striding
				B				50	50	45.5			level = 6.507
			R	A	180	00		40	40				
				B				32	33	36.2	408		
													Level readings
	Polaris		D	A	308	11*		24	24				W E
	7h 38m 47s.0			B				38	38	31.0			06.9 20.2
	7 41 40.0												23.4 10.0
	Mean 7 40 13.5		R	A	128	10		55	55				16.5 +6.3 10.2
	Diff 2 530			B			Do not write in this margin	37	38	46.2	08.6		08.6 21.9
													21.6 08.1
				A									13.0 -0.8 13.8
				B									+2.8
2	Barberton		R	A	191	01	Do not write in this margin	28	28				
				B				23	23	25.5			
			D	A	11	01		29	30				
				B				32	32	30.8	28.2		
	Polaris												W E
	7 45 10.0		R	A	139	10*		42	43				08.0 21.1
	7 46 46.5			B				27	27	34.8			22.1 08.9
	Mean 7 45 58.2												14.1 +1.9 12.2
	Diff. 1 36.5		D	A	319	09		55	55				06.8 20.0
				B				66	67	60.8	17.8		23.5 10.1
													16.7 +6.8 09.9
				A									+4.4
				B									
	* Correct minute for mean of direct and reverse												

FIGURE 126.—Example, azimuth observations on Polaris, Form 251a.

ing level should be shown. If the striding level has no number, a note should be entered of the number of the theodolite to which the striding level belongs. The value of one division of the striding level should be entered. If chronometer corrections are determined by chronometer and radio time signal comparisons, the chronometer time, the radio time, the station identification, and the radio frequency should be entered. Notes and sketches of any eccentricities of instrument or object should be entered. If there is no eccentricity it should be so stated. A separate page of the record book may be used for as many of the above notes as may be necessary to avoid crowding.

OBSERVING PROCEDURE

Azimuth observations on Polaris may be made along with the main-scheme first-order observations at a station (see p. 110) or a separate set may be observed which includes the star and two or more main-scheme stations. The sequence of the pointings should be so arranged that the pointing on the star is always the last pointing before reversal, and the first pointing after reversal of the telescope. This is done in order to make the direct and reversed pointings on the star as close together as possible so as to reduce the curvature correction due to the curvature of the star's apparent path.

The observing routine is as follows: Point on the initial and read micrometers; point on at least one additional main-scheme station and read micrometers; then point on Polaris, bringing the star near the middle of the diaphragm with the vertical wires just ahead of the star in the direction in which it is moving. Then with the alidade clamped and the striding level in position on the standards, "stand by" is called out to the recorder. When the star bisects the space between the vertical wires, "tip" is called out sharply. Both ends of the striding-level bubble are read (west end first), and the striding level is then reversed on the pivots. Next the micrometers are read, then the reversed striding level is read, again west end first. The alidade is then rotated 180° and then the striding level is temporarily lifted off the pivots while the telescope is being plunged, and pointings and readings are again repeated on the star in the same manner as before the instrument was reversed, always reading the striding level west end first.

Readings are repeated on the other stations and the initial to complete the position in the usual manner (see p. 111). Observations are continued using the circle position settings shown in table 2 on page 11 until the observations are completed in accordance with the criteria of page 237.

The striding level may be left on the instrument between plungings, but it is read only for pointings on the star, except when the inclination of the line of sight to the marked stations exceeds one degree. It is very important that the inclination of the horizontal circle be eliminated insofar as possible.

When "tip" is called out the recorder reads and records the chronometer time to the nearest half second. He should be allowed time to record this reading before the striding-level readings are called out to him. Greater accuracy is obtained when the recorder carries the half-second count mentally and uses the mental count along with his visual reading in determining the chronometer time of the "tip."

FIELD COMPUTATIONS

The formulas used in the determination of azimuth are discussed in Special Publication No. 237, "Manual of Geodetic Astronomy," page 90.

COMPUTATION OF CHRONOMETER CORRECTION

RADIO COMPARISON METHOD

If the chronometer correction was determined by comparison of the chronometer and radio time signals, the computation is made on Form 605, "Comparison of Chronometer and Radio Signals," a sample copy of which is shown in figure 127. The upper seven lines of the form are observation notes and data. The next two lines are obtained from tables of the American Ephemeris and Nautical Almanac for the current year using the Greenwich Civil Time of the time signal as argument. The sidereal time of zero hour G. C. T. comes from the Sun tables of the Ephemeris from the column headed Sidereal Time (Right Ascension of the Mean Sun + 12^h). The correction "mean solar to sidereal time" comes from table III of the Ephemeris. The rest of the computation of Form 605 is readily apparent from an examination of figure 127. The lines left blank on the sample form are not used in the field computation.

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Form 605
(Rev. June 1946)

COMPARISON OF CHRONOMETER AND RADIO SIGNALS

Station: *Redoubt Astro* Latitude: *60° 29′ 02.″57* Longitude: *152° 17′ 53.″64*

Chief of party: *H. J. Seaborg* Observer: *H. J. Seaborg*

Year: *1944* Chronometer No.: *3479* (~~mean~~, sidereal) (Strike out one)

Local date	Aug. 31	Sept. 1		
Standard time of signal, 135 Mer.	h. m. s. 23 00 00	h. m. s. 1 00 00	h. m. s.	h. m. s.
Chronometer time of signal	20 32 07.5	22 32 27.5		
Transmitting station	NPG	NPG		
Frequency of signal	9255 kc	9255 kc.		
G. C. T. of signal — Date	Sept. 1	Sept. 1		
G. C. T. of signal — Time	h. m. s. 8 00 00.0	h. m. s. 10 00 00.0	h. m. s.	h. m. s.
Sidereal time of 0^h G. C. T.	22 40 07.4	22 40 07.4		
Cor., mean solar to sidereal time	1 18.9	1 38.6		
Δ Transmission time				
Correction to signal				
G. S. T. of signal	30 41 26.3	32 41 46.0		
Longitude of station	10 09 11.6	10 09 11.6		
Local sidereal time	20 32 14.7	22 32 34.4		
Chronometer time of signal	20 32 07.5	22 32 27.5		
Chronometer correction	+ 7.2	+ 6.9		
Rate per minute		− 0.0025		
Rate per hour		− 0.15		

Remarks:

FIGURE 127.—Example, comparison of chronometer and radio signals, Form 605.

TIME STAR METHOD

Chronometer corrections from time stars are computed on Form 381a, "Computation of Time, Observations on a Star with Vertical Circle." Figure 128 is an illustration of this computation.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 381 a
Rev. Jan. 1946

COMPUTATION OF TIME, OBSERVATIONS ON A STAR WITH VERTICAL CIRCLE

STATE Ohio STATION Doyle BAROMETER READ. 28.7" C_B 0.959

DATE Dec. 3, 1943 CHRONOMETER No. 235 TEMPERATURE 3.5° C C_T 1.023

INSTRUMENT 368 LEVEL VALUE .... $C_B \times C_T$ 0.981

CHIEF OF PARTY A.P. Ratti — APPROX. ANGLE BETWEEN STAR AND POLARIS 96° 05′; CHRO. TIME OF ANGLE READING $18^h 56^m$

OBSERVER R.L. Tucker CHRO.-WATCH COMPARISON — CHRONOMETER .... WATCH (........ MER. TIME)

	STAR {East / ~~West~~}: α Arietis		STAR {East / ~~West~~}: α Arietis	
Chron. Reading, Zenith Dist.	18h 56m 58.2s	45° 00′ 20.0″	19h 01m 27.0s	44° 10′ 28.8″
Refraction		56.8		+ 55.1
Corrected Z. D. = ζ		45 01 17		44 11 24
log cos φ, φ	9.8779372	40 58 34		
log cos δ, δ	9.9633858	23 11 53		
log cos φ + log cos δ = log D, φ − δ	9.8413230	17 46 41	9.8413230	17 46 41
log sin ½[ζ+(φ−δ)], ½[ζ+(φ−δ)]	9.7168423	31 23 59	9.7116378	30 59 02.5
log sin ½[ζ−(φ−δ)], ½[ζ−(φ−δ)]	9.3720087	13 37 18	9.3587956	13 12 21.5
Sum two log sines = log N	9.0888510		9.0704334	
log N − log D = log sin² ½ t	9.2475280		9.2291104	
log sin ½ t, ½ t (arc)	9.6237640	335 08 02.3	9.6145552	335 41 23.5
t (time), t (arc)	20h 41m 04.3s	310 16 04.6	20h 45m 31.1s	311 22 47.0
Right ascension of star	2 04 00.7		2 04 00.7	
Sidereal time	22 45 05.0		22 49 31.8	
Chronometer reading	18 56 58.2		19 01 27.0	
Chronometer correction	+3 48 06.8		+3 48 04.8	

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The correction is plus if the chronometer is slow, and minus if fast.
Carry all angles to seconds only, all times to tenths of seconds, and all logarithms to seven decimal places.
In space below, compute rate of chronometer, etc.

FIGURE 128.—Example, computation of time, observations on a star with vertical circle, Form 381a.

The formula used is:

$$\sin^2 \frac{1}{2} t = \frac{\sin \frac{1}{2}[z+(\phi-\delta)] \sin \frac{1}{2}[z-(\phi-\delta)]}{\cos \phi \cos \delta}$$

where t is the hour angle of the star, z is the zenith distance of the star corrected for refraction, ϕ is the astronomic latitude of the station, and δ is the declination of the star. If the star is a west star, sin $\frac{1}{2}t$ will be positive, and $\frac{1}{2}t$ will be in the first quadrant. If the star is an east star, sin $\frac{1}{2}t$ will be negative and $\frac{1}{2}t$ will be in the fourth quadrant.

Knowing the mean barometric pressure and temperature for each star observation, the refraction is computed from tables 25, 26, and 27 on pages 318 to 320 in the manner explained below.

To obtain the refraction from tables 25, 26, and 27, first enter table 27 with the temperature reading as argument to obtain C_T. Then enter table 26 with the barometer reading as argument to obtain C_B. Take the product of C_T by C_B.

Next using the zenith distance, z, as argument enter table 25 for r_m (the mean refraction for 29.9 inches pressure and temperature 10° C.).

The refraction for the temperature and barometer readings at the time of observation will now be the product:

$$C_B \text{ times } C_T \text{ times } r_m.$$

After the hour angle has been found from the formula above, the local sidereal time is found by the formula:

$$L.S.T. = \alpha + t$$

where α is the right ascension of the star.

NOTE: When the star is east and the hour angle is in the third or fourth quadrant, it is often convenient to use the explement of the hour angle as a negative small angle. For instance, in the sample computation, for the first observation on α Arietis, $\frac{1}{2}t = -(24°51'57''.7)$ would be used in place of $\frac{1}{2}t = (335°08'02''.3)$. The sidereal time would then be $(2^h04^m00^s.7) + [-(3^h18^m55^s.7)] + 24^h = 22^h45^m05^s.0$, as in the sample computation.

The chronometer correction will now be local sidereal time minus chronometer time.

If it is desired to keep the chronometer correction always positive, 24 hours should be added to the local sidereal time before making the subtraction when the sidereal time is less than the chronometer time.

The mean chronometer time and the mean chronometer correction of each set are obtained by first taking the means for each star and then the means of these respective means. The rate of the chronometer then will be the difference in the mean corrections for the two sets divided by the difference in the mean chronometer times for the sets, or expressed symbolically, the rate of the chronometer per hour with respect to sidereal time will be:

$$R = \frac{C_t - C_o}{T_t - T_o}$$

where T_o and T_t are the mean chronometer times expressed in hours for the first and last sets, respectively, and C_o and C_t are the mean chronometer corrections at T_o and T_t, respectively.

If $C_t - C_o$ is expressed in seconds of time, R is the rate in seconds per hour. If R is positive, the chronometer is losing; if negative, the chronometer is gaining.

In the sample computation, figure 128, the east star of the first set is α Arietis. The first observed zenith distance is $45°00'20''.0$. The barometer reading is 28.7 inches and temperature is 3.5° centigrade. From table 27, we find $C_T = 1.023$, and from table 26, we obtain $C_B = 0.959$. The product of C_B by $C_T = 0.981$. Using the zenith distance

45°00′20″.0 as argument in table 25, r_m is found to be 57″.9. Hence the refraction correction is (0.981) (57″.9) = 56″.8.

Summary of Time Computation

Doyle, Ohio

Star	*Mean chronometer time*		*Mean chronometer correction*		
	First Set				
East (4 obs.)	19^h	03^m17	$+3^h$	48^m	05^s90
West (4 obs.)	19	20.74	+3	48	04.28
Mean, 1st set	19	11.96	+3	48	05.09
	Second Set				
East (4 obs.)	22	02.61	+3	48	04.38
West (3 obs.)	22	23.12	+3	48	03.40
Mean, 2nd set	22	12.86	+3	48	03.89
Difference	3	00.90			−1.20

Rate = -0^s3980 per hour or -0^s00663 per minute.

COMPUTATION OF AZIMUTH, DIRECTION METHOD

For the purpose of explaining the computation of an azimuth observed by the direction method, the azimuth from Doyle to Barberton was chosen. The observations were made on two nights, Dec. 3 and 4, 1943, sixteen observations being made on each night. The correction for rate from the observations with the theodolite has already been explained. (See p. 245.)

There follow samples of the record of the observations on Polaris and the mark on Form 251a; computation of the azimuth on Form 380; and the summary of azimuth computations.

EXPLANATION OF COMPUTATION

The first step in the computation for azimuth is to abstract on Form 380 (fig. 129) the necessary data from the horizontal angle record book Form 251a (fig. 126). Then, with the chronometer correction and rate computed as shown in figure 127 or figure 128, correct the chronometer time for each observation on Polaris to obtain the local sidereal time of the observation. (See p. 248.) Next, compute the Greenwich civil time of the mean epoch of the observations expressed in a fraction of a day, as follows:

Usually, it is sufficient to take the mean of the local sidereal times of the first and last observations of the night's observations, provided the period is not greater than 4 hours. To this mean, add the longitude of the station if west, subtract if east of Greenwich. The result is now the Greenwich sidereal time of the approximate mean epoch of the observations. Subtract from this the Greenwich sidereal time of the nearest preceding 0^h civil time and express this difference as a fractional part of a civil day, taking into account that there are 24.066 sidereal hours or 1,444 sidereal minutes in a civil day. In the example, this computation is as follows:

DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

Form 380
Ed. Jan. 1942

COMPUTATION OF AZIMUTH, DIRECTION METHOD

STATE: Ohio — STATION: Doyle — ECCENTRIC*: INSTR.: None; LIGHT: None

MARK: Barberton — INSTR.: No.: 368; LEVEL VALUE (D): 6.507"

CHRONOMETER NO.: 235 — OBSERVER: R.L. Tucker

POSITION OF STATION, φ: 40° 58' 33.668" λ: 81° 41' 29.138" — GREENWICH CIVIL DAY: 4.03

Date, 1943, position,	Dec. 3 — 1	2	3	4
Chronometer reading,	19h 40m 13.5s	19h 45m 58.2s	19h 50m 17.0s	19h 54m 13.2s
" correction,	+ 3 48 04.9	+ 3 48 04.9	+ 3 48 04.8	+ 3 48 04.8
Sidereal time,	23 28 18.4	23 34 03.1	23 38 21.8	23 42 18.0
α of Polaris,	1 46 01.6			
t of Polaris (time),	21 42 16.8	21 48 01.5	21 52 20.2	21 56 16.4
t of Polaris (arc),				
δ of Polaris,	89° 00' 01.77"			
log cot δ,	8.24171	= 8.18051		
log tan φ,	9.93880			
log cos t,	9.91636	9.92362	9.92882	9.93338
log a (to 5 places),	8.09687	8.10413	8.10933	8.11389
log cot δ,	8.241708	= 8.363770		
log sec φ,	0.122062			
log sin t,	9.752355 n	9.736036 n	9.723187 n	9.710976 n
log $\frac{1}{1-a}$ = colog (1 - a)	0.005462	0.005555	0.005622	0.005682
log (−tan A) (to 6 places),	8.121587 n	8.105361 n	8.092579 n	8.080428 n
A = Azimuth of Polaris, from north,†	+ 0° 45' 28.9"	0° 43' 48.8"	0° 42' 32.6"	0° 41' 22.2"
Difference in time between D. and R.,	2m 53.0s	1m 36.5s	1m 30.0s	1m 39.5s
Curvature correction,	- 0.1"	0.0"	0.0"	0.0"
Altitude of Polaris = h,	41° 48'			41° 50'
$\frac{d}{4}$ tan h = level factor,	1.454"			
Inclination,	+ 2.8	+ 4.4	+ 4.4	+ 4.6
Level correction,	+ 04.1	+ 06.4	+ 06.4	+ 06.7
Circle reads on Polaris,	308 11 08.6	139 10 17.8	330 10 54.9	161 10 45.8
Corrected reading on Polaris,	308 11 12.7	139 10 24.2	330 11 01.3	161 10 52.5
Circle reads on Mark,	0 00 40.8	191 01 28.2	22 03 25.1	213 04 25.0
Difference, Mark − Polaris,	51 49 28.1	51 51 04.0	51 52 23.8	51 53 32.5
Corrected azimuth of Polaris, from north,†	0 45 28.8	0 43 48.8	0 42 32.6	0 41 22.2
	180 00 00.0	180 00 00.0	180 00 00.0	180 00 00.0
Azimuth of (Clockwise from south)	232 34 56.9	232 34 52.8	232 34 56.4	232 34 54.7

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark. Carry times and angles to tenths of seconds only.

* Give volume and page of record for eccentricity, if any. † Minus, if west of north.

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FIGURE 129.—Example, computation of azimuth, direction method, Form 380.

Mean epoch of observation (L.S.T.)	0^h	00^m	$39^s.7$	
Astro. Longitude (W)	+5	26	45.9	
G.S.T. of mean epoch	5	27	25.6	
0^h Dec. 4, 1943, G.S.T	4	47	44.4	
Difference	0	39	41.2	$=39^m.687$

$39^m.687 \div 1{,}444 = 0.027$ day after 0^h Dec. 4. Hence the mean Greenwich civil time of the mean epoch of the observations is Dec. 4.027, 1943.

Although theoretically the longitude and latitude used in the above computation should be astronomic values, the use of the geodetic values will be satisfactory for field computations.

The next step is to obtain from the table in the American Ephemeris the right ascension α and declination δ of Polaris for the Greenwich civil time just computed, being careful not to overlook the corrections for the short-period terms in the table. The right ascension is taken out to one-tenth second to conform with the practice for the local sidereal time. The declination, however, is taken out to hundredths of a second.

In the designated places, the logarithms of the tangent and secant of the latitude ϕ and the logarithm of the cotangent of the declination δ of Polaris can now be entered on the form.

The right ascension α of Polaris is subtracted from the local sidereal time to obtain t, the hour angle of Polaris in units of time. If there is available a 6- or 7-place logarithm table of trigonometric functions with time as an argument, log cos t and log sin t can be taken directly from the table without first going through the tedious process of converting time to arc. (See p. 248.) Such a table is that of Shortrede, which is standard equipment on triangulation parties. It is important that the trigonometric functions be used with the proper signs.

The expression for a will have the same sign as cos t. Colog $(1-a)$ is taken from table 28 on pages 321 to 329, using log a as argument. There are two parts to this table, one for positive a, the other for negative a.

Next, we obtain log tan $(-A)$. The sign of tan $(-A)$ will depend on the sign of sin t. If sin t is positive, tan $(-A)$ will be positive; hence, the azimuth A will be negative when Polaris is west of the meridian. If sin t is negative, tan $(-A)$ will be negative; hence, the azimuth A will be positive when Polaris is east of the meridian.

The azimuth of Polaris from the north, that is, azimuth A, must now be corrected for curvature. The sign of the curvature correction is always such that it diminishes the size of the angle of Polaris from the north. The correction is found in table 29 on page 330. On page 91 of Special Publication No. 237 there is a discussion of the curvature correction.

Ordinarily, the altitude of Polaris is not observed in the field, since for the purpose of obtaining $\frac{d}{4}$ tan h, the value of the altitude, h, as taken from table I in the American Ephemeris to the nearest minute will be sufficiently accurate. The arguments are the hour angle and declination of Polaris. It will usually suffice also to compute this factor for one in every four observations and to interpolate between the computed factors. To obtain the altitude of Polaris the quantity found in table I is applied to the latitude with its sign reversed. (See p. 248.)

On page 115 of Special Publication No. 237 in connection with the micrometer method of determining azimuth in high latitudes, it is shown how the altitude can be obtained with greater accuracy.

Each circle reading on Polaris must be corrected for the inclination of the horizontal axis. This correction is obtained by multiplying the inclination, expressed in divisions of the level vial, by $\frac{d}{4}\tan h$ seconds of arc.

If H, the angle of elevation or depression of the mark from the station, is greater than 1°, level readings should be made when observing on the mark, and the correction in seconds to be applied to the circle reading on the mark is obtained by multiplying the inclination expressed in divisions of the level vial by $\frac{d}{4}\tan H$. This factor is constant for all the observations. The angle H is positive when it is an angle of elevation, negative when it is an angle of depression.

The corrected circle reading on Polaris is now subtracted from the corrected circle reading on the mark, the result being the direction of the mark measured from Polaris. To this direction, add algebraically the corrected azimuth of Polaris from the north. The result is the azimuth of the mark from the north. To reduce the azimuth from the north to an azimuth from the south, add 180° if the former is less than 180°, and subtract 180° if it is greater than 180°.

On the summary sheet the mean of the 32 results (all observations being considered of equal weight) is 232°34′55″.75.

There were no rejections, as the largest residual was +4″.9, which is less than the rejection limit of 5″ from the mean.

The probable error of the result is ±0″.29, so that the azimuth conforms to the criterion of first-order accuracy.

Other corrections including diurnal aberration, elevation of mark, and variation of the pole are not normally made in the field computation. (See pp. 106–107 of Special Publication No. 237.)

Probable error is computed by the formula

$$\text{p.e.} = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

where n is the number of accepted positions.

Summary of Azimuth Computation

Date: Dec. 3, 4, 1943
Observer: R. L. Tucker
Inst.: Theod. No. 368

Doyle to Barberton, Ohio

osition	*Azimuth* 232°34′	*v*	v^2	*Position*	*Azimuth* 232°34′	*v*	v^2
Dec. 3, 1	56″.9	−1″.1	1.21	Dec. 4, 1	50″.9	+4″.9	24.01
2	52.8	+3.0	9.00	2	57.7	−1.9	3.61
3	56.4	−0.6	0.36	3	53.0	+2.8	7.84
4	54.7	+1.1	1.21	4	51.3	+4.5	20.25
5	59.5	−3.7	13.69	5	53.7	+2.1	4.41
6	56.0	−0.2	0.04	6	58.1	−2.3	5.29
7	55.5	+0.3	0.09	7	54.4	+1.4	1.96
8	54.0	+1.8	3.24	8	55.1	+0.7	0.49
9	58.0	−2.2	4.84	9	54.1	+1.7	2.89
10	55.9	−0.1	0.01	10	60.5	−4.7	22.09
11	58.7	−2.9	8.41	11	56.8	−1.0	1.00
12	55.1	+0.7	0.49	12	54.2	+1.6	2.56
13	57.4	−1 6	2.56	13	51.2	+4.6	21.16
14	58.3	−2.5	6.25	14	54.8	+1.0	1.00
15	57.9	−2.1	4.41	15	55.9	−0.1	0.01
16	58.7	−2.9	8.41	16	56.4	−0.6	0.36
							$\Sigma v^2 = 183.15$

Mean observed azimuth 232°34′55″.75±0″.29

ABBREVIATED FIELD COMPUTATIONS

For field computations which are made primarily to insure that the observations fulfill the requirements of the specifications, the methods of computations described in the previous paragraphs may be modified as follows:

The tangent of the latitude may be used instead of tangent h. The curvature correction may be omitted for short time intervals or for observations near elongation of Polaris. The mean chronometer correction may be used for the entire set if the rate is small. The reduced time may be used to even seconds in looking up log sin t and log cos t.

SECOND-ORDER AZIMUTHS

All azimuths which are observed by the Coast and Geodetic Survey for the adjustment of triangulation are Laplace azimuths. Whenever a Laplace azimuth is observed, first-order criteria are applied. For those who may be interested in second-order azimuths, a discussion of criteria and methods is contained in the following paragraphs. These are modifications of the methods previously described for first-order azimuths.

In triangulation, a second-order azimuth is one observed with such methods as to give a probable error for the result of 0″.3 or less for class I and 0″.5 or less for class II. (See table 1, p. xv.) The observed value is not corrected for the effect of the deflection of the vertical before being used in the adjustment of triangulation. Either a direction or a repeating instrument may be used. A sufficient number of observations should be made to give results within the specified limit of probable error. Observations may be limited to one night.

Time is determined in the same manner as was described on page 239 for first-order azimuths. A sidereal watch may be used instead of a chronometer. Any theodolite having a vertical circle which reads to 30 seconds or less may be used to make time observations. If the probable error of a time set is greater than two seconds, Polaris should be observed near elongation.

OBSERVATIONS WITH A DIRECTION THEODOLITE

If a direction instrument is used, twice the number of observations on Polaris should be made as would ordinarily be taken on second-order triangulation with the class of instrument used, namely 16 positions with a first-order theodolite, or 24 positions with a second-order theodolite. An example of the record is shown in figure 126. Observations are made in the same manner as was described on page 243. If the value of one division of the striding level or sensitive plate level is not known, it should be determined in the field.

OBSERVATIONS WITH A REPEATING VERNIER THEODOLITE

If a repeating theodolite read by verniers to 10 seconds is used, from three to four sets will be sufficient, each set consisting of 6 D/R between Polaris and the mark. The record of observations should include the time as marked for each pointing upon

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 250

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HORIZONTAL ANGLES

STATION: Cut STATE: N.C. ISLAND OR COUNTY: New Hanover DATE: June 29, 1918

OBSERVER: C.L.G. (△ Station Isle used as mark) INSTRUMENT: Theo. No. 244 Sid. Watch No. 145

OBJECTS OBSERVED	TIME h. m.	TEL. D OR R	REP'S	ANGLE ° '	Do not write in this margin	A "	B "	MEAN OF VERNIERS	ANGLE MEAN D AND R ° ' "	REMARKS
Mark - Polaris	A.M. 12:57	D	0	0 00		00	55	57.5		No eccentricity of
W. E.										theodolite or mark.
26.4 08.0										Sidereal time
09.6 28.0										h. m. s.
16.8 -3.2 20.0		D	1	355 54						6 14 55.5
		D	2							16 37.5
		D	3							19 33.5
W. E.		D	4							21 37.5
26.4 08.0		D	5							23 24.0
11.0 29.5	1:07	D	6	335 25		40	40	40.0	355 54 17.1	25 13.0
15.4 -6.1 21.5										
Polaris - Mark										
W. E.	1:09	R	0	335 25		40	40	40.0		
24.5 06.0										
11.0 29.5										h. m. s.
13.5 -10.0 23.5		R	1							6 26 28.0
		R	2							28 07.5
		R	3							29 51.0
W. E.		R	4							32 38.5
26.1 07.8		R	5							34 55.0
09.5 28.0		R	6	359 51		40	35	37.5	4 04 19.6	36 43.5
16.6 -3.6 20.2										

FIGURE 130.—Example, observation on Polaris for azimuth, repetition method, Form 250.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 448
(Rev. Oct. 1942)

COMPUTATION OF AZIMUTH—REPETITION METHOD

State *Ala.* Station *Kahatchee* Eccentric {Instr. *none*; Light *none*}
Mark *BLANK* Instr. {No. *63*; Level value (d) *2".67*}
Chronometer *11* Observer *O.B.F.*
Position of station, ϕ *33° 13' 40".33* λ Greenwich civil day *6/6/98*

Date, 1898, set	June 6, 5	June 6, 6		
Chronometer reading	14h 54m 17s.7	15h 11m 48s.2	h. m. s.	h. m. s.
Chronometer correction	-31.1	-31.1		
Sidereal time	14 53 46.6	15 11 17.1		
α of Polaris	1 21 20.3	1 21 20.3		
Hour-angle of Polaris (t)	13 32 26.3	13 49 56.8		
t in arc	203° 06' 34".5	207° 29' 12".0	° ' "	° ' "
δ of Polaris	88 45 46.9			
log cot δ	8.33430	8.33430		
log tan ϕ	9.81629	9.81629		
log cos t	9.96367 n	9.94798 n		
log a (5 places)	8.11426 n	8.09857 n		
log cot δ	8.334305	8.334305		
log sec ϕ	0.077535	0.077535		
log sin t	9.593830 n	9.664211 n		
log $\frac{1}{1-a}$	9.994387	9.994584		
log ($-$tan A) (6 places)	8.000057 n	8.070635 n		
A = azimuth of Polaris from north *	0° 34' 22".8	0° 40' 26".8	° ' "	° ' "
τ and $\frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''}$	m. s. "	m. s. "	m. s. "	m. s. "
1	7 47.7 119.3	7 04.2 98.1		
2	5 09.7 52.3	4 30.2 39.8		
3	1 26.7 4.1	1 54.2 7.1		
4	1 52.3 6.9	2 26.8 11.8		
5	4 54.3 47.2	4 25.8 38.5		
6	7 37.3 114.0	6 35.8 85.4		
Sum	343.8	280.7		
Mean	57.3	46.8		
log $\frac{1}{n}\Sigma\frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''}$	1.758	1.670		
log tan A $\frac{1}{n}\Sigma\frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''}$	9.758	9.741		
Curvature correction	-0.6	-0.6		
Altitude of Polaris = h	32° 07'	° '	° '	° '
$\frac{d}{4}$ tan h = level factor	.419	.419		
Inclination	+3.6	+4.1		
Level correction †	-1".5	-1".7	"	"
Angle, star to mark (clockwise)	72° 57' 50".2	72° 51' 46".7	° ' "	° ' "
Corrected angle	72 57 48.7	72 51 45.0		
Corrected azimuth of Polaris	0 34 22.2	0 40 26.2		
	180 00 00.0	180 00 00.0	180 00 00.0	180 00 00.0
Azimuth of mark (Clockwise from south)	253 32 10.9	253 32 11.2		

*Minus if west of north.
†Sign of level correction opposite to that of inclination.

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FIGURE 131.—Example, computation of azimuth, repetition method, Form 448

Polaris, the angle readings for the beginning and ending of each half set, and the striding-level readings, direct and reversed, for the beginning and ending of each half set. If the value of one division of the striding level is not known, it should be determined in the field. (See p. 65.) A sample form of record for a repeating instrument is shown in figure 130. *It is not necessary to record the angle readings corresponding to the times of the pointings* between the first and sixth repetitions, for a curvature correction can be derived from the mean of the recorded times and the mean angle corrected accordingly.

A sample form of a computation is shown in figure 131. Values for the curvature correction used on this form may be obtained from table XII of Special Publication No. 237, "Manual of Geodetic Astronomy."

THIRD-ORDER AZIMUTHS

In triangulation, a third-order azimuth is an astronomic azimuth observed with such instruments and methods as will give a probable error for the result of not more than two seconds.

Practically the same methods are employed in observing a third-order azimuth as are used for second-order, but the larger permissible limit of error allows greater latitude in the choice of instrument and also requires a smaller number of observations. In the Northern Hemisphere, a third-order azimuth should preferably be observed upon Polaris because of the greater convenience both in the observing and in the computations. Observations upon Polaris may be made at any hour angle provided the chronometer (or watch) correction is known within four or five seconds.

If the watch correction is determined by observations upon the stars, at least two, and preferably three, observations should be made upon an east star and the same number upon a west star before the observations upon Polaris are started and a similar set after the Polaris observations are completed. Any instrument having a vertical circle reading to one minute or less may be used for the time observations.

Chapter 5.—SPECIAL SURVEYS

GENERAL STATEMENT

Geodetic surveys using the methods described in the preceding chapters are used for the triangulation networks of the United States and possessions to control mapping and charting operations and for the location of boundaries. Control surveys of various special types and methods are frequently necessary to meet special purposes. Such special surveys are usually adaptations of methods previously described with criteria modified and special instruments and equipment used to meet the requirements of a particular project.

METROPOLITAN CONTROL SURVEYS

A framework of control surveys is desirable for any metropolitan area in order to coordinate the detailed surveys for streets, water supply, sewers, boundaries, mapping, and planning. Triangulation makes possible greater precision in the surveys of large cities than can be obtained by other methods. It coordinates into a single rela ed system all local surveys connected to it and helps to assure the perpetuation of any mark established by such surveys.

The American Society of Civil Engineers Manual of Engineering Practice No. 10 entitled "Technical Procedure for City Surveys" discusses city surveys in detail.

The main-scheme network for a city survey should consist of chains of triangles arranged in geometric figures to provide a double determination of the length of each line.

SPECIFICATIONS FOR CITY SURVEYS

The specifications previously listed on pages 1, 9, and 193 also apply to city surveys. In order to obtain even greater precision of location of the control points required for detailed local and special-purpose surveys, these specifications are further strengthened by the following modifications:

1. The principal lines of the triangulation scheme should generally be from 1 to 5 miles in length. The distribution of first-order stations over the city area should be made as uniformly as possible with an average of about one for each five square miles of area.

2. In the chain of triangulation between bases, the strength of figure value ΣR_1 should not exceed 30, and ΣR_2 should not exceed 50.

3. Two or more first-order base lines located on opposite sides of the city should be used for length control of the city triangulation, and connections should be made to the national network of first-order triangulation.

4. Methods should be used on the main-scheme triangulation which will give an agreement of better than 1 part in 100,000 between the measured lengths of base lines and lengths as computed through the triangulation after the side and angle equations have been satisfied.

5. As many intersection stations as possible should be located by observations from three or more occupied first-order stations, with such methods as will give a probable error for a direction not greatly in excess of one second.

For complete horizontal control for surveying and mapping a metropolitan area, the first-order triangulation should be further broken down by first-order and lower-order traverse.

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SPECIAL PROJECT SURVEYS

Long bridges, dams, tunnels, and aqueducts all present problems of alinement and distance which may be solved by modifications of triangulation and base-measurement methods described in chapters 2 and 3. Some examples which have been described in technical magazines and other publications are the surveys for the Los Angeles Aqueduct, the James River Bridge, and others of which some of the special problems are stated briefly in the following paragraphs.

Special applications of triangulation and base-measurement methods are occasionally used to meet unusual requirements. A few such examples will be discussed briefly.

For the Wright Field all-altitudes airplane speed courses the problem was to lay out three parallel lines about 5 miles apart and about 10 miles long, monumented at both ends. This was accomplished by establishing triangulation stations in the approximate vicinity of the ends of the lines, then computing the positions of the required monuments at one end from short traverse and direction measurements, and then computing the distance and direction from triangulation stations near the other ends of the lines from which field measurements were made to lay out parallel lines.

The Cleveland Airplane Speed Trial Course of September 1948 was a 3-kilometer course for speed testing of jet airplanes during the National Air Races. The line was measured over stakes with first-order accuracy and camera stations and vertical crossing wires were alined at each end of the course.

The Amsterdam Avenue base line in New York City presented some unusual problems of measurement and of projecting the base line to stations on the roofs of buildings.

The main problem at the David Taylor Model Basin was to obtain precise alinement of carriage rails. Two different methods were used for this precise alinement. For the alinements made in 1939, intermediate points were set using a specially designed target light with an attached micrometer arrangement. Accuracy of alinement was determined by making multiple observations and micrometer readings for each point set. For the alinements made in 1945, a specially designed metal tripod was used which had a movable tribrach plate with a vernier and slow-motion screw attached, so as to allow movement of the theodolite perpendicular to the line of sight. A first-order two-micrometer theodolite was set up on the special tripod at the intermediate point to be precisely alined, and observations made on the fine-line target lamps at the two ends of the line. The deflections were read by the theodolite micrometers with the theodolite slightly off line on first one side, then the other, and the mean setting of the tripod alinement vernier was computed proportionally and the instrument set on line and the deflections again checked. The alidade of the theodolite with its inner spindle was then removed and a collimator with a special fitting inserted in the outer spindle bearing in order to aline a point on the alinement stud. By taking observations with the theodolite on both sides of the line, the determinations of the line position of the instrument were independent of the errors of the run of the micrometer. The errors of the graduated circle did not enter into the problem as only two divisions were used (0° 00′ and 0° 05′), and only one micrometer was read at each pointing, whichever was over the zero graduation. Since the deflections of the line at the instrument were very small, all the measuring in determining and checking the precise alinement was done by micrometer readings alone.

OTHER METHODS OF TRIANGULATION

There are several recent methods of triangulation, some of which were developed from scientific advances of World War II, by which the lengths of lines greater than that of the conventional ground methods of chapter 2 can be obtained. Operational procedures have been developed and used successfully but results have not yet definitely attained first-order accuracy. The principles of two of these methods will be described in the following sections.

SHORAN TRILATERATION

Shoran is an electronic method by which distance measurements are made by the determination of the travel time of a radio signal.

Observations are made by flying an airplane, equipped with the shoran transmitter-receiver-indicator units across the line about midway between two ground stations equipped with shoran transponders. Photographic recordings at regular intervals of the shoran indicator dials are used to compute the minimum sum of the distances from the airplane to the two ground stations. The accuracy of the method depends largely on the manner in which corrections are determined and applied to reduce the shoran distances to geodetic distances between the ground stations. Accurate altitude and complete meteorological data on temperature, pressure, and humidity are essential for the reductions due to the altitude of the plane and to the path of the shoran impulse. The design of the shoran equipment is important particularly in relation to the accuracy of measurement of the time interval. The final accuracy of the entire process depends on the accuracy of the accepted value of the velocity of light.

Lines of 500 to 600 miles in length are practical with airplane altitudes of 40,000 feet. Schemes of triangulation are then built up by chains of triangles in conventional figures with shoran-measured lengths of sides. The method is not suitable to short lines. It is particularly applicable to (1) triangulation connection of continents and off-lying islands where distances exceed the limit of ground triangulation methods, or where widely spaced area coverage is desired, and (2) for photogrammetric control (where rapid coverage is desired) using shoran control of the photographic plane from widely spaced shoran ground stations.

Considering the length of a shoran line as compared to that of an arc of conventional optical-method triangulation, the absolute error of determination of a line probably should be of the order of one in 100,000 or better to be considered of first-order accuracy.

From a mathematical standpoint shoran is most favorably adapted to wide-area coverage. Networks of figures composed of pentagons with all diagonals measured are comparable for adjustment purposes to conventional triangulation networks of single triangles. For extension of an arc of shoran figures intermediate azimuth control would be required. Azimuths may be introduced into the shoran figure by measuring (as one of the sides of the figure) the distance between two stations of an adjusted ground arc; or if topographic conditions are favorable for observing a ground line of 40 or more miles in length the Laplace azimuth of this line can be determined and the line included directly in a shoran figure.

The shoran instruments were originally designed for blind bombing. They are being redesigned as surveying instruments in several countries and their accuracy as a surveying instrument may be expected to be constantly improved. Additional details on shoran may be obtained from the articles "Use of Shoran in Geodetic Control" by Carl I. Aslakson and Donald A. Rice in Transactions, American Geophysical Union, Vol. 27, No. IV, August 1946, and "Geodetic Application of Shoran" by Donald A. Rice in The Journal, Coast and Geodetic Survey, April 1950. The method has been used successfully in extending a scheme of triangulation from Florida over the Bahamas and Cuba.

FLARE TRIANGULATION

Flare triangulation is a method of extending triangulation between points which exceed the sight limit distance of ground structures, up to a practical limiting distance of about 200 miles. The distance limitations, with respect to observing, depend on the height and intensity of the flare and clear weather visibility.

This method of triangulation consists of simultaneous radio-controlled theodolite observations, at three known stations and at three unknown stations, on parachute flares dropped from an airplane at three air stations about midway between the known and the unknown stations. The known stations are triangulation stations of an adjusted scheme, and a line of known azimuth is used as the initial for each flare pointing. The position of the flare is computed for each set of simultaneous pointings. The positions of the unknown stations are computed by three-point solutions on combinations of various instantaneous positions of the flares at the three air stations, or if an astronomic azimuth is observed at the unknown station, sufficient data will be available to compute the unknown position from two air stations instead of using a 3-point solution. Accuracy of final positions is obtained by meaning the independent positions obtained from multiple observations. Each set of simultaneous pointings requires separate computations. It is doubtful if greater accuracy than second-order can be obtained by this method. The flare triangulation method as used in a connection between the Florida mainland and the Bahama Islands is described in "The Journal, Coast and Geodetic Survey" of August 1948.

Clear weather is essential. A weather study should be made before a project is undertaken in order to determine the most suitable time of year for the work. The method is expensive, using airplanes with necessary ground crews and logistics support. Excessive stand-by time induced by the exacting weather requirements will greatly increase the cost per position of any work accomplished.

SHIP-TO-SHORE TRIANGULATION

Distances, azimuths, and geographic positions of adjacent stations near the shore along a precipitous coast may be determined by ship-to-shore triangulation methods. The adjacent stations along the shore must be intervisible, and azimuth is carried directly by angle measurements between shore stations. The distance is obtained by computation of triangles formed by simultaneous radio-controlled theodolite observations from three adjacent shore stations on a target on a ship which is anchored or slowly drifting off the middle shore station at a distance about equal to that between adjacent shore stations. The length of the line between shore stations on one side of the middle station has been previously determined; the length on the other side is computed from the known line through the two triangles formed by the lines to the ship. A separate length is determined from each set of simultaneous pointings. The length of the new line is the mean of the separate computed lengths obtained from multiple simultaneous observations. It is doubtful if greater than second-order accuracy can be obtained from this method.

ELECTRONIC METHODS OF MEASURING GROUND DISTANCES

There are being developed a number of systems for the measurement of distances which are based on the measurement of the travel time of a light or radio wave.

The Bergstrand system developed in Sweden is a continuous light wave system (using a Kerr electro-optic cell and a combination of polarizing prisms) that shows promise. This system is described under the title "Measurement of Distances by High Frequency Light Signalling" by Erik Bergstrand in L'Activité de La Commission Géodésique Baltique, Pendant Les Annees 1944–47, Helsinki, 1948.

The IRRAD system is an electro-optical system using pulsating light. This system has been described in an article entitled "Surveying with Pulsed Light Radar" by W. W. Hansen in Electronics, Vol. 21, No. 7, July 1948.

Chapter 6.—APPENDIX

BOARD OF SURVEYS AND MAPS SPECIFICATIONS FOR HORIZONTAL AND VERTICAL CONTROL

The following "Specifications for Horizontal and Vertical Control" were agreed upon by the various Federal surveying and mapping agencies through the approval of the Board of Surveys and Maps of the Federal Government on May 9, 1933. Although this Board was abolished in 1942 by Executive Order (No. 9094), the basic classification of triangulation is still applicable. However, the entire specifications are reproduced here for general information only and are not to be considered as modifications of the instructions contained in the preceding chapters of this manual.

Control surveys to determine the latitude, longitude, and elevation of marked points distributed throughout an area to be mapped are necessary as a basis for all topographic, hydrographic, and cadastral surveys that cover regions of considerable extent. The control surveys should also be so designed as to give the maximum benefit to those conducting general engineering operations on a large scale.

TRIANGULATION

First-order Triangulation.—Triangulation of the first order should be executed in belts about 120 miles apart over the region to be surveyed, except under such conditions as may require the substitution of first-order traverse. Triangle closures should rarely exceed 3 seconds, and the average closing error should be not greatly in excess of 1 second. The discrepancy between the measured length of a base line and its length as computed through the triangulation from the next preceding base, after the side and angle equations have been satisfied, should not exceed one part in 25,000. Laplace stations should be selected at intervals of from 6 to 10 figures along the scheme. The accuracy of azimuths to be observed at such stations should be limited to a probable error rarely exceeding 0.3 second.

Second-order Triangulation.—Triangulation of the second order (or traverse of corresponding accuracy) should be used to subdivide the areas between belts of first-order control so that no point in such an area will be farther than about 10 to 15 miles from some station of either first-order or second-order accuracy. Triangles of second-order triangulation should close with an average error not greater than 3 seconds and a maximum error seldom exceeding 5 seconds, provided that, where single triangles are necessary, the closures shall not exceed 4 seconds. Closures in length on lines of the first-order net, on second-order lines previously adjusted, or on base lines, should not exceed one part in 10,000, after the side and angle equations have been satisfied. Arcs of second-order triangulation will, in general, not exceed 150 miles in length. One Laplace station should be selected near the center of a second-order arc of triangulation which is between 120 and 150 miles in length. Should an arc be longer than that, a Laplace station should be located at intervals of from 8 to 15 figures. The accuracy of the azimuth observations at such stations should be limited to a probable error rarely exceeding 0.5 second.

Third-order Triangulation.—Triangulation of the third order is ordinarily used for the immediate control of topographic and hydrographic surveys. Extensions of third-order triangulation or of third-order traverse, from triangulation or traverse of a higher order, should be made so as to control the entire area to be surveyed. Triangles of third-order triangulation should close with a maximum error of 10 seconds and an average error seldom exceeding 5 seconds. Closures in length on lines of first-order or second-order triangulation, on lines of third-order triangulation previously adjusted, or on base lines, should not exceed one part in 5,000, after the side and angle equations have been satisfied.

Fourth-order Triangulation.—Triangulation (or traverse) of the fourth order is used to connect the control of higher grades with the detailed surveying and mapping operations in a region. It should start from control of a higher grade and should never be carried for more than a few figures without being again connected to a control station of a higher accuracy than fourth. It may be performed with a plane table, transit, or sextant. The sole requirement of accuracy in fourth-order triangulation is that positions of points must be located with error too small to be appreciable on the resulting map.

Note: The specifications listed above were in effect in 1933. They have been modified since then so that now the recommended practice is to establish at least one control station in every 7½-minute quadrangle.

BASES

Accuracy.—Bases for the control of the lengths of first-order triangulation should be measured with an accuracy represented by a probable error of not more than one part in 1,000,000 and an estimated actual error of not more than one part in 250,000 or 300,000. On second-order bases the probable error should not be greater than one part in 500,000 and the estimated actual error not greater than one part in 150,000. On third-order bases the probable error and estimated actual error should not be greater than one part in 200,000 and one part in 75,000 respectively. Any methods and instruments may be used which will secure the prescribed results. (For a statement describing what is meant by "probable error" and "estimated actual error" see appendix at the end of these specifications.)

Frequency.—The strength of figures in the triangulation between bases should be considered in determining the distance between bases, according to the method prescribed in U. S. Coast and Geodetic Survey Special Publications Nos. 93 and 120. The allowable limit for the summation of R_1 of the individual figures between bases for first-order triangulation should not exceed 80; for the second-order 100; and for the third-order 125, though, where a base site cannot readily be found these limits may be exceeded by 25 percent. The limit prescribed for the agreement between the measured length of a base and its length as computed from the preceding base is the controlling factor, and it may necessitate at times measuring an additional base or strengthening the triangulation between bases.

TRAVERSE

Traverse will be used, in general, only where the cost of triangulation would be excessive because of low relief and heavy timber. Traverse has the advantage of leaving marked points in locations where they are easily accessible, but it is inferior to triangulation in its checks against blunders, and it does not give control over so large an area as a belt of triangulation. Ordinarily, therefore, triangulation should be preferred to traverse on control of the two higher grades if its estimated cost does not exceed the estimated cost of traverse by more than 50 percent. For the third-order control the choice between triangulation and traverse will be determined by the local conditions, and the comparison of cost should be made on an equal basis.

Traverse of all grades of accuracy should be run in loops or connected at each end to triangulation or to traverse of the same grade as that which is being executed or of a higher grade. No traverse lines that are not thus connected or checked should be used for map control. On traverse of the two higher orders marked stations should be established at intervals of not more than 5 miles and with an average of 3 miles or less. Where a marked station occurs on a traverse, one of the adjacent stations should also be marked, in order that the distance and azimuth may be made permanently available, as well as the geographic position.

Accuracy.—First-, second-, third-, and fourth-order traverse should be executed with an accuracy comparable with that of triangulation of the corresponding grades. Ordinarily, the traverse methods will give a greater accuracy for the distance between stations than the triangulation method, but it is relatively weaker in the azimuth obtained. The error of closure in position of a first-order traverse line when run in a closed loop or when starting from adjusted triangulation and ending upon adjusted triangulation should not exceed one part in 25,000 of the length of the traverse line after a preliminary adjustment has been made of the azimuth discrepancies between the stations where astronomical azimuths have been observed. This error in position may be somewhat exceeded when the length of the traverse line is small compared to the length of the arc of triangulation between the points where the traverse starts and where it ends, as there may be an appreciable error remaining in the triangulation after adjustment. On first-order traverse an astronomical azimuth should be observed at intervals of 10 to 15 stations, on second-order traverse at intervals of 25 to 40 stations, and on third-order traverse at intervals of 50 to 100 stations. The astronomical azimuth should be determined with an accuracy represented by a probable error of 0.5 second for first-order traverse, 5.0 seconds for second-order traverse, and 30.0 seconds for third-order traverse.

MARKING OF TRIANGULATION AND TRAVERSE STATIONS

Stations of either first-, second-, or third-order accuracy on triangulation and traverse should be marked by tablets of some noncorrodible metal set firmly in posts of concrete, or in large boulders or outcropping bedrock, but where a station is on a building suitable marks of a different character may be used. The tablets may be set in place by means of cement, sulphur, or lead. The concrete posts should be not less than 8 inches in diameter and should extend 30 inches below the surface of the ground. They should have the shape of a truncated prism or cone, in order that the lower end may be larger than the upper and thus better able to resist the lifting effect of frost action. For the same reason the post should be smoothly molded for the upper 10 or 12 inches of the part beneath the surface. Particular care should be taken to insure that the materials used in making the concrete are clean and well mixed. The top 12 or 15 inches of the post should be at least equivalent in strength to a 1-2-3 mixture of cement, sand, and stone; the base may be made of a somewhat leaner mixture. Where a boulder is used it should be at least as large as the concrete post prescribed and should extend to a similar depth beneath the surface. Where the tablet is to be set in bedrock care should be exercised in selecting rock that is of suitable durability, and also to make sure that what is apparently bedrock is not a small detached mass of rock.

Special Marks.—Under certain conditions special marks may be used. Where no large boulder or bedrock is available at a station and where by its location it would be unduly expensive to construct a concrete mark, a metal pipe of suitable size and of noncorrodible material may be used. The base of this pipe should be so shaped as to resist extraction of the pipe and should preferably be set in concrete. In swamps a long metal pipe, set inside a drain tile filled with hydraulic cement, may be used. Where a station mark must be set on land subject to cultivation it is better to have the top of the post entirely below the depth which can be reached by a plow—that is, about 12 inches below the surface. Where a mark of this type is set it is necessary that measurements to the center of the roadways, section lines, etc., be made in sufficient number to enable one seeking to recover the mark in the future to determine its location within a few feet. The mark itself can then be found by digging or by prodding with an iron rod.

Subsurface Marks.—Where a concrete post is used a subsurface mark should be set if possible. This mark should preferably be made of concrete, not less than 6 inches thick and 10 inches in diameter, with the station point marked by a metal tablet, copper bolt, or other durable substance. The subsurface mark should be 4 or 5 inches below the base of the concrete post, and extreme care must be taken that the subsurface mark is directly underneath the surface mark.

Reference Marks.—At least one and preferably two reference marks should be set at each station. They should be metal tablets set in concrete posts, boulders, or bedrock. The metal tablets used for the reference marks should have a different inscription upon them than the station tablets and preferably should bear an arrow pointing toward the station. Where more than one reference mark is used at a station, they should be stamped and numbered serially, clockwise as viewed from the station. Particular care should be taken in selecting sites for these reference marks where they will not be subject to disturbance. Fence lines or section lines are suitable sites.

Azimuth Marks.—At each first- or second-order triangulation station at least one azimuth mark should be established in addition to the reference marks. The mark should be, if practicable, at least ¼ mile distant and visible from the ground at the triangulation station. It will not be required to measure the distance between the azimuth and station marks. However, the description of the azimuth mark should be sufficiently definite to enable one to recover it with ease. The directions to azimuth marks should be obtained from at least two positions of the circle. Where possible, additional azimuths of objects, such as church spires, cupolas, etc., visible from the ground at the station should be determined.

Third- and fourth-order triangulation usually consists of short lines and is extended into areas so that many of the stations are intervisible from the ground, thus making available azimuths directly between main-scheme stations and obviating the necessity for azimuth marks. Where main-scheme stations are not intervisible azimuth marks should be established. The accuracy of the directions to azimuth marks should be such that azimuths may be obtained with a probable error not in excess of 15 seconds.

Preservation of Station Marks.—The destruction of station marks that are in the way of public improvements is frequently unavoidable. By far the greater number of marks removed, however, are destroyed by malicious or thoughtless persons and this destruction represents a large economic loss to the Government. The restoration of the marks in even a small region will sometimes cost several thousand dollars, not counting the inconvenience caused to local engineers and surveyors by the lack of stations upon which to base their surveys.

There is a Federal law imposing a fine or imprisonment for the destruction of a Federal station mark, but it has never been enforced. Many States have laws directed to the same end, but even under State laws it is very difficult to secure a conviction. Such laws, however, have a notable restraining effect. It is very desirable that all the States should have laws imposing penalties for destruction of marks established by Federal surveys, and also prescribing the terms upon which Federal surveyors shall have the right of entry upon private property, together with methods for .eimbursing the owner of the land for any damage sustained by such entry.

The best way of protecting station and bench marks seems to be by educating the people to understand the value of these marks. To that end as great publicity as possible should be obtained when work is being conducted in a region, to make known to the inhabitants the purpose and value of the surveys and the function of the marks in perpetuating the surveys. This publicity can be obtained by the insertion of articles in local papers, by the distribution of pamphlets illustrative of the purpose of the work, and by conversation with influential people or with owners of the land upon which the marks are located.

Selection of Names.—The name of a triangulation or traverse station should be stamped upon the metal tablet, preferably before it is set into the concrete or stone. The name should not be duplicated within the confines of a county. Names for triangulation and traverse stations should have a geographic significance wherever possible. Care should be taken by the chief of party to ascertain the name which is most prevalent for a particular geographic feature, for frequently a mountain or stream will have different names in the same region. In particular, officials of the Forest Service should be consulted regarding the names of topographic features within national or State forests or in regions adjacent thereto.

Program for Control Surveys.—First- and second-order triangulation and traverse should be completed in a region before the third-order control is developed. It is also an advantage, though not always practicable, to have the third-order control completed and computed before topographic surveys are begun in a region, in order that a better distribution of the topographic working parties may be made.

Description of Station.—A clear, concise, and complete description of each triangulation and traverse station established should be made out and filed in convenient form in the district or central office of the organization making the survey. Where search is made for a triangulation or traverse station established in a previous year, a note to that effect must be entered on an appropriate form. If the station is found, the recovery note should state the condition in which the mark was found and should give any modifications or additions to the description which would make the station more easily found in the future. If the station is not found, the note should indicate the thoroughness of the search made and give recommendations as to whether or not the station should be marked "lost" in the records.

Report of Recovery of any Federal Survey Mark.—Member organizations of the Board of Surveys and Maps should instruct their field officers to report upon the condition of station and bench marks visited by them which were established by another member organization. If a properly equipped field party of a member organization finds in poor condition a Federal triangulation or traverse station mark of a third or higher order of accuracy and if its proper location can be determined with certainty and accuracy, either by a recovered underground mark or by measurements from two or more reference marks, the party should remark the station if practicable. If the tablet marking the original station is recovered, it should be reset. If an underground mark exists due care must be exercised to insure that the new surface mark is exactly centered over the subsurface mark.

A leveling bench mark should not be replaced or repaired. When a bench mark is found in poor condition by a field party from some organization represented on the Board of Surveys and Maps, a new bench mark may be established nearby, and its elevation determined by leveling from the original mark. A copy of the field notes for obtaining the elevation of the new mark should accompany the description of the new mark.

The report of the recovery of the station or bench mark, or the revised description, should be sent through official channels to the organization that established the original mark.

Elevation of Station.—The elevation above mean sea level datum should be determined for each triangulation and traverse station and for permanently marked intersection stations. The elevation of triangulation stations need be determined with only such accuracy as will enable the reduction of the directions and distances to mean sea level. It is very desirable that the elevation of first- and second-order traverse stations be determined by spirit leveling.

High Towers.—The use of high towers for instrument supports should be avoided wherever possible, both because of their expense and because usually the station cannot be used for future work without rebuilding the tower. Where local conditions make it necessary to use a tower, an azimuth mark should invariably be established in order that traverse lines may be joined to the station without the delay and expense of rebuilding the tower. If a permanent natural object is not available, an azimuth mark should be established not less than 500 feet distant from the station, in a location where it will be visible from the ground at the station. Whether a natural or artificial mark is used for the azimuth mark, it should be fully described, and either its measured or estimated distance from the station mark should be given.

Publication of Results.—Resulting positions, descriptions, and elevations of stations, together with their azimuths and distances to other stations, should be published as soon as possible and in compact form. All positions for horizontal-control stations must be given by latitude and longitude, referred to the North American datum of 1927, if possible.

Each publication containing leveling or triangulation data should include two small index maps, one of a section of the country showing the areas covered by the data in the publication, and another of the whole country showing the areas covered by each previously published report of a similar nature with reference to the numbers or names of the reports.

Density of Distribution.—The distance between belts of first- and second-order triangulation and traverse has already been specified. Third-order triangulation and traverse, which have for their function the control of detailed surveys in a region, should provide a density of distribution of stations sufficient to enable the topographer or the hydrographer to obtain the required accuracy without undue delay or expense. The nature of the terrain will determine the density of distribution of stations. No 15-minute quadrangle or area of similar extent should, however, have less than three horizontal-control stations of the third or higher order of accuracy. As many more should be established as will fulfill the requirements of the topographer. If traverse is being carried on instead of triangulation, marked traverse stations should be left at intervals averaging not more than 3 miles. The requirements of the topographer should be borne in mind on traverse as well as on triangulation.

In executing traverse or triangulation of the three higher orders particular effort should be made to locate additional stations by the intersection method. These additional stations may be either existing objects, such as water towers, church spires, or specially marked points.

The stations that are to be available for future recovery and use should be selected with two things in view—first, their use by the topographer or engineer working from the ground, and, second, their use on revision surveys by the aerial photo-topographic method. The second purpose requires as many stations as possible that could be identified from the air without special marking.

LEVELING

CLASSIFICATION:

First-order Leveling.—First-order leveling should be used in developing the main level net of the United States. The lines should be so placed that eventually no point in the country will be more than about 50 miles from a bench mark established by leveling of this order. All the lines should be divided into sections 1 to 2 kilometers in length, and each section should be run forward and backward, the two runnings of a section not to differ more than 4 mm. $\sqrt{K}$ or 0.017 foot $\sqrt{M}$, where K is the length of the section in kilometers and M its length in miles.

Second-order Leveling.—Second-order leveling should be used in subdividing loops of first-order leveling until no point within the area is much more than 12½ miles from a first- or second-order bench mark. Second-order leveling will include lines run by first-order methods, but in only one direction, between bench marks previously established by first-order leveling and all double lines of leveling whose sections, run in a backward and forward direction, check within the limits of 8.4 mm. $\sqrt{K}$ or 0.035 foot $\sqrt{M}$, where K is the length of the section in kilometers and M its length in miles.

Third-order Leveling.—Third-order leveling may be used in subdividing loops of first- or second-order leveling, where additional control may be required. Third-order lines should not be extended more than 30 miles from lines of the first or second order; they may be single-run lines but must always be loops or circuits closed upon lines of equal or higher order. Closing checks are not to exceed 12 mm. $\sqrt{\text{kilometers in circuit}}$, or 0.05 foot $\sqrt{\text{miles in circuit}}$.

Leveling of Lower Order.—Leveling that allows closure checks greater than the limit stated for third-order work, such as trigonometric leveling, barometric leveling, or "flying" levels, shall be considered as belonging to the lower order of work. No bench marks established by leveling that is less accurate than that of the third order, as above described, shall be marked by standard bench-mark tablets, except that in mountainous regions inaccessible to ordinary spirit-level lines standard marks may be used on mountain summits to mark elevations determined by trigonometric leveling; such marks should be stamped in a distinguishing manner. Elevations inferior to the third order in accuracy shall not be published in such a way as to be confused with standard work of the third or higher orders.

Bench Marks.—All first-, second-, and third-order level lines should be adequately marked by monuments, at average intervals of not more than 2 miles, preferably by metal tablets set into concrete posts, substantial buildings, outcropping rock, or large boulders. At places from which it is likely that future leveling lines will be extended, at least three bench marks should be established within a radius of about a half a mile, but far enough apart not to be affected by the same disturbing causes. In addition, supplementary marks on trees, bridge seats, and similar places should be left for each mile of line. In each city or town through which a line of levels passes, at least two permanent bench marks should be established, the number of bench marks for cities of large population being somewhat in proportion to the size of the city.

New leveling should be tied to bench marks of previous leveling by Government and other organizations wherever practicable, to furnish checks and provide means of correlating leveling results.

An effort should be made to have level lines run over the routes where traverse stations have been established, in order that the elevations of those stations may be determined.

All metal bench marks should, at the time they are established, have stamped upon them some mark which will positively identify them.

A complete description should be made of each old bench mark visited.

Elevations.—All elevations should be based on mean sea level datum.

Elevations of all bench marks required by existing law (20 Stat. L., 435) in areas to be topographically mapped should be determined by leveling of the first, second, or third order or, in mountainous country, by trigonometric leveling.

Elevations above mean sea level should be stamped on each bench mark, but not until the office computations and adjustments have been made.

Publication of Results.—The adjusted elevations and the descriptions of bench marks of first-, second-, and third-order leveling should be published as promptly as possible. The elevation in feet and, if desired, in meters also and the description of each bench mark should be printed on the same page.

Each publication containing leveling data should include two small index maps, one of a section of the country showing the area covered by the data in the publication and another of the whole country showing the areas covered by each previously published report of a similar nature, with references to the numbers or names of the reports.

The publication of the results of long spur lines of first-order leveling should await the extension of the spur line to previously established bench marks, in order to insure the results against any gross errors in the work.

CLASSIFICATION OF CONTROL

The foregoing data concerning control classification may be tabulated as follows:

	First Order	Second Order	Third Order	Fourth Order
Triangulation	Average triangle closure 1″, check on base 1/25,000.	Average triangle closure 3″, check on base 1/10,000.	Average triangle closure 5″, check on base 1/5,000.	Plane table or transit.
Traverse	Position check 1/25,000.	Position check 1/10,000.	Position check 1/5,000.	Stadia, tape, or wheel.
Leveling	Error of closure of section 0.017 ft. $\sqrt{\text{miles}}$ or 4 mm. $\sqrt{\text{kilometers}}$.	Error of closure of section 0.035 ft. $\sqrt{\text{miles}}$ or 8.4 mm. $\sqrt{\text{kilometers}}$.	Error of closure of circuit 0.05 ft. $\sqrt{\text{miles}}$ or 12 mm. $\sqrt{\text{kilometers}}$.	Flying wye levels, vertical angles.

FIELD COMPUTATIONS

In order to insure the requisite degree of accuracy for each grade of work and to avoid the necessity of going over lines a second time to correct excessive errors, certain field computations will be necessary, but these are invariably to be considered preliminary only. On triangulation, eccentric stations should be reduced to center, spherical excesses computed, and triangle closures tested as soon as possible after the angles are measured; and distances should be computed to see that the lengths check properly through the triangulation figures.

Sufficient computations should be made for base-line measures to show whether the forward and backward measures check within proper limits, and the length of base should be reduced to sea level from elevations determined in the field. In this way it can be learned whether the length as carried by the triangulation checks with the measured lengths within the limits required for the particular grade of work.

A progress sketch should invariably be made by the field party, whether engaged on triangulation, traverse, or leveling.

Least-square adjustments should never be made in the field.

In first-order and second-order traverse it is necessary to carry the computation far enough to check the azimuths carried by the traverse with the observed azimuths. If this is not done in the field, the office computation should closely follow the field work.

The field computations for leveling should be sufficiently complete to insure the requisite degree of accuracy for each grade of work. For each bench mark established or touched upon, a complete description should be written and the elevation computed on mean sea level datum.

RECOMMENDATIONS

Each organization to which may be delegated the task of carrying forward control of any of the classes designated should prepare detailed instructions for its own work, which, if approved by the Board of Surveys and Maps, should be published for general use. Each set of instructions should be complete in itself and should not refer to other sources for additional instructions. Examples of records and computations should be given for all operations.

Books of instruction for all classes of work should be furnished gratis to every technical school, college, and university that teaches engineering and should be supplied on request without charge to any interested engineer, in order to make the methods of work known to as many as possible and thus to aid in their adoption by States, counties, cities, corporations, and individuals. This will be of benefit to all concerned.

Arrangements should be made whereby any organization or individual who may wish to do control work of the first, second, or third order will be supplied by the Superintendent of Documents with field notebooks and record and computation forms at cost.

So far as practicable, the forms for records and computations used by Government surveying organizations in control surveys should be standardized.

It is just as essential to have the work of outside organizations and individuals standardized as it is to have control work by the Government bureaus standardized. The outside organizations and individuals have given very little attention to control surveys, and whatever standard specifications for control are adopted by the Government will undoubtedly be adopted to a large extent by others.

APPENDIX

To Specifications for Horizontal and Vertical Control Defining Probable and Actual Errors

Approved April 13, 1926.

The "probable error" of the measurement of a physical quantity is obtained by mathematical formulas applied to the differences between the two or more measured values of a quantity and their mean value. The probable error is a measure of the accidental errors only—that is, of those small errors which have no marked tendency to be predominantly either plus or minus. The probable error is simply a measure of the closeness of agreement among the several values of a quantity obtained by successive measurements. It will give no indication of the presence of systematic errors—for example, if a steel tape graduated at 30° C. is used at 0° C. and no temperature correction is applied, the measurements may agree within a very small limit and give a very small probable error, yet the result would be in error about 0.4 inch in every 100 feet or about one part in 3,000. Neither will the probable error give any indication of blunders, for a tape length dropped from each measure will not affect adversely the probable error of the measurement of a base.

The probable error of a measured base is found in the following manner. The base is measured in sections of about 1,000 meters each, and the probable error of each section is first obtained from the formula $0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$ where v is a residual—that is, it is the difference between each measured length and the mean of all the measured lengths of a section—and n is the number of measures of the section. The Greek letter Σ indicates that the sum of the squares of the residuals (v^2) is to be taken. If, as is usually the case, there are only two measures of a section, then the probable error of the section is 0.6745 times one-half the difference of the measured lengths of the section. The probable error of the entire base is expressed by—

$$\text{p.e.} = \pm\sqrt{h_1^2+h_2^2+\ldots h_n^2},\ \text{where } h_1, h_2, \ldots h_n$$

are the probable errors of the separate sections.

This and other formulas relating to the several classes of errors may be found in many textbooks on geodesy and least squares.

The "actual error" is the difference between the true value and the measured value of a physical quantity. It is the sum of all the systematic and accidental errors which have not been eliminated from the final adopted measured value. As the absolute value cannot ever be known, the actual error cannot be exactly determined, but its maximum value can always be estimated. The accuracy of the estimation depends directly and entirely upon knowledge of the maximum uncorrected effect of each source of error.

To illustrate again by the measurement of a base, suppose that the error in marking and the error in correcting for temperature of the tape are the only ones affecting the measurement. The error in marking the ends of a tape is partly systematic and partly accidental; the systematic error will be eliminated by taking the mean of an equal number of forward or backward measures if the person marking remains always on the same side of the base line, and experiments show that with proper methods the accidental error in marking a single tape end is about 0.1 millimeter, or 1 part in 500,000. For a kilometer section of 20 tape lengths, the probable error from marking errors alone would be 1 part in $\sqrt{20}\times 500{,}000$ or 1 part in 2,235,000.

In correcting for temperature there are three principal component sources of error—namely, the error in the calibration of the thermometer, error in reading the thermometer, and the undetermined difference between the true thermometric reading and the mean temperature of the tape. Suppose standardizations and tests show that the probable error of calibration is half a degree centigrade and that the probable error of reading is of the same magnitude. Also, that the average difference between the true thermometric readings and the mean temperature of the tape under the conditions of measurement would not exceed 2° C. but that this difference is always of one sign. The probable divergence in temperature between the tape and the thermometer readings would therefore be $2° \pm \sqrt{(0.5)^2+(0.5)^2}$ and would certainly not exceed 3°. If the tape to be used were made of invar, with a coefficient of expansion of 1 part in 1,000,000 per degree centigrade, the maximum error to be expected would be 3 parts in 1,000,000, or 1 part in 333,000.

After the error to be expected from each source is evaluated an estimate can be made of the "total actual error," which is one of the criteria for base measures.

STRENGTH OF FIGURE

The square of the probable error of the logarithm of a side of a figure is $\frac{4}{3}(d^2)\frac{D-C}{D}\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$, in which d is the probable error of an observed direction, D is the number of directions observed in the figure, C is the number of conditions to be satisfied in the figure (see p. 269), and δ_A, δ_B are the respective logarithmic differences of the sines, expressed in units of the sixth decimal place, corresponding to a change of one second in the distance angles A and B of a triangle. (See p. 268, "Computation of strength of figure.") The summation, indicated by Σ, is to be taken for the triangles used in computing the value of the side in question from the side supposed to be absolutely known.

In the preceding formula the two terms $\frac{D-C}{D}$ and $\Sigma[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$ depend entirely upon the figures chosen and are independent of the accuracy with which the angles are measured. The product of these two terms is therefore a measure of the strength of the figure with respect to length, insofar as the strength depends upon the selections of stations and of lines to be observed over.

In table 3 the values tabulated are $[\delta_A{}^2+\delta_A\delta_B+\delta_B{}^2]$. The two arguments of the table are the distance angles in degrees, the smaller distance angle being given at the top of the table. The distance angles are the angles in each triangle opposite the known side and the side required.

TABLE 3.—*Table for determining relative strength of figures in triangulation*

°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

Computation of strength of figure.—To compare with each other two alternative figures, whether triangles, quadrilaterals, or central-point figures, insofar as the strength with which the length is carried is concerned, proceed as follows:

(*a*) For each figure take out the distance angles, to the nearest degree if possible, for the best and second-best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

(*b*) For each triangle in each chain enter the table with the distance angles as the two arguments and take out the tabular value.

(*c*) For each chain, the best and second best, through each figure, take the sum of the tabular values.

(*d*) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where D is the number of directions observed and C is the number of conditions to be satisfied in the figure. (See p. 267.) The quantities so obtained, namely, $\frac{D-C}{D}\Sigma[\delta_A^2+\delta_A\delta_B+\delta_B^2]$, will for convenience be called R_1 and R_2 for the best and second-best chains, respectively.

(*e*) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the R_1 the greater the strength of the figure. The second-best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between alternative figures in reconnaissance they should be selected according to their best chains, unless said best chains are very nearly of equal strength and their second-best chains differ greatly.

Determination of C and D in strength-of-figure formula.—The number of conditions to be satisfied in any figure may be computed from the following formula:

$$C=(n'-s'+1)+(n-2s+3)$$

in which

n = total number of lines,
n' = number of lines observed in both directions,
s = total number of stations,
s' = number of occupied stations.

Thus, in a quadrilateral with one station unoccupied and one unobserved line at one of the occupied stations (see fig. 132).

$$C=(2-3+1)+(6-8+3)=1$$

(In using these formulas allowance must be made for lines or triangles previously fixed.)

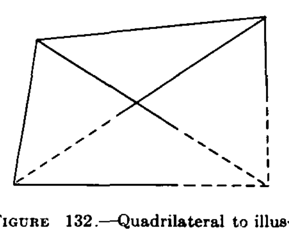

FIGURE 132.—Quadrilateral to illustrate C.

The number of conditions to be satisfied in any given figure may also be determined in another way. Starting with the fixed line or the line fixed by the preceding figure, build up the figure, station by station, computing the number of conditions at each new station. To obtain the number of conditions for the entire figure, simply add the number of conditions at all the stations. At each station the number of angle conditions is one less than the total number of full lines to the station from previously considered stations. The number of side conditions at each station is two less than the total number of lines, full and broken, to the station from previously considered stations.

As previously stated, D is the number of observed directions in a figure. It must be remembered, however, that the directions over the fixed line are not counted in obtaining D.

CONSTANTS AND FORMULAS

Dimensions of the earth according to Clarke's spheroid of reference (1866)

Equatorial radius, a, $=6{,}378{,}206.4$ meters

$\log a = 6.80469857$

Polar semi-axis, b, $=6{,}356{,}583.8$ meters

$\log b = 6.80322378$

Eccentricity, e, $=\sqrt{\dfrac{a^2-b^2}{a^2}}$,

$e^2 = 0.006768658$,

$\log e^2 = 7.83050257-10$

Base of Naperian logarithms, ϵ, $=2.71828183$

$\log \epsilon = 0.43429448$

Modulus of common logarithms, M, $=0.43429448$

$\log \mathrm{M} = 9.63778431-10$

$\pi = 3.14159265$

$\log \pi = 0.49714987$

$\log \sin 1'' = 4.68557487-10$

$\log \tan 1'' = 4.68557487-10$

1 kilometer = 0.621370 statute mile = 0.539957 nautical mile.

1 meter = 0.000621370 statute mile = 0.000539957 nautical mile.

1 statute mile = 1,609.35 meters = 1.60935 kilometers.

1 nautical mile = 1,852 meters = 1.852 kilometers.

1 nautical mile = 1.150777 statute miles.

1 statute mile = 0.868978 nautical mile.

1 meter = 39.37 inches (law of July 28, 1866).

1 meter = 3.28083333 feet.

log 3.28083333 = 0.51598417.

1 foot = 0.30480061 meter.

log 0.30480061 = 9.48401583 − 10.

Probable error of an observation, $r = 0.6745\sqrt{\dfrac{\Sigma v^2}{n-1}}$

Probable error of result, $r_0 = \dfrac{r}{\sqrt{n}} = 0.6745\sqrt{\dfrac{\Sigma v^2}{n(n-1)}}$

Probable error of an observation of unit weight, $\mu_1 = 0.6745\sqrt{\dfrac{\Sigma pv^2}{n-1}}$

Probable error of an observation of weight p_1, $r_1 = \dfrac{\mu_1}{\sqrt{p_1}} = 0.6745\sqrt{\dfrac{\Sigma pv^2}{p_1(n-1)}}$

Probable error of an observed direction, $d=0.6745\sqrt{\frac{\Sigma v^2}{c}}$ where Σv^2=sum of squares of corrections to directions, and c is the number of conditions.

Mean error of an angle, $\alpha=\sqrt{\frac{\Sigma\Delta^2}{3n}}$,

where $\Sigma\Delta^2$ is the sum of the squares of the closing errors of the triangles, and n is the number of triangles.

FORMULAS AND TABLE FOR INTERVISIBILITY

The difference between the apparent and true difference in elevation of two points is affected by two factors—the curvature of the earth's surface and the refraction of light by the earth's atmosphere. These factors are of opposite sign and of an approximately fixed relation to each other, so that the combined effect can be applied as a single factor. The effect of refraction is about one-seventh as much as the curvature. The formulas for the separate effect of each can be found in various works on geodetic surveying, but the formulas below give the approximate resultant:

$$h \text{ (in feet)} = K^2 \text{ (in miles) times } 0.574,$$

or

$$K \text{ (in miles)} = \sqrt{h \text{ (in feet)}} \text{ times } 1.32.$$

Below is a table, condensed from the one given in Appendix 9, Report for 1882, which gives the distance K (in statute miles) at which a line from the height h (in feet) will touch the horizon, taking into account terrestrial refraction with a mean assumed coefficient of refraction of 0.070.

TABLE 4.—*Correction for earth's curvature and refraction*

Distance	Correction	Distance	Correction	Distance	Correction	Distance	Correction
Miles	*Feet*	*Miles*	*Feet*	*Miles*	*Feet*	*Miles*	*Feet*
1	0.6	16	146.9	31	551.4	46	1,214.2
2	2.3	17	165.8	32	587.6	47	1,267.7
3	5.2	18	185.9	33	624.9	48	1,322.1
4	9.2	19	207.2	34	663.3	49	1,377.7
5	14.4	20	229.5	35	703.0	50	1,434.6
6	20.6	21	253.1	36	743.7	51	1,492.5
7	28.1	22	277.7	37	785.6	52	1,551.6
8	36.7	23	303.6	38	828.6	53	1,611.9
9	46.4	24	330.5	39	872.8	54	1,673.3
10	57.4	25	358.6	40	918.1	55	1,735.8
11	69.4	26	388.0	41	964.7	56	1,799.6
12	82.7	27	418.3	42	1,012.2	57	1,864.4
13	97.0	28	449.9	43	1,061.0	58	1,930.4
14	112.5	29	482.6	44	1,111.0	59	1,997.5
15	129.1	30	516.4	45	1,162.0	60	2,065.8

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To determine how much the line of sight between two stations will clear or fail to clear an intervening hill, either the table above may be used or the following formula employed:

$$h = h_1 + (h_2 - h_1)\frac{d_1}{d_1 + d_2} - 0.574 d_1 d_2,$$

where

h = height of line at obstruction, in feet,

h_1 = height of lower station, in feet,

h_2 = height of higher station, in feet,

d_1 = distance from lower station to intervening obstruction, in miles,

and

d_2 = distance from intervening obstruction to higher station, in miles.

This formula is also based on a mean assumed coefficient of refraction of 0.070.

SPHERICAL EXCESS

The spherical excess is computed by the formula:

$$\epsilon = \frac{a_1 b_1 \sin C_1 (1 - e^2 \sin^2 \phi)^2}{2a^2(1-e^2) \sin 1''} = a_1 b_1 \sin C_1 \times m.$$

In this formula ϵ is the spherical excess; a_1, b_1, and C_1 are two sides and the included angle, respectively, of the corresponding triangle; e^2 is the square of the eccentricity, and a the major semiaxis of the spheroid of reference; and ϕ is the mean latitude of the three vertices of the triangle. That part of the above expression which depends only on the latitude and the dimensions of the spheroid may be designated by a single letter, m, as shown. In the following table the logarithms of m are given with the latitude as an argument.

The above formula gives the spherical excess too small by one one-hundredth of a second for an equilateral triangle with 200-kilometer sides, or for a nonequilateral triangle of the same area. For an equilateral triangle of 100-kilometer sides, or an equivalent nonequilateral triangle, the excess as given by this formula is too small by less than one one-thousandth of a second.

In cases where a more accurate value of the spherical excess is required the formulas given on page 51 of Special Publication No. 4, The Transcontinental Triangulation, may be used. These formulas give a slightly unequal distribution of the spherical excess among the three angles of the triangle.

TABLE 5.—*Log m*

[Computed for the Clarke spheroid of 1866 as expressed in meters]

Latitude	log *m*	Latitude	log *m*	Latitude	log *m*	Latitude	log *m*
° ′		° ′		° ′		° ′	
0 00	1.40695 −10	20 00	1.40626 −10	40 00	1.40452 −10	60 00	1.40253 −10
0 30	695 −10	20 30	623 −10	40 30	446 −10	60 30	249 −10
1 00	695 −10	21 00	619 −10	41 00	441 −10	61 00	244 −10
1 30	694 −10	21 30	616 −10	41 30	436 −10	61 30	240 −10
2 00	694 −10	22 00	612 −10	42 00	431 −10	62 00	235 −10
2 30	694 −10	22 30	608 −10	42 30	426 −10	62 30	231 −10
3 00	693 −10	23 00	605 −10	43 00	421 −10	63 00	227 −10
3 30	693 −10	23 30	601 −10	43 30	416 −10	63 30	223 −10
4 00	692 −10	24 00	597 −10	44 00	411 −10	64 00	219 −10
4 30	691 −10	24 30	594 −10	44 30	406 −10	64 30	215 −10
5 00	690 −10	25 00	590 −10	45 00	400 −10	65 00	210 −10
5 30	689 −10	25 30	586 −10	45 30	395 −10	65 30	207 −10
6 00	688 −10	26 00	582 −10	46 00	390 −10	66 00	203 −10
6 30	687 −10	26 30	578 −10	46 30	385 −10	66 30	199 −10
7 00	686 −10	27 00	573 −10	47 00	380 −10	67 00	195 −10
7 30	685 −10	27 30	569 −10	47 30	375 −10	67 30	192 −10
8 00	683 −10	28 00	565 −10	48 00	369 −10	68 00	188 −10
8 30	682 −10	28 30	560 −10	48 30	364 −10	68 30	185 −10
9 00	680 −10	29 00	556 −10	49 00	359 −10	69 00	181 −10
9 30	679 −10	29 30	552 −10	49 30	354 −10	69 30	178 −10
10 00	677 −10	30 00	548 −10	50 00	349 −10	70 00	174 −10
10 30	675 −10	30 30	544 −10	50 30	344 −10	70 30	171 −10
11 00	673 −10	31 00	539 −10	51 00	339 −10	71 00	168 −10
11 30	671 −10	31 30	534 −10	51 30	334 −10	71 30	164 −10
12 00	669 −10	32 00	530 −10	52 00	329 −10	72 00	1.40161 −10
12 30	667 −10	32 30	525 −10	52 30	324 −10		
13 00	665 −10	33 00	520 −10	53 00	319 −10		
13 30	663 −10	33 30	516 −10	53 30	314 −10		
14 00	660 −10	34 00	511 −10	54 00	309 −10		
14 30	658 −10	34 30	506 −10	54 30	304 −10		
15 00	655 −10	35 00	501 −10	55 00	299 −10		
15 30	653 −10	35 30	496 −10	55 30	295 −10		
16 00	650 −10	36 00	491 −10	56 00	290 −10		
16 30	647 −10	36 30	486 −10	56 30	285 −10		
17 00	644 −10	37 00	482 −10	57 00	280 −10		
17 30	642 −10	37 30	477 −10	57 30	276 −10		
18 00	639 −10	38 00	472 −10	58 00	271 −10		
18 30	636 −10	38 30	467 −10	58 30	266 −10		
19 00	632 −10	39 00	462 −10	59 00	262 −10		
19 30	1.40629 −10	39 30	1.40457 −10	59 30	1.40257 −10		

TABLE 6.—*Natural values of m*

[Computed for the Clarke spheroid of 1866 as expressed in meters]

Latitude	$m \times 10^{10}$	Latitude	$m \times 10^{10}$	Latitude	$m \times 10^{10}$
°		°		°	
0	25.524	30	25.438	60	25.265
1	.524	31	.432	61	.260
2	.524	32	.427	62	.255
3	.523	33	.422	63	.250
4	.522	34	.416	64	.246
5	.521	35	.410	65	.241
6	25.520	36	25.405	66	25.236
7	.519	37	.399	67	.232
8	.517	38	.393	68	.228
9	.515	39	.387	69	.224
10	.513	40	.381	70	.220
11	25.511	41	25.375	71	25.216
12	.509	42	.369	72	.212
13	.506	43	.363	73	.209
14	.504	44	.357	74	.206
15	.501	45	.351	75	.203
16	25.498	46	25.345	76	25.200
17	.494	47	.339	77	.197
18	.491	48	.333	78	.194
19	.487	49	.328	79	.192
20	.484	50	.322	80	.190
21	25.480	51	25.316	81	25.188
22	.475	52	.310	82	.186
23	.471	53	.304	83	.185
24	.467	54	.298	84	.183
25	.462	55	.293	85	.182
26	25.458	56	25.287	86	25.181
27	.453	57	.281	87	.181
28	.448	58	.276	88	.180
29	.443	59	.271	89	.180
30	.438	60	.265	90	.180

Approximate spherical excess.—When needed by observers for closing triangles on station, any of the following approximate expressions may be used for determining preliminary values of the spherical excess ϵ:

Approx. ϵ in seconds $= 0.0066 \times \text{base} \times \text{altitude of triangle (in statute miles)}$.

Approx. ϵ in seconds $= \dfrac{\text{base} \times \text{altitude of triangle (in statute miles)}}{150}$.

Approx. ϵ in seconds $= \dfrac{\text{area of triangle in square statute miles}}{100} + \dfrac{\frac{1}{3}\text{ area}}{100}$.

Approx. ϵ in seconds = one second for every 75 square miles of area.

REDUCTION TO CENTER

The required reduction to center is, in seconds, $c = \dfrac{d \sin a}{s \sin 1''}$, in which d is the distance from the eccentric station to the true station, and s is the length in meters of the line between the true stations involved, and, therefore, log s is taken directly from the computation of triangle sides. a is the direction of the distant station involved, reckoned in a clockwise direction as usual but referred to the direction from the eccentric to the true station, or center, taken as zero. This definition of a is true for the case in which the object pointed upon is eccentric, as well as for the case in which the instrument is eccentric.

FORMULA FOR DIFFERENCE OF ELEVATION

The formula for the difference of elevation between stations 1 and 2 is

$$h_2-h_1=s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)\ [A\ B\ C].$$

In this formula h_1 is the elevation above mean sea level of station 1, which should be the station whose elevation is the more precisely known; h_2 is the elevation of station 2; s is the horizontal distance between the stations, reduced to sea level, log s being taken from the best available computation of triangle sides; ζ_2 is the mean corrected zenith distance of station 1, as observed from station 2; similarly, ζ_1 is the zenith distance of station 2 from station 1. The values of ζ_2 and ζ_1 are to be taken from computing Form 29. A, B, and C are correction factors whose values are nearly unity and whose logarithms may be found in tables 7 and 8 on page 276. A is the correction factor for the elevation of station 1; its formula is

$$A=1+\frac{h_1}{\rho},$$

in which ρ is the radius of curvature (see table 16 on pp. 304 to 308) of the arc between stations 1 and 2. B is the correction factor to the approximate difference of elevation, $s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)$. Its expression is

$$B=1+\frac{s}{2\rho}\tan \tfrac{1}{2}(\zeta_2-\zeta_1).$$

C is the correction factor for the distance between stations, its expression being

$$C=1+\frac{s^2}{12\rho^2}.$$

The formula for the computation of elevations from nonreciprocal observations is

$$h_2-h_1=s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right] A\ B\ C$$

in which h_2 and h_1 are elevations of the two stations, s the horizontal distance between the stations, ζ_1 the mean corrected zenith distance at the station occupied, m the coefficient of refraction, ρ the radius of curvature of the earth's surface in the mean latitude of the stations and in the azimuth of the line observed (see table 16 on pp. 304 to 308), and A, B, and C the correction factors.

The relative weights to be assigned to the various values of h_2-h_1 in the least squares adjustment are inversely proportional to s^2, and for convenience are computed by the formula $\log p=9-2 \log s$, as shown on the computation directly below h_2. By this formula a line 31.6 kilometers long is given unit weight.

After the value of h_2 and its weight, p, have been determined, the coefficient of refraction is computed by the formula,

$$0.5-m=\frac{(\zeta_1+\zeta_2-180°)\rho \sin 1''}{2s}$$

in which m, the coefficient of refraction, is the ratio of the mean angle at the two stations between the tangent to the line of sight and the chord joining the two stations, to the angle between the lines of gravity at the two stations.

FACTORS USED IN THE COMPUTATION OF ELEVATIONS FROM RECIPROCAL AND NONRECIPROCAL OBSERVATIONS

The unit of length throughout these tables is the meter.

TABLE 7.—*Log A*

Elevation of occupied station h_1	Log A units of fifth place	Elevation of occupied station h_1	Log A units of fifth place
Meters		*Meters*	
0	0.0	3009	20.5
73	.5	3156	21.5
220	1.5	3303	22.5
367	2.5	3449	23.5
514	3.5	3596	24.5
661	4.5	3743	25.5
807	5.5	3890	26.5
954	6.5	4036	27.5
1101	7.5	4183	28.5
1248	8.5	4330	29.5
1394	9.5	4477	30.5
1541	10.5	4624	31.5
1688	11.5	4770	32.5
1835	12.5	4917	33.5
1982	13.5	5064	34.5
2128	14.5	5211	35.5
2275	15.5	5357	36.5
2422	16.5	5504	37.5
2569	17.5	5651	38.5
2715	18.5	5798	39.5
2862	19.5	5945	40.5

TABLE 8.—*Log B and log C*

Log approximate difference of elevation $=\log s \tan\left(\frac{\zeta_2-\zeta_1}{2}\right)$*	Log B units of 5th place	Log s	Log C
	0.0		0.0
2.167	.5	4.875	.5
2.644	1.5	5.113	1.5
2.866	2.5	5.224	2.5
3.011	3.5	5.297	3.5
3.121	4.5	5.352	4.5
3.208	5.5	5.395	5.5
3.281	6.5	5.432	6.5
3.343	7.5	5.463	7.5
3.397	8.5		
3.445	9.5		
3.489	10.5		
3.528	11.5		
3.565	12.5		
3.598	13.5		
3.629	14.5		
3.658	15.5		
3.685	16.5		
3.711	17.5		
3.735	18.5		
3.758	19.5		
3.779	20.5		
3.800	21.5		
3.820	22.5		
3.839	23.5		
3.857	24.5		
3.874	25.5		

*Or $\log s \cot\left[\zeta_1-(0.5-m)\frac{s}{\rho \sin 1''}\right]$ for nonreciprocal observations.

Log B has the same sign as the approximate difference of elevation.
Log C is always positive.

FORMULAS FOR CORRECTION FOR RUN OF MICROMETER

Theoretically, when using instruments equipped with micrometer microscopes having single pairs of wires, each micrometer reading of the routine observations could be corrected for run by various formulas which may be found in many textbooks. In a discussion published in U. S. Coast and Geodetic Survey Report for 1884, Appendix No. 8, Assistant George Davidson presented the formulas upon which those given below are based.

On any one pointing, let:

a = the number of seconds indicated by the number of whole and fractional drum revolutions made in moving the hairs from the zero position to a centering over the next lower graduation mark,

b = the number of seconds indicated by the drum reading when the hairs are centered over the next higher graduation, this reading being considered to the same minute base as for the a reading,

r = average run of the micrometer, plus when $a>b$,

and i = distance in seconds between adjacent graduation marks.

Correction to $a = -ra/(i+r)$, or to sufficient approximation, $-ra/i$.

Correction to $b = r(i-b)/(i+r)$, or to sufficient approximation, $r(i-b)/i$.

The mean, $m = \frac{1}{2}(a+b)$.

Correction to $m = \frac{1}{2}r(i-2m)/(i+r)$, or to sufficient approximation, $\frac{1}{2}r - mr/i$.

A consideration of these formulas shows the following:

(a) When reading a single pair of wires successively on the two graduation marks adjacent to the zero point of the micrometer, the total correction for run is the same whether applied separately to the two readings or to their mean.

(b) With a single pair of wires the correction for run on observations of first-order triangulation may be disregarded, provided (1) that the run of any one micrometer is less than two seconds and the algebraic mean of the runs of all micrometers is less than one second, and (2) that the initial settings are distributed approximately uniformly throughout the space between adjacent divisions. This also applies to observations with micrometers having two pairs of wires.

(c) Where two pairs of wires are used mounted at an arbitrary distance d apart (d being considered as the apparent angular separation between pairs of wires and being less than i), the correction for run can be applied to the reading of each pair of wires separately if d is added to the b appearing in the second formula above. (Note: The reading of two pairs of wires does not eliminate run.)

(d) When conditions (1) and (2) as in (b) above are fully met, and when the two pairs of wires are always kept each on its own side of the center mark of the comb when reading, to insure a symmetrical distribution of the readings of each pair of wires throughout the graduation space, the errors due to run of the micrometer, within the limits specified above, may be neglected.

(e) The correction for a is applicable to micrometer readings of the Wild theodolite.

The run may be determined on a Wild theodolite by setting the circle coincidence on an even minute after setting the micrometer at 0 units, and then reading the micrometer in the vicinity of 0 units and 60 units. For example, the run in seconds of the Wild T–3 will be equal to 120″ plus or minus twice the difference of the two micrometer readings. The value of the run used should be the mean of several sets of readings made at intervals around the circle. The Wild theodolite cannot be adjusted for run in the field. If appreciable run is present the instrument should be returned to the shop for adjustment at the first opportunity. If observations are made with an instrument which has an appreciable run the readings of the micrometer should be corrected by solving the formula on page 277 for a.

REFERENCES TO FORMULAS IN OTHER PUBLICATIONS

For formulas and tables for logarithmic computation of geodetic positions, see Special Publication No. 8.

For formulas and tables for machine computation of geodetic positions, see Special Publication No. 241.

For formulas and tables for machine computation of elevations from zenith distances, see lithographed booklet (G-56) and revised edition of Special Publication No. 138.

LISTS OF INSTRUMENTS AND EQUIPMENT

(A) Typical list of instruments for use of a standard multiple-unit triangulation party equipped to place 4 observing units in the field:

2 altimeters, surveying
12 ammeters, pocket
12 binoculars, ordinary
2 binoculars, prismatic
1 chronometer, sidereal (azimuth observations)
6 collimators, vertical
15 compasses, azimuth
1 dividers, hairspring (drafting)
1 dividers, proportional (drafting)
1 finder, star, mariner's (azimuth observations)
1 glass, magnifying
6 heliotropes
40 lamps, signal (5-inch and 7-inch)
10 lamps, signal, small (flashlight—1½-inch)
1 lettering set (drafting)
1 level, hand
2 levels, builder's, with horizontal circle
1 machine, adding (accounts and computing)
1 machine, calculating, electric (computing)
1 pen, drop bow (drafting)
1 pen, ruling (drafting)
6 pens, fountain
12 pins, adjusting (spare)
12 plumb bobs
1 protractor, 3-arm metal (office)
6 protractors, no-arm
1 radio receiver, battery-operated short wave (azimuth observations)
4 reticles, theodolite (spare)
1 rule, slide (office)
1 scale, metric, 1-meter, diagonal
1 scale, metric, ¼-meter, diagonal
6 scales, ordinary
1 set, fixtures, beam compass, with beam
1 set, instruments, drawing
3 straightedges, assorted lengths (drafting)
8 tapes, 30-meter, steel
4 tapes, 30-meter, steel, standardized
5 tapes, 300-foot, steel
1 theodolite, second-order, with tripod
1 theodolite, 7-inch repeater
4 theodolites, first-order
1 transit, engineer's, with tripod
4 triangles, celluloid, assorted (drafting)
8 tribrach plates, aluminum (4 pairs)
1 typewriter, standard, 14-inch carriage
6 typewriters, portable

(B) Typical list of general property for use of a standard multiple-unit steel tower triangulation party equipped to place 4 observing units in the field:

8 axes
12 bags, bolt (with belt and 3 pouches)
4 bags, hoisting
4 bags, observing (canvas bags for record books, small instruments, etc.)
8 bars, digging
12 bits, drill, assorted
12 bits, wood, assorted
8 blocks, steel, single, hauling, ¾- by 4-inch
8 blocks, steel, traveling, ¾- by 2-inch
3 boxes, mortar
3 braces, carpenter's
4 buckets
6 cans, water, 10-gallon
8 chains, towing
3 chisels, carpenter's, assorted
3 chisels, cold
3 clamps, tower
1 climbers, tree
24 cots, folding, canvas

4 crowbars
7 cutters, weed
4 die sets, letters
4 die sets, numbers
3 diggers, post hole
3 drift pins
8 drills, rock, star
3 files, assorted
6 fire extinguishers, camp and spare
20 flashlights, hand
1 hacksaw
7 hammers, ball-peen
10 hammers, claw
3 hammers, sledge
16 hatchets
1 hectograph (duplicator)
3 hooks, anchor
3 hooks, brush
24 hooks, harness, double end
3 knives, corn
4 lanterns, red warning
3 levels, carpenter's
12 levels, pocket, lightkeeper's
7 lines, hauling
3 mattocks
4 mauls, stake
1 nail puller
1 oilcan, squirt
28 packboards
1 palm, sewing
12 picks
15 pliers
3 pliers, side cutting
3 punches, assorted
1 rasp, wood
1 rope assortment (pigtails and bindings)
3 rules, carpenter's, folding
1 saw, electric power, skill
10 saws, carpenter's
3 saws, crosscut (two-handled)
16 screw drivers
9 shovels, long handle, round point (holes)
13 shovels, short handle, round point (holes)
6 shovels, short handle, square point (concrete)
22 signal notices, metal
4 slings, instrument
12 spades, short handle
3 spoons, digger's
1 square, carpenter's
8 tarpaulins
4 tents, observing, ground, with frame
4 tents, observing, tower
15 tents, 9- by 9-foot, center pole, bunk
3 tents, 14-by 14-foot, ridge pole, work and storage
8 tower-bases (only), 103-foot
2 tower-bases (only), 116-foot
8 tower-extensions, 10-foot
22 towers, Bilby steel, 90-foot complete
2 trailers, office (complete with desks, chairs, files, desk lamps, stove, fire extinguisher, and flares)
3 trowels
22 trucks (with tools and equipment):
   2 1½-ton open
   10 ½- to ¾-ton panel
   8 1- to 1½-ton panel
   2 1½-ton semi-trailers
4 umbrellas, sunshade
3 winch drums (truck wheel)
7 wrenches, crescent
34 wrenches, S, end

(C) Additional instruments and general property for base line and traverse measurements:

Instruments
2 awls, marking
2 balances, dial spring, tape stretching
2 cutters, glass (rail movement check)
1 dividers, bowspring
1 dividers, hairspring
1 level, wye, with tripod
1 plumb bob
1 pulley, frictionless (balance testing)
1 rod, level (meters and feet)
2 scales, boxwood 1/10-meter (to millimeters)
2 scales, spring balance, small, for 30-meter tapes (5-kilogram)
1 stretcher set, tape, pavement-type
1 stretcher set, tape, rail-type
1 stretcher set, tape, stake-type
1 tape, 50-meter, staking
1 tape, 300-foot, checking
2 tapes, 30-meter, standardized
2 tapes, 50-meter, invar, standardized, for second-order tie traverses (4 needed for first-order base lines)
1 theodolite, 4- or 7-inch, with tripod, for lining-in
4 thermometers, base-line-tape type
1 weight, testing, 15-kilogram

General property
2 axes
2 hammers
1 hand axe
1 hand saw
1 maul, 16-pound, iron
1 punch, center marking
strips, copper (as needed)
1 sunshade

(D) Typical list of general property for use of a multiple-unit triangulation party, operating in mountainous areas, equipped to place 4 observing units in the field:

16 axes
4 bags, observing (canvas bags for record books, small instruments, etc.)
2 bars, digging
4 bits, gimlet, 3⁄16-inch
4 bits, wood, 3⁄16-inch
2 boxes, mortar
2 braces, carpenter's
12-20 cans, water, 10-gallon
4 chairs, folding
4 chisels, wood, assorted
1 climbers, tree, set
24 cots, folding, canvas
2 crowbars
4 die sets, letters
4 die sets, numbers
2 diggers, post hole
12 drills, rock, star
4 files, assorted
6 fire extinguishers (camp)
24 flashlights
10 hammers, claw
2 hammers, sledge, 2- to 4-pound
2 hammers, sledge, 8-pound
1 hectograph (duplicator)
3 hooks, brush
16 lanterns, gasoline
2 levels, carpenter's
12 levels, pocket
2 mattocks
2 mauls
1 nail puller
24 packboards
4 pails, 10-gallon
1 palm, sewing
4 picks
8 pliers, ordinary
2 pliers, wire cutting
2 rules, carpenter's, 6-foot folding
1 saw, electric power, skill
8 saws, carpenter's
2 saws, crosscut (two-handled)
6 screw drivers
4 shovels, long handle, round point
14 shovels, short handle, round point
4 shovels, short handle, square point
4 spades, short handle
2 spoons, digger's
1 square, carpenter's
1 square, tri-
16 stoves, gas, camp (heating)
24 tarpaulins, bed
8 tarpaulins, truck
4 tents, observing, ground, with frame
20 tents, 9- by 9-foot, center pole
2 tents, 14- by 14-foot, ridge pole
2 trailers, office (complete with chairs, desks, desk lamps, stoves, flares, etc.)
4 trowels
16 trucks (with tools and equipment):
  8 ½- to ¾-ton panel
  4 1- to 1½-ton panel
  2 1½-ton open
  2 1½-ton semi-trailers
2 umbrellas, sunshade

LISTS OF BOOKS AND FORMS

LIST OF BOOKS AND PUBLICATIONS NEEDED BY TRIANGULATION PARTY

Logarithmic Tables (common logs—7 places) by Vega.
Logarithms of Sines and Tangents for Every Second (7 places) by Shortrede.
Natural Sines and Cosines to Eight Decimal Places, Special Publication No. 231.
Regulations of the Coast and Geodetic Survey, Serial No. 685.
Field Book of the Stars by Olcott.
Standard Dictionary.
Tables for a Polyconic Projection of Maps and Lengths of Terrestrial Arcs of Meridian and Parallels, Special Publication No. 5.
Formulas and Tables for the Computation of Geodetic Positions, Special Publication No. 8.
Instructions to Lightkeepers on First-Order Triangulation, Special Publication No. 65.
Manual of First-Order Traverse, Special Publication No. 137.
Manual of Triangulation Computation and Adjustment, Special Publication No. 138.
Bilby Steel Tower for Triangulation, Special Publication No. 158.
Manual of Reconnaissance for Triangulation, Special Publication No. 225.
Signal Building, Special Publication No. 234.

Manual of Geodetic Astronomy, Special Publication No. 237.
Natural Tables for the Computation of Geodetic Positions, Special Publication No. 241.
Definitions of Terms Used in Geodetic and Other Surveys, Special Publication No. 242.
Manual of Geodetic Triangulation, Special Publication No. 247.
Elevations from Zenith Distances (Machine Computation) with 6-place Natural Tangent Tables 0°—45° (minute argument), lithographed booklet (G–56).

LIST OF PRINCIPAL GEODETIC FORMS NEEDED IN THE FIELD

A complete list of forms is given in Form 11, "Catalog of forms and stationery used by the U. S. Coast and Geodetic Survey."

Record Books

Form No.	Title of Form
250	Observation of horizontal angles (repeating instrument).
251a	Observations of horizontal directions, 2-micrometer theodolite.
252	Observations of double zenith distances.
590	Traverse measurements (base line).
634	Wye leveling (base line).

Field Computation Forms

Form No.	Title of Form
24A	List of directions.
25	Computation of triangles.
26	Position computation, first-order triangulation.
26a	Position computation, first-order triangulation (For calculating machine computation).
27	Position computation, third-order triangulation.
27a	Position computation, third-order triangulation (For calculating machine computation).
28B	Geographic positions.
29	Abstract of zenith distances.
29A	Computation of elevations and refractions from reciprocal observations (logarithms).
29B	Computation of elevations from nonreciprocal observations (logarithms).
29C	Computation of elevations and refractions from reciprocal observations (By calculating machine).
29D	Computation of elevations from nonreciprocal observations (By calculating machine).
380	Computation of azimuth, direction method.
381a	Computation of time, observations on a star with vertical circle.
382	Reduction to center.
448	Computation of azimuth—repetition method.
470	Abstract of horizontal directions.
589	Computation of base line.
605	Comparison of chronometer and radio signals.
635	Abstract of wye levels.
655a	Special angle computation.
662	Inverse position computation.
665	Triangle computation using two sides and included angle.

Cards and Miscellaneous Reports

Form No.	Title of Form
20	Monthly report and journal of field party.
21	Statistics, cost, and summary of field work.
525	Description of triangulation station.
525b	Description of triangulation intersection station.
526	Recovery note, triangulation station.
615	Summary of monthly reports and journals of field party and cost apportionment.
625a	Summary of motortruck record.
685	Report on condition of bench mark.
697	Monthly motortruck report.

702 Summary of monthly truck reports.
749 Daily report of building foreman.
783 Monthly report—triangulation observing unit.
785 Status of field computations.

ADDITIONAL COAST AND GEODETIC SURVEY PUBLICATIONS RELATED TO GEODESY

For a complete list of publications of this Bureau, see "List of Publications of the Coast and Geodetic Survey." This list is prepared by the Bureau Librarian in pamphlet form and is revised periodically. It shows the cost of each publication and indicates whether the various publications are available from the Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C., or from the office or field agencies of this Bureau.

Many of the older publications related to geodesy are in the form of reports covering a particular project or the results for an entire State. Some are now wholly or partially out of date and for most of them additional data are available in the form of multilithed or lithographed lists. Index maps showing the location of all geodetic control by this Bureau, including lines of leveling and traverse and arcs of triangulation, have been published for each of the 48 States. These maps are issued by this Bureau in limited quantities without charge. Anyone desiring a complete and up-to-date list of control data for a particular locality is advised to mark this locality on the index map involved and request the data from the Director, U.S. Coast and Geodetic Survey, Washington 25, D.C., rather than to procure publications which may be partially out of date. For this reason, publications involving primarily compilations of control data have been omitted from the following list, which is intended to cover publications of interest to those who may wish to investigate related subjects beyond the scope of this publication.

Triangulation and Traverse

Special Publication No. 28, Application of the theory of least squares to the adjustment of triangulation, 220 pp., 1915.

Special Publication No. 159, The Bowie method of triangulation adjustment, as applied to the first-order net in the Western part of the United States, 32 pp., 1930.

Special Publication No. 193, Manual of plane-coordinate computation, 275 pp., 1935.

Special Publication No. 194, Manual of traverse computation on the Lambert grid, 242 pp., 1935.

Special Publication No. 195, Manual of traverse computation on the transverse Mercator grid, 203 pp., 1935.

Special Publication No. 200, Formulas and tables for the computation of geodetic positions on the international ellipsoid, 120 pp., 1935.

Special Publication No. 227, Horizontal control data, 26 pp., 1941.

Special Publication No. 235, The State coordinate systems (a manual for surveyors), 62 pp., 1945.

Special Publication No. 246, Sines, cosines, and tangents, ten decimal places with ten-second interval, 0° to 6°, (natural functions), 36 pp., 1949.

Serial 347, Use of Coast and Geodetic Survey data in the survey of farms and other properties, 12 pp., 1941.

Serial 562, Plane coordinate systems, 5 pp., 1936.

Serial 583, Control surveys and their uses, 15 pp., 1935.

Serial 584, Azimuths from plane coordinates, 15 pp., 1936.

Serial 624, Computation of traverse by plane coordinates, 9 pp., 1940.

Serial 632, The preservation of triangulation station marks, 13 pp., 1941.

Leveling

Special Publication No. 129, Geodetic level and rod, 16 pp., 1935.

Special Publication No. 226, Control leveling (revised edition), 22 pp., 1948.

Special Publication No. 239, Manual of geodetic leveling, 94 pp., 1948.

Special Publication No. 240, Manual of leveling computation and adjustment, 178 pp., 1948.

General

Special Publication No. 82, The figure of the earth and isostasy from measurements in the United States, 178 pp., 1909.

Supplementary investigation in 1909 of the figure of the earth and isostasy, 80 pp., 1910.

Special Publication No. 223, Report on earth tides, 29 pp., 1940.

Special Publication No. 238, Air-line distances between cities in the United States, 246 pp., 1947.

Processed Publications, Division of Geodesy

The "G" series are mostly mimeographed or multilithed publications of limited distribution, mostly within the Bureau, to meet specific needs or to furnish preliminary information prior to formal publication.

G 16, Table of $\Delta\alpha$, transverse Mercator systems.

G 45, The ABC of triangulation adjustment.

G 58, Natural function tables for computing geographic positions on the international ellipsoid.

G 59, Philippine Islands, plane-coordinate projection tables.

STANDARD LIST OF COMMON NAMES OF OBJECTS USED AS LANDMARKS

The following classifications, which include most landmarks, are defined and accompanied by remarks to standardize their use. (Also see fig. 133.) These names shall be used so far as practicable.

BUILDING.—(See House.)

CHIMNEY.—That projecting part of a building for conveying smoke, etc., to the outer air. This term is to be used only where the building is the prominent feature and the charting of some specific part of it is desirable; for example, the chimney of a large factory.

CUPOLA.—A small turret or dome-shaped tower rising from a building, in cases where the building is the prominent object and where the cupola is small as compared to the building.

DOME.—A large cupola of rounded hemispherical form, or a roof of the same shape, whether it is actually rounded or many-sided.

FLAGPOLE.—A single staff flagpole rising from the ground and not attached to a building.

FLAGSTAFF.—A single staff flagpole rising from a building. This is not desirable as a landmark, due to its nonpermanence. Although it is desirable that the most definite part of a building (such as the flagstaff) be pointed at in making observations, this is not necessarily the most important part for charting purposes. Wherever possible give, for use on the chart, that part of the building from which the flagstaff rises, as TOWER, CUPOLA, DOME, etc.

FLAG TOWER.—Any scaffold-like tower on which flags are hoisted, such as a Coast Guard skeleton steel flagpole or a Weather Bureau signal tower. Do *not* use *Signal Tower*.

GAS TANK or OIL TANK.—Since these differ in shape and size from a water tank, the compound name will be used.

HOUSE or BUILDING.—Although it is desirable to locate a house or building by observations on a specific point, as the west gable or the flagstaff, such terms are not desirable for charting purposes, where it is the structure itself which is the landmark. Use HOUSE or BUILDING followed by a description of the point in either capitals or

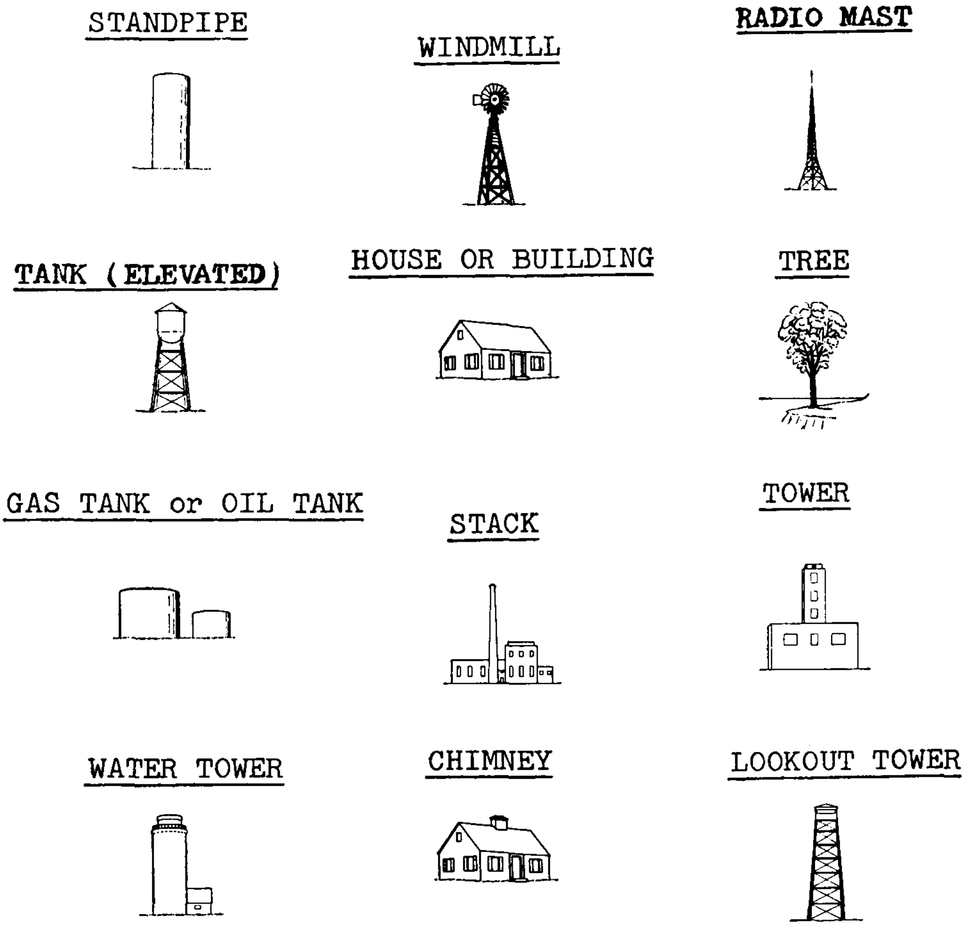

FIGURE 133.—Illustrations of objects used as landmarks.

lower-case letters, according to whether it should be used on the chart or not. Where the outline of the building should be shown on the chart, the following notation—"chart outline"—should be made on Form 567.

LOOKOUT TOWER.—Any tower surmounted by a small house in which a watch is habitually kept, such as a Coast Guard lookout tower or a fire lookout tower. Do *not* use this term in describing an observation tower, or a part of a building in which no watch is kept.

MONUMENT.—Do *not* use *Obelisk* or other terms.

OIL TANK.—(See Gas Tank.)

RADIO MAST.—A general term to include any tower, pole, or structure for elevating antennas.

SPIRE.—In general, any slender pointed structure surmounting a building. The spire is rarely less than two-thirds of the entire height and its lines are rarely broken by stages or other features. Do *not* use *Steeple*. Spire is not applicable to a short pyramid-shaped structure rising from a tower or belfry.

STACK.—Any tall smokestack or chimney, regardless of color, shape, or material, if the stack is more prominent, as a landmark, than any buildings in connection with it. Do *not* use *Chimney*.

STANDPIPE.—A tall cylindrical structure, in a waterworks system, whose height is several times greater than its diameter.

TANK.—A tank for holding water, when its base rests on the ground or other foundation, and its height is not much greater than its diameter.

TANK (ELEVATED).—A tank for holding water, where such tank is elevated high above the ground or other foundation by a tall skeleton framework.

TOWER.—(a) A part of a structure higher than the rest, but having vertical sides for the greater part of its height.

(b) An isolated structure with vertical sides (not otherwise classified), high in proportion to the size of its base, and of simple form.

(c) The top of a skyscaper, high in proportion to its horizontal size and rising above its surroundings.

(d) Any structure, whether its sides are vertical or not, with base on the ground and high in proportion to its base. Its sides may be open framework, such as a Bilby steel tower.

TREE.—Do *not* use *Lone tree* or *Conspicuous lone tree*. This is assumed, otherwise the tree would not serve as a landmark.

WATER TOWER (infrequent).—A decorative structure enclosing a tank or standpipe, or used as such, when by its appearance it would not be recognized as such.

WINDMILL.—A self-explanatory term.

EXAMPLES FOR LANDMARK LISTING ONLY

(Not in form used for triangulation name)

CHIMNEY, schoolhouse (Mt. Vernon H. S.)
CUPOLA, schoolhouse (Normal School, 98 ft. high)
FLAGPOLE (Green Hill Country Club)
LOOKOUT TOWER, fire, steel (110 ft. high)
SPIRE, church (△ Nanticoke Church Spire)
STACK (Aiea Mill)
STACK, black, metal (at Hot House)
STACK (TALLEST OF FOUR), black
STACK, white, concrete
TANK (BAY STATE CO.) (⊙ Bay)
TANK (SOUTH) (southerly of three yellow tanks)
TANK, steel (125 ft. high)
TANK, yellow (△ Hot)
TOWER (CITY HALL)

SPECIAL APPLICATIONS OF VERTICAL-ANGLE MEASUREMENTS

DETERMINATION OF HEIGHT OF STATION BY OBSERVING SEA HORIZON

At times it may be difficult to connect a triangulation scheme to a bench mark. If some of the stations are within sight of the ocean, the elevations of the stations, as determined by the vertical-angle measurements carried through the chain of triangles can be checked and made more exact by observations upon the sea horizon. Elevations determined in this manner are not as accurate as when frequent connections can be made to bench marks, for the observations are nonreciprocal and an arbitrary value must be used for the coefficient of refraction, m, which may vary for daytime observations on the sea horizon from 0.078 to 0.130. The formula for computing the height of station from the observed angle of depression is

$$h = 2\rho \frac{\sin \dfrac{\theta}{2(1-2m)} \sin \dfrac{\theta}{2}}{\cos \dfrac{(1-m)\theta}{1-2m}}$$

where h = elevation of station above sea level,

ρ = radius of curvature of the earth (in the same unit of length as h),

m = coefficient of refraction,

and θ = observed angle of depression.

For ordinary purposes the following approximate formula is all that is justified, especially as the uncertainty in the coefficient of refraction is liable to cause a considerable error in the result. The approximate formula is

$$h = \frac{1}{2} \rho \sin^2 1'' \frac{\theta^2}{1-2m},$$

in which θ is expressed in seconds of arc. If we use a mean value of log $\rho = 6.80421$ and a mean value of log $(1-2m) = 9.92428$, the approximate formula becomes simply

$$h \text{ (in meters)} = 0.000089135\ \theta^2$$

or

$$\log h = (5.95005 - 10) + 2 \log \theta.$$

This form of the formula is sufficiently exact in most cases. However, if an accurate value of m has been determined at the station by means of reciprocal observations on other triangulation stations, then it can be used to determine a better value of the constant in the approximate formula.

DETERMINATION OF DISTANCE TO BREAKER BY OBSERVING ANGLE OF DEPRESSION

In making surveys along the seacoast it quite often happens that the distance to an offshore rock or reef is desired, the position of which is indicated by breakers. If the elevation of the station from which the observations are made is known, the approximate distance may be obtained by reading the angle of depression to the breaker and computing the distance by one of the following formulas. At least two observations of the angle should be made, one with the instrument direct and one with it in the reverse position to eliminate instrumental errors.

The accuracy of the resulting distance depends principally upon the relation between the height of the station and the distance to the breaker or, in other words, upon the size of the angle of depression. If the station is high and the distance comparatively short, considerable accuracy may be obtained. Where the angle of depression is small any inaccuracy in the elevation of the station or uncertainty in the coefficient of refraction may seriously affect the accuracy of the computed distance.

The formula for the distance is as follows:

$$s=h \cot \theta+\frac{1-2m+2(1-m) \tan^2 \theta}{2\rho}h^2 \cot^3 \theta$$
$$+\frac{[1-2m+2(1-m) \tan^2 \theta]^2}{2\rho^2}h^3 \cot^5 \theta$$
$$+\frac{5[1-2m+2(1-m) \tan^2 \theta]^3}{8\rho^3}h^4 \cot^7 \theta+ \ldots .$$

in which θ is the angle of depression or the zenith distance minus 90°, ρ the radius of curvature of the earth, and m the coefficient of refraction. With some approximation the above formula will take the form,

$$s=h \cot \theta+Kh^2 \cot^3 \theta+2K^2h^3 \cot^5 \theta+5K^3h^4 \cot^7 \theta+ \ldots .$$

in which,

$$K=\frac{(1-2m) \sec^2 \theta}{2\rho},$$

or, since the value of m is so uncertain, we may use simply,

$$s=h \cot \theta+Kh^2 \cot^3 \theta.$$

TABLE 9.—*Differences of elevation and inclination corrections for varying angles of inclination*

[Length = 50 meters. Argument is inclination angle]

Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction	Angle of inclination	Difference of elevation	Inclination correction
° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.	° ′	M.	Mm.
0 00	0	0	0 30	0.44	1.9	1 00	0.87	7.6	1 30	1.31	17.1	2 00	1.74	30.5	2 30	2.18	47.6
01	.01	.0	31	.45	2.0	01	.89	7.9	31	.32	17.5	01	.76	31.0	31	.20	48.2
02	.03	.0	32	.47	2.2	02	.90	8.1	32	.34	17.9	02	.77	31.5	32	.21	48.9
03	.04	.0	33	.48	2.3	03	.92	8.4	33	.35	18.3	03	.79	32.0	33	.22	49.5
04	.06	.0	34	.49	2.4	04	.93	8.7	34	.37	18.7	04	.80	32.5	34	.24	50.2
0 05	0.07	0.1	0 35	0.51	2.6	1 05	0.95	8.9	1 35	1.38	19.1	2 05	1.82	33.0	2 35	2.25	50.8
06	.09	.1	36	.52	2.7	06	.96	9.2	36	.40	19.5	06	.83	33.6	36	.27	51.5
07	.10	.1	37	.54	2.9	07	.97	9.5	37	.41	19.9	07	.85	34.1	37	.28	52.1
08	.12	.1	38	.55	3.1	08	.99	9.8	38	.42	20.3	08	.86	34.7	38	.30	52.8
09	.13	.2	39	.57	3.2	09	1.00	10.1	39	.44	20.7	09	.88	35.2	39	.31	53.5
0 10	0.15	0.2	0 40	0.58	3.4	1 10	1.02	10.4	1 40	1.45	21.2	2 10	1.89	35.7	2 40	2.33	54.1
11	.16	.3	41	.60	3.6	11	.03	10.7	41	.47	21.6	11	.90	36.3	41	.34	54.8
12	.17	.3	42	.61	3.7	12	.05	11.0	42	.48	22.0	12	.92	36.9	42	.36	55.5
13	.19	.4	43	.63	3.9	13	.06	11.3	43	.50	22.4	13	.93	37.4	43	.37	56.2
14	.20	.4	44	.64	4.1	14	.08	11.6	44	.51	22.9	14	.95	38.0	44	.38	56.9
0 15	0.22	0.5	0 45	0.65	4.3	1 15	1.09	11.9	1 45	1.53	23.3	2 15	1.96	38.6	2 45	2.40	57.6
16	.23	.5	46	.67	4.5	16	.10	12.2	46	.54	23.8	16	.98	39.1	46	.41	58.3
17	.25	.6	47	.68	4.7	17	.12	12.5	47	.56	24.2	17	.99	39.7	47	.43	59.0
18	.26	.7	48	.70	4.9	18	.13	12.9	48	.57	24.7	18	2.01	40.3	48	.44	59.7
19	.28	.8	49	.71	5.1	19	.15	13.2	49	.59	25.1	19	.02	40.9	49	.46	60.4
0 20	0.29	0.8	0 50	0.73	5.3	1 20	1.16	13.5	1 50	1.60	25.6	2 20	2.04	41.5	2 50	2.47	61.1
21	.31	.9	51	.74	5.5	21	.18	13.9	51	.61	26.1	21	.05	42.1	51	.49	61.8
22	.32	1.0	52	.76	5.7	22	.19	14.2	52	.63	26.5	22	.06	42.7	52	.50	62.6
23	.33	1.1	53	.77	5.9	23	.21	14.6	53	.64	27.0	23	.08	43.3	53	.52	63.3
24	.35	1.2	54	.79	6.2	24	.22	14.9	54	.66	27.5	24	.09	43.9	54	.53	64.0
0 25	0.36	1.3	0 55	0.80	6.4	1 25	1.24	15.3	1 55	1.67	28.0	2 25	2.11	44.5	2 55	2.54	64.8
26	.38	1.4	56	.81	6.6	26	.25	15.6	56	.69	28.5	26	.12	45.1	56	.56	65.5
27	.39	1.5	57	.83	6.9	27	.27	16.0	57	.70	29.0	27	.14	45.7	57	.57	66.3
28	.41	1.7	58	.84	7.1	28	.28	16.4	58	.72	29.5	28	.15	46.3	58	.59	67.0
29	.42	1.8	59	.86	7.4	29	.29	16.8	59	.73	30.0	29	.17	47.0	59	.60	67.8
0 30	0.44	1.9	1 00	0.87	7.6	1 30	1.31	17.1	2 00	1.74	30.5	2 30	2.18	47.6	3 00	2.62	68.5

TABLE 10.—*Grade corrections for 50-meter tape lengths*

[For differences in elevation up to 7.50 meters. Cor. = $-0.01h^2 - 0.000001h^4$]

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meter	*Feet*	*Mm.*	*Meter*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
0	0	0	0.50	1.640	2.5	1.00	3.281	10.0	1.50	4.921	22.5
.01	.033	.0	.51	1.673	2.6	1.01	3.314	10.2	1.51	4.954	22.8
.02	.066	.0	.52	1.706	2.7	1.02	3.346	10.4	1.52	4.987	23.1
.03	.098	.0	.53	1.739	2.8	1.03	3.379	10.6	1.53	5.020	23.4
.04	.131	.0	.54	1.772	2.9	1.04	3.412	10.8	1.54	5.052	23.7
.05	.164	.0	.55	1.804	3.0	1.05	3.445	11.0	1.55	5.085	24.0
.06	.197	.0	.56	1.837	3.1	1.06	3.478	11.2	1.56	5.118	24.3
.07	.230	.0	.57	1.870	3.2	1.07	3.510	11.4	1.57	5.151	24.6
.08	.262	.1	.58	1.903	3.4	1.08	3.543	11.7	1.58	5.184	25.0
.09	.295	.1	.59	1.936	3.5	1.09	3.576	11.9	1.59	5.217	25.3
.10	.328	.1	.60	1.968	3.6	1.10	3.609	12.1	1.60	5.249	25.6
.11	.361	.1	.61	2.001	3.7	1.11	3.642	12.3	1.61	5.282	25.9
.12	.394	.1	.62	2.034	3.8	1.12	3.675	12.5	1.62	5.315	26.2
.13	.427	.2	.63	2.067	4.0	1.13	3.707	12.8	1.63	5.348	26.6
.14	.459	.2	.64	2.100	4.1	1.14	3.740	13.0	1.64	5.381	26.9
.15	.492	.2	.65	2.133	4.2	1.15	3.773	13.2	1.65	5.413	27.2
.16	.525	.3	.66	2.165	4.4	1.16	3.806	13.5	1.66	5.446	27.6
.17	.558	.3	.67	2.198	4.5	1.17	3.839	13.7	1.67	5.479	27.9
.18	.591	.3	.68	2.231	4.6	1.18	3.871	13.9	1.68	5.512	28.2
.19	.623	.4	.69	2.264	4.8	1.19	3.904	14.2	1.69	5.545	28.6
.20	.656	.4	.70	2.297	4.9	1.20	3.937	14.4	1.70	5.577	28.9
.21	.689	.4	.71	2.329	5.0	1.21	3.970	14.6	1.71	5.610	29.2
.22	.722	.5	.72	2.362	5.2	1.22	4.003	14.9	1.72	5.643	29.6
.23	.755	.5	.73	2.395	5.3	1.23	4.035	15.1	1.73	5.676	29.9
.24	.787	.6	.74	2.428	5.5	1.24	4.068	15.4	1.74	5.709	30.3
.25	.820	.6	.75	2.461	5.6	1.25	4.101	15.6	1.75	5.741	30.6
.26	.853	.7	.76	2.493	5.8	1.26	4.134	15.9	1.76	5.774	31.0
.27	.886	.7	.77	2.526	5.9	1.27	4.167	16.1	1.77	5.807	31.3
.28	.919	.8	.78	2.559	6.1	1.28	4.199	16.4	1.78	5.840	31.7
.29	.951	.8	.79	2.592	6.2	1.29	4.232	16.6	1.79	5.873	32.0
.30	.984	.9	.80	2.625	6.4	1.30	4.265	16.9	1.80	5.906	32.4
.31	1.017	1.0	.81	2.657	6.6	1.31	4.298	17.2	1.81	5.938	32.8
.32	1.050	1.0	.82	2.690	6.7	1.32	4.331	17.4	1.82	5.971	33.1
.33	1.083	1.1	.83	2.723	6.9	1.33	4.364	17.7	1.83	6.004	33.5
.34	1.115	1.2	.84	2.756	7.1	1.34	4.396	18.0	1.84	6.037	33.9
.35	1.148	1.2	.85	2.789	7.2	1.35	4.429	18.2	1.85	6.070	34.2
.36	1.181	1.3	.86	2.822	7.4	1.36	4.462	18.5	1.86	6.102	34.6
.37	1.214	1.4	.87	2.854	7.6	1.37	4.495	18.8	1.87	6.135	35.0
.38	1.247	1.4	.88	2.887	7.7	1.38	4.528	19.0	1.88	6.168	35.3
.39	1.280	1.5	.89	2.920	7.9	1.39	4.560	19.3	1.89	6.201	35.7
.40	1.312	1.6	.90	2.953	8.1	1.40	4.593	19.6	1.90	6.234	36.1
.41	1.345	1.7	.91	2.986	8.3	1.41	4.626	19.9	1.91	6.266	36.5
.42	1.378	1.8	.92	3.018	8.5	1.42	4.659	20.2	1.92	6.299	36.9
.43	1.411	1.8	.93	3.051	8.6	1.43	4.692	20.4	1.93	6.332	37.2
.44	1.444	1.9	.94	3.084	8.8	1.44	4.724	20.7	1.94	6.365	37.6
.45	1.476	2.0	.95	3.117	9.0	1.45	4.757	21.0	1.95	6.398	38.0
.46	1.509	2.1	.96	3.150	9.2	1.46	4.790	21.3	1.96	6.430	38.4
.47	1.542	2.2	.97	3.182	9.4	1.47	4.823	21.6	1.97	6.463	38.8
.48	1.575	2.3	.98	3.215	9.6	1.48	4.856	21.9	1.98	6.496	39.2
.49	1.608	2.4	.99	3.248	9.8	1.49	4.888	22.2	1.99	6.529	39.6

TABLE 10.—*Grade corrections for 50-meter tape lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
2.00	6.562	40.0	2.55	8.366	65.1	3.10	10.171	96.2	3.65	11.975	133.4
2.01	6.594	40.4	2.56	8.399	65.6	3.11	10.203	96.8	3.66	12.008	134.1
2.02	6.627	40.8	2.57	8.432	66.1	3.12	10.236	97.4	3.67	12.041	134.9
2.03	6.660	41.2	2.58	8.465	66.6	3.13	10.269	98.1	3.68	12.073	135.6
2.04	6.693	41.6	2.59	8.497	67.1	3.14	10.302	98.7	3.69	12.106	136.3
2.05	6.726	42.0	2.60	8.530	67.6	3.15	10.335	99.3	3.70	12.139	137.1
2.06	6.759	42.5	2.61	8.563	68.2	3.16	10.367	100.0	3.71	12.172	137.8
2.07	6.791	42.9	2.62	8.596	68.7	3.17	10.400	100.6	3.72	12.205	138.6
2.08	6.824	43.3	2.63	8.629	69.2	3.18	10.433	101.2	3.73	12.238	139.3
2.09	6.857	43.7	2.64	8.661	69.7	3.19	10.466	101.9	3.74	12.270	140.1
2.10	6.890	44.1	2.65	8.694	70.3	3.20	10.499	102.5	3.75	12.303	140.8
2.11	6.923	44.5	2.66	8.727	70.8	3.21	10.531	103.1	3.76	12.336	141.6
2.12	6.955	45.0	2.67	8.760	71.3	3.22	10.564	103.8	3.77	12.369	142.3
2.13	6.988	45.4	2.68	8.793	71.9	3.23	10.597	104.4	3.78	12.402	143.1
2.14	7.021	45.8	2.69	8.825	72.4	3.24	10.630	105.1	3.79	12.434	143.8
2.15	7.054	46.2	2.70	8.858	73.0	3.25	10.663	105.7	3.80	12.467	144.6
2.16	7.087	46.7	2.71	8.891	73.5	3.26	10.696	106.4	3.81	12.500	145.4
2.17	7.119	47.1	2.72	8.924	74.0	3.27	10.728	107.0	3.82	12.533	146.1
2.18	7.152	47.5	2.73	8.957	74.6	3.28	10.761	107.7	3.83	12.566	146.9
2.19	7.185	48.0	2.74	8.989	75.1	3.29	10.794	108.4	3.84	12.598	147.7
2.20	7.218	48.4	2.75	9.022	75.7	3.30	10.827	109.0	3.85	12.631	148.4
2.21	7.251	48.9	2.76	9.055	76.2	3.31	10.860	109.7	3.86	12.664	149.2
2.22	7.283	49.3	2.77	9.088	76.8	3.32	10.892	110.3	3.87	12.697	150.0
2.23	7.316	49.8	2.78	9.121	77.3	3.33	10.925	111.0	3.88	12.730	150.8
2.24	7.349	50.2	2.79	9.154	77.9	3.34	10.958	111.7	3.89	12.762	151.6
2.25	7.382	50.7	2.80	9.186	78.5	3.35	10.991	112.4	3.90	12.795	152.3
2.26	7.415	51.1	2.81	9.219	79.0	3.36	11.024	113.0	3.91	12.828	153.1
2.27	7.447	51.6	2.82	9.252	79.6	3.37	11.056	113.7	3.92	12.861	153.9
2.28	7.480	52.0	2.83	9.285	80.2	3.38	11.089	114.4	3.93	12.894	154.7
2.29	7.513	52.5	2.84	9.318	80.7	3.39	11.122	115.1	3.94	12.926	155.5
2.30	7.546	52.9	2.85	9.350	81.3	3.40	11.155	115.7	3.95	12.959	156.3
2.31	7.579	53.4	2.86	9.383	81.9	3.41	11.188	116.4	3.96	12.992	157.1
2.32	7.612	53.9	2.87	9.416	82.4	3.42	11.220	117.1	3.97	13.025	157.9
2.33	7.644	54.3	2.88	9.449	83.0	3.43	11.253	117.8	3.98	13.058	158.7
2.34	7.677	54.8	2.89	9.482	83.6	3.44	11.286	118.5	3.99	13.091	159.5
2.35	7.710	55.3	2.90	9.514	84.2	3.45	11.319	119.2	4.00	13.123	160.3
2.36	7.743	55.7	2.91	9.547	84.8	3.46	11.352	119.9	4.01	13.156	161.1
2.37	7.776	56.2	2.92	9.580	85.3	3.47	11.384	120.6	4.02	13.189	161.9
2.38	7.808	56.7	2.93	9.613	85.9	3.48	11.417	121.3	4.03	13.222	162.7
2.39	7.841	57.2	2.94	9.646	86.5	3.49	11.450	122.0	4.04	13.255	163.5
2.40	7.874	57.6	2.95	9.678	87.1	3.50	11.483	122.7	4.05	13.287	164.3
2.41	7.907	58.1	2.96	9.711	87.7	3.51	11.516	123.4	4.06	13.320	165.1
2.42	7.940	58.6	2.97	9.744	88.3	3.52	11.549	124.1	4.07	13.353	165.9
2.43	7.972	59.1	2.98	9.777	88.9	3.53	11.581	124.8	4.08	13.386	166.7
2.44	8.005	59.6	2.99	9.810	89.5	3.54	11.614	125.5	4.09	13.419	167.6
2.45	8.038	60.1	3.00	9.842	90.1	3.55	11.647	126.2	4.10	13.451	168.4
2.46	8.071	60.6	3.01	9.875	90.7	3.56	11.680	126.9	4.11	13.484	169.2
2.47	8.104	61.0	3.02	9.908	91.3	3.57	11.713	127.6	4.12	13.517	170.0
2.48	8.136	61.5	3.03	9.941	91.9	3.58	11.745	128.3	4.13	13.550	170.9
2.49	8.169	62.0	3.04	9.974	92.5	3.59	11.778	129.0	4.14	13.583	171.7
2.50	8.202	62.5	3.05	10.007	93.1	3.60	11.811	129.8	4.15	13.615	172.5
2.51	8.235	63.0	3.06	10.039	93.7	3.61	11.844	130.5	4.16	13.648	173.4
2.52	8.268	63.5	3.07	10.072	94.3	3.62	11.877	131.2	4.17	13.681	174.2
2.53	8.301	64.0	3.08	10.105	95.0	3.63	11.909	131.9	4.18	13.714	175.0
2.54	8.333	64.6	3.09	10.138	95.6	3.64	11.942	132.7	4.19	13.747	175.9

TABLE 10.—*Grade corrections for 50-meter tape lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
4.20	13.780	176.7	4.75	15.584	226.1	5.30	17.388	281.7	5.85	19.193	343.4
4.21	13.812	177.6	4.76	15.617	227.1	5.31	17.421	282.8	5.86	19.226	344.6
4.22	13.845	178.4	4.77	15.650	228.0	5.32	17.454	283.8	5.87	19.258	345.8
4.23	13.878	179.2	4.78	15.682	229.0	5.33	17.487	284.9	5.88	19.291	346.9
4.24	13.911	180.1	4.79	15.715	230.0	5.34	17.520	286.0	5.89	19.324	348.1
4.25	13.944	181.0	4.80	15.748	230.9	5.35	17.552	287.0	5.90	19.357	349.3
4.26	13.976	181.8	4.81	15.781	231.9	5.36	17.585	288.1	5.91	19.390	350.5
4.27	14.009	182.7	4.82	15.814	232.9	5.37	17.618	289.2	5.92	19.423	351.7
4.28	14.042	183.5	4.83	15.846	233.8	5.38	17.651	290.3	5.93	19.455	352.9
4.29	14.075	184.4	4.84	15.879	234.8	5.39	17.684	291.4	5.94	19.488	354.1
4.30	14.108	185.2	4.85	15.912	235.8	5.40	17.716	292.5	5.95	19.521	355.3
4.31	14.140	186.1	4.86	15.945	236.8	5.41	17.749	293.5	5.96	19.554	356.5
4.32	14.173	187.0	4.87	15.978	237.7	5.42	17.782	294.6	5.97	19.587	357.7
4.33	14.206	187.8	4.88	16.010	238.7	5.43	17.815	295.7	5.98	19.619	358.9
4.34	14.239	188.7	4.89	16.043	239.7	5.44	17.848	296.8	5.99	19.652	360.1
4.35	14.272	189.6	4.90	16.076	240.7	5.45	17.881	297.9	6.00	19.685	361.3
4.36	14.304	190.5	4.91	16.109	241.7	5.46	17.913	299.0	6.01	19.718	362.5
4.37	14.337	191.3	4.92	16.142	242.7	5.47	17.946	300.1	6.02	19.751	363.7
4.38	14.370	192.2	4.93	16.175	243.6	5.48	17.979	301.2	6.03	19.783	364.9
4.39	14.403	193.1	4.94	16.207	244.6	5.49	18.012	302.3	6.04	19.816	366.1
4.40	14.436	194.0	4.95	16.240	245.6	5.50	18.045	303.4	6.05	19.849	367.4
4.41	14.468	194.9	4.96	16.273	246.6	5.51	18.077	304.5	6.06	19.882	368.6
4.42	14.501	195.7	4.97	16.306	247.6	5.52	18.110	305.6	6.07	19.915	369.8
4.43	14.534	196.6	4.98	16.339	248.6	5.53	18.143	306.7	6.08	19.947	371.0
4.44	14.567	197.5	4.99	16.371	249.6	5.54	18.176	307.9	6.09	19.980	372.3
4.45	14.600	198.4	5.00	16.404	250.6	5.55	18.209	309.0	6.10	20.013	373.5
4.46	14.633	199.3	5.01	16.437	251.6	5.56	18.241	310.1	6.11	20.046	374.7
4.47	14.665	200.2	5.02	16.470	252.6	5.57	18.274	311.2	6.12	20.079	375.9
4.48	14.698	201.1	5.03	16.503	253.6	5.58	18.307	312.3	6.13	20.112	377.2
4.49	14.731	202.0	5.04	16.535	254.7	5.59	18.340	313.5	6.14	20.144	378.4
4.50	14.764	202.9	5.05	16.568	255.7	5.60	18.373	314.6	6.15	20.177	379.7
4.51	14.797	203.8	5.06	16.601	256.7	5.61	18.405	315.7	6.16	20.210	380.9
4.52	14.829	204.7	5.07	16.634	257.7	5.62	18.438	316.8	6.17	20.243	382.1
4.53	14.862	205.6	5.08	16.667	258.7	5.63	18.471	318.0	6.18	20.276	383.4
4.54	14.895	206.5	5.09	16.699	259.8	5.64	18.504	319.1	6.19	20.308	384.6
4.55	14.928	207.5	5.10	16.732	260.8	5.65	18.537	320.2	6.20	20.341	385.9
4.56	14.961	208.4	5.11	16.765	261.8	5.66	18.570	321.4	6.21	20.374	387.1
4.57	14.993	209.3	5.12	16.798	262.8	5.67	18.602	322.5	6.22	20.407	388.4
4.58	15.026	210.2	5.13	16.831	263.9	5.68	18.635	323.7	6.23	20.440	389.6
4.59	15.059	211.1	5.14	16.863	264.9	5.69	18.668	324.8	6.24	20.472	390.9
4.60	15.092	212.0	5.15	16.896	265.9	5.70	18.701	326.0	6.25	20.505	392.2
4.61	15.125	213.0	5.16	16.929	267.0	5.71	18.734	327.1	6.26	20.538	393.4
4.62	15.157	213.9	5.17	16.962	268.0	5.72	18.766	328.3	6.27	20.571	394.7
4.63	15.190	214.8	5.18	16.995	269.0	5.73	18.799	329.4	6.28	20.604	395.9
4.64	15.223	215.8	5.19	17.028	270.1	5.74	18.832	330.6	6.29	20.636	397.2
4.65	15.256	216.7	5.20	17.060	271.1	5.75	18.865	331.7	6.30	20.669	398.5
4.66	15.289	217.6	5.21	17.093	272.2	5.76	18.898	332.9	6.31	20.702	399.7
4.67	15.321	218.6	5.22	17.126	273.2	5.77	18.930	334.0	6.32	20.735	401.0
4.68	15.354	219.5	5.23	17.159	274.3	5.78	18.963	335.2	6.33	20.768	402.3
4.69	15.387	220.4	5.24	17.192	275.3	5.79	18.996	336.4	6.34	20.800	403.6
4.70	15.420	221.4	5.25	17.224	276.4	5.80	19.029	337.5	6.35	20.833	404.9
4.71	15.453	222.3	5.26	17.257	277.4	5.81	19.062	338.7	6.36	20.866	406.1
4.72	15.486	223.3	5.27	17.290	278.5	5.82	19.094	339.9	6.37	20.899	407.4
4.73	15.518	224.2	5.28	17.323	279.6	5.83	19.127	341.0	6.38	20.932	408.7
4.74	15.551	225.2	5.29	17.356	280.6	5.84	19.160	342.2	6.39	20.965	410.0

TABLE 10.—*Grade corrections for 50-meter tape lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
6.40	20.997	411.3	6.70	21.982	450.9	7.00	22.966	492.4	7.30	23.950	535.7
6.41	21.030	412.6	6.71	22.014	452.3	7.01	22.999	493.8	7.31	23.983	537.2
6.42	21.063	413.9	6.72	22.047	453.6	7.02	23.031	495.2	7.32	24.016	538.7
6.43	21.096	415.2	6.73	22.080	455.0	7.03	23.064	496.6	7.33	24.049	540.2
6.44	21.129	416.5	6.74	22.113	456.3	7.04	23.097	498.1	7.34	24.081	541.7
6.45	21.161	417.8	6.75	22.146	457.7	7.05	23.130	499.5	7.35	24.114	543.1
6.46	21.194	419.1	6.76	22.178	459.1	7.06	23.163	500.9	7.36	24.147	544.6
6.47	21.227	420.4	6.77	22.211	460.4	7.07	23.195	502.3	7.37	24.180	546.1
6.48	21.260	421.7	6.78	22.244	461.8	7.08	23.228	503.8	7.38	24.213	547.6
6.49	21.293	423.0	6.79	22.277	463.2	7.09	23.261	505.2	7.39	24.245	549.1
6.50	21.325	424.3	6.80	22.310	464.5	7.10	23.294	506.6	7.40	24.278	550.6
6.51	21.358	425.6	6.81	22.342	465.9	7.11	23.327	508.1	7.41	24.311	552.1
6.52	21.391	426.9	6.82	22.375	467.3	7.12	23.360	509.5	7.42	24.344	553.6
6.53	21.424	428.2	6.83	22.408	468.7	7.13	23.392	511.0	7.43	24.377	555.1
6.54	21.457	429.5	6.84	22.441	470.0	7.14	23.425	512.4	7.44	24.409	556.6
6.55	21.489	430.9	6.85	22.474	471.4	7.15	23.458	513.8	7.45	24.442	558.1
6.56	21.522	432.2	6.86	22.507	472.8	7.16	23.491	515.3	7.46	24.475	559.6
6.57	21.555	433.5	6.87	22.539	474.2	7.17	23.524	516.7	7.47	24.508	561.1
6.58	21.588	434.8	6.88	22.572	475.6	7.18	23.556	518.2	7.48	24.541	562.6
6.59	21.621	436.2	6.89	22.605	477.0	7.19	23.589	519.6	7.49	24.573	564.1
6.60	21.654	437.5	6.90	22.638	478.4	7.20	23.622	521.1	7.50	24.606	565.7
6.61	21.686	438.8	6.91	22.671	479.8	7.21	23.655	522.5			
6.62	21.719	440.2	6.92	22.703	481.2	7.22	23.688	524.0			
6.63	21.752	441.5	6.93	22.736	482.6	7.23	23.720	525.5			
6.64	21.785	442.8	6.94	22.769	484.0	7.24	23.753	526.9			
6.65	21.818	444.2	6.95	22.802	485.4	7.25	23.786	528.4			
6.66	21.850	445.5	6.96	22.835	486.8	7.26	23.819	529.9			
6.67	21.883	446.9	6.97	22.867	488.2	7.27	23.852	531.3			
6.68	21.916	448.2	6.98	22.900	489.6	7.28	23.884	532.8			
6.69	21.949	449.6	6.99	22.933	491.0	7.29	23.917	534.3			

TABLE 11.—*Grade corrections for 25-meter lengths*

[Cor. = $-0.00186h^2 - 0.000000069h^4$ (h in feet)]

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meter	*Foot*	*Mm.*	*Meter*	*Foot*	*Mm.*	*Meter*	*Foot*	*Mm.*	*Meter*	*Foot*	*Mm.*
0.0000	0.00	0.0	0.0457	0.15	0.0	0.0914	0.30	0.2	0.1372	0.45	0.4
.0030	.01	.0	.0488	.16	.0	.0945	.31	.2	.1402	.46	.4
.0061	.02	.0	.0518	.17	.1	.0975	.32	.2	.1433	.47	.4
.0091	.03	.0	.0549	.18	.1	.1006	.33	.2	.1463	.48	.4
.0122	.04	.0	.0579	.19	.1	.1036	.34	.2	.1494	.49	.4
.0152	.05	.0	0.0610	0.20	0.1	.1067	.35	.2	0.1524	0.50	0.5
.0183	.06	.0	.0640	.21	.1	.1097	.36	.2	.1554	.51	.5
.0213	.07	.0	.0671	.22	.1	.1128	.37	.3	.1585	.52	.5
.0244	.08	.0	.0701	.23	.1	.1158	.38	.3	.1615	.53	.5
.0274	.09	.0	.0732	.24	.1	.1189	.39	.3	.1646	.54	.5
0.0305	0.10	0.0	.0762	.25	.1	0.1219	0.40	0.3	.1676	.55	.6
.0335	.11	.0	.0792	.26	.1	.1250	.41	.3	.1707	.56	.6
.0366	.12	.0	.0823	.27	.1	.1280	.42	.3	.1737	.57	.6
.0396	.13	.0	.0853	.28	.1	.1311	.43	.3	.1768	.58	.6
.0427	.14	.0	.0884	.29	.2	.1341	.44	.4	.1798	.59	.6

TABLE 11.—*Grade corrections for 25-meter lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meter	*Feet*	*Mm.*	*Meter*	*Feet*	*Mm.*	*Meter*	*Feet*	*Mm.*	*Meter*	*Feet*	*Mm.*
0.1829	0.60	0.7	0.3505	1.15	2.5	0.5182	1.70	5.4	0.6858	2.25	9.4
.1859	.61	.7	.3536	.16	2.5	.5212	.71	5.5	.6888	.26	9.5
.1890	.62	.7	.3566	.17	2.5	.5243	.72	5.5	.6919	.27	9.6
.1920	.63	.7	.3597	.18	2.6	.5273	.73	5.6	.6949	.28	9.6
.1951	.64	.8	.3627	.19	2.6	.5304	.74	5.6	.6980	.29	9.7
.1981	.65	.8	0.3658	1.20	2.7	.5334	.75	5.7	0.7010	2.30	9.8
.2012	.66	.8	.3688	.21	2.7	.5364	.76	5.8	.7041	.31	9.9
.2042	.67	.8	.3719	.22	2.8	.5395	.77	5.8	.7071	.32	10.0
.2073	.68	.9	.3749	.23	2.8	.5425	.78	5.9	.7102	.33	10.1
.2103	.69	.9	.3780	.24	2.9	.5456	.79	5.9	.7132	.34	10.2
0.2134	0.70	0.9	.3810	.25	2.9	0.5486	1.80	6.0	.7163	.35	10.3
.2164	.71	.9	.3840	.26	2.9	.5517	.81	6.1	.7193	.36	10.3
.2195	.72	1.0	.3871	.27	3.0	.5547	.82	6.1	.7224	.37	10.4
.2225	.73	1.0	.3901	.28	3.1	.5578	.83	6.2	.7254	.38	10.5
.2256	.74	1.0	.3932	.29	3.1	.5608	.84	6.3	.7285	.39	10.6
.2286	.75	1.0	0.3962	1.30	3.1	.5639	.85	6.4	0.7315	2.40	10.7
.2316	.76	1.1	.3993	.31	3.2	.5669	.86	6.4	.7346	.41	10.8
.2347	.77	1.1	.4023	.32	3.2	.5700	.87	6.5	.7376	.42	10.9
.2377	.78	1.1	.4054	.33	3.3	.5730	.88	6.6	.7407	.43	11.0
.2408	.79	1.2	.4084	.34	3.3	.5761	.89	6.6	.7437	.44	11.1
0.2438	0.80	1.2	.4115	.35	3.4	0.5791	1.90	6.7	.7468	.45	11.2
.2469	.81	1.2	.4145	.36	3.4	.5822	.91	6.8	.7498	.46	11.2
.2499	.82	1.2	.4176	.37	3.5	.5852	.92	6.8	.7529	.47	11.3
.2530	.83	1.3	.4206	.38	3.5	.5883	.93	6.9	.7559	.48	11.4
.2560	.84	1.3	.4237	.39	3.6	.5913	.94	7.0	.7590	.49	11.5
.2591	.85	1.3	0.4267	1.40	3.6	.5944	.95	7.1	0.7620	2.50	11.6
.2621	.86	1.4	.4298	.41	3.7	.5974	.96	7.1	.7650	.51	11.7
.2652	.87	1.4	.4328	.42	3.7	.6005	.97	7.2	.7681	.52	11.8
.2682	.88	1.4	.4359	.43	3.8	.6035	.98	7.3	.7711	.53	11.9
.2713	.89	1.5	.4389	.44	3.8	.6066	.99	7.3	.7742	.54	12.0
0.2743	0.90	1.5	.4420	.45	3.9	0.6096	2.00	7.4	.7772	.55	12.1
.2774	.91	1.5	.4450	.46	4.0	.6126	.01	7.5	.7803	.56	12.2
.2804	.92	1.6	.4481	.47	4.0	.6157	.02	7.6	.7833	.57	12.3
.2835	.93	1.6	.4511	.48	4.1	.6187	.03	7.6	.7864	.58	12.4
.2865	.94	1.6	.4542	.49	4.1	.6218	.04	7.7	.7894	.59	12.5
.2896	.95	1.7	0.4572	1.50	4.2	.6248	.05	7.8	0.7925	2.60	12.6
.2926	.96	1.7	.4602	.51	4.3	.6279	.06	7.9	.7955	.61	12.7
.2957	.97	1.8	.4633	.52	4.3	.6309	.07	8.0	.7986	.62	12.8
.2987	.98	1.8	.4663	.53	4.4	.6340	.08	8.0	.8016	.63	12.9
.3018	.99	1.8	.4694	.54	4.4	.6370	.09	8.1	.8047	.64	13.0
0.3048	1.00	1.9	.4724	.55	4.5	0.6401	2.10	8.2	.8077	.65	13.1
.3078	.01	1.9	.4755	.56	4.6	.6431	.11	8.3	.8108	.66	13.1
.3109	.02	1.9	.4785	.57	4.6	.6462	.12	8.4	.8138	.67	13.2
.3139	.03	2.0	.4816	.58	4.7	.6492	.13	8.4	.8169	.68	13.3
.3170	.04	2.0	.4846	.59	4.7	.6523	.14	8.5	.8199	.69	13.4
.3200	.05	2.0	0.4877	1.60	4.8	.6553	.15	8.6	0.8230	2.70	13.5
.3231	.06	2.1	.4907	.61	4.8	.6584	.16	8.7	.8260	.71	13.6
.3261	.07	2.1	.4938	.62	4.9	.6614	.17	8.8	.8291	.72	13.7
.3292	.08	2.2	.4968	.63	5.0	.6645	.18	8.8	.8321	.73	13.8
.3322	.09	2.2	.4999	.64	5.0	.6675	.19	8.9	.8352	.74	13.9
0.3353	1.10	2.2	.5029	.65	5.1	0.6706	2.20	9.0	.8382	.75	14.1
.3383	.11	2.3	.5060	.66	5.2	.6736	.21	9.1	.8412	.76	14.2
.3414	.12	2.3	.5090	.67	5.2	.6767	.22	9.2	.8443	.77	14.3
.3444	.13	2.4	.5121	.68	5.3	.6797	.23	9.2	.8473	.78	14.4
.3475	.14	2.4	.5151	.69	5.3	.6828	.24	9.3	.8504	.79	14.5

TABLE 11.—*Grade corrections for 25-meter lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
0.8534	2.80	14.6	1.0211	3.35	20.9	1.1887	3.90	28.3	1.3564	4.45	36.9
.8565	.81	14.7	.0241	.36	21.0	.1918	.91	28.4	.3594	.46	37.0
.8595	.82	14.8	.0272	.37	21.1	.1948	.92	28.6	.3625	.47	37.2
.8626	.83	14.9	.0302	.38	21.2	.1979	.93	28.7	.3655	.48	37.4
.8656	.84	15.0	.0333	.39	21.4	.2009	.94	28.9	.3686	.49	37.5
.8687	.85	15.1	1.0363	3.40	21.5	.2040	.95	29.0	1.3716	4.50	37.7
.8717	.86	15.2	.0394	.41	21.6	.2070	.96	29.1	.3747	.51	37.9
.8748	.87	15.3	.0424	.42	21.8	.2101	.97	29.3	.3777	.52	38.0
.8778	.88	15.4	.0455	.43	21.9	.2131	.98	29.4	.3807	.53	38.2
.8809	.89	15.5	.0485	.44	22.0	.2162	.99	29.6	.3838	.54	38.4
0.8839	2.90	15.6	.0516	.45	22.2	1.2192	4.00	29.7	.3868	.55	38.6
.8870	.91	15.7	.0546	.46	22.3	.2223	.01	29.9	.3899	.56	38.7
.8900	.92	15.8	.0577	.47	22.4	.2253	.02	30.0	.3929	.57	38.9
.8931	.93	15.9	.0607	.48	22.5	.2283	.03	30.2	.3960	.58	39.1
.8961	.94	16.0	.0638	.49	22.7	.2314	.04	30.3	.3990	.59	39.2
.8992	.95	16.2	1.0668	3.50	22.8	.2344	.05	30.5	1.4021	4.60	39.4
.9022	.96	16.3	.0699	.51	22.9	.2375	.06	30.6	.4051	.61	39.6
.9053	.97	16.4	.0729	.52	23.1	.2405	.07	30.8	.4082	.62	39.7
.9083	.98	16.5	.0759	.53	23.2	.2436	.08	30.9	.4112	.63	39.9
.9114	.99	16.6	.0790	.54	23.3	.2466	.09	31.1	.4143	.64	40.1
0.9144	3.00	16.7	.0820	.55	23.5	1.2497	4.10	31.2	.4173	.65	40.3
.9174	.01	16.8	.0851	.56	23.6	.2527	.11	31.4	.4204	.66	40.4
.9205	.02	16.9	.0881	.57	23.7	.2558	.12	31.5	.4234	.67	40.6
.9235	.03	17.1	.0912	.58	23.8	.2588	.13	31.7	.4265	.68	40.8
.9266	.04	17.2	.0942	.59	24.0	.2619	.14	31.8	.4295	.69	40.9
.9296	.05	17.3	1.0973	3.60	24.1	.2649	.15	32.0	1.4326	4.70	41.1
.9327	.06	17.4	.1003	.61	24.2	.2680	.16	32.2	.4356	.71	41.3
.9357	.07	17.5	.1034	.62	24.4	.2710	.17	32.3	.4387	.72	41.5
.9388	.08	17.7	.1064	.63	24.5	.2741	.18	32.5	.4417	.73	41.6
.9418	.09	17.8	.1095	.64	24.6	.2771	.19	32.6	.4448	.74	41.8
0.9449	3.10	17.9	.1125	.65	24.8	1.2802	4.20	32.8	.4478	.75	42.0
.9479	.11	18.0	.1156	.66	24.9	.2832	.21	33.0	.4509	.76	42.2
.9510	.12	18.1	.1186	.67	25.0	.2863	.22	33.1	.4539	.77	42.4
.9540	.13	18.2	.1217	.68	25.1	.2893	.23	33.3	.4569	.78	42.5
.9571	.14	18.3	.1247	.69	25.3	.2924	.24	33.4	.4600	.79	42.7
.9601	.15	18.5	1.1278	3.70	25.4	.2954	.25	33.6	1.4630	4.80	42.9
.9632	.16	18.6	.1308	.71	25.5	.2985	.26	33.8	.4661	.81	43.1
.9662	.17	18.7	.1339	.72	25.7	.3015	.27	33.9	.4691	.82	43.3
.9693	.18	18.8	.1369	.73	25.8	.3045	.28	34.1	.4722	.83	43.4
.9723	.19	18.9	.1400	.74	26.0	.3076	.29	34.2	.4752	.84	43.6
0.9754	3.20	19.0	.1430	.75	26.1	1.3106	4.30	34.4	.4783	.85	43.8
.9784	.21	19.1	.1461	.76	26.2	.3137	.31	34.6	.4813	.86	44.0
.9815	.22	19.2	.1491	.77	26.4	.3167	.32	34.7	.4844	.87	44.2
.9845	.23	19.4	.1521	.78	26.5	.3198	.33	34.9	.4874	.88	44.3
.9876	.24	19.5	.1552	.79	26.7	.3228	.34	35.0	.4905	.89	44.5
.9906	.25	19.6	1.1582	3.80	26.8	.3259	.35	35.2	1.4935	4.90	44.7
.9936	.26	19.7	.1613	.81	27.0	.3289	.36	35.4	.4966	.91	44.9
.9967	.27	19.8	.1643	.82	27.1	.3320	.37	35.5	.4996	.92	45.1
.9997	.28	20.0	.1674	.83	27.3	.3350	.38	35.7	.5027	.93	45.2
1.0028	.29	20.1	.1704	.84	27.4	.3381	.39	35.8	.5057	.94	45.4
1.0058	3.30	20.2	.1735	.85	27.6	1.3411	4.40	36.0	.5088	.95	45.6
.0089	.31	20.3	.1765	.86	27.7	.3442	.41	36.2	.5118	.96	45.8
.0119	.32	20.5	.1796	.87	27.9	.3472	.42	36.3	.5149	.97	46.0
.0150	.33	20.6	.1826	.88	28.0	.3503	.43	36.5	.5179	.98	46.1
.0180	.34	20.7	.1857	.89	28.2	.3533	.44	36.7	.5210	.99	46.3

TABLE 11.—*Grade corrections for 25-meter lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
1.5240	5.00	46.5	1.6916	5.55	57.3	1.8593	6.10	69.2	2.0269	6.65	82.4
.5271	.01	46.7	.6947	.56	57.5	.8623	.11	69.4	.0300	.66	82.6
.5301	.02	46.9	.6977	.57	57.7	.8654	.12	69.7	.0330	.67	82.9
.5331	.03	47.1	.7008	.58	57.9	.8684	.13	69.9	.0361	.68	83.1
.5362	.04	47.3	.7038	.59	58.1	.8715	.14	70.1	.0391	.69	83.4
.5392	.05	47.5	1.7069	5.60	58.3	.8745	.15	70.4	2.0422	6.70	83.6
.5423	.06	47.6	.7099	.61	58.5	.8776	.16	70.6	.0452	.71	83.9
.5453	.07	47.8	.7130	.62	58.7	.8806	.17	70.8	.0483	.72	84.1
.5484	.08	48.0	.7160	.63	58.9	.8837	.18	71.0	.0513	.73	84.4
.5514	.09	48.2	.7191	.64	59.1	.8867	.19	71.3	.0544	.74	84.6
1.5545	5.10	48.4	.7221	.65	59.4	1.8898	6.20	71.5	.0574	.75	84.9
.5575	.11	48.6	.7252	.66	59.6	.8928	.21	71.7	.0605	.76	85.1
.5606	.12	48.8	.7282	.67	59.8	.8959	.22	72.0	.0635	.77	85.4
.5636	.13	49.0	.7313	.68	60.0	.8989	.23	72.2	.0665	.78	85.6
.5667	.14	49.2	.7343	.69	60.2	.9020	.24	72.5	.0696	.79	85.9
.5697	.15	49.4	1.7374	5.70	60.4	.9050	.25	72.7	2.0726	6.80	86.1
.5728	.16	49.5	.7404	.71	60.6	.9081	.26	72.9	.0757	.81	86.4
.5758	.17	49.7	.7435	.72	60.8	.9111	.27	73.2	.0787	.82	86.6
.5789	.18	49.9	.7465	.73	61.1	.9141	.28	73.4	.0818	.83	86.9
.5819	.19	50.1	.7496	.74	61.3	.9172	.29	73.7	.0848	.84	87.1
1.5850	5.20	50.3	.7526	.75	61.5	1.9202	6.30	73.9	.0879	.85	87.4
.5880	.21	50.5	.7557	.76	61.7	.9233	.31	74.1	.0909	.86	87.6
.5911	.22	50.7	.7587	.77	61.9	.9263	.32	74.4	.0940	.87	87.9
.5941	.23	50.9	.7617	.78	62.2	.9294	.33	74.6	.0970	.88	88.1
.5972	.24	51.1	.7648	.79	62.4	.9324	.34	74.8	.1001	.89	88.4
.6002	.25	51.3	1.7678	5.80	62.6	.9355	.35	75.1	2.1031	6.90	88.6
.6033	.26	51.5	.7709	.81	62.8	.9385	.36	75.3	.1062	.91	88.9
.6063	.27	51.7	.7739	.82	63.0	.9416	.37	75.5	.1092	.92	89.1
.6093	.28	51.9	.7770	.83	63.3	.9446	.38	75.7	.1123	.93	89.4
.6124	.29	52.1	.7800	.84	63.5	.9477	.39	76.0	.1153	.94	89.6
1.6154	5.30	52.3	.7831	.85	63.7	1.9507	6.40	76.2	.1184	.95	89.9
.6185	.31	52.5	.7861	.86	63.9	.9538	.41	76.4	.1214	.96	90.2
.6215	.32	52.7	.7892	.87	64.1	.9568	.42	76.7	.1245	.97	90.4
.6246	.33	52.9	.7922	.88	64.4	.9599	.43	76.9	.1275	.98	90.7
.6276	.34	53.1	.7953	.89	64.6	.9629	.44	77.2	.1306	.99	90.9
.6307	.35	53.3	1.7983	5.90	64.8	.9660	.45	77.4	2.1336	7.00	91.2
.6337	.36	53.5	.8014	.91	65.0	.9690	.46	77.6	.1367	.01	91.5
.6368	.37	53.7	.8044	.92	65.2	.9721	.47	77.9	.1397	.02	91.7
.6398	.38	53.9	.8075	.93	65.5	.9751	.48	78.1	.1427	.03	92.0
.6429	.39	54.1	.8105	.94	65.7	.9782	.49	78.4	.1458	.04	92.2
1.6459	5.40	54.3	.8136	.95	65.9	1.9812	6.50	78.6	.1488	.05	92.5
.6490	.41	54.5	.8166	.96	66.1	.9843	.51	78.9	.1519	.06	92.8
.6520	.42	54.7	.8197	.97	66.3	.9873	.52	79.1	.1549	.07	93.0
.6551	.43	54.9	.8227	.98	66.6	.9903	.53	79.4	.1580	.08	93.3
.6581	.44	55.1	.8258	.99	66.8	.9934	.54	79.6	.1610	.09	93.5
.6612	.45	55.3	1.8288	6.00	67.0	.9964	.55	79.9	2.1641	7.10	93.8
.6642	.46	55.5	.8319	.01	67.2	.9995	.56	80.1	.1671	.11	94.1
.6673	.47	55.7	.8349	.02	67.4	2.0025	.57	80.4	.1702	.12	94.3
.6703	.48	55.9	.8379	.03	67.7	.0056	.58	80.6	.1732	.13	94.6
.6734	.49	56.1	.8410	.04	67.9	.0086	.59	80.9	.1763	.14	94.9
1.6764	5.50	56.3	.8440	.05	68.1	2.0117	6.60	81.1	.1793	.15	95.2
.6795	.51	56.5	.8471	.06	68.3	.0147	.61	81.4	.1824	.16	95.4
.6825	.52	56.7	.8501	.07	68.5	.0178	.62	81.6	.1854	.17	95.7
.6855	.53	56.9	.8532	.08	68.8	.0208	.63	81.9	.1885	.18	96.0
.6886	.54	57.1	.8562	.09	69.0	.0239	.64	82.1	.1915	.19	96.2

TABLE 11.—*Grade corrections for 25-meter lengths*—Continued

Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction	Difference in elevation		Correction
Meters	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*	*Meters*	*Feet*	*Mm.*
2.1946	7.20	96.5	2.2860	7.50	104.8	2.3774	7.80	113.3	2.4689	8.10	122.2
.1976	.21	96.8	.2891	.51	105.1	.3805	.81	113.6	.4719	.11	122.5
.2007	.22	97.0	.2921	.52	105.4	.3835	.82	113.9	.4750	.12	122.8
.2037	.23	97.3	.2951	.53	105.6	.3866	.83	114.2	.4780	.13	123.1
.2068	.24	97.6	.2982	.54	105.9	.3896	.84	114.5	.4811	.14	123.4
.2098	.25	97.9	.3012	.55	106.2	.3927	.85	114.8	.4841	.15	123.7
.2129	.26	98.1	.3043	.56	106.5	.3957	.86	115.0	.4872	.16	124.0
.2159	.27	98.4	.3073	.57	106.8	.3988	.87	115.3	.4902	.17	124.3
.2189	.28	98.7	.3104	.58	107.0	.4018	.88	115.6	.4933	.18	124.6
.2220	.29	98.9	.3134	.59	107.3	.4049	.89	115.9	.4963	.19	124.9
2.2250	7.30	99.2	2.3165	7.60	107.6	2.4079	7.90	116.2	2.4994	8.20	125.2
.2281	.31	99.5	.3195	.61	107.8	.4110	.91	116.5	.5024	.21	125.5
.2311	.32	99.8	.3226	.62	108.2	.4140	.92	116.8	.5055	.22	125.8
.2342	.33	100.0	.3256	.63	108.4	.4171	.93	117.1	.5085	.23	126.1
.2372	.34	100.3	.3287	.64	108.7	.4201	.94	117.4	.5116	.24	126.4
.2403	.35	100.6	.3317	.65	109.0	.4232	.95	117.7	.5146	.25	126.8
.2433	.36	100.9	.3348	.66	109.3	.4262	.96	118.0	.5177	.26	127.1
.2464	.37	101.2	.3378	.67	109.6	.4293	.97	118.3	.5207	.27	127.4
.2494	.38	101.4	.3409	.68	109.8	.4323	.98	118.6	.5237	.28	127.7
.2525	.39	101.7	.3439	.69	110.1	.4354	.99	118.9	.5268	.29	128.0
2.2555	7.40	102.0	2.3470	7.70	110.4	2.4384	8.00	119.2	2.5298	8.30	128.3
.2586	.41	102.3	.3500	.71	110.7	.4415	.01	119.5	.5329	.31	128.6
.2616	.42	102.6	.3531	.72	111.0	.4445	.02	119.8	.5359	.32	128.9
.2647	.43	102.8	.3561	.73	111.3	.4475	.03	120.1	.5390	.33	129.2
.2677	.44	103.1	.3592	.74	111.6	.4506	.04	120.4	.5420	.34	129.5
.2708	.45	103.4	.3622	.75	111.9	.4536	.05	120.7	.5451	.35	129.9
.2738	.46	103.7	.3653	.76	112.1	.4567	.06	121.0	.5481	.36	130.2
.2769	.47	104.0	.3683	.77	112.4	.4597	.07	121.3	.5512	.37	130.5
.2799	.48	104.2	.3713	.78	112.7	.4628	.08	121.6	.5542	.38	130.8
.2830	.49	104.5	.3744	.79	113.0	.4658	.09	121.9	.5573	.39	131.1
									2.5603	8.40	131.4

TABLE 12.—*Grade corrections for lengths of 5, 10, 15, 20, 25, 30, 35, 40, and 45 meters*

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meter	*Foot*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
0.01	0.033	0	0	0	0	0	0	0	0	0
.02	.066	0	0	0	0	0	0	0	0	0
.03	.098	.1	0	0	0	0	0	0	0	0
.04	.131	.2	.1	0	0	0	0	0	0	0
.05	.164	.2	.1	.1	.1	0	0	0	0	0
.06	.197	.4	.2	.1	.1	.1	.1	0	0	0
.07	.230	.5	.2	.2	.1	.1	.1	.1	.1	.1
.08	.262	.6	.3	.2	.2	.1	.1	.1	.1	.1
.09	.295	.8	.4	.3	.2	.2	.2	.1	.1	.1
.10	.328	1.0	.5	.3	.2	.2	.2	.1	.1	.1
.11	.361	1.2	.6	.4	.3	.2	.2	.2	.2	.1
.12	.394	1.4	.7	.5	.4	.3	.2	.2	.2	.2
.13	.427	1.7	.8	.6	.4	.3	.3	.2	.2	.2
.14	.459	2.0	1.0	.7	.5	.4	.3	.3	.2	.2
.15	.492	2.3	1.1	.8	.6	.4	.3	.3	.3	.2

TABLE 12.—*Grade corrections for lengths of 5, 10, 15, 20, 25, 30, 35, 40, and 45 meters*—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meter	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
0.16	0.525	2.6	1.3	0.9	0.6	0.5	0.4	0.4	0.3	0.3
.17	.558	2.9	1.4	1.0	.7	.6	.5	.4	.4	.3
.18	.591	3.2	1.6	1.1	.8	.6	.5	.5	.4	.4
.19	.623	3.6	1.8	1.2	.9	.7	.6	.5	.5	.4
.20	.656	4.0	2.0	1.3	1.0	.8	.7	.6	.5	.4
.21	.689	4.4	2.2	1.5	1.1	.9	.7	.6	.6	.5
.22	.722	4.8	2.4	1.6	1.2	1.0	.8	.7	.6	.5
.23	.755	5.3	2.6	1.8	1.3	1.0	.9	.8	.7	.6
.24	.787	5.8	2.9	1.9	1.4	1.1	1.0	.8	.7	.6
.25	.820	6.3	3.1	2.1	1.6	1.2	1.0	.9	.8	.7
.26	.853	6.8	3.4	2.3	1.7	1.3	1.1	1.0	.8	.8
.27	.886	7.3	3.6	2.4	1.8	1.5	1.2	1.0	.9	.8
.28	.919	7.8	3.9	2.6	2.0	1.6	1.3	1.1	1.0	.9
.29	.951	8.4	4.2	2.8	2.1	1.7	1.4	1.2	1.1	.9
.30	.984	9.0	4.5	3.0	2.2	1.8	1.5	1.3	1.1	1.0
.31	1.017	9.6	4.8	3.2	2.4	1.9	1.6	1.4	1.2	1.1
.32	1.050	10.2	5.1	3.4	2.6	2.0	1.7	1.5	1.3	1.1
.33	1.083	10.9	5.4	3.6	2.7	2.2	1.8	1.6	1.4	1.2
.34	1.115	11.6	5.8	3.9	2.9	2.3	1.9	1.7	1.4	1.3
.35	1.148	12.3	6.1	4.1	3.1	2.5	2.0	1.8	1.5	1.4
.36	1.181	13.0	6.5	4.3	3.2	2.6	2.2	1.9	1.6	1.4
.37	1.214	13.7	6.8	4.6	3.4	2.7	2.3	2.0	1.7	1.5
.38	1.247	14.4	7.2	4.8	3.6	2.9	2.4	2.1	1.8	1.6
.39	1.280	15.2	7.6	5.1	3.8	3.1	2.5	2.2	1.9	1.7
.40	1.312	16.0	8.0	5.3	4.0	3.2	2.7	2.3	2.0	1.8
.41	1.345	16.8	8.4	5.6	4.2	3.4	2.8	2.4	2.1	1.9
.42	1.378	17.7	8.8	5.9	4.4	3.5	2.9	2.5	2.2	2.0
.43	1.411	18.5	9.3	6.2	4.6	3.7	3.1	2.6	2.3	2.1
.44	1.444	19.4	9.7	6.5	4.8	3.9	3.2	2.8	2.4	2.2
.45	1.476	20.3	10.1	6.8	5.1	4.1	3.4	2.9	2.5	2.3
.46	1.509	21.2	10.6	7.1	5.3	4.2	3.5	3.0	2.6	2.4
.47	1.542	22.1	11.1	7.4	5.5	4.4	3.7	3.2	2.8	2.5
.48	1.575	23.1	11.5	7.7	5.8	4.6	3.8	3.3	2.9	2.6
.49	1.608	24.1	12.0	8.0	6.0	4.8	4.0	3.4	3.0	2.7
.50	1.640	25.1	12.5	8.3	6.3	5.0	4.2	3.6	3.1	2.8
.51	1.673	26.1	13.0	8.7	6.5	5.2	4.3	3.7	3.3	2.9
.52	1.706	27.1	13.5	9.0	6.8	5.4	4.5	3.9	3.4	3.0
.53	1.739	28.2	14.1	9.4	7.0	5.6	4.7	4.0	3.5	3.1
.54	1.772	29.2	14.6	9.7	7.3	5.8	4.9	4.2	3.6	3.2
.55	1.804	30.3	15.1	10.1	7.6	6.0	5.0	4.3	3.8	3.4
.56	1.837	31.5	15.7	10.5	7.8	6.3	5.2	4.5	3.9	3.5
.57	1.870	32.6	16.2	10.8	8.1	6.5	5.4	4.6	4.1	3.6
.58	1.903	33.8	16.8	11.2	8.4	6.7	5.6	4.8	4.2	3.7
.59	1.936	34.9	17.4	11.6	8.7	7.0	5.8	5.0	4.4	3.9
.60	1.968	36.1	18.0	12.0	9.0	7.2	6.0	5.1	4.5	4.0
.61	2.001	37.3	18.6	12.4	9.3	7.4	6.2	5.3	4.7	4.1
.62	2.034	38.6	19.2	12.8	9.6	7.6	6.4	5.5	4.8	4.3
.63	2.067	39.8	19.8	13.2	9.9	8.0	6.6	5.7	5.0	4.4
.64	2.100	41.1	20.5	13.7	10.2	8.2	6.8	5.9	5.1	4.6
.65	2.133	42.4	21.1	14.1	10.6	8.4	7.0	6.0	5.3	4.7

TABLE 12.—*Grade corrections for lengths of 5, 10, 15, 20, 25, 30, 35, 40, and 45 meters*—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
0.66	2.165	43.8	21.8	14.5	10.9	8.7	7.3	6.2	5.4	4.8
.67	2.198	45.1	22.5	15.0	11.2	9.0	7.5	6.4	5.6	5.0
.68	2.231	46.5	23.2	15.4	11.6	9.2	7.7	6.6	5.8	5.2
.69	2.264	47.8	23.8	15.9	11.9	9.5	7.9	6.8	6.0	5.3
.70	2.297	49.2	24.5	16.3	12.3	9.8	8.2	7.0	6.1	5.4
.71	2.329	50.7	25.2	16.8	12.6	10.1	8.4	7.2	6.3	5.6
.72	2.362	52.1	25.9	17.3	13.0	10.4	8.6	7.4	6.5	5.8
.73	2.395	53.6	26.6	17.8	13.3	10.7	8.9	7.6	6.7	5.9
.74	2.428	55.1	27.4	18.3	13.7	11.0	9.1	7.8	6.8	6.1
.75	2.461	56.6	28.1	18.8	14.1	11.3	9.4	8.0	7.0	6.3
.76	2.493	58.1	28.9	19.3	14.4	11.6	9.6	8.3	7.2	6.4
.77	2.526	59.6	29.7	19.8	14.8	11.9	9.9	8.5	7.4	6.6
.78	2.559	61.2	30.5	20.3	15.2	12.2	10.1	8.7	7.6	6.8
.79	2.592	62.8	31.3	20.8	15.6	12.5	10.4	8.9	7.8	6.9
.80	2.625	64.4	32.1	21.4	16.0	12.8	10.7	9.1	8.0	7.1
.81	2.657	66.0	32.9	21.9	16.4	13.1	10.9	9.4	8.2	7.3
.82	2.690	67.7	33.7	22.4	16.8	13.4	11.2	9.6	8.4	7.5
.83	2.723	69.4	34.5	23.0	17.2	13.7	11.5	9.8	8.6	7.7
.84	2.756	71.1	35.3	23.5	17.6	14.1	11.8	10.1	8.8	7.8
.85	2.789	72.8	36.2	24.1	18.1	14.5	12.0	10.3	9.0	8.0
.86	2.822	74.5	37.0	24.7	18.5	14.8	12.3	10.6	9.2	8.2
.87	2.854	76.3	37.9	25.2	18.9	15.1	12.6	10.8	9.5	8.4
.88	2.887	78.0	38.8	25.8	19.4	15.5	12.9	11.1	9.7	8.6
.89	2.920	79.8	39.7	26.4	19.8	15.8	13.2	11.3	9.9	8.8
.90	2.953	81.7	40.6	27.0	20.3	16.2	13.5	11.6	10.1	9.0
.91	2.986	83.5	41.5	27.6	20.7	16.6	13.8	11.8	10.4	9.2
.92	3.018	85.4	42.4	28.2	21.2	16.9	14.1	12.1	10.6	9.4
.93	3.051	87.3	43.3	28.8	21.6	17.3	14.4	12.4	10.8	9.6
.94	3.084	89.2	44.3	29.5	22.1	17.7	14.7	12.6	11.0	9.8
.95	3.117	91.1	45.2	30.1	22.6	18.1	15.0	12.9	11.3	10.0
.96	3.150	93.0	46.2	30.8	23.1	18.5	15.4	13.2	11.5	10.2
.97	3.182	95.0	47.2	31.4	23.5	18.8	15.7	13.4	11.8	10.5
.98	3.215	97.0	48.1	32.0	24.0	19.2	16.0	13.7	12.0	10.7
.99	3.248	99.0	49.1	32.7	24.5	19.6	16.3	14.0	12.3	10.9
1.00	3.281	101.0	50.1	33.3	25.0	20.0	16.7	14.3	12.5	11.1
1.01	3.314	103.1	51.1	34.0	25.5	20.4	17.0	14.6	12.8	11.3
1.02	3.346	105.1	52.2	34.7	26.0	20.8	17.3	14.9	13.0	11.6
1.03	3.379	107.2	53.2	35.4	26.5	21.2	17.7	15.2	13.3	11.8
1.04	3.412	109.4	54.2	36.1	27.0	21.6	18.0	15.5	13.5	12.0
1.05	3.445	111.5	55.3	36.8	27.6	22.1	18.4	15.8	13.8	12.2
1.06	3.478	113.6	56.3	37.5	28.1	22.5	18.7	16.1	14.0	12.5
1.07	3.510	115.8	57.4	38.2	28.6	22.9	19.1	16.4	14.3	12.7
1.08	3.543	118.0	58.5	38.9	29.2	23.4	19.4	16.7	14.6	13.0
1.09	3.576	120.2	59.6	39.6	29.7	23.8	19.8	17.0	14.9	13.2
1.10	3.609	122.5	60.7	40.4	30.3	24.2	20.2	17.3	15.1	13.4
1.11	3.642	124.8	61.8	41.1	30.8	24.6	20.5	17.6	15.4	13.7
1.12	3.675	127.1	62.9	41.9	31.4	25.0	20.9	17.9	15.7	13.9
1.13	3.707	129.4	64.1	42.6	31.9	25.5	21.3	18.2	16.0	14.2
1.14	3.740	131.7	65.2	43.3	32.5	26.0	21.7	18.6	16.2	14.4
1.15	3.773	134.0	66.3	44.1	33.1	26.4	22.0	18.9	16.5	14.7

TABLE 12.—*Grade corrections for lengths of 5, 10, 15, 20, 25, 30, 35, 40, and 45 meters*—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
1.16	3.806	136.4	67.5	44.9	33.7	26.9	22.4	19.2	16.8	14.9
1.17	3.839	138.8	68.7	45.7	34.3	27.4	22.8	19.6	17.1	15.2
1.18	3.871	141.2	69.9	46.5	34.8	27.9	23.2	19.9	17.4	15.5
1.19	3.904	143.7	71.1	47.3	35.4	28.3	23.6	20.2	17.7	15.7
1.20	3.937	146.1	72.3	48.1	36.0	28.8	24.0	20.6	18.0	16.0
1.21	3.970	148.6	73.5	48.9	36.6	29.3	24.4	20.9	18.3	16.3
1.22	4.003	151.1	74.7	49.7	37.2	29.8	24.8	21.3	18.6	16.5
1.23	4.035	153.7	75.9	50.5	37.9	30.3	25.2	21.6	18.9	16.8
1.24	4.068	156.2	77.1	51.3	38.5	30.8	25.6	22.0	19.2	17.1
1.25	4.101	158.8	78.4	52.2	39.1	31.2	26.0	22.3	19.5	17.4
1.26	4.134	161.4	79.7	53.0	39.7	31.7	26.5	22.7	19.8	17.6
1.27	4.167	164.0	81.0	53.8	40.4	32.3	26.9	23.0	20.2	17.9
1.28	4.199	166.6	82.3	54.7	41.0	32.8	27.3	23.4	20.5	18.2
1.29	4.232	169.3	83.6	55.6	41.6	33.3	27.7	23.8	20.8	18.5
1.30	4.265	172.0	84.9	56.4	42.3	33.8	28.2	24.1	21.1	18.8
1.31	4.298	174.7	86.2	57.3	42.9	34.4	28.6	24.5	21.5	19.1
1.32	4.331	177.4	87.5	58.2	43.6	34.9	29.0	24.9	21.8	19.4
1.33	4.364	180.1	88.8	59.1	44.3	35.4	29.5	25.3	22.1	19.7
1.34	4.396	182.9	90.2	60.0	44.9	35.9	29.9	25.6	22.4	19.9
1.35	4.429	185.7	91.5	60.9	45.6	36.5	30.4	26.0	22.8	20.2
1.36	4.462	188.5	92.9	61.8	46.3	37.0	30.8	26.4	23.1	20.5
1.37	4.495	191.4	94.3	62.7	47.0	37.6	31.3	26.8	23.5	20.8
1.38	4.528	194.2	95.7	63.6	47.7	38.2	31.7	27.2	23.8	21.2
1.39	4.560	197.1	97.1	64.5	48.4	38.7	32.2	27.6	24.2	21.5
1.40	4.593	200.0	98.5	65.5	49.1	39.3	32.7	28.0	24.5	21.8
1.41	4.626	202.9	99.9	66.4	49.8	39.8	33.1	28.4	24.9	22.1
1.42	4.659	205.9	101.3	67.3	50.5	40.4	33.6	28.8	25.2	22.4
1.43	4.692	208.9	102.8	68.3	51.2	40.9	34.1	29.2	25.6	22.7
1.44	4.724	211.8	104.2	69.3	51.9	41.5	34.6	29.6	25.9	23.0
1.45	4.757	214.9	105.7	70.2	52.6	42.1	35.0	30.0	26.3	23.4
1.46	4.790	217.9	107.2	71.2	53.4	42.7	35.5	30.5	26.6	23.7
1.47	4.823	221.0	108.6	72.2	54.1	43.3	36.0	30.9	27.0	24.0
1.48	4.856	224.1	110.1	73.2	54.8	43.9	36.5	31.3	27.4	24.3
1.49	4.888	227.2	111.6	74.2	55.6	44.5	37.0	31.7	27.8	24.7
1.50	4.921	230.3	113.1	75.2	56.3	45.1	37.5	32.1	28.1	25.0
1.51	4.954	233.5	114.7	76.2	57.1	45.7	38.0	32.6	28.5	25.3
1.52	4.987	236.6	116.2	77.2	57.8	46.3	38.5	33.0	28.9	25.7
1.53	5.020	239.8	117.7	78.2	58.6	46.9	39.0	33.4	29.3	26.0
1.54	5.052	243.1	119.3	79.3	59.4	47.5	39.5	33.9	29.6	26.3
1.55	5.085	246.3	120.8	80.3	60.2	48.1	40.0	34.3	30.0	26.7
1.56	5.118	249.6	122.4	81.3	60.9	48.8	40.6	34.8	30.4	27.0
1.57	5.151	252.9	124.0	82.4	61.7	49.4	41.1	35.2	30.8	27.4
1.58	5.184	256.2	125.6	83.4	62.5	50.0	41.6	35.7	31.2	27.7
1.59	5.217	259.5	127.2	84.5	63.3	50.6	42.1	36.1	31.6	28.1
1.60	5.249	262.9	128.8	85.6	64.1	51.3	42.7	36.6	32.0	28.4
1.61	5.282	266.3	130.4	86.6	64.9	52.0	43.2	37.0	32.4	28.8
1.62	5.315	269.7	132.1	87.7	65.7	52.6	43.7	37.5	32.8	29.2
1.63	5.348	273.2	133.7	88.8	66.5	53.3	44.3	38.0	33.2	29.5
1.64	5.381	276.6	135.4	89.9	67.4	53.9	44.8	38.4	33.6	29.9
1.65	5.413	280.1	137.0	91.0	68.2	54.6	45.4	38.9	34.0	30.3

TABLE 12.—*Grade corrections for lengths of 5, 10, 15, 20, 25, 30, 35, 40, and 45 meters*—Continued

Difference in elevation		Length in meters								
		5	10	15	20	25	30	35	40	45
Meters	*Feet*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
1.66	5.446	283.6	138.7	92.1	69.0	55.2	45.9	39.4	34.4	30.6
1.67	5.479	287.1	140.4	93.2	69.8	55.9	46.5	39.8	34.9	31.0
1.68	5.512	290.7	142.1	94.4	70.7	56.5	47.0	40.3	35.3	31.4
1.69	5.545	294.3	143.8	95.5	71.5	57.2	47.6	40.8	35.7	31.7
1.70	5.577	297.9	145.5	96.6	72.4	57.9	48.2	41.3	36.1	32.1
1.71	5.610	301.5	147.3	97.8	73.2	58.6	48.7	41.8	36.6	32.5
1.72	5.643	305.2	149.0	98.9	74.1	59.3	49.3	42.3	37.0	32.9
1.73	5.676	308.8	150.8	100.1	75.0	59.9	49.9	42.8	37.4	33.2
1.74	5.709	312.5	152.5	101.2	75.8	60.6	50.5	43.3	37.8	33.6
1.75	5.741	316.2	154.3	102.4	76.7	61.3	51.0	43.8	38.3	34.0
1.76	5.774	320.0	156.1	103.6	77.6	62.0	51.6	44.3	38.7	34.4
1.77	5.807	323.8	157.9	104.8	78.5	62.7	52.2	44.8	39.2	34.8
1.78	5.840	327.6	159.7	106.0	79.4	63.5	52.8	45.3	39.6	35.2
1.79	5.873	331.4	161.5	107.2	80.3	64.2	53.4	45.8	40.1	35.6
1.80	5.906	335.2	163.3	108.4	81.2	64.9	54.0	46.3	40.5	36.0
1.81	5.938	339.1	165.1	109.6	82.1	65.6	54.6	46.8	41.0	36.4
1.82	5.971	343.0	167.0	110.8	83.0	66.3	55.2	47.3	41.4	36.8
1.83	6.004	346.9	168.8	112.0	83.9	67.1	55.8	47.8	41.9	37.2
1.84	6.037	350.9	170.7	113.3	84.8	67.8	56.4	48.4	42.3	37.6
1.85	6.070	354.8	172.6	114.5	85.7	68.5	57.0	48.9	42.8	38.0
1.86	6.102	358.8	174.5	115.8	86.7	69.2	57.7	49.4	43.2	38.4
1.87	6.135	362.9	176.4	117.0	87.6	70.0	58.3	50.0	43.7	38.9
1.88	6.168	366.9	178.3	118.3	88.6	70.8	59.0	50.5	44.2	39.3
1.89	6.201	371.0	180.2	119.6	89.5	71.5	59.6	51.0	44.7	39.7
1.90	6.234	375.1	182.2	120.8	90.5	72.3	60.2	51.6	45.1	40.1
1.91	6.266	379.2	184.1	122.1	91.4	73.1	60.8	52.1	45.6	40.5
1.92	6.299	383.3	186.1	123.4	92.4	73.9	61.4	52.7	46.1	41.0
1.93	6.332	387.5	188.0	124.7	93.3	74.6	62.1	53.2	46.6	41.4
1.94	6.365	391.7	190.0	126.0	94.3	75.4	62.7	53.8	47.0	41.8
1.95	6.398	395.9	192.0	127.3	95.3	76.2	63.4	54.3	47.5	42.2
1.96	6.430	400.2	194.0	128.6	96.3	76.9	64.0	54.9	48.0	42.7
1.97	6.463	404.4	196.0	129.9	97.3	77.7	64.7	55.4	48.5	43.1
1.98	6.496	408.7	198.0	131.2	98.3	78.5	65.3	56.0	49.0	43.6
1.99	6.529	413.1	200.0	132.6	99.3	79.3	66.0	56.6	49.5	44.0
2.00	6.562	417.4	202.0	133.9	100.3	80.1	66.7	57.2	50.0	44.4

TABLE 13.—*Factors for computing catenary correction*

Combined side and top arguments = weight of tape in grams per meter. Tabular values $=\frac{1}{24}\left(\frac{w}{t}\right)^2\times10^{10}$ when $t=15{,}000$ grams. To obtain catenary correction multiply tabular value by cube of length between supports and point off 10 decimal places. The result will be in meters]

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
20	741	748	756	763	771	778	786	794	801	809
21	817	824	832	840	848	856	864	872	880	888
22	896	904	913	921	929	938	946	954	963	971
23	980	988	997	1,005	1,014	1,023	1,031	1,040	1,049	1,058
24	1,067	1,076	1,085	1,094	1,103	1,112	1,121	1,130	1,139	1,148
25	1,157	1,167	1,176	1,185	1,195	1,204	1,214	1,223	1,233	1,242
26	1,252	1,262	1,271	1,281	1,291	1,300	1,310	1,320	1,330	1,340
27	1,350	1,360	1,370	1,380	1,390	1,400	1,411	1,421	1,431	1,442
28	1,452	1,462	1,473	1,483	1,494	1,504	1,515	1,525	1,536	1,547
29	1,557	1,568	1,579	1,590	1,601	1,612	1,623	1,634	1,645	1,656
30	1,667									

TABLE 14.—*Catenary corrections for various lengths and weights of tape*

Length of tape (meters)	Weight of tape in grams per meter									
	20.0	20.1	20.2	20.3	20.4	20.5	20.6	20.7	20.8	20.9
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	.07	.07	.08	.08	.08	.08	.08	.08	.08	.08
15	.25	.25	.26	.26	.26	.26	.27	.27	.27	.27
20	.59	.60	.60	.61	.62	.62	.63	.63	.64	.65
25	1.16	1.17	1.18	1.19	1.20	1.22	1.23	1.24	1.25	1.26
30	2.00	2.02	2.04	2.06	2.08	2.10	2.12	2.14	2.16	2.18
35	3.18	3.21	3.24	3.27	3.30	3.34	3.37	3.40	3.44	3.47
40	4.74	4.79	4.84	4.88	4.93	4.98	5.03	5.08	5.13	5.18
45	6.75	6.82	6.89	6.95	7.02	7.09	7.16	7.23	7.30	7.37
50	9.26	9.35	9.45	9.54	9.63	9.73	9.82	9.92	10.01	10.11
	21.0	21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8	21.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	.08	.08	.08	.08	.08	.09	.09	.09	.09	.09
15	.28	.28	.28	.28	.29	.29	.29	.29	.30	.30
20	.65	.66	.67	.67	.68	.68	.69	.70	.70	.71
25	1.28	1.29	1.30	1.31	1.33	1.34	1.35	1.36	1.38	1.39
30	2.21	2.23	2.25	2.27	2.29	2.31	2.33	2.35	2.38	2.40
35	3.50	3.53	3.57	3.60	3.64	3.67	3.70	3.74	3.77	3.81
40	5.23	5.28	5.33	5.38	5.43	5.48	5.53	5.58	5.63	5.68
45	7.44	7.51	7.58	7.66	7.73	7.80	7.87	7.95	8.02	8.09
50	10.21	10.31	10.40	10.50	10.60	10.70	10.80	10.90	11.00	11.10
	22.0	22.1	22.2	22.3	22.4	22.5	22.6	22.7	22.8	22.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	.09	.09	.09	.09	.09	.09	.09	.10	.10	.10
15	.30	.31	.31	.31	.31	.32	.32	.32	.32	.33
20	.72	.72	.73	.74	.74	.75	.76	.76	.77	.78
25	1.40	1.41	1.43	1.44	1.45	1.46	1.48	1.49	1.50	1.52
30	2.42	2.44	2.46	2.49	2.51	2.53	2.55	2.58	2.60	2.62
35	3.84	3.88	3.91	3.95	3.98	4.02	4.06	4.09	4.13	4.16
40	5.74	5.79	5.84	5.89	5.95	6.00	6.05	6.11	6.16	6.22
45	8.17	8.24	8.32	8.39	8.47	8.54	8.62	8.70	8.77	8.85
50	11.20	11.31	11.41	11.51	11.61	11.72	11.82	11.93	12.03	12.14
	23.0	23.1	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	.10	.10	.10	.10	.10	.10	.10	.10	.10	.11
15	.33	.33	.34	.34	.34	.35	.35	.35	.35	.36
20	.78	.79	.80	.80	.81	.82	.83	.83	.84	.85
25	1.53	1.54	1.56	1.57	1.58	1.60	1.61	1.63	1.64	1.65
30	2.65	2.67	2.69	2.71	2.74	2.76	2.78	2.81	2.83	2.86
35	4.20	4.24	4.27	4.31	4.35	4.38	4.42	4.46	4.50	4.54
40	6.27	6.32	6.38	6.43	6.49	6.55	6.60	6.66	6.71	6.77
45	8.93	9.00	9.08	9.16	9.24	9.32	9.40	9.48	9.56	9.64
50	12.25	12.35	12.46	12.57	12.68	12.78	12.89	13.00	13.11	13.22
	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8	24.9
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	.11	.11	.11	.11	.11	.11	.11	.11	.11	.11
15	.36	.36	.37	.37	.37	.38	.38	.38	.38	.39
20	.85	.86	.87	.87	.88	.89	.90	.90	.91	.92
25	1.67	1.68	1.69	1.71	1.72	1.74	1.75	1.77	1.78	1.79
30	2.88	2.90	2.93	2.95	2.98	3.00	3.03	3.05	3.08	3.10
35	4.57	4.61	4.65	4.69	4.73	4.77	4.80	4.84	4.88	4.92
40	6.83	6.88	6.94	7.00	7.06	7.11	7.17	7.23	7.29	7.35
45	9.72	9.80	9.88	9.96	10.05	10.13	10.21	10.30	10.38	10.46
50	13.33	13.44	13.56	13.67	13.78	13.89	14.01	14.12	14.24	14.35

TABLE 14.—*Catenary corrections for various lengths and weights of tape*—Continued

Length of tape (meters)	Weight of tape in grams per meter									
	25.0	25.1	25.2	25.3	25.4	25.5	25.6	25.7	25.8	25.9
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
5	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02
10	.12	.12	.12	.12	.12	.12	.12	.12	.12	.12
15	.39	.39	.40	.40	.40	.41	.41	.41	.42	.42
20	.93	.93	.94	.95	.96	.96	.97	.98	.99	.99
25	1.81	1.82	1.84	1.85	1.87	1.88	1.90	1.91	1.93	1.94
30	3.13	3.15	3.18	3.20	3.23	3.25	3.28	3.30	3.33	3.35
35	4.96	5.00	5.04	5.08	5.12	5.16	5.20	5.24	5.29	5.33
40	7.41	7.47	7.53	7.59	7.65	7.71	7.77	7.83	7.89	7.95
45	10.55	10.63	10.72	10.80	10.89	10.97	11.06	11.15	11.23	11.32
50	14.47	14.58	14.70	14.82	14.93	15.05	15.17	15.29	15.41	15.53
	26.0	26.1	26.2	26.3	26.4	26.5	26.6	26.7	26.8	26.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13
15	.42	.43	.43	.43	.44	.44	.44	.45	.45	.45
20	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07
25	1.96	1.97	1.99	2.00	2.02	2.03	2.05	2.06	2.08	2.09
30	3.38	3.41	3.43	3.46	3.48	3.51	3.54	3.56	3.59	3.62
35	5.37	5.41	5.45	5.49	5.53	5.58	5.62	5.66	5.70	5.75
40	8.01	8.07	8.14	8.20	8.26	8.32	8.39	8.45	8.51	8.58
45	11.41	11.50	11.58	11.67	11.76	11.85	11.94	12.03	12.12	12.21
50	15.65	15.77	15.89	16.01	16.13	16.26	16.38	16.50	16.63	16.75
	27.0	27.1	27.2	27.3	27.4	27.5	27.6	27.7	27.8	27.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	.14	.14	.14	.14	.14	.14	.14	.14	.14	.14
15	.46	.46	.46	.47	.47	.47	.48	.48	.48	.49
20	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15
25	2.11	2.13	2.14	2.16	2.17	2.19	2.20	2.22	2.24	2.25
30	3.64	3.67	3.70	3.73	3.75	3.78	3.81	3.84	3.86	3.89
35	5.79	5.83	5.87	5.92	5.96	6.00	6.05	6.09	6.14	6.18
40	8.64	8.70	8.77	8.83	8.90	8.96	9.03	9.09	9.16	9.23
45	12.30	12.39	12.48	12.58	12.67	12.76	12.85	12.95	13.04	13.14
50	16.88	17.00	17.13	17.25	17.38	17.51	17.63	17.76	17.89	18.02
	28.0	28.1	28.2	28.3	28.4	28.5	28.6	28.7	28.8	28.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	.15	.15	.15	.15	.15	.15	.15	.15	.15	.15
15	.49	.49	.50	.50	.50	.51	.51	.51	.52	.52
20	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.24
25	2.27	2.28	2.30	2.32	2.33	2.35	2.37	2.38	2.40	2.42
30	3.92	3.95	3.98	4.00	4.03	4.06	4.09	4.12	4.15	4.18
35	6.22	6.27	6.31	6.36	6.40	6.45	6.49	6.54	6.59	6.63
40	9.29	9.36	9.43	9.49	9.56	9.63	9.69	9.76	9.83	9.90
45	13.23	13.32	13.42	13.52	13.61	13.71	13.80	13.90	14.00	14.09
50	18.15	18.28	18.41	18.54	18.67	18.80	18.93	19.07	19.20	19.33
	29.0	29.1	29.2	29.3	29.4	29.5	29.6	29.7	29.8	29.9
5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10	.16	.16	.16	.16	.16	.16	.16	.16	.16	.17
15	.53	.53	.53	.54	.54	.54	.55	.55	.56	.56
20	1.25	1.25	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.32
25	2.43	2.45	2.47	2.48	2.50	2.52	2.54	2.55	2.57	2.59
30	4.21	4.23	4.26	4.29	4.32	4.35	4.38	4.41	4.44	4.47
35	6.68	6.72	6.77	6.82	6.86	6.91	6.96	7.00	7.05	7.10
40	9.97	10.04	10.11	10.17	10.24	10.31	10.38	10.45	10.52	10.60
45	14.19	14.29	14.39	14.49	14.59	14.69	14.79	14.89	14.99	15.09
50	19.47	19.60	19.74	19.87	20.01	20.14	20.28	20.42	20.56	20.69

TABLE 15.—*Temperature corrections for steel tapes*

[Coefficient of expansion =0.0000116 per degree centigrade. Standard temperature of tape =20 degrees centigrade. For temperatures above 20 the corrections are plus, for temperatures below 20 the corrections are minus]

Length of tape (meters)	Temperature of tape in degrees centigrade																								
	19 21	18 22	17 23	16 24	15 25	14 26	13 27	12 28	11 29	10 30	9 31	8 32	7 33	6 34	5 35	4 36	3 37	2 38	1 39	0 40	−1 41	−2 42	−3 43	−4 44	−5 45
	Mm.	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*	*Mm.*
1	0	0	0	0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3
2	0	0	.1	.1	.1	.1	.2	.2	.2	.2	.3	.3	.3	.3	.3	.4	.4	.4	.4	.5	.5	.5	.5	.6	.6
3	0	.1	.1	.1	.2	.2	.2	.3	.3	.3	.4	.4	.5	.5	.5	.6	.6	.6	.7	.7	.7	.8	.8	.8	.9
4	0	.1	.1	.2	.2	.3	.3	.4	.4	.5	.5	.6	.6	.6	.7	.7	.8	.8	.9	.9	1.0	1.0	1.1	1.1	1.2
5	.1	.1	.2	.2	.3	.3	.4	.5	.5	.6	.6	.7	.8	.8	.9	.9	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.4
6	.1	.1	.2	.3	.3	.4	.5	.6	.6	.7	.8	.8	.9	1.0	1.0	1.1	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.7	1.7
7	.1	.2	.2	.3	.4	.5	.6	.6	.7	.8	.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.5	1.6	1.7	1.8	1.9	2.0	2.0
8	.1	.2	.3	.4	.5	.6	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.9	2.0	2.1	2.2	2.3
9	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6
10	.1	.2	.3	.5	.6	.7	.8	.9	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9
11	.1	.3	.4	.5	.6	.8	.9	1.0	1.1	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.1	3.2
12	.1	.3	.4	.6	.7	.8	1.0	1.1	1.3	1.4	1.5	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.6	2.8	2.9	3.1	3.2	3.3	3.5
13	.2	.3	.5	.6	.8	.9	1.1	1.2	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.0	3.2	3.3	3.5	3.6	3.8
14	.2	.3	.5	.6	.8	1.0	1.1	1.3	1.5	1.6	1.8	1.9	2.1	2.3	2.4	2.6	2.8	2.9	3.1	3.2	3.4	3.6	3.7	3.9	4.1
15	.2	.3	.5	.7	.9	1.0	1.2	1.4	1.6	1.7	1.9	2.1	2.3	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	3.8	4.0	4.2	4.4
16	.2	.4	.6	.7	.9	1.1	1.3	1.5	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.6
17	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.9
18	.2	.4	.6	.8	1.0	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2
19	.2	.4	.7	.9	1.1	1.3	1.5	1.8	2.0	2.2	2.4	2.6	2.9	3.1	3.3	3.5	3.7	4.0	4.2	4.4	4.6	4.8	5.1	5.3	5.5
20	.2	.5	.7	.9	1.2	1.4	1.6	1.9	2.1	2.3	2.6	2.8	3.0	3.2	3.5	3.7	3.9	4.2	4.4	4.6	4.9	5.1	5.3	5.6	5.8
21	.2	.5	.7	1.0	1.2	1.5	1.7	1.9	2.2	2.4	2.7	2.9	3.2	3.4	3.7	3.9	4.1	4.4	4.6	4.9	5.1	5.4	5.6	5.8	6.1
22	.3	.5	.8	1.0	1.3	1.5	1.8	2.0	2.3	2.6	2.8	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.8	5.1	5.4	5.6	5.9	6.1	6.4
23	.3	.5	.8	1.1	1.3	1.6	1.9	2.1	2.4	2.7	2.9	3.2	3.5	3.7	4.0	4.3	4.5	4.8	5.1	5.3	5.6	5.9	6.1	6.4	6.7
24	.3	.6	.8	1.1	1.4	1.7	1.9	2.2	2.5	2.8	3.1	3.3	3.6	3.9	4.2	4.5	4.7	5.0	5.3	5.6	5.8	6.1	6.4	6.7	7.0
25	.3	.6	.9	1.2	1.4	1.7	2.0	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.6	4.9	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.2
26	.3	.6	.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	6.3	6.6	6.9	7.2	7.5
27	.3	.6	.9	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.8	4.1	4.4	4.7	5.0	5.3	5.6	6.0	6.3	6.6	6.9	7.2	7.5	7.8
28	.3	.6	1.0	1.3	1.6	1.9	2.3	2.6	2.9	3.2	3.6	3.9	4.2	4.5	4.9	5.2	5.5	5.8	6.2	6.5	6.8	7.1	7.5	7.8	8.1
29	.3	.7	1.0	1.3	1.7	2.0	2.4	2.7	3.0	3.4	3.7	4.0	4.4	4.7	5.0	5.4	5.7	6.1	6.4	6.7	7.1	7.4	7.7	8.1	8.4
30	.3	.7	1.0	1.4	1.7	2.1	2.4	2.8	3.1	3.5	3.8	4.2	4.5	4.9	5.2	5.6	5.9	6.3	6.6	7.0	7.3	7.7	8.0	8.4	8.7

TABLE 16.—*Logarithms of radii of curvature of the earth's surface (in meters)*

[Based upon Clarke's spheroid of 1866 as expressed in meters]

Azimuth (degrees)	Latitude									Azimuth (degrees)
	0°	1°	2°	3°	4°	5°	6°	7°	8°	
0	6.80175	6.80175	6.80175	6.80176	6.80177	6.80178	6.80180	6.80181	6.80183	180
5	177	177	178	178	179	180	182	184	186	175
10	184	184	184	185	186	187	188	190	192	170
15	195	195	195	196	197	198	199	201	203	165
20	209	209	210	210	211	212	214	215	217	160
25	227	228	228	228	229	230	232	233	235	155
30	248	249	249	250	250	251	252	254	256	150
35	272	272	272	273	273	274	276	277	278	145
40	296	297	297	297	298	299	300	301	303	140
45	322	322	322	323	324	324	325	326	328	135
50	348	348	348	348	349	350	351	352	353	130
55	373	373	373	373	374	374	375	376	377	125
60	396	396	396	396	397	398	398	399	400	120
65	417	417	417	418	418	418	419	420	421	115
70	435	435	436	436	436	437	437	438	439	110
75	450	450	450	450	451	451	452	452	453	105
80	461	461	461	461	462	462	463	463	464	100
85	468	468	468	468	468	469	469	470	470	95
90	470	470	470	470	471	471	472	472	473	90

Azimuth (degrees)	Latitude									Azimuth (degrees)
	8°	9°	10°	11°	12°	13°	14°	15°	16°	
0	6.80183	6.80186	6.80188	6.80191	6.80194	6.80197	6.80201	6.80204	6.80208	180
5	186	188	190	193	196	199	203	206	210	175
10	192	194	197	200	202	206	209	213	217	170
15	203	205	207	210	213	216	219	223	227	165
20	217	219	222	224	227	230	233	236	240	160
25	235	237	239	242	244	247	250	254	257	155
30	256	257	260	262	264	267	270	273	276	150
35	278	280	282	284	287	289	292	295	298	145
40	303	304	306	308	310	313	315	318	321	140
45	328	329	331	333	335	337	339	342	344	135
50	353	354	356	358	359	361	364	366	368	130
55	377	379	380	382	383	385	387	389	391	125
60	400	401	403	404	406	407	409	411	413	120
65	421	422	423	424	426	427	429	430	432	115
70	439	440	441	442	443	444	446	447	449	110
75	453	454	455	456	457	458	460	461	463	105
80	464	465	466	467	468	469	470	471	473	100
85	470	471	472	473	474	475	476	478	479	95
90	473	474	474	475	476	477	478	480	481	90

TABLE 16.—*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

Azimuth (degrees)	Latitude									Azimuth (degrees)
	16°	17°	18°	19°	20°	21°	22°	23°	24°	
0	6.80208	6.80213	6.80217	6.80222	6.80226	6.80232	6.80237	6.80242	6.80248	180
5	210	215	219	224	228	234	239	244	250	175
10	217	221	225	230	234	239	244	250	255	170
15	227	231	235	239	244	249	254	259	264	165
20	240	244	248	252	257	262	266	271	277	160
25	257	261	265	269	273	277	282	287	292	155
30	276	280	284	287	292	296	300	305	309	150
35	298	301	305	308	312	316	320	324	329	145
40	321	324	327	330	334	338	341	345	350	140
45	344	347	350	353	357	360	364	367	371	135
50	368	371	373	376	379	382	386	389	392	130
55	391	394	396	398	401	404	407	410	413	125
60	413	415	417	419	422	424	427	430	432	120
65	432	434	436	438	440	443	445	448	450	115
70	449	451	453	454	456	459	461	463	465	110
75	463	464	466	468	470	472	473	476	478	105
80	473	474	476	478	479	481	483	485	487	100
85	479	480	482	483	485	487	489	490	492	95
90	481	482	484	485	487	489	490	492	494	90

Azimuth (degrees)	Latitude									Azimuth (degrees)
	24°	25°	26°	27°	28°	29°	30°	31°	32°	
0	6.80248	6.80254	6.80260	6.80266	6.80272	6.80279	6.80285	6.80292	6.80299	180
5	250	256	262	268	274	280	287	294	300	175
10	255	261	267	273	279	285	292	298	305	170
15	264	270	276	282	288	294	300	306	313	165
20	277	282	288	293	299	305	311	317	324	160
25	292	297	302	308	313	319	325	331	337	155
30	309	314	319	324	330	335	340	346	352	150
35	329	333	338	343	348	353	358	363	369	145
40	350	354	358	362	367	372	377	382	386	140
45	371	375	379	383	387	391	396	400	405	135
50	392	396	399	403	407	411	415	419	423	130
55	413	416	420	423	426	430	434	437	441	125
60	432	435	438	442	445	448	451	455	458	120
65	450	453	455	458	461	464	467	470	473	115
70	465	468	470	473	475	478	481	484	486	110
75	478	480	482	484	487	489	492	494	497	105
80	487	489	491	493	495	498	500	502	505	100
85	492	494	496	498	501	503	505	507	510	95
90	494	496	498	500	502	504	507	509	511	90

TABLE 16.—*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

Azimuth (degrees)	Latitude									Azimuth (degrees)
	32°	33°	34°	35°	36°	37°	38°	39°	40°	
0	6.80299	6.80306	6.80313	6.80320	6.80327	6.80335	6.80342	6.80350	6.80357	180
5	300	307	314	322	329	336	344	351	359	175
10	305	312	319	326	333	340	348	355	363	170
15	313	320	326	333	340	348	355	362	369	165
20	324	330	337	343	350	357	364	371	378	160
25	337	343	349	355	362	368	375	382	388	155
30	352	358	364	370	376	382	388	394	401	150
35	369	374	380	385	391	397	402	408	414	145
40	386	392	397	402	407	412	418	423	429	140
45	405	410	414	419	424	429	434	439	444	135
50	423	428	432	436	441	445	450	454	459	130
55	441	445	449	453	457	461	465	469	474	125
60	458	462	465	469	472	476	480	484	487	120
65	473	476	480	483	486	489	493	496	500	115
70	486	489	492	495	498	501	504	507	510	110
75	497	500	502	505	508	510	513	516	519	105
80	505	507	510	512	515	517	520	523	525	100
85	510	512	514	517	519	522	524	527	529	95
90	511	514	516	518	521	523	526	528	531	90

Azimuth (degrees)	Latitude									Azimuth (degrees)
	40°	41°	42°	43°	44°	45°	46°	47°	48°	
0	6.80357	6.80365	6.80373	6.80380	6.80388	6.80396	6.80404	6.80411	6.80419	180
5	359	366	374	382	389	397	404	412	420	175
10	363	370	378	385	393	400	408	415	423	170
15	369	376	384	391	398	406	413	420	428	165
20	378	385	392	399	406	413	420	427	434	160
25	388	395	402	408	415	422	429	436	442	155
30	401	407	413	420	426	433	439	446	452	150
35	414	420	426	432	438	444	450	456	462	145
40	429	434	440	446	451	457	462	468	474	140
45	444	449	454	459	464	470	475	480	485	135
50	459	464	468	473	478	482	487	492	496	130
55	474	478	482	486	490	495	499	503	508	125
60	487	491	495	499	502	506	510	514	518	120
65	500	503	507	510	514	517	520	524	528	115
70	510	514	517	520	523	526	529	532	536	110
75	519	522	525	528	531	534	536	539	542	105
80	525	528	531	534	536	539	542	544	547	100
85	529	532	534	537	540	542	545	548	550	95
90	531	533	536	538	541	544	546	549	551	90

TABLE 16.—*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

Azimuth (degrees)	Latitude 48°	49°	50°	51°	52°	53°	54°	55°	56°	Azimuth (degrees)
0	6.80419	6.80426	6.80434	6.80442	6.80449	6.80457	6.80464	6.80471	6.80479	180
5	420	428	435	443	450	458	465	472	479	175
10	423	430	438	445	453	460	467	474	481	170
15	428	435	442	450	457	464	471	478	485	165
20	434	441	448	455	462	469	476	483	489	160
25	442	449	456	463	469	476	482	489	495	155
30	452	458	465	471	477	484	490	496	502	150
35	462	468	474	480	486	492	498	503	509	145
40	474	479	485	490	496	501	506	512	517	140
45	485	490	495	500	505	510	515	520	525	135
50	496	501	506	510	515	520	524	528	533	130
55	508	512	516	520	524	528	533	537	541	125
60	518	522	526	530	533	537	541	544	548	120
65	528	531	534	538	541	545	548	551	555	115
70	536	539	542	545	548	551	554	557	560	110
75	542	545	548	551	554	557	559	562	565	105
80	547	550	553	555	558	561	563	566	568	100
85	550	553	555	558	560	563	566	568	570	95
90	551	554	556	559	561	564	566	569	571	90

Azimuth (degrees)	Latitude 56°	57°	58°	59°	60°	61°	62°	63°	64°	Azimuth (degrees)
0	6.80479	6.80486	6.80493	6.80500	6.80506	6.80513	6.80520	6.80526	6.80532	180
5	479	486	493	500	507	514	520	526	532	175
10	481	488	495	502	509	515	522	528	534	170
15	485	492	498	505	511	518	524	530	536	165
20	489	496	502	509	515	521	527	533	539	160
25	495	501	508	514	520	526	531	537	542	155
30	502	508	514	519	525	530	536	541	546	150
35	509	515	520	525	531	536	541	546	551	145
40	517	522	527	532	537	542	546	551	556	140
45	525	530	534	539	543	548	552	556	560	135
50	533	537	542	546	550	554	558	562	565	130
55	541	545	548	552	556	560	563	567	570	125
60	548	552	555	558	562	565	568	572	575	120
65	555	558	561	564	567	570	573	576	579	115
70	560	563	566	569	572	574	577	580	582	110
75	565	568	570	573	575	578	580	583	585	105
80	568	571	573	576	578	580	583	585	587	100
85	570	573	575	578	580	582	584	586	588	95
90	571	574	576	578	580	583	585	587	589	90

TABLE 16.—*Logarithms of radii of curvature of the earth's surface (in meters)*—Continued

Azimuth (degrees)	Latitude									Azimuth (degrees)
	64°	65°	66°	67°	68°	69°	70°	71°	72°	
0	6.80532	6.80538	6.80544	6.80550	6.80555	6.80560	6.80565	6.80570	6.80575	180
5	532	538	544	550	555	561	566	570	575	175
10	534	540	545	551	556	562	566	571	576	170
15	536	542	547	553	558	563	568	572	577	165
20	539	544	550	555	560	565	570	574	578	160
25	542	548	553	558	562	567	572	576	580	155
30	546	551	556	561	565	570	574	578	582	150
35	551	556	560	564	569	573	577	581	584	145
40	556	560	564	568	572	576	580	583	587	140
45	560	564	568	572	576	579	583	586	589	135
50	565	569	573	576	579	583	586	589	592	130
55	570	574	577	580	583	586	589	591	594	125
60	575	578	581	584	586	589	591	594	596	120
65	579	582	584	587	589	592	594	596	598	115
70	582	585	587	590	592	594	596	598	600	110
75	585	587	590	592	594	596	598	600	601	105
80	587	589	591	593	595	597	599	601	602	100
85	588	590	592	594	596	598	600	601	603	95
90	589	591	593	595	597	598	600	602	603	90

TABLE 17.—*Length of 1 degree of the meridian at different latitudes*

[Based upon Clarke's spheroid of 1866 as expressed in meters]

Latitude (degrees)	Meters	Statute miles	Nautical miles	Latitude (degrees)	Meters	Statute miles	Nautical miles
0–1	110,567.3	68.703	59.702	45–46	111,140.8	69.060	60.011
1–2	110,568.0	68.704	59.702	46–47	111,160.5	69.072	60.022
2–3	110,569.4	68.705	59.703	47–48	111,180.2	69.084	60.033
3–4	110,571.4	68.706	59.704	48–49	111,199.9	69.096	60.043
4–5	110,574.1	68.707	59.705	49–50	111,219.5	69.108	60.054
5–6	110,577.6	68.710	59.707	50–51	111,239.0	69.121	60.064
6–7	110,581.6	68.712	59.709	51–52	111,258.3	69.133	60.075
7–8	110,586.4	68.715	59.712	52–53	111,277.6	69.145	60.085
8–9	110,591.8	68.718	59.715	53–54	111,296.6	69.156	60.095
9–10	110,597.8	68.722	59.718	54–55	111,315.4	69.168	60.106
10–11	110,604.5	68.726	59.722	55–56	111,334.0	69.180	60.116
11–12	110,611.9	68.731	59.726	56–57	111,352.4	69.191	60.125
12–13	110,619.8	68.736	59.730	57–58	111,370.5	69.202	60.135
13–14	110,628.4	68.741	59.735	58–59	111,388.4	69.213	60.145
14–15	110,637.6	68.747	59.740	59–60	111,405.9	69.224	60.154
15–16	110,647.5	68.753	59.745	60–61	111,423.1	69.235	60.164
16–17	110,657.8	68.759	59.750	61–62	111,439.9	69.246	60.173
17–18	110,668.8	68.766	59.756	62–63	111,456.4	69.256	60.182
18–19	110,680.4	68.773	59.763	63–64	111,472.4	69.266	60.190
19–20	110,692.4	68.781	59.769	64–65	111,488.1	69.275	60.199
20–21	110,705.1	68.789	59.776	65–66	111,503.3	69.285	60.207
21–22	110,718.2	68.797	59.783	66–67	111,518.0	69.294	60.215
22–23	110,731.8	68.805	59.790	67–68	111,532.3	69.303	60.223
23–24	110,746.0	68.814	59.798	68–69	111,546.2	69.311	60.230
24–25	110,760.6	68.823	59.806	69–70	111,559.5	69.320	60.237
25–26	110,775.6	68.833	59.814	70–71	111,572.2	69.328	60.244
26–27	110,791.1	68.842	59.822	71–72	111,584.5	69.335	60.251
27–28	110,807.0	68.852	59.831	72–73	111,596.2	69.343	60.257
28–29	110,823.3	68.862	59.840	73–74	111,607.3	69.349	60.263
29–30	110,840.0	68.873	59.849	74–75	111,617.9	69.356	60.269
30–31	110,857.0	68.883	59.858	75–76	111,627.8	69.362	60.274
31–32	110,874.4	68.894	59.867	76–77	111,637.1	69.368	60.279
32–33	110,892.1	68.905	59.877	77–78	111,645.9	69.373	60.284
33–34	110,910.1	68.916	59.887	78–79	111,653.9	69.378	60.288
34–35	110,928.3	68.928	59.896	79–80	111,661.4	69.383	60.292
35–36	110,946.9	68.939	59.907	80–81	111,668.2	69.387	60.296
36–37	110,965.6	68.951	59.917	81–82	111,674.4	69.391	60.299
37–38	110,984.5	68.962	59.927	82–83	111,679.9	69.395	60.302
38–39	111,003.7	68.974	59.937	83–84	111,684.7	69.398	60.305
39–40	111,023.0	68.986	59.948	84–85	111,688.9	69.400	60.307
40–41	111,042.4	68.998	59.958	85–86	111,692.3	69.402	60.309
41–42	111,061.9	69.011	59.969	86–87	111,695.1	69.404	60.311
42–43	111,081.6	69.023	59.979	87–88	111,697.2	69.405	60.312
43–44	111,101.3	69.035	59.990	88–89	111,698.6	69.406	60.312
44–45	111,121.0	69.047	60.001	89–90	111,699.3	69.407	60.313

TABLE 18.—*Length of 1 degree of the parallel at different latitudes*

[Based upon Clarke's spheroid of 1866 as expressed in meters]

Latitude (degrees)	Meters	Statute miles	Nautical miles	Latitude (degrees)	Meters	Statute miles	Nautical miles
0	111,321	69.172	60.108	46	77,466	48.136	41.828
1	111,304	69.162	60.099	47	76,058	47.261	41.068
2	111,253	69.130	60.072	48	74,628	46.372	40.296
3	111,169	69.078	60.027	49	73,174	45.469	39.511
4	111,051	69.005	59.963	50	71,698	44.552	38.714
5	110,900	68.911	59.881	51	70,200	43.621	37.905
6	110,715	68.795	59.781	52	68,680	42.676	37.084
7	110,497	68.660	59.663	53	67,140	41.719	36.252
8	110,245	68.504	59.527	54	65,578	40.749	35.409
9	109,959	68.326	59.373	55	63,996	39.766	34.555
10	109,641	68.129	59.201	56	62,395	38.771	33.691
11	109,289	67.910	59.011	57	60,774	37.764	32.816
12	108,904	67.670	58.803	58	59,135	36.745	31.930
13	108,486	67.410	58.578	59	57,478	35.716	31.035
14	108,036	67.131	58.334	60	55,802	34.674	30.131
15	107,553	66.830	58.073	61	54,110	33.623	29.217
16	107,036	66.510	57.795	62	52,400	32.560	28.294
17	106,487	66.169	57.499	63	50,675	31.488	27.362
18	105,906	65.808	57.185	64	48,934	30.406	26.422
19	105,294	65.427	56.854	65	47,177	29.315	25.474
20	104,649	65.026	56.506	66	45,407	28.215	24.518
21	103,972	64.606	56.140	67	43,622	27.106	23.554
22	103,264	64.166	55.758	68	41,823	25.988	22.583
23	102,524	63.706	55.359	69	40,012	24.862	21.605
24	101,754	63.228	54.942	70	38,188	23.729	20.620
25	100,952	62.729	54.510	71	36,353	22.589	19.629
26	100,119	62.212	54.060	72	34,506	21.441	18.632
27	99,257	61.676	53.594	73	32,648	20.287	17.629
28	98,364	61.122	53.112	74	30,781	19.127	16.620
29	97,441	60.548	52.614	75	28,903	17.960	15.607
30	96,488	59.956	52.099	76	27,017	16.788	14.588
31	95,506	59.345	51.569	77	25,123	15.611	13.565
32	94,495	58.716	51.023	78	23,220	14.428	12.538
33	93,455	58.071	50.462	79	21,311	13.242	11.507
34	92,387	57.407	49.885	80	19,394	12.051	10.472
35	91,290	56.725	49.293	81	17,472	10.857	9.434
36	90,166	56.027	48.686	82	15,545	9.659	8.393
37	89,014	55.311	48.064	83	13,612	8.458	7.350
38	87,835	54.579	47.427	84	11,675	7.255	6.304
39	86,629	53.829	46.776	85	9,735	6.049	5.256
40	85,396	53.063	46.110	86	7,792	4.842	4.207
41	84,137	52.281	45.431	87	5,846	3.632	3.157
42	82,853	51.483	44.737	88	3,898	2.422	2.105
43	81,543	50.669	44.030	89	1,949	1.211	1.053
44	80,208	49.840	43.309	90	0	0	0
45	78,849	48.995	42.575				

TABLE 19.—*Proportional change in a number corresponding to a change in its logarithm*

[Computed from the formula $\frac{\Delta N}{N}=\frac{\Delta \log N}{\mu}$, where μ = modulus of common logarithms = 0.4343]

$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—	$\Delta \log N$ units of seventh decimal place	$\frac{\Delta N}{N}$ one part in—
1	4,343,000	26	167,000	51	85,000	76	57,100
2	2,171,000	27	161,000	52	84,000	77	56,400
3	1,448,000	28	155,000	53	82,000	78	55,700
4	1,086,000	29	150,000	54	80,000	79	55,000
5	869,000	30	145,000	55	79,000	80	54,300
6	724,000	31	140,000	56	78,000	81	53,600
7	620,000	32	136,000	57	76,000	82	53,000
8	543,000	33	132,000	58	75,000	83	52,300
9	483,000	34	128,000	59	74,000	84	51,700
10	434,000	35	124,000	60	72,000	85	51,100
11	395,000	36	121,000	61	71,000	86	50,500
12	362,000	37	117,000	62	70,000	87	49,900
13	334,000	38	114,000	63	69,000	88	49,400
14	310,000	39	111,000	64	68,000	89	48,800
15	290,000	40	109,000	65	67,000	90	48,300
16	271,000	41	106,000	66	66,000	91	47,700
17	255,000	42	103,000	67	65,000	92	47,200
18	241,000	43	101,000	68	64,000	93	46,700
19	229,000	44	99,000	69	63,000	94	46,200
20	217,000	45	97,000	70	62,000	95	45,700
21	207,000	46	94,000	71	61,000	96	45,200
22	197,000	47	92,000	72	60,000	97	44,800
23	189,000	48	90,000	73	59,000	98	44,300
24	181,000	49	89,000	74	58,700	99	43,900
25	174,000	50	87,000	75	57,900	100	43,400

TABLE 20.—*Fractional change in a number corresponding to a change in its logarithm*

[Computed from the formula $\frac{\Delta N}{N}=\frac{\Delta \log N}{\mu}$, where μ = modulus of common logarithms = 0.43429448]

For $\Delta \log N$ = 1 unit in	$\frac{\Delta N}{N}$	For $\Delta \log N$ = 4 units in	$\frac{\Delta N}{N}$ (in round numbers)
4th place	$\frac{1}{4343}$	4th place	$\frac{1}{1000}$
5th place	$\frac{1}{43429}$	5th place	$\frac{1}{10000}$
6th place	$\frac{1}{434294}$	6th place	$\frac{1}{100000}$
7th place	$\frac{1}{4342945}$	7th place	$\frac{1}{1000000}$

TABLE 21.—*Lengths–Feet to meters (from 1 to 1,000 units)*

[Reduction factor: 1 foot = 0.3048006096 meter]

Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters
0		100	30.48006	200	60.96012	300	91.44018	400	121.92024	500	152.40030	600	182.88037	700	213.36043	800	243.84049	900	274.32055
1	0.30480	1	30.78486	1	61.26492	1	91.74498	1	122.22504	1	152.70511	1	183.18517	1	213.66523	1	244.14529	1	274.62535
2	.60960	2	31.08966	2	61.56972	2	92.04978	2	122.52985	2	153.00991	2	183.48997	2	213.97003	2	244.45009	2	274.93015
3	.91440	3	31.39446	3	61.87452	3	92.35458	3	122.83465	3	153.31471	3	183.79477	3	214.27483	3	244.75489	3	275.23495
4	1.21920	4	31.69926	4	62.17932	4	92.65939	4	123.13945	4	153.61951	4	184.09957	4	214.57963	4	245.05969	4	275.53975
5	1.52400	5	32.00406	5	62.48412	5	92.96419	5	123.44425	5	153.92431	5	184.40437	5	214.88443	5	245.36449	5	275.84455
6	1.82880	6	32.30886	6	62.78893	6	93.26399	6	123.74905	6	154.22911	6	184.70917	6	215.18923	6	245.66929	6	276.14935
7	2.13360	7	32.61367	7	63.09373	7	93.57379	7	124.05385	7	154.53391	7	185.01397	7	215.49403	7	245.97409	7	276.45415
8	2.43840	8	32.91847	8	63.39853	8	93.87859	8	124.35865	8	154.83871	8	185.31877	8	215.79883	8	246.27889	8	276.75895
9	2.74321	9	33.22327	9	63.70333	9	94.18339	9	124.66345	9	155.14351	9	185.62357	9	216.10363	9	246.53369	9	277.06375
10	3.04801	110	33.52807	210	64.00813	310	94.48819	410	124.96825	510	155.44831	610	185.92837	710	216.40843	810	246.88849	910	277.36855
1	3.35281	1	33.83287	1	64.31293	1	94.79299	1	125.27305	1	155.75311	1	186.23317	1	216.71323	1	247.19329	1	277.67336
2	3.65761	2	34.13767	2	64.61773	2	95.09779	2	125.57785	2	156.05791	2	186.53797	2	217.01803	2	247.49809	2	277.97816
3	3.96241	3	34.44247	3	64.92253	3	95.40259	3	125.88265	3	156.36271	3	186.84277	3	217.32283	3	247.80290	3	278.28296
4	4.26721	4	34.74727	4	65.22733	4	95.70739	4	126.18745	4	156.66751	4	187.14757	4	217.62764	4	248.10770	4	278.58776
5	4.57201	5	35.05207	5	65.53213	5	96.01219	5	126.49225	5	156.97231	5	187.45237	5	217.93244	5	248.41250	5	278.89256
6	4.87681	6	35.35687	6	65.83693	6	96.31699	6	126.79705	6	157.27711	6	187.75718	6	218.23724	6	248.71730	6	279.19736
7	5.18161	7	35.66167	7	66.14173	7	96.62179	7	127.10185	7	157.58192	7	188.06198	7	218.54204	7	249.02210	7	279.50216
8	5.48641	8	35.96647	8	66.44653	8	96.92659	8	127.40665	8	157.88672	8	188.36678	8	218.84684	8	249.32690	8	279.80696
9	5.79121	9	36.27127	9	66.75133	9	97.23139	9	127.71146	9	158.19152	9	188.67158	9	219.15164	9	249.63170	9	280.11176
20	6.09601	120	36.57607	220	67.05613	320	97.53620	420	128.01626	520	158.49632	620	188.97638	720	219.45644	820	249.93650	920	280.41656
1	6.40081	1	36.88087	1	67.36093	1	97.84100	1	128.32106	1	158.80112	1	189.28118	1	219.76124	1	250.24130	1	280.72136
2	6.70561	2	37.18567	2	67.66574	2	98.14580	2	128.62586	2	159.10592	2	189.58598	2	220.06604	2	250.54610	2	281.02616
3	7.01041	3	37.49047	3	67.97054	3	98.45060	3	128.93066	3	159.41072	3	189.89078	3	220.37084	3	250.85090	3	281.33096
4	7.31521	4	37.79528	4	68.27534	4	98.75540	4	129.23546	4	159.71552	4	190.19558	4	220.67564	4	251.15570	4	281.63576
5	7.62002	5	38.10008	5	68.58014	5	99.06020	5	129.54026	5	160.02032	5	190.50038	5	220.98044	5	251.46050	5	281.94056
6	7.92482	6	38.40488	6	68.88494	6	99.36500	6	129.84506	6	160.32512	6	190.80518	6	221.28524	6	251.76530	6	282.24536
7	8.22962	7	38.70968	7	69.18974	7	99.66980	7	130.14986	7	160.62992	7	191.10998	7	221.59004	7	252.07010	7	282.55017
8	8.53442	8	39.01448	8	69.49454	8	99.97460	8	130.45466	8	160.93472	8	191.41478	8	221.89484	8	252.37490	8	282.85497
9	8.83922	9	39.31928	9	69.79934	9	100.27940	9	130.75946	9	161.23952	9	191.71958	9	222.19964	9	252.67971	9	283.15977
30	9.14402	130	39.62408	230	70.10414	330	100.58420	430	131.06426	530	161.54432	630	192.02438	730	222.50445	830	252.98451	930	283.46457
1	9.44882	1	39.92888	1	70.40894	1	100.88900	1	131.36906	1	161.84912	1	192.32918	1	222.80925	1	253.28931	1	283.76937
2	9.75362	2	40.23368	2	70.71374	2	101.19380	2	131.67386	2	162.15392	2	192.63399	2	223.11405	2	253.59411	2	284.07417
3	10.05842	3	40.53848	3	71.01854	3	101.49860	3	131.97866	3	162.45872	3	192.93879	3	223.41885	3	253.89891	3	284.37897
4	10.36322	4	40.84328	4	71.32334	4	101.80340	4	132.28346	4	162.76353	4	193.24359	4	223.72365	4	254.20371	4	284.68377
5	10.66802	5	41.14808	5	71.62814	5	102.10820	5	132.58827	5	163.06833	5	193.54839	5	224.02845	5	254.50851	5	284.98857
6	10.97282	6	41.45288	6	71.93294	6	102.41300	6	132.89307	6	163.37313	6	193.85319	6	224.33325	6	254.81331	6	285.29337
7	11.27762	7	41.75768	7	72.23774	7	102.71781	7	133.19787	7	163.67793	7	194.15799	7	224.63805	7	255.11811	7	285.59817
8	11.58242	8	42.06248	8	72.54255	8	103.02261	8	133.50267	8	163.98273	8	194.46279	8	224.94285	8	255.42291	8	285.90297
9	11.88722	9	42.36728	9	72.84735	9	103.32741	9	133.80747	9	164.28753	9	194.76759	9	225.24765	9	255.72771	9	286.20777
40	12.19202	140	42.67209	240	73.15215	340	103.63221	440	134.11227	540	164.59233	640	195.07239	740	225.55245	840	256.03251	940	286.51257
1	12.49682	1	42.97689	1	73.45695	1	103.93701	1	134.41707	1	164.89713	1	195.37719	1	225.85725	1	256.33731	1	286.81737
2	12.80163	2	43.28169	2	73.76175	2	104.24181	2	134.72187	2	165.20193	2	195.68199	2	226.16205	2	256.64211	2	287.12217
3	13.10643	3	43.58649	3	74.06655	3	104.54661	3	135.02667	3	165.50673	3	195.98679	3	226.46685	3	256.94691	3	287.42697
4	13.41123	4	43.89129	4	74.37135	4	104.85141	4	135.33147	4	165.81153	4	196.29159	4	226.77165	4	257.25171	4	287.73178
5	13.71603	5	44.19609	5	74.67615	5	105.15621	5	135.63627	5	166.11633	5	196.59639	5	227.07645	5	257.55652	5	288.03658
6	14.02083	6	44.50089	6	74.98095	6	105.46101	6	135.94107	6	166.42113	6	196.90119	6	227.38125	6	257.86132	6	288.34138
7	14.32563	7	44.80569	7	75.28575	7	105.76581	7	136.24587	7	166.72593	7	197.20599	7	227.68606	7	258.16612	7	288.64618
8	14.63043	8	45.11049	8	75.59055	8	106.07061	8	136.55067	8	167.03073	8	197.51080	8	227.99086	8	258.47092	8	288.95098
9	14.93523	9	45.41529	9	75.89535	9	106.37541	9	136.85547	9	167.33553	9	197.81560	9	228.29566	9	258.77572	9	289.25578
50	15.24003	150	45.72009	250	76.20015	350	106.68021	450	137.16027	550	167.64034	650	198.12040	750	228.60046	850	259.08052	950	289.56058
1	15.54483	1	46.02489	1	76.50495	1	106.98501	1	137.46507	1	167.94514	1	198.42520	1	228.90526	1	259.38532	1	289.86538
2	15.84963	2	46.32969	2	76.80975	2	107.28981	2	137.76988	2	168.24994	2	198.73000	2	229.21006	2	259.69012	2	290.17018
3	16.15443	3	46.63449	3	77.11455	3	107.59462	3	138.07468	3	168.55474	3	199.03480	3	229.51486	3	259.99492	3	290.47498
4	16.45923	4	46.93929	4	77.41935	4	107.89942	4	138.37948	4	168.85954	4	199.33960	4	229.81966	4	260.29972	4	290.77978
5	16.76403	5	47.24409	5	77.72416	5	108.20422	5	138.68428	5	169.16434	5	199.64440	5	230.12446	5	260.60452	5	291.08458
6	17.06883	6	47.54890	6	78.02896	6	108.50902	6	138.98908	6	169.46914	6	199.94920	6	230.42926	6	260.90932	6	291.38938
7	17.37363	7	47.85370	7	78.33376	7	108.81382	7	139.29388	7	169.77394	7	200.25400	7	230.73406	7	261.21412	7	291.69418
8	17.67844	8	48.15850	8	78.63856	8	109.11862	8	139.59868	8	170.07874	8	200.55880	8	231.03886	8	261.51892	8	291.99898
9	17.98324	9	48.46330	9	78.94336	9	109.42342	9	139.90348	9	170.38354	9	200.86360	9	231.34366	9	261.82372	9	292.30378
60	18.28804	160	48.76810	260	79.24816	360	109.72822	460	140.20828	560	170.68834	660	201.16840	760	231.64846	860	262.12852	960	292.60859
1	18.59284	1	49.07290	1	79.55296	1	110.03302	1	140.51308	1	170.99314	1	201.47320	1	231.95326	1	262.43332	1	292.91339
2	18.89764	2	49.37770	2	79.85776	2	110.33782	2	140.81788	2	171.29794	2	201.77800	2	232.25806	2	262.73813	2	293.21819
3	19.20244	3	49.68250	3	80.16256	3	110.64262	3	141.12268	3	171.60274	3	202.08280	3	232.56287	3	263.04293	3	293.52299
4	19.50724	4	49.98730	4	80.46736	4	110.94742	4	141.42748	4	171.90754	4	202.38760	4	232.86767	4	263.34773	4	293.82779
5	19.81204	5	50.29210	5	80.77216	5	111.25222	5	141.73228	5	172.21234	5	202.69241	5	233.17247	5	263.65253	5	294.13259
6	20.11684	6	50.59690	6	81.07696	6	111.55702	6	142.03708	6	172.51715	6	202.99721	6	233.47727	6	263.95733	6	294.43739
7	20.42164	7	50.90170	7	81.38176	7	111.86182	7	142.34188	7	172.82195	7	203.30201	7	233.78207	7	264.26213	7	294.74219
8	20.72644	8	51.20650	8	81.68656	8	112.16662	8	142.64669	8	173.12675	8	203.60681	8	234.08687	8	264.56693	8	295.04699
9	21.03124	9	51.51130	9	81.99136	9	112.47142	9	142.95149	9	173.43155	9	203.91161	9	234.39167	9	264.87173	9	295.35179
70	21.33604	170	51.81610	270	82.29616	370	112.77623	470	143.25629	570	173.73635	670	204.21641	770	234.69647	870	265.17653	970	295.65659
1	21.64084	1	52.12090	1	82.60097	1	113.08103	1	143.56109	1	174.04115	1	204.52121	1	235.00127	1	265.48133	1	295.96139
2	21.94564	2	52.42570	2	82.90577	2	113.38583	2	143.86589	2	174.34595	2	204.82601	2	235.30607	2	265.78613	2	296.26619
3	22.25044	3	52.73051	3	83.21057	3	113.69063	3	144.17069	3	174.65075	3	205.13081	3	235.61087	3	266.09093	3	296.57099
4	22.55525	4	53.03531	4	83.51537	4	113.99543	4	144.47549	4	174.95555	4	205.43561	4	235.91567	4	266.39573	4	296.87579
5	22.86005	5	53.34011	5	83.82017	5	114.30023	5	144.78029	5	175.26035	5	205.74041	5	236.22047	5	266.70053	5	297.18059
6	23.16485	6	53.64491	6	84.12497	6	114.60503	6	145.08509	6	175.56515	6	206.04521	6	236.52527	6	267.00533	6	297.48539
7	23.46965	7	53.94971	7	84.42977	7	114.90983	7	145.38989	7	175.86995	7	206.35001	7	236.83007	7	267.31013	7	297.79020
8	23.77445	8	54.25451	8	84.73457	8	115.21463	8	145.69469	8	176.17475	8	206.65481	8	237.13487	8	267.61494	8	298.09500
9	24.07925	9	54.55931	9	85.03937	9	115.51943	9	145.99949	9	176.47955	9	206.95961	9	237.43967	9	267.91974	9	298.39980
80	24.38405	180	54.86411	280	85.34417	380	115.82423	480	146.30429	580	176.78435	680	207.26441	780	237.74448	880	268.22454	980	298.70460
1	24.68885	1	55.16891	1	85.64897	1	116.12903	1	146.60909	1	177.08915	1	207.56922	1	238.04928	1	268.52934	1	299.00940
2	24.99365	2	55.47371	2	85.95377	2	116.43383	2	146.91389	2	177.39395	2	207.87402	2	238.35408	2	268.83414	2	299.31420
3	25.29845	3	55.77851	3	86.25857	3	116.73863	3	147.21869	3	177.69876	3	208.17882	3	238.65888	3	269.13894	3	299.61900
4	25.60325	4	56.08331	4	86.56337	4	117.04343	4	147.52350	4	178.00356	4	208.48362	4	238.96368	4	269.44374	4	299.92380
5	25.90805	5	56.38811	5	86.86817	5	117.34823	5	147.82830	5	178.30836	5	208.78842	5	239.26848	5	269.74854	5	300.22860
6	26.21285	6	56.69291	6	87.17297	6	117.65304	6	148.13310	6	178.61316	6	209.09322	6	239.57328	6	270.05334	6	300.53340
7	26.51765	7	56.99771	7	87.47777	7	117.95784	7	148.43790	7	178.91796	7	209.39802	7	239.87808	7	270.35814	7	300.83820
8	26.82245	8	57.30251	8	87.78258	8	118.26264	8	148.74270	8	179.22276	8	209.70282	8	240.18288	8	270.66294	8	301.14300
9	27.12725	9	57.60732	9	88.08738	9	118.56744	9	149.04750	9	179.52756	9	210.00762	9	240.48768	9	270.96774	9	301.44780
90	27.43205	190	57.91212	290	88.39218	390	118.87224	490	149.35230	590	179.83236	690	210.31242	790	240.79248	890	271.27254	990	301 75260
1	27.73686	1	58.21692	1	88.69698	1	119.17704	1	149.65710	1	180.13716	1	210.61722	1	241.09728	1	271.57734	1	302.05740
2	28.04166	2	58.52172	2	89.00178	2	119.48184	2	149.96190	2	180.44196	2	210.92202	2	241.40208	2	271.88214	2	302.36220
3	28.34646	3	58.82652	3	89.30658	3	119.78664	3	150.26670	3	180.74676	3	211.22682	3	241.70688	3	272.18694	3	302.66701
4	28.65126	4	59.13132	4	89.61138	4	120.09144	4	150.57150	4	181.05156	4	211.53162	4	242.01168	4	272.49174	4	302.97181
5	28.95606	5	59.43612	5	89.91618	5	120.39624	5	150.87630	5	181.35636	5	211.83642	5	242.31648	5	272.79655	5	303.27661
6	29.26086	6	59.74092	6	90.22098	6	120.70104	6	151.18110	6	181.66116	6	212.14122	6	242.62129	6	273.10135	6	303.58141
7	29.56566	7	60.04572	7	90.52578	7	121.00584	7	151.48590	7	181.96596	7	212.44602	7	242.92609	7	273.40615	7	303.88621
8	29.87046	8	60.35052	8	90.83058	8	121.31064	8	151.79070	8	182.27076	8	212.75083	8	243.23089	8	273.71095	8	304.19101
9	30.17526	9	60.65532	9	91.13538	9	121.61544	9	152.09550	9	182.57557	9	213.05563	9	243.53569	9	274.01575	9	304.49581

1 inch = 0.02540 meter	4 inches = 0.10160 meter	7 inches = 0.17780 meter	10 inches = 0.25400 meter
2 inches = .05080 meter	5 inches = .12700 meter	8 inches = .20320 meter	11 inches = .27940 meter
3 inches = .07620 meter	6 inches = .15240 meter	9 inches = .22860 meter	12 inches = .30480 meter

TABLE 22.—*Lengths-Meters to feet (from 1 to 1,000 units)*

[Reduction factor: 1 meter = 3.280833333 feet]

Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet	Meters	Feet
0		100	328.08333	200	656.16667	300	984.25000	400	1,312.33333	500	1,640.41667	600	1,968.50000	700	2,296.58333	800	2,624.66667	900	2,952.75000
1	3.28083	1	331.36417	1	659.44750	1	987.53083	1	1,315.61417	1	1,643.69750	1	1,971.78083	1	2,299.86417	1	2,627.94750	1	2,956.03083
2	6.56167	2	334.64500	2	662.72833	2	990.81167	2	1,318.89500	2	1,646.97833	2	1,975.06167	2	2,303.14500	2	2,631.22833	2	2,959.31167
3	9.84250	3	337.92583	3	666.00917	3	994.09250	3	1,322.17583	3	1,650.25917	3	1,978.34250	3	2,306.42583	3	2,634.50917	3	2,962.59250
4	13.12333	4	341.20667	4	669.29000	4	997.37333	4	1,325.45667	4	1,653.54000	4	1,981.62333	4	2,309.70667	4	2,637,79000	4	2,965.87333
5	16.40417	5	344 48750	5	672.57083	5	1,000.65417	5	1,328.73750	5	1,656.82083	5	1,984.90417	5	2,312.98750	5	2,641.07083	5	2,969.15417
6	19.68500	6	347.76833	6	675.85167	6	1,003.93500	6	1,332.01833	6	1,660.10167	6	1,988.18500	6	2,316.26833	6	2,644.35167	6	2,972.43500
7	22.96583	7	351.04917	7	679.13250	7	1,007.21583	7	1,335.29917	7	1,663.38250	7	1,991.46583	7	2,319.54917	7	2,647.63250	7	2,975.71583
8	26.24667	8	354.33000	8	682.41333	8	1,010.49667	8	1,338.58000	8	1,666.66333	8	1,994.74667	8	2,322.83000	8	2,650.91333	8	2,978.99667
9	29.52750	9	357.61083	9	685.69417	9	1,013.77750	9	1,341.86083	9	1,669.94417	9	1,998.02750	9	2,326.11083	9	2,654.19417	9	2,982.27750
10	32.80833	110	360.89167	210	688.97500	310	1,017.05833	410	1,345.14167	510	1,673.22500	610	2,001.30833	710	2,329.39167	810	2,657.47500	910	2,985.55833
1	36.08917	1	364.17250	1	692.25583	1	1,020 33917	1	1,348.42250	1	1,676.50583	1	2,004.58917	1	2,332.67250	1	2,660.75583	1	2,988.83917
2	39.37000	2	367.45333	2	695.53667	2	1,023.62000	2	1,351.70333	2	1,679.78667	2	2,007.87000	2	2,335.95333	2	2,664.03667	2	2,992.12000
3	42.65083	3	370.73417	3	698.81750	3	1,026.90083	3	1,354.98417	3	1,683.06750	3	2,011.15083	3	2,339.23417	3	2,667.31750	3	2,995.40083
4	45.93167	4	374.01500	4	702.09833	4	1,030.18167	4	1,358.26500	4	1,686.34833	4	2,014.43167	4	2,342.51500	4	2,670.59833	4	2,998.68167
5	49.21250	5	377.29583	5	705.37917	5	1,033.46250	5	1,361.54583	5	1,689.62917	5	2,017.71250	5	2,345.79583	5	2,673.87917	5	3,001.96250
6	52.49333	6	380.57667	6	708.66000	6	1,036.74333	6	1,364.82667	6	1,692.91000	6	2,020.99333	6	2,349.07667	6	2,677.16000	6	3,005.24333
7	55.77417	7	383.85750	7	711.94083	7	1,040.02417	7	1,368.10750	7	1,696.19083	7	2,024.27417	7	2,352.35750	7	2,680.44083	7	3,008.52417
8	59.05500	8	387.13833	8	715.22167	8	1,043.30500	8	1,371.38833	8	1,699.47167	8	2,027.55500	8	2,355.63833	8	2,683.72167	8	3,011.80500
9	62.33583	9	390.41917	9	718.50250	9	1,046.58583	9	1,374.66917	9	1,702.75250	9	2,030.83583	9	2,358.91917	9	2,687.00250	9	3,015.08583
20	65.61667	120	393.70000	220	721.78333	320	1,049.86667	420	1,377.95000	520	1,706.03333	620	2,034.11667	720	2,362.20000	820	2,690.28333	920	3,018.36667
1	68.89750	1	396.98083	1	725.06417	1	1,053.14750	1	1,381.23083	1	1,709.31417	1	2,037.39750	1	2,365.48083	1	2,693.56417	1	3,021.64750
2	72.17833	2	400.26167	2	728.34500	2	1,056.42833	2	1,384.51167	2	1,712.59500	2	2,040.67833	2	2,368.76167	2	2,696.84500	2	3,024.92833
3	75.45917	3	403.54250	3	731.62583	3	1,059.70917	3	1,387.79250	3	1,715.87583	3	2,043.95917	3	2,372.04250	3	2,700.12583	3	3,028.20917
4	78.74000	4	406.82333	4	734.90667	4	1,062.99000	4	1,391.07333	4	1,719.15667	4	2,047.24000	4	2,375.32333	4	2,703.40667	4	3,031.49000
5	82.02083	5	410.10417	5	738.18750	5	1,066.27083	5	1,394.35417	5	1,722.43750	5	2,050.52083	5	2,378.60417	5	2,706.68750	5	3,034.77083
6	85.30167	6	413.38500	6	741.46833	6	1,069.55167	6	1,397.63500	6	1,725.71833	6	2,053.80167	6	2,381.88500	6	2,709.96833	6	3,038.05167
7	88.58250	7	416.66583	7	744.74917	7	1,072.83250	7	1,400.91583	7	1,728.99917	7	2,057.08250	7	2,385.16583	7	2,713.24917	7	3,041.33250
8	91.86333	8	419.94667	8	748.03000	8	1,076.11333	8	1,404.19667	8	1,732.28000	8	2,060.36333	8	2,388.44667	8	2,716.53000	8	3,044.61333
9	95.14417	9	423.22750	9	751.31083	9	1,079.39417	9	1,407.47750	9	1,735.56083	9	2,063.64417	9	2,391.72750	9	2,719.81083	9	3,047.89417
30	98.42500	130	426.50833	230	754.59167	330	1,082.67500	430	1,410,75833	530	1,738.84167	630	2,066.92500	730	2,395.00833	830	2,723.09167	930	3,051.17500
1	101.70583	1	429.78917	1	757.87250	1	1,085.95583	1	1,414.03917	1	1,742.12250	1	2,070.20583	1	2,398.28917	1	2,726.37250	1	3,054.45583
2	104.98667	2	433.07000	2	761.15333	2	1,089.23667	2	1,417.32000	2	1,745.40333	2	2,073.48667	2	2,401.57000	2	2,729.65333	2	3,057.73667
3	108.26750	3	436.35083	3	764.43417	3	1,092.51750	3	1,420.60083	3	1,748.68417	3	2,076.76750	3	2,404.85083	3	2,732,93417	3	3,061.01750
4	111.54833	4	439.63167	4	767.71500	4	1,095.79833	4	1,423.88167	4	1,751.96500	4	2,080.04833	4	2,408.13167	4	2,736.21500	4	3,064.29833
5	114.82917	5	442.91250	5	770.99583	5	1,099.07917	5	1,427.16250	5	1,755.24583	5	2,083.32917	5	2,411.41250	5	2,739.49583	5	3,067.57917
6	118.11000	6	446.19333	6	774.27667	6	1,102.36000	6	1,430.44333	6	1,758.52667	6	2,086.61000	6	2,414.69333	6	2,742.77667	6	3,070.86000
7	121.39083	7	449.47417	7	777.55750	7	1,105.64083	7	1,433.72417	7	1,761.80750	7	2,089.89083	7	2,417.97417	7	2,746.05750	7	3,074.14083
8	124.67167	8	452.75500	8	780.83833	8	1,108.92167	8	1,437.00500	8	1,765.08833	8	2,093.17167	8	2,421.25500	8	2,749.33833	8	3,077.42167
9	127.95250	9	456.03583	9	784.11917	9	1,112.20250	9	1,440.28583	9	1,768.36917	9	2,096.45250	9	2,424.53583	9	2,752.61917	9	3,080.70250
40	131.23333	140	459.31667	240	787.40000	340	1,115.48333	440	1,443.56667	540	1,771.65000	640	2,099.73333	740	2,427.81667	840	2,755.90000	940	3,083.98333
1	134.51417	1	462.59750	1	790.68083	1	1,118.76417	1	1,446.84750	1	1,774.93083	1	2,103.01417	1	2,431.09750	1	2,759.18083	1	3,087.26417
2	137.79500	2	465.87833	2	793.96167	2	1,122.04500	2	1,450.12833	2	1,778.21167	2	2,106.29500	2	2,434.37833	2	2,762.46167	2	3,090.54500
3	141.07583	3	469.15917	3	797.24250	3	1,125.32583	3	1,453.40917	3	1,781.49250	3	2,109.57583	3	2,437.65917	3	2,765.74250	3	3,093.82583
4	144.35667	4	472.44000	4	800.52333	4	1,128.60667	4	1,456.69000	4	1,784.77333	4	2,112.85667	4	2,440.94000	4	2,769.02333	4	3,097.10667
5	147.63750	5	475.72083	5	803.80417	5	1,131.88750	5	1,459.97083	5	1,788.05417	5	2,116.13750	5	2,444.22083	5	2,772.30417	5	3,100.38750
6	150.91833	6	479.00167	6	807.08500	6	1,135.16833	6	1,463.25167	6	1,791.33500	6	2,119.41833	6	2,447.50167	6	2,775.58500	6	3,103.66833
7	154.19917	7	482.28250	7	810.36583	7	1,138.44917	7	1,466.53250	7	1,794.61583	7	2,122.69917	7	2,450.78250	7	2,778.86583	7	3,106.94917
8	157.48000	8	485.56333	8	813.64667	8	1,141.73000	8	1,469.81333	8	1,797.89667	8	2,125.98000	8	2,454.06333	8	2,782.14667	8	3,110.23000
9	160.76083	9	488.84417	9	816.92750	9	1,145.01083	9	1,473.09417	9	1,801.17750	9	2,129.26083	9	2,457.34417	9	2,785.42750	9	3,113.51083

50	164.04167	150	492.12500	250	820.20833	350	1,148.29167	450	1,476.37500	550	1,804.45833	650	2,132.54167	750	2,460.62500	850	2,788.70833	950	3,116.79167
1	167 32250	1	495.40583	1	823.48917	1	1,151.57250	1	1,479.65583	1	1,807.73917	1	2,135.82250	1	2,463.90583	1	2,791.98917	1	3,120.07250
2	170.60333	2	498.68667	2	826.77000	2	1,154.85333	2	1,482.93667	2	1,811.02000	2	2,139.10333	2	2,467.18667	2	2,795.27000	2	3,123.35333
3	173.88417	3	501.96750	3	830.05083	3	1,158.13417	3	1,486.21750	3	1,814.30083	3	2,142.38417	3	2,470.46750	3	2,798.55083	3	3,126.63417
4	177.16500	4	505.24833	4	833.33167	4	1,161.41500	4	1,489.49833	4	1,817.58167	4	2,145.66500	4	2,473.74833	4	2,801.83167	4	3,129.91500
5	180.44583	5	508.52917	5	836.61250	5	1,164.69583	5	1,492.77917	5	1,820.86250	5	2,148.94583	5	2,477.02917	5	2,805.11250	5	3,133.19583
6	183.72667	6	511.81000	6	839.89333	6	1,167.97667	6	1,496.06000	6	1,824.14333	6	2,152.22667	6	2,480.31000	6	2,808.39333	6	3,136.47667
7	187.00750	7	515.09083	7	843.17417	7	1,171.25750	7	1,499.34083	7	1,827.42417	7	2,155.50750	7	2,483.59083	7	2,811.67417	7	3,139.75750
8	190.28833	8	518.37167	8	846.45500	8	1,174.53833	8	1,502.62167	8	1,830.70500	8	2,158.78833	8	2,486.87167	8	2,814.95500	8	3,143.03833
9	193.56917	9	521.65250	9	849.73583	9	1,177.81917	9	1,505.90250	9	1,833.98583	9	2,162.06917	9	2,490.15250	9	2,818.23583	9	3,146.31917
60	196.85000	160	524.93333	260	853.01667	360	1,181.10000	460	1,509.18333	560	1,837.26667	660	2,165.35000	760	2,493 43333	860	2,821.51667	960	3,149.60000
1	200.13083	1	528.21417	1	856.29750	1	1,184.38083	1	1,512.46417	1	1,840.54750	1	2,168.63083	1	2,496.71417	1	2,824.79750	1	3,152.88083
2	203.41167	2	531.49500	2	859.57833	2	1,187.66167	2	1,515.74500	2	1,843.82833	2	2,171.91167	2	2,499.99500	2	2,828.07833	2	3,156.16167
3	206.69250	3	534.77583	3	862.85917	3	1,190.94250	3	1,519.02583	3	1,847.10917	3	2,175.19250	3	2,503.27583	3	2,831.35917	3	3,159.44250
4	209.97333	4	538.05667	4	866.14000	4	1,194.22333	4	1,522.30667	4	1,850.39000	4	2,178.47333	4	2,506.55667	4	2,834.64000	4	3,162.72333
5	213.25417	5	541.33750	5	869.42083	5	1,197.50417	5	1,525.58750	5	1,853.67083	5	2,181.75417	5	2,509.83750	5	2,837.92083	5	3,166.00417
6	216.53500	6	544.61833	6	872.70167	6	1,200.78500	6	1,528.86833	6	1,856.95167	6	2,185.03500	6	2,513.11833	6	2,841.20167	6	3,169.28500
7	219.81583	7	547.89917	7	875.98250	7	1,204.06583	7	1,532.14917	7	1,860.23250	7	2,188.31583	7	2,516.39917	7	2,844.48250	7	3,172.56583
8	223.09667	8	551.18000	8	879.26333	8	1,207.34667	8	1,535.43000	8	1,863.51333	8	2,191.59667	8	2,519.68000	8	2,847.76333	8	3,175.84667
9	226.37750	9	554.46083	9	882.54417	9	1,210.62750	9	1,538.71083	9	1,866.79417	9	2,194.87750	9	2,522.96083	9	2,851.04417	9	3,179.12750
70	229.65833	170	557.74167	270	885.82500	370	1,213.90833	470	1,541.99167	570	1,870.07500	670	2,198.15833	770	2,526.24167	870	2,854.32500	970	3,182.40833
1	232.93917	1	561.02250	1	889.10583	1	1,217.18917	1	1,545.27250	1	1,873.35583	1	2,201.43917	1	2,529.52250	1	2,857.60583	1	3,185.68917
2	236.22000	2	564.30333	2	892.38667	2	1,220.47000	2	1,548.55333	2	1,876.63667	2	2,204.72000	2	2,532.80333	2	2,860.88667	2	3,188.97000
3	239.50083	3	567.58417	3	895.66750	3	1,223.75083	3	1,551.83417	3	1,879.91750	3	2,208.00083	3	2,536.08417	3	2,864.16750	3	3,192.25083
4	242.78167	4	570.86500	4	898.94833	4	1,227.03167	4	1,555.11500	4	1,883.19833	4	2,211.28167	4	2,539.36500	4	2,867.44833	4	3,195.53167
5	246.06250	5	574.14583	5	902.22917	5	1,230.31250	5	1,558.39583	5	1,886.47917	5	2,214.56250	5	2,542.64583	5	2,870.72917	5	3,198.81250
6	249.34333	6	577.42667	6	905.51000	6	1,233.59333	6	1,561.67667	6	1,889.76000	6	2,217.84333	6	2,545.92667	6	2,874.01000	6	3,202.09333
7	252.62417	7	580.70750	7	908.79083	7	1,236.87417	7	1,564.95750	7	1,893.04083	7	2,221.12417	7	2,549.20750	7	2,877.29083	7	3,205.37417
8	255.90500	8	583.98833	8	912.07167	8	1,240.15500	8	1,568.23833	8	1,896.32167	8	2,224.40500	8	2,552.48833	8	2,880.57167	8	3,208.65500
9	259.18583	9	587.26917	9	915.35250	9	1,243.43583	9	1,571.51917	9	1,899.60250	9	2,227.68583	9	2,555.76917	9	2,883.85250	9	3,211.93583
80	262.46667	180	590.55000	280	918.63333	380	1,246.71667	480	1,574.80000	580	1,902.88333	680	2,230.96667	780	2,559.05000	880	2,887.13333	980	3,215.21667
1	265.74750	1	593.83083	1	921.91417	1	1,249.99750	1	1,578.08083	1	1,906.16417	1	2,234.24750	1	2,562.33083	1	2,890.41417	1	3,218.49750
2	269.02833	2	597.11167	2	925.19500	2	1,253.27833	2	1,581.36167	2	1,909.44500	2	2,237.52833	2	2,565.61167	2	2,893.69500	2	3,221.77833
3	272.30917	3	600.39250	3	928.47583	3	1,256.55917	3	1,584.64250	3	1,912.72583	3	2,240.80917	3	2,568.89250	3	2,896.97583	3	3,225.05917
4	275.59000	4	603.67333	4	931.75667	4	1,259.84000	4	1,587.92333	4	1,916.00667	4	2,244.09000	4	2,572.17333	4	2,900.25667	4	3,228.34000
5	278.87083	5	606.95417	5	935.03750	5	1,263.12083	5	1,591.20417	5	1,919.28750	5	2,247.37083	5	2,575.45417	5	2,903.53750	5	3,231.62083
6	282.15167	6	610.23500	6	938.31833	6	1,266.40167	6	1,594.48500	6	1,922.56833	6	2,250.65167	6	2,578.73500	6	2,906.81833	6	3,234.90167
7	285.43250	7	613.51583	7	941.59917	7	1,269.68250	7	1,597.76583	7	1,925.84917	7	2,253.93250	7	2,582.01583	7	2,910.09917	7	3,238.18250
8	288.71333	8	616.79667	8	944.88000	8	1,272.96333	8	1,601.04667	8	1,929.13000	8	2,257.21333	8	2,585.29667	8	2,913.38000	8	3,241.46333
9	291.99417	9	620.07750	9	948.16083	9	1,276.24417	9	1,604.32750	9	1,932.41083	9	2,260.49417	9	2,588.57750	9	2,916.66083	9	3,244.74417
90	295.27500	190	623.35833	290	951.44167	390	1,279.52500	490	1,607.60833	590	1,935.69167	690	2,263.77500	790	2,591.85833	890	2,919.94167	990	3,248.02500
1	298.55583	1	626.63917	1	954.72250	1	1,282.80583	1	1,610.88917	1	1,938.97250	1	2,267.05583	1	2,595.13917	1	2,923.22250	1	3,251.30583
2	301.83667	2	629.92000	2	958.00333	2	1,286.08667	2	1,614.17000	2	1,942.25333	2	2,270.33667	2	2,598.42000	2	2,926.50333	2	3,254.58667
3	305.11750	3	633.20083	3	961.28417	3	1,289.36750	3	1,617.45083	3	1,945.53417	3	2,273.61750	3	2,601.70083	3	2,929.78417	3	3,257.86750
4	308.39833	4	636.48167	4	964.56500	4	1,292.64833	4	1,620.73167	4	1,948.81500	4	2,276.89833	4	2,604.98167	4	2,933.06500	4	3,261.14833
5	311.67917	5	639.76250	5	967.84583	5	1,295.92917	5	1,624.01250	5	1,952.09583	5	2,280.17917	5	2,608.26250	5	2,936.34583	5	3,264.42917
6	314.96000	6	643.04333	6	971.12667	6	1,299.21000	6	1,627.29333	6	1,955.37667	6	2,283.46000	6	2,611.54333	6	2,939.62667	6	3,267.71000
7	318.24083	7	646.32417	7	974.40750	7	1,302.49083	7	1,630.57417	7	1,958.65750	7	2,286.74083	7	2,614.82417	7	2,942.90750	7	3,270.99083
8	321.52167	8	649.60500	8	977.68833	8	1,305.77167	8	1,633.85500	8	1,961.93833	8	2,290.02167	8	2,618.10500	8	2,946.18833	8	3,274.27167
9	324.80250	9	652.88583	9	980.96917	9	1,309.05250	9	1,637.13583	9	1,965.21917	9	2,293.30250	9	2,621.38583	9	2,949.46917	9	3,277.55250

TABLE 23.—*Corrections to log s and log Δλ for difference in arc and sine for position computation*

Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)	Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)	Log s (−)	Log difference (units of eighth decimal place)	Log Δλ (+)
3.3760	1	1.8850	4.8270	799	3.3360	5.1780	4025	3.6870
3.5260	2	2.0350	4.8380	841	3.3470	5.1830	4119	3.6920
3.6140	3	2.1230	4.8500	889	3.3590	5.1880	4215	3.6970
3.6770	4	2.1860	4.8620	939	3.3710	5.1940	4333	3.7030
3.7250	5	2.2340	4.8710	979	3.3800	5.1990	4434	3.7080
3.7650	6	2.2740	4.8820	1030	3.3910	5.2040	4537	3.7130
3.7980	7	2.3070	4.8920	1078	3.4010	5.2090	4643	3.7180
3.8270	8	2.3360	4.9040	1140	3.4130	5.2140	4751	3.7230
3.8530	9	2.3620	4.9130	1188	3.4220	5.2190	4862	3.7280
3.8760	10	2.3850	4.9220	1238	3.4310	5.2240	4975	3.7330
4.0260	20	2.5350	4.9330	1303	3.4420	5.2290	5091	3.7380
4.1140	30	2.6230	4.9420	1358	3.4510	5.2330	5186	3.7420
4.1770	40	2.6860	4.9520	1422	3.4610	5.2380	5306	3.7470
4.2250	50	2.7340	4.9590	1468	3.4680	5.2420	5405	3.7510
4.2650	60	2.7740	4.9680	1530	3.4770	5.2470	5531	3.7560
4.2980	70	2.8070	4.9780	1603	3.4870	5.2520	5660	3.7610
4.3270	80	2.8360	4.9860	1663	3.4950	5.2560	5765	3.7650
4.3530	90	2.8620	4.9930	1717	3.5020	5.2600	5872	3.7690
4.3760	100	2.8850	5.0020	1790	3.5110	5.2650	6009	3.7740
4.3960	110	2.9050	5.0100	1857	3.5190	5.2690	6121	3.7780
4.4150	120	2.9240	5.0170	1918	3.5260	5.2740	6263	3.7830
4.4330	130	2.9420	5.0250	1990	3.5340	5.2780	6380	3.7870
4.4490	140	2.9580	5.0330	2064	3.5420	5.2820	6498	3.7910
4.4640	150	2.9730	5.0400	2132	3.5490	5.2860	6619	3.7950
4.4780	160	2.9870	5.0480	2212	3.5570	5.2900	6742	3.7990
4.4910	170	3.0000	5.0550	2285	3.5640	5.2940	6868	3.8030
4.5030	180	3.0120	5.0620	2359	3.5710	5.2990	7027	3.8080
4.5260	200	3.0350	5.0680	2425	3.5770	5.3030	7158	3.8120
4.5560	230	3.0650	5.0750	2505	3.5840	5.3070	7291	3.8160
4.5750	250	3.0840	5.0820	2587	3.5910	5.3110	7427	3.8200
4.5910	270	3.1000	5.0890	2672	3.5980	5.3150	7565	3.8240
4.6140	300	3.1230	5.0950	2747	3.6040	5.3190	7705	3.8280
4.6350	330	3.1440	5.1020	2837	3.6110	5.3230	7849	3.8320
4.6540	360	3.1630	5.1080	2916	3.6170	5.3270	7995	3.8360
4.6710	390	3.1800	5.1140	2998	3.6230	5.3310	8143	3.8400
4.6870	420	3.1960	5.1210	3096	3.6300	5.3350	8295	3.8440
4.7020	450	3.2110	5.1270	3183	3.6360	5.3390	8449	3.8480
4.7160	480	3.2250	5.1330	3272	3.6420	5.3430	8606	3.8520
4.7340	521	3.2430	5.1390	3364	3.6480	5.3470	8766	3.8560
4.7500	561	3.2590	5.1450	3458	3.6540			
4.7610	590	3.2700	5.1500	3538	3.6590			
4.7750	629	3.2840	5.1560	3637	3.6650			
4.7890	671	3.2980	5.1610	3722	3.6700			
4.8010	709	3.3100	5.1670	3826	3.6760			
4.8130	750	3.3220	5.1720	3916	3.6810			

TABLE 24.—*Arc-sine corrections for inverse position computation*

Log s_1	Arc-sine correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$	Log s_1	Arc-sine correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$	Log s_1	Arc-sine correction in units of seventh decimal of logarithms	Log $\Delta\phi$ or log $\Delta\lambda$
4.177	1	2.686	5.223	124	3.732	5.525	497	4.034
4.327	2	2.836	5.234	130	3.743	5.530	508	4.039
4.415	3	2.924	5.243	136	3.752	5.534	519	4.043
4.478	4	2.987	5.253	142	3.762	5.539	530	4.048
4.526	5	3.035	5.260	147	3.769	5.543	541	4.052
4.566	6	3.075	5.269	153	3.778	5.548	553	4.057
4.599	7	3.108	5.279	160	3.788	5.553	565	4.062
4.628	8	3.137	5.287	166	3.796	5.557	577	4.066
4.654	9	3.163	5.294	172	3.803	5.561	588	4.070
4.677	10	3.186	5.303	179	3.812	5.566	600	4.075
4.697	11	3.206	5.311	186	3.820	5.570	613	4.079
4.716	12	3.225	5.318	192	3.827	5.575	625	4.084
4.734	13	3.243	5.326	199	3.835	5.579	637	4.088
4.750	14	3.259	5.334	206	3.843	5.583	650	4.092
4.765	15	3.274	5.341	213	3.850	5.587	663	4.096
4.779	16	3.288	5.349	221	3.858	5.591	674	4.100
4.792	17	3.301	5.356	228	3.865	5.595	687	4.104
4.804	18	3.313	5.363	236	3.872	5.600	702	4.109
4.827	20	3.336	5.369	243	3.878	5.604	716	4.113
4.857	23	3.366	5.376	251	3.885	5.608	729	4.117
4.876	25	3.385	5.383	259	3.892	5.612	743	4.121
4.892	27	3.401	5.390	267	3.899	5.616	757	4.125
4.915	30	3.424	5.396	275	3.905	5.620	771	4.129
4.936	33	3.445	5.403	284	3.912	5.624	785	4.133
4.955	36	3.464	5.409	292	3.918	5.628	800	4.137
4.972	39	3.481	5.415	300	3.924	5.632	814	4.141
4.988	42	3.497	5.422	309	3.931	5.636	829	4.145
5.003	45	3.512	5.428	318	3.937	5.640	845	4.149
5.017	48	3.526	5.434	327	3.943	5.644	861	4.153
5.035	52	3.544	5.440	336	3.949	5.648	877	4.157
5.051	56	3.560	5.446	345	3.955	5.652	893	4.161
5.062	59	3.571	5.451	354	3.960	5.656	909	4.165
5.076	63	3.585	5.457	364	3.966	5.660	925	4.169
5.090	67	3.599	5.462	373	3.971	5.663	941	4.172
5.102	71	3.611	5.468	383	3.977	5.667	957	4.176
5.114	75	3.623	5.473	392	3.982	5.671	973	4.180
5.128	80	3.637	5.479	402	3.988	5.674	989	4.183
5.139	84	3.648	5.484	412	3.993	5.678	1005	4.187
5.151	89	3.660	5.489	422	3.998			
5.163	94	3.672	5.495	433	4.004			
5.172	98	3.681	5.500	443	4.009			
5.183	103	3.692	5.505	453	4.014			
5.193	108	3.702	5.510	464	4.019			
5.205	114	3.714	5.515	474	4.024			
5.214	119	3.723	5.520	486	4.029			

TABLE 25.—*Mean refraction*, r_m

[Pressure=760 *mm*; Temperature=10° *C*; Relative humidity=60%]

z	00′	10′	20′	30′	40′	50′	60′
°	″	″	″	″	″	″	″
0	0.0	0.2	0.3	0.5	0.7	0.8	1.0
1	1.0	1.2	1.3	1.5	1.7	1.9	2.0
2	2.0	2.2	2.4	2.5	2.7	2.9	3.0
3	3.0	3.2	3.4	3.5	3.7	3.9	4.0
4	4.0	4.2	4.4	4.6	4.7	4.9	5.1
5	5.1	5.2	5.4	5.6	5.7	5.9	6.1
6	6.1	6.3	6.4	6.6	6.8	6.9	7.1
7	7.1	7.3	7.5	7.6	7.8	8.0	8.1
8	8.1	8.3	8.5	8.7	8.8	9.0	9.2
9	9.2	9.3	9.5	9.7	9.9	10.0	10.2
10	10.2	10.4	10.6	10.7	10.9	11.1	11.3
11	11.3	11.4	11.6	11.8	12.0	12.1	12.3
12	12.3	12.5	12.7	12.8	13.0	13.2	13.4
13	13.4	13.5	13.7	13.9	14.1	14.3	14.4
14	14.4	14.6	14.8	15.0	15.2	15.3	15.5
15	15.5	15.7	15.9	16.1	16.2	16.4	16.6
16	16.6	16.8	17.0	17.2	17.3	17.5	17.7
17	17.7	17.9	18.1	18.3	18.4	18.6	18.8
18	18.8	19.0	19.2	19.4	19.6	19.8	19.9
19	19.9	20.1	20.3	20.5	20.7	20.9	21.1
20	21.1	21.3	21.5	21.7	21.8	22.0	22.2
21	22.2	22.4	22.6	22.8	23.0	23.2	23.4
22	23.4	23.6	23.8	24.0	24.2	24.4	24.6
23	24.6	24.8	25.0	25.2	25.4	25.6	25.8
24	25.8	26.0	26.2	26.4	26.6	26.8	27.0
25	27.0	27.2	27.4	27.6	27.8	28.0	28.2
26	28.2	28.4	28.7	28.9	29.1	29.3	29.5
27	29.5	29.7	29.9	30.1	30.4	30.6	30.8
28	30.8	31.0	31.2	31.4	31.7	31.9	32.1
29	32.1	32.3	32.5	32.8	33.0	33.2	33.4
30	33.4	33.6	33.9	34.1	34.3	34.6	34.8
31	34.8	35.0	35.2	35.5	35.7	35.9	36.2
32	36.2	36.4	36.6	36.9	37.1	37.4	37.6
33	37.6	37.8	38.1	38.3	38.6	38.8	39.0
34	39.0	39.3	39.5	39.8	40.0	40.3	40.5
35	40.5	40.8	41.0	41.3	41.5	41.8	42.1
36	42.1	42.3	42.6	42.8	43.1	43.3	43.6
37	43.6	43.9	44.1	44.4	44.7	44.9	45.2
38	45.2	45.5	45.8	46.0	46.3	46.6	46.9
39	46.9	47.1	47.4	47.7	48.0	48.3	48.6
40	48.6	48.8	49.1	49.4	49.7	50.0	50.3
41	50.3	50.6	50.9	51.2	51.5	51.8	52.1
42	52.1	52.4	52.7	53.0	53.3	53.6	54.0
43	54.0	54.3	54.6	54.9	55.2	55.5	55.9
44	55.9	56.2	56.5	56.9	57.2	57.5	57.9
45	57.9	58.2	58.5	58.9	59.2	59.6	59.9
46	59.9	60.2	60.6	61.0	61.3	61.7	62.0
47	62.0	62.4	62.8	63.1	63.5	63.9	64.2
48	64.2	64.6	65.0	65.4	65.7	66.1	66.5
49	66.5	66.9	67.3	67.7	68.1	68.5	68.9
50	68.9	69.3	69.7	70.1	70.6	71.0	71.4
51	71.4	71.8	72.2	72.7	73.1	73.6	74.0
52	74.0	74.4	74.9	75.3	75.8	76.2	76.7
53	76.7	77.2	77.6	78.1	78.6	79.1	79.5
54	79.5	80.0	80.5	81.0	81.5	82.0	82.5
55	82.5	83.0	83.5	84.1	84.6	85.1	85.6
56	85.6	86 2	86.7	87.3	87.8	88.4	88.9
57	88.9	89.5	90.1	90.7	91.2	91.8	92.4
58	92.4	93.0	93.6	94.2	94.8	95.5	96.1
59	96.1	96.7	97.4	98.0	98.6	99.3	100.0

TABLE 25.—*Mean refraction*, r_m—Continued

[Pressure=760 *mm*; Temperature=10° *C*; Relative humidity=60%]

z	00′	10′	20′	30′	40′	50′	60′
°	″	″	″	″	″	″	″
60	100.0	100.6	101.3	102.0	102.7	103.4	104.1
61	104.1	104.8	105.5	106.3	107.0	107.7	108.5
62	108.5	109.2	110.0	110.8	111.6	112.4	113.2
63	113.2	114.0	114.8	115.6	116.5	117.3	118.2
64	118.2	119.0	119.9	120.8	121.7	122.6	123.5
65	123.5	124.5	125.4	126.4	127.4	128.3	129.3
66	129.3	130.3	131.4	132.4	133.4	134.5	135.6
67	135.6	136.7	137.8	138.9	140.0	141.2	142.3
68	142.3	143.5	144.7	145.9	147.2	148.4	149.7
69	149.7	151.0	152.3	153.6	155.0	156.4	157.8
70	157.8	159.2	160.6	162.1	163.6	165.1	166.6
71	166.6	168.2	169.7	171.4	173.0	174.7	176.3
72	176.3	178.1	179.8	181.6	183.4	185.3	187.2
73	187.2	189.1	191.0	193.0	195.1	197.1	199.2
74	199.2	201.4	203.6	205.8	208.1	210.4	212.8
75	212.8	215.2	217.7	220.2	222.8	225.5	228.2
76	228.2	230.9	233.7	236.6	239.6	242.6	245.7
77	245.7	248.9	252.1	255.4	258.9	262.3	265.9
78	265.9	269.6	273.4	277.2	281.2	285.3	289.5
79	289.5	293.8	298.2	302.8	307.5	312.3	317.3
80	317.3	322.4	327.7	333.2	338.8	344.6	350.6
81	350.6	356.8	363.2	369.8	376.6	383.7	391.1
82	391.1	398.7	406.6	414.8	423.3	432.1	441.3
83	441.3	450.9	460.9	471.2	482.0	493.3	505.1
84	505.1	517.4	530.3	543.8	558.0	572.8	588.4
85	588.4	604.4	621.6	639.7	658.8	678.9	700.2
86	700.2	722.7	746.6	771.8	798.7	827.2	857.6
87	857.6	890.0	924.7	961.6	1,001.3	1,043.9	1,089.7
88	1,089.7	1,138.9	1,192.0	1,249.2	1,311.4	1,378.6	1,452.0
89	1,452.0	1,531.7	1,618.8	1,714.0	1,818.4	1,933.1	2,059.5

TABLE 26.—*Pressure correction factor*, C_B

Apply to mean refraction in table 25

[$r=(r_m)(C_B)(C_T)$]

Barometer		C_B	Barometer		C_B	Barometer		C_B	Barometer		C_B	Barometer		C_B
Inches	*mm*		*Inches*	*mm*		*Inches*	*mm*		*Inches*	*mm*		*Inches*	*mm*	
20.0	508	0.670	22.4	569	0.749	24.8	630	0.829	27.2	691	0.909	29.6	752	0.989
20.1	511	0.673	22.5	572	0.752	24.9	632	0.832	27.3	693	0.912	29.7	754	0.992
20.2	513	0.676	22.6	574	0.755	25.0	635	0.835	27.4	696	0.916	29.8	757	0.996
20.3	516	0.679	22.7	576	0.759	25.1	637	0.838	27.5	699	0.920	29.9	759	0.999
20.4	518	0.682	22.8	579	0.762	25.2	640	0.842	27.6	701	0.923	30.0	762	1.003
20.5	521	0.685	22.9	582	0.766	25.3	643	0.846	27.7	704	0.926	30.1	765	1.007
20.6	523	0.688	23.0	584	0.770	25.4	645	0.849	27.8	706	0.929	30.2	767	1.010
20.7	526	0.692	23.1	587	0.773	25.5	648	0.853	27.9	709	0.933	30.3	770	1.013
20.8	528	0.696	23.2	589	0.776	25.6	650	0.856	28.0	711	0.936	30.4	772	1.016
20.9	531	0.699	23.3	592	0.779	25.7	653	0.859	28.1	714	0.939	30.5	775	1.020
21.0	533	0.703	23.4	594	0.783	25.8	655	0.862	28.2	716	0.942	30.6	777	1.023
21.1	536	0.706	23.5	597	0.786	25.9	658	0.866	28.3	719	0.946	30.7	780	1.026
21.2	538	0.709	23.6	599	0.789	26.0	660	0.869	28.4	721	0.949	30.8	782	1.029
21.3	541	0.712	23.7	602	0.792	26.1	663	0.872	28.5	724	0.953	30.9	785	1.033
21.4	544	0.716	23.8	605	0.796	26.2	665	0.875	28.6	726	0.956	31.0	787	1.036
21.5	546	0.719	23.9	607	0.799	26.3	668	0.879	28.7	729	0.959			
21.6	549	0.722	24.0	610	0.803	26.4	671	0.882	28.8	732	0.963			
21.7	551	0.725	24.1	612	0.806	26.5	673	0.885	28.9	734	0.966			
21.8	554	0.729	24.2	615	0.809	26.6	676	0.889	29.0	737	0.970			
21.9	556	0.732	24.3	617	0.813	26.7	678	0.892	29.1	739	0.973			
22.0	559	0.735	24.4	620	0.816	26.8	681	0.896	29.2	742	0.976			
22.1	561	0.739	24.5	622	0.820	26.9	683	0.899	29.3	744	0.979			
22.2	564	0.742	24.6	625	0.823	27.0	686	0.902	29.4	747	0.983			
22.3	566	0.746	24.7	627	0.826	27.1	688	0.905	29.5	749	0.986			

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TABLE 27.—*Temperature correction factor, C_T*

[Apply to mean refraction in table 25]

[$r=(r_m)\ (C_B)\ (C_T)$]

Temperature		C_T	Temperature		C_T	Temperature		C_T	Temperature		C_T	Temperature		C_T
Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade		Fahrenheit	Centigrade	
°	°		°	°		°	°		°	°		°	°	
−25	−31.7	1.172	8	−13.3	1.089	41	5.0	1.018	74	23.3	0.955	107	41.7	0.900
−24	−31.1	1.169	9	−12.8	1.087	42	5.6	1.016	75	23.9	0.953	108	42.2	0.899
−23	−30.6	1.166	10	−12.2	1.085	43	6.1	1.014	76	24.4	0.952	109	42.8	0.897
−22	−30.0	1.164	11	−11.7	1.082	44	6.7	1.012	77	25.0	0.950	110	43.3	0.895
−21	−29.4	1.161	12	−11.1	1.080	45	7.2	1.010	78	25.6	0.948	111	43.9	0.894
−20	−28.9	1.158	13	−10.6	1.078	46	7.8	1.008	79	26.1	0.946	112	44.4	0.892
−19	−28.3	1.156	14	−10.0	1.076	47	8.3	1.006	80	26.7	0.945	113	45.0	0.891
−18	−27.8	1.153	15	− 9.4	1.073	48	8.9	1.004	81	27.2	0.943	114	45.6	0.890
−17	−27.2	1.151	16	− 8.9	1.071	49	9.4	1.002	82	27.8	0.941	115	46.1	0.888
−16	−26.7	1.148	17	− 8.3	1.069	50	10.0	1.000	83	28.3	0.939	116	46.7	0.886
−15	−26.1	1.145	18	− 7.8	1.067	51	10.6	0.998	84	28.9	0.938	117	47.2	0.885
−14	−25.6	1.143	19	− 7.2	1.064	52	11.1	0.996	85	29.4	0.936	118	47.8	0.884
−13	−25.0	1.140	20	− 6.7	1.062	53	11.7	0.994	86	30.0	0.934	119	48.3	0.882
−12	−24.4	1.138	21	− 6.1	1.060	54	12.2	0.992	87	30.6	0.933	120	48.9	0.881
−11	−23.9	1.135	22	− 5.6	1.058	55	12.8	0.990	88	31.1	0.931	121	49.4	0.880
−10	−23.3	1.133	23	− 5.0	1.056	56	13.3	0.988	89	31.7	0.929	122	50.0	0.878
− 9	−22.8	1.130	24	− 4.4	1.054	57	13.9	0.986	90	32.2	0.928	123	50.6	0.877
− 8	−22.2	1.128	25	− 3.9	1.051	58	14.4	0.985	91	32.8	0.926	124	51.1	0.876
− 7	−21.7	1.125	26	− 3.3	1.049	59	15.0	0.983	92	33.3	0.924	125	51.7	0.874
− 6	−21.1	1.123	27	− 2.8	1.047	60	15.6	0.981	93	33.9	0.923	126	52.2	0.873
− 5	−20.6	1.120	28	− 2.2	1.045	61	16.1	0.979	94	34.4	0.921	127	52.8	0.871
− 4	−20.0	1.118	29	− 1.7	1.043	62	16.7	0.977	95	35.0	0.919	128	53.3	0.870
− 3	−19.4	1.115	30	− 1.1	1.041	63	17.2	0.975	96	35.6	0.917	129	53.9	0.868
− 2	−18.9	1.113	31	− 0.6	1.039	64	17.8	0.973	97	36.1	0.916	130	54.4	0.867
− 1	−18.3	1.111	32	0.0	1.036	65	18.3	0.972	98	36.7	0.914			
0	−17.8	1.108	33	+ 0.6	1.034	66	18.9	0.970	99	37.2	0.912			
+ 1	−17.2	1.106	34	1.1	1.032	67	19.4	0.968	100	37.8	0.911			
2	−16.7	1.103	35	1.7	1.030	68	20.0	0.966	101	38.3	0.909			
3	−16.1	1.101	36	2.2	1.028	69	20.6	0.964	102	38.9	0.908			
4	−15.6	1.099	37	2.8	1.026	70	21.1	0.962	103	39.4	0.906			
5	−15.0	1.096	38	3.3	1.024	71	21.7	0.961	104	40.0	0.905			
6	−14.4	1.094	39	3.9	1.022	72	22.2	0.959	105	40.6	0.903			
7	−13.9	1.092	40	4.4	1.020	73	22.8	0.957	106	41.1	0.902			

TABLE 28.—*Log* $\frac{1}{1-a}$ [$=$*colog* $(1-a)$]

(1) When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
9.00	0.045758	5869	5980	6092	6204	6317	6429	6542	6656	6769	6883
8.99	0.044660	4769	4878	4987	5096	5205	5315	5425	5536	5647	5758
98	3591	3697	3803	3909	4016	4122	4229	4337	4444	4552	4660
97	2549	2652	2755	2858	2962	3066	3171	3275	3380	3486	3591
96	1532	1633	1733	1834	1936	2037	2139	2241	2343	2446	2549
95	0.040541	0639	0737	0836	0935	1034	1133	1232	1332	1432	1532
94	0.039575	9670	9766	9862	9959	*0055	*0152	*0249	*0346	*0443	*0541
93	8633	8726	8819	8913	9007	9101	9195	9290	9385	9480	9575
92	7714	7805	7896	7987	8079	8171	8263	8355	8447	8540	8633
91	6818	6907	6996	7085	7174	7263	7353	7443	7533	7624	7714
8.90	0.035944	6031	6118	6204	6291	6379	6466	6554	6642	6730	6818
89	5092	5177	5261	5346	5431	5516	5601	5687	5772	5858	5944
88	4261	4343	4426	4508	4591	4674	4757	4841	4924	5008	5092
87	3451	3531	3611	3692	3772	3853	3934	4016	4097	4179	4261
86	2660	2738	2816	2895	2974	3053	3132	3211	3291	3371	3451
85	0.031888	1965	2041	2118	2195	2272	2349	2426	2504	2582	2660
84	1136	1210	1285	1360	1435	1510	1585	1660	1736	1812	1888
83	0.030402	0474	0547	0620	0693	0766	0840	0914	0987	1061	1136
82	0.029685	9756	9827	9898	9970	*0041	*0113	*0185	*0257	*0329	*0402
81	8987	9056	9125	9194	9264	9334	9404	9474	9544	9615	9685
8.80	0.028305	8372	8440	8508	8576	8644	8712	8780	8849	8918	8987
79	7640	7705	7771	7838	7904	7970	8037	8103	8170	8237	8305
78	6990	7055	7119	7183	7248	7313	7378	7443	7509	7574	7640
77	6357	6420	6482	6545	6608	6672	6735	6799	6862	6926	6990
76	5739	5800	5861	5923	5984	6046	6108	6170	6232	6294	6357
75	0.025136	5195	5255	5315	5375	5435	5496	5556	5617	5678	5739
74	4547	4605	4664	4722	4781	4840	4899	4958	5017	5076	5136
73	3973	4029	4086	4143	4201	4258	4316	4373	4431	4489	4547
72	3412	3467	3523	3579	3635	3691	3747	3803	3859	3916	3973
71	2865	2919	2973	3027	3082	3137	3191	3246	3301	3357	3412
8.70	0.022331	2383	2436	2489	2543	2596	2649	2703	2757	2811	2865
69	1809	1861	1913	1964	2016	2068	2121	2173	2225	2278	2331
68	1301	1351	1401	1452	1503	1553	1604	1655	1707	1758	1809
67	0804	0853	0902	0952	1001	1051	1100	1150	1200	1250	1301
66	0.020319	0367	0415	0463	0512	0560	0609	0657	0706	0755	0804
65	0.019846	9893	9940	9987	*0034	*0081	*0128	*0176	*0223	*0271	*0319
64	9384	9430	9475	9521	9567	9613	9660	9706	9752	9799	9846
63	8933	8978	9022	9067	9112	9157	9202	9247	9293	9338	9384
62	8493	8536	8580	8624	8667	8711	8755	8800	8844	8888	8933
61	8063	8105	8148	8191	8233	8276	8319	8363	8406	8449	8493
8.60	0.017643	7685	7726	7768	7810	7852	7894	7936	7978	8020	8063
59	7233	7274	7315	7355	7396	7437	7478	7519	7560	7602	7643
58	6833	6873	6913	6952	6992	7032	7072	7112	7153	7193	7233
57	6443	6482	6520	6559	6598	6637	6676	6715	6755	6794	6833
56	6062	6099	6137	6175	6213	6251	6289	6328	6366	6404	6443
55	0.015689	5726	5763	5800	5837	5874	5912	5949	5986	6024	6062
54	5326	5362	5398	5434	5470	5507	5543	5579	5616	5653	5689
53	4971	5006	5041	5077	5112	5147	5183	5218	5254	5290	5326
52	4624	4659	4693	4727	4762	4797	4831	4866	4901	4936	4971
51	4286	4319	4353	4387	4420	4454	4488	4522	4556	4590	4624
8.50	0.013955	3988	4021	4054	4087	4120	4153	4186	4219	4253	4286

Proportional parts

	111	108	105	102	99
1	11.1	10.8	10.5	10.2	9.9
2	22.2	21.6	21.0	20.4	19.8
3	33.3	32.4	31.5	30.6	29.7
4	44.4	43.2	42.0	40.8	39.6
5	55.5	54.0	52.5	51.0	49.5
6	66.6	64.8	63.0	61.2	59.4
7	77.7	75.6	73.5	71.4	69.3
8	88.8	86.4	84.0	81.6	79.2
9	99.9	97.2	94.5	91.8	89.1
	96	93	90	87	84
1	9.6	9.3	9.0	8.7	8.4
2	19.2	18.6	18.0	17.4	16.8
3	28.8	27.9	27.0	26.1	25.2
4	38.4	37.2	36.0	34.8	33.6
5	48.0	46.5	45.0	43.5	42.0
6	57.6	55.8	54.0	52.2	50.4
7	67.2	65.1	63.0	60.9	58.8
8	76.8	74.4	72.0	69.6	67.2
9	86.4	83.7	81.0	78.3	75.6
	81	78	75	72	69
1	8.1	7.8	7.5	7.2	6.9
2	16.2	15.6	15.0	14.4	13.8
3	24.3	23.4	22.5	21.6	20.7
4	32.4	31.2	30.0	28.8	27.6
5	40.5	39.0	37.5	36.0	34.5
6	48.6	46.8	45.0	43.2	41.4
7	56.7	54.6	52.5	50.4	48.3
8	64.8	62.4	60.0	57.6	55.2
9	72.9	70.2	67.5	64.8	62.1
	66	63	60	57	55
1	6.6	6.3	6.0	5.7	5.5
2	13.2	12.6	12.0	11.4	11.0
3	19.8	18.9	18.0	17.1	16.5
4	26.4	25.2	24.0	22.8	22.0
5	33.0	31.5	30.0	28.5	27.5
6	39.6	37.8	36.0	34.2	33.0
7	46.2	44.1	42.0	39.9	38.5
8	52.8	50.4	48.0	45.6	44.0
9	59.4	56.7	54.0	51.3	49.5
	53	51	49	47	45
1	5.3	5.1	4.9	4.7	4.5
2	10.6	10.2	9.8	9.4	9.0
3	15.9	15.3	14.7	14.1	13.5
4	21.2	20.4	19.6	18.8	18.0
5	26.5	25.5	24.5	23.5	22.5
6	31.8	30.6	29.4	28.2	27.0
7	37.1	35.7	34.3	32.9	31.5
8	42.4	40.8	39.2	37.6	36.0
9	47.7	45.9	44.1	42.3	40.5
	43	41	39	37	35
1	4.3	4.1	3.9	3.7	3.5
2	8.6	8.2	7.8	7.4	7.0
3	12.9	12.3	11.7	11.1	10.5
4	17.2	16.4	15.6	14.8	14.0
5	21.5	20.5	19.5	18.5	17.5
6	25.8	24.6	23.4	22.2	21.0
7	30.1	28.7	27.3	25.9	24.5
8	34.4	32.8	31.2	29.6	28.0
9	38.7	36.9	35.1	33.3	31.5

TABLE 28.—*Log* $\frac{1}{1-a}$ [$= colog\ (1-a)$]—Continued

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.50	0.013955	3988	4021	4054	4087	4120	4153	4186	4219	4253	4286
49	3633	3665	3697	3729	3761	3793	3825	3858	3890	3923	3955
48	3318	3349	3380	3411	3443	3474	3506	3537	3569	3601	3633
47	3010	3040	3071	3101	3132	3163	3194	3225	3256	3287	3318
46	2709	2739	2769	2799	2829	2859	2889	2919	2949	2979	3010
45	0.012416	2445	2474	2503	2532	2562	2591	2621	2650	2680	2709
44	2129	2158	2186	2215	2243	2272	2300	2329	2358	2387	2416
43	1849	1877	1905	1933	1961	1989	2017	2045	2073	2101	2129
42	1576	1603	1630	1657	1685	1712	1739	1767	1794	1822	1849
41	1309	1335	1362	1388	1415	1442	1468	1495	1522	1549	1576
8.40	0.011048	1074	1100	1126	1152	1178	1204	1230	1256	1283	1309
39	0794	0819	0844	0869	0895	0920	0946	0971	0997	1023	1048
38	0545	0570	0594	0619	0644	0669	0694	0718	0743	0769	0794
37	0302	0326	0350	0374	0399	0423	0447	0472	0496	0520	0545
36	0.010065	0088	0112	0135	0159	0183	0207	0230	0254	0278	0302
35	0.009833	9856	9879	9902	9925	9948	9972	9995	*0018	*0041	*0065
34	9607	9629	9652	9674	9697	9719	9742	9765	9787	9810	9833
33	9386	9408	9430	9452	9474	9496	9518	9540	9562	9584	9607
32	9170	9191	9213	9234	9256	9277	9299	9320	9342	9364	9386
31	8959	8980	9001	9022	9043	9064	9085	9106	9127	9149	9170
8.30	0.008753	8773	8794	8814	8835	8855	8876	8897	8917	8938	8959
29	8552	8572	8592	8612	8632	8652	8672	8692	8712	8733	8753
28	8355	8375	8394	8414	8433	8453	8473	8492	8512	8532	8552
27	8163	8182	8201	8220	8239	8259	8278	8297	8316	8336	8355
26	7976	7994	8013	8031	8050	8069	8088	8106	8125	8144	8163
25	0.007792	7811	7829	7847	7865	7884	7902	7920	7939	7957	7976
24	7614	7631	7649	7667	7685	7702	7720	7738	7756	7774	7792
23	7439	7456	7473	7491	7508	7526	7543	7561	7578	7596	7614
22	7268	7285	7302	7319	7336	7353	7370	7387	7404	7421	7439
21	7101	7118	7134	7151	7167	7184	7201	7218	7234	7251	7268
8.20	0.006938	6954	6971	6987	7003	7019	7036	7052	7068	7085	7101
19	6779	6795	6811	6826	6842	6858	6874	6890	6906	6922	6938
18	6624	6639	6654	6670	6685	6701	6716	6732	6748	6763	6779
17	6472	6487	6502	6517	6532	6547	6562	6578	6593	6608	6624
16	6323	6338	6353	6367	6382	6397	6412	6427	6442	6457	6472
15	0.006178	6193	6207	6221	6236	6250	6265	6279	6294	6309	6323
14	6037	6051	6065	6079	6093	6107	6121	6135	6150	6164	6178
13	5898	5912	5926	5940	5953	5967	5981	5995	6009	6023	6037
12	5763	5777	5790	5803	5817	5830	5844	5857	5871	5885	5898
11	5631	5644	5657	5670	5684	5697	5710	5723	5737	5750	5763
8.10	0.005502	5515	5528	5541	5553	5566	5579	5592	5605	5618	5631
09	5376	5389	5401	5414	5426	5439	5451	5464	5477	5489	5502
08	5253	5265	5277	5290	5302	5314	5327	5339	5351	5364	5376
07	5133	5145	5157	5169	5181	5193	5205	5217	5229	5241	5253
06	5015	5027	5038	5050	5062	5074	5085	5097	5109	5121	5133
05	4900	4912	4923	4935	4946	4957	4969	4980	4992	5004	5015
04	4788	4799	4810	4822	4833	4844	4855	4866	4878	4889	4900
03	4679	4690	4700	4711	4722	4733	4744	4755	4766	4777	4788
02	4572	4582	4593	4603	4614	4625	4636	4646	4657	4668	4679
01	4467	4477	4488	4498	4509	4519	4529	4540	4550	4561	4572
8.00	0.004365	4375	4385	4395	4405	4416	4426	4436	4446	4457	4467

Proportional parts

	34	33	32	31	30
1	3.4	3.3	3.2	3.1	3.0
2	6.8	6.6	6.4	6.2	6.0
3	10.2	9.9	9.6	9.3	9.0
4	13.6	13.2	12.8	12.4	12.0
5	17.0	16.5	16.0	15.5	15.0
6	20.4	19.8	19.2	18.6	18.0
7	23.8	23.1	22.4	21.7	21.0
8	27.2	26.4	25.6	24.8	24.0
9	30.6	29.7	28.8	27.9	27.0
	29	28	27	26	25
1	2.9	2.8	2.7	2.6	2.5
2	5.8	5.6	5.4	5.2	5.0
3	8.7	8.4	8.1	7.8	7.5
4	11.6	11.2	10.8	10.4	10.0
5	14.5	14.0	13.5	13.0	12.5
6	17.4	16.8	16.2	15.6	15.0
7	20.3	19.6	18.9	18.2	17.5
8	23.2	22.4	21.6	20.8	20.0
9	26.1	25.2	24.3	23.4	22.5
	24	23	22	21	20
1	2.4	2.3	2.2	2.1	2.0
2	4.8	4.6	4.4	4.2	4.0
3	7.2	6.9	6.6	6.3	6.0
4	9.6	9.2	8.8	8.4	8.0
5	12.0	11.5	11.0	10.5	10.0
6	14.4	13.8	13.2	12.6	12.0
7	16.8	16.1	15.4	14.7	14.0
8	19.2	18.4	17.6	16.8	16.0
9	21.6	20.7	19.8	18.9	18.0
	19	18	17	16	15
1	1.9	1.8	1.7	1.6	1.5
2	3.8	3.6	3.4	3.2	3.0
3	5.7	5.4	5.1	4.8	4.5
4	7.6	7.2	6.8	6.4	6.0
5	9.5	9.0	8.5	8.0	7.5
6	11.4	10.8	10.2	9.6	9.0
7	13.3	12.6	11.9	11.2	10.5
8	15.2	14.4	13.6	12.8	12.0
9	17.1	16.2	15.3	14.4	13.5
	14	13	12	11	10
1	1.4	1.3	1.2	1.1	1.0
2	2.8	2.6	2.4	2.2	2.0
3	4.2	3.9	3.6	3.3	3.0
4	5.6	5.2	4.8	4.4	4.0
5	7.0	6.5	6.0	5.5	5.0
6	8.4	7.8	7.2	6.6	6.0
7	9.8	9.1	8.4	7.7	7.0
8	11.2	10.4	9.6	8.8	8.0
9	12.6	11.7	10.8	9.9	9.0

TABLE 28 —Log $\frac{1}{1-a}$ [$=$colog $(1-a)$]—Continued

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.00	0.004365	4375	4385	4395	4405	4416	4426	4436	4446	4457	4467
7.99	4265	4275	4285	4295	4305	4315	4325	4335	4345	4355	4365
98	4167	4177	4187	4196	4206	4216	4226	4235	4245	4255	4265
97	4072	4082	4091	4100	4110	4119	4129	4139	4148	4158	4167
96	3979	3988	3997	4007	4016	4025	4035	4044	4053	4063	4072
95	0.003888	3897	3906	3915	3924	3933	3942	3951	3961	3970	3979
94	3799	3808	3817	3826	3834	3843	3852	3861	3870	3879	3888
93	3712	3721	3729	3738	3747	3755	3764	3773	3782	3790	3799
92	3627	3636	3644	3653	3661	3670	3678	3687	3695	3704	3712
91	3545	3553	3561	3569	3577	3586	3594	3602	3611	3619	3627
7.90	0.003463	3472	3480	3488	3496	3504	3512	3520	3528	3536	3545
89	3384	3392	3400	3408	3416	3424	3432	3440	3448	3456	3463
88	3307	3315	3322	3330	3338	3345	3353	3361	3369	3377	3384
87	3231	3239	3246	3254	3261	3269	3277	3284	3292	3299	3307
86	3158	3165	3172	3180	3187	3194	3202	3209	3217	3224	3231
85	0.003086	3093	3100	3107	3114	3121	3129	3136	3143	3150	3158
84	3015	3022	3029	3036	3043	3050	3057	3064	3071	3078	3086
83	2946	2953	2960	2967	2974	2980	2987	2994	3001	3008	3015
82	2879	2886	2892	2899	2906	2912	2919	2926	2933	2939	2946
81	2813	2820	2826	2833	2839	2846	2852	2859	2866	2872	2879
7.80	0.002749	2755	2762	2768	2774	2781	2787	2794	2800	2807	2813
79	2686	2692	2699	2705	2711	2717	2724	2730	2736	2743	2749
78	2625	2631	2637	2643	2649	2655	2661	2668	2674	2680	2686
77	2565	2571	2577	2583	2589	2595	2601	2607	2613	2619	2625
76	2506	2512	2518	2524	2530	2535	2541	2547	2553	2559	2565
75	0.002449	2455	2460	2466	2472	2478	2483	2489	2495	2501	2506
74	2393	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449
73	2339	2344	2349	2355	2360	2366	2371	2377	2382	2388	2393
72	2285	2290	2296	2301	2306	2312	2317	2322	2328	2333	2339
71	2233	2238	2243	2249	2254	2259	2264	2269	2275	2280	2285
7.70	0.002182	2187	2192	2197	2202	2207	2213	2218	2223	2228	2233
69	2132	2137	2142	2147	2152	2157	2162	2167	2172	2177	2182
68	2084	2088	2093	2098	2103	2108	2113	2118	2122	2127	2132
67	2036	2041	2046	2050	2055	2060	2065	2069	2074	2079	2084
66	1990	1994	1999	2003	2008	2013	2017	2022	2027	2031	2036
65	0.001944	1949	1953	1958	1962	1967	1971	1976	1980	1985	1990
64	1900	1904	1909	1913	1918	1922	1926	1931	1935	1940	1944
63	1857	1861	1865	1869	1874	1878	1882	1887	1891	1896	1900
62	1814	1818	1823	1827	1831	1835	1840	1844	1848	1852	1857
61	1773	1777	1781	1785	1789	1793	1798	1802	1806	1810	1814
7.60	0.001732	1736	1740	1744	1748	1753	1757	1761	1765	1769	1773
59	1693	1697	1701	1705	1709	1713	1716	1720	1724	1728	1732
58	1654	1658	1662	1666	1670	1673	1677	1681	1685	1689	1693
57	1617	1620	1624	1628	1632	1635	1639	1643	1647	1650	1654
56	1580	1583	1587	1591	1594	1598	1602	1605	1609	1613	1617
55	0.001544	1547	1551	1554	1558	1562	1565	1569	1572	1576	1580
54	1508	1512	1515	1519	1522	1526	1529	1533	1537	1540	1544
53	1474	1477	1481	1484	1488	1491	1495	1498	1502	1505	1508
52	1440	1444	1447	1450	1454	1457	1461	1464	1467	1471	1474
51	1408	1411	1414	1417	1421	1424	1427	1431	1434	1437	1440
7.50	0.001376	1379	1382	1385	1388	1391	1395	1398	1401	1404	1408

Proportional parts

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

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TABLE 28.—$Log \frac{1}{1-a}$ [$= colog\ (1-a)$]—Continued

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.50	0.001376	1379	1382	1385	1388	1391	1395	1398	1401	1404	1408
49	1344	1347	1350	1354	1357	1360	1363	1366	1369	1372	1376
48	1314	1317	1320	1323	1326	1329	1332	1335	1338	1341	1344
47	1284	1287	1290	1292	1295	1298	1301	1304	1307	1311	1314
46	1254	1257	1260	1263	1266	1269	1272	1275	1278	1281	1284
45	0.001226	1229	1231	1234	1237	1240	1243	1246	1249	1251	1254
44	1198	1201	1203	1206	1209	1212	1214	1217	1220	1223	1226
43	1170	1173	1176	1179	1181	1184	1187	1190	1192	1195	1198
42	1144	1146	1149	1152	1154	1157	1160	1162	1165	1168	1170
41	1118	1120	1123	1126	1128	1131	1133	1136	1139	1141	1144
7.40	0.001092	1095	1097	1100	1102	1105	1107	1110	1113	1115	1118
39	1067	1070	1072	1075	1077	1080	1082	1085	1087	1090	1092
38	1043	1045	1048	1050	1053	1055	1058	1060	1062	1065	1067
37	1019	1022	1024	1026	1029	1031	1033	1036	1038	1041	1043
36	0996	0998	1001	1003	1005	1008	1010	1012	1015	1017	1019
35	0.000973	0976	0978	0980	0982	0985	0987	0989	0991	0994	0996
34	951	953	956	958	960	962	964	967	969	971	973
33	929	932	934	936	938	940	942	945	947	949	951
32	908	910	913	915	917	919	921	923	925	927	929
31	888	890	892	894	896	898	900	902	904	906	908
7.30	0.000867	869	871	873	875	877	879	882	884	886	888
29	848	850	852	854	855	857	859	861	863	865	867
28	828	830	832	834	836	838	840	842	844	846	848
27	809	811	813	815	817	819	821	823	825	826	828
26	791	793	795	796	798	800	802	804	806	808	809
25	0.000773	775	777	778	780	782	784	786	787	789	791
24	755	757	759	761	762	764	766	768	769	771	773
23	738	740	742	743	745	747	748	750	752	754	755
22	721	723	725	726	728	730	731	733	735	736	738
21	705	707	708	710	711	713	715	716	718	720	721
7.20	0.000689	690	692	694	695	697	698	700	702	703	705
19	673	675	676	678	679	681	683	684	686	687	689
18	658	659	661	662	664	665	667	669	670	672	673
17	643	644	646	647	649	650	652	653	655	656	658
16	628	630	631	633	634	635	637	638	640	641	643
15	0.000614	615	617	618	620	621	622	624	625	627	628
14	600	601	603	604	605	607	608	610	611	612	614
13	586	588	589	590	592	593	594	596	597	599	600
12	573	574	576	577	578	580	581	582	584	585	586
11	560	561	562	564	565	566	568	569	570	572	573
7.10	0.000547	548	550	551	552	553	555	556	557	559	560
09	535	536	537	538	540	541	542	543	545	546	547
08	522	524	525	526	527	529	530	531	532	533	535
07	511	512	513	514	515	516	518	519	520	521	522
06	499	500	501	502	504	505	506	507	508	509	511
05	0.000488	489	490	491	492	493	494	495	497	498	499
04	476	478	479	480	481	482	483	484	485	486	488
03	466	467	468	469	470	471	472	473	474	475	476
02	455	456	457	458	459	460	461	462	463	465	466
01	445	446	447	448	449	450	451	452	453	454	455
7.00	0.000435	436	437	438	439	440	441	442	443	444	445

Proportional parts

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	2	1
1	0.2	0.1
2	0.4	0.2
3	0.6	0.3
4	0.8	0.4
5	1.0	0.5
6	1.2	0.6
7	1.4	0.7
8	1.6	0.8
9	1.8	0.9

TABLE 28.—*Log* $\frac{1}{1-a}$ [$= colog\ (1-a)$]—Continued

When a is positive:

Log a	0	1	2	3	4	5	6	7	8	9	10
7.00	0.000435	436	437	438	439	440	441	442	443	444	445
6.9	345	353	361	370	378	387	396	405	415	425	435
8	274	280	287	294	301	308	315	322	330	337	345
7	218	223	228	233	239	244	250	256	262	268	274
6	173	177	181	185	190	194	199	203	208	213	218
5	0.000137	141	144	147	151	154	158	161	165	169	173
4	109	112	114	117	120	122	125	128	131	134	137
3	087	089	091	093	095	097	100	102	104	107	109
2	69	70	72	74	75	77	79	81	83	85	87
1	55	56	57	59	60	61	63	64	66	67	69
6.0	0.000043	44	45	47	48	49	50	51	52	53	55
5.9	34	35	36	37	38	39	40	41	41	42	43
8	27	28	29	29	30	31	31	32	33	34	34
7	22	22	23	23	24	24	25	26	26	27	27
6	17	18	18	19	19	19	20	20	21	21	22
5	0.000014	14	14	15	15	15	16	16	17	17	17
4	11	11	11	12	12	12	13	13	13	13	14
3	09	09	09	09	10	10	10	10	10	11	11
2	7	7	7	7	8	8	8	8	8	8	9
1	5	6	6	6	6	6	6	6	7	7	7
5.0	0.000004	4	5	5	5	5	5	5	5	5	5
4	0.000000	1	1	1	1	1	2	2	3	3	4

(2) When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
4 n	0.000000	*9999	*9999	*9999	*9999	*9999	*9998	*9998	*9997	*9997	*9996
5.0 n	9.999996	96	95	95	95	95	95	95	95	95	95
1 n	95	94	94	94	94	94	94	94	93	93	93
2 n	93	93	93	93	92	92	92	92	92	92	91
3 n	91	91	91	91	90	90	90	90	90	89	89
4 n	89	89	89	88	88	88	87	87	87	87	86
5 n	9.999986	86	86	85	85	85	84	84	83	83	83
6 n	83	82	82	81	81	81	80	80	79	79	78
7 n	78	78	77	77	76	76	75	74	74	73	73
8 n	73	72	71	71	70	69	69	68	67	66	66
9 n	66	65	64	63	62	61	60	59	59	58	57
6.0 n	9.999957	56	55	53	52	51	50	49	48	47	45
1 n	45	44	43	41	40	39	37	36	34	33	31
2 n	31	30	28	26	25	23	21	19	17	15	13
3 n	913	911	909	907	905	903	901	898	896	893	891
4 n	891	888	886	883	880	878	875	872	869	866	863
5 n	9.999863	859	856	853	849	846	842	839	835	831	827
6 n	827	823	819	815	810	806	802	797	792	787	782
7 n	782	777	772	767	761	756	750	744	738	732	726
8 n	726	720	713	706	700	693	685	678	671	663	655
9 n	655	647	639	631	622	613	604	595	585	576	566
7.00 n	9.999566	565	564	563	562	561	560	559	558	557	556

Proportional parts

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

	2	1
1	0.2	0.1
2	0.4	0.2
3	0.6	0.3
4	0.8	0.4
5	1.0	0.5
6	1.2	0.6
7	1.4	0.7
8	1.6	0.8
9	1.8	0.9

TABLE 28.—$Log \frac{1}{1-a}$ [$=colog\ (1-a)$]—Continued

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
7. 00 n	9. 999566	565	564	563	562	561	560	559	558	557	556
01 n	556	555	554	553	552	551	550	549	548	547	545
02 n	545	544	543	542	541	540	539	538	537	536	535
03 n	535	534	533	532	531	530	528	527	526	525	524
04 n	524	523	522	521	520	519	517	516	515	514	513
05 n	9. 999513	512	511	510	508	507	506	505	504	503	502
06 n	502	501	499	498	497	496	495	494	492	491	490
07 n	490	489	488	487	485	484	483	482	481	479	478
08 n	478	477	476	475	473	472	471	470	469	467	466
09 n	466	465	464	462	461	460	459	457	456	455	454
7. 10 n	9. 999454	452	451	450	449	447	446	445	443	442	441
11 n	441	440	438	437	436	434	433	432	430	429	428
12 n	428	427	425	424	423	421	420	419	417	416	415
13 n	415	413	412	410	409	408	406	405	404	402	401
14 n	401	400	398	397	395	394	393	391	390	388	387
15 n	9. 999387	386	384	383	381	380	378	377	376	374	373
16 n	373	371	370	368	367	365	364	363	361	360	358
17 n	358	357	355	354	352	351	349	348	346	345	343
18 n	343	342	340	339	337	336	334	333	331	329	328
19 n	328	326	325	323	322	320	319	317	315	314	312
7. 20 n	9. 999312	311	309	307	306	304	303	301	299	298	296
21 n	296	295	293	291	290	288	286	285	283	282	280
22 n	280	278	277	275	273	272	270	268	266	265	263
23 n	263	261	260	258	256	255	253	251	249	247	246
24 n	246	244	242	241	239	237	235	234	232	230	228
25 n	9. 999228	227	225	223	221	219	218	216	214	212	210
26 n	210	209	207	205	203	201	199	198	196	194	192
27 n	192	190	188	186	185	183	181	179	177	175	173
28 n	173	171	169	168	166	164	162	160	158	156	154
29 n	154	152	150	148	146	144	142	140	138	136	134
7. 30 n	9. 999134	132	130	128	126	124	122	120	118	116	114
31 n	114	112	110	108	106	104	102	100	098	096	094
32 n	094	091	089	087	085	083	081	079	077	075	072
33 n	072	070	068	066	064	062	060	057	055	053	051
34 n	051	049	047	044	042	040	038	036	033	031	029
35 n	9. 999029	9027	9024	9022	9020	9018	9015	9013	9011	9009	9006
36 n	9006	9004	9002	8999	8997	8995	8992	8990	8988	8985	8983
37 n	8983	8981	8978	8976	8974	8971	8969	8967	8964	8962	8959
38 n	8959	8957	8955	8952	8950	8947	8945	8943	8940	8938	8935
39 n	8935	8933	8930	8928	8925	8923	8920	8918	8915	8913	8910
7. 40 n	9. 998910	8908	8905	8903	8900	8898	8895	8893	8890	8888	8885
41 n	8885	8883	8880	8877	8875	8872	8870	8867	8864	8862	8859
42 n	8859	8857	8854	8851	8849	8846	8843	8841	8838	8835	8833
43 n	8833	8830	8827	8825	8822	8819	8816	8814	8811	8808	8805
44 n	8805	8803	8800	8797	8794	8792	8789	8786	8783	8781	8778
45 n	9. 998778	8775	8772	8769	8766	8764	8761	8758	8755	8752	8749
46 n	8749	8746	8744	8741	8738	8735	8732	8729	8726	8723	8720
47 n	8720	8717	8714	8711	8708	8705	8702	8699	8696	8693	8690
48 n	8690	8687	8684	8681	8678	8675	8672	8669	8666	8663	8660
49 n	8660	8657	8654	8651	8648	8644	8641	8638	8635	8632	8629
7. 50 n	9. 998629	8626	8622	8619	8616	8613	8610	8607	8603	8600	8597

Proportional parts

	1	2
1	0. 1	0. 2
2	0. 2	0. 4
3	0. 3	0. 6
4	0. 4	0. 8
5	0. 5	1. 0
6	0. 6	1. 2
7	0. 7	1. 4
8	0. 8	1. 6
9	0. 9	1. 8

	3	4
1	0. 3	0. 4
2	0. 6	0. 8
3	0. 9	1. 2
4	1. 2	1. 6
5	1. 5	2. 0
6	1. 8	2. 4
7	2. 1	2. 8
8	2. 4	3. 2
9	2. 7	3. 6

TABLE 28.—*Log* $\frac{1}{1-a}$ [$=colog\ (1-a)$]—Continued

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
7. 50 n	9. 998629	8626	8622	8619	8616	8613	8610	8607	8603	8600	8597
51 n	8597	8594	8590	8587	8584	8581	8577	8574	8571	8568	8564
52 n	8564	8561	8558	8554	8551	8548	8544	8541	8538	8534	8531
53 n	8531	8528	8524	8521	8517	8514	8511	8507	8504	8500	8497
54 n	8497	8493	8490	8486	8483	8479	8476	8472	8469	8465	8462
55 n	9. 998462	8458	8455	8451	8448	8444	8440	8437	8433	8430	8426
56 n	8426	8422	8419	8415	8411	8408	8404	8400	8397	8393	8389
57 n	8389	8386	8382	8378	8375	8371	8367	8363	8360	8356	8352
58 n	8352	8348	8344	8341	8337	8333	8329	8325	8321	8318	8314
59 n	8314	8310	8306	8302	8298	8294	8290	8286	8282	8278	8274
7. 60 n	9. 998274	8271	8267	8263	8259	8255	8251	8246	8242	8238	8234
61 n	8234	8230	8226	8222	8218	8214	8210	8206	8202	8197	8193
62 n	8193	8189	8185	8181	8177	8172	8168	8164	8160	8156	8151
63 n	8151	8147	8143	8139	8134	8130	8126	8121	8117	8113	8108
64 n	8108	8104	8100	8095	8091	8087	8082	8078	8073	8069	8064
65 n	9. 998064	8060	8055	8051	8047	8042	8038	8033	8028	8024	8019
66 n	8019	8015	8010	8006	8001	7997	7992	7987	7983	7978	7973
67 n	7973	7969	7964	7959	7955	7950	7945	7941	7936	7931	7926
68 n	7926	7922	7917	7912	7907	7902	7898	7893	7888	7883	7878
69 n	7878	7873	7868	7863	7859	7854	7849	7844	7839	7834	7829
7. 70 n	9. 997829	7824	7819	7814	7809	7804	7799	7794	7789	7783	7778
71 n	7778	7773	7768	7763	7758	7753	7748	7742	7737	7732	7727
72 n	7727	7722	7716	7711	7706	7700	7695	7690	7685	7679	7674
73 n	7674	7669	7663	7658	7652	7647	7642	7636	7631	7625	7620
74 n	7620	7614	7609	7603	7598	7592	7587	7581	7576	7570	7565
75 n	9. 997565	7559	7553	7548	7542	7537	7531	7525	7519	7514	7508
76 n	7508	7502	7497	7491	7485	7479	7473	7468	7462	7456	7450
77 n	7450	7444	7438	7433	7427	7421	7415	7409	7403	7397	7391
78 n	7391	7385	7379	7373	7367	7361	7355	7349	7343	7337	7330
79 n	7330	7324	7318	7312	7306	7300	7293	7287	7281	7275	7268
7. 80 n	9. 997268	7262	7256	7250	7243	7237	7231	7224	7218	7211	7205
81 n	7205	7199	7192	7186	7179	7173	7166	7160	7153	7147	7140
82 n	7140	7134	7127	7120	7114	7107	7100	7094	7087	7080	7074
83 n	7074	7067	7060	7053	7047	7040	7033	7026	7019	7013	7006
84 n	7006	6999	6992	6985	6978	6971	6964	6957	6950	6943	6936
85 n	9. 996936	6929	6922	6915	6908	6901	6894	6887	6880	6872	6865
86 n	6865	6858	6851	6844	6836	6829	6822	6814	6807	6800	6792
87 n	6792	6785	6778	6770	6763	6755	6748	6740	6733	6725	6718
88 n	6718	6710	6703	6695	6688	6680	6672	6665	6657	6650	6642
89 n	6642	6634	6626	6619	6611	6603	6595	6587	6580	6572	6564
7. 90 n	9. 996564	6556	6548	6540	6532	6524	6516	6508	6500	6492	6484
91 n	6484	6476	6468	6460	6452	6444	6435	6427	6419	6411	6403
92 n	6403	6394	6386	6378	6369	6361	6353	6344	6336	6328	6319
93 n	6319	6311	6302	6294	6285	6277	6268	6260	6251	6242	6234
94 n	6234	6225	6217	6208	6199	6190	6182	6173	6164	6155	6146
95 n	9. 996146	6138	6129	6120	6111	6102	6093	6084	6075	6066	6057
96 n	6057	6048	6039	6030	6021	6012	6003	5993	5984	5975	5966
97 n	5966	5956	5947	5938	5929	5919	5910	5900	5891	5882	5872
98 n	5872	5863	5853	5844	5834	5825	5815	5805	5796	5786	5777
99 n	5777	5767	5757	5747	5738	5728	5718	5708	5698	5689	5679
8. 00 n	9. 995679	5669	5659	5649	5639	5629	5619	5609	5599	5589	5578

Proportional parts

	4	5
1	0. 4	0. 5
2	0. 8	1. 0
3	1. 2	1. 5
4	1. 6	2. 0
5	2. 0	2. 5
6	2. 4	3. 0
7	2. 8	3. 5
8	3. 2	4. 0
9	3. 6	4. 5

	6	7
1	0. 6	0. 7
2	1. 2	1. 4
3	1. 8	2. 1
4	2. 4	2. 8
5	3. 0	3. 5
6	3. 6	4. 2
7	4. 2	4. 9
8	4. 8	5. 6
9	5. 4	6. 3

	8	9
1	0. 8	0. 9
2	1. 6	1. 8
3	2. 4	2. 7
4	3. 2	3. 6
5	4. 0	4. 5
6	4. 8	5. 4
7	5. 6	6. 3
8	6. 4	7. 2
9	7. 2	8. 1

	10	11
1	1. 0	1. 1
2	2. 0	2. 2
3	3. 0	3. 3
4	4. 0	4. 4
5	5. 0	5. 5
6	6. 0	6. 6
7	7. 0	7. 7
8	8. 0	8. 8
9	9. 0	9. 9

TABLE 28.—$Log \frac{1}{1-a}$ [$=colog\ (1-a)$]—Continued

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
8. 00 n	9. 995679	5669	5659	5649	5639	5629	5619	5609	5599	5589	5578
01 n	5578	5568	5558	5548	5538	5528	5517	5507	5497	5486	5476
02 n	5476	5466	5455	5445	5434	5424	5413	5403	5392	5382	5371
03 n	5371	5361	5350	5339	5329	5318	5307	5296	5286	5275	5264
04 n	5264	5253	5242	5231	5220	5209	5198	5187	5176	5165	5154
05 n	9. 995154	5143	5132	5121	5110	5098	5087	5076	5065	5053	5042
06 n	5042	5031	5019	5008	4996	4985	4973	4962	4950	4939	4927
07 n	4927	4916	4904	4892	4881	4869	4857	4845	4833	4822	4810
08 n	4810	4798	4786	4774	4762	4750	4738	4726	4714	4702	4690
09 n	4690	4677	4665	4653	4641	4628	4616	4604	4591	4579	4567
8. 10 n	9. 994567	4554	4542	4529	4517	4504	4492	4479	4466	4454	4441
11 n	4441	4428	4415	4403	4390	4377	4364	4351	4338	4325	4312
12 n	4312	4299	4286	4273	4260	4247	4234	4220	4207	4194	4181
13 n	4181	4167	4154	4141	4127	4114	4100	4087	4073	4060	4046
14 n	4046	4032	4019	4005	3991	3978	3964	3950	3936	3922	3908
15 n	9. 993908	3894	3880	3866	3852	3838	3824	3810	3796	3782	3767
16 n	3767	3753	3739	3725	3710	3696	3681	3667	3652	3638	3623
17 n	3623	3609	3594	3579	3565	3550	3535	3521	3506	3491	3476
18 n	3476	3461	3446	3431	3416	3401	3386	3371	3356	3340	3325
19 n	3325	3310	3295	3279	3264	3248	3233	3218	3202	3186	3171
8. 20 n	9. 993171	3155	3140	3124	3108	3092	3077	3061	3045	3029	3013
21 n	3013	2997	2981	2965	2949	2933	2917	2900	2884	2868	2852
22 n	2852	2835	2819	2803	2786	2770	2753	2736	2720	2703	2687
23 n	2687	2670	2653	2636	2619	2603	2586	2569	2552	2535	2518
24 n	2518	2501	2483	2466	2449	2432	2414	2397	2380	2362	2345
25 n	9. 992345	2327	2310	2292	2275	2257	2239	2222	2204	2186	2168
26 n	2168	2150	2132	2114	2096	2078	2060	2042	2024	2006	1987
27 n	1987	1969	1951	1932	1914	1896	1877	1858	1840	1821	1803
28 n	1803	1784	1765	1746	1727	1709	1690	1671	1652	1633	1613
29 n	1613	1594	1575	1556	1537	1517	1498	1478	1459	1440	1420
8. 30 n	9. 991420	1400	1381	1361	1341	1322	1302	1282	1262	1242	1222
31 n	1222	1202	1182	1162	1142	1122	1101	1081	1061	1040	1020
32 n	1020	0999	0979	0958	0938	0917	0896	0875	0855	0834	0813
33 n	0813	0792	0771	0750	0729	0708	0686	0665	0644	0622	0601
34 n	0601	0580	0558	0537	0515	0493	0472	0450	0428	0406	0385
35 n	9. 990385	0363	0341	0319	0297	0274	0252	0230	0208	0186	0163
36 n	9. 990163	0141	0118	0096	0073	0051	0028	0005	*9982	*9960	*9937
37 n	9. 989937	9914	9891	9868	9845	9821	9798	9775	9752	9728	9705
38 n	9705	9682	9658	9634	9611	9587	9563	9540	9516	9492	9468
39 n	9468	9444	9420	9396	9372	9348	9323	9299	9275	9250	9226
8. 40 n	9. 989226	9201	9177	9152	9127	9103	9078	9053	9028	9003	8978
41 n	8978	8953	8928	8903	8877	8852	8827	8801	8776	8750	8725
42 n	8725	8699	8673	8647	8622	8596	8570	8544	8518	8492	8465
43 n	8465	8439	8413	8386	8360	8334	8307	8280	8254	8227	8200
44 n	8200	8173	8147	8120	8093	8066	8038	8011	7984	7957	7929
45 n	9. 987929	7902	7874	7847	7819	7791	7764	7736	7708	7680	7652
46 n	7652	7624	7596	7568	7539	7511	7483	7454	7426	7397	7369
47 n	7369	7340	7311	7282	7253	7224	7195	7166	7137	7108	7079
48 n	7079	7049	7020	6990	6961	6931	6902	6872	6842	6812	6782
49 n	6782	6752	6722	6692	6662	6631	6601	6571	6540	6510	6479
8. 50 n	9. 986479	6448	6418	6387	6356	6325	6294	6263	6232	6200	6169

Proportional parts

	10	11	12	13	14
1	1. 0	1. 1	1. 2	1. 3	1. 4
2	2. 0	2. 2	2. 4	2. 6	2. 8
3	3. 0	3. 3	3. 6	3. 9	4. 2
4	4. 0	4. 4	4. 8	5. 2	5. 6
5	5. 0	5. 5	6. 0	6. 5	7. 0
6	6. 0	6. 6	7. 2	7. 8	8. 4
7	7. 0	7. 7	8. 4	9. 1	9. 8
8	8. 0	8. 8	9. 6	10. 4	11. 2
9	9. 0	9. 9	10. 8	11. 7	12. 6

	15	16	17	18	19
1	1. 5	1. 6	1. 7	1. 8	1. 9
2	3. 0	3. 2	3. 4	3. 6	3. 8
3	4. 5	4. 8	5. 1	5. 4	5. 7
4	6. 0	6. 4	6. 8	7. 2	7. 6
5	7. 5	8. 0	8. 5	9. 0	9. 5
6	9. 0	9. 6	10. 2	10. 8	11. 4
7	10. 5	11. 2	11. 9	12. 6	13. 3
8	12. 0	12. 8	13. 6	14. 4	15. 2
9	13. 5	14. 4	15. 3	16. 2	17. 1

	20	21	22	23	24
1	2. 0	2. 1	2. 2	2. 3	2. 4
2	4. 0	4. 2	4. 4	4. 6	4. 8
3	6. 0	6. 3	6. 6	6. 9	7. 2
4	8. 0	8. 4	8. 8	9. 2	9. 6
5	10. 0	10. 5	11. 0	11. 5	12. 0
6	12. 0	12. 6	13. 2	13. 8	14. 4
7	14. 0	14. 7	15. 4	16. 1	16. 8
8	16. 0	16. 8	17. 6	18. 4	19. 2
9	18. 0	18. 9	19. 8	20. 7	21. 6

	25	26	27	28	29
1	2. 5	2. 6	2. 7	2. 8	2. 9
2	5. 0	5. 2	5. 4	5. 6	5. 8
3	7. 5	7. 8	8. 1	8. 4	8. 7
4	10. 0	10. 4	10. 8	11. 2	11. 6
5	12. 5	13. 0	13. 5	14. 0	14. 5
6	15. 0	15. 6	16. 2	16. 8	17. 4
7	17. 5	18. 2	18. 9	19. 6	20. 3
8	20. 0	20. 8	21. 6	22. 4	23. 2
9	22. 5	23. 4	24. 3	25. 2	26. 1

	30	31	32
1	3. 0	3. 1	3. 2
2	6. 0	6. 2	6. 4
3	9. 0	9. 3	9. 6
4	12. 0	12. 4	12. 8
5	15. 0	15. 5	16. 0
6	18. 0	18. 6	19. 2
7	21. 0	21. 7	22. 4
8	24. 0	24. 8	25. 6
9	27. 0	27. 9	28. 8

TABLE 28.—*Log* $\frac{1}{1-a}$ [$=colog\ (1-a)$]—Continued

When a is negative:

Log a	0	1	2	3	4	5	6	7	8	9	10
8.50 n	9.986479	6448	6418	6387	6356	6325	6294	6263	6232	6200	6169
51 n	6169	6138	6106	6075	6043	6011	5980	5948	5916	5884	5852
52 n	5852	5820	5788	5756	5723	5691	5659	5626	5593	5561	5528
53 n	5528	5495	5462	5429	5396	5363	5330	5297	5263	5230	5197
54 n	5197	5163	5129	5096	5062	5028	4994	4960	4926	4892	4858
55 n	9.984858	4823	4789	4755	4720	4685	4651	4616	4581	4546	4511
56 n	4511	4476	4441	4406	4370	4335	4300	4264	4228	4193	4157
57 n	4157	4121	4085	4049	4013	3977	3941	3904	3868	3831	3795
58 n	3795	3758	3721	3684	3648	3611	3573	3536	3499	3462	3424
59 n	3424	3387	3349	3312	3274	3236	3198	3160	3122	3084	3046
8.60 n	9.983046	3007	2969	2930	2892	2853	2814	2776	2737	2698	2658
61 n	2658	2619	2580	2541	2501	2462	2422	2382	2343	2303	2263
62 n	2263	2223	2183	2142	2102	2062	2021	1981	1940	1899	1858
63 n	1858	1817	1776	1735	1694	1653	1611	1570	1528	1486	1444
64 n	1444	1403	1361	1319	1276	1234	1192	1149	1107	1064	1022
65 n	9.981022	0979	0936	0893	0850	0807	0763	0720	0677	0633	0589
66 n	0589	0546	0502	0458	0414	0370	0325	0281	0237	0192	0147
67 n	9.980147	0103	0058	0013	*9968	*9923	*9878	*9832	*9787	*9741	*9695
68 n	9.979695	9650	9604	9558	9512	9466	9420	9373	9327	9280	9234
69 n	9234	9187	9140	9093	9046	8999	8952	8904	8857	8809	8762
8.70 n	9.978762	8714	8666	8618	8570	8522	8473	8425	8376	8328	8279
71 n	8279	8230	8181	8132	8083	8034	7985	7935	7885	7836	7786
72 n	7786	7736	7686	7636	7586	7535	7485	7434	7384	7333	7282
73 n	7282	7231	7180	7128	7077	7026	6974	6922	6870	6818	6766
74 n	6766	6714	6662	6610	6557	6505	6452	6399	6346	6293	6240
75 n	9.976240	6187	6133	6080	6026	5972	5918	5864	5810	5756	5702
76 n	5702	5647	5593	5538	5483	5428	5373	5318	5262	5207	5152
77 n	5152	5096	5040	4984	4928	4872	4816	4759	4703	4646	4589
78 n	4589	4532	4475	4418	4361	4304	4246	4188	4131	4073	4015
79 n	4015	3957	3898	3840	3781	3723	3664	3605	3546	3487	3428
8.80 n	9.973428	3368	3309	3249	3189	3129	3069	3009	2949	2888	2828
81 n	2828	2767	2706	2645	2584	2523	2461	2400	2338	2276	2215
82 n	2215	2153	2090	2028	1966	1903	1840	1777	1714	1651	1588
83 n	1588	1525	1461	1398	1334	1270	1206	1141	1077	1013	0948
84 n	0948	0883	0818	0753	0688	0623	0557	0492	0426	0360	0294
85 n	9.970294	0228	0161	0095	0028	*9962	*9895	*9828	*9760	*9693	*9626
86 n	9.969626	9558	9490	9422	9354	9286	9218	9149	9081	9012	8943
87 n	8943	8874	8804	8735	8666	8596	8526	8456	8386	8316	8245
88 n	8245	8175	8104	8033	7962	7891	7819	7748	7676	7604	7532
89 n	7532	7460	7388	7316	7243	7170	7097	7024	6951	6878	6804
8.90 n	9.966804	6731	6657	6583	6509	6435	6360	6285	6211	6136	6061
91 n	6061	5985	5910	5834	5759	5683	5607	5531	5454	5378	5301
92 n	5301	5224	5147	5070	4992	4915	4837	4759	4681	4603	4525
93 n	4525	4446	4368	4289	4210	4130	4051	3972	3892	3812	3732
94 n	3732	3652	3571	3491	3410	3329	3248	3167	3086	3004	2922
95 n	9.962922	2840	2758	2676	2594	2511	2428	2345	2262	2179	2095
96 n	2095	2012	1928	1844	1760	1675	1591	1506	1421	1336	1251
97 n	1251	1165	1080	0994	0908	0822	0735	0649	0562	0475	0388
98 n	9.960388	0301	0213	0126	0038	*9950	*9862	*9773	*9685	*9596	*9507
99 n	9.959507	9418	9329	9239	9149	9059	8969	8879	8789	8698	8607
9.00 n	9.958607	8516	8425	8334	8242	8150	8058	7966	7874	7781	7689

Proportional parts

	32	34	36	38	40
1	3.2	3.4	3.6	3.8	4.0
2	6.4	6.8	7.2	7.6	8.0
3	9.6	10.2	10.8	11.4	12.0
4	12.8	13.6	14.4	15.2	16.0
5	16.0	17.0	18.0	19.0	20.0
6	19.2	20.4	21.6	22.8	24.0
7	22.4	23.8	25.2	26.6	28.0
8	25.6	27.2	28.8	30.4	32.0
9	28.8	30.6	32.4	34.2	36.0
	42	**44**	**46**	**48**	**50**
1	4.2	4.4	4.6	4.8	5.0
2	8.4	8.8	9.2	9.6	10.0
3	12.6	13.2	13.8	14.4	15.0
4	16.8	17.6	18.4	19.2	20.0
5	21.0	22.0	23.0	24.0	25.0
6	25.2	26.4	27.6	28.8	30.0
7	29.4	30.8	32.2	33.6	35.0
8	33.6	35.2	36.8	38.4	40.0
9	37.8	39.6	41.4	43.2	45.0
	52	**54**	**56**	**58**	**60**
1	5.2	5.4	5.6	5.8	6.0
2	10.4	10.8	11.2	11.6	12.0
3	15.6	16.2	16.8	17.4	18.0
4	20.8	21.6	22.4	23.2	24.0
5	26.0	27.0	28.0	29.0	30.0
6	31.2	32.4	33.6	34.8	36.0
7	36.4	37.8	39.2	40.6	42.0
8	41.6	43.2	44.8	46.4	48.0
9	46.8	48.6	50.4	52.2	54.0
	62	**64**	**66**	**68**	**70**
1	6.2	6.4	6.6	6.8	7.0
2	12.4	12.8	13.2	13.6	14.0
3	18.6	19.2	19.8	20.4	21.0
4	24.8	25.6	26.4	27.2	28.0
5	31.0	32.0	33.0	34.0	35.0
6	37.2	38.4	39.6	40.8	42.0
7	43.4	44.8	46.2	47.6	49.0
8	49.6	51.2	52.8	54.4	56.0
9	55.8	57.6	59.4	61.2	63.0
	72	**74**	**76**	**78**	**80**
1	7.2	7.4	7.6	7.8	8.0
2	14.4	14.8	15.2	15.6	16.0
3	21.6	22.2	22.8	23.4	24.0
4	28.8	29.6	30.4	31.2	32.0
5	36.0	37.0	38.0	39.0	40.0
6	43.2	44.4	45.6	46.8	48.0
7	50.4	51.8	53.2	54.6	56.0
8	57.6	59.2	60.8	62.4	64.0
9	64.8	66.6	68.4	70.2	72.0
	82	**84**	**86**	**88**	**90**
1	8.2	8.4	8.6	8.8	9.0
2	16.4	16.8	17.2	17.6	18.0
3	24.6	25.2	25.8	26.4	27.0
4	32.8	33.6	34.4	35.2	36.0
5	41.0	42.0	43.0	44.0	45.0
6	49.2	50.4	51.6	52.8	54.0
7	57.4	58.8	60.2	61.6	63.0
8	65.6	67.2	68.8	70.4	72.0
9	73.8	75.6	77.4	79.2	81.0

TABLE 29.—*Curvature correction*

For azimuth observations on Polaris

Azimuth of Polaris \ 2τ	1 m	2 m	3 m	4 m	5 m	6 m	7 m	8 m	9 m	10 m
° ′	″	″	″	″	″	″	″	″	″	″
0 00	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
10	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1
20	.0	.0	.0	.0	.1	.1	.1	.2	.2	.3
30	.0	.0	.0	.1	.1	.2	.2	.3	.3	.4
40	.0	.0	.1	.1	.1	.2	.3	.4	.5	.6
50	.0	.0	.1	.1	.2	.3	.3	.5	.6	.7
1 00	.0	.0	.1	.1	.2	.3	.4	.5	.7	.9
10	.0	.0	.1	.2	.2	.4	.5	.6	.8	1.0
20	.0	.0	.1	.2	.3	.4	.6	.7	.9	1.1
30	.0	.0	.1	.2	.3	.5	.6	.8	1.0	1.3
40	.0	.1	.1	.2	.4	.5	.7	.9	1.2	1.4
50	.0	.1	.1	.3	.4	.6	.8	1.0	1.3	1.6
2 00	.0	.1	.2	.3	.4	.6	.8	1.1	1.4	1.7
10	.0	.1	.2	.3	.5	.7	.9	1.2	1.5	1.9
20	.0	.1	.2	.3	.5	.7	1.0	1.3	1.6	2.0
30	.0	.1	.2	.3	.5	.8	1.1	1.4	1.7	2.1
40	.0	.1	.2	.4	.6	.8	1.1	1.5	1.9	2.3
50	.0	.1	.2	.4	.6	.9	1.2	1.6	2.0	2.4
3 00	.0	.1	.2	.4	.6	.9	1.3	1.6	2.1	2.6
10	.0	.1	.2	.4	.7	1.0	1.3	1.7	2.2	2.7

NOTE.—2τ = Difference in time between direct and reverse pointings on Polaris.

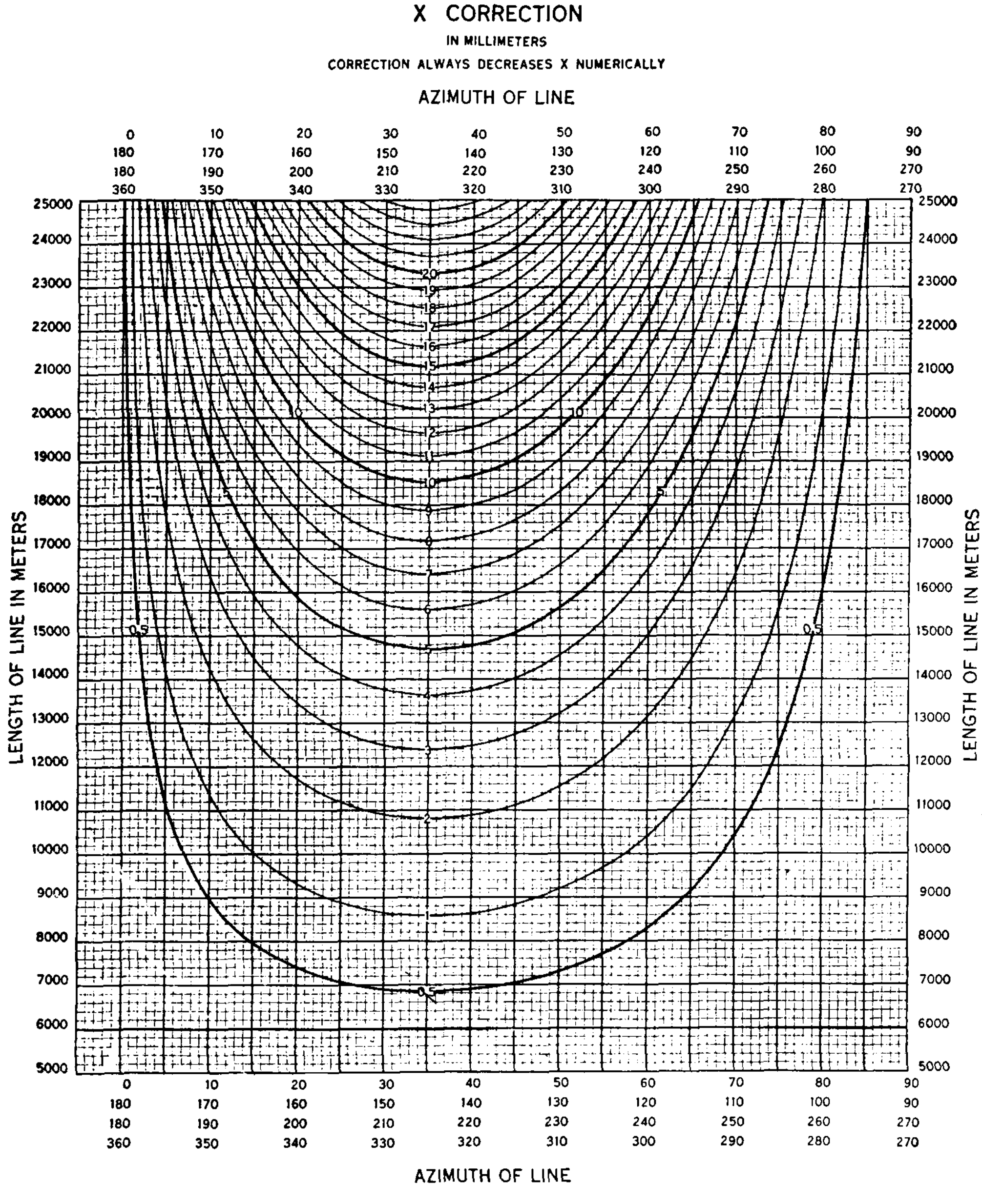

FIGURE 134.—Nomogram which may be used to determine x-correction of Form 26a. Separate copies of this and the following two nomograms may be obtained from the Washington Office on request.

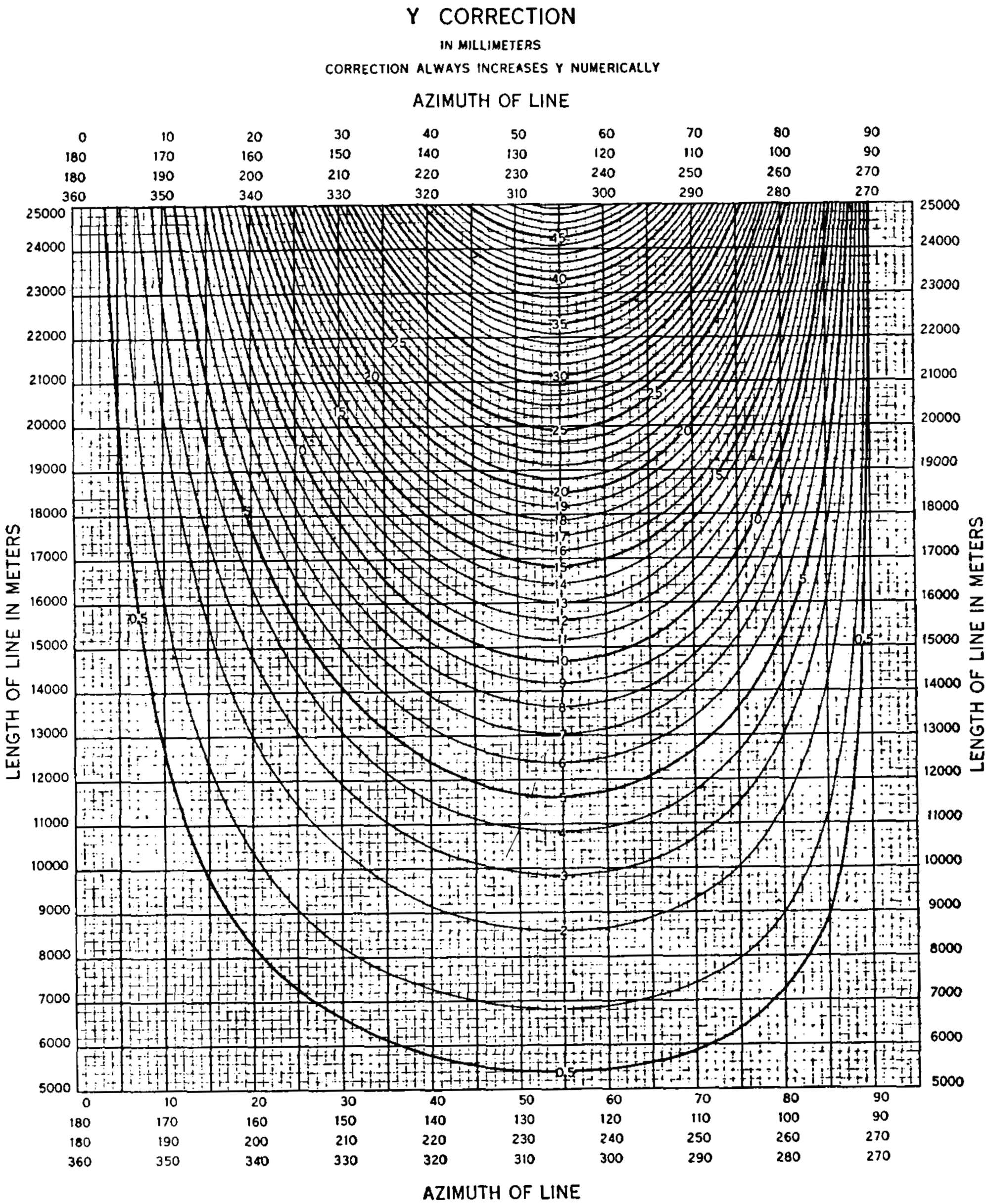

Figure 135.—Nomogram which may be used to determine *y*-correction of Form 26a.

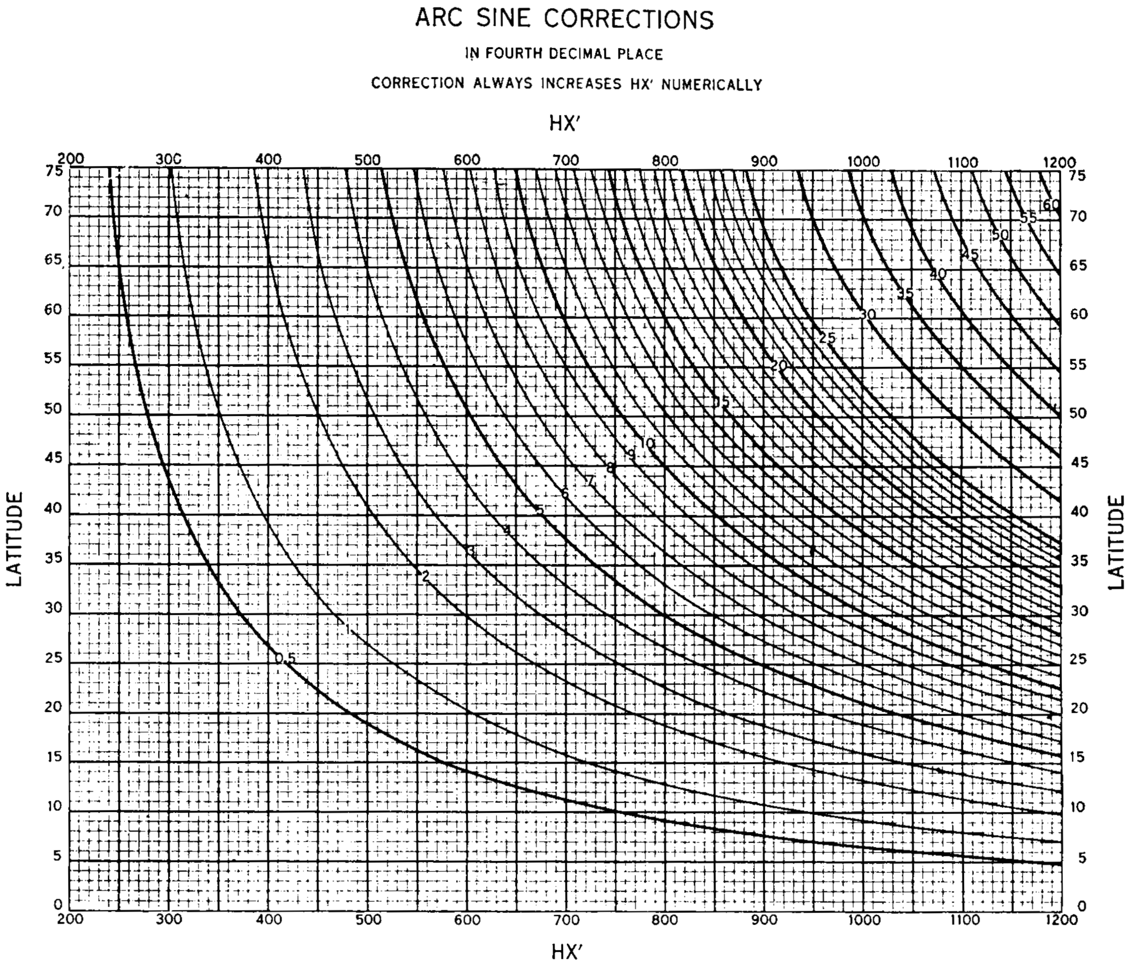

FIGURE 136.—Nomogram which may be used to determine arc-sine correction of Form 26a.

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