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THE DETERMINATION OF TRUE GROUND MOTION FROM SEISMOGRAPH RECORDS

By
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THE DETERMINATION OF TRUE GROUND MOTION FROM SEISMOGRAPH RECORDS

INTRODUCTION

This publication contains four papers that describe the results of investigations made to determine the performance of U. S. Coast and Geodetic Survey accelerographs used in measuring destructive earthquake motions, and to appraise the accuracy of integration methods employed in reducing the recorded acceleration to corresponding velocity and displacement curves. The Survey accelerographs were designed by the National Bureau of Standards in cooperation with the Coast and Geodetic Survey. Through the courtesy of the Department of Civil and Sanitary Engineering of the Massachusetts Institute of Technology a shaking table built for engineering-seismological research was made available for the instrument tests. For testing computational methods that organization also made available its mechanical differential analyzer. In the following pages a comparison is made between the results obtained on the analyzer and those obtained using a method of numerical integration developed in the Washington Office of the Coast and Geodetic Survey.

The investigations and their significance are described in four chapters as follows:

1. Tests of Earthquake Accelerometers on a Shaking Table.—A. C. Ruge and H. E. McComb.
2. Discussion of Principal Results from the Engineering Viewpoint.—A. C. Ruge.
3. An Appraisal of Numerical Integration Methods as Applied to Strong-Motion Data.—Frank Neumann.
4. Analysis of Accelerograms by Means of the Massachusetts Institute of Technology Differential Analyzer.—A. C. Ruge.

At the time of the tests, in 1941, Professor A. C. Ruge was a Research Associate in the Department of Civil and Sanitary Engineering, Massachusetts Institute of Technology; Mr. H. E. McComb was Chief, Section of Operations, Division of Geomagnetism and Seismology, U. S. Coast and Geodetic Survey; and Mr. Frank Neumann was Chief, Section of Seismology, of the same Division. Messrs. Ruge and McComb chose the records for testing and jointly conducted all of the shaking-table laboratory work. Professor Ruge directed the analytical work on the differential analyzer and discusses the engineering aspects of the investigation. Mr. Neumann describes in detail the methods of numerical integration developed in connection with the routine analysis of strong-motion seismograph records.

These investigations have special significance in the problem of designing structures in seismic areas. They establish the accuracy with which destructive earthquake motions, in terms of acceleration, velocity, and displacement, can actually be measured or computed. Accelerographs of the type discussed are in wide use on the Pacific coast as part of the Coast and Geodetic

Survey's program of seismological investigations. While most of the discussion refers to the interpretation of accelerograph records the method and conclusions are applicable to the interpretation of all seismograms obtained with direct-recording pendulums, that is, seismographs in which the motion of the pendulum is recorded directly on the record and not through a galvanometer or similar device.

As a result of these studies unifilar suspensions were installed on all accelerometer units of the Coast and Geodetic Survey. The pendulum frequencies are 10 cps and greater, the higher frequencies being used where less sensitivity is required. Experience has demonstrated that such suspensions are stable and that their natural transverse vibrations (more than 200 per second) do not interfere with earthquake recording. The results being obtained are comparable with those described in this series of papers for suspensions of the quadrifilar type. In precise analytical work distortion of the film through ordinary temperature and humidity changes is more in evidence than fluctuations that might be ascribed to pendulum instability.

Chapter I

TESTS OF EARTHQUAKE ACCELEROMETERS ON A SHAKING TABLE

By A. C. Ruge and H. E. McComb

An outline of a cooperative program on the investigation of tests of earthquake accelerometers on a shaking table and the proposed interpretation of the results was published in the October 1937 issue of the Bulletin of the Seismological Society of America under the title *Tests of Earthquake Accelerometers on a Shaking Table* by H. E. McComb and A. C. Ruge. A complete description of the shaking table and method of operation of its component parts was given in the July 1936 number of the Bulletin of the Seismological Society of America under the title *A Machine for Reproducing Earthquake Motions Direct from a Shadowgraph of the Earthquake* by A. C. Ruge. Briefly stated this program was planned:

1) To investigate the efficiency of strong-motion accelerographs when subjected to irregular forced vibrations such as may be expected during a destructive earthquake.

2) To determine the degree of independence of the component parts of the accelerograph.

3) To determine the order or degree of agreement which may be expected between actual displacements imposed upon the accelerograph by the shaking table and those obtained by integrating the recorded acceleration, using numerical integration methods developed at the U.S. Coast and Geodetic Survey.

4) To ascertain whether it is possible to detect with certainty, by integration processes, the presence of long-period waves of small amplitude—waves having periods of 30 to 90 sec. and with maximum accelerations not exceeding $0.001g$.

5) To investigate the possibility of using the differential analyzer at the Massachusetts Institute of Technology for performing the task of integrating the accelerograms and to compare the results thus obtained with those obtained by numerical integration of the same accelerograms.

A 6-inch accelerograph mounted on the shaking table to be described is shown in figure 1. A standard 12-inch accelerograph is shown in figure 2.

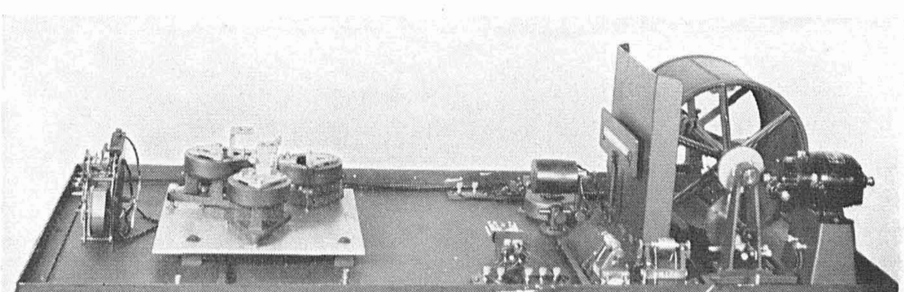


FIGURE 1.—Original 6-inch accelerograph equipped with accelerometers having quadrifilar suspensions.

In figure 6 is shown the microtilt mechanism used in this investigation for imposing upon the accelerometer small accelerations of long periods.

The nominal, static magnifications of the systems used in these tests were calculated from the dimensions of the optical and mechanical levers involved.

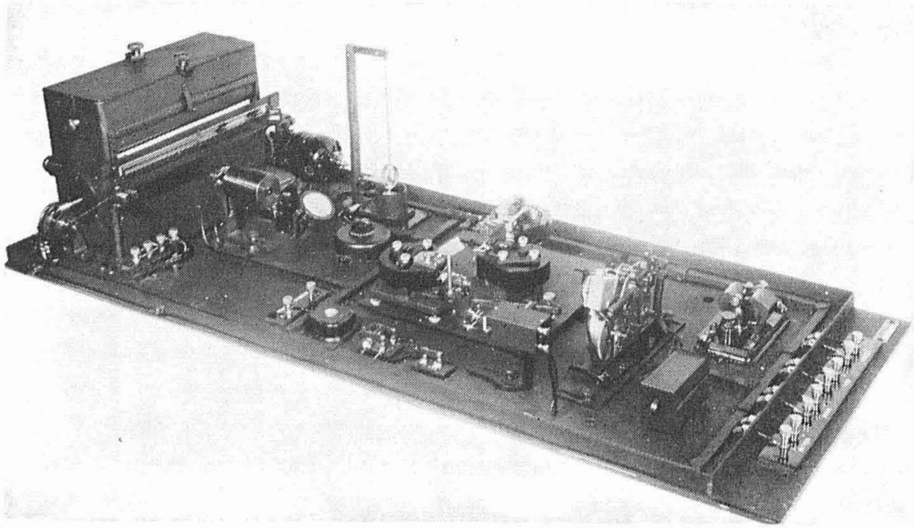


FIGURE 2.—Accelerograph, later model, equipped with 12-inch tape recorder, pendulum starter, and pivot accelerometers designed for normal and low magnifications.

The mechanical lever used in connection with the registration of the table motions is shown in the foreground of figure 5. One end of this mechanical lever rests upon a flat glass plate set at a predetermined angle to the direction of motion of the shaking table. The degree of magnification of the lever is a function of this angle and is variable over a wide range.

Fifty-one tests were made, in all. Many of these were duplicates. Of the accelerograms obtained, it was decided that a minimum of six should be analyzed in detail to cover adequately the essential aspects of the investiga-

TABLE 1.—*Descriptive list of records analyzed*

Record No.	Type of motion	Instrument	Remarks
17	Earthquake, without tilting mechanism.	Pivot no. 61 and quadrifilar.	Recording drum translating.
25	Earthquake, without tilting instrument.	Three components, all pivots	Recording drum translating.
32	Earthquake, with small tilting of instrument.	Pivot no. 61	Recording drum not translating.
39	Earthquake, with large tilting of instrument.	Quadrifilar	Recording drum not translating.
44	Smooth, arbitrary motion with large tilting of instrument.	Pivot no. 61	Recording drum not translating.
46*a*	Smooth, arbitrary motion without tilt.	Pivot no. 61	Recording drum not translating.
46*b*	Smooth, arbitrary motion without tilt but with instrument set at approximately 45 degrees to direction of motion.	Pivot no. 62	Recording drum not translating.

tion. These records are listed in table 1. Later, another record, No. 17, was used in supplemental studies.

The reasons for selecting certain records for study are obvious from the descriptions given in table 1. There is no reason to believe that the results obtained from the selected records would be materially different from any other similar group obtained under the same conditions. Every effort has been made to select the accelerograms for study on an impartial basis, attention being given, of course, to the selection of those for which very complete data existed with respect to testing conditions, sensitivity, recorded table motion, and so on, so that a maximum of useful information could be obtained.

In order to make clear the nature of the problems involved in accelerometer testing and to provide an introduction to the discussion of integration and analyses which follow this paper, the first few seconds of each accelerogram are reproduced together with the corresponding table motions. Records 25 and 32 are so nearly alike that the slightest differences are noticeable only when magnified; the recorded table motions are exactly alike so far as they can be measured. The effect of tilt is of course not visible on any of the accelerograms, the maximum tilt being only about 4 minutes of arc, which

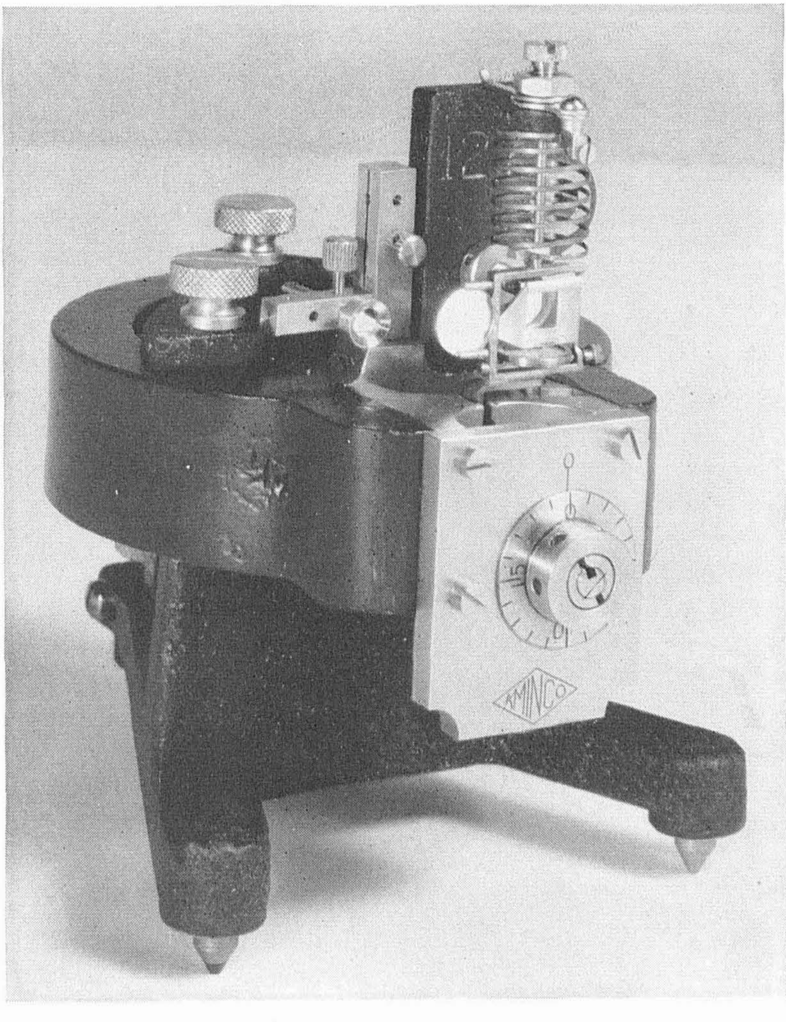


FIGURE 3.—Enlarged view of single-component accelerometer with pivoted vane.

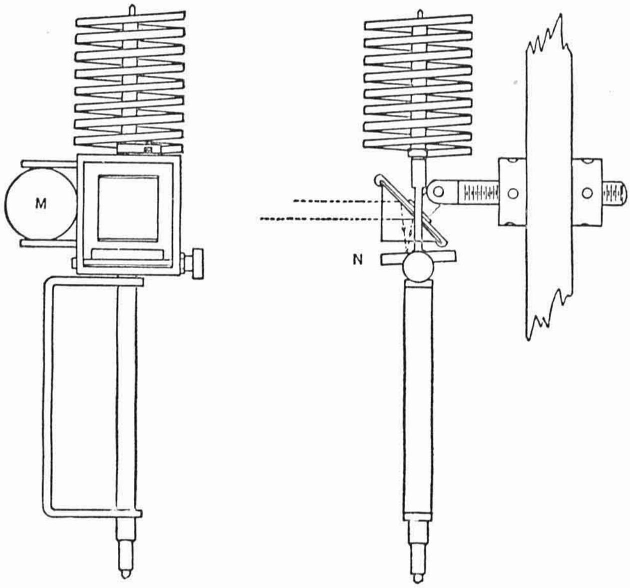


FIGURE 4.—Front and side views of the pivoted vane, showing the method of obtaining normal and reduced magnification. For normal magnification the light is reflected directly from the mirror M to the recorder. For reduced magnification the light is reflected downward by a fixed prism to a mirror N, the axis of which is approximately parallel to the axis of rotation of the vane. After reflection upward from this mirror it passes again through the prism, and thence to the recorder. The degree of magnification is a function of the angle of inclination of this mirror, being zero when the axis of the mirror is strictly parallel to the axis of the vane.

produces a deflection of the axis of the spot on the accelerogram of about 0.13 mm.

The very active character of records 25 and 32, as contrasted with the relatively smooth character of the recorded table motion, results from the ex-

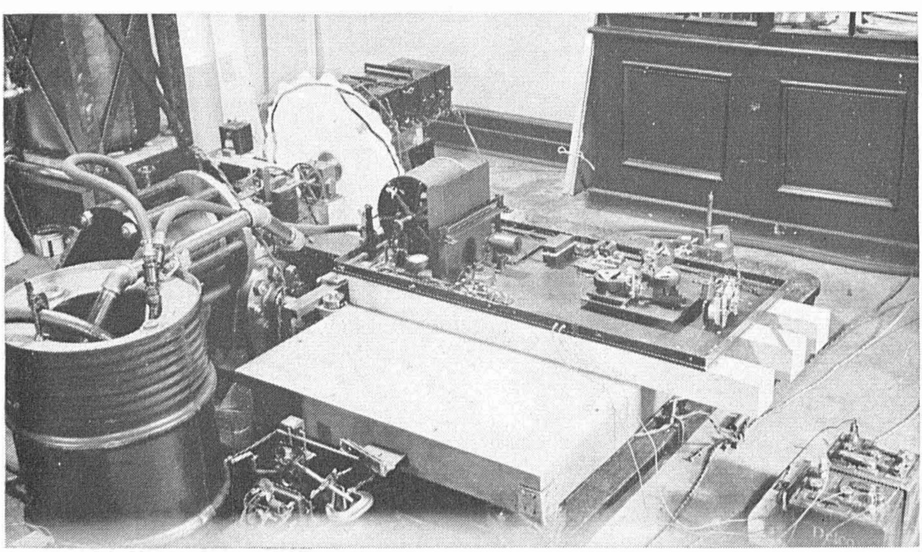


FIGURE 5.—View of 6-inch accelerograph mounted on a shaking table, showing photocell cam, table lever, and driving mechanism.

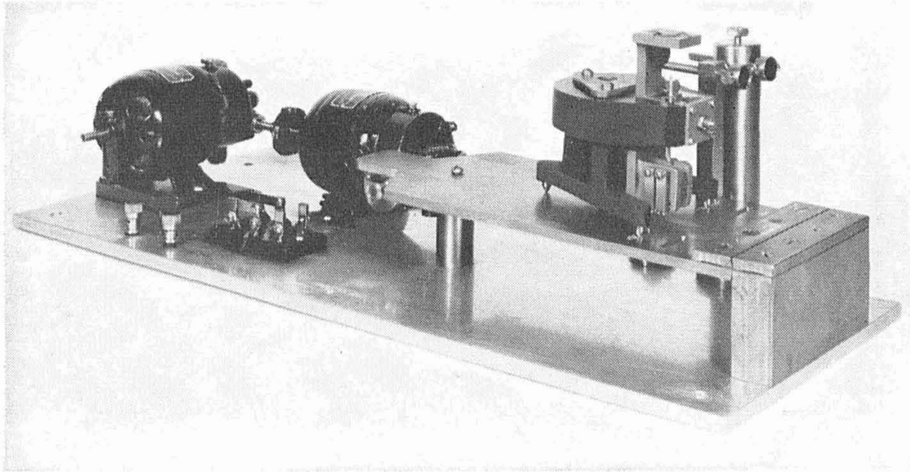


FIGURE 6.—Microtilt mechanism for applying arbitrary, small-amplitude, long-period tilt to the accelerometer.

tremely small parasitic vibrations of the shaking machine as it reproduces the motion from the cam. To the touch, the table motion feels extremely smooth, as indeed it is. The mean amplitude of the larger vibrations is estimated to be of the order of 0.003 cm., and they have a frequency of about 12 cycles per second. The approximate magnification for waves of this frequency is about 100.

Records 44, 46*a*, and 46*b* are quite smooth and show good definition of form. The conditions here are more favorable for integration and analysis than an actual earthquake record would be. In 46*a* and 46*b* the accelerometers recorded simultaneously, 46*a* being the direct accelerogram and 46*b* the component at 45 degrees to the direction of motion of the table, thus making possible a study of the fidelity of the recordings obtained when the instrument is oriented at an angle to the true direction of ground motion.

No. 25 is a record of three components, and indicates that they are, for all practical purposes, independent of each other.

Measurements of the zero lines at the beginning and end of the records were made on enlargements and there is some indication of small, semipermanent zero shifts of the pivot instruments. The maximum over-all shift on any one instrument never reached the equivalent of 0.001 g. In one record there was no evidence of any shift whatsoever. On the original accelerograms the maximum shift corresponds to 0.2 mm. From an engineer's standpoint such effects are negligible, and it is only in the integration over long sections of the record that they assume an apparent importance out of all proportion to their significance. For instance, a constant error in the acceleration of 1/4000 gravity over a period of 50 seconds produces an apparent displacement of more than 3 meters. This, as compared with real motions of a few centimeters, gives the impression of a large error in the record, and it is important to recognize this fact in connection with the precise analyses of the accelerograms.

Some part of the apparent axis shifts may be due to causes other than real shifts of the instrument pivots in their bearings. In order of importance, they are: (*a*) failure of the recording light spot to be a perfect "point"; (*b*) irregular developing and fixing of the accelerogram; and (*c*) differential shrinkage of the accelerogram. Furthermore, errors may be due to irregular shape of the light spot and to optical effects arising from a lack of uniformity of illumination over the spot itself that causes a variable position of the apparent center line of the trace, depending upon the speed of the spot and the form of the trace.

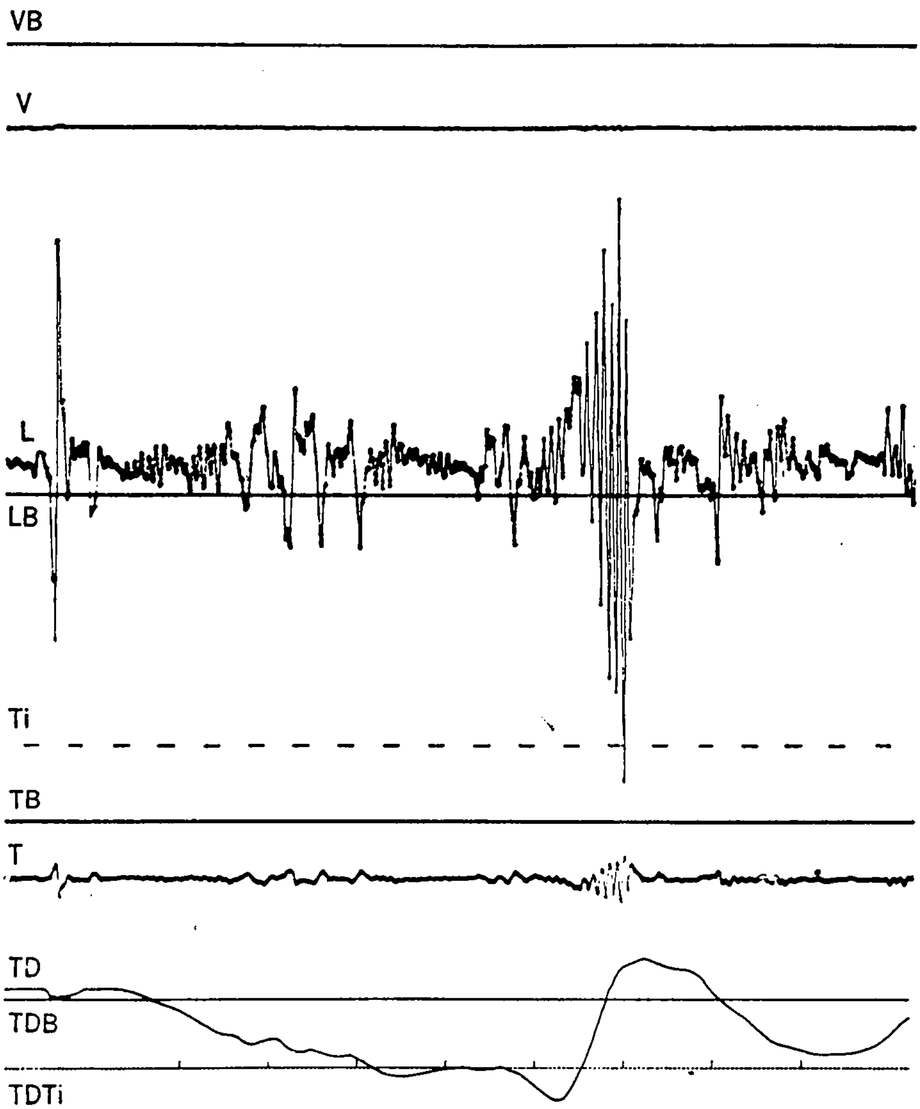


FIGURE 7.—Part of typical accelerogram, test No. 25, recorded by three accelerometers mounted on a shaking table. VB, vertical-component baseline; V, vertical component; L, longitudinal component; LB, longitudinal-component baseline; Ti, time marks, every half-second; TB, transverse-component baseline; T, transverse component; TD, table displacement; TDB, table-displacement baseline; TDTi, table-displacement time marks, every half-second.

The effects due to irregular developing and fixing are closely connected with optical effects. If a perfect spot is recording on a perfectly uniformly sensitized surface, the developing and fixing probably will introduce no serious defects so long as a readable accelerogram results. Any deviation from perfect optics will, however, be reflected directly, or may even be exaggerated in the finishing process.

Measurements of differential shrinkage on bromide paper of the type used on the accelerograms have shown that it may be as much as 0.2 per cent. When the fixed baselines are not greater than 1 centimeter from the trace axis, such differential shrinkages will cause apparent axis shifts equivalent to about 0.0001 *g.*, which, relative to other effects which have been discussed, is small.

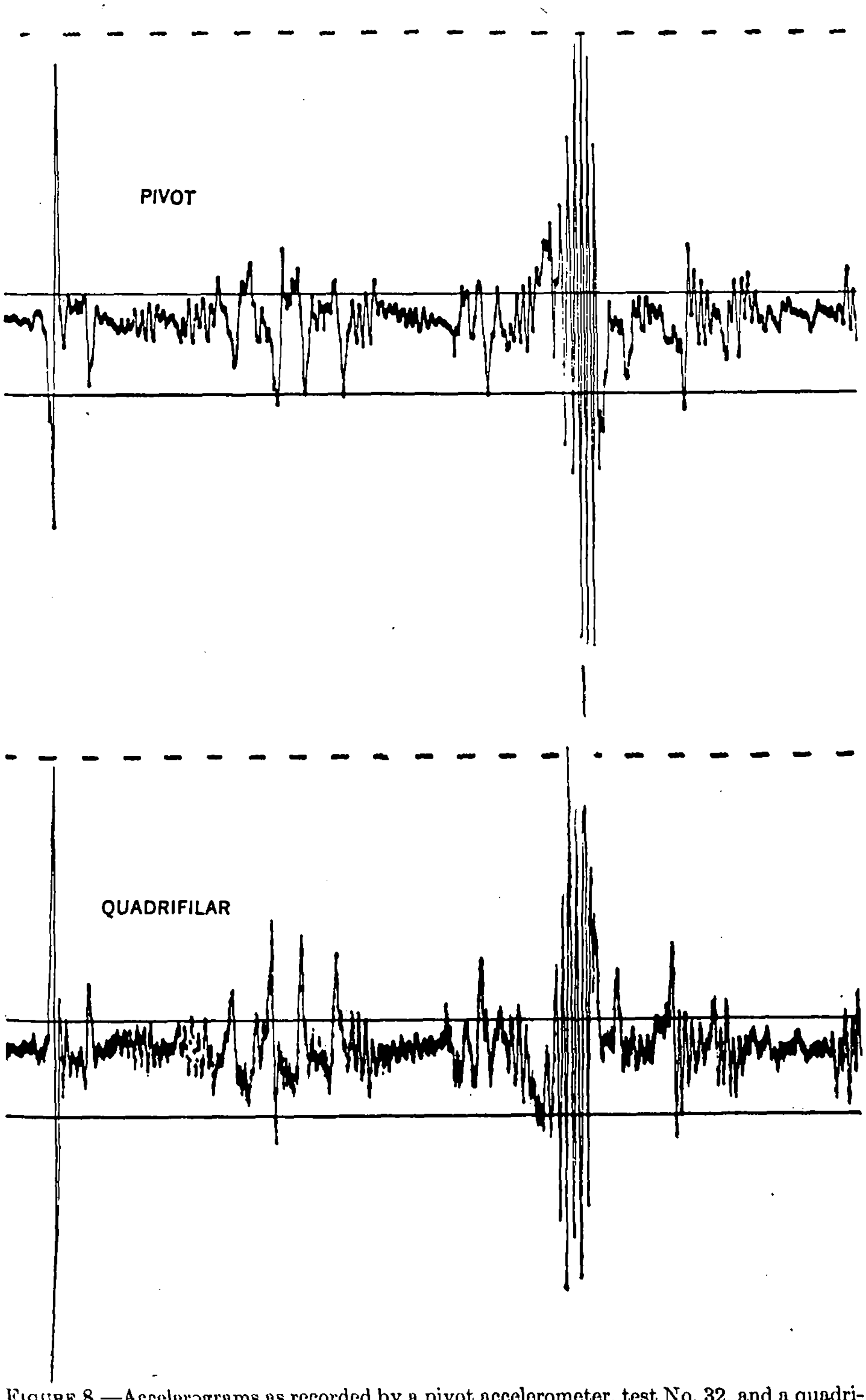


FIGURE 8.—Accelerograms as recorded by a pivot accelerometer, test No. 32, and a quadrifilar accelerometer, test No. 39. Recorded table motion same as in figure 7.

Whether or not errors of like magnitude occur as a result of flexure of instrumental parts during a severe earthquake has not been ascertained, because such errors are too small to be determined with any degree of precision by any convenient means. Errors due to variation in drum speed are negligible if the time marks on the accelerograms are reliable and are used, provided the backlash is removed from the drum by some kind of brake.

It seems that one must conclude from the results of these tests and subsequent exhaustive studies that accelerographs of the type now in use are satisfactory so far as practical field operation is concerned. It is true, of course, that as experience is gained in operating technique and in the processing of the records, minor changes will be made. In fact, many improvements have been made since this investigation was begun.

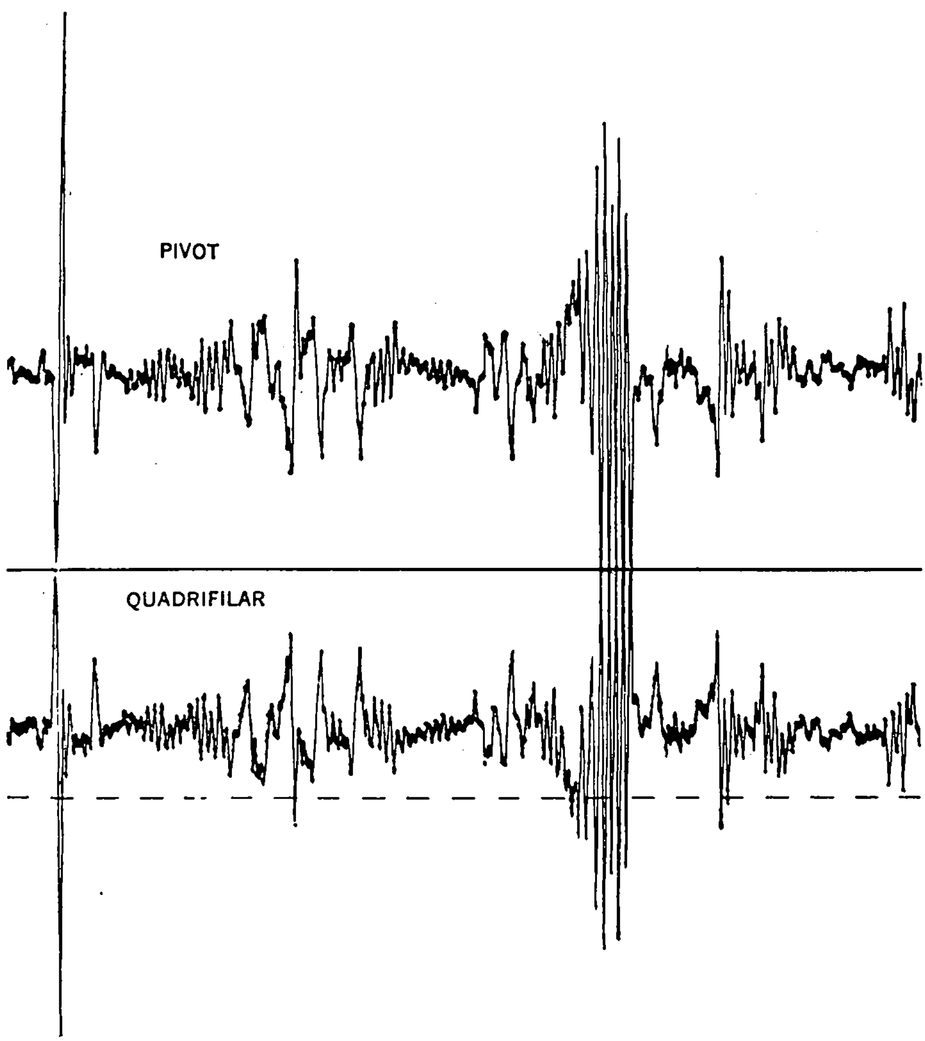


FIGURE 9.—Accelerograms recorded simultaneously by pivot and quadrifilar accelerometers mounted on the shaking table as test No. 17. Recorded table motion same as in figure 7.

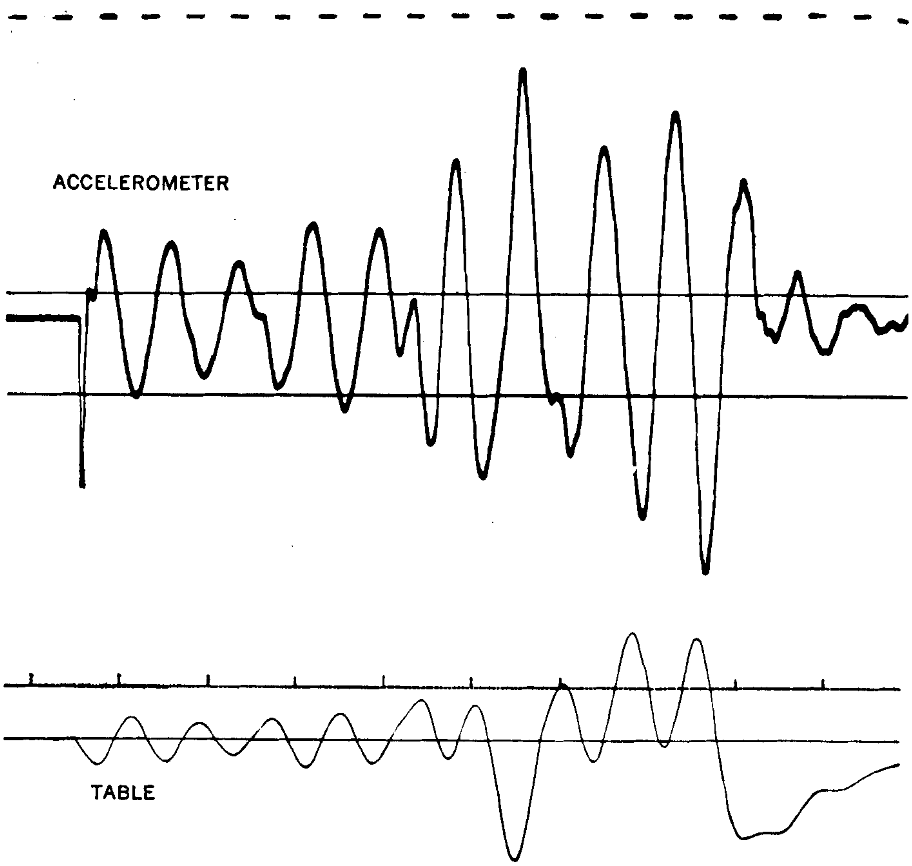


FIGURE 10.—Response of pivot accelerometer, test No. 44, to irregular motions of the table.

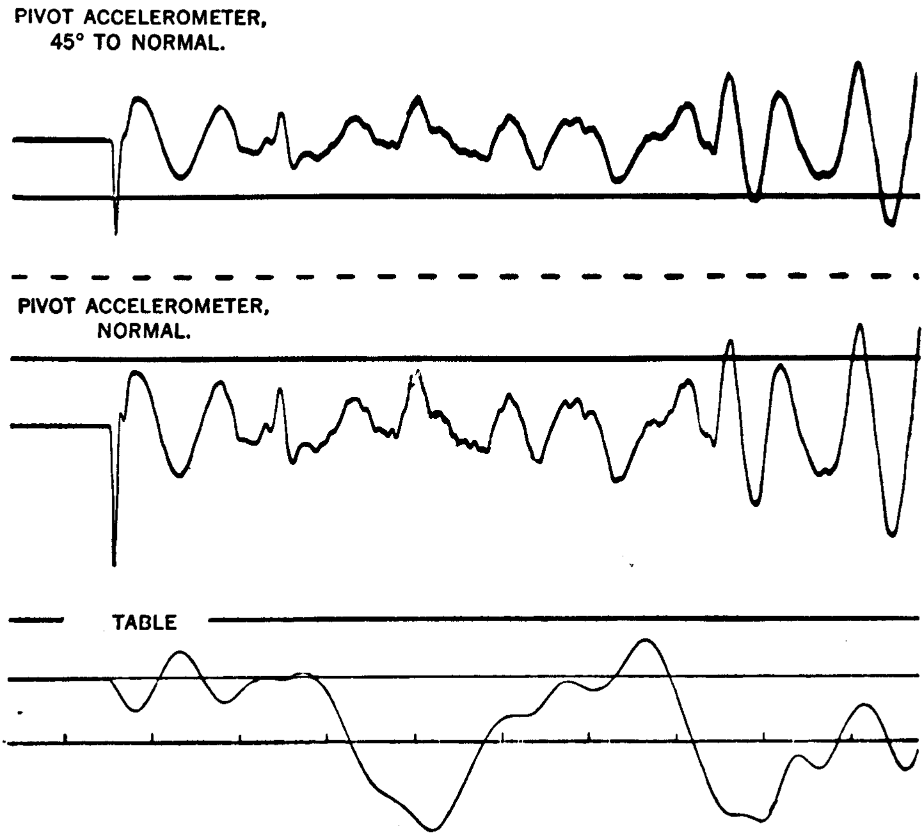


FIGURE 11.—Records from two pivot accelerometers, test Nos. 46a and 46b, one accelerometer set approximately 45 degrees to the direction of motion. Corresponding table motion at the bottom.

Chapter II

DISCUSSION OF PRINCIPAL RESULTS FROM THE ENGINEERING STANDPOINT

By A. C. Ruge

A CRITERION FOR ACCURACY

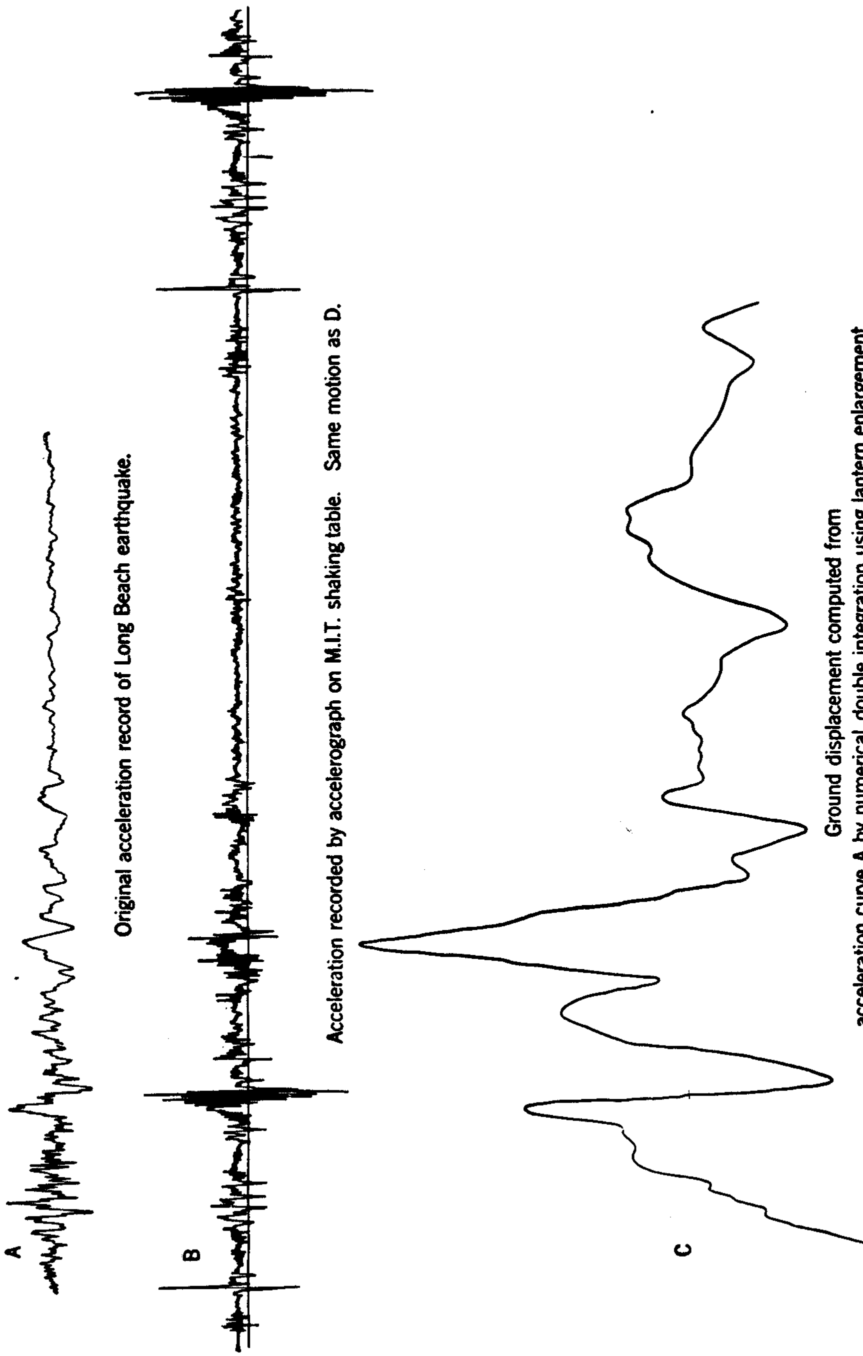
THE PURPOSE of the present paper is to discuss the results of the research program from the engineering standpoint. That the results are also important from the scientific standpoint is obvious, and Mr. Neumann gives careful consideration to that aspect of the program.

The engineer is primarily interested in one and only one question: How accurate are the results obtained from the strong-motion recording devices now in the field? In order to answer this question we have first to set up an acceptable criterion for "accuracy." Clearly, one cannot apply the elementary engineering concept of "accuracy of measurement" to define the accuracy of a complex function of time like an earthquake motion. For example, the accuracy of displacement or velocity or acceleration at any one instant of time has no significance whatever as a criterion of accuracy from the engineering standpoint; nor has the average accuracy at all instants of time any real significance. Still less can one apply intangible criteria based upon how nearly the displacement, velocity, or acceleration on the record "looks like" that of the true earthquake.

It appears, then, that the true criterion for accuracy must depend not alone upon the function under consideration, but also upon the *effect* of that function upon the particular phenomenon under investigation. For example, given a true earthquake motion and the "calculated" earthquake motion based on instrumental records, the "accuracy" of the calculated motion depends upon how it would affect some particular engineering structure as compared with how the true motion would affect the same structure. This criterion is far broader than, and includes, the elementary engineering concept of accuracy; thus, whether or not there be any vibration of the structure resulting from the earthquake the criterion sets a sound basis for determining the accuracy.

This criterion, based upon *effect*, leads to the obvious inference that a certain calculated earthquake motion might be highly accurate for one class of structures and at the same time grossly inaccurate for another; and this is indeed the case. If the calculated motion was derived by double integration of the record made by an accelerometer of, say, 1/10-second period, then one would not expect the motion to be "accurate" when applied to a structure the period of which is below about ¼ second; nor would one expect accuracy if the motion were applied to an exceedingly long-period oscillating system such as the water of a large lake.

From the engineering standpoint, then, we are concerned with the accuracy of the derived earthquake motions as they would affect engineering structures; very fortunately, practically all important engineering structures fall within the range of periods over which the present strong-motion instruments give reliable performance, as will be seen from examination of the results



A. Original acceleration record of Long Beach earthquake.

B. Acceleration recorded by accelerograph on M.I.T. shaking table. Same motion as D.

C. Ground displacement computed from acceleration curve A by numerical double integration using lantern enlargement. Coast and Geodetic Survey.

FIGURE 12.

displayed in figures 12 and 13. These two figures comprise an engineering summary of the characteristic results of the research program and show at a glance the remarkable fidelity with which the actual motion can be calculated from strong-motion records.

DISCUSSION OF RESULTS

Figure 12 shows the results obtained with a motion copied from the 1933 Long Beach earthquake as recorded in Los Angeles. Curve C (computed

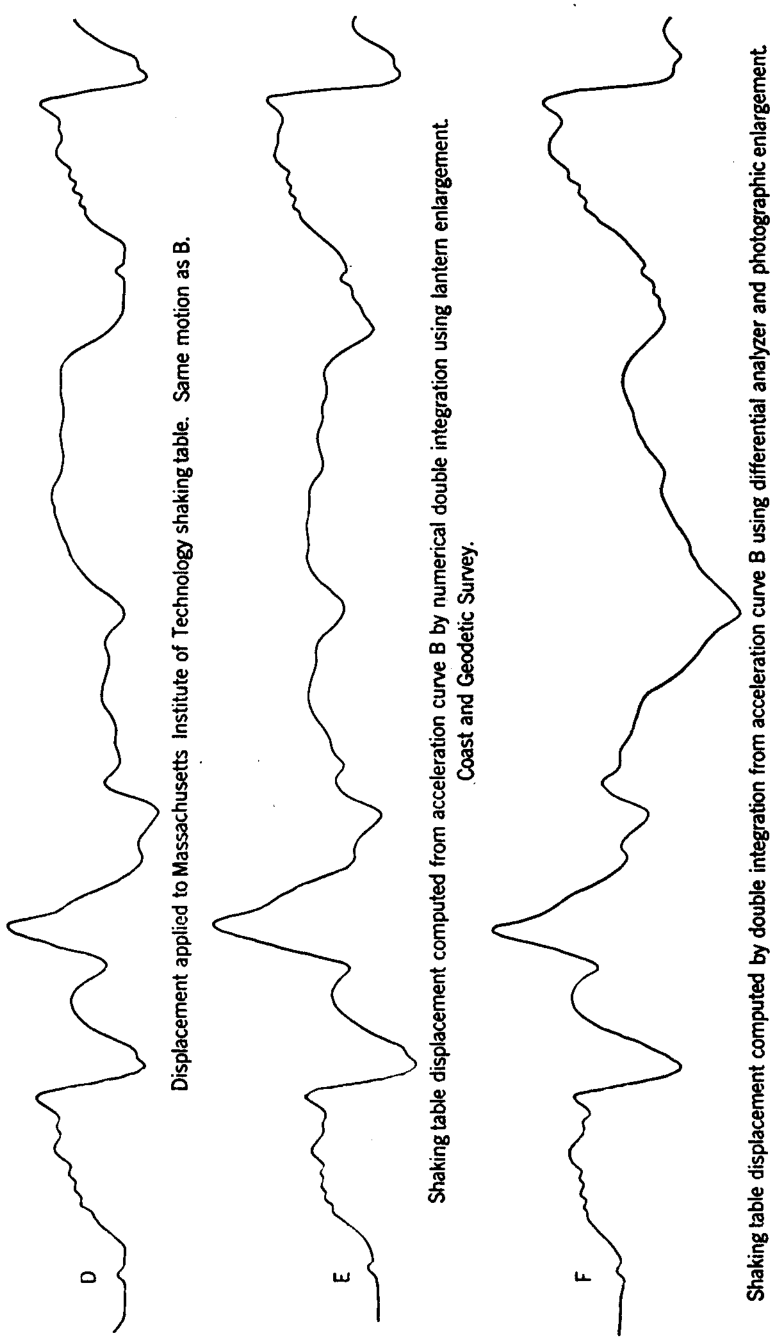
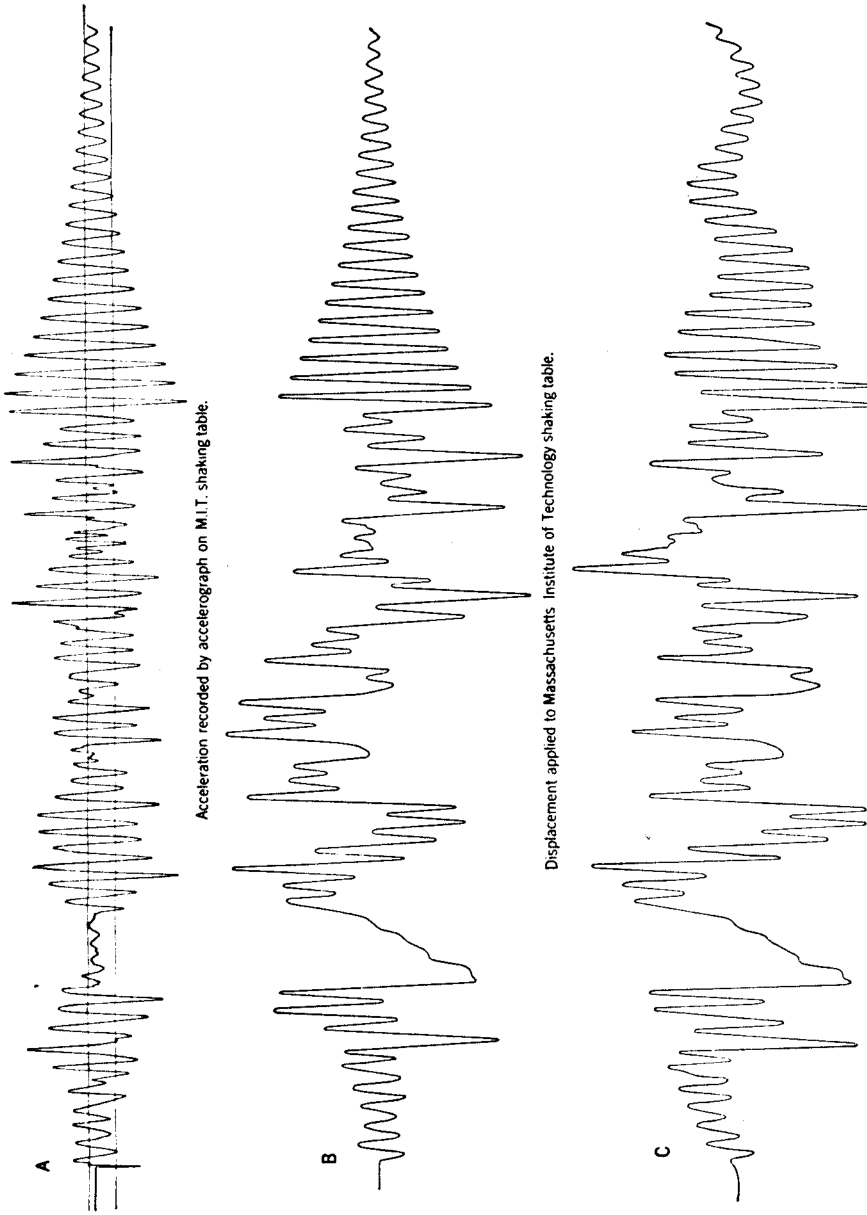


FIGURE 12.—Results of Massachusetts Institute of Technology shaking-table test using a table motion simulating the ground motion at the Los Angeles Subway Terminal building during the Long Beach earthquake of 1933.

from original accelerogram, curve A) was used as a template to drive the shaking table, the motion of which is shown in curve D. It is obvious that the motion of C and D differs only in scale. Curve B is a typical accelerogram recorded by a strong-motion instrument subjected to the motion of the shaking table, curve D. Curves E and F show the computed shaking-table displacement as derived by numerical and mechanical integration, respectively.

The agreement between curves D, E and F is indeed remarkable, considering the obvious difficulties involved in double-integrating a record of the com-



Shaking table displacement computed from acceleration curve by numerical double integration using lantern enlargement. Coast and Geodetic Survey.

FIGURE 13.

plexity of curve B. This test of accuracy is an exceedingly severe one, as is evidenced by comparison with the relatively open and smooth character of the original earthquake accelerogram of curve A from which curve C was computed. It is doubtful if any actual earthquake would ever impose integration conditions more difficult than those illustrated in figure 12, and therefore it may be concluded that the errors of curves E and F relative to curve D are expectable maxima.

It is interesting to compare the accelerograms, curves A and B, in view of the identity of motion in curves C and D. At first glance curve B appears very dissimilar to curve A, but upon careful inspection the major features may be identified on the two curves. A study of curves A, B, C, and D brings out clearly why the engineer needs much more than the accelerogram alone in order to visualize the possible effect of the earthquake upon a given structure.

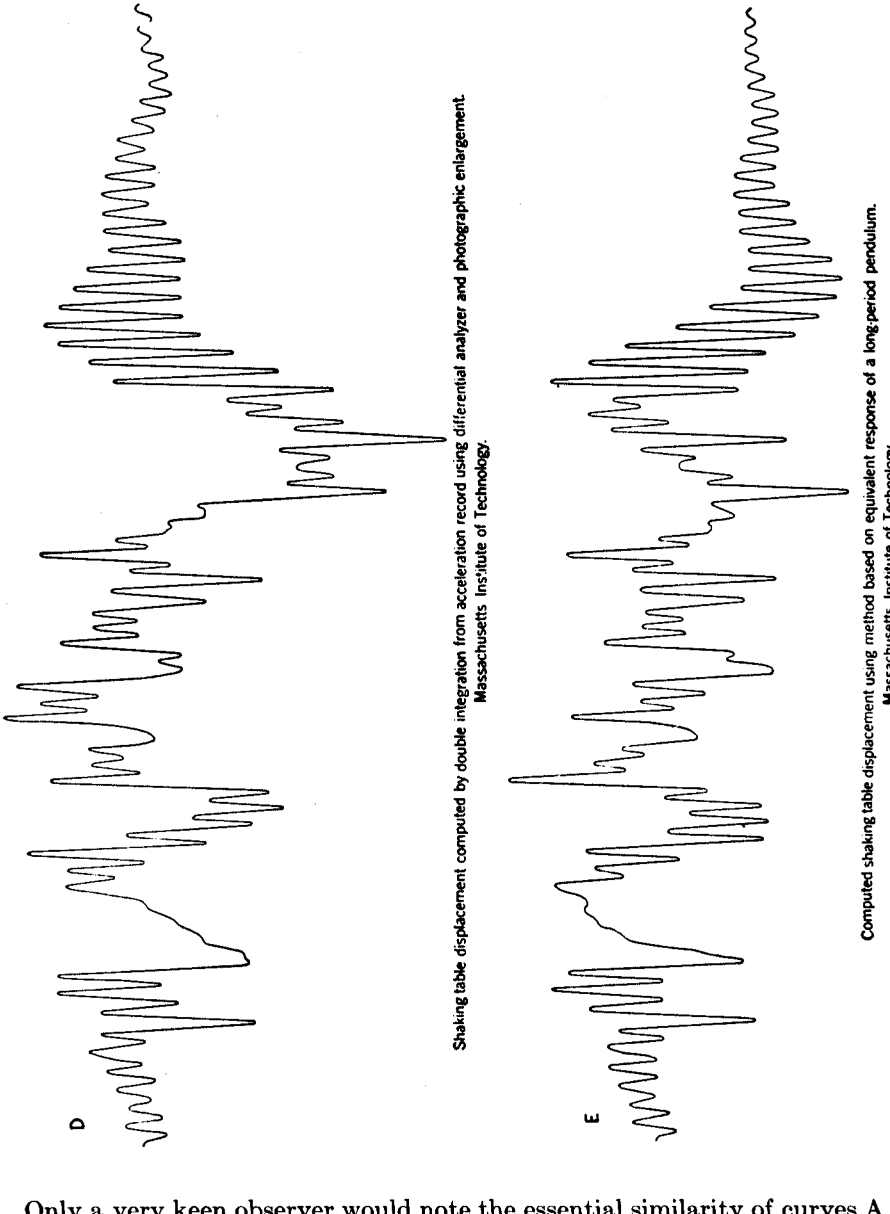


FIGURE 13.—Results of Massachusetts Institute of Technology shaking-table test when a smooth idealized earthquake motion is applied to the table.

Only a very keen observer would note the essential similarity of curves A and B without having something besides the accelerograms alone to guide him. And it is safe to say that even the most experienced observer cannot directly visualize even the general form of curve C or D from an inspection of curve A or B.

The only significant difference between curves F and E is in the form of a sort of fold about the center of the record. This difference arises from different treatments of a slight instrumental axis shift which occurred in the course of the record analyzed in figure 12; full details will be found in the succeeding papers. In curve E the axis shift was corrected for in the computations, while in curve F it was deliberately left uncorrected. Applying our criterion for true accuracy, it is clear that for any practical engineering structure the difference in the effect of curves E and F would be completely negligible from the engineering standpoint.

Turning now to figure 13, we see the agreement between computed and actual motions for a smooth "idealized" earthquake motion. As in figure 12, the only significant difference between the numerical and mechanical integrations is in the treatment of a small instrumental axis shift. The essential identity of curves B, C, and D as related to engineering structures is obvious. The slight wavering of the axis in the final damped wave train may possibly have resulted from a small accidental tilting of the shaking-table platform which of course would not appear in the record of the table motion, curve B; or it could have resulted from irregular shrinkage of the record paper. In general, the agreement between the curves is excellent.

Curve E, showing the response of a 10-second-period damped pendulum to the same acceleration (curve A) has been added as a matter of interest. Despite the distortion introduced by the pendulum response, as shown by the difference between curves D and E, the effects of the two curves upon an engineering structure having a natural period anywhere between ⅓ second and 5 seconds would differ by less than 10 per cent, while for the greater part of the period range the difference would be well within 5 per cent. Therefore we may properly regard curve E as a representation of the motion shown in curve D to an accuracy of 5 to 10 per cent, or better, over a period range of ⅓ second to 5 seconds, which is adequate for engineering purposes. The significance of curve E is that it can be obtained more easily than curves C or D, as explained in the third and fourth papers, where the Torsion Pendulum Analyzer and Differential Analyzer methods are discussed.

The reader is referred to the succeeding papers for complete details regarding the material presented in figures 12 and 13 as well as for results of the analysis of several other records.

CONCLUSIONS

The following conclusions relate to the engineering features of the results of the research:

1. The performance and accuracy of the present U. S. Coast and Geodetic Survey accelerometers under simulated earthquake motions are more than adequate for engineering purposes.
2. The components of acceleration measured by the three elements of the accelerograph are independent, so far as can be determined from the records studied.
3. The displacements and velocities computed by numerical integration agree with the actual displacements and velocities closely enough for engineering work of the highest quality.
4. The integration of accelerograms by the M.I.T. Differential Analyzer leads to results agreeing very closely with the numerical integration method. The choice of method of integration needs only to depend upon questions of economy of time and expense.

Chapter III

AN APPRAISAL OF NUMERICAL INTEGRATION METHODS AS APPLIED TO STRONG-MOTION DATA

By Frank Neumann

INTRODUCTION

THE SEISMOLOGICAL investigations described in this paper are a part of the strong-motion program inaugurated by the Coast and Geodetic Survey in 1932[1] with the active cooperation of Pacific Coast engineers and others interested in the practical aspects of the earthquake menace. Since the beginning of the program, the Survey has published analyses of instrumental records on the assumption that the instruments, especially accelerographs, performed according to theoretical expectations. Furthermore, the records were subjected to methods of analysis which were new, difficult, and not without certain controversial features. General acceptance of results could therefore not be expected without some proof of their real value. This has now been obtained through shaking-table tests conducted by other institutions cooperating with the Coast and Geodetic Survey. Consequently, a large part of the present paper deals with shaking-table projects at the Massachusetts Institute of Technology[2] and at the National Bureau of Standards.[3]

In the analysis of seismograms, the field in which the writer is primarily interested, much of the computational work involves the process of integration—meaning that elemental areas or ordinates of the curve under study are subjected to a process of summation producing another curve, which is called the first integral. Some error is certain to enter into this computation when it is applied to accelerograph records which originally were not intended for such rigid treatment. When a second integral is called for, the problem is obviously one requiring extraordinary caution. Under these circumstances, it was necessary, in the past, to keep an open mind concerning the validity of the results obtained by such methods. It is now believed, however, that with results that have been substantially verified by Professor Ruge, working independently at the Massachusettes Institute of Technology, a proper appraisal can be made of this phase of the Survey's work. Professor Ruge's results are presented in another paper in this symposium.

The basic equation calling for the use of integration is one that expresses the response of a damped pendulum to displacements imposed on its points of support, such as occurs in an earthquake or a shaking-table test. It is normally expressed in the following form, in which x is the displacement of the support, y the displacement of the pendulum relative to the support, ε the damping factor, and T_0 the pendulum period:

$$-\frac{d^2 x}{d t^2} = \frac{d^2 y}{a t^2} + 2\varepsilon \frac{dy}{dt} + \left(\frac{2\pi}{T_0}\right)^2 y$$

It may also be expressed in the following form, which, though unorthodox, clarifies the numerical integration process as outlined in this paper; $y\Delta t$ represents an increment of area on the acceleration curve, Δt being an arbitrary constant and y the variable mean ordinate in each successive time increment:

$$-D = y + 2h\frac{2\pi}{T_0}(\Sigma_0{}^t y\Delta t + C_1) + \left(\frac{2\pi}{T_0}\right)^2 [\Sigma_0{}^t(\Sigma_0{}^t y\Delta t + C_1)\,\Delta t + C_2]$$

D is the displacement of the ground, y the displacement of the center of oscillation of the seismograph pendulum as shown on the seismograph record, T_0 the free pendulum period, h the damping factor, and t the time. In an accelerograph record, only the third term, involving the double integral is significant. For a displacement-meter record, only the first term is significant. For the record of a pendulum of intermediate period, all terms must be taken into account.

The investigations described here are restricted to the problem of calculating the displacement curve corresponding to a recorded acceleration curve. Displacement is especially important in engineering research, and the seismologist uses it to investigate seismic wave theory. The curves obtained are also invaluable in accumulating period data, as only the very short-period waves can be recognized on an accelerogram. Relatively few displacement meters are in use (none recording vertical motion), and hence the economical and technical advantages of being able to compute it from the many acceleration records available are obvious. Integrating machines were not used in the Survey investigation, because none was readily available. They may or may not be superior to adding-machine calculations in efficiency and precision, but conclusions should not be hastily drawn in view of the precision demonstrated and an accelerometer zero shift which requires a flexible computational method to overcome it. The numerical method of integration is being described in order to reveal some little-known details of its efficiency and the nature of the axis adjustments which may be criticized as not being in accord with rigid mathematical practice but which nevertheless give satisfactory results when applied to the special case of seismogram analysis.

DESCRIPTION OF DOUBLE-INTEGRATION METHOD AS APPLIED TO ACCELEROGRAMS

The curve recorded by the accelerograph is first enlarged and divided into small, equally spaced time increments of less than 0.1 second. The mean ordinate of each increment is then measured from a baseline, a step which is equivalent to measuring the areas of the increments. These readings are the raw material used in the integration, or summation, processes which are carried out on an ordinary adding machine, or, preferably and more efficiently, on a double-register machine capable of printing actual negative totals and subtotals instead of their complementary equivalents. As no digits are dropped in any of the summations, and as a system of checks practically eliminates errors of operation, this part of the work may be considered mathematically correct. In practice, the number of increments may vary from 200 to 1,000, depending upon the length and complexity of the record. Essential but not necessarily complete details of the process will be given in their proper order. The reader is referred to figure 14 for a graphical representation of the axis adjustments explained in the text.

1) Enlarge the accelerogram. This is necessary because the original record is too small, especially in time scale, to make measurements which are sufficiently accurate for precise integration. In the first method developed by the Survey,[4] enlargements were made on high-grade cross-section paper shellacked to aluminum plate, thus providing a ready and accurate way of scaling the mean ordinates. In this method and in most of the work described in this report a lantern projector ("Balopticon") capable of projecting opaque objects was used. Magnification was usually about seven diameters. A thin pencil line drawn exactly in the center of the image of the curve constituted the en-

largement. With the aid of a small white card, on which two parallel lines were ruled with a space between them slightly larger than the image of the baseline, a high degree of accuracy was obtained in setting the baseline image on the chosen baseline of the cross-section paper. As these settings could be repeated with differences of the order of 0.1 millimeter, and as checks were made frequently, the over-all accuracy of the settings provided a minimum of error in a most important phase of the work, as such errors are doubly cumulative. Slow-motion adjustments were made by manipulating a pair of

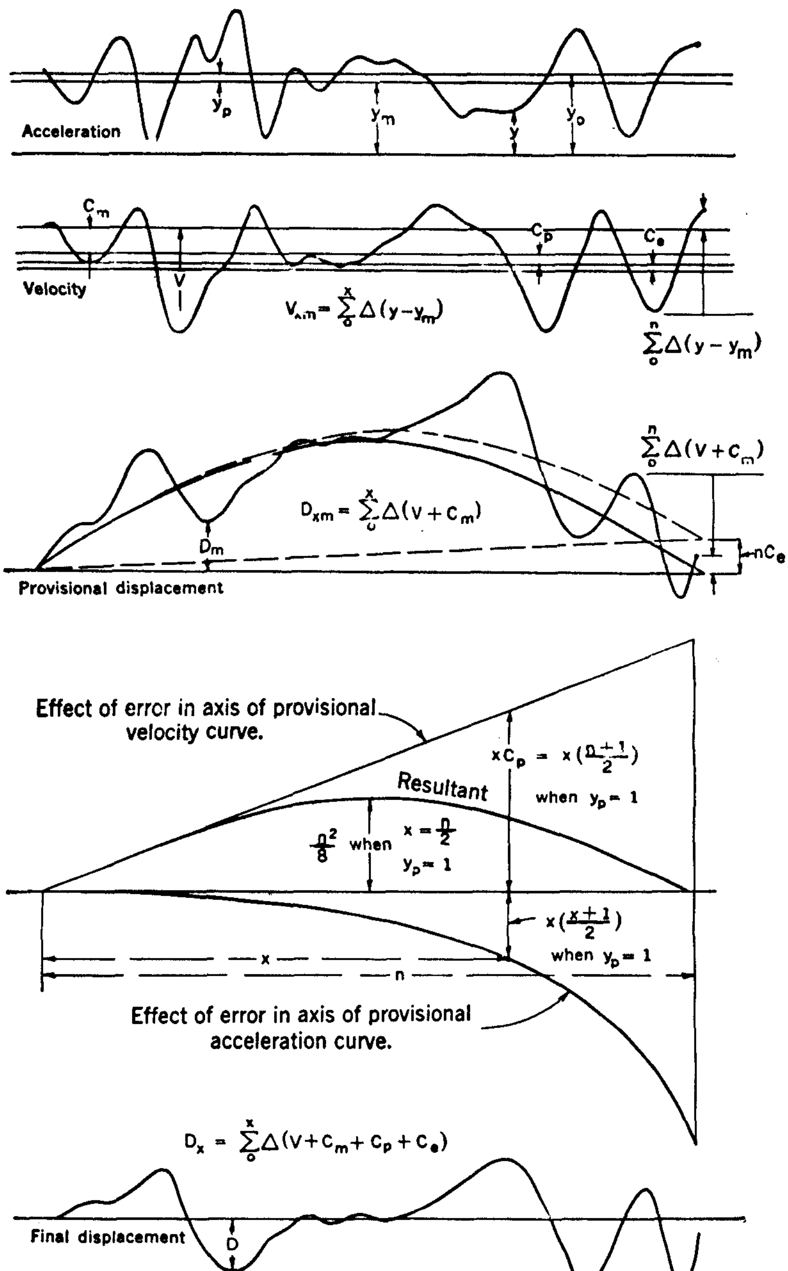


FIGURE 14.—Diagrams illustrating how parabolic correction is made in the numerical double-integration process by shifting axes of acceleration and velocity curves.

turnbuckles which controlled the elevation of the board frame on which the paper was mounted. Before enlargements were made, the lantern was always adjusted so that the image of a machine-ruled grid, on aluminum plate, set exactly square with the cross-section paper to avoid asymmetry in the image. Compressed air was used to reduce heat distortion from the 1,000-watt projector lamp.

A mechanical enlarging apparatus now in use is described in a later section of the paper. This was necessitated by the discovery in the Massachusetts Institute of Technology shaking-table tests that heat distortion was still present. There was also a desire to eliminate certain other undesirable features of lantern enlarging.

2) To scale the ordinates, select a time increment which will correspond to some convenient interval on the cross-section paper, usually 1, 2, or 5 millimeters, or 0.2, 0.4, or 1.0 inch, depending upon the character and enlargement of the record. (For discussion of the magnitude of the time increment see "System of Checks" at end of this section, and discussion of test No. 17 on page 38.) Successive mean ordinates of the increments are measured from an optional baseline and tabulated on an adding-machine slip, usually in groups of ten, the sums of each group being used later for checks on the summations. In lantern enlargements it was sometimes found necessary to expand unusually active portions of a curve because the steep slopes made accurate scaling practically impossible. In many cases straight lines were drawn between the turning points when the motion was smooth and rapid. The tabulated ordinates, y, are the data used in the next step, the first of the numerical integration process.

3) Determine mean ordinate y_m as measured from the baseline. The tentative algebraic ordinates are $y-y_m$.

4) Compute the tentative velocity curve (the first integral) by obtaining running subtotals of successive values of the algebraic acceleration ordinates. V (velocity) is numerically equal to $\Sigma(y-y_m)$. The axis of this curve is coincident with the first point on the curve, as the constant of integration is tentatively zero.

5) Obtain the algebraic sum of V ordinates and divide by number of ordinates. This is C_m, the constant necessary to apply to all values of V to make the total sum nearly zero so that the last ordinate of the second integral (the tentative displacement curve) will be near zero. C_m is a tentative value of the true constant of integration for the first integral, or velocity curve.

6) Compute running totals (subtotals) of $V + C_m$. D (displacement) $= \Sigma(V + C_m)$. This is the tentative second integral or displacement curve, with axis coinciding with the start of the curve, and the last ordinate near zero. It will nearly always be bent symmetrically upward or downward in the form of a parabola, owing to inaccuracies in the tentative positions of the acceleration- and velocity-curve axes previously indicated by y_m and C_m. The next five steps explain how this parabolic deviation is eliminated.

7) Construct a parabola, equal in length to the tentative displacement curve, for an acceleration-curve axis shift of one unit of integration, that is, one unit of the original ordinate readings. If the base of the parabola is divided into ten equal parts, the successive ordinates will be $n^2/2$ times the following factors, n being the number of ordinates: 0, 0.09, 0.16, 0.21, 0.24, 0.25, 0.24, 0.21, 0.16, 0.09, and 0. A parabola of any other magnitude can be quickly constructed by increasing or decreasing all nine ordinates in the same ratio. Select the one that most nearly resembles the bend in the tentative displacement curve and will serve as an axis regardless of where the beginning and end ordinates of the curve may fall. The corrections to the acceleration- and velocity-curve axes will then be greater or less than those for unit shift of acceleration-curve axis in the same ratio that the parabola is greater or less in amplitude than the parabola based on unit shift of acceleration-curve axis.

The correction to the *velocity-curve* axis for unit shift of acceleration-curve axis is $(n + 1)/2$.

If the parabola in the displacement curve is 0.7 of the magnitude of the "unit" parabola, the acceleration-curve axis correction, y_p, is 0.7 of one unit of integration; and the velocity-curve axis correction, C_p, is $0.7 \left(\frac{n+1}{2}\right)$ units of integration. If the bend is in a positive direction, the acceleration correction will be positive and the velocity negative. If the bend is negative, the signs are reversed.

8) In the preceding operation it will usually be necessary to tilt the parabola in order to define the central axis, because the last ordinate of the displacement curve does not ordinarily coincide with its axis. If the base of the parabola falls above the last ordinate, a constant negative correction must be applied to the ordinates of the velocity curve in order to lower the axis of the final displacement curve to a horizontal position. An opposite sign is used if the reverse is true. To find the correction, first measure the distance, in units of integration, between the end of the tilted parabola and the horizontal axis of the tentative displacement curve. This, divided by the number of ordinates, n, is designated C_e, the correction to the constant of integration, C_m, and takes care of end conditions in the final displacement curve. Any correction due to failure of the beginning of the curve to coincide with the axis is merged with the C_e correction, the resultant slope of the base of the parabola being the measure of the total correction to be applied.

9) The final axis reading of the acceleration curve, as measured from the baseline in units of integration, is $y = y_m + y_p$, signifying the mean ordinate and the parabolic factors. See steps 3, 4, and 7, above.

10) The axis of the final velocity curve, expressed in the form of a correction to the first ordinate, is $C = C_m + C_p + C_e$, signifying the mean ordinate and the parabolic and end ordinate factors. C is the final constant of integration for the first integral.

11) The axis of the final displacement curve is determined for all practical purposes when the parabolic and end ordinate corrections are made. In the final computation, however, the figures are algebraic ordinates measured from an axis which coincides with the beginning of the curve. This can be changed as desired. It would be legitimate, should further adjustment seem necessary, to impose an axis correction of parabolic form, or of constant slope on any displacement curve computed by this method.

12) The final computation is made preferably on a double-register adding machine, using (*a*) the original ordinate, (*b*) the ordinate representing the final acceleration-curve axis as summarized in step 9, and (*c*) the constant of integration for the velocity curve summarized in step 10 above. In the following example of the machine computation they are indicated by y, y_o, and C, respectively.

R	C	22									
R	y	278	271	265	249	286	251	203	195	150	100
R	y_0	−226	−225	−226	−225	−226	−225	−226	−225	−226	−225
R	Velocity	74	120	159	183	243	269	246	216	140	15
B	*Velocity*	*74*	*120*	*159*	*183*	*243*	*269*	*246*	*216*	*140*	*15*
B	*Displacement*	*74*	*194*	*353*	*536*	*779*	*1,048*	*1,294*	*1,510*	*1,650*	*1,665*

C is put into the machine only once, as a correction to the first ordinate of the acceleration curve—a step equivalent to applying it to each ordinate of the velocity curve. The algebraic ordinates are not printed, as the subtotal key immediately gives the first summation, which is the result desired. The original ordinate, y_o, is made effective to one or two decimal places by varying the last digit systematically by one digit; thus, if $y_o = 225.7$, the operator uses 225 three times and 226 seven times. It is not necessary to use fractions in C. R and B indicate red and black registers. For single-register machines the "red" work may be done first, then the "black." After y are the ordinates originally scaled on the enlarged accelerogram. The figures after "velocity" are $\Sigma\ (y-y_o)$, the running subtotals. In a similar way the displacements are $\Sigma\Sigma(y-y_o)$. In practice, the machine obviously produces one long column of figures instead of the separate groups shown here.

13) The velocity and displacement curves are drawn by use of the velocity and displacement ordinates just computed, all expressed in units of integration.

14) Conversion of units of integration to units of velocity and displacement is accomplished as follows:

$$\Delta V = \Delta y \times \Delta t$$
$$\Delta D = \Delta y \times \Delta t \times \Delta t$$

Δy is one unit of integration, which necessarily has a definite value in terms of acceleration depending upon the sensitivity of the accelerometer and the enlargement of the original accelerogram. Δt is the time increment, selected before the ordinates were scaled, in seconds. In the case of test 25 described in another section, $\Delta V = 0.3423 \times 0.02882 = 0.009865$ cm/sec., and $\Delta D = 0.3423 \times (0.02882)^2 = 0.000281$ cm. On the velocity and displacement curves just drawn the equivalents of 1 cm/sec. and 1 cm. are then simply the reciprocals of ΔV and ΔD.

System of checks.—Numerical integration would be impractical were it not for a system of checks to aid the machine operator in discovering errors. This is achieved by first listing all ordinates in groups of ten (usually) and using the group sums and their subtotals as checks on the first summation (velocity curve), for the group subtotals must check with the last figures in each group of summations of the individual ordinates. There is thus a check on every tenth computation. When computations are repeated, using new constants, it is also a simple matter to compute by differences the values of every tenth ordinate of the new curve, and use this as a check. These two principles can be applied to practically all phases of the work.

Because it is so evident here, it is interesting to note that, while the first integral (velocity) is a summation of *mean* ordinates of the acceleration curve, the first integral itself is expressed merely as a series of equally spaced ordinates, or points. The second integral (displacement) is therefore a summation of these ordinates and not the mean ordinates between them. There is a way to treat this problem mathematically, but it is not believed practicable to attempt it because of the additional labor. One object of the tests described later is to learn the effect of the size of the time increment on the final result, it being desirable to reduce the number of increments as much as possible without introducing serious errors. See the results of a partial investigation in test 17Q, page 38.

Practicability.—Assistants with limited training can do 95 per cent of the work. The time required to execute all summations, including readjustments, is estimated to be about one-half that required to enlarge and scale an acceleration curve and construct the computed velocity and displacement curves.

Numerical integration possesses a desirable flexibility in that definite figures are always available for making essential adjustments on a quantitative basis, an especially important factor when the work is complicated by shifting of the zero position of the accelerometer pendulum.

A considerable saving of labor would be effected if the final displacement curve were simply scaled from the tentative displacement curve after the parabolic axis had been determined, thus eliminating the final double-integration computation described in step 12. The errors would not approach those of processing and, as the engineer is evidently not interested in errors of such small magnitude, there seems to be no reason why such procedure would not be satisfactory, especially after the general accuracy of the work has been determined.

RESULTS OF MASSACHUSETTS INSTITUTE OF TECHNOLOGY SHAKING-TABLE TESTS

These were the first tests of Survey accelerometers ever made on a shaking table. The laboratory work conducted by Professor A. C. Ruge, of the Massachusetts Institute of Technology, and Mr. H. E. McComb, of the U. S. Coast and Geodetic Survey, has been described in other papers.[5] The test records processed were selected by them, the author being given a choice of one record in each group of tests. As the primary purpose was to learn how accurately the table motion could be computed by double-integrating the acceleration curve, it was decided to withhold the table-displacement records from the authors in order to avoid possible bias in making axis adjustments through previous knowledge of the character of the curves to be computed. As the specified tests were all completed without effort to investigate sources of error, the errors found are applicable to all the author's integration results obtained prior to 1937. Subsequent study of the errors resulted in the development of a new and more accurate type of enlarging apparatus, as is explained in later sections.

The three "Long Beach type" test records, Nos. 25, 32, and 39, were not identified as such by the writer, because the shaking-table acceleration records were apparently in no way similar to the earthquake accelerations recorded in 1933. In spite of the more open time scale they were more difficult to integrate than anything previously attempted, owing to uncontrollable parasitic vibrations in the shaking-table. It also developed later that the table displacements were only about one-third those originally computed from the 1933 record, and the time scales were also different. Caution should therefore be used not to mistake the percentage error in the computed table displacement for the percentage of error in the earlier computation of ground displacement at the Los Angeles Subway Terminal Building, where the 1933 record was obtained. Figure 25 shows the relative magnitude of the shaking-table motion and the actual ground motion.

Tests 44 and 46 were not typical of earthquake motions except that wave forms of the type recorded are no doubt at times present in earthquake motions but in more complex patterns.

An outstanding feature of the tests was the use of a special apparatus which, during most of the tests, imposed a slow simple harmonic tilting motion on the accelerometer, causing the light-spot "zero position" to move over ranges of approximately 0.2 and 0.4 millimeter. The purpose of this was to obtain light-spot deflections which would produce long-period waves of high amplitude on the computed displacement curve, thus reproducing the questionable waves of similar character which were so prominent on the first displacement curves computed from the Subway Terminal record in 1933.[6] Although not an indispensable part of the tests, the tilt apparatus provided a practical and novel way of duplicating errors resulting from the absence of baseline controls on the earlier records. This type of error is discussed in detail on pages 39 and 40.

The tilt apparatus completed a cycle in 64 seconds, which, taking the light-spot deflections into account, corresponded to horizontal motion displacements of 25 and 50 centimeters from the position of rest. One complete cycle was needed in order to have a trial displacement curve long enough to define a central axis, but this requirement was not found practicable in the tests. Professor Ruge suggested a mathematical way out of the resulting difficulty, but it was found more practical by him and the writer to use tilt-apparatus measurements to compute the deflections of the light spot due to the tilt and then eliminate the equivalent displacement from that obtained by double-integrating the accelerograms. The result was then a displacement curve with all tilt effects eliminated. Absence of exact data on the phase of the tilt, absence of a complete cycle, the presence of zero shifts due to slightly unstable accelerometer pendulums, and the complexity of some of the records due to parasitic vibrations all combined to create a computational problem never realized in practice.

Enlargements of test records 25, 32, 39, 44, and 46 were made with the lantern projector. In the first three tests the time increment used was 0.029 second, and in the last two, 0.072 second and 0.052 second, respectively. The number of increments used in each case was 1,475, 1,620, 1,640, 680, and 1,030.

The unadjusted velocity curves, obtained by one integration of the acceleration curves, are shown in figure 18 for the first four tests. When no shift in the acceleration axis occurs, the axis of the velocity curve is theoretically linear except for the effect of the sinusoidal tilt imposed on the accelerometer. A change in the direction of the velocity-curve axis indicates a semipermanent shift in the zero position of the accelerometer pendulum, due, it is believed, to minute shifting of the pivots in the agate bearings. The figures on the axes are those ordinates of the original acceleration curve (measured from the base lines) which mark the positions of the finally adopted axes, all figures being expressed in units of integration.

As abrupt shifts of the velocity curve like those in figure 18 have never appeared in processing actual earthquake accelerograms, they must be considered as something peculiar to the shaking-table tests. The theoretical implication is that an acceleration of the order of 0.01 gravity was imposed on the accelerometer in one direction only and lasted but a small fraction of a second. The physical implication is that the accelerometer was suddenly tilted, or the pendulum mirror disturbed. It is possible that unavoidable motions of the observer on the shaking table during the tests may have caused minute tilting of the table, or even air currents. Such effects would not, of course, affect the recorded table displacement. On the other hand, a large error in scaling the acceleration curve would produce the observed effect, but errors of the magnitude shown by the corrections applied (in units of integration) could hardly have escaped detection in revisions which were motivated by the presence of such discrepancies. Moreover, both Professor Ruge and the writer were unable to reconcile the computed velocity curve for test 39 with any normal type of curve. For this reason Professor Ruge did not complete the computation of the no. 39 test record, assuming that the quadrifilar accelerometer was not in satisfactory adjustment, a conclusion in which Mr. McComb concurred. The writer carried the computation through, making adjustments which are discussed in the following section. It is believed that the computation has value in showing what accuracy to expect when and if such discrepancies appear in the processing of actual earthquake records.

When the computed displacements for tests 25, 32, and 39 were compared, it was evident that they were similar in general form, but the author did not know whether the table motion had been purposely varied or whether the differences represented real errors. No attempt was made to investigate the cause of the differences before the comparison with the table-motion records, as it was thought that this could be done to much better advantage afterward.

Figures 16 and 17 show the comparisons between the computed and recorded table displacements in the form of error curves. The similarities between the computed and recorded curves show up to better advantage when they are placed side by side. Although the errors seem relatively large compared with the total displacement, the latter is undoubtedly below the range of even slightly destructive motion. The errors found have but little or no engineering significance, because the accelerations involved are small. As previously stated, they have been considerably reduced through the development of a new enlarging apparatus.

Figure 15 is one of the few illustrations which show the acceleration, velocity and displacement of the same motion on the same time scale. To those not familiar with the relationship between them it will be interesting to note how positive algebraic ordinates on the acceleration and velocity curves always coincide with upward slopes of the velocity and displacement curves, respec-

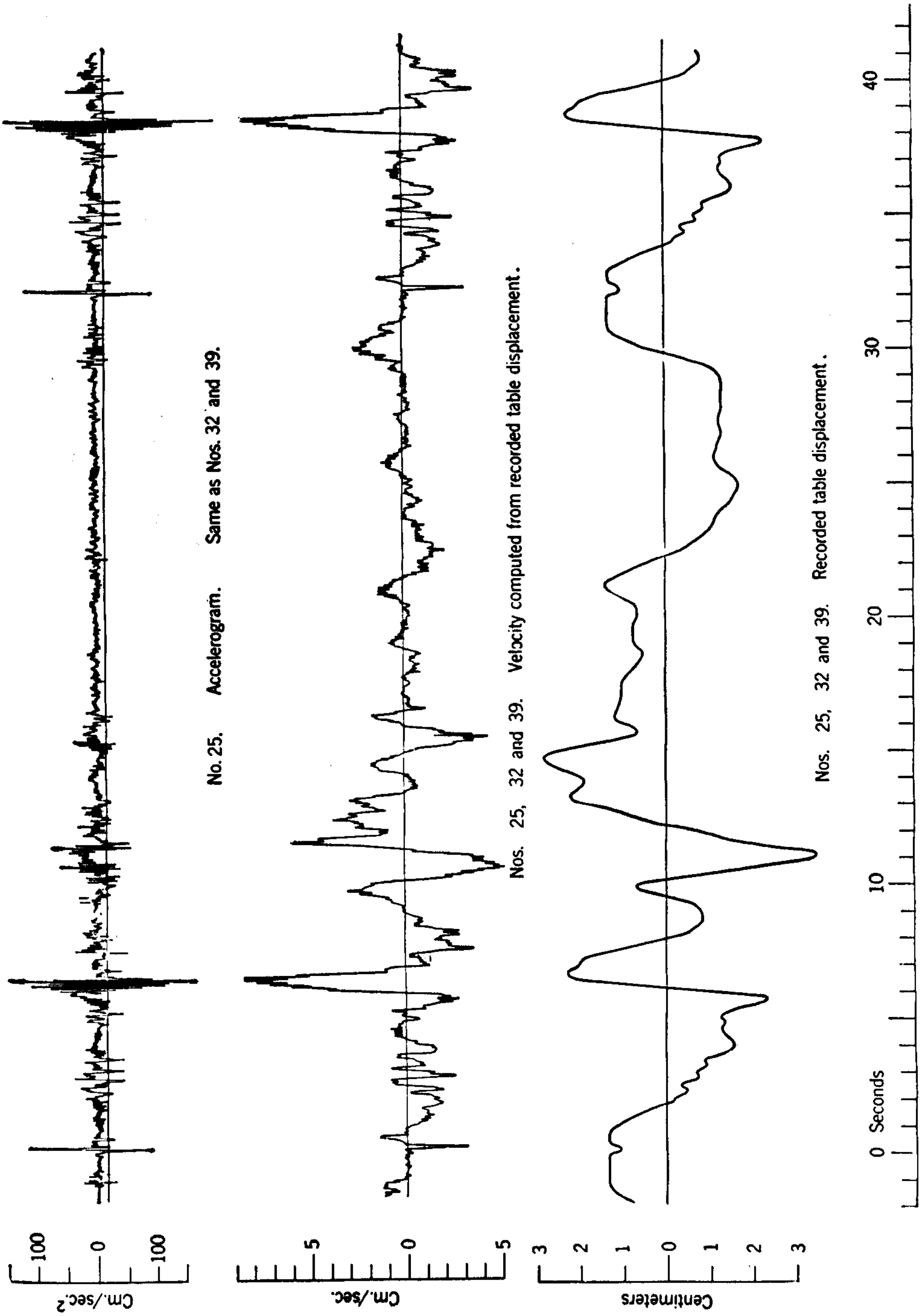


FIGURE 15.—Acceleration, velocity, and displacement in Massachusetts Institute of Technology shaking-table test simulating one component of Long Beach earthquake motion in downtown Los Angeles. The table motion was about one-third the actual earthquake motion.

tively; and vice versa. In reversing the process (from integration to differentiation) it can be seen how the magnitude and direction of the slopes of the displacement and velocity curves govern the magnitude and the algebraic signs of the velocity and acceleration curves, respectively. The velocity curve has special significance in this paper because it shows the true form of the three curves shown only in crude form in figure 5, and it is the curve used in computing most of the "velocity error curves" discussed in later sections.

No. 25 error curve (fig. 16) shows a total range of about 3 centimeters. Where the amplitude of the curve is greatest, just before 10 seconds on the time scale, the acceleration error is about 1.7 cm/sec², which is equivalent to about 0.25 millimeter on the original accelerogram. It is probably due in large part to heat distortion during enlargement and an axis adjustment necessitated by an apparent shift of the zero position of the accelerometer pendulum. Two axis adjustments were made on the basis of the evidence shown in the trial velocity curve in figure 18. There is always, also, the possibility of errors in the original scaling of the enlarged acceleration curve.

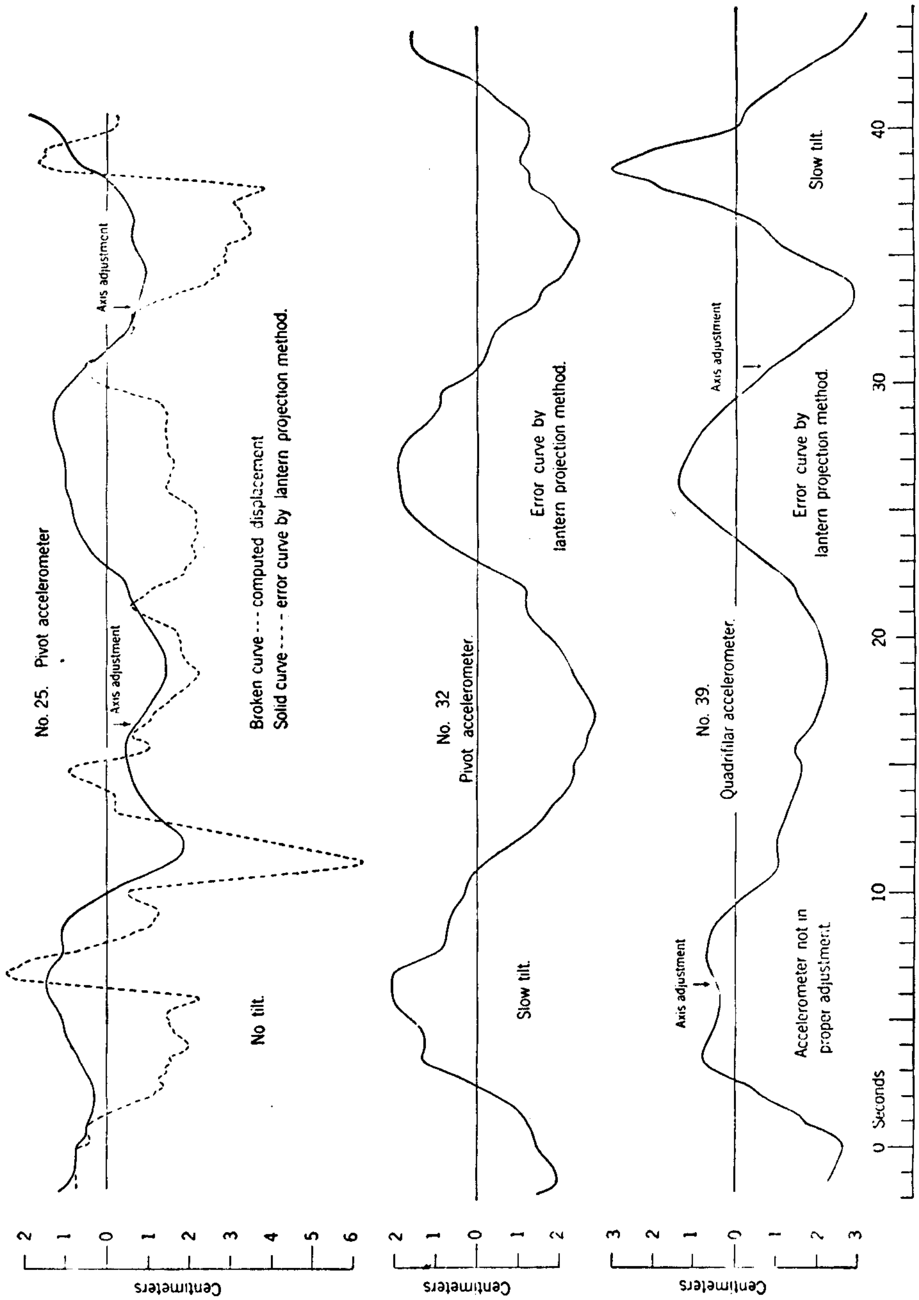


FIGURE 16.—Error curves obtained in three tests using lantern enlargements (now obsolete) and numerical integration.

No. 32 error curve is notable for three reasons. It is the only record of a pivot accelerometer which, in the author's experience, did not show, in the course of integration, evidence of semipermanent shifts in the acceleration-curve axis. It is therefore the only test which definitely shows elimination of tilt effects without errors due to axis adjustments. Secondly, if the tilt effect had not been removed, the curve would have been superposed on a sinusoidal wave of 25 centimeters single amplitude; so the curve represents the error to

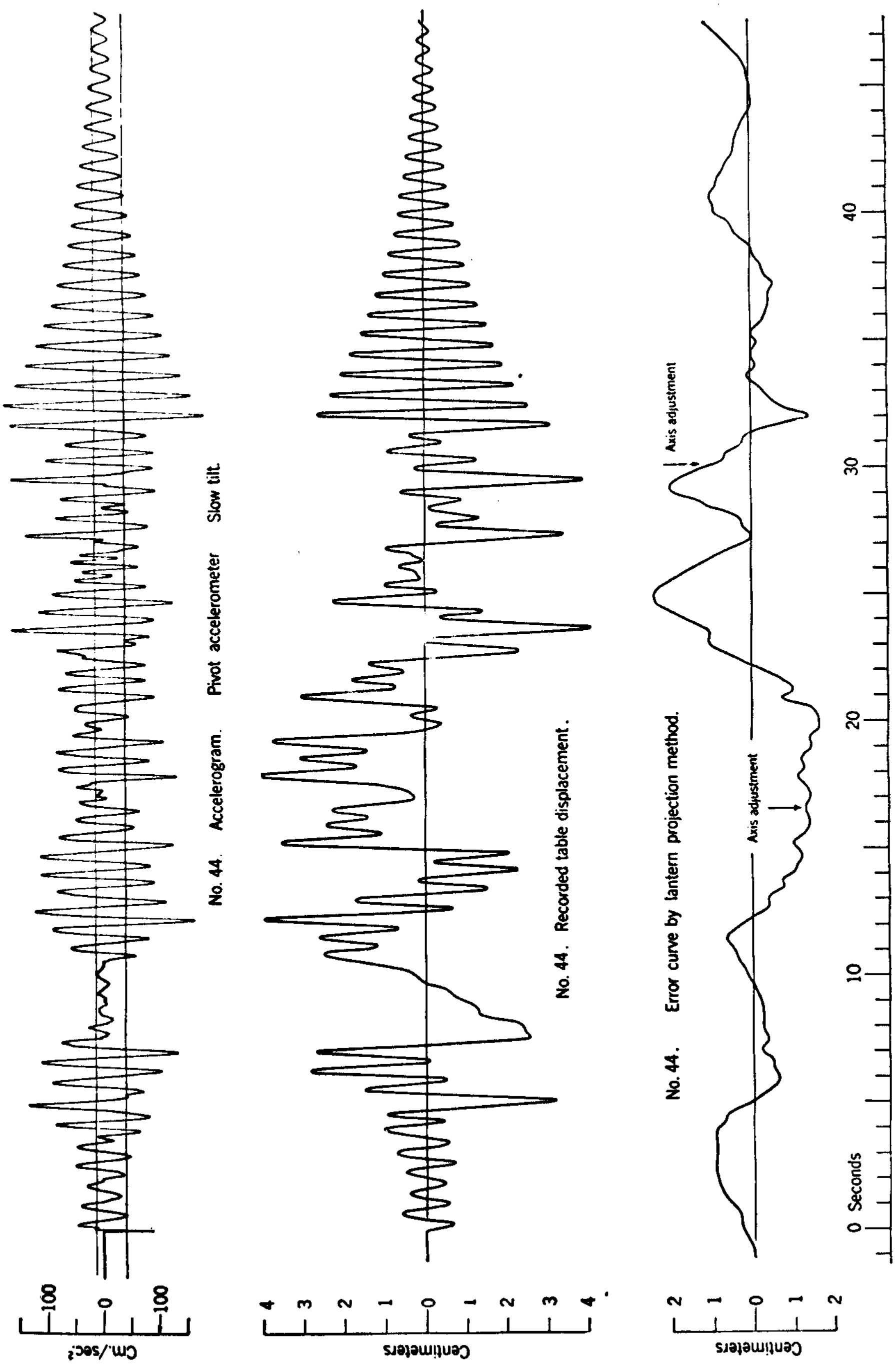
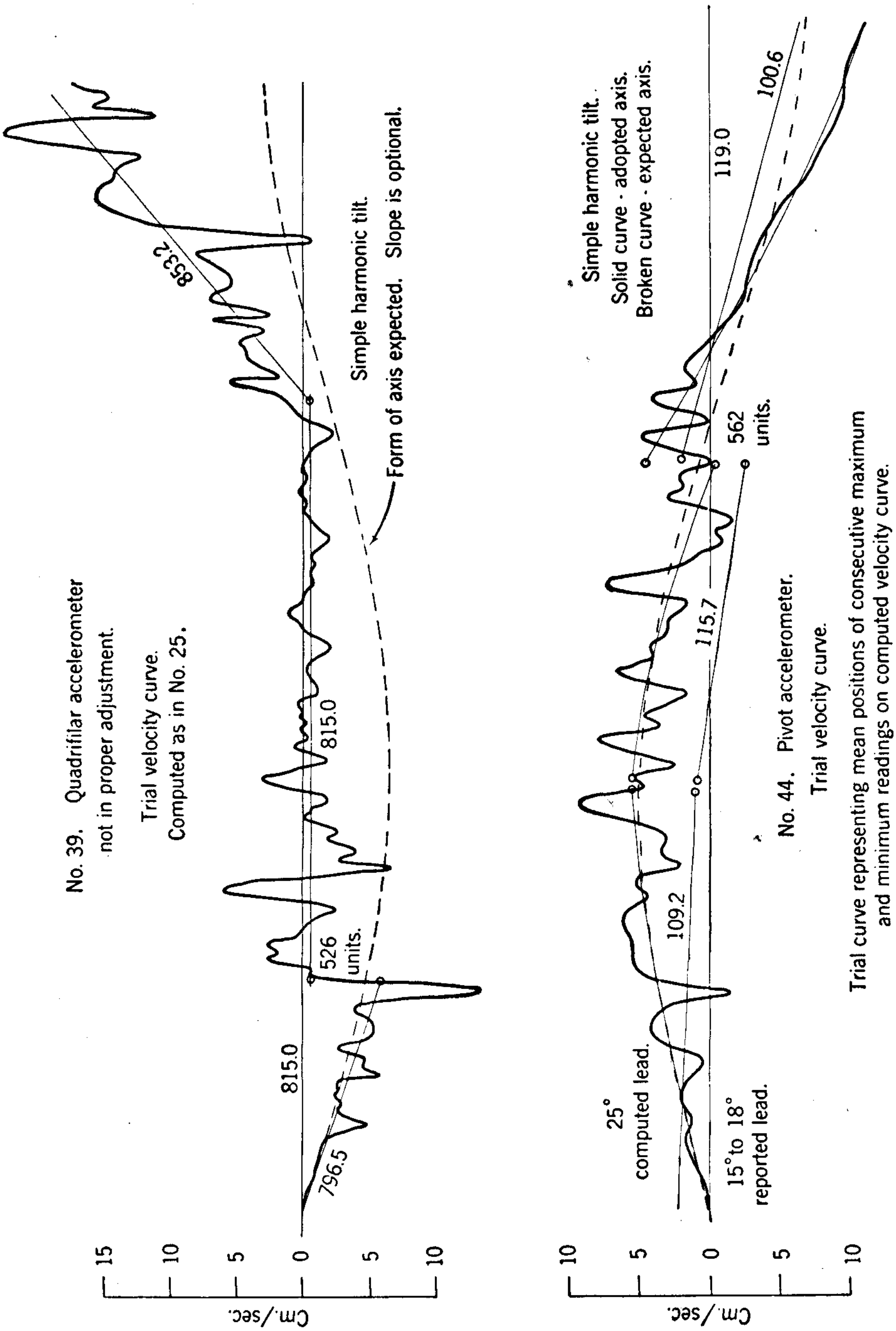


FIGURE 17.—Result of Massachusetts Institute of Technology shaking-table test using lantern enlargement and numerical integration.



be expected in computing a ground wave of similar period (64 seconds) and displacement. Third, because of the absence of axis adjustments the uniform and periodic character of errors due to paper distortion is clearly shown. The other error curves show evidence of this too, but it is distorted presumably by axis adjustments. The distortion is undoubtedly due to unequal heating of the accelerogram in the enlarging lantern, because there are just as many cycles of sinusoidal character as sectional exposures of the original accelerogram. In each section exposed it is presumed that the distortion was greater

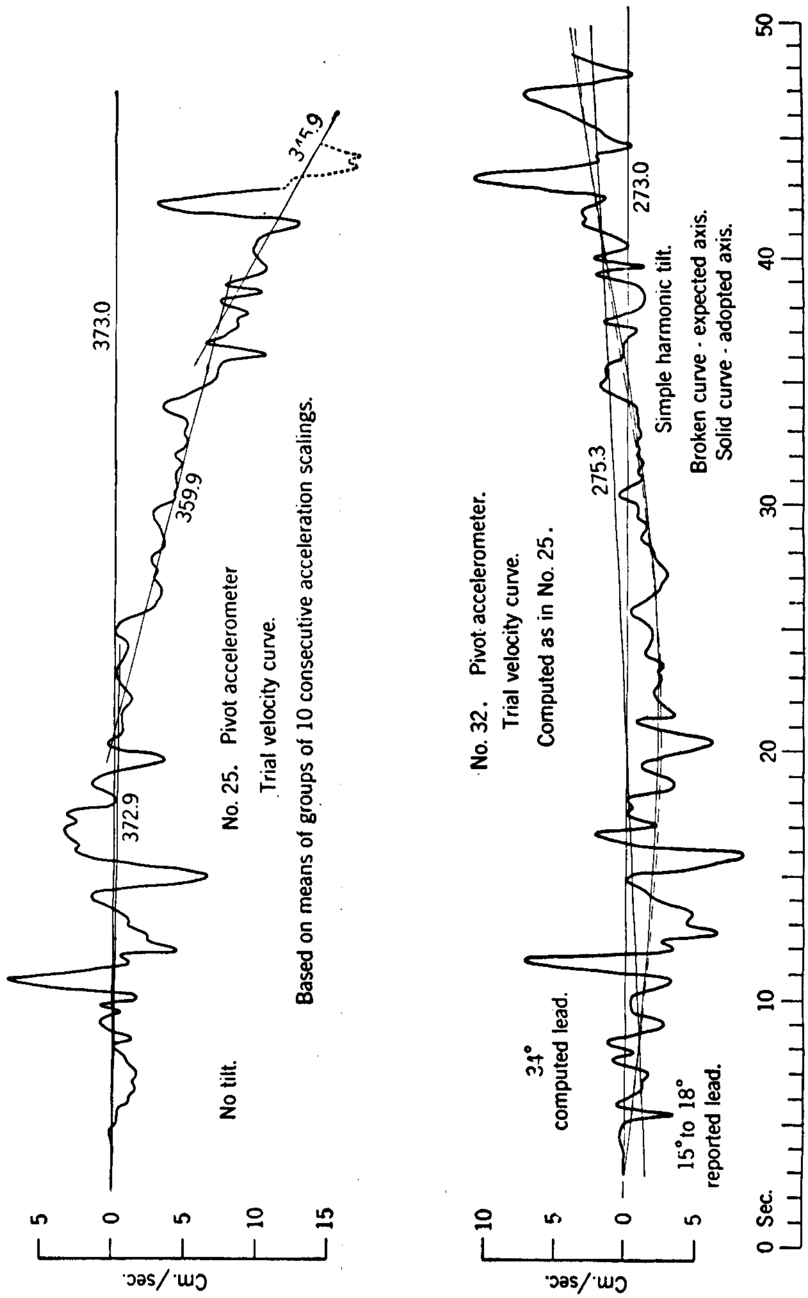


FIGURE 18.—Unadjusted velocity curves showing drift of axes due to shifting of the zero positions of the accelerometer pendulums. Figures on the various segments are readings of the original and revised acceleration-curve axes as measured from the baselines on the enlarged curves.

in the center than at the sides in spite of a stream of compressed air which blew continuously over the entire exposed surface. The 4-centimeter range of error corresponds to a lack of symmetry of about 0.15 millimeter on the original record, this value presumably representing the variation in distance between the baseline and the true axis of the curve.

No. 39 error curve embodies an axis shift of the type not previously found and clearly shown in the trial velocity curve, figure 18. The special features of the shift were discussed in one of the earlier paragraphs of this section. The writer knew that the instrument had been subjected to a simple harmonic tilt of 64 seconds period and 50 centimeters displacement, but it will be seen from the unadjusted velocity curve in figure 18 that there was little else to do but ignore the tilt and consider the three segments as of linear character. Although the accelerometer was considered out of adjustment, there seemed to be only one disturbance of a magnitude that could not be attributed to normal errors of accelerometer zero shifting. It will be shown in the discussion in the next section that there were actually two such disturbances (fig. 19), both in the same direction and of about equal magnitude, and both occurring at the same phase of table motion. The second, however, was not clear enough to justify anything more than another acceleration-curve axis adjustment, because of distortion effects, the steep gradient of the tilt curve, and the limited number of ordinates used. As an abrupt shift of velocity-curve axis is equivalent to an erroneous reading of an acceleration-curve ordinate, the first break in the velocity curve was corrected by simply changing one of the acceleration ordinates the necessary amount, namely, 526 units of integration.

TABLE 2.—*Test 46. Comparison of Double Amplitudes*

Table motion	Full motion	Ratio*	45° Component	Ratio*
mm.	*mm.*		*mm.*	
28.8	27.0	1.07	27.9	1.03
33.5	30.5	1.12	31.7	1.06
16.5	16.0	1.03	15.1	1.09
56.8	57.0	1.00	59.0	.96
37.0	36.0	1.03	39.2	.94
32.0	32.0	1.00	33.0	.97
25.7	25.9	.99	25.8	1.00
19.8	18.7	1.06	20.9	.95
15.0	14.0	1.07	15.5	.97
11.5	10.3	1.12	12.1	.95
88.0	92.0	.96	90.0	.98
76.0	74.0	1.03	76.0	1.00
64.0	65.0	.98	65.0	.98
52.8	53.5	.99	52.0	1.02
43.2	43.0	1.00	42.0	1.03
34.6	34.5	1.00	34.0	1.02
26.9	29.0	.93	27.8	.97
20.7	21.0	.99	20.5	1.01
15.2	15.3	1.00	15.1	1.01
11.1	12.0	.92	11.0	1.01
Means		1.01		1.00

* Ratio between computed and actual table motions.

As this was the only quadrifilar record of the test series processed, a special study of it was eventually made. The results are described in the next section. No. 39 was the only quadrifilar record ever found to contain large zero shifts. Ultimately another quadrifilar record, no. 17, was processed, with good results, because it was considered necessary to make a satisfactory appraisal of the quadrifilar type of instrument, especially in view of the fact that some important records, including all for the Long Beach earthquake of 1933, had been obtained with that type.

No. 44 error curve shows what to expect in the case of a smooth type of wave. The 4-centimeter range of error is about the same as that for the more complex type of record used in the preceding tests, but, measured in terms of acceleration, some of the discrepancies are considerably greater. The superposing of tilt equivalent to 50 centimeters displacement, two apparent axis shifts, and another of the unusual type of break in the velocity curve, all tended to make the adjustment a most laborious one. As may be inferred from figure 18, the amplitudes of the trial velocity curve were too great to define the axis by inspection, and hence a curve representing the means of consecutive maximum and minimum ordinates was used.

No. 46 error curves are not shown, because the desired results of the test can be expressed numerically. Two pivot accelerometers simultaneously recorded a table motion similar in type to test 44, both records being made on the same sheet. One recorded the full motion of the table; the other, a 45° component of it. The readings of the 45° component were divided by the sine of 45° to make the two computed displacement curves comparable. The main object was to learn how well the 45° component functioned in comparison with its theoretical performance. This is important because most of the motion which an accelerometer records during an earthquake is some component of a motion which is constantly changing in direction. A comparison is best made by comparing the recorded table amplitudes of waves selected at random with the computed amplitudes. The results are shown in the table 2 on page 32. In the integration processes two axis adjustments were required in each curve.

DISCUSSION OF ERRORS FOUND IN MASSACHUSETTS INSTITUTE OF TECHNOLOGY SHAKING-TABLE TESTS

While it was evident from inspection that much of the error in the tests described above was due to heat distortion during enlargement, it was necessary to find a method of expressing the error in terms of acceleration, if possible, to distinguish between errors due to instrumental performance and those due to processing of the records. After much investigation a compromise was found which answers practically all fundamental requirements of a critical analysis. It is based on obvious properties of the velocity curve (the first integral of the recorded acceleration) and the fact that it should be identical with a similar velocity curve obtained by differentiation of the recorded displacement, the derivative being devoid of all drift and therefore quite suited to serve as a "correct" velocity curve. We may then accept the difference between the two curves, the *velocity error curve,* as error due either to instrumental performance or to processing of the record. Any change in slope of the axis of this curve represents an acceleration-axis error.

In the Massachusetts Institute of Technology tests the table-displacement curves were so open that there was no difficulty in determining the first derivative; in fact, the precision with which the light spot of Professor Ruge's apparatus would retrace a previous recording was uncanny. The velocity curve in figure 15 is based on one of these records, but in determining the error curves it was expanded to a scale of approximately 7 centimeters to the second. In view of the ability of the shaking table to repeat motions with such precision, it was decided that the velocity curve computed from No. 25 displacement record could safely be used in tests 17, 32, and 39 also.

One would expect to find in these velocity error curves a check on the precision with which axis adjustments were made in the M.I.T. tests, but they were so irregular (owing to heat distortion) that the choice of axes still remained a matter of judgment.

A different type of problem is presented in test 39 and to a smaller degree in test 44. The velocity error curve for no. 39 is shown in figure 16. As previously stated, the accelerometer was considered out of adjustment. In the original processing the author adjusted for an abrupt shift of both acceleration and velocity curve axes at 10 seconds on the time scale, but only for a shift of acceleration-curve axis at 33 seconds, as shown in figure 18. The velocity-error curve (fig. 19) shows that there were actually two abrupt velocity axis shifts of practically equal magnitude, and that in all probability a semi-permanent shift of the acceleration-curve axis occurred only in the first instance. This discrepancy need not influence an appraisal of the validity of the method of making such adjustments, because velocity-curve shifts of this kind never occur in practice; and neither do the tilt motions, the presence of which largely obscured the disturbance at 33 seconds. There is also no doubt that heat-distortion effects complicated this particular problem. With the new mechanical enlarger such distortion is eliminated. Some probable

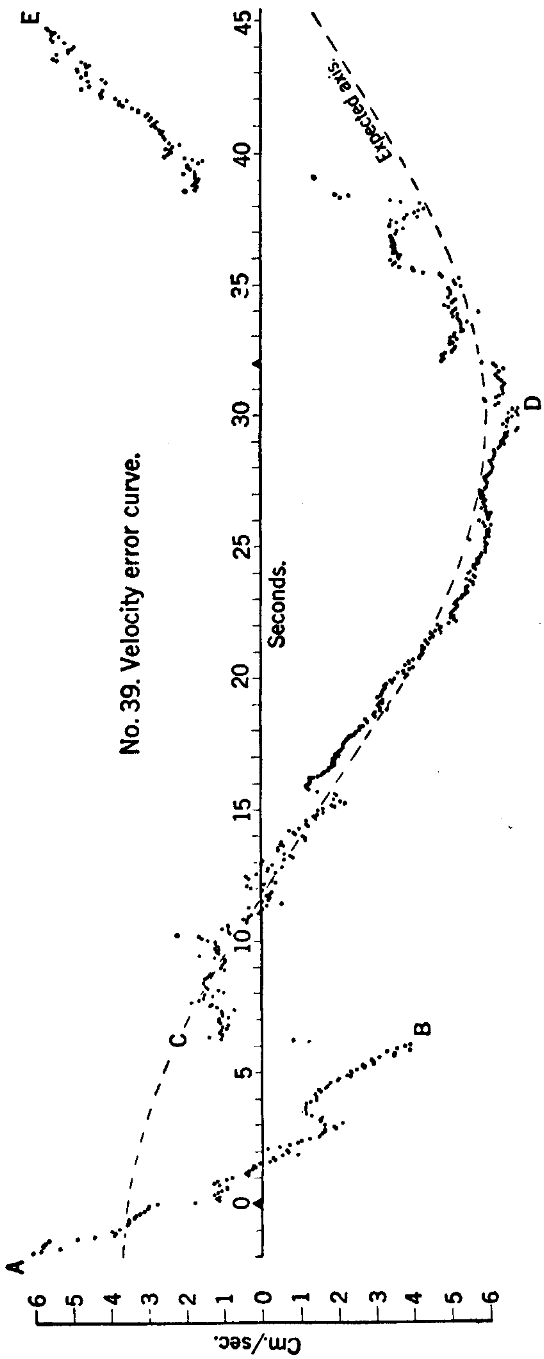


FIGURE 19.—Difference between velocity obtained by one integration of the accelerograph curve and that obtained by one differentiation of the recorded table-displacement curve. The quadrifilar accelerometer was subsequently considered not to be in proper adjustment during test No. 39. The broken curve shows the nature of the distortion expected as a result of the periodic tilt imposed on the accelerometer.

causes of the abrupt displacements in the axis of the velocity curve were given in the preceding section because they were critically studied during the original processing in order to justify the adjustments made.

Barring the two abrupt shifts, the velocity error curve apparently shows that the instrument functioned in a normal manner, as will be seen by comparing the error curve with those obtained in test 17 (fig. 21). The broken line in figure 19 shows the expected deviation of the velocity-curve axis due to the periodic tilt imposed on the accelerometer. In the original adjustment the axes selected were equivalent to straight lines drawn between *AB*, *CD*, and *DE*. If these lines are drawn, it will be seen by inspection that the resulting curve is practically the first derivative of the displacement error curve for no. 39 (fig. 18).

In order to investigate further the behavior of the quadrifilar accelerometer a study was made of the most active portion of another quadrifilar record, no. 17, a record similar to nos. 25, 32, and 39. A combination of verniers was used to read the horizontal and vertical coordinates of many points on the curve, the readings being made to the nearest 0.02 or 0.03 millimeter. The points were plotted on a greatly enlarged scale, the curve was drawn, and the velocity curve computed in the usual manner. The velocity error curve for this is shown in figure 21, together with other error curves obtained from the same record, using the new mechanical enlarging apparatus.

The velocity error curve obtained by the vernier method is remarkable in several respects. Its unusual smoothness shows very small error in the enlarging and scaling processes, so that for limited stretches the accelerometer must function almost perfectly. It shows that the motion of the accelerometer pendulum is accurately recorded by the light spot when the center of the trace is used, a matter which has been a subject of speculation. Because of this performance the sudden shift of the velocity error curve seems to be real. Although error curves based on apparently less accurate methods are shown for both the quadrifilar and pivot instruments in test 17 (fig. 21), there is, nevertheless, evidence in them of the same type of error as is found by the vernier method. But the offset of the velocity curve of 1 cm/sec. in test 17 does not explain the two axis offsets of 5 cm/sec. in Test 39. They also may be different in that the test 39 shifts are permanent whereas in test 17 they apparently are not.

It seems necessary to conclude that some forces other than those recorded on the table displacement curve, and the known tilt, must have affected the accelerometers in the course of the shaking-table tests. In test 17 the effect was within the range of ordinary discrepancies, but in test 39 it was not and therefore was more serious.

RESULTS OBTAINED WITH NEW MECHANICAL ENLARGING APPARATUS

Partial elimination of errors revealed in the shaking-table tests was not difficult once the chief source was known. Although some special apparatus was constructed to provide for continuous movement of the original record, and the cross-section paper on which the enlarged image was projected (thus avoiding unequal heating of the very small section of the record being used), it was never used extensively and was soon replaced by a new mechanical enlarging apparatus. The lantern never was wholly satisfactory in practice, for several reasons. When the accelerogram trace was weak, as often happened, the enlarged image was even weaker, or invisible. Tracing the image was also an enervating task which required about a day of darkroom work for every record of a strong earthquake; and it called for a certain amount of drafting skill, which was not always available. Worst of all, magnification was uniform in all directions, so that on an enlargement the transverse magnification was too great; and the longitudinal magnification was so small that it was frequently necessary to reconstruct active portions of a record on a more open time scale. The vernier method previously discussed was accurate, but too laborious for routine use.

The device finally adopted and now in use is the mechanical enlarging apparatus shown in figure 20. The principles on which it operates are outlined briefly in the legend. Enlargement is obtained through two pulley systems, which magnify the displacement of a point index as it is moved over the accelerogram trace. Longitudinal and transverse magnifications are independent, as each is controlled by a separate pulley system. The chief mechanical problem was to reduce friction to a minimum in the two table motions and the pulley system which controlled them. This was accomplished satisfactorily by using a total of thirty-six high-grade ball bearings and high-grade fishline. Friction is so small that, even though the longitudinal magnification is 25, it is quite possible to repeat readings within the small diameter of the pinholes which outline the enlarged curve.

The following are some of the advantages over lantern enlargements: (1) The record may be expanded longitudinally at will while transverse enlarge-

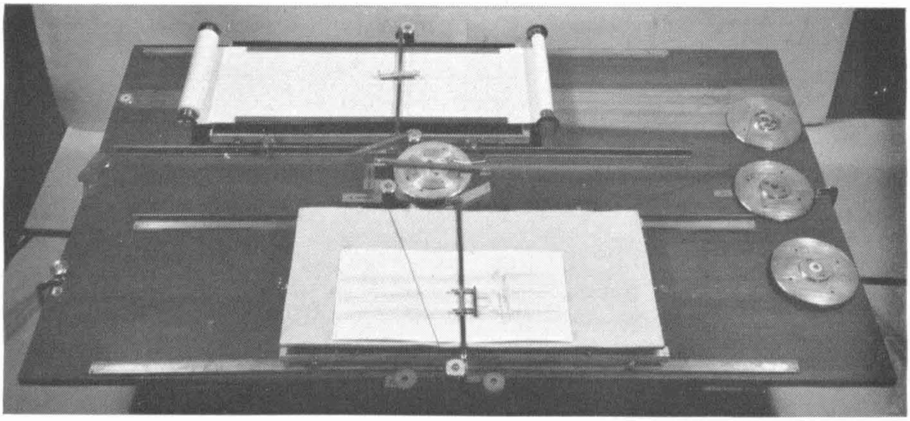


FIGURE 20.—New mechanical enlarging apparatus designed to expand an accelerograph record 3, 8, or 25 times longitudinally and 1.5 or 2.5 times transversely. The two tables roll on ball bearings along metal tracks in opposite but parallel directions through a pulley-and-string hookup. The ratio between the table motions is controlled by the pulley system at the end of the board. The pulley in the center of the board controls two sliding elements, which accomplish the transverse enlargement. Longitudinal and transverse motions are independent and are controlled by two small wheels on the front center edge of the board. With these wheels the operator sets a sliding pointer, or index, on successive points of the original accelerograph curve fastened on top of the slow-moving front table. At each setting, an assistant on the opposite side of the board punches a pinhole in the cross-section paper on the rear high-speed table by means of a pin fixed to the carriage which slides transversely above the table. The pinpoints outline the expanded curve. Slow-motion screws on the rear table provide for frequent transverse table adjustments necessary to keep the baseline on the enlargement in precise alignment with the accelerogram baseline.

ment is kept within convenient bounds, thus producing a curve which is easy to read and which never requires further expansion. (2) Weak traces are never lost if they are at all visible on the original record. (3) The mechanical method is less fatiguing and requires less skill. (4) The results are at least as

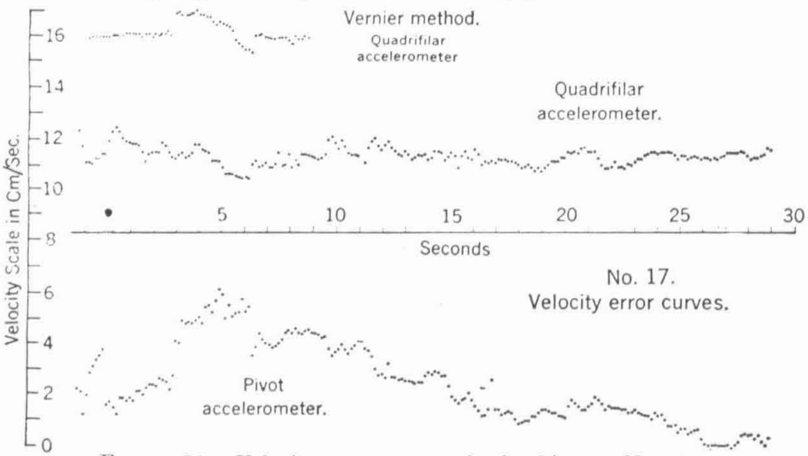


FIGURE 21.—Velocity error curves obtained in test No. 17.

accurate as the best that might be obtained by improved lantern enlargements. (5) The record is not subjected to heat or any other type of distortion except that due to changes in humidity.

Record no. 17 was used to appraise the performance of the mechanical enlarger. Only the first part of the record was used, as it was of the so-called "Long Beach type" and the true velocity curve, needed for critical analysis, was already available. The last part of the record was an irregular motion made through special controls. Simultaneous records of a quadrifilar and a pivot accelerometer were registered on the same sheet.

As a first step, the velocity curves were computed for both pivot and quadrifilar instruments. The velocity error curves in figure 21 show that, so far as can be determined by inspection, the zero position of the quadrifilar pendulum was maintained throughout the test. In the pivot accelerometer curve, at least one axis shift is clearly evident and there is a certain amount of wandering. A notable feature is that neither curve appears as smooth (not as smooth in spots) as no. 39 error curve, figure 19, which is based on a lantern enlargement. This is probably due in part to the fact that the mechanical enlargement was made by personnel without previous experience, a procedure which, it was hoped, would compensate in some measure for the fact that the table motion was known to the author, a condition which was avoided in all other tests. After no. 17 was processed, a possible source of error was eliminated by making the enlarged curve on one long strip of cross-section paper instead of on individual table-size sheets as previously. This reduces the manipulations of the operators considerably. The new spools can be seen in figure 20.

The displacement error curve based on the record of the quadrifilar accelerometer is shown in figure 22. The range of error is about 1 centimeter, approximately a third of that believed to be due to heat distortion. The displacement was not computed for the pivot accelerometer record, as the velocity error curve in figure 21 shows that zero shifting would in all probability influence the magnitude of the error and thus invalidate the effort to obtain a

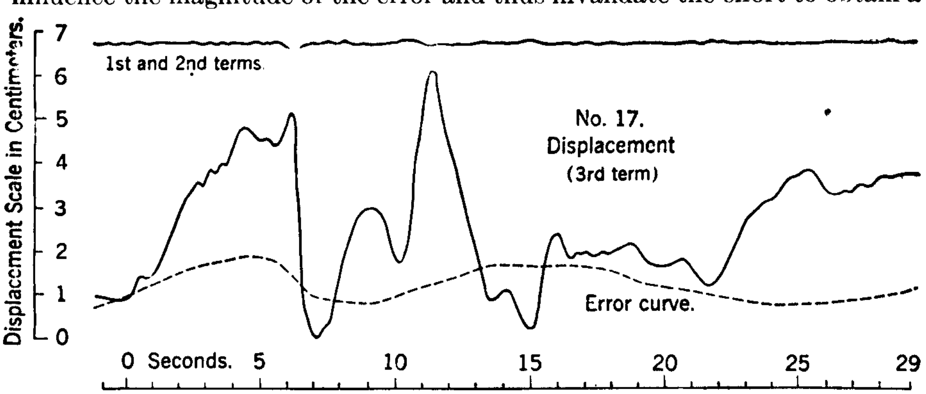


FIGURE 22.—Computed displacement based on test No. 17 quadrifilar accelerometer record and an enlargement made with the new enlarging apparatus. The top curve shows the combined magnitude of the first and second terms of the equation of motion, which are disregarded in computing displacement from an accelerograph record. The error curve represents the minimum error yet obtained, a range of 1 cm.

curve which embodies only normal processing errors.

No. 17 quadrifilar test was ideal for determining the magnitude of the first two terms of the equation of motion, which are ordinarily omitted in computing displacement as being insignificant. The equation is explained in the introductory paragraphs of this paper. In test 17Q the magnitudes of the coordinates are shown in the following equation:

$$2{,}359\ D = 0.224\ y + 0.558 \sum_{0}^{900} y + \sum_{0}^{900}\left(\sum_{0}^{900} y\right)$$

The third term is dominant, not because of the magnitude of its coefficient (unity), but because the numbers resulting from the second summation are roughly ten times as large as those of the first summation (second term); and these, in turn, are roughly ten times as large as the algebraic ordinates of the acceleration curve. The factor required to convert units of integration to centimeters of displacement is 2,359.

The first two terms were computed and the resultant curve was inserted in figure 22, where it can be compared with the third term, designated the computed displacement. The first term is, for all practical purposes, the acceleration curve in figure 15 after 100 cm/sec.^{2} is made equivalent to 0.025 cm.; the second term is the corresponding velocity curve with 5 cm/sec. equivalent to 0.088 cm. The curves in figure 22 should be combined to obtain the true computed displacement, but it is apparent that no serious discrepancy is introduced by not doing so. If greater accuracy were necessary, a more practical solution would be to reduce the period of the accelerometer pendulums. This would result in a desirable decrease of sensitivity and at the same time reduce the type of error just discussed. The instrument would then function as an accelerometer over a greater range of periods and reduce the relative weight of the first two terms. A less desirable solution would be to correct for the second term only, as the first is only about 10 per cent of the second in magnitude.

Test 17Q was also used to determine the effect of varying the magnitude of Δt, the increment of time. This is important in numerical integration because it is desirable to keep the total number of increments at a minimum in order to reduce the cost of processing. As stated in an earlier section under "System of Checks" (see p. 24), a certain amount of error might be expected from the use of individual ordinates instead of mean ordinates in the integration of the velocity curve, and that it might seem desirable to increase the number of increments. The results, however, seem to show that the number of increments can be reduced without materially changing the result. The smoother character of the velocity curve seems to compensate for the theoretical deficiency.

The displacement curve for test 17Q was computed in four different ways, the smallest increment (0.0338 sec.) being used first, and then increments five and ten times as large. The results are shown in figure 23. It appears that some of the error may be due to the use of time increments (in the acceleration

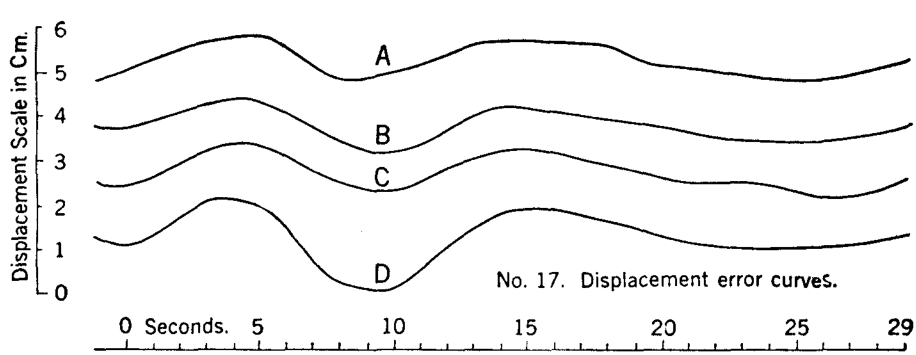


FIGURE 23.—Displacement error curves obtained in test No. 17 with use of different time increments in the double-integration process. Curve A was obtained when an increment of 0.0338 sec. was used in both integrations; curve B, 0.0338 sec. in the first and 0.1690 sec. in the second; curve C, 0.1690 sec. in both integrations; and curve D, 0.3380 sec. in both integrations.

curve) which are too large, inasmuch as the same type of error curve is obtained in exaggerated form when the increment is increased. This does not appear to be a serious problem, however. It seems that some labor might be saved without sacrificing accuracy appreciably, by reducing the number of increments in the second summation.

DISCUSSION AND REPROCESSING OF SUBWAY TERMINAL RECORD OF THE LONG BEACH EARTHQUAKE

The Los Angeles Subway Terminal accelerogram of the Long Beach earthquake of 1933 was the first record used to investigate the practicability of obtaining displacement by numerical integration. As the active portion of the displacement computed at that time was simulated in the Massachusetts Institute of Technology shaking-table tests, and used by a number of engineering institutions in laboratory investigations, it is important to appraise the earlier results in the light of the higher degree of accuracy now attained. But it should be repeated that the shaking-table displacement was only about one-third the amplitude originally computed, so that the percentage error indicated in the computed error curves (fig. 16) is probably three times greater than that indicated in a comparison with the computed earthquake motion.

The following are some of the difficulties and uncertainties encountered in the earlier integration. Baselines were not included in the original accelerograph design, so it was necessary to use either the time marks as a base from which to scale ordinates, or acceleration traces recorded when the instrument was set in motion by weak aftershocks. The distances between the time marks and the earthquake traces were so large that paper distortion of considerable magnitude could be taken for granted. For the quiet traces it was possible to reduce the distance factor, but there was then the question of accelerometer pendulum stability between the periods of operation, and another concerning possible lack of parallelism between traces recorded on different turns of the drum. The paper, too, had a thick gelatine film, which suffered severely under the intense heat of the lantern projector. To make matters worse, the time scale was only half as open as that now in use, with the result that paper distortion effects were four times greater since each lantern exposure covered a time interval twice as long. In the earlier computation, near-by quiet traces were used as baselines. Because of the pressure of other work, investigation of the various sources of error was postponed from time to time until prospects of shaking-table tests finally provided assurance that the problem would be solved in the most desirable way.

The velocity and displacement curves obtained in 1933 are published in Coast and Geodetic Survey *Special Publication 201*, "Earthquake Investigations in California, 1934–1935," pages 38 and 39. The displacement curves are marked by long-period excursions with single amplitudes as high as 45 centimeters. Having periods of from 60 to 80 seconds, they correspond to deviations of the original acceleration curve of about 0.1 millimeter.

The M.I.T. tests show that the traces can be measured to a much greater over-all accuracy than 0.1 millimeter, and that if such long-period waves are present they can certainly be detected if baselines are available. It is thus clear that any errors of this order in the original computation of the Subway Terminal record must have been owing either to distortion of the paper or to lack of parallelism in the traces due to minute variations in the pitch of the screw drive. There was evidence of both in recent investigations of the original record. The distance between supposedly parallel lines of time marks over four turns of the drum was found to vary slowly, in an irregular pattern, over a range of 0.1 millimeter when placed in the lantern projector, and the range was only slightly less when measured on the new enlarging apparatus with total absence of heat. Some lines 1 to 2 centimeters apart, made on the same turn of the drum, deviated about as much as those recorded on different turns of the drum. There is thus strong evidence that the heat of the lantern must have added greatly to the discrepancies of the first analysis.

During the shaking-table tests the same Subway Terminal drum was used to determine screw-thread variation by having a baseline mirror record during four turns of the drum. The measured distances between consecutive turns are shown in figure 24. This test proves the inadequacy of acceleration records without baselines, a situation which was recognized and corrected as soon as possible after the first double integration of the Subway Terminal record of the 1933 Long Beach earthquake.

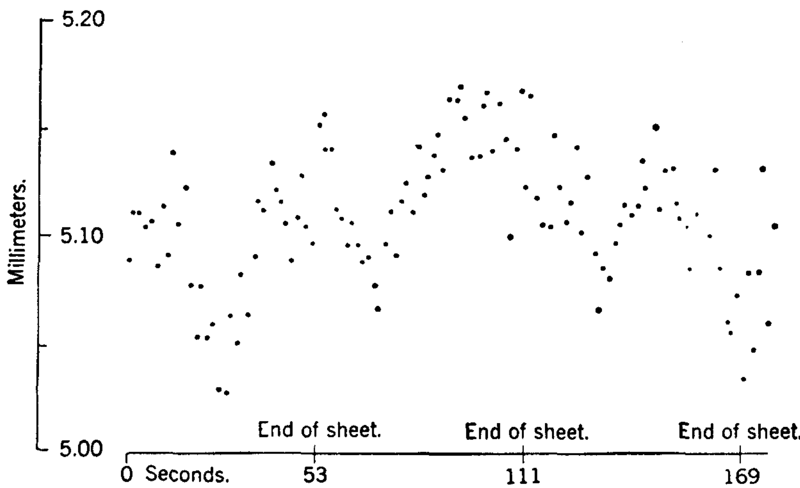


FIGURE 24.—Variations in space between baselines recorded on successive turns of a recording drum. The record was made with the same recorder that registered the Long Beach earthquake at the Los Angeles Subway Terminal to check the validity of computations based on the assumption of true parallelism. The variation is sufficient to account for a large part of the long-period waves of large amplitude obtained in the 1933 computation, which was made without baseline control.

The active portion of the NE–SW component of the Subway Terminal record was double-integrated again, using an enlargement made with the new apparatus. The result, shown in figure 25, is in substantial agreement with the result of the original computation shown in the same figure. Any engineering results obtained on the basis of the earlier computation may presumably be considered valid. The use of different traces as baselines will change the slope of the axis by amounts of the order shown in figure 25, but they all represent insignificant accelerations.

Although it is impossible to establish the existence of long-period waves with any degree of assurance on the Subway Terminal record, there is some recent evidence, based on a different type of analysis, that waves of 25-second period may be present in epicentral areas, but with amplitudes of the same order as those of other waves.

COMPARISON OF SIMULTANEOUS RECORDINGS ON AN ACCELEROGRAPH AND A DISPLACEMENT METER

The Eureka strong-motion records of the earthquake of December 20, 1940, off Cape Mendocino, California, were used to compare the displacement computed from an accelerogram with that recorded on a displacement meter. The displacement meter has a damped 10-second pendulum and unit magnification, and theoretically records true ground displacement for all ground waves under about 3 or 4 seconds' period. A primary purpose in double integrating acceleration records is to obtain displacement data at the great majority of stations where displacement meters are not operated.

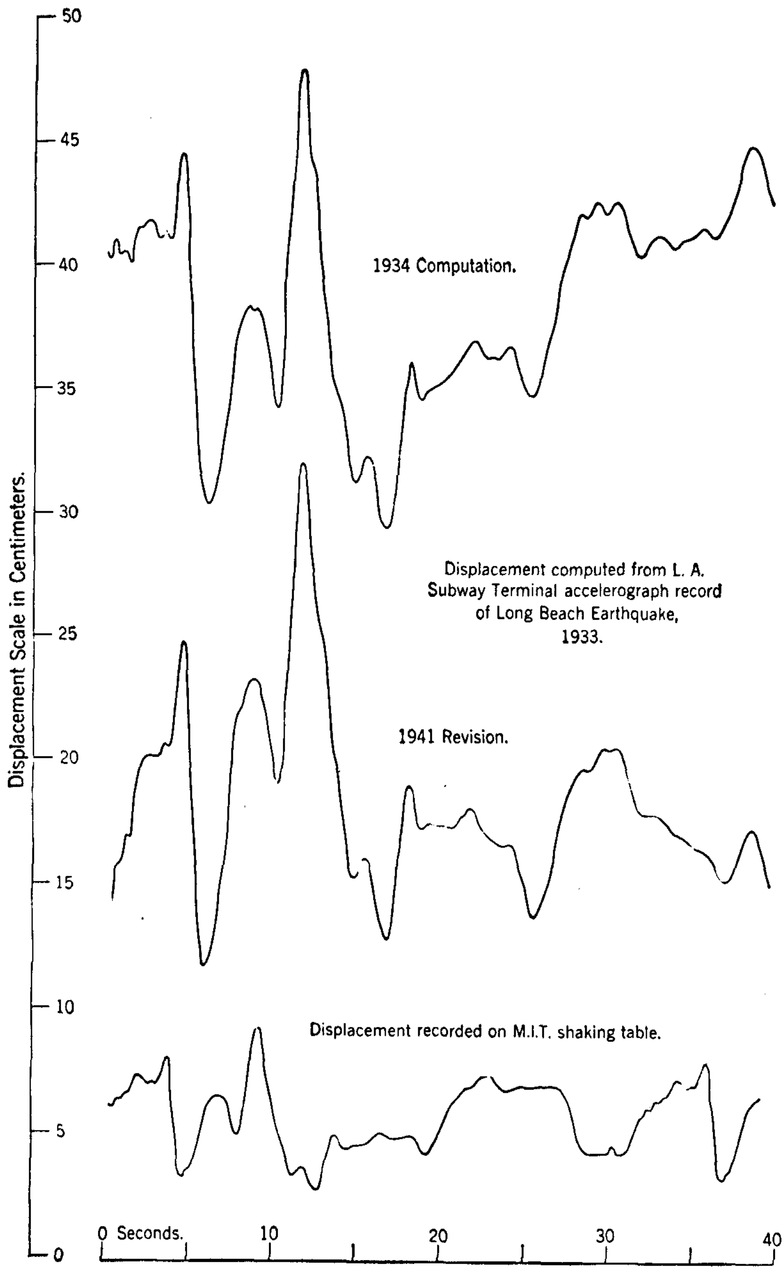


FIGURE 25.—Comparison between shaking-table displacement and those computed from the Los Angeles Subway Terminal record of the Long Beach earthquake, all on the same time scale.

Figure 26 shows the entire section of the Eureka accelerogram which was used in the computation. Only parts of the computed and recorded displacements are reproduced, as they are sufficient to make a comparison between the wave forms, which are of relatively small displacement. As the earthquake motion is only moderately active, it would be expected that errors of mensuration would be a minimum and the resulting error small; but over the entire length of the record there are differences of the same order as found in processing more difficult records such as the shaking-table records. A single correction for pendulum zero shift was not enough to eliminate the spurious deviations, which covered a range of nearly 3 centimeters in the latter part of the curve, not shown in the illustration.

A velocity error curve was computed to investigate the nature of the difference between the "true" velocity obtained by differentiating the recorded displacement curve and the unadjusted velocity obtained in the first summation of the acceleration ordinates. It is shown as part of figure 26. As there can be no appreciable drift in the curve derived from the displacement, all the

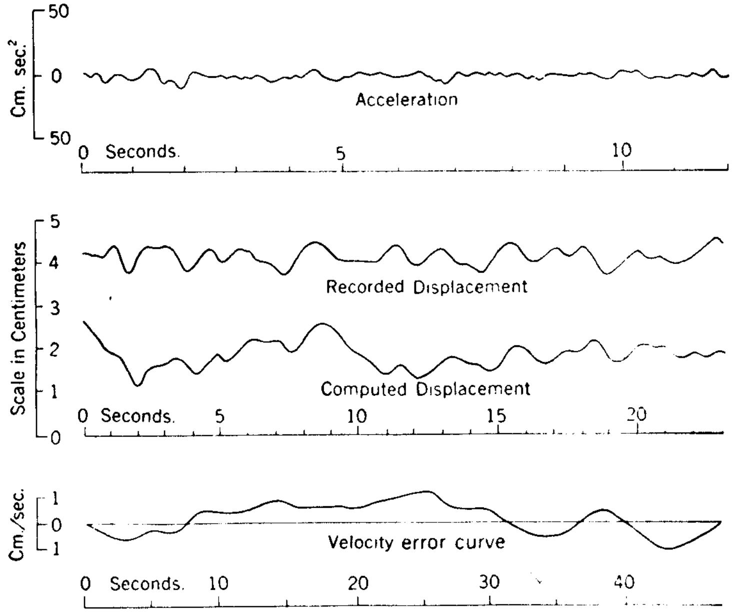


FIGURE 26.—Comparison between the displacement-meter record obtained at Eureka, California, on December 21, 1940, and the displacement computed from the accelerograph record obtained at the same station. The difference is of the same general character as obtained in the shaking-table test with pivot accelerometers.

abnormal drift must be charged either to instrumental performance or to processing operations. The error curve does not shift suddenly at specific points, but drifts in such an aimless way that it is impossible to correct it entirely by making the usual semipermanent shifts in the position of the acceleration-curve axis. This type of drift is very similar in type to that observed in other pivot-accelerometer results, and it seems reasonable to suppose that in addition to abrupt shifts, which can be readily detected as definite changes in the slopes of the trial velocity curves, there is sometimes also a gradual creep which defies detection in those curves. So far as errors of trace measurement are concerned, it is estimated that the deviations of the computed displacement curve would correspond to errors of 0.08 inch (8 units of integration) on the enlarged trace made with the mechanical enlarger. This must be ruled out as a source of error, because similar errors are not evident in other work, not even that (no. 17Q, for instance) in which inexperienced personnel were employed.

A COMPARISON BETWEEN TORSION PENDULUM ANALYZER AND NUMERICAL INTEGRATION RESULTS

In seismological engineering research a fundamental problem is to determine the response of a structure to a known ground motion in order to predict probable earthquake stresses. In its simplest form this is properly a seismological problem because it is, mathematically, simply a reversal of the equation of motion of a damped pendulum discussed in the Introduction. The seismograph is considered as simulating the motion of a structure, each having the same free periods and damping. Its bearing on the problem of computing displacement lies in the fact that a practical solution would enable the seismologist to determine displacement from an accelerograph record by simply obtaining the response of a 10-second pendulum. A mathematical solution, however, is so laborious as to be ordinarily impractical. Professor A. C. Ruge, of the Massachusetts Institute of Technology, employed this principle in analyzing the shaking-table records in addition to processing them by double integration methods, but in this he had available the very efficient differential analyzer of that institution. Mathematical solutions have been attempted by other investigators,[7] but with only partially successful results. The equation, in the form recently used by M. A. Biot,[8] is,

$$a = 2\pi \int_0^t a_0 (\theta) \sin \frac{2\pi}{T} (t - \theta) \, d\theta$$

in which a is the angular displacement of the pendulum mass.

In 1935 and 1936 the author gave much thought to the physical aspect of the problem, seeking first a practical way of imposing a varying acceleration on a horizontal (seismograph) pendulum by subjecting it to varying tilts. In searching for mechanical equivalents which would simulate this somewhat impractical solution, the idea of using a simple torsion pendulum eventually developed. An experimental pendulum (fig. 27) was built in the Washington

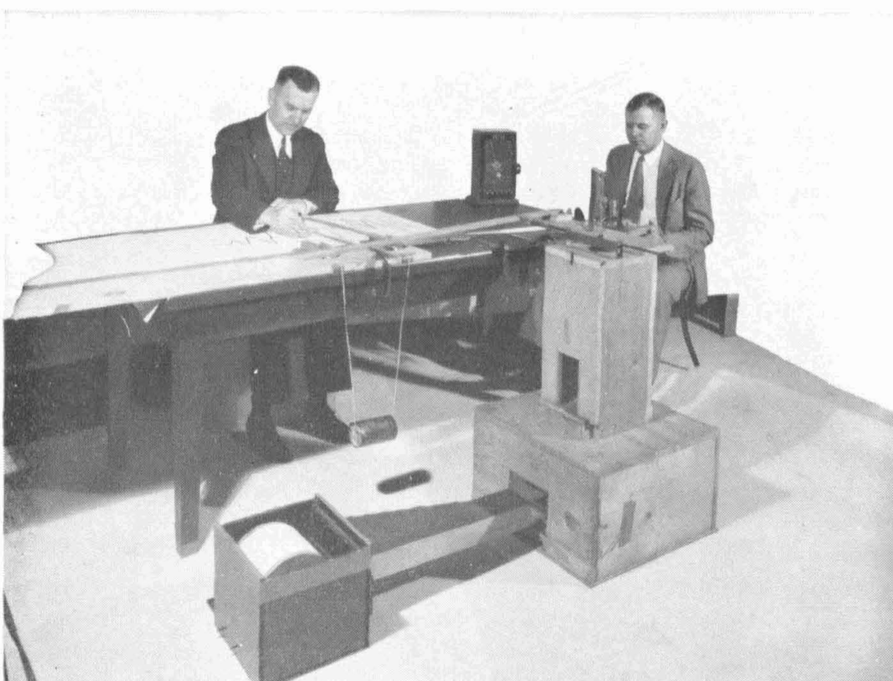


FIGURE 27.—Experimental torsion-pendulum analyzer made at the U. S. Coast and Geodetic Survey office in 1936 to determine displacement from acceleration mechanically. The 1000-sec. damped pendulum suspended in the tall box in the right foreground simulates the response of a 5-sec. seismograph pendulum. The torsion head is rotated through a system of levers manipulated by the operator on the left, who follows the acceleration curve with a pointer attached to the end of the system of levers. An assistant pulls the accelerogram across the table. The operator listens to the metronome at the corner of the table, to control the tracing speed. The pendulum motion is recorded photographically.

Office, and two papers[9] on it were presented before seismological organizations, which naturally were interested in its seismological rather than its engineering aspects. The immediate purpose of the experiment was to explore the practicability of determining displacement from acceleration by a purely mechanical method.

The physical aspect of the torsion-pendulum solution is rather simple. First, the free period and damping must correspond to the period and damping of the oscillator under consideration, except that the same result will be obtained if the pendulum period and the rate of applying the acceleration are slowed up, or speeded up, in the same ratio. If the torsion (suspension) head of the pendulum is rotated through a definite angle, the effect on the pendulum mass will be similar to that of a hypothetical circular field of force acting on the pendulum mass, the field of force increasing and decreasing in the same degree that the torsion head rotation is increased and decreased. It is analogous to tilting a horizontal pendulum sidewise so that a component of the earth's gravitational field of force causes the seismograph pendulum to rotate. It is also analogous to the response of a galvanometer pendulum, except that the field of force is a controlled magnetic field acting on a suspended magnet instead of a field produced by a twisting of the torsion head. In 1936, Dr. Blake of the Survey developed the fundamental equations relating to the use of a galvanometer in determining the response of a simple oscillator to impose accelerations, but his results were not published.

The present purpose is to show a previously unpublished comparison between the pendulum displacement obtained in the 1936 experiment and the revised displacement computed by numerical integration from the same accelerogram used in the pendulum work. The record was the east–west component of the accelerogram of the destructive Helena, Montana, aftershock of October 31, 1935. A computed displacement was published,[10] but it was definitely unsatisfactory because no corrections were made for what is now recognized as imperfect instrumental performance due in part to the emergency nature of the project. The comparison is shown in figure 28. The pendulum curve represents the response of a 5-second damped pendulum to the earthquake accelerations, this period being long enough to record displacement for the relatively short-period earthquake waves. The acceleration was

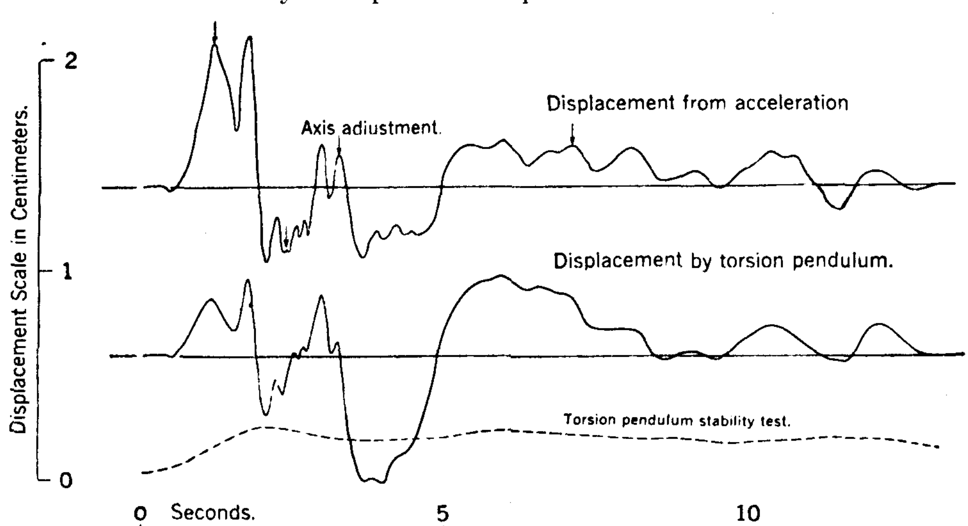


FIGURE 28.—Comparison between the displacement computed by double integration of the north-south component of the Helena, Montana, accelerogram of October 31, 1935, and the displacement obtained with the torsion-pendulum analyzer. The stability test was made by repeating the torsion-pendulum operation in all details except that the axis of the enlarged curve was used instead of the curve itself.

applied to the torsion head manually at a speed only 1/200th that of the actual earthquake recording, and the pendulum period was increased 200 times, to 1000 seconds, to obtain a pendulum response equivalent to that of a 5-second pendulum. The following formula is used, when the torsion pendulum functions as a displacement meter, to express the equivalent of static magnifications of a seismograph:

$$V = \frac{8\pi^2 L\theta}{T^2}$$

L is the distance from pendulum mirror to recorder, and θ the arbitrary (sensitivity) angle through which the torsion head is turned to have the equivalent effect of one unit of linear acceleration. T is the free period of the torsion pendulum.

The comparison may not seem impressive at first glance, but it should be noted that the displacement scale is far more open than any other used in this

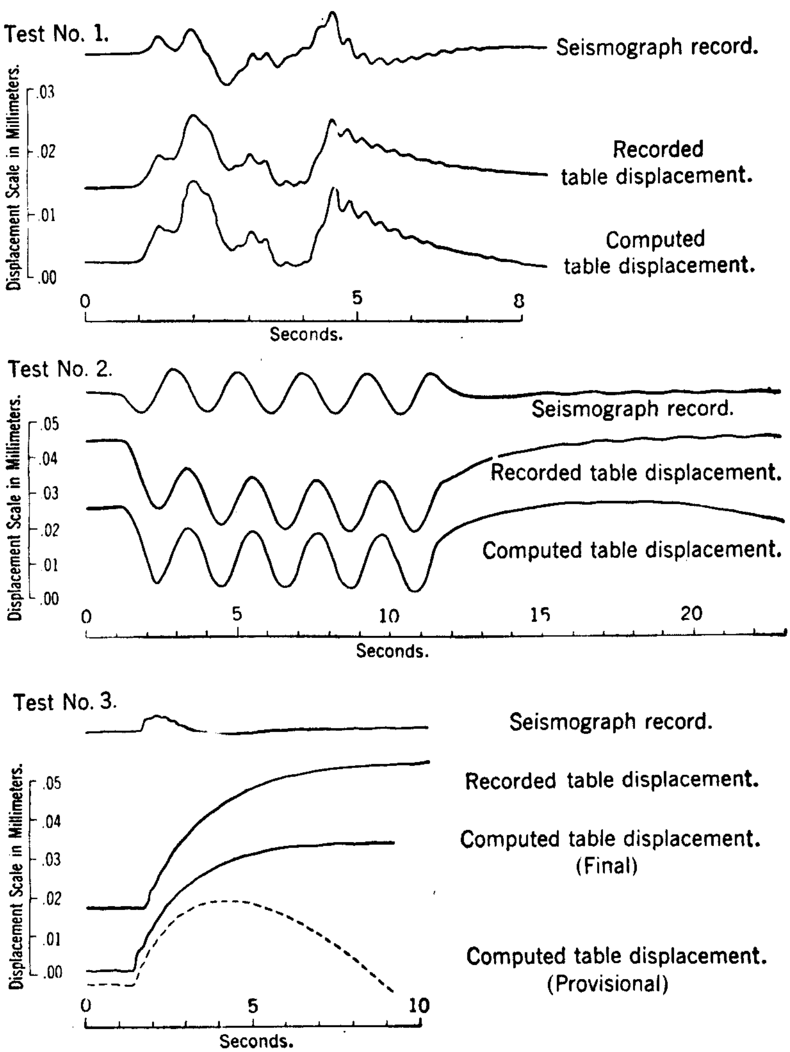


FIGURE 29.—Results of three shaking-table tests with a Wood-Anderson seismograph at the National Bureau of Standards in 1937. The computed displacements were obtained by numerical integration, all three terms of the equation of motion being used. $T_0=6$ seconds; $V=390$.

paper for a similar purpose, and that the fluctuations range over a band of only about 0.5 centimeter. Some of the discrepancy is undoubtedly due to accelerometer performance, as is shown in the number of adjustments necessary. A certain amount of error also enters into the pendulum record for this reason (in addition to that discussed in the next paragraph), but, as a pendulum has a zero position of its own, it automatically smooths out the effect of errors due to unstable accelerometer pendulums.

The torsion pendulum would seem to furnish an ideal method of determining displacement from acceleration, were it not for the fact that a pendulum record necessarily assumes that a condition of rest, zero acceleration, and velocity, exists at the start of the record. But curves obtained by integration show that the earth motion may be considerable by the time the accelerograph begins to record. Some error is therefore certain to appear in the early part of a pendulum displacement record—unfortunately a most important part of the record. There are ways of handling this situation, but they have not yet been explored for practicability.

RESULTS OF SHAKING-TABLE TESTS AT THE NATIONAL BUREAU OF STANDARDS

In 1936, Dr. Frank Wenner, of the National Bureau of Standards, and Mr. H. E. McComb, of the Coast and Geodetic Survey, conducted a series of shaking-table tests to study the performances of certain types of teleseismic instruments. The range of the table motion was extremely small because of the high sensitivity of the seismographs. Of the four instruments used, the records of only one, a Wood-Anderson, were considered satisfactory for processing by methods of integration, because the instrument was the only one which recorded, without some modification, the actual motion of the pendulum.

Computation of displacement from the seismograph records required the use of all three terms of the equation of motion referred to previously, because (unlike accelerometers) the pendulum period was of the so-called "intermediate" type, 6 seconds, and all terms could be expected to be of the same order of magnitude for a fairly wide range of imposed periods. In this work, therefore, any inaccuracy in determining the second integral of the third term would influence the final result only in a relatively small degree because of the weight of the first two terms. It was, consequently, not so rigid a test of precision as one finds in processing accelerograms. The only stipulation made by the author was that the table motion should represent oscillatory motion about a central axis, as such an assumption was considered necessary in making the required axis adjustments. That type of motion is characteristic of practically all earthquake records. The recorded table displacement records were withheld and all measurements and computations were made by Dr. As Blake, of the Survey's seismological staff, under the general supervision of the author. It will be evident from the records shown in figure 29 that the test presented a simple problem so far as the difficulties of mensuration were concerned. The obvious purpose was to test the seismologist's ability to adjust correctly the axes of the integrated curves.

In making the computations it was decided that the axis adjustments should be such that the zero position of the table should be the same at the beginning and end of each test, as would be the case in oscillations about a central axis. The computed curves in figure 29 show that in the first case the solution was correct on this basis even though the table motion was not a typical earthquake motion. In the second case the correct displacement was obtained without axis information either at the beginning or end of the record.

Following the tentative stipulation that the starting and stopping points should be identical, Dr. Blake produced, for the first solution of the third test, the curve marked "Provisional." This indicated that the table was still in a state of motion at the end of the test (because of the steep slope at the end), and it was presumed that either the solution must be incorrect or that the table kept moving off in one direction after the end of the test. The solution shown by the "Final" curve was made on the assumption that the table had to be in a state of rest at the end of the test; that is, the displacement curve had to be horizontal. The permanent displacement thus obtained proved to be the correct as well as the only logical solution.

The tests were of value in demonstrating, under unfavorable circumstances, the soundness of the basis on which all axis adjustments are made in the numerical integration process. They prove that by making certain logical assumptions, which could almost be called axioms, only one solution within a restricted range of error is possible. The adjustments necessary are therefore believed to be devoid of any serious uncertainty due to what might be considered unjustifiable guesswork.

SUMMARY

Accuracy of displacement curves computed from accelerograph records by numerical double integration.—The following results were obtained from Coast and Geodetic Survey accelerograph records in shaking-table tests made at the Massachusetts Institute of Technology:

1. With the standard type of pivot accelerometer now in use and accelerogram enlargements made with a lantern ("Balopticon") projector, maximum displacement errors of 2 cm. (4-cm. range) were found. The error curves represent slow motion of insignificant acceleration and are therefore of little importance in engineering investigations. The wave forms of only the longer period waves are involved. This error is believed to be close to the maximum in Coast and Geodetic Survey double-integration results reported prior to 1937. See paragraph 5, below, for one exception.

2. With the same accelerometer, using accelerogram enlargements made with a specially designed mechanical enlarging apparatus, errors of mensuration were reduced about 75 per cent, but minute shifts in the zero positions of the pivoted pendulums resulted in errors as large as those stated in paragraph 1, the actual magnitude depending largely upon the individual instrumental performance.

3. With a quadrifilar accelerometer record an error of 0.5 cm. (1.0-cm. range) was obtained when the specially designed mechanical enlarger, and personnel without previous experience in operating it, were employed. This may tentatively be considered the error of mensuration (including light-spot and paper distortion) and computation.

4. A considerably greater accuracy than stated in paragraph 3 was obtained when special vernier scales were used for reading the original acceleration record, but the method was too laborious to be practical.

5. The errors in the 1934 processing of the Los Angeles Subway Terminal accelerograph record of the Long Beach earthquake of 1933 were much greater than those found in the shaking-table tests because of the absence of baseline controls on the earlier records, failure to find satisfactory substitutes, and an exaggerated effect of heat distortion in the lantern enlarger due to a smaller time scale. A revision of the earlier work, including rescaling of one of the original acceleration curves, revealed that the active part of the curve computed in 1934 was substantially correct and satisfactory for engineering investigations. Ultra-long-period waves of the magnitude reported in the 1934 computation must be ruled out. Special shaking-table tests proved that such waves, if they exist, can be detected with certainty with proper instrumental control.

6. A comparison between a displacement-meter record in the field and the displacement computed from a pivot accelerometer record obtained at the same station revealed the same order of error in the computed displacement as found in the M.I.T. shaking-table tests.

7. The complexity and magnitude of the motion imposed on the accelerometer appear to have but little influence on the magnitude of the error. For major shocks the percentage of error in computed displacement is relatively small, but for light shocks the computed displacement obtained from pivot-accelerometer records is often badly distorted.

Accelerograph performance.—The preceding paragraphs show that the pivot type of accelerometer now in use is satisfactory from the engineering viewpoint and that wave forms in terms of displacement can be satisfactorily computed for all but the longer-period waves. In transferring from the quadrifilar type of pendulum suspension to the pivot type to obtain a sturdier and more readily adjustable instrument, some sacrifice was made in accuracy of performance, but it is not serious. Although the pivot suspensions embody the highest quality of workmanship, they nevertheless undergo (when recording an earthquake) a certain amount of minute shifting, and this is greatly amplified in the double-integration process. This necessitates a high standard of servicing, and some adjusting in the mathematical treatment.

The present drum speed of 1 cm/sec. seems satisfactory enough for the present. Any expected increase in the accuracy of computed displacements through opening up the time scale would, at the present time, be nullified by errors resulting from pendulum instability. A more immediate advantage would be greater ease in disentangling overlapping curves and extrapolating those which go off the sheet entirely. Reduction of accelerometer sensitivity solves this problem, which in practice is serious. Errors due to imperfections in the uniformity of the paper speed are of secondary importance.

A test with one accelerometer recording a 45 degree component of the true table motion indicated that accelerographs correctly record the components of an impressed motion according to theoretical expectations, but obviously within the limits of normal instrumental performance.

Numerical integration.—The shaking-table tests prove the validity of the basis on which axis adjustments are made when one is double-integrating an accelerograph record to obtain displacement. All shaking-table motions were computed from the recorded acceleration (or seismograph) records without advance knowledge of the table motion, and no preliminary tests were made to investigate possible sources of error. They demonstrated that even permanent displacements can be detected under favorable conditions; but with most accelerograph records this is problematical.

In the accelerometer tests a systematic error was found to be due to heat distortion of the accelerogram in the lantern enlargement process. After the tests, a specially designed mechanical enlarging apparatus eliminated this and incorporated many other practical advantages.

With respect to the more complex type of shaking-table accelerograph record, it was found that a time increment five times larger than the 1/30 second actually used would have given practically the same result in computation of the shaking-table displacement. This means that the time employed on the summation processes could safely have been reduced to one-fifth that required for the smaller increment. Caution is necessary, however, if the velocity curve is to be used for period investigations or other special purposes, as the increment must be small enough to outline correctly all important waves. Time increments between 0.07 and 0.15 second would appear to serve satisfactorily for active types of accelerograms.

The effect of omitting the first two terms of the fundamental equation of pendulum motion was determined for a complex type of shaking-table motion and was found to be rather insignificant. Current practice assumes that an accelerometer registers true acceleration for very rapid motions as well as for the slower ones, but there are limitations. The effect would be even less if the accelerometer pendulum period should be shortened, a step which would also effect a desirable decrease in sensitivity.

The time required to process accelerograms is not prohibitive. The actual summation processes require less time than enlarging and scaling the acceleration curves and constructing the computed curves, but a considerable amount of additional work is usually involved because of adjustments and recomputations made necessary by accelerometer-pendulum zero shifts.

Displacement with a torsion-pendulum analyzer.—An actual earthquake accelerograph record was used to test the practicability of determining displacement by making an experimental torsion pendulum simulate the response of a long-period seismograph pendulum. A comparison between the pendulum curve and the displacement computed by double-integrating the accelerograph record revealed a difference which was only half the smallest displacement error found in the M.I.T. shaking-table tests. Pendulum results, however, are subject to some uncertainty at the beginning of the motion, because acceleration records lose a certain amount of the initial ground motion in getting started. They "smooth out" rather than correct the effects of unstable accelerometer pendulums. The torsion pendulum, nevertheless, is well suited to play an important part in the practical solution of seismological as well as engineering problems.

Chapter IV

ANALYSIS OF ACCELEROGRAMS BY MEANS OF THE M.I.T. DIFFERENTIAL ANALYZER

By Arthur C. Ruge

THE DIFFERENTIAL ANALYZER

The differential analyzer is fundamentally a precise mechanical integrating device comprising a number of integrating units which can be so coupled that differential equations may be integrated mechanically, the solutions being given either in the form of plots or tabulations of numerical ordinates, or both. The machine and its operation has been described in detail in a paper by V. Bush published in the *Journal* of the Franklin Institute, vol. 212, no. 4. In the present investigation the differential analyzer was used principally as a precision integraph for the calculation of velocities and displacements from the accelerograms listed in the paper by A. C. Ruge and H. E. McComb. For such work the machine is accurate to about 0.1 per cent in double integration.

The function to be integrated is fed into the machine by an operator who follows the function by means of a hand crank controlling the vertical position of an index which is driven horizontally by the machine. The input table allows the use of a function up to 18 inches high by 24 inches long. This height permits one to make full use of the accuracy of the machine by reducing errors of following to a negligible amount. A function longer than 24 inches may be handled by cutting it into sections and putting them on the input table in succession; or, since four input tables are available, several sections may be put in place at a time, the operator simply moving from one table to the next at the end of a section.

PROCEDURE

In this investigation the original accelerograms were carefully enlarged about 4½ times by the U. S. Coast and Geodetic Survey, resulting in a total length of nearly 10 feet. The magnification used gave a trace large enough to make accurate following easy, and is about the maximum useful magnification for ordinary records on bromide paper because of the limits of optical definition and of the paper itself.

The machine was set up to give the first and second integrals of the acceleration; these integrals were of course in the form of areas which were converted into velocity and displacement by easily arrived at constants. In a problem of this sort the final result is given in the form of tabulations of ordinates at any selected intervals of time, the machine automatically stamping any desired information contained in it on a long strip of paper without interrupting the operation. In this case, the tabulator was made to stamp the time, velocity, and displacement at intervals of time corresponding to 1/20 second on the record, in addition to other data discussed in the next paragraph. The tabulated ordinates retain the full accuracy of which the machine is capable since they are printed from mechanical counters driven directly by it. For convenience in following the progress of the machine solution, and in the later reading of the tabulations, the machine was also made to plot the velocity and displacement on an output table.

In addition to giving the first and second integrals of acceleration the machine was arranged simultaneously to calculate and stamp the response of a 10-second-period damped pendulum. This calculation is mathematically identical with that given by the Torsion Pendulum Analyzer described by Mr. Neumann. The principal interest in obtaining the damped pendulum response curves lies in the fact that it is unnecessary to apply axis corrections since the pendulum does not accumulate large axis drifts, whereas the double-integration method makes axis corrections imperative. See figure 13, curve E, page 17, which is entirely without axis correction.

The tabulated displacements were plotted to a small scale and a smooth curve was drawn through the points as an axis correction. It was found that simple parabolas were sufficiently close fits for records made without tilt. The corrected displacements were then plotted to enlarged scales to provide curves such as those shown in figures 12 and 13, pages 14–17. Velocity plots were also made, but none are reproduced here since they are practically the same as Mr. Neumann's calculated velocity curves.

RESULTS AND CONCLUSIONS

The results shown in figures 12 and 13, pages 14 to 17, are typical. It is concluded that the mechanical integration method is accurate enough for all engineering purposes and that the choice between numerical and mechanical integration will depend upon questions of convenience and of economy of time and expense. As compared with Mr. Neumann's improved methods of enlarging the accelerograms and computing the integrals, it seems probable that mechanical integration offers no advantage in accuracy and little if any advantage in time.

There would be little point in reproducing the complete set of curves obtained from the differential analyzer, because it is impractical to employ a scale large enough to enable the reader to measure the differences in detail between them and the curves given in Mr. Neumann's paper. The only significant differences arise from different treatments of axis corrections and instrumental axis shifts. For example, in figure 12, had the axis of the differential analyzer curve F been corrected on the same basis as that applied to the numerically computed curve E, the differences in final result would scarcely be visible at the scale used. It should be emphasized that the deviations between curves E and F result from axis corrections involving accelerations of the order of 1/1,000 gravity, a quantity totally negligible from the engineering standpoint.

As to the significance of the pendulum response curves, the following table shows that for engineering purposes the distortion introduced by the pendulum is unimportant. This suggests that the pendulum response as calculated either by the differential analyzer or by the torsion-pendulum analyzer would be a satisfactory substitute for the true displacement.

TABLE 3.—*Comparison of 10-second damped pendulum response with true motion: Maximum amplitude (inches) of simple undamped structure when subjected to both motions*

Motion applied to structure	Period of structure		
	1 sec.	2½ sec.	4 sec.
No. 25. True motion	1.7	4.0	2.8
No. 25. Pendulum response	1.8	3.9	2.7
No. 46. True motion	1.7	2.4	8.8
No. 46. Pendulum response	1.8	2.4	8.3

The data given in table 3 were calculated by means of the differential analyzer. The greatest error introduced by the pendulum response is only 6 per cent. In actual structures the presence of damping would reduce these errors considerably.

More work needs to be done in order to establish the relative merits of the differential analyzer and torsion-pendulum analyzer methods of calculating the long-period pendulum response from accelerograms. The accuracies are probably about equal if the torsion pendulum device is carefully built and operated, but the differential analyzer appears to be somewhat more practical for solving this particular problem.

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