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E. LESTER JONES, DIRECTOR

MODERN METHODS FOR MEASURING THE INTENSITY OF GRAVITY

By

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Geodetic Computer

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PREFACE.

Since the discovery in 1671 of the variation of gravity in different parts of the world, the measurement of its intensity has been a problem of absorbing interest. It has engaged the attention of eminent scientists in many countries, and numerous publications have been written to describe the different methods employed. Much of this literature is now of historical interest only, for the instruments and methods in present use are so far superior to those used before 1885 as to render practically all determinations of gravity previous to that time of little value.

The form of gravity apparatus developed by the U. S. Coast and Geodectic Survey ranks as one of the best in the world and has been adopted by Canada, Mexico, Siam, and several countries in South America. The description of this apparatus and of the methods for making the measurements and computations are given in various publications of this Bureau; but some of these publications are out of print, and many of them are obsolete or misleading, due to changes of methods since they were printed. It seemed desirable, therefore, at this time to publish a report on gravity work giving up-to-date information in regard to the instruments and methods now employed, together with the necessary instructions for making the observations and computations. Detailed descriptions of the instruments are given both as a matter of general interest and as an aid to the new observer in thoroughly understanding the various parts of the apparatus.

Free use has been made of valuable material from the old gravity publications of this Bureau and from a gravity report in manuscript form by Prof. William Burger, Northwestern University, Evanston, Ill., written at the close of a long gravity campaign just before he resigned from the Survey in 1910. The historical facts in the first part of chapter 1 were mostly taken from an article on the figure of the earth, by G. B. Airy, given in volume 5 of the Encyclopædia Metropolitana. Many helpful suggestions and criticisms have been received from Maj. William Bowie, chief of the division of geodesy, from Miss Sarah Beall, goedetic computer, and from other members of this Bureau.

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By Clarence H. Swick, *Geodetic Computer, United States Coast and Geodetic Survey.*

INTRODUCTION.

The gravity work of the United States Coast and Geodetic Survey may be divided into three main epochs depending upon the kind of apparatus used and the number and value of the results obtained.

Active interest of the Survey in gravity determinations began in 1873. From that time until 1891 many kinds of apparatus were tried, and extensive research was carried on in the theory and practice of gravity determinations. The various forms of pendulums used were similar in at least one respect, namely, that of length. All of them were about a meter in virtual length with a vibration period of about one second. It was very difficult to obtain a high degree of accuracy with any of these early forms of apparatus. In spite of elaborate care used in making the observations the results obtained were not of sufficient accuracy to be of value in the modern investigations of gravity and isostasy.

The second epoch in gravity work began in 1891 when a radical change was made in the type of apparatus. The virtual length of the pendulum was changed to one-quarter of a meter, an air-tight case was constructed in which the pendulums could be swung at low pressure, and a chronometer was substituted for the clock with which the period of the pendulum had been obtained in previous work. The general type of the apparatus is similar to the gravity apparatus designed by Von Sterneck, of Austria, 5 or 10 years previously.[1] The results secured with this new apparatus were much more accurate than could be obtained with the old meter pendulums, and the cost of the work was considerably less on account of greater ease in transporting and setting up the apparatus and in preparing the stations.

Many minor changes have been made in the apparatus since 1891, but no radical change comparable with the one just described. This is a tribute to the care and skill used in designing and constructing what is known as the Mendenhall invariable half-second pendulum apparatus. At the beginning of 1909 there had been 47 stations in the United States determined with this new type of pendulum.

[1] See p. 83, Mittheilungen des K. K. Militär-Geographischen Institutes, vol. 7, Vienna, 1887.

The period from 1909 to 1920 constitutes the third epoch in gravity work in this country. It is marked by the large number of stations observed and by at least one important improvement in the apparatus in the form of the interferometer, an instrument used for measuring the flexure of the pendulum support. (See p. 26.) Of the 276 gravity stations in the United States 230 were established during this period. Much interest was added to gravity work in the early part of this period by the application, for the first time, of the theory of isostasy [2] to gravity investigations.

It is not unlikely that the year 1920 marks the beginning of a fourth epoch of gravity work in this country. New pendulums of a nickel-steel alloy, called invar, have just been constructed. The temperature coefficient of this metal is only about one-fifteenth as much as that of the bronze previously used for this purpose. Apparently the only difficulty with the invar pendulums is that they are magnetic. Experimental work is being done by this Bureau to ascertain the effect of the earth's magnetism on the period of the invar pendulum and, if possible, to discover some way of eliminating it from the result. Another important change at this time is the use of wireless for receiving the Naval Observatory time signals by which the chronometer rates are obtained. It should soon be possible to measure the intensity of gravity, without undue cost, in almost any locality even if it happens to be far removed from a telegraph line and where there are no cellars or other constant-temperature rooms.

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[2] See U. S. Coast and Geodetic Survey Special Publications Nos. 10, 12, and 40.

Chapter 1.—HISTORICAL OUTLINE.

THE BEGINNING OF GRAVITY DETERMINATIONS.

Gravity may be defined as the force which tends to pull bodies toward the earth or the force that gives weight to bodies. It is the resultant of two opposing forces, gravitation or the attractive force of the earth and the centrifugal or repelling force due to the earth's rotation. The intensity of gravity is usually expressed either in dynes of force or centimeters of acceleration.

Newton's Principia, published in 1687,[3] contained his announcement of the theory of universal gravitation and the corollary of the oblate spheroidal form of the earth. Previous to this, however, the fact that the intensity of gravity is different at different places on the earth was demonstrated by Richer in 1671 while on an astronomic expedition to the island of Cayenne, South America. The clock which he took with him had been regulated to keep almost perfect time at Paris, but when it was set up in South America it lost two minutes a day due to the decrease of gravity between latitude 49° and latitude 5°.

Huygens, the Dutch astronomer and mathematician, was the first to use the pendulum for the measurement of gravity, as well as the first to apply pendulums to clocks. No satisfactory method for measuring the intensity of gravity has been devised since the time of Huygens which does not depend upon the principle of the pendulum. There have been, however, a great many different types of pendulums and forms of apparatus devised and used since then.

THE SIMPLE PENDULUM.

It was thought at first that for gravity determinations some form of pendulum must be used which at least approximated a simple pendulum, namely, a heavy particle suspended from a fixed point by a fine thread inextensible and without weight. A simple pendulum is possible, of course, in theory only, and the actual pendulums used had to have corrections applied to reduce them to the equivalent simple pendulum.

The well-known theory of the simple pendulum is that when it is allowed to oscillate with a small arc in a vacuum, the time of oscillation is expressed by the formula,

$$t \text{ (in secs.)} = \pi\sqrt{\frac{l}{g}}\left(1+\frac{a^2}{16}\right),$$

[3] See Life of Sir Isaac Newton, by David Brewster, published by John Murray, London, 1831.

where l is the length of the pendulum, g is the accelleration due to gravity, and a is the maximum inclination of the thread to the vertical. If a is small, say less than 1°, the term in a^2 is almost insensible. When t and l have been determined, g can be computed.

The usual manner of expressing the result in all the early experiments was to state what would be the length of a simple pendulum which would oscillate in exactly one second at the place of observation or, as usually expressed, the "length of the seconds pendulum." The values of l and t in the formula above were then known and, the correction for amplitude $\left(\frac{a^2}{16}\right)$ having already been applied to l, the value of g was readily computed when desired.

COMPARISON OF ABSOLUTE AND RELATIVE METHODS FOR GRAVITY DETERMINATIONS.

There are two general methods for determining the intensity of gravity at a station, one called the absolute method and the other the relative method. In the absolute method some form of pendulum approximating a simple pendulum or else a compound reversible pendulum (see p. 10) is used. With either type of pendulum, the length of an equivalent simple pendulum which will vibrate in a certain definite time, say one second, is determined with the greatest possible accuracy, and from this the intensity of gravity is computed.

In the relative method a pendulum of invariable length is swung first at some station where the value of gravity is already known and then at the new station. The intensity of gravity at the two stations is inversely proportional to the square of the vibration periods; that is,

$$\frac{g_n}{g_b} = \frac{t_b^2}{t_n^2}$$

or,

$$g_n = \frac{t_b^2 \, g_b}{t_n^2}$$

where, as before, g is the intensity of gravity, and t is the period of oscillation of the pendulum. The subscripts b and n refer to the base station and the new station, respectively. Any form of pendulum may be used for relative determinations, the only requirement being that the virtual length of the pendulum remain absolutely constant.

The first measurements of gravity were necessarily made by the absolute method as there were then no base stations. It was not long, however, before the relative method came to be used to a large extent, especially by the English, as much greater accuracy could be attained. (See p. 11.) The Germans, however, advocated absolute determinations, and they and the English formed rival schools in this respect for a great many years.

EARLY FORMS OF APPARATUS.

The type of pendulum commonly used for making gravity determinations during the first part of the eighteenth century consisted of a small weight suspended by a thin thread or a fiber of the aloe plant. The weight was usually in the shape of two frustums of cones with their bases in contact. The pendulum was made of such a length that it vibrated a trifle slower or a trifle faster than the pendulum of a clock, and the observations were made by noting coincidences, that is, by noting the times when the two pendulums were vibrating exactly together. It is easily seen that in the interval between two consecutive coincidences one pendulum had lost one beat on the other; and since the number of vibrations of the clock pendulum could be read from the face of the clock, the number of vibrations of the other pendulum was obtained by simply adding or subtracting 1. An adaptation of this coincidence method is still used in making gravity observations. (See p. 56.)

The first French pendulum was designed by Borda. It consisted of a platinum ball which was suspended by means of a small brass cap accurately ground to fit the ball and covering about one-fifth of its surface. When a thin coating of oil was put on the inner surface of the cap and the cap placed on the ball, the latter would be held in place by the atmospheric pressure. The ball could thus be suspended in various positions with regard to the direction of gravity and irregularities of its density eliminated from the results. The stem of the pendulum consisted of a fine wire with the cap just described attached at the lower end and at the upper end a special device for holding the knife-edge. The latter could be adjusted by means of a small weight above the knife-edge so as to have the same period of vibration as the whole pendulum. It could thus be neglected in the calculation of the equivalent simple pendulum.

During the latter part of the eighteenth century Bessel introduced a new method for gravity measurements. His pendulum consisted of a heavy ball suspended by a fine wire and was vibrated first with one length of the wire and then with a different length. The difference between the two lengths used could be easily and accurately measured, and the difference in the lengths of the equivalent simple pendulums isochronous to the observed pendulums, and finally the intensity of gravity, could be computed with great accuracy. The chief value of this method was in the fact that any small inaccuracies, as for example, in computing the correction for the diameter of the ball, would affect both lengths of the pendulum about equally and thus be practically eliminated when the differences were taken and the equivalent simple pendulums were computed.

An important improvement in making the coincidence observations was also introduced by Bessel. He projected the image of the clock pendulum onto the gravity pendulum by means of a lens and was thus able to place the two pendulums at a considerable distance apart and prevent any interdisturbance. It is well known that when two pendulums are oscillated near each other each one will influence the period of vibration of the other unless these periods are exactly equal and the two pendulums are vibrating together.

In a few of the early experiments the pendulum of a clock was itself used as a gravity pendulum. This is the simplest of all methods for determining gravity as the mechanism of the clock registers the number of vibrations. The accuracy that could be obtained was not as great as with other forms of apparatus, still many determinations of considerable value were made by this method. When the escapement of the clock was so constructed that the pendulum received its impulse at the middle of the oscillation, the effect upon the period of vibration was reduced to a minimum.

VARIOUS TYPES OF COMPOUND REVERSIBLE PENDULUMS.

Kater, the English scientist, was the first to apply to gravity measurements the theorem of Huygens that the centers of suspension and oscillation of a compound pendulum are reciprocal. According to this theorem if a body be suspended at its center of oscillation its former point of suspension becomes the center of oscillation and the vibrations in the two positions are executed in equal times. In every pendulum not simple the parts near the center of suspension tend to move faster than those farther away and force the latter to move more rapidly than they otherwise would. Between these there is a particle that moves of its own accord, at the rate forced upon the others. This particle fulfills all the conditions of a simple pendulum that has the period of the compound pendulum. Its position is called the center of oscillation or percussion.

Kater's pendulum consisted of a thin bar of brass with a bob at one end and with two knife-edges so placed that one was at the center of oscillation when the other was used as the point of suspension. The period of the pendulum on one knife-edge was made equal to the period on the other knife-edge by means of two sliding adjustable weights. When these weights had been adjusted the period of the pendulum was the same as for an equivalent simple pendulum having a length equal to the distance between the knife-edges. By measuring this distance and the period of vibration the absolute value of the intensity of gravity could be computed. In actual use it was never possible to adjust the pendulum with such refinement that the periods would be exactly the same on either knife-edge, and small corrections had to be applied to take account of the difference. The most difficult part of the observations was to measure the distance between the knife-edges with the necessary accuracy. Another measurement which had to be made in order that certain corrections might be computed was to ascertain the position of the center of gravity of the pendulum in relation to the knife-edges, but it was not necessary to use such great precision for this measurement.

Many different types of the compound or reversible pendulum have been constructed since this first one by Kater. All of them depend upon the principle stated in Huygens theorem. They constitute the most accurate known means for measuring the absolute intensity of gravity. Nearly all of the various reversible pendulums were made heavier at one end, but in the later types the form of the pendulum was made symmetrical at the two ends in order that the resistance of the air would be the same when swung on either knife-edge. If a weight to act as a bob was attached to one end of the pendulum, as in the case of the Repsold pendulum (see below), a shell of exactly the same dimensions and shape was attached in an exactly symmetrical position at the other end of the pendulum. (See *R*, fig. 1.) Many troublesome measurements and computations were avoided in this manner, and the accuracy of the results was correspondingly greater.

It must be remembered that the relative gravity of two stations can be measured with much greater precision than the absolute gravity at a single station. For this reason nearly all determinations of the intensity of gravity have been made by the relative method. The reversible pendulums, however, have performed a very useful service in the absolute measurements at the various base stations.

FIRST DETERMINATIONS OF GRAVITY IN THE UNITED STATES.

Active interest of the Coast and Geodetic Survey in gravity determinations began in 1873. From then until 1890 the work was done, to a large extent, either by or under the direction of C. S. Peirce who made many contributions to the theory and practice of gravity research.

KATER, REPSOLD, AND PEIRCE PENDULUMS.

The pendulums used during this period were of three general types known as the Kater, Repsold, and Peirce pendulums. They are shown in figures 1 and 2. The Repsold and one of the Peirce pendulums (*R* and *P*, fig. 1) were of the reversible type with two knife-edges and could be used for absolute determinations. The other Peirce pendulum (*I*, fig. 1) and all of the Kater pendulums (fig. 2) were of the invariable type with one knife-edge and could be used only for relative determinations. The pendulums were made of two different lengths. Some were approximately a yard in virtual length and the rest approximately a meter. The latter, of course, had a period of vibration of about one second.

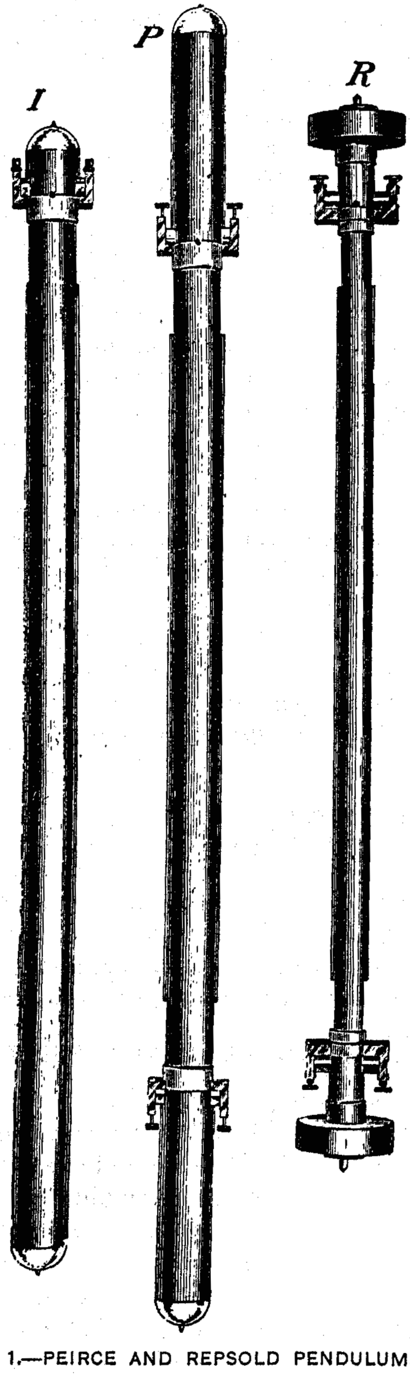

Fig. 1.—PEIRCE AND REPSOLD PENDULUMS.

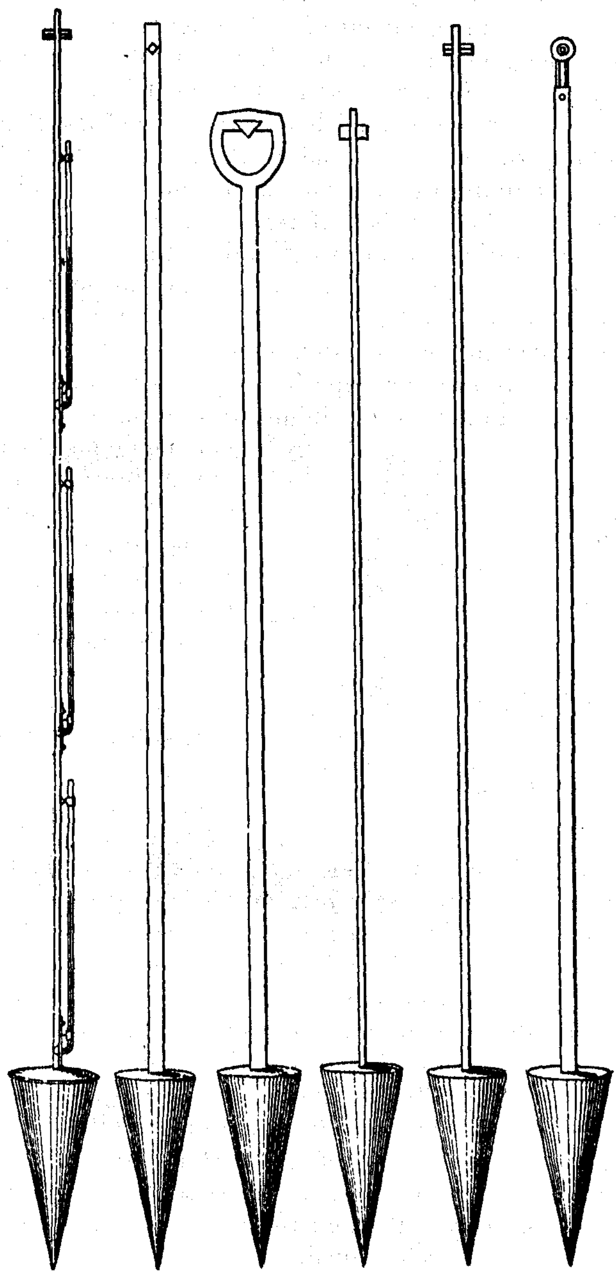

Fig. 2.—KATER PENDULUMS.

At many of the stations, part of the observations were made with the yard pendulum and part with the meter. Two determinations of gravity, entirely independent of each other, were thus obtained.

The reversible pendulums, as usual, were made heavier at one end than the other, but the form was kept symmetrical at the two ends. (See p. 11.) This was accomplished in the case of the Peirce pendulums by adding the extra material on the inside of the tube and for the Repsold pendulum by attaching a disk of solid metal at one end of the tube and a hollow shell of the same outside dimensions in a corresponding position at the other end of the tube. The ratio of the distances from the center of mass to the two knife-edges was 7 to 3 for the Repsold and 3 to 1 for the Peirce reversible.

The tube which formed the stem of both the Peirce and Repsold pendulums was cut away on opposite sides just below the knife-edges and thus gave room for the pendulum to be slipped on over the supporting plane which was built out in bracket shape. (See fig. 3.) The pendulums were made entirely of brass except for the knife-edges which were of hardened steel. The latter could be detached from the pendulum when the apparatus was shipped.

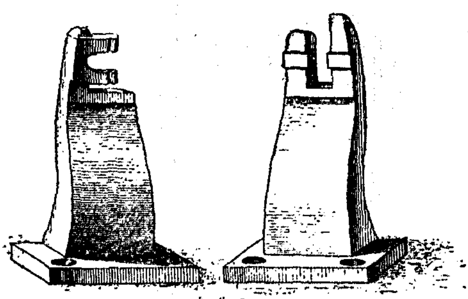

Fig. 4.—BRACKET SUPPORT FOR KATER PENDULUMS.

It will be noticed in figure 2 that the characteristic feature of the various Kater pendulums is the cone-shaped bob. The pendulums are distinguished from each other chiefly by the shape and mounting of the knife-edges.

Four distinct forms are shown in figure 2. Beginning at the left, the first one is a silver pendulum. It has three thermometers attached to the stem for obtaining the temperature of the pendulum when in use. The second and fifth are two views of another form similar to the silver pendulum but made of brass and with no thermometers attached to the stem. The third and fourth are two views of one of the yard pendulums having a specially designed head to permit the supporting plane to be made in one piece. The last one to the right is another form of the meter pendulum which has a small cylinder in place of the usual knife-edge. This pendulum did not give satisfactory results, and the use of a cylinder in place of a knife-edge had to be discontinued.

The supports upon which the pendulums were swung were of various forms. The Kater pendulums were suspended from a brass bracket shown in figure 4. This was secured to a wall or any solid support by means of bolts. The Repsold support, shown in figure 5,

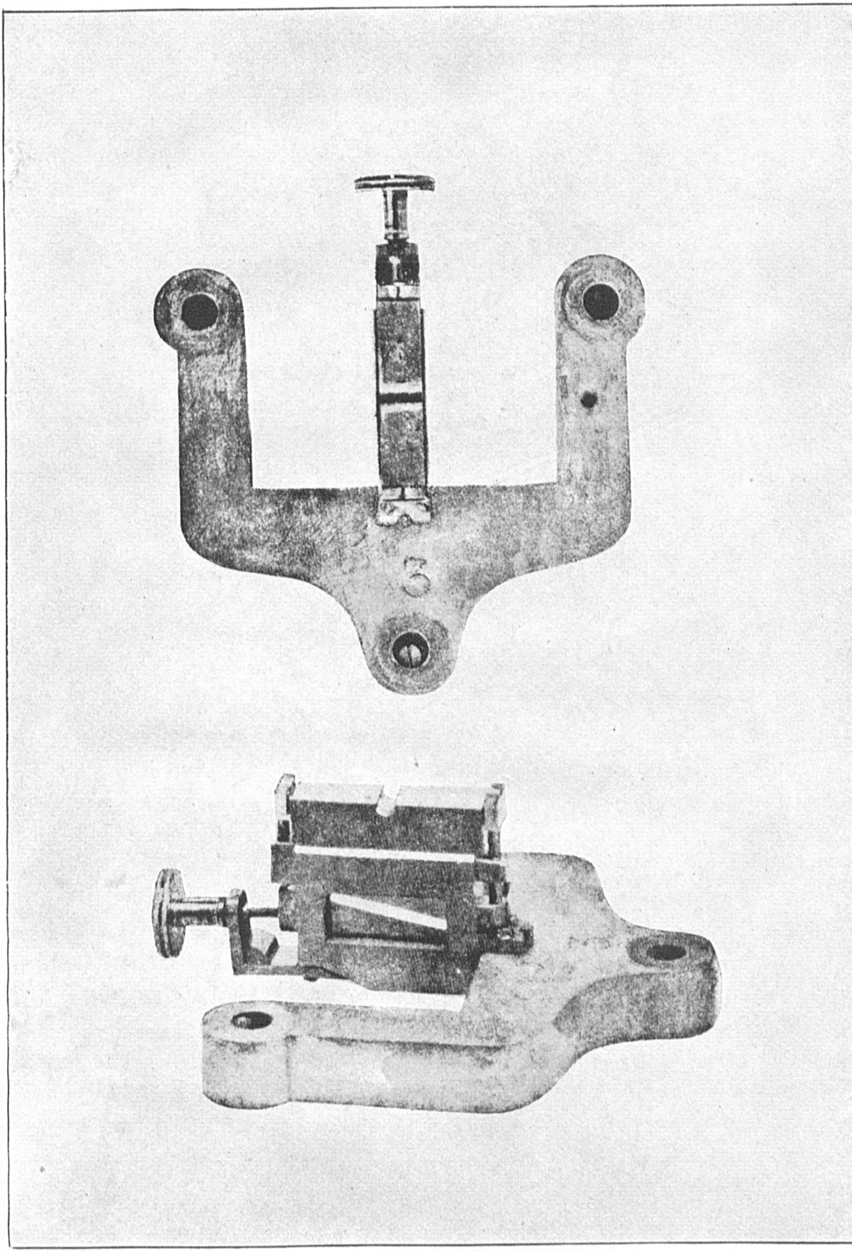

Fig. 3.—PLANE USED AS A SUPPORT FOR THE PEIRCE PENDULUMS.

consisted of a tripod which could be taken apart for shipment. The Peirce pendulums were supported on a head shown in figure 3, usually secured to some form of stand built of timber.

VERTICAL COMPARATOR.

For measuring the distance between the knife-edges of the reversible pendulums, a special apparatus devised by Repsold, called a vertical comparator, was used. It consisted of two micrometer microscopes attached to opposite ends of a tube about a meter in length. One of the microscopes could be adjusted horizontally until it was in the same vertical plane as the other and also vertically to make the distance between the two microscopes practically the same as the distance between the knife-edges of the pendulum being measured. The whole instrument could be turned about a vertical axis.

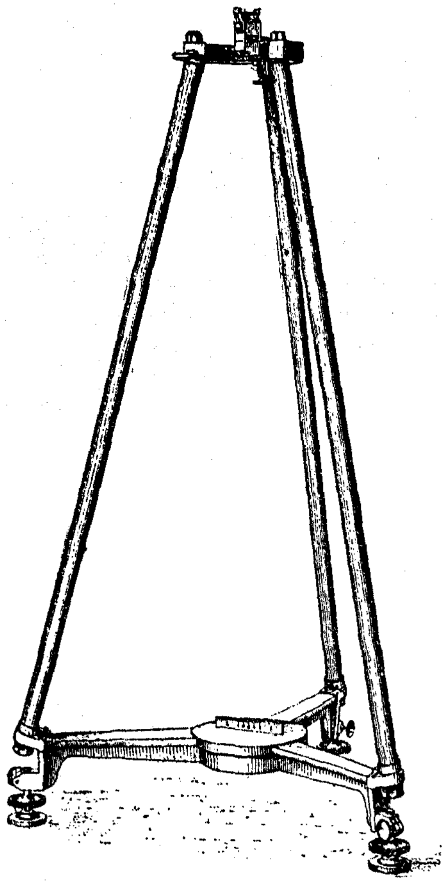

FIG. 5.—TRIPOD SUPPORT FOR REPSOLD PENDULUMS.

After the microscopes had been adjusted to the correct position as described above, they were pointed first on the standard bar of known length and then on the knife-edges of the pendulum. The difference between the length of the standard bar and the distance between the knife-edges was thus determined by means of the micrometer eyepieces of the microscopes.

Figure 6 shows a Peirce reversible pendulum, a standard meter bar, and the vertical comparator in position on the special stand ready for the measurements to be made. Any lengthening of the pendulum due to its own weight was taken account of in the measurements made in this manner, and except for the change in length due to temperature the length thus found was the same as when the pendulum was used for making the gravity determination. The length was measured in both positions of the pendulum, namely, with the heavy end down and the heavy end up.

Between 1873 and 1890 many determinations of the intensity of gravity were made by observers of the U. S. Coast and Geodetic Survey both in this and other countries. All of these determinations were made by means of the long, heavy pendulums just described. These pendulums were usually swung in the open air, protected merely against air currents, and were seriously affected by atmospheric conditions and variations of temperature.

Despite the great care used in making the observations, the results obtained were not very accurate as compared with present-day standards although they were probably as good as the results obtained by any other country up to that time. The cost of the work was excessive, due to heavy transportation and the care needed in preparing the stations.

Viewed from only one angle it might almost seem as if the gravity work during this period was largely wasted effort. This is far from the truth, however. A great deal was learned about the theory of the pendulum and methods of observation without which the later developments of the apparatus and the present improved methods for making gravity determinations would not have been possible.

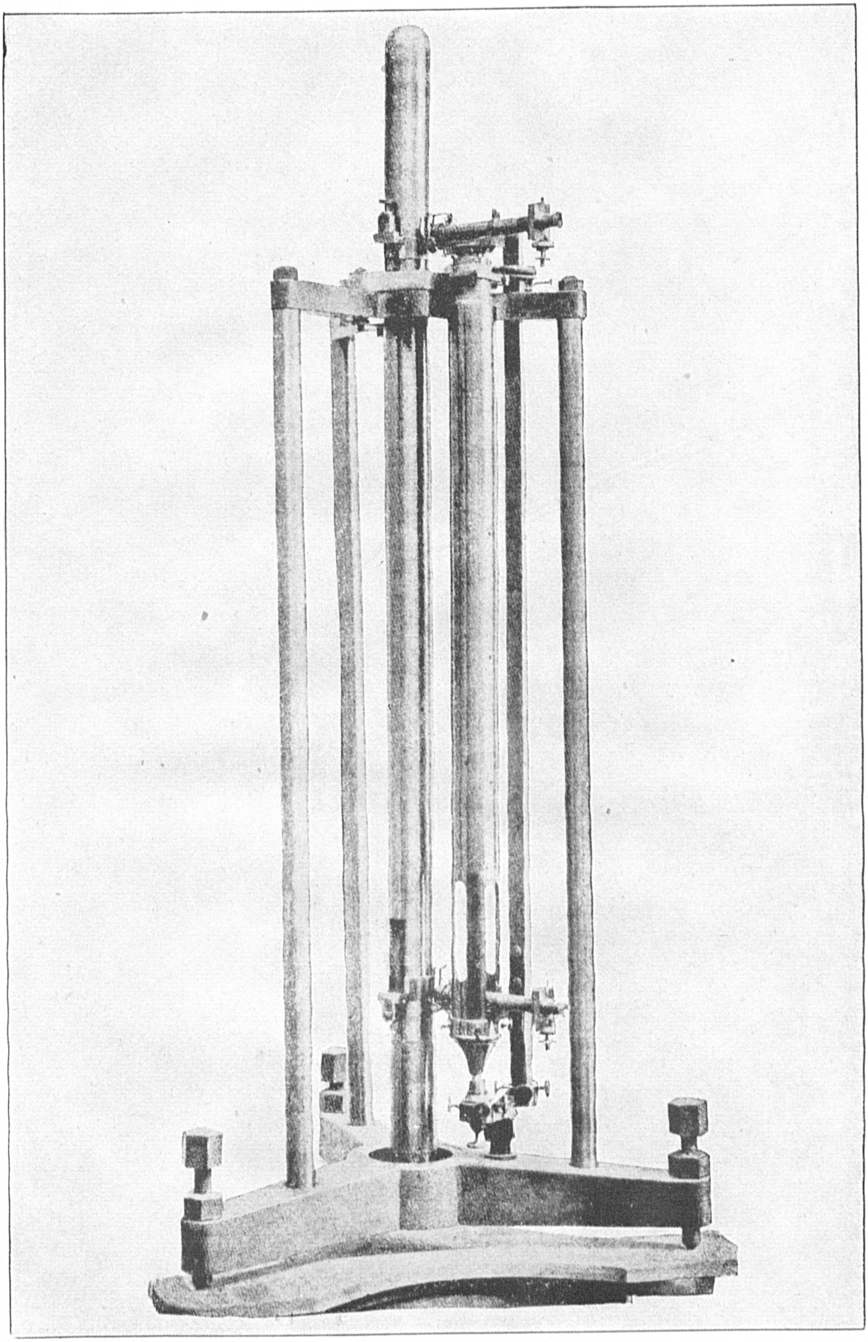

Fig. 6.—VERTICAL COMPARATOR IN USE.

Chapter 2.—DESCRIPTION OF THE MODERN GRAVITY APPARATUS.

In 1890 the Mendenhall invariable half-second (quarter-meter) pendulum apparatus was designed and constructed by members of the U. S. Coast and Geodetic Survey and has been used for all determinations of gravity in this country since that time. The change in the length of the pendulum from the yard and meter previously used to the quarter-meter length, and the various other changes in the form of the apparatus made at the same time, undoubtedly constitute the most important step in gravity work ever taken in this country. The idea of using a short pendulum for gravity determinations was first suggested by Von Sterneck, of Austria, who designed and supervised the construction of the first apparatus of this type ever made. Credit for the design and construction of the Mendenhall apparatus is divided among several members of the Survey, but special mention should be made of T. C. Mendenhall, then Superintendent, E. G. Fischer, chief mechanician, C. A. Schott, Edwin Smith, E. D. Preston, and G. R. Putnam.

Due to the superiority of the apparatus, the results obtained since 1890 are far better than those obtained previous to that time and in fact are the only results of sufficient accuracy to be used in the modern investigations of the theory of gravity and isostasy.

Many minor changes have been made in the apparatus since it was first constructed, as, for example, the substitution of an electric light in place of the oil lamp in the flash apparatus and the use of a manometer inside the receiver instead of a manometer attached to the outside which received the inside pressure through a packing box and indicated the difference between the inside and outside pressures.[4] In the main, though, the apparatus in present use is practically the same as when first designed. The following descriptions of the various parts will deal principally with the present form of the apparatus and only incidentally with the changes made since 1890.

The modern gravity apparatus consists of the following essential parts: Four pendulums, an air-tight receiver in which the three working pendulums are swung, a telescope and flash apparatus for making the observations, three chronometers, one or two chronographs, an interferometer, thermometers, manometers, etc.

[4] With this arrangement of the apparatus the pressure readings had to be corrected for atmospheric pressure. This required the use of a mercurial barometer, a very difficult instrument to transport.

QUARTER-METER PENDULUMS.

The pendulum itself is quite simple in construction. (See *B*, fig. 8.) It consists of a thin stem with a heavy bob at one end and an inverted stirrup at the other end to hold the agate plane on which the pendulum swings.

The bob is lenticular in shape, about 9 cm. in diameter, and 4 cm. thick at the center, its faces being spherical surfaces. The stem is rectangular in section, 4 by 14 mm., with rounded corners, and the bob and head are connected rigidly to it by means of rivets. The stem and bob are designed to offer little resistance to the air when in motion. The length of the pendulum from the plane to the center of the bob is about 248 mm. and the total weight about 1.2 kg.

The head of the pendulum is made in the form of a stirrup. This permits the agates, namely, the knife-edge and plane, by means of which the pendulum is supported, each to be made as one piece. For the first pendulums the head was made in the form of a T (see fig. 7), and the knife-edge instead of the plane was fastened to it. This necessitated that the plane be made in two parts with enough room between for the stem of the pendulum to pass freely when swinging. There were two difficulties with this form of construction. One was that a slight wearing of the knife-edge changed the virtual length of the pendulum and the other that it was almost impossible to make the two parts of the agate plane lie in exactly the same plane and remain so. These difficulties were overcome by changing the head of the pendulum to the stirrup form and by attaching the plane instead of the knife-edge to the head. The difference in the two forms of pendulum can be seen by comparison of figures 7 and 8.

For use in making the observations a small rectangular mirror of speculum metal is set in each side of the pendulum head. These mirrors are made of metal, so that they will reflect the light from the front surface instead of from the back surface as in an ordinary glass mirror, and thus any difficulty due to refraction will be avoided. The two mirrors on each pendulum are carefully adjusted until a ray of light will be reflected in the same direction when the pendulum is hanging freely with either side toward the front.

The pendulum is placed in the receiver by means of a jointed handle (see *H*, fig. 8), which has leather-lined hooks fitting under lugs on either side of the head.

A small projection at the bottom of the bob with a white vertical line ruled on it is used as the reference mark in making the observations for amplitude of vibration.

DUMMY PENDULUM.

With each set of apparatus there are three working pendulums as just described, and one other called the dummy pendulum (*A*, fig. 8).

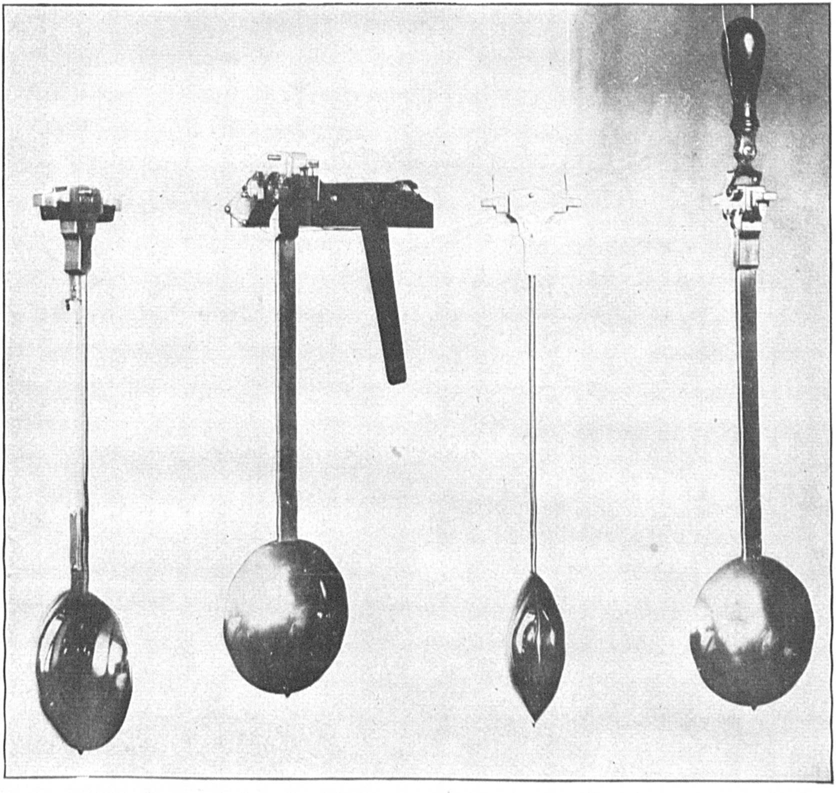

Fig. 7.—VARIOUS VIEWS OF THE FIRST QUARTER-METER PENDULUMS OF THE U. S. COAST AND GEODETIC SURVEY.

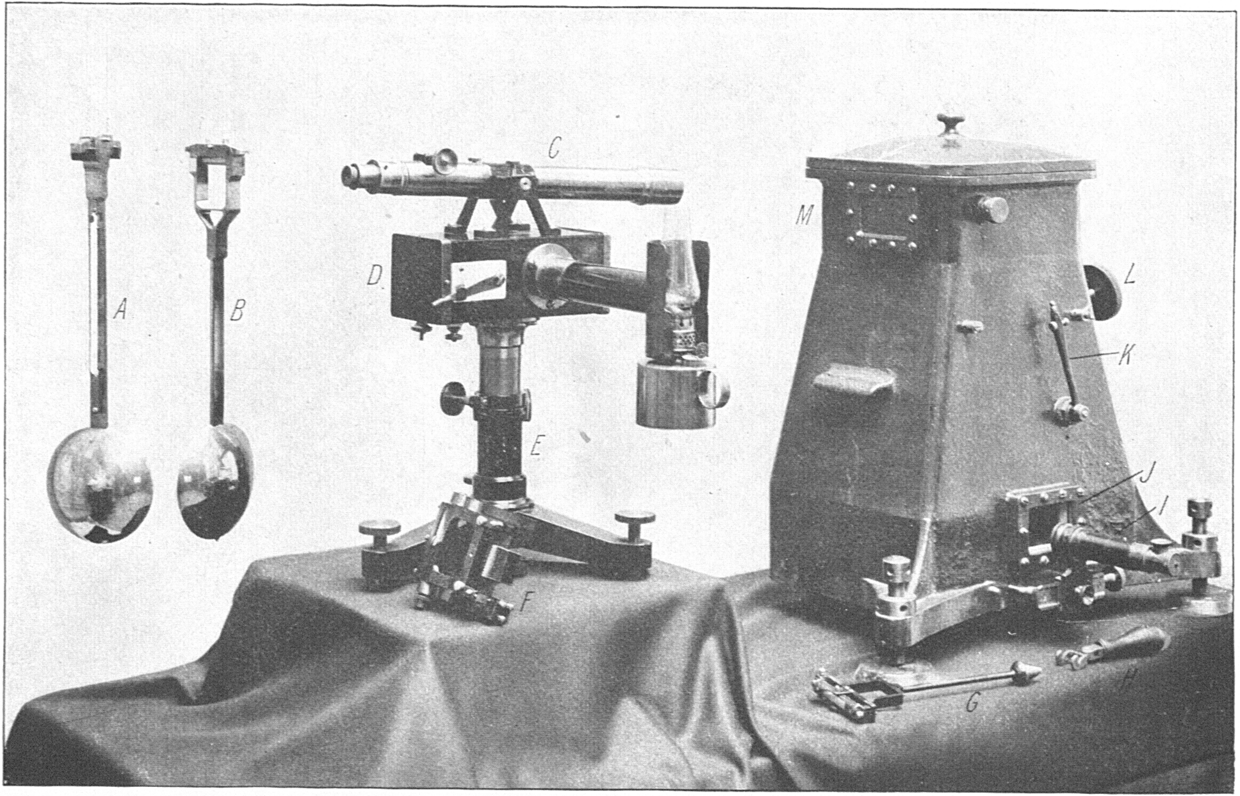

Fig. 8.—MODERN GRAVITY APPARATUS.

The latter is similar to the others in material and dimensions except that it has no mirrors, is supported upon two hard-rubber knife-edges parallel to the plane of the bob, and has a thermometer fastened to one side of its stem. The bulb of the thermometer, bent back for the purpose, is packed with filings in a small metal box, and this box, after being covered with tin foil, is pressed into a slot cut in the stem of the dummy immediately above the bob, a very close metallic contact between the bob and the surrounding metal being thus attained. The dummy is suspended near the working pendulum and is prevented from oscillating by the nature of its support and by a fork projecting from one side of the receiver and holding it firmly near the bob.

Due to the similarity of construction of the two pendulums, the temperature as read from the thermometer on the dummy will be a close approximation to the temperature of the stem of the working pendulum, which is the value sought. Some form of insulation should be used between the fork mentioned above and the dummy in order that there may be no direct metallic contact between the receiver and the dummy, thus making the conditions as regards temperature as nearly as possible the same as for the working pendulum.

INVAR PENDULUMS.

All of the quarter-meter pendulums constructed previous to 1920 were made of a bronze consisting of one part aluminum to nine parts copper. During the early part of 1920 there were finished two new sets of pendulums made of invar metal, an alloy of approximately one part nickel to two parts iron, having a temperature coefficient only one-fifteenth as great as the bronze previously used. In other respects the invar pendulums are similar to the bronze pendulums except that the bob is about 4 mm. thicker at the center and the weight about 0.3 kg. greater, making the total weight about 1.5 kg. There is only one mirror attached to the head of the invar pendulum, and this mirror is made of platinum iridium. (See p. 6.)

KNIFE-EDGE.

Like the plane in the head of the pendulum, the knife-edge on which the pendulum is supported is made of agate. It is rigidly secured to a brass plate, *m*, figure 9, which in turn can be attached by means of a screw to a shelf, *n*, inside the receiver. A lever, *ol*, called the lifting lever, is pivoted to the plate and is held by a spring, *p*, against the end of the screw, *q*. When the pendulum is placed in the receiver it is first suspended on two lugs carried on the end of the lever, *ol*, which lugs fit into corresponding sockets in projections on either side of the pendulum head. The lever is manipulated from the outside of the receiver by turning the screw, *q*, and the pendulum can thus be lowered gently to prevent injury to the knife-edge or plane. The lugs on the lever insure that the pendulum will come down on the knife-edge in exactly the correct position and that this position will not be altered after the pendulum has been raised and lowered.

Stops are provided to limit the motion of the lifting lever. It is necessary, of course, that the lever should be lowered far enough each time that there will be no interference with the pendulum when the latter has been started to vibrate.

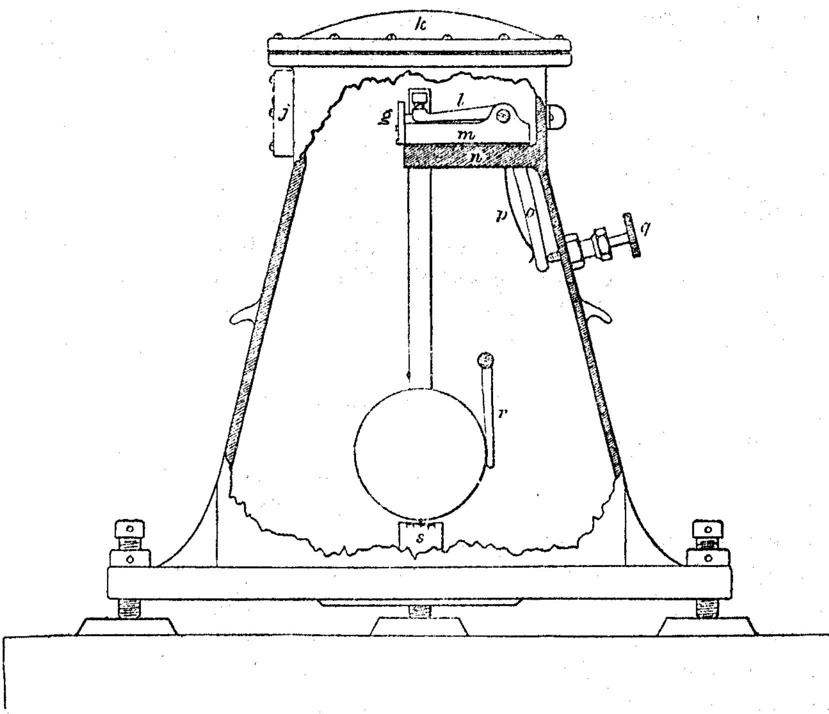

Fig. 9.—DETAILED SKETCH OF RECEIVER.

A small brass cap, which can be attached by a screw, is provided to fit above the knife-edge and prevent injury when the knife-edge is not in use.

The knife-edge itself and the plane in the head of the pendulum are two very important parts of the pendulum apparatus. Great skill must be exercised to get them as nearly perfect as possible, that is, to make the plane an optical plane and the knife-edge a geometric line. The final grinding of these pieces of agate is usually done by E. G. Fischer, chief of the instrument section, who is responsible for the unique design of the knife-edge, a sketch of which is shown in figure 10. The two planes forming the knife-edge meet at an angle of 130°, and this not only insures against chipping or breaking under ordinary usage but also makes considerably smaller than it would otherwise be any distortion or compression caused by the weight of the pendulum. This large angle is just as effective for the knife-edge as a much smaller angle would be, since the maximum arc through which the pendulum vibrates is only about 1°.

It is, of course, impossible to make a knife-edge which is a true geometric line. The most nearly perfect knife-edge that can be made will be found when tested to have what has been called the missing

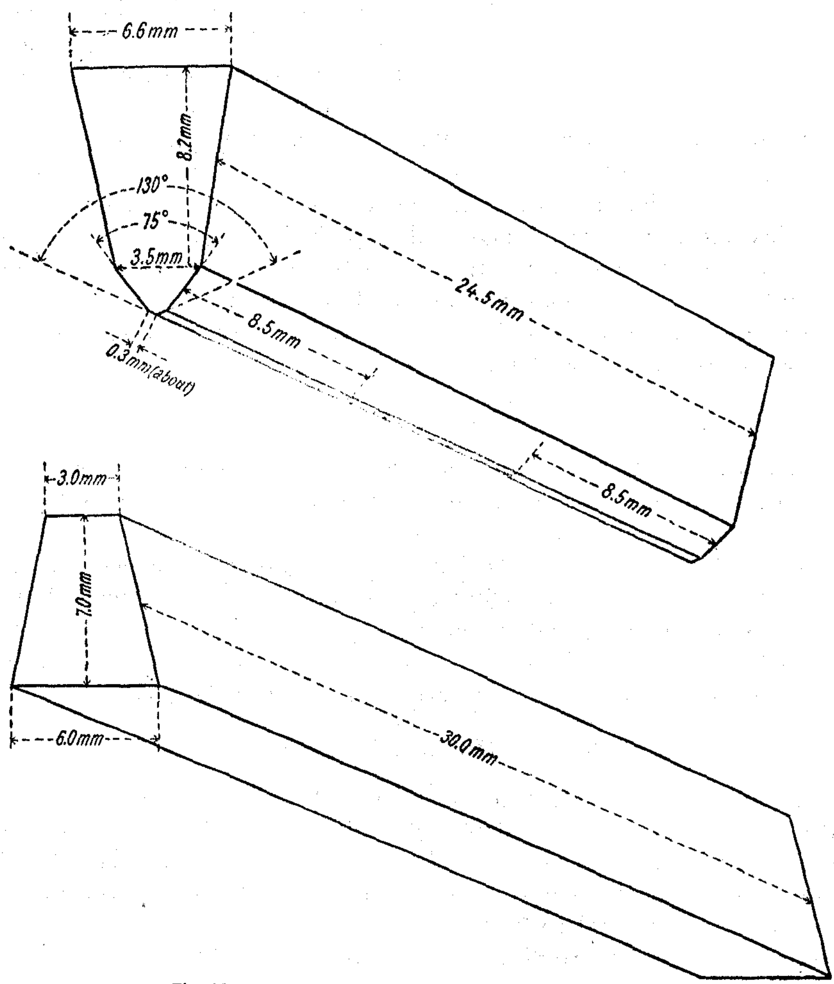

Fig. 10.—DIAGRAM OF KNIFE-EDGE AND PLANE.

triangle, that is, the two planes forming the edge will not quite meet. The base of this missing triangle on the knife-edges for the pendulum apparatus has been found by optical means to be less than 1 micron, and these knife-edges are undoubtedly the nearest perfect of any of their kind in the world.

The dimensions of the knife-edge are given in figure 10. The total length is about 24.5 mm., but only about two-thirds of this length is used in supporting the pendulum as the middle third of the edge is removed in order to place the load upon it farther from its center, thus defining the line of oscillation more definitely and increasing the constancy of this line in a rotary sense.

On the front edge of the plate *m*, figure 9, is attached a small mirror *g*, called the stationary mirror, similar to the pendulum mirrors described on page 18, but held in position by three adjusting screws opposed by small coiled springs. This mirror is in such a position that it forms practically one plane with the pendulum mirror when the pendulum is hanging freely on the knife-edge. After the stationary mirror has been properly adjusted, the slit of light emitted from the flash apparatus (see p. 24) will be reflected from each of the two mirrors, the stationary mirror and the one on the pendulum, back into the telescope and the two images will appear there as slightly overlapping, the end of one covering the end of the other. A more detailed account of this adjustment will be found on page 41.

The apparatus consisting of the knife-edge, lifting lever, and stationary mirror are shown at *F*, figure 8.

LEVEL.

The horizontality of the knife-edge is tested by means of a sensitive level mounted over an agate plane, similar to those of the pendulums, and having a small stem and bob to bring the center of gravity below the knife-edge. (See *G*, fig. 8.) The level is placed on the knife-edge by means of the lifting lever in the same way as a pendulum. The level tube itself is held at one end by adjusting screws by means of which it can be made parallel with the knife-edge. The ordinary reversing method is used in adjusting and testing the level.

RECEIVER.

The body of the receiver (see figs. 8 and 16) is a heavy brass casting with walls 7 mm. thick, and of inside dimensions 17 cm. square at the top, 21 by 28 cm. at the bottom, and 38 cm. high. The flared edge at the top of the receiver and the bottom of the edge of the cover, *k*, figure 9, are ground carefully to plane surfaces, and when a small amount of grease is applied to these contact surfaces an air-tight joint is obtained. Screws or clamps are used to hold the cover in place during shipment to prevent injury to the contact surfaces. A portion of the main casting forms a solid shelf, *n*, figure 9, extending entirely across the receiver on one side but having in it openings for the pendulum, dummy, and lever *ol*, and two small holes in the edge for attaching the manometer.

The receiver is supported by three heavy foot screws resting on footplates. Any looseness in these screws is counteracted by means of lock nuts which are tightened after the receiver has been leveled.

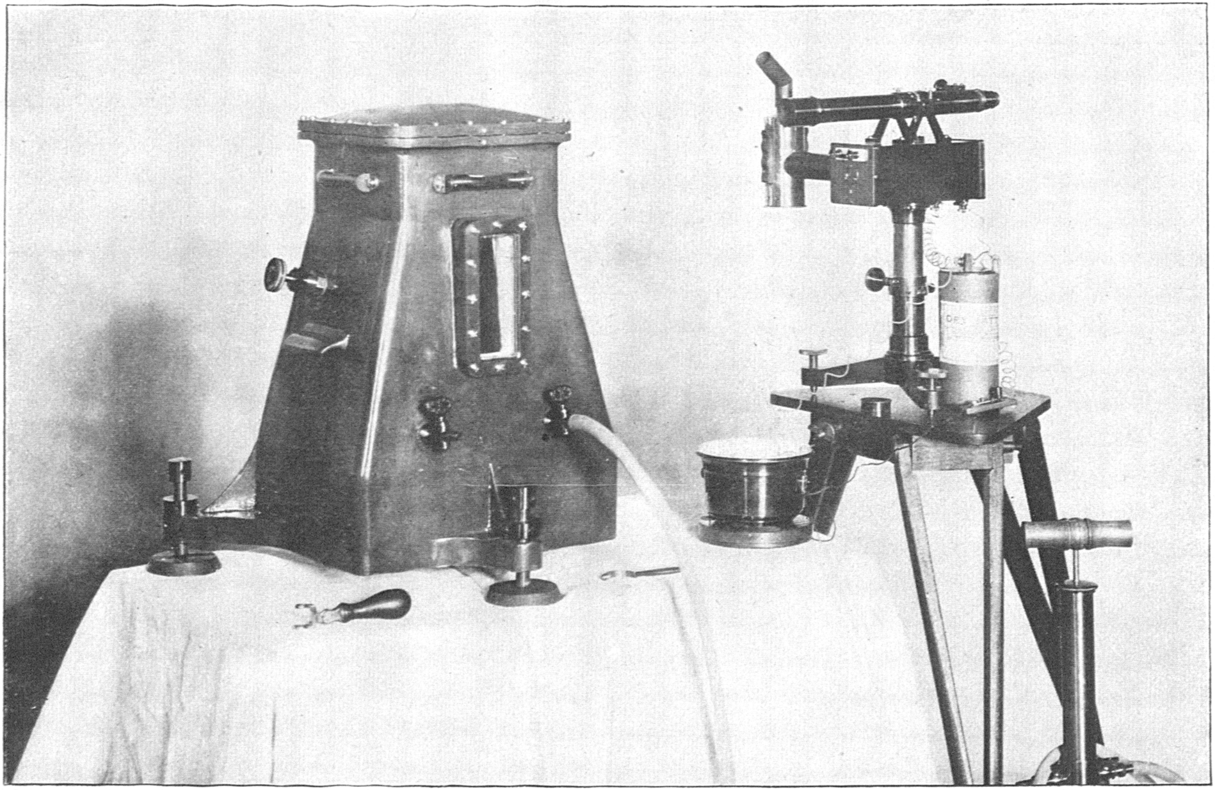

Fig. 11.—ORIGINAL FORM OF QUARTER-METER PENDULUM APPARATUS.

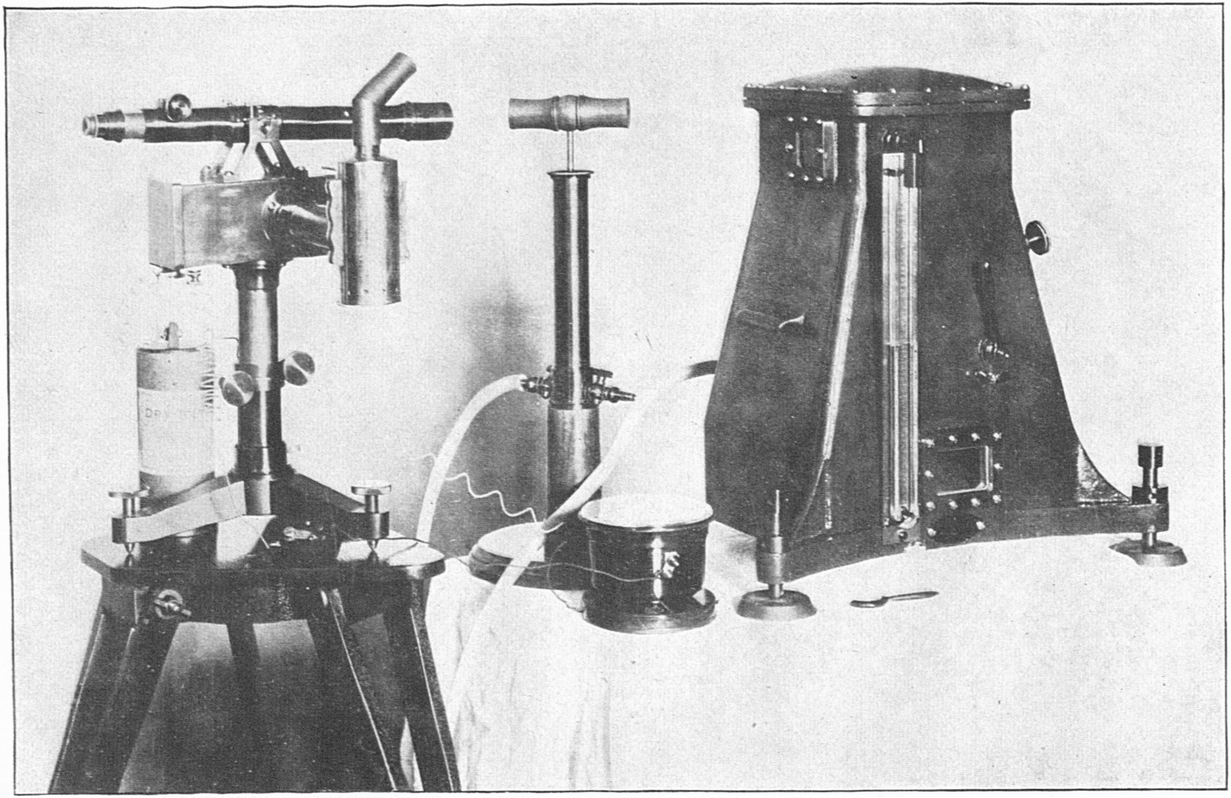

Fig. 12.—ORIGINAL FORM OF QUARTER-METER PENDULUM APPARATUS.

Opposite side to that shown in Fig. 11.

There are two levels mounted on the outside of the receiver at right angles to each other (see fig. 11) which are used to bring the receiver to an approximately correct position.

There are three windows in the receiver, one for the coincidence observations (*M*, fig. 8), one for observing the amplitude of vibration (*J*, fig. 8), and one for the temperature and pressure readings. (See fig. 11.) The windows are of heavy plate glass resting against flanged projections on the receiver which are ground to plane surfaces. A thin layer of a preparation made of tallow, beeswax, and rosin is used to make an air-tight joint between the glass and the flange, and the glass is held in position and protected from injury by a brass frame attached to the receiver with screws.

All manipulating of the pendulum after the receiver has been sealed and pumped out is done by means of the screw *q*, and the lever *r*, figure 9. The former operates the lifting lever and the latter, called the starting lever, is used to start the pendulum vibrating. There is a small piece of cork fastened to the starting lever where it comes in contact with the bob to prevent injury to the pendulum.

The screw *q* (fig. 9) works through an ordinary packing case which prevents any leakage of air into the receiver. The axis of the lever *r*, where it passes through the receiver, is made cone shape and fits a cone-shaped opening in a nut screwed into the hole in the receiver. A lock nut holds the cone sufficiently tight in the opening to prevent leakage. The tightness of this lock nut is adjusted by means of a special wrench provided for the purpose.

A millimeter scale (*s*, fig. 9) for reading the arc of oscillation is fastened to a projection in the bottom of the receiver immediately under the point of the pendulum. The readings of the amplitude are made by means of a microscope (*I*, fig. 8) the pointing of which is controlled by a quick-motion screw. The microscope is held in position by two lugs which fit in holes near the bottom of the receiver.

In order to protect the pendulum against temperature variations the receiver is incased in a thick cover, shown in figure 13, made of heavy wool felt covered with leather. There are openings in this cover corresponding in position with the windows and levers in the receiver, but these are kept closed with plugs of the same material as the cover except during the time the observations are being made.

FLASH APPARATUS.

The flash apparatus consists of a light metal box, called a flash box, supported on a stand and with a telescope mounted on its upper surface. Figures 11 and 12 show the flash apparatus as used a few years ago, and figure 13 shows it with two modern improvements, one a prism at the objective end of the telescope for use when the receiver is mounted on a concrete floor and is too low for observations to be made comfortably with the telescope horizontal, and the other an electric light inside the flash box in place of the oil lamp previously used.

The flash box contains an electromagnet, *a*, figure 14, whose coils are connected with the chronometer circuit through the binding posts *f*, and whose armature carries a long arm *d* projecting through an opening in the end of the box. This arm moves two shutters, *t* and *v*, figure 15, and by an ingenious device a flash of light

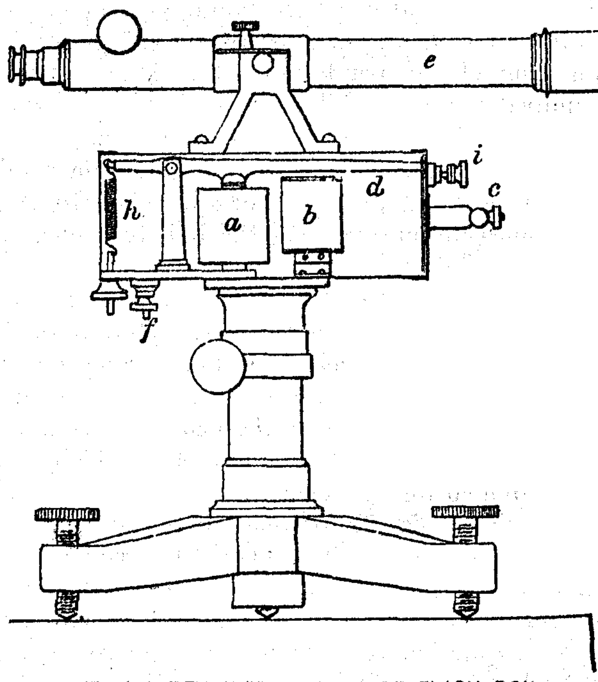

Fig. 14.—DETAILED SKETCH OF FLASH BOX.

is emitted from the box when the circuit is broken but not when it is closed. The left-hand part of figure 15 shows a vertical section of the end of the box and the shutters just after the circuit has been broken and the arm is rising. The right-hand part of the same figure shows how light is prevented from passing through the shutters when the arm is decending.

OPERATION OF SHUTTERS.

The end of the box has an opening containing a piece of finely ground glass for diffusing the light, and in front of this are fastened two pieces of metal, *z* (fig. 15), with a narrow horizontal slit between them. The shutters move in suitable guides and have horizontal slits in them the same size as that between the pieces *z*. The arm *d*

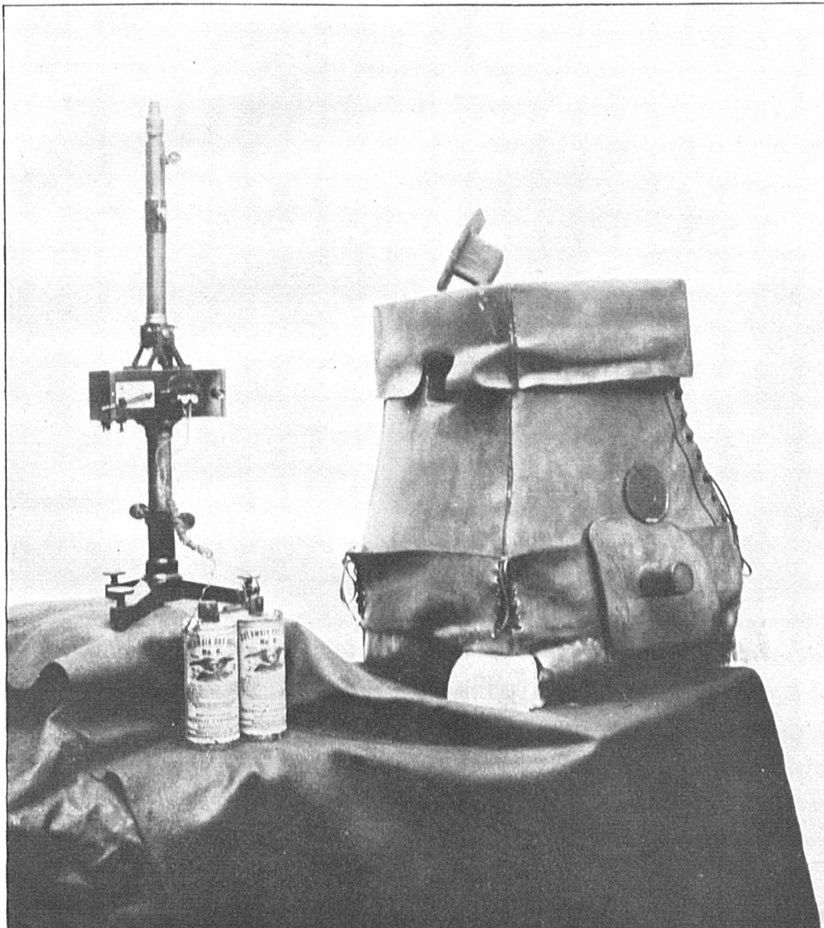

Fig. 13.—LATEST FORM OF GRAVITY APPARATUS, SHOWING FELT-AND-LEATHER COVER AND VERTICAL TELESCOPE.

passes through the upper end of these two shutters. The shutter *t* has no play on the arm but moves directly with it. The shutter *v*, however, has an opening for the arm somewhat longer than the thickness of the arm and so does not move until the arm is near the middle of its stroke. A friction spring holds the shutter *v* so that it moves only when actuated by the arm, and a stop prevents it from descending below the point where its slit is directly in front of the one in *z*.

When the circuit is closed, the arm *d* is down, the slit in *t* is below the line of the other two slits, and no light is emitted. As soon as the circuit is broken the spring *h* causes the arm to rise and the slit in *t* passes the line of the other slits emitting a flash (position shown in left-hand part of fig. 15). Before the end of its stroke the arm also lifts the shutter *v* until its slit is no longer in line with the one in *z*. When the circuit is again closed the arm *d* is pulled down, but the slit in *t* is opposite the one in *z* when the shutter *v* just commences to move (position shown in right-hand part of fig. 15). It is thus seen that only one flash of light is given out each time the circuit is broken and closed by the chronometer. (See p. 27.)

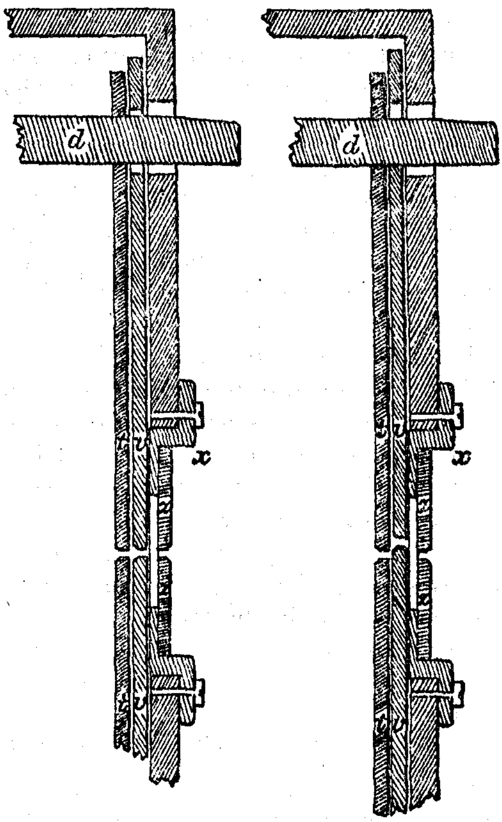

FIG. 15.—DETAILED SKETCH OF SHUTTERS IN FLASH BOX.

The small electric lamp which furnishes the light for the flash is in the flash box immediately behind the shutters. The cells for running this lamp are connected to binding posts on one side of the flash box almost directly under the pivot of the telescope. There are also two switches on the same side of the flash box, one for turning off the current through the lamp during intervals in the observations and the other for breaking the circuit through the armature when the flash is not being used.

On the end of the flash box which contains the shutters there is a small cam attached to a milled-head nut, which operates a small L-shaped lever. When the cam is turned to a certain position the lever presses down the end of the arm *d* (fig. 14) and then lets it start a short distance on its upstroke or until the slits in the shutters are opposite the one in the box and a continuous beam of light is obtained. This continuous beam is necessary in making the adjustment of the fixed mirror inside the receiver. (See p. 41.) After the mirror has been adjusted the cam is turned to another position which throws the lever away from the arm in order that the latter may be operated by the armature.

OBSERVING TELESCOPE.

The telescope is an ordinary observing telescope which may be focused for objects as near as 4 feet. It has a scale in the eyepiece end which is used in making flexure observations. This scale is mounted on a ring and can be rotated until it is at right angles to the direction of the fringes. (See p. 442, Appendix 6, Report for 1910.) It is thrown out of focus during the pendulum observations in order that it may not interfere with the vision of the flashes.

The prism which is used with the telescope to facilitate the observations when the receiver is mounted low (see fig. 13) is fastened in a frame which has a ring to fit on the objective end of the telescope and pivots to fit the wyes on top of the flash box. The telescope can therefore be changed from the horizontal to the vertical position by merely placing the prism on the objective end and setting it in a vertical position in the wyes.

The flash apparatus is supported on a stand which rests on three foot screws and has both vertical and azimuthal movements and clamps.

INTERFEROMETER.

The interferometer [5] is a special instrument used to measure the slight movement of the receiver caused by the oscillation of the pendulum. It depends in principle on the interference of light waves. The general theory of the instrument is exactly the same as in the various forms used by Michelson, the inventor of the interferometer, but the details are of a special nature to adapt it for use in gravity work. These details were worked out by members of the U. S. Coast and Geodetic Survey in 1908 and 1909.

Appendix 6 of the Report for 1910 gives a detailed description of the interferometer and complete instructions for its use, and it will not be necessary, therefore, to go into further detail in the present publication (See fig. 16.)

[5] A most interesting series of lectures on the principle and use of the interferometer will be found in "Light Waves and Their Uses," by A. A. Michelson, University of Chicago Press, 1903.

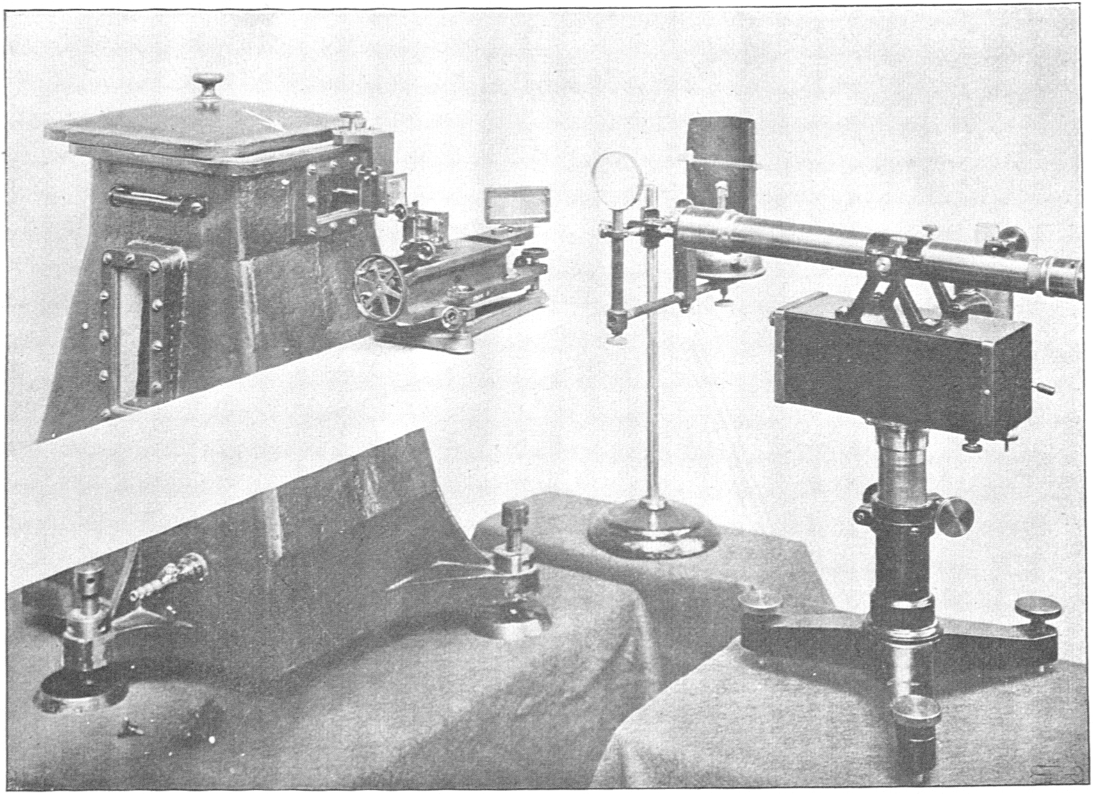

Fig. 16.—INTERFEROMETER IN POSITION FOR MAKING FLEXURE OBSERVATIONS.

SWITCHBOARD.

The gravity switchboard consists of three 20-ohm relays and a system of switches attached to a board, as shown diagramatically in figure 17. The concealed wiring is indicated in the figure by dotted lines. The two chronometers used in making the coincidence observations, the flash apparatus, the chronograph, and several cells are connected to the switchboard as indicated in the diagram. The box containing the switchboard has two compartments, one in the bottom and the other in the lid, in which the required number of cells for the various circuits can be placed. The electric bell, shown in the figure, is used for signaling purposes between the observatory and the pendulum room at those stations where astronomic observations are made for determining the chronometer rates. (See p. 55.) It can be cut out of the circuit by connecting the wires from the chronograph to the binding posts F and F'. Directions for the use of the switchboard will be found on page 50.

MISCELLANEOUS INSTRUMENTS.

Various other instruments, in addition to those already described, are needed in making gravity determinations. Most important of these are the chronometers, of which three are needed. They should be of the one-second break-circuit type, so that they will break the circuit in which they are connected at one-second intervals with the exception of one two-second interval each minute for identifying the even minute on the chronograph sheet.

CHRONOGRAPH.

Another important instrument in gravity work is the chronograph (see fig. 18) on which are recorded the noon time signals and the comparison of the chronometers. A description of the chronograph will be found on pages 11 to 13 of U. S. Coast and Geodetic Survey Special Publication No. 14. When the pendulum room is at some distance from the telegraph office; that is, when it is not possible or convenient to have a temporary wire connection between the two—it is necessary to have two chronographs, one for recording the noon signal and the other for the chronometer comparison. If wireless time signals or star observations are used to obtain the rates of the chronometers, only one chronograph is needed.

MANOMETER.

The manometer used for gravity work is a simple U-tube closed at one end, graduated to millimeters, and capable of indicating pressures up to about 90 mm. The closed end is filled with mercury to the exclusion of all air. The manometer is hung from the shelf

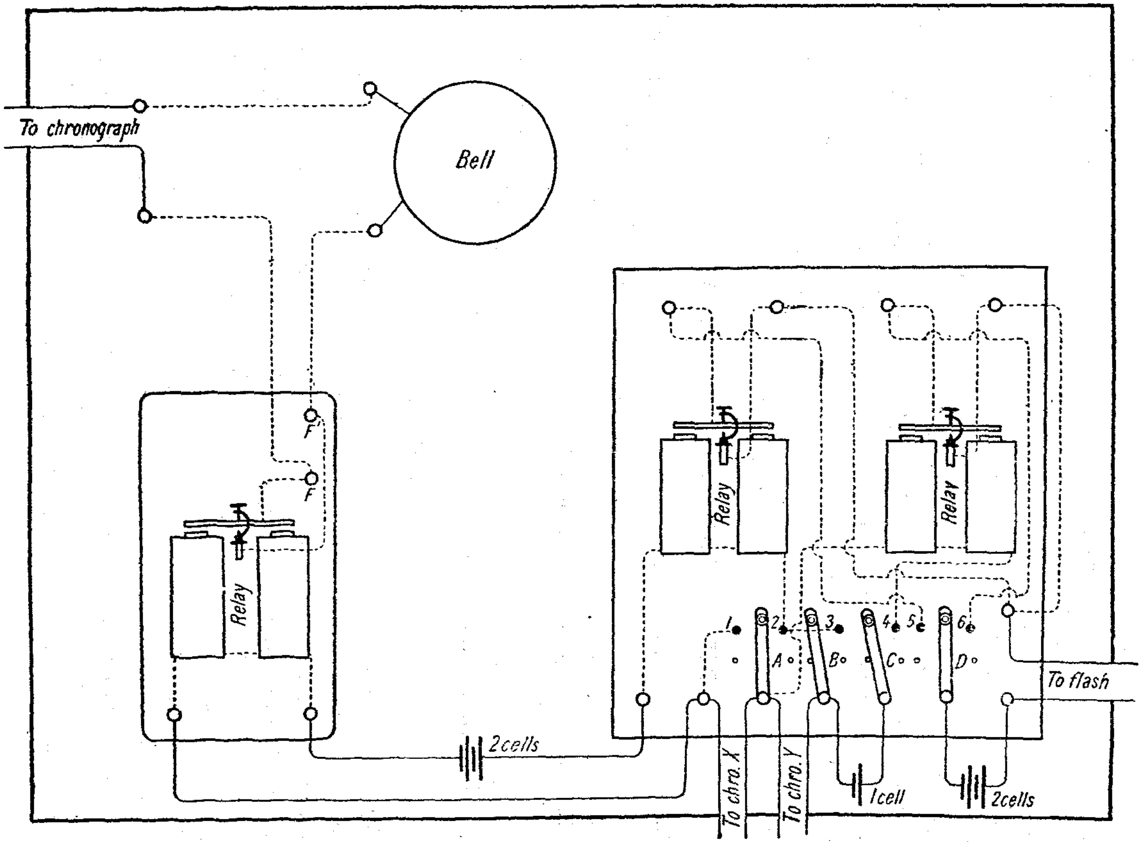

Fig. 17.—DIAGRAM OF SWITCHBOARD SHOWING ELECTRICAL CONNECTIONS.

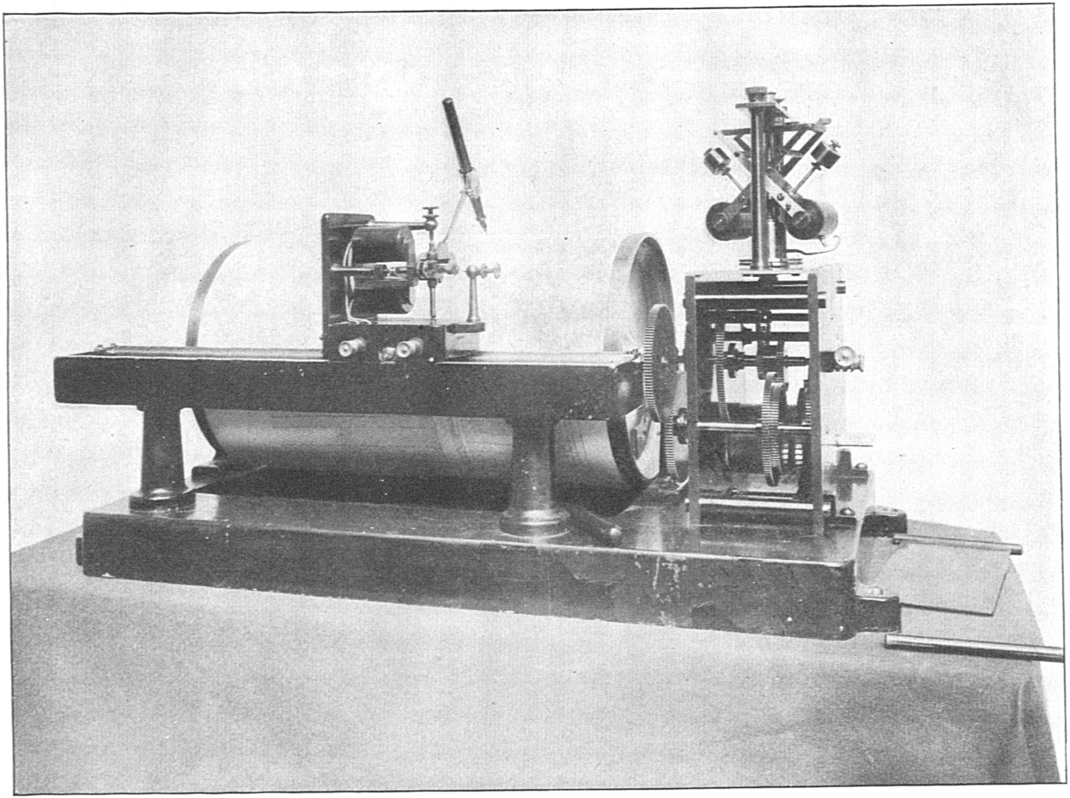

Fig. 18.—CHRONOGRAPH.

inside the receiver by means of a small frame, which has two pins that fit in corresponding holes in the shelf. Sealing wax is used to fasten the frame to the manometer. The readings are made through the same window in the receiver that is used for reading the thermometer. (See fig. 11.) The zero graduations on the two limbs of the manometer must be in the same horizontal line, and the amount of mercury in the tube should be such that the readings on the two limbs will be about equal. The graduations extend up from the zero line on the closed end of the manometer and down from the zero line on the open end. In a perfect vacuum the mercury would stand at the zero line in both sides of the manometer if the amount of mercury in the tube was exactly right. For any other low pressures the mercury would be as much depressed in the open end as it would be raised in the closed end of the tube, and the two readings would still be equal.

The thermometer has already been described in connection with the dummy pendulum. (See p. 19.)

AIR PUMP.

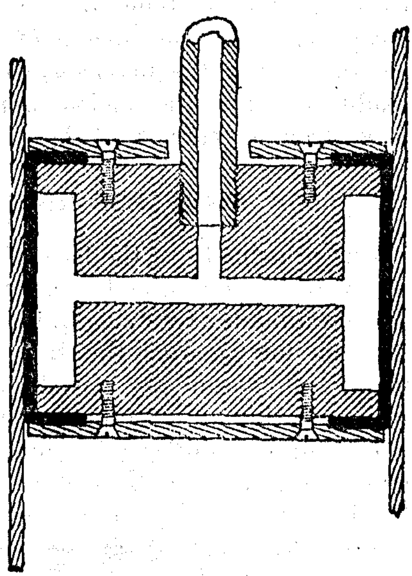

FIG. 19.—CROSS SECTION OF AIR-PUMP PISTON.

The air pump for exhausting the receiver is shown in figure 12. It weighs only about 6 pounds and has a very ingenious self-packing piston, invented by E. G. Fischer, chief of the instrument section of this Bureau. The piston, a cross section of which is shown in figure 19, consists of a leather cylinder stretched over a metal frame, which is so constructed that the outside air is admitted to the inside of the leather cylinder through the tubular piston rod and a cavity in the frame. As soon as the pump is started, the air becomes partially exhausted from the pump cylinder, which causes the atmospheric pressure on the inside of the piston to force the leather out against the cylinder wall. This produces a perfect elastic packing with minimum friction.

There are two stopcocks at the bottom of the pump, each connected with a valve so arranged that air is drawn in at one cock and forced out at the other. When one side of the pump is connected to the receiver, it will exhaust the air, and when the other side is connected it will force air into the receiver. The latter arrangement is used when it becomes necessary to locate a leak in the receiver.

In addition to the two valves just mentioned there is an escape valve at the top of the pump cylinder which permits air to escape when the pressure above the piston exceeds atmospheric pressure. This causes a partial vacuum above the piston when the piston descends and not only insures the proper working of the leather cylinder when the piston is near the bottom of its stroke and the pressure below it becomes greater than atmospheric, but also tends to equalize the force necessary for the up and down strokes of the pump. If the top of the cylinder were open to the outside pressure of the air, practically all the effort required to operate the pump would have to be applied on the upstroke, as there would be a partial vacuum below the piston, which would cause it to spring back to within a short distance of the bottom of the cylinder. Since the escape valve in the bottom of the pump does not open until the pressure below the piston exceeds atmospheric pressure, it can be readily seen that the efficient working of the pump requires that each downstroke be carried all the way to the bottom of the cylinder.

WIRELESS RECEIVING APPARATUS.

Due principally to the lack of a recording device suitable for field use, wireless apparatus has not been employed in the field up to the present time for receiving Naval Observatory time signals. During the last few months the Bureau of Standards, at the request of the U. S. Coast and Geodetic Survey, has been studying and experimenting on a method for recording wireless signals which will not require equipment too delicate to be readily transported and quickly set up. Dr. E. A. Eckhardt has been in charge of this work. He has succeeded in devising an apparatus by means of which the wireless signals can be recorded directly on a chronograph together with the beats of a break-circuit chronometer. This is exactly what is desired for gravity and longitude work. It is probable that the apparatus can be used to receive the Naval Observatory time signals sent out from Annapolis, Md., anywhere in the United States.

The apparatus has not been entirely perfected at this time and, of course, has not been used in the field. It will not be advisable, therefore, to attempt to describe it in this report.

TRANSIT.

As stated on page 55, it was the general practice on gravity work until quite recently to obtain the chronometer rates by making star observations at each station with the astronomic transit. At present the chronometer rates are obtained by comparison with the Naval

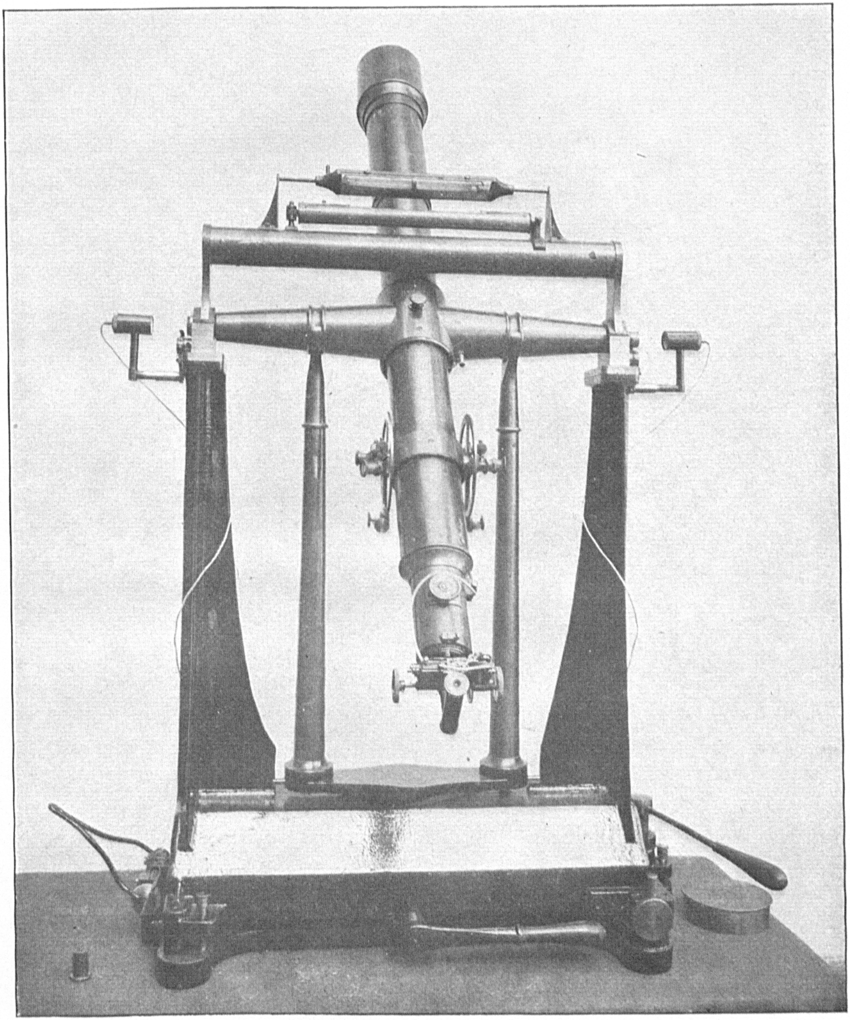

Fig. 20.—ASTRONOMIC TRANSIT.

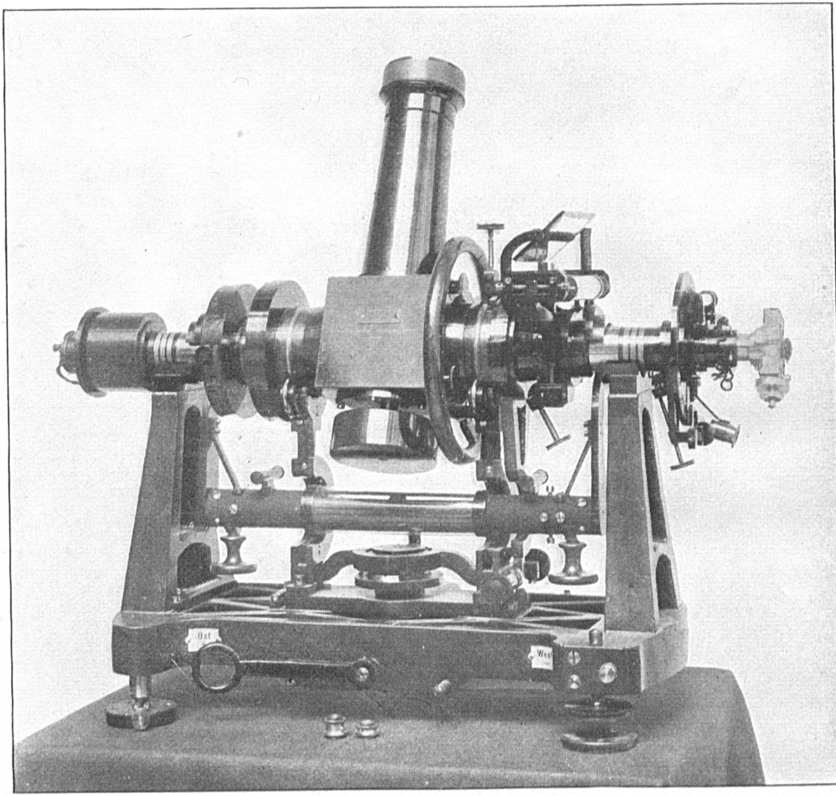

Fig. 21.—BROKEN-TELESCOPE TYPE OF ASTRONOMIC TRANSIT.

Observatory time signals which are sent out over the telegraph lines. There may be an occasional station, however, where, due to some unusual condition, it may be necessary again to make time observations. A description of the transit instrument (see figs. 20 and 21) and the methods employed for making the observations and computations will be found in U. S. Coast and Geodetic Survey Special Publication No. 14.

Chapter 3.—GRAVITY OBSERVATIONS.

The various operations at a gravity station may be summarized as follows: Location and preparation of station, unpacking and setting up of instruments, making the observations, and packing and shipping instruments. These operations are taken up in order in the present chapter, but first there is given a set of general instructions which cover the requirements that must be met by the observer to make the determination of gravity acceptable.

GENERAL INSTRUCTIONS.

1. All determinations of the intensity of gravity are to be made with the half-second pendulum apparatus, and are to be relative determinations with the pendulum room at the Coast and Geodetic Survey office at Washington as the base station. The absolute value of gravity at the base station shall be assumed to be 980.112 dynes.[6]

2. A standard determination of the intensity of gravity shall consist of not less than six swings, of which the probable error of the mean result shall be less than ±0.004 dyne. Each swing will be from 11 to 12 hours in duration.

3. At least two different pendulums shall be used at each station, one for two swings and the other for the remaining four swings. The three pendulums of a set shall be used alternately at different stations in such a manner that any one pendulum will be idle at every third station, and that about one-third the total number of swings during a season's work will be made with each pendulum. For example: At the first station, pendulum *A*4 may be used for two swings and *A*5 for four swings; at the second station, *A*5 for two swings and *A*6 for four; at the third station, *A*6 for two swings and *A*4 for four; and so on. This program may need modification if there is a failure of the time signals. (See footnote on p. 52.)

4. Each pendulum shall be swung in the direct position only and on the same knife-edge as was used during the standardization at the base station. A pendulum is said to be in the direct position on the knife-edge when the lettering on the top of the head of the pendulum is right side up as read from the front of the receiver. The standard conditions for each swing shall be: Pressure reduced to about 60 mm. and the total arc at the beginning of a swing about 8 mm. These conditions should be approached as nearly as feasible.

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[6] See p. 49 of U. S. Coast and Geodetic Survey Special Publication No. 40.

5. The second pendulum to be used at each station should be placed in the corner of the receiver while the first pendulum is being swung in order that it may assume the temperature of the inside of the receiver before it is used. In order to make the *conditions exactly* the same for the second pendulum as for the first, a pendulum should also be placed in the corner of the receiver during the time that the second pendulum is swinging.

6. The errors of the chronometers must be determined at the beginning of the pendulum observations at a station and again at the end. If the Naval Observatory time signals are used for this purpose the errors of the chronometers should be determined also on each intervening day, if possible. These additional determinations are not required where it is necessary to make time observations with the astronomic transit.

7. The time intervening between the end of one swing and the beginning of the next should be made as short as possible. The first swing should be started as soon as possible after the first determination of the chronometer errors, and the last swing should be continued as nearly as possible up to the time of the final determination of the chronometer errors. Any variation of the chronometer rate from the mean daily rate during these short intervals when no pendulum is swinging affects the final derived value of gravity at the station.

8. Observations of coincidences and readings of the pressure and temperature shall be made at the beginning and end of each swing. One or more additional readings of the temperature should be made during a swing if there is much fluctuation of temperature, especially when the bronze pendulums are being used. The record must show the time these additional readings were taken.

9. Complete observations of coincidences between the pendulum and each of two chronometers, designated as the gravity chronometers, shall ordinarily be made in connection with each swing. A set of coincidence observations must include at least two "downs" and one "up," or one "down" and two "ups." (See p. 56.) The readings should be made on a hack chronometer, which can be compared with the two gravity chronometers by means of the minute breaks.

10. The flexure shall be determined with such a degree of accuracy that the probable error of the computed flexure correction will be less than ±0.0000005 second.

11. The latitude of each gravity station must be determined within 1 minute, either by scaling from some accurate large-scale map, such as a Geological Survey quadrangle sheet, or by star observations with a theodolite. The star observations should be made by the method outlined in Table 1 of the American Ephemeris and Nautical Almanac, that is, by observations on Polaris.

12. The elevation of the knife-edge above mean sea level at each gravity station must be determined within 20 feet, unless to secure this degree of accuracy would require the expenditure of considerable time and money. In every case the uncertainty of the elevation should be noted in the record.

13. The observer should strive to keep as well within the accuracy specified in paragraphs 2, 10, 11, and 12 of these instructions as is possible by the use of good judgment and skill; but the time and money spent on each operation should be restricted substantially to that required to keep barely within these limits.

14. Such a description of each gravity station must be furnished as will enable a later observer to place his pendulum apparatus at the same place and same elevation if it should become necessary to reoccupy the station. The description should state how the receiver was mounted, the approximate direction of oscillation of the pendulum, and, in case the station is located in a building, the difference in elevation of the knife-edge and the general level of the ground just outside the building.

15. A rough contour map extending out about 200 or 250 m. in all directions from the station should be furnished with the description of the station. This is needed by the office in computing the topographic corrections. No topographic details except the contours need be shown on the map, and a hand level is the only instrument needed to obtain the data, as the distances may be stepped off. If the station is located in a fair-sized town, it may be possible to obtain a blue print of the city map or to make a tracing of the required section if the contours or a sufficient number of elevations are shown on it. There should also be furnished with the description of station the U. S. Geological Survey map of the quadrangle with the station plotted on it.

16. To insure against the loss of records in the mails, the following computations should be made at each station: Reading of the chronograph sheets, computation of the rate correction, and filling in on the computation sheets of all data that come from the record book, namely, the total coincidence intervals, the mean time of one coincidence for each swing, the correction for flexure, and all temperature, pressure, and arc readings. The computations should not be carried beyond this point if to do so would delay the party or put an undue burden on the observer. The records and computations should be forwarded by registered mail on different days in conformity with the general practice of this office.

17. In case it becomes necessary at any station to make time observations with the astronomic transit, the methods described on pages 14 to 22 of Special Publication No. 14 should be followed.

LOCATION AND PREPARATION OF STATION.

The first thing to be done at a station is to find a room in which the pendulum observations can be made. If possible, a room with a solid concrete floor should be selected in order that the receiver may be set on small concrete blocks or bricks cemented to the floor with plaster of Paris. Otherwise some form of pier will have to be built. The proximity of any heavy machinery which would cause excessive vibration should be avoided, as it interferes with flexure observations. A small and uniform range of temperature is a prime requisite for a good pendulum room, that is, the room should approach as nearly as possible a constant temperature room, especially when the bronze pendulums are used.

If the Naval Observatory noon signals, sent by telegraph, are to be used for rating the chronometers, it will be very convenient and conduce to greater accuracy if the station can be located in the same building as the telegraph office, or at least so near that a temporary wire connection can be made easily between the two. If time observations with the astronomic transit are to be made, the matter of an available site for the observatory must be looked into. It is important that the observatory should be near enough to the pendulum room that the necessary wire connection between the two can be made by the observer without the necessity of hiring a lineman.

Another matter which should not be overlooked in locating the station is the nearness of available accommodations for the observer. Extra temperature readings or the observations late at night may cause the observer considerable inconvenience if his quarters are located too far from the station.

It is often possible to find a good pendulum room in the basement of some public building such as a post office, courthouse, city hall, or schoolhouse, and ordinarily no difficulty is experienced in obtaining permission to use such a room. There is no objection to locating the station in a store, residence, or cyclone cellar, or other private building if no public building is available. Often a cave or tunnel can be used to good advantage for a pendulum room if the station is not in a town.

The pendulum room must not be smaller than about 4 by 9 feet unless some arrangement can be made to place the flash apparatus outside the room and make the observations through an opening. With a large basement having no partitions, it may be necessary to construct a small room in one corner by putting up a temporary partition of boards or building paper in order to insure against too great fluctuation of temperature and to prevent any disturbance of the instruments by persons having access to the basement. If there is a choice, the middle of the north side or the northeast corner of the basement should be selected for the pendulum room, as it will be in the shady part of the building during the hottest part of the day and consequently will maintain a more uniform temperature. The room must be made dark enough to cause the flashes to show clearly and distinctly. This can be accomplished by tacking boards or black muslin over the windows.

CONSTRUCTION OF GRAVITY ROOM.

It is sometimes impossible to find a suitable room for pendulum observations and it becomes necessary, if the bronze pendulums are being used, to put up a special building for the purpose. This building should be about 6 by 9 feet inside dimensions and is usually constructed with double walls and double roof, with the space thus inclosed packed with some insulating material such as sawdust, hay, or earth. One of the gravity observers has recently recommended that the building be made with a single wall, with building paper tacked to the studding on the inside to form a dead air space, and with a shield made of brush, straw, or other suitable material, separated from the building by at least 2 or 3 feet, to serve as a protection from the sun. Such a building is much cheaper and less troublesome to construct than the one with the double walls and will probably give a temperature control nearly as satisfactory.

CONSTRUCTION OF GRAVITY PIER.

After having selected the location for the station, the next thing to be done, if there is no concrete floor or solid rock on which to mount the receiver, is to construct a pier. The pier should be about 16 by 24 inches on top with the long dimension in the direction of the flash apparatus and should project slightly above the ground or floor. No boxing should be used for the underground part of the pier as a better connection with the earth and thus a firmer foundation will be obtained without it. A rich mixture should be used for the concrete, say, 1 part cement to 1½ parts sand and 3 parts gravel, in order that it may harden quickly. Under the best conditions, at least 24 hours must be allowed for the concrete to set before it is used. In case the best available room has a wooden floor it may be possible to obtain permission to cut a hole in it large enough for the pier.

A design is being prepared in this office for a portable knockdown pier made of a frame work of steel or aluminum strongly braced and resting on metal footplates buried in the ground. If such a pier can be made stable enough to be satisfactory it will mean a considerable saving in time and labor especially when a large proportion of the stations are located outside of towns as is likely to be the case when the invar pendulums are used.

TENT FOR USE WITH INVAR PENDULUMS.

With the invar pendulums the temperature control is not nearly so important as with the bronze pendulums and an ordinary tent, protected by a fly from the direct rays of the sun, will undoubtedly give satisfactory conditions. A small inner room of black muslin, about 5 feet square, to fit in one corner of the tent, will make a pendulum room sufficiently dark for the flashes to be seen distinctly. Figure 22 shows the plan of a tent, 10 by 12 feet, arranged as a combination pendulum room and living tent for the observer and one assistant.

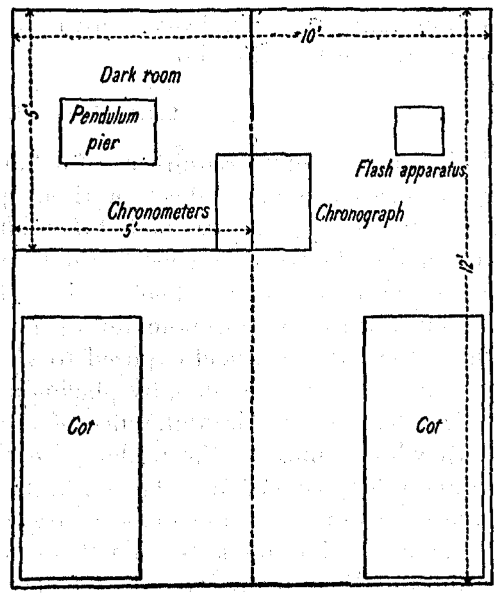

Fig. 22.—PLAN OF TENT FOR GRAVITY WORK WITH INVAR PENDULUMS.

When a large number of the stations of a season's work are located in out-of-the-way places at some distance from railroads, and the invar pendulums and a wireless receiving set are being used for the work, it will probably be found that an automobile truck will be the most efficient and economical means of transportation. If such is the case, it will be necessary for the observer to have some one to assist him in the work. The assistant should be able to drive the truck and keep it in good repair and also help the observer pack and unpack the instruments besides doing the greater share of the camp work.

UNPACKING AND SETTING UP INSTRUMENTS.

Unpacking the instruments requires considerable care to prevent injury to any of the delicate parts. The pendulums themselves, although apparently of rugged construction, must be handled very carefully. A slight change in the virtual length of a pendulum, such as might be caused by a fall of a few inches on a hard surface, would make necessary the restandardization of the pendulum at the base station before using it for any further determinations.

The box in which the pendulums are packed is so constructed that they are held rigidly but with no possibility of strain on any part. This box, called the pendulum box, is placed for shipment inside of a large packing box lined with thick pads of excelsior covered with canvas to protect the pendulums against severe shocks. Except for the time they are in the receiver, the pendulums should be kept in the pendulum box to protect them from dust and injury.

Other delicate parts of the pendulum apparatus are the manometers, thermometers, and chronometers. The manometers should always be kept in an upright position so there will be no opportunity for air bubbles to get into the closed end of the tube. They are packed in a small box which is carried by the observer as personal luggage from station to station and always kept right side up. The thermometers are carried in the same box.

CHRONOMETERS.

The chronometers require careful handling. When shipped, each is wrapped in a large sheet of tissue paper to keep out the dust. After the tissue paper has been taken off the corks must be removed from under the balance wheel. First unscrew the glass from the face of the chronometer and then, holding the tips of the figners against the face, turn the chronometer over and remove the bowl. This leaves the balance wheel exposed to view. The cork wedges should be removed, one at a time, by placing a knife blade flat down on the wedge close to the circumference of the balance wheel and pressing down while removing the wedge. In this manner no undue strain is put on the pivots of the balance wheel. As soon as the chronometers have been unpacked and made ready for use they should be placed in the special box made to hold them during the observations, which permits of their being wound each day without being lifted or disturbed. (See p. 95 of U. S. Coast and Geodetic Survey Special Publication No. 14.)

RECEIVER AND INCLUDED APPARATUS.

Before lifting the receiver out of its packing case, the two three-cornered boxes which hold the knife edges should be removed from the corner of the case. The receiver should then be taken out and placed on the pier or concrete floor in the desired position with a footplate under each foot screw and, when mounted on a concrete floor, with a small stone or concrete block about 3 inches high under each footplate. These blocks should be secured to the floor and the footplates held in place with a little plaster of Paris mixed rather thin.

The knife-edge to be used should be attached to the shelf inside the receiver with the screw, which holds it in place, set up fairly hard to give a rigid connection. Do not remove the small protecting cap over the knife-edge itself until the knife-edge is in position.

LEVELING RECEIVER.

As soon as the plaster of Paris under the footplates has hardened sufficiently, the receiver should be wiped out with a piece of clean cheesecloth and then leveled, the level tubes attached to the outside of the receiver being used to bring it to an approximately correct position. This will be accurate enough in the direction in which the pendulum oscillates unless it should be desired to have the arc readings equal on each side of the center, in which case a pendulum should be put in the receiver and lowered onto the knife-edge, in order that the relation between the line on the bottom of the pendulum and the index line of the arc scale can be watched through the microscope while the leveling screws are being adjusted. The leveling in the direction of the knife-edge, at right angles to the direction of oscillation of the pendulum, is a much more important matter and should be done as accurately as possible. The final adjustment in this direction should be made by means of the sensitive level described on page 22. This level should be placed in the receiver and lowered on the knife-edge in the same manner as a pendulum; that is, by making use of the lifting lever described on page 19, and should never be placed by hand directly on the knife-edge. Any error of the level itself should either be adjusted out or else allowed for by the usual method of reversals. This final leveling should all be done by means of the one foot screw which controls the levels in this direction—that is, by the one on the same side of the receiver as the pet cocks—to avoid any disturbance of the levels at right angles to this direction.

In order to make the receiver as stable as possible, and thus minimize the flexure effect, the foot screws should be allowed to project only a short distance below the receiver, and after the leveling has been done the lock nuts should be set up rather firmly.

DUMMY PENDULUM.

The dummy pendulum with the thermometer attached to its stem is the next thing to be put in the receiver. As stated on page 19, a good metallic contact between the bulb of the thermometer and the stem of the dummy pendulum is secured by packing the bulb with filings in a small metal box of the same material as the pendulum, which fits into a slot in the stem of the dummy. A piece of tin foil should be wrapped around the outside of the box before pushing it into the slot in order to give a better contact. This may make the fit rather snug, and care must be used not to strain or break the thermometer. When the dummy is placed in the receiver, see that it rests squarely on the two hard-rubber knife-edges and that the stem is held by the fork projecting on the inside of the receiver. It must, of course, have the right side of the stem turned toward the window to permit the reading of the thermometer.

MANOMETER.

Before the manometer is put in the receiver it should be examined closely for air bubbles and should be tested by holding it firmly in an upright position in one hand and striking that hand downward against the other, thus giving it a slight jar. This will cause the mercury to spring slightly away from the closed end of the tube and come back again with a snap. If there are no air bubbles in the end of the tube there will be a sharp metallic click which can not be mistaken after it has been heard a few times.

As stated on page 27, the manometer is held in position in the receiver by means of a small frame having two pins that fit in corresponding holes in the shelf inside of the receiver. The manometer is read through the same window as the thermometer, one part of the manometer appearing on one side of the thermometer and the other part on the opposite side. Pieces of white paper should be pasted on the back of each limb of the manometer, that is, on the opposite side from the scale, to make the scale and the top of the mercury show more clearly. It is very important that the two zero graduations of the scale should be in the same horizontal line, and it is desirable that there should be the right amount of mercury in the tube to make the two readings about equal. (See p. 29.) The latter is not necessary, however, as the sum of the two readings will not be affected by a small difference in the amount of mercury. Instructions for varying the amount of mercury in the manometer will be found on page 63.

If it becomes necessary to make any change in the amount of mercury in the manometer or do any other handling of mercury, extreme care must be used that none of it comes in contact with the bronze pendulums, as amalgamation would take place very quickly, thus changing the virtual length of the pendulum and destroying its usefulness, at least until the mercury could be removed and the pendulum restandardized. The invar pendulums, fortunately, are not susceptible of amalgamation.

PLACING OF PENDULUMS IN RECEIVER.

As stated in the general instructions, an idle pendulum is always kept in the corner of the receiver during the observations at a station. This pendulum is held by a small wooden block which is cut to fit the lower half of the bob and is made of the proper shape to fit in the corner of the receiver. The idle pendulum is placed in the corner nearest the dummy pendulum in such a position that it does not touch the receiver at any point.

The last thing to be put in the receiver is the working pendulum. It should be taken out of the pendulum box by means of the special lifting handle and carefully dusted with a clean camel's-hair brush and then examined closely with a flash light to see that there are no bristles or other foreign matter adhering to it. The agate plane and knife-edge should be wiped with a piece of clean chamois moistened with alcohol and should be examined with a flash light to see that they are perfectly clean. The pendulum must not come in contact with the receiver or other part of the apparatus when it is being lowered into place, and the lifting lever must be raised so that the pendulum is suspended from the end of this lever and can not be placed directly on the knife-edge.

Except for the dummy, none of the pendulums should ever be handled with the bare hands and the dummy no more than necessary. All manipulating of the pendulums should be done by means of the lifting-handle provided for this purpose.

ADJUSTMENT OF STATIONARY MIRROR.

Before sealing and pumping out the receiver, the mirror attached to the knife-edge frame, called the stationary mirror, must be adjusted to the proper position relative to the mirror on the pendulum. After lowering the pendulum onto the knife-edge an approximate adjustment should be made at first by placing the eye directly in front of the mirrors and about 6 or 8 inches distant and then adjusting the stationary mirror until the image of the eye, or of a flash light, will look undistorted when a part of the image is seen in one mirror and a part in the other mirror at the same time. The final adjustment must be made by making use of the flash apparatus. The shutter should be opened to allow a continuous beam of light to fall on the mirrors and be reflected back into the telescope. Two images of the slit of light will then be seen in the telescope, one stationary, and the other moving up and down with a half-second period, for no matter how carefully the pendulum is lowered onto the knife-edge it will always have a slight vibration. The two reflected slits should be made to overlap for about one-third of their lengths with the stationary one, preferably, always to the left as seen in the telescope in order that the observer will not become confused when making the observations. In an up-and-down direction, the stationary slit should occupy a position midway between the extreme positions of the moving slit in order that in the observations of coincidences the time interval between an "up" coincidence and the next "down" will be equal to that between a "down" coincidence and the next "up."

The adjustment of the mirror can be made most conveniently by two persons, one to watch the slits in the telescope while the other turns the adjusting screws. If the observer is working alone he should turn one of the screws a slight amount and then look through the telescope to note the effect, repeating the process until the necessary refinement has been attained.

PUMPING OUT RECEIVER.

The receiver is now ready to be sealed and pumped out. The contact surfaces of the receiver and cover should be wiped carefully with a piece of clean cheesecloth and then covered with a thin coating of grease made of a mixture of tallow and oil. The gravity outfit includes several small cans of this grease made of different consistencies varying from one part oil and six parts tallow to equal parts of the two ingredients. The thick grease is for warm weather and the thin for cold. A little experience will indicate what consistency to use for any given temperature. The grease should be just thin enough to allow of its being rubbed out readily into a very thin even layer on the contact surfaces.

After putting the cover on the receiver, twist it slightly back and forth a few times to smooth out the grease and bring the surfaces together, using care not to disturb the position of the receiver. Listen for grit between the surfaces while doing this, and if there seems to be any grinding remove the cover, wipe the contact surfaces, and apply a new coating of grease. After 30 or 40 strokes with the pump, twist the cover back and forth a few times as before to improve the contact between the surfaces, and then rub grease around the edge of the cover where it comes in contact with the receiver.

Always make sure that the arrow painted on the top of the cover points toward the flash apparatus, as that is the position which gives the best fit between the surfaces. Be very careful never to mar or scratch the contact surfaces of the receiver or cover. When removing the cover from the receiver always turn it upside down before laying it down. Be careful that the cover does not fall. A light blow on the edge is likely to make the contact surface bulge slightly so that it will no longer be a true plane.

Pump out the receiver to about 50 mm. at least an hour or two, if possible, before time to begin the observations in order that there may be time enough to make sure that the receiver is not leaking. Allow about 30 minutes after pumping for conditions to become settled inside the receiver, but after that, if the pressure is increasing more than about 2 mm. per hour, it will be necessary to locate the leak.

LEAKS IN RECEIVER.

Make sure that the packing around the screw that operates the lifting lever is sufficiently tight and also that the nut holding the cone of the starting lever in place has not become loosened. If the pressure still continues to increase rapidly, the leak is quite probably between the contact surfaces of the receiver and cover. If it can not be stopped by rubbing grease around the edge of the cover, let air into the receiver, remove the cover, wipe off the grease, and then repeat the whole process of sealing and pumping out the receiver, using a little greater care than before in the process of sealing.

The rate of increase of the pressure will soon indicate whether the trouble has been found. If this rate is about the same as it was the first time the receiver was exhausted, the trouble is probably around a window or some other opening. To find a leak of this kind requires considerable patience. It is necessary to let the air into the receiver and then, after fastening down the cover with screws or clamps, to pump air into the receiver, connecting the hose for this purpose to the opposite side of the pump as explained on page 29. Make a strong solution of soapsuds, and brush it around the edges of the windows and openings until the leak is found by the bubbling of the suds. It is possible for a leak to develop around the plate in the bottom of the receiver, around the knob in the cover, or even through pores in the metal. The heavy paint on the outside of the receiver is for the purpose of closing any pores, but this paint may become broken or cracked. Remember that the leak is probably a very small one and that the bubbles will therefore form very slowly. Watch carefully, work slowly, and be persistent. Pump more air into the receiver if necessary.

A leak around one of the windows is perhaps the hardest to fix. The wax used for sealing the windows is made of tallow, rosin, and beeswax, and at ordinary temperatures is quite stiff and hard. It is necessary to heat the contact surfaces before applying the wax. First take all the apparatus out of the receiver, then turn it down on its side so that the window will be on top. Remove the frame and glass and clean the contact surfaces carefully with gasoline or some other solvent. Heat the receiver around the opening by laying on a hot iron, warm the piece of glass used for the window, being careful not to break it, and warm the wax until it becomes soft. Then apply a thin coating of the wax to the warm contact surfaces. When the glass is placed over the opening and worked down to a good contact with the receiver, the presence of air bubbles between the surfaces can be readily detected through the glass. Unless these bubbles are very small they must be worked out or else another attempt made with a fresh coating of the wax. The frame which fits around the window should be put on and screwed down before the wax hardens.

FLASH APPARATUS.

The flash apparatus should be mounted on a heavy box or solid block of some kind in such a position that the telescope is about on a level with the top of the receiver and about 6 feet distant, directly in front of the observing window. The switchboard should be placed close beside the flash apparatus on the right-hand side to make it convenient for the observer to manipulate the switches while watching

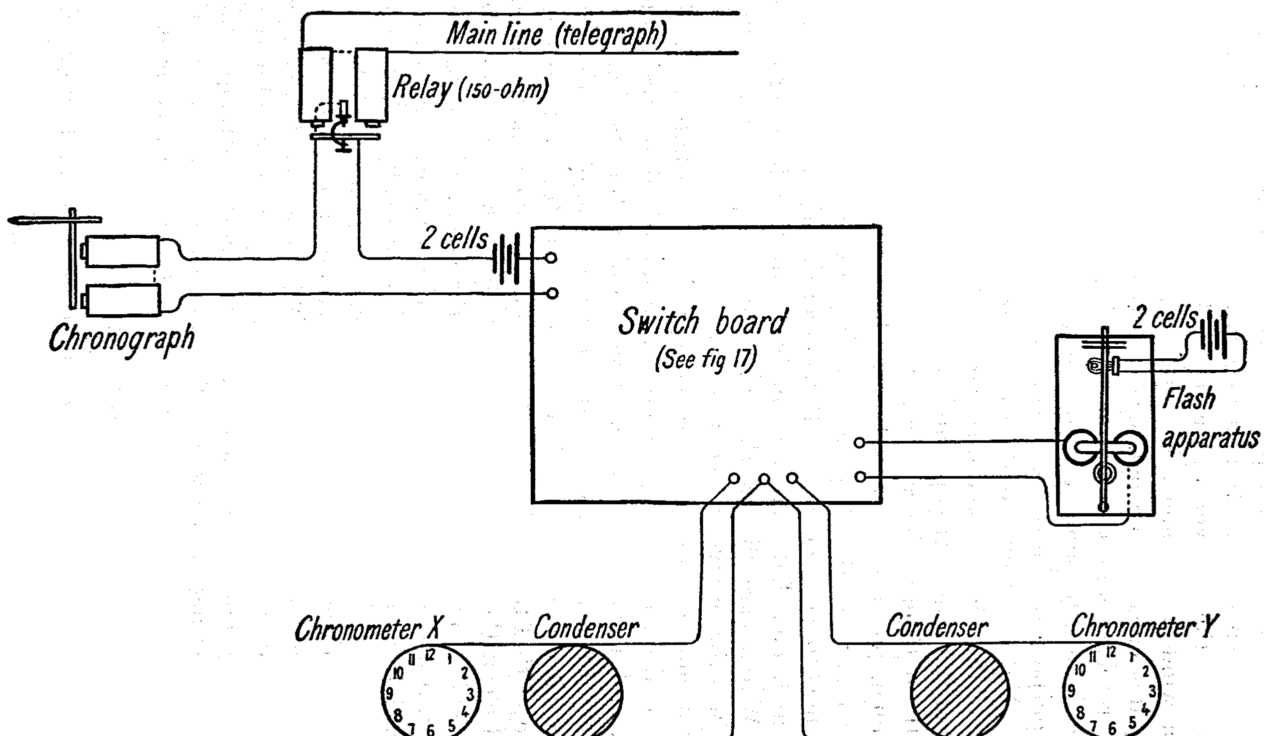

Fig. 23.—DIAGRAM OF ELECTRICAL CONNECTIONS IN PENDULUM ROOM WHEN DIRECT CONNECTION CAN BE MADE WITH TELEGRAPH LINE.

the flashes through the telescope. The electrical connections are shown in figures 17 and 23.

As the mirrors in the receiver which reflect the slit of light from the flash box back into the telescope are quite small, the slits will not be seen in the telescope until the flash apparatus is almost exactly in the correct position. To find this position it is best to use the telescope directly, that is, without the prism on the objective end. Hold a hand lamp near the telescope in such a position that the light will shine on the mirrors in the receiver, and then shift the eye until the image of the light can be seen in the mirrors. Now move the flash apparatus horizontally until it is midway between the hand lamp and the eye. Next hold the lamp just below the flash box, and again find its image in the mirrors. Adjust the height of the apparatus until the image of the light will be seen when sighting along the top of the telescope. Since the angle of incidence and reflection are equal, the apparatus to be in perfect adjustment must be at such a height that a point midway between the objective of the telescope and the opening in the flash box will be on a level with the center of the mirrors in the receiver.

The final adjustment of the position of the flash apparatus should be made while looking through the telescope. Open the shutter in the front of the flash box and focus the telescope approximately by running out the eyepiece carriage a little more than halfway. Then move the telescope slowly up and down and sideways, a very little at a time, and keep the telescope pointed at the mirrors until the slits can be seen. The slits will probably appear very much blurred at first, due to the telescope not being accurately focused. Do not mistake for the real images the secondary image caused by reflection from the window in the front of the receiver. The latter is usually quite dim and may be distinguished by the fact that it is always stationary.

After the slits have been found in the telescope and the telescope has been focused on them, the final adjustment of the stationary mirror should be made as explained on page 41. Then focus the telescope on the mirrors themselves. This will cause the slit images to become broad and blurred and almost to cover the face of the mirrors. Adjust the position of the flash apparatus both horizontally and vertically until the blurred images fall on the two mirrors symmetrically. When the telescope is refocused on the slits they should now appear equally bright, and the moving slit will have the same range of movement above and below the stationary slit before passing out of the field of view. This means that in the observations for coincidences the moving slit will appear in the telescope about the same interval of time before a coincidence no matter whether moving up or down, and this interval will be long enough in either case that

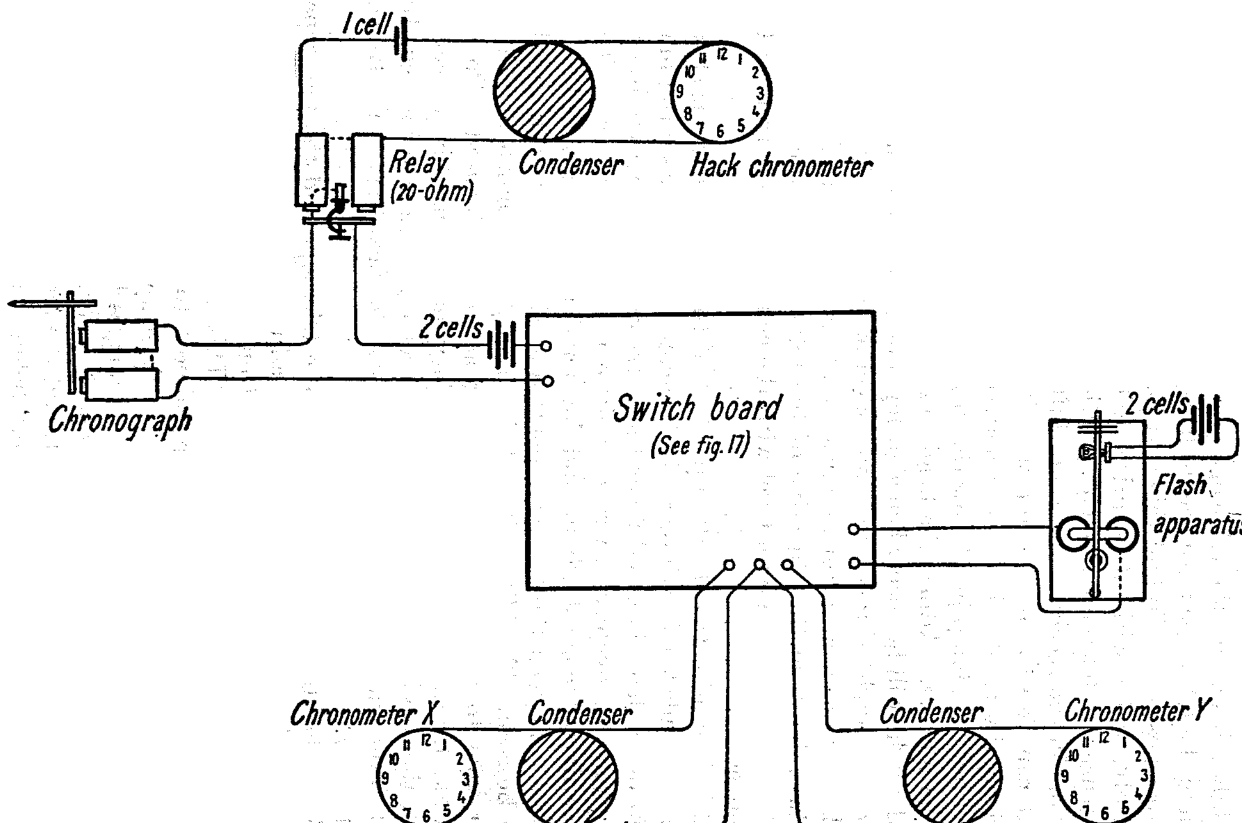

Fig. 24.—DIAGRAM OF ELECTRICAL CONNECTIONS IN PENDULUM ROOM DURING COMPARISON OF CHRONOMETERS.

there will be little danger of missing the coincidence. The apparatus should be clamped very firmly after the adjustment has been satisfactorily completed.

ELECTRIC CONNECTIONS.

Use a good grade of rather heavy insulated wire for the various electric circuits. It will save much confusion if wire with a different color of insulation can be used for each of the main circuits. Study the diagrams (figs. 17 and 23 to 26), and make the connections exactly as indicated. A little extra care in the wiring is well worth while. A short circuit or a bad connection is sometimes very difficult to find and may interrupt the observations long enough to spoil a whole day's work.

Keep the platinum points of the relays clean and bright by rubbing them occasionally with very fine emery paper. All the relays except one are arranged as break-circuit relays, the circuit through the points being broken whenever the current through the coils is interrupted. The exception is the 150-ohm relay connected in the telegraph line. (See figs. 23 and 25.) The ticks of the Naval Observatory clock come over the wires as make-circuits, and in order to produce a satisfactory record on the chronograph they must be made to cause breaks in the chronograph circuit. The motion of the armature of the relay is limited by two screws, one with a platinum tip and the other with an insulated tip. These two screws must be in reversed positions in the 150-ohm relay to what they are in the other relays, in order to change the make-circuit to a break-circuit.

When the gravity station is too far from the telegraph office for a direct connection by wire between them, it is necessary to set up a chronograph in the telegraph office to be used in recording the noon signals. A diagram of the wiring for this chronograph is given in figure 25. Note that the chronometer is not put directly in the chronograph circuit but operates that circuit through a 20-ohm break-circuit relay. Two cells are used for the chronograph but only one for the chronometer circuit. The wiring of the pendulum room remains the same as shown in figure 23 except that the chronograph is connected directly to the switchboard instead of through the points of the 150-ohm relay as indicated in the diagram. Figure 24 shows the connections during the chronometer comparisons.

Always be sure to connect a condenser across the terminals of a chronometer before putting it into a circuit. This is to prevent sparking and consequent injury to the contact points of the break-circuit contact in the chronometer. Never use more than one strong cell in a chronometer circuit which contains but one 20-ohm relay. If the cells are weak, two may be used without damage, but it is always preferable to use one good cell. Remember in connecting a chronometer that one of the binding posts is insulated from the bowl, and be sure this insulation is not rendered ineffective by a faulty connection.

TESTING THE CIRCUITS.

The number of cells indicated in figures 17 and 23 to 26 for the various circuits is the correct number when new or strong cells are used. More will be needed if the cells become very weak, but it is usually much better to discard the old ones and put in new. A small pocket voltameter is very useful for testing the cells.

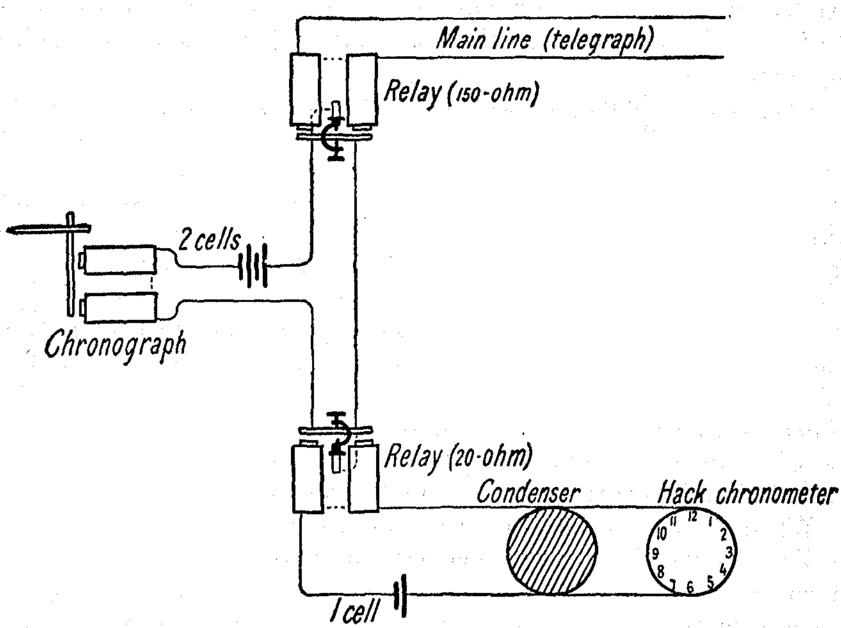

Fig. 25.—DIAGRAM OF ELECTRICAL CONNECTIONS IN TELEGRAPH OFFICE WHEN THE PENDULUM ROOM IS TOO FAR AWAY FOR CONNECTION BY WIRE.

When the circuits have all been connected, test them to make sure they are working properly. Adjust the tension of the springs on the various relays until there is a strong sharp click in each case, and adjust the screws which control the motion of the armature of each relay until the maximum motion is not more than one-sixteenth of an inch with the armature coming as close as possible to the ends of the magnets without actually touching them. See that the screws which hold the pivots of the armature are just tight enough to allow the armature to work freely without undue looseness.

Start the pendulum swinging, and watch the moving slit for a minute or two with first one chronometer and then the other operating the shutter. If the slit has an erratic motion it may be due to dust or undue friction in the shutter slides, to tightness or looseness

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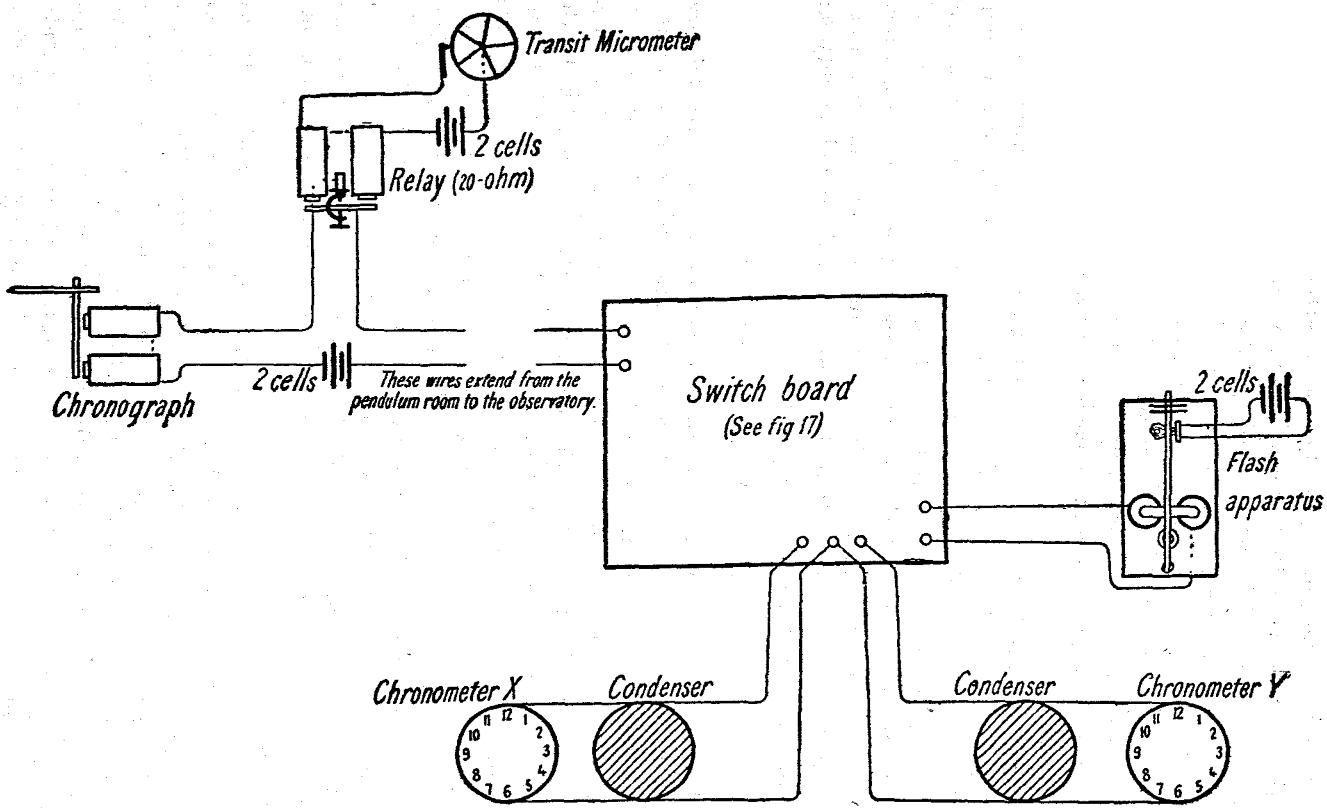

Fig. 26.—DIAGRAM OF ELECTRICAL CONNECTIONS WHEN STAR OBSERVATIONS ARE USED TO OBTAIN THE CHRONOMETER RATES.

in the pivots of the armature operating the shutters, or to a bad adjustment of the tension of the spring attached to this armature. Another possibility is that, due to excessive wear, the armature may come in contact with the ends of the magnets and thus prevent a quick and uniform release when the circuit is broken by the chronometer. The distance between the armature and the ends of the magnets can not be adjusted for this relay as for the relays on the switchboard, and the easiest way to remedy the trouble is to paste a piece of paper on the end of each magnet or on the face of the armature.

If not absolutely necessary, do not change the tension of the spring of the armature in the flash apparatus after the observations have been started unless it can be done after finishing the last set of readings on one swing and before beginning the next swing. A change in the tension of this spring changes the speed with which the shutters move and affects the results in the same manner as an error in the chronometer rate if the change is made any time between the first and last observations of a swing.

MANIPULATION OF SWITCHES.

The four switches which control the various circuits are lettered *A*, *B*, *C*, and *D* in figure 17. Switches *A* and *D* have two contacts apiece and *B* and *C* one apiece. When *A* is to the right, chronometer *X* will operate the left hand and middle relays, and switch *B* can be either on or off without affecting anything. When *A* is to the left and *B* to the right, chronometer *Y* will operate the left-hand and middle relays. When *A* is at neutral and *B* to the right, both chronometers *X* and *Y* will operate these same two relays. Since the left-hand relay is connected with the chronograph circuit, either chronometer or both can be made to record on the chronograph by manipulation of the two switches *A* and *B*. Using these same switches in the manner described and having switch *D* to the left will cause either chronometer or both to operate the circuit through the flash apparatus.

At those stations where time observations are made with the transit it is sometimes desirable to keep the *X* chronometer operating the chronograph circuit and at the same time to use either chronometer to operate the flash apparatus. To do this it is necessary to keep switch *A* turned to the right and use switches *C* and *D* for the flash apparatus. When *C* is to the right, chronometer *Y* will operate the right-hand relay. The flash apparatus will then be operated by chronometer *Y* when *D* is to the right and by chronometer *X* when *D* is to the left. It is impossible with this arrangement of the switches to operate the flash apparatus with both chronometers at the same time.

CHRONOGRAPH.

A description of the chronograph and some of its main adjustments is given in U. S. Coast and Geodetic Survey Special Publication No. 14. The chronograph should be mounted on a level shelf or stand 1 foot wide and about 2½ feet long, elevated about 3 feet above the floor. The shelf should have a hole 2 or 3 inches square near the left-hand end for the cable which supports the driving weights. For convenience in writing the necessary data on the moving chronograph sheet, the chronograph should be placed with the pen armature and its driving mechanism toward the back of the shelf. See that all pivots, especially those of the drum and governor, are free from dust and grit. Use a very little watch oil where necessary. Keep the pivots of the armature which operates the pen and the pivots of the penholder just tight enough so that they work freely but without looseness.

REGULATING THE CHRONOGRAPH.

The speed of the drum is controlled by the governor and should be regulated so that the drum will make exactly one complete revolution per sidereal minute at single speed or two revolutions at double speed. Additional driving weights are always needed for the double speed. When the governor is first released, the speed continually increases until the governor balls have moved far enough away from the axis of revolution to cause a small knife-edge projection on one of them to strike a small hook. The effect of the friction at the base of the weight attached to the hook and the inertia of the weight cause the speed to decrease continually until the hook is released. The speed then increases again until the hook is engaged, decreases until it is released, and so on. The total range of variation of the speed is, however, surprisingly small, so small in fact that the speed can be assumed to be uniform in reading the chronograph sheet. The speed is regulated by screwing or unscrewing the weights which are above the governor balls and attached to the same arm. This moves them nearer to or farther from the axis, and thus decreases or increases, respectively, the critical speed at which the hook is engaged. The speed at which the hook is engaged may also be changed by loosening the nuts which hold the governor ball having the knife-edge projection and turning the ball slightly on its axis. The most uniform motion of the drum is obtained when the adjustment is such that the hook is engaged and released at intervals varying from a half second to two seconds of time.

To put a sheet of paper on the drum, first release the gears by means of the little plug in the side of the gear box, then turn the drum to the desired position and wind on the paper. The sheet should be tight and smooth on the drum and with the right edge on top to permit the pen to run over it smoothly. A little talcum on the hands will prevent soiling the sheet while putting it on the drum.

MAKING THE OBSERVATIONS.

The observations required for a determination of gravity are made in the following order: First, the errors of the gravity chronometers are obtained either by star observations or by comparison with the time signals from the Naval Observatory. Then the first pendulum is started swinging, the coincidence observations are made, and the temperature, pressure, and arc readings are obtained. About 11½ hours later another set of coincidence, temperature, pressure, and arc readings is taken, thus completing one swing. The pendulum is then stopped, raised off the knife-edge by means of the lifting screw at the back of the receiver, lowered again carefully, and started swinging. The readings for the second swing are made in exactly the same manner as for the first swing, one set being taken immediately after the pendulum is started and the other 11 hours or more later. When the Naval Observatory time signals are used to get the chronometer corrections, the second set of observations of the second swing should be started just early enough so as not to interfere with comparison of the chronometers and the recording of the signals, which completes the first day's observations.

If the same pendulum is used for the second day's observations, it will not be necessary to open the receiver, but simply to raise and lower the pendulum, then start it swinging and make the observations exactly as for the first day. After the time comparisons at the end of the second day, it will be necessary to change the pendulum, if that was not done at the end of the first day.[7] This means, of course, that the receiver must be opened. The pendulum in the corner of the receiver is then put on the knife-edge in place of the one previously observed, and another pendulum is put in the corner of the receiver. The second pendulum to be swung may require a slightly different adjustment of the stationary mirror from that used for the first one, and this adjustment should be made before the receiver is sealed.

The observations for the third day are the same as for the other two days, and the whole series for the station is completed by a final determination of the chronometer errors at the end of the third day's work. If, however, star observations are used in place of the time signals and cloudy weather interferes, it will be necessary to continue the pendulum observations until the final time determination can be made.

This completes a general outline of the observations at a gravity station. A few of these various operations will be taken up in more detail in the following pages.

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[7] The pendulums should not be changed when there is a failure of the time signals. In other words, when Naval Observatory time signals are used, the work with each pendulum should begin and end with a determination of the chronometer errors.

TIME OBSERVATIONS AND COMPARISON OF CHRONOMETERS.

The object to be accomplished by the time observations, or by the comparison of the chronometers with the time signals of the Naval Observatory, is to find the errors of the two gravity chronometers on the different days that the pendulums are observed, so that their rates may be computed. This fact must be kept clearly in mind, whatever method may be used for finding these errors. Any lag in the time signals will not affect the computed chronometer rates if it is the same amount each day, and the personal equation in recording the wireless signals is also automatically eliminated from the results if it remains constant from day to day.

COMPARISON OF CHRONOMETERS.

When time signals by telegraph are used and the telegraph office is too far from the pendulum room for a direct wire connection, the procedure is as follows: Each of the two gravity chronometers is compared with the hack chronometer by having each record on the chronograph at the same time as the hack chronometer. If they are not interfering, that is, beating at the same instant, all three chronometers may be put on the chronograph at the same time, and three or four minutes of such a record will be sufficient. In case of interference, however, two of the chronometers should be recorded on the chronograph for two or three minutes, then another two, and so on until a record is obtained from which the necessary data can be read. A little experience will indicate the best method for combining the chronometers to get the best results.

The face reading of each chronometer for at least one of the minute breaks appearing on the chronograph sheet should be written on the margin of the sheet with a leader line to indicate which break is meant. In case of interference there should also be shown the exact point where the record of each chronometer begins and ends. The record for several days may be put on the same sheet by shifting the pen carriage a couple of inches each time, but the date must be shown without fail on each day's record.

It is difficult to write on the moving chronograph sheet, especially when the drum is revolving at double speed. The pressure on the drum of the pen or pencil used for making the notations must not be heavy enough to retard the motion of the drum. For this reason it is usually more satisfactory to use a fountain pen for this purpose as well as for the automatic record, since slight pressure will produce legible marks. If a pencil is used and the marks are dim, go over them again with a pen or pencil at the end of the comparison when the chronograph has been stopped. Examine each chronograph sheet before sending it to the office to see that all the notations are legible and intelligible and that the dates are properly shown. Nothing is much harder to interpret than a chronograph sheet on which the notations are lacking or illegible.

As soon as the chronometers have been compared as described above, the hack chronometer and condenser should be disconnected and taken to the telegraph office to be connected in the circuit there as shown in figure 25. The hack chronometer must be handled very carefully on this trip to guard against sudden jars, and especially must any sudden twist be avoided as it is likely to cause the chronometer to lose or gain a fraction of a second. The chronometer should be placed in the special carrying case made of heavy felt and leather to protect it from extreme variations of temperature while on the way to and from the telegraph office. It is very important that the rate of this chronometer remain constant until it has been returned to the pendulum room and been compared again with the gravity chronometers.

TIME SIGNALS.

The noon signals from the Naval Observatory start at 11:55:01 on the seventy-fifth meridian time. The twenty-ninth and fifty-fifth to fifty-ninth seconds are omitted for each minute and the fiftieth to fifty-ninth seconds are omitted on the last minute before the noon signal. The noon signal itself is a whole second or more in duration which makes it easily distinguished on the chronograph record.

It is well to have all connections made and the chronograph started before the time signals begin coming in, in order that there will be a complete record of the signals, together with the breaks of the hack chronometer, for the whole five minutes. This is not absolutely necessary, however, as the last minute or two of the signals will be sufficient in an emergency. The noon tick itself is the most important of all and ordinarily is the only one read from the sheet unless there is some trouble with the record. All the necessary notations on the chronograph sheet should be made clearly and legibly.

After the hack chronometer has been thus compared with the noon signals it should be taken back to the pendulum room, and another comparison between it and the gravity chronometers should be recorded on the chronograph there. This will give all the necessary data for computing the relative rates of the three chronometers for the interval between the two comparisons, and from these rates can be interpolated the time of the noon signal as it would have been recorded had the two gravity chronometers been connected with the chronograph in the telegraph office. (See sample computations on p. 67.)

Similar comparisons of the chronometers and the noon signals are made each day that the pendulum observations are continued. All this time work should be done as quickly as possible in order that there may be the shortest possible interval between the pendulum swings.

If the pendulum room is in the telegraph office, or close enough for a connection by wire between the two, the time comparisons are very much simplified. All that is necessary is to put the two gravity chronometers on the chronograph at the same time the noon signals are being recorded, and all the necessary data are obtained by the one operation. If there is interference between the two chronometers their record should be continued two or three minutes after the end of the time signals, alternating the chronometers on the chronograph if that is necessary in order to make sure which one is ahead. If there is no switch provided to cut the telegraph relay out of the chronograph circuit, the armature of the relay can be wedged over to make a continuous contact during this subsequent comparison of the chronometers. Follow carefully the instructions given on page 53 for marking the chronograph sheet.

ERRORS OF CHRONOMETERS BY STAR OBSERVATIONS.

In case the errors of the chronometers are determined by star observations, which was the usual method until a few years ago, the chronograph should be placed in the observatory and a wire connection should be made between the switchboard and the chronograph exactly as if the latter were in the pendulum room. (See fig. 26.) The chronograph should be operated by one of the gravity chronometers while the star observations are being recorded, and a comparison of the two gravity chronometers should be made both at the beginning and at the end of the time observations. During these comparisons one observer should watch the chronograph while the other operates the switchboard. The bell on the switchboard can be rung by making a short circuit in the observatory, and in this way signals can be sent to the observer in the pendulum room to inform him when to change the switches. Signals in the reverse direction can be sent if necessary by rattling the armature of the left-hand relay on the switchboard. The observer operating the switches should record the time as given by the hack chronometer each time the switches are changed and later should make a face comparison of the three chronometers. This is necessary in order to interpret the chronograph sheet, since the chronometers will be too far away for the usual notations to be made on the chronograph sheet while the comparison is being recorded.

COINCIDENCE OBSERVATIONS.

A coincidence observation consists merely in noting the time on the hack chronometer when the slit of light reflected from the moving mirror of the pendulum is in the same horizontal line with the slit reflected from the stationary mirror. If the moving slit is apparently moving in an up direction as seen in the telescope the letter *U* should be inserted in the record, and if in a down direction the letter *D*. (See sample record on p. 59.) A set of coincidence readings must consist of not less than two "downs" and one "up" or two "ups" and one "down."

Each pendulum is made of such a length that its period of vibration is slightly greater than one-half of a sidereal second or approximately 0.5008 sidereal second. It will therefore lose one beat on the chronometer every four or five minutes. Let it be supposed that at the instant the shutter is opened by the chronometer the pendulum is swinging toward the observer and has just reached the middle point of its vibration. The two slits of light will appear as one straight line in the telescope and will constitute a coincidence. One second later when the shutter is again opened the pendulum will have made the equivalent of slightly less than two complete vibrations and will be swinging toward the observer again but will not quite have reached its mid-position. The moving mirror will therefore reflect the slit of light slightly lower than will the stationary mirror. The telescope is an inverting telescope, and so the moving slit will appear slightly above the stationary one, and this will indicate that the coincidence observed on the preceding second should be designated with a *U*. On each succeeding second the moving slit will appear a little farther above the stationary slit until it passes out of the field of view. Three or four minutes later it will again appear in the field of view but moving in the opposite direction or down. Each time the shutter is opened the pendulum is now a little past its mid-position and swinging away from the observer. In the telescope the moving slit will appear above the stationary one but will move down a little with each second until the two coincide to give a "*D*" coincidence.

In the interval between the "up" and "down" coincidences there has been one less oscillation of the pendulum than the number of half-seconds registered by the chronometer, and it is a simple matter to compute the number of oscillations of the pendulum during this time. The interval from an "up" to a "down" may be several seconds different than the interval from a "down" to an "up," especially if the stationary mirror has not been carefully adjusted, but the mean of the two intervals remains fairly constant during a swing. The total number of coincidences during a 12-hour swing may be computed by simply dividing the total time between the extreme coincidences of the swing by the average time of one coincidence, and thus will be obtained the total number of times the pendulum has lost a beat on the chronometer during that period.

Unless the two gravity chronometers are beating exactly together their coincidences with the pendulum will not occur at the same time, and it will be possible to make the readings for the second chronometer during the intervals of waiting for the first. The readings for the two chronometers should be recorded on opposite pages of the record book as shown in the sample record on page 59.

The hack chronometer should be near the flash apparatus so that the face can be easily read. Just before the coincidence occurs, glance at the second hand of the chronometer, and pick up the count, using the click of the shutter of the flash apparatus, however, in place of the tick of the chronometer to hold the count. Then look through the telescope and watch the slits while keeping up the count. There may be several seconds, especially near the end of the swing, when the slit from the pendulum mirror is moving slowly, during which the two slits are apparently in the same straight line. Take the mean of the first and last second of the continuance of the coincidence, or else on the second that seems to give a perfect coincidence count aloud, as that will help in remembering, but continue to count and watch a few seconds longer, and correct your first judgment if that seems desirable. First record the number of the second at which the coincidence occurred, then look at the chronometer to get the hours and minutes. Beware of parallax through the thick glass covering the face of the chronometer, and remember in reading the minutes that several seconds have elapsed since the coincidence and the minute may have changed.

The times of coincidences are actually read on the hack chronometer as stated above, but these readings must be corrected for the relative rate between the hack chronometer and the gravity chronometer operating the shutter in order that the computed value of the coincidence interval will be for the latter. The simplest method of finding this relative rate close enough to correct the readings is to note the second on the hack chronometer when the minute break of the gravity chronometer occurs by listening to the shutter of the flash apparatus and at the same time watching the hack chronometer. A record of the time of this minute break should be made for both of the gravity chronometers for each set of coincidence observations. (See sample record on p. 59.) The method of making the correction will be explained later in the chapter on computations.

TEMPERATURE, PRESSURE, AND ARC READINGS.

For the first set of observations after pumping out the receiver, a little time should be allowed for conditions to become settled before the temperature, pressure, and arc readings are made; therefore the coincidence observations should be made first. At other times, however, these readings may be made before or after or even during the coincidence observations. They should be made as quickly as possible so that the heat from the hand lamp or the body of the observer will not disturb the temperature conditions. The thermometer should be read to the nearest five-hundredths of a degree and the manometer to tenths of a millimeter. Have the eye nearly on the level of the mercury each time so that there will be no parallax, and use the top of the mercury meniscus as the point to be read. When the case is first pumped out, the convex meniscus will be in the open end of the manometer tube, but as soon as there has been a slight increase of pressure the convex meniscus will be transferred to the closed end.

Before attempting to make the arc readings, be sure that the microscope used for this purpose is properly adjusted and focused. There are two sliding adjustments near the eyepiece for focusing the microscope, one to give a clear focus of the cross hair, and the other to produce a distinct image of the projection on the pendulum bob and of the scale just under it. If the cross hair is not parallel with the graduations, it can be made so by turning the eyepiece end of the microscope.

After the microscope has been adjusted it should be moved to one side of the center by means of the screw which controls its motion until the cross hair coincides with the center of the white line on the projection at the bottom of the bob at the instant the pendulum reaches the limit of its vibration. Due to the quick motion of the pendulum it will be necessary to watch it four or five seconds to get a good setting of the microscope. The position of the cross hair on the scale should then be read to the nearest tenth of a millimeter, calling the center of the scale zero for reading in either direction. The microscope should then be shifted by means of the screw until the cross hair is at the opposite end of the motion of the pendulum and another reading made. The sum of the two readings is the amplitude of vibration of the pendulum.

Much the same procedure is used when the pendulum is started swinging. If it is desired to have an initial arc of about 8 mm. the microscope is moved to the left until the scale reading is 4 mm. or slightly more. Then the pendulum is pushed carefully to one side by means of the starting lever, while being watched through the microscope, until the white line on the bottom of the bob coincides with the cross hair. Then if the pendulum is released quickly the total arc of vibration will be about 8 mm. Be careful not to jar the receiver when releasing the pendulum.

Sample pages of the record book for gravity observations follow.

(Left-hand page.)

(Right-hand page.)

DEPARTMENT OF COMMERCE,
U. S. COAST AND GEODETIC SURVEY,
FORM 497.

Gravity. *Observations.*

[Station: *Newcastle, Wyo.* Date: *Oct. 20, 1919.* Swing: *1.*]

[Observer: *E. W. E.* Pendulum: *A4, Direct.*]

Chronometer: *1836.*				Arc.	Pressure.	Temperature.		Chronometer: *1842.*				Mean time and remarks.
U or D.	Coincidence.	Interval.	Minute break.					U or D.	Coincidence.	Interval.	Minute break.	
	h. m. s.	*m. s.*	*s.*	*mm.*	*mm.*	° *C.*			*h. m. s.*	*m. s.*	*s.*	
U	*22 00 31*			*3.9*	*26.5*			*D*	*22 02 55*			
D	*22 05 29*	*4 58*	*47*	*3.8*	*26.0*	*16.70*		*U*	*22 07 53*	*4 58*	*27*	*11:45 a. m.*
U	*22 10 23*	*4 54*						*D*	*22 12 55*	*5 03*		
		4 56		*7.7*	*52.5*					*5 00.5*		
	3 10					[8] *16.95*						*4:50 p. m.*
U	*8 23 16 (17)*			*0.9*	*33.5*			*U*	*8 21 26 (25)*			
D	*8 28 18 (19)*	*5 02*	*46*	*0.8*	*33.5*	*17.30*		*D*	*8 26 36 (35)*	*5 10*	*28*	*10:05 p. m.*
U	*8 33 09 (10)*	*4 51*						*U*	*8 31 26 (25)*	*4 50*		
		4 56		*1.7*	*67.0*					*5 00*		

[8] Additional temperature reading. (See par. 8, p. 33.)

FLEXURE OBSERVATIONS.

The flexure observations at a station may be made either before or after the regular gravity observations, although the usual practice is to make them after. The essential consideration is that the stability of the receiver must be the same as during the gravity observations. There will be no difficulty in making the flexure determination if the instructions given in Appendix 6 of the Report for 1910 are followed carefully. It must be remembered that the fringes are affected not only by the rhythmic motion of the receiver due to the vibration of the pendulum, but also by any jar in the ground around the pier on which the receiver is supported; and so the observations must be made when disturbances caused by teams, autotrucks, or street cars are not too frequent. At the base station in Washington, for instance, about the only time the flexure observations can be made successfully is between midnight and 3 a. m., but of course the conditions here are much worse than at the usual field station. A sample record and computation of flexure is given below. The gravity record book can be used for this purpose by changing the headings.

Flexure observations and computations.

[Newcastle, Wyo., Oct. 23, 1919. E. W. Eickelberg, observer.]

Semiarcs.		Total arc.	Width of fringe.			Movement of fringe.			Displacement per 5-mm arc.
			A	B	Diff.	A	A	Diff.	
mm	*mm*	*mm*	*Scale divisions.*			*Scale divisions.*			*Wave lengths.*
8.1	8.1	16.2	0.0	2.0	2.0	0.1	0.5	0.4	
			0.8	3.0	2.2	0.4	0.8	0.4	$\frac{0.36}{2.16}\times\frac{5}{15.7}=0.053$
			0.5	2.5	2.0	0.2	0.5	0.3	
			1.4	4.0	2.6	0.2	0.6	0.4	
		15.2	1.0	3.0	2.0	0.0	0.3	0.3	
Mean		15.7			2.16			0.36	
		15.2	0.5	2.2	1.7	0.4	0.6	0.2	
			0.0	1.4	1.4	0.0	0.2	0.2	$\frac{0.20}{1.38}\times\frac{5}{14.6}=0.050$
			0.7	2.0	1.3	0.7	0.9	0.2	
			0.3	1.5	1.2	0.2	0.4	0.2	
7.0	7.1	14.1	0.8	2.1	1.3	0.4	0.6	0.2	
Mean		14.6			1.38			0.20	
3.1	3.0	6.1	0.1	1.4	1.3	0.3	0.5	0.2	
			0.6	1.8	1.2	0.6	0.7	0.1	$\frac{0.14}{1.28}\times\frac{5}{6.0}=0.091$
			0.3	1.7	1.4	0.4	0.5	0.1	
			0.8	2.1	1.3	0.0	0.2	0.2	
		5.9	0.0	1.2	1.2	0.8	0.9	0.1	
Mean		6.0			1.28			0.14	
		5.9	0.6	2.8	2.2	0.6	0.9	0.3	
			0.1	2.3	2.2	0.4	0.6	0.2	$\frac{0.20}{2.18}\times\frac{5}{5.8}=0.079$
			0.0	2.1	2.1	0.0	0.2	0.2	
			0.4	2.5	2.1	0.7	0.8	0.1	
2.8	2.9	5.7	0.5	2.8	2.3	0.3	0.5	0.2	
Mean		5.8			2.18			0.20	Mean = 0.068

Correction to period = 0.068 × 173 = 11.8 in seventh decimal place.

MISCELLANEOUS OBSERVATIONS AND INSTRUCTIONS.

The chronometers should be wound every 24 hours at about the same time of day. Be very careful in winding the gravity chronometers not to disturb them in any way. The winding can be done by letting down the front side of the chronometer box. It is well to have a felt covering over the chronometer box as a protection against temperature variation for, although the chronometers are supposed to be compensated, their rates are apt to vary considerably with changes of temperature.

Put extra blankets over the chronometer box and also over the receiver if there is a sudden and extreme variation of temperature.

As soon as possible after reaching a station, place the chronometers, pendulums, and receiver in the pendulum room, in order that they may assume approximately the temperature of the room before the observations are started.

Use all three foot screws in leveling the receiver, so that each one will come into close contact with its footplate.

Make sure that the arc-reading scale in the bottom of the receiver is not loose and that it is far enough away from the bob that there is no chance of the slightest friction. See that there are no fibers on this scale which might touch the bob.

Do not handle the knife-edge plate without the guard cap in place over the knife-edge, and be sure the cap is securely attached before packing for shipment.

See that all parts of the knife-edge plate are tight and that the lifting lever works easily, but without any loose motion.

Be sure to use the correct knife-edge at each station, that is, the same one that was used during the standardization. At intervals examine carefully with a hand glass both the knife-edge and the agate planes for injuries and greasy dirt.

Do not lower the pendulum too rapidly onto the knife-edge, as this is likely to injure the knife-edge and also cause the pendulum to jar away from its correct position. Never raise the pendulum off the knife-edge while it is swinging appreciably, as that is apt to cause the agates to scrape against each other. First stop the pendulum carefully with the starting lever.

Examine occasionally the cork on the starting lever where it comes in contact with the pendulum bob to see that it is not loose and that it is free from dirt and fibers.

Be sure that the mercury column of the thermometer is unbroken and that there are no globules of mercury at the top of the tube. It is a good plan to test by using a second thermometer. If the thermometers are placed near together and under the same conditions they should register the same after allowing for the standardization correction. Sometimes the mercury column may be made continuous by shaking down, but this must be done very carefully. Never attempt to accomplish this result by heating the thermometer, but instead use a new thermometer if necessary.

Do not fail to remove the thermometer from the dummy pendulum before shipping. The breaking of the thermometer in the pendulum box would be very serious, especially with the bronze pendulums, as the mercury would amalgamate with the pendulums and alter their periods.

The pendulum thermometers should be standardized at the Bureau of Standards every year or so, to determine any variation of the zero point.

If the temperature inside the receiver during the observations is compared with the outside temperature, it should be remembered that the reduced air pressure in the receiver causes an expansion of the bulb of the thermometer, and that consequently the reading may be as much as one-third of a degree centigrade too small. This does not affect the gravity results, however, as the same conditions exist during standardization.

Do not touch with the hand the pendulum mirrors or the mirror attached to the knife-edge plate. These mirrors are made of highly polished metal and tarnish easily. Never attempt to polish them or apply any polishing material. Each mirror is an optical plane, and the polishing is an exceedingly delicate operation, which must be done only at the office by a skilled instrument maker. If absolutely necessary to remove dust, do this with a piece of dry soft chamois, using a very light touch.

Do not use mineral oil on any of the leather washers or gaskets, nor oil of any kind on rubber.

MAGNITUDE OF ERRORS.

An idea of the relative precision of the various observations at a gravity station may be obtained from the following list. An error of 0.00000025 second in the computed period of the pendulum corresponds to an error of 0.001 dyne in the computed value of g, or one-fourth the allowable probable error of a gravity determination, and may be produced by any one of the following errors:

1. An error of 10 seconds (average) in the total observed time of a 12-hour swing.

2. An error of 0.04 second in the observed daily rate of the chronometer.

3. A change of 0.02 second in the armature or slit time of the flash apparatus between the beginning and end of a 12-hour swing.

4. An error of 0.06° C. with the bronze pendulums and 0.9° C. with the invar pendulums in the mean temperature of the pendulum.

5. An error of 2½ mm. with the bronze pendulums or 3 mm. with the invar pendulums in the mean pressure inside the case.

6. An error of 0.8 mm. in the observed arc at the beginning of a 12-hour swing.

7. An error of 0.4 mm. in the observed arc at the end of a 12-hour swing.

8. An error of 0.014 fringe of sodium light in the observed flexure of the pier.

PACKING AND SHIPPING INSTRUMENTS.

The gravity apparatus as a whole is probably the most delicate set of instruments in use by this Bureau for field observations. Good results can not be obtained with this apparatus with anything less than the most careful and skillful handling. This must be kept in mind in packing the instruments for shipment. Pack every part carefully and securely, and on each box put the usual warning notice "Delicate instruments, handle carefully." Always send the apparatus by express unless specific instructions to do otherwise are received from the office.

It is a good plan to have a list of the instruments or parts that belong in each box pasted on the inside of the box in order that nothing will be left out or mislaid. Some of the boxes have special blocks to fit on the inside to hold the instruments in place. Be sure that none of these are left out or improperly placed.

Before packing the receiver, every detachable part should be removed and the top fastened down with screws or clamps. The packing box for the receiver has special holders for the microscope used in reading the arc scale and for the two three-cornered boxes containing the knife-edge plates.

For the pendulums themselves there is a special box mentioned on page 37, which in turn is packed for shipment in a padded compartment at one end of the tool box. Do not fail to remove the thermometer from the dummy pendulum and be sure that the block which fits on top of the pendulums in the pendulum box is screwed down tight.

The manometer must always be kept right side up to prevent getting any air bubbles in the closed end of the tube. It is well to put a small rubber cork or plug of cotton in the open end of the tube to keep out any dust. The manometer should then be placed, together with the thermometers, in the special carrying case. (See p. 38.) Some observers prefer to fill the open end of the manometer with mercury and seal it before carrying the manometer to the next station. The mercury should be cleaned by straining it through chamois before pouring it into the tube. A small piece of clean cheesecloth folded two or three times and tied over the end of the tube before the sealing wax is applied will greatly facilitate the removal of the wax at the next station. When pouring out the surplus mercury, care must be used not to allow too much to run out and also to prevent air bubbles from getting into the closed end of the tube. When tipping the manometer, always keep the closed end lower than the open end. If any air bubbles get in the mercury in the open part of the tube, they may be worked out by inserting a very fine iron wire and drawing the bubble out. The end of the wire must be kept in contact with the glass and also with the bubble. Bubbles in the closed end of the manometer must be boiled out, but this should not be attempted in the field as considerable experience is necessary to do this successfully without breaking the tube.

Before the chronometers are packed they must be stopped and corked. Open the case in the manner described on page 38, then stop the balance wheel by letting the end of a clean strip of paper rub against it. The same care must be used in putting the cork wedges under the balance wheel as was used in removing them. (See p. 38.) Use two thin clean wedges, one under each spoke, and insert them in such a way as not to injure or strain the pivots of the wheel. If possible, each observer should have personal instruction at the office before attempting to cork a chronometer.

After closing the case, wrap each chronometer in a large sheet of clean tissue paper to keep out the dust, then wrap it in a piece of cloth to prevent the paper from becoming torn, and finally pack securely in the shipping box with cotton and excelsior in such a way that there is a thick cushion on all sides between the chronometer and the box and between it and the adjacent chronometer.

Below is a list of instruments, spare parts, and tools which make up the gravity outfit:

GRAVITY OUTFIT LIST.

Adjusting pins, assorted, bundle	1
Air pump	1
Alarm clock	1
Alcohol lamp with stand and lens	1
Alcohol, pint	1
Aneroid barometer	1
Asbestos fiber, envelope	1
Beeswax, piece	1
Bevel	1
Bits and drills, assorted	10
Box for holding chronometers	1
Boxes to fit in stem of dummy pendulum, metal	2
Brace	1
Brush for pendulums, camel's hair	1
Bulbs for hand lamps and flash apparatus, small	12
Case, felt and leather, for carrying chronometer	1
Case for receiver, felt and leather	1
Cells for hand lamps, spare	12
Chamois skin, roll	1
Cheesecloth, yards	10
Chisel, cold	1
Chisel, wood	1
Chronographs (see p. 27)	1 or 2
Chronometers with keys	3
Condensers	3
Copper wire, good grade insulated, bundle	1
Dividers (old), pair	1
Drill, stone	1
Dry cells	15
Felt for covering chronometer box, etc., pieces	3
Filings, same material as pendulum, bottle	1
Flash apparatus complete	1

Folder for chronograph sheets	1
Footplates for receiver	3
Fountain pen for each chronograph	1
Glass, plate, of proper sizes for receiver windows, pieces	3
Glass tubes with wire through center for sodium flame, can	1
Hammer, claw	1
Hand lamps	3
Hand level	1
Handle for manometer	1
Handle for pendulums, lifting	1
Hatchet	1
Ink for fountain pen, bottle	1
Interferometer	1
Knife-edges	2
Lantern	1
Level, carpenter's	1
Lever, extension, for starting lever	1
Lever for receiver foot screws	1
Lever for stopcock	1
Manometers	3
Mercury, small bottle	1
Microscope for arc readings	1
Packing floss, bundle	1
Pendulum, dummy	1
Pendulum level	1
Pendulums, working	3
Pliers, pair	1
Plummet, brass	1
Receiver, brass	1
Relay, 150-ohm	1
Relays, 20-ohm	2
Repairs, miscellaneous, box	1
Rubber tubing, thick-walled, piece 4 to 6 feet	1
Saw, hand	1
Saw, keyhole	1
Scale for reading chronograph sheets, boxwood	1
Scale for reading chronograph sheets, glass	1
Screw driver bit	1
Screw driver, large	1
Screw driver, small	1
Screw drivers for instruments, small, assorted	3
Sealing mixture (tallow and oil), different consistencies (see p. 42) for sealing receiver, cans	6
Sealing wax, stick	1
Sealing wax (tallow, rosin, and beeswax) for receiver windows, can	1
Shellac, bottle	1
Sodium chloride, chemically pure, bottle	1
Stick with hook for manipulating wooden block (see p. 40)	1
Stoppers, rubber, for manometer, bottle	1
Switchboard, gravity	1
Switches, double-point	4
Switches, single-point	6
Tallow, package	1
Tape, 50-foot, cloth	1
Tape, 100-foot, graduated to feet and meters, steel	1
Theodolite, 7-inch (see par. 11, p. 33)	1
Thermometers	2
Tin foil, fold	1
Tissue paper, large sheets	25
Voltameter	1
Washers for air pump, oiled silk	6
Washers for openings in the receiver, assorted, leather	12
Watch oil, bottles	2
Watch, sidereal (if desired)	1
Weights for each chronograph, [9] set	1
Wicks for alcohol lamp, bundle	1
Wire for each chronograph, flexible connecting	1
Wooden block for holding extra pendulum in receiver	1
Wrench, adjusting, for starting lever	1
Wrench for round nuts	1
Wrench, monkey	1

[9] One of the recent observers used makeshift weights, such as pieces of scrap iron, bricks, or a pail of sand to avoid shipping the heavy chronograph weights.

Chapter 4.—GRAVITY COMPUTATIONS.

It is not necessary to make a complete computation of each gravity station in the field. In paragraph 16 of the general instructions on page 34 is stated the minimum amount of field computing. Much will be added to the efficiency of the work if the observer will follow rather closely the general form of the sample computations given in this chapter. It is less desirable to have the whole computation for a station finished than it is to have the work done carefully and neatly, though as a consequence only partly completed.

The sample computations in this chapter are all for the same station and are arranged consecutively so that anyone desiring to do so may follow them all the way through and see how each value is obtained. This has made it necessary to incorporate a few rather simple details, but it is hoped that even these may prove useful at times. The object of the computations is to obtain the period of each pendulum reduced to standard conditions of arc, pressure, and temperature, and corrected for the rate of the chronometer and the flexure of the receiver. From this corrected period the intensity of gravity relative to the base station is easily obtained.

COMPUTATION OF CHRONOMETER RATES.

The first case to be taken up under this heading is for time signals by telegraph when the gravity station is at some distance from the telegraph office and it is necessary to transport the hack chronometer from one to the other. The following table shows the chronometer readings as scaled from the chronograph sheet for the two comparisons, one just before and the other just after the time signals. There are also shown the time of the noon signal as read from the chronograph operated by the hack chronometer in the telegraph office and the interpolated readings of the noon signal for the two gravity chronometers.

The first thing to do in reading a chronograph sheet is to number the minutes and seconds so as to avoid confusion and lessen the possibility of erroneous readings. (See sample chronograph record facing p. 13 of Spec. Pub. No. 14.) It is well to use a different color of ink for marking the minutes and seconds of each chronometer. A convenient arrangement is to use black for the hack chronometer, red for the gravity chronometer with the smaller designating number, and green for the other gravity chronometer. Any other combination of colors may be used, but it is convenient to hold to the same combination throughout the season's work so that any one color will always refer to the same chronometer.

The beginning of each break made by the chronometers is the exact point to be used in reading a chronograph record, as the break of the circuit is sharp and definite while the make is indefinite. There is usually a slight projection at the beginning of each break caused by the momentum of the pen when it is jerked to one side, and this makes a very definite point to use in making the measurements. A glass scale with converging lines which make it possible to divide various lengths of seconds into 10 equal parts should be used for reading the fractions of seconds.

It will be noted from the following table that five readings are made for each comparison, and then the means are taken. Each line in the table gives the readings of the gravity chronometers corresponding to a certain second of the hack chronometer.

Chronometer comparisons and computations.

[Newcastle, Wyo., Oct. 21, 1919.]

	Simultaneous readings (scaled from chronograph sheet).								
	Chro. 1818.			Chro. 1836.			Chro. 1842.		
	h.	*m.*	*s.*	*h.*	*m.*	*s.*	*h.*	*m.*	*s.*
	19	50	59	22	29	13.07	23	12	30.12
		51	29		29	43.06		13	00.11
		51	59		30	13.07		13	30.11
		52	29		30	43.06		14	00.10
		52	59		31	13.07		14	30.11
Mean	19	51	59	22	30	13.066	23	13	30.110
Noon	20	31	52.61	(23	10	06.699)[10]	(23	53	23.684)[10]
	20	52	59	23	31	13.10	24	14	30.05
		53	29		31	43.11		15	00.05
		53	59		32	13.10		15	30.05
		54	29		32	43.10		16	00.06
		54	59		33	13.10		16	30.06
Mean	20	53	59	23	32	13.102	24	15	30.054

[10] Interpolated.

Interpolation for noon,

FIRST METHOD.

H. M. S.   H. M. S.   M. S.   S.

(20 31 52.61)−(19 51 59)=39 53.61 =2393.61.

(20 53 59)−(19 51 59)=62 =3720

(23 32 13.102)−(22 30 13.066)=62 00.036=3720.036

(24 15 30.054)−(23 13 30.110)=61 59.944=3719.944

M. S.

$$\frac{2393.61}{3720}\times 3720.036 = 2393.633 = 39\ 53.633$$

$$\frac{2393.61}{3720}\times 3719.944 = 2393.574 = 39\ 53.574$$

H. M. S.   M. S.   H. M. S.

(22 30 13.066)+(39 53.633)=23 10 06.699, noon on chro. 1836.

(23 13 30.110)+(39 53.574)=23 53 23.684, noon on chro. 1842.

SECOND METHOD.

H.	M.	S.		H.	M.	S.		H.	M.	S.		
(20	31	52.61)	−	(19	51	59)	=		39 m.	53.61 s.	=	39.9 m.
(20	53	59)	−	(19	51	59)	=		62 m.			
(22	30	13.066)	−	(19	51	59)	=	2	38	14.066		
(23	13	30.110)	−	(19	51	59)	=	3	21	31.110		
(23	32	13.102)	−	(20	53	59)	=	2	38	14.102		
(24	15	30.054)	−	(20	53	59)	=	3	21	31.054		

(2 38 14.102) − (2 38 14.066) = +0.036 s.

(3 21 31.054) − (3 21 31.110) = −0.056 s.

$$\frac{39.9}{62} \times 0.036 = 0.023 \text{ s.}$$

$$\frac{39.9}{62} \times (-0.056) = -0.036 \text{ s.}$$

H.	M.	S.		S.		H.	M.	S.
(2	38	14.066)	+	0.023	=	2	38	14.089
(3	21	31.110)	−	0.036	=	2	21	31.074

				H.	M.	S.		H.	M.	S.
(20	31	52.61)	+	(2	38	14.089)	=	23	10	06.699, noon on chro. 1836.
(20	31	52.61)	+	(2	21	31.074)	=	22	53	23.684, noon on chro. 1842.

EXPLANATION OF COMPUTATION FOR NOON INTERPOLATION.

The first method of computation shown above is simple interpolation. The ratio of the interval between the first comparison and noon to the total interval between the two comparisons for each of the gravity chronometers, Nos. 1836 and 1842, is made equal to the corresponding ratio for the hack chronometer, No. 1818. The full number of decimal places shown must be used throughout, and the multiplications and divisions must be carried out to seven or eight significant figures in order to obtain the necessary accuracy.

The second method, while it looks a little longer, is quicker and more accurate. The multiplications and divisions can ordinarily be made on a slide rule with sufficient accuracy, as the quantity interpolated is much smaller. The amount that each gravity chronometer has gained or lost on the hack chronometer during the interval between the two comparisons is first obtained, and then this amount is multiplied by the same ratio used in the first method, to obtain the amount the gravity chronometer has gained or lost on the hack chronometer by the time the noon signal occurred. When this proportional part is applied to the difference between the hack chronometer and the gravity chronometer at the first comparison, there is obtained the computed difference between the two at the time of the noon signal and finally the interpolated reading of the gravity chronometer.

The computation may be considerably simplified by dealing only with the seconds and writing in the hours and minutes of the result by inspection.

When the chronometers are interfering it is sometimes necessary to record on the chronograph only two chronometers at a time. It is then impossible, of course, to have simultaneous readings of all three chronometers. The table of comparisons must be made four columns wide with simultaneous readings of two of the chronometers in the first two columns and simultaneous readings of one of the first two chronometers and the third chronometer in the last two columns. A case like this is shown in the following table where there is interference between chronometers 1818 and 1842.

Chronometer comparisons and computations when there is interference.

[Newcastle, Wyo., Oct. 23, 1919.]

	Simultaneous readings.		Simultaneous readings.	
	Chro. 1818.	Chro. 1836.	Chro. 1836.	Chro. 1842.
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*
	20 10 31	22 48 46.63	22 52 00	23 35 11.38
	11 01	49 16.64	52 30	35 41.38
	11 31	49 46.63	53 00	36 11.38
	12 01	50 16.63	53 30	36 41.38
	12 31	50 46.64	54 00	37 11.39
Means	20 11 31	22 49 46.634	22 53 00	23 36 11.382
Noon	20 39 44.33	(23 17 59.969)[11]	23 17 59.969	(24 01 11.312)[11]
	20 55 01	23 33 16.65	23 39 00	24 22 11.30
	55 31	33 46.64	39 30	22 41.31
	56 01	34 16.64	40 00	23 11.31
	56 31	34 46.64	40 30	23 41.31
	57 01	35 16.64	41 00	24 11.31
Means	20 56 01	23 34 16.642	23 40 00	24 23 11.308

[11] Interpolated.

The following computation to obtain the interpolated noon readings in the table above is made by the second method described on page 68. As the hours and minutes are obtained by inspection, only the seconds are considered except in computing the ratio factor.

Interpolation for noon.

H. M. S. H. M. S. M. S. M.

(20 39 44.33)−(20 11 31)=28 13.33=28.2

(20 56 01)−(20 11 31)=44 30 =44.5

s. s. s.

46.634−31 =15.634

16.642−01 =15.642

15.642−15.634= 0.008

$\frac{28.2}{44.5}\times 0.008=0.005$

s. s. s.

15.634+ 0.005=15.639

44.33 +15.639=59.969, noon on chro. 1836.

H. M. S. H. M. S. M. S. M.

(23 17 59.969)−(22 53 00)=24 59.969=25.0

H. M. S. H. M. S. M.
(23 40 00)−(22 53 00)=47

s. s. s.
11.382−00 = 11.382
11.308−00 = 11.308
11.308−11.382=− 0.074

$\frac{25.0}{47}\times(-0.074)=-0.039$

11.382− 0.039=11.343
59.969+11.343=11.312, noon on chro. 1842.

If the telegraph office and pendulum room are near enough together to be connected by an electric circuit, the time signals can be recorded on the same chronograph as the breaks of the two gravity chronometers, and the comparisons and computations just described will be eliminated since the time of the noon signal on each of the two gravity chronometers can be scaled directly from the chronograph sheet.

CHRONOMETER RATES.

The method used in computing the daily rates from the time of the noon signals is shown by the sample computation below. The rate of the hack chronometer does not enter into the computation of the pendulum periods and so is not computed.

Computation of rates of gravity chronometers.

[Newcastle, Wyo.]

CHRONOMETER 1836.

Date.	Noon readings.	Correction for error of time signal.	Corrected noon readings.	Rate of chronometer on mean time per mean day.	Correction for gain of sidereal time on mean time per mean day.	Rate of chronometer per mean day on sidereal time.	Rate of chronometer per sidereal day on sidereal time.	Rate correction.
1919.	h. m. s.	s.	h. m. s.	m. s.	m. s.	s.	s.	
Oct. 20..	23 06 10.204	−0.18	23 06 10.024					
21..	10 06.699	−0.22	10 06.479	−3 56.455	3 56.555	+0.100	+0.100	+6
22..	(12)							
23..	17 59.969	−0.24	17 59.729	−3 56.625	3 56.555	−0.070	−0.070	−4

CHRONOMETER 1842.

Date.	Noon readings.	Correction for error of time signal.	Corrected noon readings.	Rate of chronometer on mean time per mean day.	Correction for gain of sidereal time on mean time per mean day.	Rate of chronometer per mean day on sidereal time.	Rate of chronometer per sidereal day on sidereal time.	Rate correction.
Oct. 20..	23 49 29.594	−0.18	23 49 29.414					
21..	53 23.684	−0.22	53 23.464	−3 54.050	3 56.555	+2.505	+2.498	+145
22..	(12)							
23..	24 01 11.312	−0.24	24 01 11.072	−3 53.804	3 56.555	+2.751	+2.743	+159

[12] The noon signal on Oct. 22 was too uncertain to be used in the computations.

The corrections in column 3 of the table above is for the error of the sending clock at the Naval Observatory. A tabulation of these errors for both the noon and the midnight signals for each day of the month are forwarded to this office from the Naval Observatory about the 10th of the following month.

The values in the fifth column above are obtained by taking the difference between the corrected noon readings (fourth column) for consecutive days. Due to trouble with the telegraph line on October 22 and the omission of the noon readings for that day, it was necessary to take the difference between October 21 and October 23 and divide by 2.

The correction in the sixth column, $3^m\ 56^s.555$, is to take account of the gain of sidereal on mean time and must be applied to the values in the fifth column to get the rate of the chronometer on sidereal time.

The rates given in the seventh column are the rates of the chronometer on sidereal time, but they are the rates per mean day instead of per sidereal day. In order to obtain the rates per sidereal day, given in next to the last column, it is necessary to multiply the rates per mean day by the ratio $\frac{23.934}{24}$ (or 0.99725), which is the ratio of the length of a sidereal day to that of a mean day. This ratio is so near 1 that it need not be considered for small rates, and even when the rates are as high as 3 or 4 seconds a day the effect on the period of neglecting to make use of this ratio will be only about 1 in the seventh decimal place.

The rate corrections to be applied to the period of the pendulum are computed by the formula,

$$\text{Rate correction} = +0.00001157\ R\ P,$$

where R is the rate of the chronometer in seconds on sidereal time per sidereal day (+ if losing, − if gaining) and P is the period of the pendulum. For use in the field the following approximate formula is sufficiently accurate:

$$\text{Rate correction} = +57.9\ R\ \text{(approximate)},$$

the result in this case being in the seventh decimal place of the period.

If no field computation of the periods of the pendulums has been made it will be necessary in making the office computation to find the uncorrected periods of the pendulums (see p. 72) before computing the rate corrections. The approximate periods of the pendulums used in computing the rate corrections in the preceding table are as follows:

Pendulum $A4$ (Oct. 20–21), period = 0.5008 sec.
Pendulum $A6$ (Oct. 21–23), period = 0.5006 sec.

Then,

0.5008 (+0.00001157) R = +0.000005794 R (or 57.94 R in seventh decimal place).

0.5006 (+0.00001157) R = +0.000005792 R (or 57.92 R in seventh decimal place).

COMPUTATION OF PERIODS OF PENDULUMS.

Before computing the periods of the pendulums, all additions and subtractions in the record book should be checked carefully and the readings corrected to take account of the relative rate of the hack and the gravity chronometers. The amount of this relative rate will be indicated by the recorded time of the minute breaks. In the sample record on page 59 the minute break of chronometer 1836 occurred at 47 seconds on the hack chronometer for the first set of readings and at 46 seconds for the second set of readings. The second set of readings must therefore be corrected by +1 second to take account of the relative rate. For chronometer 1842 (right-hand page of record) the minute break occurred at 27 seconds in the first set of readings and at 28 in the second set, so the correction to be applied to the second set is −1 second. The corrected readings (seconds only) are shown in parentheses.

The data to be copied from the record book on the computation sheets (see sample below) include the arc, pressure, and temperature readings, the average time between consecutive coincidences and the total intervals between the first "up" of the first set and the last "up" of the second set and between the first "down" of the first set and the last "down" of the second set. The computation of the periods is then made in the manner indicated below.

DEPARTMENT OF COMMERCE,
U. S. COAST AND GEODETIC SURVEY,
FORM 443.

Computation of period—Half-second pendulums.

[Station: *Newcastle, Wyo.* Pendulum: *A4.* Knife-Edge: *A1.* Date: *Oct. 20 and 21, 1919.*
Position: *Direct.*]

	Swing: *1.*		Swing: *2.*	
	Chronometer No. *1836.*	Chronometer No. *1842.*	Chronometer No. *1836.*	Chronometer No. *1842.*
(1) 1 coincidence,	*296* sec.	*300* sec.	*296* sec.	*300* sec.
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*
(2) Coincidence interval,	U *10 32 39*	*10 23 32*	*11 02 08*	*10 52 57*
(3) Coincidence interval,	*37959* sec.	*37412* sec.	*39728* sec.	*39177* sec.
(4) No. coincidences,	*128*	*124*	*134*	*130*
(5) 10 coincidences,	*2965.5* sec.	*3017.1* sec.	*2964.8* sec.	*3013.6* sec.
	h. m. s.	*h. m. s.*	*h. m. s.*	*h. m. s.*
(6) Coincidence interval,	D *10 22 50*	*10 23 40*	*10 52 19*	*10 53 05*
(7) Coincidence interval,	*37370* sec.	*37420* sec.	*39139* sec.	*39185* sec.
(8) No. coincidences,	*126*	*124*	*132*	*130*
(9) 10 coincidences,	*2965.9* sec.	*3017.7* sec.	*2965.1* sec.	*3014.2* sec.
(10) Mean, U and D,	*2965.7* sec.	*3017.4* sec.	*2965.0* sec.	*3013.9* sec.
	s	*s*	*s*	*s*
(11) Uncor. period,	0.50 *08444*	0.50 *08299*	0.50 *08446*	0.50 *08308*
(12) Corr. for arc,	*−16*	*− 16*	*−16*	*− 16*
(13) Corr. for pressure,	*+ 4*	*+ 4*	*+ 2*	*+ 2*
(14) Corr. for temperature,	*−76*	*− 76*	*−83*	*− 83*
(15) Sub-total,	0.50 *08356*	0.50 *08211*	0.50 *08349*	0.50 *08211*
(16) Corr. for rate,	*+ 6*	*+145*	*+ 6*	*+145*
(17) Corr. for flexure,	*−12*	*− 12*	*−12*	*− 12*
(18) Period,	0.50 *08350*	0.50 *08344*	0.50 *08343*	0.50 *08344*

Computation of period—Half-second pendulums—Continued.

	Pressure.	Temperature.		Pressure.	Temperature.
		°C. Means			°C. Means
mm	*mm.*	16.70 } 16.85 } Mean total 17.00	*mm.*	*mm.*	
Arc, { 7.7	52.5 } Mean 59.8	17.00 }	Arc, { 7.7	55.5 } Mean 62.0	17.28 } Mean total 17.18
1.7	67.0	} 17.15 } 17.30	1.7	68.5	17.08
		Therm. Corr. —.16			Therm. Corr. —.16
		Corrected temp. 16.84			Corrected temp. 17.02

It will be found that the computation above can be made more rapidly by working in horizontal lines, preferably through all six swings, than by computing each column by itself. It should be noticed also that line 3 is the same as line 7 except that the former is for the *U* interval and the latter for the *D* interval, and so these two lines should be computed at the same time. The same thing is true of lines 4 and 8 and also of lines 5 and 9.

Line 3 is the same as line 2 except that the interval is given in seconds instead of hours, minutes, and seconds. The table on page 80 will be found convenient in making this conversion.

Line 4 is derived by dividing line 3 by line 1 and taking the nearest even number. The coincidences which occur during a swing are never all observed, but if they were it would be found that they are alternately "ups" and "downs." Therefore, if the elapsed time between an "up" at the beginning of a swing and an "up" at the end of the same swing were divided by the average interval between consecutive "ups," the result necessarily would be an integer. The divisor in line 1, however, is one-half the interval between consecutive "ups," so the result obtained by dividing line 3 by line 1 will be twice as much as if the divisor were the total time between consecutive "ups" and consequently must be an even number. The same reasoning, of course, applies to the "down" coincidences. (See lines 7 and 8.) It sometimes happens that the result (line 4 or line 8) comes out very nearly an odd number, and it is difficult to decide whether it is correct to take the nearest even number above or the one below. If a wrong decision is made the computation of the periods will fail to check within the required limits; that is, the mean period of the pendulum between time determinations as derived from one chronometer will fail to check with the mean period as derived from the other chronometer. This is assuming, of course, that there are no compensating errors. It is a good plan to take an extra set of coincidence readings near the middle of the 12-hour swing at those stations where the temperature control is bad or other conditions cause the interval between coincidences to fluctuate considerably. Remember that the interval between coincidences depends not only on the period of the pendulum but also on the rates of the gravity chronometers, and so these rates must be kept as uniform as possible by handling the chronometers carefully and protecting them from temperature changes.

Line 5 of the preceding computation is obtained by dividing line 3 by line 4. This gives the average time of 10 coincidences, which is combined with the corresponding quantity in line 9 to obtain the mean of the "ups" and "downs" shown in line 10.

The uncorrected period of the pendulum in line 11 is obtained from the table on page 84 by using as an argument the average time of 10 coincidences given in line 10 of the computation. The table was computed by the formula

$$\text{Uncorrected period} = \frac{c}{2c-10},$$

where c is the average time of 10 coincidences.

ARC CORRECTION.

The correction for arc in line 12 of the computation is taken from the table on page 89, using as arguments the total arcs of oscillation at the beginning and end of the swing. The formula for computing the arc correction is known as Borda's formula and is given by

$$\text{Arc correction} = -\frac{PM}{32}\,\frac{\sin(\alpha_o+\alpha_n)\sin(\alpha_o-\alpha_n)}{\log\sin\alpha_o-\log\sin\alpha_n},$$

where P is the period of the pendulum, M is the modulus of the common logarithmic system (=0.43429), α_o and α_n are the initial and final semiarcs expressed as angles.

The scale in the receiver which is used for reading the arcs is graduated to millimeters and gives the actual distance through which the bottom of the bob moves. It is convenient, therefore, to convert the above formula as follows:

Let R = distance from knife-edge to arc scale
= 296.93 mm. for receivers A and B.

Let a_o and a_n denote the initial and final amplitudes in millimeters.

Then $a_o = 2\,R\alpha_o$

$a_n = 2\,R\alpha_n$

By substituting these values in Borda's formula, using the above value of R and putting $P=\frac{1}{2}$ as a sufficient approximation, there is obtained as a working formula,

$$\text{Arc correction} = -0.1924\,\frac{a_o^2-a_n^2}{\log_{10}a_o-\log_{10}a_n},$$

expressed in units of the seventh decimal place of the period.

PRESSURE CORRECTION.

The correction for pressure in line 13 of the computation of period (p. 72) is taken from the table on page 90 which is the pressure-correction table for the bronze pendulums. For the invar pendulums the table on page 91, which has slightly different values should be used. In either case the standard pressure to which the period of the pendulum is reduced is 60 mm. The arguments for using these tables are the mean pressure and the mean corrected temperature for the swing. The formulas used in computing the tables are as follows:

For the bronze pendulums,

$$\text{Pres. Cor.} = 0.000000101 \left(60 - \frac{Pr}{1 + 0.00367\ T^\circ}\right).$$

For the invar pendulums,

$$\text{Pres. Cor.} = 0.000000089 \left(60 - \frac{Pr}{1 + 0.00367\ T^\circ}\right),$$

where Pr is observed pressure in millimeters and T° is the temperature in centigrade degrees. The quantities outside the parentheses in the two formulas above are constants which are derived experimentally. The quantity 0.00367 is a theoretical constant used in reducing the observed pressure to a temperature of 0° C.

The presence of air in the receiver lengthens the period of the pendulum by its buoyancy, by its hydrodynamic effect, and by its viscosity. The buoyancy effect is the most important and depends not only upon the pressure of the air but also upon the amount of water vapor in the air. The pressure of water vapor is greater in relation to its density than is the case for air and so the density or buoyancy effect is less for saturated air than for dry air at the same pressure. It is safe to assume, however, that the maximum error in the computed gravity involved by not taking account of the humidity of the air within the receiver is never more than 1 in the third decimal place of dynes and probably never more than half that amount.

TEMPERATURE CORRECTION.

Next to be considered is the correction for temperature shown in line 14 of the computation of period (p. 72). This correction depends upon the material of the pendulum and is about 15 times as large for the bronze as for the invar at corresponding temperatures. The standard temperature to which the period of the pendulum is reduced is 15° C., and the formula may be stated as follows:

$$\text{Temperature correction} = K\ (15^\circ - T^\circ)$$

where T° is the temperature of the pendulum in degrees centigrade and K is a constant depending upon the material of the pendulum.

The value of K has been determined experimentally with the following values:

For the bronze pendulums,

$K = 0.00000413$ for pendulum A4.
$= 0.00000418$ for pendulums A5 and A6.
$= 0.00000419$ for pendulum B4.
$= 0.00000418$ for pendulum B5.
$= 0.00000415$ for pendulum B6.

For the invar pendulums,

$K = 0.00000028$ for pendulums B7, B8, and B9.

It should be noted that the temperature as read must be corrected for the error of graduation of the thermometer before using it in the formula above.

FINAL CORRECTED PERIOD.

The correction for rate shown in line 16 of the computation of period (p. 72) has already been computed and explained. (See p. 70.)

The correction for flexure in line 17 is taken from page 60, which shows a sample record of the flexure observations and the computation of the flexure correction. A detailed explanation of the computation of the flexure correction will be found on pages 439 to 441 of Appendix 6 of the Report of the Superintendent for 1910.

The final corrected period of the pendulum is given in line 18 of the computation (p. 72). It is derived by applying to the uncorrected period in line 11 the algebraic sum of all the various corrections. The mean of the corrected periods as given in the second and fourth columns should agree with the mean of the periods as given in the third and fifth columns. In other words, the mean period of the pendulum for the interval between time determinations should come out practically the same as computed for either one of the two gravity chronometers. This is a valuable check on the computations, for if the two means fail to agree within 3 or 4 in the seventh decimal place the computations are pretty sure to be in error unless there is a gross blunder in the observations. Theoretically, there would be no discrepancy between the mean periods as computed for the two chronometers if there were no lost time between the last observation of one swing and the first observation of the next and if the time determinations could be made exactly at the beginning of the first swing and end of the second swing. Any variation of the chronometer rates from the mean rates during these idle intervals of the pendulum affects the computed period of the pendulum, and so, unless the rates of the two gravity chronometers vary in exactly the same proportion and direction during these intervals, the mean periods of the pendulum as computed for the two chronometers will not quite agree.

COMPUTATION OF THE INTENSITY OF GRAVITY.

After the corrected periods of the pendulums have been obtained as just explained it becomes a simple matter to compute the intensity of gravity at the station by means of the formula

$$g_s = \frac{P_w^2}{P_s^2} g_w,$$

where g_s and g_w are the intensities of gravity at the new station and base station (Washington), respectively, and P_s and P_w are the periods of the same invariable pendulum at these respective stations.

The adopted value of g_w is 980.112 dynes (log = 2.9912757). The P_w used in the final computation is the mean of the two standardizations at the base station, one just before and the other just after the field season. In order not to delay the computations until the second standardization has been made, it is customary to make a preliminary computation of g_s at each station, using the P_w determined by the first standardization. This preliminary computation is shown below. The method used in correcting for the final standardization will be explained later.

Preliminary computation of the intensity of gravity.

[Newcastle, Wyo.]

Pendulum.	Number of swing.	Date.	Period of oscillation.			log P_s.
			By chro. 1836.	By chro. 1842.	Mean = P_s.	
		1919.	*s.*	*s.*	*s.*	
A4	1	Oct. 20	0.5008350	0.5008344	0.5008347	9.6996044
A4	2	Oct. 20–21	.5008343	.5008344	.5008344	9.6996041
Mean			8346	8344	8346	
A5	3	Oct. 21	.5006663	.5006678	.5006670	9.6995490
A5	4	Oct. 21–22	.5006653	.5006673	.5006663	9.6995484
A6	5	Oct. 22	.5006286	.5006252	.5006269	9.6995142
A6	6	Oct. 22–23	.5006278	.5006272	.5006275	9.6995147
Mean			6470	6469	6469	

Pendulum.	Number of swing.	log P_s^2.	log $P_w^2 g_w$.	log g_s.	g_s.	v.	v^2.
					dynes.		
A4	1	9.3993888	2.3906744	2.9912856	980.134	+0.005	0.000025
A4	2	9.3993882	2.3906744	2.9912862	980.136	+ .003	.000009
A5	3	9.3990980	2.3903825	2.9912845	980.132	+ .007	.000049
A5	4	9.3990968	2.3903825	2.9912857	980.135	+ .004	.000016
A6	5	9.3990284	2.3903209	2.9912925	980.150	− .011	.000121
A6	6	9.3990294	2.3903209	2.9912915	980.148	− .009	.000081
Mean					980.139	Sum	.000301
Probable error					± 0.002		

The preceding computation can be readily followed and needs no detailed explanation. The two sets of means in the three columns under the heading "Period of oscillation" are merely for the purpose of checking to make sure that the two chronometers give the required agreement during the intervals between time determinations. The probable error of the result is computed in the usual manner by the formula:

$$r_o = 0.67\sqrt{\frac{\Sigma v^2}{n(n-1)}},$$

where v is the residual obtained by subtracting the individual value from the mean of all, and n is the number of quantities used in getting the mean. The Σ, as usual, indicates the sum.

CORRECTION FOR RESTANDARDIZATION.

After the standardization at the close of the field season has been made the preliminary computations for the various stations may be corrected and the final values of the intensity of gravity obtained. The corrections to be applied to the preliminary values are computed as follows:

Let P_U = the value of the Washington period which was used in the preliminary computation.

And P_A = the final adopted value of the Washington period.

Then

$$\text{Correction} = +0.391\ (P_A - P_U)\ \text{(approximate)},$$

where $P_A - P_U$ is expressed in units of the seventh decimal place of period and the correction is in units of the third decimal place of dynes. The formula will give the correction within 0.0005 dyne if $P_A - P_U$ is less than 200 in units of the seventh decimal place.

Computation of restandardization corrections.

Pendulum.	Standardization periods.			$P_A - P_U$ (in units of the seventh decimal place).	Correction to g (third decimal place of dynes).
	Preseason P_U.	Post-season.	Mean P_A.		
	s.	*s.*	*s.*		
A4	0.5008404	0.5008428	0.5008416	+12	+5
A5	.5006721	.5006730	.5006725½	+ 4½	+2
A6	.5006366	.5006379	.5006372½	+ 6½	+3

Final computation of the intensity of gravity.

[Newcastle, Wyo.]

Pendulum.	Swing.	Preliminary value of *g*.	Restandardization correction.	Final value of *g*.	*V*.	V^2
		dynes.	*dynes.*	*dynes.*		
*A*4	1	980.134	+0.005	980.139	+0.003	0.000009
*A*4	2	980.136	+ .005	980.141	+ .001	.000001
*A*5	3	980.132	+ .002	980.134	+ .008	.000064
*A*5	4	980.135	+ .002	980.137	+ .005	.000025
*A*6	5	980.150	+ .003	980.153	− .011	.000121
*A*6	6	980.148	+ .003	980.151	− .009	.000081
Mean				980.142	Sum....	.000301
Probable error				±0.002		

TABLES.

The following tables will be found very useful in the gravity computations. The explanation of these tables, with the formulas used in computing them, will be found on the pages referred to at the top of each table. It should be noted that there are two tables for the pressure correction, one for the bronze pendulums, and the other for the invar pendulums. Be sure to use the right one.

NUMBER OF SECONDS IN AN INTERVAL OF TIME.

[See p. 73.]

Minutes.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
0	000	3600	7200	10800	14400	18000	21600	25200	28800	32400	36000	39600	43200
1	060	3660	7260	10860	14460	18060	21660	25260	28860	32460	36060	39660	43260
2	120	3720	7320	10920	14520	18120	21720	25320	28920	32520	36120	39720	43320
3	180	3780	7380	10980	14580	18180	21780	25380	28980	32580	36180	39780	43380
4	240	3840	7440	11040	14640	18240	21840	25440	29040	32640	36240	39840	43440
5	300	3900	7500	11100	14700	18300	21900	25500	29100	32700	36300	39900	43500
6	360	3960	7560	11160	14760	18360	21960	25560	29160	32760	36360	39960	43560
7	420	4020	7620	11220	14820	18420	22020	25620	29220	32820	36420	40020	43620
8	480	4080	7680	11280	14880	18480	22080	25680	29280	32880	36480	40080	43680
9	540	4140	7740	11340	14940	18540	22140	25740	29340	32940	36540	40140	43740
10	600	4200	7800	11400	15000	18600	22200	25800	29400	33000	36600	40200	43800
11	660	4260	7860	11460	15060	18660	22260	25860	29460	33060	36660	40260	43860
12	720	4320	7920	11520	15120	18720	22320	25920	29520	33120	36720	40320	43920
13	780	4380	7980	11580	15180	18780	22380	25980	29580	33180	36780	40380	43980
14	840	4440	8040	11640	15240	18840	22440	26040	29640	33240	36840	40440	44040
15	900	4500	8100	11700	15300	18900	22500	26100	29700	33300	36900	40500	44100
16	960	4560	8160	11760	15360	18960	22560	26160	29760	33360	36960	40560	44160
17	1020	4620	8220	11820	15420	19020	22620	26220	29820	33420	37020	40620	44220
18	1080	4680	8280	11880	15480	19080	22680	26280	29880	33480	37080	40680	44280
19	1140	4740	8340	11940	15540	19140	22740	26340	29940	33540	37140	40740	44340
20	1200	4800	8400	12000	15600	19200	22800	26400	30000	33600	37200	40800	44400
21	1260	4860	8460	12060	15660	19260	22860	26460	30060	33660	37260	40860	44460
22	1320	4920	8520	12120	15720	19320	22920	26520	30120	33720	37320	40920	44520
23	1380	4980	8580	12180	15780	19380	22980	26580	30180	33780	37380	40980	44580
24	1440	5040	8640	12240	15840	19440	23040	26640	30240	33840	37440	41040	44640
25	1500	5100	8700	12300	15900	19500	23100	26700	30300	33900	37500	41100	44700
26	1560	5160	8760	12360	15960	19560	23160	26760	30360	33960	37560	41160	44760
27	1620	5220	8820	12420	16020	19620	23220	26820	30420	34020	37620	41220	44820
28	1680	5280	8880	12480	16080	19680	23280	26880	30480	34080	37680	41280	44880
29	1740	5340	8940	12540	16140	19740	23340	26940	30540	34140	37740	41340	44940
30	1800	5400	9000	12600	16200	19800	23400	27000	30600	34200	37800	41400	45000
31	1860	5460	9060	12660	16260	19860	23460	27060	30660	34260	37860	41460	45060
32	1920	5520	9120	12720	16320	19920	23520	27120	30720	34320	37920	41520	45120
33	1980	5580	9180	12780	16380	19980	23580	27180	30780	34380	37980	41580	45180
34	2040	5640	9240	12840	16440	20040	23640	27240	30840	34440	38040	41640	45240
35	2100	5700	9300	12900	16500	20100	23700	27300	30900	34500	38100	41700	45300
36	2160	5760	9360	12960	16560	20160	23760	27360	30960	34560	38160	41760	45360
37	2220	5820	9420	13020	16620	20220	23820	27420	31020	34620	38220	41820	45420
38	2280	5880	9480	13080	16680	20280	23880	27480	31080	34680	38280	41880	45480
39	2340	5940	9540	13140	16740	20340	23940	27540	31140	34740	38340	41940	45540
40	2400	6000	9600	13200	16800	20400	24000	27600	31200	34800	38400	42000	45600
41	2460	6060	9660	13260	16860	20460	24060	27660	31260	34860	38460	42060	45660
42	2520	6120	9720	13320	16920	20520	24120	27720	31320	34920	38520	42120	45720
43	2580	6180	9780	13380	16980	20580	24180	27780	31380	34980	38580	42180	45780
44	2640	6240	9840	13440	17040	20640	24240	27840	31440	35040	38640	42240	45840
45	2700	6300	9900	13500	17100	20700	24300	27900	31500	35100	38700	42300	45900
46	2760	6360	9960	13560	17160	20760	24360	27960	31560	35160	38760	42360	45960
47	2820	6420	10020	13620	17220	20820	24420	28020	31620	35220	38820	42420	46020
48	2880	6480	10080	13680	17280	20880	24480	28080	31680	35280	38880	42480	46080
49	2940	6540	10140	13740	17340	20940	24540	28140	31740	35340	38940	42540	46140
50	3000	6600	10200	13800	17400	21000	24600	28200	31800	35400	39000	42600	46200
51	3060	6660	10260	13860	17460	21060	24660	28260	31860	35460	39060	42660	46260
52	3120	6720	10320	13920	17520	21120	24720	28320	31920	35520	39120	42720	46320
53	3180	6780	10380	13980	17580	21180	24780	28380	31980	35580	39180	42780	46380
54	3240	6840	10440	14040	17640	21240	24840	28440	32040	35640	39240	42840	46440
55	3300	6900	10500	14100	17700	21300	24900	28500	32100	35700	39300	42900	46500
56	3360	6960	10560	14160	17760	21360	24960	28560	32160	35760	39360	42960	46560
57	3420	7020	10620	14220	17820	21420	25020	28620	32220	35820	39420	43020	46620
58	3480	7080	10680	14280	17880	21480	25080	28680	32280	35880	39480	43080	46680
59	3540	7140	10740	14340	17940	21540	25140	28740	32340	35940	39540	43140	46740
60	3600	7200	10800	14400	18000	21600	25200	28800	32400	36000	39600	43200	46800

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	1800	1900	2000	2100
0	0.5013928	0.5013193	0.5012531	0.5011933
5	889	158	500	905
10	850	123	469	876
15	812	089	438	848
20	774	055	407	820
25	736	.5013021	376	792
30	699	.5012987	346	765
35	661	953	315	737
40	624	920	285	710
45	587	887	255	682
50	.5013550	.5012854	.5012225	.5011655
55	514	820	195	628
60	477	788	166	601
65	441	755	136	574
70	405	723	106	547
75	369	690	077	521
80	333	658	048	494
85	298	626	.5012019	468
90	263	594	.5011990	442
95	228	563	962	416
100	.5013193	.5012531	.5011933	.5011390

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	2200	2300	2400	2500	2600	2700	2800
0	0.5011390	0.5010893	0.5010438	0.5010020	0.5009634	0.5009276	0.5008944
1	84	89	34	16	30	73	41
2	79	84	30	12	26	70	38
3	74	79	25	08	23	66	35
4	69	74	21	04	19	63	32
5	64	70	17	.5010000	15	59	29
6	58	65	12	.5009996	12	56	25
7	53	60	08	92	08	52	22
8	48	55	.5010404	88	04	49	19
9	43	51	.5010399	84	.5009601	46	16
10	.5011338	.5010846	.5010395	.5009980	.5009597	.5009242	.5008913
11	33	41	91	76	93	39	10
12	28	37	86	72	90	35	06
13	22	32	82	68	86	32	03
14	17	27	78	64	82	28	.5008900
15	12	22	73	60	78	25	.5008897
16	07	18	69	56	75	22	94
17	.5011302	13	65	52	71	18	91
18	.5011297	08	61	48	68	15	87
19	92	.5010804	56	44	64	12	84
20	.5011287	.5010799	.5010352	.5009940	.5009560	.5009208	.5008881
21	82	94	48	36	57	05	78
22	76	90	43	32	53	.5009201	75
23	72	85	39	28	49	.5009198	72
24	66	80	35	25	46	95	68
25	61	76	31	21	42	91	65
26	56	71	26	17	38	88	62
27	51	67	22	13	35	84	59
28	46	62	18	09	31	81	56
29	41	57	14	05	27	78	53
30	.5011236	.5010753	.5010309	.5009901	.5009524	.5009174	.5008850
31	31	48	05	.5009897	20	71	46
32	26	44	.5010301	93	17	68	43
33	21	39	.5010297	89	13	64	40
34	16	34	92	85	09	61	37
35	11	30	88	81	06	58	34
36	06	25	84	78	.5009502	54	31
37	.5011201	20	80	74	.5009498	51	28
38	.5011196	16	75	70	95	48	24
39	91	11	71	66	91	44	21
40	.5011186	.5010707	.5010267	.5009862	.5009488	.5009141	.5008818
41	81	.5010702	63	58	84	38	15
42	76	.5010698	58	54	81	34	12
43	71	93	54	50	77	31	09
44	66	88	50	46	73	27	06
45	61	84	46	42	70	24	03
46	56	79	42	39	66	21	.5008800
47	51	75	38	35	62	18	.5008797
48	46	70	33	31	59	14	94
49	41	66	29	27	55	11	90
50	.5011136	.5010661	.5010225	.5009823	.5009452	.5009108	.5008787

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—*Continued.*

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	2200	2300	2400	2500	2600	2700	2800
50	0.5011136	0.5010661	0.5010225	0.5009823	0.5009452	0.5009108	0.5008787
51	31	56	21	19	48	04	84
52	26	52	17	16	45	.5009101	81
53	21	47	12	12	41	.5009098	78
54	16	43	08	08	38	94	75
55	11	38	04	04	34	91	72
56	06	34	.5010200	.5009800	30	88	69
57	.5011101	29	.5010196	.5009796	27	84	66
58	.5011096	25	92	92	23	81	63
59	91	20	88	88	20	78	60
60	.5011086	.5010616	.5010183	.5009785	.5009416	.5009074	.5008757
61	82	11	79	81	13	71	54
62	77	07	75	77	09	68	51
63	72	.5010602	71	73	06	65	47
64	67	.5010598	67	69	.5009402	61	44
65	62	93	63	66	.5009398	58	41
66	57	89	59	62	95	55	38
67	52	84	54	58	92	51	35
68	47	80	50	54	88	48	32
69	42	75	46	50	84	45	29
70	.5011038	.5010571	.5010142	.5009747	.5009381	.5009042	.5008726
71	33	66	38	43	77	38	23
72	28	62	34	39	74	35	20
73	23	57	30	35	70	32	17
74	18	53	26	31	67	29	14
75	13	48	22	28	63	25	11
76	08	44	17	24	60	22	08
77	.5011004	40	13	20	56	19	05
78	.5010999	35	09	16	53	16	.5008702
79	94	31	05	12	49	12	.5008699
80	.5010989	.5010526	.5010101	.5009709	.5009346	.5009009	.5008696
81	84	22	.5010097	05	42	06	93
82	79	18	93	.5009701	39	.5009002	90
83	74	13	89	.5009697	35	.5008999	87
84	70	09	85	94	32	96	84
85	65	04	81	90	28	93	81
86	60	.5010500	77	86	25	90	78
87	55	.5010495	73	82	21	86	75
88	51	91	68	79	18	83	72
89	46	87	64	75	14	80	69
90	.5010941	.5010482	.5010060	.5009671	.5009311	.5008977	.5008665
91	36	78	56	68	08	74	62
92	31	73	52	64	04	70	60
93	27	69	48	60	.5009301	67	56
94	22	65	44	56	.5009297	64	54
95	17	60	40	53	94	61	50
96	12	56	36	49	90	57	48
97	08	52	32	45	87	54	44
98	.5010903	47	28	41	83	51	42
99	.5010898	43	24	38	80	48	39
100	.5010893	.5010438	.5010020	.5009634	.5009276	.5008944	.5008636

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	2900	3000	3100	3200	3300	3400	3500
0	0.5008636	0.5008347	0.5008078	0.5007825	0.5007587	0.5007364	0.5007153
1	33	44	75	22	85	62	51
2	30	42	72	20	83	59	49
3	27	39	70	17	80	57	47
4	24	36	67	15	78	55	45
5	21	33	64	12	76	53	43
6	18	30	62	10	74	51	41
7	15	28	59	08	71	49	39
8	12	25	57	05	69	46	37
9	09	22	54	03	67	44	35
10	.5008606	.5008320	.5008052	.5007800	.5007564	.5007342	.5007133
11	03	17	49	.5007798	62	40	31
12	.5008600	14	46	96	60	38	29
13	.5008597	11	44	93	58	36	27
14	94	08	41	91	55	34	25
15	91	06	39	88	53	31	23
16	88	03	36	86	51	29	21
17	85	.5008300	33	83	48	27	18
18	82	.5008297	31	81	46	25	16
19	79	95	28	78	44	23	14
20	.5008576	.5008292	.5008026	.5007776	.5007542	.5007321	.5007112
21	73	89	23	74	39	19	10
22	70	86	20	71	37	16	08
23	68	84	18	69	35	14	06
24	64	81	15	66	32	12	04
25	62	78	13	64	30	10	02
26	59	75	10	62	28	08	.5007100
27	56	73	08	59	26	06	.5007098
28	53	70	05	57	23	04	96
29	50	67	03	54	21	.5007301	94
30	.5008547	.5008264	.5008000	.5007752	.5007519	.5007299	.5007092
31	44	62	.5007997	50	16	97	90
32	41	59	95	47	14	95	88
33	38	56	92	45	12	93	86
34	35	53	90	42	10	91	84
35	32	51	87	40	08	89	82
36	30	48	85	38	05	86	80
37	27	45	82	35	03	84	78
38	24	43	80	33	.5007501	82	76
39	21	40	77	30	.5007498	80	74
40	.5008518	.5008237	.5007974	.5007728	.5007496	.5007278	.5007072
41	15	34	72	26	94	76	70
42	12	32	69	23	92	74	68
43	09	29	67	21	90	72	66
44	06	26	64	18	87	70	64
45	03	24	62	16	85	67	62
46	.5008500	21	59	14	83	65	60
47	.5008498	18	57	11	80	63	58
48	95	16	54	09	78	61	56
49	92	13	52	06	76	59	54
50	.5008489	.5008210	.5007949	.5007704	.5007474	.5007257	.5007052

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	2900	3000	3100	3200	3300	3400	3500
50	0.5008489	0.5008210	0.5007949	0.5007704	0.5007474	0.5007257	0.5007052
51	86	08	47	.5007702	72	55	50
52	83	05	44	.5007699	69	53	48
53	80	.5008202	42	97	67	51	46
54	78	.5008199	39	95	65	48	44
55	75	97	36	92	63	46	42
56	72	94	34	90	60	44	40
57	69	91	32	88	58	42	38
58	66	89	29	85	56	40	36
59	63	86	26	83	54	38	34
60	.5008460	.5008183	.5007924	.5007680	.5007452	.5007236	.5007032
61	57	81	21	78	49	34	30
62	54	78	19	76	47	32	28
63	52	75	16	73	45	30	26
64	49	73	14	71	43	28	24
65	46	70	11	69	40	25	22
66	43	67	09	66	38	23	20
67	40	65	06	64	36	21	18
68	37	62	04	62	34	19	17
69	34	59	.5007901	59	32	17	15
70	.5008432	.5008157	.5007899	.5007657	.5007429	.5007215	.5007013
71	29	54	96	55	27	13	11
72	26	51	94	52	25	11	09
73	23	49	91	50	23	09	07
74	20	46	89	48	21	07	05
75	18	43	86	45	18	05	03
76	15	41	84	43	16	02	.5007001
77	12	38	81	41	14	.5007200	.5006999
78	09	35	79	38	12	.5007198	97
79	06	33	76	36	10	96	95
80	.5008403	.5008130	.5007874	.5007634	.5007407	.5007194	.5006993
81	.5008401	28	72	31	05	92	91
82	.5008398	25	69	29	03	90	89
83	95	22	67	27	.5007401	88	87
84	92	20	64	24	.5007399	86	85
85	89	17	62	22	96	84	83
86	86	14	59	20	94	82	81
87	84	12	57	17	92	80	79
88	81	09	54	15	90	78	77
89	78	06	52	13	88	76	75
90	.5008375	.5008104	.5007849	.5007610	.5007386	.5007174	.5006974
91	72	.5008101	47	08	83	72	72
92	70	.5008098	44	06	81	70	70
93	67	96	42	03	79	67	68
94	64	93	40	.5007601	77	65	66
95	61	91	37	.5007599	75	63	64
96	58	88	34	96	72	61	62
97	56	85	32	94	70	59	60
98	53	83	30	92	68	57	58
99	50	80	27	90	66	55	56
100	.5008347	.5008078	.5007825	.5007587	.5007364	.5007153	.5006954

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	3600	3700	3800	3900	4000	4100	4200
0	0.5006954	0.5006766	0.5006588	0.5006418	0.5006258	0.5006105	0.5005960
1	52	64	86	17	56	04	58
2	50	62	84	15	55	02	57
3	48	60	82	14	53	.5006101	55
4	46	59	81	12	52	.5006099	54
5	44	57	79	10	50	98	52
6	42	55	77	09	48	96	51
7	41	53	76	07	47	95	50
8	39	51	74	05	45	93	48
9	37	49	72	04	44	92	47
10	.5006935	.5006748	.5006570	.5006402	.5006242	.5006090	.5005945
11	33	46	69	.5006400	41	89	44
12	31	44	67	.5006399	39	87	42
13	29	42	65	97	38	86	41
14	27	40	63	96	36	84	40
15	25	38	62	94	34	83	38
16	23	37	60	92	33	81	37
17	21	35	58	91	31	80	35
18	19	33	56	89	30	78	34
19	18	31	55	87	28	77	33
20	.5006916	.5006730	.5006553	.5006386	.5006227	.5006075	.5005931
21	14	28	51	84	25	74	30
22	12	26	50	82	24	72	28
23	10	24	48	81	22	71	27
24	08	22	46	79	20	70	26
25	06	20	44	78	19	68	24
26	04	19	43	76	17	66	23
27	02	17	41	74	16	65	21
28	.5006900	15	39	73	14	64	20
29	.5006898	13	38	71	13	62	19
30	.5006897	.5006711	.5006536	.5006369	.5006211	.5006061	.5005917
31	95	10	34	68	10	59	16
32	93	08	32	66	08	58	14
33	91	06	31	64	07	56	13
34	89	04	29	63	05	55	12
35	87	02	27	61	04	53	10
36	85	.5006701	26	60	02	52	09
37	83	.5006699	24	58	.5006200	50	07
38	81	97	22	56	.5006199	49	06
39	80	95	21	55	97	47	05
40	.5006878	.5006693	.5006519	.5006353	.5006196	.5006046	.5005903
41	76	92	17	52	94	44	02
42	74	90	16	50	93	43	.5005900
43	72	88	14	48	91	42	.5005899
44	70	86	12	47	90	40	98
45	68	84	10	45	88	39	96
46	66	83	09	44	87	37	95
47	64	81	07	42	85	36	93
48	62	79	05	40	84	34	92
49	61	77	04	39	82	33	91
50	.5006859	.5006676	.5006502	.5006337	.5006180	.5006031	.5005889

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	3600	3700	3800	3900	4000	4100	4200
50	0.5006859	0.5006676	0.5006502	0.5006337	0.5006180	0.5006031	0.5005889
51	57	74	.5006500	36	79	30	88
52	55	72	.5006499	34	77	28	86
53	53	70	97	32	76	27	85
54	51	69	95	31	74	26	84
55	49	67	94	29	73	24	82
56	47	65	92	28	71	23	81
57	46	63	90	26	70	21	80
58	44	61	88	24	68	20	78
59	42	60	87	23	67	18	77
60	.5006840	.5006658	.5006485	.5006321	.5006165	.5006017	.5005875
61	38	56	83	20	64	15	74
62	36	54	82	18	62	14	73
63	34	52	80	16	61	12	71
64	32	51	78	15	59	11	70
65	31	49	77	13	58	10	68
66	29	47	75	12	56	08	67
67	27	45	73	10	55	07	66
68	25	44	72	08	53	05	64
69	23	42	70	07	52	04	63
70	.5006821	.5006640	.5006468	.5006305	.5006150	.5006002	.5005862
71	19	38	67	04	49	01	60
72	18	37	65	02	47	.5006000	59
73	16	35	63	.5006300	46	.5005998	58
74	14	33	61	.5006299	44	97	56
75	12	31	60	97	42	95	55
76	10	30	58	96	41	94	53
77	08	28	57	94	40	92	52
78	06	26	55	92	38	91	51
79	05	24	53	91	36	89	49
80	.5006803	.5006622	.5006452	.5006289	5006135	.5005988	.5005848
81	.5006801	21	50	88	34	87	47
82	.5006799	19	48	86	32	85	45
83	97	17	47	85	30	84	44
84	95	16	45	83	29	82	42
85	94	14	43	81	28	81	41
86	92	12	42	80	26	80	40
87	90	10	40	78	24	78	38
88	88	09	38	77	23	77	37
89	86	07	37	75	22	75	36
90	.5006784	.5006605	.5006435	.5006274	.5006120	.5005974	.5005834
91	82	03	33	72	18	72	33
92	81	02	32	70	17	71	32
93	79	.5006600	30	69	16	69	30
94	77	.5006598	28	67	14	68	29
95	75	96	27	66	12	67	28
96	73	95	25	64	11	65	26
97	71	93	23	62	10	64	25
98	70	91	22	61	08	62	23
99	68	89	20	59	06	61	22
100	.5006766	.5006588	.5006418	.5006258	.5006105	.5005960	5005821

PERIOD OF QUARTER-METER PENDULUM WHEN PENDULUM IS SLOWER THAN CHRONOMETER—Continued.

[Top and left-hand arguments combined give the time interval of 10 coincidences. For explanation of table, see p. 74.]

	4300	4400	4500	4600	4700	4800	4900	5000	5100
0.....	0.5005821	0.5005688	0.5005562	0.5005441	0.5005325	0.5005214	0.5005107	0.5005005	0.5004907
5.....	14	82	56	35	19	08	.5005102	.5005000	.5004902
10....	07	75	49	29	14	.5005203	.5005097	.5004995	.5004897
15....	.5005800	69	43	23	08	.5005198	92	90	92
20....	.5005794	62	37	17	.5005302	92	86	85	88
25....	87	56	31	11	.5005297	87	81	80	83
30....	80	50	25	05	91	81	76	75	78
35....	74	43	19	.5005400	85	76	71	70	73
40....	67	37	13	.5005394	80	71	66	65	68
45....	60	31	07	88	74	65	61	60	64
50....	.5005754	.5005624	.5005501	.5005382	.5005269	.5005160	.5005056	.5004955	.5004859
55....	47	18	.5005494	76	63	55	50	50	54
60....	40	12	88	71	58	49	45	46	50
65....	34	.5005605	82	65	52	44	40	41	45
70....	27	.5005599	76	59	47	39	35	36	40
75....	21	93	70	53	41	34	30	31	36
80....	14	87	64	48	36	28	25	26	31
85....	08	80	58	42	30	23	20	21	26
90....	.5005701	74	53	36	25	18	15	16	22
95....	.5005695	68	47	30	19	12	10	12	17
100...	.5005688	.5005562	.5005441	.5005325	.5005214	.5005107	.5005005	.5004907	.5004812

	5200	5300	5400	5500	5600	5700	5800	5900	6000
0.....	0.5004812	0.5004721	0.5004634	0.5004550	0.5004468	0.5004390	0.5004314	0.5004241	0.5004170
5.....	08	17	30	46	64	86	4310	37	67
10....	.5004803	12	25	41	60	82	07	34	63
15....	.5004798	08	21	37	56	78	.5004303	30	60
20....	94	.5004704	17	33	52	74	.5004299	26	56
25....	89	.5004699	12	29	48	71	96	23	53
30....	85	95	08	25	44	67	92	19	49
35....	80	90	04	21	40	63	88	16	46
40....	76	86	.5004600	17	37	59	84	12	42
45....	71	82	.5004596	13	33	55	81	09	39
50....	.5004766	.5004677	.5004591	.5004509	.5004429	.5004352	.5004277	.5004205	.5004136
55....	61	73	87	04	25	48	74	.5004202	32
60....	57	68	83	.5004501	21	44	70	.5004198	29
65....	53	64	79	.5004496	17	40	66	95	25
70....	48	60	75	92	13	36	63	91	22
75....	44	56	70	88	09	33	59	88	19
80....	39	51	66	84	05	29	55	84	15
85....	35	47	62	80	.5004401	25	52	81	12
90....	30	42	58	76	.5004398	22	48	77	08
95....	26	38	54	72	94	18	44	74	05
100...	.5004721	.5004634	.5004550	.5004468	.5004390	.5004314	.5004241	.5004170	.5004102

ARC CORRECTION.

[Correction $=-0.1924 \frac{a_o^2-a_n^2}{\log_{10}a_o-\log_{10}a_n}$. Correction is in seventh decimal place of period. See p. 74.]

Total arc at beginning of swing in millimeters.	Total arc at end of swing in millimeters.								
	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
2.5	......	......	......	......	......	......	......	−3	−2
2.6	......	......	......	......	......	......	......	4	3
2.7	......	......	......	......	......	......	......	4	3
2.8	......	......	......	......	......	......	......	4	3
2.9	......	......	......	......	......	......	......	4	3
3.0	......	......	......	......	......	......	......	4	3
3.1	......	......	......	......	......	......	−6	4	3
3.2	......	......	......	......	......	......	6	5	3
3.3	......	......	......	......	......	......	6	5	4
3.4	......	......	......	......	......	−8	6	5	4
3.5	......	......	......	......	......	8	7	5	4
3.6	......	......	......	......	......	8	7	5	4
3.7	......	......	......	......	......	9	7	6	4
3.8	......	......	......	......	......	9	7	6	4
3.9	......	......	......	......	......	9	7	6	5
4.0	......	......	......	......	−11	9	8	6	5
4.1	......	......	......	......	11	10	8	6	5
4.2	......	......	......	......	12	10	8	7	5
4.3	......	......	......	......	12	10	8	7	5
4.4	......	......	......	......	12	10	9	7	5
4.5	......	......	......	......	12	11	9	7	5
4.6	......	......	......	−15	13	11	9	7	6
4.7	......	......	......	15	13	11	9	8	6
4.8	......	......	−17	15	13	12	10	8	6
4.9	......	......	18	16	14	12	10	8	6
5.0	......	−20	18	16	14	12	10	8	7
5.1	......	20	18	16	14	12	10	8	7
5.2	......	21	19	17	15	13	11	9	7
5.3	......	21	19	17	15	13	11	9	7
5.4	......	21	20	18	16	13	11	9	8
5.5	−24	22	20	18	16	14	12	10	8
5.6	25	22	20	18	16	14	12	10	8
5.7	25	23	21	18	16	14	12	10	8
5.8	26	23	21	18	17	14	12	11	8
5.9	26	24	21	18	17	15	12	11	8
6.0	27	24	22	19	17	15	13	11	9
6.1	27	24	22	19	17	15	13	11	9
6.2	28	25	28	20	18	16	13	11	9
6.3	28	26	23	20	18	16	13	11	9
6.4	29	26	24	21	19	17	14	12	10
6.5	29	26	24	21	19	17	15	12	10
6.6	30	27	24	22	19	17	15	12	10
6.7	30	27	25	22	20	17	15	13	10
6.8	31	28	25	23	20	18	15	13	10
6.9	31	28	26	23	21	18	16	13	11
7.0	32	29	26	23	21	18	16	13	11
7.1	32	29	27	24	21	19	16	14	11
7.2	33	30	27	24	22	19	17	14	11
7.3	33	30	27	25	22	19	17	14	12
7.4	34	31	28	25	22	20	17	15	12
7.5	34	31	28	26	23	20	18	15	12
7.6	35	32	29	26	23	21	18	15	12
7.7	35	32	29	26	24	21	18	15	13
7.8	36	33	30	27	24	21	18	16	13
7.9	36	33	30	27	24	22	19	16	13
8.0	37	34	31	28	25	22	19	16	13
8.1	37	34	31	28	25	22	20	17	14
8.2	38	35	32	29	26	23	20	17	14
8.3	38	35	32	29	26	23	20	17	14
8.4	39	36	33	30	27	24	21	18	14
8.5	39	36	33	30	27	24	21	18	15

PRESSURE CORRECTION—BRONZE PENDULUMS.

[Correction $=0.000000101\left(60-\frac{Pr}{1+0.00367\,T^{\circ}}\right)$. Correction is given in seventh decimal place of period. See p. 75.]

Temperature in degrees centigrade.	Pressure in millimeters of mercury.												
	40	45	50	55	60	65	70	75	80	85	90	95	100
0	+20	+15	+10	+ 5	0	− 5	−10	−15	−20	−25	−30	−35	−40
1	20	15	10	+ 5	0	− 5	10	15	20	25	30	35	40
2	20	15	10	+ 5	0	− 5	10	15	20	25	30	35	40
3	21	16	11	6	+ 1	− 4	9	14	19	24	29	34	39
4	21	16	11	6	+ 1	− 4	9	14	19	24	29	34	39
5	+21	+16	+11	+ 6	+ 1	− 4	− 9	−14	−19	−24	−29	−34	−39
6	21	16	11	6	+ 1	− 4	9	14	18	23	28	33	38
7	21	16	11	6	+ 2	− 3	8	13	18	23	28	33	38
8	21	16	12	7	+ 2	− 3	8	13	18	23	28	33	38
9	21	17	12	7	+ 2	− 3	8	13	18	23	27	32	37
10	+22	+17	+12	+ 7	+ 2	− 3	− 8	−12	−17	−22	−27	−32	−37
11	22	17	12	7	+ 2	− 3	7	12	17	22	27	32	36
12	22	17	12	7	+ 3	− 2	7	12	17	22	26	31	36
13	22	17	12	8	+ 3	− 2	7	12	17	21	26	31	36
14	22	17	13	8	+ 3	− 2	7	11	16	21	26	31	35
15	+22	+18	+13	+ 8	+ 3	− 2	− 6	−11	−16	−21	−26	−30	−35
16	22	18	13	8	+ 3	− 1	6	11	16	20	25	30	35
17	23	18	13	8	+ 4	− 1	6	11	15	20	25	30	34
18	23	18	13	8	+ 4	− 1	6	10	15	20	25	29	34
19	23	18	13	9	+ 4	− 1	5	10	15	20	24	29	34
20	+23	+18	+14	+ 9	+ 4	− 1	− 5	−10	−15	−19	−24	−29	−33
21	23	18	14	9	+ 4	0	− 5	10	14	19	24	28	33
22	23	19	14	9	+ 5	0	− 5	10	14	19	24	28	33
23	23	19	14	9	+ 5	0	− 5	9	14	19	23	28	33
24	23	19	14	10	+ 5	0	− 4	9	14	18	23	28	32
25	+24	+19	+14	+10	+ 5	0	− 4	− 9	−13	−18	−23	−27	−32
26	24	19	14	10	5	+ 1	− 4	9	13	18	22	27	32
27	24	19	15	10	5	+ 1	− 4	8	13	18	22	27	31
28	24	19	15	10	6	+ 1	− 4	8	13	17	22	26	31
29	24	20	15	10	6	+ 1	− 3	8	12	17	22	26	31
30	+24	+20	+15	+11	+ 6	+ 1	− 3	− 8	−12	−17	−21	−26	−30
31	24	20	15	11	6	+ 2	− 3	7	12	16	21	26	30
32	24	20	15	11	6	+ 2	− 3	7	12	16	21	25	30
33	25	20	16	11	7	+ 2	− 2	7	11	16	20	25	30
34	25	20	16	11	7	+ 2	− 2	7	11	16	20	25	29
35	+25	+20	+16	+11	+ 7	+ 2	− 2	− 7	−11	−15	−20	−24	−29
36	25	20	16	12	7	+ 3	− 2	6	11	15	20	24	29
37	25	21	16	12	7	+ 3	− 2	6	11	15	19	24	28
38	25	21	16	12	7	+ 3	− 1	6	10	15	19	24	28
39	25	21	16	12	8	+ 3	− 1	6	10	15	19	23	28
40	+25	+21	+17	+12	+ 8	+ 3	− 1	− 5	−10	−14	−19	−23	−27

PRESSURE CORRECTION—INVAR PENDULUMS.

[Correction = $0.000000089 \left(60-\frac{Pr}{1+0.00367\,T^{\circ}}\right)$. Correction is given in seventh decimal place of period. See p. 75.]

Temperature in degrees centigrade.	Pressure in millimeters of mercury.												
	40	45	50	55	60	65	70	75	80	85	90	95	100
0	+18	+13	+ 9	+ 4	0	− 4	− 9	−13	−18	−22	−27	−31	−36
1	18	14	9	+ 5	0	− 4	9	13	18	22	26	31	35
2	18	14	9	+ 5	0	− 4	8	13	17	22	26	31	35
3	18	14	9	5	+ 1	− 4	8	13	17	21	26	30	35
4	18	14	10	5	+ 1	− 4	8	12	17	21	26	30	34
5	+18	+14	+10	+ 5	+ 1	− 3	− 8	−12	−17	−21	−25	−30	−34
6	19	14	10	6	+ 1	− 3	8	12	16	21	25	29	34
7	19	14	10	6	+ 1	− 3	7	12	16	20	25	29	33
8	19	14	10	6	+ 2	− 3	7	11	16	20	24	20	33
9	19	15	10	6	+ 2	− 3	7	11	16	20	24	28	33
10	+19	+15	+10	+ 6	+ 2	− 2	− 7	−11	−15	−20	−24	−28	−32
11	19	15	11	6	+ 2	− 2	6	11	15	19	24	28	32
12	19	15	11	7	+ 2	− 2	6	11	15	19	23	28	32
13	19	15	11	7	+ 2	− 2	6	10	15	19	23	27	32
14	20	15	11	7	+ 3	− 2	6	10	14	19	23	27	31
15	+20	+15	+11	+ 7	+ 3	− 1	− 6	−10	−14	−18	−23	−27	−31
16	20	16	11	7	+ 3	− 1	5	10	14	18	22	26	31
17	20	16	12	7	+ 3	− 1	5	9	14	18	22	26	30
18	20	16	12	7	+ 3	− 1	5	9	13	18	22	26	30
19	20	16	12	8	+ 3	− 1	5	9	13	17	21	26	30
20	+20	+16	+12	+ 8	+ 4	0	− 5	− 9	−13	−17	−21	−25	−30
21	20	16	12	8	+ 4	0	− 4	9	13	17	21	25	29
22	20	16	12	8	+ 4	0	− 4	8	12	17	21	25	29
23	21	16	12	8	+ 4	0	− 4	8	12	16	20	25	29
24	21	17	12	8	+ 4	0	− 4	8	12	16	20	24	28
25	+21	+17	+13	+ 9	+ 4	0	− 4	− 8	−12	−16	−20	−24	−28
26	21	17	13	9	5	+ 1	− 3	8	12	16	20	24	28
27	21	17	13	9	5	+ 1	− 3	7	11	15	19	24	28
28	21	17	13	9	5	+ 1	− 3	7	11	15	19	23	27
29	21	17	13	9	5	+ 1	− 3	7	11	15	19	23	27
30	+21	+17	+13	+ 9	+ 5	+ 1	− 3	− 7	−11	−15	−19	−23	−27
31	21	17	13	9	5	+ 1	− 3	7	11	15	19	23	27
32	22	18	14	10	6	+ 2	− 2	6	10	14	18	22	26
33	22	18	14	10	6	+ 2	− 2	6	10	14	18	22	26
34	22	18	14	10	6	+ 2	− 2	6	10	14	18	22	26
35	+22	+18	+14	+10	+ 6	+ 2	− 2	− 6	−10	−14	−18	−22	−25
36	22	18	14	10	6	+ 2	− 2	6	9	13	17	21	25
37	22	18	14	10	6	+ 2	− 1	5	9	13	17	21	25
38	22	18	14	10	7	+ 3	− 1	5	9	13	17	21	25
39	22	18	14	11	7	+ 3	− 1	5	9	13	17	21	24
40	+22	+18	+15	+11	+ 7	+ 3	− 1	− 5	− 9	−13	−16	−20	−24

APPENDIX.

The observer in the field is often asked by the people with whom he comes in contact to explain his work. Ordinarily he must use simple nontechnical language for this explanation if he wishes to be readily understood. This is a difficult task and especially so if he is asked, "Of what use are these gravity observations?"

It is of the greatest importance for every scientist to be able to explain the purpose of his investigations in such a careful, tactful manner as to insure the laymen's respect and cooperation. There is a tendency in human nature to regard with a slight feeling of distrust and antagonism any work, especially of a purely scientific nature, that seems unnecessarily refined and exact and for which the purpose is not readily apparent. Every citizen helps to pay for all investigations carried on by the Federal Government and, when any of these investigations come to his attention, he usually feels a personal interest in them. He will undoubtedly become a better, more intelligent, citizen if he is convinced that the work has a practical value which will be of benefit to him, to the community in which he lives, or to the country as a whole. The scientific observer may therefore devote a part of his energies to instruction and explanation and feel that the time thus used is well invested.

The purpose of this appendix is to render, if possible, some slight assistance in this respect to the gravity observer, by outlining very briefly a few of the fundamental reasons why gravity observations are necessary and valuable.

PURPOSE OF GRAVITY DETERMINATIONS.

Before a large area like the United States can be properly mapped, it is necessary to make accurate and extended surveys. The first surveys to be made are called control surveys and must be carried out with the greatest possible precision. Since these control surveys are extended over very long distances, a thousand or two thousand miles or even more, they can not be computed as if they were on a plane surface. The curvature of the earth must be taken into account. If the earth were a true sphere, its curvature could be computed from the length of the radius only, and this length could be found by making certain astronomic observations in connection with the control surveys. The earth is not, however, a true sphere and so its shape must be determined as well as its size.

By making use of gravity results it is possible to compute the shape of the earth with a very high degree of precision. Computations were completed in 1917 which included all gravity determinations in the United States available at that time, and it was found that the radius of the earth at the pole is approximately one part in 297.4 shorter than the radius at the Equator. As soon as a sufficient number of new gravity stations have been determined a new computation will be made which will give still more accurate data in regard to the shape of the earth.

Gravity is made up of two opposing forces, gravitation or the attractive pull of the earth, and the centrifugal or repelling force due to the earth's rotation. The surface of the earth moves fastest at the Equator, consequently the centrifugal force is greatest there. The shortest radius of the earth is at the poles, and therefore gravitation, or the attractive force, is greatest at the poles. As a result of these two conditions a mass weighing 200 pounds at sea level at the Equator will weigh approximately 201 pounds at sea level at either pole provided the weighing is done with a spring balance. With beam scales the weights would be affected in the same proportion as the mass weighed, and no difference in weight would be apparent. While this variation in the force of gravity at different places on the earth can be neglected for all ordinary purposes, there are certain physical measurements for which the local intensity of gravity must be known.

If gravity stations are reasonably close together, the force of gravity at any point may be interpolated with a fair degree of accuracy from the nearest stations after certain corrections for local conditions have been applied. This requires the use of a formula, called the gravity formula, which is derived from the results of all gravity determinations and which, therefore, depends for its accuracy upon the number and precision of these determinations.

Recent developments in the study of gravity results seem to indicate that there is a decided relation between the intensity of gravity and the density of the rock underlying and surrounding the station. This should have an important bearing on the study of the geology of the country, and it is possible that it may have an economic value in the location of oil and other mineral deposits.

Finally, it should be remembered that very often too great stress is put upon the demands of utility. In addition to their other uses gravity determinations are very important in connection with many scientific problems, and it may be safely stated that the correct solution of almost any one of these problems will result ultimately in the betterment of man's condition. The question of immediate utility must not be allowed to obscure altogether the higher, although more remote, object.

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