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E. LESTER JONES, SUPERINTENDENT

PHYSICAL LAWS UNDERLYING THE SCALE OF A SOUNDING TUBE

By

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Geodetic Computer

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PREFATORY NOTE.

The use of the sounding tube, or depth recorder, has long been recognized as a convenient and rapid method for getting approximate soundings in depths of 100 fathoms or less. It has never been considered an instrument for accurate surveying, and, indeed, the depths shown by two tubes of different patterns thrown overboard at the same point, or even by two tubes of the same pattern, have often exhibited surprising discrepancies.[1] Moreover, it does not appear that the method used in graduating tubes in current use has ever been published in detail. It is the purpose of this paper: (1) To provide as correct a scale as possible for the sounding tubes of the new Coast and Geodetic Survey type, the scale to be computed on assumptions definitely stated and by a stated formula; (2) to provide a method of correcting the depths as read directly from the scale for variations in temperature and in atmospheric pressure, so that the tube may be used in surveying as an instrument the precision of which is comparable with that of the lead; (3) to provide a compilation of physical data likely to prove convenient in a further study of the subject.

This paper was nearly ready for publication two years ago, when the war delayed its completion. In the meantime, the first draft of the manuscript was read by the late Dr. R. A. Harris, of this Survey, and several suggestions were derived from his memoranda. Many of the tables were prepared with the help of the Section of Tides and Currents.

[1] The more extreme discrepancies are due partly to accidents in the working of the tubes, partly to irregularities in their bore.

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PHYSICAL LAWS UNDERLYING THE SCALE OF A SOUNDING TUBE.

By Walter D. Lambert, *Geodetic Computer.*

GENERAL STATEMENT.

The idea of measuring the depth of water by the pressure it exerts is one that must have occurred to inventors, even in early times. It has been embodied in many forms of apparatus,[1] none of which, however, were ever widely and continuously used until Lord Kelvin (then Sir William Thomson) put his depth recorder on the market in 1871. Improvements in detail have since been made, but the fundamental principle remains the same. The Tanner-Blish tube, now used by the United States Navy, was patented in 1899. The main advantage of these contrivances is that the sounding can be taken while the ship is under way.

The volume of air follows, approximately at least, Boyle's law that the volume is inversely proportional to the pressure, and therefore at great depths and pressures the volume is so small and varies so little in absolute amount, even with considerable change of depth, that the scale divisions become very minute, and, therefore, this type of instrument has seldom been used for depths over 100 fathoms. It is believed that by increasing the size of the instrument the sounding tubes of the United States Coast and Geodetic Survey type could be made useful up to 250 fathoms.

To avoid the difficulty mentioned above, various devices have been invented, among them the following:

(1) Apparatus similar in principle to an aneroid barometer.

(2) Apparatus for measuring pressure by the change in volume of sea water itself or of some other fluid.

(3) Apparatus in which the gas to be compressed has an initial pressure of several times the prevailing atmospheric pressure.

(4) Apparatus in which, instead of measuring the variable volume of air under compression, a constant volume of more or less compressed air is allowed to expand till the pressure is reduced to the prevailing atmospheric pressure. The expanded volume is a measure of the original compression.

None of these forms of apparatus has attained any wide use among navigators or hydrographers, and in this paper only those sounding tubes are considered in which the pressure of the water is exerted against air initially at atmospheric pressure.

[1] Ericsson invented in 1836 a device similar to Sir William Thomson's.

GENERAL PRINCIPLES OF THE SOUNDING TUBE.

All sounding tubes are alike in fundamental conception, which is that of balancing the pressure of water against the pressure of compressed air. The tube is full of air of the same constitution, temperature, and pressure as the air near the surface of the water. It is inclosed in a protective metal case in which it is dropped to the bottom where the depth is to be determined. The tube being open at the lower end, the pressure of the water compresses the air in the tube and the water enters to occupy space left as the air recedes under compression; from the relative amounts of air and water in the tube the water pressure, and therefore the depth, may be deduced.

The fundamental differences between tubes of different makes lie in the method of ascertaining the amount of water in the tube. There are tubes in which the water rising in the tube leaves a record of the maximum height reached by it (which corresponds to the lowest depth). In one type the inner surface of the tube is of ground glass, which is wetted by the rising water and remains wet when the water *falls in the tube as the latter is raised to the surface again. It has* been found that the wetted area sometimes spreads upward a little in the tube after the water has withdrawn. This effect is probably due to capillary action in the roughened surface of the tube. In another type of tube the inner surface is coated with a substance that changes color when touched by sea water, so that the latter leaves an automatic record of the height to which it has risen. In the new United States Coast and Geodetic Survey tube the opening in the tube is closed by a valve and spring, the resistance of the latter being equal to the pressure of 1 fathom of water. When the depth of the tube is greater than 1 fathom, the water pressure overcomes the resistance of the spring behind the valve and the water enters and continues to enter until the tube has reached its lowest point. When the tube is pulled up again, the pressure outside the tube is less than the air pressure within, and the valve remains closed, thus trapping the water. The water is thus brought to the surface in the tube, and its amount may be measured in any convenient way. The way provided for ordinary surveying purposes is by means of a graduated rod of uniform diameter. The rod, which is of known dimensions, is plunged into the tube until the water in it is about to overflow. The depth to which the rod must be inserted gives the amount of air space in the tube, and thus the depth. The depth may be read directly from the scale marked on the rod, or by marking the point to be measured with a sliding marker provided for the purpose, and laying the rod along a suitably graduated scale. For accurate experimental work the water could be measured by weighing it with care.

This paper has been prepared with the new Coast and Geodetic Survey tube especially in view. Most of the considerations are, however, of a general nature and will apply to any tube in which the volume of atmospheric air is used as a measure of the water pressure. Whenever the point under discussion does not concern sounding tubes of older types, attention is drawn to the fact.

APPROXIMATE FORMULAS.

To graduate the scale of a sounding tube, a relation must be found between the volume of the air in the tube under compression and the corresponding depth. A fair approximation to the desired relation is readily obtained. Assume that the air at the surface exerts a pressure[1] p_1 and has the absolute temperature t_1. As the tube descends this air is compressed by the water pressure, and the temperature is thereby raised. However, the whole apparatus will quickly take on the temperature of the surrounding water,[2] and we shall assume that the compressed air in the tube has, in fact, the temperature of the water at the point whose depth is to be measured. Let this temperature be t_2 on the absolute scale and let δ be the mean density of the water between the bottom and surface, and let p_2 be the pressure due to the weight of the water.

Then

$$p_2 = k\delta gh. \tag{1}$$

In this equation g is the acceleration of gravity, and k is a constant dependent on the units used. Suppose that the air obeys the laws of a perfect gas; then at the surface

$$p_1 v_1 = Rt_1;\ \text{or},\ v_1 = \frac{Rt_1}{p_1}. \tag{2}$$

In this equation R is the gas constant, its value depending on the units used. At the depth h, where the total pressure is $p_1 + p_2$ and the temperature t_2, the corresponding volume v_2 is given by

$$v_2 = \frac{Rt_2}{p_1 + p_2}. \tag{3}$$

The ratio of the volume occupied by the compressed air to the original volume of the tube is, therefore,[3]

$$\frac{v_2}{v_1} = \frac{t_2}{t_1} \cdot \frac{p_1}{p_1 + k\delta gh}. \tag{4}$$

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[1] For brevity in this publication the word pressure is used in the sense of intensity of pressure or force per unit area.

[2] For the contrary case see note 3, p. 20.

[3] The following reasoning may make the point clearer: Suppose for the moment that R is so chosen and that v is the volume in cubic centimeters occupied by 1 gram of air under standard conditions (temperature 0°C, pressure 1 atmosphere). If the pressure be made p_1 and the temperature t_1, each gram will occupy v_1 cubic centimeters, and the tube containing V cubic centimeters will contain $\frac{V}{v_1}$ grams of air under these conditions. The air in the tube being now subjected to pressure $p_1 + p_2$ and temperature t_2, each gram occupies volume v_2 cubic centimeters, or $\frac{V}{v_1}$ grams of the compressed air in the tube will occupy $V\frac{v_2}{v_1}$ cubic centimeters. $\frac{v_2}{v_1}$ is, therefore, the fraction of the total volume that is occupied by the air. $1-\frac{v_2}{v_1}$ is the fraction of the volume occupied by the sea water.

This simple calculation omits several rather obvious considerations: (1) At depths still within the range of the instrument the behavior of the air departs perceptibly from that of a perfect gas; (2) vapor tension of sea water has not been allowed for; (3) the fact has not been considered that the water in the tube may dissolve some of the compressed air in the tube, thus diminishing its apparent volume.

MORE ACCURATE FORMULAS AND CORRECTIONS.

DEPARTURE FROM PERFECT GAS.

Within a range wider than is here needed the behavior of air under different conditions of temperature (t), pressure (p), and volume (v) may be represented by Van der Waals's equation:

$$\left(p+\frac{a}{v^2}\right)(v-b)=Rt. \tag{5}$$

In this equation a and b are quantities small in comparison with p and v. The numerical values of a and b depend on the gas under consideration and the units used in measuring p and v. In deriving equation (5) Van der Waals assigned the following physical interpretations to a and b: b represents four times the volume of all molecules concerned, and $\frac{a}{v^2}$ is the pressure due to the mutual attraction of the molecules, which Van der Waals takes as inversely proportional to the square of the volume of the gas, so that a is the factor of proportionality.

To square with the observed facts over wide ranges of temperature and pressure, a and b must be made variable, but for the comparatively limited ranges involved in the present case it is quite sufficient to take them as constant. In the computation of the tables the values of a and b used were deduced such as to represent the results of physical experiment over about the range of conditions to which the normal sounding tube would be subject; that is, Van der Waals's equation was used as an empirical formula. The experimental data were taken from Winkelmann's Handbuch der Physik, volume 1, part 2, page 1259, and the values deduced are on p. 25 of this paper. Equation (5) by a simple transformation gives the value of p when v is known, but for our purposes it is better to have v given in terms of p. This would require the solution of a cubic equation; but by taking advantage of the smallness of a and b, which are taken as small quantities of the first order, equation (5) may be transformed into the following approximate form, which retains small quantities of the second order. The equation is readily derived by the method of successive approximations.

$$v=\frac{Rt}{p}+b-\frac{a}{Rt}+\left[\frac{2ab}{(Rt)^2}-\frac{a^2}{(Rt)^3}\right]p. \tag{6}$$

The term in square brackets, even when multiplied by p, is nearly always negligible in our calculation, except when extreme refinement is required at great depths.

VAPOR PRESSURE OF WATER.

Equation (5) or (6) applies to dry air only. The pressure of the water vapor is added to the pressure of the air and is the same as if the latter were absent. Let us define the quantity p_1 more strictly and let it signify the pressure at the surface due to the air only and let us call p'_1 the pressure of the water vapor in the atmosphere immediately above the surface, be that what it may. At sea it will probably be close to the tension of saturated vapor corresponding to the temperature of the air. When the air is compressed by water pressure, the water vapor is made to occupy a smaller space, and the air undoubtedly becomes over-saturated and the vapor condenses, the condensed moisture being added to the sea water in the tube. Even if the air in the tube were initially quite dry, it would take up moisture from the sea water in the tube and become saturated. The water given up by the air, or taken up by it, replaces or is replaced by an equal amount of water from outside of the tube, but in any case the amount is very small. The tension of saturated vapor over a salt solution is somewhat lower than it is over pure water. (For numerical values, see p. 39.) The pressure on the water may also affect the saturation tension of the water vapor above it, but only by a very minute amount.

The pressure to be used in equation (5) or (6) for computing the volume under conditions at the surface is p_1 only. Let p' denote the pressure of saturated vapor above water corresponding to the salinity and temperature of the sea water in question. The pressure that compresses the air is:

Total atmospheric pressure + pressure due to weight of water = $p_1+p'_1+p_2$, which is equal to the counterpressure exerted by the air and by the saturated vapor: that is, it equals p_2+p', so that the air pressure by itself without the vapor tension, which is to be used in equation (6) to determine the volume of the compressed air, is $p=p_1+p'_1+p_2-p'$. Using the more exact relation (6) in the same way as equation (1) was used to obtain equation (4), we have, instead of (4), the more exact relation

$$\frac{v_2}{v_1}=\left(\frac{Rt_2}{p_1+p'_1+p_2-p'}-\frac{a}{Rt_2}+b\right)\div\left(\frac{Rt_1}{p_1}-\frac{a}{Rt_1}+b\right)$$
$$=\frac{t_2}{t_1}\left[\frac{p_1}{p_1+p'_1+p_2-p'}-\left(\frac{a}{(Rt_2)^2}-\frac{b}{Rt_2}\right)p_1\right]\div\left[1-\left(\frac{a}{(Rt_1)^2}-\frac{b}{Rt_1}\right)p_1\right] \quad (7)$$

With an assumed set of conditions of temperature, etc., and with the depth as the only variable, the formula (7) is readily adapted for tabulation, being less complicated than it looks.

It may be written

$$\frac{v_2}{v_1}=\frac{C_1}{p_2+C_2}-C_3, \tag{8}$$

in which

$$C_1=\frac{t_2}{t_1}\cdot\frac{p_1}{1-\left(\frac{a}{(Rt_1)^2}-\frac{b}{Rt_1}\right)p_1}, \tag{9}$$

$$C_2=p_1+p'_1-p', \tag{10}$$

$$C_3=\frac{\frac{t_2}{t_1}\left(\frac{a}{(Rt_2)^2}-\frac{b}{Rt_2}\right)p_1}{1-\left(\frac{a}{(Rt_1)^2}-\frac{b}{Rt_1}\right)p_1}. \tag{11}$$

The C's, though of complicated form, may be computed once for all for the given set of assumed conditions. It should be noted that equations (8) to (11) do not take into account the small term $\left[\frac{2ab}{(Rt)^2}-\frac{a^2}{(Rt)^3}\right]$ in equation (6). For depths less than 110 fathoms the effect of this term is only a few units of the fifth place of decimals, and may be allowed for by making C_3 slightly variable. The correction term to $\frac{v_2}{v_1}$ is

$$+\left[\frac{2ab}{(Rt_2)^2}-\frac{a^2}{(Rt_2)^3}\right]\left(\frac{p_2+C_2}{v_1}\right)$$

or

$$+\left[\frac{2ab}{(Rt_2)^2}-\frac{a^2}{(Rt_2)^3}\right](p_2+C_2)\frac{Rt_1}{p_1}. \tag{11a}$$

This term is to be added to the expression (8) for $\frac{v_2}{v_1}$, or may be allowed for by combining it with C_3. The quantity p_2 is nearly proportional to the depth. The small correction terms proportional to the square of the depth are worked out later. (See p. 24.)

When there is a valve spring of known strength, as in the tubes of the Coast and Geodetic Survey pattern, its effect may be allowed for by noticing that it acts like the vapor tension of water (p') in reducing the resistance that must be sustained by the compressed air alone. To allow for the spring, formula (10) may be rewritten

$$C_2=p_1+p_1'-p'-s, \tag{11b}$$

in which s is the pressure of the spring expressed in the same units as the other quantities. A method practically equivalent to the foregoing is to compute the table using (10) in its original form, but taking p_2 in (8) for the depth $h-n$ instead of for depth h, and to tabulate the results under depth h, in which n denotes the number of units in the depth of water exerting a pressure equal to the resistance of the spring. In the tubes of the Coast and Geodetic Survey pattern $n=1$ fathom.

CORRECTION OF THE READING OF THE TUBE FOR DEPARTURE FROM ASSUMED NORMAL CONDITIONS.

When the conditions vary much from those assumed and it is desired to make accurate allowance for that fact, as would be the case in experimental work, it will probably be most satisfactory to compute directly from formulas (7) to (11a). However, for the degree of accuracy required under ordinary working conditions, differential formulas suffice. These may be derived from the simpler basic formula (4) instead of from the more complex formulas (8) to (11). The effect of the omitted terms would seldom amount to more than 0.5 fathom for depths less than 100 fathoms. For completeness, however, the formula derived from the basic formulas (8) to (11) is also given.

From (4),

$$\log\left(\frac{v_2}{v_1}\right)=\log t_2-\log t_1+\log p_1-\log\,(p_1+k\delta gh)$$

Differentiate this expression, keeping $\frac{v_2}{v_1}$ constant.

$$0=\frac{dt_2}{t_2}-\frac{dt_1}{t_1}+\frac{dp_1}{p_1}-\frac{dp_1+k\delta g dh}{p_1+k\delta gh},$$

which solved for dh gives

$$dh=\left(h+\frac{p_1}{k\delta g}\right)\left(\frac{dt_2}{t_2}-\frac{dt_1}{t_1}\right)+h\frac{dp_1}{p_1}. \tag{12}$$

The quantity $\frac{p_1}{k\delta g}$ is the height of a column of water exerting a pressure p_1 and is about $5\frac{1}{2}$ fathoms, since p_1 is always about 1 atmosphere.

In deriving (12), dt_1, dh_2, dt_1, and dp_1 were treated as infinitesimals. The corresponding finite increments Δh, Δt_2, etc., may be substituted for them without serious error so long as $\frac{\Delta h}{h}$, $\frac{\Delta t_2}{t_2}$, etc., are small. In accordance with a well-known principle, increased accuracy will be obtained by adding to each original quantity half the corresponding increment; that is, by replacing h by $h+\frac{\Delta h}{2}$, t_2 by $t_2+\frac{\Delta t_2}{2}$, etc. In this way (12) becomes

$$\Delta h=\left(h+\frac{\Delta h}{2}+\frac{p_1+\frac{\Delta p_1}{2}}{k\delta g}\right)\left(\frac{\Delta t_2}{t_2+\frac{\Delta t_2}{2}}-\frac{\Delta t_1}{t_1+\frac{\Delta t_1}{2}}\right)+\frac{\Delta p_1}{p_1+\frac{\Delta p_1}{2}}\left(h+\frac{\Delta h}{2}\right). \tag{13}$$

Equation (13) gives a second approximation after an approximate value of Δh has been found by any method.

A simplified form which will give this approximate value with sufficient accuracy, and may usually be used to give the definitive correction, is obtained by omitting the increments on the right-hand side; and by replacing both t_2 and t_1 by T, equal to a mean between them $\left(T=\frac{t_1+t_2}{2}\right)$.

This gives

$$\Delta h=\frac{1}{T}\left(h+\frac{p_1}{k\delta g}\right)(\Delta t_2-\Delta t_1)+h\frac{\Delta p_1}{p_1}. \qquad (14)$$

In the Coast and Geodetic Survey tube the standard conditions are:

$t_1=60°$ F. $=519°$ F. above absolute zero.
$t_2=50°$ F. $=509°$ F. above absolute zero.
$p_1=$ pressure of 30 inches of mercury.

Therefore, $T=514°$, and (14) becomes

$$\Delta h=0.001946\ [h+5.46]\ [t_1-t_2-10°]\ +0.0333\ \Delta B\times h. \qquad (15)$$

h and Δh are in fathoms and $30+\Delta B$ is the reading of the barometer in inches. From (15) Table 9, page 45, has been computed. The above formula is for any tube the scale of which is based on the assumptions stated. In strictness for the Coast and Geodetic Survey tube h should be replaced by $h-1$, since the valve spring takes up the pressure due to 1 fathom of depth.

The more exact formula deduced from equations (8) to (11) is stated below without proof. The proof is similar to that of (12), but is tedious owing to the length of the formulas. The formula reads

$$dp_2=\frac{dt_2}{t_2}\left[p_2+C_2-\frac{ap_1}{R^2t_1t_2}\,\frac{(p_2+C_2)^2}{C_1}\right]-\frac{dt_1}{t_1}\left[(p_2+C_2)\left(\frac{1+\frac{ap_1}{R^2t_1^2}}{1-z_1p_1}\right)\right.$$
$$\left.-\frac{C_3}{C_1}(p_2+C_2)^2\right]+\frac{dp_1}{p_1}\left[\frac{(p_2+C_2)}{1-z_1p_1}-\frac{C_3}{C_1}(p_2+C_2)^2-p_1\right]-d(p'_1-p'). \qquad (16)$$

As before, the differentials may be replaced by the corresponding finite increments, and each finite quantity increased by half its increment, as in (13).

In deriving (16) some terms, in which a or b occurs multiplied by other quantities likewise small, have been dropped. The symbol z_1 stands for $\frac{a}{R^2t^2_1}-\frac{b}{Rt_1}\cdot$ Water pressure p_2 may be converted into depth by noting that

$$p_2=0.18141945h+0.000000780h^2.$$

(See eq. (35), p. 24.) It is seen that p_2+C_2 in (16) corresponds to $h+\frac{p_1}{kg\delta h}$ in (12), so that an estimate may be made of the effect of the additional small terms in (16). As a numerical example, take $p_2+C_2=20$ atmospheres, corresponding to about 105 fathoms depth, $\left|\frac{dt_1}{t_1}\right|^*>\frac{1}{10}$, corresponding to $\left|dt_1\right|>27°$ C., also $\left|\frac{dt_2}{t_2}\right|>\frac{1}{10}$ and $\left|\frac{dp_1}{p_1}\right|=\frac{1}{30}$, corresponding to one inch difference in the barometer reading. The additional terms are (1) $\frac{ap_1}{R^2t_1t_2}\frac{(p_2+C_2)^2}{C_1}$ in the correction for t_2, (2) the factors $1+\frac{ap_1}{R^2t_1^2}$ and $1-z_1p_1$, which in the approximate formulas are each replaced by unity, and (3) the terms in C_3 in the corrections for t_1 and p_1, which together with the vapor pressure terms $d\ (p_1'-p')$ are omitted in the approximate formulas. The terms in $\frac{ap_1}{R^2t_1t_2}\frac{(p_2+C_2)^2}{C_1}$ affects the correction for t_2 by about 1/20 of its amount in the extreme case supposed or by nearly half a fathom. The factor $1+\frac{ap_1}{R^2t_1^2}$ for $p_2+C_2=20$ atmospheres, affects the correction for air temperature by less than $\frac{1}{400}$ of its amount, or less than 0.03 fathom. The terms in C_3 amount to about $\frac{1}{100}$ of the total corrections for air temperature and air pressure, or about 0.1 fathom and 0.04 fathom, respectively. The factor $1-z_1p_1$ has less effect than the factor $1+\frac{ap_1}{R^2t_1^2}$. The effect of vapor-pressure terms $d(p_1'-p')$ is practically independent of the depth. The vapor pressures are functions of the temperatures of the air and water, respectively, but do not vary proportionally to the temperature over any considerable range, and so have not been included with the temperature terms. An inspection of Table 3 will show that p_1' or p', particularly the former, might vary by 20 mm. of mercury $=\frac{1}{38}$ atmosphere corresponding to 0.14 fathom.

AMOUNT OF COMPRESSED AIR DISSOLVED IN THE WATER.

It is difficult to make a satisfactory allowance for the amount of compressed air dissolved in the water, because in practice there is never time for the process of solution to be completed. In order, however, to ascertain the maximum possible correction due to this cause, we shall work out the effect of absorption of air by water when continued till the latter is saturated.

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*The vertical bars indicate numerical value regardless of algebraic sign.

At a given temperature the mass of gas absorbed per unit volume of water is, by Henry's law, proportional to the pressure of the gas; and since the density of the gas is very nearly inversely proportional to its pressure, the volume of gas absorbed per unit volume of water, or the ratio between the volume of the gas absorbed and the volume of the absorbing water, is, within wide limits, independent of the pressure of the gas. At ordinary temperatures water will absorb about 2 per cent of its bulk of air. (See p. 40 for more precise data.)

When there is no shaking together of air and water, the process of saturating the water with the air is a decidedly slow one. (See note 1, p. 26.) Just how much this process would be hastened by the motion and jarring of the sounding tube incidental to taking a sounding, it would be impossible to predict from theory. The correction to be applied in practice could be determined by experiment only for a rough average for ordinary working conditions. Table 10 (p. 45) shows the limiting values (1) when no absorption of air has taken place and (2) when the absorption of gas is complete. The scale to be used in practice would be very nearly that of the first limiting case for small depths and perhaps less near for larger depths for which the graduation is unfortunately more sensitive.

The two scales on sounding tubes in general use are the Thomson scale and the Parmenter scale. The former scale is used on the tubes designed by Sir William Thomson (Lord Kelvin) and the second on the Tanner-Blish tubes made in the United States by D. Ballauf. No statement of the formula by which the Thomson scales were originally made has been found. One of the scales was measured and the measurements examined to see whether any allowance was made for absorption, but apparently there was none, or, if any was made, it was very small. Lieut. Parmenter of the U. S. Navy, who computed the scales for the Ballauf tubes, has left an unpublished memorandum of his method. Unfortunately it is rather hard to follow. Apparently, however, allowance was made for absorption. His statement is "That at 100 fathoms the absorption of air was 0.13 per cent, and at 5 fathoms the absorption was 0.04 per cent." It is not clear whether 0.13 per cent of the original air space is intended or 0.13 per cent of the volume of water in the tube, the latter mode of statement being in accordance with our usual conception of the physical phenomenon. A study of the figures of his table would seem to indicate that 13 per cent (not 0.13 per cent) of the air space itself may be meant. This would be about 0.13 inch at 100 fathoms. This allowance for absorption seems somewhat excessive. Tests of the new United States Coast and Geodetic Survey tubes under various pressures were made at the Bureau of Standards, and the results of these tests were examined to discover the amounts of absorption.

The tests, however, were not made with this phenomenon especially in view, and nothing definite could be learned from them except that, even at pressures equivalent to 100 fathoms of water, the effect of absorption on the size of the air space is probably less than 1 millimeter (0.04 inch), even when considerable time is allowed. The tubes, however, were not used under working conditions, but simply put into the testing apparatus and subjected to the required pressure. There was none of the moving or jarring incidental to the usual process of taking the sounding.

A careful preliminary test of the new Coast and Geodetic Survey tubes made by the Coast and Geodetic Survey in Florida Straits in May, 1919, indicates that a scale computed on the assumption that absorption may be neglected gives good results, even at depths of 100 fathoms and more.

Lieut. Parmenter's statement of the amount of the absorption is given as the result of 36 tests on the U. S. S. *Prairie* in 1900. Details of these tests are not given, so they can not be examined to see if the apparently large absorption can be otherwise accounted for. However, it may be that Lieut. Parmenter's table is substantially correct on this point. Further experiment on this point is desirable.

The method by which the table for the second limiting case, that of complete absorption, was computed is as follows:

We may assume that the water, being continuously subject to the air pressure p_1 of about 1 atmosphere, has already absorbed the corresponding amount of gas. This assumption is found to be approximately correct. (See Krümmel, Handbuch der Ozeanographie, vol. 1, p. 296.) Under compression some of the air is absorbed into the water, which would tend to reduce the pressure; but the latter is maintained by the inflow of water to take the place of the air absorbed, and this process continues until equilibrium is attained. The earlier forms of sounding tube communicated freely with the water outside without any check valve intervening; and if unlimited time were allowed, there was no reason why all the air might not be absorbed into the water of the tube and ultimately be diffused into the surrounding ocean. In the United States Coast and Geodetic Survey tube this diffusion into the outside water is prevented by the valve. Even with this form of apparatus, the volume of air might be so small that in time the water in the tube alone would absorb all the air. This can not happen, however, at depths of less than 250 or 300 fathoms, and for such depths the tube is not used.

In computing the volume of air remaining after absorption is complete it will be assumed that Henry's law is exact and that the density of the air is the same as for an ideal gas. These approximations are amply accurate for the purpose in hand, since they are in themselves near the truth and are used merely to calculate a small correction, and not the principal quantity. Let ρ_2 be the density due to the pressure of p_1+p_2. Let m be the number of grams of air absorbed by 1 cubic unit of water under unit pressure, and let Δv be the diminution in volume of air due to absorption, so that $v-\Delta v$ is the reduced volume. Let w be the volume of water when equilibrium has been established. The water already contains the amount of air due to the atmospheric pressure p_1, so that the additional mass of air absorbed is that due to the additional pressure p_2 and is given by the equation

$$\rho_2 \Delta v = w m p_2 \tag{17}$$

and since the water flows in to replace the absorbed air,

$$v - \Delta v + w = V, \tag{18}$$

in which V is the entire volume of the tube.

Let ρ_1 be the density of the gas at pressure p_1, then

$$\frac{\rho_2}{\rho_1} = \frac{p_2 + p_1}{p_1}. \tag{19}$$

Then from (17), (18), and (19), by eliminating w and ρ_2 and solving for Δv, we get

$$\Delta v = \frac{(V-v)\, m p_2}{\left(\frac{p_2}{p_1}+1\right)\rho_1 - m p_2}$$

or,

$$\Delta v = \frac{(V-v)\frac{m p_1}{\rho_1} p_2}{p_1 + p_2 - \frac{m p_1}{\rho_1} p_2}.$$

The quantity $\frac{m p_1}{\rho_1}$ is a constant for a given temperature and is equal to the fraction of its volume of air that water will absorb. Put $\frac{m p_1}{\rho_1}$ equal to α; then

$$\Delta v = \frac{(V-v)\alpha}{1 + \frac{p_1}{p_2} - \alpha}. \tag{20}$$

The numerical value of α is about 0.02.

It may be noted that the oxygen and nitrogen of the air are not absorbed into the water in the same proportion in which these gases exist in the atmosphere, the proportion of nitrogen in the dissolved air being less than that in the atmospheric air. The undissolved air is, therefore, of slightly different composition from ordinary atmospheric air, and in strictness its physical constants would be slightly different. This effect is obviously too minute to need further consideration.

The constant α in equation (20) is 0.01942 and is derived as follows: From Table 5 on page 40 it appears that for a salinity 32.5 parts per thousand and a temperature of 10° C., 1 liter of sea water will absorb 6.51 c. c. of oxygen and 12.22 of nitrogen, or 18.73 c. c. in all. This figure refers to the volume at 0°. To reduce the volume to 10° with sufficient accuracy for the purpose in hand, multiply it by $\frac{283}{273}$, which is the ratio of the absolute temperatures. The result is 19.42 c. c. per liter; that is, sea water at 10° will absorb 0.01942 of its own volume of air, or about 2 per cent, as previously stated.

The effect of the air dissolved in the water on the vapor tension of the latter is also negligible. (See J. J. Thomson, Application of Dynamics to Physics and Chemistry, p. 173.) The effect of the dissolved air on the volume of the dissolving water is likewise negligible. The increase in volume of the water is about equal to the volume that the air would occupy under a pressure of 2,500 atmospheres. (See Winkelmann, Handbuch der Physik, vol. 1, pt. 2, p. 1521.)

In the tubes of the new Coast and Geodetic Survey pattern there is a quasi absorption that can be easily guarded against. In measuring the amount of water contained in a tube it is inverted and the air bubble rises through the water. In this way minute air bubbles may be formed and remain in the water for some little time, thus increasing its apparent volume. To remove the bubbles the water may be thoroughly stirred with a fine wire.

ABSORPTION BY THE WALLS OF THE TUBE.

There is still to be considered the possibility that, under the pressure up to 20 atmospheres, the water or compressed air might permeate the walls of the tube to an extent sufficient to vitiate the measurements. This possibility is to be feared much more in the case of brass than of glass, which has long been used in accurate physical experiments at high pressure. With reference to brass or bronze the United States Bureau of Standards furnishes the following information.

> The rate of air or water passing through brass or bronze would depend upon many factors, of which the following might be named: Composition of material; condition of material, cast, rolled, forged, etc.; temperature; pressure; depth immersion; etc. The bureau has been unable to find any specific data on the rate of flow of air or water hrough brasses or bronzes.
>
> Carpenter and Edwards (Proceedings Institution of Mechanical Engineers, 1910, p. 1597) state that from their investigations, a pure copper-aluminum bronze containing from 9 to 11 per cent of aluminum has the best ability to withstand high pressures (14 to 20 tons per square inch). This material when properly cast did not leak until just before rupture; the original article gives the necessary precautions which should be taken in casting. This alloy is also only very slightly attacked by fresh or salt water.

The possibilities of error from this source would be diminished by giving as little time for the pressure to act as may be consistent with other requirements. The experience of the Coast and Geodetic Survey does not indicate that the error arising from the permeability of the tube is serious, but no data have been obtained as to the exact amount or the rate of permeation.

CHANGE IN VOLUME OF THE TRAPPED WATER.

There is one particular in which an accurate calculation for a tube of the Coast and Geodetic Survey type differs from the calculation for a tube of the recording type mentioned on page 6. In the latter type the height of the water in the tube is recorded automatically in situ; in the Coast and Geodetic Survey tube the volume of the water is measured at the surface and under surface conditions of temperature and pressure (but see p. 20), whereas we are concerned with its volume under the conditions that prevailed at the depth measured. As on page 16 let w denote the volume of the water at the given depth, v the volume of air in the tube, and V the entire volume of the tube. Call Δw the total increment of w due to change of temperature and decrease of pressure experienced in going from the given depth to the surface and put

$$\Delta w = \Delta_1 w + \Delta_2 w, \tag{21}$$

in which $\Delta_1 w$ is the increment due to change in temperature and $\Delta_2 w$ the increment due to the decrease of pressure. These increments are so small that they may be computed independently. Since $v+w=V=\text{constant}$, $\Delta w=-\Delta v$.

Let δ_1 and δ_2 denote, respectively, the densities of the water at air temperature t_1 and water temperature t_2. Then, since the mass of the water is unchanged by the change in temperature,

$$w\delta_2 = (w+\Delta_2 w)\delta_1$$

$$\text{or}\quad \Delta_2 w = w\frac{(\delta_2-\delta_1)}{\delta_1}. \tag{22}$$

Since the δ's are nearly unity, we may put in rough calculations

$$\Delta_2 w = w(\delta_2-\delta_1). \tag{23}$$

We have further,

$$\Delta_1 w = w\mu p_2,$$

μ denoting the coefficient of compressibility for the temperature and salinity of the water, and p_2 the water pressure. The vapor pressures p_1' and p' are much too small to need consideration. (See

Table 3, p. 39.) Since $\frac{v_2}{v_1}$ (formula (8), p. 10) is the ratio of the volume occupied by the compressed air to the total volume of the tube, we have

$$v=\frac{v_2}{v_1}V \text{ and } w=\left(1-\frac{v_2}{v_1}\right)V.$$

Therefore,

$$\Delta w=\Delta_1 w+\Delta_2 w=w\left(\mu p_2+\frac{\delta_2-\delta_1}{\delta_1}\right)=V\left(1-\frac{v_2}{v_1}\right)\left(\mu p_2+\frac{\delta_2-\delta_1}{\delta_1}\right). \quad (24)$$

The corresponding correction to h, Δh, is found by

$$\Delta h=-\Delta v\div\frac{\partial v}{\partial h}=\Delta w\div\left[V\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)\right]=\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}\left(\mu p_2+\frac{\delta_2-\delta_1}{\delta_1}\right). \quad (25)$$

The minus sign is used before Δv because we are reducing back from the volumes at the surface to the volumes under water, whereas the Δ's have been defined as the changes caused by going in the opposite direction. $\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)$ is essentially negative. Δh may be separated into $\Delta_1 h+\Delta_2 h$.
where

$$\Delta_1 h=\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}\times\mu p_2, \quad (26)$$

which is the the correction due to change in pressure, and

$$\Delta_2 h=\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}\times\frac{\delta_2-\delta_1}{\delta_1}, \quad (27)$$

which is the correction due to change in temperature.

$\Delta_1 h$ has been allowed for in computing Table 8 for the scale of the Coast and Geodetic Survey tube. Its value is small, being less than 2 fathoms for a depth of 100 fathoms. $\Delta_2 h$ has not been allowed for in computing Table 8, as a further correction of the same sort has to be introduced when the actual temperatures differ from the standard assumed temperatures, and the introduction of part of the necessary correction into the table itself would complicate rather than simplify matters. The quantity $\frac{\delta_2-\delta_1}{\delta_1}$ is a function of the temperature, but is not a linear one even approximately, so that this term does not lend itself to combination with the other temperature corrections in equations (13) or (16). The quantity $\dfrac{1-\dfrac{v_2}{v_1}}{\dfrac{\partial}{\partial h}\left(\dfrac{v_2}{v_1}\right)}$ is tabulated for each depth on pages 42–44. Although the expression for $\dfrac{\partial}{\partial h}\left(\dfrac{v_2}{v_1}\right)$ can readily be deduced from equations (8) and (20), in practice it is more convenient to get its numerical value from the differences in the tabulated values of $\dfrac{v_2}{v_1}$ by some one of the formulas connecting finite differences and derivatives.

Under certain circumstances it may not be necessary to make the correction for change in volume of water due to change in temperature. If the difference in temperature is not great and the measurement of volume is made very promptly, the water may be assumed to have maintained its original temperature. If this assumption is made, it will be advisable to keep the graduated rods that are introduced to measure the volume at somewhere near water temperature. On the other hand, if the correction is to be applied, sufficient time should be allowed for the water to take on the temperature of the air.

CORRECTION FOR VARIATION IN THE DENSITY OF THE WATER AND FOR THE ACCELERATION OF GRAVITY.

Tables 7 and 8 were computed with a standard surface density of water equal to 1.025 and a standard acceleration of gravity (g_{45}) equal to theoretical gravity at sea level in latitude 45°. When actual conditions depart much from the assumed conditions, as when soundings are taken in fresh or nearly fresh water, a correction must be applied.

The water pressure is proportional to the product (density) × (acceleration of gravity) × (depth), or, with the notation previously used,

$$p_2 = k\delta gh. \tag{28}$$

For a given reading of the tube scale, p_2 is constant, although each of its factors may vary. Logarithmic differentiation of (28) gives

$$\frac{dh}{h} = -\left(\frac{d\delta}{\delta} + \frac{dg}{g}\right).$$

By substituting finite increments for differentials, and denoting by $\Delta_8 h$ the correction for difference between the actual and standard values of density and gravity, we get

$$\Delta_8 h = -h\left(\frac{\Delta\delta}{1.025} + \frac{\Delta g}{g_{45}}\right). \tag{29}$$

$\Delta\delta$ is to be taken so that $1.025+\Delta\delta$ shall represent the mean density from the surface down to depth h for the actual distribution in depth of salinity and water temperature and a pressure of 1 atmosphere; $\Delta\delta$ can be deduced from Table 2 for the assumed conditions. No allowance is to be made for increase in the density of the water due to the pressure of the water above it, since this effect is small and has already been allowed for with sufficient accuracy (p. 24). The term $\frac{\Delta g}{g_{45}}$ may be dropped, except in computations of unusual refinement. In such cases take

$$\frac{\Delta g}{g_{45}} = -0.0026 \cos 2\varphi. \tag{30}$$

In (30) φ is the geographic latitude. Tables of $\frac{\Delta g}{g_{45}}$ or related quantities are commonly given in collections of meteorological tables for the purpose of reducing the height of the barometer to latitude 45°.

CORRECTION FOR FLUCTUATIONS OF THE WATER SURFACE.

In finding the depth to which the sounding tube has been submerged, no correction need or should be made for the fact that the crest of a wave has passed over the spot where the tube lies, and that its maximum depth below the instantaneous surface is therefore greater than the depth below the mean surface of the water, which is the depth commonly desired. This statement applies to surface waves such as are raised by the wind, and whose length (distance from crest to crest, or from trough to trough) is less than the depth of the water, and whose amplitude is small compared with the depth; for it is well known that the oscillatory effects of such waves die out very quickly as the depth below the surface increases, and that the water pressure at the bottom under the trough, or under the crest is the same, being equal to the static pressure due to the depth below the undisturbed mean surface. The statement does not apply to long waves like tidal waves, "tidal wave" being used in its proper sense of a periodic wave produced by the attraction of the sun or moon. Depths found by the sounding tube are to be corrected for the stage of the tide in the same way as are soundings taken with the lead.

SITUATION OF THE POINT WHOSE DEPTH IS RECORDED.

In making an accurate comparison of the depth shown by the tube with that shown by the lead at the same point, two points must be remembered: (1) As the tube sounding is usually made, the tube may not go to the bottom, but the protective metal case is attached to a weighted rod several feet from the end. This is done in order to prevent the case and tube from being injured by striking the bottom. The rod remains upright after striking bottom, the weight being at the lower end, and a constant allowance must be made for the distance between the end of the rod and the lower end of the tube. (2) Equilibrium between the air pressure within and the water pressure without is established, not at a fixed point in the tube, but at the boundary surface between the air and water in the tube.[1]

This would require a variable correction the amount of which could be easily deduced from Table 7 or 8, column 2, pages 42–44, and from the dimensions of the tube. To find the exact point where equilibrium would be established requires the solution of a quadratic equation even when Boyle's law is assumed to be exact. If a denotes the length of the tube, b the height of a column of water exerting a pressure equal to that of the atmosphere, h the depth of the bottom of the tube,[2] x the height to which the water rises in the tube, then

$$\frac{a}{a-x}=\frac{b+h-x}{b}, \text{ or } x=\frac{a+b+h}{2}-\sqrt{\left(\frac{a+b+h}{2}\right)^2-ah}. \qquad (31)$$

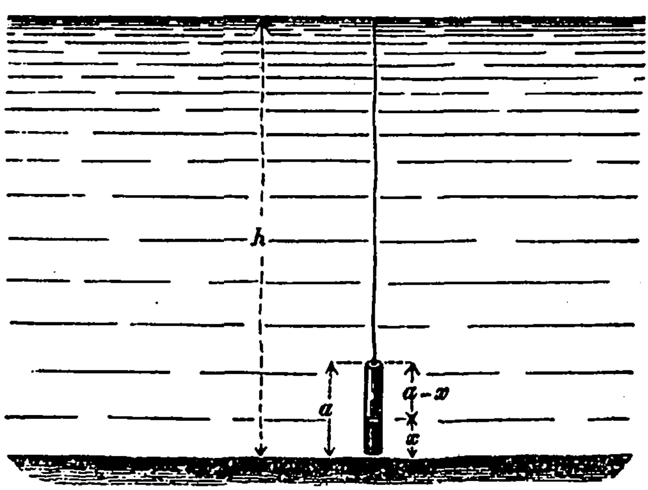

FIG. 1.—Diagram showing position of point where depth is recorded.

This is the equation frequently given for graduating the tube. In the Coast Survey tube the correction for distance between end of tube and water surface, which is precisely the quantity x, is (to the nearest tenth of a fathom) 0.3 fathom for depths of 16 fathoms and over. The correction due to the position of the tube on the weighted rod is found by direct measurement.

[1] The fact that in the Coast and Geodetic Survey tube there is a spring valve that takes up the pressure due to 1 fathom of water does not affect the correctness of this statement.

[2] The quantity h elsewhere denotes the depth of the boundary surface between the air space and the water in the tube.

NUMERICAL DATA AND CONSTRUCTION OF THE TABLES.

The tables for the physical properties of sea water are based principally on the data and tables in Krümmel, Handbuch der Ozeanographie, second edition. The table for the density of water of different temperatures and degrees of salinity is constructed as follows: Knudsen's empirical formula, cited by Krümmel (vol. 1, p. 237) for the density of water at 0° C.[1] and different degrees of salinity was first used, and then the densities at other temperatures were interpolated from the table on pages 232 and 233 of Krümmel's work. The vapor pressure for different temperatures and degrees of salinity were computed indirectly from the formulas and tables on pages 241 and 242. The tables give the effect of the salinity of the water on the boiling point. The alteration in the boiling point was converted into the equivalent alteration of vapor pressure by means of the Smithsonian Physical Tables (sixth revised edition, 1914). The ratio of the vapor pressure of water of given salinity to the vapor pressure of pure water, as deduced from a consideration of their boiling points, was assumed to be independent of the temperature. (See Thomson, Application of Dynamics to Physics and Chemistry, pp. 175–177; also Chwolson, Traité de Physique, Vol. III, p. 956–969.) The vapor pressure of pure water was taken from the Smithsonian Tables. In this way Table 3, page 39, was prepared.

In constructing the tables of water pressure corresponding to a given depth, the density of the sea water was taken as 1.025. For a temperature of 10° C. this corresponds to a salinity of 32.5 parts per thousand. The assumed density seems to represent fairly well conditions in American coastal waters. The density of the sea water is as a rule a trifle higher elsewhere than the value here adopted, but the difference is not important in practical work.

The compressive force exerted by the water is proportional to the acceleration of gravity. This quantity varies between the Equator and poles by about one two-hundredths part of itself. In accurate investigations of the scale of the tube the appropriate local gravity should be used, but for ordinary purposes we may use the normal acceleration for sea level in latitude 45°, which has been taken as 980.62 centimeters-per-second per second. The density of mercury at 0° C. is 13.595945 (Landoldt and Börnstein, 4th ed., p. 45). One standard atmosphere, which is the pressure of 76 centimeters of mercury under the above gravity, is therefore $76 \times 980.62 \times 13.595945 =$ 1,013,210 dynes per square centimeter. One fathom = 182.8804 centimeters. Therefore, the weight of each fathom of sea water exerts a pressure of 0.18141945 standard atmosphere. This is the quantity $k\delta g$ in equation (4), if h be in fathoms. There are slight corrections

[1] $\sigma_0 = -0.093 + 0.8149S - 0.000482S^2 + 0.0000068S^3$. For σ_0 see p. 33. S is salinity in parts per thousand, by weight.

to this figure owing to the increase in density of water under the compression of the water above it and the increase in gravity as the center of the earth is approached. These effects are small, but, being easily evaluated, have been included in making up the tables.

Let $\delta=\delta_0+\Delta\delta$ and $g=g_0+\delta g$ be the values of density and gravity, respectively, at any depth h, δ_0 and g_0 being their values at the surface. If p be the pressure and k a constant depending on the units used, then

$$dp=k\delta g dh=k(g_0+\Delta g)\ (\delta_0+\Delta\delta)dh=k(\delta_0 g+\delta_0\Delta g+g_0\Delta\delta)dh. \qquad (32)$$

In (23) the small term of the second order in $\Delta g\ \Delta\delta$, has been omitted. If β be the coefficient of compressibility, by definition $\frac{1}{\delta}\frac{d\delta}{dp}=\beta$; or, approximately neglecting the variation in δ, $\Delta\delta=\beta\delta_0 p$; and with the approximate value $p=k\delta_0 g_0 h$, we have

$$\Delta\delta=\beta k\delta_0^2 g_0 h. \qquad (33)$$

In approaching the center of the earth through free air by h meters, the acceleration of gravity increases 0.0003086 h centimeters per second per second.[1] In this case, however, the approach toward the center is not through free air but through water, the attraction of which, if it be treated as an indefinitely extended layer of thickness h, is $2\pi f\delta h$, in which f is the gravitation constant 667.3×10^{-10} C. G. S. units. Since, when the point is below this layer of water, the attraction of the water is reversed in direction, double the above effect must be subtracted from the rate of increase of gravity in free air.

The numerical expression of $2\pi f\delta_0 h$ is 0.0000429 h; therefore the increase of gravity in approaching the center of the earth through h meters of water is 0.0002228 h centimeters per second per second, or $\Delta g=0.000407h$, when h is in fathoms instead of in meters. For brevity write $\Delta g=ch$, c standing for the above numerical value 0.000407. Resuming equation (32) we find for the corrected value of the increment of water pressure,

$$dp_2=(k\delta_0 g_0+k\delta_0 ch+\beta k^2\delta_0^2 g_0^2 h)dh, \qquad (34)$$

whence

$$p_2=k\delta_0 g_0 h+(k\delta_0 g_0)\frac{ch^2}{g_0^2}+\beta(k\delta_0 g_0)^2\frac{h^2}{2}.$$

The quantity $\beta=451\times10^{-7}$ (see p. 40); therefore, on substituting numerical values,

$$p_2=0.18141945h+0.0000000377h^2+0.000000742h^2, \qquad (35)$$

or

$$p_2=0.18141945h+0.000000780h^2.$$

The unit of p is the standard atmosphere and of h is the fathom.

[1] Investigations of Gravity and Isostasy. Spec. Pub. No. 40, U. S. Coast and Geodetic Survey. See p. 93.

The constants of Van der Waals's equation (5) are taken as follows:

$$a = 0.00246,\ b = 0.00192,\ R = 0.0036650 = 1/272.85.$$[1]

Absolute zero is taken at $-273°$ C. The unit of v is the volume of the mass of air which will just fill the chamber of the tube under standard conditions; namely, a pressure of one atmosphere and a temperature of 0° centigrade. The above numbers were deduced by trial to fit approximately the data given in Winkelmann, Handbuch der Physik (vol. 1, pt. 2, p. 1259). They are intended to apply to pressures between 1 and 30 atmospheres and temperatures between 0° and 100° C.

SUMMARY OF CONCLUSIONS.

To get the best results from a sounding tube, allowance must be made for the temperature of the air and of the water and for the barometric pressure; also for the fact that air does not exactly conform to Boyle's law and for other possible sources of error. The tables for the scale of the tube given in this work contain these allowances and corrections as far as it was practicable to include them. Other tables given are intended to facilitate the calculation of the necessary corrections.

If the sounding tube is carefully made and used and the corrections for temperature, etc., are properly applied, the accuracy of the results should be practically the same as that of soundings carefully taken with a lead. The present tubes are available up to depths of 100 fathoms. It is believed that a larger tube could be made that would give satisfactory results up to 250 fathoms.

The advantages of a tube over the lead are the rapidity with which the sounding can be made and the fact that it is unnecessary to stop the vessel in order to get a good sounding. The sounding tube gives the mean depth of the water unaffected by fluctuations of level due to short surface waves.

In taking a sounding sufficient time should be allowed for equilibrium to be established, both as to the inflow of water and as to the equality of temperature of the tube and of the surrounding water. On the other hand, the time must not be unduly prolonged so as to give time for the air to diffuse into the water or for the air and water to permeate the walls of the tube. Care should be taken to avoid the quasi absorption (p. 17) in the Coast and Geodetic Survey tube. In measuring the volume of the air space above the water brought to the surface in a Coast and Geodetic Survey tube, conditions of time and temperature should be such that the temperature of the water in the tube may be definitely assumed to be either that of the water at the depth where the sounding was taken or else that of the surrounding air. (See p. 20.) In the latter case, the correction for change in volume of water with change in temperature should be applied.

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[1] Since a and b enter the equation, R is no longer exactly $\frac{1}{273}$, as for a perfect gas; otherwise p and v would not be unity together.

Further study is desirable on the absorption of air by water under working conditions and of the permeation of the walls of the tube by air and water. It would be desirable also to consider whether the reading of the tube is affected by the kinetic pressure due to its descent through the water or by the shock of its striking the bottom.

NOTE 1.—RATE OF ABSORPTION OF AIR BY WATER.

Let a chamber of gas be maintained at a given temperature and at constant pressure against a column of liquid of uniform cross section capable of absorbing the gas into solution. Let this column be closed at the end opposite the gas chamber, and let the length of the column be denoted by l, and let x denote the distance of any cross section of liquid from the surface of separation of liquid and gas. Let ρ be the density of the gas dissolved in the liquid at this cross section at the time t.

The partial differential equation which ρ must satisfy is

$$\frac{\partial\rho}{\partial t}=k\frac{\partial^2\rho}{\partial x^2}. \qquad (36)$$

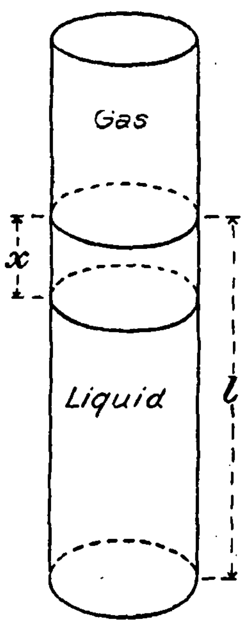

FIG. 2.—Diagram illustrating absorption of gas by a liquid.

(See Winkelmann, Handbuch der Physik, vol. 1, pt. 2, p. 1446.) The quantity k is a constant, the coefficient of diffusion. It is required to build up a solution of this equation $\rho=f(x, t)$, in which $f(x, t)$ must satisfy the following conditions:

(1) $f(x, \infty)=\rho_s$; in which ρ_s is the gas density in the liquid when the latter is saturated.

(2) $f(x, 0)=0$ except when $x=0$, in which case $f(x, t)$ is indeterminate.

(3) $\frac{\partial f}{\partial x}=0$ when $x=l$ for all values of t, since the further end of the tube is closed and there is no flow of gas across it.

To simplify the printing of exponential quantities with complicated exponents we shall use the notation exp (z) for ϵ^z, z standing for any expression simple or complicated and ϵ being the base of the natural logarithms.

A simple particular solution of (36) is

$$\rho=\exp(-kc^2t)\sin cx \qquad (37)$$
$$\text{or } \rho=\exp(-kc^2t)\cos cx$$

In (37) c is an arbitrary constant. If we choose the sine function and put $c=\frac{n\pi}{2l}$, where n is an odd integer, we satisfy condition (3); and by making use of the Fourier expansion,

$$1=\frac{4}{\pi}\left[\sin y+\frac{\sin y}{3}+\frac{\sin 5y}{5}\cdots\cdots\cdots\cdots\right] \tag{38}$$

we can build up a solution of (36) that satisfies the required conditions; namely,

$$\begin{aligned}\rho=\rho_s\Big\{1-\frac{4}{\pi}\Big[&\exp\left(\frac{-k\pi^2t}{4l^2}\right)\sin\frac{\pi x}{2l}+\frac{1}{3}\exp\left(\frac{-9k\pi^2t}{4l^2}\right)\sin\frac{3\pi x}{2l}\\&+\frac{1}{5}\exp\left(\frac{-25k\pi^2t}{4l^2}\right)\sin\frac{5\pi x}{2l}\cdots\cdots\Big]\Big\}\end{aligned} \tag{39}$$

The mean gas density over the whole column of liquid ρ_m is defined by

$$\rho_m=\frac{1}{l}\int_0^l\rho\,dx \tag{40}$$

Therefore, at any time, t,

$$\rho_m=\rho_s\Big\{1-\frac{8}{\pi^2}\Big[\exp\left(\frac{-k\pi^2t}{4l^2}\right)+\frac{1}{9}\exp\left(\frac{-9k\pi^2t}{4l^2}\right)+\frac{1}{25}\exp\left(\frac{-25k\pi^2t}{4l^2}\right)\cdots\Big]\Big\} \tag{41}$$

According to this equation ρ_m reduces to zero when $t=0$, as it should, since $\frac{\pi^2}{8}=1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}\cdots\cdots\cdots\cdots$; also when $t=\infty$, ρ_m becomes ρ_s.

Winkelmann gives the following numerical values of k in c. g. s. units for temperatures of 16° C. (Handbuch der Physik, vol. 1, pt. 2, p. 1450.)

Gas.	Coefficient of diffusion k.
CO_2	0.0000159
N	.0000200
O	.0000187
N_2O	.0000156
Cl	.0000127
NH_3	.0000128

For a numerical example let us take $l=50$ centimeters, which is rather shorter than the column of liquid in the sounding tube when the latter is used at a considerable depth. Let us consider also the case of nitrogen, which has a larger coefficient of diffusion than any other gas in the preceding table. Accordingly, $k=2\times10^{-5}$. In order for $\frac{k\pi^2t}{4l^2}$ to be of such size that ρ_m may approximate even roughly to its limiting value ρ_s, t must be large. Take $t=\frac{5\times10^7}{\pi^2}$ seconds, or over 58 days; by computing from (41) we find that, even with this large value of t, ρ_m is less than one-fourth (23 per cent) of the possible density of saturation.

The values of the coefficients of diffusion given in the preceding table are for pure water. Although they appear to be somewhat larger for sea water, absorption, whether of fresh water or of sea-water, when effected by diffusion alone, is an extremely slow process.

In sea water, under natural conditions, absorption is aided by convection currents due to differences of temperature or to differences of salinity produced by evaporation. It is also supposed that particles of dust act as nuclei of condensation and as carriers for minute quantities of gas, and so hasten absorption at lower depths. It has been suggested also that there are nuclei of condensation of a electrolytic nature present in sea water which act as carriers and may serve to account for the observed difference in the rate of diffusion between air into sea water and air into pure water.[1]

In the sounding tube absorption would be hastened by all the influences just mentioned, and probably much more by the motion of the water itself in entering the tube, and by the other motions and concussions incidental to the process of taking a sounding. The information available leads us to conclude that if the correction for the quantity of air absorbed be omitted the resulting error in depth will be small, and that it will be better not to attempt to make the correction until further tests have been made.

NOTE 2.—CERTAIN QUESTIONS IN MECHANICS CONNECTED WITH THE SOUNDING TUBE.

The problem has been treated hitherto as a statical problem; that is, as if either the tube descended to the bottom with extreme slowness or else as if the valve were not released to admit the water until the tube had reached its lowest point, and as if, furthermore, the water lost all momentum immediately after passing the valve opening.

It is found that the tube will descend to a depth of about 105 fathoms in about 45 seconds. A freely falling body would cover this distance in about 6 seconds. The buoyant effect of water will diminish the apparent acceleration of gravity and increase the time of descent, but not greatly. Most of the difference between 45 and 6 seconds is to be explained by the resistance of the water. This resistance implies a pressure which would be additional to the statical pressure of the water. Formulas for the motion of a body subject to a constant acceleration and to a resistance proportional to the square of the velocity (Newton's assumption, which is probably nearly correct) may be found in many works on mechanics.[2]

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[1] For an account of diffusion under natural conditions and of experiments dealing with the rate of diffusion of gases through liquids, also references to the literature of the subject, see Krümmel, Handbuch der Ozeanographie (vol. 1, p. 298).

[2] Some idea of the total resistance encountered may be derived from the following general considerations. If a body of mass m in a resisting medium like water is acted on by a constant force like gravity and by the resistance of the medium, which is some function $f(v)$ of the velocity v, it being understood that resistance

In this case, however, the acceleration is not constant, since water is continually entering the tube, thus diminishing the buoyant effect of the water outside. The problem is thus rendered very complicated and requires for a numerical solution further experimental data, especially on the rate at which water under a given pressure would pass through the valve opening.

It might be found that the opening was so large that, owing to the pressure arising from the motion, an amount of water would enter the tube in excess of the amount determined for the depth. If the valve opening is too small, it will require an excessive time to establish equilibrium between the pressures inside and outside the tube. On this account care should be taken not to raise the tube too soon to the surface. The matter seems to be one for experimental investigation.

When the tube reaches the bottom, it is traveling with considerable velocity and may be stopped with more or less suddenness, depending on the nature of the bottom. The first effect of the shock would be to throw the water in the tube against the valve at the bottom, thus preventing the entrance of any more water. Possibly when the water in the tube rebounds from the lower end there might be a chance for more water to enter, but, if the valve opening is small, this does not seem likely. Much would also depend on how nearly *instantaneous is the adjustment* of the amount of water in the tube to the depth. If the opening is small, this adjustment would require time, and it would be well to investigate the effect of letting the tube remain at the depth to be sounded for a longer or shorter period.

NOTE 3.—THE HEAT EVOLVED BY THE COMPRESSION OF THE AIR IN THE TUBE.

In all formulas for the volume occupied by the compressed air it has been assumed that sufficient time had been allowed for the air to take on the temperature of the surrounding water. As the tube in practice is lowered and quickly raised again, it may be of interest to estimate how much heat must be given out by the tube in this short time in order that our assumption may be justified.

The amount of heat evolved in compressing a gas depends on how the compression is brought about. We shall calculate the amounts on three simple suppositions. The formulas used will be found in almost any elementary book on thermodynamics, or may be readily

increases with velocity, then the velocity will increase more and more slowly as a certain limiting velocity V, called the terminal velocity, is approached. The resistance $f(v)$ depends, among other things, on the size and shape of the body. When the acceleration is practically zero as the terminal velocity is approached, terminal resistance $f(V)=mg$, g being the acceleration of gravity. This gives the limit to which the resistance or total pressure on the body approximates. What would be the pressure intensity on a particular point—for instance, on the valve—can not be estimated, even approximately, without a knowledge of the form of body.

deduced from the ones there given. The air is treated as a perfect gas, an assumption which greatly simplifies the formulas and gives more than sufficient accuracy for the purpose in hand, which is illustrative only.

Case 1.—The compression is so gradual that the heat of compression is absorbed by the water as fast as it is involved and the air is all the while at the temperature of the surrounding water. This is called isothermal compression.

Denote by J the mechanical equivalent of heat, by W' the amount of work done compressing the gas isothermally at temperature t_1, from volume v_1 and pressure p_1 to volume v_2 and pressure p_2, and H' the heat given out in the process.

Then

$$H'=\frac{W'}{J}=\frac{Rt_1}{J}\log_e\frac{p_2}{p_1}=\frac{p_1v_1}{J}\log_e\frac{p_2}{p_1}=\frac{p_1v_1}{J}\log_e\frac{v_1}{v_2} \tag{42}$$

In this case the heat given out is less than will be given out on any other admissible supposition.

Case 2.—Suppose the air to be surrounded by matter impervious to heat, so that heat of compression is retained. Suppose that the air retains its heat till the pressure p_2 is attained, and that the nonconducting layer is then removed. The temperature of the air, and the corresponding amount of heat and final volume attained under pressure p_2 will depend on how the pressure is applied. If the pressure is increased gradually, so that the volume and temperature are at every instant adjusted to the pressure, the compression is called adiabatic. If t''_2 and v''_2 denote the absolute temperature and the volume corresponding to pressure p_2 and if k denote the ratio of the specific heat of the air at constant pressure C_p to its specific heat at constant volume, C_v, that is, $k=\frac{C_p}{C_v}$ the formulas for adiabatic compression are

$$t_2''=t_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \tag{43}$$

$$v_2''=v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \tag{44}$$

When the nonconducting layer is removed, the air cools at constant pressure p_2 to the temperature t_1 of the surrounding water and in so doing shrinks from volume v_2'' to v_2, the volume it finally attained in Case 1.

In so doing the heat H'' given out is expressed by

$$H''=C_p(t_2''-t_1)$$

$$=\frac{k}{k-1}\,\frac{p_1v_1}{J}\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}-1\right] \tag{45}$$

Case 3.—In this case, as in Case 2, the heat is to be retained by a nonconducting layer, but the pressure, instead of being increased gradually, is suddenly increased from p_1 to p_2. When the volume v_2''' and the temperature t_2''' corresponding to pressure p_2 have been reached, the nonconducting layer is removed, the air cools to the temperature t_1, and shrinks to the volume v_1 of Case 1. The formulas for H''', the heat given out, are

$$t_2''' = t_1\left[\frac{p_2}{p_1} - \frac{p_2 - p_1}{p_1 k}\right] \tag{46}$$

$$v_2''' = v_1\left[1 - \frac{p_2 - p_1}{p_2 k}\right] \tag{47}$$

$$H''' = C_p[t_2''' - t_1]$$

$$= \frac{v_1}{J}(p_2 - p_1) \tag{48}$$

As a numerical illustration we take $p_1 = 1$ atmosphere, $p_2 = 20$ atmospheres, corresponding to a water pressure of 105 fathoms nearly, and $t_1 = 273°$ absolute $= 0°$ C. If the C. G. S. system of units be used, we must take the pressure in dynes (1 atmosphere $= 1.0132 \times 10^6$ dynes), and the unit volume will be that of a gram of air under standard conditions $= 773.5$ c. c. We have further, by experiment, $C_p = 0.238$, and

$$k = \frac{C_p}{C_v} = 1.4;$$

and from theory,

$$J = \frac{1.0132 \times 10^6 \times 773.5}{273 \times 0.238\left(1 - \frac{1}{1.4}\right)} = 4.22 \times 10^7 \text{ ergs.} \tag{49}$$

The values of H will be expressed in gram calories per unit volume of air, and, to find the heat given out by each cubic centimeter of air at atmospheric pressure, we must divide the respective values of H by 773.5.

Case 1. (Isothermal compression. Compression gradual, heat given out as fast as produced.) Each cubic centimeter of air at atmospheric pressure gives out enough heat to raise the temperature of 1 gram of pure water by 0.°072 C.

Case 2. (Adiabatic compression. Compression gradual, heat is retained till pressure of 20 atmospheres is reached and is then given out.) The temperature of the air is raised by 369.°5, but each cubic centimeter of air at the original atmospheric pressure gives out only enough heat in cooling to raise the temperature of one gram of pure water by 0.°114 C.

Case 3. (Compression sudden; heat is retained till equilibrium is reached at a pressure of 20 atmospheres; heat then given off.) The temperature of the compressed air is raised by 1482°, but each cubic centimeter of air at the orignal volume gives out in cooling only enough heat to raise the temperature of 1 gram of pure water by 0.°456 C.

Sea water has a specific heat slightly less than pure water (0. 936 for water of salinity 32.5 parts per thousand, density 1.026 at 0°); and if instead of reckoning by grams of pure water we reckon the heating effect in a cubic centimeter of pure water the figures in Cases 1, 2, and 3 will be, respectively, 0.°077, 0.°122, and 0.°487.

The actual heating effect evidently lies between Case 1 and Case 3, probably much nearer to the former. In any event, it seems plain that, in spite of the limited time spent in taking a sounding, the air can take on practically the temperature of the surrounding water, as has been assumed.

NOTE 4.—COMPRESSIBILITY OF SEA WATER.

The mean coefficient of compressibility μ of a substance between the pressures P and $P+p$ is defined by the equation

$$\mu=\frac{v_0-v}{pv_0} \qquad 50$$

In this equation v_0 and v are the volumes corresponding to pressures P and $P+p$, respectively. The true coefficient of compressibility (μ_0) at pressure P is the limit of the mean coefficient as p approaches zero, or

$$\mu_0=-\frac{1}{v}\frac{dv}{dp}=-\frac{d}{dp}(\log v). \qquad (51)$$

We have also, since $\frac{v}{v_0}=\frac{\rho_0}{\rho}$, the ρ's being densities corresponding to the v's,

$$\mu_0=\frac{d}{dp}(\log \rho)=\frac{1}{\rho}\frac{d\rho}{dp}. \qquad (52)$$

The most thorough investigation on the compressibility of sea water is by W. V. Ekman, in paper No. 43 of the "Publications de Circonstance" of the "Conseil Permanent International pour l'exploration de la mer" entitled, "Die Zusammendrueckbarkeit des Meerwassers." His final result in his own notation is:

$$10^8\mu=\frac{4886}{1+0.000186p}-[227+28.33t-0.551t^2+0.004t^3]+\frac{p}{1000}[105.5$$
$$+9.50t-0.158t^2]-\frac{1.5p^2t}{1000000}-\frac{\sigma_0-28}{10}[147.3-2.72t+0.04t^2-\frac{p}{1000}(32.4$$
$$-0.87t+0.02t^2)]+\left(\frac{\sigma_0-28}{10}\right)^2[4.5-0.1t-\frac{p}{1000}(1.8-0.06t)]. \qquad (53)$$

In this equation μ represents the mean compressibility between atmospheric pressure and p additional units of pressure, so that when p is zero the pressure is 1 atmosphere. The unit of p is the bar, that is, 1,000,000 dynes per square centimeter.[1] One standard atmosphere = 1.01323 bars, or 1 bar = 0.98694 atmosphere; t represents the temperature in degrees centigrade; σ_0 is a quantity connected with the density δ_0 of the water at 0° C., in such a way that density at 0°C. $= 1 + \frac{\sigma_0}{1000}$.

On the basis of formula (53) Tables 4a and 4b have been calculated. It seemed more convenient, however, to tabulate the true compressibility for $p=0$, to make the unit of pressure the atmosphere instead of the bar and to use as one of the arguments the salinity instead of the density at 0°C. This has accordingly been done.

In strictness the tabulated true conpressibility, μ_0, applies only when $p=0$; practically, it may be taken for the mean compressibility for all pressures within the range of present sounding tubes. To take into account, however, the terms of formula (53) that do not appear in μ_0, we may proceed as follows: Suppose the change in relative volumes to be expanded in a series of powers of the increment of pressure, p. From (50) and (51), the first term is evidently $p\mu_0$; that is,

$$\frac{v_0 - v}{v_0} = \mu_0 p + a_2 p^2 + a_3 p^3 \cdots\cdots, \tag{54}$$

the a's being coefficients independent of p, but for sea water dependent on the temperature and salinity. A brief table of the values of a_2 is given on page 40. The unit of pressure is the atmosphere. a_3 is very small, indeed; according to (53), it is, when reduced to the atmosphere as unit of pressure, $\left(1.76 - \frac{1.5t}{100}\right) \times 10^{-12}$. If it is desired to compute the change in relative density, the expression to be used is

$$\frac{\rho - \rho_0}{\rho_0} = \mu_0 p + (a_2 + \mu_0^2) p^2 \cdots\cdots \tag{55}$$

SCHEDULE OF MATHEMATICAL NOTATION.

ENGLISH CHARACTERS.

a Constant of Van der Waals's equation; definition, p. 8; numerical value, p. 25.

a In equation on p. 22 only, special meaning.

a_2 Coefficient in formula (55), p. 33, for change of volume under pressure. Numerical values in table, p. 40.

a_3 See p. 33.

b Constant of Van der Waals's equation, p. 8, numerical value, p. 25.

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[1] This unit is also called the megabar, the bar being then defined as a pressure of one dyne per square centimeter.

b In equation on p. 22 only, special meaning.

B Used only in ΔB, p. 12.

c Special meaning, p. 24; different special meaning in note 1, p. 26.

C_1, C_2, C_3 Defined by equations (9), (10), (11), and (11b), p. 10.

C_p, C_v Specific heats at constant pressure and constant volume, respectively, in note 3, p. 29.

f Used only on p. 24, and defined there.

g Acceleration of gravity in general.

g_0 Acceleration of gravity at sea level.

h Depth to which tube is submerged below sea surface; more specifically depth of bottom of air space, see p. 22. Working unit of h, the fathom.

H', H'', H''' Quantities of heat, note 3, p. 29 only.

J Mechanical equivalent of heat, note 3, p. 29 only.

k General meaning, coefficient of proportionality between depth and water pressure; special meaning in note 1, p. 26, coefficient of diffusion of gas into liquid; second special meaning, note 3, p. 29 only, is $\frac{C_p}{C_v}$.

l In note 1, p. 26 only.

m Mass in general, note 2, p. 28; elsewhere number of grams of air absorbed by a cubic unit of water.

n See p. 10, following equation (11b).

p Pressure in general. Working unit of pressure the atmosphere.

P and p Indicate particular pressures defined in note 4, p. 32.

p_1 Atmospheric pressure at sea surface, exclusive of pressure due to water vapor, also a somewhat different meaning in note 3, p. 29.

p'_1 Pressure at the surface due to water vapor.

p' Pressure due to water vapor in the air space of the tube, and assumed to be the vapor pressure of saturation for water of the salinity and temperature of that surrounding the tube.

p_2 Pressure due to the weight of the water, and also a somewhat different special meaning in note 3, p. 29.

R The gas constant of Charles's law or of Van der Waals's equation.

s See equation (11b), p. 10.

S Salinity of water, see Table 1, pts. 1 and 2, p. 37.

t Time in note 1, p. 26 only, elsewhere temperature. In the formulas temperatures are on the absolute scale unless otherwise stated.

t_1 Temperature of the air, always on absolute scale in formulas.

t_2 Temperature of the water, always on absolute scale in formulas.

t_2''', t_2'' Temperatures defined in note 3, p. 29.

T Defined, p. 12.

v To suggest volume in general, and in particular volume of air in tube as on p. 16, except in note 2, p. 28, where v is used for velocity.

v_0 Initial volume under compression. Note 4, p. 32.

v_1 Volume of unit mass of air under pressure p_1 and temperature t_1; denotes a different special volume in note 3, p. 29.

v_2 Volume of unit mass of air when tube is submerged; denotes a different special volume in note 3, p. 29.

v_2', v_2'', v_2''' Volumes defined note 3, p. 29.

V Entire volume of the tube except in note 2, p. 28, where V denotes terminal velocity.

w Volume of water in the tube.

W' Mechanical work equivalent to H'.

x Two distances, defined where they occur, pp. 22 and 26.

z_1 Defined, p. 12.

GREEK CHARACTERS.

α Volume of gas which unit volume of water will absorb.
β Coefficient of compressibility of sea water.
δ Density of sea water in general.
δ_0 Density of sea water at the surface.
δ_1 Density of sea water at air temperature and atmospheric pressure.
δ_2 Density of sea water at water temperature and atmospheric pressure.
Δ Finite increment of quantity following, which see.
$\Delta_1, \Delta_2, \Delta_3$ Special finite increments, see pp. 18–20.
ϵ Base of natural logarithms = 2.71828 . . . . .
μ Mean coefficient of compressibility of sea water. See note 4, p. 32.
μ_0 True coefficient of compressibility of sea water, at atmospheric pressure, see note 4, p. 32.
ρ Denotes density in general in note 4; elsewhere a gas density, particularly in note 1, p. 26.
ρ_1, ρ_2 Densities of air due to different pressures, p. 16.
ρ_m, ρ_s See note 1, p. 26.
σ_0 Defined in note 4, p. 32.
φ Geographic latitude, p. 38.

EXAMPLES IN THE USE OF THE TABLES.

Tables 1, 3, 5, 9, and 10 are self-explanatory. The following examples may illustrate the use of the other tables:

Example 1.—The volume of a quantity of water is 100 c. c. at atmospheric pressure, temperature being 5° C. and salinity 30. What will be its volume at the same temperature under an additional pressure of 20 atmospheres?

In the notation of formula (54), page 33, $v_0 = 100$, and it is required to find v. From Table 4, for the given salinity and temperature,

$$\mu = 4{,}644 \times 10^{-8},\ a_2 = -0.774 \times 10^{-8}$$

From (54),
$$\frac{v_0 - v}{v_0} = 4{,}644 \times 10^{-8} \times 20 - 0.774 \times 10^{-8} \times 20^2$$
$$= 10^{-8} \times 92{,}570.$$

Therefore, $v_0 - v = 0.09257$, and $v = v_0 - 0.09257 = 99.90743$ c. c.—*Answer.*

Example 2.—Under the conditions supposed in ex. 1, the density of the water at atmospheric pressure is found to be 1.02375. What will be its density under the additional 20 atmospheres pressure?

Since density varies inversely as volume, the new density will be

$$1.02375 \times \frac{100}{99.90743} = 1.0246983.$$

The increase in density, $\rho - \rho_0$, is $1.0246983 - 1.02375 = 0.0009486$.

The increase in density may also be found directly from formula (55), page —, without bringing in the volumes. In (55) $\rho_0 = 1.02375$

$$\rho - \rho_0 = \rho_0[\mu_0 p + (\mu_0^2 + a_2)p^2]$$
$$= 1.02375\ \{4{,}644 \times 10^{-8} \times 20 + [(4{,}644 \times 10^{-8})^2 - 0.774 \times 10^{-8}]\ 20^2\}$$

$\rho - \rho_0 = 0.0009485$, which agrees substantially with the result found by the first method.

Example 3.—What is the correction to the depth for the difference of temperature between air and water due to the change in volume of the water with temperature? (Table 9 gives only the effect of temperature on the volume of air.)

The first-mentioned correction is needed only in a table in which the water is brought to the surface for measurement. (See p. 18.) Take, for example, air temperature, 24° C. water temperature, 10° C. salinity, 36, and depth, 27 fathoms. In the notation of formula (27), page 19, for 24°, $\delta_1 = 1.02442$, and for 10°, $\delta_2 = 1.02775$. From Table 8,

$$\frac{\left(1-\frac{v_2}{v_1}\right)}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)} = -165.$$

Therefore, the required correction

$$=\Delta_2 h = -165 \times \frac{1.02775 - 1.02442}{1.02442} \text{ or } \Delta_2 h = -165 \times \frac{0.00333}{1.024} = -0.5$$

fathom.

This example shows the necessity of applying the correction in accurate work, as the difference in temperature is by no means extreme, and, for a given difference of temperature, the correction varies directly as

$$\frac{1-\left(\frac{v_2}{v_1}\right)}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)},$$

which increases rapidly with the depth.

Example 4.—Suppose that the expansion of water due to the release of pressure on coming to the surface has not been allowed for in computing the scale. (The allowance has been made in Table 8.) What correction must be applied to the heights read from the scale; that is, what is $\Delta_1 h$ formula (26), page 19?

Take depth and salinity as in example 3. For depth 27 fathoms, $p_2 = 4.90$ atmospheres (Table 6). For 10° and salinity 36, $\mu_0 = 4474 \times 10^{-8}$, and, as before,

$$\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)} = -165.$$

Therefore, $\Delta_2 h = -165 \times 4{,}474 \times 10^{-8} \times 4.90 = -0.036$ fathoms.

At this depth, the correction is small, but $\Delta_2 h$ increases even more rapidly with the depth than $\Delta_1 h$, so that toward the end of the table $\Delta_2 h$ should not be neglected in accurate work.

TABLE 1.—*Miscellaneous physical data compiled from various sources.*

1. COMPOSITION OF SEA WATER.

The salinity of sea water is defined as the number of grams of salts contained in 1,000 grams of sea water. The relative proportions of the various salts in sea water is almost constant the world over, except under obviously peculiar conditions. The following may be taken as representative:

Amounts of various salts in 1,000 grams of sea water; salinity, 35.

Name.	Chemical symbol.	Amount.	Per cent of all salts.
		Grams.	
Common salt	$NaCl$	27.21	77.75
Magnesium chloride	$Mg\,Cl_2$	3.81	10.88
Magnesium sulphate	$Mg\,SO_4$	1.66	4.74
Calcium sulphate	$Ca\,SO_4$	1.26	3.60
Potassium sulphate	K_2SO_4	.86	2.47
Magnesium bromide	$Mg\,Br_2$	.08	.22
Calcium carbonate and traces of other substances	$Ca\,CO_3$	.12	.34
Total		35.00	100.00

Since, in a dilute solution like sea water the various salts are partially dissociated into their ions, it is better to give simply the amount of the separate elements and composite ions.

Amounts of various elements in 1,000 grams of sea water; salinity 35.

Name.	Chemical symbol.	Amount.	Name.	Chemical symbol.	Amount.
		Grams.			*Grams.*
Chlorine	Cl	19.32	Sulphuric acid ions	SO_4	2.69
Sodium	Na	10.72	Carbonic acid ions and traces other matter.	CO_3	.08
Magnesium	Mg	1.32			
Calcium	Ca	.42			
Potassium	K	.38	Total		35.00
Bromine	Br	.07			

2. The salinity, the density, and the chlorine content of sea water are connected with one another by the three following empirical formulas, the last of which is derived from the two preceding:

$$S = 0.030 + 1.805\,\mathrm{Cl}$$
$$\sigma_0 = -0.069 + 1.4708\,\mathrm{Cl} - 0.00157\,(\mathrm{Cl})^2 + 0.0000398\,(\mathrm{Cl})^3$$
$$\sigma_0 = -0.093 + 0.8149\,S - 0.000482\,S^2 + 0.0000068\,S^3$$

S represents the salinity, Cl the chlorine content in grams of 1,000 grams of sea water, and σ_0 is a quantity such that density at 0° C. $= 1 + \frac{\sigma_0}{1000}$

The limiting case when Cl or S is zero is not that of distilled water, but that of natural water, just contiguous to water that is barely brackish, and for this extreme case the formulas should not be pressed too closely.

The value of σ_0 may be found from the salinity by interpolation on the second line of Table 2.

3. PHYSICAL CONSTANTS OF SEA WATER DEPENDENT ON THE SALINITY.

Salinity.	5	10	15	20	25	30	35	40
Boiling point, pressure 760 mm	100.08°C.	100.16°C	100.23°C	100.31°C	100.39°C	100.47°C	100.56°C	100.64°C
Freezing point, pressure 760 mm	−0.27°C	−0.53°C	−0.80°C	−1.07°C	−1.35°C	−1.63°C	−1.91°C	−2.20°C
Temperature of maximum density	2.93°C	1.86°C	0.77°C	−0.31°C	−1.40°C	−2.47°C	−3.52°C	−4.54°C
Maximum density [1]	1.00415	1.00818	1.001213	1.001607	1.002010	1.002415	1.002822	1.003232
Specific heat at 17.5°C	0.982	0.968	0.958	0.951	0.945	0.939	0.932	0.926
Thermal conductivity (C. G. S. units) at 17.5°C	0.00138	0.00137	0.00136	0.00135	0.00135	0.00135	0.00134	0.00134
Index of refraction, *D*-line at 18° C	1.33405	1.33502	1.33598	1.33694	1.33790	1.33885	1.33981	1.34077

[1] For the higher salinities the maximum density is attained only by undercooled water.

Surface tension in dynes per cm. $= 77.09 - 0.1788t + 0.0221\,S$.

Coefficient of viscosity (internal friction) in C. G. S. units $= \frac{0.0180(1 + 0.00180S - 0.000009S^2)}{(1 + 0.0355t + 0.000175t^2)}$.

In these equations S is the salinity and t is the temperature centigrade.

4. PHYSICAL CONSTANTS OF AIR.

Atmospheric air is a mixture containing about 77 per cent nitrogen, 21 per cent oxygen, less than 1 per cent of argon and allied rare gases, and about 1 per cent, on the average, of water vapor. Carbon dioxide is only 0.03 per cent.

One liter of air, temperature 0° C., pressure 1 atmosphere, weighs 1.2928 grams.

Coefficient of viscosity (C. G. S. units) $= (173.3 + 0.46t) \times 10^{-6}$.

Thermal conductivity (C. G. S. units), $0.0000568\ (1 + 0.00190t)$.

Specific heat at constant pressure, 0.2377.

Ratio, specific heat at constant pressure to specific heat at constant volume = 1.405.

5. ACCELERATION OF GRAVITY AT SEA LEVEL IN LATITUDE φ.

(cm. per sec. per sec.)[1]

$$g = 978.039\ (1 + 0.005294 \sin^2 \varphi - 0.000007 \sin^2 2\varphi)$$

$$= 980.621\ (1 - 0.002640 \cos 2\varphi + 0.000007 \cos^2 2\varphi)$$

The coefficients of the parentheses in the two forms of the expression for gravity are, respectively, gravity at the Equator and at latitude 45°.

TABLE 2.—*Density of sea water.*

[By salinity is meant the number of grams of salts dissolved in 1,000 grams of sea water. For basis of table, see Table 1. Density of pure water at 4° C. is unity, so that tabulated density is weight in grams of a cubic centimeter of sea water.]

Temp., °C. \ Salinity	6	8	10	12	14	16	18	20	22
−2	1.00466	1.00629	1.00792	1.00955	1.01117	1.01280	1.01442	1.01604	1.01767
0	1.00478	1.00640	1.00801	1.00963	1.01124	1.01285	1.01446	1.01607	1.01767
2	1.00484	1.00644	1.00804	1.00964	1.01124	1.01284	1.01443	1.01603	1.01762
4	1.00483	1.00642	1.00801	1.00959	1.01118	1.01276	1.01435	1.01593	1.01751
6	1.00476	1.00634	1.00791	1.00949	1.01106	1.01263	1.01421	1.01578	1.01735
8	1.00463	1.00620	1.00776	1.00933	1.01089	1.01245	1.01401	1.01557	1.01713
10	1.00445	1.00601	1.00756	1.00912	1.01067	1.01222	1.01377	1.01532	1.01687
12	1.00422	1.00577	1.00731	1.00886	1.01040	1.01194	1.01348	1.01502	1.01657
14	1.00394	1.00548	1.00701	1.00855	1.01008	1.01162	1.01315	1.01468	1.01622
16	1.00361	1.00514	1.00667	1.00820	1.00973	1.01125	1.01278	1.01430	1.01583
18	1.00324	1.00476	1.00628	1.00780	1.00932	1.01084	1.01236	1.01388	1.01540
20	1.00283	1.00434	1.00586	1.00737	1.00888	1.01039	1.01190	1.01342	1.01493
22	1.00237	1.00388	1.00539	1.00690	1.00840	1.00991	1.01141	1.01292	1.01442
24	1.00188	1.00338	1.00488	1.00639	1.00789	1.00938	1.01088	1.01238	1.01388
26	1.00135	1.00285	1.00434	1.00584	1.00733	1.00883	1.01032	1.01181	1.01331
28	1.00078	1.00227	1.00376	1.00525	1.00674	1.00823	1.00972	1.01121	1.01270
30	1.00018	1.00166	1.00315	1.00463	1.00612	1.00760	1.00908	1.01057	1.01205

Temp., °C. \ Salinity	24	26	28	30	32	34	36	38	40
−2	1.01929	1.02091	1.02253	1.02415	1.02577	1.02739	1.02902	1.03065	1.03227
0	1.01928	1.02089	1.02250	1.02410	1.02571	1.02732	1.02894	1.03055	1.03217
2	1.01922	1.02081	1.02240	1.02400	1.02560	1.02720	1.02880	1.03040	1.03200
4	1.01909	1.02068	1.02226	1.02384	1.02543	1.02702	1.02861	1.03020	1.03179
6	1.01892	1.02049	1.02206	1.02364	1.02521	1.02679	1.02837	1.02995	1.03153
8	1.01869	1.02026	1.02182	1.02338	1.02495	1.02651	1.02808	1.02965	1.03123
10	1.01842	1.01998	1.02153	1.02308	1.02464	1.02619	1.02775	1.02932	1.03088
12	1.01810	1.01965	1.02119	1.02274	1.02429	1.02584	1.02739	1.02894	1.03050
14	1.01775	1.01929	1.02082	1.02236	1.02390	1.02544	1.02698	1.02853	1.03007
16	1.01735	1.01888	1.02041	1.02194	1.02347	1.02500	1.02654	1.02807	1.02962
18	1.01692	1.01844	1.01996	1.02148	1.02300	1.02453	1.02606	1.02759	1.02912
20	1.01644	1.01795	1.01947	1.02099	1.02250	1.02402	1.02554	1.02707	1.02860
22	1.01593	1.01744	1.01895	1.02046	1.02197	1.02348	1.02500	1.02652	1.02804
24	1.01538	1.01689	1.01839	1.01989	1.02140	1.02291	1.02442	1.02594	1.02745
26	1.01480	1.01630	1.01780	1.01930	1.02080	1.02230	1.02381	1.02532	1.02683
28	1.01419	1.01568	1.01717	1.01867	1.02017	1.02167	1.02317	1.02468	1.02619
30	1.01354	1.01503	1.01652	1.01801	1.01950	1.02100	1.02250	1.02400	1.02551

[1] (Spec. pub. No. 40, U. S. Coast and Geodetic Survey.)

TABLE 3.—*Vapor pressure of sea water for various salinities and temperatures.*

The vapor pressures are given in millimeters of mercury. For basis of table see p. 23.]

Temp., °C. \ Salinity.	6	8	10	12	14	16	18	20	22
−1	4.24	4.24	4.23	4.23	4.22	4.22	4.21	4.21	4.20
0	4.56	4.56	4.55	4.55	4.54	4.54	4.53	4.53	4.52
1	4.91	4.90	4.90	4.89	4.89	4.88	4.88	4.87	4.87
2	5.28	5.27	5.27	5.26	5.25	5.25	5.24	5.24	5.23
3	5.67	5.66	5.65	5.65	5.64	5.64	5.63	5.62	5.62
4	6.08	6.07	6.07	6.06	6.05	6.05	6.04	6.03	6.03
5	6.52	6.51	6.51	6.50	6.49	6.49	6.48	6.47	6.46
6	6.99	6.98	6.98	6.97	6.96	6.95	6.94	6.94	6.93
7	7.49	7.48	7.47	7.46	7.46	7.45	7.44	7.43	7.42
8	8.02	8.01	8.00	7.99	7.98	7.97	7.97	7.96	7.95
9	8.58	8.57	8.56	8.55	8.54	8.53	8.52	8.51	8.50
10	9.18	9.17	9.16	9.15	9.14	9.13	9.12	9.11	9.10
11	9.81	9.80	9.79	9.78	9.77	9.76	9.75	9.74	9.72
12	10.48	10.47	10.46	10.45	10.44	10.43	10.41	10.40	10.39
13	11.20	11.18	11.17	11.16	11.15	10.14	11.12	11.11	11.10
14	11.95	11.94	11.92	11.91	11.90	11.88	11.87	11 86	11.84
15	12.75	12.73	12.72	12.71	12.70	12.68	12.66	12.65	12.63
16	13.59	13.58	13.56	13.55	13.53	13.52	13.50	13.49	13.47
17	14.49	14.47	14.45	14.44	14.42	14.40	14.39	14.37	14.36
18	15.43	15.41	15.39	15.38	15.36	15.34	15.33	15.31	15.29
19	16.43	16.41	16.39	16.37	16.35	16.34	16.32	16.30	16.28
20	17.48	17.46	17.44	17.42	17.40	17.38	17.36	17.34	17.32
21	18.59	18.57	18.55	18.53	18.51	18.49	18.47	18.45	18.43
22	19.77	19.74	19.72	19.70	19.68	19.66	19.63	19.61	19.59
23	21.00	20.98	20.96	20.93	20.91	20.89	20.86	20.84	20.82
24	22.31	22.28	22.26	22.23	22.21	22.18	22.16	22.13	22.11
25	23.68	23.66	23.63	23.61	23.58	23.55	23.53	23.50	23.47
26	25.18	25.11	25.08	25.05	25.02	24.99	24.97	24.94	24.91
27	26.66	26.63	26.60	26.57	26.54	26.51	26.48	26.45	26.42
28	28.26	28.23	28.20	28.17	28.14	28.11	28.07	28.04	28.01
29	29.95	29.92	29.89	29.85	29.82	29.78	29.75	29.72	29.68
30	31.73	31.69	31.66	31.62	31.59	31.55	31.52	31.48	31.44

Temp., °C. \ Salinity.	*24*	*26*	*28*	*30*	*32*	*34*	*36*	*38*	*40*
−1	4.20	4.19	4.19	4.18	4.18	4.17	4.17	4.16	4.16
0	4.52	4.51	4.51	4.50	4.50	4.49	4.49	4.48	4.48
1	4.86	4.85	4.85	4.84	4.84	4.83	4.83	4.82	4.82
2	5.22	5.22	5.21	5.21	5.20	5.19	5.19	5.18	5.17
3	5.61	5.60	5.60	5.59	5.58	5.58	5.57	5.56	5.56
4	6.02	6.01	6.01	6.00	5.99	5.98	5.98	5.97	5.96
5	6.46	6.45	6.44	6.43	6.43	6.42	6.41	6.40	6.40
6	6.92	6.91	6.90	6.90	6.89	6.88	6.87	6.86	6.86
7	7.41	7.40	7.40	7.39	7.38	7.37	7.36	7.35	7.34
8	7.94	7.93	7.92	7.91	7.90	7.89	7.88	7.87	7.86
9	8.49	8.48	8.47	8.46	8.45	8.44	8.43	8.42	8.41
10	9.09	9.08	9.07	9.06	9.05	9.03	9.02	9.01	9.00
11	9.71	9.70	9.69	9.68	9.67	9.66	9.65	9.63	9.62
12	10.38	10.37	10.35	10.34	10.33	10.32	10.31	10.29	10.28
13	11.08	11.07	11.06	11.04	11.08	11.02	11.01	10.99	10.98
14	11.83	11.82	11.80	11.79	11.77	11.76	11.75	11.73	11.72
15	12.62	12.60	12.59	12.58	12.56	12.55	12.53	12.52	12.50
16	13.45	13.44	13.42	13.41	13.39	13.78	13.36	13.35	13.33
17	14.34	14.32	14.31	14.29	14.27	14.26	14.24	14.22	14.20
18	15.27	15.26	15.24	15.22	15.20	15.18	15.17	15.15	15.13
19	16.26	16.24	16.22	16.20	16.19	16.17	16.15	16.13	16.11
20	17.30	17.29	17.27	17.25	17.22	17.20	17.18	17.16	17.14
21	18.41	18.38	18.36	18.34	18.32	18.30	18.28	18.26	18.23
22	19.57	19.54	19.52	19.50	19.48	19.45	19.43	19.41	19.38
23	20.79	20.77	20.74	20.72	20.70	20.67	20.65	20.62	20.60
24	22.08	22.06	22.03	22.01	21.98	21.96	21.93	21.90	21.88
25	23.45	23.42	23.39	23.36	23.34	23.31	23.28	23.25	23.23
26	24.88	24.85	24.82	24.79	24.77	24.74	24.71	24.68	24.65
27	26.39	26.36	26.33	26.30	26.27	26.24	26.21	26.17	26.14
28	27.98	27.95	27.92	27.88	27.85	27.82	27.78	27.75	27.72
29	29.65	29.62	29.58	29.55	29.52	29.48	29.44	29.41	29.37
30	31.41	31.37	31.34	31.30	31.26	31.23	31.19	31.15	31.12

TABLE 4.—*Compressibility of sea water.*[1]

[Table 4a gives $10^8\times\mu_0$, μ_0 being the true coefficient of compressibility at atmospheric pressure, for water of various salinities and temperatures. Table 4b gives $10^8\times a_2$, a_2 being the coefficient of the second term in the formula (55), p. 33 for the change in relative volume, for water of various salinities and temperatures. The unit of pressure in both tables is the atmosphere. Temperatures are in degrees centigrade.]

TABLE 4a.—$10^8\times\mu_0$.

Temp., ° C. \ Salinity.	5	10	15	20	25	30	35	40
0	5,106	5,037	4,970	4,905	4,842	4,779	4,719	4,659
5	4,942	4,880	4,819	4,759	4,701	4,644	4,589	4,534
10	4,808	4,751	4,695	4,640	4,587	4,535	4,484	4,434
15	4,701	4,648	4,596	4,546	4,497	4,448	4,400	4,354
20	4,617	4,568	4,520	4,473	4,426	4,381	4,336	4,292
25	4,554	4,508	4,462	4,418	4,374	4,330	4,287	4,245
30	4,509	4,465	4,421	4,378	4,335	4,293	4,251	4,210

TABLE 4b.—$10^8\times a_2$.

Temp., ° C. \ Salinity.	5	10	15	20	25	30	35	40
0	−0.897	−0.880	−0.864	−0.849	−0.834	−0.820	−0.806	−0.793
5	.841	.826	.812	.799	.786	.774	.762	.750
10	.796	.783	.771	.759	.747	.736	.725	.715
15	.761	.749	.738	.727	.717	.707	.697	.687
20	.737	.726	.716	.706	.696	.686	.676	.667
25	.723	.713	.703	.693	.683	.674	.664	.655
30	−0.720	−0.710	−0.700	−0.690	−0.680	−0.670	−0.660	−0.650

TABLE 5.—*Absorption of atmospheric gases by sea water.*

[The tabular values are the number of cubic centimeters of gas that can be absorbed by sea water of the given salinity at the given temperature. The volumes are reduced to a temperature of 0°C., and a pressure of one atmosphere. Carbon dioxide (CO_2) is absorbed rather freely by sea water, but, since the total vapor pressure of CO_2 is only about 0.0003 atmosphere, its absorption need not be considered in connection with the sounding tube.]

1. OXYGEN.

Temp., ° C. \ Salinity.	0	5	10	15	20	25	30	35	40
	c. c.	c. c.	c. c.	c. c.	c. c.	c. c.	c. c.	c. c.	c. c.
−2	10.88	10.53	10.18	9.84	9.50	9.16	8.82	8.47	8.12
0	10.29	9.97	9.65	9.33	9.01	8.68	8.36	8.03	7.71
5	9.03	8.75	8.48	8.21	7.94	7.67	7.40	7.13	6.86
10	8.02	7.79	7.56	7.33	7.10	6.87	6.63	6.40	6.17
15	7.22	7.03	6.83	6.63	6.43	6.23	6.04	5.84	5.64
20	6.57	6.40	6.22	6.05	5.88	5.70	5.53	5.35	5.18
25	6.04	5.88	5.72	5.56	5.40	5.24	5.08	4.93	4.77
30	5.57	5.42	5.27	5.12	4.96	4.80	4.65	4.50	4.35

2. NITROGEN.

−2	19.45	18.83	18.18	17.61	16.90	16.27	15.63	15.00	14.36
0	18.56	17.97	17.37	16.77	16.18	15.58	14.99	14.40	13.80
5	16.60	16.10	15.60	15.10	14.59	14.09	13.59	13.08	12.58
10	14.97	14.55	14.13	13.70	13.27	12.85	12.43	12.00	11.57
15	13.63	13.27	12.91	12.55	12.20	11.84	11.48	11.12	10.76
20	12.54	12.24	11.93	11.63	11.32	11.02	10.71	10.40	10.09
25	11.66	11.40	11.13	10.86	10.59	10.32	10.05	9.78	9.51
30	10.94	10.70	10.46	10.22	9.98	9.74	9.50	9.26	9.02

[1] See note 4, p. 32.

TABLE 6.—*Pressure of sea water at various depths, by formula (35), page 24.*

[Density 1.025; temp. 10°C.; gravity as in lat. 45°.]

Depth, h (fathoms).	Water pressure, p_2 (atmospheres).	Depth, h (fathoms).	Water pressure, p_2 (atmospheres).	Depth, h (fathoms).	Water pressure, p_2 (atmospheres).
1	0.18142	41	7.43951	81	14.70009
2	.36284	42	7.62099	82	14.88164
3	.54426	43	7.80248	83	15.06319
4	.72569	44	7.98397	84	15.23544
5	0.90712	45	8.16545	85	15.42629
6	1.08855	46	8.34694	86	15.60784
7	1.26997	47	8.52844	87	15.78939
8	1.45141	48	8.70993	88	15.97095
9	1.63284	49	8.89142	89	16.15251
10	1.81427	50	9.07292	90	16.33407
11	1.99571	51	9.25442	91	16.51563
12	2.17714	52	9.43592	92	16.69719
13	2.35858	53	9.61742	93	16.87875
14	2.54002	54	9.79892	94	17.06032
15	2.72147	55	9.98043	95	17.24189
16	2.90291	56	10.16193	96	17.42345
17	3.08436	57	10.34344	97	17.60442
18	3.26580	58	10.52495	98	17.78660
19	3.44725	59	10.70646	99	17.96817
20	3.62870	60	10.88797	100	18.14974
21	3.81015	61	11.06949	101	18.33132
22	3.99160	62	11.25100	102	18.51290
23	4.17306	63	11.43252	103	18.69448
24	4.35452	64	11.61404	104	18.87606
25	4.53597	65	11.79556	105	19.05764
26	4.71743	66	11.97708	106	19.23922
27	4.89889	67	12.15860	107	19.42081
28	5.08036	68	12.34013	108	19.60240
29	5.26182	69	12.52166	109	19.78698
30	5.44329	70	12.70318	110	19.96558
31	5.62476	71	12.88471	111	20.14717
32	5.80622	72	13.06624	112	20.32876
33	5.98769	73	13.24778	113	20.51035
34	6.16916	74	13.42931	114	20.69195
35	6.35064	75	13.61084	115	20.87355
36	6.53211	76	13.79238	116	21.05515
37	6.71359	77	13.97392	117	21.23675
38	6.89506	78	14.15546	118	21.41835
39	7.07654	79	14.33700	119	21.59996
40	7.25803	80	14.51855	120	21.78156

TABLE 7.—*Table for scale of sounding tube.*

[Temperature of air, 15° C.; of water, 10°; humidity of air, 100%; surface density of water, 1.025; gravity as in latitude 45°.[1] $\frac{v_2}{v_1}$ is volume of air when volume of tube is taken as unity. $\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$ is given to not more than three significant figures and is always greater than unity.]

Depth, h (fathoms).	Volume of air space $\frac{v_2}{v_1}$	$\frac{\partial\left(\frac{v_2}{v_1}\right)}{\partial h}$ (Units of fifth decimal place).	$\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$	Depth, h (fathoms).	Volume of air space $\frac{v_2}{v_1}$	$\frac{\partial\left(\frac{v_2}{v_1}\right)}{\partial h}$ (Units of fifth decimal place).	$\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$
1	0.82630	....	....	61	0.07980	− 121	− 763
2	.71529	....	....	62	.07869	− 117	− 787
3	.63057	....	....	63	.07753	− 114	− 811
4	.56376	....	....	64	.07641	− 110	− 836
5	.50978	....	....	65	.07532	− 107	− 862
6	.46521	....	....	66	.07426	− 104	− 887
7	.42781	....	....	67	.07323	− 102	− 913
8	.39596	....	....	68	.07223	− 99	− 940
9	.36853	....	....	69	.07126	− 96	− 966
10	.34465	−2233	− 29	70	.07031	− 94	− 994
11	.32367	−1970	− 34	71	.06939	− 91	−1020
12	.30510	−1751	− 40	72	.06849	− 89	−1050
13	.28854	−1566	− 45	73	.06761	− 87	−1080
14	.27369	−1409	− 52	74	.06676	− 84	−1110
15	.26029	−1274	− 58	75	.06593	− 82	−1140
16	.24813	−1158	− 65	76	.06511	− 80	−1160
17	.23706	−1057	− 72	77	.06432	− 78	−1190
18	.22694	− 969	− 80	78	.06355	− 76	−1220
19	.21764	− 892	− 88	79	.06279	− 75	−1250
20	.20908	− 823	− 96	80	.06205	− 73	−1280
21	.20116	− 762	− 105	81	.06133	− 71	−1310
22	.19382	− 707	− 114	82	.06063	− 70	−1350
23	.18699	− 658	− 124	83	.05994	− 68	−1380
24	.18063	− 614	− 133	84	.05927	− 67	−1410
25	.17469	− 575	− 144	85	.05861	− 65	−1450
26	.16912	− 539	− 154	86	.05796	− 64	−1480
27	.16390	− 506	− 165	87	.05733	− 62	−1510
28	.15899	− 476	− 177	88	.05672	− 61	−1550
29	.15436	− 449	− 188	89	.05611	− 60	−1580
30	.15000	− 424	− 200	90	.05552	− 58	−1610
31	.14587	− 401	− 213	91	.05494	− 57	−1650
32	.14197	− 380	− 226	92	.05438	− 56	−1690
33	.13827	− 360	− 239	93	.05382	− 55	−1730
34	.13476	− 342	− 253	94	.05328	− 54	−1760
35	.13142	− 326	− 267	95	.05275	− 53	−1790
36	.12824	− 310	− 281	96	.05223	− 52	−1830
37	.12521	− 296	− 296	97	.05171	− 51	−1870
38	.12232	− 282	− 311	98	.05121	− 50	−1900
39	.11956	− 270	− 326	99	.05072	− 49	−1940
40	.11692	− 258	− 342	100	.05023	− 48	−1980
41	.11440	− 247	− 359	101	.04976	− 47	−2020
42	.11198	− 237	− 375	102	.04929	− 46	−2060
43	.10966	− 227	− 392	103	.04884	− 45	−2100
44	.10744	− 218	− 409	104	.04839	− 44	−2140
45	.10530	− 209	− 427	105	.04795	− 44	−2180
46	.10325	− 201	− 445	106	.04752	− 43	−2220
47	.10128	− 194	− 464	107	.04709	− 42	−2260
48	.09938	− 187	− 483	108	.04667	− 41	−2310
49	.09755	− 180	− 502	109	.04626	− 41	−2350
50	.09578	− 173	− 522	110	.04586	− 40	−2390
51	.09408	− 167	− 542	111	.04547	− 39	−2430
52	.09244	− 161	− 562	112	.04508	− 39	−2470
53	.09085	− 156	− 583	113	.04469	− 38	−2520
54	.08931	− 151	− 604	114	.04432	− 37	−2560
55	.08783	− 146	− 626	115	.04395	− 37	−2610
56	.08640	− 141	− 648	116	.04359	− 36	−2650
57	.08501	− 137	− 670	117	.04223	− 36	−2690
58	.08367	− 132	− 693	118	.04288	− 35	−2730
59	.08236	− 128	− 716	119	.04253	− 34	−2780
60	.08110	− 124	− 739	120	.04219	− 34	−2840

[1] See formulas (7) to (11a), pp. 9 and 10, and (21) and following, pp. 18–20.

TABLE 8.—*Special table for scale of Coast and Geodetic Survey tube.*

[Temperature of air, 60° F., of water, 50° F.=10° C.; humidity of air, 100%; surface density of water, 1.025; gravity as in lat. 45°.[1] $\frac{v_2}{v_1}$ is volume of air when volume of tube is taken as unity. $\dfrac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$ is given to not more than three significant figures and is always greater than unity.

The last three columns give the length of rod of the diameter stated that must be inserted in order to fill the air space, thus bringing the water to the top of the tube. The numbers are computed for a tube 24 inches long and ½ inch in diameter.]

Depth, h (fathoms).	Volume of air space $\frac{v_2}{v_1}$	$\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)$ units of fifth decimal place.	$\dfrac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$	Length of rod to bring water to top (inches). ¼ inch diameter.	$\frac{5}{16}$ inch diameter.	⅜ inch diameter.
1	0.97561					
2	.82455					
3	.71399					
4	.62955					
5	.56287					24.016
6	.50911					21.722
7	.46465					19.825
8	.42733					18.232
9	.39554				24.302	16.876
10	.36815	−2549	− 25		22.619	15.708
11	.34431	−2229	− 29		21.154	14.691
12	.32330	−1967	− 34		19.868	13.797
13	.30481	−1749	− 40		18.728	13.005
14	.28827	−1565	− 45		17.711	12.300
15	.27343	−1408	− 52		16.799	11.666
16	.26004	−1273	− 58	24.964	15.977	11.095
17	.24790	−1158	− 65	23.798	15.231	10.577
18	.23683	−1057	− 72	22.736	14.551	10.105
19	.22671	− 969	− 80	21.764	13.929	9.673
20	.21741	− 892	− 88	20.872	13.358	9.276
21	.20885	− 823	− 96	20.049	12.832	8.911
22	.20093	− 762	− 105	19.289	12.345	8.573
23	.19359	− 708	− 114	18.584	11.894	8.260
24	.18676	− 659	− 123	17.929	11.475	7.969
25	.18040	− 616	− 133	17.318	11.084	7.697
26	.17445	− 575	− 143	16.747	10.718	7.443
27	.16888	− 539	− 154	16.213	10.376	7.206
28	.16366	− 506	− 165	15.711	10.055	6.983
29	.15874	− 477	− 176	15.239	9.753	6.773
30	.15411	− 450	− 188	14.795	9.469	6.575
21	.14974	− 425	− 200	14.375	9.200	6.389
32	.14561	− 402	− 212	13.979	8.946	6.213
33	.14170	− 381	− 225	13.603	8.706	6.046
34	.13800	− 361	− 239	13.247	8.478	5.888
35	.13448	− 343	− 252	12.910	8.262	5.738
36	.13113	− 326	− 266	12.589	8.057	5.595
37	.12795	− 311	− 280	12.283	7.861	5.459
38	.12491	− 296	− 295	11.992	7.675	5.330
39	.12202	− 383	− 310	11.713	7.497	5.206
40	.11925	− 270	− 326	11.448	7.327	5.088
41	.11661	− 259	− 341	11.194	7.164	4.975
32	.11408	− 248	− 357	10.951	7.010	4.867
43	.11165	− 237	− 374	10.718	6.860	4.764
44	.10933	− 228	− 391	10.405	6.717	4.664
45	.10709	− 219	− 408	10.281	6.580	4.569
46	.10495	− 210	− 426	10.075	6.448	4.478
47	.10289	− 202	− 444	9.878	6.322	4.390
48	.10091	− 194	− 462	9.687	6.200	4.305
49	.09900	− 187	− 481	9.504	6.083	4.224
50	.09716	− 180	− 500	9.328	5.970	4.146
51	.09539	− 174	− 520	9.158	5.861	4.070
52	.09368	− 168	− 539	8.994	5.756	3.997
53	.09203	− 162	− 559	8.835	5.654	3.927
54	.09044	− 157	− 579	8.682	5.556	3.859
55	.08890	− 151	− 601	8.534	5.462	3.793

[1] See formulas (7) to (11b), pp. 9 and 10, and remark on p. 19.

TABLE 8.—*Special table for scale of Coast and Geodetic Survey tube*—Continued.

Depth, h (fathoms).	Volume of air space $\frac{v_2}{v_1}$	$\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)$ units of fifth decimal place.	$\frac{1-\frac{v_2}{v_1}}{\frac{\partial}{\partial h}\left(\frac{v_2}{v_1}\right)}$	Length of rod to bring water to top (inches).		
				¼ inch diameter.	$\frac{5}{16}$ inch diameter.	⅜ inch diameter.
56	0.08741	− 147	− 622	8.391	5.370	3.729
57	.08596	− 142	− 644	8.253	5.282	3.668
58	.08457	− 137	− 666	8.119	5.196	3.609
59	.08322	− 133	− 688	7.989	5.113	3.551
60	.08191	− 129	− 711	7.863	5.032	3.495
61	.08063	− 125	− 734	7.741	4.954	3.440
62	.07940	− 121	− 758	7.623	4.878	3.388
63	.07821	− 118	− 782	7.508	4.805	3.337
64	.07704	− 114	− 806	7.396	4.734	3.287
65	.07592	− 111	− 830	7.288	4.664	3.239
66	.07482	− 108	− 855	7.183	4.597	3.192
67	.07375	− 105	− 881	7.080	4.531	3.147
68	.07272	− 102	− 906	6.981	4.468	3.103
69	.07171	− 99	− 932	6.884	4.406	3.060
70	.07073	− 97	− 959	6.790	4.345	3.018
71	.06977	− 94	− 986	6.698	4.287	2.977
72	.06884	− 92	−1010	6.608	4.230	2.937
73	.06793	− 90	−1040	6.521	4.174	2.898
74	.06705	− 87	−1070	6.436	4.119	2.861
75	.06618	− 85	−1100	6.354	4.066	2.824
76	.06534	− 83	−1130	6.273	4.015	2.788
77	.06452	− 81	−1150	6.194	3.964	2.753
78	.06372	− 79	−1180	6.117	3.915	2.719
79	.06294	− 77	−1210	6.042	3.867	2.685
80	.06218	− 76	−1240	5.969	3.820	2.653
81	.06143	− 74	−1270	5.897	3.774	2.621
82	.06070	− 72	−1300	5.827	3.729	2.590
83	.05999	− 70	−1330	5.759	3.686	2.560
84	.05929	− 69	−1360	5.692	3.643	2.530
85	.05861	− 67	−1400	5.627	3.601	2.501
86	.05794	− 66	−1430	5.563	3.560	2.472
87	.05729	− 64	−1460	5.500	3.520	2.444
88	.05666	− 63	−1490	5.439	3.481	2.417
89	.05603	− 62	−1530	5.379	3.443	2.391
90	.05542	− 60	−1560	5.320	3.405	2.365
91	.05482	− 59	−1590	5.263	3.368	2.339
92	.05424	− 58	−1630	5.207	3.332	2.314
93	.05366	− 57	−1660	5.151	3.297	2.290
94	.05310	− 56	−1700	5.097	3.262	2.266
95	.05255	− 55	−1730	5.044	3.228	2.242
96	.05200	− 54	−1770	4.992	3.195	2.219
97	.05147	− 53	−1800	4.941	3.163	2.196
98	.05095	− 52	−1840	4.892	3.131	2.174
99	.05044	− 51	−1880	4.843	3.099	2.152
100	.04994	− 50	−1910	4.794	3.068	2.131
101	.04945	− 49	−1950	4.747	3.038	2.110
102	.04897	− 48	−1990	4.701	3.009	2.089
103	.04850	− 47	−2030	4.656	2.980	2.069
104	.04803	− 46	−2070	4.611	2.951	2.049
105	.04757	− 45	−2100	4.567	2.923	2.030
106	.04713	− 44	−2140	4.524	2.895	2.011
107	.04669	− 44	−2180	4.482	2.868	1.992
108	.04625	− 43	−2220	4.440	2.842	1.973
109	.04583	− 42	−2260	4.399	2.816	1.955
110	.04541	− 41	−2300	4.359	2.790	1.937
111	.04500	− 41	−2340	4.320	2.765	1.920
112	.04459	− 40	−2380	4.281	2.740	1.903
113	.04420	− 39	−2430	4.243	2.716	1.886
114	.04381	− 39	−2470	4.206	2.692	1.869
115	.04342	− 38	−2510	4.169	2.668	1.853
116	.04304	− 37	−2560	4.132	2.645	1.837
117	.04267	− 37	−2600	4.096	2.622	1.821
118	.04231	− 36	−2640	4.061	2.599	1.805
119	.04195	− 36	−2680	4.027	2.577	1.790
120	.04159	− 35	−2720	3.993	2.555	1.775

TABLE 9.—*Corrections to sounding tube readings for temperature and pressure.*

[Computed for air temperature=60° F.=15½° C.; water temperature=50° F.=10° C. Barometer=30 in.=76.200 cm. See equation (15), p. 12.]

1. CORRECTION FOR TEMPERATURE.

Depth from scale, h (fathoms).	Temperature of air minus temperature of water (Fahrenheit).										
	−40°	−30°	−20°	−10°	0°	+10°	+20°	+30°	+40°	+50°	+60°
	Fathoms.	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*
5	+ 1.0	+0.8	+0.6	+0.4	+0.2	0.0	−0.2	−0.4	−0.6	−0.8	− 1.0
10	+ 1.5	+1.2	+ .9	+ .6	+ .3	.0	− .3	− .6	− .9	−1.2	− 1.5
20	+ 2.5	+2.0	+1.5	+1.0	+ .5	.0	− .5	−1.0	−1.5	−2.0	− 2.5
30	+ 3.4	+2.8	+2.1	+1.4	+ .7	.0	− .7	−1.4	−2.1	−2.8	− 3.4
40	+ 4.4	+3.5	+2.6	+1.8	+ .9	.0	− .9	−1.8	−2.6	−3.5	− 4.4
50	+ 5.4	+4.3	+3.2	+2.2	+1.1	.0	−1.1	−2.2	−3.2	−4.3	− 5.4
60	+ 6.4	+5.1	+3.8	+2.6	+1.3	.0	−1.3	−2.6	−3.8	−5.1	− 6.4
70	+ 7.3	+5.9	+4.4	+2.9	+1.5	.0	−1.5	−2.9	−4.4	−5.9	− 7.3
80	+ 8.3	+6.6	+5.0	+3.3	+1.7	.0	−1.7	−3.3	−5.0	−6.6	− 8.3
90	+ 9.3	+7.4	+5.6	+3.7	+1.9	.0	−1.9	−3.7	−5.6	−7.4	− 9.3
100	+10.3	+8.2	+6.2	+4.1	+2.0	.0	−2.0	−4.1	−6.2	−8.2	−10.3
110	+11.2	+9.0	+6.7	+4.5	+2.2	.0	−2.2	−4.5	−6.7	−9.0	−11.2

2. CORRECTION FOR PRESSURE.

Depth from scale, h (fathoms).	Barometer reading in inches.										
	29.0	29.2	29.4	29.6	29.8	30.0	30.2	30.4	30.6	30.8	31.0
	Fathoms.	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*	*Fathoms.*
10	−0.3	−0.3	−0.2	−0.1	−0.1	0.0	+0.1	+0.1	+0.2	+0.3	+0.3
20	− .7	− .5	− .4	− .3	− .1	.0	+ .1	+ .3	+ .4	+ .5	+ .7
30	−1.0	− .8	− .6	− .4	− .2	.0	+ .2	+ .4	+ .6	+ .8	+1.0
40	−1.3	−1.1	− .8	− .5	− .3	.0	+ .3	+ .5	+ .8	+1.1	+1.3
50	−1.7	−1.3	−1.0	− .7	− .3	.0	+ .3	+ .7	+1.0	+1.3	+1.7
60	−2.0	−1.6	−1.2	− .8	− .4	.0	+ .4	+ .8	+1.2	+1.6	+2.0
70	−2.3	−1.9	−1.4	− .9	− .5	.0	+ .5	+ .9	+1.4	+1.9	+2.3
80	−2.7	−2.1	−1.6	−1.1	− .5	.0	+ .5	+1.1	+1.6	+2.1	+2.7
90	−3.0	−2.4	−1.8	−1.2	− .6	.0	+ .6	+1.2	+1.8	+2.4	+3.0
100	−3.3	−2.7	−2.0	−1.3	− .7	.0	+ .7	+1.3	+2.0	+2.7	+3.3
110	−3.7	−2.9	−2.2	−1.5	− .7	.0	+ .7	+1.5	+2.2	+2.9	+3.7

TABLE 10.—*Effect of absorption of air on scale of sounding tube.*

[Column 1 shows the depth in fathoms. Column 2 shows the relative volume of air at the given depth. Column 2 has been taken from Table 7, and column 3 shows the relative volume of air if the sea water in the tube were saturated with atmospheric gases.[1] Column 4 shows the correction to be applied to the depths read from a scale computed on the supposition of no absorption, when, in point of fact, absorption had gone on to the saturation point.]

1	2	3	4	1	2	3	4
Depth (fathoms).	v_2/v_1 No absorption.	v_2/v_1 Complete saturation.	Correction to depth (fathoms).	Depth (fathoms).	v_2/v_1 No absorption.	v_2/v_1 Complete saturation.	Correction to depth (fathoms).
5	0.5098	0.5052	− 0.07	55	0.0878	0.0714	− 9.2
10	.3447	.3363	− .3	60	.0811	.0644	−10.8
15	.2603	.2496	− .8	65	.0753	.0584	−12.4
20	.2091	.1968	− 1.4	70	.0703	.0533	−14.2
25	.1747	.1613	− 2.1	75	.0659	.0487	−16.1
30	.1500	.1358	− 3.0	80	.0621	.0447	−18.1
35	.1314	.1166	− 4.0	85	.0586	.0411	−20.1
40	.1169	.1016	− 5.1	90	.0555	.0379	−22.2
45	.1053	.0896	− 6.3	95	.0528	.0350	−24.5
50	.0958	.0797	− 7.7	100	.0502	.0324	−26.7

[1] See equation (7), p. 9. t_2=10° C.; t_1=15° C.; $p_1+p'_1$=1 atmosphere. [2] See equation (20), p. 16.