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DEPARTMENT OF COMMERCE AND LABOR
COAST AND GEODETIC SURVEY
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GEODESY

THE FIGURE OF THE EARTH AND ISOSTASY FROM MEASUREMENTS IN THE UNITED STATES

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BY

JOHN F. HAYFORD

Inspector of Geodetic Work, and Chief, Computing Division
Coast and Geodetic Survey

Mr. Lambert

WASHINGTON
GOVERNMENT PRINTING OFFICE
1909

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LETTER OF TRANSMITTAL.

DEPARTMENT OF COMMERCE AND LABOR,
COAST AND GEODETIC SURVEY,
Washington, April 30, 1909.

Hon. CHARLES NAGEL,
Secretary of Commerce and Labor, Washington.

SIR: I have the honor to submit for your approval for publication the accompanying report by the Chief of the Computing Division of this Service, entitled "The Figure of the Earth and Isostasy from Measurements in the United States."

It gives the necessary details as to principles and method of an investigation of which the results have been for some time before the world.

All the geodetic work in this country prior to the year 1906 which was sufficiently far advanced to lend itself to the purposes of this discussion has been utilized. The effect of the accumulation of further data on the values obtained by this investigation will be shown in a subsequent publication.

Very respectfully,

O. H. TITTMANN,
Superintendent.

Approved:
CHARLES NAGEL,
Secretary.

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THE FIGURE OF THE EARTH AND ISOSTASY FROM MEASUREMENTS IN THE UNITED STATES.

By JOHN F. HAYFORD,
Inspector of Geodetic Work and Chief of Computing Division, Coast and Geodetic Survey.

The investigation of the figure and size of the earth, treated in this publication, has been in progress in the computing division of the Coast and Geodetic Survey more than five years. The progress made in the extensive computations connected with it has been very rapid at times and at other times has been stopped for several months in succession. The rate of progress has been fixed mainly by the fact that the energy spent upon this problem has been merely the small available surplus left after that work of the division had been done which must be completed promptly in answer to official calls and to facilitate the progress of current field work. The computing division (thirteen persons upon an average) is engaged principally in obtaining from the field observations the best available lengths, azimuths, and positions of lines and points fixed by triangulation, and in furnishing these values to persons who are to use them as controls for surveys. Research work, such as that here reported upon, is but a small part of the activity of the division.

The writer has been engaged at various times for more than ten years in developing the ideas concerned in this investigation. He has been steadily engaged, during this whole period, in other duties which took much of his time. Attention has been given to this problem during hundreds of short periods of a few hours each, or even of a few minutes each, in the intervals between the other duties.

The writer has, during the investigation, talked freely to many people about it. He gladly acknowledges that, as a result, he received many valuable suggestions and criticisms from various persons, both within the Survey and outside of it. Special mention should be made, in this connection, of Messrs. M. H. Doolittle and H. C. Mitchell and Miss Lilian Pike, members of the computing division, and Miss L. J. Harvie (now Mrs. S. J. Barnett) a former member of the division. The rapidity and accuracy with which the difficult, extensive, and unusual computations concerned in this investigation have been made is largely due to the skill and zeal of the many members of the computing division who have taken part in them. While the investigation was in progress, various preliminary public statements have been made by the writer before scientific gatherings. The following statements have been published:

Geodetic Operations in the United States, printed in the report of O. H. Tittmann, Superintendent, Coast and Geodetic Survey, to the Fourteenth General Conference of the International Geodetic Association, Volume I, pages 193-206.

The Form of the Geoid as Determined by Measurements in the United States; Report of the Eighth International Geographic Congress, held in the United States, 1904, Washington, Government Printing Office, 1905, pages 535-540.

The Geodetic Evidence of Isostasy; Proceedings of the Washington Academy of Sciences, 1906, Volume VIII, pages 25-40.

Geodetic Operations in the United States in 1903-1906; A Report to the Fifteenth General Conference of the International Geodetic Association (printed as a separate and also as a part of the Association's report).

This investigation of the figure of the earth and of the reality of the condition called isostasy is based entirely upon observed deflections of the vertical in the United States.

No use has been made in the investigation of determinations of gravity. The writer believes that determinations of the intensity of gravity will furnish much additional evidence of great weight in connection with this and similar investigations. It was believed to be best, however, to deal thoroughly with one phase of the investigation before taking up the other; to deal first with deflections of the vertical within the United States, and later, with this new light upon the problem, to make and study gravity observations in the same area. If anyone will carefully consider the great extent of the computations and of the theoretical studies connected with the investigation as here published, curtailed as it is by the omission of all consideration of the intensity of gravity, the wisdom of the decision to limit the investigation will be appreciated.

Some of the noteworthy features of the investigation are the following:

1. The area treated extends over a wide range in latitude and in longitude—18° 50′ in latitude and 57° 07′ in longitude.

2. A large number of astronomic determinations have been used (507).

3. All the astronomic determinations are connected by continuous primary triangulation. The triangulation does not consist of separate and distinct belts of triangulation or arcs.

4. Unusual methods of computation have been used.

5. The effects of all topographic irregularities within 4 126 kilometers of each astronomic station have been taken into account.

6. The effect of the possible distribution of densities beneath the surface of the earth corresponding to the condition called isostasy has been carefully taken into account and the existence of said condition established.

7. The investigation leads to values of the equatorial and polar dimensions of the earth, based on observations in a single country, the United States, which are of a very high degree of accuracy.

The order of presentation in this publication is as follows:

First. A statement is made of the data used and of the methods of investigation and computation, accompanied by sufficient illustration and explanation to enable the reader to understand what was done and to understand, in a general way only, why it was done. In this portion, to preserve the continuity and to avoid confusion of thought, but little space is devoted to showing the accuracy of the data and of the methods or the reasons for various features of the method of treatment.

Second. The accuracy of the data and of each step of the method is discussed in detail and the advantages of the adopted methods are set forth.

Third. Various minor questions and supplementary investigations are treated. Though these questions are such as naturally arise in connection with the main investigation, it is believed to be conducive to clearness to deal with them separately, rather than to interrupt the continuity of the main statement by dealing with these questions when they first arise.

Fourth. The conclusions reached and the degree of certainty of the conclusions are summarized at the end.

DATA USED IN THE INVESTIGATION.

The triangulation used in this investigation includes the transcontinental triangulation from New Jersey to California,* the western oblique arc covering three-fourths of the length of California,† the eastern oblique arc from Maine to Louisiana,‡ the triangulation by the Lake

* The Transcontinental Triangulation, Special Publication No. 4 of the Coast and Geodetic Survey. The positions as given in this publication are not upon the United States Standard Datum.

† Triangulation in California, Part I, Appendix 9 of the Coast and Geodetic Survey Report for 1904. This publication includes the primary triangulation forming the western oblique arc, as well as all secondary and tertiary triangulation in California south of the latitude of Monterey Bay. The positions given are upon the United States Standard Datum.

‡ The Eastern Oblique Arc of the United States, Special Publication No. 7 of the Coast and Geodetic Survey. The positions given in this publication are not upon the United States Standard Datum.

Survey in the vicinity of the Great Lakes,* and comprised mainly within the States of New York, Ohio, Indiana, Illinois, Wisconsin, and Michigan, and triangulation not included in the preceding items,† but extending over various portions of New England, New York, southern Maryland, eastern Virginia, North Carolina, and Tennessee.

All this triangulation has been reduced to the United States Standard Datum; that is, the latitudes, longitudes, and azimuths have been computed continuously through it on the basis of the assumption that the latitude of the triangulation station, Meades Ranch (in Kansas), is 39° 13′ 26″.686, its longitude is 98° 32′ 30″.506, and the azimuth of the line, Meades Ranch to Waldo, is 75° 28′ 14″.52. The computations are all based upon the Clarke spheroid of 1866.

In the investigation 507 astronomic determinations have been used—265 of latitude, 79 of longitude, and 163 of azimuth. Eleven of the determinations of longitude were made at stations practically coincident with stations at which determinations of azimuth were made. Hence the 507 astronomic determinations furnish that component of the deflection of the vertical which lies in the meridian at 265 stations, and the prime vertical component at 231 stations.

These astronomic determinations are distributed, though not uniformly, over the whole area covered by the triangulation referred to above. They are scattered over 33 States. The extremes in latitude are 48° 47′ at St. Ignace, on the northern shore of Lake Superior, and 29° 57′ at New Orleans, Louisiana. The extremes in longitude are 67° 16′ at Calais, Maine, and 124° 24′ at Cape Mendocino, California.

The astronomic observations used in the investigation are shown in the following two tables.

Each line in the table headed "Deflections in meridian" represents a determination of the astronomic latitude.

The name of the triangulation station is given in the first column, and in the second is given a serial number by which the station is identified in later portions of this publication, and on illustration No. 13, in the pocket at the end of this volume. The columns headed "Geodetic latitude" and "Geodetic longitude" give the position of the station on the United States Standard Datum, as determined by the triangulation. The column headed "Astronomic latitude" gives the seconds only of the astronomic latitude, the degrees and minutes being the same as for the geodetic latitude.

The column headed A−G is the astronomic minus the geodetic latitude. It is, therefore, that component of the deflection of the vertical at the station which lies in the meridian. By "deflection" is meant, in this connection, the angle between the actual line of gravity at the station and the line which is normal to the Clarke spheroid at the station, upon the supposition that the United States Standard Datum is correct. A plus sign in the column A−G means that the zenith of the station defined by the actual line of gravity is farther north on the celestial sphere than the zenith defined by the line normal to the Clarke spheroid.

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* Report upon the Adjustment of the Lake Survey Triangulation and its Adaptation to the United States Standard Datum of the Coast and Geodetic Survey. This forms a part (pages 2883–3031) of Appendix EEE of the Annual Report of the Chief of Engineers, U. S. Army, for 1902. Also, for further details in regard to this triangulation, see Professional Papers No. 24, Corps of Engineers, U. S. Army, Report Upon the Primary Triangulation of the United States Lake Survey, Washington, Government Printing Office, 1882. This publication is now rare.

† Appendix 9 of the Coast and Geodetic Survey Report for 1884, "Connection at Lake Ontario of the Primary Triangulation of the Coast and Geodetic Survey with that of the Lake Survey," states a few principal facts in regard to a portion of this triangulation in New York State. Appendices 8 of the Coast and Geodetic Survey Report for 1885, 8 of 1888, and 10 of 1894, give the principal facts in regard to portions of this triangulation which lie in Rhode Island, in Connecticut, and in Massachusetts, respectively. The remainder of the triangulation concerned in this investigation is unpublished.

Deflections of the vertical in the United States expressed in terms of the Clarke spheroid of 1866 (a=6 378 206.4, b=6 356 583.8 meters) and the United States Standard Geodetic Datum.

DEFLECTIONS IN MERIDIAN.

Station.	No.	Geodetic latitude.			Geodetic longitude.			Astronomic latitude.	A−G.
		°	′	″	°	′	″	″	″
Point Arena	1	38	55	18.93	123	43	36.91	10.16	−08.77
Mendocino City	2	39	18	13.46	123	48	16.64	05.50	−07.96
Bodega	3	38	18	29.84	123	03	15.64	20.11	−09.73
Point Reyes	4	37	59	44.30	122	58	34.31	33.62	−10.68
Ukiah	5	39	08	58.88	123	12	28.19	54.59	−04.29
Sulphur Peak	6	38	45	54.28	122	50	40.22	44.42	−09.86
Ross Mountain	7	38	30	20.58	123	07	08.82	09.96	−10.62
Mount Helena	8	38	40	10.13	122	37	57.17	01.05	−09.08
Mount Tamalpais	9	37	55	27.55	122	35	45.47	19.18	−08.37
San Francisco:									
New Presidio	10	37	47	51.38	122	27	05.17	48.35	−03.03
Old Presidio	11	37	47	39.15	122	27	13.55	35.96	−03.19
Lafayette Park	12	37	47	31.92	122	25	37.16	28.31	−03.61
Washington Square	13	37	47	64.71	122	24	31.51	56.90	−07.81
Monticello	14	38	39	50.96	122	11	22.22	46.26	−04.70
Vaca	15	38	22	33.44	122	05	01.21	23.27	−10.17
Mount Diablo	16	37	52	55.45	121	54	50.45	49.60	−05.85
Yolo Northwest Base	17	38	40	44.94	121	51	28.73	37.25	−07.69
Yolo Southeast Base	18	38	31	42.63	121	47	56.59	34.55	−08.08
Mount Hamilton, Lick Observatory, C. and G. S. Station	19	37	20	34.81	121	38	14.42	28.85	−05.96
Marysville	20	39	08	19.64	121	35	39.01	12.27	−07.37
Mocho	21	37	28	39.70	121	33	18.71	36.71	−02.99
Mount Lola	22	39	26	00.28	120	21	51.32	57.37	−02.91
Round Top	23	38	39	50.30	120	00	01.57	46.27	−04.03
Mount Conness	24	37	57	59.25	119	19	07.47	55.98	−03.27
Lake Tahoe Southeast	25	38	57	16.71	119	56	41.04	19.37	+02.66
Verdi	26	39	31	06.11	119	57	51.90	04.29	−01.82
Carson City Observatory, zenith telescope	27	39	09	52.26	119	45	48.68	47.25	−05.01
Carson Sink	28	39	35	00.70	118	14	04.36	57.67	−03.03
Toiyabe Dome	29	38	49	59.50	117	21	09.16	53.91	−05.59
Mount Callahan	30	39	42	35.42	116	56	59.30	31.92	−03.50
Diamond Peak	31	39	35	07.19	115	49	03.94	03.65	−03.54
Pioche, U. S. Engineers	32	37	55	38.53	114	26	47.80	25.80	−12.73
Pioche	33	37	59	11.18	114	03	05.17	06.80	−04.38
Pilot Peak	34	41	01	17.19	114	04	35.75	07.83	−09.36
Ibepah	35	39	49	42.68	113	55	09.77	38.97	−03.71
Oasis	36	39	17	38.00	112	37	44.44	35.29	−02.71
Beaver, U. S. Engineers	37	38	16	25.62	112	38	27.26	22.90	−02.72
Deseret	38	40	27	34.99	112	37	32.78	31.25	−03.74
Promontory	39	41	17	53.67	112	25	09.98	47.77	−05.90
Antelope	40	40	57	44.04	112	12	55.65	40.16	−03.88
Waddoup	41	40	54	24.00	111	53	04.11	21.73	−02.27
Ogden Observatory, longitude pier	42	41	13	12.54	111	59	37.97	08.33	−04.21
Salt Lake City	43	40	46	12.38	111	53	27.30	03.36	−09.02
Ogden Peak	44	41	12	00.78	111	52	53.64	59.22	−01.56
Gunnison, Utah	45	39	09	31.04	111	49	13.66	25.46	−05.58
Mount Nebo	46	39	48	38.52	111	45	56.32	32.31	−06.21
Wasatch	47	39	06	57.43	111	27	08.77	53.83	−03.60
Mount Ellen	48	38	07	16.65	110	48	50.71	24.17	+07.52
Patmos Head	49	39	30	10.38	110	18	57.53	56.86	−13.52
Green River	50	38	59	30.28	110	09	55.73	23.63	−06.65
Mount Waas	51	38	32	20.59	109	13	38.00	29.00	+08.41
Tavaputs	52	39	32	24.57	109	00	19.32	17.12	−07.45
Grand Junction	53	39	03	55.22	108	33	53.40	59.04	+03.82
Uncompahgre	54	38	04	18.13	107	27	41.27	15.74	−02.39
Gunnison, Colo	55	38	32	47.09	106	55	26.73	44.39	−02.70
Treasury Mountain	56	39	00	51.11	107	05	54.09	47.25	−03.86
Mount Ouray	57	38	25	22.86	106	13	27.21	18.00	−04.86
Pikes Peak	58	38	50	26.32	105	02	37.57	27.28	+00.96
Colorado Springs (1873)	59	38	50	03.24	104	49	35.17	59.98	−03.26
El Paso East Base	60	38	57	22.38	104	27	41.99	16.50	−05.88
Adobe	61	38	40	40.84	103	33	16.12	37.42	−03.42
Wallace	62	38	54	44.35	101	35	31.50	44.25	−00.10
Russell Southeast Base	63	38	51	22.30	98	47	07.81	22.73	+00.43
Ellsworth	64	38	43	48.76	98	13	44.98	47.49	−01.27
Salina West Base	65	38	51	07.07	97	36	10.51	03.52	−04.15

Deflections of the vertical in the United States expressed in terms of the Clarke spheroid of 1866 (a=6 378 206.4, b=6 356 583.8 meters) and the United States Standard Geodetic Datum—Continued.

DEFLECTIONS IN MERIDIAN—Continued.

Station.	No.	Geodetic latitude.			Geodetic longitude.			Astronomic latitude.	A-G.
		°	′	″	°	′	″	″	″
Adams	66	39	02	41.03	96	04	23.94	41.80	+00.77
Kansas City Astronomic Station	67	39	05	50.40	94	35	22.16	51.12	+00.72
Hunter	68	38	25	45.21	92	46	24.64	48.00	+02.79
Jefferson City	69	38	33	41.16	92	09	45.55	43.95	+02.79
St. Louis University	70	38	37	58.37	90	12	16.50	62.78	+04.41
Bording	71	38	36	45.30	89	20	25.17	50.93	+05.63
Newton	72	38	55	28.59	88	09	50.96	31.10	+02.51
Olney West Base	73	38	51	38.57	88	06	08.38	41.28	+02.71
Parkersburg Triangulation Station	74	38	34	51.52	88	01	49.00	53.05	+01.53
Vincennes	75	38	40	35.70	87	31	35.05	36.80	+01.10
Weed Patch	76	39	10	00.64	86	13	01.76	00.68	+00.04
Reizin	77	39	02	53.84	85	08	23.72	53.76	−00.08
Cincinnati, Mount Lookout, C. and G. S., 1881, transit pier	78	39	08	21.87	84	25	21.93	20.61	−01.26
Minerva	79	38	42	31.16	83	55	06.98	30.89	−00.27
Gould	80	38	38	29.57	82	49	56.72	29.78	+00.21
Piney	81	38	26	40.03	82	03	29.00	41.40	+01.37
Charleston, W. Va.	82	38	21	02.29	81	37	59.19	06.95	+04.66
Keeney	83	37	46	25.59	80	42	19.15	23.07	−02.52
Elliott Knob	84	38	09	59.61	79	18	51.38	57.08	−02.53
Long Mountain	85	37	17	27.61	79	05	10.14	28.84	+01.23
Charlottesville, McCormick Observatory transit	86	38	01	58.03	78	31	20.62	61.09	+03.06
Clark	87	38	18	41.35	78	00	10.85	39.60	−01.75
Strasburg	88	38	59	29.94	78	21	39.21	31.56	+01.62
Bull Run	89	38	52	54.21	77	42	12.40	56.72	+02.51
Maryland Heights	90	39	20	28.43	77	42	59.51	32.19	+03.76
Sugar Loaf	91	39	15	45.79	77	23	36.91	49.54	+03.75
Rockville	92	39	05	11.21	77	09	36.65	10.42	−00.79
Georgetown College Observatory	93	38	54	29.94	77	04	39.05	25.79	−04.15
Causten	94	38	55	34.96	77	04	26.02	32.02	−02.94
Naval Observatory, clock room	95	38	55	17.03	77	04	02.24	13.74	−03.29
Naval Observatory, old site, station east of dome	96	38	53	42.31	77	03	05.81	38.83	−03.48
C. and G. S. Observatory, lat. pier	97	38	53	12.15	77	00	32.24	07.35	−04.80
Seaton	98	38	53	28.96	76	59	59.54	25.12	−03.84
Soper	99	39	05	11.94	76	57	00.08	10.61	−01.33
Hill	100	38	53	54.38	76	52	49.95	52.36	−02.02
Webb	101	39	05	26.31	76	40	29.69	25.35	−00.96
Marriott, 1849	102	38	52	27.83	76	36	35.69	25.05	−02.78
Taylor	103	38	59	48.49	76	27	55.91	46.07	−02.42
Calvert	104	38	21	34.31	76	23	34.92	31.71	−02.60
Pooles Island	105	39	17	15.66	76	15	44.06	17.52	+01.86
Principio	106	39	35	36.69	76	00	16.10	32.75	−03.94
Dover	107	39	09	20.75	75	31	23.96	13.47	−07.28
Cape Henlopen	108	38	46	42.13	75	05	03.06	40.07	−02.06
Cape May	109	38	55	48.69	74	55	47.49	44.63	−04.06
Cape Henry Light-House (old)	110	36	55	32.75	76	00	30.59	30.25	−02.50
Wolftrap	111	37	24	04.77	76	14	43.04	02.02	−02.75
Tangier Island	112	37	47	57.73	75	59	15.06	56.57	−01.16
North End Knott Island	113	36	33	57.61	75	55	28.79	55.37	−02.24
Staunton	114	38	08	46.51	79	04	19.09	50.7	+04.19
Moore	115	36	23	53.52	80	16	59.16	55.05	+01.53
Young	116	35	44	14.34	80	38	49.62	21.69	+07.35
King	117	35	12	11.65	81	18	56.37	13.42	+01.77
Nashville	118	36	09	57.57	86	47	00.25	61.35	+03.78
Lebanon North Base	119	36	12	46.89	86	18	24.56	49.64	+02.75
Paris	120	34	56	29.94	82	24	41.34	32.05	+02.11
Currahee	121	34	31	38.68	83	22	25.13	37.74	−00.94
Lavender	122	34	19	18.00	85	17	29.62	15.75	−02.25
Sawnee	123	34	14	04.95	84	09	41.02	04.03	−00.92
Aurora	124	34	08	47.22	86	11	00.66	47.63	+00.41
Atlanta Middle Base	125	33	54	21.06	84	16	35.38	21.66	+00.60
Atlanta	126	33	44	58.03	84	23	18.98	59.28	+01.25
Kahatchee	127	33	13	37.88	86	21	36.16	39.74	+01.86
Montgomery	128	32	22	39.34	86	18	00.41	45.24	+05.90
Lower Peach Tree	129	31	50	20.45	87	32	42.81	21.00	+00.55
Coon	130	31	14	50.32	88	05	43.59	47.74	−02.58
Tanyard	131	34	39	11.20	87	23	02.25	13.43	+02.23

Deflections of the vertical in the United States expressed in terms of the Clarke spheroid of 1866 (a=6 378 206.4, b=6 356 583.8 meters) and the United States Standard Geodetic Datum—Continued.

DEFLECTIONS IN MERIDIAN—Continued.

Station.	No.	Geodetic latitude.			Geodetic longitude.			Astronomic latitude.	A−G.
		°	′	″	°	′	″	″	″
Mobile	132	30	41	30.87	88	02	33.36	33.52	+02.65
East Pascagoula	133	30	20	35.52	88	32	45.21	40.91	+05.39
Fort Morgan azimuth station	134	30	13	42.24	88	01	23 26	47.88	+05.64
New Orleans	135	29	57	19.94	90	04	24.64	25.36	+05.42
Yard	136	39	58	24.80	75	23	13.74	29.39	+04.59
Mount Rose	137	40	22	03.41	74	43	25.82	05.30	+01.89
Beacon Hill	138	40	22	26.55	74	13	41.85	27.93	+01.38
New York	139	40	43	51.24	73	59	13.82	48.26	−02.98
West Hills	140	40	48	55.35	73	25	32.63	49.92	−05.43
Sandford	141	41	27	43.19	72	56	59.85	40.57	−02.62
Manomet	142	41	55	38.70	70	35	28.13	35.41	−03.29
Mount Tom	143	42	14	29.89	72	38	55.61	27.62	−02.27
Cambridge, Cloverden Obs., lat. sta.	144	42	22	46.29	71	07	17.60	40.96	−05.33
Cambridge, Harvard Observatory	145	42	22	54.04	71	07	43.88	48.59	−05.45
Wachusett	146	42	29	19.84	71	53	12.23	16.02	−03.82
Thompson	147	42	36	41.92	70	43	49.16	37.98	−03.94
Unkonoonuc	148	42	58	59.86	71	35	19.38	59.34	−00.52
Gunstock	149	43	31	02.98	71	22	10.88	03.71	+00.73
Isles of Shoals	150	42	59	14.98	70	36	50.28	12.97	−02.01
Agamenticus	151	43	13	24.63	70	41	32.91	24.92	+00.29
Mount Independence	152	43	45	33.78	70	19	15.06	34.56	+00.78
Cape Small	153	43	46	43.43	69	50	44.54	43.70	+00.27
Mount Pleasant	154	44	01	36.64	70	49	22.20	36.24	−00.40
Sabattus	155	44	08	37.98	70	04	43.24	37.67	−00.31
Ragged Mountain	156	44	12	44.98	69	09	04.83	42.93	−02.05
Mount Desert	157	44	21	07.11	68	13	36.17	06.52	−00.59
Howard	158	44	37	47.14	67	23	46.16	49.17	+02.03
Mount Harris	159	44	39	54.66	69	08	55.58	54.68	+00.02
Farmington	160	44	40	22.77	70	09	17.37	19.35	−03.42
Bangor	161	44	48	16.14	68	47	00.15	12.89	−03.25
Humpback	162	44	51	51.12	68	06	37.49	47.37	−03.75
Cooper	163	44	59	13.43	67	28	02.89	12.49	−00.94
Calais	164	45	11	05.68	67	16	52.76	09.39	+03.71
Burlington	165	44	28	09.05	73	12	34.13	09.46	+00.41
Rensselaer Polytechnic Astr. Sta.	166	42	43	49.60	73	41	06.36	50.69	+01.09
Rouses Point	167	45	00	23.27	73	21	30.40	27.22	+03.95
Mount Merino	168	42	14	05.06	73	49	03.54	04.78	−00.28
Howlett	169	42	59	57.67	76	17	20.09	59.59	+01.92
Tassel	170	42	56	15.76	75	18	56.82	19.37	+03.61
Prospect	171	43	25	17.95	73	46	04.57	12.60	−05.35
Cheever	172	44	04	54.06	73	27	02.85	53.42	−00.64
Ogdensburg Light-House	173	44	41	52.44	75	30	13.8	53.18	+00.74
Watertown Court-House	174	43	58	32.31	75	54	52.96	32.87	+00.56
Sandy Creek North Base	175	43	40	43.67	76	12	00.92	41.51	−02.16
Oswego	176	43	26	37.31	76	30	49.28	39.56	+02.25
Rochester City Hall	177	43	09	18.05	77	36	49.60	22.43	+04.38
Tonawanda	178	43	00	03.73	78	53	20.88	07.82	+04.09
Buffalo, intersection of Exchange and Michigan streets	179	42	52	40.98	78	52	13.25	44.63	+03.65
Dunkirk Light-House	180	42	29	37.68	79	21	14.81	44.27	+06.59
Erie Range Light No. 1	181	42	09	11.66	80	04	39.22	17.46	+05.80
Cleveland Light-House	182	41	30	01.40	81	42	08.68	06.10	+04.70
Sandusky West Base	183	41	29	02.11	82	40	57.57	04.56	+02.45
Toledo, stone longitude post, 1881	184	41	39	04.36	83	32	31.00	03.62	−00.74
Monroe Court-House	185	41	54	52.44	83	23	48.93	48.62	−03.82
Cleveland, C. and G. S. Lat. Sta.	186	41	30	20.87	81	41	29.21	25.65	+04.78
Case School Observatory	187	41	30	10.36	81	36	27.60	14.50	+04.14
Detroit, L. S. east pier	188	42	19	60.56	83	03	06.65	58.62	−01.94
Thones Hill	189	46	31	54.12	87	27	03.30	60.57	+06.45
Taquamenon Island	190	46	31	55.22	84	56	54.80	56.87	+01.65
Marquette Light-House	191	46	32	48.12	87	22	33.72	54.50	+06.38
Sault Ste. Marie Observatory, west pier	192	46	30	05.77	84	20	55.64	06.24	+00.47
Round Island, observatory post	193	45	50	07.46	84	36	43.92	05.87	−01.59
Iroquois Point Light House (old)	194	46	29	03.61	84	37	53.67	04.81	+01.20
Ford River 2	195	45	41	12.32	87	06	08.44	05.31	−07.01
Burnt Bluff 1 (old)	196	45	41	09.95	86	42	38.90	03.85	−06.10
Beaver Island Light-House	197	45	34	32.85	85	34	23.84	28.75	−04.10
Cedar River	198	45	25	48.76	87	19	34.12	43.10	−05.66

Deflections of the vertical in the United States expressed in terms of the Clarke spheroid of 1866 (a=6 378 206.4, b=6 356 583.8 meters) and the United States Standard Geodetic Datum—Continued.

DEFLECTIONS IN MERIDIAN—Continued.

Station.	No.	Geodetic latitude.			Geodetic longitude.			Astronomic latitude.	A - G.
		°	′	″	°	′	″	″	″
Rock Island Light-House	199	45	25	39.38	86	49	39.83	35.46	−03.92
Boyers Bluff (old)	200	45	25	12.97	86	56	10.52	09.27	−03.70
Door Bluff	201	45	17	47.11	87	03	53.67	45.66	−01.45
Menominee	202	45	05	12.90	87	35	33.71	12.75	−00.15
Green Island Light-House	203	45	03	23.24	87	29	33.7	24.77	+01.53
Fort Howard	204	44	30	30.40	88	02	33.21	30.25	−00.15
Minnesota Junction	205	43	28	28.54	88	43	46.78	31.78	+03.24
South Haven, azimuth station	206	42	16	09.37	86	20	42.48	13.45	+04.08
Willow Springs	207	41	43	36.90	87	51	05.63	38.60	+01.70
Fairmount	208	40	01	35.72	87	50	48.10	36.67	+00.95
Chicago Light-House	209	41	53	20.50	87	36	51.46	22.46	+01.96
Escanaba Light-House	210	45	44	41.43	87	02	39.80	35.02	−06.41
Aminicon	211	46	41	32.47	91	51	42.84	36.29	+03.82
Minnesota Point South Base	212	46	42	49.77	92	01	54.39	51.39	+01.62
Minnesota Point North Base	213	46	45	28.22	92	04	42.52	28.29	+00.07
Brule River	214	46	45	18.21	91	35	18.02	20.08	+01.87
Whitefish Point, latitude post	215	46	46	06.79	84	57	25.75	07.17	+00.38
Porcupine Mountains	216	46	47	01.33	89	43	49.97	03.77	+02.44
Keweenaw Point South Base	217	46	52	18.01	88	29	15.82	22.31	+04.30
Huron Mountains	218	46	52	42.54	87	55	16.62	53.04	+10.50
Buchanan	219	46	56	28.41	91	47	19.05	24.42	−03.99
Crebassa Light-House	220	46	58	41.32	88	24	52.66	41.18	−00.14
Outer Island	221	47	04	17.40	90	26	26.40	14.48	−02.92
Wheal Kate	222	47	04	20.53	88	39	44.27	18.09	−02.44
Sawteeth East	223	47	23	19.36	91	10	14.97	08.97	−10.39
Mount Houghton	224	47	24	26.27	87	56	28.58	15.80	−10.47
Vulcan	225	47	26	46.92	87	47	37.16	44.55	−02.37
Copper Harbor, stone post	226	47	28	09.05	87	51	56.53	03.12	−05.93
Gargantua	227	47	34	42.83	84	58	53.93	38.77	−04.06
Michipicotan	228	47	45	21.84	85	52	57.59	20.55	−01.29
Farquhars Knob	229	47	52	40.52	89	59	22.86	35.24	−05.28
Isle Royal East	230	48	07	43.90	88	33	36.91	55.19	+11.29
Tip Top	231	48	16	24.87	86	00	11.61	25.75	+00.88
St. Ignace	232	48	47	19.23	87	51	06.27	28.62	+09.39
Mount Toro	233	36	31	35.17	121	36	32.82	40.34	+05.17
Arguello	234	34	34	58.64	120	33	40.02	53.44	−05.20
Gaviota	235	34	29	68.36	120	11	53.02	55.78	−12.58
Santa Cruz West	236	34	04	24.15	119	55	02.17	28.04	+03.89
New San Miguel	237	34	02	23.60	120	23	09.34	21.55	−02.05
Santa Barbara	238	34	23	77.66	119	42	52.88	59.28	−18.38
San Diego 1851	239	32	41	63.94	117	14	31.17	57.65	−06.29
San Diego 1892	240	32	43	29.96	117	09	28.51	20.80	−09.16
Fort Gratiot Observatory	241	42	59	52.76	82	25	44.49	52.18	−00.58
Lospe	242	34	53	38.48	120	36	19.94	32.89	−05.59
Tepusquet	243	34	54	38.02	120	11	10.03	26.85	−11.17
Santa Lucia	244	36	08	45.62	121	25	05.54	38.08	−07.54
Los Angeles Normal School	245	34	02	62.20	118	15	16.51	44.21	−17.99
Dominguez Hill	246	33	51	55.63	118	14	11.45	55.18	−00.45
University of Illinois	247	40	06	18.84	88	13	33.48	20.10	+01.26
Provincetown	248	42	03	09.37	70	11	19.10	04.15	−05.22
Crescent City	249	41	44	48.56	124	12	01.35	43.27	−05.29
Red Bluff	*251	40	44	44.18	124	12	56.75	40.16	−04.02
Cape Mendocino	252	40	26	21.02	124	24	11.23	16.18	−04.84
Santa Ana	253	36	54	19.60	121	13	58.10	15.66	−03.94
Point Pinos	254	36	37	59.19	121	55	31.63	59.48	+00.29
Hepsedam	255	36	18	53.60	120	49	26.30	42.16	−11.44
Castle Mount	256	35	56	21.34	120	20	22.84	19.57	−01.77
Rocky Butte	257	35	39	56.03	121	03	32.13	43.95	−12.08
San Simeon 1852	258	35	38	36.3	121	11	26.4	24.38	−11.92
San Simeon 1874	259	35	38	33.96	121	11	52.03	19.58	−14.38
San Luis Obispo	260	35	10	43.56	120	44	45.26	37.25	−06.31
Avila	261	35	10	40.20	120	43	17.32	32.73	−07.47
Point Conception	262	34	26	69.56	120	26	45.43	56.34	−13.22
San Buenaventura	*264	34	15	59.20	119	15	57.15	48.27	−10.93
San Pedro	265	33	43	21.04	118	17	02.18	18.80	−02.24
Santa Catalina Island	266	33	26	29.87	118	29	50.26	35.19	+05.32
Soledad	267	32	50	24.52	117	15	07.24	21.78	−02.74

* Serial Nos. 250 and 263 do not exist in this list.

Each line in the following table headed "Deflections in prime vertical" represents a determination of the astronomic azimuth or astronomic longitude at a station.

In connection with this table the same explanations apply to the first four columns as have already been given in connection with the preceding lists of deflections in the meridian.

Lines representing azimuth observations are distinguished from lines representing longitude observations by the fact that there are entries in the fifth and sixth columns. For an azimuth station the entry in the fifth column shows the geodetic azimuth at the station named in the first column to the station named in the sixth column, as fixed by triangulation. The seventh column shows the seconds only of the astronomic azimuth of the same line observed at the station named in the first column. For example, at Paxton (No. 2) the geodetic azimuth to Mount Sanhedrin, as determined by triangulation, was 203° 47′ 14″.65, and the astronomic azimuth of the same line, as observed at Paxton, was 203° 47′ 05″.77.

The eighth column shows the astronomic minus the geodetic value of the azimuth or longitude as the case may be.

The ninth column shows the factor necessary, in each case, to reduce the value in the eighth column to the prime vertical component of the deflection of the vertical at the station, expressed in seconds of the prime vertical great circle. This factor is, for each longitude station, the cosine of the latitude ($\cos \phi'$) and for each azimuth station it is $-\cot \phi'$.

The factor $\cos \phi'$, used in connection with the longitude stations, arises from the fact that the determination of the astronomic longitude is affected by the amount of the prime vertical component of the deflection as measured along a small circle, a latitude circle passed through the zenith, not as measured along a prime vertical great circle.

A measured astronomic azimuth is necessarily referred to the meridian plane as defined by the actual, that is the deflected, line of gravity at the station and a line through the station parallel to the earth's axis of rotation. The angle measured in the horizon of the station between this deflected plane and the true meridian plane is $\tan \phi'$ times the prime vertical component of the deflection of the vertical. This angle, measured in the horizon, is the difference between the astronomic and the geodetic azimuth (A − G). Hence, to convert the A − G of the eighth column, as observed, to the required prime vertical component of the deflection of the vertical measured in seconds of the prime vertical great circle, it is necessary to multiply by $\cot \phi'$, as indicated in the ninth column. It is desired to have a plus sign in the last column mean, uniformly, that the zenith of the station defined by the actual line of gravity at the station is farther west on the celestial sphere than the zenith fixed by the line normal to the Clarke spheroid at the station. Hence, the minus sign is necessary with the factor $\cot \phi'$ used with azimuth observations, for a westward deflection of the zenith makes the observed astronomic azimuth of any line (expressed in the clockwise direction) too small.

The last column is, therefore, in each case, the prime vertical component of the deflection of the vertical expressed in seconds of a great circle and is directly comparable with the last column of the table headed "Deflections in meridian." The plus sign means, uniformly, that the zenith is displaced to the westward at the station.

Deflections in prime vertical.

Station.	No.	Geodetic latitude.	Geodetic longitude.	Geodetic azimuth.	To station.	Ast. Long. or Az.	A − G.	Cos ϕ' or −Cot ϕ'	A − G (P. V.).
		° ′ ″	° ′ ″	° ′ ″		″	″		″
Point Arena	1	38 54 36.16	123 41 24.44	... .		46.26	+21.82	+0.7781	+16.98
Paxton	2	39 08 09.20	123 18 43.25	203 47 14.65	Mount Sanhedrin	05.77	−08.88	−1.2289	+10.90
Ukiah	3	39 08 58.88	123 12 28.09	... .		33.51	+05.42	+0.7755	+04.20
Mount Helena	4	38 40 11.08	122 37 57.82	324 01 34.82	Mount Diablo	24.96	−09.86	−1.2496	+12.32
Mount Tamalpais	5	37 55 27.51	122 35 45.24	274 15 19.46	Mount Diablo	15.04	−04.42	−1.2834	+05.67
San Francisco:									
New Presidio	6	37 47 51.38	122 27 05.23	... .		12.20	+06.97	+0.7902	+05.51
Lafayette Park	7	37 47 31.92	122 25 37.22	... .		42.92	+05.70	+0.7902	+04.50
Washington Square	8	37 47 64.71	122 24 31.51	... .		36.62	+05.11	+0.7902	+04.04
Mount Hamilton, Lick Observatory, C. and G. S. Longitude Station	9	37 20 34.81	121 38 14.50	... .		27.14	+12.64	+0.7950	+10.05
Marysville	10	39 08 29.25	121 35 10.34	... .		17.79	+07.45	+0.7756	+05.78
Monticello	11	38 39 50.64	122 11 22.33	91 04 27.64	Mount Helena	25.30	−02.34	−1.2498	+02.92
Vaca	12	38 22 33.81	122 05 01.99	235 38 37.32	Yolo Southeast Base	36.55	−00.77	−1.2628	+00.97
Mount Diablo	13	37 52 55.48	121 54 48.36	144 28 18.91	Mount Helena	16.03	−02.88	−1.2854	+03.70
Yolo Northwest Base	14	38 40 44.81	121 51 28.55	343 05 07.93	Yolo Southeast Base	02.07	−05.86	−1.2491	+07.32
Yolo Southeast Base	15	38 31 42.18	121 47 58.52	163 07 18.98	Yolo Northwest Base	13.11	−05.87	−1.2559	+07.37
Sacramento	16	38 34 34.79	121 29 30.46	... .		35.80	+05.34	+0.7818	+04.17
Mocho	17	37 28 39.70	121 33 18.78	144 57 40.03	Mount Diablo	35.71	−04.32	−1.3043	+05.63
Mount Lola	18	39 26 00.06	120 21 51.59	67 22 03.42	Mount Helena	02.36	−01.06	−1.2160	+01.29
Round Top	19	38 39 50.32	120 00 01.13	90 58 57.20	Mount Helena	53.89	−03.31	−1.2498	+04.14
Lake Tahoe Southeast, longitude sta.	20	38 57 16.71	119 56 41.04	... .		45.90	+04.86	+0.7777	+03.78
Lake Tahoe Southeast, azimuth sta.	21	38 58 02.84	119 56 55.01	177 56 26.75	Folsom Peak	19.13	−07.62	−1.2364	+09.42
Verdi	22	39 31 14.49	119 58 58.15	... .		52.90	−05.25	+0.7714	−04.05
Genoa	23	39 00 13.71	119 50 46.35	... .		22.34	−24.01	+0.7771	−18.65
Carson City, Observatory transit, 1889	24	39 09 52.22	119 45 49.03	... .		44.60	−04.43	+0.7753	−03.43
Virginia City	25	39 18 39.68	119 38 49.98	... .		42.08	−07.90	+0.7737	−06.11
Mount Conness	26	37 58 02.59	119 19 14.23	142 39 27.37	Round Top	19.46	−07.91	−1.2815	+10.14
Carson Sink	27	39 35 00.23	118 14 05.09	262 20 28.69	Mount Callahan	25.50	−03.19	−1.2095	+03.86
Toiyabe Dome	28	38 49 58.81	117 21 08.63	77 20 55.93	Mount Grant	49.29	−06.64	−1.2423	+08.25
Mount Callahan	29	39 42 35.00	116 57 00.64	83 09 39.39	Carson Sink	34.84	−04.55	−1.2041	+05.48
Diamond Peak	30	39 35 06.91	115 49 04.96	98 27 17.09	Mount Callahan	13.82	−03.27	−1.2094	+03.95
Eureka	31	39 30 48.34	115 57 30.65	... .		37.56	+06.91	+0.7715	+05.33
Pilot Peak	32	41 01 17.24	114 04 36.29	303 40 16.61	Mount Nebo	14.15	−02.46	−1.1495	+02.83
Austin	33	39 29 32.59	117 04 12.21	... .		27.24	+15.03	+0.7717	+11.60
Pioche	34	37 59 10.90	114 03 04.82	250 58 56.09	Tushar	50.29	−05.80	−1.2806	+07.43
Ibepah	35	39 49 42.46	113 55 08.93	81 11 33.73	Diamond Peak	28.49	−05.24	−1.1990	+06.28
Oasis	36	39 17 38.00	112 37 44.38	... .		55.41	+11.03	+0.7739	+08.54
Deseret	37	40 27 35.30	112 37 32.55	314 14 12.20	Mount Nebo	01.38	−10.82	−1.1725	+12.69
Promontory	38	41 17 53.69	112 25 09.27	283 24 00.52	Ogden Peak	02.64	+02.12	−1.1383	−02.41
Antelope	39	40 57 44.37	112 12 55.55	31 59 11.07	Deseret	04.14	−06.93	−1.1519	+07.98
Ogden Peak	40	41 12 00.76	111 52 53.07	356 19 41.73	Mount Nebo	30.37	−11.36	−1.1423	+12.98
Ogden Observatory, longitude pier	41	41 13 12.54	111 59 37.97	... .		55.59	+17.62	+0.7522	+13.25
Ogden, azimuth station	42	41 13 12.40	111 59 37.97	283 08 58.94	Ogden Peak	44.70	−14.24	−1.1415	+16.25
Waddoup	43	40 54 26.00	111 53 10.50	180 42 54.07	Ogden Peak	32.55	−21.52	−1.1541	+24.84
Salt Lake City, longitude sta.	44	40 46 12.38	111 53 27.30	... .		47.60	+20.30	+0.7573	+15.37
Salt Lake City, azimuth station	45	40 46 11.58	111 53 27.34	192 02 66.15	City Creek	50.50	−15.65	−1.1598	+18.15
Mount Nebo	46	39 48 39.11	111 45 56.92	20 05 38.64	Tushar	23.06	−15.58	−1.1998	+18.69
Wasatch	47	39 06 54.36	111 27 11.92	160 54 09.84	Mount Nebo	02.73	−07.11	−1.2299	+08.74
Mount Ellen	48	38 07 17.13	110 48 50.66	195 35 62.08	Patmos Head	57.89	−04.19	−1.2744	+05.34
Patmos Head	49	39 30 08.57	110 18 57.95	66 41 27.36	Wasatch	18.70	−08.66	−1.2130	+10.50
Green River	50	38 59 30.28	110 09 55.68	... .		53.08	−02.60	+0.7772	−02.02
Mount Waas	51	38 32 21.47	109 13 38.34	72 00 28.31	Mount Ellen	16.62	−11.69	−1.2554	+14.68
Tavaputs	52	39 32 23.95	109 00 19.20	88 17 44.58	Patmos Head	40.85	−03.73	−1.2114	+04.52
Grand Junction, longitude sta.	53	39 03 55.22	108 33 53.40	... .		54.02	+00.62	+0.7764	+00.48
Grand Junction, azimuth sta.	54	39 04 18.95	108 33 36.51	23 57 29.49	Chiquita	23.98	−05.51	−1.2318	+06.79
Gunnison, Colo., longitude sta.	55	38 32 47.09	106 55 26.73	... .		30.68	+03.95	+0.7821	+03.09
Gunnison, Colo., azimuth sta.	56	38 32 46.63	106 55 26.73	41 55 09.44	Uncompahgre	00.39	−09.05	−1.2551	+11.36
Uncompahgre	57	38 04 18.42	107 27 41.78	196 42 61.82	Treasury Mountain	55.84	−05.98	−1.2767	+07.63
Treasury Mountain	58	39 00 51.96	107 05 54.78	74 45 15.99	Mount Waas	04.71	−11.28	−1.2343	+13.92
Mount Ouray	59	38 25 22.20	106 13 27.33	70 35 52.02	Uncompahgre	51.27	−00.75	−1.2607	+00.95
Pikes Peak	60	38 50 26.32	105 02 37.34	66 05 08.82	Mount Ouray	16.70	+07.88	−1.2420	−09.79
Colorado Springs (1885)	61	38 49 59.28	104 49 34.71	... .		10.65	−24.06	+0.7790	−18.74
El Paso East Base	62	38 57 22.53	104 27 41.99	102 48 01.03	El Paso West Base	04.62	+03.59	−1.2309	−04.44
Overland	63	39 02 20.35	103 10 15.65	284 10 31.45	Eureka	32.62	+01.17	−1.2331	−01.44
Wallace	64	38 54 44.34	101 35 31.50	... .		25.96	−05.54	+0.7781	−04.31
Russell Southeast Base	65	38 51 22.30	98 47 08.07	140 42 65.0	Russell NW. Base	59.79	−05.21	−1.2413	+06.47
Ellsworth	66	38 43 48.76	98 13 44.98	... .		36.36	−08.62	+0.7801	−06.72
Salina West Base	67	38 51 07.67	97 36 10.84	248 36 22.26	Salina East Base	18.32	−03.94	−1.2815	+04.89
Adams	68	39 02 40.97	96 04 24.37	11 46 10.48	Clark	11.94	+01.46	−1.2330	−01.80
Kansas City, astr. sta.	69	39 05 50.40	94 35 22.16	... .		21.06	−01.10	+0.7761	−00.85
Hunter	70	38 25 45.21	92 46 24.37	221 48 21.41	Christian	20.49	−00.92	−1.2604	+01.16
Jefferson City	71	38 33 41.16	92 09 45.55	199 55 34.19	Cedar	37.47	+03.28	−1.2544	−04.11
Berger	72	38 35 58.12	91 17 27.98	39 12 01.33	Winter	05.64	+04.31	−1.2527	−05.40
Kleinschmidt	73	38 30 19.82	90 19 29.66	200 09 29.05	Insane Asylum	31.81	+02.76	−1.2569	−03.47
St. Louis, Washington University, transit pier, 1881-82	74	38 37 58.59	90 12 17.44	... .		18.84	+01.40	+0.7812	+01.09
Bording	75	38 36 45.30	89 20 25.60	53 25 04.11	Geoffrey	07.53	+03.42	−1.2521	−04.28
Newton	76	38 55 28.59	88 09 50.32	321 29 04.59	Claremont	05.30	+00.71	−1.2382	−00.88
Parkersburg, longitude station	77	38 34 51.52	88 01 49.00	... .		48.30	−00.70	+0.7817	−00.55
Parkersburg, azimuth station	78	38 34 51.52	88 01 49.00	143 16 15.64	Denver	15.55	−00.09	−1.2534	+00.11

Deflections in prime vertical—Continued.

Station.	No.	Geodetic latitude.			Geodetic longitude.			Geodetic azimuth.			To station.	Ast. Long. or Az.	A−G.	Cos φ′ br −Cot φ′.	A−G (P. V.).
		°	′	″	°	′	″	°	′	″		″	″		
Vincennes	79	38	40	35.70	87	31	35.05	...	..	.....		30.14	−04.91	+0.7807	−03.83
Osborn	80	38	51	23.46	86	52	35.79	192	16	16.78	Calvary	17.59	+00.81	−1.2413	−01.01
Weed Patch	81	39	10	00.63	86	13	00.77	7	33	20.48	Fountain	21.28	+00.80	−1.2276	−00.98
Reizin	82	39	02	53.86	85	08	24.08	276	56	47.50	Tanner	46.02	−01.48	−1.2326	+01.82
Cincinnati, Mt. Lookout, C. and G. S. transit pier, 1881	83	39	08	21.87	84	25	21.93	...	..	.....		21.52	−00.41	+0.7756	−00.31
Minerva	84	38	42	31.16	83	55	06.65	210	54	46.17	Ash Ridge	42.38	−03.79	−1.2478	+04.73
Gould	85	38	38	27.58	82	49	56.73	84	49	09.90	Howland	13.61	+03.71	−1.2508	−04.64
Piney	86	38	26	39.59	82	03	29.00	119	04	31.38	Gebhardt	31.84	+00.46	−1.2597	−00.58
Charleston, W. Va.	87	38	21	02.29	81	37	59.19	...	..	.....		61.95	+02.76	+0.7842	+02.16
Keeney	88	37	46	24.84	80	42	19.18	257	04	32.52	Bald Knob	35.89	+03.37	−1.2904	−04.35
Elliott Knob	89	38	09	59.33	79	18	51.33	303	25	21.08	Humpback	24.46	+03.38	−1.2723	−04.30
Charlottesville, McCormick Observatory	90	38	01	58.03	78	31	20.62	...	..	.....		20.10	−00.52	+0.7877	−00.41
Clark	91	38	18	41.10	78	00	11.48	202	19	27.68	Bull Run	27.98	+00.30	−1.2657	−00.38
Strasburg	92	38	59	29.95	78	21	39.01	...	..	.....		35.70	−03.31	+0.7772	−02.57
Long Mount	93	37	17	27.59	79	05	10.23	223	28	45.44	Spear	41.64	−03.80	−1.3131	+04.99
Bull Run	94	38	52	53.58	77	42	12.60	263	53	29.48	Peach Grove	28.49	−00.99	−1.2401	+01.23
Maryland Heights	95	39	20	27.69	77	42	59.92	358	43	09.41	Bull Run	07.18	−02.23	−1.2200	+02.72
Sugarloaf	96	39	15	44.54	77	23	36.88	32	29	21.17	Bull Run	16.97	−04.20	−1.2234	+05.14
Causten	97	38	55	35.29	77	04	23.81	210	54	36.78	Soper	41.65	+04.87	−1.2381	−06.03
Naval Observatory, clock room	*99	38	55	17.03	77	03	62.24	...	..	.....		56.76	−05.48	+0.7780	−04.26
Naval Observatory, old site, center of dome	100	38	53	42.27	77	03	06.10	...	..	.....		02.30	−03.80	+0.7783	−02.96
C. and G. S. Observatory, transit	101	38	53	12.15	77	00	32.15	...	..	.....		25.64	−06.51	+0.7784	−05.07
Soper	102	39	05	11.84	76	57	00.73	268	49	17.06	Webb	23.60	+06.54	−1.2311	−08.05
Hill	103	38	53	54.91	76	52	49.76	219	46	50.05	Webb	58.11	+08.06	−1.2394	−09.99
Webb	104	39	05	26.56	76	40	30.17	88	59	41.63	Soper	49.38	+07.75	−1.2309	−09.54
Marriott	105	38	52	27.56	76	36	35.15	96	37	33.97	Hill	43.40	+09.43	−1.2404	−11.70
Calvert	106	38	21	34.29	76	23	34.92	252	06	00.08	Meekins Neck	09.18	+09.10	−1.2635	−11.50
Principio	107	39	35	36.69	76	00	16.41	1	34	36.41	Turkey Point	43.50	+07.09	−1.2091	−08.57
Dover	108	39	09	20.75	75	31	23.91	...	..	.....		18.45	−05.46	+0.7754	−04.23
Cape Henlopen Light-House	109	38	46	51.55	75	05	02.94	173	45	17.37	Brandywine Shoals L. H.	17.64	+00.27	−1.2446	−00.34
Cape May, transit	110	38	55	48.69	74	55	47.41	...	..	.....		45.68	−01.73	+0.7779	−01.35
Cape Henry Light-House (old)	111	36	55	32.82	76	00	30.23	205	38	19.62	Cape Charles L. H. (old)	19.31	−00.31	−1.3307	+00.41
Roslyn, longitude station	†113	37	14	28.35	77	23	51.10	...	..	.....		46.61	−04.49	+0.7961	−03.57
Staunton	114	38	08	46.51	79	04	19.09	...	..	.....		16.24	−02.85	+0.7864	−02.24
North End Knott Island	115	36	33	57.61	75	55	18.58	172	34	03.76	Ragged Island	08.67	+04.91	−1.3481	−06.62
Wolftrap	116	37	24	04.75	76	14	42.98	14	28	27.63	New Point Comfort	26.0	−01.63	−1.3079	+02.13
Tangier Island	117	37	47	57.70	75	59	15.00	282	48	32.2	Watts Island L. H.	36.5	+04.30	−1.2892	−05.54
Seaton, longitude station	118	38	53	28.96	76	59	59.54	...	..	.....		52.73	−06.81	+0.7783	−05.37
Statesville	119	35	46	56.41	80	53	39.91	...	..	.....		41.31	+01.40	+0.8112	+01.14
Moore	120	36	23	53.49	80	16	59.22	158	33	30.90	Buffalo	31.19	+00.29	−1.3565	−00.39
Young	121	35	44	14.36	80	38	51.12	126	52	51.43	Poore	53.69	+02.26	−1.3898	−03.14
King	122	35	12	27.72	81	18	45.52	141	33	38.08	Benn	36.9	−01.20	−1.4173	+01.70
Nashville	123	36	09	57.57	86	47	00.25	...	..	.....		01.24	+00.99	+0.8073	+00.80
Lebanon North Base	124	36	12	46.89	86	18	24.56	6	00	48.39	Lebanon South Base	48.76	+00.37	−1.3657	−00.51
Paris	125	34	56	29.06	82	24	39.96	267	18	14.63	Wofford	15.17	+00.54	−1.4312	−00.77
Currahee	126	34	31	44.89	83	22	33.20	188	10	24.70	Rabun	27.89	+03.19	−1.4534	−04.64
Lavender	127	34	19	19.24	85	17	18.25	300	11	59.70	Kenesaw	59.11	−00.59	−1.4647	+00.86
Sawnee	128	34	14	11.84	84	09	38.70	245	34	27.60	Currahee	26.12	−01.48	−1.4695	+02.17
Atlanta Middle Base	129	33	54	21.46	84	16	37.63	312	22	31.39	Stone Mountain	28.94	−02.45	−1.4878	+03.65
Atlanta	130	33	44	58.11	84	23	18.91	...	..	.....		20.07	+01.16	+0.8315	+00.96
Montgomery	131	32	22	39.34	86	17	60.41	...	..	.....		59.19	−01.22	+0.8445	−01.03
Lower Peach Tree	132	31	50	20.45	87	32	42.87	...	..	.....		40.94	−01.93	+0.8495	−01.64
Aurora	133	34	08	47.48	86	11	00.66	236	06	30.01	Brandon	29.35	−00.66	−1.4744	+00.97
Kahatchee	134	33	13	38.27	86	21	36.53	253	32	14.06	Horn	12.80	−01.26	−1.5265	+01.92
Ethridge	135	32	04	45.89	87	03	29.07	245	52	50.62	Lovers Leap	49.31	−01.31	−1.5954	+02.09
Fort Morgan, azimuth station	136	30	13	42.24	88	01	23.26	143	59	49.26	Cedar Point	45.61	−03.65	−1.7162	+06.26
Mobile	137	30	41	30.89	88	02	33.40	...	..	.....		37.37	+03.97	+0.8599	+03.41
Tanyard	138	34	39	11.36	87	23	01.06	349	59	08.92	Penit	04.78	−04.14	−1.4467	+05.99
East Pascagoula	139	30	20	35.52	88	32	45.21	296	30	51.75	Bayou Casotte	51.27	−00.48	−1.7084	+00.82
New Orleans	†141	29	56	53.40	90	04	11.66	...	..	.....		11.44	−00.22	+0.8665	−00.19
Yard	142	39	58	24.80	75	23	13.79	347	17	38.94	Lippincott	38.57	−00.37	−1.1929	+00.44
Mount Rose	143	40	22	03.41	74	43	25.77	7	46	60.18	Mount Holly	55.59	−04.59	−1.1764	+05.40
Beacon Hill	144	40	22	26.55	74	13	41.91	183	35	31.24	Weasel	29.89	−01.35	−1.1761	+01.59
West Hills	145	40	48	55.19	73	25	32.40	174	57	35.76	Wooster	38.32	+02.56	−1.1579	−02.96
Sandford	146	41	27	42.80	72	56	59.30	5	50	16.88	Ruland	25.28	+08.40	−1.1318	−09.51
Cambridge, Harvard Observatory, west transit pier	147	42	22	53.48	71	07	44.48	356	22	56.27	Blue Hill	57.95	+01.68	−1.0958	−01.84
Mount Tom	148	42	14	30.80	72	38	55.06	212	37	17.23	Monadnock	21.74	+04.51	−1.1012	−04.97
Spencer	149	41	40	43.26	71	29	40.68	185	57	36.34	Beaconpole	33.23	−03.11	−1.1232	+03.49
Beaconpole	150	41	59	42.47	71	27	01.26	228	55	19.77	Blue Hill	17.23	−02.54	−1.1108	+02.82
Copecut	151	41	43	17.24	71	03	36.64	175	17	06.30	Blue Hill	06.47	+00.17	−1.1216	−00.19
Indian	152	41	25	46.74	70	40	40.70	135	35	64.33	Copecut	58.82	−05.51	−1.1331	+06.24
Shootflying	153	41	41	07.33	70	20	49.74	143	03	20.31	Manomet	22.74	+02.43	−1.1230	−02.72
Blue Hill	154	42	12	43.94	71	06	52.64	305	57	32.16	Manomet	30.06	−02.10	−1.1024	+02.32
Wachusett	155	42	29	20.78	71	53	13.98	24	17	34.50	Bald Hill	41.32	+06.82	−1.0917	−07.45
Thompson	156	42	36	41.92	70	43	49.16	351	21	42.68	Manomet	41.85	−00.83	−1.0871	+00.90
Cambridge, Harvard Observatory, center of dome	157	42	22	53.49	71	07	43.88	...	..	.....		45.69	+01.81	+0.7387	+01.34
Duxbury, transit	158	42	02	55.18	70	40	10.96	...	..	.....		12.87	+01.91	+0.7426	+01.42
Unkonoonuc	159	42	58	59.86	71	35	19.38	196	35	18.88	Gunstock	20.40	+01.52	−1.0730	−01.63
Gunstock	160	43	31	04.32	71	22	11.38	217	43	29.56	Mount Pleasant	33.53	+03.97	−1.0532	−04.18
Agamenticus	161	43	13	24.63	70	41	32.91	2	36	58.26	Thompson	55.53	−02.73	−1.0640	+02.90
Mount Pleasant	162	44	01	36.69	70	49	21.94	205	59	19.58	Mount Blue	21.51	+01.93	−1.0345	−02.00
Mount Independence	163	43	45	33.75	70	19	14.34	26	55	50.74	Agamenticus	48.64	−02.10	−1.0443	+02.19

* Serial number 98 does not exist in this list.

† Serial numbers 112 and 140 do not exist in this list.

Deflections in prime vertical—Continued.

Station.	No.	Geodetic latitude.	Geodetic longitude.	Geodetic azimuth.	To station.	Ast. Long. or Az.	A − G.	Cos ϕ' or −Cot ϕ'.	A − G (P. V.).
		° ′ ″	° ′ ″	° ′ ″		″	″		″
Sabattus	164	44 08 37.83	70 04 43.74	24 31 22.95	Mount Independence	23.58	+00.63	−1.0303	−00.65
Cape Small	165	43 46 43.22	69 50 44.22	155 19 01.96	Sabattus	03.51	+01.55	−1.0436	−01.62
Ragged Mountain	166	44 12 45.38	69 09 05.33	81 48 43.95	Mount Pleasant	44.98	+01.03	−1.0279	−01.06
Mount Harris	167	44 39 54.87	69 08 54.67	254 35 08.43	Humpback	10.73	+02.30	−1.0118	−02.33
Mount Desert	168	44 21 05.23	68 13 37.65	78 30 48.09	Ragged Mountain	46.77	−01.32	−1.0229	+01.35
Humpback	169	44 51 50.69	68 06 37.78	254 42 29.88	Cooper	32.30	+02.42	−1.0048	−02.43
Howard	170	44 37 46.58	67 23 45.36	63 54 45.07	Pigeon	45.11	+00.04	−1.0130	−00.04
Cooper	171	44 59 13.48	67 28 02.25	351 53 12.04	Howard	12.05	+00.01	−1.0004	−00.01
Bangor	172	44 48 16.14	68 47 00.15	... .		02.60	+02.45	+0.7095	+01.74
Calais	173	45 11 05.68	67 16 52.76	... .		57.86	+05.10	+0.7048	+03.59
Mount Merino	174	42 14 05.06	73 49 03.54	74 44 42.3	Catskill	47.26	+04.96	−1.1015	−05.46
Howlett	175	42 59 52.96	76 17 26.62	275 41 31.18	Fenner	31.43	+00.25	−1.0724	−00.27
Tassel	176	42 56 29.13	75 19 00.51	92 55 41.75	Fenner	44.13	+02.38	−1.0746	−02.56
Prospect	177	43 25 17.95	73 46 04.57	298 02 50.28	Mount Equinox	60.34	+10.06	−1.0567	−10.63
Cheever	178	44 04 54.06	73 27 02.85	275 10 56.66	Whitford	70.96	+14.30	−1.0326	−14.77
Ogdensburg Light-House	179	44 41 52	75 30 14.71	... .		18.24	+03.53	+0.7108	+02.51
Ogdensburg Light-House	180	44 41 52.44	75 30 14.29	206 54 47.85	Windmill	47.00	−00.85	−1.0106	+00.86
Watertown Court-House	181	43 58 32.31	75 54 52.96	... .		01.29	+08.33	+0.7196	+05.99
Mannsville	182	43 42 54.48	76 03 13.50	... .		26.09	+12.59	+0.7228	+09.10
Oswego	183	43 26 37.31	76 30 49.28	... .		52.89	+03.61	+0.7261	+02.62
Rochester City Hall	184	43 09 18.05	77 36 49.60	... .		50.34	+00.74	+0.7295	+00.54
Tonawanda	185	43 00 03.73	78 53 20.88	... .		20.49	−00.39	+0.7313	−00.29
Tonawanda	186	43 00 03.73	78 53 20.88	314 22 37.18	Buffalo Plains	40.61	+03.43	−1.0723	−03.68
Buffalo, intersection Exchange and Michigan streets	187	42 52 40.98	78 52 13.25	... .		17.34	+04.09	+0.7328	+03.00
Dunkirk Light-House	188	42 29 37.68	79 21 14.81	... .		25.14	+10.33	+0.7374	+07.62
Sandy Creek North Base	189	43 40 43.67	76 12 00.92	357 14 23.29	Sandy Creek South Base	26.95	+03.66	−1.0472	−03.83
Erie Range Light No. 1	190	42 09 11.66	80 04 39.22	... .		46.29	+07.07	+0.7414	+05.24
Cleveland Light-House	191	41 30 01.40	81 42 08.68	... .		12.54	+03.86	+0.7490	+02.89
Toledo	192	41 39 04.36	83 32 31.00	... .		29.94	−01.06	+0.7472	−00.79
Sandusky West Base	193	41 29 02.11	82 40 57.57	319 36 42.87	Sandusky East Base	38.60	−04.27	−1.1309	+04.83
Cleveland, transit	194	41 30 20.88	81 41 29.21	... .		28.90	−00.31	+0.7489	−00.23
Monroe Court-House	195	41 54 52.44	83 23 48.93	... .		45.69	−03.24	+0.7441	−02.41
Detroit	196	42 20 00.56	83 03 06.65	... .		02.94	−03.71	+0.7392	−02.74
Sault Ste. Marie Observatory, west pier.	*198	46 30 05.77	84 20 55.64	... .		48.09	−07.55	+0.6884	−05.20
Mackinac A-West Base	199	45 47 14.43	84 46 21.65	213 53 25.25	D-St. Ignace	29.05	+03.80	−0.9729	−03.70
Sault Ste. Marie Observatory, west pier	200	46 30 05.77	84 20 55.64	178 06 34.04	Azimuth	38.87	+04.83	−0.9489	−04.58
Ford River 2	201	45 41 12.32	87 06 08.44	... .		00.69	−07.75	+0.6986	−05.41
Thones Hill	202	46 31 54.12	87 27 03.30	... .		49.29	−14.01	+0.6879	−09.64
Ford River 2	203	45 41 12.32	87 06 08.44	31 35 55.81	Cedar River	00.28	+04.47	−0.9763	−04.36
Keweenaw South Base	204	46 52 18.01	88 29 15.82	199 10 59.69	Keweenaw North Base	67.29	+10.60	−0.9367	−09.93
Gargantua	205	47 34 42.83	84 58 53.93	335 06 23.73	Mamainse	19.93	−03.80	−0.9138	+03.47
Menominee	206	45 05 12.90	87 35 33.71	... .		27.09	−06.62	+0.7060	−04.67
Fort Howard	207	44 30 30.40	88 02 33.21	... .		33.39	+00.18	+0.7132	+00.13
Aminicon	208	46 41 32.47	91 51 42.84	153 36 56.53	Lester	66.15	+09.62	−0.9426	−09.07
Bruce	209	44 31 39.49	87 53 57.29	139 16 06.90	Longtail Point L. H.	04.90	−02.00	−1.0166	+02.03
Chicago Light-House	210	41 53 20.50	87 36 51.46	... .		48.24	−03.22	+0.7444	−02.40
Willow Springs	211	41 43 36.90	87 51 05.63	... .		06.09	+00.46	+0.7463	+00.34
Willow Springs	212	41 43 36.90	87 51 05.63	224 29 40.94	Shot Tower	41.24	+00.30	−1.1273	−00.34
Minnesota Point North Base	213	46 45 28.22	92 04 42.52	... .		32.34	−10.18	+0.6851	−06.97
Minnesota Point North Base	214	46 45 28.22	92 04 42.52	323 52 15.65	Minnesota Point S. B.	24.32	+08.67	−0.9404	−08.15
Minnesota Junction	215	43 28 28.54	88 43 46.78	272 30 48.99	Horicon	45.03	−03.96	−1.0547	+04.18
San Diego	216	32 43 29.96	117 09 28.46	... .		41.22	+12.76	+0.8413	+10.73
Los Angeles Normal School	217	34 03 02.20	118 15 16.46	... .		23.42	+06.96	+0.8285	+05.77
Santa Ana	218	36 54 19.37	121 13 57.78	38 44 33.91	Mount Toro	26.21	−07.70	−1.3316	+10.25
Mount Toro	219	36 31 34.71	121 36 32.28	162 12 31.32	Loma Prieta	26.63	−04.69	−1.3501	+06.33
Hepsedam	220	36 18 53.60	120 49 26.36	150 57 28.97	Santa Ana	18.58	−10.39	−1.3606	+14.14
Santa Lucia	221	36 08 45.33	121 25 05.94	157 58 56.29	Mount Toro	51.50	−04.79	−1.3690	+06.56
Castle Mount	222	35 56 21.34	120 20 22.91	133 51 16.62	Hepsedam	11.08	−05.54	−1.3795	+07.64
Lospe	223	34 53 38.48	120 36 19.94	267 10 02.14	Tepusquet	51.78	−10.36	−1.4338	+14.85
Tepusquet	224	34 54 37.43	120 11 09.65	87 24 26.30	Lospe	12.24	−14.06	−1.4329	+20.15
Arguello	225	34 34 58.96	120 33 39.01	284 59 48.69	Gaviota	37.73	−10.96	−1.4505	+15.90
Gaviota	226	34 30 07.45	120 11 53.43	283 30 11.77	Santa Barbara	01.69	−10.08	−1.4549	+14.67
New San Miguel	227	34 02 23.75	120 23 09.47	165 05 23.69	Arguello	09.76	−13.93	−1.4804	+20.62
Santa Cruz West	228	34 04 24.02	119 55 02.76	206 51 42.78	Santa Barbara	29.45	−13.33	−1.4785	+19.71
Santa Barbara	229	34 24 17.60	119 42 53.09	103 46 36.35	Gaviota	26.05	−10.30	−1.4602	+15.04
Los Angeles Northwest Base	230	33 55 05.65	118 03 23.78	322 33 21.50	Los Angeles SE. Base	08.35	−13.15	−1.4871	+19.56
Los Angeles Southeast Base	231	33 47 34.65	117 56 30.32	142 37 11.84	Los Angeles NW. Base	57.42	−14.42	−1.4942	+21.55
Fort Gratiot Observatory	232	42 59 52.76	82 25 44.49	... .		35.13	−09.36	+0.7314	−06.85
Provincetown	233	42 03 09.37	70 11 19.02	... .		17.21	−01.81	+0.7425	−01.34
Dominguez Hill	234	33 51 55.64	118 14 11.64	95 17 48.03	West Beach	41.48	−06.55	−1.4901	+09.76
Davis	235	38 20 28.53	75 06 21.63	13 37 06.10	Quilling	05.95	−00.15	−1.2643	+00.19
Sulphur Peak	236	38 45 54.28	122 50 40.22	81 28 50.74	Mark	35.63	−15.11	−1.2453	+18.82
Ross Mountain	237	38 30 20.58	123 07 09.22	164 24 53.44	Mark	38.98	−14.46	−1.2570	+18.18
Point Avisadero	238	37 43 33.88	122 21 47.48	195 03 36.5	Elongation Mark	39.96	+03.40	−1.2927	−04.47
Monterey Bay	239	36 35 30.81	121 52 59.20	116 01 15.9	Elongation Mark	11.8	−04.10	−1.3469	+05.52
Santa Cruz	240	36 58 42.02	122 03 18.69	165 14 28.9	Azimuth Mark	18.33	−10.57	−1.3280	+14.04
Avila	241	35 10 40.20	120 43 17.32	281 43 31.3	San Luis Obispo W. B.	10.2	−21.10	−1.4187	+29.93
San Buenaventura	242	34 15 54.80	119 15 57.15	114 04 42	San Buenaventura Az. Mk.	28.80	−13.20	−1.4679	+19.38
Buenavista	243	34 03 18.60	118 14 34.38	... .		41.25	+06.87	+0.8284	+05.69
Soledad	244	32 50 24.52	117 15 07.24	178 32 48.37	Soledad Azimuth Mark	30.26	−18.11	−1.5493	+28.06
San Diego 1851	245	32 42 03.95	117 14 31.29	187 53 35.0	Fitchs Hill	14.18	−20.82	−1.5576	+32.43
San Diego 1871	246	32 43 12.31	117 09 27.83	... .		42.30	+14.47	+0.8413	+12.17

* Serial number 197 does not exist in this list.

COMPUTATION OF TOPOGRAPHIC DEFLECTIONS.

By the expression "topographic deflection at a station" is meant the deflection which would be produced by the irregular distribution of the masses corresponding to the known irregularities of the surface—the topography.

Every cubic foot of the mass of a hill to the northward of a station must necessarily attract the plumb bob at that station, and so tend to produce a southward displacement of the zenith at the station. The density of the mass of the hill is approximately known. Its volume may be calculated from the information given in the form of contour lines on a good topographic map. The distance and direction of each part of its mass from the station may also be obtained from the map. It is therefore possible, from the known law of gravitation, namely, that the attraction varies inversely as the square of the distance between the attracting masses and directly as the product of their masses, to compute the southward displacement of the zenith of the station produced by the mass of the hill.

This may be done for each feature of the topography at all distances and in every direction from the station so as to include, if necessary, the whole surface of the earth.

If such a computation were successfully made with absolute accuracy, the resulting computed deflection would be what is here called the topographic deflection at the station.

Very early in the investigation it was realized that it would be necessary to compute the topographic deflection for each station, and that the computation to serve its full purpose must extend to a great distance from the station. It was also realized that to make such computations by any method known to have been used heretofore would be impossible on account of the great expenditure of time and money involved. It was necessary, therefore, to devise some new method of computation, or to modify old methods, so as to make these computations feasible. The method described in the following pages was developed, and has served its purpose admirably.

The formula upon which the computation is based is well known.* It is

$$D = 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\epsilon \frac{r'}{r_1}$$

D is the meridian component of the deflection at the station produced by a mass of the surface material of the earth which is a stratum h statute miles thick, lying within a four-sided compartment limited by two radial lines drawn from the station and by two arcs of circles with their common center at the station and having the radii r' and r_1.

a' and a_1 are the angles between each of the two radial lines and the meridian.

δ is the mean surface density of the earth.

Δ is the mean density of the earth as a whole.

The constant 12″.44 depends upon the supposition that for the present purpose the earth may be considered a sphere of which the radius is 6 370 kilometers, or 3 960 miles.

The whole of the attracting stratum is assumed to be in the horizon of the station.

If the prime vertical component of the deflection at the station is desired, the only change necessary is to measure the angles a' and a_1 from the prime vertical instead of from the meridian.

If the stratum considered within any compartment be that which is limited below at sea level and above at the actual irregular surface of the earth, then with considerable accuracy the following statement, based upon the formula, may be made: For compartments bounded by circles whose radii are in geometric progression, and by radial lines the sines of whose angles with a reference line are in arithmetic progression, the deflections produced at a station at the center of the circles, in a direction parallel to the reference line, are, for each compartment, proportional to the mean elevation of the land surface within this compartment. The use of this method of division into compartments makes the computation much shorter and more convenient than it would otherwise be.

* For the derivation of this formula, see Geodesy, A. R. Clarke, Oxford, 1880, pp. 294-296.

The statement that for such compartments the deflections produced are proportional to the mean elevations is subject to three principal reservations affecting its accuracy. These reservations refer—

(*a*) To compartments so far from the station that the curvature of the earth's surface must be considered. This matter has been attended to by increasing the radii of certain of the outer circles, as indicated on page 22.

(*b*) To compartments near the station of which the mean surface lies so far above or below the station as to make a slope correction necessary. The method of computing the slope correction is given under the appropriate heading later.

(*c*) To compartments of which some part lies far above or far below the mean elevation for the compartment. This matter will be discussed later in connection with the other errors of computation.

The value $\frac{1}{2.09}$ was adopted for the ratio $\frac{\delta}{\Delta}$ which appears in the formula. This value is based upon the values of Δ and δ derived by Prof. William Harkness, namely, $\Delta = 5.576$ and $\delta = 2.67$.*

Any values for the constant ratio $\frac{r'}{r_1}$ and for the constant difference $(\sin a' - \sin a_1)$ may be adopted arbitrarily. Advantage was taken of this fact to adopt such values as would make the computation as simple and rapid as possible. The values $\frac{r'}{r_1} = 1.426$ and $(\sin a' - \sin a_1) = 0.25$ were adopted. With these values, together with the value $\frac{1}{2.09}$ for $\frac{\delta}{\Delta}$ referred to above, and it being decided that h is to be expressed in feet, for convenience, instead of statute miles, the formula becomes

$$D = 12''.44\left(\frac{1}{2.09}\right)\frac{(\text{h, in feet})}{5\,280}\,(0.25)\,\log_\epsilon 1.426$$
$$= 0.''0001000\ (\text{h, in feet})$$

As a result, then, of this particular selection of arbitrary constants defining the limits of the compartments, the deflection produced at the station by the material lying above sea level in any compartment is, expressed in hundredths of seconds of arc, the same as the mean elevation of the surface within the compartment expressed in hundreds of feet. This particular selection of constants saved a large number of multiplications which would otherwise have been necessary.

It should be carefully noted that the formula for D is not homogeneous in either of the forms given above. In each form D, as obtained, is in seconds of arc. In the first form, h is necessarily expressed in statute miles and in the second in feet.

Since the maps used in this investigation showed elevations in feet and the charts showed depths in feet and fathoms, it was necessary to adapt the formula to the foot as the unit. There would be no essential difficulty in adapting it to the meter as the unit, if desired, by a new choice of arbitrary constant values for $\frac{r'}{r_1}$ and $(\sin a' - \sin a_1)$. Throughout this report the foot, fathom, and mile used have the following metric values:

	Meters.
1 foot =	0.3048006
1 fathom =	1.828804
1 mile (statute) =	1 609.347

* For this value of Δ and the statement of the data on which it is based, see The Solar Parallax and its Related Constants, by William Harkness, Washington, Government Printing Office, 1891, pp. 89-91, 139. Similarly for this value of δ and the basis upon which it rests, consult the same publication, pp. 91-92.

SELECTION OF RADII.

The arbitrary selection of the adopted values of $(\sin a' - \sin a_1)$ and $\frac{r'}{r_1}$ was guided by three considerations. It was important to avoid the loss of time involved in making many multiplications. The compartments must be small enough to bring the accuracy of the method within the required limits and yet large enough to avoid excessive amount of detail in the computation. The compartments should be compact areas, not long and narrow, in order to facilitate the estimation of the mean elevation within each compartment. It was evidently necessary that the adopted value of $(\sin a' - \sin a_1)$ should be an aliquot part of unity, the sine of 90°.

The radii of the adopted circles separating the compartments are shown in the following table. Each group of 16 compartments, 4 in each quadrant, forms a ring. The rings were, for convenience of designation, assigned serial numbers, commencing with the outer ring.

Ring.	Outer radius.		Ring.	Outer radius.	
	Miles.	*Kilometers.*		*Miles.*	*Kilometers.*
1	2 564	4 126	20	2.899	4.665
2	1 757	2 828	21	2.033	3.272
3	1 219	1 962	22	1.426	2.295
4	850.8	1 369	23	1.0000	1.609
5	595.2	957.9	24	.7013	1.129
6	416.8	670.8	25	.4918	.7915
7	292.2	470.3	26	.3449	.5551
8	204.9	329.8	27	.2419	.3893
9	143.7	231.3	28	.1696	.2729
10	100.77	162.27	29	.1190	.1915
11	70.67	113.73	30	.0834	.1342
12	49.56	79.76	31	.0585	.0941
13	34.75	55.92	32	.0410	.0660
14	24.37	39.22	33	.0288	.0463
15	17.09	27.50	34	.0202	.0325
16	11.987	19.29	35	.0142	.0228
17	8.406	13.53	36	.0099	.0160
18	5.895	9.487	37	.0070	.0112
19	4.134	6.653	38	.0049	.0079

After the arbitrary ratio, $\frac{r'}{r_1}$, between the radii of successive circles had been adopted, it was still necessary to adopt a radius for some one circle. The outer radius of ring 23 was arbitrarily made exactly 1 statute mile, as shown in the above table, and the radii of the other circles, with the exception of the outer radii of rings 1 to 6, were calculated from this one by the use of the fixed ratio, 1.426.

The outer radii of rings 1 to 6, if computed in the same manner as the remaining radii, would have been, in miles: 2 456, 1 722, 1 207, 847.1, 594.0, and 416.6. But for the larger rings it is necessary, in order to secure the required degree of accuracy, to take account of the fact that for such rings, at a great distance from the station, the curvature of the earth's surface throws the compartment surface considerably below the horizon of the station. The exact formula for D, which then must be substituted for that given on page 20, is as follows:*

$$D = 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \left(\log_\varepsilon \frac{\tan \frac{1}{4} \theta'}{\tan \frac{1}{4} \theta_1} + \cos \tfrac{1}{2} \theta' - \cos \tfrac{1}{2} \theta_1 \right)$$

in which θ' is the r′ reduced to an arc, that is, θ' expressed in radians $= \frac{r' \text{ in miles}}{3960}$, 3960 miles

* See Geodesy, A. R. Clarke, Oxford, 1880, page 296.

being the adopted mean radius of the earth for the region under consideration. θ_1 bears this same relation to r_1. The quantity

$$\log_\varepsilon \frac{\tan \frac{1}{4}\theta'}{\tan \frac{1}{4}\theta_1} + \cos \tfrac{1}{2}\theta' - \cos \tfrac{1}{2}\theta_1$$

will be, in each case, smaller than $\log_\varepsilon \frac{r'}{r_1}$ and hence D, computed by the exact formula, is in every case less than D computed by the approximate formula. By trial it was found, however, that for all rings smaller than ring 6 the error of D, computed from the approximate formula, was less than one-thousandth part, and the approximate formula was, therefore, sufficiently exact. For rings 1 to 6 values for the radii r' and r_1 were found by successive trials and approximations such that the value of

$$\log_\varepsilon \frac{\tan \frac{1}{4}\theta'}{\tan \frac{1}{4}\theta_1} + \cos \tfrac{1}{2}\theta' - \cos \tfrac{1}{2}\theta_1$$

would, for each ring, be equal to $\log_\varepsilon 1.426$ and hence make it true for these rings, as for all smaller rings, that $D = 0.''0001000$ (h, in feet). The resulting radii as adopted are those shown in the table above.

The amounts by which the outer radii of rings 1 to 6 were increased to take account of the curvature were as follows:

Ring.	Increase in outer radius.	
	Miles.	*Kilometers.*
1	108	174
2	35	56
3	12	19
4	3.7	6
5	1.2	1.9
6	0.2	0.3

USE OF TEMPLATES.

For each scale of map or chart to be used in the computations the circles and radial lines defining the limits of the compartments were drawn to the proper scale on a sheet of transparent celluloid. Such a celluloid sheet, with compartment boundaries on it, has, for convenience, been called a template.

Such a template is shown in illustration No. 1. The circles are marked with the proper numbers corresponding to the table on the preceding page, and the scale of the template is marked upon it, as indicated. No attempt has been made to reproduce the illustration to the proper scale.

Each template consists of a sheet similar to that shown in illustration No. 1, of sufficient size to reach to the extreme limits of the largest map to be used, and carrying the lines indicating compartments in one quadrant, only.*

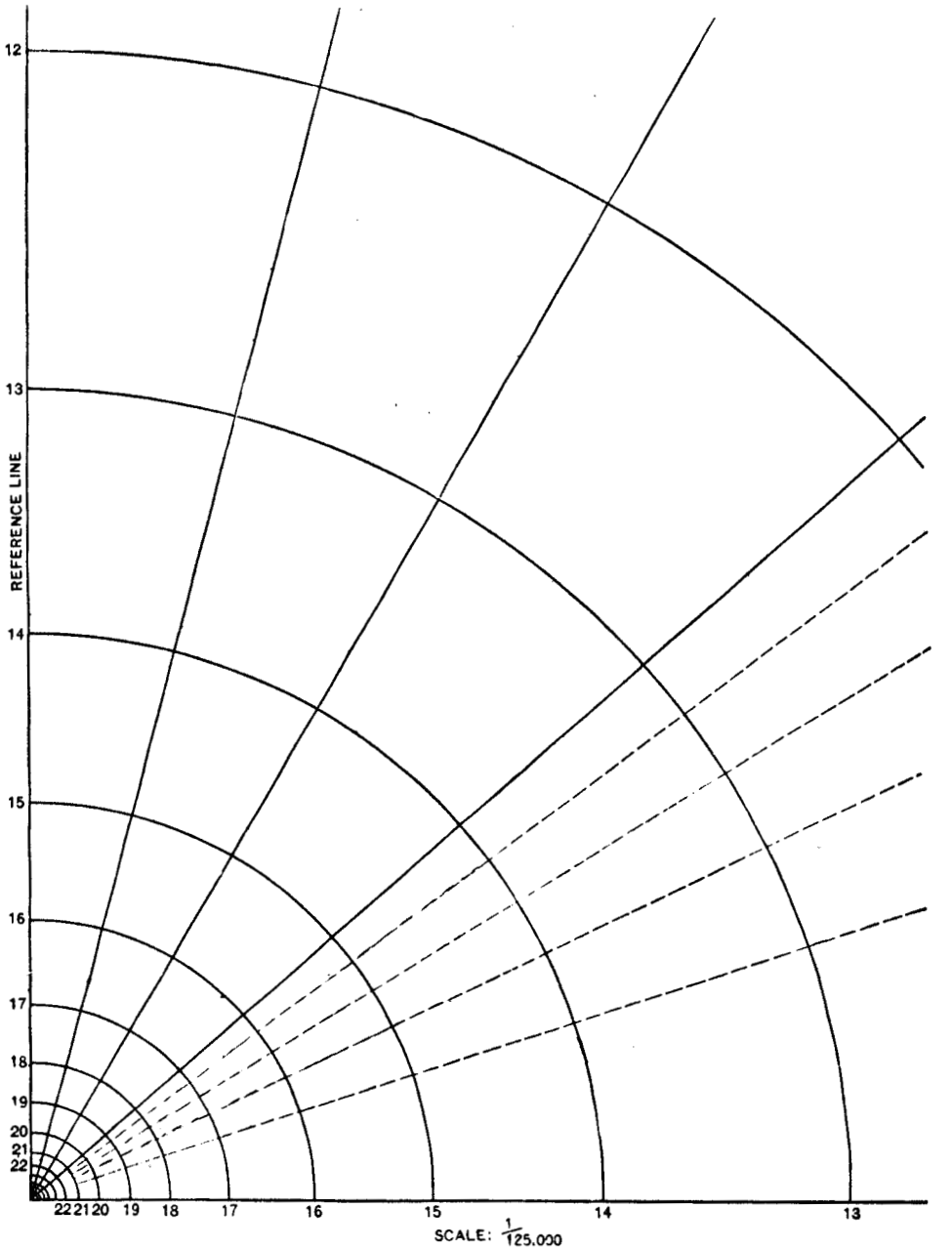
A complete template (not ordinarily used) is shown in illustration No. 2, in which the distinguishing numbers are marked upon the sectors limited by radial lines. For a computation of the meridian component of the topographic deflection the reference line is pointed toward the north and the sectors are numbered clockwise, commencing with the first which is to the southward of east from the station. The common center of the circles and the radial lines is placed, in each case, at the point on the map corresponding to the station.

A comparison of illustration No. 1 with illustration No. 2 will show why the templates actually used were drawn for one quadrant only. Such a sheet of transparent celluloid as that shown in illustration No. 1 may be placed in four successive positions on a map, corresponding to the four quadrants of illustration No. 2, since it may be turned with either face toward the

* The dotted radial lines on the template are ordinarily ignored. They subdivide certain compartments into five parts for a special purpose, indicated later under the heading "Errors in topographic deflections due to method of computation."

map. In order to avoid confusion the sector numbers were omitted from the quadrant templates used, as the numbers would be correct in only one-fourth of the cases.

The complete template for a computation of the prime vertical component of the topographic deflection is exactly like that shown in illustration No. 2, except that the reference line is now pointed toward the west. The sectors are again numbered clockwise, commencing with



No. 1.

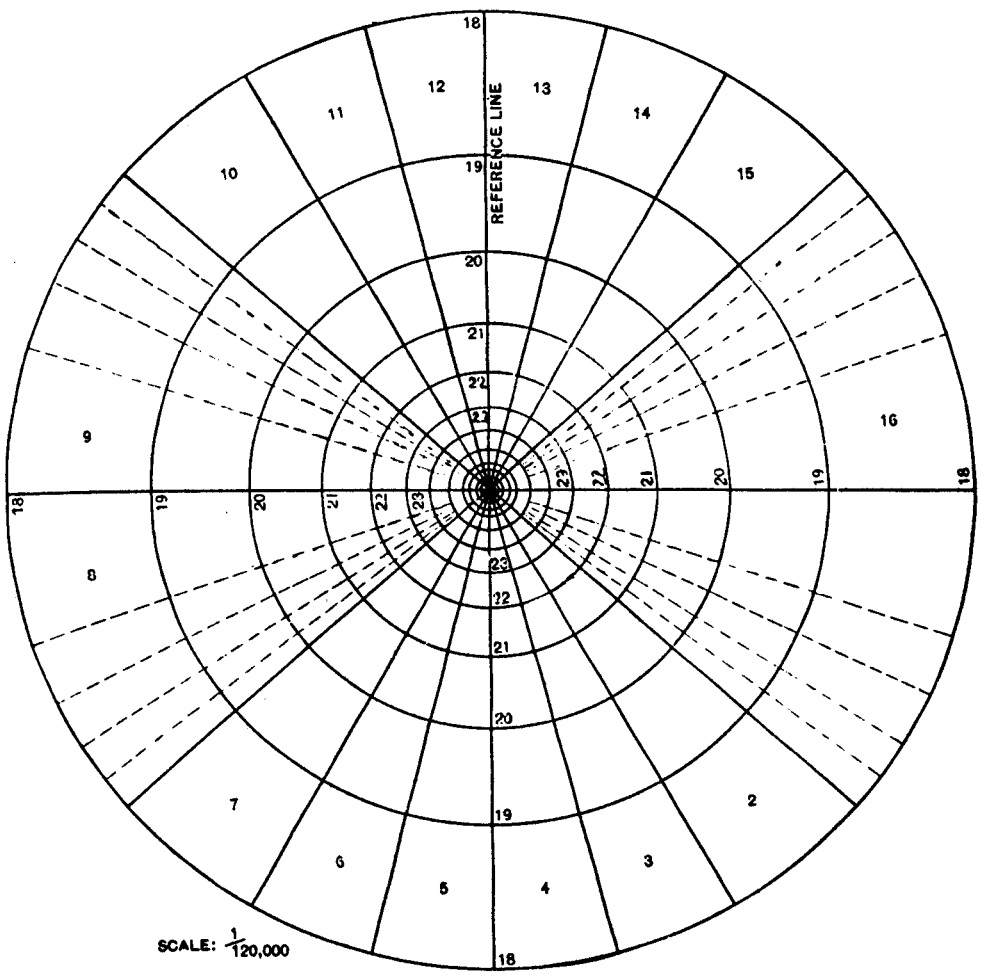
the first which is to the east of north from the station. The same quadrant templates are used for both computations of the meridian components and the prime vertical components.

Each compartment is designated by the number of the ring and the number of the sector within which it falls.

By the use of these transparent celluloid templates, the many circles and radial lines, fixing the limits of the compartments on a given map, for any station, were superposed on the map by

the mere process of laying the template on the map in the proper position. The use of the templates saved a very considerable amount of labor which would otherwise be necessary in drawing many thousands of compartments on many hundreds of maps. It also left the maps without damage or defacement.

To compute the component in the meridian of the topographic deflection at a given station, the computer places the template in the proper position on a contour map, namely, with the intersection of the radial lines at the station and with the reference line of the template in the meridian of the station and pointing northward. He then estimates from the contour lines on the map seen through the template the mean elevation of the surface within each



No. 2

compartment. He records in the proper blank on his computation form 0.″01 of deflection for each 100 feet of elevation above sea level, and assigns to each the proper sign, according to whether the compartment in question is to the northward or the southward of the station. The algebraic sum of all such recorded quantities is the meridian component of the topographic deflection at each station.

If the component of the topographic deflection in the prime vertical is required, the only changes in the process are to place the reference line of the template in the direction of the prime vertical instead of in the meridian and to make the corresponding assignment of algebraic signs.

The following is a reproduction of a computation of the meridian component of a topographic deflection, practically in the form in which it came from the computer:

Computation of topographic deflection, latitude station No. 164, Calais, Maine.

Ring.	Number of sector.																Horizontal sum.	Continuous sum.
	Plus unless italics.								Minus unless italics.									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
28	.010	.011	.011	.010	.010	.010	.010	.010	.009	.008	.007	.007	.006	.007	.007	.008	+ .02	+ .02
27	.010	.011	.011	.010	.011	.011	.011	.010	.008	.007	.006	.004	.005	.005	.006	.007	+ .04	+ .06
26	.009	.010	.009	.010	.010	.009	.009	.009	.007	.005	.004	.002	.002	.003	.004	.005	+ .04	+ .10
25	.008	.012	.011	.012	.010	.007	.005	.003	.003	.000	.000	.000	.000	.000	.000	.002	+ .06	+ .16
24	.008	.012	.012	.012	.010	.012	.010	.002	.006	.004	.003	.005	.002	.002	.000	.000	+ .06	+ .22
23	.007	.117	.015	.013	.015	.015	.005	.005	.010	.008	.008	.007	.005	.004	.004	.000	+ .05	+ .27
22	.005	.020	.024	.02	.02	.01	.01	.02	.02	.02	.02	.01	.004	.004	.004	.003	+ .04	+ .31
21	.006	.022	.02	.01	.01	.01	.01	.02	.01	.02	.02	.02	.01	.01	.02	.007	− .01	+ .30
20	.009	.025	.02	.01	.01	.01	.02	.01	.01	.02	.02	.01	.01	.01	.02	.016	.00	+ .30
19	.012	.02	.01	.02	.02	.02	.01	.02	.01	.02	.02	.01	.01	.02	.02	.01	+ .01	+ .31
18	.012	.01	.02	.01	.01	.02	.01	.01	.01	.02	.01	.01	.01	.02	.01	.00	+ .01	+ .32
17	.01	.01	.01	.02	.01	.01	.01	.01	.01	.01	.01	.01	.01	.02	.01	.00	+ .01	+ .33
16	.01	.01	.01	.01	.01	.00	.01	.01	.01	.01	.01	.02	.02	.02	.01	.01	− .04	+ .29
15	.00	.00	.02	.02	.02	.02	.02	.03	.02	.02	.02	.03	.02	.01	.01	.01	− .01	+ .28
14	.00	.00	.01	.01	.02	.02	.03	.04	.03	.03	.02	.03	.02	.01	.02	.01	− .04	+ .24
13	*.01*	*.01*	.01	.01	.01	.02	.03	.05	.05	.04	.03	.04	.02	.03	.02	.01	− .13	+ .11
12	*.02*	*.00*	*.01*	*.01*	.00	.01	.03	.05	.05	.03	.03	.04	.03	.02	.02	.01	− .18	− .07
11	*.02*	*.03*	*.03*	*.02*	*.03*	*.01*	.02	.05	.03	.04	.05	.04	.03	.02	.01	.01	− .30	− .37
10	.01	.01	*.02*	*.03*	*.04*	*.03*	.00	.02	.07	.08	.06	.04	.09	.07	.02	.05	− .56	− .93
9	.03	*.01*	*.01*	*.04*	*.04*	*.04*	*.01*	.03	.11	.10	.06	.05	.10	.07	.06	.04	− .68	− 1.61
8	*.00*	*.01*	*.02*	*.05*	*.04*	*.04*	*.03*	.07	.13	.11	.08	.08	.07	.05	.01	.00	− .65	− 2.26
7	*.18*	*.25*	*.44*	*.02*	*.01*	*.03*	*.00*	.09	.07	.07	.04	.03	.10	.05	*.01*	*.01*	−1.18	− 3.44
6	*.52*	.77	*.95*	*.47*	*.36*	*.10*	.00	.10	.08	.10	.12	10	.01	*.01*	*.04*	*.03*	−3.40	− 6.84
5	*.90*	*1.01*	*1.01*	*1.02*	*.94*	*.65*	*.09*	.12	.09	.12	.13	.14	.11	.10	.01	.05	−6.25	−13.09
4	*1.00*	*1.00*	*.98*	*.95*	*1.00*	*.90*	*.22*	.12	.05	.05	.11	.12	.13	.09	.02	*.02*	−6.48	−19.57
3	*1.00*	*1.00*	*1.07*	*.95*	*1.07*	*1.03*	*.07*	.10	.08	*.02*	.03	.02	.20	*.62*	*.70*	*.45*	−4.63	−24.20
2	*.85*	*1.03*	*1.18*	*1.15*	*1.00*	*.55*	*.02*	.03	.11	.03	*.01*	.10	*.05*	*.05*	*.60*	.77	−4.51	−28.71
1	*.82*	*.80*	*.89*	*.50*	*.49*	*.37*	*.30*	.06	.37	.04	.01	*.07*	.25	*.04*	*.45*	*.67*	−3.55	−32.26

The sector corresponding to each column of the above form may be identified from illustration No. 2. The sector numbered 1 is limited on its northern side by the eastern line from the station; that numbered 5 is limited on its eastern side by the southern line from the station, and so on. Sectors 1 to 8 represent areas farther south than the station; therefore the deflections corresponding to land areas within these sectors have the plus sign. For land areas within sectors 9-16, and therefore north of the station, the deflection has the minus sign.

A plus sign means that the zenith is deflected to the north and the astronomic latitude increased.

The italic figures* represent water compartments in which the mean elevations are negative. The signs of these italic quantities are the reverse of those for other quantities in the same sectors, as indicated by the heading. An unusual process is necessary in deriving the italic figures, which will be explained later.

For each compartment in rings 28 to 14, comprising all topography at distances from 0.19 to 39.22 kilometers from the station, the deflection is between ″.000 and ″.04, and no italics occur—that is, the mean elevation for each compartment is positive and not greater than 400 feet.

The italic figures representing negative mean elevations begin to occur in ring 13. The italic number 1.18 in ring 2 and sector 3 expresses the fact that in that compartment, which lies in the western part of the south Atlantic, the mean depth is more than 3 000 fathoms.

Each entry in the column headed "Horizontal sum" is the algebraic sum of the quantities in that line, and represents, therefore, the deflection due to a whole ring. At this station no ring of topography smaller than ring 13 produces a deflection greater than ″.06. Ring 4 has the largest effect, −6″.48. This ring includes oceanic compartments having mean depths of nearly 3 000 fathoms, and a land compartment with a mean elevation of 1 300 feet.

In each line of the column headed "Continuous sum" is shown the sum down to that point of the column headed "Horizontal sum." Hence each of these values is the topographic deflection due to all rings of topography from the station out to and including the ring indicated. This column serves (though that is not its main purpose) to indicate how important

* In the computations as made these were red figures, but, for convenience in printing, italics, instead of red, have been used.

it is—at this station, for example—to extend the computation to a great distance if it is desired to secure even a fair approximation to the topographic deflection.

The compartment which is in ring 13 and sector 1, southeastward from the station, from 24 to 35 kilometers from it, is the nearest compartment which has a negative mean elevation and therefore is represented by an italic figure in the table. It comprises a part of the Bay of Fundy.

A large group of italic figures, which are principally in rings 11 to 1 and sectors 1 to 6, correspond to the Atlantic Ocean compartments. These italic figures for ocean compartments are much larger for this station than are the values for the land compartments. That is the case for many stations. The maximum italic number is 1″.18. The maximum value (not in italics) for any land compartment is ″.37 for ring 1, sector 9, a compartment comprising practically all of the Cordilleras, the western mountain system of North America.

The compartment in ring 3, sector 13, for which the computed deflection is ″.20, comprises the high mountain mass east of Hudson Bay and southeast of Hudson Strait.

The compartment in ring 3, sector 10, contains part of Hudson Bay, as indicated by the italic number ″.02.

The compartment in ring 6, sector 15 (in italics ″.04), contains the Gulf of St. Lawrence, and the compartment in ring 5, sector 16 (in italics, ″.05), contains Newfoundland and the Great Bank of Newfoundland.

There is a group of italic numbers in rings 3 to 1 and sectors 14 to 16. These compartments cover the northern part of the Atlantic Ocean.

WATER COMPARTMENTS.

To obtain the italic figures representing the deflections for compartments which include oceanic areas, a variation from the procedure for land compartments is necessary for two reasons. First, the depths are, as a rule, expressed in fathoms rather than feet on the charts which were used. No depths were expressed in feet on these charts, as a rule, unless they were less than 18 feet. These small depths are, for the purpose of computing topographic deflections, almost or quite negligible. Second, to treat the depths below mean sea level in the same manner as the land elevations above mean sea level, with only a change in the algebraic sign, would be equivalent to assuming the space between sea level and the ocean bottom to be void, whereas, in fact, it is filled with sea water having a density of 1.027.

If the sea water were to increase in density from 1.027 to 2.67 (the mean surface density of the earth) by simply decreasing in volume and depth, without any horizontal transfer of material, and remaining in contact with the original sea bottom, the new depth of material would everywhere become .385 $\left(=\frac{1.027}{2.67}\right)$ of the former depth and the new surface would everywhere be .615 (=1−.385) below the original sea level. Hence, in the computation of topographic deflections, each ocean compartment has been considered as if it were a void from sea level down to a depth .615 of the actual mean depth of that compartment. Taking also into account the reduction of fathoms to feet the special factor necessary to put oceanic compartments on the same basis as land compartments is 3.69 (=six times .615), or, in other words, for a sea compartment the deflection produced is

$$D = 0''.0003690 \text{ (depth in fathoms).}$$

That is, the mean depth of each oceanic compartment expressed in fathoms was multiplied by 3.69 and the result then treated as if it were a negative elevation of a land surface, expressed in feet.

For compartments which are partly oceanic areas and partly land areas especial care must be taken in estimating the mean elevation to keep in mind the negative sign and the factor 3.69, in connection with the water portion of the compartment. In practice these mixed compartments were found to give little trouble.

In compartments containing deep fresh water lakes a process somewhat similar to that followed for ocean areas is necessary. Account must be taken of the fact that the density of the lake water is 1.000 and that the lake surface is at a certain known elevation above mean

sea level. The Great Lakes were the only important cases of this kind which occurred in connection with the present investigation. For each of these lakes a special table was used, giving the values in seconds of arc to be entered in the computation for various depths in that lake.

The computation of the topographic deflection was commenced, in each case, with the areas nearest the station which were shown clearly on the contour maps of the largest scale available. As the computation progressed outward to larger rings, when the limits of the maps of this largest scale were passed, other contour maps of the next smaller scale available were used and the templates of corresponding scale applied and the process continued until the limiting distance, 4 126 kilometers, was reached.

For example, in the computation of the topographic deflection in the meridian at Calais (No. 164) the maps and charts used were the following, in order proceeding outward:

1. Coast and Geodetic Survey chart No. 300, scale $\frac{1}{40000}$, showing principally the St. Croix River and Passamaquoddy Bay.

2. The post-route map of Maine, scale $\frac{1}{414000}$, approximately, showing drainage, but no contours, and used in connection with a few known elevations.

3. Coast and Geodetic Survey chart No. 301, scale $\frac{1}{40000}$, showing Cobscook Bay and part of the Bay of Fundy and contours of land in the immediate vicinity.

4. Hydrographic Office chart No. 1412, Mercator projection, scale 1° of longitude = 1.55 inches. This was used for depths southeast of Nova Scotia.

5. U. S. Geological Survey contour map of the United States in three parts, scale $\frac{1}{2500000}$. This was used for all land compartments in the United States not covered by the foregoing.

6. Coast and Geodetic Survey chart No. 101, scale $\frac{1}{80000}$, used for the vicinity of Grand Manan Channel.

7. Coast and Geodetic Survey chart No. 6, scale $\frac{1}{400000}$, used for some of the depths in the Gulf of Mexico.

8. Map of the United States and Canada, published by the Canadian Geological Survey, scale 1 inch = 242 miles; British Admiralty chart No. 2059, Mercator projection, scale 1° of longitude = 0.3 inch; and Century Atlas map of the region around the North Pole, scale 1 inch = 290 miles. These three were used for Canada and contiguous waters and islands, the last being used especially to obtain the approximate elevations for Greenland and other regions near the Pole.

9. British Admiralty charts Nos. 2936 and 2935, Mercator projection, scale 1° of longitude = 0.2 inch. These were used for ocean depths in the open sea, far from the coast in the Atlantic.

10. The small U. S. Geological Survey map of the United States, scale 1 inch = 111.1 miles, and the Century Atlas were used for Mexico.

Difficulties were encountered when it was attempted to apply the templates to charts on a small scale, covering very large areas, constructed on a Mercator projection. On such charts no one scale of distances applies to the whole chart, and the radial lines of the ordinary template, representing arcs of great circles on the earth's surface, are not straight lines on the chart. These difficulties were overcome by constructing special templates to fit such charts, the templates being distorted in the same manner as the charts. On these special templates the radial lines are curved, and the lines which on ordinary templates are arcs of circles were curves of varying radii of curvature. These special templates were constructed by plotting by latitudes and longitudes a sufficient number of the points representing the corners of compartments to enable one to draw the two sets of curves connecting them with a free hand with sufficient accuracy. The required latitudes and longitudes of corners were obtained by scaling from a large globe, or map with a polyconic projection, on either of which an ordinary template could be used with sufficient accuracy in locating the corners. Such a special template, once constructed for a given Mercator chart and for a given station on the chart, could be used with sufficient accuracy for any other station within one or two degrees of the same latitude. For other stations, even on the same chart, differing much in latitude from the one for which the template was constructed, it was necessary to construct a new special template.

This difficulty in regard to small scale Mercator charts was not found to be serious for the reason that, as shown later, the direct estimation of mean depths on such charts was found to be necessary in a relatively small number of cases only. For the many other cases the values were derived directly by interpolation.

Three considerations operated to fix upon the ring which has for its outside radius 4 126 kilometers (2 564 miles) as the largest ring to be taken into the computation of the topographic deflections:

(*a*) In the next larger ring considerable areas would be included for which our knowledge of the elevations is very limited, as, for example, the unexplored Arctic regions and the interior of South America.

(*b*) The largest ring included in the computation has a sufficiently great outer radius to insure that for stations in the extreme eastern part of the United States the ring reaches to the Pacific, and that for extreme western stations it reaches to the Atlantic. Hence the whole width of the continent is taken in by the computation. In the next larger ring, for any station, portions of each ocean would be included, and in general would tend to balance each other. Hence the total computed effect for each larger ring omitted will be in general considerably less than for each of the last few rings included in the computation.

(*c*) The larger the ring considered the more nearly the computed topographic deflection corresponding to that ring tends to approach a constant value for the whole United States, and, therefore, the less serious is the effect of omitting said ring.

EXAMPLES OF TOPOGRAPHIC DEFLECTION COMPUTATIONS.

The following four additional examples of computations of topographic deflections are given for widely separated stations.

In contrast to the example given on page 26 of the computation of the meridian component of the deflection at Calais, Maine, a station in a comparatively flat country and near the Atlantic Ocean, the following example represents a computation for the station Uncompahgre, Colorado, in a region of steep, high mountains, and far from the nearest ocean.

Computation of topographic deflection, latitude station No. 54, Uncompahgre, Colorado.

Ring.	Number of sector.																Horizontal sum.	Continuous sum.
	Plus unless italics.								Minus unless italics.									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
	+.02	+.02	+.01	+.01					+.02	+.03	+.03	+.04	+.03	+.03	+.02	+.02		
26	1.32	1.33	1.34	1.36	1.38	1.39	1.38	1.38	1.32	1.30	1.30	1.29	1.30	1.30	1.32	1.32	+ .27	+ .27
	+.03	+.01	+.01	+.01	+.01	+.01	+.01		+.03	+.04	+.03	+.03	+.02	+.02	+.02	+.02		
25	1.28	1.32	1.34	1.34	1.34	1.34	1.34	1.37	1.27	1.26	1.27	1.28	1.29	1.29	1.30	1.30	+ .29	+ .56
	+.03	+.01	+.01	+.01	+.01	+.01	+.01	+.01	+.02	+.05	+.04	+.02	+.02	+.02	+.02	+.02		
24	1.24	1.30	1.31	1.30	1.30	1.29	1.30	1.31	1.26	1.20	1.22	1.25	1.28	1.28	1.27	1.26	+ .22	+ .78
	+.02	+.01	+.04	+.02	+.02	+.02	+.02	+.02	+.01	+.04	+.04	+.02	+.01	+.01	+.01	+.01		
23	1.22	1.29	1.25	1.24	1.22	1.22	1.24	1.28	1.23	1.17	1.17	1.22	1.26	1.26	1.26	1.26	+ .15	+ .93
	+.01	.00	+.01	+.02	+.02	+.02	+.01	+.01	+.01	+.03	+.02	+.01			+.01			
22	1.20	1.25	1.20	1.17	1.15	1.18	1.21	1.25	1.20	1.12	1.15	1.22	1.26	1.26	1.23	1.26	− .07	+ .86
	+.01	00	+.01	+.02	+.02				+.01	+.02	+.01	+.01			+.01			
21	1.16	1.23	1.12	1.10	1.08	1.25	1.29	1.29	1.14	1.09	1.14	1.17	1.26	1.24	1.21	1.24	+ .03	+ .89
	+.01		+.01	+.01					+.01	+.01	+.01	+.01						
20	1.14	1.20	1.10	1.10	1.20	1.25	1.22	1.26	1.15	1.06	1.08	1.16	1.26	1.25	1.20	1.24	+ .06	+ .95
	+.01		+.01	+.01		+.01				+.01	+.01							
19	1.06	1.11	1.04	1.08	1.12	1.08	1.19	1.25	1.16	1.08	1.04	1.22	1.24	1.19	1.20	1.20	− .38	+ .57
18	1.09	.98	1.02	1.04	1.05	1.20	1.18	1.22	1.10	1.14	1.09	1.20	1.15	1.15	1.17	1.22	− .44	+ .13
17	1.00	1.10	1.20	1.26	1.20	1.15	1.24	1.16	1.13	1.09	1.10	1.10	1.04	1.14	1.18	1.24	+ .29	+ .42
16	1.00	1.16	1.20	1.16	1.20	1.27	1.28	1.10	.95	1.04	1.02	1.14	1.10	1.08	1.10	.95	+ .99	+1.41
15	1.10	1.13	1.10	1.18	1.20	1.15	1.23	1.20	.77	.95	.96	1.05	1.04	1.04	.95	1.04	+1.49	+2.90
14	1.17	1.07	1.09	1.22	1.19	1.10	1.15	1.14	.79	.90	.91	.85	.88	.90	.88	1.17	+1.85	+4.75
13	1.09	.99	1.10	1.20	1.20	1.08	1.10	1.04	.82	.66	.80	.86	.88	.80	.85	.78	+2.35	+7.10
12	1.05	1.06	1.02	1.00	.98	.95	1.05	1.00	.79	.58	.72	.80	1.00	1.00	.85	.92	+1.45	+8.55
11	.96	1.00	.80	.73	.76	.77	.93	.85	.74	.65	.72	.85	.94	.95	.92	.90	+ .13	+8.68
10	.82	1.03	.78	.70	.65	.65	.65	.62	.73	.62	.76	.88	.86	1.00	1.10	.95	−1.00	+7.68
9	.87	.85	.73	.71	.68	.59	.56	.55	.50	.60	.83	.90	.90	.94	1.10	.84	−1.07	+6.61
8	.69	.86	.74	.70	.74	.66	.70	.67	.69	.58	.62	.69	.84	.94	.83	.61	− .04	+6.57
7	.52	.53	.63	.53	.73	.65	.57	.57	.72	.92	.70	.70	.76	.77	.50	.50	− .84	+5.73
6	.36	.42	.54	.50	.62	.52	.49	.49	.49	.64	.83	.70	.61	.50	.42	.35	− .60	+5.13
5	.19	.30	.41	.40	.40	.38	.14	.24	.58	.56	.71	.53	.44	.32	.27	.22	−1.17	+3.96
					.34	.14	.05	.27										
4	.08	.14	.26	.43	.03	.06	.22	.10	.45	.35	.65	.38	.26	.23	.15	.13	−1.64	+2.32
	.11	.00		.40	.04	.04			.45	.03								
3	.02	.33	.50	.06	.40	.51	.66	.77	.05	.18	.48	.20	.12	.08	.12	.09	−2.08	−0.36
	.01	.05	.32							.26								
2	.27	.25	.18	.70	.68	.74	.84	.84	.75	.18	.40	.15	.11	.02	.05	.10	−4.07	−4.43
		.14								.27						.03		
1	.24	.33	.77	.77	.82	.95	1.07	1.00	.98	.07	.30	.04	.03	.02	.12	.20	−5.07	−9.50

The elevation of the station is 14 289 feet (4 355 meters) and the slopes in its vicinity are very steep.

In each compartment space in rings 26 to 19 in which two numbers are given, the upper one is the slope correction. There are slope corrections in every ring from 26 out to 19 (outer radius 6.653 kilometers). The explanation of such corrections will be found on page 34.

There are a few compartments in rings 4 to 1 in which two numbers are given, of which one is an italic number. It was found to be more convenient in many cases for compartments which involve both oceanic and land areas to estimate the mean depth for the oceanic area and the mean elevation for the land area separately, than to combine them in a single estimate. This has accordingly been done in many cases and both of the separate estimates entered on the computation form, as, for example, for the compartment in ring 4, sector 6, including parts of Mexico and of Lower California. For this compartment the estimated mean elevation of the land portion is 2 000 feet, corresponding to ″.20. This, multiplied by 0.7, the ratio of the land area to the total area of the compartment, gave the value ″.14 (not in italics), entered in the computation for the compartment. Similarly the mean depth of the oceanic portion of the compartment was estimated to be 550 fathoms, corresponding to ″.20. This, multiplied by 0.3, the ratio of the oceanic area to the total area of the compartment, gave ″.06, the italic number entered in the computation for the compartment. The ″.06 (italic) combined by algebraic addition with the ″.14 (not italic) gives ″.08 (not italic), which is precisely the quantity which would have been obtained by making a single estimate for the whole compartment at once.

Mount Uncompahgre is near the northern edge of a large mountain mass, extending above the 10 000 foot contour. The Gunnison River lies at the northern edge of this mountain mass. The valley of the river to the northeastward of the station is represented by the values ″.88, ″.90, ″.88, and ″.80 in rings 14 and 13 and sectors 13 and 14. The larger values 1″.00 to 1″.04 in the same sectors in rings 15 and 12 represent the mountains rising above the 10 000 foot contour on the south and north sides of the valley, respectively.

Rings 17 to 11, inclusive, lie with their southern parts on the mountain mass and their northern sides on the northward slope of the mountain mass or in the river valley. Hence for each of these rings the total is positive (a deflection of the zenith to the north) as shown in the column headed "Horizontal sum." For the larger rings 10 to 1, which lie nearly or quite outside of the mountain mass of which Mount Uncompahgre forms a part, the deflection is negative in each case (zenith deflected to the southward).

The italic figures in sectors 1 to 4 and 16 represent portions of the Atlantic Ocean and the Gulf of Mexico. Those in sectors 5 to 10 represent portions of the Pacific Ocean. As shown in the last column, the algebraic sign as well as the amount of the computed topographic deflection depends upon the distance to which the computation is carried. If the computation be stopped with ring 4, outer radius 1 369 kilometers, or at a shorter distance from the station, the computed deflection of the zenith is to the northward, otherwise it is to the southward.

The following examples of the computation of the prime vertical component of the topographic deflection are for three especially interesting stations. For the first, No. 115, North End Knott Island, the depression occupied by the Atlantic Ocean has its greatest influence on the computed deflection. For the second, No. 1, Point Arena, the Pacific Ocean depression has its greatest influence. At the third, No. 59, Mount Ouray, the computed deflection is comparatively small, as the two oceanic depressions and the different parts of the continental mass nearly balance each other.

Computation of topographic deflection, azimuth station No. 115, North End Knott Island, Virginia.

Ring.	Number of sector.																Horizontal sum.	Continuous sum.
	Plus unless italics.								Minus unless italics.									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
25	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	.00
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	.00
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	.00
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	.00
21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	.00
20	.000	.001	.002	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	+ .01	+ .01
19	.001	.000	*.001*	*.001*	*.001*	*.000*	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00	+ .01
18	*.001*	*.002*	*.002*	*.002*	*.002*	*.002*	*.001*	*.000*	.000	.000	.000	.000	.001	.001	.000	.000	− .01	.00
17	*.002*	*.003*	*.003*	*.003*	*.003*	*.003*	*.002*	*.000*	*.000*	.000	.000	.000	.000	.001	.001	.000	− .02	− .02
16	*.003*	*.004*	*.004*	*.004*	*.004*	*.003*	*.004*	*.000*	*.000*	.002	.000	.001	.001	.001	.001	.001	− .03	− .05
15	*.003*	*.004*	*.004*	*.004*	*.004*	*.004*	*.004*	*.002*	.002	.003	.001	.001	.001	.001	.001	.001	− .04	− .09
14	*.004*	*.004*	*.004*	*.004*	*.004*	*.005*	*.005*	*.003*	.002	.003	.003	.002	.001	.002	.002	.002	− .05	− .14
13	*.004*	*.004*	*.004*	*.004*	*.005*	*.005*	*.005*	*.003*	.001	.003	.004	.002	.001	.001	001	.000	− .05	− .19
12	*.004*	*.006*	*.006*	*.006*	*.007*	*.007*	*.007*	*.003*	.000	.003	.003	.003	.005	.005	.001	*.000*	− .07	− .26
11	*.004*	*.008*	*.010*	*.011*	*.031*	*.031*	*.009*	*.004*	.002	.002	.003	.003	.008	.007	.003	.000	− .14	− .40
10	*.00*	*.02*	*.13*	*.22*	*.33*	*.28*	*.20*	*.02*	.00	.00	.00	.01	.01	.01	.01	.00	−1.24	− 1.64
9	*.00*	*.11*	*.44*	*.53*	*.55*	*.55*	*.61*	*.42*	.00	.00	.00	.00	.03	.04	.03	.01	−3.32	− 4.96
8	*.00*	*.29*	*.59*	*.64*	*.84*	*.87*	*.85*	*.76*	*.01*	.01	.03	.05	.06	.07	.07	.04	−5.16	−10.12
7	*.00*	*.39*	*.66*	*.81*	*.90*	*.92*	*.95*	*.78*	*.05*	.01	.04	.09	.18	.20	.23	.12	−6.23	−16.35
6	.03	*.30*	*.74*	*1.07*	*.95*	*.99*	*1.07*	*.94*	*.12*	.01	.06	.18	.18	.14	.12	.14	−6.74	−23.09
5	.04	*.11*	*.84*	*.99*	*1.03*	*1.05*	*1.07*	*1.07*	*.14*	.02	.08	.17	.10	.08	.09	.07	−6.62	−29.71
4	.05	*.09*	*.84*	*1.01*	*.98*	*.92*	*1.01*	*1.05*	*.03*	*.04*	.03	.05	.04	.07	.07	.08	−6.12	−35.83
3	.04	*.12*	*.98*	*1.05*	*.98*	*1.12*	*1.14*	*.94*	*.29*	*.47*	*.02*	.03	.10	.11	.13	.09	−5.97	−41.80
2	.03	*.21*	*.89*	*1.03*	*.95*	*1.10*	*1.07*	*.88*	*.24*	*.46*	.13	.20	.42	.52	.28	.07	−7.02	−48.82
1	*.35*	*.75*	*.70*	*.76*	*.84*	*.88*	*.92*	*.11*	*.55*	*.45*	*.17*	*.08*	.28	.55	.43	.16	−5.48	−54.30

No. 115, North End Knott Island, is situated on a low-lying island in Currituck Sound, Virginia, surrounded by shoal waters and low lands. There is no large change in the topography, until the submerged edge of the continent, as represented by the 1 000-fathom line, is reached, 130 kilometers from the station. The first effect of this is shown by the values in italics, ″.22, ″.33, ″.28, and ″.20, in ring 10, sectors 4 to 7, inclusive.

In ring 7, sectors 13, 14, 15, the values ″.18, ″.20, ″.23 show the effect of the main mass of the Blue Ridge Mountains.

In ring 4, sectors 11, 12, 13, the values ″.03, ″.05, ″.04 correspond to the Mississippi Valley.

In rings 1 and 2, sector 14, the values ″.55 and ″.52 show the effect of the Cordilleras, the western mountain system of North America.

Computation of topographic deflection, longitude station No. 1, Point Arena, California.

Ring.	Number of sector.																Horizontal sum.	Continuous sum.
	Plus unless italics.								Minus unless italics.									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
30	.020	.021	.021	.020	.019	.018	.016	.014	.013	.014	.015	.015	.018	.018	.019	.019	+ .02	+ .02
29	.020	.021	.021	.020	.021	.019	.015	.010	.009	.010	.010	.011	.016	.018	.019	.019	+ .04	+ .06
28	.020	.021	.021	.020	.018	.017	.017	.018	.010	.009	.009	.010	.015	.018	.018	.020	+ .04	+ .10
27	.021	.022	.022	.021	.019	.018	.017	.021	.010	.009	.008	.007	.015	.021	.021	.021	+ .05	+ .15
26	.021	.022	.021	.020	.019	.018	.018	.026	.020	.020	.018	.015	.012	.020	.025	.020	+ .02	+ .17
25	.020	.022	.022	.021	.021	.018	.019	.027	.023	.024	.024	.019	.009	.015	.022	.012	+ .02	+ .19
24	.02	.02	.02	.02	.022	.020	.022	.026	.022	023	.025	.024	.010	.015	.022	.018	+ .01	+ .20
23	.02	.02	.02	.02	.02	.022	.026	.025	.000	.000	.000	.000	.019	.018	.022	.012	+ .10	+ .30
22	.02	.02	.02	.02	.02	.02	.018	.022	*.003*	*.003*	*.002*	*.002*	*.001*	.010	.019	.012	+ .13	+ .43
21	.02	.02	.02	.02	.02	.02	.023	.008	*.006*	*.007*	*.007*	*.007*	*.003*	.001	.012	.010	+ .16	+ .59
20	.01	.02	.02	.02	.02	.02	.01	*.00*	*.009*	*.011*	*.011*	*.011*	*.004*	*.002*	.001	.012	+ .16	+ .75
19	.01	.02	.03	.04	.04	.03	.01	*.00*	*.01*	*.01*	*.02*	*.02*	*.01*	*.01*	*.00*	.00	+ .26	+ 1 01
18	.00	.02	.04	.05	.05	.03	.01	*.00*	*.01*	*.02*	*.02*	*.02*	*.02*	*.01*	*.01*	*.00*	+ .31	+ 1.32
17	.00	.03	.05	.06	.05	.04	.01	*.01*	*.03*	*.03*	*.03*	*.02*	*.01*	*.01*	*.01*	*.01*	+ .38	+ 1.70
16	.05	.06	.06	.05	.06	.05	.02	*.01*	*.06*	*.07*	*.07*	*.04*	*.01*	*.01*	*.01*	*.01*	+ .64	+ 2.34
15	.08	.12	.12	.14	.10	.09	.02	*.02*	*.22*	*.22*	*.17*	*.11*	*.03*	*.03*	*.02*	*.01*	+ 1.46	+ 3.80
14	.10	.15	.15	.15	.15	.10	.03	*.04*	*.37*	*.37*	*.35*	*.18*	*.09*	*.11*	*.07*	*.02*	+ 2.35	+ 6.15
13	.15	.19	.16	.15	.15	.09	.04	*.06*	*.66*	*.66*	*.60*	*.55*	*.18*	*.18*	*.18*	*.04*	+ 3.92	+ 10.07
12	.17	.17	.17	.18	.15	.07	.07	*.06*	*.62*	*.70*	*.73*	*.73*	*.44*	*.44*	*.29*	*.11*	+ 4.98	+ 15.05
11	.20	.18	.17	.12	.15	.09	.03	*.06*	*.73*	*.77*	*.77*	*.81*	*.70*	*.62*	*.51*	*.11*	+ 5.90	+ 20.95
10	.29	.07	.02	.03	.06	.05	.04	.01	*.81*	*.77*	*.77*	*.77*	*.73*	*.66*	*.47*	*.21* .02	+ 5.74	+ 26.69
9	.17	.11	.09	.04	.02	.01	.05	.02	*.77*	*.84*	*.84*	*.84*	*.73*	*.77*	*.47*	*.25* .02	+ 6.00	+ 32.69
8	.36	.50	.50	.50	.50	.17	.19	.01	*87*	*.88*	*.92*	*.92*	*.84*	*.84*	*.73*	*.44* .01	+ 9.16	+ 41.85
7	.47	.53	.48	.45	.75	.55	.03	*.24* .06	*.92*	*.92*	*.95*	*.95*	*.92*	*.84*	*.44*	*.44* .02	+ 9.44	+ 51.29
6	.51	.54	.53	.74	.59	.55	.27	*.27* .08	*.92*	*.92*	*.92*	*.95*	*.92*	*.81*	*.66*	*.47* .01	+10.10	+ 61.39
5	.38	.49	.58	.57	.58	.33	.25	*.50* .01	*.92*	*1.01*	*1.00*	*1.01*	*.84*	*.70*	*.70*	*.47* .01	+ 9.33	+ 70.72
4	.37	.66	.70	.68	.63	.48	.14	.05 *.54*	*.92*	*.88*	*1.00*	*1.07*	*.92*	*.85*	*.77*	.02 *.51*	+10.07	+ 80.79
3	.40	.40	.55	.68	.67	.54	*.06* .31	.04 *.61*	*.91*	*.92*	*1.00*	*1.06*	*1.03*	*.92*	*.81*	.07 *.55*	+10.04	+ 90.83
2	.12	.15	.13	.12	.13	.17	*.14* .46	*.69*	*1.03*	*1.00*	*1.03*	*1.10*	*1.09*	*1.03*	*.92*	.07 *.60*	+ 8.18	+ 99.01
1	.02	.05	.09	.06	*.15* .02	*.37*	*.34* .18	*.88*	*.97*	*1.07*	*1.00*	*1.11*	*1.07*	*1.03*	*.77*	*.01* .09	+ 5.62	+104.63

At this station the computed prime vertical component of the deflection of the vertical is greater than at any other station concerned in the investigation. The station is so near the water's edge that italic figures occur in ring 22 (outer radius 2.295 km.). The 1 000-fathom curve approaches within 35 kilometers of the station—note the italic figures ″.37 and ″.37 in ring 14, sectors 9 and 10, to the southwestward of the station. The large computed deflection depends mainly upon the italic figures corresponding to ocean compartments, of which 19 are each as great as 1″.00, all being to the westward of the station. An italic 1″.00 corresponds to a depth of 2 710 fathoms.

The land surface slopes upward to the east from the station, as shown by the values in sectors 1 to 7, out to ring 11, in which there is one compartment, in sector 1, in which the mean elevation is 2 000 feet. The small values, all less than ″.10, in sectors 3 to 8 of rings 10 and 9, correspond to the great valley of the Sacramento and the depression occupied by San Francisco Bay.

The effect of practically all of Alaska is represented by the value ″.09 (*not* in italics) in ring 1, sector 16.

Computation of topographic deflection, azimuth station No. 59, Mount Ouray, Colorado.

Ring.	Number of sector. Plus unless italics.								Minus unless italics.								Horizontal sum.	Continuous sum.
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
			+.02	+.02	+.02	+.01			+.01	+.01	+.01	+.01	+.01	+.01	+.01	+.01		
26	1.35	1.34	1.30	1.30	1.30	1.32	1.34	1.35	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	+ .03	+ .03
	+.01	+.01	+.02	+.02	+.02	+.02	+.01	+.01	+.02	+.02	+.02	+.01	+.01	+.01	+.01	+.02		
25	1.30	1.30	1.28	1.27	1.26	1.26	1.30	1.30	1.28	1.28	1.28	1.30	1.30	1.29	1.29	1.28	− .03	.00
	+.01	+.01	+.01	+.02	+.02	+.01	+.01	+.01	+.02	+.01	+.01	+.01	+.01	+.02	+.02	+.03		
24	1.28	1.28	1.26	1.23	1.24	1.26	1.27	1.26	1.24	1.26	1.28	1.29	1.29	1.24	1.22	1.20	+ .03	+ .03
	+.02	+.01	+.02	+.02	+.02	+.01	+.01	+.01	+.02	+.01	+.01	+.01	+.01	+.01	+.02	+.03		
23	1.20	1.23	1.20	1.18	1.18	1.22	1.23	1.22	1.21	1.25	1.26	1.27	1.27	1.24	1.19	1.16	− .19	− .16
	+.02	+.02	+.01	+.02	+.03	+.02	+.02	+.02	+.01	+.01						+.01		
22	1.13	1.14	1.17	1.12	1.10	1.16	1.16	1.16	1.17	1.22	1.24	1.25	1.26	1.28	1.26	1.20	− .61	− .77
	+.02	+.02	+.02	+.02	+.02	+.02	+.02	+.02	+.01	+.01								
21	1.10	1.06	1.10	1.07	1.07	1.09	1.09	1.08	1.14	1.16	1.20	1.25	1.25	1.23	1.25	1.27	− .95	− 1.72
	+.01	+.02	+.02	+.02	+.02	+.02	+.02	+.01	+.01	+.01						+.01		
20	1.14	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.09	1.13	1.20	1.23	1.20	1.17	1.20	1.09	−1.00	− 2.72
	+.01	+.01	+.01	+.01	+.01	+.01												
19	1.00	.95	.94	.95	.94	.94	1.02	1.10	1.12	1.10	1.14	1.18	1.14	1.13	1.19	1.15	−1.25	− 3.97
	+.01	+.01	+.01	+.01	+.01	+.01	+.01											
18	.90	.87	.92	.88	.85	.92	.98	1.04	1.11	1.04	1.06	1.06	1.14	1.07	1.16	1.10	−1.31	− 5.28
	+.01	+.01	+.01	+.01														
17	.81	.82	.81	.85	.86	.96	1.08	1.13	1.12	.98	.96	1.00	1.02	1.00	1.11	.94	− .77	− 6.05
16	.79	.75	.93	.99	.87	.83	1.00	1.08	1.05	1.05	.95	.89	.94	.96	1.08	1.00	− .68	− 6.73
15	.77	.70	.70	.90	1.05	.82	.82	.93	.97	1.10	1.02	.86	.90	.93	1.04	1.15	−1.28	− 8.01
14	.83	.92	.75	.70	.75	1.06	.80	.90	.90	1.00	1.10	.90	.82	.87	1.05	1.12	−1.05	− 9.06
13	.90	.95	1.00	.85	.67	1.00	.82	.79	.96	1.00	.88	.88	.82	.90	1.08	1.06	− .60	− 9.66
12	1.00	.99	.89	.70	.80	.86	1.05	.78	1.10	1.10	1.00	.90	.83	.87	.96	1.10	− .73	−10.39
11	1.01	.73	.88	.66	.67	.88	.97	.78	.97	1.10	1.08	.88	.83	1.06	1.00	1.07	−1.41	−11.80
10	.92	.72	.79	.58	.50	.66	.61	.80	.95	1.10	1.15	.77	.70	.75	.85	.80	−1.49	−13.29
9	.68	.60	.63	.55	.47	.53	.66	.88	.69	.80	.88	.70	.72	.62	.75	.82	− .98	−14.27
8	.56	.50	.52	.48	.42	.49	.60	.67	.68	.56	.58	.66	.55	.55	.65	.73	− .72	−14.99
7	.52	.40	.38	.38	.35	.40	.45	.43	.69	.64	.55	.60	.55	.65	.74	.71	−1.82	−16.81
6	.40	.31	.26	.25	.25	.27	.32	.43	.61	.60	.52	.62	.57	.48	.62	.73	−2.26	−19.07
5	.24	.18	.17	.13	.11	.12	.16	.32	.37	.19	.34	.44	.66	.57	.52	.62	−2.28	−21.35

	Interpolation.				Extrapolation.				Interpolated values for Station 59.	
	Station 57.	Station 62.	Difference.	Difference times .404.	Station X.	Station 52.	Difference.	Difference times −.013.		
	″	″	″	″	″	″	″	″	″	″
12	+ 95	−1.62	−2.57	−1.04	− .09	+ .44	+ .53	−.01	− .10	
11	+1.66	−2.31	−3.97	−1.60	+ .06	+1.31	+1.25	−.02	+ .04	
10	+2.19	−3.62	−5.81	−2.35	− .16	+1.10	+1.26	−.02	− .18	
9	+2.08	−3.95	−6.03	−2.44	− .36	+1.16	+1.52	−.02	− .38	
8	+ .81	−4.00	−4.81	−1.94	−1.13	+2.23	+3.36	−.04	−1.17	
7	−1.24	−2 88	−1.64	− .66	−1.90	+ .82	+2.72	−.04	−1.94	
6	−1.30	−3.49	−2.19	− .88	−2.18	− .43	+1.75	−.02	−2.20	
5	−1.59	−3.21	−1.62	− .65	−2.24	−1.02	+1.22	−.02	−2.26	
4	− .65	−2.39	−1.74	− .70	−1.35	+ .76	+2.19	−.03	−1.38	−22.73
3	+4.14	+1.95	−2.19	− .88	+3.26	+5.14	+1.88	−.02	+3.24	−19.49
2	+5.28	+4.72	− .56	− .23	+5.05	+5.62	+ .57	−.01	+5.04	−14.45
1	+3.59	+2.66	− .93	− .38	+3.21	+4.12	+ .91	−.01	+3.20	−11.25

This is the station which is farthest west of those having negative values for the computed topographic deflection.

The elevation of the station is 14 043 feet (4 280 meters).

In each compartment space in which two figures are given the upper one is the slope correction. There are slope corrections in every ring from 26 out to 17 (outer radius 13.528 kilometers).

For nearly all of the rings out to and including No. 4 the topographic deflection for each ring is negative, the zenith being deflected to the eastward corresponding to greater elevations to the westward than to the eastward of the station. In rings 3 to 1 the influence of the Pacific (which is much nearer than the Atlantic) is shown by the large positive values.

The interpolations* (and extrapolations) for rings 12 to 1 are shown at the foot of the computation. For rings 7, 6, and 5, the agreement between interpolated and computed values was within the interpolation limit, being ″.12 for ring 7, ″.06 for ring 6, ″.02 for ring 5. Hence, under rules 1 and 2, page 43, the interpolated values were accepted for rings 4 to 1 and no direct computation made.

*This is explained under the heading "Method of interpolating for outer rings," p. 39.

CORRECTION FOR SLOPE.

Reference has already been made to the fact that, for compartments near the station for which the mean surface lies either far above or far below the station, a slope correction may be necessary, since the approximate formula used depends upon the assumption that the attracting material all lies in the horizon of the station.

Let the slope correction be defined as the correction which it is necessary to apply to a deflection computed by the approximate formula to obtain the true deflection which would be obtained from an exact formula.

In order to derive the necessary slope correction, the more exact formula may be compared with the approximate formula which has been used. The approximate formula is:

$$D = 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\varepsilon \frac{r'}{r_1}.$$

The more exact formula is:*

$$D = 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\varepsilon \frac{r' + \sqrt{(r')^2 + h^2}}{r_1 + \sqrt{r_1^2 + h^2}}.$$

In the more exact formula it is necessary, according to its derivation, that h be reckoned from the staion, that is, h is the distance of the upper surface of the compartment above or below the station, not above or below sea level, or any other arbitrarily selected level. The upper and lower limiting surfaces of the mass considered in this formula are necessarily the level surface passing through the station and the actual surface of the ground.

This restriction is necessary because it is the deflection of the vertical at the station of observation which is desired, not the deflection at a point at sea level. The necessity of the slope correction arises from the fact that any inclination of the lines connecting the station with parts of the attracting mass reduces the horizontal component of the attraction of the parts of the mass. It is the horizontal component only of the attraction which produces deflections of the vertical. The vertical component affects the intensity of gravity, not its direction.

The direct and obvious method of dealing with the problem is to use the exact formula and to reckon all mean elevations of compartment surfaces from the station as a zero, not from sea level. But to do so would involve long and difficult computations. As a matter of fact, the process of computation used with the approximate formula and with sea level as a reference surface gives values in very close agreement with what would be secured if the exact formula were used and elevations referred to the station. The so-called slope corrections necessary to reduce the former values to the latter are so small, as a rule, as to be negligible and, when not negligible, may be obtained quickly from a table.

Suppose that the approximate formula had been used, but that h had been reckoned, even in the approximate formula, from the station, not from sea level. Then the required slope correction, C, for any compartment, would evidently be

$$C = D \text{ (exact)} - D \text{ (approximate)}$$

$$= 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\varepsilon \frac{r' + \sqrt{(r')^2 + h^2}}{r_1 + \sqrt{r_1^2 + h^2}} - 12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\varepsilon \frac{r'}{r_1}.$$

* Geodesy, A. R. Clarke, Oxford, 1880, pp. 294–296.

C has been computed directly from the above formula for various of the adopted rings and for various assumed values of h, and the results are shown in the following table:

Slope correction.

[Quantities in table are differences of elevations in feet between the station and the mean surface of the compartment.]

Slope correction.	=Negligible.	=0″.01	=0″.02	=0″.03	=0″.04	=0″.05
Ring.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
29	0 to 300	400 to 500				
28	0 to 400	500 to 600				
27	0 to 500	600 to 700	800			
26	0 to 600	700 to 900	1 000 to 1 100	1 200 to 1 300	1 400	
25	0 to 800	900 to 1 100	1 200 to 1 400	1 500 to 1 600	1 700 to 1 800	1 900 to 2 000
24	0 to 1 000	1 100 to 1 400	1 500 to 1 800	1 900 to 2 000	2 100 to 2 200	2 300
23	0 to 1 200	1 300 to 1 800	1 900 to 2 200	2 300 to 2 500	2 600 to 2 700	
22	0 to 1 600	1 700 to 2 300	2 400 to 2 800	2 900 to 3 100		
21	0 to 2 000	2 100 to 2 900	3 000 to 3 500			
20	0 to 2 500	2 600 to 3 700				
19	0 to 3 200	3 300 to 4 700				
18	0 to 4 000	4 100 to 6 500				
17	0 to 5 000	5 100 to 7 500				
16	0 to 6 600	6 700 to 8 000				
15	0 to 7 000					

The correction is to be added to the arithmetical value of D, as entered in the computation (so as numerically to increase it), if the compartment surface is below the station and is not below sea level. The correction is to be subtracted from the arithmetical value of D, as entered in the computation (and thus numerically to decrease it), if the compartment surface is above the station or if it is a sea compartment represented by italic figures in the computation.

As all computations were carried to hundredths of seconds only, the correction was considered negligible if less than ″.005.

The table is not complete. Only such values were computed and inserted as were expected to occur or did occur in the computations of topographic deflections made in connection with the present investigation. It is not safe to extrapolate from the figures given in the table.

The table is computed as if h were reckoned from the level of the station, but the approximate formula has actually been used with h reckoned from sea level, because the maps and charts are ordinarily so drawn. In other words, instead of the true h, which should have been used, there has actually been used in the approximate formula $h+h_s$, in which h_s is the elevation of the station above sea level. The effect of depressing the reference plane from the station to sea level has been to add to the value shown in the body of such a computation as that on page 26, for each compartment, the quantity ″.0001 (h_s, in feet). But each value shown in the column headed "Horizontal sum" has been unaffected, since it is the algebraic sum of eight values taken as positive (sectors 1 to 8) and eight values taken as negative (sectors 9 to 16).

The depression of the reference plane to sea level has, in effect, added to the mass which should be considered in each compartment according to the exact formula (namely that between the actual surface of the ground and the level of the station), a mass filling that compartment from the level of the station to sea level. The effects of such added masses exactly counterbalance each other in pairs, even when computed by the exact formula. For, if opposite compartments be considered (that is, compartments in the same ring and of sector numbers differing by 8) it will be seen that for the added masses in question for two such compartments the exact formulæ are identical. The h's are the same for the two compartments (each being equal to h_s, the elevation of the station), the values of r_1 and r' are the same, because the compartments are in the same ring, and the values of a' and a_1 are the same, because the sectors are opposite. This is evidently as it should be, for every elementary portion of one mass is exactly counterbalanced by an equal elementary portion of the other mass at the same distance from the station on the opposite side and at the same angle of depression from the station.

Hence, as these masses virtually added in each computation by depressing the plane of reference from the station to sea level would counterbalance each other fully in fact in their effect on deflection at the station, and as their effects also counterbalance as computed either by the approximate formula, as used. or by the exact formula, it is evident that they need not be considered in deriving the slope corrections. As shown by the statement at the top of the table of slope corrections, they were not considered, the value used for h being, in each case, the difference of elevation between the station and the mean surface of the compartment.

The numerical value of the slope correction, as given by the table, represents the decrease in the computed value of the deflection at the station due to the inclination of lines joining the station to different parts of the attracting mass. The slope correction C is always, therefore, essentially negative, as shown by the formula from which it was computed. But for compartments in which the land surface is below the station the mass considered is really a negative one, namely, that which would be necessary to fill the compartment up to the level of the station. Hence, for such a compartment the slope correction is added numerically to the arithmetical value of D, entered in the computation form, as indicated in the statement following the table indicating the manner in which the correction must be applied in each case.

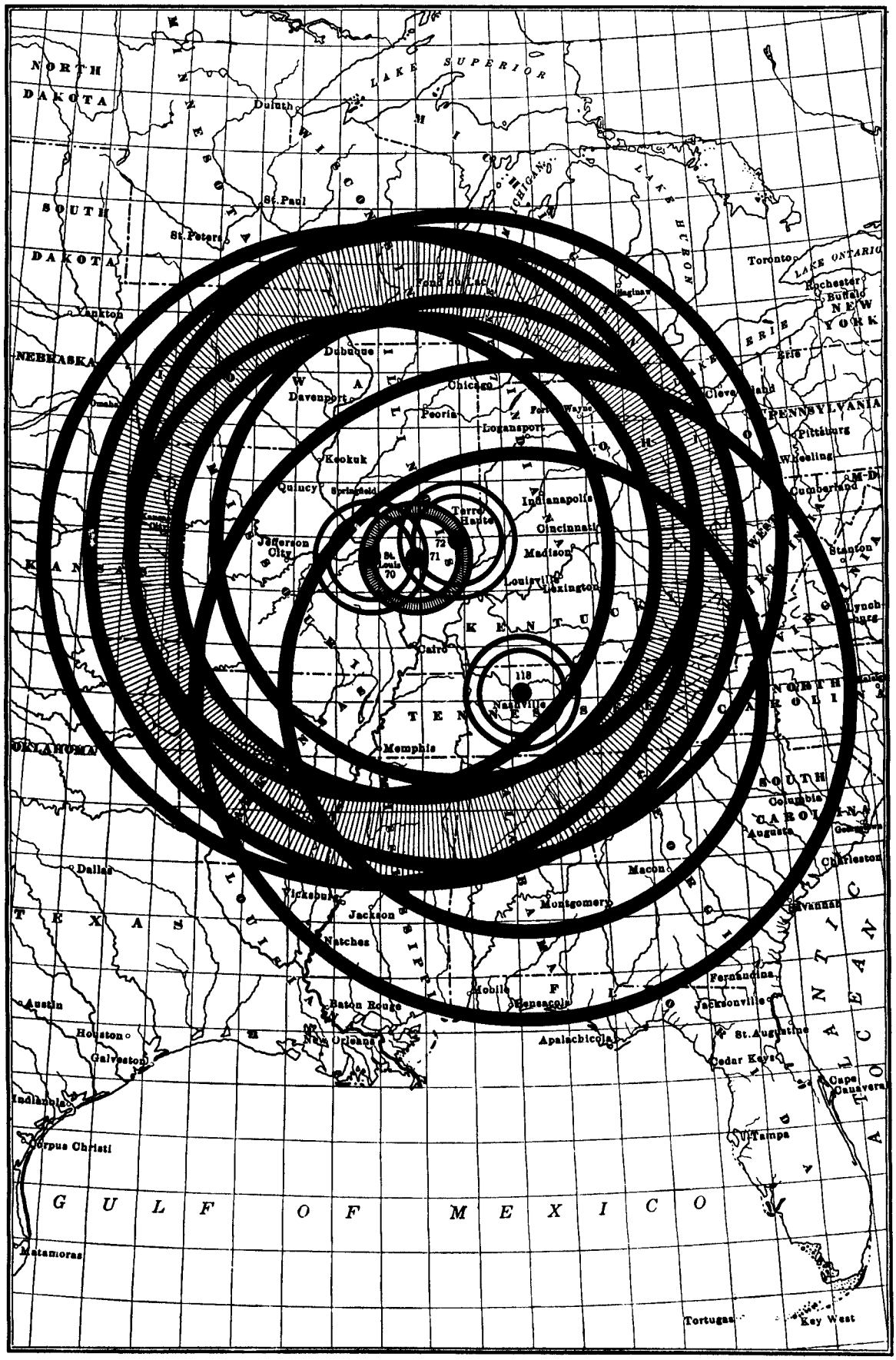
In the 496 computations of topographic deflections which have been made, appreciable slope corrections have been found in 43 computations. In each of these computations the slope corrections were found to be appreciable for only a few compartments, and no correction exceeded ″.05 for any compartment. Hence, the work of applying the slope corrections was light. There are few if any computations in which the slope correction table was used more extensively than that at Mount Uncompahgre, shown on page 29.

POSSIBLE INTERPOLATION FOR OUTER RINGS.

To compute the deflections for all rings at all stations by the methods thus far described would be an unnecessary waste of time. In computations so made each value in the column headed "Horizontal sum" expresses the deflection at the station due to an entire ring of topography. A comparison of the similar computations at any two stations comparatively near each other shows that the deflections produced by corresponding rings tend to be more nearly alike for the two stations the larger the ring considered. If the comparison be extended to include several stations in a group it becomes evident that it is possible to obtain the deflection with considerable accuracy for any large ring for a station near the center of the group by interpolation from the computed deflections for that ring at surrounding stations near it. This is most readily understood from a concrete illustration.

In illustration No. 3 the four solid black dots marked 70, 71, 72, and 118, represent, respectively, the four latitude stations, St. Louis, Bording, Newton, and Nashville. (See illustration No. 13 in the pocket at the end of the volume for the location of these astronomic stations.) Station No. 71, Bording, is within the triangle defined by the other three stations as vertices. For this station, No. 71, it was found to be possible to interpolate the deflections for the six outer rings (Nos. 6 to 1) with sufficient accuracy from the deflections for corresponding rings at the three stations, Nos. 70, 72, and 118. The distance from station No. 71 to either No. 70 or No. 72 is less than 100 kilometers (60 miles) and to No. 118 is more than 320 kilometers (200 miles). The four small rings on the illustration represent the ring numbered 11 in the computation of topographic deflections and having for its inner radius 80 kilometers and for its outer radius 114 kilometers (see p. 22). The four large rings on the illustration represent ring No. 6 for each of the four stations. The inner radius of ring No. 6 is 470 kilometers and the outer radius 671 kilometers. In each case the ring corresponding to station No. 71 is shaded.

The degree of resemblance between the computed deflections for corresponding rings at stations No. 72 and No. 118 is shown in the following table, in which all of the values were


NORTH DAKOTA
SOUTH DAKOTA
NEBRASKA
KANSAS
OKLAHOMA
TEXAS
LAKE SUPERIOR
LAKE MICHIGAN
LAKE ONTARIO
LAKE ERIE
Duluth
St. Paul
St. Peters
Yankton
Dubuque
Davenport
Chicago
Peoria
Keokuk
Quincy
Springfield
Jefferson City
St. Louis
70
71
72
Terre Haute
Indianapolis
Cincinnati
Madison
Louisville
Lexington
Cairo
118
Nashville
Memphis
Dallas
Vicksburg
Jackson
Natchez
Baton Rouge
New Orleans
Galveston
Houston
Austin
Indianola
Corpus Christi
Matamoras
Mobile
Montgomery
Macon
Pensacola
Apalachicola
Tampa
Tortugas
Key West
Cedar Keys
St. Augustine
Jacksonville
Fernandina
Savannah
Charleston
Columbia
Augusta
Toronto
Rochester
Buffalo
NEW YORK
Erie
Cleveland
PENNSYLVANIA
Pittsburg
Wheeling
Cumberland
Stanton
Lynchburg
NORTH CAROLINA
SOUTH CAROLINA
Cape Canaveral
GULF OF MEXICO
ATLANTIC OCEAN

No. 3.

computed directly by the complete method, the mean elevation of each compartment in each ring being estimated:

Topographic deflection.

Ring.	Station No. 72.	Station No. 118.
	″	″
16	− .03	+ .06
15	− .04	− .03
14	− .10	+ .09
13	− .10	+ .11
12	− .08	+ .03
11	− .10	+ .11
10	− .13	+ .26
9	− .16	+ .34
8	− .11	+ .11
7	+ .01	− .21
6	− .03	− .49
5	− .45	−1.00
4	−1.47	−2.84
3	− 3.44	−3.87
2	−2.74	−2.91
1	−3.63	−3.78

For ring 11 and smaller rings there is little resemblance between the computed deflections for the two stations, the algebraic signs being opposite, as a rule. For rings 6 and larger rings there is no contradiction in signs. Rings 11, for these two stations, do not touch or intersect each other. Rings 6 do intersect and overlap. In fact, in no part of these two rings is their distance apart more than two-thirds as great as the width of either ring. For the still larger rings 5, 4, 3, 2, and 1, the percentage of overlapping of corresponding rings steadily increases, and so, also, does the resemblance of values tend to increase. It is evident, without drawing them, that for these two stations the rings 1 (of which the outer radius is 4 126 kilometers and the width 1 298 kilometers) have a very large percentage of overlapping. This is the reason for the close agreement in the two values for these outermost rings, namely, −3″.63 and −3″.78.

In general, for corresponding successively larger rings for any two given stations, the resemblance of values must tend to increase. For the larger the rings the greater is the percentage of overlapping of the two rings and the more insignificant becomes the fixed distance between their centers (the two stations) in comparison with the widths of rings (which widths become greater as the rings become larger, being for each ring at least .42 of the inner radius of that ring). It is obvious, also, that the same considerations show that the tendency to a more and more close resemblance with increasing size of rings exists for all four stations and, hence, if the deflection for successive rings for station No. 71 be interpolated from the values for corresponding rings at stations Nos. 70, 72, and 118, these interpolated values will tend to agree with directly computed values for station No. 71 more closely as the rings are successively larger.

These ideas were, at first, a matter of pure theory. They have, however, been tested very fully during the course of the present investigation and their truth established. The nature and extent of the tests will be set forth later in connection with the discussion of the degree of accuracy attained in the computation of the topographic deflections.

For station No. 71 and for rings 13 to 7 the deflections were computed directly and were also interpolated (by the method to be explained later) from the three deflections for corresponding rings at the three surrounding stations, Nos. 70, 72, and 118. The following table shows the computed and interpolated values and their difference:

Station No. 71.

Ring.	Computed deflections.	Interpolated deflections.	Computed minus interpolated.
	″	″	″
13	−.08	−.05	−.03
12	−.12	−.05	−.07
11	−.14	−.01	−.13
10	−.11	+.01	−.12
9	−.09	−.03	−.06
8	−.10	−.06	−.04
7	−.13	−.09	−.04

According to the evidence given by rings 9, 8, and 7, it was decided, in accordance with certain criteria given later, that it would be safe to stop the direct computation at ring 7, proceeding outward, and to accept the interpolated values for rings 6 to 1 as sufficiently close to the truth.

The interpolation, based on the values for stations No. 72 and No. 118, virtually serves to determine the slight difference between the desired values for station No. 71 and the known values for station No. 70 (the nearest of the three stations from which the interpolation was made).. The following table shows how small are these differences between deflections for corresponding rings at station No. 71 and its nearest neighbor. station No. 70:

Topographic deflections.

Ring.	Station No. 70.	Station No. 71.	Difference.
	″	″	″
16	+.03	−.05	+.08
15	+.01	−.03	+.04
14	+.02	−.05	+.07
13	−.03	−.08	+.05
12	−.04	−.12	+.08
11	+.04	−.14	+.18
10	+.08	−.11	+.19
9	+.01	−.09	+.10
8	−.04	−.10	+.06
7	−.15	−.13	−.02
6	−.26	−.18	−.08
5	−.50	−.51	+.01
4	−1.47	−1.56	+.09
3	−3.10	−3.26	+.16
2	−2.94	−2.85	−.09
1	−3.82	−3.74	−.08

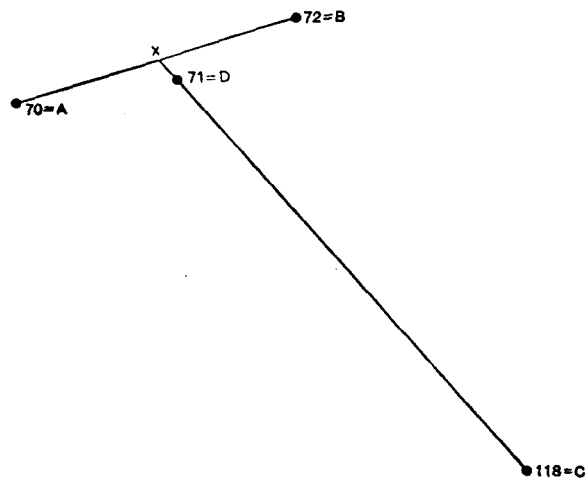
For rings 16 to 7 the values for both stations were computed. For rings 6 to 1 the values for station No. 71 were interpolated from surrounding stations and the column of differences shows, therefore, the small quantities fixed by interpolation as being the differences between station No. 70 and station No. 71.

METHOD OF INTERPOLATING FOR OUTER RINGS.

The purpose of obtaining the deflections for certain rings by interpolation is to save time in computation. The greater the number of rings for which the interpolation is made the greater is the saving accomplished. On the other hand, if the amount of interpolation is made too great the accuracy will fall below the required standard. It was necessary, therefore, to fix the amount and method of interpolation carefully in order to save as much time as possible and yet hold the accuracy up to the required standard. The following method of interpolation, and criteria for determining when interpolations should be made, were adopted and used after a gradual evolution during the computations for the first few stations. The degree of accuracy

secured will be indicated in connection with the discussion of the accuracy of the computations of topographic deflections.

The decision having been made to interpolate the deflections for some of the rings for station No. 71, from the corresponding values for the three stations, No. 70, No. 72, and No. 118, such a figure as that shown in illustration No. 4 was drawn upon a map, on which the astronomic stations had been plotted in their proper relative positions. Let the three stations from which the interpolation is to be made be called, in the general case, A, B, and C. In this case they are, respectively, No. 70, No. 72, and No. 118. Let the station for which the interpolation is to be made be called D (in this case No. 71). The figure, such as is indicated in illustration No. 4, is drawn in each case by first connecting two of the stations, A and B, by a straight line and then drawing the straight line CD until it intersects AB in X. A linear interpolation is first made between A and B (stations No. 70 and No. 72, in this case) to obtain a value corresponding to X for each ring, and then a second linear interpolation is made between X and C to obtain the required value for each ring at D.



No. 4.

This process may be called interpolation along a plane. If the three values at A, B, and C were represented graphically by ordinates above a reference plane on which A, B, and C were located in their proper relative positions, and if a plane were passed through the three points in space fixed by these ordinates, then the interpolated value for D is represented by the ordinate at D limited by this plane.

The numerical work of the interpolation for station No. 71 is shown in the following table. The factor .438 is the ratio $\frac{AX}{AB}$. The factor .063 is the ratio $\frac{XD}{XC}$. These may, for convenience, be called interpolation factors.

The decision may be made arbitrarily as to which two of the three stations shall be called A and B, and shall be utilized first by making a linear interpolation directly between them. Except for the effects of inaccuracy in scaling interpolation factors from the map, and inaccuracies in numerical work, the final results will be independent of the choice among three possible decisions. The effects of the small unavoidable inaccuracies in the scaling of interpolation factors will, in general, be smaller the nearer the angle CXA approaches to a right angle. Hence, it is advisable to choose among three possible decisions so as to make CXA as nearly as possible a right angle.

Interpolation of deflections for outer rings at latitude station No. 71, Bording, Illinois.

Ring.	Station No. 70.	Station No. 72.	Difference.	Difference times .438.	Station X.	Station No. 118.	Difference.	Difference times .063.	Deflection.
	″	″	″	″	″	″	″	″	″
13	− .03	− .10	−.07	−.03	− .06	+ .11	+ .17	+.01	− .05
12	− .04	− .08	−.04	−.02	− .06	+ .03	+ .09	+.01	− .05
11	+ .04	− .10	−.14	−.06	− .02	+ .11	+ .13	+.01	− .01
10	+ .08	− .13	−.21	−.09	− .01	+ .26	+ .27	+.02	+ .01
9	+ .01	− .16	−.17	−.07	− .06	+ .34	+ .40	+.03	− .03
8	− .04	− .11	−.07	−.03	− .07	+ .11	+ .18	+.01	− .06
7	− .15	+ .01	+.16	+.07	− .08	− .21	− .13	−.01	− .09
6	− .26	− .03	+.23	+.10	− .16	− .49	− .33	−.02	− .18
5	− .50	− .45	+.05	+.02	− .48	−1.00	− .52	−.03	− .51
4	−1.47	−1.47	.00	.00	−1.47	−2.84	−1.37	−.09	−1.56
3	−3.10	−3.44	−.34	−.15	−3.25	−3.47	− .22	−.01	−3.26
2	−2.94	−2.74	+.20	+.09	−2.85	−2.91	− .06	.00	−2.85
1	−3.82	−3.63	+.19	+.08	−3.74	−3.78	− .04	.00	−3.74

In applying this method of interpolation the order of procedure was, first, to compute the topographic deflections completely for three or four stations at the edges of the areas to be covered, so selected that all, or nearly all, of the remaining stations were included within the lines joining these stations. Then successive stations were selected for computation and for each in turn the computation was made complete up to the ring for which the adopted criteria, stated later, showed the interpolation to be safe. Then the interpolated values were depended upon for the remaining rings.

By inspection of the map on which the astronomic stations were all plotted the order of computation was so selected as to insure, as far as possible, that each new station computed should be near the center of an area containing no stations for which the computation of the same kind (meridian or prime vertical) had already been made. The interpolation was then made (or attempted) from the three stations among those already computed which lay nearest to it. The interpolations for the first few stations within a new region were thus in general made (or attempted) from stations at a considerable distance. Later interpolations were made from much nearer stations as the area became more thickly covered with stations for which the computations were already made.

In a few cases the point X of illustration No. 4 fell between D and C, and the last step of the interpolation process was really an extrapolation. This is the case for the azimuth station Mount Ouray, No. 59, see page 33. In the sample computation as there printed it will be noted that the interpolation factor, .013, was given the minus sign and the resulting correction represents an extrapolation. Similarly, in some cases the point X fell beyond A or beyond B, instead of falling between them, and the first interpolation factor became negative and really represented an extrapolation.

The same criteria of safety were applied to these cases as to the others in which only direct interpolations were involved. The cases of extrapolation most frequently occur at stations lying near the edge of the area covered by the investigation. The total number of cases was small.

The following table will serve as an illustration of the degree of agreement between values for corresponding rings at adjacent stations and of the amount of computation saved by interpolation.

The group of prime vertical stations indicated in the table is in California, at Point Arena and to the eastward and southward of that point. (See illustration No. 13 in the pocket at the end of the volume.) The stations are given in the table in geographic order.

Topographic deflections in prime vertical, separate rings.

[Italic numbers were interpolated.]

Ring.	Point Arena. 1.	Ross Mountain. 237.	Paxton. 2.	Ukiah. 3.	Sulphur Peak. 236.	Mount Helena. 4.	Monticello. 11.	Vaca. 12.	Yolo SE. Base. 15.
	″	″	″	″	″	″	″	″	″
30	+ .02								
29	+ .04								
28	+ .04							+ .07	
27	+ .05							+ .01	
26	+ .02							+ .01	
25	+ .02							+ .06	
24	+ .01							+ .04	
23	+ .10							.00	
22	+ .13							− .06	
21	+ .16							+ .10	
20	+ .16		− .03	− .22	+ .20	.00	− .13	− .12	
19	+ .26		+ .02	− .33	+ .39	+ .08	− .16	− .32	
18	+ .31		− .05	− .27	+ .43	+ .13	− .03	− .45	− .04
17	+ .38	+ .37	− .05	− .08	+ .60	− .12	− .14	− .80	− .08
16	+ .64	+ .39	+ .18	+ .07	+ .43	− .25	− .24	− .52	− .13
15	+ 1.46	+ .38	+ .54	+ .39	+ .12	− .17	− .50	− .15	− .23
14	+ 2.35	+ .58	+ *.79*	+ .46	+ .12	− .01	− .59	− .32	− .30
13	+ 3.92	+ 1.29	+ *1.52*	+ .98	+ .18	− .34	− .65	− .39	− .34
12	+ 4.98	+ 2.55	+ *1.69*	+ .99	+ *.37*	− .40	− .62	− .49	− .18

Topographic deflections in prime vertical, separate rings—Continued.

Ring.	Point Arena. 1.	Ross Mountain. 237.	Paxton. 2.	Ukiah. 3.	Sulphur Peak. 236.	Mount Helena. 4	Monticello. 11.	Vaca. 12.	Yolo SE. Base. 15.
	″	″	″	″	″	″	″	″	″
11	+ 5.90	+ 3.89	+ 2.73	+ 2.03	+ 1.21	+ .23	− .28	− .07	+ .58
10	+ 5.74	+ 4.73	+ 4.76	+ 4.50	+ 3.70	+ 3.12	+ 1.72	+ 2.51	+ 2.35
9	+ 6.00	+ 5.74	+ 6.05	+ 6.08	+ 6.42	+ 6.72	+ 6.30	+ 6.69	+ 6.01
8	+ 9.16	+ 8.49	+ 8.88	+ 8.81	+ 8.64	+ 8.62	+ 8.37	+ 8.23	+ 7.77
7	+ 9.44	+ 9.39	+ 9.31	+ 9.30	+ 9.42	+ 9.47	+ 9.33	+ 9.37	+ 9.44
6	+10.10	+ 9.91	+10.15	+10.15	+ 9.97	+ 9.91	+ 9.87	+ 9.74	+ 9.73
5	+ 9.33	+ 9.28	+ 9.30	+ 9.30	+ 9.30	+ 9.30	+ 9.34	+ 9.30	+ 9.35
4	+10.07	+10.07	+10.16	+10.18	+10.16	+10.17	+10.23	+10.19	+10.25
3	+10.04	+ 9.90	+ 9.94	+ 9.91	+ 9.82	+ 9.77	+ 9.65	+ 9.63	+ 9.56
2	+ 8.18	+ 8.07	+ 8.17	+ 8.16	+ 8.07	+ 8.04	+ 7.99	+ 7.95	+ 7.94
1	+ 5.62	+ 5.63	+ 5.63	+ 5.63	+ 5.64	+ 5.64	+ 5.65	+ 5.65	+ 5.66

The following table shows certain facts as to attempted and accepted interpolation for the nine prime vertical stations shown in the preceding table and also for the stations from which interpolations for these stations were attempted. The stations are placed in the table in the order in which the computations were made. At the points indicated by the blank lines in the table, computations were made for stations which are here omitted because they have no connection by attempted interpolations with the nine stations in question.

Name of station and State.	No. of station.	Stations from which interpolation was made or attempted.	Number of rings for which interpolation was accepted.	Distance to nearest station from which interpolation was made or attempted.	Inner radius of smallest ring for which the interpolation was accepted
				Kilometers.	*Kilometers.*
Wallace, Kansas	64	.. ..	..	...	.....
San Diego 1892, California	216	.. ..	..	...	.....
Point Arena, California	1	.. ..	..	...	.....
Promontory, Utah	38	64 1 216	0	940	.....
Uncompahgre, Colorado	57	64 216 38	0	520	.....
El Paso East Base, Colorado	62	64 57 38	0	250	.....
Tavaputs, Colorado	52	57 62 38	0	211	.....
Pioche, Nevada	34	38 216 52	0	392	.....
Wasatch, Utah	47	34 52 38	1	217	2 828
Pilot Peak, Nevada	32	38 1 47	0	142	.....
Round Top, California	19	1 34 32	0	323	.....
Mount Callahan, Nevada	29	19 32 34	2	286	1 962
Mount Conness, California	26	19 216 29	6	98	470
New San Miguel, California	227	216 1 26	0	334	.....
Mount Toro, California	219	1 227 26	4	257	958
Mount Diablo, California	13	1 219 19	4	152	958
Marysville, California	10	19 1 13	3	142	1 369
Mount Tamalpais, California	5	1 219 13	5	60	671
Yolo SE. Base, California	15	13 10 1	4	71	958
Ukiah, California	3	1 10 5	4	51	958
Mount Helena, California	4	3 15 5	6	73	470
Ross Mountain, California	237	1 5 4	6	46	470
Sulphur Peak, California	236	4 3 237	12	21	56
Paxton, California	2	3 1 236	14	9	28
Monticello, California	11	4 15 10	5	37	671
Vaca, California	12	15 13 237	6	30	470

CRITERIA OF ACCEPTED INTERPOLATIONS.

The computation for any station commenced with the small inner rings and proceeded outward. The three rules used by the computers in deciding at what ring it was allowable to begin to accept the interpolated values and to accept them for all larger rings were as follows:

Rule 1.—Commence to accept interpolated values as final with the first ring for which such interpolation is allowable under either rule 2 or rule 3 and which is beyond the one containing the nearest of the three stations from which the interpolation is made.

Rule 2.—Let 1″.00 divided by the number of a ring be called the interpolation limit for that ring. Subject to rule 1, acceptance of the interpolation may begin with a given ring if the three rings next within it each shows an agreement between the interpolated and computed values which is within the interpolation limit.

Rule 3.—Subject to rule 1, acceptance of the interpolation may begin with a given ring if the next ring inside of it shows an agreement within the interpolation limit and if at the nearest of the three stations from which the interpolation is made the agreement was also within the interpolation limit for the corresponding ring and for all rings farther out for which the comparison was made.

Rule 1 insures that the interpolation shall not be accepted for very small rings because of a chance agreement between interpolated and computed values when there is no reason for such agreement. It insures that the first ring for which the interpolation is accepted shall be one which overlaps, to a large extent, the corresponding ring at the nearest station.

To understand the nature of the so-called interpolation limit, referred to in rule 2, one must remember that the rings are numbered with 1 at the outside, the largest ring dealt with in the computation, and that the numbers increase inward (see table on p. 22). The interpolation limit is, therefore, 1″.00 for ring 1, 0″.50 for ring 2, 0″.33 for ring 3, and so on, becoming smaller the greater the number of rings which lie outside the one considered. The interpolation limit is smaller, the greater is the number of rings for which it is proposed to accept the interpolation.

Under rules 2 and 3 the total error made by accepting interpolated values would always be less than 1″.00 if the error of interpolation, I−C (interpolated minus computed), was of the same sign and magnitude for all larger rings, as on the last ring for which the comparison was made.

It was believed, however, that the agreement between the interpolated and computed values (commencing with rings not smaller than those contemplated under rule 1) would tend strongly to be closer and closer for successive rings proceeding outward. It was also believed that there would be a strong tendency for the various differences between interpolated and computed values, for several rings such as are interpolated under the rule, to include values having both the plus and minus signs, and, therefore, for the errors in the accepted interpolations to tend to be eliminated from the final result for the station. Both of these beliefs were based, at first, on theory only.

Their theoretical basis is indicated in illustration No. 3 and the accompanying text. (See p. 36.) If the beliefs indicated above are correct, the total error introduced at any station by accepting the interpolated values was, in general, much less than 1″.00. The correctness of the beliefs is established by the evidence obtained during the progress of the computations. This evidence will be shown in connection with the discussion of the accuracy of the computation of topographic deflections.

Rule 2 requires an agreement within the interpolation limit for three successive rings, because experience shows that a chance agreement between interpolated and computed values may occur for a single ring or even for two rings. It is believed that very rarely will a chance agreement within the small interpolation limit occur for three successive rings.

In connection with rule 3 it should be recalled that the accepted interpolations, if any, for the "nearest of the three stations from which the interpolation is made" are interpolations over a longer interval between stations, as a rule, than are the proposed interpolations for the new station. (See p. 41.) The order of computation was so selected as to insure, as far as possible, that each new station computed should be near the center of an area containing no

stations for which the computation of the same kind (meridian or prime vertical) had already been made. Rule 3 is, therefore, a recognition that the fact that interpolation has been found to be safe for a given ring over a long interval at an adjacent station, is valid evidence that it is probably safe over a shorter interval at the station in question.

An illustration of the application of rules 1 and 2 at meridian station No. 71 is shown on page 39. Station 72 was the nearest of the three (70, 72, and 118) from which it was proposed to attempt an interpolation. Station 72 lay in ring 11. Hence, in so far as rule 1 is concerned, the interpolation might have commenced with ring 10, but the difference between the interpolated and computed values for ring 11 (−″.13) was not within the interpolation limit, which is ″.09 for this ring. Similarly, for ring 10 the difference between interpolated and computed values was −″.12, outside of the interpolation limit, which is ″.10 for this ring. For the three successive rings, 9, 8, and 7, the agreement was within the interpolation limit and, therefore, under rule 2, the interpolated values were accepted for rings 6 to 1.

Another illustration of the application of rules 1 and 2 is shown for the prime vertical station, Mount Ouray, No. 59, in the table on page 33, and is commented upon in the fifth paragraph after the table.

In neither of these cases was an interpolation acceptable, under rule 3, for a smaller ring than under rule 2. On the other hand, at each of the prime vertical stations, 47, 29, 219, 13, 10, 4, 237, 11, and 12, of the table on page 42, rules 3 and 1 control instead of rules 2 and 1.

At prime vertical station No. 237, in so far as Rule 1 was concerned, the interpolation might have been accepted for ring 12 and all larger rings. The nearest of the three stations from which the interpolation was to be made was No. 4, Mount Helena, at a distance of 46 kilometers, and, therefore, falling in the thirteenth ring. The agreement of the interpolated and computed values was outside of the interpolation limit for rings 15, 14, 13, 11, 10, 9, and 8, and inside of the interpolation limit for ring 12 (I−C= +″.01) and for ring 7 (I−C=″.00). At station No. 4, the nearest of the three stations from which the interpolation was made, the agreement for ring 7 also fell within the interpolation limit, I−C being −″.12. No comparison of the interpolated and computed values had been made for larger rings at station No. 4, the acceptance of interpolation having commenced at that station with ring 6. Hence, under rule 3, the interpolation was accepted at station No. 237, Ross Mountain, for ring 6 and all larger rings.

SAVING BY INTERPOLATION OF OUTER RINGS.

The following table indicates how much labor was saved by interpolation:

Ring.	Outer radius of ring.	Number of computations out of the total of 496 in which the interpolated value was accepted for the ring specified.
	Kilometers.	
25	0.79	1
24	1.1	2
23	1.6	4
22	2.3	9
21	3.3	10
20	4.7	11
19	6.7	13
18	9.5	14
17	13	14
16	19	16
15	28	20
14	39	40
13	56	48
12	80	84
11	114	103
10	162	129
9	231	154
8	330	179
7	470	211
6	671	263
5	958	312
4	1 369	357
3	1 962	383
2	2 828	412
1	4 126	428

There are only 68 computations out of 496, less than one-seventh, for which no interpolations were accepted. In other words, the interpolation was accepted for at least ring 1 in more than sixth-sevenths of the computations made.

For more than one-half of the computations, 263 out of 496, the interpolation was accepted for rings 6 to 1, and thus no direct computation made for any topography at a greater distance from the station than 470 kilometers (292 miles), this being the outer radius of ring 7.

Similarly, for more than one-fourth of all the computations, 129 out of 496, the interpolation was accepted for rings 10 to 1, and no direct computation was made for any topography more than 114 kilometers (71 miles) from the station.

The interpolation was accepted for ring 25 (outer radius 0.79 kilometer, or 0.49 mile) and for all larger rings at the meridian station No. 11, Old Presidio, San Francisco, California, the nearest station from which the interpolation was made being No. 10, New Presidio, distant only .430 kilometer.

The interpolation was accepted for 3 217 rings out of a total of 12 395, or for more than one-fourth. The proportional part of the labor saved by interpolation is much larger than this, for it requires much more time to make the direct computation of the topographic deflection for an outer (large) ring than for an inner (small) ring. For a few of the largest rings, the ones for which the interpolated values are more frequently accepted, difficulties are encountered in applying the templates which serve to fix the compartment boundaries, because, on the Mercator charts, which must be used for these rings, the distances are much distorted. Another reason why it takes longer to make a computation for an outer ring than for an inner ring is that a larger number of authorities must be consulted, sometimes three of four maps or charts for a single compartment in an outer ring. So, also, it takes more time, as a rule, to estimate the mean elevation for a very large compartment of an outer ring than for a small compartment of an inner ring, because within the larger compartment there is a greater total range of elevation. The only notable exception to this rule occurs when the large compartment happens to fall entirely in the deep parts of the Pacific where the depths are comparatively uniform. After allowing for the fact that the computation takes more time for large rings than for small rings, and also for the fact that the interpolation itself takes some time, though not much, it is estimated that the scheme of interpolation saved between one-third and one-half of the time which would otherwise have been necessary to make direct computations of the topographic deflections complete out to the distance 4 126 kilometers from the station.

VALUE OF THE METHOD OF COMPUTING TOPOGRAPHIC DEFLECTIONS.

The three features of this method of computing topographic deflections which are especially valuable in reducing the time required for the computations, are (*a*) the use of compartments so chosen that for each compartment the deflection produced, as expressed in seconds of arc, is 0.0001 of the mean elevation within the compartment expressed in feet; (*b*) the use of transparent celluloid templates having the compartment boundaries marked upon them; (*c*) the interpolation of the deflections for certain outer rings for certain stations, from corresponding values for surrounding stations already computed.

The first of the three features mentioned saves more time than might be supposed by the casual reader. It saves time in another way in addition to the mere saving of multiplications. Because the computer knows that it is true for each compartment that an error of 100 feet in estimating the mean elevation produces an error of precisely ″.01 in the computed deflection he has a clear and ever-present conception of the accuracy required in his estimates. For each compartment he takes time merely to estimate with the required degree of accuracy. For many compartments a single glance at the map gives the estimated elevation with the required degree of accuracy, and the computer writes the result on his computation sheet without delay. For example, there are many compartments for which a glance shows that the total range of variation of elevation within the compartment is less than 100 feet.

If the compartments were so selected that the reduction factor by which the mean elevation must be multiplied to secure the deflection in seconds were different for different compart-

ments, the necessary degree of accuracy of estimating mean elevations would also be variable. The computer in such a case, not being able without delay to ascertain the degree of accuracy necessary for the particular compartment he is examining at any instant, would inevitably be inclined to make his estimate for each compartment with the highest degree of accuracy which is necessary for any compartment. He would lose much time, therefore, in making estimates with an unnecessarily high degree of accuracy.

The combination of the graphic and numerical methods involved in the process possesses great advantages. Any process of computing the topographic deflections by considering the land forms to be represented by regular geometrical figures, such as cones, disks, prisms, etc., loses greatly in accuracy at the outset by such assumptions, because such regular figures can not be made to fit the extremely irregular land forms. Any apparent accuracy gained later by strict adherence to exact formulæ is fallacious. The process used in this investigation makes the mathematical surfaces dealt with in the computation fit closely to the actual irregular land forms.

Moreover, the process has the great advantage of fitting the mathematical surfaces actually represented by the computation to the irregular land forms of nature most closely for the near topography, where a failure to fit closely would produce great errors in the computation, and leaves it fitting loosely upon the distant topography, where the effect on the computed results of the failure to fit exactly is comparatively insignificant. The mathematical surface actually dealt with and accurately represented by the computation is discontinuous, being made up of a large number of horizontal surfaces, each covering one compartment and each placed by the computer by estimation at the mean elevation of the actual land surface within the compartment. Obviously this discontinuous mathematical surface, made up of many horizontal surfaces at different levels, can be and is fitted to actual land surfaces much more closely near the station, where the areas of the compartments are very small (being a few square meters only for some compartments in ring 37), than it is at a distance from the station, where the areas of the compartments are large, being millions of square kilometers for the largest compartment of ring 1.

By the method of computation used in this investigation and described on the preceding pages, one computation for either a meridian or a prime vertical component of the deflection at a station, taking into account all the topography within 4 126 kilometers of the station, was found to take, on an average, the equivalent of 9.4 working hours for one computer.

THE ECONOMICS OF THIS RESEARCH.

Every known device has been utilized for reducing the time required for the computation of topographic deflections without allowing the accuracy to fall below the necessary high standard. If a mere matter of saving dollars and time were involved, and no more, this would probably not be fully justified. A finite and rather definitely limited amount of money and time is available for geodetic investigation in the United States. The complete solving of the geodetic problems awaiting solution would require millions in money and scores of years. The real problem before the geodesist is, therefore, to make as much progress in his attack on a sensibly infinite problem as can be made with his finite available money and time. Every device which saves money and time without serious loss of accuracy is a means to a greater advance into the unknown, and therein rests its real value. Conversely, every unnecessary refinement or complication which is allowed to remain in the methods of computation produces not simply a comparatively useless expenditure of money and time, but is also a serious drag, preventing real advance. Such unnecessary refinements or complications may give satisfaction to the computer and the investigator and may be frankly sanctioned if the purpose of the investigation is to furnish such satisfaction. They should not be sanctioned if the real purpose is the much broader, better one of securing as much as is feasible of real increase in the sum of human knowledge.

At the beginning of this investigation of the figure of the earth from measurements in the United States it was believed that real progress could be made mainly by ascertaining, with a

considerable degree of accuracy, the relation between observed deflections of the vertical and the known topography. It was also believed that the topography, even at a great distance from the station, must be used. To make the computations of the topographic deflections for all the stations by the methods hitherto used was evidently impossible with the finite amount of time and money available. The development of new and quicker methods of sufficient accuracy has made the computation possible. The computations have been made and the expected advance in knowledge has resulted. Continuous and close attention to the economics of the problem has led to real advance in the research.

Attention is called in this particular place in this publication to the importance of attention to the economics of geodesy because such attention has led to the greatest aggregate saving in this particular part of the research, the computation of topographic deflections. Such attention has, however, been given to the economics of geodesy in connection with each part of the investigation.* The spirit of the investigation and the motives which control in fixing methods will be missed by the reader who does not realize this. The methods used represent an effort to utilize available finite resources in the ways which are most effective in increasing the sum of human knowledge in regard to the figure and size of the earth.

Such are the reasons for the unusual prominence given to time-saving devices and methods in this publication.

VALUES OF TOPOGRAPHIC DEFLECTIONS.

The computed topographic deflections, due to all topography within 4 126 kilometers of each station, are given in the third column of the tables which follow. The observed deflections are given in the second column. They are repeated from the tables on pages 12–19.

The first column gives the serial number of the station as printed in the tables on pages 12–19. The location of each station is shown on illustration No. 13, in the pocket at the end of the volume, on which the stations are identified by the serial numbers. The remaining columns of the tables should be ignored for the present. They will be of interest in connection with later portions of this publication and will be explained there.

The many astronomic stations involved in this investigation will, for convenience, be considered in four principal geographic groups, known as the northeastern group, southeastern group, central group, and western group. The limits of these four groups are shown on illustration No. 13.

Within each of these four geographic groups the latitude stations, longitude stations, and azimuth stations are each placed in a separate group in the following tables, thus making 12 groups in all.

The observed deflections and the topographic deflections given in connection with the latitude stations are, of course, the meridian components of the deflections. A plus sign on the observed deflection means that the astronomic latitude is greater than the geodetic latitude, that the zenith is displaced to the northward and the nadir to the southward. Similarly, a plus sign on the topographic deflection means that the effective excess of mass represented by greater mean elevations of the surface is to the southward of the station.

The observed deflections and the topographic deflections, given in connection with the longitude and azimuth stations, are the prime vertical components of the deflections expressed in seconds of the prime vertical great circle. A plus sign on the observed deflection means that the zenith is deflected to the westward and the nadir to the eastward. Similarly, a plus sign on the topographic deflection means that the effective excess of mass represented by greater mean elevations of the surface is to the eastward of the station.

* The total time spent on all computations connected with this investigation (commencing with known values of the 507 astronomic and geodetic latitudes, longitudes, and azimuths) was the equivalent of only 20.6 working hours for one computer for each of the 507 deflections utilized.

Deflections in meridian.

LATITUDE STATIONS OF NORTHEASTERN GROUP.

Station.	Observed deflection A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.			
			Depth of compensation (kilometers).			
			329.8	162.2	120.9	113.7
	″	″	″	″	″	″
241	− .58	−11.28	+ .20	+ .23	+ .22	+ .22
188	−1.94	−12.21	− .59	− .46	− .39	− .39
185	−3.82	−11.95	− .28	− .24	− .22	− .22
184	− .74	−11.82	− .09	− .05	− .05	− .05
183	+2.45	−11.05	+1.11	+ .93	+ .81	+ .79
182	+4.70	− 9.89	+2.73	+2.39	+2.19	+2.15
186	+4.78	− 9.82	+2.80	+2.46	+2.26	+2.22
187	+4.14	− 9.75	+2.95	+2.60	+2.39	+2.35
181	+5.80	−10.33	+3.34	+2.96	+2.73	+2.67
180	+6.59	− 9.80	+4.20	+3.66	+3.35	+3.27
179	+3.65	−12.57	+1.73	+1.25	+ .99	+ .92
178	+4.09	−11.90	+2.32	+1.87	+1.58	+1.51
177	+4.38	−11.43	+3.80	+3.28	+2.92	+2.83
176	+2.25	−14.34	+2.18	+2.09	+1.93	+1.90
175	−2.16	−16.51	+ .47	+ .53	+ .46	+ .43
174	+ .56	−13.96	+3.07	+3.00	+2.82	+2.78
173	+ .74	−16.45	+ .42	+ .52	+ .45	+ .42
169	+1.92	−12.96	+3.86	+3.62	+3.40	+3.33
170	+3.61	−17.62	+2.22	+2.56	+2.50	+2.46
168	− .28	−28.74	−2.97	−1.25	− .60	− .54
166	+1.09	−24.60	− .37	+ .72	+ .91	+ .92
171	−5.35	−25.77	−3.41	−2.38	−2.09	−2.04
172	− .64	−21.88	−1.22	− .80	− .77	− .80
165	+ .41	−18.81	+1.28	+1.39	+1.22	+1.19
167	+3.95	−18.60	+ .68	+ .92	+ .69	+ .66
164	+3.71	−32.26	−2.80	−1.02	− .62	− .55
163	− .94	−33.16	−3.21	−1.35	− .93	− .85
158	+2.03	−35.94	−4.05	−1.98	−1.51	−1.42
162	−3.75	−32.49	−3.50	−1.68	−1.22	−1.16
157	− .59	−34.44	−4.28	−2.23	−1.69	−1.60
161	−3.25	−30.78	−3.17	−1.44	−1.01	− .94
159	+ .02	−30.69	−3.21	−1.53	−1.10	−1.04
156	−2.05	−32.80	−3.66	−1.77	−1.30	−1.24
160	−3.42	−30.58	−4.34	−2.59	−2.09	−1.99
155	− .31	−31.44	−3.61	−1.80	−1.32	−1.25
153	+ .27	−33.71	−4.18	−2.22	−1.73	−1.65
152	+ .78	−32.95	−4.28	−2.38	−1.87	−1.80
154	− .40	−31.68	−4.69	−2.93	−2.41	−2.31
149	+ .73	−29.61	−2.36	− .59	− .13	− .03
151	+ .29	−33.73	−4.20	−2.23	−1.78	−1.69
150	−2.01	−34.26	−3.89	−1.83	−1.37	−1.30
148	− .52	−31.82	−3.51	−1.60	−1.14	−1.05
147	−3.94	−34.71	−3.48	−1.36	− .92	− .87
144	−5.33	−34.48	−3.37	−1.22	− .78	− .72
145	−5.45	−34.50	−3.43	−1.28	− .84	− .78
142	−3.29	−36.54	−2.80	− .32	+ .15	+ .22
248	−5.22	−37.32	−3.26	− .87	− .44	− .38
146	−3.82	−32.28	−3.76	−1.11	− .62	− .55
143	−2.27	−33.16	−4.64	−2.51	−1.94	−1.83
141	−2.62	−36.89	−6.35	−3.71	−3.01	−2.89
140	−5.43	−35.63	−4.65	−1.92	−1.27	−1.16
139	−2.98	−33.44	−4.52	−2.17	−1.57	−1.49
138	+1.38	−31.88	−3.13	− .86	− .30	− .22
137	+1.89	−33.33	−4.81	−2.48	−1.88	−1.79
136	+4.59	−33.22	−5.35	−3.05	−2.43	−2.35
107	−7.28	−29.19	−2.82	−1.01	− .61	− .56
109	−4.06	−30.34	−3.22	−1.30	− .85	− .81
108	−2.06	−29.64	−2.76	− .87	− .45	− .41

Deflections in prime vertical.

LONGITUDE STATIONS OF NORTHEASTERN GROUP.

Station.	Observed deflection A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered. Depth of compensation (kilometers).			
			329.8	162.2	120.9	113.7
	″	″	″	″	″	″
232	− 6.85	−20.25	− .59	− .33	− .27	− .27
196	− 2.74	−20.04	− .74	− .47	− .38	− .38
195	− 2.41	−20.06	− .95	− .66	− .57	− .57
192	− .79	−19.86	− .84	− .57	− .48	− .48
191	+ 2.89	−19.76	+ 1.34	+1.32	+1.19	+1.19
194	− .23	−19.61	+ 1.47	+1.45	+1.32	+1.32
190	+ 5.24	−20.17	+ 1.79	+1.60	+1.44	+1.39
188	+ 7.62	−20.15	+ 2.36	+2.25	+2.09	+2.04
187	+ 3.00	−21.86	+ 1.05	+1.13	+1.07	+1.05
185	− .29	−22.37	+ .38	+ .53	+ .49	+ .47
184	+ .54	−25.39	− 1.05	− .64	− .56	− .56
183	+ 2.62	−22.81	+ 1.92	+1.92	+1.74	+1.70
182	+ 9.10	−18.68	+ 5.80	+5.51	+5.16	+5.07
181	+ 5.99	−20.34	+ 3.90	+3.67	+3.38	+3.30
179	+ 2.51	−23.77	+ .18	+ .35	+ .30	+ .29
173	+ 3.59	−32.33	− 2.85	−1.50	−1.14	−1.09
172	+ 1.74	−33.30	− 2.39	−1.05	− .73	− .70
157	+ 1.34	−40.08	− 4.93	−2.66	−2.04	−1.95
158	+ 1.42	−40.77	− 4.68	−2.47	−1.91	−1.83
233	− 1.34	−40.39	− 3.87	−1.65	−1.13	−1.03
108	− 4.23	−42.92	− 4.34	−1.75	−1.22	−1.14
110	− 1.35	−46.26	− 5.18	−1.96	−1.24	−1.13

AZIMUTH STATIONS OF NORTHEASTERN GROUP.

	″	″	″	″	″	″
193	+ 4.83	−20.66	− .55	− .38	− .38	− .38
188	− 3.68	−22.37	+ .38	+ .53	+ .49	+ .47
189	− 3.83	−20.68	+ 3.81	+3.59	+3.28	+3.20
180	+ .86	−23.77	+ .18	+ .35	+ .30	+ .29
175	− .27	−25.51	+ .45	+ .75	+ .68	+ .66
176	− 2.56	−28.36	− .71	− .14	− .10	− .12
174	− 5.46	−36.29	− 3.73	−1.99	−1.55	−1.45
177	−10.63	−33.33	− 4.80	−3.67	−3.35	−3.31
178	−14.77	−37.46	−10.32	−9.20	−8.80	−8.73
171	− .01	−33.33	− 3.04	−1.63	−1.28	−1.23
170	− .04	−34.48	− 2.72	−1.28	− .96	− .92
169	− 2.43	−33.52	− 2.77	−1.38	−1.06	−1.02
168	+ 1.35	−34.52	− 3.80	−2.41	−2.08	−2.04
167	− 2.33	−33.49	− 2.71	−1.32	− .97	− .94
166	− 1.06	−34.35	− 3.08	−1.52	−1.11	−1.07
164	− .65	−35.28	− 4.11	−2.31	−1.77	−1.69
165	− 1.62	−35.01	− 3.19	−1.47	− .98	− .92
163	+ 2.19	−38.01	− 6.01	−4.03	−3.43	−3.33
162	− 2.00	−36.11	− 5.39	−3.55	−2.93	−2.82
160	− 4.18	−35.23	− 3.59	−1.75	−1.22	−1.12
161	+ 2.90	−38.52	− 5.23	−3.07	−2.44	−2.32
159	− 1.63	−39.61	− 6.34	−4.15	−3.48	−3.36
156	+ .90	−39.45	− 4.95	−2.81	−2.20	−2.12
147	− 1.84	−40.08	− 4.93	−2.66	−2.04	−1.95
154	+ 2.32	−39.58	− 4.13	−1.90	−1.31	−1.23
150	+ 2.82	−40.56	− 5.08	−2.86	−2.26	−2.17
149	+ 3.49	−41.81	− 5.85	−3.61	−3.04	−2.95
151	− .19	−40.29	− 4.09	−1.87	−1.36	−1.27
152	+ 6.24	−40.25	− 3.42	−1.25	− .81	− .72
153	− 2.72	−40.69	− 3.86	−1.67	−1.21	−1.13
155	− 7.45	−41.14	− 6.91	−4.84	−4.25	−4.15
148	− 4.97	−37.52	− 3.70	−1.78	−1.26	−1.18
146	− 9.51	−41.83	− 6.75	−4.64	−4.10	−4.03
145	− 2.96	−40.80	− 4.28	−2.00	−1.48	−1.39
144	+ 1.59	−40.91	− 3.91	−1.51	− .96	− .86
143	+ 5.40	−40.72	− 4.29	−2.04	−1.53	−1.43
142	+ .44	−41.82	− 4.85	−2.44	−1.89	−1.79
109	− .34	−46.97	− 5.66	−2.38	−1.66	−1.54

Deflections in meridian.

LATITUDE STATIONS OF SOUTHEASTERN GROUP.

Station.	Observed deflection A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.				
			Depth of compensation (kilometers).				
			329.8	162.2	120.9	113.7	79.8
	″	″	″	″	″	″	″
80	+ .21	−13.63	− .34	− .40	− .40	− .42	− .41
81	+1.37	−12.90	+1.49	+1.17	+ .99	+ .95	+ .43
82	+4.66	−12.23	+2.77	+2.36	+2.09	+2.02	+1.42
83	−2.52	−19.88	−2.79	−2.39	−2.29	−2.29	−2.25
84	−2.53	−22.73	−3.47	−2.30	−1.87	−1.79	−1.31
114	+4.19	−22.33	−2.51	−1.31	− .89	− .84	−1.01
86	+3.06	−24.87	−3.40	−1.97	−1.52	−1.45	−1.06
87	−1.75	−24.61	−2.23	− .77	− .42	− .35	− .01
89	+2.51	−25.53	−3.58	−2.39	−2.06	−2.00	−1.68
88	+1.62	−21.59	−1.03	− .16	− .01	.00	+ .12
90	+3.76	−25.67	−3.67	−2.43	−2.10	−2.07	−1.81
91	+3.75	−26.36	−3.47	−2.04	−1.67	−1.61	−1.28
92	− .79	−27.17	−3.73	−2.22	−1.85	−1.78	−1.44
99	−1.33	−27.86	−3.81	−2.19	−1.79	−1.72	−1.35
94	−2.94	−27.53	−3.98	−2.51	−2.16	−2.10	−1.73
95	−3.29	−27.64	−4.08	−2.61	−2.26	−2.20	−1.84
93	−4.15	−27.50	−3.99	−2.52	−2.16	−2.11	−1.79
96	−3.48	−26.99	−3.42	−1.97	−1.62	−1.57	−1.25
97	−4.80	−26.93	−3.28	−1.82	−1.46	−1.41	−1.11
98	−3.84	−26.71	−3.04	−1.58	−1.22	−1.17	− .87
100	−2.02	−26.52	−2.54	−1.01	− .64	− .59	− .29
102	−2.78	−27.28	−2.64	−1.01	− .66	− .60	− .32
103	−2.42	−27.95	−3.00	−1.30	− .92	− .86	− .57
101	− .96	−27.77	−3.14	−1.46	−1.05	− .98	− .65
105	+1.86	−28.95	−3.46	−1.66	−1.18	−1.11	− .78
106	−3.94	−31.30	−5.19	−3.23	−2.68	−2.61	−2.20
104	−2.60	−26.32	−2.61	− .95	− .61	− .57	− .35
112	−1.16	−29.34	−3.01	−1.06	− .63	− .58	− .29
111	−2.75	−29.43	−3.17	−1.15	− .70	− .65	− .33
110	−2.50	−30.30	−3.22	− .96	− .44	− .38	− .05
113	−2.24	−31.30	−3.72	−1.31	− .77	− .70	− .34
85	+1.23	−24.77	−3.66	−2.05	−1.55	−1.47	−1.00
115	+1.53	−25.12	−5.07	−3.23	−2.60	−2.49	−1.81
116	+7.35	−23.62	−3.45	−1.71	−1.14	−1.06	− .60
117	+1.77	−24.47	−4.21	−2.33	−1.74	−1.64	−1.07
120	+2.11	−27.01	−6.95	−4.72	−3.85	−3.69	−2.78
121	− .94	−26.29	−6.92	−4.86	−4.06	−3.91	−3.07
123	− .92	−23.23	−4.90	−3.34	−2.80	−2.72	−1.09
125	+ .60	−22.59	−3.74	−2.23	−1.73	−1.67	−1.27
126	+1.25	−22.00	−2.96	−1.51	−1.06	−1.02	− .66
122	−2.25	−20.96	−3.43	−2.41	−2.14	−2.11	−1.83
119	+2.75	−13.14	+ .71	+ .79	+ .71	+ .71	+ .56
118	+3.78	−13.21	+ .65	+ .73	+ .70	+ .69	+ .53
131	+2.23	−16.64	+ .30	+ .21	+ .29	+ .29	+ .30
124	+ .41	−21.92	−4.11	−3.10	−2.80	−2.78	−2.48
127	+1.86	−20.61	− .94	+ .26	+ .52	+ .57	+ .79
128	+5.90	−23.39	−2.32	− .98	− .69	− .63	− .37
129	+ .55	−23.29	−2.25	− .92	− .64	− .60	− .37
130	−2.58	−24.54	−2.62	−1.01	− .64	− .59	− .35
132	+2.65	−26.26	−3.70	−1.57	−1.05	− .98	− .57
134	+5.64	−27.58	−4.45	−1.90	−1.26	−1.17	− .69
133	+5.39	−27.08	−3.89	−1.63	−1.07	−1.00	− .59
135	+5.42	−27.35	−3.13	−1.10	− .67	− .61	− .31

Deflections in prime vertical.

LONGITUDE STATIONS OF SOUTHEASTERN GROUP.

Station.	Observed deflection A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.				
			Depth of compensation (kilometers).				
			329.8	162.2	120.9	113.7	79.8
	″	″	″	″	″	″	″
87	+2.16	−21.75	+2.39	+2.14	+1.86	+1.79	+1.38
114	−2.24	−41.98	−8.39	−5.66	−4.76	−4.60	−3.67
90	− .41	−43.61	−8.43	−5.45	−4.53	−4.35	−3.37
92	−2.57	−40.99	−7.80	−5.23	−4.38	−4.21	−3.35
99	−4.26	−41.66	−5.63	−3.16	−2.53	−2.44	−1.86
100	−2.96	−41.24	−5.04	−2.53	−1.89	−1.80	−1.24
101	−5.07	−40.68	−4.34	−1.82	−1.20	−1.10	− .57
118	−5.37	−40.88	−4.53	−2.02	−1.39	−1.30	− .78
113	−3.57	−44.31	−5.27	−2.43	−1.77	−1.66	−1.16
119	+1.14	−35.24	−4.59	−2.56	−1.97	−1.88	−1.37
130	+ .96	−22.20	− .53	− .14	− .07	− .07	− .05
123	+ .80	−14.21	+ .89	+ .53	+ .37	+ .33	+ .18
131	−1.03	−16.57	+ .43	+ .37	+ .31	+ .31	+ .23
132	−1.64	−14.40	+ .24	+ .20	+ .16	+ .16	+ .09
137	+3.41	−15.11	− .57	− .31	− .24	− .24	− .19
141	− .19	−14.59	−1.09	− .44	− .29	− .27	− .17

AZIMUTH STATIONS OF SOUTHEASTERN GROUP.

	″	″	″	″	″	″	″
85	− 4.64	−21.29	+ .54	+ .41	+ .35	+ .32	+ .12
86	− .58	−22.41	+ .98	+ .80	+ .62	+ .58	+ .31
88	− 4.35	−29.48	−1.81	−1.08	− .99	−1.03	−1.06
89	− 4.30	−39.07	−6.29	−3.81	−3.01	−2.87	−2.09
91	− .38	−42.16	−6.28	−3.41	−2.57	−2.42	−1.65
94	+ 1.23	−39.07	−4.55	−2.28	−1.68	−1.58	−1.09
95	+ 2.72	−38.80	−5.14	−2.90	−2.27	−2.17	−1.65
96	+ 5.14	−37.47	−3.17	− .93	− .36	− .27	+ .14
102	− 8.05	−40.70	−5.17	−2.78	−2.14	−2.06	−1.52
97	− 6.03	−41.35	−5.35	−2.88	−2.27	−2.17	−1.58
103	− 9.99	−40.83	−4.23	−1.71	−1.10	−1.02	− .52
105	−11.70	−42.00	−4.70	−2.25	−1.67	−1.58	−1.14
104	− 9.54	−41.34	−4.94	−2.47	−1.87	−1.77	−1.28
107	− 8.57	−40.67	−4.16	−1.83	−1.30	−1.21	− .79
235	+ .19	−48.68	−6.05	−2.39	−1.55	−1.43	− .83
106	−11.50	−44.48	−5.02	−2.28	−1.69	−1.61	−1.19
117	− 5.54	−47.86	−5.65	−2.12	−1.31	−1.20	− .65
116	+ 2.13	−48.89	−6.00	−2.27	−1.44	−1.31	− .72
111	+ .41	−52.31	−7.34	−3.00	−1.96	−1.81	−1.07
115	− 6.62	−54.30	−7.96	−3.23	−2.09	−1.92	−1.09
93	+ 4.99	−40.00	−5.57	−2.91	−2.14	−2.00	−1.28
120	− .39	−38.11	−5.58	−3.19	−2.47	−2.33	−1.62
121	− 3.14	−36.03	−4.48	−2.27	−1.63	−1.52	− .94
122	+ 1.70	−33.59	−3.92	−2.00	−1.45	−1.36	− .90
125	− .77	−31.19	−4.59	−2.99	−2.42	−2.33	−1.73
126	− 4.64	−28.33	−4.17	−3.14	−2.79	−2.73	−2.24
128	+ 2.17	−22.67	− .59	− .20	− .15	− .15	− .13
129	+ 3.65	−22.45	− .52	− .11	− .04	− .04	− .01
127	+ .86	−18.81	− .75	− .91	−1.00	−1.02	−1.10
124	− .51	−14.36	+1.44	+1.00	+ .80	+ .74	+ .51
138	+ 5.99	−13.18	+1.02	+ .72	+ .56	+ .52	+ .33
133	+ .97	−15.93	+ .63	+ .39	+ .30	+ .24	+ .08
134	+ 1.92	−15.62	+ .98	+ .84	+ .74	+ .71	+ .55
135	+ 2.09	−15.10	+ .44	+ .38	+ .32	+ .32	+ .23
136	+ 6.26	−15.16	− .52	− .19	− .11	− .09	− .02
139	+ .82	−14.93	− .61	− .23	− .14	− .14	− .06

Deflections in meridian.

LATITUDE STATIONS OF CENTRAL GROUP.

Station.	Observed deflection. A-G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.				
			Depth of compensation (kilometers).				
			329.8	231.8	162.2	120.9	113.7
	″	″	″	″	″	″	″
227	− 4.06	− 9.06	−1.54	−1.36	−1.17	−1.02	− .98
228	− 1.29	− 7.88	− .90	− .70	− .44	− .27	− .23
231	+ .88	− 8.71	−2.06	−1.78	−1.44	−1.17	−1.12
232	+ 9.39	− 8.94	−3.53	−3.24	−2.85	−2.53	−2.47
230	+11.29	− 6.48	−1.19	− .98	− .76	− .54	− .51
229	− 5.28	− 9.82	−4.88	−4.60	−4.24	−3.90	−3.81
223	−10.39	− 7.98	−3.78	−3.63	−3.40	−3.11	−3.05
219	− 3.99	− 6.51	−2.40	−2.29	−2.16	−1.97	−1.94
213	+ .07	− 6.27	−2.14	−2.04	−1.90	−1.74	−1.73
212	+ 1.62	− 4.64	− .58	− .54	− .46	− .37	− .37
211	+ 3.82	− 3.29	+ .73	+ .71	+ .68	+ .68	+ .68
214	+ 1.87	− 2.93	+1.17	+1.16	+1.11	+1.07	+1.06
221	− 2.92	− 3.13	+1.20	+1.14	+1.01	+ .90	+ .88
216	+ 2.44	− .53	+3.98	+3.70	+3.30	+2.93	+2.85
226	− 5.93	− 3.33	+2.12	+2.04	+1.88	+1.72	+1.69
225	− 2.37	− 3.99	+1.53	+1.43	+1.29	+1.15	+1.12
224	−10.47	− 3.82	+1.65	+1.54	+1.35	+1.20	+1.17
222	− 2.44	− 2.53	+2.51	+2.27	+1.95	+1.65	+1.59
220	− .14	− 3.01	+2.24	+2.00	+1.67	+1.36	+1.30
217	+ 4.30	− 2.01	+3.15	+2.90	+2.55	+2.22	+2.13
218	+10.50	− 1.90	+3.61	+3.38	+3.02	+2.68	+2.60
189	+ 6.45	− 3.50	+2.83	+2.68	+2.43	+2.21	+2.14
191	+ 6.38	− 3.67	+2.70	+2.54	+2.32	+2.10	+2.03
215	+ .38	− 8.08	− .24	− .15	− .03	+ .07	+ .07
190	+ 1.65	− 7.92	+ .10	+ .18	+ .24	+ .29	+ .29
194	+ 1.20	− 7.68	+ .52	+ .64	+ .75	+ .83	+ .83
192	+ .47	− 9.22	− .86	− .73	− .58	− .46	− .45
193	− 1.59	− 9.48	− .68	− .54	− .42	− .32	− .31
197	− 4.10	− 9.05	− .45	− .39	− .35	− .31	− .30
196	− 6.10	− 9.24	−1.23	−1.11	− .97	− .84	− .81
210	− 6.41	− 8.78	−1.07	− .98	− .85	− .72	− .70
195	− 7.01	− 9.02	−1.30	−1.18	−1.05	− .93	− .90
198	− 5.66	− 8.94	−1.15	−1.03	− .89	− .74	− .72
200	− 3.70	− 8.89	− .84	− .73	− .60	− .47	− .45
199	− 3.92	− 8.73	− .61	− .49	− .34	− .22	− .20
201	− 1.45	− 8.81	− .65	− .51	− .38	− .26	− .24
202	− .15	− 9.20	−1.08	− .92	− .74	− .62	− .59
203	+ 1.53	− 9.18	−1.00	− .84	− .67	− .55	− .52
204	− .15	− 8.74	− .47	− .33	− .21	− .11	− .10
205	+ 3.24	− 9.07	− .44	− .35	− .25	− .18	− .18
206	+ 4.08	− 9.49	+ .61	+ .64	+ .61	+ .56	+ .57
209	+ 1.96	−10.01	− .05	+ .03	+ .07	+ .11	+ .10
207	+ 1.70	− 9.91	+ .05	+ .11	+ .17	+ .18	+ .20
247	+ 1.26	−11.43	− .39	− .28	− .21	− .16	− .16
208	+ .95	−11.71	− .60	− .47	− .37	− .30	− .29
62	− .10	−11.74	− .01	+ .05	+ .09	+ .13	+ .13
63	+ .43	−12.00	+ .06	+ .20	+ .25	+ .24	+ .24
64	− 1.27	−12.33	− .09	+ .03	+ .06	+ .06	+ .06
65	− 4.15	−12.22	− .04	+ .09	+ .15	+ .16	+ .16
66	+ .77	−11.37	+ .66	+ .82	+ .87	+ .87	+ .87
67	+ .72	−11.63	+ .39	+ .54	+ .62	+ .66	+ .66
68	+ 2.79	−11.96	− .08	− .04	− .08	− .16	− .19
69	+ 2.79	−11.38	+ .54	+ .61	+ .60	+ .56	+ .56
70	+ 4.41	−12.41	− .47	− .34	− .29	− .25	− .25
71	+ 5.63	−13.04	− .85	− .68	− .53	− .41	− .41
72	+ 2.51	−12.60	− .79	− .62	− .47	− .35	− .35
73	+ 2.71	−12.69	− .81	− .65	− .50	− .38	− .38
74	+ 1.53	−12.70	− .62	− .47	− .35	− .26	− .26
75	+ 1.10	−12.63	− .55	− .42	− .29	− .21	− .21
76	+ .04	−12.56	− .65	− .53	− .41	− .30	− .30
77	− .08	−13.62	−1.43	−1.27	−1.13	− .98	− .97
78	− 1.26	−12.89	− .75	− .66	− .56	− .46	− .43
79	− .27	−12.90	− .17	− .14	− .12	− .07	− .07

Deflections in prime vertical.

LONGITUDE STATIONS OF CENTRAL GROUP.

Station.	Observed deflection. A-G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.				
			Depth of compensation (kilometers).				
			329.8	231.3	162.2	120.9	113.7
	″	″	″	″	″	″	″
213	−6.97	−16.37	−3.72	−3.33	−2.99	−2.72	−2.67
202	−9.64	−18.64	−3.34	−2.96	−2.56	−2.20	−2.14
198	−5.20	−15.55	+.81	+.81	+.78	+.67	+.65
201	−5.41	−17.73	−2.04	−1.69	−1.36	−1.10	−1.07
206	−4.67	−17.96	−2.31	−1.89	−1.55	−1.26	−1.22
207	+.13	−17.21	−1.81	−1.44	−1.10	−.86	−.82
210	−2.40	−16.71	−.87	−.71	−.57	−.45	−.45
211	+.34	−16.65	−.98	−.82	−.69	−.59	−.59
64	−4.31	−21.56	−7.76	−5.70	−4.06	−3.02	−2.84
66	−6.72	−18.80	−4.64	−3.36	−2.41	−1.80	−1.70
69	−.85	−15.90	−1.20	−.61	−.22	+.01	+.04
74	+1.09	−16.70	−1.69	−1.39	−1.16	−.98	−.98
77	−.55	−15.67	−.05	+.01	+.04	+.05	+.05
79	−3.83	−15.49	+.29	+.32	+.32	+.29	+.29
83	−.31	−18.21	+.10	+.04	−.03	−.11	−.12

AZIMUTH STATIONS OF CENTRAL GROUP.

	″	″	″	″	″	″	″
205	+3.47	−11.94	+3.02	+2.78	+2.46	+2.17	+2.11
214	−8.15	−16.37	−3.72	−3.33	−2.99	−2.72	−2.67
208	−9.07	−14.28	−1.63	−1.29	−1.03	−.81	−.79
204	−9.93	−15.75	−1.35	−1.06	−.82	−.61	−.61
200	−4.58	−15.55	+.81	+.81	+.78	+.67	+.65
199	−3.70	−16.78	−.06	+.07	+.10	+.10	+.10
203	−4.36	−17.73	−2.04	−1.69	−1.36	−1.10	−1.07
209	+2.03	−17.20	−1.70	−1.34	−1.00	−.75	−.71
215	+4.18	−15.57	−.64	−.36	−.11	.00	+.03
212	−.34	−16.65	−.98	−.82	−.69	−.59	−.59
65	+6.47	−19.20	−5.05	−3.69	−2.64	−2.02	−1.89
67	+4.89	−17.87	−3.64	−2.55	−1.81	−1.31	−1.24
68	−1.80	−17.37	−2.73	−2.00	−1.49	−1.20	−1.16
70	+1.16	−15.72	−1.20	−.83	−.59	−.45	−.46
71	−4.11	−15.95	−1.29	−.92	−.67	−.51	−.51
72	−5.40	−16.30	−1.46	−1.10	−.86	−.67	−.67
73	−3.47	−16.28	−1.31	−1.01	−.80	−.65	−.65
75	−4.28	−15.52	−.22	−.05	+.06	+.13	+.13
76	−.88	−16.02	−.30	−.22	−.14	−.15	−.15
78	+.11	−15.67	−.05	+.01	+.04	+.05	+.05
80	−1.01	−14.97	+.97	+.93	+.85	+.72	+.71
81	−.98	−15.27	+.93	+.87	+.78	+.62	+.62
82	+1.82	−17.63	−.09	−.13	−.17	−.19	−.19
84	+4.73	−18.94	+.64	+.58	+.48	+.35	+.34

Deflections in meridian.

LATITUDE STATIONS OF WESTERN GROUP.

Station.	Observed deflection, A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.		
			Depth of compensation (kilometers)		
			162.2	120.9	113.7
	″	″	″	″	″
240	− 9.16	−55.32	− 7.23	− 5.79	− 5.51
239	− 6.29	−54.22	− 6.17	− 4.73	− 4.45
267	− 2.74	−52.78	− 4.65	− 3.20	− 2.91
266	+ 5.32	−52.58	− 2.67	− .97	− .65
265	− 2.24	−60.85	−10.65	− 8.66	− 8.26
246	− .45	−59.48	− 9.31	− 7.24	− 6.83
245	−17.99	−64.34	−13.71	−11.40	−10.95
236	+ 3.89	−52.73	− 3.09	− 1.17	− .82
237	− 2.05	−57.35	− 7.24	− 5.21	− 4.83
264	−10.93	−64.22	−13.35	−11.11	−10.67
238	−18.38	−64.97	−14.91	−12.78	−12.35
235	−12.58	−59.02	− 9.98	− 8.02	− 7.65
262	−13.22	−62.17	−12.65	−10.60	−10.20
234	− 5.20	−59.17	−10.40	− 8.50	− 8.15
243	−11.17	−53.52	− 6.26	− 4.70	− 4.42
242	− 5.59	−56.67	− 8.23	− 6.47	− 6.14
261	− 7.47	−57.69	−10.40	− 8.74	− 8.45
260	− 6.31	−58.72	−11.29	− 9.61	− 9.33
257	−12.08	−53.81	− 7.82	− 6.35	− 6.08
258	−11.92	−57.36	−10.58	− 8.95	− 8.62
259	−14.38	−57.42	−10.61	− 8.88	− 8.55
256	− 1.77	−45.51	− 2.64	− 1.82	− 1.69
255	−11.44	−46.96	− 4.19	− 3.23	− 3.08
244	− 7.54	−51.09	− 5.16	− 3.66	− 3.39
233	+ 5.17	−47.17	− 2.29	− .97	− .72
254	+ .29	−50.71	− 4.89	− 3.30	− 2.98
253	− 3.94	−46.26	− 3.25	− 2.25	− 2.09
19	− 5.96	−46.91	− 4.27	− 3.16	− 2.98
21	− 2.99	−39.64	+ 1.50	+ 2.31	− 2.41
16	− 5.85	−41.22	− .31	+ .62	− .75
13	− 7.81	−46.02	− 3.39	− 2.08	− 1.89
12	− 3.61	−45.72	− 3.02	− 1.70	− 1.51
10	− 3.03	−45.74	− 3.01	− 1.68	− 1.48
11	− 3.19	−45.66	− 2.90	− 1.58	− 1.38
9	− 8.37	−48.83	− 6.03	− 4.67	− 4.46
4	−10.68	−49.00	− 5.91	− 4.35	− 4.09
3	− 9.73	−50.03	− 6.73	− 5.13	− 4.84
7	−10.62	−50.65	− 7.53	− 5.91	− 5.61
1	− 8.77	−54.03	−10.15	− 8.19	− 7.81
2	− 7.96	−48.02	− 6.28	− 4.78	− 4.51
252	− 4.84	−38.90	− 4.51	− 3.93	− 3.86
251	− 4.02	−33.37	− .37	+ .16	+ .25
249	− 5.29	−31.38	− 2.29	− 1.77	− 1.71
5	− 4.29	−45.32	− 4.84	− 3.45	− 3.21
6	− 9.86	−49.61	− 8.12	− 6.65	− 6.37
8	− 9.08	−47.05	− 6.15	− 4.84	− 4.60
14	− 4.70	−43.78	− 4.15	− 3.04	− 2.86
15	−10.17	−46.50	− 6.48	− 5.39	− 5.23
17	− 7.69	−42.01	− 3.22	− 2.20	− 2.03
18	− 8.08	−41.62	− 2.78	− 1.82	− 1.68
20	− 7.37	−42.00	− 4.21	− 3.04	− 2.85
24	− 3.27	−40.15	− 3.83	− 2.86	− 2.70
23	− 4.03	−37.82	− 3.36	− 2.64	− 2.51
25	+ 2.66	−26.67	+ 5.09	+ 5.39	+ 5.42
27	− 5.01	−29.37	+ .32	+ .53	+ .52
22	− 2.91	−31.19	+ .16	+ .63	+ .69
26	− 1.82	−25.85	+ 2.45	+ 2.52	+ 2.47
28	− 3.03	−20.49	+ 3.15	+ 3.04	+ 3.02
29	− 5.59	−29.75	− 1.60	− 1.22	− 1.14
30	− 3.50	−17.49	+ 2.48	+ 2.16	+ 2.06
31	−13.54	−15.82	+ 3.75	+ 3.49	+ 3.42
32	−12.73	−34.60	− 3.63	− 2.66	− 2.49

Deflections in meridian—Continued.

LATITUDE STATIONS OF WESTERN GROUP—Continued.

Station.	Observed deflection. A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered. Depth of compensation (kilometers).		
			162.2	120.9	113.7
	″	″	″	″	″
33	− 4.38	−33.47	− 4.04	− 3.22	− 3.09
37	− 2.72	−20.50	+ 1.71	+ 1.65	+ 1.61
48	+ 7.52	−12.17	+ 8.30	+ 8.36	+ 8.35
35	− 3.71	−16.05	+ .86	+ .74	+ .68
34	− 9.36	−18.90	− 5.92	− 5.87	− 5.87
39	− 5.90	−14.10	− 2.75	− 2.59	− 2.56
42	− 4.21	−12.42	− 1.09	− 1.04	− 1.03
44	− 1.56	−11.29	+ .06	+ .09	+ .11
40	− 3.88	−11.42	+ .85	+ .85	+ .84
41	− 2.27	−12.02	.00	− .06	− .09
43	− 9.02	−15.62	− 2.98	− 2.98	− 3.01
38	− 3.74	−11.66	+ 1.79	+ 1.65	+ 1.61
46	− 6.21	−14.93	− .04	− .09	− .13
36	− 2.71	−14.21	+ 2.66	+ 2.45	+ 2.37
45	− 5.58	−15.61	+ .82	+ .63	+ .54
47	− 3.60	−20.74	− 3.16	− 3.00	− 2.99
49	−13.52	−27.20	− 9.42	− 8.66	− 8.53
50	− 6.65	−22.76	− 4.15	− 3.55	− 3.42
51	+ 8.41	− 8.70	+ 7.10	+ 6.76	+ 6.69
52	− 7.45	−21.75	− 7.33	− 6.89	− 6.79
53	+ 3.82	− 8.55	+ 4.97	+ 4.87	+ 4.84
54	− 2.39	− 9.50	+ 5.97	+ 5.65	+ 5.54
55	− 2.70	−14.96	− 1.03	− .96	− .97
56	− 3.86	− 6.52	+ 4.52	+ 4.29	+ 4.24
57	− 4.86	−16.61	− 2.34	− 2.07	− 2.00
58	+ .96	− 8.76	+ 3.29	+ 3.44	+ 3.47
59	− 3.26	−14.59	− 2.50	− 2.26	− 2.21
60	− 5.88	−14.76	− 3.64	− 3.53	− 3.49
61	− 3.42	−15.59	− 2.74	− 2.43	− 2.36

Deflections in prime vertical.

LONGITUDE STATIONS OF WESTERN GROUP.

	″	″	″	″	″
246	+12.17	+ 74.66	+13.11	+10.75	+10.28
216	+10.73	+ 74.42	+12.90	+10.54	+10.07
243	+ 5.69	+ 66.95	+ 6.01	+ 4.40	+ 4.12
217	+ 5.77	+ 67.29	+ 6.26	+ 4.65	+ 4.36
9	+10.05	+ 87.89	+12.91	+10.65	+10.26
8	+ 4.04	+ 84.30	+ 7.23	+ 4.76	+ 4.33
7	+ 4.50	+ 84.84	+ 7.58	+ 5.05	+ 4.62
6	+ 5.51	+ 84.98	+ 7.69	+ 5.17	+ 4.73
1	+16.98	+104.63	+20.39	+16.45	+15.69
3	+ 4.20	+ 86.04	+ 7.97	+ 5.45	+ 5.02
16	+ 4.17	+ 79.24	+ 6.32	+ 4.32	+ 3.99
10	+ 5.78	+ 81.53	+ 7.84	+ 5.67	+ 5.29
20	+ 3.78	+ 74.66	+ 8.50	+ 6.77	+ 6.50
23	−18.65	+ 53.01	−11.05	−12.32	−12.50
24	− 3.43	+ 58.83	− 3.36	− 4.31	− 4.43
25	− 6.11	+ 53.60	− 6.81	− 7.55	− 7.65
22	− 4.05	+ 62.16	− .42	− 1.59	− 1.78
33	+11.60	+ 50.23	+ 3.40	+ 2.74	+ 2.65
31	+ 5.33	+ 46.89	+ 5.33	+ 4.88	+ 4.82
41	+13.25	+ 44.72	+11.84	+11.37	+11.05
44	+15.37	+ 48.49	+16.12	+14.46	+14.10
36	+ 8.54	+ 35.58	+ 3.33	+ 2.74	+ 2.64
50	− 2.02	+ 22.62	− .28	− .49	− .51
53	+ .48	+ 19.10	.00	− .84	− .99
55	+ 3.09	+ 10.31	+ 1.75	+ 1.43	+ 1.33
61	−18.74	− 28.78	−16.35	−14.32	−13.90

Deflections in prime vertical—Continued.

AZIMUTH STATIONS OF WESTERN GROUP.

Station.	Observed deflection, A−G.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered. Depth of compensation (kilometers).		
			162.2	120.9	113.7
	″	″	″	″	″
245	+32.43	+74.63	+12.17	+ 9.71	+ 9.22
244	+28.06	+75.68	+14.18	+11.87	+11.41
231	+21.55	+67.65	+ 7.85	+ 6.11	+ 5.80
230	+19.56	+67.94	+ 7.69	+ 5.94	+ 5.62
234	+ 9.76	+67.77	+ 6.62	+ 4.91	+ 4.58
242	+19.38	+72.94	+ 9.04	+ 7.28	+ 6.96
229	+15.04	+73.75	+ 7.13	+ 5.10	+ 4.76
228	+19.71	+76.41	+ 9.54	+ 7.52	+ 7.20
227	+20.62	+80.27	+11.32	+ 8.80	+ 8.33
226	+14.67	+78.05	+ 9.10	+ 6.81	+ 6.38
224	+20.15	+80.78	+10.79	+ 8.44	+ 8.03
225	+15.90	+86.09	+14.36	+11.54	+10.99
223	+14.85	+86.56	+14.05	+11.30	+10.79
241	+29.93	+84.44	+12.15	+ 9.46	+ 8.94
222	+ 7.64	+73.39	+ 4.43	+ 2.79	+ 2.54
220	+14.14	+79.85	+ 8.76	+ 6.82	+ 6.46
221	+ 6.56	+88.30	+12.33	+ 9.36	+ 8.81
219	+ 6.33	+85.30	+ 9.57	+ 6.83	+ 6.31
239	+ 5.52	+94.98	+16.04	+12.77	+12.13
240	+14.04	+87.24	+ 9.48	+ 6.54	+ 6.02
218	+10.25	+82.89	+ 9.57	+ 7.50	+ 7.14
17	+ 5.63	+82.78	+ 8.89	+ 6.83	+ 6.49
13	+ 3.70	+79.08	+ 4.64	+ 2.70	+ 2.39
238	− 4.47	+84.37	+ 7.28	+ 4.78	+ 4.34
5	+ 5.67	+83.62	+ 5.89	+ 3.31	+ 2.87
237	+18.18	+90.66	+11.61	+ 8.69	+ 8.16
2	+10.90	+89.69	+10.48	+ 7.69	+ 7.20
236	+18.82	+85.19	+ 8.61	+ 6.35	+ 5.98
4	+12.32	+79.91	+ 4.71	+ 2.81	+ 2.50
11	+ 2.92	+75.11	+ 2.18	+ .64	+ .42
12	+ .97	+75.86	+ 2.25	+ .56	+ .29
15	+ 7.37	+77.34	+ 4.08	+ 2.33	+ 2.05
14	+ 7.32	+75.66	+ 3.28	+ 1.65	+ 1.40
26	+10.14	+77.05	+11.90	+ 9.99	+ 9.64
19	+ 4.14	+78.58	+ 9.23	+ 6.89	+ 6.45
21	+ 9.42	+73.38	+ 7.42	+ 5.74	+ 5.47
18	+ 1.29	+76.73	+ 9.40	+ 7.29	+ 6.91
27	+ 3.86	+64.35	+ 9.74	+ 8.48	+ 8.24
28	+ 8.25	+51.35	+ 2.01	+ 1.33	+ 1.19
29	+ 5.48	+47.85	+ 1.94	+ 1.38	+ 1.29
30	+ 3.95	+42.85	+ 2.27	+ 1.89	+ 1.84
34	+ 7.43	+39.57	+ 1.81	+ 1.53	+ 1.49
35	+ 6.28	+28.54	− 2.76	− 2.39	− 2.28
32	+ 2.83	+23.61	− 6.44	− 5.95	− 5.81
38	− 2.41	+29.48	− 1.00	− 1.75	− 1.89
42	+16.25	+44.72	+12.84	+11.35	+11.05
40	+12.98	+45.96	+14.05	+12.48	+12.17
43	+24.84	+54.71	+22.11	+20.38	+20.00
45	+18.15	+48.49	+16.12	+14.45	+14.10
39	+ 7.98	+37.85	+ 6.30	+ 5.16	+ 4.93
37	+12.69	+34.08	+ 3.38	+ 2.81	+ 2.72
46	+18.69	+42.03	+10.93	+ 9.75	+ 9.51
47	+ 8.74	+29.30	+ 1.84	+ 1.48	+ 1.42
48	+ 5.34	+23.85	− .82	− .55	− .44
49	+10.50	+27.24	+ 4.38	+ 3.16	+ 3.15
51	+14.68	+31.12	+ 7.42	+ 6.30	+ 6.08
52	+ 4.52	+20.84	+ 1.52	+ .98	+ .87
54	+ 6.79	+19.90	+ .86	+ .02	− .13
57	+ 7.63	+18.70	+ 4.96	+ 4.17	+ 3.99
56	+11.36	+10.31	+ 1.76	+ 1.43	+ 1.33
58	+13.92	+20.00	+ 8.89	+ 8.06	+ 7.84
59	+ .95	−11.25	−11.07	−10.21	−10.01
60	− 9.79	−28.23	−17.00	−15.02	−14.62
62	− 4.44	−22.20	− 8.72	− 7.01	− 6.67
63	− 1.44	−19.00	− 4.18	− 3.03	− 2.82

The meridian components of the topographic deflection are all negative, varying from $-0''.53$ at the latitude station Porcupine (No. 216), on the south shore of Lake Superior, to $-64''.97$ at the latitude station Santa Barbara, California, (No. 238). All these computed topographic deflections have one sign, because the United States is near the southern point of the continent of North America. At the latitude station New Orleans, Louisiana, (No. 135), near the northern shore of the Gulf of Mexico, the meridian component of the topographic deflection is $-27''.35$ and at the latitude station Howard, Maine, (No. 158), it is $-35''.94$.

An unusually rapid change of the meridian component of the topographic deflection between adjacent points is illustrated by the latitude stations Green River, Utah, (No. 50), and Mount Waas, Utah, (No. 51). Although these stations are only 96 kilometers apart the topographic deflections differ by $14''.06$, being $-22''.76$ at the former and $-8''.70$ at the latter. These stations are in a region of high, steep mountains, and the deep, broad valley of the Grand River lies between them.

The maximum negative prime vertical component of the topographic deflection occurs at the azimuth station North End Knott Island, Virginia, (No. 115), namely, $-54''.30$. This station is only 130 kilometers from the submerged edge of the continent as fixed by the one thousand fathom line.

The maximum positive prime vertical component of the topographic deflection occurs at the longitude station Point Arena, California, (No. 1), namely, $+104''.63$. This station is only 35 kilometers from the submerged edge of the continent as fixed by the one thousand fathom line.

For the azimuth station Mount Ouray, Colorado, (No. 59), in longitude 106° 13′ and for all prime vertical stations to the eastward of it, the prime vertical component of the topographic deflection is negative. For all stations to the westward of Mount Ouray it is positive.

An unusually rapid change of the prime vertical component of the topographic deflection between adjacent stations is illustrated by the azimuth stations Gunnison, Colorado, (No. 56), and Mount Ouray, Colorado, (No. 59). Although these stations are only 63 kilometers apart the prime vertical components of the topographic deflections differ by $21''.56$, being $+10''.31$ at the former and $-11''.25$ at the latter. These stations are in a region of high, steep mountains.

These computed topographic deflections must necessarily exist as actual deflections if the material comprising the surface of the earth and down to the level of the lowest point of the ocean floor has a density of 2.67 and if the densities below that level have no relation to the topography of the surface. The irregularities of the surface constituting the topography certainly exist. The computations of the topographic deflections depend upon the well-established law of gravitation that the attraction between two masses is proportional to their product and inversely proportional to the square of the distance between them.

On the other hand, a comparison of these computed topographic deflections, shown in the third column of the preceding tables, with the observed deflections of the vertical as shown in the second column, shows clearly that the latter are much smaller than the former. After a comparison in detail, one finds it difficult to see easily what relation exists between the two sets of quantities. One may possibly begin to doubt that any close relation does exist between them.

As the observed deflections evidently do not correspond to the deflections due to the known topography, it is evident that they must be due, in part at least, to variations in density beneath the surface.

CONSTRUCTION OF CONTOURS OF THE GEOID.

Before proceeding to a study of the possible relation of the distribution of the subsurface densities to observed deflections of the vertical, it is desirable to show the outcome of a subsidiary investigation which was made to develop the extent to which the observed deflections of the vertical are related to the topography. This investigation was made by constructing the contour lines of the geoid graphically, starting with the observed deflections of the vertical as a basis.

By contour lines of the geoid are meant lines of equal elevation on the geoid surface, referred to the Clarke spheroid of 1866 as a reference surface, the spheroid being supposed to be in the position fixed by the adopted United States Standard (Geodetic) Datum. The contour lines serve to indicate clearly to the eye, and in a comprehensive manner, the departure of the geoid from the spheroid.

The geoid surface is a surface which is everywhere normal to the direction of gravity (an equipotential surface), and it is that particular one of many such surfaces, lying at different elevations, which coincides with the mean sea surface over the oceans. Obviously, the mean sea surface must, with considerable accuracy, be everywhere normal to the direction of gravity.

The mean sea surface is an existing physical representation of the geoid surface for the three-fourths of the earth covered by the oceans. No similar physical representation exists for the areas covered by continents. One may conceive of such a physical representation by supposing that narrow canals, say one foot wide, were cut down to a depth somewhat below mean sea level along the township boundaries of the land system over the United States. Such canals would form a rectangular system following approximately along meridians and parallels and approximately six miles apart in each direction. If the sea water were allowed free access to all these canals and were protected from all disturbances, the surface of the water in the canals would everywhere become normal to the direction of gravity and would be at sea level and, therefore, would be a part of the surface of the geoid. One may think of the surface of the water in these hypothetical canals as forming a concrete representation of that portion of the geoid which lies under the United States.

The restriction in the preceding statement that the supposed canals must be very narrow (1 foot wide) is introduced because it the canals were supposed to be of considerable width the supposed removal of masses to make the canals would change the direction of gravity at various points and so produce a new geoid.

The problem at present under consideration is that of constructing the contour lines which will represent the relation of the irregular geoid to the regular ellipsoid of revolution known as the Clarke spheroid of 1866, which is supposed to be in the position fixed by the adopted United States Standard Datum.

The deflections of the vertical, as observed and shown in the tables on pages 12–19, are slopes of the geoid, at the points of observation, with reference to the spheroid. Having given these slopes in the direction of the meridian and the prime vertical at these few points the problem in hand is to construct the contour lines of the geoid. It is a problem similar to that which would be before the topographic draftsman if the topographer in the field furnished to him observed values of the slopes of the land surface in the direction of the meridian and of the prime vertical at a few points.

The separate steps in the construction of the contour lines of the geoid were as follows:

1. A series of drawings covering the area in question were made on a large scale, showing simply a few meridians and parallels drawn according to the polyconic projection.

2. The figures showing the observed deflections in the meridian were placed on each drawing in their proper positions.

3. Lines of equal deflection in the meridian were drawn by eye, after first locating a few points on each line by assuming that on the straight lines joining adjacent observed values the rate of change of the deflection in the meridian is constant.

4. The figures showing observed deflections in the prime vertical, resulting from both longitude and azimuth observations, were placed on the drawing in their proper positions. These deflections as placed on the drawing were of course in seconds of arc of the prime vertical great circle, so as to be directly comparable with the deflections in the meridian.

5. Lines of equal deflection in the prime vertical were then drawn in the same manner as indicated above for deflections in the meridian.

6. A rectangular system of lines was placed on the drawing in such a manner as to divide the earth's surface into portions which were as nearly as possible squares, 20.6 kilometers on

each side and with their sides always placed in the meridian and prime vertical. The particular length, 20.6 kilometers, was chosen because a line having a slope of one second of arc rises or falls one decimeter in 20.6 kilometers ($\sin 1'' = \frac{1}{206000}$). In other words, to convert the slope of such a line expressed in seconds of arc into the difference of elevation of its two ends in meters, all that is necessary is to move the decimal point one place to the left. This simple device eliminated the necessity for multiplications.

7. On the middle of each of the 20.6-kilometer lines of the rectangular system there were then placed the figures indicating with the proper sign the difference of elevation of the two ends of this line, supposing it to be a line on the geoid. For any 20.6-kilometer line in the direction of the meridian the required difference was obtained by estimation from the lines of equal deflection in the meridian which had already been placed upon the drawing, the estimated deflection in the meridian being converted into a difference of elevation by moving the decimal point one place to the left. Similarly, the difference to be placed on the middle of each 20.6-kilometer line which lay in the direction of the prime vertical was obtained from the lines of equal prime vertical deflection.

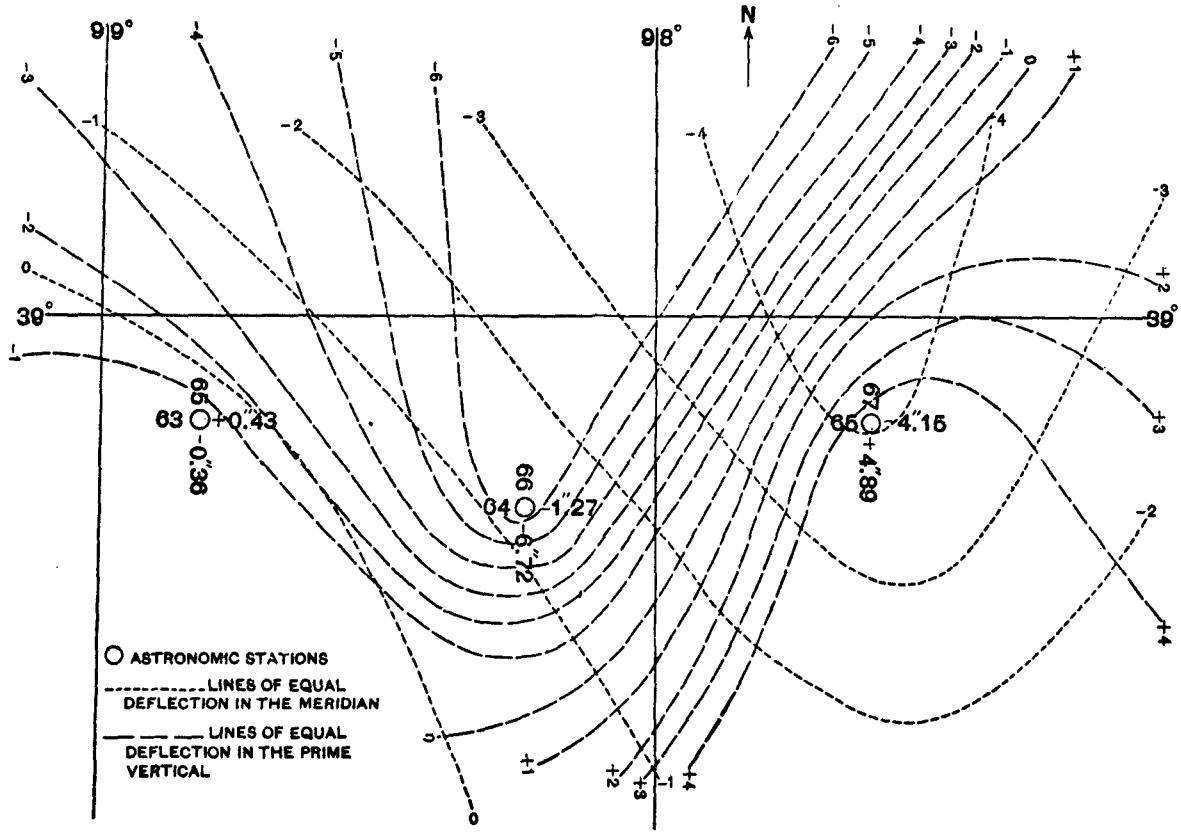
8. Starting with an assumed elevation for one point on the rectangular system, the elevation of all other points of the rectangular system (corners of the squares) were computed by use of the differences of elevation which had already been placed on the drawing, as indicated in the preceding paragraph. These elevations were placed on the drawing as rapidly as they were obtained. The elevations of one row of points were computed at a time. Let it be supposed that the elevations of the row of points along a meridian have been fixed and those of the next row to the westward are to be computed. For each new point three values can in general be obtained for its elevation. First, from the fixed point due eastward by applying the prime vertical difference of elevation. Second, from a fixed point due southeast by applying first a prime vertical difference and then a meridional difference of elevation. Third, from a fixed point due northeast by applying first a prime vertical and then a meridional difference of elevation. The mean of these three values was taken as the required elevation. Thus the process of computing the elevations of successive rows of points went on until the elevation of every intersection of the rectangular system had been fixed and marked on the drawing. For points on the margin only two values for the elevation could be so determined in general, and in some cases only one. When the successive rows of points established were in an oblique line (northeast and southwest or northwest and southeast), instead of a north and south or east and west line, but two values were available for the elevation of each new point, and the mean was taken.

9. From the figures expressing elevations now upon the drawing the contour lines, or lines of equal elevation of the geoid above the spheroid, were drawn in exact accordance with the figures, without any generalization or smoothing.

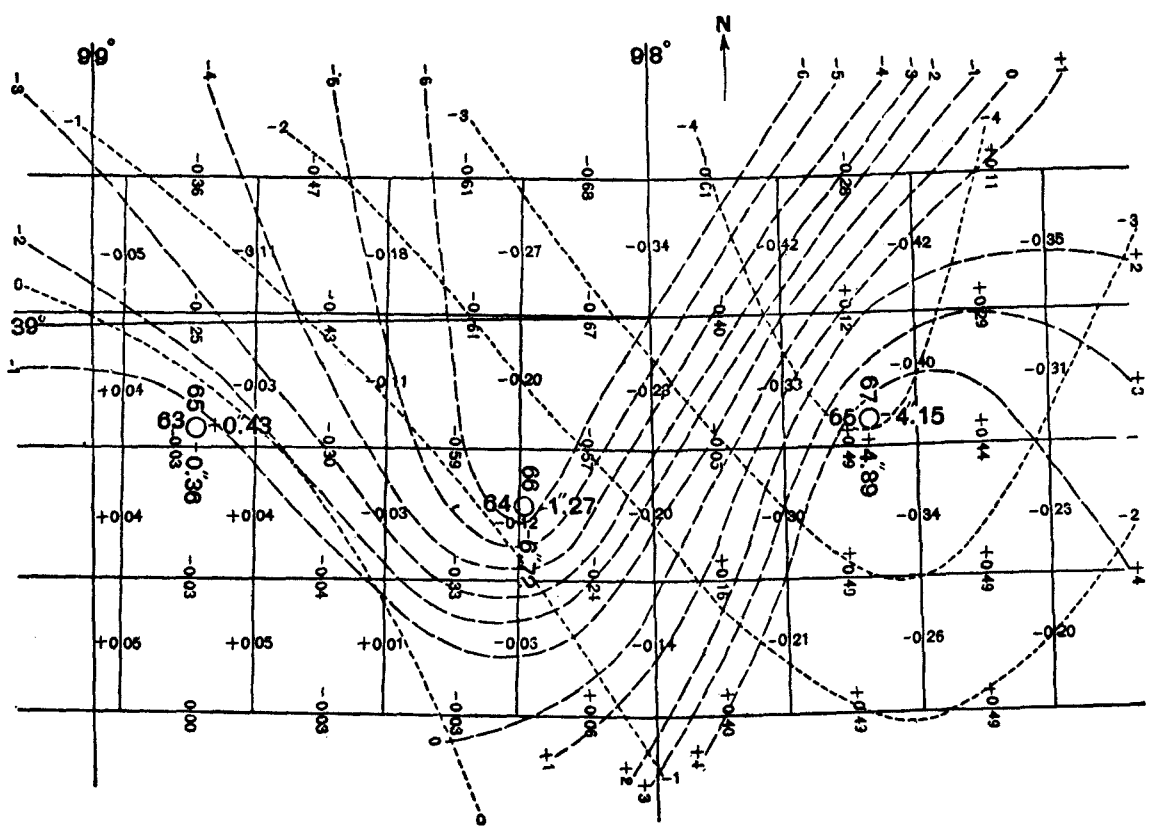
Illustrations Nos. 5, 6, and 7 show the details of the construction of the contour lines of the geoid for a small area in Kansas in the vicinity of latitude stations Russell Southeast Base, Ellsworth, and Salina West Base (Nos. 63, 64, and 65).

Illustration No. 5 shows steps 1–5, inclusive, of the process of constructing the contour lines. Two meridians (98° and 99°) are shown and one parallel (39°). The astronomic stations, at each of which both the meridian and the prime vertical components of the deflection were observed, are indicated by small circles. The serial numbers of the latitude stations (63, 64, and 65) are shown at the left of the respective circles. The observed meridian component of the deflection is shown at the right of each circle. Similarly, if the illustration is held so that the west is at the bottom, the serial number of each longitude or azimuth station shows at the left of each circle and the observed prime vertical component of each deflection of the vertical shows at the right.* The lines of equal deflection in the meridian and of equal deflection in

* The observed deflections are taken directly from pages 12–19 or 48–56, with the exception that a correction has been applied to the prime vertical component of the deflection at azimuth station No. 65 to take account of an accumulated error in the geodetic azimuth. This correction will be explained later.


99°
98°
N
39°
39°
65
63 +0.43
-0.36
66
64 -1.27
-6.12
67
65 -4.15
+4.89
ASTRONOMIC STATIONS
LINES OF EQUAL DEFLECTION IN THE MERIDIAN
LINES OF EQUAL DEFLECTION IN THE PRIME VERTICAL

No. 5.

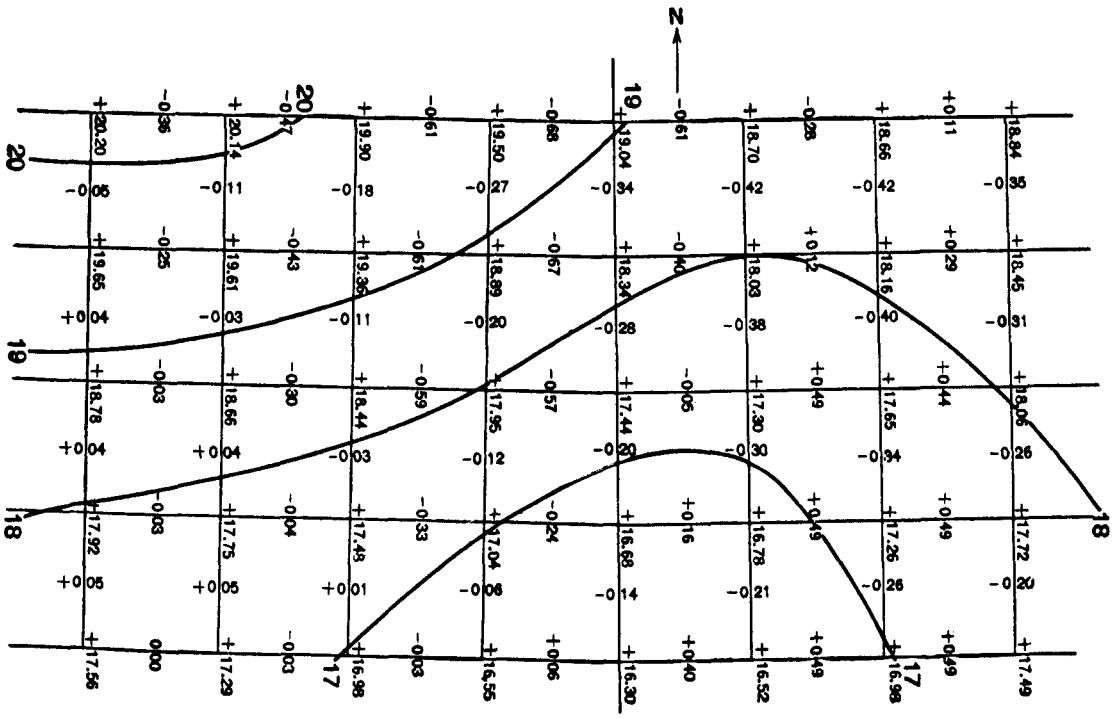

99°
98°
N
39°
65
63 +0.43
-0.36
66
64 -1.27
-6.12
67
65 -4.15
+4.89

No. 6.

the prime vertical, drawn as indicated in the description of steps 3 and 5, are shown. The shapes of these lines depend in part upon information given by adjacent stations which are beyond the limits of this illustration.

Illustration No. 6 contains all that is on illustration No. 5, and also shows the rectangular system of lines corresponding to step 6 and the figures corresponding to step 7. It will be noted that some of the areas defined by the rectangular system of lines are not perfect squares, and that some of the sides of these areas do not lie exactly in the meridian or the prime vertical. These approximations, referred to in the description of step 6, are necessary on account of the fact that the earth's surface is spheroidal, not plane.

Illustration No. 7 is like illustration No. 6 in showing steps 6 and 7, but, for the sake of clearness, steps 1–5 have been omitted. The row of elevations at the eastern edge of the



No. 7.

illustration, shown by the figures +18.84, +18.45, +18.06, +17.72, +17.49, at the corners of the squares, were fixed from the eastward by constructions which are beyond the eastern limit of this illustration. The numerical work of fixing the next row of elevations, namely, +18.66, +18.16, +17.65, +17.26, and +16.98, is as follows:

18.84	18.45	18.06	17.72	17.49	Row already fixed.
18.73	18.16	17.62	17.23	17.00	From eastward.
18.58	18.02	17.57	17.26	.....	From southeastward.
.....	18.31	17.76	17.28	16.97	From northeastward.
18.66	18.16	17.65	17.26	16.98	New fixed row.

In the computation the second, third, and fourth rows of figures marked "From eastward," "From southeastward," and "From northeastward" were obtained from the first row in turn, as indicated in the description of step 8. A plus sign on the difference of elevation written on the middle of any 20.6-kilometer line of the rectangular system which lies in the east and west direction means that the eastern end of the line is higher than the western end. Similarly, a plus sign on the middle of any 20.6-kilometer line in the direction of the meridian indicates a slope upward to the south. The elevation + 18.73 in the row marked "From eastward"

was obtained by subtracting + 0.11 (see illustration No. 7) from + 18.84. The elevation + 18.58 in the row marked "From southeastward" was obtained by subtracting + 0.29 and − 0.42 (see illustration No. 7) from + 18.45. The elevation + 18.31, shown in the row marked "From northeastward," was obtained by subtracting + 0.11 from + 18.84 and then adding − 0.42. (See illustration 7.) Each of the elevations of the new fixed row was obtained, as indicated in the description of step 8, by taking the mean of the two or three values just above it; that is, 18.66 is the mean of 18.73 and 18.58, and 18.16 is the mean of 18.16, 18.02, and 18.31, and so on.

All elevations being fixed in the manner indicated, row by row, proceeding from east to west, the contour lines of the geoid were then drawn as indicated in the description of step 9. In illustration 7 portions of contours 17, 18, 19, and 20 meters are shown. On the 18-meter contour ten points were fixed directly by interpolation at the ten points within the limits of the illustration at which this contour crosses the lines of the rectangular system.

As the purpose of the general process just described is to detect, if possible, whether any relation exists between that portion of the geoid which underlies the United States, on the one hand, and the topography of the United States, on the other hand, it is important to note that each of the nine steps of the process is independent of any consideration of the topography and may be taken without any information whatever in regard to the topography. In the process there are few and unimportant opportunities for bias or prejudgment of the draftsman to affect the location of the contours. It is almost entirely an automatic process. The drawing was not compared with the topography until it was complete.

The process insures that the surface represented shall be continuous, and shall have no sudden changes of slope, even though the deflections in the direction of the meridian and the prime vertical indicated by the observations are apparently inconsistent with the assumption of a continuous surface. In other words, the method of construction is in itself a method of adjustment of the discrepancies in observations. Because it is an adjustment process the slopes indicated by the constructed contour lines of the geoid do not agree exactly with the observed slopes (that is, observed deflections of the vertical) at the observation stations, the disagreement being greater or less according to the degree of inconsistency of the observations with each other in a given locality and with the requirement that the geoid surface shall be continuous. The degree of inconsistency is indicated mainly by the discrepancies between the two or three derived values of which the mean is taken for each elevation in such a computation as that shown on page 61.

It is not claimed that the method of construction of the contour lines of the geoid is even approximately perfect. It has grave defects. For example, for points on the margin of the belt covered by the construction there are but one or two determinations, in general, whereas for interior points there are three determinations. Hence the outer portions of the geoid contours frequently show a tendency to sharp curvature which is fictitious, being due to a defect in method, not to the facts of nature. So, too, it will be found that the results secured will be somewhat different if the construction of the contours proceeds from west to east along a given belt instead of from east to west. On account of these and other defects in the process, together with the fact that the problem of constructing a surface, having given a few observed slopes only at widely scattered points, is essentially indeterminate,* the surface represented by the geoid contours, as constructed, is only a rough approximation to the actual geoid surface. It is, however, the best approximation available. Because it is an approximation constructed by methods independent of the known topography it is believed to be valuable for the purpose for which it is to be used—namely, to study the relation between the geoid and the topography.

* The problem is essentially indeterminate, because there are only a finite number of observed values of the slopes, whereas the geoid, an irregular surface, has an infinite number of unknown slopes at its infinite number of points within the United States, which slopes must conform to only a few known conditions. But the process of constructing the geoid contours which has been used gives one solution which is believed to be reasonable because it insures continuity of the surface, insures continuity in the change of slope from point to point, and insures that the rate of change of slope is not much more rapid in any case than the minimum rate of change consistent with the observed slopes.

The contour lines of the geoid, as resulting from this construction based on 496 observed deflections in the United States, are shown on illustration No. 17 at the end of this publication. The contour lines of the geoid are shown in red. Certain selected contour lines of the land surface are shown in black on the same illustration, so that a comparison of the two kinds of contours may readily be made in order to detect what relation, if any, exists between the shape of the geoid and the topography.

The observed deflections used in constructing the contour lines of the geoid are shown on pages 12–19 and again on pages 48–56. The prime vertical components of the deflections, as determined at stations west of Meades Ranch, Kansas, were, however, first corrected in each case by subtracting from 6″.16 to 7″.52, to take account of the known accumulated error in geodetic azimuths, as will be explained later.

In constructing the contour lines of the geoid the initial point at the extreme northeastern part of the United States, near Calais, Maine, was arbitrarily taken as +10 meters in order to avoid negative contours.

COMPARISON OF GEOID AND TOPOGRAPHY.

A comparison shows the following relations between the shape of the geoid and the topography:

1. The 10-meter geoid contour is approximately parallel to the Atlantic shore line from the extreme north in Maine to latitude 37°.

2. The effect of the Adirondack Mountains upon the geoid is clearly indicated by the 20½-meter oval among the geoid contours. This oval is somewhat too far west to correspond to the summit of the Adirondacks, but it should be remarked that but few astronomic observations were taken in this locality.

3. The effect of Lake Erie upon the geoid contours is clearly shown and the position of Lake Ontario is also indicated by the geoid contours.

4. The lowest point on the geoid occurs in the eastern portion of Lake Superior. The lowest point of the bottom of Lake Superior is in its eastern portion and is more than 100 meters below sea level.

5. The divide between the Ohio River and the Great Lakes in Illinois is roughly indicated by a closed 16-meter contour line of the geoid.

6. The highest point on the geoid along the transcontinental triangulation east of the Mississippi River (17+meters) occurs in latitude 37½° and longitude 81½° in the Alleghenies, not far from the position in which it should be found if the contour lines of the geoid were controlled entirely by the general features of the topography.

7. After omitting the exceptional region indicated in the following paragraph, the highest point on the geoid east of the Mississippi River is in latitude 35° and longitude 84°, at the southwestern end of the highest portion of the Alleghenies. It is indicated by the contour 20½ meters.

8. South of latitude 33° and west of longitude 86° the contour lines on the geoid show a steep slope upward toward the southeast, to which there is nothing corresponding in the topography. This constitutes a most interesting exception to the general rule that the contours of the geoid show a relation to the general features of the topography. No adequate explanation has yet been found for this exception.

9. West of the Mississippi River, in longitude 91° to 93° along the thirty-ninth parallel, the upward slope of the geoid to the southward and the long curves of contours 19, 20, 21, and 22 meters correspond to the fact that there is a large region, comprising one-quarter of the State of Missouri and a part of Arkansas, lying close to the belt of triangulation on the southward side and having an elevation of more than 1 000 feet, whereas to the northward the elevations are less.

10. From longitude 98° in central Kansas, nearly to longitude 107° in central Colorado, the slope of the geoid is continuously upward to the westward. This statement is also true of the topography. The upward slope of the geoid to the westward gradually increases from a very

gentle slope near longitude 98° to a maximum in longitude 105°, near Colorado Springs and Pikes Peak, in Colorado, and then decreases gradually to zero (at a summit) in longitude 106½°, northeast of Gunnison, Colorado. The slope of the land surface in central Kansas is upward to the westward, but very small. It increases steadily as one proceeds westward through western Kansas and eastern Colorado. Between Colorado Springs and Pikes Peak, in a few miles, there is a comparatively sudden rise from elevations of 5 000 to 6 000 feet on the plains to more than 10 000 feet on the mountain tops. Of this group of mountains reaching above the 10 000-foot contour and forming the greatest mountain mass in the United States the center is approximately in the same location as the summit on the geoid indicated by the 39½-meter contour. Thus from longitude 98° to longitude 106½° there is a close relation between the geoid and the topography.

11. The valley on the geoid surface in longitude 108° to 112°, indicated by the 34, 35, 36, and 37 meter contours, coincides in position with the valley of the Colorado and Grand rivers, the most important general feature of the topography in this region.

12. The center in latitude 40° to 41° and longitude 110° to 111°, around which contours 40 to 44 meters show a regular curvature, coincides with the summit of the great Uintah group of mountains reaching above the 10 000-foot contour over a large area. The 44-meter contour on the geoid, which indicates the highest part of the geoid within the area covered by this investigation, falls upon one of the greatest of the mountain masses in the United States.

13. The nearly closed oval formed by the 30-meter contour of the geoid, in latitude 39°, longitude 114°, corresponds to the southern portion of the great depression of which the Great Salt Lake occupies the lowest part. The depression on the geoid is less definite than on the land surface and is somewhat to the southwestward of the depression on the land surface.

14. Along the Pacific coast line, from Point Arena in latitude 39° southward to latitude 33½°, the geoid contours vary from 20 meters to 24 meters only. There is a general tendency for the contours at the coast to be parallel to the coast.

15. The valley on the geoid in latitude 34° and longitude 118° to 120° corresponds to the fact that there are high islands (San Miguel, Santa Cruz, Santa Catalina, and others) 50 kilometers or more from the coast in this region and that there is, therefore, virtually a valley between these islands and the coast line.

16. From longitude 118° to longitude 122° and between latitudes 38° and 40° the geoid has a decided upward slope to the northward of which no counterpart exists in the topography. This constitutes a second important exception to the general rule that the contours of the geoid show a relation to the general features of the topography.

Certain local features of the geoid contours have no apparent counterpart in the topographic contours. This is to be expected for three reasons. First, the geoid contours being based on observations at a few scattered points only may be expected to be true to the facts in their general features only, not in their local features. Second, in some cases a local feature of topography may have had considerable influence in producing local deflection at a given station and this, in turn, may have affected the geoid contours, as constructed, over a considerable area, and the local features of the topography may thus have been exaggerated into a general feature of the geoid. Third, the other acknowledged defects in the method of construction of the geoid contours have probably produced some of the marked irregularities in the geoid contours which appear as local features. Among the local features of the geoid contours which are not believed to be of much significance, for the reasons stated, are the following:

(*a*) The break in the 10-meter contour in latitude 40° and longitude 75° and the sharp bend to the westward of the broken ends;

(*b*) The sharp curvature in the 17, 18, and 19 meter contours in longitude 96° to 98°;

(*c*) The sharp curvature in the 28-meter contour in the vicinity of longitude 117°;

(*d*) The sharp curvature of the 24-meter contour in the immediate vicinity of San Francisco, California, and the presence of the 22½-meter contour in latitude 37½°, longitude 122°, breaking the 23-meter contour. These are both believed to be due to an effect of earthquakes in disturbing geodetic azimuths which was not recognized until after this computation was made;

(*e*) The closed 23-meter contour in latitude 35°, longitude 120°.

The average geoid contour at the Atlantic coast line from Maine to Virginia is about 9 meters. The average geoid contour at the Pacific coast line, from latitude 39° at Point Arena, to latitude 32½° at San Diego, is about 23 meters, or about 14 meters higher than at the Atlantic coast line. This difference of 14 meters is believed, however, to be of little significance. It may have been due largely to errors in the prime vertical deflections used in constructing the geoid contours and to the acknowledged defects in the method of constructing the geoid contours. An average change of +0″.7 in the prime vertical components of the deflections would cause the 14-meter difference to disappear. Moreover, it is not necessarily true that the geoid must bear the same relation to the spheroid along the two coast lines.

A study in detail of the geoid contours as shown on illustration No. 17 shows conclusively that, though the irregularities in the geoid are much too small to correspond to the computed topographic deflections, yet the geoid is not independent of the topography. The general features of the topography which cover large areas are indicated by the geoid contours. On the geoid the greatest elevations correspond approximately in position to the greatest mountain masses. Depressions and valleys in the geoid correspond to the greater depressions and valleys in the land surfaces. The steepest slopes of the geoid tend to correspond in position to the steepest general slopes of the land surface. A contour of the geoid tends to follow each coast line.

The smaller features of the topography are not shown in the geoid contours. It is possible that, if a larger number of observed deflections were available and were used in constructing the geoid contours, smaller features of the topography might show an effect on the geoid contours.

The contradictions between the geoid and the topography are few in comparison with the agreements. That is, the directions of slopes on the geoid agree generally with the general slopes of the topography. There are only two important contradictions, to both of which especial attention has been called.

ISOSTASY MUST BE CONSIDERED.

The logical conclusion from the study of the geoid contours for the United States, taken in connection with the fact already noted that the computed topographic deflections are much larger than the observed deflections of the vertical, is that some influence must be in operation which produces an incomplete counterbalancing of the deflections produced by the topography, leaving much smaller deflections in the same direction.

There is abundant evidence in the literature of geodesy indicating that this relation of observed deflections of the vertical to the topography is not peculiar to the United States; that, in fact, it exists elsewhere.

Any computation, even though quite rough, of the deflections of the vertical which must be produced by the masses constituting the continents considered as excesses of mass, and of the oceans considered as representing defects of mass, shows that said computed deflections are much greater than those which have been observed.

Several such computations treating certain continents as approximations to geometric figures have been made.* These computations indicate that deflections of the vertical greater than 30″ should be common. The observers do not find them to be so.

On the other hand, whenever the directions and magnitudes of observed deflections are carefully studied it becomes evident that, as a rule, the directions of the deflections and their relative magnitudes evidently bear some relation to the topography surrounding the stations. The deflections, as a rule, are in the directions which correspond roughly to those which would be expected on the supposition that they are produced by the topography. Similarly, as to relative magnitudes, in the areas of low relief and slight slope the observed deflections are, in

* For example, see Höhere Geodäsie, F. R. Helmert, Part II, commencing on page 313, and Bulletin 48, U. S. Geological Survey, On the Form and Position of Sea Level, R. S. Woodward, pp. 80–85.

general, small, and the areas of high relief and steep slopes and their immediate vicinity are characterized by larger observed deflections. These are the general facts in regard to observed deflections. Many individual exceptions may be cited, but the general statement as made is true to such an extent as to be a strong indication of the direction in which one must look for some general law connecting the observed facts.

Deflections of the vertical must be due to irregularities in the distribution of the masses composing the earth. Such irregularities may occur either as a result of irregularities in the surface of the earth (topography) or as a result of irregularities in the distribution of the densities beneath the surface. The deflections can be produced in no other way.

The irregularities in the surface of the earth (the topography) are visible on land and are detected by soundings at sea. In either case they are known. In this investigation the deflections which must be produced by these known irregularities have been computed, namely, the topographic deflections. The distribution of the density below the surface of the earth is invisible and unknown. Both the general approximate studies for the whole world of the necessary effects of the known topography in producing deflections of the vertical, and the detailed exact study already made for the United States alone, by means of computed topographic deflections and geoid contours, indicate that one must look to the distribution of the subsurface densities for an explanation of the discrepancies between observed deflections of the vertical and the deflections which must inevitably be produced by the topography. Moreover, from the general considerations set forth in the preceding paragraphs, it seems that there must be some general law of distribution of subsurface densities which fixes a relation between subsurface densities and the surface elevations such as to bring about an incomplete balancing of deflections produced by topography on the one hand against deflections produced by variation in subsurface densities on the other hand.

The theory of isostasy postulates precisely such a relation between subsurface densities and surface elevations.

These are, briefly, the considerations which led to the determination to investigate thoroughly the possible relations between the theory of isostasy and deflections of the vertical in connection with the present investigation.

ISOSTASY DEFINED.

If the earth were composed of homogeneous material, its figure of equilibrium, under the influence of gravity and its own rotation, would be an ellipsoid of revolution.

The earth is composed of heterogeneous material which varies considerably in density. If this heterogeneous material were so arranged that its density at any point depended simply upon the depth of that point below the surface, or, more accurately, if all the material lying at each equipotential surface (rotation considered) was of one density, a state of equilibrium would exist and there would be no tendency toward a rearrangement of masses.

If the heterogeneous material composing the earth were not arranged in this manner at the outset, the stresses produced by gravity would tend to bring about such an arrangement; but as the material is not a perfect fluid, as it possesses considerable viscosity, at least near the surface, the rearrangement will be imperfect. In the partial rearrangement some stresses will still remain, different portions of the same horizontal stratum may have somewhat different densities, and the actual surface of the earth will be a slight departure from the ellipsoid of revolution in the sense that above each region of deficient density there will be a bulge or bump on the ellipsoid, and above each region of excessive density there will be a hollow, relatively speaking. The bumps on this supposed earth will be the mountains, the plateaus, the continents; and the hollows will be the oceans. The excess of material represented by that portion of the continent which is above sea level will be compensated for by a defect of density in the underlying material. The continents will be floated, so to speak, because they are composed of relatively light material; and, similarly, the floor of the ocean will, on this supposed earth, be depressed because it is composed of unusually dense material. This particular condition of approximate equilibrium has been given the name isostasy.

The adjustment of the material toward this condition, which is produced in nature by the stresses due to gravity, may be called the isostatic adjustment.

The compensation of the excess of matter at the surface (continents) by the defect of density below, and of surface defect of matter (oceans) by excess of density below, may be called the isostatic compensation.

Let the depth within which the isostatic compensation is complete be called the depth of compensation. At and below this depth the condition as to stress of any element of mass is isostatic; that is, any element of mass is subject to equal pressures from all directions as if it were a portion of a perfect fluid. Above this depth, on the other hand, each element of mass is subject in general to different pressures in different directions—to stresses which tend to distort it and to move it.

In terms of masses, densities, and volumes, the conditions above the depth of compensation may be expressed as follows: The mass in any prismatic column which has for its base a unit area of the horizontal surface which lies at the depth of compensation, for its edges vertical lines (lines of gravity) and for its upper limit the actual irregular surface of the earth (or the sea surface if the area in question is beneath the ocean) is the same as the mass in any other similar prismatic column having any other unit area of the same surface for its base.* To make the illustration concrete, if the depth of compensation is 114 kilometers below sea level, any column extending down to this depth below sea evel and having 1 square kilometer for its base has the same mass as any other such column. One such column, located under a mountainous region, may be 3 kilometers longer than another located under the seacoast. On the other hand, the solid portion of such a column under one of the deep parts of the ocean may be 5 kilometers shorter than the column at the coast. Yet, if isostatic compensation is complete at the depth 114 kilometers, all three of these columns have the same mass. The water above the suboceanic column is understood to be included in this mass. The masses being equal and the lengths of the columns different, it follows that the mean density of the column beneath the mountainous region is three parts in 114 less than the mean density of the column under the seacoast. So, also, the mean density of the solid portion of the suboceanic column must be greater than the mean density of the seacoast column, the excess being somewhat less than five parts in 114 on account of the sea water being virtually a part of the column.

This relation of the masses in various columns, and consequently of the densities, follows from the requirement of the definition of the expression "depth of compensation" that, at that depth, each element of mass is subject to equal pressures from all directions. In order that this may be true the vertical pressures, due to gravity, on the various units of area at that depth must be the same.

If this condition of equal pressures, that is of equal superimposed masses, is fully satisfied at a given depth the compensation is said to be complete at that depth. If there is a variation from equality of superimposed masses the differences may be taken as a measure of the degree of incompleteness of the compensation.

In the above definitions it has been tacitly assumed that g, the intensity of gravity, is everywhere the same at a given depth. Equal superincumbent masses would produce equal pressures only in case the intensity of gravity is the same in the two cases. The intensity of gravity varies with change of latitude and is subject also to anomalous variations which are to some extent associated with the relation to continents and oceanic areas. But even the extreme variations in the intensities of gravity are small in comparison with the variations in density postulated. The extreme variation of the intensity of gravity at sea level on each side of its mean value is only one part in 400. Even this small range of variation does not occur except between points which are many thousands of kilometers apart. As will be shown later, the postulated variations in mean densities are about one part in 30 on each side of an average value. Hence, it is not advisable to complicate the conception of isostasy and intro-

*It would be more accurate to use the words "inverted truncated pyramid" instead of "prismatic column." The latter expression has been selected because it is sufficiently exact for the purpose and corresponds to the allowable approximations actually made in the mathematical part of the investigation.

duce long circumlocutions into its definition in order to introduce the refinement of considering the variations in the intensity of gravity.

The variation of the intensity of gravity with change of depth below the surface need not be considered, as its effect in the various columns of material considered will be substantially the same.

The idea implied in this definition of the phrase "depth of compensation," that the isostatic compensation is complete within some depth much less than the radius of the earth, is not ordinarily expressed in the literature of the subject, but it is an idea which it is difficult to avoid if the subject is studied carefully from any point of view. It is proposed, therefore, in this investigation to assume that the depth of compensation is much less than the radius and to treat it as an unknown to be determined.

In this investigation the attempt is made to ascertain with as much precision as possible the extent to which the condition called isostasy exists, and the manner in which the isostatic compensation is distributed. This attempt is made primarily because it appears that, if successful, it would lead to considerable increase in the accuracy with which the size and figure of the earth may be derived from geodetic observations. It has been kept in mind that for entirely different reasons it is also important to establish thoroughly, or to disprove, the theory that the condition called isostasy exists. There are many other theories of geology and geophysics which are so intimately related to this theory that they must stand, or fall, or be greatly modified, according to its fate.

In passing, it is interesting to note that, though the beginning of the ideas involving the theory of isostasy are found in primitive form at least as long ago as the discussions by Pratt and Airy in connection with deflections of the vertical apparently produced by the Himalayas, the ideas were first presented in such a clear and forceful manner as to attract general attention in an address in 1889 before the Philosophical Society* of Washington by Maj. C. E. Dutton. Since this address was printed the theory has had its present definite name, and its validity, as well as its relation to various geological problems, has been vigorously discussed.

COMPUTATIONS OF DEFLECTIONS, ISOSTATIC COMPENSATION CONSIDERED.

In the principal investigation it has been assumed that the isostatic compensation is complete and uniformly distributed with respect to depth from the surface down to an unknown depth of compensation which is to be determined from the observations.

Let h_1 be an assumed depth of compensation. Various values for h_1 are assumed in the investigation and the most probable value derived from the comparison of the computations made on different assumptions. Let δ_1 be the compensating defect of the density. Then the assumption stated in the preceding paragraph may be expressed mathematically for any particular compartment used in the computation of topographic deflections by the equation $\delta h = -\delta_1 h_1$. The symbols δ and h have the same significance as in connection with the computation of topographic deflections. δ is the mean surface density of the earth; that is, the mean density for the first few miles below the surface, and h is the mean elevation above mean sea level of the surface of the earth within the compartment. The area of the compartment times δh is evidently the total mass in the compartment above sea level. The area of the compartment times $\delta_1 h_1$ is the compensating defect of mass assumed to lie below the compartment.

In the equation $\delta h = -\delta_1 h_1$, expressing complete and uniformly distributed compensation extending to depth h_1, δ is a constant for all compartments and is assumed to be 2.67. The depth of compensation h_1 is assumed to be the same for all compartments. Hence, the equation shown above may be written

$$\frac{\delta}{h_1} = (\text{a constant}) = -\frac{\delta_1}{h}$$

* On some of the greater problems of physical geology, C. E. Dutton, Bulletin of the Philosophical Society of Washington, Vol. XI, pp. 51-64.

expressing the fact that, under the adopted assumption, the defect of density below a compartment is directly proportional to the mean elevation of the surface of the compartment above sea level. For a compartment at the sea coast, for which the mean elevation is zero, the compensating defect of density (δ_1) is also zero. For an oceanic compartment in which h is negative, δ_1 is also negative. For such a compartment δ_1 is a compensating excess of density instead of a compensating defect of density. For an oceanic compartment h is not the negative elevation of the bottom; that is, the depth, but is, instead, .615 of the depth. This modification is necessary to take account of the mass of sea water, as explained on page 27. The compensating defect of density is, according to the adopted assumption, a maximum under the high mountains; is zero under the seashore, and is a maximum compensating excess of density under the deepest parts of the ocean.

If greater refinement in statement is required, it is necessary to state that the compensation is assumed to extend from the actual surface of the land and from the bottom of the sea down to the depth of compensation, which is assumed to be at the same distance below sea level in each case.

It is assumed that below the depth of compensation no excess or defect of density exists or, in other words, that below that depth the density is simply a function of the depth below sea level and has no relation to surface conditions.

The assumption that the isostatic compensation is uniformly distributed through a depth which is everywhere the same was adopted in the main investigation from among various reasonable assumptions for two reasons. As far as the writer can determine, this assumption lends itself most readily to computation, that is, gives rise to simple computations which may be most quickly made. Moreover, it seems to the writer to be the most probable one of the simple assumptions. Certain more complicated and less easily stated assumptions may be slightly more probable. A discussion of certain of the many possible assumptions will be found later in this paper, among various other discussions of subsidiary considerations.

The deflection due to the defect or excess of mass beneath the surface which constitutes the isostatic compensation may be computed by the same formula that was used for computing the topographic deflection. It will, however, be necessary in this case to write the formula in its more exact form, as shown on page 34, in which appears the difference of elevation between the station and the surface of the mass considered.

The formula, as adapted to this case, is:

$$D_c = 12''.44 \frac{\delta_1}{\Delta} h_1 (\sin a' - \sin a_1) \log_\epsilon \frac{r' + \sqrt{(r')^2 + h_1^2}}{r_1 + \sqrt{r_1^2 + h_1^2}}$$

D_c is one component of the deflection at the station produced by the compensating defect of mass comprised within a stratum h_1 statute miles thick, lying within a compartment limited as before. The symbols common to this formula and those on pages 20 and 34, Δ, a', a_1, r', and r_1, have identical meanings.

h_1 is the depth of compensation. The fact that this depth is large in comparison with many of the values of r' and r_1 makes it necessary as above to introduce the radicals involving h_1^2 in the formula.*

The derivation for the above formula is precisely the same as for the formula on page 34.

Under the adopted assumption as to compensation it has already been shown that $\delta h = -\delta_1 h_1$. Making this substitution in the formula for D_c it becomes

$$D_c = -12''.44 \frac{\delta}{\Delta} h (\sin a' - \sin a_1) \log_\epsilon \frac{r' + \sqrt{(r')^2 + h_1^2}}{r_1 + \sqrt{r_1^2 + h_1^2}}$$

This is the same as the approximate formula for D given on page 34 except in the last factor (the logarithmic factor) and in having a minus sign.

* See Clarke's Geodesy, p. 295.

Hence
$$\frac{-D_c}{D}=\frac{\log\dfrac{r'+\sqrt{(r')^2+h_1^2}}{r_1+\sqrt{r_1^2+h_1^2}}}{\log\dfrac{r'}{r_1}}$$

It is not necessary in this formula to specify that the logarithms refer to the base ϵ, for the ratio of the logarithms is the same whatever the base.

Let F be the factor by which D, the topographic deflection, must be multiplied to secure the resultant deflection $D+D_c$ due to both the topography and the compensating defect or excess of mass below the surface.

Then
$$F=\frac{D+D_c}{D}=1-\frac{\log\dfrac{r'+\sqrt{(r')^2+h_1^2}}{r_1+\sqrt{r_1^2+h_1^2}}}{\log\dfrac{r'}{r_1}}$$

If this formula be restricted in application to the particular case in hand in which the compartments have been so selected that the ratio $\frac{r'}{r_1}$ is 1.426 (and its logarithm 0.1541),* the formula becomes

$$F=1-\frac{\log\dfrac{r'+\sqrt{(r')^2+h_1^2}}{r_1+\sqrt{r_1^2+h_1^2}}}{0.1541}$$

The factors F have been computed from this formula for the rings used in computing the topographic deflections and for various assumed depths of compensation (h_1).

Reduction factors, F, corresponding to various depths of compensation in kilometers.

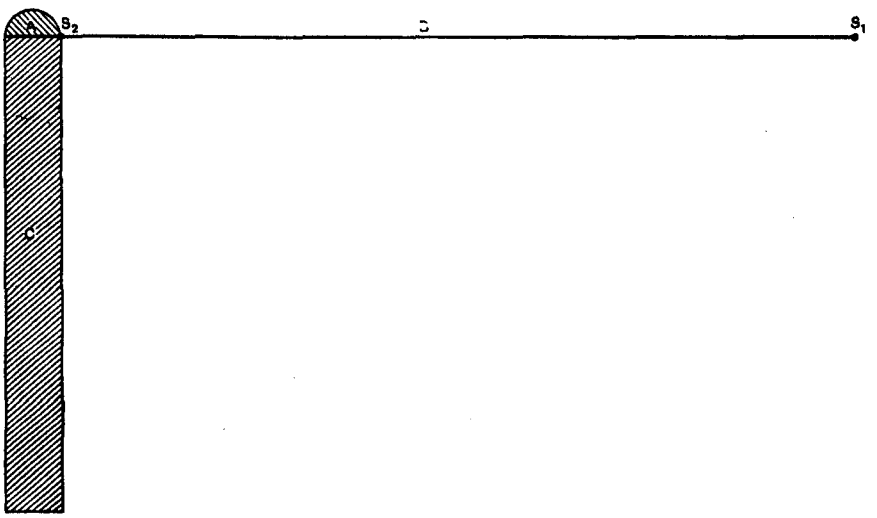
[Depths in kilometers.]

Ring.	329.8	231.3	162.2	120.9	113.7	79.76	55.92	96.0
29	.....	.....	.....	.....	.....	.....	.997	.998
28	.....	.....	.....	.....	.....	.997	.996	.998
27	.....	.....	.....	.997	.997	.996	.995	.996
26	.....	.....	.997	.996	.996	.995	.992	.995
25	.....	.997	.996	.995	.995	.992	.988	.993
24	.997	.996	.995	.993	.992	.988	.983	.990
23	.996	.995	.992	.989	.988	.983	.976	.986
22	.995	.992	.988	.984	.983	.976	.965	.980
21	.992	.988	.983	.979	.976	.965	.951	.971
20	.988	.983	.976	.967	.965	.951	.930	.959
19	.983	.976	.965	.954	.951	.930	.900	.942
18	.976	.965	.951	.935	.930	.900	.859	.917
17	.965	.951	.930	.906	.900	.859	.801	.882
16	.951	.930	.900	.867	.859	.801	.721	.833
15	.930	.900	.859	.813	.801	.721	.618	.766
14	.900	.859	.801	.736	.721	.618	.493	.675
13	.859	.801	.721	.638	.618	.493	.358	.561
12	.801	.721	.618	.517	.493	.358	.234	.428
11	.721	.618	.493	.382	.358	.234	.139	.297
10	.618	.493	.358	.253	.234	.139	.077	.183
9	.493	.358	.234	.153	.139	.077	.040	.106
8	.358	.234	.139	.086	.077	.040	.020	.056
7	.234	.139	.077	.045	.040	.020	.010	.028
6	.139	.077	.040	.022	.020	.010	.005	.014
5	.077	.040	.020	.011	.010	.005	.003	.007
4	.040	.020	.010	.006	.005	.003	.001	.003
3	.020	.010	.005	.003	.003	.001	.001	.002
2	.010	.005	.003	.001	.001	.001	.000	.001
1	.005	.003	.001	.001	.001	.000	.000	.000

* See page 21.

The factor, F, is evidently the same for a whole ring, since in the derivation of the formula for F for a given compartment the factor ($\sin a' - \sin a_1$) has disappeared. The reason for this may be perceived in another way. The mass above sea level in any compartment and the compensating defect of mass are equal. The latter has an effect in producing a deflection at the station which is slightly smaller than the former, simply because much of the compensating defect of mass lies far below the horizon of the station. The angles of depression below the horizon of the station to different parts of the compensating defect of mass being the same for all compartments in the same ring, evidently F, depending implicitly as it does upon the depression angles, should be the same.

The following simple case serves to illustrate the meaning of the factors: In illustration No. 8 let the area A, shaded by lines running downward to the right, be the vertical section of a mountain standing on a plain of indefinite extent, B, which is practically at sea level. Suppose the depth of compensation to be 113.7 kilometers. Then, according to the adopted assumption, within the space 113.7 kilometers deep under the mountain, which is represented in vertical section by the area marked C, shown by lines running downward to the left, the density at every level is less than the density at the same level under the plain by a constant amount δ_1, such that



No. 8.

the whole defect of mass in C, as compared with a corresponding space under B, is exactly equal to the mass of the mountain, A, above sea level.

If the mountain is at a great distance, 3 400 kilometers, for example (in ring 1), from the station for which the deflection of the vertical is being computed, the positive mass A and the negative mass C are in nearly the same direction and at nearly the same distance from the stations and, therefore, their effects in producing deflections will be nearly of the same size and, as they are of opposite signs, will nearly counterbalance each other. The effect of C will necessarily be slightly less than that of A. The factor, F, in the last line of the table, in the column headed 113.7 kilometers, expresses the fact that, for this case, the negative deflection produced by C is 0.999 of the positive deflection produced by A, and that, therefore, the resulting deflection is 0.001 of that which would be produced by A alone.

If the station is at a distance of 200 kilometers from the mountain, as at S_1 (the mountain being, therefore, in ring 9), the angles of depression in various parts of the negative mass C, especially the lower parts, will be sufficiently great to reduce considerably the effects of these parts in producing deflections at the station, whereas all parts of the mass, A, are still practically in the horizon of the station. Moreover, the lower portions of C are at a considerably greater distance from the station than are any parts of A. For both these reasons the negative deflection produced by C is only 0.861 as great as the positive deflection produced by A, and the

resultant deflections is 0.139 of that produced by A alone. (See factor for ring 9 in the column headed 113.7 in the table.)

If the station is at the foot of the mountain at a horizontal distance of 7 kilometers from its summit, as at S_2, some portions of the mountain may be at a sufficient angle of elevation from the station to require a slope correction. (See p. 34.) But the mass A will have nearly full effect in producing the deflection of the vertical at the station. On the other hand, much of the negative mass C is at such a large angle of depression from S_2 that it is under S_2 rather than in its horizon. These portions of C have little effect in producing a deflection of the vertical at the station S_2. Their principal effect at S_2 is to decrease the intensity of gravity rather than to change the direction of gravity. For this case, the table of reduction factors shows that if the attention be limited to the compartment of ring 18 (outer radius 9.5 kilometers, inner radius 6.7 kilometers), which includes the summit, the resultant deflection is 0.930 of that produced by A alone, or, in other words, the negative deflection produced by C is only 0.070 of the positive deflection produced by A. For portions of the mountain still nearer to the station the ratio of negative effect of C to the positive effect of A is still smaller and the reduction factor larger.

The table of reduction factors serves to enable one, having made an assumption as to the depth of compensation corresponding to the heading of any one column in the table, to derive from the computed topographic deflections, due directly to the topography, such as those shown in the examples on pages 26, 29, 31, 32, and 33, the resultant computed deflections due to both topography and the assumed isostatic compensation. Each "horizontal sum," in the examples cited, is the deflection for one ring. Each is multiplied by its appropriate factor, F, from the table, and the new sum taken, as illustrated below, for the assumed depth of compensation 113.7. The following example is for azimuth station No. 115, North End Knott Island:

Ring.	Topographic deflection. (See p. 31.)	Reduction factor, F. (See p. 70.)	Computed deflection, uniform isostatic compensation considered, extending to depth 113.7 kilometers.
	"		"
25	.00	.995	.00
24	.00	.992	.00
23	.00	.988	.00
22	.00	.983	.00
21	.00	.976	.00
20	+ .01	.965	+ .01
19	.00	.951	.00
18	− .01	.930	− .01
17	− .02	.900	− .02
16	− .03	.859	− .03
15	− .04	.801	− .03
14	− .05	.721	− .04
13	− .05	.618	− .03
12	− .07	.493	− .04
11	− .14	.358	− .05
10	−1.24	.234	− .29
9	−3.32	.139	− .46
8	−5.16	.077	− .40
7	−6.23	.040	− .25
6	−6.74	.020	− .14
5	−6.62	.010	− .07
4	−6.12	.005	− .03
3	−5.97	.003	− .02
2	−7.02	.001	− .01
1	−5.48	.001	− .01
			Sum=−1.92

It may be noticed that in the table of reduction factors F, with the exception of the column headed 120.9, the figures are the same in the various columns. The columns differ from each other simply in having the figures displaced vertically. This arises from the fact that the suc-

cessive assumed depths of compensation, with the one exception stated, are the same as the outer radii of the successive rings. An inspection of the formula shows that the relation stated is true when such a selection has been made. The arbitrary selection of these particular depths, therefore, saved considerable time in computing the factors F, and was allowable in the beginning of the investigation when little was known as to the most probable depth of compensation. The selection of the depth, 120.9, to which this time-saving device does not apply, was made later in the investigation after the most probable depth was approximately known.

TOPOGRAPHIC DEFLECTIONS WITHOUT AND WITH ISOSTATIC COMPENSATION.

The tables on pages 48–56 show in parallel columns, for convenience in comparison, all of the observed deflections, all of the computed topographic deflections, and all of the available values of the computed deflections with uniform isostatic compensation considered, for various assumed depths of compensation. The assumed depths of compensation, 162.2, 120.9, and 113.7 kilometers, are common to all four of the geographical groups, designated as the northeastern group, southeastern group, central group, and western group. These are the only assumed depths for the western group. For the other three groups certain other assumed depths also appear, these depths having been used in preliminary stages of the investigation.

The following nine cases have been selected from the tables on pages 48–56. They are extreme or unusual cases. Taken together they indicate the general tendencies which are exhibited in the complete tables.

Number of station.	Name of station.	Observed deflection.	Topographic deflection.	Computed deflection, uniform isostatic compensation considered.					
				Depth of compensation (kilometers).					
				329.8	231.3	162.2	120.9	113.7	79.8
		″	″	″	″	″	″	″	″
238 Mer.	Santa Barbara, California	−18.38	− 64.97			−14.91	−12.78	−12.35	
1 P. V.	Point Arena, California	+16.98	+104.63			+20.39	+16.45	+15.69	
'15 P. V.	North End Knott Island, Virginia	− 6.62	− 54.30	− 7.96		− 3.23	− 2.09	− 1.92	−1.09
43 P. V.	Waddoup, Utah	+24.84	+ 54.71			+22.11	+20.38	+20.00	
49 Mer.	Patmos Head, Utah	−13.52	− 27.20			− 9.42	− 8.66	− 8.53	
178 P. V.	Cheever, New York	−14.77	− 37.46	−10.32		− 9.20	− 8.80	− 8.73	
216 Mer.	Porcupine, Michigan	+ 2.44	− 0.53	+ 3.98	+ 3.70	+ 3.30	+ 2.93	+ 2.85	
169 Mer.	Howlett, New York	+ 1.92	− 12.96	+ 3.86		+ 3.62	+ 3.40	+ 3.33	
205 P. V.	Gargantua, Canada	+ 3.47	− 11.94	+ 3.02	+2.78	+ 2.46	+ 2.17	+ 2.11	
209 Mer.	Chicago L. H., Illinois	+ 1.96	− 10.01	− 0.05	+0.03	+ 0.07	+ 0.11	+ 0.10	

The computed deflections, with isostatic compensation considered, are, as a rule, much smaller than, and of the same sign as, the topographic deflections. There are some exceptions, such as those shown in the last four lines of the above table, but they are not numerous.

The computed deflections, with isostatic compensation considered, ordinarily decrease numerically as the assumed depth of compensation decreases. There are rare exceptions like the one shown in the last line of the above table.

The computed deflection, with isostatic compensation considered, ordinarily agrees much more closely with the observed deflection than does the topographic deflection.

THE AREA METHOD.

An unusual method has been followed in this investigation in forming the observation equations connecting the observed deflections of the vertical, on the one hand, with the constants expressing the figure and size of the earth, on the other hand. As the method is unusual, it has seemed best to show it in considerable detail.

The method here used is called the area method to contrast it with the usual method, which may be called the arc method.

In the area method no attention whatever is paid to the question whether the various astronomic stations are placed approximately along arcs. The only condition required, other than the necessary degree of accuracy in the observations, is that all the astronomic stations shall be connected with continuous triangulation all computed on one basis, that is, on one assumption as to the equatorial and polar dimensions of the reference spheroid and as to the starting latitude, longitude, and azimuth at some one point. Astronomic latitudes, longitudes, and azimuths are all used in one set of equations.

The arc method of deducing the figure of the earth may be illustrated by supposing that a skilled model maker is given several stiff wires, each representing a geodetic arc, either of a parallel or a meridian, each bent to the radius deduced from the astronomic observations on that arc, and is told in what latitude each is located on the geoid, and then requested to construct the ellipsoid of revolution which will conform most closely to the bent wires. Similarly, the area method is illustrated by supposing that the model maker is given a piece of sheet metal cut to the outline of the continuous triangulation which is supplied with the necessary astronomic observations, and accurately molded to fit the curvatures of the geoid as shown by the astronomic observations, and that he is then requested to construct the ellipsoid of revolution which will conform most accurately to the bent sheet. Such a bent sheet essentially includes within itself the bent wires referred to in the first illustration, and, moreover, the wires are now held rigidly in their proper relative positions. The sheet is much more, however, than this rigid system of bent lines, for each arc usually treated as a line is really a belt of considerable width, which is now utilized fully. It is obvious that the model maker would succeed much better in constructing accurately the required ellipsoid of revolution from the one bent sheet than from the several bent wires.

In the area method as used, observation equations of the following form were written:

For each observation of the astronomic latitude:

$$k_1(\phi)+l_1(\lambda)+m_1(\alpha)+n_1\left(\frac{a}{100}\right)+o_1(10\ 000\ e^2)+(\phi_A-\phi')=D_M$$

For each observation of the astronomic longitude:

$$k_2(\phi)+l_2(\lambda)+m_2(\alpha)+n_2\left(\frac{a}{100}\right)+o_2(10\ 000\ e^2)+\cos\phi'(\lambda_A-\lambda')=D_P$$

For each observation of the astronomic azimuth:

$$k_3(\phi)+l_3(\lambda)+m_3(\alpha)+n_3\left(\frac{a}{100}\right)+o_3(10\ 000\ e^2)-\cot\phi'(\alpha_A-\alpha')=D_P$$

In these equations the meanings of the symbols are as follows:

The quantities ϕ_A, λ_A, and α_A are observed astronomic values of the latitude and longitude and azimuth, respectively, at the astronomic stations.

The quantities ϕ', λ', and α' are the values of the geodetic latitude, longitude, and azimuth at the astronomic stations as computed on the United States Standard Datum and the Clarke spheroid of 1866.

The statement that the geodetic latitudes, longitudes, and azimuths are upon the United States Standard Datum means that the computation was carried continuously through all the triangulation upon the assumption that the latitude of the triangulation station at Meades Ranch, Kansas, is 39° 13′ 26″.686; its longitude, 98° 32′ 30″.506, and the azimuth of the line Meades Ranch to Waldo, 75° 28′ 14″.52.

$\phi_A-\phi'$, the absolute term in each latitude observation equation is, therefore, the apparent meridian component of the deflection of the vertical at a latitude station. It is the quantity called A–G, shown in the last column of the tables on pages 12–15, and called the observed deflection in the tables on pages 48–56.

$\lambda_A-\lambda'$ is the difference between the astronomic longitude and the geodetic longitude.

Cos $\phi'(\lambda_A-\lambda')$ is this difference reduced from the parallel of latitude to the prime vertical great circle, and is, therefore, the apparent prime vertical component of the deflection of the vertical at a longitude station.

Similarly $-\cot\phi'(\alpha_A-\alpha')$ is the apparent prime vertical component of the deflection of the vertical at an azimuth station as derived from the azimuth observation.

Cos $\phi'(\lambda_A-\lambda')$ and $-\cot\phi'(\alpha_A-\alpha')$ are the quantities shown in the last column of the tables on pages 17–19 and also shown in the tables on pages 48–56 under the heading "Observed deflections."

For the explanation of the factors called $\cos\phi'$ and $-\cot\phi'$ in the preceding three paragraphs, see page 16.

The three quantities, (ϕ), (λ), and (α) are the required most probable corrections (to be derived from these observation equations) to the initial latitude, longitude, and azimuth (ϕ, λ, α), respectively, at the initial station, Meades Ranch, Kansas. Similarly, $\left(\frac{a}{100}\right)$ is the one-hundredth part of the required most probable correctior to the Clarke 1866 value of the equatorial radius and $(10\ 000\ e^2)$ is 10 000 times the required most probable correction to the Clarke 1866 value of the square of the eccentricity, e^2. *

k_1 is a numerical coefficient, computed by the formula shown later, such that if the initial latitude (at Meades Ranch) were corrected by the amount (ϕ) the change produced in $\phi_A-\phi'$ would be $k_1(\phi)$. Or, in other words k_1 is a numerical coefficient such that if, instead of starting the computation with the initial latitude ϕ, it had been started with the initial latitude $\phi+(\phi)$, the computed value of the latitude at the station considered (at which an astronomic latitude has been observed) would be $\phi'-k_1(\phi)$ instead of ϕ'.

Similarly, k_2 is a numerical coefficient such that if the initial latitude were corrected by (ϕ) the change produced in $\cos\phi'(\lambda_A-\lambda')$ would be $k_2(\phi)$.

So, too, k_3 expresses the relation between (ϕ) and $-\cot\phi'(\alpha_A-\alpha')$.

The coefficients l_1, l_2, and l_3 express corresponding relations between (λ), the correction to the initial longitude, and $(\phi_A-\phi')$, $\cos\phi'(\lambda_A-\lambda')$, and $-\cot\phi'(\alpha_A-\alpha')$ in the observation equations referring, respectively, to latitudes, longitudes, and azimuths.

So, too, the coefficients m_1, m_2, m_3, n_1, n_2, n_3, o_1, o_2, o_3 express similar relations between (α), $\left(\frac{a}{100}\right)$ and $(10\ 000\ e^2)$ and the quantities $(\phi_A-\phi')$, $\cos\phi'(\lambda_A-\lambda')$, and $-\cot\phi'(\lambda_A-\lambda')$, forming the absolute terms of the observation equations.

The quantities D_M, representing the residuals of the latitude observation equations, are the final unexplained meridian components of the deflections of the vertical. The quantities D_P, representing the residuals of the longitude and azimuth equations, are the final unexplained prime vertical components of the deflections of the vertical.

The least square solution of the problem consists in finding such values for the required quantities (ϕ), (λ), (α), $\left(\frac{a}{100}\right)$, and $(10\ 000\ e^2)$ as will make $\Sigma D_M^2+\Sigma D_P^2$ a minimum—that is, the solution makes the sum of the squares of the unexplained deflections of the vertical a minimum.

The quantities $(\phi_A-\phi')$, $\cos\phi'(\lambda_A-\lambda')$, and $-\cot\phi'(\alpha_A-\alpha')$, the observed apparent components of the deflections of the vertical, as given in the second column of the tables on pages 48–56, arise from four principal sources, namely:

(1) From the errors in the initial latitude, longitude, and azimuth (at Meades Ranch) used in computing the geodetic positions and from the errors in the assumed elements (a and e^2) of the Clarke spheroid of 1866 on which the geodetic positions were computed.

(2) From the errors in the astronomic observations, excluding effects of deflections of the vertical.

(3) From the errors in the triangulation—that is, in the lengths and angles fixed by the triangulation.

(4) From the deflections of the vertical.

* In each of these cases a parenthesis is used to indicate the correction to the quantity contained within the parenthesis.

It is proposed to reduce the effects of errors from source (1) to a minimum by deriving from the computation which is to be made the best possible values for the corrections to the initial data and to the elements of the spheroid.

A careful examination for the actual case in hand, involving the astronomic observations and triangulation in the United States used in this investigation, shows that the effects of the deflections of the vertical (4) upon the quantities $(\phi_A - \phi')$, $\cos\phi'(\lambda_A - \lambda')$, and $-\cot\phi'(\alpha_A - \alpha')$ greatly exceed the effects of the errors in the astronomic observations (2) and of errors in the triangulation (3).

Hence, it is proper to proceed with a least square solution on the basis stated above—that is, to make $\Sigma D_M^2 + \Sigma D_P^2$ a minimum.

The discussion of the evidence that the effects of errors in the astronomic observations and in the triangulation are small in comparison with the errors due to unexplained deflections of the vertical, will be given later in connection with the general discussions of the accuracy of the various steps of this investigation.

In the observation equations as written, and in the above statements, it is assumed that there are but five unknowns to be determined. As, however, it is assumed in this investigation that isostatic compensation exists, extending to a depth to be determined, said depth of compensation is a sixth unknown. It is possible to introduce it as a sixth unknown in the observation equations, but it was believed that another method of procedure was advisable, and it has been followed. A separate solution has been made of the observation equations for each of several assumed depths of compensation. In accordance with the general principle that the most probable values of the unknowns are those from the solution which makes the sum of the squares of the residuals a minimum, that one of the solutions for which said sum is least has been adopted, in this investigation, as the most probable solution. Each set of observation equations differs from the others only in the absolute terms. The differences will be shown clearly later.

FORMULÆ FOR COEFFICIENTS.

For convenience of reference, the formulæ from which the coefficients $k_1, k_2, k_3, l_1, \ldots\ldots o_1, o_2$, and o_3, were computed are given here, together with the meanings of the symbols which enter these formulæ. Their derivation will be shown later.

$$k_1 = -1 + \frac{s}{R}(1+Q)\frac{\sin^2\omega}{2\sin^2\frac{1}{2}(\alpha_B - \alpha_F)\sin\theta} \qquad l_1 = \text{zero}$$

$$k_2 = -\sin\phi'\sin\omega \qquad l_2 = -\cos\phi'$$

$$k_3 = -\frac{\cot\phi'\sin^2\omega}{\sin\alpha_F\sin\theta} \qquad l_3 = \text{zero}$$

$$m_1 = \frac{s}{R}(1+Q)\frac{\sin\alpha_B(1+\cos\omega)}{2\sin^2\frac{1}{2}(\alpha_B - \alpha_F)} \qquad n_1 = \frac{100}{a\sin 1''}\frac{N}{R}\theta\cos\alpha_B$$

$$m_2 = \frac{\cos\phi'\cos\alpha_B\sin\omega}{\sin\alpha_F} \qquad n_2 = -\frac{100}{a\sin 1''}\theta\sin\alpha_B$$

$$m_3 = -\frac{\cot\phi'\sin\alpha_B\cos\omega}{\sin\alpha_F} \qquad n_3 = -\frac{100}{a\sin 1''}\theta\sin\alpha_B$$

$$o_1 = \frac{N}{R}\frac{\sin^2\phi}{20000\sin 1''(1-e^2\sin^2\phi)}\theta\cos\alpha_B - \frac{R}{N}\frac{1}{40000\sin 1''(1-e^2)^2}(\phi'-\phi)[1+2\sin^2\phi+3\cos(\phi+\phi')]$$

$$o_2 = -\frac{\sin^2\phi}{20\,000\sin 1''(1-e^2\sin^2\phi)}\theta\sin\alpha_B + \frac{(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi}{60\,000\,a\sin 1''(1-e^2)^2}s\theta^2\cos^2\alpha_F\sin\alpha_B$$

$$o_3 = -\frac{\sin^2\phi}{20\,000\sin 1''(1-e^2\sin^2\phi)}\theta\sin\alpha_B + \frac{(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi}{60\,000\,a\sin 1''(1-e^2)^2}s\theta^2\cos^2\alpha_F\sin\alpha_B$$

The symbols in these formulæ have the following meanings:

ϕ, λ, α and ϕ', λ', α' have already been defined as referring, respectively, to the adopted latitude, longitude, and azimuth at the initial station (Meades Ranch, in this case), and the computed geodetic latitude, longitude, and azimuth at a station at which astronomic observations have been made.

a is the equatorial radius of the earth, and e is the eccentricity of the ellipse which is the earth's cross section through the axis of rotation. The adopted values of a and e, used in this connection, are necessarily the Clarke values of 1866, as the geodetic latitudes, longitudes, and azimuths were computed on that spheroid.

s is the distance in linear units measured along the surface of the Clarke spheroid of 1866 between the point (ϕ, λ) and the point (ϕ', λ'), that is, between the initial station Meades Ranch and the astronomic station under consideration.

$\omega = \lambda' - \lambda$, that is, the difference of longitude of the astronomic station under consideration and Meades Ranch.

α_F is the azimuth from the point (ϕ, λ) Meades Ranch, to the point (ϕ', λ'), the astronomic station under consideration, and α_B is the back azimuth between these same points.

R is the radius of curvature of the meridian in latitude $\frac{1}{2}(\phi + \phi')$.

$$N = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}$$

$$\theta = \frac{s}{N}\left[1 + \frac{e^2 \theta^2}{6(1 - e^2)} \cos^2 \phi \cos^2 \alpha_F\right]$$

$$Q = \frac{\theta^2}{12} \cos^2 \tfrac{1}{2}(\alpha_B + \alpha_F)$$

DERIVATION OF FORMULÆ FOR COEFFICIENTS.

Let (ϕ), (λ), (α), (a), and (e^2), respectively, be corrections to the latitude, longitude, and azimuth, ϕ, λ, and α, at Meades Ranch (the initial point) on the United States Standard Datum, to a (the equatorial radius of the earth), and to e^2 (the square of the eccentricity) as fixed by the Clarke spheroid of 1866.

Let ϕ', λ', α' be the geodetic latitude, longitude, and azimuth at *any* point occupied as an astronomic station, as computed on the United States Standard Datum and the Clarke spheroid of 1866.

It is required, as a preliminary to the derivation of the formulæ for the coefficients of the observation equations, to derive expressions for ϕ'', λ'', α'' the latitude, longitude, and azimuth at any point occupied as an astronomic station, after the above corrections (ϕ), (λ), (α), (a), and (e^2) have been applied. The *lengths and the angles at each triangulation station*, as fixed by triangulation, are assumed to remain unchanged.

The required expressions may be written in symbolic form as follows:

$$\phi'' = \phi' + f_1(\phi) + g_1(\lambda) + h_1(\alpha) + i_1(a) + j_1(e^2) \quad (1)$$
$$\lambda'' = \lambda' + f_2(\phi) + g_2(\lambda) + h_2(\alpha) + i_2(a) + j_2(e^2) \quad (2)$$
$$\alpha'' = \alpha' + f_3(\phi) + g_3(\lambda) + h_3(\alpha) + i_3(a) + j_3(e^2) \quad (3)$$

in which the f's, g's, h's, i's, and j's are coefficients for which expressions are yet to be derived.

Let it be assumed that the equations numbered (36), on page 249 in the "Account of Principal Triangulation," Capt. A. R. Clarke, London, 1858,* correctly express the relations between ϕ, λ, α (referring to the initial station) and ϕ', λ', α', a, and e^2. In these equations, as written by Clarke, the α and α' stand for the azimuths of the line joining the two points, always counted

* The title page of this volume reads as follows: "Ordnance Trigonometrical Survey of Great Britain and Ireland. Account of the observations and calculations, of the principal triangulation; and of the figure, dimensions, and mean specific gravity of the earth as derived therefrom. Published by order of the Master-General and Board of Ordnance. Drawn up by Capt. Alexander Ross Clarke . . . under the direction of Lieut.-Col. H. James . . . Superintendent of the Ordnance Survey . . . London, Printed by G. E. Eyre and W. Spottiswoode, 1858."

in the nearest way from the north, sometimes clockwise and sometimes counterclockwise. In rewriting the equations for our use and putting them in the notation ordinarily used in the Coast and Geodetic Survey, it will be assumed that all azimuths are to be counted from the south around by west, clockwise, in the usual manner, and the necessary changes in signs of terms involving α and α' will be made. To avoid conflicts in notation the symbol α_F will be substituted for α, and α_B for α'.

The equations as thus rewritten in our usual notation are:

$$\cot \tfrac{1}{2}(\alpha_B+\omega+\zeta) = -\frac{\cos \frac{1}{2}(90^\circ-\phi-\theta)}{\cos \frac{1}{2}(90^\circ-\phi+\theta)} \tan\frac{\alpha_F}{2} \tag{4}$$

$$\cot \tfrac{1}{2}(\alpha_B-\omega+\zeta) = -\frac{\sin \frac{1}{2}(90^\circ-\phi-\theta)}{\sin \frac{1}{2}(90^\circ-\phi+\theta)} \tan\frac{\alpha_F}{2} \tag{5}$$

$$\phi'-\phi = -\frac{s \sin \frac{1}{2}(\alpha_B+\alpha_F+\zeta)}{R \sin \frac{1}{2}(\alpha_B-\alpha_F+\zeta)}(1+Q) \tag{6}$$

In these equations $\omega=\lambda'-\lambda$, the difference of longitude between the astronomic station and the initial station.

s = the distance in linear units (meters) between the point (ϕ, λ) and the point (ϕ', λ'), that is, between the initial station and the astronomic station.

α_F is the azimuth from the point (ϕ, λ) to the point (ϕ', λ') and α_B is the back azimuth.

R is the radius of curvature of the meridian in latitude $\frac{\phi+\phi'}{2}$.

The general expression for R is

$$R = \frac{a(1-e^2)}{[1-e^2 \sin^2 \frac{1}{2}(\phi+\phi')]^{\frac{3}{2}}} \tag{7}$$

θ, ζ, and Q have the following values with sufficient accuracy for our purpose.*

$$\theta = \frac{s}{N}\left[1+\frac{1}{6}\frac{e^2\theta^2}{(1-e^2)}\cos^2\phi\cos^2\alpha_F\right] \tag{8}$$

in which N is the length of the normal at latitude ϕ limited by the minor axis, or axis of revolution.

$$N = \frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}} \tag{9}$$

$$\zeta = -\frac{1}{4}\frac{e^2\theta^2}{(1-e^2)}\cos^2\phi\sin 2\alpha_F \tag{10}$$

$$Q = \frac{\theta^2}{12}\cos^2\tfrac{1}{2}(\alpha_B+\alpha_F) \tag{11}$$

The geometrical meaning of ζ and θ may be made clear by the following figure, which will also be useful in later work in deriving required formulæ.

The figure here shown is a spherical (not spheroidal) triangle upon a sphere of which the radius is the normal C H (= N) at the point (ϕ, λ) of the spheroid, as limited by the minor axis of the spheroid at H. The point H is the center of the sphere. The sphere is, therefore, tangent to the spheroid at (ϕ, λ), but they have few other common points on their surfaces.

The bold faced letters ***A, B, C, a, b, c,*** are used to designate the parts of the spherical triangle in the manner which is customary in text-books on spherical trigonometry.

The arc ***BC*** or ***a*** joining ***C*** or (ϕ, λ) with the pole of the sphere is made equal to $x = 90^\circ - \phi$.

The angle ***C*** of the spherical triangle is made equal to $180^\circ - \alpha_F$.

The angle ***B*** at the pole is made equal to $\omega = (\lambda' - \lambda)$.

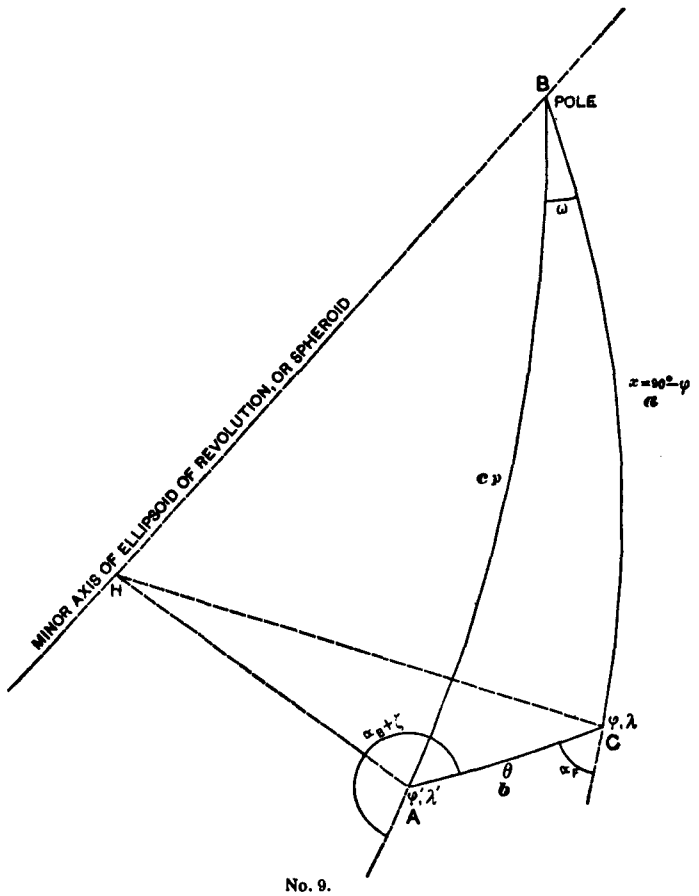
These statements fix the position and size of the sphere and fix all parts of the spherical triangle, since three parts of the spherical triangle have been fixed.

The angle ***A*** is evidently nearly equal to $\alpha_B - 180^\circ$, or the angle indicated on the figure is nearly equal to α_B. This latter angle has been therefore placed equal to $\alpha_B+\zeta$, in which ζ is a very small angle of which the value has been derived by Clarke as expressed in (10),

*See "Account of Principal Triangulation," etc., p. 249.

and of which he makes the statement that its value does not "amount to a tenth of a second even in a distance of a hundred miles."

θ is an arc on the sphere. Its radius of curvature is necessarily N, that being the radius of the sphere. If its length (linear) were equal to s, the linear distance between the points (ϕ, λ) and (ϕ', λ') as fixed by triangulation, we would have $\theta = \frac{s}{N}$. But the point A representing the point (ϕ', λ') on the sphere was fixed as being the intersection of the two arcs c and b (or θ), which were started in fixed directions from B and C, whereas the true point (ϕ', λ') lies on the spheroid at a certain distance, s, measured on the surface of the spheroid, from (ϕ, λ). Hence the length (linear) of θ is not equal to s, and θ differs slightly from $\frac{s}{N}$, as shown in (8), which has been derived by Clarke.



No. 9.

The arc c (or p) is approximately (not exactly) equal to $90° - \phi'$.

The preceding formulæ have all been proved by Clarke (See "Account of Principal Triangulation," etc., p. 249, and preceding), and will here be accepted without question. The derivation of the formulæ which follow is, in the main, that given by Clarke on pages 616–620 of the "Account," somewhat amplified. The latter part of the work of using the formulæ, or rather the coefficients expressed by them, in making a least square adjustment, differs radically from Clarke.

The relations between ϕ, λ, α_F, ϕ', λ', α_B being those shown in (4), (5), and (6), and on the figure above, it is required to derive expressions for the coefficients in (1), (2), and (3).

Let the correction (λ) be supposed applied to the initial longitude, λ, while α_F, ϕ, and s remain unchanged. In the figure above, it is evident that the spherical triangle will simply be rotated about B, the pole, as a center without changing its dimensions. c, A, B, and b

preserve the same values as before, and evidently preserve the relations to the quantities ϕ', α_B, ω, and θ which they had before. ϕ', α_B, ω and θ remain unchanged. The change in λ' must be equal to (λ). Looking now to the formulæ (4), (5), and (6) for corroboration we find that λ does not appear in any of them. The only longitude function which appears is $\omega = (\lambda' - \lambda)$. Hence a change (λ) in λ produces an equal change in λ' and affects nothing else.

Hence in (1), (2), and (3),

$$g_1 = \quad 0.000 \tag{12}$$
$$g_2 = +1.000 \tag{13}$$
$$g_3 = \quad 0.000 \tag{14}$$

In obtaining the expressions for the coefficients f_1, f_2, f_3, h_1, h_2, and h_3 it is important to note that a small change (ϕ) in ϕ, or (α_F) in α_F will cause a much smaller change in θ, see (8), which may be neglected.

Hence in the figure on page 79, if ϕ be corrected by (ϕ) while λ, α_F and s remain constant, $\boldsymbol{C}$ and $\boldsymbol{b}$ remain constant while $\boldsymbol{a}$, $\boldsymbol{B}$, $\boldsymbol{c}$ and $\boldsymbol{A}$ vary.

Similarly, if α_F be corrected by (α_F) while ϕ, λ and s remain constant, $\boldsymbol{a}$ and $\boldsymbol{b}$ remain constant while $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{c}$ vary.

Turning to the fundamental equations numbered (4) and (5) it is evident that if α_F varies both α_B and ω must vary. θ, as already indicated, remains practically constant. Similarly, ζ remains practically constant and zero, see (10). By differentiation of (4) there is obtained

$$\frac{d\alpha_B}{d\alpha_F} + \frac{d\omega}{d\alpha_F} = \frac{\sin^2\frac{1}{2}(\alpha_B + \omega)\cos\frac{1}{2}(90° - \phi - \theta)}{\cos^2\frac{\alpha_F}{2}\cos\frac{1}{2}(90° - \phi + \theta)} \tag{15}$$

The general relation for any spherical triangle

$$\sin(\boldsymbol{A} + \boldsymbol{B})\cos^2\tfrac{1}{2}\boldsymbol{c} = \cos\tfrac{1}{2}(\boldsymbol{a} - \boldsymbol{b})\cos\tfrac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\sin\boldsymbol{C} \tag{16}$$

which can be derived from the equation on line 25 of page 161 of Chauvenet's Trigonometry, 9th edition (using the general formula $\sin 2\theta = 2\sin\theta\cos\theta$), and the general relation for any spherical triangle

$$\cos\tfrac{1}{2}(\boldsymbol{A} + \boldsymbol{B}) = \frac{\cos\frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})}{\cos\frac{1}{2}\boldsymbol{c}}\sin\tfrac{1}{2}\boldsymbol{C} \tag{17}$$

become, for the spherical triangle shown on page 79, neglecting the very small angle ζ,

$$-\sin(\alpha_B + \omega)\cos^2\frac{p}{2} = \cos\tfrac{1}{2}(90° - \phi - \theta)\cos\tfrac{1}{2}(90° - \phi + \theta)\sin\alpha_F \tag{18}$$

and

$$\sin\tfrac{1}{2}(\alpha_B + \omega) = \frac{\cos\frac{1}{2}(90° - \phi + \theta)}{\cos\frac{p}{2}}\cos\frac{\alpha_F}{2} \tag{19}$$

Using (18) and (19) to simplify (15) there is obtained

$$\frac{d\alpha_B}{d\alpha_F} + \frac{d\omega}{d\alpha_F} = -\frac{\sin(\alpha_B + \omega)}{\sin\alpha_F} \tag{20}$$

By differentiation of (5) in accordance with the statements above, there is obtained

$$\frac{d\alpha_B}{d\alpha_F} - \frac{d\omega}{d\alpha_F} = \frac{\sin^2\frac{1}{2}(\alpha_B - \omega)\sin\frac{1}{2}(90° - \phi - \theta)}{\cos^2\frac{\alpha_F}{2}\sin\frac{1}{2}(90° - \phi + \theta)} \tag{21}$$

The general relation for any spherical triangle

$$\sin(\boldsymbol{A} - \boldsymbol{B})\sin^2\frac{\boldsymbol{c}}{2} = \sin\tfrac{1}{2}(\boldsymbol{a} - \boldsymbol{b})\sin\tfrac{1}{2}(\boldsymbol{a} + \boldsymbol{b})\sin\boldsymbol{C} \tag{22}$$

which can be obtained in the same manner as (16), and the general relation for any spherical triangle

$$\cos \tfrac{1}{2}(A-B) = \frac{\sin \tfrac{1}{2}(a+b)}{\sin \frac{c}{2}} \sin \frac{C}{2} \tag{23}$$

become for the spherical triangle shown on page 79, neglecting the very small angle ζ,

$$-\sin(\alpha_B - \omega) \sin^2 \frac{p}{2} = \sin \tfrac{1}{2}(90° - \phi - \theta) \sin \tfrac{1}{2}(90° - \phi + \theta) \sin \alpha_F \tag{24}$$

and

$$\sin \tfrac{1}{2}(\alpha_B - \omega) = \frac{\sin \tfrac{1}{2}(90° - \phi + \theta)}{\sin \frac{p}{2}} \cos \frac{\alpha_F}{2} \tag{25}$$

Using (24) and (25) to simplify (21), there is obtained

$$\frac{d\alpha_B}{d\alpha_F} - \frac{d\omega}{d\alpha_F} = -\frac{\sin(\alpha_B - \omega)}{\sin \alpha_F} \tag{26}$$

Adding equations (20) and (26), member by member, there is obtained, after simplification,

$$\frac{d\alpha_B}{d\alpha_F} = -\frac{\sin \alpha_B \cos \omega}{\sin \alpha_F} = h_3 \text{ [which is a required coefficient in (3)]} \tag{27}$$

Similarly, by subtracting (26) from (20) member by member, and simplifying, there is obtained

$$\frac{d\omega}{d\alpha_F} = -\frac{\cos \alpha_B \sin \omega}{\sin \alpha_F} = h_2 \text{ [which is a required coefficient in (2)]} \tag{28}$$

Turning now to (4) and (5), if ϕ varies while α_F remains constant, α_B and ω must vary. θ will remain practically constant, see (8). In the following derivation of the values of $\frac{d\alpha_B}{d\phi}$ and $\frac{d\omega}{d\phi}$, ζ will be assumed to be zero.

Differentiating (4) under the above stated conditions, there is obtained

$$\frac{d\alpha_B}{d\phi} + \frac{d\omega}{d\phi} = -\frac{\sin^2 \tfrac{1}{2}(\alpha_B + \omega) \sin \theta \tan \frac{\alpha_F}{2}}{\cos^2 \tfrac{1}{2}(90° - \phi + \theta)} \tag{29}$$

After simplifying (29) by the use of (19) and of the general relation $\sin 2\theta = 2 \sin\theta \cos\theta$, it becomes

$$\frac{d\alpha_B}{d\phi} + \frac{d\omega}{d\phi} = -\frac{\sin \theta \sin \alpha_F}{2 \cos^2 \frac{p}{2}} \tag{30}$$

Differentiating (5) under the conditions stated above, there is obtained

$$\frac{d\alpha_B}{d\phi} - \frac{d\omega}{d\phi} = -\frac{\sin^2 \tfrac{1}{2}(\alpha_B - \omega) \sin \theta \tan \frac{\alpha_F}{2}}{\sin^2 \tfrac{1}{2}(90° - \phi + \theta)} \tag{31}$$

After simplifying (31) by the use of (25) and the general relation $\sin 2\theta = 2 \sin\theta \cos\theta$, it is

$$\frac{d\alpha_B}{d\phi} - \frac{d\omega}{d\phi} = -\frac{\sin\theta \sin\alpha_F}{2 \sin^2 \frac{p}{2}} \tag{32}$$

The general relation for any spherical triangle

$$\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c} \tag{33}$$

becomes for the spherical triangle shown on page 79

$$\frac{\sin\omega}{\sin\theta}=\frac{\sin\alpha_F}{\sin p} \tag{34}$$

Adding (30) and (32), member by member, and simplifying by means of the general relation $\sin 2\theta = 2\sin\theta\cos\theta$ and (34), there is obtained

$$\frac{d\alpha_B}{d\phi}=-\frac{\sin^2\omega}{\sin\alpha_F\sin\theta}=f_3 \text{ [which is a required coefficient in (3)]} \tag{35}$$

Subtracting (32) from (30), member by member, and simplifying by means of (34) and the general relation $\cos 2\theta=\cos^2\theta-\sin^2\theta$, there is obtained

$$\frac{d\omega}{d\phi}=\cot p\sin\omega \tag{36}$$

Assuming that with sufficient accuracy for the purpose in hand $\phi'=90°-p$, (36) becomes

$$\frac{d\omega}{d\phi}=\tan\phi'\sin\omega=f_2 \text{ [which is a required coefficient in (2)]} \tag{37}$$

In (6) it is evident that if α_F be corrected by the amount (α_F) while ϕ and s are held fixed, α_B and ϕ' will be changed. In deriving an expression for $\frac{d\phi'}{d\alpha_F}$ it will be assumed that with sufficient accuracy $(1+Q)$ may be considered constant and the small angle ζ neglected.

By differentiation of (6) and simplification there is obtained

$$\frac{d\phi'}{d\alpha_F}=-\frac{s}{R}(1+Q)\frac{\sin\alpha_B-\sin\alpha_F\frac{d\alpha_B}{d\alpha_F}}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)} \tag{38}$$

Substituting in this the value of $\frac{d\alpha_B}{d\alpha_F}$ from (27) there is obtained

$$\frac{d\phi'}{d\alpha_F}=-\frac{s}{R}(1+Q)\frac{\sin\alpha_B(1+\cos\omega)}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)}=h_1 \text{ [which is one of the required coefficients in (1)]} \tag{39}$$

Similarly, if ϕ be corrected by (ϕ) while α_F and s are held fixed, α_B and ϕ' will both be changed. Making the same assumptions as were made when deriving $\frac{d\phi'}{d\alpha_F}$, there is obtained by differentiation of (6)

$$\frac{d\phi'}{d\phi}=1+\frac{s}{R}(1+Q)\frac{\sin\alpha_F\frac{d\alpha_B}{d\phi}}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)} \tag{40}$$

Substituting in this the value of $\frac{d\alpha_B}{d\phi}$ from (35), there is obtained

$$\frac{d\phi'}{d\phi}=1-\frac{s}{R}(1+Q)\frac{\sin^2\omega}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)\sin\theta}=f_1 \text{ [which is a required coefficient in (1)]} \tag{41}$$

To determine the coefficients i_1, i_2, i_3, j_1, j_2, and j_3, which serve to express the effects upon ϕ', λ', and α_B of changes (a) (e^2) in a and e^2, the constants which fix the size and shape of the spheroid, it is important to note, first, that in the fundamental equations (4), (5), and (6), neglect-

ing the very small quantity ζ, θ is the only quantity directly involving a and e^2 which enters the equations explicitly.

From (8) and (9), and neglecting the term in (8) containing θ^2, there is obtained

$$\theta = \frac{s}{a}(1 - e^2 \sin^2 \phi)^{\frac{1}{2}} \tag{42}$$

Differentiating (42) with respect to θ and a, e^2 being supposed constant,

$$\frac{d\theta}{da} = -\frac{\theta}{a} \tag{43}$$

Differentiating (42) with respect to θ and e^2, a being supposed constant,

$$\frac{d\theta}{d[e^2]} = -\frac{\theta}{2} \frac{\sin^2 \phi}{(1 - e^2 \sin^2 \phi)} \tag{44}$$

In (4) and (5), when θ varies α_B and ω also vary.

Neglecting ζ and differentiating (4), there is obtained

$$\frac{d\alpha_B}{d\theta} + \frac{d\omega}{d\theta} = \frac{\sin^2 \frac{1}{2}(\alpha_B + \omega) \tan \frac{\alpha_F}{2} \cos \phi}{\cos^2 \frac{1}{2}(90° - \phi + \theta)} \tag{45}$$

Combining (19) with this, and simplifying, there is obtained

$$\frac{d\alpha_B}{d\theta} + \frac{d\omega}{d\theta} = \frac{\sin \alpha_F \cos \phi}{2 \cos^2 \frac{p}{2}} \tag{46}$$

Similarly, neglecting ζ and differentiating (5), there is obtained

$$\frac{d\alpha_B}{d\theta} - \frac{d\omega}{d\theta} = -\frac{\sin^2 \frac{1}{2}(\alpha_B - \omega) \tan \frac{\alpha_F}{2} \cos \phi}{\sin^2 \frac{1}{2}(90° - \phi + \theta)} \tag{47}$$

Combining (25) with this, and simplifying, there is obtained

$$\frac{d\alpha_B}{d\theta} - \frac{d\omega}{d\theta} = -\frac{\sin \alpha_F \cos \phi}{2 \sin^2 \frac{p}{2}} \tag{48}$$

Adding (46) and (48), member by member, there is obtained

$$\frac{d\alpha_B}{d\theta} = -\sin \alpha_F \cos \phi \frac{\cot p}{\sin p} \tag{49}$$

In the spherical triangle shown on page 79 the law of proportional sines gives

$$\frac{\sin \alpha_F}{\sin p} = -\frac{\sin \alpha_B}{\cos \phi} \tag{50}$$

Substituting the value of sin p from (50) in (49), and assuming that with sufficient accuracy $\cot p = \tan \phi'$ (that is, that $p = 90° - \phi'$), there is obtained

$$\frac{d\alpha_B}{d\theta} = \tan \phi' \sin \alpha_B \tag{51}$$

Subtracting (48) from (46), member by member, and substituting the value of sin p from (50), and assuming $p = 90° - \phi'$, there is obtained

$$\frac{d\omega}{d\theta} = -\frac{\sin \alpha_B}{\cos \phi'} \tag{52}$$

By combining (51) with (43), there is obtained

$$\frac{d\alpha_B}{da} = -\frac{\theta}{a}\tan\phi' \sin\alpha_B = i_3 \text{ [which is a required coefficient in (3)]} \tag{53}$$

Similarly, from (52) and (43) there is obtained

$$\frac{d\omega}{da} = \frac{\theta}{a}\frac{\sin\alpha_B}{\cos\phi'} = i_2 \text{ [which is a required coefficient in (2)]} \tag{54}$$

Substituting in (8) the value of N from (9), differentiating with respect to e^2, and neglecting terms of higher order, there is obtained

$$\frac{d\theta}{d[e^2]} = -\frac{\theta}{2}\frac{\sin^2\phi}{1-e^2\sin^2\phi} + \frac{s\theta^2(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\cos^2\alpha_F}{6a(1-e^2)^2} \tag{55}$$

which is a value for $\frac{d\theta}{d[e^2]}$ somewhat more accurate than that shown in (44), in deriving which the term in (8) involving θ^2 was neglected.

Combining (55) and (52) there is obtained

$$\frac{d\omega}{d[e^2]} = -\frac{\sin\alpha_B}{\cos\phi'}\left[-\frac{\theta}{2}\frac{\sin^2\phi}{1-e^2\sin^2\phi} + \frac{s\theta^2(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\cos^2\alpha_F}{6a(1-e^2)^2}\right] = j_2 \tag{56}$$

[which is a required coefficient in (2)].

Similarly, combining (55) and (51), there is obtained

$$\frac{d\alpha_B}{d[e^2]} = \tan\phi'\sin\alpha_B\left[-\frac{\theta}{2}\frac{\sin^2\phi}{1-e^2\sin^2\phi} + \frac{s\theta^2(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\cos^2\alpha_F}{6a(1-e^2)^2}\right] = j_3 \tag{57}$$

[which is a required coefficient in (3)].

$(\phi'-\phi)$ R is (with sufficient accuracy) the difference of latitude expressed in meters. [See meaning of R as shown in (7).]

In the spherical triangle shown on page 79, let it be assumed that $\phi' = 90° - p$. Then evidently $(90° - p - \phi)N$ is also an expression for the difference of latitude expressed in meters. Hence,

$$(\phi'-\phi)R = (90° - p - \phi)N$$

or

$$(\phi'-\phi) = \frac{N}{R}(90° - p - \phi) \tag{58}$$

If $\frac{N}{R}$ varies while ϕ remains constant, ϕ' and p will vary in (58). Differentiating (58) accordingly, there is obtained

$$d\phi' = -\frac{N}{R}dp + (90° - p - \phi)d\left[\frac{N}{R}\right] \tag{59}$$

For the spherical triangle shown on page 79, the following general formulæ for any spherical triangle

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\sin c \cos A = \sin b \cos a - \cos b \sin a \cos C$$

become, respectively,

$$\cos p = \sin\phi\cos\theta - \cos\phi\sin\theta\cos\alpha_F \tag{60}$$

$$-\sin p \cos\alpha_B = \sin\theta\sin\phi + \cos\theta\cos\phi\cos\alpha_F \tag{61}$$

Differentiating (60), ϕ and α_F being constant, there is obtained

$$-\sin p\, dp = -\sin\phi\sin\theta\, d\theta - \cos\phi\cos\alpha_F\cos\theta\, d\theta \tag{62}$$

By multiplying (61) by $d\theta$, member by member, and comparing with (62), there is obtained

$$-\sin p\,dp = \sin p \cos\alpha_B d\theta, \text{ whence}$$

$$dp = -\cos\alpha_B d\theta \tag{63}$$

From (7) and (9)

$$\frac{N}{R} = \frac{[1-e^2\sin^2\frac{1}{2}(\phi+\phi')]^{\frac{3}{2}}}{(1-e^2)(1-e^2\sin^2\phi)^{\frac{1}{2}}} \tag{64}$$

Differentiating (64) and neglecting terms of higher orders,

$$\frac{d\left[\frac{N}{R}\right]}{d[e^2]} = \frac{1}{(1-e^2)^{\frac{3}{2}}}[1+\tfrac{1}{2}\sin^2\phi - \tfrac{3}{2}\sin^2\tfrac{1}{2}(\phi+\phi')] \tag{65}$$

or in slightly more convenient form for computing

$$\frac{d\left[\frac{N}{R}\right]}{d[e^2]} = \frac{1}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')] \tag{66}$$

Substituting the values of dp and $d\left[\frac{N}{R}\right]$ from (63) and (66) in (59), there is obtained

$$d\phi' = \frac{N}{R}\cos\alpha_B d\theta + \frac{90^\circ - p - \phi}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')]d[e^2] \tag{67}$$

By using (43) and (44) this may be written

$$d\phi' = -\frac{N}{R}\frac{\theta}{a}\cos\alpha_B da - \frac{N}{R}\frac{\theta}{2}\frac{\sin^2\phi\cos\alpha_B d[e^2]}{1-e^2\sin^2\phi} + \frac{90^\circ - p - \phi}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')]d[e^2]$$

Whence, the required coefficients i_1 and j_1 in (1) are

$$i_1 = -\frac{N}{R}\frac{\theta}{a}\cos\alpha_B \tag{68}$$

$$j_1 = -\frac{N}{R}\frac{\theta}{2}\frac{\sin^2\phi\cos\alpha_B}{1-e^2\sin^2\phi} + \frac{90^\circ - p - \phi}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')] \tag{69}$$

(69) may be put in a slightly more convenient form by substituting for $90^\circ - p - \phi$ its value from (58). j_1 then becomes

$$j_1 = -\frac{N}{R}\frac{\theta}{2}\frac{\sin^2\phi\cos\alpha_B}{1-e^2\sin^2\phi} + \frac{R}{N}\frac{\phi'-\phi}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')] \tag{70}$$

This completes the derivation of the coefficients required in (1), (2), and (3). The references to the equations showing their values are as follows:

f_1, (41); g_1, (12); h_1, (39); i_1, (68); j_1, (70).
f_2, (37); g_2, (13); h_2, (28); i_2, (54); j_2, (56).
f_3, (35); g_3, (14); h_3, (27); i_3, (53); j_3, (57).

The coefficients, collected for convenience of reference, are

$$
\left.
\begin{aligned}
f_1 &= 1 - \frac{s}{R}(1+Q)\frac{\sin^2\omega}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)\sin\theta}\\
f_2 &= \tan\phi'\sin\omega\\
f_3 &= -\frac{\sin^2\omega}{\sin\alpha_F\sin\theta}\\
g_1 &= 0.000\\
g_2 &= +1.000\\
g_3 &= 0.000\\
h_1 &= -\frac{s}{R}(1+Q)\frac{\sin\alpha_B(1+\cos\omega)}{2\sin^2\frac{1}{2}(\alpha_B-\alpha_F)}\\
h_2 &= -\frac{\cos\alpha_B\sin\omega}{\sin\alpha_F}\\
h_3 &= -\frac{\sin\alpha_B\cos\omega}{\sin\alpha_F}\\
i_1 &= -\frac{N}{R}\frac{\theta}{a}\cos\alpha_B\\
i_2 &= \frac{\theta}{a}\frac{\sin\alpha_B}{\cos\phi'}\\
i_3 &= -\frac{\theta}{a}\tan\phi'\sin\alpha_B\\
j_1 &= -\frac{N}{R}\frac{\theta}{2}\frac{\sin^2\phi\cos\alpha_B}{1-e^2\sin^2\phi} + \frac{R}{N}\frac{\phi'-\phi}{4(1-e^2)^2}[1+2\sin^2\phi+3\cos(\phi+\phi')]\\
j_2 &= -\frac{\sin\alpha_B}{\cos\phi'}\left[-\frac{\theta}{2}\frac{\sin^2\phi}{1-e^2\sin^2\phi} + \frac{s\theta^2(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\cos^2\alpha_F}{6a(1-e^2)^2}\right]\\
j_3 &= \tan\phi'\sin\alpha_B\left[-\frac{\theta}{2}\frac{\sin^2\phi}{1-e^2\sin^2\phi} + \frac{s\theta^2(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\cos^2\alpha_F}{6a(1-e^2)^2}\right]
\end{aligned}
\right\} \quad (71)
$$

According to pages 74, 75, k_1 is a numerical coefficient such that if the initial latitude were corrected by the amount (ϕ) the change produced in $\phi_A - \phi'$ would be $k_1(\phi)$. Or, in other words, k_1 is a numerical coefficient such that if, instead of starting the computation with the initial latitude, ϕ, it had been started with the initial latitude $\phi+(\phi)$, the computed value of the latitude at the station considered (at which an astronomic latitude has been observed) would be $\phi' - k_1(\phi)$ instead of ϕ'.

According to equation (1), page 77, f_1 is a numerical coefficient such that if, instead of starting the computation with the initial latitude ϕ, it had been started with the initial latitude $\phi+(\phi)$, the computed value of the latitude at the station being considered would be $\phi' + f_1(\phi)$, instead of ϕ'.

Hence, f_1 is the negative of k_1, or

$$k_1 = -f_1 \quad (72)$$

Similarly,

$$l_1 = -g_1 \quad (73)$$

and

$$m_1 = -h_1 \quad (74)$$

To obtain n_1 from i_1, the sign must be changed for the same reason as for k_1, l_1, and m_1. Also, i_1 must be multiplied by 100 because n_1 is a coefficient for $\left(\frac{a}{100}\right)$ (see page 74), whereas i_1 is a coefficient for (a). Moreover, as the absolute terms of the observation equations (p. 74) are all to be expressed in seconds of arc, as being most convenient for the computer, all other terms must be expressed in the same units. Hence, as θ is in radians * in the expression for i_1, i_1 must also be divided by sin 1″.

*According to the way in which formulæ (71) have been derived, involving differentiation, θ and $\phi'-\phi$, wherever they occur, are necessarily in radians.

Hence

$$n_1 = -\frac{100}{\sin 1''} i_1 \tag{75}$$

Similarly,

$$o_1 = -\frac{j_1}{10\,000 \sin 1''} \tag{76}$$

In obtaining k_2, l_2, m_2, n_2, and o_2 from f_2, g_2, h_2, i_2, and j_2 the same considerations apply as those noted in connection with k_1, l_1, m_1, n_1, and o_1, but, in addition, it must be noted that in the observation equations for astronomic longitudes (shown on page 74) $-\lambda' \cos\phi'$ enters, instead of simply $-\lambda'$. Hence, each of the coefficients f_2, g_2, h_2, i_2, j_2, must be multiplied by $\cos\phi'$. This factor, $\cos\phi'$, arises from the fact that to derive the deflection of the vertical expressed in terms of the prime vertical great circle, one must multiply the difference between the astronomic and geodetic longitude by $\cos\phi'$ (see page 74).

For a similar reason, the factor $-\cot\phi'$ enters in obtaining k_3, l_3, m_3, n_3, and o_3 from f_3, g_3, h_3, i_3, and j_3.

The relations are, therefore:

$$k_2 = -f_2 \cos\phi' \tag{77}$$

$$l_2 = -g_2 \cos\phi' \tag{78}$$

$$m_2 = -h_2 \cos\phi' \tag{79}$$

$$n_2 = -\frac{100}{\sin 1''} i_2 \cos\phi' \tag{80}$$

$$o_2 = -\frac{j_2 \cos\phi'}{10\,000 \sin 1''} \tag{81}$$

$$k_3 = f_3 \cot\phi' \tag{82}$$

$$l_3 = g_3 \cot\phi' \tag{83}$$

$$m_3 = h_3 \cot\phi' \tag{84}$$

$$n_3 = \frac{100}{\sin 1''} i_3 \cot\phi' \tag{85}$$

$$o_3 = \frac{j_3 \cot\phi'}{10\,000 \sin 1''} \tag{86}$$

Using the relations (72) to (86), inclusive, to make the conversions, the formulæ of (71) become the formulæ for the coefficients k_1, k_2, k_3, ... o_1, o_2, o_3, printed on page 76, of which the derivation was desired.

As a check on the correctness of the derivation of these formulæ, the dimensions of the so-called coefficients should be examined. The formulæ on page 76 show that the k's, l's, and m's are all abstract numbers. This is as it should be, since in the observation equations on page 74 the absolute terms are all in seconds of arc and so, also, are (ϕ), (λ), and (α). The formulæ show that the n's are each of the dimensions seconds of arc divided by a length. This is as it should be, since each n is to be multiplied by a length, the required correction to the equatorial radius, and the product must be seconds of arc to correspond to the absolute terms. Similarly, each of the o's is in seconds of arc simply, which is as it should be, since it is to be multiplied by the required correction to e^2, an abstract number, in order to give products which are seconds of arc corresponding to the absolute terms.

The quantities $\left(\frac{a}{100}\right)$ and $(10\,000\ e^2)$ were used arbitrarily in the observation equations rather than (a) and (e^2), simply for the purpose of making the average values of the n and o coefficients of about the same magnitude as the average values of the k, l, and m coefficients. Such approximate equality, by insuring that the relation of the decimal point to the significant figures shall be about the same in the different coefficients, facilitates the least square computation, especially when a summation term is used as a check in the formation and solution of the normal equations.

EXAMPLES OF COMPUTATIONS OF COEFFICIENTS.

For use in computing the numerical values of the coefficients in the observation equations, the formulæ for which are shown on page 76, it is necessary first to compute values of the length of the arc θ connecting the initial station with each astronomic station and to compute the forward and back azimuths along this arc. To make computations of θ and of the azimuths named with the highest attainable degree of accuracy would be a long and difficult process. For the present purpose only a moderate degree of accuracy is necessary and the simple and short process of computation here described was found to be sufficient.

Imagine a sphere of which the radius is

$$N_M = \frac{a}{[1-e^2\sin^2\frac{1}{2}(\phi+\phi')]^{\frac{1}{2}}}$$

and of which the center is the point at which the normal to the spheroid at latitude $\frac{1}{2}(\phi+\phi')$ intersects the axis of revolution of the spheroid. Since the value given above for N_M (compare with formula (9) on page 78) is the length of the normal at latitude $\frac{1}{2}(\phi+\phi')$ limited by the axis of revolution, the small circle on the sphere having the latitude $\frac{1}{2}(\phi+\phi')$ will coincide in space with the small circle of that latitude on the spheroid. Near that latitude the surface of the sphere and the surface of the spheroid are nearly in coincidence. In the approximate computations which were made the two points concerned in latitudes ϕ and ϕ' were transferred from the spheroid to the sphere by making the linear distance from latitude $\frac{1}{2}(\phi+\phi')$ to each point approximately the same on the sphere as on the spheroid, while their longitudes remain the same on the sphere as on the spheroid. It was then assumed that θ and the forward and back azimuths between the points as computed from a spherical triangle on the sphere are sufficiently close approximations to the true values of these quantities on the spheroid. The approximations are close because the parts of the sphere and spheroid which are concerned nearly coincide.

Let y be an arc of the meridian on the sphere described above which has the same length in meters as the arc of the meridian on the spheroid from latitude ϕ to latitude ϕ'.

The mean radius of curvature of the arc of the meridian between these latitudes on the spheroid is given by formula (7) for R on page 78. Hence the ratio of this radius of curvature to the radius of curvature of the sphere, N_M, is

$$\frac{R}{N_M} = \frac{1-e^2}{1-e^2\sin^2\frac{1}{2}(\phi+\phi')}$$

Hence

$$y = \frac{(1-e^2)(\phi-\phi')}{1-e^2\sin^2\frac{1}{2}(\phi+\phi')}$$

Let x be $\frac{y}{2}-\frac{1}{2}(\phi-\phi')$, that is, x is the correction to the arc $\frac{1}{2}(\phi-\phi')$ on the spheroid to obtain the arc $\frac{y}{2}$ of the same length in meters on the sphere.

Then

$$x = -\tfrac{1}{2}(\phi-\phi')\left(1-\frac{1-e^2}{1-e^2\sin^2\frac{1}{2}(\phi+\phi')}\right)$$
$$= -\tfrac{1}{2}(\phi-\phi')\frac{e^2\cos^2\frac{1}{2}(\phi+\phi')}{1-e^2\sin^2\frac{1}{2}(\phi+\phi')}$$

Let

$$a = 90^\circ-\phi \text{ and } c = 90^\circ-\phi'$$

Then, after calling the denominator $1-e^2\sin^2\frac{1}{2}(\phi+\phi')$ equal to unity, the above expression for x becomes

$$x = \tfrac{1}{2}(a-c)e^2\sin^2\tfrac{1}{2}(a+c)$$

If, then, in a spherical triangle on the sphere described, $B = \lambda'-\lambda = \omega$ is the angle at the pole and a' and c' are the adjacent sides

$$a' = a-x = 90^\circ-\phi-x$$
$$c' = c+x = 90^\circ-\phi'+x$$

The remaining parts of this spherical triangle, A', C', and b in the ordinary notation of spherical trigonometry, may be computed by Napier's analogies, namely,

$$\tan \tfrac{1}{2}(A' - C') = \sin \tfrac{1}{2}(a' - c') \csc \tfrac{1}{2}(a' + c') \cot \tfrac{1}{2}B$$
$$\tan \tfrac{1}{2}(A' + C') = \cos \tfrac{1}{2}(a' - c') \sec \tfrac{1}{2}(a' + c') \cot \tfrac{1}{2}B$$

and the law of sines, namely,

$$\sin b = \sin B \sin a' \csc A' = \sin B \sin c' \csc C'$$

using both forms for a check.

The required quantities are

$$\theta = b$$
$$\alpha_F = 180° - C'$$
$$\alpha_B = 180° + A'$$

in which C' and A' are to be considered positive or negative according as $B = \lambda' - \lambda = \omega$ is positive or negative.

The following example illustrates the method as applied to the longest line in this investigation, Meades Ranch, Kansas, to Calais, Maine, (latitude station No. 164, longitude station No. 173):

	° ′ ″		° ′ ″		
ϕ	39 13 26.7	$a' = a - x$	50 45 54	log sin B	9.71511
λ	98 32 30.5	$c' = c + x$	44 49 34	log sin a'	9.88905
ϕ'	45 11 05.7	$\frac{1}{2}(a' - c')$	2 58 10	log csc A'	0.00075
λ'	67 16 52.8	log sin $\frac{1}{2}(a' - c')$	8.71436	log sin θ	9.60491
$a = 90° - \phi$	50 46 33.3	log csc $\frac{1}{2}(a' + c')$	0.13033	log sin B	9.71511
$c = 90° - \phi'$	44 48 54.3	log cot $\frac{1}{2}B$	0.55319	log sin c'	9.84816
$\frac{1}{2}(a + c) = \frac{1}{2}(a' + c')$	47 47 43.8	log tan $\frac{1}{2}(A' - C')$	9.39788	log csc C'	0.04165
$\frac{1}{2}(a - c)$	2 58 49.5	log cos $\frac{1}{2}(a' - c')$	9.99942	log sin θ	9.60492
$B = \lambda' - \lambda = \omega$	−31 15 37.7	log sec $\frac{1}{2}(a' + c')$	0.17277	$\alpha_F = 180° - C'$	245° 18′ 26″
$\frac{1}{2}B$	−15 37 48.9	log cot $\frac{1}{2}B$	0.55319	$\alpha_B = 180° + A'$	86 37 26
$\frac{1}{2}(a - c)$ in seconds	10729.″5	log tan $\frac{1}{2}(A' + C')$	0.72538		
log $\frac{1}{2}(a - c)$ in seconds	4.0306	$\frac{1}{2}(A' - C')$	−14° 02′ 04″	θ	23° 44′ 36″
log sin² $\frac{1}{2}(a + c)$	9.7393	$\frac{1}{2}(A' + C')$	−79 20 30	θ in seconds	85476″
log e²	7.8305	A'	−93 22 34	log(θ in seconds)	4.93184
log x	1.6004	C'	−65 18 26	log A	8.50906
x	+39.″8			log s	6.42278

The length in meters of the arc θ on the sphere in question is $s = N_M \theta$, or if θ be expressed in seconds of arc,

$$s = N_M\ (\theta \text{ in seconds}) \sin 1'' = (\theta \text{ in seconds})\left(\frac{1}{A}\right)$$

in which

$$A = \frac{[1 - e^2 \sin^2 \frac{1}{2}(\phi + \phi')]^{\frac{1}{2}}}{a \sin 1''}$$

The logarithm of A is tabulated for $\frac{1}{2}(\phi + \phi')$ as an argument in Appendix 9 of the Coast and Geodetic Survey Report for 1894.

No complete proof is here offered as to the closeness with which the θ and s so computed approach to the true θ and s. The considerations given on page 88 indicate that the approximation is close. Moreover, for the longest line concerned in this investigation the values of log s, α_F, and α_B were computed by accurate formulæ. Their values as computed were

$$\alpha_F = 245°\ 18'\ 28''.01$$
$$\alpha_B = 86°\ 37'\ 32''.87$$
$$\log s = 6.4227742$$

Hence the errors of the approximate computation of these quantities, shown above, are:

For α_F, $-2''$, for α_B, $-7''$, and for log s, less than one unit in the last decimal place computed, the fifth.

These errors are so small as to have effects on the computed coefficients of the observation equations which are negligible in the present investigation.

From formula (11) page 78

$$\log (1 + Q) = \log \left[1 + \frac{\theta^2}{12} \cos^2 \tfrac{1}{2}(\alpha_B + \alpha_F) \right]$$

Expanding the right-hand member by the formula

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots\cdots$$

which is an expansion by Maclaurin's Theorem, and neglecting higher powers than the first,

$$\log(1+Q) = \frac{\theta^2}{12}\cos^2\tfrac{1}{2}(\alpha_B+\alpha_F)M\sin^2 1''$$

where M is the modulus of the common system of logarithms and $\sin^2 1''$ is introduced because θ is to be expressed in seconds.

Therefore

$$\log[\log(1+Q)] = \log\theta^2 + \log\cos^2\tfrac{1}{2}(\alpha_B+\alpha_F) + 7.92975 - 20$$

where $7.92975 - 20$ is $\log\frac{M\sin^2 1''}{12}$

Applied to the particular example in hand this becomes

$\alpha_B+\alpha_F$	331° 55′ 52″
$\frac{1}{2}(\alpha_B+\alpha_F)$	165 57 56
$\log\cos^2\frac{1}{2}(\alpha_B+\alpha_F)$	9.9737
log [θ^2 in seconds]	9.8637
log constant	7.9298
log [log (1 + Q)]	7.7672
log (1 + Q)	0.00585

Let $\frac{s(1+Q)}{2R\sin^2\frac{1}{2}(\alpha_B-\alpha_F)}$ be denoted by W, then

$$k_1+1 = \frac{\sin^2\omega}{\sin\theta}W \text{ and } m_1 = \sin\alpha_B(1+\cos\omega)W$$

in which

$$\frac{1}{R} = B\sin 1'' \text{ and } B = \frac{[1-e^2\sin^2\frac{1}{2}(\phi+\phi')]^{\frac{3}{2}}}{a(1-e^2)\sin 1''}$$

The logarithm of B is tabulated for $\frac{1}{2}(\phi+\phi')$ as an argument in Appendix 9 of the Coast and Geodetic Survey Report for 1894.

Therefore $\log\frac{1}{2R} = \log B + 4.38454 - 10$

The coefficients k_1, m_1, and n_1, for the case in hand, may now be computed as follows:

$\alpha_B-\alpha_F$	−158° 41′ 00″	cos ω	0.85482
$\frac{1}{2}(\alpha_B-\alpha_F)$	− 79 20 30	log (1 + cos ω)	0.26830
log $\csc^2\frac{1}{2}(\alpha_B-\alpha_F)$	0.01512	log W	9.33897
log B	8.51068	log $\sin\alpha_B$	9.99925
log s	6.42278	log m_1	9.60652
log (1 + Q)	0.00585	m_1	0.4041
log constant	4.38454		
log W	9.33897	log [θ in seconds]	4.93184
log $\sin^2\omega$	9.43022	log $\cos\alpha_B$	8.77004
log csc θ	0.39508	log B	8.51068
log (k_1 + 1)	9.16427	log (constant*)	6.68616
k_1	−0.8540	log n_1	8.89872
		n_1	0.0792

The first term of θ_1 is n_1 times a constant, which constant is

$$\frac{a\sin^2\phi}{(1-e^2\sin^2\phi)\,2000000} = 1.2787$$

* This constant is $\frac{100\,N\sin 1''}{a}$.

The second term may be computed as follows:

$$\text{2nd term} = -[1+2\sin^2\phi+3\cos(\phi'+\phi)]\,\tfrac{1}{2}(\phi'-\phi)\,\frac{1}{B}\,(\text{constant*})$$

$$= -[1.79975+3\cos(\phi'+\phi)]\,\tfrac{1}{2}(\phi'-\phi)\,\frac{1}{B}\,(\text{constant*})$$

$\phi'+\phi$	84° 24′ 32″
$\frac{1}{2}(\phi'-\phi)=\frac{1}{2}(a-c)$	10729″.5
log cos $(\phi'+\phi)$	8.98869
log 3 cos $(\phi'+\phi)$	9.46581
3 cos $(\phi'+\phi)$	0.29229
log $[1+2\sin^2\phi+3\cos(\phi'+\phi)]$	0.32057
log $[\frac{1}{2}(\phi'-\phi)$ in seconds$]$	4.03058
log $\frac{1}{B}$	1.48932
log constant	4.21401
log 2nd term	0.05448
2nd term	−1.1337
1.2787 n_1 = 1st term	+0.1013
o_1	−1.0324

The following is the computation of the coefficients k_2, l_2, m_2, n_2, and o_2 for this same station, Calais:

log sinϕ'	9.85088
log sin ω	9.71511
log k_2	9.56599
k_2	+0.3681
$l_2 = -\cos\phi'$	−0.7048
log cosϕ'	9.84808
log sin ω	9.71511
log cos α_B	8.77004
log csc α_F	0.04165
log m_2	8.37488
m_2	+0.0237

log $[\theta$ in seconds$]$	4.93184
log sin α_B	9.99925
log (constant†)	5.19530
log n_2	0.12639
n_2	−1.3378
log cos² α_F	9.2418
log sin α_B	9.9992
log s	6.4228
log $[\theta^2$ in seconds$]$	9.8637
log (constant‡)	2.8863
log 2nd term of o_2	8.4138
2nd term of o_2	+0.0259
1.2787 n_2 = 1st term of o_2	−1.7107
o_2	−1.6848

As an example of the computation of the coefficients k_3, l_3, m_3, n_3, and o_3, take the azimuth station Cooper, Maine, for which

$$\phi' = 44°\ 59'\ 13''.48$$
$$\lambda' = 67\ \ 28\ \ 02.25$$
$$\omega = -31°\ 04'\ 28''$$

---

* This constant is $\dfrac{1}{20000\,N\,(1-e^2)^2 \sin 1''}$ and its logarithm is 4.21401-10.

† This constant is $\dfrac{100}{a}$.

‡ This constant is $\dfrac{(1-e^2\sin^2\phi)^{\frac{1}{2}}\cos^2\phi\,\sin 1''}{60000\,(1-e^2)^2\,a}$.

α_F, α_B, θ and s, computed in the same way as for Calais, are, in this case,

$$\alpha_F = 245^\circ\ 46'\ 48''$$
$$\alpha_B = 86\ \ 55\ \ 30$$
$$\theta = 23\ \ 36\ \ 04 = 84964''$$
$$\log s = 6.42018$$

$$l_3 \text{ is zero and } k_3 = -\frac{\cot\phi' \sin^2\omega}{\sin\alpha_F \sin\theta} = -\frac{\cos\phi' \sin\omega}{\sin\alpha_F \sin\theta}\ \frac{\sin\omega}{\sin\phi'}$$

In the spherical triangle shown on p. 79, the law of sines gives

$$\frac{\sin\omega}{\sin\theta} = \frac{\sin\alpha_F}{\sin c}$$

If c is replaced in this by its approximate value $90 - \phi'$ this becomes

$$\frac{\sin\omega}{\sin\theta} = \frac{\sin\alpha_F}{\cos\phi'} \text{ or } \frac{\cos\phi' \sin\omega}{\sin\alpha_F \sin\theta} = 1$$

Therefore, $k_3 = -\frac{\sin\omega}{\sin\phi'}$ with sufficient accuracy.

log sin ω	9.71278
log sin ϕ'	9.84939
log k_3	9.86339
k_3	+0.7301

log cos ω	9.93272
log cot ϕ'	0.00020
log csc α_F	0.04001
log sin α_B	9.99937
log m_3	9.97231
m_3	+0.9382

log [θ in seconds]	4.92924
log sin α_B	9.99937
log constant*	5.19530
log n_3	0.12391
n_3	−1.3302

log cos² α_F	9.2261
log sin α_B	9.9994
log s	6.4202
log [θ^2 in seconds]	9.8585
log constant †	2.8863
log 2nd term	8.3905
2nd term	+0.0246
1.2787n_3	−1.7009
o_3	−1.6763

FIVE LEAST SQUARE SOLUTIONS.

Five complete least square solutions were made of the problem of determining the most probable values of the constants representing the figure and size of the earth. From a comparison of the five solutions the most probable depth of compensation has also been derived.

For convenience, the five solutions have been designated by the letters B, E, H, G, and A. For each solution the 507 observation equations, corresponding to the 507 astronomic observations, were written in a form similar to that shown on page 74, which is the form for solution A. The forms for the equations for the other four solutions differ only in the absolute term, the last term on the left-hand side of each equation.

In solution B, the absolute term of each observation equation was the observed apparent deflection of the vertical, as shown in the second column of the tables on pages 48–56, minus the topographic deflection, as shown in the third column of those tables. If the computed topographic deflection were the actual deflection and if there were no errors of observation and no errors in the assumed values of the latitude, longitude, and azimuth at the initial station, Meades Ranch, and if the a and e² of the Clarke spheroid of 1866 expressed the figure and size of the earth precisely, each one of the absolute terms in these observation equations of the solution B would be zero. Solution B is, therefore, made upon the assumption that no isostatic compensation exists, that the portions of the continent above sea level are excesses of mass, and the oceans represent defects of mass. It may conveniently be considered to be a solution based upon the

* This constant is the same as that in n_2. † This constant is the same as that in o_2.

supposition that if isostatic compensation exists it is uniformly distributed through an infinite depth.

In solution E, the absolute term of each observation equation is the observed apparent deflection minus the deflection computed on the assumption that the isostatic compensation is complete and uniformly distributed through the depth 162.2 kilometers, as shown in the column so headed in the tables on pages 48–56.

In solution H, the assumption is that the compensation is complete and uniformly distributed through the depth 120.9 kilometers, and in solution G that the depth is 113.7 kilometers. These solutions are similar to solution E, the computed deflection which is subtracted from the observed apparent deflection being taken from a different column of the tables on pages 48–56.

If the isostatic compensation were complete and uniformly distributed through the depth assumed in connection with any one of the solutions E, H, or G, the residuals from that solution would be very small, being due simply to the errors of observation. The excess of the residuals above the average magnitude due to errors of observation is a measure of the degree of agreement between the assumption as to isostatic compensation and the fact.

In solution A, the absolute terms are simply the observed apparent deflections as shown in the tables on pages 48–56. The observation equations were, therefore, precisely as indicated on page 74, including the absolute terms. This is the solution usually made in connection with the arc method. No relation is postulated between the deflections of the vertical and the topography. This is equivalent to the assumption that there is complete isostatic compensation at depth zero; that there exists immediately below every elevation (either mountain or continent) the full compensating defect of density, and that at the very surface of the ocean floor there lies material of the excessive density necessary to compensate for the depression of this floor. Under no other condition can it be true that the observed deflections of the vertical are independent of the known topography.

The particular depths of compensation assumed in solutions E, H, and G depend mainly upon extensive preliminary investigations made largely for the purpose of obtaining an approximate idea of the most probable depth of compensation.

THE OBSERVATION EQUATIONS.

The observation equations for solution G are given below. In each column the symbols (ϕ), (λ), (α_E), (α_W), $\left(\frac{a}{100}\right)$ and $(10000e^2)$ should be considered as repeated down the column. The observation equations for the other four solutions differ from these only in the absolute terms, as already stated. The statements of the preceding paragraphs will enable one to reproduce the absolute terms of any equation of any solution as desired by using the tables on pages 12–19 and 48–56.

The equations are printed in four groups. The same geographic order is used in each group as in the tables on pages 48–56. The number of each station as printed enables one to identify the station in the tables on pages 12–19 and on illustration No. 13 at the end of this publication.

Observation Equations.

NORTHEASTERN GROUP—LATITUDES.

Station.					″
241	−0.961(ϕ)	+0.216(α_E)	+0.147($\frac{a}{100}$)	−0.569(10 000e²)	− 0.80 =DM
188	−0.964	+0.208	+0.115	−0.486	− 1.55 =DM
185	−0.965	+0.203	+0.094	−0.433	− 3.60 =DM
184	−0.966	+0.201	+0.081	−0.399	− 0.69 =DM
183	−0.962	+0.213	+0.065	−0.386	+ 1.66 =DM
182	−0.957	+0.225	+0.058	−0.399	+ 2.55 =DM
186	−0.957	+0.225	+0.058	−0.400	+ 2.56 =DM
187	−0.956	+0.227	+0.057	−0.400	+ 1.79 =DM
181	−0.948	+0.246	+0.079	−0.498	+ 3.13 =DM
180	−0.944	+0.256	+0.091	−0.547	+ 3.32 =DM
179	−0.941	+0.262	+0.108	−0.597	+ 2.73 =DM
178	−0.941	+0.262	+0.114	−0.611	+ 2.58 =DM
177	−0.934	+0.278	+0.110	−0.645	+ 1.55 =DM
176	−0.927	+0.292	+0.113	−0.692	+ 0.35 =DM
175	−0.925	+0.296	+0.123	−0.722	− 2.59 =DM

Observation Equations—Continued.

NORTHEASTERN GROUP—LATITUDES—Continued.

Station.					″	
174	$-0.923(\phi)$	$+0.300(\alpha_E)$	$+0.136(\frac{a}{100})$	$-0.757(10\,000e^2)$	-2.22	$=D_M$
173	-0.920	$+0.305$	$+0.170$	-0.836	$+0.32$	$=D_M$
169	-0.925	$+0.295$	$+0.086$	-0.646	-1.41	$=D_M$
170	-0.919	$+0.307$	$+0.072$	-0.654	$+1.15$	$=D_M$
168	-0.908	$+0.325$	$+0.015$	-0.595	$+0.26$	$=D_M$
166	-0.907	$+0.327$	$+0.040$	-0.656	$+0.17$	$=D_M$
171	-0.908	$+0.326$	$+0.079$	-0.732	-3.31	$=D_M$
172	-0.905	$+0.330$	$+0.111$	-0.807	$+0.16$	$=D_M$
165	-0.903	$+0.333$	$+0.129$	-0.850	-0.78	$=D_M$
167	-0.904	$+0.331$	$+0.160$	-0.900	$+3.29$	$=D_M$
164	-0.854	$+0.404$	$+0.079$	-1.032	$+4.26$	$=D_M$
163	-0.856	$+0.402$	$+0.072$	-1.010	-0.09	$=D_M$
158	-0.855	$+0.403$	$+0.052$	-0.976	$+3.45$	$=D_M$
162	-0.862	$+0.394$	$+0.076$	-0.984	-2.59	$=D_M$
157	-0.863	$+0.393$	$+0.050$	-0.931	$+1.01$	$=D_M$
161	-0.867	$+0.386$	$+0.083$	-0.965	-2.31	$=D_M$
159	-0.871	$+0.382$	$+0.081$	-0.944	$+1.06$	$=D_M$
156	-0.871	$+0.382$	$+0.057$	-0.899	-0.81	$=D_M$
160	-0.879	$+0.370$	$+0.097$	-0.926	-1.43	$=D_M$
155	-0.879	$+0.371$	$+0.067$	-0.874	$+0.94$	$=D_M$
153	-0.877	$+0.374$	$+0.044$	-0.840	$+1.92$	$=D_M$
152	-0.881	$+0.368$	$+0.050$	-0.829	$+2.58$	$=D_M$
154	-0.885	$+0.362$	$+0.072$	-0.848	$+1.91$	$=D_M$
149	-0.889	$+0.355$	$+0.052$	-0.784	$+0.76$	$=D_M$
151	-0.884	$+0.364$	$+0.027$	-0.763	$+1.98$	$=D_M$
150	-0.883	$+0.364$	$+0.013$	-0.738	-0.71	$=D_M$
148	-0.891	$+0.353$	$+0.026$	-0.720	$+0.53$	$=D_M$
147	-0.884	$+0.363$	-0.006	-0.693	-3.07	$=D_M$
144	-0.887	$+0.358$	-0.013	-0.658	-4.61	$=D_M$
145	-0.887	$+0.358$	-0.012	-0.658	-4.67	$=D_M$
142	-0.883	$+0.365$	-0.044	-0.612	-3.51	$=D_M$
248	-0.880	$+0.369$	-0.044	-0.635	-4.84	$=D_M$
146	-0.893	$+0.349$	$+0.004$	-0.657	-3.27	$=D_M$
143	-0.899	$+0.340$	$+0.001$	-0.615	-0.44	$=D_M$
141	-0.901	$+0.336$	-0.038	-0.513	$+0.27$	$=D_M$
140	-0.905	$+0.330$	-0.067	-0.421	-4.27	$=D_M$
139	-0.909	$+0.323$	-0.064	-0.401	-1.49	$=D_M$
138	-0.911	$+0.320$	-0.081	-0.348	$+1.60$	$=D_M$
137	-0.914	$+0.314$	-0.076	-0.340	$+3.68$	$=D_M$
136	-0.919	$+0.306$	-0.089	-0.275	$+6.94$	$=D_M$
107	-0.920	$+0.304$	-0.132	-0.155	-6.72	$=D_M$
109	-0.916	$+0.312$	-0.152	-0.129	-3.25	$=D_M$
108	$-0\ 917$	$+0.310$	-0.158	-0.104	-1.65	$=D_M$

NORTHEASTERN GROUP—LONGITUDES.

Station.						″	
232	$+0.189(\phi)$	$-0.731(\lambda)$	$+0.045(\alpha_E)$	$-0.701(\frac{a}{100})$	$-0.894(10\,000e^2)$	-6.58	$=D_P$
196	$+0.180$	-0.739	$+0.035$	-0.674	-0.860	-2.36	$=D_P$
195	$+0.174$	-0.744	$+0.029$	-0.659	-0.842	-1.84	$=D_P$
192	$+0.172$	-0.747	$+0.025$	-0.653	-0.834	-0.31	$=D_P$
191	$+0.192$	-0.749	$+0.018$	-0.732	-0.935	$+1.70$	$=D_P$
194	$+0.192$	-0.749	$+0.018$	-0.732	-0.936	-1.55	$=D_P$
190	$+0.213$	-0.741	$+0.024$	-0.802	-1.023	$+3.85$	$=D_P$
188	$+0.222$	-0.737	$+0.028$	-0.833	-1.062	$+5.58$	$=D_P$
187	$+0.229$	-0.733	$+0.033$	-0.853	-1.088	$+1.95$	$=D_P$
185	$+0.229$	-0.731	$+0.035$	-0.852	-1.086	-0.76	$=D_P$
184	$+0.244$	-0.730	$+0.033$	-0.907	-1.155	$+1.10$	$=D_P$
183	$+0.258$	-0.726	$+0.034$	-0.954	-1.213	$+0.92$	$=D_P$
182	$+0.264$	-0.723	$+0.037$	-0.973	-1.237	$+4.03$	$=D_P$
181	$+0.267$	-0.720	$+0.041$	-0.979	-1.244	$+2.69$	$=D_P$
179	$+0.275$	-0.711	$+0.052$	-0.996	-1.264	$+2.22$	$=D_P$
173	$+0.368$	-0.705	$+0.024$	-1.338	-1.685	$+4.68$	$=D_P$
172	$+0.350$	-0.710	$+0.025$	-1.276	-1.612	$+2.44$	$=D_P$
157	$+0.310$	-0.739	-0.004	-1.179	-1.500	$+3.29$	$=D_P$
158	$+0.313$	-0.743	-0.011	-1.198	-1.525	$+3.25$	$=D_P$
233	$+0.318$	-0.742	-0.013	-1.218	-1.550	-0.31	$=D_P$
108	$+0.247$	-0.775	-0.040	-0.996	-1.273	-3.09	$=D_P$
110	$+0.252$	-0.778	-0.046	-1.021	-1.306	-0.22	$=D_P$

NORTHEASTERN GROUP—AZIMUTHS.

Station.					″	
193	$+0.413(\phi)$	$+1.125(\alpha_E)$	$-0.690(\frac{a}{100})$	$-0.881(10\,000e^2)$	$+5.21$	$=D_P$
186	$+0.493$	$+1.069$	-0.852	-1.086	-4.15	$=D_P$
189	$+0.550$	$+1.037$	-0.967	-1.230	-7.03	$=D_P$
180	$+0.556$	$+1.013$	-0.996	-1.264	$+0.57$	$=D_P$
175	$+0.555$	$+1.051$	-0.963	-1.226	-0.93	$=D_P$

Observation Equations—Continued.

NORTHEASTERN GROUP—AZIMUTHS—Continued.

Station.					″
176	+0.579(ϕ)	+1.045(α_E)	−1.004($\frac{a}{100}$)	−1.278(10 000e²)	− 2.44 =Dp
174	+0.622	+1.047	−1.067	−1.359	− 4.01 =Dp
177	+0.610	+1.023	−1.069	−1.359	− 7.32 =Dp
178	+0.610	+1.008	−1.082	−1.374	− 6.04 =Dp
171	+0.730	+0.938	−1.330	−1.678	+ 1.22 =Dp
170	+0.736	+0.944	−1.333	−1.683	+ 0.88 =Dp
169	+0.718	+0.947	−1.304	−1.646	− 1.41 =Dp
168	+0.722	+0.956	−1.299	−1.643	+ 3.39 =Dp
167	+0.698	+0.960	−1.261	−1.594	− 1.39 =Dp
166	+0.704	+0.968	−1.261	−1.596	+ 0.01 =Dp
164	+0.684	+0.978	−1.223	−1.549	+ 1.04 =Dp
165	+0.694	+0.982	−1.232	−1.562	− 0.70 =Dp
163	+0.684	+0.987	−1.213	−1.538	+ 5.52 =Dp
162	+0.669	+0.986	−1.192	−1.522	+ 0.82 =Dp
160	+0.663	+1.001	−1.169	−1.494	− 3.06 =Dp
161	+0.682	+1.000	−1.197	−1.530	+ 5.22 =Dp
159	+0.665	+1.013	−1.160	−1.482	+ 1.73 =Dp
156	+0.689	+1.012	−1.196	−1.528	+ 3.02 =Dp
147	+0.683	+1.020	−1.179	−1.507	+ 0.11 =Dp
154	+0.686	+1.023	−1.180	−1.502	+ 3.55 =Dp
150	+0.681	+1.031	−1.166	−1.485	+ 4.99 =Dp
149	+0.684	+1.037	−1.164	−1.483	+ 6.44 =Dp
151	+0.693	+1.032	−1.182	−1.506	+ 1.08 =Dp
152	+0.706	+1.035	−1.198	−1.526	+ 6.96 =Dp
153	+0.710	+1.026	−1.212	−1.543	− 1.59 =Dp
155	+0.664	+1.025	−1.148	−1.460	− 3.30 =Dp
148	+0.650	+1.036	−1.116	−1.421	− 3.79 =Dp
146	+0.652	+1.055	−1.103	−1.407	− 5.48 =Dp
145	+0.649	+1.073	−1.084	−1.383	− 1.57 =Dp
144	+0.636	+1.090	−1.050	−1.341	+ 2.45 =Dp
143	+0.624	+1.094	−1.029	−1.314	+ 6.83 =Dp
142	+0.612	+1.109	−1.001	−1.280	+ 2.23 =Dp
109	+0.636	+1.135	−1.014	−1.298	+ 1.20 =Dp

SOUTHEASTERN GROUP—LATITUDES.

Station.					″
80	−0.963(ϕ)	+0.211(α_E)	−0.092($\frac{a}{100}$)	+0.011(10 000e²)	+ 0.63 =DM
81	−0.959	+0.221	−0.109	+0.033	+ 0.42 =DM
82	−0.957	+0.226	−0.118	+0.043	+ 2.64 =DM
83	−0.952	+0.238	−0.158	+0.124	− 0.23 =DM
84	−0.944	+0.256	−0.148	+0.046	− 0.74 =DM
114	−0.943	+0.259	−0.151	+0.047	+ 5.03 =DM
86	−0.939	+0.266	−0.163	+0.058	+ 4.51 =DM
87	−0.936	+0.273	−0.153	+0.008	− 1.40 =DM
89	−0.934	+0.277	−0.124	−0.084	+ 4.51 =DM
88	−0.938	+0.268	−0.112	−0.092	+ 1.62 =DM
90	−0.934	+0.276	−0.099	−0.152	+ 5.83 =DM
91	−0.932	+0.281	−0.106	−0.145	+ 5.36 =DM
92	−0.931	+0.284	−0.119	−0.122	+ 0.99 =DM
99	−0.930	+0.286	−0.121	−0.125	+ 0.39 =DM
94	−0.930	+0.285	−0.128	−0.099	− 0.84 =DM
95	−0.930	+0.285	−0.129	−0.098	− 1.09 =DM
93	−0.930	+0.285	−0.129	−0.096	− 2.04 =DM
96	−0.930	+0.285	−0.130	−0.094	− 1.91 =DM
97	−0.930	+0.286	−0.131	−0.094	− 3.39 =DM
98	−0.930	+0.286	−0.131	−0.094	− 2.67 =DM
100	−0.929	+0.287	−0.132	−0.097	− 1.43 =DM
102	−0.927	+0.290	−0.136	−0.097	− 2.18 =DM
103	−0.926	+0.292	−0.132	−0.119	− 1.56 =DM
101	−0.928	+0.290	−0.123	−0.129	+ 0.02 =DM
105	−0.925	+0.295	−0.117	−0.164	+ 2.97 =DM
106	−0.923	+0.298	−0.103	−0.212	− 1.33 =DM
104	−0.926	+0.293	−0.166	−0.021	− 2.03 =DM
112	−0.923	+0.298	−0.202	+0.062	− 0.58 =DM
111	−0.925	+0.295	−0.221	+0.130	− 2.10 =DM
110	−0.923	+0.298	−0.249	+0.207	− 2.12 =DM
113	−0.923	+0.299	−0.270	+0.267	− 1.54 =DM
85	−0.943	+0.259	−0.198	+0.185	+ 2.70 =DM
115	−0.949	+0.244	−0.237	+0.350	+ 4.02 =DM
116	−0.951	+0.239	−0.271	+0.470	+ 8.41 =DM
117	−0.955	+0.231	−0.295	+0.574	+ 3.41 =DM
120	−0.960	+0.216	−0.301	+0.633	+ 5.80 =DM
121	−0.965	+0.204	−0.317	+0.720	+ 2.97 =DM
123	−0.968	+0.194	−0.328	+0.782	+ 1.80 =DM
125	−0.969	+0.192	−0.346	+0.847	+ 2.27 =DM

Observation Equations—Continued.

SOUTHEASTERN GROUP—LATITUDES—Continued.

Station.						″	
126	−0.970(ϕ)		+0.191(α_E)	−0.354($\frac{n}{100}$)	+0.878(10 000e²)	+ 2.27	=DM
122	−0.973		+0.179	−0.316	+0.775	− 0.14	=DM
119	−0.977		+0.165	−0.205	+0.437	+ 2.04	=DM
118	−0.979		+0.159	−0.205	+0.448	+ 3.09	=DM
131	−0.981		+0.151	−0.286	+0.726	+ 1.94	=DM
124	−0.977		+0.167	−0.321	+0.815	+ 3.19	=DM
127	−0.977		+0.164	−0.372	+0.997	+ 1.29	=DM
128	−0.977		+0.165	−0.419	+1.171	+ 6.53	=DM
129	−0.982		+0.149	−0.443	+1.292	+ 1.15	=DM
130	−0.983		+0.142	−0.474	+1.423	− 1.99	=DM
132	−0.983		+0.142	−0.505	+1.545	+ 3.63	=DM
134	−0.983		+0.143	−0.531	+1.650	+ 6.81	=DM
133	−0.985		+0.136	−0.523	+1.627	+ 6.39	=DM
135	−0.989		+0.115	−0.538	+1.722	+ 6.03	=DM

SOUTHEASTERN GROUP—LONGITUDES.

Station.						″	
87	+0.180(ϕ)	−0.784(λ)	−0.036(α_E)	−0.735($\frac{a}{100}$)	−0.941(10 000e²)	+ 0.37	=DP
114	+0.206	−0.786	−0.046	−0.845	−1.081	+ 2.36	=DP
90	+0.211	−0.788	−0.050	−0.868	−1.112	+ 3.94	=DP
92	+0.217	−0.777	−0.034	−0.875	−1.120	+ 1.64	=DP
99	+0.230	−0.778	−0.039	−0.930	−1.190	− 1.82	=DP
100	+0.230	−0.778	−0.040	−0.931	−1.191	− 1.16	=DP
101	+0.230	−0.778	−0.040	−0.933	−1.193	− 3.97	=DP
118	+0.230	−0.778	−0.040	−0.933	−1.194	− 4.07	=DP
113	+0.218	−0.796	−0.066	−0.917	−1.173	− 1.91	=DP
119	+0.177	−0.811	−0.082	−0.768	−0.982	+ 3.02	=DP
130	+0.136	−0.832	−0.108	−0.618	−0.788	+ 1.03	=DP
123	+0.120	−0.807	−0.063	−0.513	−0.656	+ 0.47	=DP
131	+0.114	−0.844	−0.128	−0.535	−0.681	− 1.34	=DP
132	+0.101	−0.850	−0.136	−0.481	−0.612	− 1.80	=DP
137	+0.093	−0.860	−0.154	−0.460	−0.583	+ 3.65	=DP
141	+0.074	−0.866	−0.165	−0.372	−0.470	+ 0.08	=DP

SOUTHEASTERN GROUP—AZIMUTHS.

Station.						″	
85	+0.434(ϕ)		+1.194(α_E)	−0.683($\frac{a}{100}$)	−0.875(10 000e²)	− 4.96	=DP
86	+0.456		+1.195	−0.717	−0.918	− 1.16	=DP
88	+0.500		+1.204	−0.775	−0.992	− 3.32	=DP
89	+0.533		+1.184	−0.835	−1.068	− 1.43	=DP
91	+0.566		+1.170	−0.890	−1.140	+ 2.04	=DP
94	+0.567		+1.153	−0.903	−1.156	+ 2.81	=DP
95	+0.561		+1.142	−0.902	−1.154	+ 4.89	=DP
96	+0.570		+1.142	−0.916	−1.172	+ 5.41	=DP
102	+0.584		+1.142	−0.935	−1.196	− 5.99	=DP
97	+0.582		+1.147	−0.930	−1.190	− 3.86	=DP
103	+0.587		+1.147	−0.938	−1.200	− 8.97	=DP
105	+0.595		+1.145	−0.950	−1.215	−10.12	=DP
104	+0.591		+1.140	−0.947	−1.211	− 7.77	=DP
107	+0.601		+1.123	−0.975	−1.247	− 7.36	=DP
235	+0.641		+1.146	−1.013	−1.297	+ 1.62	=DP
106	+0.608		+1.156	−0.959	−1.227	− 9.89	=DP
117	+0.626		+1.167	−0.976	−1.250	− 4.34	=DP
116	+0.625		+1.180	−0.966	−1.236	+ 3.44	=DP
111	+0.638		+1.191	−0.976	−1.249	+ 2.22	=DP
115	+0.646		+1.200	−0.980	−1.254	− 4 70	=DP
93	+0.550		+1.206	−0.845	−1.081	+ 6.99	=DP
120	+0.528		+1.240	−0.794	−1.016	+ 1.94	=DP
121	+0.526		+1.262	−0.778	−0.995	− 1.62	=DP
122	+0.514		+1.284	−0.750	−0.959	+ 3.06	=DP
125	+0.485		+1.300	−0.703	−0.898	+ 1.56	=DP
126	+0.462		+1.319	−0.661	−0.844	− 1.91	=DP
128	+0.442		+1.334	−0.627	−0.801	+ 2.32	=DP
129	+0.442		+1.346	−0.622	−0.794	+ 3.69	=DP
127	+0.407		+1.338	−0.578	−0.738	+ 1.88	=DP
124	+0.359		+1.282	−0.534	−0.682	− 1.25	=DP
138	+0.340		+1.337	−0.487	−0.622	+ 5.47	=DP
133	+0.381		+1.348	−0.540	−0.689	+ 0.73	=DP
134	+0.385		+1.382	−0.532	−0.678	+ 1.21	=DP
135	+0.375		+1.430	−0.502	−0.639	+ 1.77	=DP
136	+0.363		+1.514	−0.461	−0.584	+ 6.35	=DP
139	+0.344		+1.511	−0.438	−0.555	+ 0.96	=DP

Observation Equations—Continued.

CENTRAL GROUP—LATITUDES.

Station.							
						″	
227	−0.972(ϕ)		+0.183(α_E)	+0.422($\frac{h}{100}$)	−0.955(10 000e²)	− 3.08	=DM
228	−0.976		+0.171	+0.438	−0.958	− 1.06	=DM
231	−0.976		+0.169	+0.468	−0.990	+ 2.00	=DM
232	−0.982		+0.145	+0.508	−1.005	+11.86	=DM
230	−0.985		+0.135	+0.475	−0.961	+11.80	=DM
229	−0.989		+0.116	+0.468	−0.936	− 1.47	=DM
223	−0.992		+0.100	+0.446	−0.897	− 7.34	=DM
219	−0.993		+0.092	+0.423	−0.862	− 2.05	=DM
213	−0.994		+0.088	+0.414	−0.846	+ 1.80	=DM
212	−0.994		+0.088	+0.411	−0.844	+ 1.99	=DM
211	−0.993		+0.091	+0.410	−0.843	+ 3.14	=DM
214	−0.993		+0.094	+0.412	−0.849	+ 0.81	=DM
221	−0.990		+0.110	+0.425	−0.878	− 3.80	=DM
216	−0.988		+0.119	+0.406	−0.861	− 0.41	=DM
226	−0.983		+0.144	+0.434	−0.923	− 7.62	=DM
225	−0.982		+0.145	+0.433	−0.922	− 3.49	=DM
224	−0.983		+0.144	+0.431	−0.918	−11.64	=DM
222	−0.985		+0.134	+0.417	−0.889	− 4.03	=DM
220	−0.984		+0.137	+0.410	−0.884	− 1.44	=DM
217	−0.985		+0.136	+0.404	−0.876	+ 2.17	=DM
218	−0.983		+0.144	+0.402	−0.880	+ 7.90	=DM
189	−0.981		+0.150	+0.380	−0.858	+ 4.31	=DM
191	−0.981		+0.151	+0.380	−0.859	+ 4.35	=DM
215	−0.972		+0.183	+0.376	−0.897	+ 0.31	=DM
190	−0.972		+0.183	+0.363	−0.879	+ 1.36	=DM
194	−0.970		+0.187	+0.358	−0.878	+ 0.37	=DM
192	−0.969		+0.191	+0.357	−0.882	+ 0.92	=DM
193	−0.970		+0.188	+0.322	−0.825	− 1.28	=DM
197	−0.974		+0.175	+0.314	−0.794	− 3.80	=DM
196	−0.979		+0.160	+0.328	−0.794	− 5.29	=DM
210	−0.980		+0.155	+0.333	−0.797	− 5.71	=DM
195	−0.980		+0.154	+0.330	−0.791	− 6.11	=DM
198	−0.981		+0.152	+0.317	−0.767	− 4.94	=DM
200	−0.979		+0.157	+0.314	−0.769	− 3.25	=DM
199	−0.979		+0.158	+0.314	−0.771	− 3.72	=DM
201	−0.980		+0.155	+0.308	−0.757	− 1.21	=DM
202	−0.982		+0.148	+0.299	−0.734	+ 0.44	=DM
203	−0.981		+0.149	+0.297	−0.732	+ 2.05	=DM
204	−0.983		+0.142	+0.270	−0.677	− 0.05	=DM
205	−0.985		+0.133	+0.215	−0.567	+ 3.42	=DM
206	−0.977		+0.164	+0.134	−0.449	+ 3.51	=DM
209	−0.982		+0.148	+0.120	−0.394	+ 1.86	=DM
207	−0.983		+0.144	+0 113	−0.373	+ 1.50	=DM
247	−0.984		+0.139	+0.024	−0.158	+ 1.42	=DM
208	−0.983		+0.144	+0.017	−0.150	+ 1.24	=DM
62	−0.999		−0.041(α_W)	−0.020	+0.043	− 0.23	=DM
63	−1.000		−0.003	−0.021	+0.054	+ 0.19	=DM
64	−1.000		+0.004	−0.028	+0.073	− 1.33	=DM
65	−1.000		+0.013	−0.021	+0.055	− 4.31	=DM
66	−0.999		+0.034	−0.012	+0.024	− 0.10	=DM
67	−0.998		+0.054	−0.011	+0.014	+ 0.06	=DM
68	−0.995		+0.078	−0.053	+0.109	+ 2.98	=DM
69	−0.994		+0.086	−0.047	+0.086	+ 2.23	=DM
70	−0.989		+0.113	−0.048	+0.069	+ 4.66	=DM
71	−0.987		+0.124	−0.055	+0.065	+ 6.04	=DM
72	−0.984		+0.140	−0.043	+0.011	+ 2.86	=DM
73	−0.983		+0.141	−0.047	+0.020	+ 3.09	=DM
74	−0.983		+0.142	−0.063	+0.062	+ 1.79	=DM
75	−0.982		+0.149	−0.060	+0.044	+ 1.31	=DM
76	−0.977		+0.166	−0.040	−0.039	+ 0.34	=DM
77	−0.973		+0.180	−0.053	−0.030	+ 0.89	=DM
78	−0.970		+0.190	−0.053	−0.050	− 0.83	=DM
79	−0.967		+0.196	−0.081	+0.010	− 0.20	=DM

CENTRAL GROUP—LONGITUDES.

Station.							
						″	
213	+0.082(ϕ)	−0.685(λ)	+0.127(α_E)	−0.283($\frac{h}{100}$)	−0.359(10 000e²)	− 4.30	=DP
202	+0.140	−0.688	+0.116	−0.485	−0.614	− 7.50	=DP
198	+0.178	−0.688	+0.109	−0.619	−0.784	− 5.85	=DP
201	+0.142	−0.699	+0.101	−0.500	−0.634	− 4.34	=DP
206	+0.134	−0.706	+0.092	−0.478	−0.608	− 3.45	=DP
207	+0.128	−0.713	+0.083	−0.458	−0.583	+ 0.95	=DP
210	+0.127	−0.744	+0.037	−0.477	−0.609	− 1.95	=DP
211	+0.124	−0.746	+0.034	−0.466	−0.596	+ 0.93	=DP

Observation Equations—Continued.

CENTRAL GROUP—LONGITUDES—*Continued.*

Station.							″	
64	$-0.033(\phi)$	$-0.778(\lambda)$		$-0.006(\alpha_W)$	$+0.133(\frac{a}{100})$	$+0.170(10\,000e^2)$	-1.47	$=D_P$
66	$+0.003$	-0.780	$-0.009(\alpha_E)$		-0.014	-0.018	-5.02	$=D_P$
69	$+0.044$	-0.776	-0.003		-0.173	-0.221	-0.89	$=D_P$
74	$+0.090$	-0.781	-0.015		-0.364	-0.466	$+2.07$	$=D_P$
77	$+0.114$	-0.782	-0.019		-0.459	-0.586	-0.60	$=D_P$
79	$+0.119$	-0.781	-0.018		-0.481	-0.614	-4.12	$=D_P$
83	$+0.154$	-0.776	-0.016		-0.615	-0.786	-0.19	$=D_P$

CENTRAL GROUP—AZIMUTHS.

Station.						″	
205	$+0.318(\phi)$	$+1.020(\alpha_E)$		$-0.592(\frac{a}{100})$	$-0.748(10\,000e^2)$	$+1.36$	$=D_P$
214	$+0.155$	$+1.056$		-0.283	-0.359	-5.48	$=D_P$
208	$+0.160$	$+1.058$		-0.293	-0.371	-8.28	$=D_P$
204	$+0.239$	$+1.045$		-0.440	-0.557	-9.32	$=D_P$
200	$+0.338$	$+1.035$		-0.619	-0.784	-5.23	$=D_P$
199	$+0.332$	$+1.049$		-0.601	-0.763	-3.80	$=D_P$
203	$+0.277$	$+1.061$		-0.500	-0.634	-3.29	$=D_P$
209	$+0.263$	$+1.085$		-0.465	-0.592	$+2.74$	$=D_P$
215	$+0.248$	$+1.109$		-0.429	-0.546	$+4.15$	$=D_P$
212	$+0.279$	$+1.144$		-0.466	-0.596	$+0.25$	$=D_P$
65	-0.007		$+1.235(\alpha_W)$	$+0.011$	$+0.014$	$+8.36$	$=D_P$
67	$+0.026$	$+1.235$		-0.041	-0.052	$+6.13$	$=D_P$
68	$+0.068$	$+1.229$		-0.108	-0.138	-0.64	$=D_P$
70	$+0.162$	$+1.240$		-0.252	-0.322	$+1.62$	$=D_P$
71	$+0.178$	$+1.235$		-0.279	-0.356	-3.60	$=D_P$
72	$+0.202$	$+1.232$		-0.317	-0.405	-4.73	$=D_P$
73	$+0.230$	$+1.232$		-0.359	-0.459	-2.82	$=D_P$
75	$+0.256$	$+1.225$		-0.402	-0.514	-4.41	$=D_P$
76	$+0.287$	$+1.213$		-0.453	-0.579	-0.73	$=D_P$
78	$+0.292$	$+1.221$		-0.459	-0.586	$+0.06$	$=D_P$
80	$+0.322$	$+1.209$		-0.509	-0.650	-1.72	$=D_P$
81	$+0.338$	$+1.198$		-0.537	-0.687	-1.60	$=D_P$
82	$+0.368$	$+1.196$		-0.583	-0.745	$+2.01$	$=D_P$
84	$+0.404$	$+1.199$		-0.637	-0.814	$+4.39$	$=D_P$

WESTERN GROUP—LATITUDES.

Station.					″	
240	$-0.947(\phi)$	$-0.249(\alpha_W)$	$-0.443(\frac{a}{100})$	$+1.043(10\,000e^2)$	-3.65	$=D_M$
239	-0.947	-0.250	-0.445	$+1.047$	-1.84	$=D_M$
267	-0.947	-0.250	-0.438	$+1.019$	$+0.17$	$=D_M$
266	-0.940	-0.266	-0.415	$+0.883$	$+5.97$	$=D_M$
265	-0.941	-0.263	-0.398	$+0.830$	$+6.02$	$=D_M$
246	-0.941	-0.263	-0.390	$+0.802$	$+6.38$	$=D_M$
245	-0.941	-0.263	-0.380	$+0.765$	-7.04	$=D_M$
236	-0.931	-0.284	-0.394	$+0.741$	$+4.71$	$=D_M$
237	-0.928	-0.290	-0.400	$+0.742$	$+2.78$	$=D_M$
264	-0.935	-0.276	-0.377	$+0.712$	-0.26	$=D_M$
238	-0.932	-0.281	-0.374	$+0.680$	-6.03	$=D_M$
235	-0.929	-0.288	-0.373	$+0.655$	-4.93	$=D_M$
262	-0.927	-0.291	-0.378	$+0.661$	-3.02	$=D_M$
234	-0.927	-0.292	-0.372	$+0.635$	$+2.95$	$=D_M$
243	-0.929	-0.287	-0.350	$+0.578$	-6.75	$=D_M$
242	-0.926	-0.293	-0.356	$+0.576$	$+0.55$	$=D_M$
261	-0.926	-0.294	-0.341	$+0.521$	$+0.98$	$=D_M$
260	-0.926	-0.294	-0.341	$+0.521$	$+3.02$	$=D_M$
257	-0.923	-0.298	-0.318	$+0.428$	-6.00	$=D_M$
258	-0.922	-0.300	-0.321	$+0.430$	-3.30	$=D_M$
259	-0.922	-0.300	-0.321	$+0.430$	-5.83	$=D_M$
256	-0.921	-0.289	-0.296	$+0.388$	-0.08	$=D_M$
255	-0.925	-0.295	-0.280	$+0.315$	-8.36	$=D_M$
244	-0.921	-0.303	-0.295	$+0.337$	-4.15	$=D_M$
233	-0.920	-0.305	-0.277	$+0.268$	$+5.89$	$=D_M$
254	-0.918	-0.309	-0.274	$+0.245$	$+3.27$	$=D_M$
253	-0.922	-0.300	-0.252	$+0.208$	-1.85	$=D_M$
19	-0.920	-0.305	-0.232	$+0.129$	-2.98	$=D_M$
21	-0.920	-0.304	-0.224	$+0.108$	-5.40	$=D_M$
16	-0.918	-0.309	-0.206	$+0.037$	-6.60	$=D_M$
13	-0.914	-0.315	-0.216	$+0.043$	-5.92	$=D_M$
12	-0.914	-0.315	-0.217	$+0.044$	-2.10	$=D_M$
10	-0.914	-0.315	-0.217	$+0.043$	-1.55	$=D_M$
11	-0.914	-0.315	-0.217	$+0.044$	-1.81	$=D_M$
9	-0.913	-0.317	-0.211	$+0.021$	-3.91	$=D_M$
4	-0.910	-0.322	-0.212	$+0.004$	-6.59	$=D_M$
3	-0.910	-0.323	-0.196	-0.047	-4.89	$=D_M$
7	-0.909	-0.324	-0.186	-0.079	-5.01	$=D_M$

Observation Equations—Continued.

WESTERN GROUP—LATITUDES—Continued.

Station.						″	
1	−0.905(ϕ)		−0.331(α_W)	−0.171($\frac{n}{1000}$)	−0.152(10 000e²)	− 0.96	=DM
2	−0.904		−0.332	−0.151	−0.210	− 3.45	=DM
252	−0.900		−0.339	−0.097	−0.382	− 0.98	=DM
251	−0.901		−0.337	−0.078	−0.421	− 4.27	=DM
249	−0.901		−0.337	−0.023	−0.550	− 3.58	=DM
5	−0.908		−0.325	−0.152	−0.178	− 1.08	=DM
6	−0.911		−0.320	−0.168	−0.115	− 3.49	=DM
8	−0.912		−0.318	−0.171	−0.097	− 4.48	=DM
14	−0.916		−0.312	−0.166	−0.089	− 1.84	=DM
15	−0.916		−0.311	−0.181	−0.043	− 4.94	=DM
17	−0.918		−0.308	−0.162	−0.087	− 5.66	=DM
18	−0.918		−0.307	−0.169	−0.063	− 6.40	=DM
20	−0.920		−0.305	−0.134	−0.152	− 4.52	=DM
24	−0.935		−0.276	−0.174	+0.059	− 0.57	=DM
23	−0.930		−0.285	−0.143	−0.059	− 1.52	=DM
25	−0.931		−0.284	−0.126	−0.102	− 2.76	=DM
27	−0.932		−0.282	−0.113	−0.131	− 5.53	=DM
22	−0.928		−0.289	−0.104	−0.179	− 3.60	=DM
26	−0.931		−0.284	−0.095	−0.186	− 4.29	=DM
28	−0.941		−0.262	−0.074	−0.173	− 6.05	=DM
29	−0.946		−0.251	−0.108	−0.052	− 4.45	=DM
30	−0.949		−0.246	−0.055	−0.176	− 5.56	=DM
31	−0.955		−0.231	−0.052	−0.146	−16.96	=DM
32	−0.962		−0.213	−0.134	+0.120	−10.24	=DM
33	−0.963		−0.208	−0.127	+0.114	− 1.29	=DM
37	−0.970		−0.190	−0.101	+0.082	− 4.33	=DM
48	−0.977		−0.165	−0.098	+0.121	− 0.83	=DM
35	−0.964		−0.206	−0.024	−0.160	− 4.39	=DM
34	−0.963		−0.208	+0.041	−0.324	− 3.49	=DM
39	−0.971		−0.187	+0.069	−0.344	− 3.34	=DM
42	−0.972		−0.181	+0.068	−0.330	− 3.18	=DM
44	−0.973		−0.180	+0.067	−0.327	− 1.67	=DM
40	−0.972		−0.184	+0.052	−0.299	− 4.72	=DM
41	−0.973		−0.180	+0.051	−0.289	− 2.18	=DM
43	−0.973		−0.180	+0.043	−0.271	− 6.01	=DM
38	−0.970		−0.189	+0.021	−0.235	− 5.35	=DM
46	−0.973		−0.178	−0.010	−0.139	− 6.08	=DM
36	−0.970		−0.189	−0.044	−0.072	− 5.08	=DM
45	−0.973		−0.179	−0.046	−0.045	− 6.12	=DM
47	−0.975		−0.174	−0.046	−0.036	− 0.61	=DM
49	−0.979		−0.159	−0.018	−0.083	− 4.99	=DM
50	−0.979		−0.157	−0.046	−0.008	− 3.23	=DM
51	−0.983		−0.144	−0.066	+0.067	+ 1.72	=DM
52	−0.983		−0.141	−0.009	−0.080	− 0.66	=DM
53	−0.985		−0.135	−0.033	−0.008	− 1.02	=DM
54	−0.988		−0.121	−0.084	+0.150	− 7.93	=DM
55	−0.989		−0.113	−0.055	+0.080	− 1.73	=DM
56	−0.989		−0.116	−0.030	+0.008	− 8.10	=DM
57	−0.991		−0.104	−0.059	+0.102	− 2.86	=DM
58	−0.994		−0.088	−0.032	+0.044	− 2.51	=DM
59	−0.994		−0.085	−0.032	+0.046	− 1.05	=DM
60	−0.995		−0.080	−0.024	+0.028	− 2.39	=DM
61	−0.996		−0.068	−0.037	+0.073	− 1.06	=DM

WESTERN GROUP—LONGITUDES.

Station.						″	
246	−0.172(ϕ)	−0.841(λ)	−0.135(α_W)	+0.811($\frac{n}{1000}$)	+1.034(10 000e²)	+ 1.89	=DP
216	−0.173	−0.841	−0.135	+0.811	+1.034	+ 0.66	=DP
243	−0.189	−0.828	−0.115	+0.857	+1.094	+ 1.57	=DP
217	−0.189	−0.829	−0.115	+0.857	+1.095	+ 1.41	=DP
9	−0.238	−0.795	−0.070	+0.999	+1.279	− 0.21	=DP
8	−0.248	−0.790	−0.065	+1.032	+1.321	− 0.29	=DP
7	−0.248	−0.790	−0.066	+1.033	+1.322	− 0.12	=DP
6	−0.248	−0.790	−0.066	+1.034	+1.323	+ 0.78	=DP
1	−0.267	−0.778	−0.052	+1.085	+1.388	+ 1.29	=DP
3	−0.264	−0.796	−0.046	+1.065	+1.362	− 0.82	=DP
16	−0.243	−0.782	−0.050	+0.993	+1.270	+ 0.18	=DP
10	−0.247	−0.776	−0.040	+0.997	+1.275	+ 0.49	=DP
20	−0.229	−0.778	−0.038	+0.927	+1.186	− 2.72	=DP
23	−0.229	−0.777	−0.037	+0.923	+1.181	− 6.15	=DP
24	−0.229	−0.775	−0.034	+0.919	+1.176	+ 1.00	=DP
25	−0.228	−0.774	−0.031	+0.914	+1.170	+ 1.54	=DP
22	−0.233	−0.771	−0.029	+0.929	+1.188	− 2.27	=DP
33	−0.202	−0.772	−0.021	+0.804	+1.029	+ 8.95	=DP
31	−0.190	−0.772	−0.018	+0.757	+0.968	+ 0.51	=DP

Observation Equations—Continued.

WESTERN GROUP—LONGITUDES—Continued.

Station.						″	
41	−0.153(ϕ)	−0.752(λ)	+0.021(α_W)	+0.586($\frac{a}{100}$)	+0.749(10 000e²)	+ 2.20	=Dp
44	−0.151	−0.757	+0.013	+0.582	+0.744	+ 1.27	=Dp
36	−0.154	−0.774	−0.014	+0.614	+0.785	+ 5.90	=Dp
50	−0.127	−0.777	−0.014	+0.507	+0.649	− 1.51	=Dp
53	−0.110	−0.776	−0.010	+0.437	+0.560	+ 1.47	=Dp
55	−0.091	−0.782	−0.017	+0.366	+0.468	+ 1.76	=Dp
61	−0.069	−0.779	−0.010	+0.275	+0.352	− 4.84	=Dp

WESTERN GROUP—AZIMUTHS.

Station.						″	
245	−0.593(ϕ)		+1.359(α_W)	+0.814($\frac{a}{100}$)	+1.039(10 000e²)	+23.21	=Dp
244	−0.592		+1.354	+0.815	+1.039	+16.65	=Dp
231	−0.597		+1.314	+0.844	+1.078	+15.75	=Dp
230	−0.599		+1.309	+0.849	+1.084	+13.94	=Dp
234	−0.605		+1.309	+0.856	+1.094	+ 5.18	=Dp
242	−0.628		+1.287	+0.900	+1.151	+12.42	=Dp
229	−0.639		+1.279	+0.919	+1.175	+10.28	=Dp
228	−0.651		+1.288	+0.928	+1.186	+12.51	=Dp
227	−0.665		+1.285	+0.948	+1.212	+12.29	=Dp
226	−0.652		+1.271	+0.940	+1.202	+ 8.29	=Dp
224	−0.644		+1.258	+0.939	+1.201	+12.12	=Dp
225	−0.660		+1.266	+0.955	+1.222	+ 4.91	=Dp
223	−0.657		+1.255	+0.957	+1.224	+ 4.06	=Dp
241	−0.655		+1.245	+0.962	+1.230	+20.99	=Dp
222	−0.633		+1.226	+0.945	+1.209	+ 5.10	=Dp
220	−0.640		+1.211	+0.965	+1.236	+ 7.68	=Dp
221	−0.659		+1.210	+0.991	+1.268	− 2.25	=Dp
219	−0.658		+1.198	+0.999	+1.278	+ 0.02	=Dp
239	−0.665		+1.193	+1.010	+1.293	− 6.61	=Dp
240	−0.663		+1.181	+1.017	+1.302	+ 8.02	=Dp
218	−0.642		+1.190	+0.982	+1.258	+ 3.11	=Dp
17	−0.642		+1.172	+0.996	+1.275	− 0.86	=Dp
13	−0.646		+1.158	+1.011	+1.294	+ 1.31	=Dp
238	−0.660		+1.158	+1.030	+1.318	− 8.81	=Dp
5	−0.663		+1.151	+1.040	+1.330	+ 2.80	=Dp
237	−0.668		+1.132	+1.062	+1.358	+10.02	=Dp
2	−0.664		+1.114	+1.069	+1.367	+ 3.70	=Dp
236	−0.657		+1.128	+1.050	+1.343	+12.84	=Dp
4	−0.653		+1.132	+1.041	+1.332	+ 9.82	=Dp
11	−0.642		+1.136	+1.022	+1.308	+ 2.50	=Dp
12	−0.643		+1.144	+1.018	+1.302	+ 0.68	=Dp
15	−0.634		+1.143	+1.006	+1.287	+ 5.32	=Dp
14	−0.633		+1.138	+1.008	+1.290	+ 5.92	=Dp
26	−0.577		+1.177	+0.901	+1.153	+ 0.50	=Dp
19	−0.586		+1.154	+0.930	+1.189	− 2.31	=Dp
21	−0.580		+1.147	+0.927	+1.186	+ 3.95	=Dp
18	−0.585		+1.132	+0.945	+1.208	− 5.62	=Dp
27	−0.529		+1.145	+0.854	+1.093	− 4.38	=Dp
28	−0.514		+1.170	+0.817	+1.045	+ 7.06	=Dp
29	−0.494		+1.150	+0.799	+1.022	+ 4.19	=Dp
30	−0.466		+1.161	+0.751	+0.960	+ 2.11	=Dp
34	−0.434		+1.213	+0.675	+0.864	+ 5.94	=Dp
35	−0.414		+1.166	+0.669	+0.856	+ 8.56	=Dp
32	−0.408		+1.137	+0.676	+0.864	+ 8.64	=Dp
38	−0.363		+1.139	+0.604	+0.773	− 0.52	=Dp
42	−0.353		+1.143	+0.586	+0.749	+ 5.20	=Dp
40	−0.350		+1.144	+0.581	+0.743	+ 0.81	=Dp
43	−0.352		+1.151	+0.581	+0.743	+ 4.84	=Dp
45	−0.354		+1.154	+0.582	+0.744	+ 4.05	=Dp
39	−0.361		+1.148	+0.596	+0.762	+ 3.05	=Dp
37	−0.375		+1.158	+0.613	+0.784	+ 9.97	=Dp
46	−0.357		+1.178	+0.576	+0.737	+ 9.18	=Dp
47	−0.354		+1.197	+0.563	+0.720	+ 7.32	=Dp
48	−0.344		+1.226	+0.535	+0.684	+ 5.78	=Dp
49	−0.321		+1.192	+0.513	+0.657	+ 7.35	=Dp
51	−0.298		+1.222	+0.466	+0.597	+ 8.60	=Dp
52	−0.285		+1.197	+0.456	+0.584	+ 3.65	=Dp
54	−0.276		+1.210	+0.437	+0.560	+ 6.92	=Dp
57	−0.251		+1.241	+0.389	+0.498	+ 3.64	=Dp
56	−0.234		+1.230	+0.366	+0.468	+10.03	=Dp
58	−0.236		+1.217	+0.374	+0.478	+ 6.08	=Dp
59	−0.215		+1.235	+0.336	+0.429	+10.96	=Dp
60	−0.181		+1.227	+0.284	+0.364	+ 4.83	=Dp
62	−0.164		+1.226	+0.259	+0.331	+ 2.23	=Dp
63	−0.128		+1.226	+0.202	+0.259	+ 1.38	=Dp

WHY TWO CORRECTIONS TO THE INITIAL AZIMUTH.

Thus far in this publication there has been no apparent recognition of the Laplace equation connecting longitude and azimuth observations. The usual form of expression of this equation is (astronomic azimuth—geodetic azimuth) + sin ϕ (astronomic longitude—geodetic longitude) = 0. In the present investigation the same principle may be expressed by the statement that the two observation equations, one a longitude equation and the other an azimuth equation, for a given station at which both the astronomic longitude and astronomic azimuth were observed, should show the same residual, or unexplained prime vertical component of the deflection of the vertical. Small differences between such pairs of residuals will exist on account of the errors of the astronomic observations and other accidental errors. If, however, there has been a considerable accumulation of error in the geodetic azimuth, as carried through the triangulation by the adjusted angles, it will be put in evidence by differences between the longitude and azimuth residuals at coincident stations too large to be accounted for by the accidental errors, and by a systematic tendency for such differences to be of one sign for considerable areas.

There are 11 stations concerned in the present investigation at which a longitude station coincides with an azimuth station. A preliminary examination made in the manner just indicated showed that there had been, somewhere between Illinois and Colorado, in the transcontinental triangulation, an accumulation of about 5″ of error in the geodetic azimuth, carried through the adjusted angles. The evidence available was not sufficient to determine the places at which the accumulation occurred with any greater accuracy than indicated in the preceding sentence. The station already adopted as the initial, Meades Ranch, happened to be in the middle portion of this section within which it was apparent that the accumulation took place. Under these conditions it seemed that the simple device of introducing into the equations two unknowns, representing corrections to the initial azimuth at Meades Ranch, instead of one, would, with little additional work, take account automatically of the supposed twist in azimuth, and determine its amount substantially as well as any more complicated method. The required correction (α_E) to the initial azimuth was introduced into all equations pertaining to stations east of Meades Ranch, and the required correction (α_W) into all equations pertaining to points west of Meades Ranch. This is equivalent to assuming that at Meades Ranch, in carrying the computation of the azimuth through the triangulation from east to west, an error of $(\alpha_E)-(\alpha_W)$ was suddenly introduced into the geodetic azimuth at Meades Ranch.

The necessary introduction of an extra unknown into the equations in the manner indicated, thus virtually inserting a hinge in the triangulation midway between the Atlantic and the Pacific, has, of course, made the determination of the figure and size of the earth weaker than it otherwise would have been. The weakening has been properly taken into account in deriving the probable errors. These probable errors show this to be a very strong determination in spite of this weakening.

MEANINGS OF COEFFICIENTS IN OBSERVATION EQUATIONS.

The meanings of the coefficients in the observation equations have already been explained in a general way (pp. 74, 75). With the numerical values of the coefficients before one in the printed observation equations it is possible to make this explanation more definite and concrete.

It is assumed in fixing the form of the observation equations that a small correction (ϕ) is to be applied to the initial geodetic latitude at Meades Ranch, Kansas, and it is required to express by the proper coefficients k_1, k_2, k_3, the effect which this change would have on the apparent deflections of the vertical, such as are expressed in the absolute terms of the observation equations. A similar treatment is to be given to the initial longitude, initial azimuth, the assumed equatorial radius, and the assumed flattening.

The relations expressed by these coefficients may be seen by examining the formulæ from which the coefficients are computed (see p. 76). To visualize these relations and to obtain a more concrete conception of their meanings and laws of variation, imagine a model spheroid to be made to scale to represent the earth and imagine the 507 observation stations corre-

sponding to the observation equations to be plotted on the surface of this spheroid in their proper relative positions. Imagine also that a frame or cage is made to scale of wires so as to fit closely over this model spheroid, each meridian and each parallel of latitude being represented by a wire. By the adoption in the computations of geodetic positions of certain values for the initial latitude, initial longitude, and initial azimuth at Meades Ranch, thus fixing the United States Standard Datum, and the adoption of the Clarke spheroid of 1866, the exact position of the supposed wire cage and its size and shape have been fixed.

Imagine the wire cage to be in position on the model spheroid. Suppose, now, that without any other change being made the cage is so moved that the wire representing the meridian through Meades Ranch is moved southward by the amount (ϕ), without change of direction of this wire (which would correspond to a change of the initial azimuth) and without any lateral shift (which would correspond to a change of the initial longitude). Such a motion will consist, evidently, of a rotation of the cage about one of its diameters terminating in its equator at two points which are 90° from Meades Ranch in longitude. The geodetic latitude of Meades Ranch, read from the cage, will be increased by (ϕ). Evidently the geodetic latitude of any other point in the meridian of Meades Ranch will be increased by the same amount. So, also, it is evident that for other points on either side of this meridian, the change will be smaller the greater the distance from this meridian, to longitude differences as great as 90°. Note that, in the latitude observation equations, the coefficient k_1 is sensibly unity (-1.000) at latitude stations Nos. 63, 64, and 65 (p. 97), which are nearly in the same meridian as Meades Ranch, and that k_1 decreases numerically in proceeding either to the eastward or westward. The minimum value of k_1 ($-.854$) occurs at latitude station 164, Calais, Maine (p. 94), and it is also small ($-.900$) at latitude station No. 252, Cape Mendocino, California (p. 99).

The minus sign in the coefficients arises from the fact that $-\phi'$ enters in the observation equations, not ϕ' directly.

The wire cage device gives one a good idea of the essential meaning of each coefficient and an approximate conception of its laws of variation. In this device it is not easy to take into account the small departures of the spheroid from the sphere, but said departures are accurately taken into account in the formulæ for the coefficients.

The small displacement of the wire cage, as described, will evidently not change the longitude of any point on the spheroid which lies in the same meridian as Meades Ranch, and will not change the longitude of any point on the equator (because every meridian line will move across the equator parallel to that portion of itself). Virtually the meridians will be crowded more closely together everywhere in the United States as the north pole of the cage is made to approach Meades Ranch and all differences of longitude, reckoned from Meades Ranch, will be increased, the changes tending to be numerically greater the greater the longitude difference from Meades Ranch and the greater the latitude of the station. Hence, k_2, expressing the relation between (ϕ) and $\lambda' \cos \phi'$ (see form of longitude equation, p. 74), changes sign through zero in the meridian of Meades Ranch, as indicated by the longitude observation equations for longitude stations 64 and 66, in Kansas (p. 98), has a maximum positive value ($+.368$) at longitude station 173, Calais, Maine (see p. 94), far to the eastward and in a high latitude, and has a maximum negative value ($-.267$) at longitude station No. 1, Point Arena, California (p. 99).

Similar relations hold with respect to the coefficient k_3, expressing the relation between (ϕ), the correction to the initial latitude, and $\alpha' \cot \phi'$, except that on account of the factor $\cot \phi'$ there is a tendency for k_3 to increase with decrease of latitude. Between azimuth stations 65 and 67, in Kansas (p. 98), k_3 changes sign at the meridian of Meades Ranch; it has a maximum positive value ($+.736$) at azimuth station No. 170, Howard, Maine (p. 95), the azimuth station farthest to the eastward, although this station is in a high latitude, and has a maximum negative value ($-.668$) at azimuth station No. 237, Ross Mountain, California (p. 100).

Let the wire cage be imagined to be again in its position as fixed by the adopted United States Standard Datum. Suppose, now, that, without any other change being made, the cage

is so moved that the wire representing the parallel of latitude through Meades Ranch is moved eastward, along itself, by the amount (λ), without change of direction of the meridian through Meades Ranch (corresponding to a change of the initial azimuth), and without any lateral shift in the parallel of latitude (corresponding to a change in the initial latitude). Such a motion will consist, evidently, of a rotation of the wire cage about its polar axis through an angle (λ). Such a motion will produce no change in latitude nor in azimuth at any point. Hence, the coefficients l_1 and l_3 are zero for all stations. The longitudes of all points will be changed by the same amount (λ). Hence, l_2, expressing the relation between (λ), the correction to the initial longitude, and $-\lambda'\cos\phi'$, is simply $-\cos\phi'$. This coefficient, l_2, is therefore negative in all parts of the area covered by this investigation, is a maximum ($-.866$) at the longitude station which is farthest south, No. 141, New Orleans, Louisiana (p. 96), and a minimum ($-.685$) at the longitude station which is farthest north, No. 213, Minnesota Point North Base, Minnesota (p. 97).

To shift the wire cage from its position as fixed by the United States Standard Datum in such a way as to correspond to a change (α) in the initial azimuth without any other change, one must evidently rotate the cage counter-clockwise about its diameter passing through Meades Ranch. This will decrease all latitudes to the eastward, and increase all latitudes to the westward of Meades Ranch, the change being greater the greater the distance and the difference of longitude from Meades Ranch. The coefficient m_1, expressing the relation between (α) and $-\phi'$, is found, therefore, to pass through zero between latitude stations 63 and 64, in Kansas (p. 97), to have a positive maximum ($+.404$) at latitude station No. 164, Calais, Maine (p. 94), this being the station farthest east and most distant from Meades Ranch, and to have a negative maximum ($-.339$) at latitude station 252, Cape Mendocino, California (p. 99).

So, too, by considering the motion of the wire cage described, it is evident that one should expect the coefficient m_2, expressing the relation between (α) and $-\lambda'\cos\phi'$, to have a positive maximum ($+.127$) at the longitude station farthest north, No. 213, Minnesota Point North Base, Minnesota (p. 97), and to have a negative maximum ($-.165$) at the longitude station farthest south, No. 141, New Orleans, Louisiana (p. 96). The stations at which the values of m_2 should be zero are evidently those at which the arc of a great circle from Meades Ranch to the station is perpendicular to the meridian at the station, or, in other words, at which the azimuth of Meades Ranch from the station is either 90° or 270°. Such stations are all farther north than Meades Ranch, the excess of latitude being greater the greater the difference of longitude between the station and Meades Ranch. For these changes of sign, see page 94 for the northeastern group, pages 97 and 98 for the central group, and pages 99 and 100 for the western group. Consult also illustration No. 13 at the end of the volume.

Similarly, if one considers the motion of the wire cage corresponding to a change (α) in the initial azimuth, it is evident that the azimuth at all stations tends to change by amounts nearly equal to (α), but decreasing somewhat as the distance from Meades Ranch increases, since for points at 90° on a great circle from Meades Ranch the change becomes zero. There all wires of the cage move to new positions parallel to their old ones. On account of the factor $\cot\phi'$ the coefficient m_3, expressing the relation between (α) and $\alpha'\cot\phi'$, tends to be smaller the greater the latitude. One should expect, therefore, that m_3 would vary nearly as $\cot\phi'$ but tend to be smaller for the more distant stations. The minimum value ($+.938$) is found at azimuth station No. 171, Cooper, Maine (p. 95), though there are other azimuth stations, such as No. 205, Gargantua, Canada (p. 98), (for which $m_3 = +1.020$), in much higher latitudes but nearer to Meades Ranch. The maximum value ($+1.514$) occurs at azimuth station No. 136, Fort Morgan, Alabama (p. 96). This is the azimuth station which is farthest south.

An increase in a, the equatorial radius of the spheroid, without change of flattening and without any change in the initial latitude, longitude, or azimuth, is evidently expressed by an increase in size of the wire cage without change of shape or position. Such an increase in size would evidently tend to make all geodetic latitudes approach that of Meades Ranch. Hence, one should expect that n_1, expressing the relation between $\left(\frac{a}{100}\right)$ and $-\phi'$, to have a maximum positive value ($+.508$) for the latitude station which is farthest north, No. 232, St. Ignace,

Canada (p. 97), and a maximum negative value ($-.538$) for the latitude station which is farthest south, No. 135, New Orleans, Louisiana (p. 96). The zero values of n_1 should be expected at stations at which the parallel of latitude points directly toward Meades Ranch; that is, at which the azimuth of Meades Ranch from the station is 90° or 270°. For the points at which n_1 changes sign, see page 94 for the northeastern group, page 97 for the central group, and page 99 for the western group.

Similarly, from a consideration of the wire cage it is evident that an increase in the equatorial radius would tend to make the longitude of all stations approach that of Meades Ranch. One should expect, therefore, n_2, expressing the relation of $\left(\frac{a}{100}\right)$ to $-\lambda'\cos\phi'$, to pass through zero at the meridian of Meades Ranch (see longitude stations 64 and 66 in Kansas, p. 98), to be a negative maximum (-1.338) for the longitude station farthest to the northeast, No. 173, Calais, Maine (p. 94), and to be a positive maximum ($+1.085$) for the longitude station farthest to the west, No. 1, Point Arena, California (p. 99).

The effect of a change in the equatorial radius upon the azimuth at any station is $\sin\phi'$ times the change in longitude produced at that station, the factor $\sin\phi'$ serving always to convert the difference of longitude of two meridians into their convergence. The change from the factor $\cos\phi'$, in the absolute term of the longitude observation equation, to the factor $\cot\phi'$, in the absolute term of the azimuth equation, neutralizes the factor $\sin\phi'$ and, therefore, the law of variation of n_3, expressing the relation between $\left(\frac{a}{100}\right)$ and $\alpha'\cot\phi'$, is precisely the same as that for n_2. Hence, it is found that n_3 changes sign between azimuth stations Nos. 65 and 67, in Kansas (p. 98), has a negative maximum (-1.333) at the azimuth station farthest to the northeast, No. 170, Howard, Maine (p. 95), and has a positive maximum ($+1.069$) at the azimuth station farthest west, No. 2, Paxton, California (p. 100).

An increase in the flattening of the spheroid without change of equatorial radius and without any change in the initial latitude, longitude, or azimuth, is evidently expressed by a corresponding flattening of the wire cage. But in this case, the conception of the wire cage will help one but little in forming a concrete idea of the meaning of the coefficients o_1, o_2, and o_3. It may, however, aid one in recognizing the complexity of the relations which they express, for if one attempts to study the nature of the change of shape of the cage corresponding to an increase in the flattening it is soon evident that the radii of curvature of its different wires are all changed and the parallels of latitude are all shifted in position. Some radii of curvature are increased, as, for example, of given parallels of latitude near the equator and of arcs of the meridian near the pole, and some radii of curvature are decreased, as, for example, of arcs of the meridian near the equator.

The laws of variation of o_1, o_2, o_3, as determined by an inspection of their numerical values in the observation equations and checked by an examination of the formulæ from which these coefficients were computed, are here stated for comparison with the preceding paragraphs.

The coefficient o_1 depends mainly upon the latitude, but also tends to have greater negative or smaller positive values the greater the departure in longitude from Meades Ranch in either direction. For the latitude station No. 232, St. Ignace, Canada (p. 97), which has the highest latitude of any station, o_1 is -1.005, but this is slightly exceeded by o_1 (-1.032) at latitude station No. 164, Calais, Maine (p. 94), the fact that this station is in a considerably lower latitude than No. 232, being offset by the fact that it is much farther east. The maximum positive value of o_1 ($+1.722$) occurs at the latitude station which is farthest south, No. 135, New Orleans, Louisiana (p. 96). In the longitude of Meades Ranch the sign of o_1 changes at the latitude of Meades Ranch (39° 13′), but for stations to the eastward or to the westward it changes sign at a lower latitude, the extreme case being at the Pacific coast, where it changes sign in latitude 38° 01′.

The coefficients o_2 and o_3 follow nearly the same law of variation as the coefficients n_2 and n_3, respectively. They have their positive maximum and negative maximum values at the same stations and change sign between the same stations as n_2 and n_3. The extreme values for o_2 are -1.685 at longitude station No. 173, Calais, Maine (p. 94), and $+1.388$ at longitude station

No. 1, Point Arena, California (p. 99); the extreme values for o_3 are -1.683 at azimuth station No. 170, Howard, Maine (p. 95), and $+1.367$ at azimuth station No. 2, Paxton, California (p. 100).

THE NORMAL EQUATIONS AND VALUES OF UNKNOWNS.

The normal equations formed in the usual way from the observation equations for solution G are as follows:

Normal equations.—Solution G.

$$+286.69(\phi)-3.32(\lambda)+17.63(\alpha_E)-18.06(\alpha_W)-77.58\left(\frac{a}{100}\right)-71.75(10\,000e^2)-22.95=0$$

$$-3.32(\phi)+46.75(\lambda)+0.28(\alpha_E)+0.97(\alpha_W)+12.03\left(\frac{a}{100}\right)+15.25(10\,000e^2)-0.78=0$$

$$+17.63(\phi)+0.28(\lambda)+137.29(\alpha_E)-88.43\left(\frac{a}{100}\right)-127.73(10\,000e^2)-29.28=0$$

$$-18.06(\phi)+0.97(\lambda)+101.61(\alpha_W)+64.13\left(\frac{a}{100}\right)+73.25(10\,000e^2)+543.63=0$$

$$-77.58(\phi)+12.03(\lambda)-88.43(\alpha_E)+64.13(\alpha_W)+185.15\left(\frac{a}{100}\right)+187.82(10\,000e^2)+319.99=0$$

$$-71.75(\phi)+15.25(\lambda)-127.73(\alpha_E)+73.25(\alpha_W)+187.82\left(\frac{a}{100}\right)+371.18(10\,000e^2)+492.79=0$$

For convenience of comparison, the absolute terms of the normal equations for all five solutions are given here together.

Absolute terms.

∞ 162.2 120.9 113.7 0

Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
−1738.89	− 43.14	− 24.85	− 22.95	− 8.59
+ 142.81	+ 23.55	+ 3.26	− 0.78	− 45.87
+3962.61	+ 23.95	− 22.63	− 29.28	−168.10
−4292.95	+385.74	+519.94	+543.63	+964.19
−8596.18	+ .93.85	+286.97	+319.99	+879.34
−9861.02	+274.01	+461.86	+492.79	+998.15

The values of the unknowns derived from the five solutions are given below. The solutions gave directly the values of $\left(\frac{a}{100}\right)$ and $(10\,000e^2)$, but for convenience of reference there are given in these tables (a) and (e^2), the corrections to the Clarke equatorial radius and square of the eccentricity.

	(ϕ)	(λ)	(α_E)	(α_W)	(a)	(e^2)
	″	″	″	″	*Meters.*	
Solution B	+21.04	−16.54	+6.10	+10.53	+4 890	+0.000659
Solution E	+ .25	− .76	+ .63	− 4.68	+ 222	− .000064
Solution H	− .16	− .01	+ .21	− 5.29	+ 98	− .000066
Solution G	− .22	+ .13	+ .13	− 5.40	+ 76	− .000065
Solution A	− 1.20	+ 1.85	− .63	− 7.82	− 261	− .000035

Applying the corrections (a) and (e^2) to the Clarke 1866 values, and converting the results into the more common forms of expression, there are obtained the following values expressing the figure and size of the earth.

	Equatorial radius.	Reciprocal of flattening.	Polar semi-diameter.
	Meters.		*Meters.*
Solution B (extreme rigidity)	6 383 096	268.7	6 359 344
Solution E (depth of compensation 162.2 kilometers)	6 378 428	297.7	6 357 006
Solution H (depth of compensation 120.9 kilometers)	6 378 304	297.9	6 356 890
Solution G (depth of compensation 113.7 kilometers)	6 378 283	297.8	6 356 868
Solution A (depth of compensation zero)	6 377 945	296.5	6 356 435

THE RESIDUALS.

For the five solutions, the residuals corresponding to the quantities D_P and D_M in the second member of the observation equations, as indicated on page 74, are given in the following tables. These residuals are the portions of the deflections of the vertical which remain unexplained after each solution. The comparison of the residuals from different solutions furnishes the basis for a selection of the most probable solution. The residuals of the selected most probable solution contain the information as to the manner and extent to which the assumptions of that solution differ from the facts of nature.

A plus sign on the residual at a latitude station for a given solution indicates that the zenith at that station is deflected to the northward of the position in which it would be if the assumptions on which that solution is based were correct. Similarly a plus sign on the residual at a longitude station or an azimuth station for a given solution indicates that the zenith at that station is deflected to the westward of the position in which it would be if the assumptions on which that solution is based were a correct representation of all the facts.

Residuals at latitude stations of northeastern group.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	0	162.2	120.9	113.7	0
	″	″	″	″	″
241	− 4.76	− .22	− .08	− .08	+ .25
188	− 6.32	−1.02	− .92	− .91	−1.04
185	− 9.19	−3.21	−3.03	−3.01	−2.88
184	− 6.69	− .37	− .15	− .13	+ .22
183	− 4.81	+1.81	+2.16	+2.20	+3.43
182	− 3.97	+2.60	+3.03	+3.09	+5.70
186	− 3.96	+2.61	+3.04	+3.10	+5.78
187	− 4.69	+1.83	+2.27	+2.33	+5.14
181	− 1.73	+3.25	+3.68	+3.76	+6.75
180	− 1.07	+3.41	+3.89	+3.99	+7.52
179	− .63	+2.95	+3.36	+3.44	+4.54
178	− .66	+2.80	+3.23	+3.31	+4.97
177	− 1.02	+1.70	+2.20	+2.30	+5.26
176	− .17	+ .81	+1.09	+1.13	+3.13
175	− 2.05	−2.00	−1.82	−1.78	−1.30
174	− 1.41	−1.69	−1.42	−1.38	+1.39
173	+ 2.50	+1.10	+1.22	+1.24	+1.50
169	− 2.83	−1.14	− .76	− .68	+2.85
170	+ 2.98	−1.59	+1.82	+1.87	+4.56
168	+ 8.15	+1.36	+ .94	+ .90	+ .78
166	+ 6.23	+ .86	+ .86	+ .87	+2.10
171	+ 2.36	−2.35	−2.49	−2.53	−4.41
172	+ 4.32	+ .91	+ .98	+1.01	+ .23
165	+ 2.96	− .16	+ .09	+ .12	+1.25
167	+ 7.44	+3.95	+4.22	+4.24	+4.72
164	+17.53	+5.61	+5.31	+5.23	+4.64
163	+13.53	+1.26	+ .94	+ .87	.00
158	+18.55	+4.79	+4.45	+4.37	+3.02
162	+10.24	−1.24	−1.59	−1.65	−2.82
157	+14.40	+2.38	+1.98	+1.90	+ .39
161	+ 9.34	− .98	−1.31	−1.38	−2.33
159	+12.45	+2.36	+2.04	+1.98	+ .94
156	+11.62	+ .45	+ .11	+ .06	−1.08
160	+ 9.56	− .01	− .41	− .51	−2.53
155	+12.41	+2.21	+1.87	+1.80	+ .64
153	+14.42	+3.14	+2.81	+2.74	+1.27
152	+14.42	+3.82	+3.46	+3.40	+1.77
154	+12.80	+3.24	+2.85	+2.76	+ .54
149	+11.18	+1.94	+1.64	+1.55	+1.73
151	+13.93	+3.08	+2.81	+2.74	+1.32
150	+11.66	+ .33	+ .07	+ .02	− .95
148	+11.23	+1.60	+1.34	+1.26	+ .51
147	+ 9.52	−2.14	−2.36	−2.38	−2.85
144	+ 7.70	−3.71	−3.91	−3.95	−4.23
145	+ 7.65	−3.77	−3.97	−4.01	−4.35
142	+10.71	−2.67	−2.86	−2.90	−2.13
248	+ 9.50	−4.03	−4.19	−4.22	−4.06
146	+ 7.67	−2.28	−2.55	−2.60	−2.75
143	+10.04	+ .63	+ .29	+ .20	−1.19

Residuals at latitude stations of northeastern group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
141	+12.12	+1.32	+ .90	+ .82	−1.47
140	+ 7.12	−3.41	−3.74	−3.80	−4.23
139	+ 7.53	− .72	−1.00	−1.03	−1.78
138	+ 9.79	+2.26	+2.04	+2.01	+2.60
137	+11.95	+4.39	+4.13	+4.09	+3.11
136	+14.18	+7.58	+7.32	+7.29	+5.83
107	− 3.07	−6.50	−6.49	−6.48	−5.97
109	+ .63	−3.05	−3.06	−3.04	−2.72
108	+ 1.77	−1.51	−1.49	−1.46	− .70

Residuals at longitude stations of northeastern group.

	″	″	″	″	″
232	−10.43	−6.87	−6.70	−6.66	− 6.31
196	− 5.10	−2.59	−2.47	−2.44	− 2.28
195	− 3.98	−2.05	−1.95	−1.92	− 2.00
192	− 2.23	− .51	− .42	− .39	− .40
191	− 2.77	+1.17	+1.57	+1.62	+ 3.50
194	− 6.01	−2.08	−1.67	−1.64	+ .38
190	− 3.67	+3.15	+3.66	+3.77	+ 6.05
188	− 2.93	+4.84	+5.39	+5.50	+ 8.52
187	− 6.88	+1.31	+1.78	+1.87	+ 3.96
185	− 9.62	−1.38	− .93	− .84	+ .67
184	− 8.62	+ .54	+ .94	+1.02	+ 1.65
183	−11.57	.00	+ .71	+ .84	+ 3.86
182	−10.21	+2.86	+3.77	+3.95	+10.40
181	−11.96	+1.59	+2.44	+2.61	+ 7.30
179	−12.89	+1.40	+2.04	+2.14	+ 3.88
173	−21.07	+3.85	+4.47	+4.59	+ 5.91
172	−18.72	+1.64	+2.23	+2.35	+ 3.89
157	− 7.40	+2.98	+3.16	+3.21	+ 3.21
158	− 7.63	+2.85	+3.11	+3.16	+ 3.34
233	−11.84	− .76	− .44	− .39	+ .64
108	− .63	−3.25	−3.18	−3.18	− 2.89
110	+ 4.27	− .19	− .30	− .31	+ .06

Residuals at azimuth stations of northeastern group.

	″	″	″	″	″
193	+ 1.49	+5.06	+5.28	+5.31	+ 5.73
186	−13.23	−4.61	−4.15	−4.06	− 2.34
189	−20.64	−7.99	−7.12	−6.95	− 2.29
180	−14.53	− .11	+ .54	+ .64	+ 2.60
175	−11.84	−1.57	− .96	− .85	+ 1.35
176	−13.16	−3.03	−2.49	−2.36	− .84
174	−10.83	−4.15	−3.95	−3.94	− 3.60
177	−19.46	−7.66	−7.33	−7.25	− 8.74
178	−20.29	−6.30	−6.02	−5.97	−12.83
171	−21.69	+ .52	+1.14	+1.26	+ 2.68
170	−20.59	+ .14	+ .79	+ .92	+ 2.55
169	−22.64	−2.11	−1.49	−1.37	+ .09
168	−17.45	+2.71	+3.31	+3.44	+ 3.85
167	−20.46	−2.01	−1.46	−1.34	+ .08
166	−18.17	− .53	− .05	+ .06	+ 1.34
164	−15.03	+ .73	+1.03	+1.10	+ 1.65
165	−16.56	−1.09	− .73	− .64	+ .69
163	− 8.84	+5.31	+5.54	+5.57	+ 4.45
162	−14.12	+ .67	+ .86	+ .89	+ .22
160	−15.90	−3.27	−3.02	−2.99	− 2.03
161	− 6.74	+5.09	+5.27	+5.29	+ 5.12
159	− 8.34	+1.70	+1.79	+1.80	+ .48
156	− 7.53	+2.85	+3.03	+3.09	+ 3.09
147	− 8.75	− .02	+ .14	+ .18	+ .30
154	− 5.03	+3.38	+3.56	+3.61	+ 4.46
150	− 2.81	+4.86	+5.02	+5.05	+ 4.92
149	− .68	+6.29	+6.47	+6.50	+ 5.57
151	− 6.75	+ .85	+1.10	+1.14	+ 1.94

Residuals at azimuth stations of northeastern group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
152	− .98	+6.64	+6.98	+7.02	+ 8.40
153	−10.27	−1.93	−1.59	−1.53	− .51
155	−11.85	−3.41	−3.26	−3.24	− 5.38
148	−11.39	−3.94	−3.76	−3.72	− 3.00
146	−10.74	−5.59	−5.45	−5.41	− 7.58
145	− 4.08	−1.64	−1.52	−1.50	− 1.10
144	+ 2.35	+2.48	+2.53	+2.53	+ 3.35
143	+ 6.94	+6.84	+6.91	+6.91	+ 7.11
142	+ 4.52	+2.33	+2.32	+2.31	+ 2.07
109	+ 8.80	+1.50	+1.31	+1.28	+ 1.28

Residuals at latitude stations of southeastern group.

	″	″	″	″	″
80	− 9.56	+ .29	+ .71	+ .79	+1.48
81	− 9.67	− .16	+ .45	+ .56	+2.64
82	− 7.37	+1.91	+2.63	+2.76	+5.96
83	− 8.13	− .65	− .26	− .19	−1.16
84	− 5.03	− .66	− .63	− .64	−1.19
114	+ 1.19	+5.06	+5.11	+5.13	+5.53
86	+ 2.21	+4.56	+4.59	+4.59	+4.43
87	− 2.60	−1.39	−1.28	−1.28	− .40
89	+ 3.46	+4.62	+4.71	+4.71	+3.81
88	− .97	+1.52	+1.79	+1.84	+2.90
90	+ 5.62	+6.01	+6.07	+6.09	+5.02
91	+ 6.08	+5.59	+5.62	+5.62	+5.02
92	+ 1.90	+1.18	+1.23	+1.22	+ .50
99	+ 1.97	+ .62	+ .63	+ .62	− .03
94	− .15	− .70	− .63	− .63	−1.64
95	− .43	− .96	− .88	− .88	−1.98
93	− 1.42	−1.91	−1.85	−1.83	−2.84
96	− 1.29	−1.79	−1.72	−1 71	−2.17
97	− 2.72	−3.26	−3.20	−3.19	−3.49
98	− 1.98	−2.54	−2.48	−2.47	−2.53
100	− .39	−1.29	−1.24	−1.23	− .72
102	− .52	−2.06	−1.98	−1.98	−1.47
103	+ .59	−1.39	−1.34	−1.34	−1.11
101	+ 2.19	+ .26	+ .26	+ .25	+ .34
105	+ 6.35	+3.32	+3.24	+3.23	+3.15
106	+ 3.32	− .85	−1.01	−1.03	−2.68
104	− 2.23	−2.05	−1.93	−1.90	−1.23
112	+ 1.13	− .63	− .56	− .53	+ .27
111	− .93	−2.22	−2.14	−2.11	−1.30
110	− .61	−2.27	−2.23	−2.20	−1.00
113	+ .02	−1.75	−1.70	−1.68	− .71
85	− .72	+2.65	+2.67	+2.67	+2.66
115	− 1.11	+3.93	+3.87	+3.85	+3.02
116	+ 2.26	+8.07	+8.12	+8.14	+8.89
117	− 3.09	+2.98	+3.05	+3.05	+3.34
120	− .31	+5.65	+5.45	+5.40	+3.69
121	− 4.47	+2.64	+2.53	+2.50	+ .67
123	− 7.76	+1.07	+1.24	+1.28	+ .71
125	− 7.36	+1.40	+1.63	+1.69	+2.24
126	− 7.52	+1.29	+1.58	+1.67	+2.90
122	−11.02	−1.17	− .73	− .65	− .64
119	−10.80	+1.08	+1.74	+1.84	+4.21
118	− 9.71	+2.16	+2.78	+2.88	+5.23
131	−10.05	+ .77	+1.37	+1.48	+3.81
124	− 7.53	+2.14	+2.55	+2.65	+2.02
127	− 8.71	− .01	+ .51	+ .59	+3.55
128	− 3.03	+5.06	+5.60	+5.68	+7.65
129	− 9.06	− .49	+ .10	+ .20	+2.35
130	−11.66	−3.69	−3.15	−3.05	− .75
132	− 5.42	+1.95	+2.38	+2.47	+4.52
134	− 1.68	+5.14	+5.48	+5.56	+7.54
133	− 2.28	+4.65	+5.07	+5.16	+7.28
135	− 2.30	+4.04	+4.62	+4.73	+7.34

Residuals at longitude stations of southeastern group.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
87	− 1.70	− .39	+ .17	+ .27	+2.76
114	+ 8.35	+2.85	+2.36	+2.27	−1.33
90	+10.59	+4.44	+3.96	+3.85	+ .57
92	+ 5.46	+2.06	+1.65	+1.55	−1.57
99	+ 1.55	−1.77	−1.90	−1.91	−3.10
100	+ 2.37	−1.12	−1.24	−1.25	−1.80
101	− .41	−3.93	−4.04	−4.06	−3.91
118	− .52	−4.03	−4.15	−4.16	−4.21
113	+ 5.52	−1.82	−1.97	−2.00	−2.46
119	+ 8.99	+3.24	+2.96	+2.92	+1.83
130	+ 3.71	+ .83	+ .90	+ .92	+1.22
123	+ 1.09	+ .15	+ .43	+ .37	+ .77
131	+ .47	−1.56	−1.34	−1.46	−1.01
132	+ .56	−1.93	−1.80	−1.92	−1.78
137	+ 7.43	+3.64	+3.65	+3.53	+3.21
141	+ 7.99	+ .30	+ .10	− .05	− .64

Residuals at azimuth stations of southeastern group.

	″	″	″	″	″
85	− 6.10	−5.14	− 4.91	− 4.85	− 3.82
86	− 2.40	−1.52	− 1.12	− 1.05	+ .31
88	− 1.44	−3.47	− 3.30	− 3.22	− 3.34
89	+ 5.34	− .78	− 1.24	− 1.33	− 3.13
91	+ 9.79	+2.67	+ 2.22	+ 2.13	+ .93
94	+ 7.49	+3.12	+ 2.93	+ 2.90	+ 2.59
95	+ 8.58	+5.22	+ 5.01	+ 4.98	+ 4.09
96	+ 9.05	+5.65	+ 5.52	+ 5.50	+ 6.54
102	− 1.70	−5.71	− 5.90	− 5.90	− 6.61
97	+ 1.24	−3.58	− 3.73	− 3.77	− 4.61
103	− 3.59	−8.72	− 8.87	− 8.88	− 8.55
105	− 4.66	−9.91	−10.02	−10.03	−10.23
104	− 3.10	−7.53	− 7.66	− 7.68	− 8.07
107	− 4.30	−7.25	− 7.27	− 7.28	− 7.02
235	+11.27	+2.05	+ 1.74	+ 1.70	+ 1.80
106	− 2.16	−9.68	− 9.81	− 9.80	−10.02
117	+ 6.65	−3.89	− 4.22	− 4.25	− 4.04
116	+15.99	+3.95	+ 3.58	+ 3.53	+ 3.59
111	+17.45	+2.95	+ 2.38	+ 2.31	+ 1.88
115	+12.41	−3.84	− 4.52	− 4.61	− 5.15
93	+15.47	+7.62	+ 7.17	+ 7.09	+ 6.15
120	+10.87	+2.60	+ 2.15	+ 2.04	+ .62
121	+ 7.05	−1.03	− 1.43	− 1.52	− 2.19
122	+10.94	+3.59	+ 3.23	+ 3.17	+ 2.57
125	+ 8.26	+2.18	+ 1.73	+ 1.67	− .02
126	+ 3.57	−1.48	− 1.74	− 1.79	− 4.00
128	+ 6.34	+2.44	+ 2.43	+ 2.42	+ 2.72
129	+ 7.96	+3.85	+ 3.81	+ 3.81	+ 4.17
127	+ 3.27	+1.91	+ 1.98	+ 2.01	+ 1.30
124	− 1.38	−1.36	− 1.17	− 1.12	− .12
138	+ 6.57	+5.52	+ 5.59	+ 5.60	+ 6.23
133	+ 2.19	+ .77	+ .81	+ .86	+ 1.31
134	+ 3.59	+1.30	+ 1.33	+ 1.34	+ 2.21
135	+ 5.04	+2.00	+ 1.94	+ 1.91	+ 2.27
136	+11.90	+6.85	+ 6.56	+ 6.50	+ 6.28
139	+ 7.13	+1.47	+ 1.16	+ 1.11	+ .79

Residuals at latitude stations of central group.

	″	″	″	″	″
227	+ .01	− 1.47	− 1.81	− 1.90	− 3.78
228	+ 2.20	+ .60	+ .23	+ .14	− 1.03
231	+ 6.45	+ 3.86	+ 3.35	+ 3.24	+ 1.07
232	+16.77	+13.86	+13.26	+13.14	+ 9.50
230	+14.77	+13.56	+13.11	+13.02	+11.48
229	+ 1.16	+ .43	− .13	− .27	− 5.06
223	− 6.77	− 5.61	− 6.07	− 6.18	−10.11

Residuals at latitude stations of central group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
219	− 2.81	− .53	− .86	− .93	− 3.66
213	+ .63	+ 3.24	+ 2.95	+ 2.90	+ .42
212	+ .42	+ 3.34	+ 3.12	+ 3.08	+ 1.98
211	+ 1.27	+ 4.40	+ 4.27	+ 4.23	+ 4.18
214	− .97	+ 2.03	+ 1.94	+ 1.91	+ 2.23
221	− 4.95	− 2.60	− 2.65	− 2.67	− 2.60
216	− 2.91	+ .42	+ .65	+ .69	+ 2.79
226	− 7.26	− 6.41	− 6.43	− 6.45	− 5.65
225	− 3.06	− 2.26	− 2.30	− 2.32	− 2.09
224	−11.43	−10.43	−10.46	−10.48	−10.18
222	− 5.29	− 3.05	− 2.91	− 2.90	− 2.12
220	− 2.77	− .49	− .33	− .32	+ .20
217	+ .40	+ 3.05	+ 3.24	+ 3.28	+ 4.65
218	+ 6.46	+ 8.78	+ 8.98	+ 9.02	+10.85
189	+ 3.15	+ 5.26	+ 5.36	+ 5.40	+ 6.84
191	+ 3.25	+ 5.30	+ 5.40	+ 5.44	+ 6.77
215	+ 1.60	+ 1.69	+ 1.46	+ 1.42	+ .76
190	+ 2.19	+ 2.65	+ 2.49	+ 2.45	+ 2.06
194	+ 1.33	+ 1.69	+ 1.49	+ 1.45	+ 1.62
192	+ 2.11	+ 2.29	+ 2.05	+ 2.01	+ .89
193	− 1.06	− .05	− .22	− .26	− 1.09
197	− 4.35	− 2.68	− 2.77	− 2.80	− 3.58
196	− 5.68	− 4.04	− 4.23	− 4.29	− 5.60
210	− 6.27	− 4.46	− 4.65	− 4.70	− 5.92
195	− 6.75	− 4.87	− 5.05	− 5.11	− 6.51
198	− 5.99	− 3.72	− 3.92	− 3.96	− 5.14
200	− 4.16	− 2.06	− 2.23	− 2.27	− 3.17
199	− 4.55	− 2.53	− 2.70	− 2.74	− 3.39
201	− 2.24	− .05	− .20	− .25	− .91
202	− .92	+ 1.57	+ 1.43	+ 1.38	+ .41
203	+ .68	+ 3.18	+ 3.04	+ 2.99	+ 2.09
204	− 2.48	+ .94	+ .85	+ .83	+ .47
205	− .83	+ 4.17	+ 4.19	+ 4.19	+ 3.98
206	− 2.39	+ 3.91	+ 4.14	+ 4.14	+ 4.96
209	− 4.52	+ 2.26	+ 2.41	+ 2.44	+ 2.87
207	− 5.13	+ 1.87	+ 2.06	+ 2.06	+ 2.62
247	− 7.03	+ 1.47	+ 1.73	+ 1.78	+ 2.34
208	− 7.30	+ 1.30	+ 1.55	+ 1.59	+ 2.05
62	−10.51	− .32	+ .10	+ .18	+ 1.46
63	− 9.31	− .14	+ .31	+ .37	+ 1.69
64	−10.84	− 1.69	− 1.24	− 1.18	− .03
65	−13.56	− 4.62	− 4.20	− 4.14	− 2.92
66	− 9.10	− .37	+ .04	+ .10	+ 1.97
67	− 8.76	− .15	+ .21	+ .27	+ 1.91
68	− 7.56	+ 2.48	+ 3.00	+ 3.10	+ 4.03
69	− 7.95	+ 1.84	+ 2.31	+ 2.37	+ 4.02
70	− 5.19	+ 4.37	+ 4.75	+ 4.81	+ 5.63
71	− 3.60	+ 5.83	+ 6.13	+ 6.19	+ 6.86
72	− 6.77	+ 2.72	+ 3.00	+ 3.05	+ 3.71
73	− 6.59	+ 2.94	+ 3.22	+ 3.27	+ 3.92
74	− 8.26	+ 1.54	+ 1.87	+ 1.92	+ 2.76
75	− 8.67	+ 1.08	+ 1.41	+ 1.47	+ 2.33
76	− 9.16	+ .25	+ .52	+ .57	+ 1.22
77	− 8.62	+ .82	+ 1.06	+ 1.10	+ 1.12
78	−10.54	− .91	− .62	− .60	− .06
79	−10.42	− .45	− .09	− .03	+ .97

Residuals at longitude stations of central group.

	″	″	″	″	″
213	+ 7.03	−3.76	−4.27	−4.37	−7.55
202	− 3.73	−7.13	−7.50	−7.57	−9.67
198	− 9.30	−6.22	−5.96	−5.92	−4.86
201	− 1.14	−4.12	−4.38	−4.42	−5.41
206	+ .97	−3.16	−3.48	−3.53	−4.73
207	+ 6.09	+1.21	+ .93	+ .87	+ .01
210	+ 2.18	−1.88	−2.02	−2.04	−2.49
211	+ 5.43	+1.00	+ .86	+ .84	+ .22

Residuals at longitude stations of central group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
64	+36.98	+ .55	−1.23	−1.54	−6.07
66	+24.18	−3.74	−4.92	−5.12	−8.12
69	+18.88	− .27	− .88	− .99	−1.81
74	+11.64	+2.35	+2.01	+1.97	+ .66
77	+ 4.03	− .62	− .68	− .70	− .72
79	− .60	−4.21	−4.20	−4.22	−3.94
83	− 1.38	− .52	− .31	− .28	− .04

Residuals at azimuth stations of central group.

	″	″	″	″	″
205	− 5.55	+ .90	+1.37	+1.46	+4.25
214	+ 1.72	−4.85	−5.28	−5.36	−8.14
208	− 1.74	−7.74	−8.11	−8.16	−9.03
204	− 7.96	−9.01	−9.21	−9.21	−9.53
200	−11.04	−5.50	−5.18	−5.13	−3.75
199	− 7.95	−3.90	−3.72	−3.70	−2.92
203	− 2.96	−2.97	−3.16	−3.18	−3.83
209	+ 4.74	+3.13	+2.90	+2.86	+2.45
215	+ 7.16	+4.45	+4.31	+4.27	+4.50
212	+ 2.44	+ .49	+ .38	+ .37	+ .03
65	+39.16	+3.34	+1.96	+1.69	−3.21
67	+28.49	+7.43	+6.45	+6.29	+4.21
68	+18.31	+ .33	− .37	− .49	−2.33
70	+13.41	+2.22	+1.81	+1.76	+ .96
71	+ 7.13	−3.01	−3.41	−3.46	−4.25
72	+ 4.50	−4.16	−4.55	−4.59	−5.45
73	+ 4.58	−2.34	−2.65	−2.68	−3.42
75	+ 1.05	−4.07	−4.25	−4.28	−4.13
76	+ 2.59	− .54	− .59	− .60	− .60
78	+ 3.06	+ .27	+ .20	+ .19	+ .39
80	− 1.06	−1.73	−1.60	−1.60	− .60
81	− 2.08	−1.67	−1.48	−1.48	− .50
82	+ 1.07	+2.02	+2.12	+2.13	+2.42
84	+ 2.97	+4.21	+4.48	+4.50	+5.44

Residuals at latitude stations of western group.

	″	″	″	″	″
240	+ 8.82	− 2.66	− 3.02	− 3.12	− 5.29
239	+10.51	− .85	− 1.21	− 1.31	− 2.40
267	+12.78	+ 1.22	+ .83	+ .73	+ 1.14
266	+20.85	+ 7.51	+ 6.86	+ 6.72	+ 9.30
265	+22.05	+ 7.99	+ 7.03	+ 6.80	+ 1.69
246	+22.68	+ 8.47	+ 7.42	+ 7.19	+ 3.47
245	+10.24	− 4.62	− 5.92	− 6.20	−14.08
236	+19.66	+ 6.72	+ 5.84	+ 5.66	+ 8.00
237	+18.05	+ 4.95	+ 3.96	+ 3.76	+ 2.12
264	+16.97	+ 2.18	+ .95	+ .68	− 6.92
238	+10.21	− 3.66	− 4.78	− 5.04	−14.34
235	+ 9.94	− 2.73	− 3.68	− 3.88	− 8.47
262	+12.25	− .70	− 1.74	− 1.96	− 9.08
234	+17.38	+ 5.10	+ 4.21	+ 4.03	− 1.06
243	+ 6.46	− 4.95	− 5.53	− 5.64	− 7.10
242	+14.90	+ 2.62	+ 1.85	+ 1.69	− 1.46
261	+14.40	+ 2.98	+ 2.30	+ 2.17	− 3.35
260	+16.59	+ 5.03	+ 4.33	+ 4.21	− 2.19
257	+ 6.44	− 4.08	− 4.60	− 4.71	− 7.96
258	+10.02	− 1.16	− 1.83	− 2.00	− 7.78
259	+ 7.62	− 3.59	− 4.36	− 4.53	−10.24
256	+ 9.25	+ 1.09	+ 1.18	+ 1.21	+ 2.24
255	+ 1.34	− 6.92	− 6.98	− 6.98	− 7.40
244	+ 8.77	− 2.06	− 2.64	− 2.76	− 3.41
233	+17.99	+ 7.87	+ 7.45	+ 7.35	+ 9.28
254	+16.65	+ 5.63	+ 4.94	+ 4.77	+ 4.44
253	+ 8.81	− .21	− .34	− .35	+ .10

Residuals at latitude stations of western group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
19	+ 7.89	− 1.09	− 1.35	− 1.39	− 1.91
21	+ 3.85	− 3.87	− 3.84	− 3.80	+ 1.04
16	+ 2.97	− 4.80	− 4.91	− 4.91	− 1.82
13	+ 5.38	− 3.69	− 4.16	− 4.21	− 3.70
12	+ 9.24	+ .15	− .34	− .40	+ .50
10	+ 9.84	+ .72	+ .22	+ .16	+ 1.08
11	+ 9.60	+ .45	− .04	− .10	+ .92
9	+ 8.73	− 1.56	− 2.10	− 2.17	− 4.66
4	+ 5.44	− 3.96	− 4.69	− 4.82	− 6.52
3	+ 7.86	− 2.12	− 2.91	− 3.06	− 5.58
7	+ 7.88	− 2.16	− 2.98	− 3.15	− 6.48
1	+13.37	+ 2.42	+ 1.25	+ .99	− 4.60
2	+ 8.78	− .55	− 1.29	− 1.44	− 3.81
252	+ 4.29	+ 1.06	+ 1.18	+ 1.22	− .72
251	+ .26	− 2.21	− 2.05	− 2.04	+ .05
249	− 1.17	− 1.35	− 1.26	− 1.22	− 1.32
5	+ 9.90	+ 1.62	+ 1.00	+ .88	− .20
6	+ 8.24	− .77	− 1.45	− 1.61	− 5.79
8	+ 6.43	− 1.99	− 2.52	− 2.63	− 5.02
14	+ 7.82	+ .37	+ .04	− .02	− .70
15	+ 4.65	− 2.84	− 3.13	− 3.17	− 6.15
17	+ 3.27	− 3.56	− 3.81	− 3.86	− 3.72
18	+ 2.31	− 4.43	− 4.62	− 4.63	− 4.11
20	+ 4.51	− 2.16	− 2.60	− 2.67	− 3.47
24	+ 6.18	+ 1.19	+ .99	+ .95	+ .45
23	+ 3.84	+ .15	+ .17	+ .15	− .29
25	− .08	− 1.55	− 1.13	− 1.05	+ 6.37
27	− 4.61	− 4.41	− 3.92	− 3.80	− 1.34
22	− .57	− 2.06	− 1.84	− 1.80	+ .80
26	− 4.43	− 3.27	− 2.66	− 2.50	+ 1.83
28	− 9.86	− 5.24	− 4.49	− 4.37	+ .40
29	− 4.00	− 3.26	− 2.97	− 2.93	− 2.19
30	−12.42	− 5.07	− 4.14	− 3.95	− .23
31	−23.74	−16.47	−15.61	−15.45	−10.40
32	− 6.37	− 8.72	− 9.01	− 9.06	− 9.60
33	+ 1.18	+ .04	− .10	− .13	− 1.31
37	− 9.03	− 4.06	− 3.35	− 3.22	+ .17
48	− 6.60	− .55	+ .01	+ .12	+10.20
35	−12.33	− 3.80	− 3.13	− 2.98	− .82
34	−13.05	− 2.41	− 1.99	− 1.91	− 6.57
39	−13.10	− 2.14	− 1.86	− 1.84	− 3.33
42	−12.99	− 2.15	− 1.76	− 1.72	− 1.69
44	−11.51	− .66	− .26	− .22	+ .96
40	−14.28	− 3.80	− 3.35	− 3.28	− 1.31
41	−12.03	− 1.37	− .86	− .77	+ .27
43	−15.46	− 5.17	− 4.71	− 4.61	− 6.46
38	−15.00	− 4.69	− 4.06	− 3.95	− 1.07
46	−15.03	− 5.51	− 4.94	− 4.82	− 3.57
36	−13.52	− 4.78	− 3.99	− 3.83	+ .07
45	−14.87	− 5.88	− 5.13	− 4.94	− 2.88
47	− 7.69	+ .05	+ .45	+ .53	− .94
49	−10.02	− 3.59	− 3.83	− 3.88	−11.03
50	− 8.44	− 2.11	− 2.15	− 2.20	− 4.12
51	− 7.88	+ 1.55	+ 2.47	+ 2.62	+10.87
52	− 8.83	+ .33	+ .39	+ .36	− 5.12
53	−11.43	− .83	− .20	− .09	+ 6.15
54	−18.07	− 8.32	− 7.42	− 7.22	− .09
55	−11.90	− 1.56	− 1.08	− 1.00	− .51
56	−20.79	− 8.16	− 7.42	− 7.28	− 1.69
57	−12.42	− 2.48	− 2.21	− 2.19	− 2.74
58	−13.39	− 2.27	− 1.91	− 1.87	+ 2.91
59	−11.74	− .71	− .45	− .43	− 1.33
60	−13.88	− 2.19	− 1.81	− 1.77	− 4.01
61	−10.84	− .74	− .56	− .55	− 1.62

Residuals at longitude stations of western group.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
246	− 7.15	+1.42	+2.29	+2.49	+ 9.40
216	− 8.37	+ .19	+1.06	+1.26	+ 7.96
243	− 3.64	+2.00	+2.06	+2.07	+ 2.66
217	− 3.87	+1.83	+1.89	+1.90	+ 2.74
9	−13.16	− .59	− .04	+ .04	+ 6.36
8	−13.92	− .90	− .18	− .06	+ .23
7	−13.96	− .79	− .01	+ .11	+ .69
6	−13.03	+ .12	+ .88	+1.01	+ 1.70
1	−18.75	−1.12	+1.00	+1.45	+12.95
3	−13.66	−1.53	− .80	− .69	+ .15
16	−10.85	.00	+ .30	+ .33	+ .37
10	−11.38	+ .05	+ .51	+ .59	+ 1.90
20	−10.08	−2.71	−2.61	−2.63	+ .08
23	−11.10	−5.60	−5.96	−6.07	−22.35
24	− 1.93	+1.91	+1.24	+1.07	− 7.14
25	+ .37	+2.65	+1.78	+1.59	− 9.82
22	− 5.41	−1.67	−2.13	−2.23	− 7.81
33	+15.77	+9.96	+9.13	+8.95	+ 8.12
31	+10.42	+1.66	+ .69	+ .49	+ 1.95
41	+11.56	+2.67	+1.88	+1.98	+10.09
44	+ 9.72	+ .54	+ .96	+1.09	+12.27
36	+17.57	+6.68	+6.00	+5.86	+ 5.52
50	+14.46	− .41	−1.36	−1.54	− 4.75
53	+16.86	+1.70	+1.46	+1.41	− 2.08
55	+24.60	+2.50	+1.82	+1.74	+ .77
61	+37.14	−1.38	−4.31	−4.89	−20.86

Residuals at azimuth stations of western group.

	″	″	″	″	″
245	+ 6.28	+14.89	+15.74	+15.94	+20.03
244	+ .88	+ 8.54	+ 9.24	+ 9.41	+15.69
231	+ 3.55	+ 8.58	+ 8.71	+ 8.73	+ 9.41
230	+ 1.46	+ 6.78	+ 6.91	+ 6.94	+ 7.45
234	− 7.89	− 1.94	− 1.85	− 1.82	− 2.37
242	− 1.63	+ 5.42	+ 5.52	+ 5.55	+ 7.32
229	− 6.00	+ 3.05	+ 3.41	+ 3.45	+ 2.99
228	− 3.64	+ 5.28	+ 5.61	+ 5.63	+ 7.58
227	− 5.77	+ 4.45	+ 5.26	+ 5.43	+ 8.47
226	− 9.83	+ .77	+ 1.38	+ 1.50	+ 2.64
224	− 7.10	+ 4.63	+ 5.29	+ 5.40	+ 8.21
225	−15.99	− 3.22	− 2.09	− 1.85	+ 3.87
223	−17.45	− 3.90	− 2.85	− 2.64	+ 2.90
241	− .03	+13.14	+14.13	+14.34	+18.04
222	−11.98	− 1.36	− 1.40	− 1.45	− 4.08
220	−11.09	+ .90	+ 1.15	+ 1.21	+ 2.49
221	−26.05	−10.21	− 8.96	− 8.71	− 5.14
219	−22.93	− 7.61	− 6.59	− 6.38	− 5.30
239	−32.98	−14.86	−13.31	−12.98	− 6.10
240	−16.40	+ .29	+ 1.50	+ 1.72	+ 2.49
218	−17.31	− 3.68	− 3.30	− 3.25	− 1.29
17	−21.21	− 7.51	− 7.16	− 7.12	− 5.81
13	−18.81	− 5.11	− 4.88	− 4.87	− 7.67
238	−31.48	−15.89	−15.12	−14.99	−15.88
5	−20.16	− 4.32	− 3.47	− 3.34	− 5.71
237	−13.73	+ 2.59	+ 3.76	+ 3.98	+ 6.88
2	−19.75	− 3.46	− 2.42	− 2.25	− .28
236	− 8.12	+ 6.24	+ 6.76	+ 6.82	+ 7.58
4	− 9.73	+ 3.61	+ 3.77	+ 3.78	+ 1.07
11	−15.14	− 3.31	− 3.48	− 3.57	− 8.32
12	−18.01	− 5.37	− 5.39	− 5.43	−10.32
15	−13.60	− .81	− .76	− .78	− 3.88
14	−11.88	− .03	− .11	− .16	− 3.90

Residuals at azimuth stations of western group—Continued.

Station.	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
26	−15.09	− 6.15	− 5.85	− 5.80	− 1.13
19	−21.30	− 9.34	− 8.62	− 8.48	− 7.02
21	−10.94	− 2.22	− 2.16	− 2.18	− 1.69
18	−21.66	−12.23	−11.76	−11.67	− 9.75
27	−10.60	−10.18	−10.47	−10.51	− 7.07
28	+ 5.24	+ 1.78	+ .93	+ .80	− 2.78
29	+ 5.15	− .85	− 1.79	− 1.97	− 5.36
30	+ 6.57	− 2.82	− 3.90	− 4.11	− 6.87
34	+10.20	+ .78	− .35	− .56	− 3.60
35	+19.66	+ 4.41	+ 2.66	+ 2.31	− 4.39
32	+21.36	+ 4.79	+ 2.93	+ 2.54	− 7.64
38	+ 7.10	− 5.99	− 6.54	− 6.64	−12.73
42	+ 9.73	− 1.21	− 1.01	− .94	+ 5.94
40	+ 5.01	− 5.70	− 5.41	− 5.33	+ 2.68
43	+ 8.15	− 1.93	− 1.49	− 1.34	+14.48
45	+ 7.73	− 2.65	− 2.26	− 2.15	+ 7.77
39	+ 8.79	− 2.95	− 3.11	− 3.11	− 2.39
37	+18.06	+ 4.66	+ 3.90	+ 3.75	+ 2.21
46	+14.58	+ 2.96	+ 2.85	+ 2.86	+ 8.14
47	+16.87	+ 2.00	+ 1.06	+ .89	− 1.92
48	+17.83	+ 1.09	− .46	− .80	− 5.47
49	+18.47	+ 1.18	+ 1.16	+ .95	− .01
51	+16.88	+ 2.12	+ 2.03	+ 2.03	+ 4.06
52	+16.44	− 2.04	− 2.68	− 2.79	− 5.89
54	+18.88	+ .81	+ .47	+ .41	− 3.68
57	+19.02	− 2.66	− 3.01	− 3.03	− 2.96
56	+30.06	+ 4.30	+ 3.51	+ 3.41	+ .90
58	+23.21	− .20	− .49	− .47	+ 3.54
59	+39.94	+ 6.66	+ 4.71	+ 4.31	− 9.48
60	+43.84	+ 1.82	− 1.19	− 1.78	−20.04
62	+42.06	− 1.14	− 3.85	− 4.37	−14.62
63	+39.36	− 2.75	− 4.85	− 5.23	−11.49

REASONS FOR ADOPTION OF SOLUTION G.

The sums of the squares of the residuals of the different solutions were as follows:

Solution B (extreme rigidity)	65 434
Solution E (depth of compensation 162.2 kilometers)	8 220
Solution H (depth of compensation 120.9 kilometers)	8 020
Solution G (depth of compensation 113.7 kilometers)	8 013
Solution A (depth of compensation zero)	13 922

Solution G, having the smallest sum of the squares of residuals, is probably the closest approximation to the truth.

Solution B is evidently far from the truth. The sum of the squares of the residuals is more than eight times as large as for solution G. This comparison constitutes a very strong proof that the assumption of an isostatic compensation which is complete and uniformly distributed within the depth 113.7 kilometers is a very much closer approximation to the truth than the assumption of extreme rigidity.

The very wide departure of the equatorial radius, flattening, and polar semidiameter, as derived from solution B (see page 105), from all previous derived values is in itself a strong indication that the assumption of extreme rigidity is far from the truth. In fact, observations of gravity show positively that the value of the flattening given by solution B, $\frac{1}{268.7}$, can not exist.

Solution A is also evidently a considerably wider departure from the truth than solution G. The introduction of the assumption of complete isostatic compensation uniformly distributed within the depth 113.7 kilometers in the place of the assumption that no relation exists between deflections of the vertical and topography, has reduced the sum of the squares of the residuals from 13 922 to 8 013. In other words, the introduction of the assumption of isostasy in a

definite and reasonable form has eliminated 42 per cent from the sum of the squares of the residuals.

Solution G is apparently a closer approximation to the truth than solution E.

Solution G is apparently slightly nearer the truth than solution H, but there is little basis here shown for a choice between these two.

The following table shows other means of comparison among the five solutions:

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
Maximum residual	+43.84	−16.47	+15.74	+15.94	−22.35
Percentage of residuals greater than 5″.00	66	18	18	18	29
Percentage of residuals less than 2″.00	15	41	43	43	35
Mean residual, without regard to sign	8.86	3.06	3.04	3.04	3.92

These comparisons confirm the statements already made, based on the sums of the squares of the residuals.

The results of the five solutions make it clear that if the assumed depth of compensation is made to vary from infinity to zero the sum of the squares of the residuals decreases from 65 434 for the assumed depth infinity to a minimum value of about 8 010 for some assumed depth not differing greatly from 114 kilometers, and then increases again to 13 922 as the assumed depth is decreased to zero.

The depth for which the sum of the squares of the residuals would be a minimum is the ideal most probable depth of compensation.

To ascertain this ideal most probable depth with great accuracy is not important, for two reasons: (a) It is evident from a comparison of the sums of the squares of the residuals for solutions E, H, and G that the ideal minimum sum, when ascertained, would be found to be very little less than 8 013, the sum corresponding to solution G, and that therefore the corresponding solution would be a very slight improvement on solution G. (b) It is evident from a comparison of the values of the equatorial radius, flattening, and polar semidiameter, as derived from solutions G and H, that a change in the depth of compensation adopted as most probable introduces but little change into the corresponding most probable values for the equatorial radius, flattening, and polar semidiameter.

From the residuals of solutions E, H, and G, the conclusion was reached that the most probable depth of compensation is 112.9 kilometers, which agrees so closely with the depth used in solution G, 113.7 kilometers, that it is not certain that solution G can be improved upon. The approximate process by which the value 112.9 was derived is easy of application. But the explanation of it is so long that it is deemed best not to insert it here because of the break in the continuity which would result. It is given later in this publication with the discussion of various subsidiary questions.

ADOPTED VALUES.

For these reasons solution G is adopted as the most probable solution. This fixes upon the following values as the most probable which can be derived at present from observations in the United States:

	°	′	″	″
Latitude of Meades Ranch	39	13	26.47	±0.17
Longitude of Meades Ranch	98	32	30.64	±0.40
Azimuth of line Meades Ranch to Waldo, to be used in computations extending eastward from Meades Ranch	75	28	14.65	±0.32
Azimuth of line Meades Ranch to Waldo, to be used in computations extending westward from Meades Ranch	75	28	09.12	±0.33

Equatorial radius of the earth, meters, 6 378 283±34.

Reciprocal of flattening, 297.8±0.9.

Polar semidiameter, meters, 6 356 868.

The probable errors were computed in the usual way. They are somewhat too small, as they are based upon the assumption that the residuals are all accidental in character and independent of each other. It is evident from an examination of the residuals of solution G, as shown on pages 106–114, that there is a sufficient tendency to geographic grouping of residuals of like sign to indicate that the residuals are not entirely independent of each other. It is extremely difficult to estimate the amount by which the computed probable errors should be increased to represent the real uncertainties in the values. It is believed, however, that the real probable errors are not very much larger than those shown above.

The United States Standard Datum is evidently a very near approach to the ideal. The derived correction to the initial latitude (ϕ) is but little greater than its own probable error, and the derived corrections to the initial longitude (λ) and to the initial azimuth for computations extending eastward (α_E) are much smaller than their respective probable errors. The sign of each of these three required corrections is therefore still in doubt.

For the correction (α_W) to the initial azimuth for computations extending westward, the relatively large value $-5''.40$ is in accordance with the preliminary investigation made, which indicated an accumulated error in geodetic azimuth in the transcontinental triangulation somewhere between Illinois and Colorado. As stated on page 101, the preliminary investigation indicated this accumulated error to be about $5''$. Solution G makes it $5''.53$, $(\alpha_W)-(\alpha_E)$.

ERRORS DUE TO ALL CAUSES.

Thus far this publication has consisted, in the main, of a statement of the data used and of the methods of investigation and computation, accompanied by barely sufficient illustration and explanation to enable the reader to understand what was done and, in a general way only, to understand why it was done. In the pages which follow will be found a systematic discussion of the accuracy of the data used and of the accuracy of the various parts of the computation. In connection with this discussion the reasons for various features of the methods of computation will appear. This separation of the bare statement of what was done, on the one hand, from the discussion of accuracy and of reasons for methods, on the other hand, has two advantages. It enables the writer to secure greater continuity in the statement of what was done, with consequent gain in clearness; it also enables him to make the discussions of accuracy and of reasons for methods, more concrete than would otherwise be possible, as by virtue of the separation they are, in form, discussions of matters already presented, of something concrete already done, rather than of something abstract to be done.

Because the methods of this investigation are unusual, it is especially important to consider carefully the accuracy of each step of the process and to indicate the reasons for decisions as to methods. Moreover, as a definite and sustained effort has been made to keep the economics of the problem in view and to simplify and shorten each step of the process as much as possible without appreciable loss of ultimate accuracy, it is especially important, on the one hand, to ascertain whether this has been carried too far and, on the other hand, to ascertain whether it is advisable in any future investigation to simplify and shorten still more.

The combined errors due to all causes are represented by the residuals in the observation equations. The residuals of the adopted solution—G—will be taken as the basis of the following discussion. As the residuals in this solution are smaller on an average than in any of the other four solutions, it sets a higher standard and furnishes a more severe test of the methods than any of the other solutions.

As already stated (p. 115), in solution G the mean residual without regard to sign is $3''.04$, 43 per cent of the residuals are less than $2''.00$, 18 per cent are greater than $5''.00$, and the maximum residual is $+15''.94$.

For the residuals of solution G, considered in three separate classes, the principal statistics are as follows:

	Mean without regard to sign.	Maximum.
For ϕ residuals	$2''.76$	$-15''.45$
For λ residuals	2 .34	+ 8 .95
For α residuals	3 .85	+15 .94

EFFECTS OF ERRORS IN ASTRONOMIC OBSERVATIONS.

A summary of the latitude observations for each of 109 of the latitude stations involved in this investigation has been published.* These may fairly be taken as representative of the 265 latitude stations involved in this investigation, at 261 of which the observations were made with zenith telescopes. At the 4 remaining stations the observations were made with Airy's zenith sector. Of these 109 latitudes there are only 29 for which the probable error of the derived latitude is as great as $\pm 0''.10$, the largest probable error is $\pm 0''.66$,† and the average probable error is $\pm 0''.09$. The average error of observation of an astronomic latitude without regard to sign is probably $0''.11$,‡ or one twenty-fifth of the average residual without regard to sign for latitude stations in solution G, namely, $2''.76$. It is reasonably certain that the error of no astronomic latitude among these 109 is greater than $2''.31$,§ which is but little more than one-seventh of the maximum latitude residual in solution G. These statements include the errors due to inaccuracies in the declinations of the stars observed.

There are many astronomic latitudes used in this investigation to which no correction has been applied for variation of the pole. Out of 109 such corrections in the list above referred to, 60 are less than $0''.10$, and the maximum is $+0''.27$. Hence the fact that such corrections have been omitted at many stations is of little significance in the present investigation. The omitted corrections constitute errors of the accidental class, which have evidently contributed but a small percentage of the mean residual, $2''.76$.

The astronomic longitude of Fort Gratiot, Michigan, No. 232, was determined by transportation of chronometers between that point and Detroit, Michigan. All of the other astronomic longitudes, including Detroit, were determined by the telegraphic method. Many of the longitude stations are in the longitude net of the United States, and the remainder are each connected with one or more stations of the net. The probable error of the average longitude determination in the telegraphic net is $\pm 0^s.024 = \pm 0''.36$.‖ This probable error is derived from the adjustment, and includes the systematic error peculiar to each station and not exhibited in the probable errors computed from the discrepancies between determinations of the same longitude difference on various nights. For many of the longitude determinations concerned in this investigation but not included in the telegraphic net, summaries of the results, showing their accuracy, have been published.¶ Any error of the astronomic longitudes which is common to all the stations does not affect the accuracy of the results obtained from the present investigation, since this investigation is concerned simply with differences of longitude between points connected by continuous triangulation. The mean error made in transferring the longitude from Greenwich across the Atlantic, therefore, has no effect on this investigation.

On account of the complicated manner in which the various longitude determinations are interrelated in the longitude net and with other determinations connected with it, to determine the probable error of each derived longitude accurately would require too great an expenditure of time to be warranted by the value of the result. Accordingly, an approximate computation of the probable errors of the differences of longitude between Kansas City, a point in the central

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*See The Transcontinental Triangulation, Special Publication No. 4, Coast and Geodetic Survey, pp. 626-737. Others of the astronomic latitudes used in this investigation are also published in a similar manner in The Eastern Oblique Arc, Special Publication No. 7, Coast and Geodetic Survey, pp. 253-316.

†This large probable error occurred at latitude station No. 13, Washington Square, San Francisco. There is no other probable error among the 109 greater than $\pm 0''.23$.

‡According to the theory of errors of observation the average error without regard to sign is 1.2 times the probable error. Thus, if the probable error is $\pm 0''.09$, the average error without regard to sign is $0''.11$.

§In this and in various other parts of this investigation, it is assumed to be reasonably certain that the error does not in any case exceed 3.5 times the maximum probable error of any observation. According to the theory of errors, an error as great as 3.5 times the probable error should occur on an average once in 55 times. In the above case it is stated as reasonably certain that the maximum error is not greater than $2''.31$, which is 3.5 times the maximum probable error, $\pm 0''.66$.

‖See p. 255 of The Telegraphic Longitude Net of the United States and its Connection with that of Europe, by C. A. Schott, Appendix 2, Coast and Geodetic Survey Report for 1897.

¶See the Transcontinental Triangulation, pp. 807-826, and The Eastern Oblique Arc, pp. 317-326.

portion of the longitude net, and each of ten stations selected at random out of the 79 concerned in this investigation, has been made. For these ten stations the probable error varied from $\pm 0^s.010$ to $\pm 0^s.124$, with a mean of $\pm 0^s.036$. Hence the average error, without regard to sign of the longitudes as referred to Kansas City, is probably about $0^s.043$. Reducing this to arc and multiplying by $\cos\phi'$ for latitude 39°, this being the mean latitude of the longitude stations, there is obtained for the average error in the apparent prime vertical deflection at longitude stations 0″.50, which is slightly more than one-fifth of the average residual without regard to sign for longitude stations in solution G. It is reasonably certain that the maximum error of any observed astronomic longitude is not greater than $0^s.43$, which corresponds to a maximum error in the apparent prime vertical deflection of 5″.0, which is somewhat more than one-half of the maximum residual for longitude stations in solution G, namely, +8″.95.

For 73 of the 163 azimuth stations involved in this investigation, a summary for each station has been published.* These may fairly be taken as representative of the whole 163 stations. Of the 73 there are only 12 for which the probable error of the result is greater than ±0″.30, the largest probable error is ±0″.79, and the average probable error is ±0″.23. The observations were all made upon stars with a variety of theodolites, some repeating theodolites and some direction theodolites. The average error of the observed astronomic azimuth, without regard to sign, is probably about 0″.28. This, reduced to a prime vertical component of the deflection of the vertical, by multiplying by 1.23, the factor cot ϕ' for the assumed mean latitude, 39°, is 0″.34, or less than one-eleventh of the average residual without regard to sign for azimuth stations in solution G, namely, 3″.85. It is reasonably certain that the error of no observed astronomic azimuth among these 73 is greater than 2″.77, corresponding to a prime vertical deflection of 3″.48 at that station; which is less than one-fourth of the maximum azimuth residual in solution G, namely, +15″.94.

From the preceding three paragraphs it appears that but a small portion, certainly less than one-tenth part on an average, of the residuals of the latitude and azimuth observation equations in solution G is due to errors in the astronomic determinations. The errors in the observed astronomic longitudes are relatively larger—probably furnishing about one-fifth part of the residuals of the longitude observation equations in solution G. Even in connection with the residuals of the longitude equations it is evident that the errors of the astronomic observations have but a minor, though not an insignificant, part.

The portion of the residuals produced by errors of the observed astronomic latitudes should have no relation to the geographic distribution of the stations.

On the other hand, there should be a slight tendency for residuals produced in part by errors in the observed astronomic longitudes to be greater for extreme eastern or extreme western stations than for stations in the middle part of the country. Also, there should be a slight tendency for the effects of errors in the astronomic longitudes to be greater in the southern part of the country than in the northern part, on account of the variation of the factor cos ϕ' from .866 for the extreme southern station (No. 141) to .685 for the extreme northern station (No. 213).

The errors in the observed astronomic azimuths have no relation to geographic distribution, but their effects in producing residuals should tend to be considerably greater for southern than for northern stations on account of the variation of the factor cot ϕ' from 1.72 at the extreme southern station (No. 136) to .91 at the extreme northern station (No. 205).

No such laws of distribution are observable in the residuals.

EFFECTS OF ERRORS IN DISTANCES.

The errors in the computed lengths of the lines joining triangulation stations depend upon the errors in the angle measurements and the errors in the measurements of base lines. The errors in length due to the second cause are much smaller than those due to the first.

*See The Transcontinental Triangulation, pp. 743–801. Similar summaries for 56 astronomic stations also involved in this investigation are published in The Eastern Oblique Arc, pp. 328–366, 375–376.

To make a careful estimate in detail of the effect of the errors in length upon the present investigation would be a complicated, difficult, and tedious matter. The present purpose will be served by a brief consideration of the more important principles and facts concerned, from which it will appear that errors in length, that is, in the computed distances between triangulation stations, contribute a minor part only to the errors of the final results of this investigation.

The laws of accumulation of error in triangulation are such that the error of distance, expressed in meters, between any two triangulation stations tends to be proportional to the square root of the distance between the stations. The error in distance, when expressed as a proportional part of the distance, therefore, tends to vary inversely as the square root of the distance.

The transcontinental triangulation may be taken as typical of all the triangulation involved in this investigation. The probable errors of length in the various parts of this triangulation have been computed and published.* The largest probable error in length, when expressed as a proportional part, occurs between the line Hartley-Kent, in Maryland, and the line joining Cape May and Cape Henlopen light-houses, at the eastern end of the triangulation. The probable error of length of this section of the triangulation is ± 1.1 meters on 56 kilometers, or one part in 50 000. This probable error corresponds to a probable error of the difference of longitude between the two ends of the section of only $\pm''.05$. In such a section of triangulation, lying along a meridian, the probable error of the difference of latitude of the two ends produced by such an error in length would be only $\pm''.04$.

Of the 11 sections into which the transcontinental triangulation is divided by bases, there are only two in which the probable error of length exceeds one part in 100 000.

Combining the probable errors of the various sections of the transcontinental triangulation as published, by taking the square root of the sum of their squares, it becomes evident that the probable error of the distance from the initial station Meades Ranch to either the eastern or the western end of the triangulation is only ± 6 meters, corresponding to a probable error in the difference of longitude of $\pm''.25$, or to a probable error of a difference of latitude of $\pm''.20$ in a similar triangulation lying along a meridian. It must be kept in mind, also, that the effect on differences of longitude is somewhat reduced in its effect on apparent deflections of the vertical by multiplication by the factor $\cos\phi'$.

A comparison of these small possible effects of errors of length upon the apparent deflections of the vertical at latitude and longitude stations with the average residual of more than 2″ (see p. 116) which has been found in this investigation, shows that the errors of length have but a small part in producing those residuals. The average residual would be reduced by less than one-tenth part if absolutely exact lengths were substituted for the best lengths now available.

The effects upon the apparent deflections of the vertical of errors in the computed geodetic azimuths as produced by errors of length are of the same magnitude as the similar effects produced through the computed geodetic longitudes.

The dimensions of the earth derived from this investigation, the equatorial radius and the polar semidiameter, are necessarily in error by the same proportional part as the average distance between the astronomic stations involved in this investigation. By the methods indicated above, it is found that the probable error of the total length of the transcontinental triangulation is only ± 8 meters, or one part in 500 000. The writer estimates from this and other considerations that the probable error of the average distance involved in this investigation is about one part in 500 000. From this cause, then, the computed equatorial radius may have a probable error of this proportional part, or ± 12 meters. The computed probable error of the equatorial radius, derived from this investigation, is ± 34 meters (see p. 115), indicating that errors due to other causes are much larger than those arising from errors in the computed lengths.

There may be constant errors as great as one part in 500 000 in the measurement of each of several of the bases concerned in this triangulation. But as the various bases were measured under a variety of conditions with sets of base apparatus varying widely in design, the

*See The Transcontinental Triangulation, pp. 368, 395, 417, 434, 451, 480, 514, 551, 567, 592, and 611.

errors which were constant for each base were probably different in sign, as well as magnitude, for different bases. These errors are therefore in the accidental class in so far as this investigation, as a whole, is concerned.

EFFECTS OF ERRORS IN GEODETIC AZIMUTHS.

The errors in the computed geodetic azimuths used in this investigation are due mainly to the errors in the angle measurements, as errors in the angles used in the computation enter with full value into the computed azimuth. As already indicated, the effects of errors of length upon the computed geodetic azimuths are very small.

The errors in the geodetic azimuths affect directly the absolute terms in the azimuth observation equations and the corresponding residuals. Through these equations the final results of the investigation are affected.

The errors in the geodetic azimuths also necessarily affect the computed geodetic latitudes and geodetic longitudes; but the effects upon the latitude and longitude observation equations are very small in comparison with the effects upon the azimuth observation equations.

The accuracy of the angle measurements has been determined from the adjustment of the triangulation. The probable errors of the computed geodetic azimuths may be computed. Let d be the probable error of an observed direction in any section of triangulation. This has been computed from the results of the adjustment of various sections of triangulation* involved in this triangulation by the formula

$$d = .675\sqrt{\frac{\Sigma v^2}{c}}$$

in which each v is the correction to an observed direction as derived from the adjustment, and c is the number of rigid conditions satisfied by the adjustment.

The probable error of an adjusted direction which will be called d_a is necessarily less than the probable error of an observed direction. The relation between the two is given by the formula

$$d_a^2 = d^2\left(\frac{n-c}{n}\right)$$

in which n is the number of observed directions in the section of triangulation considered. Or, expressed in general terms in words, the average value of the ratio of the weight of the observed value of a quantity to that of its adjusted value equals the ratio of the number of independent unknowns to the number of observed quantities.†

The geodetic azimuth may be computed through a chain of adjusted triangulation by using any series of lines between terminal points of the section, each selected line being one over which observations were made in both directions. The same computed azimuth will be secured, whatever series of lines is selected, the triangulation having been completely adjusted. The accuracy with which the azimuth is carried through the triangulation will depend, therefore, upon the least number of lines that can be arbitrarily selected which will connect the terminal points of the section and over which observations have been made in both directions. Let l be this minimum number of lines. Then the number of adjusted directions involved in carrying the computed geodetic azimuth through the section is 2l, and the error introduced into the computed geodetic azimuth is

$$d_a\sqrt{2l}$$

or its square is

$$d^2\left(\frac{n-c}{n}\right)2l$$

* See The Transcontinental Triangulation, p. 613; The Eastern Oblique Arc, p. 235; and Triangulation in California, Part I, Appendix 9, of the Report of the Coast and Geodetic Survey for 1904, p. 520.

† See the Adjustment of Observations, by T. W. Wright, second edition, p. 143.

This formula has been applied to successive adjusted sections of nearly all of the triangulation involved in this investigation.* If the geodetic azimuth be considered as absolutely fixed at Meades Ranch, the initial point used in this investigation, the computed probable error of the geodetic azimuth is $\pm 2''.9$ at San Diego, California, $\pm 3''.4$ at the point in Virginia at which the eastern oblique arc triangulation to the southward leaves the transcontinental triangulation, $\pm 3''.9$ at Cape May, $\pm 4''.1$ at Calais, Maine, and $\pm 5''.0$ at New Orleans, Louisiana.

These are so large as to indicate that the major portion of the residuals from the azimuth observation equations may be due to the errors in the geodetic azimuths resulting from the errors of measurement of the horizontal angles in the triangulation, the average value of such residuals without regard to sign being only $3''.85$. (See p. 116.)

But dependence need not be placed on these computed probable errors alone to determine the accuracy of the geodetic azimuths. Other strong evidence is available. Wherever both the astronomic longitude and the astronomic azimuth are determined at the same triangulation station in such a triangulation as that under consideration, of the primary grade of accuracy and of large extent, it is possible to determine with considerable accuracy the true geodetic azimuth and, therefore, the accumulated error in the geodetic azimuth. For each such point it is evident that the prime vertical component of the deflection of the vertical should be the same, except for errors of observation, whether derived from the observed astronomic longitude or from the observed astronomic azimuth. Expressed algebraically, this is (see pp. 74, 75)

$$\cos\phi'(\lambda_A - \lambda') = -\cot\phi'(\alpha_A - \alpha')$$

This relation is evidently the same as that expressed by the well-known Laplace equation (astronomic azimuth − geodetic azimuth) + $\sin\phi'$ (astronomic longitude − geodetic longitude) = 0. A point at which coincident astronomic longitude and astronomic azimuth observations have been made, has already been given by others the appropriate name of Laplace point.

There are 11 Laplace points in the present investigation. It has already been stated (see p. 101) that a preliminary investigation making use of these points indicated that there had been, somewhere between Illinois and Colorado, in the transcontinental triangulation, an accumulation of about 5″ of error in the geodetic azimuth, carried through the adjusted angles. The evidence available was not sufficient to determine the places at which the accumulation occurred with any greater accuracy than indicated in the preceding sentence. The investigation did not show, with certainty, that any error in geodetic azimuth had accumulated in any other part of the triangulation. The device was, therefore, adopted of virtually introducing a hinge into the triangulation at Meades Ranch by putting into the observation equations and, therefore, also into the normal equations, two separate required corrections to the initial azimuth at Meades Ranch, one (α_E) pertaining to triangulation to the eastward of Meades Ranch, and the other (α_W) pertaining to triangulation to the westward. This is equivalent to assuming that the error in geodetic azimuth all occurred suddenly at Meades Ranch, instead of gradually at unknown points between Illinois and Colorado.

The values of (α_E) and (α_W) at Meades Ranch, as derived from the adopted best solution, G, are (see p. 105) $+''.13$ and $-5''.40$, differing by $5''.53$, thus confirming the conclusion reached from the preliminary examination that the accumulated error in geodetic azimuth between Illinois and Colorado was about 5″.

As an indication of the errors which still remain in the results due to errors in the corrected geodetic azimuths, the values of the residuals at the 11 Laplace points, as given separately by the longitude and the azimuth observation equations in solution G, are given in the table which follows. These residuals are the unexplained portions of the prime vertical components of the deflection of the vertical, and in each case their difference, shown in the sixth

* The formula has not been applied to the Lake Survey triangulation as the values of d are not available for it.

column, should be zero to satisfy the Laplace equation and thereby indicate that there were no errors of observation and no errors in the geodetic azimuths.

Place.	Longitude station.	Azimuth station.	Longitude residual.	Azimuth residual.	Difference long—az.
			″	″	″
Cambridge, Massachusetts	157	147	+3.21	+0.18	+3.03
Ogdensburg, New York	179	180	+2.14	+ .64	+1.50
Tonawanda, New York	185	186	− .84	−4.06	+3.22
Minnesota Point N. B., Minnesota	213	214	−4.37	−5.36	+ .99
Sault Ste. Marie, Michigan	198	200	−5.92	−5.13	− .79
Ford River 2, Michigan	201	203	−4.42	−3.18	−1.24
Willow Springs, Illinois	211	212	+ .84	+ .37	+ .47
Parkersburg, Illinois	77	78	− .70	+ .19	− .89
Gunnison, Colorado	55	56	+1.74	+3.41	−1.67
Salt Lake City, Utah	44	45	+1.09	−2.15	+3.24
Ogden, Utah	41	42	+1.98	− .94	+2.92

The differences show too little tendency to grouping of signs, and nearly all of them are too small, to warrant the conclusion that they are due to anything else than accidental errors in the astronomic observations and triangulation. It is possible that the four values greater than 2″ are indications of twist in azimuth, but it is not certain that even these are not due to accidental errors.

The differences are so small as to show that only a minor part of the residuals of the azimuth observation equations are due to uncorrected errors of geodetic azimuth. They are of such a magnitude as to indicate, however, that the small excess of the average residual of the azimuth equations (3″.85) over the average residuals of either the latitude equations (2″.76) or the longitude equations (2″.34) (see p. 116) is probably due to the errors of the geodetic azimuths.

An approximate examination shows that the possible accumulated errors of the geodetic azimuth corresponding to the differences in the table are so small that their effects on the computed geodetic latitudes and longitudes are probably everywhere less than 1″ and, as a rule, are less than 0″.2. Hence, the errors in the geodetic latitudes and longitudes, due to this cause, contribute a minor, but not negligible, part of the residuals of the latitude and longitude equations.

It is evidently desirable that more Laplace points be introduced into this triangulation before it is again used for a study of the figure of the earth.

The fact that the average residual in the azimuth equation is considerably larger than in either the latitude or longitude equation, and the evidence indicated above that this is probably due to accumulated errors in the geodetic azimuths used, make it certain that to secure the theoretically best results the azimuth equations should be assigned less weight than the latitude and longitude equations.

The equations have all been given equal weight because it was not certain until the investigation was nearly complete that the azimuth equations should be given less weight than the others, and it is even now very difficult to decide how much the weights of the azimuth equations should be reduced. Considering it granted that they should be reduced somewhat in weight, has the failure to make the reduction seriously affected the adopted results? To test this question the G solution has been repeated after assigning to each azimuth equation east of Meades Ranch the weight 0.7 and to each azimuth equation west of Meades Ranch the weight 0.4, the weights of the latitude and longitude equations remaining unity, as before. The assigned weights 0.7 and 0.4 are based upon ratios of mean squares of residuals in the different groups. The amount of change in the derived results produced by this reduction of the weights of the azimuth equations is shown below.

	Adopted solution.	Solution with reduced weights.	Difference.
	″	″	″
Correction to initial latitude	− 0.22	− 0.20	0.02
Correction to initial longitude	+ .13	+ .17	.04
Correction to initial azimuth for eastward computations	+ .13	− .06	.07
Correction to initial azimuth for westward computations	− 5.40	− 5.81	.41
Correction to Clarke 1866 equatorial radius	+76 meters	+72 meters	4 meters
Correction to Clarke 1866 square of eccentricity	− .000065	− .000072	.000007

The changes in the final results produced by the reduction of weights of the azimuth equations are so small as to be of little consequence. For only one of the six unknowns (α_W) is the change as great as the probable error of the unknown as derived from solution G. The changes produced in the other five unknowns are from $\frac{1}{10}$ to $\frac{1}{3}$ of the probable errors of those quantities derived from solution G.

Hence, it is immaterial, in so far as the final results are concerned, whether the weights of the azimuth equations are reduced or not.

As it is evident, however, that the azimuth equations are affected by greater errors than the latitude and longitude equations, in drawing each conclusion which is based in part on the residuals of the azimuth equations (as to the depth of compensation, for example) it is important to note the degree to which the evidence given by the azimuth residuals is corroborated by the evidence from the latitude and longitude residuals. Later in this publication it will appear that this precaution has been taken.

THE ACCURACY OF THE COMPUTATIONS OF TOPOGRAPHIC DEFLECTIONS.

The accuracy of the results from solution B (see p. 92), in which solution the absolute term of the observation equation is, in each case, the observed apparent deflection minus the topographic deflection, depends directly upon the accuracy of the computation of topographic deflections. All errors in the computed topographic deflections enter with full value into the observation equations.

But solution B is relatively unimportant as compared with solution G, the adopted best solution. In solution G, which has been ascertained to represent the truth more accurately than any of the other solutions, the absolute term of the observation equation is in each case (see p. 93) the observed apparent deflection minus the deflection computed upon the supposition that the isostatic compensation is complete and uniformly distributed throughout the depth 113.7 kilometers. These computed deflections are obtained from the computations of topographic deflections by multiplying the topographic deflection derived separately for each ring of topography by a factor pertaining to that ring. These reduction factors (see p. 70), are nearly unity for inner rings, are smaller the larger the ring, and become nearly zero for the largest ring used, No. 1, being only .001 for that ring. The errors of the computed topographic deflection, therefore, enter into solution G with sensibly full value for inner rings, with a decreasing percentage of effect for successively larger rings (as fixed by the reduction factors), and for the outer ring (No. 1) have practically no effect.

Errors in the computed topographic deflections arise from a variety of causes and follow laws having a similar degree of variety.

In discussing the separate classes of errors affecting the computed topographic deflections it is important to note for each, not only its full effect on the comparatively unimportant solution B, but also its reduced effect on solution G, from which the more important conclusions of this investigation are drawn.

It is important to note, also, with reference to each class of errors, whether they are of the accidental type or of the systematic or constant types. The effects of accidental errors are, to a large extent, eliminated from the final results and conclusions in such an investigation as this, which depends upon a large number of observations. The uneliminated effects of the

accidental errors are properly measured by the probable errors attached to the final results. On the other hand, the effects of constant or systematic errors may or may not be eliminated, and the computed probable errors do not furnish a reliable measure of their effects. Therefore it is especially important to scrutinize carefully possible constant or systematic errors.

ERRORS IN TOPOGRAPHIC DEFLECTIONS DUE TO MAPS.

The accuracy with which contour lines are located on a given map depends fundamentally upon (a) the number of points within the area of which both the elevation and horizontal position are fixed by instrumental determinations, (b) the accuracy with which the instrumental observations are made, and (c) the accuracy with which the contours as drawn conform to the instrumental determinations.

It is probable that the errors arising in the computed deflections from errors in the instrumental observations referred to (b) are negligible; that they affect the hundredths of seconds only.

For some areas touched by this investigation the number of points instrumentally determined is so small that there is considerable uncertainty in the mean elevations of those areas. Greenland is an extreme example, and Guatemala is a typical example.

As a rule, the contours as drawn on the original field topographic sheets probably conform to the instrumental determinations so closely as to introduce no appreciable errors into the computed deflections, except for topography very close to the station. But many of the maps actually used are on a much smaller scale than the field sheets. As a necessary accompaniment of the reduction in scale the contours are generalized. In the process of generalization the apparent mean elevation of a given area, as read from the contours, may possibly be changed, and probably is changed in some cases, enough to affect the computed deflections appreciably. The errors in the computed topographic deflections so produced are of the accidental class. As their principal effect is upon the inner rings they affect solution G with substantially full value, as well as solution B.

Similarly, errors in the mean elevations, as shown by the maps, for areas near the station, due to an insufficient number of points fixed directly by instrumental determinations, are of the accidental class and affect solution G as well as solution B with full value.

On the other hand, an error in the mean elevation, as read from the map, for an area at a considerable distance from all of the stations at which astronomic observations were made, and outside the area covered by these stations, produces errors in the computed deflections which tend to be small. But such errors are of the systematic kind, the effects of which are not easily eliminated from the results of an investigation, even though it is based on observations at many stations. For example, from the best information available, it was decided to treat Guatemala as having a mean elevation of 3 000 feet (914 meters). It is possible that this mean elevation is actually as small as 2 000 feet (610 meters). If so, the computed prime vertical components of the topographic deflections are in error by about 0″.01 near the Atlantic and Gulf coasts and by about 0″.01 with the reverse sign at San Diego, California, these being the extreme values of the error. Similarly, at New Orleans, in the supposed case, the meridian component of the topographic deflection is in error by 0″.04, this being the maximum error from this cause in the United States. The minimum is 0″.01, with the same sign, at Calais, Maine.

The reduction factors (see p. 70) are, however, so small for all compartments which include portions of Guatemala, that errors from this cause do not affect the hundredths of seconds in the observation equations of solution G. In general, it is believed that the errors in mean elevation of distant topography produce small but systematic errors in solution B and inappreciable errors in solution G.

The nearness of approach of the computations of topographic deflection to the station is limited by the scale or contour interval of the best map or field sheet available. The scale of the map fixes the size of the smallest ring of topography which can be definitely located on the map by means of the template and for each compartment of which an estimate of the

mean elevation can be made. If the contour interval is large, or the topography is nearly flat, or both, it may happen that, though a small ring and the compartments comprising it may be definitely located on the map, the whole ring may fall between one pair of contours and, therefore, in so far as that map is concerned, may all seem to lie at one elevation. In this case the computation is virtually limited to larger rings by the contour interval of the map, rather than its scale. In such a case, if a map of smaller contour interval, and possibly also larger scale, later becomes available the computation may be extended inward for several more rings, and for these rings the computed topographic deflection may be found to differ considerably from zero. This limitation of nearness of approach to the station, as fixed by available maps and field sheets, introduces much larger errors than arise in any other way from the incompleteness or inaccuracy of the mapping.

At the station Cape Mendocino (No. 252) the computation of the meridian component of the topographic deflection was carried to ring 37, of which the inner radius is 0.0079 kilometer. At Rouse Point (No. 167) the computation of the meridian component was stopped at ring 13, of which the inner radius is 39.22 kilometers. These are the extremes in this respect between which all other cases lie. In 322 cases out of 496 the computation was carried inward toward the station at least to ring 26, of which the inner radius is 0.389 kilometer. The question arises: How large are the errors in the computed deflections due to the omitted topography nearer to the station than the nearest ring used in the computation?

There are 16 cases out of 496 in which the computation was carried inward at least to ring 35. For these cases a small absolute limit can be set to the possible error due to omitted topography in smaller rings. Let it be assumed that, in ring 35, the mean elevation of each of the eight compartments south of the station is 15 000 feet (4 572 meters) above the station and for each of the eight compartments north of the station, the mean elevation is 15 000 feet below the station (or the equivalent of that after the density of sea water is considered). As the outer radius of ring 35 is only 22.8 meters, the supposed condition represents a more abrupt slope than it is possible to find on the earth. But the meridian component of the topographic deflection at the station computed by the more exact formula on page 34 corresponding to this extreme condition is only 0″.0065 for each compartment, or 0″.10 for the whole ring. Obviously the deflection produced by the topography in ring 35 at any station may approach but not exceed 0″.10. If the same computation be made for successively smaller rings it will be found that the successive limits computed are in decreasing geometric progression in which the fixed ratio is $\frac{1}{1.43}$. It follows from this law that the absolute limit to the sum of the topographic deflections for ring 35 and all smaller rings is 0″.25. The actual neglected topography inside ring 35 probably corresponds to topographic deflections which are on an average much smaller than this absolute limit.

All of the computations of topographic deflections were examined to ascertain the maximum computed value for each ring in the 496 computations. The following table shows these maxima for rings 37 to 30 in comparison with the absolute limits computed as indicated in the preceding paragraph:

Ring.	Maximum value in computations of meridian deflection.	Maximum value in computations of prime vertical deflection.	Computed absolute limit.
37	″.01	----	″.05
36	.01	----	.07
35	.03	″.01	.10
34	.04	.02	.15
33	.07	.03	.20
32	.11	.05	.29
31	.13	.04	.42
30	.18	.40	.59

With one exception the observed maxima are less than one-half the absolute limit. In the exceptional case it is only two-thirds the absolute limit.

For the hypothetical case of a compartment in ring 35 (outer radius 22.8 meters) having a mean elevation of 15 000 feet above the station, the computed deflection (see p. 125) is only 0″.0065, corresponding to an effective mean elevation of only 65 feet above the station. The reason for the very large discrepancy between the effective elevation and the actual elevation is that in such a compartment much of the attracting material is above the station at such a large angle of elevation from the station that it is ineffective in producing a deflection of the vertical.*

Computations for various rings made as indicated on page 125 have, as there stated, shown that the absolute limit varies from ring to ring in geometric progression with a constant ratio of 1.43. This is the ratio between the outer radii of successive rings. (See p. 21.) Hence the maximum effective elevation of a compartment is proportional to the outer radius of the ring in which it is located.† The absolute limit 0″.59 for ring 30, increasing in geometric ratio 1.43, becomes 43″ for ring 18, of which the outer radius is only 2 295 meters. The maximum value of the topographic deflection for ring 18 in this investigation was 1″.80 in the meridian computations, and in the prime vertical computations, 2″.88 In general it is found that deflections for large rings as actually computed from the maps fall much farther below the absolute limit than do those for small rings. Hence, in estimating the effect of neglected topography for larger rings than ring 35, it is desirable to find some closer limit, some limit approaching actual conditions more closely, than does the absolute limit.

The following table is the basis of such an attempt to find a closer limit:

Ring.	Maximum value in any one of the 231 prime vertical computations.	Station at which said maximum was observed.	Sum of maxima *without regard to* sign to and including this ring.
35	− ″.01	No. 8, San Francisco, Washington Square, California	″.01
34	− .02	No. 241, Avila, California	.03
33	− .03		.06
32	− .05		.11
31	− .04	No. 225, Arguello, California	.15
30	+ .40	No. 228, Santa Cruz West, California	.55
29	+ .31		.86
28	+ .29		1.2
27	+ .26	No. 26, Mount Conness, California	1.4
26	+ .32		1.7
25	+ .46		2.2
24	− .66	No. 25, Virginia City, Nevada	2.8
23	− 1.02		3.9
22	− 1.39	No. 23, Genoa, Nevada	5.3
21	− 1.85		7. 1
20	− 2.50		9.6
19	− 2.32		12
18	+ 2.88	No. 43, Waddoup, Utah	15
17	+ 2.93		18
16	+ 2.86	No. 44, Salt Lake City, Utah	21
15	+ 2.73		23
14	+ 2.39		26
13	+ 3.92	No. 1, Point Arena, California	30
12	+ 4.98		35
11	+ 5.90		41
10	+ 5.74		46

* It would be effective in producing a change of the intensity of gravity at the station.

† This conclusion is reached here by the use of the regular formula used in this investigation. One who cares to do so may prove it more elegantly by deriving the formula which is directly applicable to the limiting case.

Ring.	Maximum value in any one of the 231 prime vertical computations.	Station at which said maximum was observed.	Sum of maxima without regard to sign to and including this ring.
9	+ 7″.24	No. 222, Castle Mount, California	53″
8	+ 9 .72	No. 239, Monterey Bay, California	63
7	+ 9 .47	No. 4, Mount Helena, California	73
6	+10 .15	No. 3, Ukiah, California	83
5	+ 9 .96	No. 19, Round Top, California	93
4	+10 .64		103
3	+10 .04	No. 1, Point Arena, California	113
2	− 8 .57	No. 85, Gould, Ohio	122
1	− 7 .40	No. 233, Provincetown, Massachusetts	129

For the present purpose three facts are especially significant in this table: First, the values in the second column increase downward much more slowly than the absolute limits already discussed, which are in geometric progression with a constant ratio 1.43. Second, among the maxima both algebraic signs are found. Third, the different maxima occur at different stations.

The sums in the last column may be taken as furnishing a limit of the deflections due to neglected topography which will probably not be reached in any actual case. Thus, for example, the deflection corresponding to the neglected topography for all rings smaller than ring 17 will probably in no case exceed 15″, for in order to be 15″ each ring smaller than ring 17 must produce a deflection as great as the maximum observed for that ring at any station, and moreover all of these extraordinarily large values must have one algebraic sign. Such a coincidence is not likely to occur in the United States.*

To obtain a reliable estimate of the errors probably introduced by the neglected topography in the inner rings it is evidently desirable to go one step further, for it is probable that the limits fixed in the preceding paragraph are approached but seldom. It is desirable to secure some estimate of the average effect, as distinguished from the maximum effect, of the neglected topography.

Ten prime vertical stations were selected at random by taking the twenty-third, forty-sixth, sixty-ninth, etc., stations in the prime vertical list as arranged in geographic order in the observation equations. The numbers of the selected stations in the prime vertical list are 193, 147, 113, 106, 198, 65, 3, 229, 11, and 47. At No. 193 the smallest ring computed was ring 30, and the algebraic sum of the computed topographic deflections for rings 30 to 18, inclusive, was −0″.20. The sum without regard to sign of the values in the second column of the table above for these rings is 14″.66, of which the observed −0″.20 is but 0.01 part. In other words, the actual computed deflection for these rings at station No. 193 is only 0.01 of the probable maximum for these rings. Similar comparisons were made for the other nine stations selected at random named above. In no case was the ratio of the actual value to the probable maximum greater than 0.11, and the average ratio was 0.03. It is fair to estimate, then, that though the probable maximum topographic deflection due to all topography in rings smaller than ring 17 is 15″, the probable average effect of such topography is only 0″.45 (0.03 of 15″).

There were 117 of the 496 computations of topographic deflections which included no ring smaller than ring 17.

There were 322 of the 496 computations which were carried inward toward the station at least to ring 26. By the same reasoning as in the preceding paragraphs it may be estimated that for computations carried to ring 26 the probable maximum topographic deflection corresponding to the neglected smaller rings is 1″.4, and the probable average topographic deflection only 0″.04.

* It should be noted that in such a table as that above, constructed from the meridian components of the deflections of the vertical, instead of the prime vertical components, the values are in general much smaller than in the table shown. For example, the 15″ limit for all rings smaller than ring 17 is, in the table, based on meridian deflections only 9″.

Taking into account all of the considerations which have been given, and guided by judgment based upon much study of the details of the problem, it is estimated by the writer that the topographic deflection corresponding to the topography of neglected inner rings is about 0″.3 on an average; that it is reasonably certain that in about two-thirds of the 496 separate cases it does not exceed 1″.5; that it probably exceeds 5″ in a few cases only; and that it is improbable that it is as great as 15″ in any case.

The effect of errors from this source is nearly as great on solution G as on solution B, but in either case the errors are in the accidental class and their effect upon the final results is very small. If it were possible to take all of the topography into the computation, to neglect no inner rings, the average of the residuals would probably be reduced by about 10 per cent.

Errors due to the shrinkage and distortion of the maps used are negligible.

The errors of each of the classes of errors discussed in the preceding paragraphs, and due to the errors of maps, have been made as small as possible by using the best maps for the purpose which are available. The errors of these classes still remaining could not be made smaller by any method of computation. They are inherent in the data, not due to the method of computation.

ERRORS IN TOPOGRAPHIC DEFLECTIONS DUE TO ERRORS IN ASSUMED MEAN DENSITIES.

So, too, the errors in the computed deflections due to the error in the adopted ratio of the surface density of the earth to the mean density—namely, 1 to 2.09*—are inherent in the data and can not be reduced by adopting any more refined methods of computation. It is unlikely that this ratio is in error by as much as one-fifteenth of itself. Any error, expressed as a proportional part in the adopted value of this ratio, produces an error of the same proportional part in the computed topographic deflections and in the reduced deflections when compensation is considered (see the formulæ on pp. 20 and 69). Hence it is unlikely that the computed deflections involved in solutions B and G are in error by as much as one-fifteenth part. The computed deflections involved in solution G, corresponding to the assumption that the depth of compensation is 113.7 kilometers, are, at more than two-thirds of the stations, less than 3″.00 and the maximum is 20″.00 (at prime vertical station No. 43, Waddoup, Utah). Hence an assumed change of one-fifteenth part in the adopted ratio 1 to 2.09 would produce changes in the residuals of solution G which would be less than $0''.20 \left(=\frac{3''.00}{15}\right)$ for more than two-thirds of the stations, and the maximum change would be about $1''.3 \left(=\frac{20''.00}{15}\right)$.

An error in this ratio introduces errors of the systematic class into the observation equations which will not be effectively eliminated from the final results. For example, if the adopted ratio is one-fifteenth part too large the computed prime vertical deflections west of Colorado, which are all positive, are each too large by one-fifteenth, and those for stations east of Colorado, which are all negative, are also too large by one-fifteenth. These will both tend to make the derived equatorial radius and polar semidiameter from solution G too large.

If the adopted ratio as used in solution G is changed from 1 to 2.09 to zero to 2.09, that is, is reduced by 100 per cent of itself, the computed deflection would become zero as in solution A. In other words, such a reduction of 100 per cent in the assumed ratio in solution G reduces that solution to solution A. Hence, a reduction of one-fifteenth in the adopted ratio would change the final results from solution G by approximately one-fifteenth of the difference between those results and the corresponding ones from solution A, making them approach the results from solution A. Solution A gave an equatorial radius 337 meters less than solution G, and a polar semidiameter 433 meters less. Hence, an error of one-fifteenth in the adopted ratio,

*This ratio is based upon the supposition that the surface density of the solid portion of the earth is 2.67 and the mean density of the earth is 5.576. For the data and considerations upon which these values are based, see The Solar Parallax and its Related Constants, by William Harkness, Washington, Government Printing Office, 1891, pp. 89–92, 139. The data there given is also a part of the basis for the above estimate of the uncertainty in the adopted ratio 1 to 2.09.

which is improbable, would produce changes of only about 22 and 29 meters, respectively, in the equatorial radius and polar semidiameter, as derived from solution G. It appears, therefore, that the error arising from this source is probably insignificant.

ERRORS IN TOPOGRAPHIC DEFLECTIONS DUE TO METHOD OF COMPUTATION.

The remaining errors affecting the computed topographic deflections, which are discussed herein, are capable of being further diminished by using a more precise method of computation or by introducing greater refinements into the method which has been used.

In assuming, as has been assumed in the computation, that the deflection produced by the masses within a given compartment is proportional to the mean elevation of the surface in that compartment, the fact is neglected that equal masses in different portions of the compartment produce different deflections at the station. Of two equal masses within a given compartment and lying on the same radial line from the station, the one farthest from the station produces least effect. Of two equal masses within a given compartment and at the same distance from the station, the one which lies more nearly at right angles to the reference line of the template (more nearly in the prime vertical if the meridian component of the deflection is being computed) produces least effect. The errors arising from this source are evidently smaller the smaller the compartments used. In this investigation it is believed that the compartments are sufficiently small to insure that this error will, as a rule, be less than 0″.02 for each compartment, though in rare cases it may be greater than 0″.05. Moreover, in occasional cases when it was noticed, during the progress of the computation, that large and abrupt variations of elevations occurred within a given compartment, especially within one of the larger compartments shown on the template, errors from this cause were guarded against by subdividing this particular compartment into smaller compartments.

The smaller compartments, or subdivisions, were bounded by radial lines and circles which were located in accordance with the principles used in fixing the boundaries of the large compartments.*

The errors under discussion arising from variations of elevations within each compartment belong mainly in the accidental class as there is great variety in the direction and steepness of slopes within the area covered by the computation for each station. Though there are a large number of compartments involved in the computation for a station, nearly 600 in the extreme case, it is believed that the errors are so small for each compartment and so nearly accidental in character that their effect upon the computed deflection for a station is ordinarily less than 0″.10, though it may occasionally exceed 0″.20. The basis for this estimate is an examination in detail of the differences of computed deflections for adjacent compartments as shown in computations such as are reproduced on pages 29–33. The errors from this source tend to be less for inner rings with small compartments than for outer rings. Hence they are much smaller in their effect on solution G than on solution B, the reduction factor F (see p. 70), being smaller for outer rings, and therefore effective in reducing the errors for those rings in solution G.

Errors arise from the computer's inability to estimate the mean elevation within a compartment with absolute accuracy. The difficulty of making the estimate increases with the increase in the size of the compartment, with increase in the total range of variation of elevation within the compartment, and with increased irregularity of the contours. In making the computation for any given station there is no difficulty in estimating the mean elevation within less than 100 feet for the greater number of compartments, and thus securing each of the deflections corresponding to these compartments within less than 0″.01. There were, as a rule, however, a few compartments concerned in each computation in which there was difficulty in making the estimate. To secure greater accuracy in connection with these difficult compartments, to obtain a check on the results, and to obtain a measure of the accuracy attained, a second computer was required to make independent estimates for each station of the mean

*See footnote on p. 33.

elevation of at least 10 per cent of the compartments estimated by the original computer. The compartments for which it was presumably most difficult to make the estimates were selected, by inspection of the maps, to be subjected to this second estimate. Slight differences, of course, developed between the two computers for many of the compartments. If the estimate of the second computer agreed with that of the first computer within less than 0″.20 on each compartment, and also within 0″.20 on the total topographic deflection for the station, the work of the first computer was allowed to stand unchanged. Otherwise, the computers revised their work together with extra care, subdividing into smaller compartments, if necessary, until they had agreed within the specified limits. In 58 per cent of all the cases it was not found necessary to reexamine together any compartments.

It is believed that these precautions insure that the errors due to inaccuracies in estimates are so small that the computed topographic deflection for any station is ordinarily in error from this cause by less than 0″.20, though occasionally the error may be as great at 0″.30. These errors are, apparently, of the accidental class. They tend to be larger for outer rings than for inner rings, and therefore have much less effect on solution G than on solution B.

How large are the errors introduced into the computed topographic deflections by the interpolation of values corresponding to outer rings? The complete computation was made for 68 stations. Each new station to be computed was so chosen, if possible, as to lie within the triangle defined by the nearest three stations for which the computation had already been made, and near the center of the said triangle. From these three surrounding stations the interpolation, if any, was made.

The computation was commenced with the inner smaller rings and proceeded outward. The three rules used by the computers in deciding at what ring it was allowable to begin to accept the interpolated values and to accept them for all larger rings were, as stated on page 43, as follows:

Rule 1. Commence to accept the interpolated values as final with the first ring for which such interpolation is allowable under either rule 2 or rule 3, and which is beyond the one containing the nearest of the three stations from which the interpolation is made.

Rule 2.—Let 1″.00 divided by the number of a ring be called the interpolation limit for that ring. Subject to rule 1, acceptance of the interpolation may begin with a given ring if the three rings next within it each shows an agreement between the interpolated and computed values which is within the interpolation limit.

Rule 3.—Subject to rule 1, acceptance of the interpolation may begin with a given ring, if the next ring inside of it shows an agreement within the interpolation limit and if at the nearest of the three stations from which the interpolation is made, the agreement was also within the interpolation limit for the corresponding ring and for all rings farther out for which the comparison was made.

Under these rules the total error made by accepting interpolated values would always be less than 1″.00, if the error of interpolation was of the same sign and magnitude for all larger rings as was I–C (interpolated minus computed) on the last ring for which the comparison was made.

It was believed, however, that the agreement between the interpolated and computed values (commencing with rings not smaller than those contemplated under rule 1) would tend strongly to be closer and closer for successive rings proceeding outward. It was also believed that there would be a strong tendency for the various differences between interpolated and computed values for several rings such as are interpolated under the rules to include values having both the plus and minus signs, and, therefore, for the errors in the accepted interpolations to tend to be eliminated from the final result for the station. Both of these beliefs were based, at first, on theory only. If they are correct the total error introduced at any station by accepting interpolated values will be, in general, much less than 1″.00.

The correctness of these beliefs is established by the results secured during the progress of the computations. During the progress of 479 computations a comparison between the computed and interpolated values was secured on from 2 to 15 rings. In 73 per cent of the cases the average value, without regard to sign, of I–C (interpolated minus computed) was less for the outer one-half of the rings on which both interpolation and computation was made at that

station than for the inner half of such rings. So, too, in 84 per cent of the cases there were found to be both plus and minus signs of the values of I–C at the station.

These tests confirm the theory to such an extent that it is believed that the total error introduced into the computed topographic deflection at a station by the acceptance of interpolated values is seldom greater than 0″.50 and is, as a rule, less than 0.″25.

More than 90 per cent of the accepted interpolations were for rings larger than ring 13. For these the reduction factors (see p. 70), used in connection with solution G, are all less than 0″.50. Hence the effect upon the absolute terms in the observation equations of solution G of errors of interpolation is probably seldom greater than 0″.20, and is, as a rule, less than 0″.10.

The errors due to the accepted interpolation must be almost entirely in the accidental class.

The possible magnitude of the following errors has been considered and in each case the conclusion reached that these errors are negligible; in other words, that they affect the hundredths of seconds only, not the tenths:

(1) Errors due to inaccuracy in the construction of templates.

(2) Errors due to inaccuracy in placing the templates on the maps.

(3) Errors due to the omission of slope corrections known to be each less than 0″.01. The slope corrections which were larger than 0″.01 were computed. The slope corrections referred to are those necessary to take account of the fact that in certain cases the mean surface within the compartment is so far above or below the station that the masses concerned in the computation can not be considered to be sensibly in the horizon of the station.

(4) Errors due to omitted decimal places.

The comprehensive conclusion is that the total error in the computed value of one component of the topographic deflection at a station, due to all errors of such a character that they are capable of being further diminished by using a more precise method of computation or by introducing additional refinements into the method which has been used, is seldom more than 0″.60 and is probably less than 0″.30 in about one-half of the cases. This total error affects each absolute term of the observation equation in solution B. Similarly, it is believed that the total error of this character which affects solution G is seldom greater than 0″.30 and is less than 0″.15 in about one-half of the observation equations. The average residual from solution G being more than 3″, it is believed that the accuracy with which the topographic deflections have been computed is amply sufficient and that any changes or refinements in this part of the computations which would increase the time required to make the computations would not be warranted by the slight additional accuracy secured.

Would a gain in accuracy result from extending the computation of topographic deflections to a greater distance from each station? The considerations which led to stopping the computations at the ring (No. 1) having an outer radius of 4 126 kilometers (2 564 miles) instead of extending it to the antipodes have been given on page 29. Since the computations have been completed it has been estimated that not more than four more such rings as have been used would be necessary to extend the computations to the antipodes. In this connection it must be recalled that the rings rapidly increase in width at great distances from the station, partly on account of curvature (see p. 23). It was also estimated that the sum of the computed topographic deflections for these three or four rings would not in any case exceed 20″, and would probably be less than 10″, as a rule, for each station. The basis for this estimate is indicated on page 29, and again in the table on page 127, in which it will be noted that the observed maximum for ring 1 is much less than for either ring 4 or ring 3. The omitted portion, probably less than 10″, of each computed topographic deflection introduces errors into solution B which are of the systematic class since, for example, the omitted portion of the prime vertical deflection will have a decided tendency to be of one sign for stations near the Atlantic coast and to be of the opposite algebraic sign for stations near the Pacific coast. The computed results from solution B might, therefore, be appreciably altered by extending the computation to the antipodes.

It is reasonably certain, however, that such an extension of the computation would not affect any residual in solution G by more than 0″.01, and that it would have no effect upon the last significant figure retained in each final result from this solution. This certainty arises

from the fact that the reduction factor F (see p. 70) for ring 1 is only 0″.001, and would be still smaller for larger rings. The reduced topographic deflection with isostatic compensation extending to a depth of 113.7 kilometers is only 0″.01 at any station for ring 1, the largest ring for which the computation was made, and would be still less for each of the larger rings which have been omitted.

EFFECTS OF ERRORS IN COEFFICIENTS.

The coefficients are used in forming the normal equations and in computing the residuals of the observation equations. The accuracy necessary in connection with these uses depends mainly upon the magnitude of the derived values of the unknown quantities and of the absolute terms in the observation equations. In solution G the derived values of the unknowns are so small as to contain two significant figures only, except (α_W), which is 5″.40 (see p. 105), and the absolute terms of the observation equations are as a rule less than 10″. Under these conditions an examination of the manner in which the coefficients are used shows that if each were correct to the nearest unit in the second decimal place the derived values of the unknowns would probably be correct to the last decimal place now retained and the computed residuals would ordinarily differ from their true values by hundredths of seconds only, not tenths. As each unknown as now computed has two uncertain figures (two significant figures in its probable error), and as the average residual is more than 2″, this grade of accuracy in the coefficients would be sufficient. As a matter of fact, however, each coefficient was computed to three or more decimal places and was so used. Therefore the effect of omitted decimal places in the coefficients, in the computation as made, is so small as to be of no consequence.

The coefficients have been computed by formulæ which are known to be approximate. An exhaustive examination has not been made of the magnitude of the quantities neglected in this approximation, but the extensive investigation which has been made enables the writer to state with considerable confidence that the errors in the coefficients due to approximations in deriving the formulæ from which they are computed probably do not affect the second decimal place in any case and, as a rule, do not affect even the third decimal place. Therefore the errors due to this cause are of no consequence.

As an illustration it may be mentioned that the errors of $-2''$ in α_F, $-7''$ in α_B, and of one in the fifth decimal place in log s, as computed for latitude station No. 164, from the adopted approximate formulæ for that purpose (see p. 89), produce errors in the fifth decimal place only in any of the coefficients for that station.

GENERAL CONCLUSION AS TO CAUSE OF RESIDUALS.

Various sources of error, each of which contributes something to the residuals of the adopted solution G, have been discussed somewhat in detail. Among these sources have been the astronomic observations of latitude, longitude, and azimuth, the observations of angles, the measurement of base lines in the triangulation, the errors in the computations of topographic deflections due to the errors and incompleteness of available maps and also to the method of computation, the errors in the assumed mean densities, and, finally, errors in the computed coefficients used in the equations. The detailed examination of these separate sources of error has shown that no one class of errors nor the combination of all classes enumerated is sufficient to account for such large residuals as those in solution G. These residuals must be due mainly to some other cause.

Neither does an examination in detail of these various classes of errors show any reason why the residuals of solution G should be smaller than those from solution A.

From a long and careful study of the details of the evidence the writer is firmly convinced on two main points.

The first is that the residuals of solution G are smaller than those of any other solution made, simply because the assumption upon which solution G is based is a closer approximation to the truth, as a statement of a general law controlling the variation of subsurface densities, than any of the other assumptions made. The assumption for solution G is that the isostatic compensation is complete and uniformly distributed throughout the depth 113.7 kilometers.

The second main point is that the residuals of solution G are mainly due to irregular variations of subsurface densities from the law indicated in the preceding paragraph, the densities being excessive for various areas of greater or less extent and deficient for various other areas. No general law controlling these variations has, as yet, been discovered by the writer.

As these two conclusions are of fundamental importance, some of the evidence on which they are based will now be set forth in considerable detail.

Illustration No. 10, at the end of the volume, shows the residuals of solution G, of which the numerical values are tabulated on pages 106–114.

The arrows are drawn to the scale indicated on the sketch, to represent the residuals of solution G. Arrows representing residuals which are almost zero have been lengthened sufficiently to make them show.

The butt of each arrow is placed as nearly as possible in the position of the station of observation. In some cases it has been necessary to displace the arrows from their true position in order to avoid confusion with other arrows.

The number on each arrow is the same as the number given to the corresponding observation in the list printed on pages 12–19.

An arrow pointing to the southward indicates that the observed astronomic latitude is greater than the computed latitude, and an arrow pointing to the northward indicates a difference of the reverse sign. In other words, the arrow indicates the direction of the nadir point from its normal position as fixed by solution G. The arrow must tend, therefore, to point toward regions in which the density is in excess of that postulated in solution G, and away from regions in which it is less than that postulated.

Similarly, an arrow pointing to the eastward or westward indicates the prime vertical component of the deflection of the nadir point, as fixed by a determination of astronomic longitude or astronomic azimuth, said deflection being expressed with reference to the normal position of the nadir as fixed by solution G. At stations where both longitude and azimuth were observed, only the residual corresponding to the longitude determination was plotted.

The residuals represented by these arrows are due, in part, to errors of observation and computation. Subject to this reservation, they represent the effects of the departures of the actual distribution of densities from that postulated in solution G.

To facilitate a comparison of the residuals with the topography with a view to detecting any possible relation, certain contour lines have been drawn, namely, the shore line, the 100-foot contour, the 1 000-foot contour, and the 4 000-foot contour, in the eastern part of the United States; and in the western part, the shore line, the 1 000-foot, 5 000-foot, and 10 000-foot contours. In each ocean the 1 000-fathom curve has also been drawn. This serves to indicate the position of the submerged edges of the continent.

This illustration No. 10 contains a great mass of material which is available for extensive study.

Even a hasty examination indicates that for certain areas there is a decided tendency for all of the residuals to be of one sign. For example, in a large continuous area comprising nearly all of Utah, all of Nevada, and nearly all of that part of California which lies between latitudes 37° and 40°, all of the meridian residuals are negative, indicating either an excess of density to the northward or a defect of density to the southward, or both.

The grouping together of residuals of the same sign is more clearly shown in illustrations Nos. 11 and 12, at the end of the volume.

Within each area on illustration No. 11 which is inclosed by a solid black line and marked $\phi+$, the meridian residual of solution G is positive for every latitude station, corresponding to an excess of density to the southward of the station. Similarly, in areas marked $\phi-$ all meridian residuals in solution G are negative, corresponding to an excess of density to the northward. Note the large $\phi-$ area in Utah, Nevada, and California to which attention has already been called,* and the very large $\phi+$ area covering Missouri, Illinois, Indiana, and a part of Wisconsin

*A single latitude station, No. 10, New Presidio, San Francisco, Cal., within this area has a small positive meridian residual $+''.16$. The illustration, No. 11, is so small that this could not be conveniently shown as distinct from other San Francisco latitude stations.

Note also that three $\phi+$ areas together make a nearly continuous $\phi+$ area extending from northern Maryland to Louisiana, along the Allegheny Mountains and the Gulf coast. These three areas are separated by two $\phi-$ areas, each of which contains but a single latitude station. Aside from these exceptionally large groups each group shown in illustration No. 11 is too large, as a rule, to be probably due merely to accidental errors. For example, it is improbable that the large $\phi+$ group containing 10 stations on the shores of Lake Ontario and Lake Erie is due to accidental errors. Neither will any of the classes of errors previously discussed to which the observations and computations are subject, produce such a geographic grouping of residuals. The grouping must be due to irregular variations of densities from the general law postulated as the basis of solution G.

Within each area on illustration No. 12 which is inclosed by a solid black line and marked PV+, the prime vertical residual of solution G is positive for every longitude and every azimuth station, corresponding to an excess of density to the eastward of the station. Similarly, in areas marked PV− all prime vertical residuals are negative, corresponding to an excess of density to the westward.* Two PV+ areas together extend almost without interruption from western Maryland to southern Mississippi; another PV+ area contains nearly all of the stations on the shores of Lake Erie,Lake Ontario, and the St. Lawrence River; and 8 stations are grouped in one PV− area in northern Wisconsin and the northern peninsula of Michigan. There are various other groups too large to be due to accidental errors. They must be due to variations in density from the law postulated as the basis of solution G, the departures being of one sign for considerable areas.

Illustrations Nos. 11 and 12 are a graphic proof that the residuals of like sign are grouped together geographically to such an extent as to show that they are due largely not to accidental errors but to systematic disturbances having a regional distribution.

An analytical proof of the same point follows. The 507 astronomic stations concerned in this investigation have been divided into ten groups, each geographically as compact as possible and each containing about 50 stations. The limits of these groups are indicated on illustration No. 13 at the end of the volume. In each group the mean of the latitude residuals with regard to sign was taken. The probable error of that mean was also computed upon the supposition that the residuals represent accidental errors only, the basis of the computation being the probable error of one latitude observation equation as computed from the 265 latitude residuals. The longitude residuals and the azimuth residuals were treated in the same manner. The results are given below in tabular form. The probable error of a single latitude observation equation is $\pm 2''.38$, of a single longitude equation $\pm 2''.00$, and of a single azimuth observation equation $\pm 3''.26$.

Group.	Latitude residuals.			Longitude residuals.			Azimuth residuals.		
	Number of residuals.	Mean with regard to sign.	Probable error	Number of residuals.	Mean with regard to sign.	Probable error.	Number of residuals.	Mean with regard to sign.	Probable error.
		″	″		″	″		″	″
1	24	+0.30	±.49	5	+2.58	±.89	23	+1.40	±.68
2	28	+0.87	.45	15	+0.63	.52	11	−2.94	.98
3	26	−0.44	.47	6	−2.48	.82	17	−2.13	.79
4	24	+2.63	.49	9	+0.68	.67	17	+2.19	.79
5	39	+0.44	.38	6	−4.16	.82	8	−3.80	1.15
6	25	+2.36	.48	9	−0.04	.67	17	−0.37	.79
7	22	−1.10	.51	7	−1.56	.75	17	−0.09	.79
8	21	−3.45	.52	5	+3.67	.89	17	−1.72	.79
9	29	−1.97	.44	13	−0.42	.55	15	−3.35	.84
10	27	+0.31	.46	4	+1.93	1.00	21	+2.20	.71

*There are five prime vertical stations, viz, Nos. 8, 42, 44, 78, and 194, each of which has a residual of opposite sign from that for other stations in the group surrounding it. In each of these cases the exceptional station is either coincident with another station or so nearly so that no distinction can be made on the small scale of illustration No. 12.

If the residuals were entirely due to accidental errors, only about one-half of the means with regard to sign would exceed the corresponding probable errors. In the above table in 23 cases out of 30 the mean exceeds its probable error, and in 3 cases it is more than 5 times its probable error, namely, groups 4 and 8 of the latitude residuals and group 5 of the longitude residuals. This test agrees, therefore, with the graphic test in showing that the residuals are due to regional disturbances.

TESTS OF RELIABILITY OF CONCLUSIONS.

Are the conclusions that the E, H, and G solutions are much nearer the truth than the B and A solutions, and that solution G is nearest the truth, dependent on the effects of a few large residuals? Are these conclusions dependent upon and vitiated by the geographic grouping of residuals of like sign to which attention has been directed? Do astronomic observations in all parts of the area treated and all three classes of astronomic observations cooperate in confirming the conclusions reached? In answer to these questions the following tables are presented with comments:

*Mean values of the squares of the residuals in various groups.**

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group.—All the observations, 507 residuals	129.06	16.21	15.82	15.81	27.46
All latitude observations, 265 residuals	79.37	13.34	12.91	12.86	18.96
All longitude observations, 79 residuals	132.91	8.47	8.81	9.01	35.51
All azimuth observations, 163 residuals	207.98	24.63	23.94	23.89	37.38
Group 1 (Maine, New Hampshire, Massachusetts, Rhode Island), 52 residuals	168.18	8.98	9.14	9.18	9.46
Group 2 (Connecticut, New York, Pennsylvania, Ohio, Michigan), 54 residuals	72.26	9.45	9.65	9.71	21.32
Group 3 (New Jersey, Pennsylvania, Delaware, Maryland, Virginia), 49 residuals	36.62	17.38	17.49	17.48	17.71
Group 4 (Virginia, North Carolina, Tennessee, Georgia, Alabama, Mississippi, Louisiana), 50 residuals	50.23	10.83	10.83	10.86	13.80
Group 5 (Michigan, Minnesota, Wisconsin), 53 residuals	31.52	23.94	23.96	24.00	28.44
Group 6 (Virginia, West Virginia, Kentucky, Ohio, Indiana, Illinois, Missouri, Wisconsin), 51 residuals	35.45	8.16	8.52	8.58	10.29
Group 7 (Missouri, Kansas, Colorado, Utah), 46 residuals	453.35	8.92	8.53	8.70	45.50
Group 8 (Utah, Nevada, California), 43 residuals	157.25	25.25	21.83	21.28	34.09
Group 9 (California, northern part), 57 residuals	138.85	16.63	15.97	15.94	37.64
Group 10 (California, southern part), 52 residuals	182.16	33.10	32.29	32.28	57.39

* Certain numerical values printed in this table and on the following pages differ slightly from those given in "Geodetic Operations in the United States, 1903–1906. A Report to the Fifteenth General Conference of the International Geodetic Association," because of the correction of an error in computation which was discovered after that report went to the printer.

Mean value of residuals without regard to sign.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
United States group.—All the observations, 507 residuals	8.86	3.06	3.04	3.04	3.92
All latitude observations, 265 residuals	7.29	2.78	2.76	2.76	3.34
All longitude observations, 79 residuals	8.85	2.22	2.30	2.34	4.18
All azimuth observations, 163 residuals	11.42	3.91	3.85	3.85	4.72
Group 1, 52 residuals	11.95	2.50	2.50	2.50	2.46
Group 2, 54 residuals	6.83	2.43	2.51	2.53	3.69
Group 3, 49 residuals	4.15	3.35	3.34	3.33	3.33
Group 4, 50 residuals	6.00	2.69	2.71	2.72	2.94
Group 5, 53 residuals	4.33	3.83	3.85	3.86	4.32
Group 6, 51 residuals	5.11	2.31	2.36	2.37	2.66
Group 7, 46 residuals	18.47	2.19	2.21	2.26	4.83
Group 8, 43 residuals	11.59	3.95	3.54	3.48	4.41
Group 9, 57 residuals	9.83	2.86	2.82	2.83	4.42
Group 10, 52 residuals	11.56	4.55	4.56	4.57	6.17

Percentage of residuals less than 2″.00.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group.—All the observations, 507 residuals	15	41	43	43	34
All latitude observations, 265 residuals	18	43	45	45	40
All longitude observations, 79 residuals	18	57	56	56	39
All azimuth observations, 163 residuals	10	29	34	34	23
Group 1, 52 residuals	4	40	46	48	46
Group 2, 54 residuals	15	52	48	48	33
Group 3, 49 residuals	45	35	41	41	39
Group 4, 50 residuals	16	46	48	44	38
Group 5, 53 residuals	30	26	28	28	25
Group 6, 51 residuals	20	53	49	51	37
Group 7, 46 residuals	0	54	57	57	33
Group 8, 43 residuals	2	28	40	40	37
Group 9, 57 residuals	11	46	46	46	44
Group 10, 52 residuals	10	27	29	29	12

Percentage of residuals greater than 5″.00.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group.—All the observations, 507 residuals	66	18	18	18	29
All latitude observations, 265 residuals	61	14	13	13	23
All longitude observations, 79 residuals	66	8	9	10	30
All azimuth observations, 163 residuals	75	29	30	30	38
Group 1, 52 residuals	94	10	12	12	10
Group 2, 54 residuals	52	11	13	13	28
Group 3, 49 residuals	27	22	22	22	24
Group 4, 50 residuals	58	14	16	16	18
Group 5, 53 residuals	36	25	28	28	34
Group 6, 51 residuals	53	10	8	6	14
Group 7, 46 residuals	100	9	7	11	30
Group 8, 43 residuals	95	30	21	19	40
Group 9, 57 residuals	72	14	12	12	40
Group 10, 52 residuals	83	37	38	40	54

Maximum residual in each group.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
United States group.—All the observations, 507 residuals	+43.84	−16.47	+15.74	+15.94	−22.35
All latitude observations, 265 residuals	−23.74	−16.47	−15.61	−15.45	−14.34
All longitude observations, 79 residuals	+37.14	+ 9.96	+ 9.13	+ 8.95	−22.35
All azimuth observations, 163 residuals	+43.84	−15.89	+15.74	+15.94	−20.04
Group 1, 52 residuals	−22.64	+ 6.64	+ 6.98	+ 7.02	+ 8.40
Group 2, 54 residuals	−20.64	− 7.99	− 7.33	− 7.25	−12.83
Group 3, 49 residuals	+17.45	− 9.91	−10.02	−10.03	−10.23
Group 4, 50 residuals	+15.47	+ 8.07	+ 8.12	+ 8.14	+ 8.89
Group 5, 53 residuals	+16.77	+13.86	+13.26	+13.14	+11.48
Group 6, 51 residuals	+11.64	+ 6.01	+ 6.13	+ 6.19	+ 6.86
Group 7, 46 residuals	+43.84	− 8.32	− 7.42	− 7.28	−20.86
Group 8, 43 residuals	−23.74	−16.47	−15.61	−15.45	+14.48
Group 9, 57 residuals	−31.48	−15.89	−15.12	−14.99	−22.35
Group 10, 52 residuals	−32.98	+14.89	+15.74	+15.94	+20.03

The above tables are placed in the order of the reliability of the tests furnished by them, the most reliable tests being placed first.

The geographic limits of groups 1 to 10 are shown on illustration No. 13 at the end of the volume.

In the first table, giving mean values of the squares of the residuals, every geographic group and every class of astronomic observations all cooperate in indicating that the E, H, and G solutions, representing isostatic compensation, are much nearer the truth than either solution B, on the basis of extreme rigidity, or solution A, based upon the assumption that the deflections of the vertical are independent of the topography.

The first table indicates but slight differences between the E, H, and G solutions. Though the mean square is less for solution G than for E or H in the group comprising all observations (507 residuals) and also in the group of latitude observations (265 residuals) and the group of azimuth observations (163 residuals), this is not true for the smaller group of longitude observations (79 residuals) and the geographic groups 1 to 7, inclusive (46 to 54 residuals each). It is noticeable, however, that the differences in favor of solution G, as against solution E, in groups 8, 9, and 10 are much larger (from .69 to 3.97) than the differences against solution G in groups 1 to 7 (from .03 to .42). While the preponderance of evidence is in favor of solution G, the evidence contains contradictions.

The evidence shown in the second table, of mean values of residuals without regard to sign, corroborates closely that shown in the first table. The only exceptional features are that in group 1 the mean residual for solution A is slightly smaller than for the G, H, and E solutions, and in group 3 it is the same as for the G solution.

The evidence in the third table, the percentage of residuals less than 2″.00, corroborates, in a general way, that in the first table. It is, however, stronger in favor of solution G than is the first table. Of the 14 groups there are but three in which the percentage of residuals less than 2″.00 is not at least as large in the G solution as in the H and E solutions. An exceptional feature of the table is that for groups 3 and 5 the percentage of residuals less than 2″.00 is greater for solution B than for any other solution. This is, however, offset by the fact that in group 7 not a single residual is less than 2″.00 in solution B, all being in fact greater than 5″.00.

The fourth table, showing the percentage of residuals greater than 5″.00, furnishes a general corroboration of the first table and contains no exceptional features.

The last table, showing the maximum residual in each group, corroborates the first table. It is slightly stronger than the first table in favor of solution G as against solutions E and H. Its exceptional features are that in groups 5 and 8 and in the group of latitude residuals the maximum residual is, in each case, smaller for solution A than for any other solution.

The residuals have also been studied in four geographic groups, known as the northeastern, southeastern, central, and western groups, of which the limits are shown on illustration No. 13.

The tables for these groups are given below in the same form as the tables already shown. For convenience of comparison the line for "all the observations" is repeated from above in each table. In the first table, moreover, the subdivision of each geographic group into latitude, longitude, and azimuth observations is also shown.

Mean value of the squares of the residuals in various groups.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group, all the observations, 507 residuals.	129.06	16.21	15.82	15.81	27.46
Northeastern group, 118 residuals	112.46	10.03	10.14	10.17	15.37
Latitude observations, 58 residuals	81.16	7.71	7.77	7.78	10.99
Longitude observations, 22 residuals	93.14	7.26	8.52	8.82	20.97
Azimuth observations, 38 residuals	171.41	15.19	14.70	14.61	18.80
Southeastern group, 105 residuals	41.57	13.02	13.09	13.11	15.09
Latitude observations, 53 residuals	29.35	8.84	9.36	9.47	13.11
Longitude observations, 16 residuals	29.30	6.39	5.91	5.83	5.28
Azimuth observations, 36 residuals	65.03	22.12	21.78	21.69	22.37
Central group, 102 residuals	86.22	16.01	16.17	16.23	20.07
Latitude observations, 63 residuals	43.64	16.84	16.81	16.84	19.45
Longitude observations, 15 residuals	179.07	11.66	13.05	13.39	23.64
Azimuth observations, 24 residuals	139.94	16.55	16.42	16.38	19.48
Western group, 182 residuals	214.31	22.18	20.88	20.78	46.57
Latitude observations, 91 residuals	132.09	17.14	15.56	15.32	27.10
Longitude observations, 26 residuals	203.70	8.93	8.41	8.59	73.26
Azimuth observations, 65 residuals	333.66	34.53	33.31	33.30	63.16

Mean value of residuals without regard to sign.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	″	″	″	″	″
United States group, all the observations, 507 residuals.	8.86	3.06	3.04	3.04	3.92
Northeastern group, 118 residuals	8.97	2.57	2.60	2.60	3.09
Southeastern group, 105 residuals	5.07	2.88	2.89	2.89	3.10
Central group, 102 residuals	6.44	3.02	3.06	3.07	3.54
Western group, 182 residuals	12.34	3.50	3.40	3.40	5.14

Percentage of residuals less than 2″.00.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group, all the observations, 507 residuals.	15	41	43	43	34
Northeastern group, 118 residuals	11	44	45	46	38
Southeastern group, 105 residuals	30	44	48	46	39
Central group, 102 residuals	22	40	40	41	32
Western group, 182 residuals	7	37	41	41	30

Percentage of residuals greater than 5''.00.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
United States group, all the observations, 507 residuals.	66	18	18	18	29
Northeastern group, 118 residuals	69	12	14	14	19
Southeastern group, 105 residuals	47	18	18	17	20
Central group, 102 residuals	49	15	17	18	24
Western group, 182 residuals	85	24	21	21	44

Maximum residual in each group.

	Solution B.	Solution E.	Solution H.	Solution G.	Solution A.
	''	''	''	''	''
United States group, all the observations, 507 residuals.	+43.84	−16.47	+15.74	+15.94	−22.35
Northeastern group, 118 residuals	−22.64	− 7.99	− 7.33	+ 7.29	−12.83
Southeastern group, 105 residuals	+17.45	− 9.91	−10.02	−10.03	−10.23
Central group, 102 residuals	+39.16	+13.86	+13.26	+13.14	+11.48
Western group, 182 residuals	+43.84	−16.47	+15.74	+15.94	−22.35

These tables corroborate the conclusions already drawn.

The exceptional features of the tables are that, in the longitude observations of the southeastern group, the mean value of the square of the residuals is less in solution A than in any of the other solutions, and that in the central group the maximum residual is less for solution A than for any other solution.

From these statistical studies of the residuals taken in various groups the following three conclusions may be drawn.

(1) That solution B (extreme rigidity) is much farther from the truth than any of the other solutions and that this conclusion is not dependent upon nor vitiated by a few large residuals or by the geographic grouping of the residuals and is a necessary conclusion regardless of what class of observations is utilized (latitudes, longitudes, or azimuths). The evidence is practically unanimous on this point.

(2) That solution A (depth of compensation zero) is farther from the truth than any of the three solutions G, H, and E (depth of compensation 113.7, 120.9, and 162.2 kilometers, respectively), but much nearer the truth than solution B (extreme rigidity), and that this conclusion is nearly free from doubt due to the influence of exceptionally large residuals, to geographic grouping of residuals, or to contradiction between different classes of astronomic observations.

(3) That the preponderance of evidence is in favor of solution G (depth of compensation 113.7), being nearer the truth than either solutions H or E (depths of compensation 120.9 and 162.2 kilometers), but that this conclusion is drawn from conflicting evidence indicating that the nearness of approach to the truth is so nearly the same in these three solutions that the choice between them is made uncertain by the influence of a few unusually large residuals, and by the influence of the geographic grouping of residuals. Moreover, the three classes of astronomic observations do not agree in the choice among these three. In other words, though it is certain that an approach to perfect isostatic compensation exists extending to a moderate depth (certainly not greater than 200 kilometers) if it is uniformly distributed, the precise depth is difficult to determine from the data in hand.

SOLUTIONS OF EQUATIONS IN FOUR SEPARATE GEOGRAPHIC GROUPS.

During the progress of this investigation as the data gradually became available the normal equations corresponding in form to those shown on page 105 were formed separately for each of the four geographic groups known as the northeastern, southeastern, central, and western groups (see illustration No. 13). For some of these, other depths of compensation were assumed than those involved in solutions G, H, and E. These various solutions served as a reconnaissance of the problem.

They are now superseded by the much stronger solutions which involve all of the data at one time. They are much weaker than the final solutions not simply or mainly because each involved only about one-quarter of all the data. The fact that each was restricted, roughly speaking, to one-quarter of the total area is much more important in reducing the strength of the conclusions drawn from each solution.

The results of the various solutions are, however, of value as an indication of the degree of instability in derived results and conclusions which may be expected whenever studies are made of the figure of the earth and isostasy from data confined to a small area, and as an indication of possible characteristic differences between different parts of the United States as to the depth and completeness of isostatic compensation.

Comparison of various solutions made separately for four groups.

[Corrections to the assumed United States Standard Datum and Clarke 1866 values.]

	(ϕ)	(λ)	(α)	(a)	(e^2) in units of the sixth decimal place.	Mean square of residuals.
NORTHEASTERN GROUP.	*Seconds.*	*Seconds.*	*Seconds.*	*Meters.*		
Solution B (extreme rigidity)	+19.13±1.60	+ 1.87±4.13	− 9.34±3.34	+2 276±358	+859±212	34.53
Solution D (depth of compensation 329.8 km.)	+ 1.32±0.77	− 6.53±1.99	+ 5.04±1.61	+ 379±172	+298±102	10.31
Solution E (depth of compensation 162.2 km.)	− 0.13±0.73	− 4.02±1.89	+ 4.63±1.52	+ 126±164	+280± 97	9.31
Solution A (depth of compensation zero)	− 0.63±0.88	− 0.38±2.28	+ 3.43±1.84	− 110±197	+209±117	13.52
SOUTHEASTERN GROUP.						
Solution B (extreme rigidity)	+19.14±0.63	−11.24±2.22	+ 2.25±1.31	+4 494±165	+922± 82	23.40
Solution D (depth of compensation 329.8 km.)	+ 1.47±0.46	+ 2.26±1.61	− 1.61±0.95	+ 343±120	−216± 60	12.33
Solution E (depth of compensation 162.2 km.)	+ 0.23±0.43	+ 5.92±1.52	− 3.57±0.89	− 142±113	−317± 56	10.89
Solution F (depth of compensation 79.8 km.)	− 0.30±0.42	+ 7.82±1.46	− 4.65±0.86	− 401±108	−365± 54	10.10
Solution A (depth of compensation zero)	− 1.20±0.42	+ 9.08±1.47	− 5.44±0.86	− 594±109	−438± 54	10.16
CENTRAL GROUP.						
Solution B (extreme rigidity)	+11.59±0.59	+18.50±1.51	−14.42±0.93	+ 856±163	−473± 87	16.80
Solution D (depth of compensation 329.8 km.)	+ 1.13±0.56	+ 2.70±1.44	− 2.91±0.89	− 376±155	−381± 83	15.28
Solution E (depth of compensation 162.2 km.)	+ 1.05±0.55	0.00±1.43	− 1.32±0.88	− 183±154	−267± 82	15.05
Solution A (depth of compensation zero)	+ 1.07±0.59	− 3.67±1.52	+ 0.60±0.94	+ 72±164	−126± 88	17.04
WESTERN GROUP.						
Solution B (extreme rigidity)	+21.15±0.62	+53.74±2.58	−31.87±1.42	+11 540±189	+608±111	49.67
Solution E (depth of compensation 162.2 km.)	− 1.80±0.38	+ 5.42±1.58	− 8.02±0.87	+ 866±116	−226± 68	18.72
Solution G (depth of compensation 113.7 km.)	− 2.39±0.38	+ 2.80±1.57	− 6.66±0.87	+ 374±115	−208± 67	18.47
Solution A (depth of compensation zero)	− 3.25±0.58	− 2.26±2.40	− 5.06±1.33	− 740±176	− 43±103	43.29

It is to be noted that, in each group, some solution for a moderate depth of compensation gave a smaller mean square of the residuals than did solution A. This shows that it is possible to derive the depth of compensation separately from each group. From the results here given, supplemented by certain approximate computations, the most probable depth of compensation was derived from each of the first three groups in June, 1905, before the solution for the western group or for all groups combined had been made.

Most probable depth of compensation.

From northeastern group,	146 kilometers,	estimated weight 1
From southeastern group,	48 kilometers,	estimated weight $\frac{1}{2}$
From central group,	127 kilometers,	estimated weight $\frac{1}{2}$
Weighted mean,	117 kilometers	

The weight estimated for the most probable depth of compensation from the northeastern group was made double that for each of the other two groups because the solutions for different depths showed a tendency to a more rapid variation in the mean square of the residuals with respect to assumed depth of compensation.

These three values of the most probable depth of compensation differ considerably. But the weighted mean, which was a prediction, is very close to the most probable value derived a year later from the complete investigation, namely, 112.9 kilometers.

No estimate of the depth of compensation was made from the western group. If made it would apparently have been not far from 117 kilometers.

Though the derived depth of compensation is much smaller for the southeastern group than for the others, it is not certain that this difference is real. It may be merely a result of the unavoidable errors in the derivation. The writer considers it merely an indication, not a proof, that the depth of compensation is smaller for this region than for the remainder of the United States.

The degree of instability in the determination of the equatorial radius and the square of the eccentricity is shown by the four corrections to the Clarke 1866 values as derived from the best solution for each group. The best solution is considered to be in each case that for which the mean square of the residuals is least.

	(a)	(e^2)
From the northeastern group	$+126 \pm 164$ meters	$+.000280 \pm .000097$
From the southeastern group	-401 ± 108 meters	$-.000365 \pm .000054$
From the central group	-183 ± 154 meters	$-.000267 \pm .000082$
From the western group	$+374 \pm 115$ meters	$-.000208 \pm .000067$

The adopted final results from the complete investigation involving all the observations are $(a) = +76 \pm 34$ meters and $(e^2) = -.000065 \pm .000021$.

If weights be fixed, as is usual, in inverse proportion to the squares of the probable errors, no one of the values of the equatorial radius derived from separate groups is to be assigned a weight as great as $\frac{1}{10}\left(\frac{34^2}{108^2}=\frac{1}{10.9}\right)$ of that assigned to the final adopted value. Similarly, no one of the values of the correction to the square of the eccentricity derived from a separate group is to be assigned a weight as great as $\frac{1}{6}\left(\frac{.000021^2}{.000054^2}=\frac{1}{6.6}\right)$ of that of the final adopted value. In other words, though the combined group on which the final results depend contains only about four times as many observations as each of the four separate groups, because of the effect of increased area, the weights of its results are from 6 to 23 $\left(\frac{34^2}{164^2}=\frac{1}{23}\right)$ times as great as the weights of those from the separate groups.

The total range of the four values of the equatorial radius from the four groups is 775 meters. The difference between the Bessel and the Clarke 1866 values of the equatorial radius is 809 meters. The total range in the four values of e^2 is .000645, corresponding to a range of 28 in the reciprocal of the flattening. The difference between the Bessel and the Clarke 1866 values of the reciprocal of flattening is only 4.2.

CONSTANCY OF THE DEPTH OF COMPENSATION.

It has been assumed in this investigation that the depth of compensation is the same in all parts of the area concerned in this investigation. This area includes all of the United States and large adjacent areas. The area concerned is that over which the computation of topographic deflections was extended, the areas covered by rings of moderate radius (neither very small or very large) being concerned to the greater extent, while the outer larger rings are concerned to a progressively smaller extent. It matters little in this investigation whether the actual depth of compensation is great or small in the distant areas covered only by outer rings in the computation of topographic deflections. This investigation can furnish little or no information as to the depth of compensation in such distant areas. The investigation does, however, furnish some indications of the possible variation of depth of compensation within the limits of the United States.

In connection with the preceding paragraph, note that in the table on page 70 of reduction factors a change in the assumed depth of compensation produces a very small change in the factor for the outer ring (No. 1). Note also that for such a ring as No. 11 (outer radius 113.7 kilometers) a change in the assumed depth produces a comparatively large change in the factor F. Thus, for the two cases cited a change of the assumed depth from 120.9 to 113.7 kilometers makes changes in F of less than .001 and of .024, respectively. Similarly, it should be noted that, for the small ring No. 24 (outer radius 1.13 kilometers), this change of assumed depth makes a change of only .001 in F, and for smaller rings the corresponding change is still smaller. The more sensitive are the factors to change of assumed depth of compensation the more sensitive are the computed deflections to such changes of assumption, and therefore the stronger the determination of the depth of compensation in the area covered by the ring in question.

The most reliable evidence obtained in this investigation in regard to the possible variation in the depth of compensation is contained in the residuals of the A, G, H, and E solutions for all of the observations treated as one group. These residuals are tabulated on pages 106–114 and statistics in regard to them are given in the tables on pages 135–139. For any given region the most probable depth of compensation is that for which the mean square of the residuals of the corresponding solution is least.

In the table on page 138 showing the statistics of the residuals indicated above, as separated into four geographic groups, solution E has the smallest mean square of the residuals in the first three groups, and for the western group solution G has the smallest. As solution E was made for an assumed depth 162.2 kilometers, and solution G for 113.7 kilometers, this indicates that the depth of compensation is greater for the other three groups than for the western group. The evidence is very weak, however, since the mean square of the residuals varies less than 2 per cent among solutions E, H, and G, in the northeastern group, and still less in the southeastern and central groups. The determination of the depth of compensation in these three groups is, therefore, weak. The depth of compensation is much more strongly determined in the western group, in which the mean square of the residuals shows a range of variation among solutions E, H, and G of more than 6 per cent.

If in the same table on page 138 the mean squares of the residuals for the latitude, longitude, and azimuth equations of each group separately are studied, the weakness of the evidence becomes still more apparent. Of the nine tests possible in this way for the three groups, northeastern, southeastern, and central, there are five which show the H or G solution having smaller values than the E solution and thus contradicting the result for the group as a whole.

Similarly, if the evidence given by the tables on pages 138, 139 of mean residuals without regard to sign, of percentage of residuals less than 2″.00, of percentage of residuals greater than 5″.00, and of maximum residuals, is studied, it will be found to be weak and to contain many contradictions within itself though its general tendency is to indicate that the depth of compensation is somewhat greater for the other three groups than for the western group.

If the evidence from the ten geographic groups given in the tables on pages 135–137 is examined, it will be found to lead to the same conclusion as above. In the first table, giving the mean squares of the residuals, groups 8, 9, and 10,in the far west, show a minimum for solution G and each of the remaining groups show a minimum for either solution E or solution H. The evidence is weak, as it consists of small differences. Moreover, the evidence from the four tables following this first one is both weak and contradictory, though in general trend it corroborates that from the first table.

Finally, it may be noted that, as shown on page 141, from the preliminary solutions made separately for the northeastern, southeastern, and central groups, it was predicted that the depth of compensation was 117 kilometers and that this afterwards proved to agree almost exactly with the final result. These solutions indicated, therefore, that the mean depth of compensation is substantially the same for these three groups as for the western group. The evidence from these preliminary solutions for separate groups is weaker than that already discussed, depending upon the final solutions, because in the preliminary solutions the values of the five unknowns, including (a) and (e^2), were determined with much less accuracy than in the final solution.

The general conclusion is that while there are indications that the depth of compensation is greater in the eastern and central portions of the United States than in the western portion, the evidence is not strong enough to prove that there is a real difference in depth of compensation in the different regions. Possibly such a difference may exist, but it is not safe now to assert that it exists.

METHOD OF COMPUTING THE MOST PROBABLE DEPTH OF COMPENSATION.

On page 115, after calling attention to the relative values of the sums of the squares of the residuals for various solutions and to the fact that this sum is less for solution G than for any of the others, it was stated that the most probable depth of compensation, as derived from these solutions, is 112.9 kilometers. The method by which this particular value, 112.9 kilometers, was derived was not there stated. It will now be given.

Having made three complete solutions for three assumed depths of compensation, and having obtained the sums of the squares of the residuals for each, the problem is to compute the most probable depth of compensation, or that depth for which the sums of the squares of the residuals would be a minimum. The concrete case in hand is, having made the three solutions E, H, and G, for which the sums of the squares of the residuals are, respectively, 8 220, 8 020, and 8 013, and the corresponding assumed depths are 162.2, 120.9, and 113.7, to compute the most probable depth.

The fundamental assumption made for this purpose is that the curve which has for abscissæ the logarithms of the depths corresponding to the three solutions and for ordinates the sums of the squares of the residuals in those solutions, is a parabola, with its axis vertical. This assumption is based on three considerations. First, for small changes of assumed depth of compensation the computed deflection varies nearly proportionally to the variation in the logarithm of the depth; second, the consideration just stated being approximately true, it follows that when the assumed depth is changed the residual for any one station will vary nearly in proportion to the change in the computed deflection, and the curve having logarithms of depth for abscissæ and that residual squared for ordinates will therefore be a parabola with its axis vertical; third, any short portion of the curve referred to at the beginning of this paragraph, which is a composite of many such parabolas, will be nearly a parabola with its axis vertical. This assumption is a generalization of which no proof is offered. Its safety depends upon the fact that it is applied to short portions only of the curve.

The proposition made above, that for small changes of assumed depth of compensation the computed deflection varies nearly proportionally to the variation in the logarithm of the depth, is one which it is not easy to prove from the formulæ. It may be made evident in another way. In the table of reduction factors, see page 70, three columns are headed 162.2, 113.7, and 79.76 kilometers. The logarithms of these three depths are 2.210, 2.056,

and 1.902, with the successive differences .154. The two successive differences of the factors for corresponding rings, in these columns are

Differences of Factors F.

Ring.	Factor for depth 162.2 minus that for depth 113.7.	Factor for depth 113.7 minus that for depth 79.76.
26	.001	.001
25	.001	.003
24	.003	.004
23	.004	.005
22	.005	.007
21	.007	.011
20	.011	.014
19	.014	.021
18	.021	.030
17	.030	.041
16	.041	.058
15	.058	.080
14	.080	.103
13	.103	.125
12	.125	.135
11	.135	.124
10	.124	.095
9	.095	.062
8	.062	.037
7	.037	.020
6	.020	.010
5	.010	.005
4	.005	.002
3	.002	.002
2	.002	.000
1	.000	.001

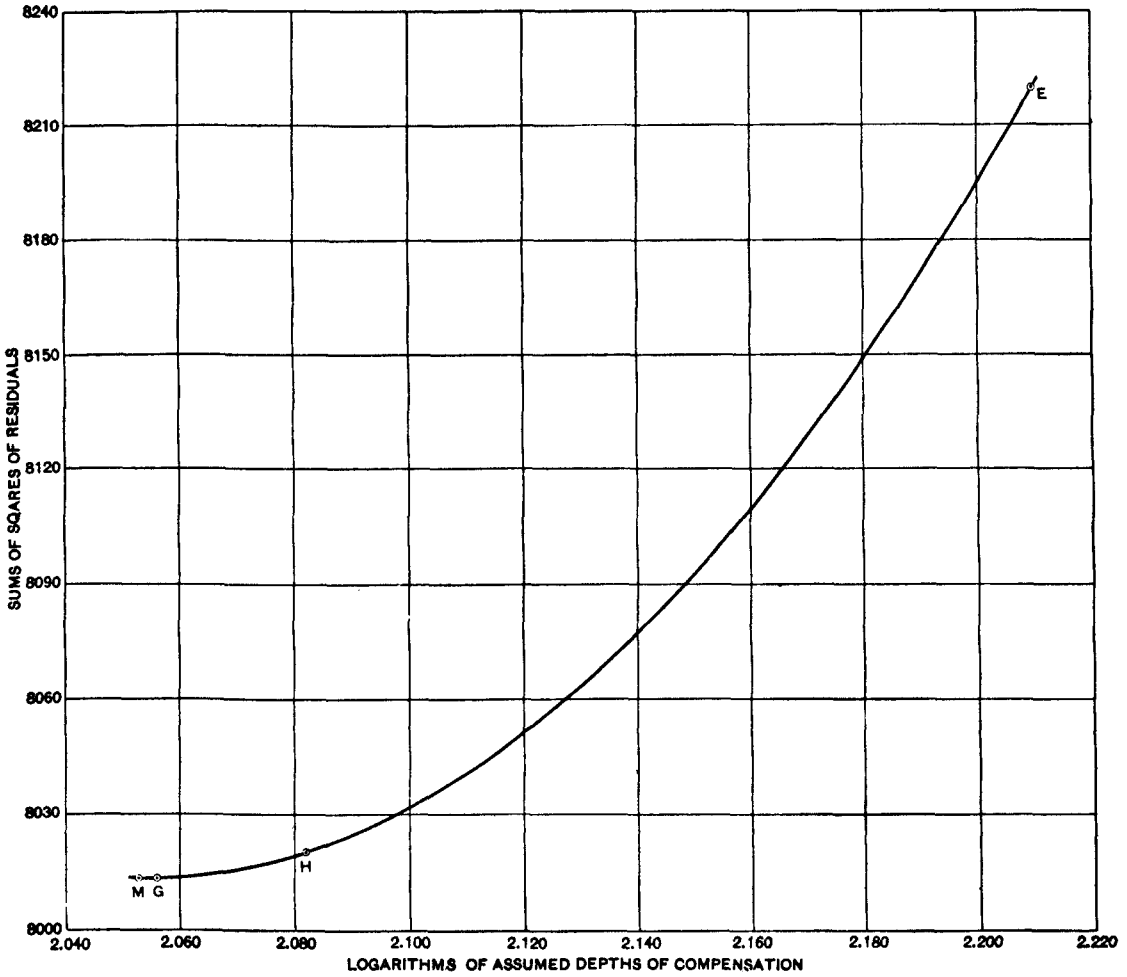
These two columns of differences are identical except that in the second column each difference is raised one line higher than in the first.* Hence the mean difference in the second column is identical with the mean difference in the first column. If the values of the computed topographic deflections for separate rings, such as are shown on page 72, into which these factors are multiplied to obtain the computed deflection when isostatic compensation is considered, were all equal, it is evident that the two successive differences between the computed deflections corresponding to these three depths would be equal, just as the differences of the logarithms of the depths are equal. In other words, the computed deflections would be proportional to the logarithms of the depths. But the computed topographic deflections for the separate rings at a station are not equal. They ordinarily vary somewhat irregularly from ring to ring, with a tendency to increase, as a rule, with increase in the size of the rings, except for the last few outer rings. Hence in the table above, in addition to comparing mean differences, the actual corresponding differences must be compared. The greatest disparity between corresponding differences occurs on ring 9, for which the smaller difference (.062) is nearly two-thirds as great as the larger difference (.095). For other rings the disparity is less. Hence, even though the computed topographic deflection is different for different rings, it is still approximately true that for small changes of assumed depth of compensation the computed deflection varies nearly proportionally to the logarithm of the depth. Of course the smaller the assumed change of depth the closer is the approach to proportionality. Finally, it should be stated that for many cases the law stated has been tested by comparing the deflections computed for various assumed depths of compensation and has been found upon an average to be approximately true (see tables on pp. 48–56).

* This arises from the peculiar relation of the factors, in such columns as the three selected, which has already been pointed out on p. 72.

The proposition which has been made, that when the assumed depth of compensation is changed the residual at a station varies nearly in proportion to the change in the computed deflection, may be tested by comparing the computed deflections on pages 48–56 with the residuals on pages 106–114.

The abscissa of the lowest point on the actual curve having logarithms of depths for abscissæ and sums of squares of residuals for ordinates is obviously the logarithm of the most probable depth of compensation, as for that depth the sum of the squares of the residuals is a minimum.

To find the abscissa of the lowest point of a parabola with its axis vertical, having given three points on the curve, two properties of such a parabola were used. The slope of the chord joining two points on the curve is the same as the slope of the curve at the point of which the



No. 14.

abscissa is the mean of the abscissæ of the two points. The slope of the curve at any point is proportional to the distance measured along the horizontal from the lowest point of the curve to that point.

The method may be illustrated numerically by the case at hand, namely, the sums of the squares of the residuals in the three solutions E, H, and G being 8 220, 8 020, and 8 013, respectively, what is the most probable depth of compensation?

On the curve shown in illustration No. 14 having for abscissæ the logarithms of the assumed depths of compensation and for ordinates the sum of the squares of the corresponding residuals, the three abscissæ corresponding, respectively, to the ordinates 8 220, 8 020, and 8 013 are 2.210, 2.082, and 2.056, these being the logarithms of the depths 162.2, 120.9, and 113.7 kilometers, assumed in solutions E, H, and G.

Hence the slope of the chord from the point H to the point E on the curve is

$$\frac{8\,220-8\,020}{2.210-2.082}=\frac{+\ 200}{+.128}=+1\,560$$

and this is assumed to be the slope of the curve itself at the point of which the abscissæ is 2.146 (the mean of 2.210 and 2.082).

Similarly the slope of the chord from the point G to the point H is

$$\frac{8\,020-8\,013}{2.082-2.056}=\frac{+\ 7}{+.026}=+269,$$

and this is assumed to be the slope of the curve at the point of which the abscissæ is 2.069.

Hence the rate of change of slope of the curve is

$$\frac{1\,560-269}{2.146-2.069}=\frac{+1\,291}{+.077}=+16\,800$$

The distance from the point having the abscissæ 2.069 back to the point at which the slope of the curve is zero is, therefore, $\frac{269}{16\,800}=.016$, and the abscissæ of this minimum point (M) on the curve is $2.069-0.016=2.053$. The most probable depth of compensation has this logarithm and is 113.0 kilometers.*

A consideration of the approximations involved in such a computation shows that the accuracy of the determination of the most probable depth will be greater the nearer are the three points (such as E, H, and G) for which the solutions are made, to each other and to the point sought, and that it is desirable that the point sought shall be included between two of the three points. For these reasons it is important to make a good prediction as early as possible in the investigation of the most probable depth of compensation.

The method of determining the most probable depth here described was evolved gradually during the progress of this investigation. The best proof of the general soundness of the method is the fact that successive computations of the most probable depth showed a rapid convergence upon one value, that which has been adopted as final.

To obtain an idea of the accuracy with which the most probable depth of compensation is determined by this investigation, seven other computations of it were made in addition to that shown above, using in each case a part of the residuals only, as indicated in the following table:

	Probable depth of compensation (kilometers).
From all latitude residuals	98
From all longitude residuals	156
From all azimuth residuals	105
From all residuals of the central group	174
From all residuals of the northeastern group	187
From all residuals of the southeastern group	(†)
From all residuals of the western group	107

This indicates that the probable error of the derived depth of compensation is a few kilometers, not tens of kilometers nor tenths of kilometers.

Other tests were made with still smaller groups of residuals which confirmed this conclusion.

The general conclusion reached on this point, from all the evidence available, is that for the United States and adjacent areas, if the isostatic compensation is uniformly distributed with respect to depth, the most probable value of the limiting depth is 70 miles (113 kilometers), and it is practically certain that the limiting depth is not less than 50 miles (80 kilometers) nor more than 100 miles (160 kilometers).

* The difference, 0.1 kilometer, between this value and the value 112.9 kilometers (see p. 115) obtained from the original computation is too small to be of any importance and is due to a slight difference in the arrangement of the two computations and to the effect of omitted decimal places.

† The observations indicated this part of the curve, near the points corresponding to E, H, and G of illustration No. 14, to be nearly straight, but convex upward instead of downward. Hence, no determination of the probable depth of compensation could be made from these three points alone. The A and B solutions (depth zero and depth infinite) for the southeastern group show the general shape of the curve to be concave upward.

UNIFORM DISTRIBUTION OF COMPENSATION, WHY ASSUMED.

In order to make it feasible to compute the deflections of the vertical, taking into account both the topography and the isostatic compensation, it has been assumed in the principal portion of the investigation that the compensation is uniformly distributed with respect to depth from the surface to the limiting depth of compensation; that is, it is assumed that the defect of density below any given portion of an elevated continental area, as compared with the density at the same level below a coast area lying at sea level, is a constant for all levels between the surface and the limiting depth of compensation. It is assumed that below that depth no defect or excess of density exists.

This assumption was adopted as a working hypothesis, because it happens to be that one of the reasonable assumptions which lends itself most readily to computation, and because it seemed to be the most probable simple assumption.

It was necessary to make some assumption as to the distribution of the compensation with respect to depth. Various considerations should influence one in deciding which of various possible assumptions is probably nearest the truth.

Near the surface of the earth to the depth to which the observations of temperature have been extended, the temperature has been found to increase about 1° C., upon an average, for each 30 meters increase in depth. The temperature probably continues to increase down to the limiting depth of compensation, though possibly the average rate of increase in the lower half of the interval is less than that stated. At 100 kilometers below the surface very high temperatures must exist, so high as to tend to decrease the strength of the material, assuming for the moment that the material is all solid. With increasing temperatures and, consequently, decreasing strength as the depth below the surface increases, one should expect stress differences of a given intensity to produce motion, or stresses to produce a change of volume, at the greater depths rather than near the surface. If this consideration alone be taken into account, the readjustment of material to eliminate stress differences and equalize stresses, and, consequently, the isostatic compensation, should be expected to increase with increase of depth.

If, however, a consideration of the probable increase of temperature with increase of depth leads one to believe that, in spite of the tendency of increase of pressure to raise the melting points of the various materials composing the earth, the material below a critical depth is liquid while that above is solid, his belief as to the probable distribution of compensation with respect to depth must be correspondingly modified. In this case one may conceive that a light solid crust is floating on a heavier liquid substratum. On this basis the thinner portions of the crust (oceanic areas) should have their upper surfaces lower and their lower surfaces higher than the corresponding parts of the thicker portions of the crust (continents), this being the necessary condition of stability, as for an indefinitely extended floe of ice. It is here assumed that the crust itself has the same density at all parts at a given level, this being the assumption ordinarily made by those who deal with a crust floating in a liquid substratum. The isostatic compensation in this case all occurs near the bottom of the crust. It extends, below a continental mass, from the level of the lower surface of the crust under the seacoast down to the bottom of the actual crust under the continental mass. It is represented by the protuberance on the under side of the crust, which corresponds for equilibrium to the continental protuberance above sea level. The isostatic compensation on this basis is therefore zero down to a deep level, and from that point is uniform down to a still deeper level, at which it suddenly changes to zero again.

G. H. Darwin, in his memoir entitled "On the Stresses Caused in the Interior of the Earth by the Weight of Continents and Mountains,"* assumes that the earth is a competent, elastic structure, solid throughout. Upon that assumption the stresses which must exist in it, due to the weight of the continents and mountains, are computed. The computation indicates that continents of such dimensions and form as those now in existence would produce stress-differences which would increase from the surface downward to a maximum at a depth of from 600 to 1 000 miles, said maximum being as great as 4 tons per square inch. On this basis the failure of the material on account of the stress-differences, due to the continents, would be more apt to

* Philosophical Transactions of the Royal Society of London, 1882, vol. 173, pp. 187-230.

occur at greater depths than near the surface, and therefore the readjustment of material and the isostatic compensation should increase with increase of depth for the first 600 or 1 000 miles below the surface. Darwin's computation also indicates that the stress differences produced by parallel mountain chains, separated by valleys, increases rapidly with increase of depth, down to a maximum at a depth which is $\frac{1}{6.28}$ part of the distance between adjacent mountain chains. Thus, if two parallel mountain chains are 500 kilometers apart, the depth at which the maximum occurs will be 80 kilometers. Two such mountain chains, rising 4 000 meters above the intervening valley bottom, would produce maximum stress differences of 2.6 tons per square inch on the assumption stated. According to this point of view, the stress differences due to the existing inequalities of the earth's surface must increase with increasing depth to several tens of kilometers below the surface. Beyond this depth at which the maximum stress differences must occur, according to Darwin's computation, and which is directly proportional to the distance between successive ridges, the stress differences must decrease slowly with increase of depth. On this basis the failures under stress, readjustment, and consequent isostatic compensation, due to the inequalities in the surface of the continents, should increase with increase of depth to a level far below the surface and then slowly decrease.

The distribution of stress differences was computed by Darwin, with the results stated above, upon the assumption that every part of the earth is a competent, elastic structure, that is, that no failure under stress occurs at any point. If failure does occur at any point, permanent deformation of the material takes place, there is readjustment in form and position, and the stress differences below that point are reduced in amount, the stress-differences produced by the inequalities of load at the surface not being transmitted below that level with their full value corresponding to purely elastic deformation. Hence, in so far as failure and readjustment occur at a given level, they tend to prevent similar failure and readjustment at lower levels. From this point of view it appears that isostatic compensation should increase with increase of depth to a short distance only below the surface and beyond that should decrease with increase of depth.

In the preceding paragraphs it is tacitly assumed that failure (yielding) under the stress differences at certain levels is accompanied by isostatic readjustment produced either by bodily transfer of material or by changes of density, or by both, at that level.

Prof. T. C. Chamberlin has reached the conclusion that, according to the planetesimal theory of the formation of the earth, the isostatic compensation would be greatest at a level slightly below the surface, and from that point would decline at a varying rate, which rate increases rapidly at first and at greater depths decreases slowly, approaching zero at great depths. This peculiar distribution is indicated as being due to differential weathering and vulcanism during the process of growth of the earth.*

These different considerations as to the probable distribution of isostatic compensation with respect to depth are not mutually exclusive. It may be that the actual distribution is a resultant of several or all of the actions and modifying influences which have been indicated briefly. Some of these tend to produce a uniform distribution of isostatic compensation with respect to depth, some to produce a maximum of isostatic compensation near the surface, some to produce it at moderate depths neither very near the surface nor very near the limiting depth of compensation, and some tend to produce a maximum near the limiting depth. Therefore it has seemed that the most probable simple assumption is that the compensation is uniformly distributed from the surface to the limiting depth.

It is not supposed that at the limiting depth of compensation there exists a perfectly abrupt change of conditions with respect to compensation. But it is believed to be possible that the decrease of isostatic compensation from its mean value to zero may all take place within so small a range of depth that the difference between the actual mode of distribution and the abrupt change postulated in the stated assumption may not be capable of detection by the geodetic observations. Hence, it is deemed justifiable to make the assumption in the form stated, which is such as to lend itself most readily to computation.

* Journal of Geology, 1907, p. 76, and Geology, by T. C. Chamberlin and R. D. Salisbury, Vol. II, pp. 107–111.

It should be noted that each of the considerations brought forward in the preceding paragraphs indicates that the isostatic compensation must be sensibly limited to some finite depth much less than the radius of the earth. The writer knows of no plausible conception of the conditions within the earth that would lead one to believe that isostatic compensation extends to the center. Hence, throughout this investigation it is assumed that, whatever the mode of distribution of the isostatic compensation with respect to depth, it extends to a limiting depth which is but a small fraction of the radius.

While it seems desirable, in connection with the present investigation, that the first assumption made as to the distribution of compensation should be a reasonable one and, preferably, that it should be the most probable simple assumption, it was evident, at the outset, that a failure to make the best selection would not be fatal to ultimate success and would probably not even hamper the investigation. It appeared on the preliminary reconnaissance of the problem that the most efficient method of attack is probably to make some one assumption as to the distribution of the isostatic compensation with respect to depth, to make full computations on this assumption, and then to test other assumptions by comparison with this one; not by complete new computations, but by a computation of the small differences in computed deflections produced by the change from one assumption to the other. As various assumptions were to be tested, it was not of paramount importance which should be tested first. Relative ease of computation was properly one of the controlling elements in making the choice.

POSSIBLE COMPENSATION IN A 10-MILE STRATUM.

It seemed desirable to test the assumption that possibly the compensation is complete and uniformly distributed through a comparatively thin stratum, lying at a considerable depth below the surface. It was decided to assume the thickness of said stratum to be 10 miles (16 kilometers) and to derive the most probable depth for it.

Such an assumption corresponds roughly (see p. 147) to the hypothesis that the outer portion of the earth is a light solid crust floating on a heavier liquid substratum. The correspondence is not exact, for under said hypothesis the thickness of the stratum within which the compensation occurs is variable, and the mean depth of compensation is variable, as indicated in the discussion later in this publication under the heading, "The floating crust hypothesis," page 163, whereas, under the assumption to be tested, the thickness is constant (10 miles), and the depth (to be derived) is assumed to be constant.

As a first approximation, it was assumed that the compensation occurs in a stratum 10 miles thick of which the bottom lies at the depth 35 miles. Let A be a uniform distribution of defect (or excess) of density through the depth 35 miles from the surface and with the defect (or excess) 3.5 times as great as that necessary for complete compensation within said depth. Let B be a uniform distribution of defect (or excess) of density through the depth 25 miles from the surface and with the defect (or excess) 2.5 times as great as that necessary for complete compensation within said depth. Then the uniform distribution of density which will produce complete compensation within the specified stratum lying between depths 25 and 35 miles, and which will for convenience be called X, is the exact equivalent of the difference, A–B, of the two distributions, A and B, or X = A–B.

This may be made clear by a reference to page 68. It is there shown that for complete compensation between the surface and depth h_1, $\delta h = -\delta_1 h_1$, or $\delta_1 = -\frac{h}{h_1}\delta$ in which δ_1 is the compensating defect of density, h is the mean elevation above mean sea level of the surface in the area under consideration, and δ is the mean surface density of the earth (2.67). In the assumed distribution A, $h_1 = 35$ miles and the corresponding value of δ_1 is $-\frac{h}{35}\delta$. The assumed defect of density in distribution A is stated to be 3.5 times this and is, therefore,

$$\delta_A = -3.5\frac{h}{35}\delta = -\frac{h}{10}\delta$$

Similarly, the assumed defect of density in distribution B is

$$\delta_B = -2.5\frac{h}{25}\delta = -\frac{h}{10}\delta$$

Therefore, when the distribution B is subtracted from distribution A, it leaves no compensation between the surface and the depth 25 miles, but does leave the compensating defect of density, $\delta_A = -\frac{h}{10}\delta$, uniformly distributed between depth 25 and depth 35 miles. This is exactly the value of δ_A necessary for complete compensation in a stratum 10 miles thick, as may be seen by comparing with the formula $\delta_1 = -\frac{h}{h_1}\delta$ with $h_1 = 10$ miles.

If the area in question is a plateau at an elevation of one mile above mean sea level $\delta_A = \delta_B = -\frac{\delta}{10} = \frac{-2.67}{10} = -.267$. This compensating defect of density in the stratum ten miles thick between depth 25 and depth 35 miles, under one square mile of the surface, is exactly equivalent to a defect of mass of one cubic mile of material of density 2.67, which is equal to the excess of mass above mean sea level. The compensation is, therefore, complete.

Let F_X be a reduction factor, similar to the reduction factors F shown on page 70, such that if the topographic deflection D is multiplied by F_X the product DF_X is the resultant deflection $D + D_X$, due to both the topography and the compensating defects or excesses of mass supposed to lie between depths 25 and 35 miles, according to the assumed distribution of density which has been called X. As indicated on pages 69, 70, the deflection due to the compensation is necessarily opposite in sign to the topographic deflection. As there the deflection due to compensation alone is $-D(1-F)$, so, in the case in hand, it is $D_X = -D(1-F_X)$.

Let F_{35} and F_{25} be such factors F as are shown on page 70 for complete compensation extending from the surface down to the depths 35 and 25 miles, respectively. Then the deflection due to complete uniform compensation extending to depths 35 miles would be $-D(1-F_{35})$ and that due to the assumed compensation called A would be 3.5 times this, or $-3.5D(1-F_{35})$. Similarly the deflection due to the assumed compensation called B would be $-2.5D(1-F_{25})$. Hence the deflection due to the assumed compensation called X, which is equivalent to A − B, is

$$D_X = -\{3.5D(1-F_{35}) - 2.5D(1-F_{25})\} = -D\{1-(3.5F_{35} - 2.5F_{25})\}$$

By placing this equal to the value of D_X, already derived, namely, $-D(1-F_X)$, it appears that

$$F_X = 3.5F_{35} - 2.5F_{25}$$

The factor F_X was computed by this formula for each ring of topography. To do this it was necessary first to obtain F_{35} and F_{25} for each ring of topography, as they had not yet been computed, though F had been computed for various other depths, as shown on page 70. It was possible to compute F_{35} and F_{25} directly from the formula for F, shown on page 70. The final computations of their values were made, however, by a much shorter process, given later in this publication under the heading, "Various reduction factors obtained graphically," see page 154.

The reduction factors F_X so computed are shown below in comparison with the factors F, taken from page 70, for uniform compensation extending to the depth 113.7 kilometers (70.67 miles).

Ring.	Factor F_X.	Factor F for depth 113.7 kilometers.	Difference.	Ring.	Factor F_X.	Factor F for depth 113.7 kilometers.	Difference.
27	1.000	.997	+.003	13	.656	.618	+.038
26	1.000	.996	+.004	12	.464	.493	−.029
25	1.000	.995	+.005	11	.292	.358	−.066
24	.999	.992	+.007	10	.168	.234	−.066
23	.998	.988	+.010	9	.092	.139	−.047
22	.996	.983	+.013	8	.045	.077	−.032
21	.995	.976	+.019	7	.023	.040	−.017
20	.994	.965	+.029	6	.010	.020	−.010
19	.993	.951	+.042	5	.008	.010	−.002
18	.988	.930	+.058	4	.004	.005	−.001
17	.982	.900	+.082	3	.002	.003	−.001
16	.966	.859	+.107	2	.000	.001	−.001
15	.912	.801	+.111	1	.000	.001	−.001
14	.816	.721	+.095				

It appears that the factor F_x for each ring of topography is not very different from the factor F for depth 113.7 kilometers for that same ring. If the agreement were exact for each ring, the computed total deflection for a station with the compensation assumed to be complete and uniformly distributed to depth 113.7 kilometers—namely, the sum of the quantities DF (one for each ring), see page 72—would be identical with the computed total deflection for a station with the assumed compensation X—namely, the sum of the quantities DF_x. It would then be absolutely impossible by observations of deflections of the vertical to determine which of these two assumed compensations is nearer the truth. As the agreement of the two factors for each ring is close, it is evident that it will be difficult to discriminate by observation between the two kinds of compensation.

To obtain more concrete evidence on this point, it was decided to compute the resultant deflections due to topography and to the assumed compensation X for ten meridian and ten prime vertical stations, selected as being fair representatives of the 496 such stations. The following table shows the selection of stations made, the computed deflections with the X compensation considered, the computed deflections corresponding to solution G—that is, with complete compensation uniformly distributed from the surface to depth 113.7 kilometers—and the small differences between the two sets of computed deflections.

Computed deflection.

MERIDIAN STATIONS.

No.	Name.	X compensation.	Compensation uniform to depth 113.7 kilometers.	Difference.
152	Mount Independence, Maine	− 1.69	− 1.80	+.11
169	Howlett, New York	+ 3.48	+ 3.33	+.15
100	Hill, Maryland	− .45	− .59	+.14
123	Sawnee, Georgia	− 2.66	− 2.72	+.06
189	Thones Hill, Michigan	+ 2.27	+ 2.14	+.13
76	Weed Patch, Indiana	− .29	− .30	+.01
60	El Paso E. B., Colorado	− 3.80	− 3.49	−.31
35	Ibepah, Utah	+ .56	+ .68	−.12
17	Yolo NW. B., California	− 1.51	− 2.03	+.52
238	Santa Barbara, California	−12.13	−12.35	+.22

PRIME VERTICAL STATIONS.

163	Mount Independence, Maine	−3.14	−3.33	+ .19
175	Howlett, New York	+ .70	+ .66	+ .04
103	Hill, Maryland	− .75	−1.02	+ .27
128	Sawnee, Georgia	− .14	− .15	+ .01
202	Thones Hill, Michigan	−2.23	−2.14	− .09
81	Weed Patch, Indiana	+ .61	+ .62	− .01
62	El Paso E. B., Colorado	−6.03	−6.67	+ .64
35	Ibepah, Utah	−2.37	−2.28	− .09
14	Yolo NW. B., California	+ .27	+1.40	−1.13
229	Santa Barbara, California	+3.95	+4.76	− .81
	Sum without regard to sign,	49.03	52.46	

Of the twenty differences for these representative stations, only three are greater than 0″.50, and only one is greater than 1″.00, whereas the average residual without regard to sign from the G solution, in which the isostatic compensation is assumed to be complete and uniformly distributed from the surface to the depth 113.7 kilometers, is 3″.04 (see p. 136), and but 43 per cent of these residuals are less than 2″.00 (see p. 136). It is evident that the agreement of the deflections computed with the X compensation with those computed with the G compensation is well within the limits of the accidental errors, and that it is therefore very difficult to ascertain which of the assumed compensations is nearer the truth.

The assumption of the particular depth 35 miles for the bottom of the stratum 10 miles thick within which compensation X is assumed to lie, was made after a preliminary reconnais-

sance of the problem, in which an attempt was made to predict the assumed depth for which the agreement with the G compensation would be closest. An examination of the preceding table shows that the deflections computed with the X compensation are on an average slightly smaller than those computed with the G compensation. Since, with any assumed distribution of compensation, the greater the assumed depth the larger are the computed deflections (see pp. 70–72), it appears that a closer agreement would be obtained if the assumed depth were made slightly greater. It was decided as a second approximation to increase the assumed depth by 2 miles—that is, to try a compensation, which will be called X_2, complete and uniformly distributed through a stratum 10 miles thick with its bottom at the depth 37 miles.

The computations were then made for this compensation, X_2, precisely as outlined above for compensation X. The factors F_{X_2} and the corresponding deflections were computed and the deflections compared with those for compensation G. The factors F_{X_2} were found to be as follows:

Ring.	Factor F_{X_2}.	Factor F for depth 113.7 kilometers.	Difference.	Ring.	Factor F_{X_2}.	Factor F for depth 113.7 kilometers.	Difference.
27	1.000	.997	+.003	13	.693	.618	+.075
26	1.000	.996	+.004	12	.504	.493	+.011
25	1.000	.995	+.005	11	.315	.358	−.043
24	1.000	.992	+.008	10	.187	.234	−.047
23	1.000	.988	+.012	9	.101	.139	−.038
22	1.000	.983	+.017	8	.049	.077	−.028
21	1.000	.976	+.024	7	.025	.040	−.015
20	1.000	.965	+.035	6	.011	.020	−.009
19	.998	.951	+.047	5	.006	.010	−.004
18	.993	.930	+.063	4	.003	.005	−.002
17	.987	.900	+.087	3	.001	.003	−.002
16	.976	.859	+.117	2	.000	.001	−.001
15	.925	.801	+.124	1	.000	.001	−.001
14	.840	.721	+.119				

These factors are also shown graphically on illustration No. 15 at the end of the volume.

The differences, taken in the same manner as in the table on page 151, between the deflections computed with compensation X_2 and the corresponding deflections computed with compensation G, were found to be as follows:

	X_2 deflection minus G deflection.		X_2 deflection minus G deflection.
Meridian stations:		Prime vertical stations:	
No. 152	+.08	No. 163	+.09
No. 169	+.26	No. 175	+.09
No. 100	+.13	No. 103	+.22
No. 123	.00	No. 128	+.02
No. 189	+.19	No. 202	−.16
No. 76	+.02	No. 81	+.01
No. 60	−.37	No. 62	+.27
No. 35	−.07	No. 35	−.26
No. 17	+.44	No. 14	−1.04
No. 238	−.20	No. 229	−.57

The sum without regard to sign of the twenty X_2 deflections was found to be 51″.05, which is slightly smaller than the corresponding sum of G deflections—namely, 52″.46. This indicated that the assumed depth should have been slightly greater, apparently about 1 mile. It was now possible to determine approximately for each station the assumed depth which would make the X deflection agree with the G deflection at that station, since there was available the change in the X deflection at that station produced by a small change in assumed depth—2 miles—from 35 to 37. A study in detail for the separate stations led to the conclusion that for no other assumed depth would the agreement of the X deflections with the G deflections be closer than for the depth 37 miles.

Therefore the general conclusion from this special investigation is that if complete compensation is assumed to be uniformly distributed through a stratum 10 miles (16 kilometers)

thick the depth for the bottom of this stratum which will make the effects of this compensation most nearly agree with the G compensation is 37 miles, and, moreover, that 37 miles is the most probable depth for the bottom of such a compensation.

The basis of the second part of the above conclusion will appear more clearly if the comparison of the F_{x_2} and F factors, shown in the table on page 152 and in illustration No. 15 at the end of the volume, is considered. The two kinds of factors agree closely for all corresponding rings—that is, the two curves in illustration No. 15 follow each other closely. In either case a slight change in assumed depth requires simply that the new factors are to be secured (as shown under the heading "Various reduction factors obtained graphically," p. 154) by reading them from a slightly different part of the proper curve. Corresponding changes in the assumed depths, as indicated in the abscissæ of illustration No. 15, will produce nearly equal changes in the F and F_{x_2} factors and therefore nearly equal changes in the computed deflections and computed residuals. Hence, having proved that the sum of the squares of the residuals is a minimum for the G solution (depth, 113.7 kilometers) as among solutions for assumed compensation extending from the surface to a limiting depth, and having proved that for the X_2 compensation the computed deflections and therefore the computed residuals are practically the same as for the G compensation, it follows that for the X_2 compensation the sum of the squares of the residuals is probably less than for any other assumed compensation in a stratum 10 miles thick. Therefore 37 miles is the most probable depth for the bottom of an assumed stratum 10 miles thick through which complete compensation is assumed to be uniformly distributed.

The general conclusion is also reached from this special investigation that it is not possible to ascertain whether the X_2 compensation is more probable than the G compensation, since the two sets of computed deflections agree so closely that their differences are much smaller than the accidental errors.

This conclusion might be tested further after a study of the curves on illustration No. 15 and the computed topographic deflections for separate rings for each station, such as those on pages 26–33, by selecting the stations at which the computed X_2 deflections would differ most from the computed G deflections and ascertaining whether the sum of the squares of the X_2 residuals is much smaller at such stations than the corresponding sum of the squares of the G residuals.

The theoretically complete test would be to compute the X factors and make complete X solutions for various assumed depths in precisely the manner in which the solutions E, H, and G have been made.

It is the judgment of the writer, based on a study of details, as well as upon the general considerations indicated in the preceding pages, that neither of the additional tests suggested would probably make much change in the present conclusions and that therefore it is inadvisable at present to make the test. At some future time, when more data are available, one or both of these tests may be advisable.

To make this special investigation complete in form, other assumptions should be made as to the thickness of the stratum in question; that is, for the assumed thickness 10 miles there should be substituted 5 miles, 15 miles, 20 miles, etc. The results of computations, based upon such assumptions as to the thickness of the stratum, may, however, be anticipated closely. If the stratum is assumed to be 10 miles thick, it is shown above that its bottom is at the depth 37 miles, its top at the depth 27 miles, and its mean depth is 32 miles. The G compensation is uniformly distributed through a stratum 71 miles thick, of which the bottom is at the depth 71 miles and the top at depth zero, the mean depth being 35½ miles. This may be considered as a special X compensation. Hence for any assumed thickness of the stratum, considered in the X compensation, between 10 miles and 71 miles, it is evident that the complete solution would show the mean depth to be between 32 and 35½ miles. Similarly, by extrapolation, it appears that if the stratum is assumed to be less than 10 miles thick its mean depth will be found to be slightly less than 32 miles.

The method of testing the assumption that possibly the compensation is complete and uniformly distributed through a thin stratum, lying at a considerable depth below the surface, has been set forth fully in the preceding pages, because it is a general method, capable of application to any assumed mode of distribution of the compensation with respect to depth, provided only that the limiting depth of compensation is assumed constant for all of the area considered. Any assumed distribution of compensation with respect to depth may be treated, as in the preceding case, as made up of sums and differences of several compensations each assumed to be uniformly distributed from the surface down to the limiting depth. It then follows that from the reduction factors F, computed as indicated on page 70, it is possible to compute the factors corresponding to the particular case, and to make the test by the general method indicated on the preceding pages for the assumed compensation X.

This general method has been followed, as shown later in this publication, in testing the assumption that possibly the compensation is greatest at the surface and decreases uniformly downward to zero at some limiting depth, and to test the assumption that the compensation is distributed with respect to depth according to a particular law postulated by Prof. T. C. Chamberlin.

VARIOUS REDUCTION FACTORS OBTAINED GRAPHICALLY.

The reduction factors F, shown in the table on page 70, corresponding to isostatic compensation uniformly distributed from the surface to depth 113.7 kilometers (70.67 miles), are also shown in the curve drawn as a solid line on illustration No. 15 at the end of the volume. The ordinates of the curve are the values of F for the depth 113.7 kilometers, corresponding to the rings indicated by the first line of abscissæ at the bottom of the figure

It will now be shown that the use of this curve is not restricted to the assumed depth 113.7 kilometers, but that it may be used to obtain the reduction factors F for any ring and for any assumed depth.

The formula for F, as derived on pages 69, 70, is

$$F = 1 - \frac{\log \dfrac{r' + \sqrt{(r')^2 + h_1^2}}{r_1 + \sqrt{r_1^2 + h_1^2}}}{0.1541}$$

and is subject to the restriction that $\frac{r'}{r_1} = 1.426$.

Let $R = \frac{r'}{h_1}$, that is, R is defined as the ratio of the outer radius of a ring r' to the assumed depth of compensation, h_1.

Then,

$$r' = h_1 R \text{ and } r_1 = \frac{h_1 R}{1.426}$$

By substituting these values of r' and r_1 in the above formula for F and simplifying there is obtained

$$F = 1 - 6.489 \log \frac{1.426\ (R + \sqrt{R^2 + 1})}{R + \sqrt{R^2 + (1.426)^2}}$$

This formula for F contains but one variable, R. Therefore for any ring and any depth of compensation uniform from the surface down, F depends simply upon R, the ratio of the outer radius of the ring to the depth of compensation.

It is important to note that the preceding statement and formula are subject to the restriction that the ratio of radii of successive rings, $\frac{r'}{r_1}$, is 1.426, the ratio used throughout the present investigation.

On illustration No. 15, in the second line of abscissæ at the bottom of the figure, are the values of log R, placed in proper relation to the curve which is drawn as a solid line. Therefore

if this second line of abscissæ is used the curve gives the values of F for any ring and any depth of uniform compensation, subject to the restriction of the preceding paragraph as to the ratio of radii of rings.

As the ratio of radii of successive rings is constant, being 1.426,* and the corresponding logarithms of the radii, therefore, have a constant difference 0.1541, the successive corresponding values of R $(=\frac{r'}{h_1})$ are in the ratio 1.426, and the successive corresponding differences of log R are 0.1541, as indicated at the bottom of illustration No. 15.

The curve was used to determine the factors F_{35}, see page 150, for a compensation assumed to be uniformly distributed from the surface to depth 35 miles. For example, for this case and for ring 11, $R=\frac{r'}{h_1}=\frac{70.67}{35.00}$, and $\log R=0.305$. The value read from the curve corresponding to the abscissa $\log R=0.305$ is $F_{35}=.141$. Similarly, for ring 12, $R=\frac{49.56}{35.00}$, $\log R=0.151$, and the value of F_{35}, read from the curve, is .236. Note that, on account of the relation to which attention was called in the preceding paragraph, the two values of log R differ by 0.154, the difference between successive numbers on the second line of abscissæ. As that for ring 11 falls .02 space to the right of 0.308 on the second line of abscissæ, so, also, that for ring 12 falls .02 space to the right of 0.154, and the values for rings 13, 14, 15, etc., are necessarily at intervals of one space (0.154 in log R). It was necessary, therefore, to compute log R for a single ring only to locate the proper point on the curve to be read to obtain the values of F_{35} for all rings.

The readings of the curve thus obtained are identical with those which might be obtained by displacing the whole plotted curve 1.98 spaces to the right and using the first line of abscissæ at the bottom of the figure.

Similarly, in obtaining the factors, F_{37}, for a compensation assumed to be uniformly distributed from the surface to depth 37 miles, log R was first computed for ring 11 and was found to be 0.281 (.18 space to the right of 0.308 on the second line of abscissæ on illustration No. 15) and the corresponding F_{37} as scaled from the curve was found to be .153. The remaining values of F_{37} for various rings were then read off from the curve at points an integral number of spaces (each corresponding to 0.154 in log R) to the right and left of this point. This was equivalent to displacing the curve 1.82 spaces to the right and then using the first line of abscissæ at the bottom of the figure.

Required values of the reduction factors F, for the assumed depths of uniform compensation, 25 miles, 27 miles, and various other depths, were also obtained quickly from this curve.

As indicated on pages 150, 152, reduction factors, F_{X_2}, were computed by the formula

$$F_{X_2}=3.7\ F_{37}-2.7\ F_{27}$$

in which F_{37} and F_{27} were obtained as indicated above.

F_{X_2} is the reduction factor corresponding to complete compensation uniformly distributed through a stratum 10 miles thick, with its bottom at the depth 37 miles. The values of F_{X_2} so computed for the rings indicated in the first line of abscissæ at the bottom of illustration No. 15 are plotted on the curve drawn as a series of dashes in that figure. The corresponding values of $\log R=\log\frac{r'}{h_1}$ in which h_1 has the value 37 miles, are shown in the first line of abscissæ at the top of the illustration.

As F_{37} and F_{27} are each functions of R only, so F_{X_2} must necessarily be a function of R only. Therefore it follows that the curve marked X_2 and drawn as a series of dashes may be used to determine the reduction factors F_X for complete compensation uniformly distributed through a stratum 10 miles thick for any ring and for any assumed depth of the bottom of the stratum.

* This statement is not strictly true, though nearly so, for rings 6 to 1, see p. 22. The values of log R on illustration No. 15 have, for convenience, been entered there as if it were strictly true, as the small errors so introduced are negligible.

The process is similar to that already indicated for using the curve drawn as a solid line to derive the F factors for any depth and any ring. Thus, if the bottom of the 10-mile stratum be assumed to be at depth 35 miles for ring 11, $R = \frac{70.67}{35.00}$ and $\log R = 0.305$. The corresponding value of F_x read from the curve of dashes using the first line of abscissæ at the top of the illustration No.15 is 0.292. For other rings and this depth the readings are taken from the curve at integral numbers of spaces (each equal to 0.154 in log R) to the right or left of this point. This corresponds to displacing the curve of dashes 0.16 space to the right and then using the first line of abscissæ at the bottom of the illustration.

The curve on illustration No. 15, marked D and drawn with alternate dashes and dots, gives the reduction factors corresponding to complete compensation which is greatest at the surface and diminishes uniformly with respect to depth until it becomes zero at the limiting depth. If this curve is read with the first line of abscissæ below the illustration, it gives such factors for the assumed limiting depth 109.0 miles. If it is read with the second set of abscissæ above the illustration, it gives such factors for any depth. Information in regard to the test of this mode of distribution of compensation is given below under the heading "Uniformly decreasing compensation."

The curve on illustration No. 15, marked C and drawn by dots only, gives the reduction factors corresponding to complete compensation distributed with respect to depth according to a law postulated by Prof. T. C. Chamberlin. If this curve is read with the first line of abscissæ below the illustration, it gives such factors for the assumed depth 178.6 miles. If it is read with the third set of abscissæ above the illustration, it gives such factors for any depth. Information in regard to the test of this mode of compensation is given later (p. 159) under the heading, "The Chamberlin compensation."

On the scale of publication of illustration No. 15 it is difficult to read the third decimal place of a factor. The numerical values of each set of factors are given in this publication, pages 70, 152, 158, 161: One may, therefore, reconstruct the illustration to any desired scale and then use it in deriving factors for various depths. It is believed that few, if any, of the numerical values of the factors given are in error by more than .002.

UNIFORMLY DECREASING COMPENSATION.

It seemed desirable, for reasons indicated on page 148, to test the assumption that possibly the compensation is complete within a certain limiting depth, but that it is greatest at the surface, and decreases uniformly with respect to depth until it becomes zero at the limiting depth.

The same general plan was followed in making this test as that outlined on pages 149–154, in connection with a test of possible compensation in a stratum ten miles thick.

As before, let h = the mean elevation above mean sea level of the surface in the area under consideration, let h_1 = the limiting depth of compensation, and let δ = the mean surface density of the earth (2.67).

Then, under the assumptions stated, the compensating defect of density at the surface is $-\frac{2h}{h_1}\delta$ and is zero at depth h_1. For any intermediate depth d, the compensating defect of density is $\delta_d = -\frac{h_1 - d}{h_1}\frac{2h}{h_1}\delta$; that is, it is proportional at any point to the elevation of that point above the limiting depth of compensation h_1. The average compensating defect of density or average value of δ_d, which will be called δ_U, is $-\frac{h}{h_1}\delta$, which is that necessary for complete compensation uniformly distributed to the depth h_1 (see pages 68, 149).

As a convenient approximation to this assumed distribution of compensation, decreasing uniformly with respect to depth from the surface to depth h_1, the following assumed values were adopted:

From depth zero to depth $.1h_1$ let $\delta_d = 2.0\delta_U$
From depth $.1h_1$ to depth $.3h_1$ let $\delta_d = 1.6\delta_U$
From depth $.3h_1$ to depth $.5h_1$ let $\delta_d = 1.2\delta_U$
From depth $.5h_1$ to depth $.7h_1$ let $\delta_d = .8\delta_U$
From depth $.7h_1$ to depth $.9h_1$ let $\delta_d = .4\delta_U$
From depth $.9h_1$ to depth $1.0h_1$ let $\delta_d =$ zero

These arbitrary assumptions insure that δ_d has its proper value at the surface and at depth h_1. Between those points the assumption makes δ_d vary by sudden changes of $.4\delta_U$ at such points as to insure that the average value of δ_d is correct (namely, $=\delta_U$) and that, at any depth, the assumed value of δ_d does not differ much from the value $-\frac{h_1 - d}{h_1}\frac{2h}{h_1}\delta$, which it would have if it decreased continuously and uniformly from the surface to depth h_1.

These arbitrary assumptions serve to substitute for a uniformly decreasing compensation another compensation which approximates closely to it and consists of the sum of several compensations, each uniformly distributed from the surface down to its own limiting depth. The separate uniform compensations of which the sum is utilized are

No. 1 $\delta_d = .4\delta_U$ from the surface to depth $.9h_1$
No. 2 $\delta_d = .4\delta_U$ from the surface to depth $.7h_1$
No. 3 $\delta_d = .4\delta_U$ from the surface to depth $.5h_1$
No. 4 $\delta_d = .4\delta_U$ from the surface to depth $.3h_1$
No. 5 $\delta_d = .4\delta_U$ from the surface to depth $.1h_1$

If uniform compensation from the surface to depth $.9h_1$ were complete, the compensating defect of density would be $\delta_1 = -\frac{h}{.9h_1}\delta$ (see p. 68). In the first of the five compensations indicated above, the compensating defect of density $\delta_d = .4\delta_U = -\frac{.4h}{h_1}\delta$, and is therefore .36 of that necessary (namely, $-\frac{h}{.9h_1}\delta$) for complete compensation uniformly distributed from the surface to depth $.9h_1$

Similarly, in the second of the five compensations indicated above, δ_d is .28 of that necessary for complete compensation within depth $.7h_1$, and the corresponding quantities for the third, fourth, and fifth compensations are .20, .12, and .04.

By the same line of reasoning as that followed on pages 149, 150 in proving the formula $F_X = 3.5F_{35} - 2.5F_{25}$ it follows that

$$F_{D5} = .36F_{(.9h_1)} + .28F_{(.7h_1)} + .20F_{(.5h_1)} + .12F_{(.3h_1)} + .04F_{(.1h_1)}$$

In this formula F_{D5} is a compensation factor by which the computed topographic deflection must be multiplied to secure the resultant deflection if the sum of the five compensations indicated on the preceding page exists. $F_{(.9h_1)}$ is the factor, such as those shown in the table on page 70, or on the solid curve on illustration No. 15 at the end of the volume, for complete compensation, extending from the surface to depth $.9h_1$. $F_{(.7h_1)}$, $F_{(.5h_1)}$, etc., each has the corresponding meaning with respect to depths $.7h_1$, $.5h_1$, etc., respectively.

Let F_D be the compensation factor corresponding to complete compensation which decreases uniformly from a maximum at the surface to zero at depth h_1. Then, since the sum of the five compensations indicated above is nearly equivalent to this uniformly decreasing compensation, F_{D_5} is a close approximation to F_D.

In the concrete case to be investigated h_1 was first assumed to be 141.3 miles. F_{D5} was then computed for each ring from the above formula. The necessary values of $F_{(.9h_1)}$, $F_{(.7h_1)}$ etc., in the formula were those indicated by the solid curve marked F on illustration No. 15, for depths 127.2, 98.9, 70.7, 42.4, and 14.1 miles.

In order to ascertain the closeness with which F_{D5} approximates to F_D, ten (instead of five) separate compensations, each with $\delta_d = .2\delta_U$ (instead of $.4\delta_U$), were next assumed, extending to depths $\frac{1}{20}h_1$, $\frac{3}{20}h_1$, $\frac{5}{20}h_1$ - - - - $\frac{19}{20}h_1$. The formula for computing the compensation factors then became

$$F_{D10} = .19F_{(.95h_1)} + .17F_{(.85h_1)} + .15F_{(.75h_1)} + \text{- - - - - -} + .01F_{(.05h_1)}$$

The values of F_{D10} were computed from this formula for seven selected rings. No difference between F_{D10} and F_{D5} was greater than .003. Hence it appeared that if the number of separate assumed uniform compensations were increased indefinitely beyond ten to secure a closer approximation of F_{D10} to F_D, probably no computed factor would be changed by more than .001. Therefore it was assumed that, with sufficient accuracy, F_{D10} may be assumed to be equal to F_D. By inspection the necessary corrections, not exceeding .003, were applied to the values of F_{D5} for all rings to make them agree with F_{D10}, and the corrected values were taken as F_D for each ring corresponding to the limiting depth, 141.3 miles.

These values for F_D were then compared with the corresponding values of F for depth 70.67 miles, see page 70, and it appeared that the two sets of factors would agree much more closely if the limiting depth for the decreasing compensation were made 118.3 miles instead of 141.3 miles. Accordingly new factors F_D, corresponding to this limiting depth, were computed.

The resultant deflections due to both the topography and assumed complete compensation decreasing uniformly with respect to depth from the surface down to the depth 118.3, were then computed for the same ten meridian stations and ten prime vertical stations which were taken as representative in the investigation of possible compensation complete within a stratum ten miles thick, see page 151. These computed deflections were compared with corresponding values for these stations computed for complete compensation uniformly distributed from the surface to depth 70.67 miles. The maximum difference was found to be only 0″.71. But an examination in detail showed that the differences would be still smaller and probably a minimum if the limiting depth for the decreasing compensation were changed to 109.0 miles. Accordingly, the factors and the resultant deflections for the 20 representative stations were computed for the depth 109.0 miles and are shown below in comparison with corresponding quantities for uniform compensation extending to depth 70.67 miles.

Ring.	Factor F_D for depth 175.4 kilometers, or 109.0 miles.	Factor F for depth 113.7 kilometers, or 70.67 miles.	Difference.	Ring.	Factor F_D for depth 175.4 kilometers, or 109.0 miles.	Factor F for depth 113.7 kilometers, or 70.67 miles.	Difference.
27	.996	.997	—.001	13	.588	.618	—.030
26	.995	.996	—.001	12	.472	.493	—.021
25	.993	.995	—.002	11	.353	.358	—.005
24	.989	.992	—.003	10	.240	.234	+.006
23	.984	.988	—.004	9	.150	.139	+.011
22	.978	.983	—.005	8	.086	.077	+.009
21	.969	.976	—.007	7	.047	.040	+.007
20	.956	.965	—.009	6	.024	.020	+.004
19	.939	.951	—.012	5	.012	.010	+.002
18	.914	.930	—.016	4	.006	.005	+.001
17	.876	.900	—.024	3	.003	.003	.000
16	.828	.859	—.031	2	.002	.001	+.001
15	.766	.801	—.035	1	.001	.001	.000
14	.686	.721	—.035				

The factors F_D are shown graphically on illustration No. 15 at the end of the volume on the curve marked D and drawn with alternate dots and dashes. If this curve is read with the first line of abscissæ below the illustration it gives such factors for the assumed limiting depth 109.0 miles. If it is read with the second set of abscissæ above the illustration it gives such factors for any depth.

Computed deflection.

MERIDIAN STATIONS.

No.	Name.	Compensation decreasing uniformly to depth 109.0 miles.	Compensation uniform to depth 113.7 kilometers, or 70.67 miles.	Difference.
		″	″	″
152	Mount Independence, Maine	− 1.84	− 1.80	−.04
169	Howlett, New York	+ 3.25	+ 3.33	−.08
100	Hill, Maryland	− .65	− .59	−.06
123	Sawnee, Georgia	− 2.73	− 2.72	−.01
189	Thones Hill, Michigan	+ 2.03	+ 2.14	−.11
76	Weed Patch, Indiana	− .30	− .30	.00
60	El Paso E. B., Colorado	− 3.39	− 3.49	+.10
35	Ibepah, Utah	+ .63	+ .68	−.05
17	Yolo NW. B., California	− 2.15	− 2.03	−.12
238	Santa Barbara, California	−12.23	−12.35	+.12

PRIME VERTICAL STATIONS.

163	Mount Independence, Maine	−3.38	−3.33	−.05
175	Howlett, New York	+ .60	+ .66	−.06
103	Hill, Maryland	−1.13	−1.02	−.11
128	Sawnee, Georgia	− .19	− .15	−.04
202	Thones Hill, Michigan	−2.07	−2.14	+.07
81	Weed Patch, Indiana	+ .60	+ .62	−.02
62	El Paso E. B., Colorado	−6.65	−6.67	+.02
35	Ibepah, Utah	−2.05	−2.28	+.23
14	Yolo NW. B., California	+1.78	+1.40	+.38
229	Santa Barbara, California	+4.93	+4.76	+.17

Twelve of the 20 differences are each less than 0″.10, and the maximum difference is only 0″.38. The average residual without regard to sign from the G solution, in which the isostatic compensation is assumed to be complete and uniformly distributed from the surface to the depth 113.7 kilometers, is 3″.04 (see p. 136) and but 43 per cent of these residuals are less than 2″.00 (see p. 136). The above differences are small in comparison with the residuals of the G solution.

By the lines of reasoning indicated on pages 152, 153 the following conclusions have been reached: First, the most probable limiting depth is 109.0 miles for complete compensation, which is assumed to be greatest at the surface and to diminish uniformly with respect to depth until it becomes zero at the limiting depth. Second, it is not possible to ascertain whether this compensation is more probable than the G compensation, uniformly distributed from the surface to the depth 70.67 miles, since the two sets of computed deflections agree so closely that their differences are much smaller than the accidental errors.

THE CHAMBERLIN COMPENSATION.

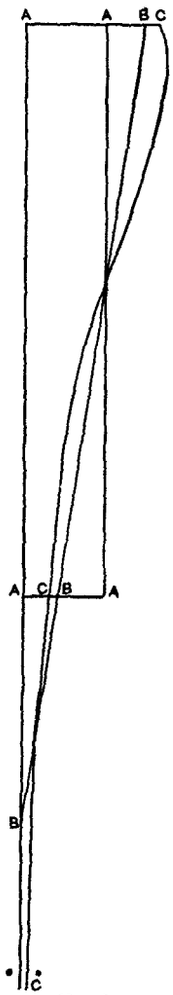
Prof. T. C. Chamberlin has reached the conclusion that, according to the planitesimal theory of the formation of the earth, the isostatic compensation would be greatest at a level slightly below the surface and from that point would decline at a varying rate, which rate increases rapidly at first and at greater depth decreases slowly, approaching zero at great depth. This peculiar distribution is indicated as being due to differential weathering and vulcanism during the process of growth of the earth.*

It seemed desirable to test the geodetic evidence for any indication that this mode of distribution of the compensation with respect to depth is more probable than the G compensation, which is assumed to be uniformly distributed from the surface to the limiting depth.

* Journal of Geology, 1907, p. 76; and Geology, by T. C. Chamberlin and R. D. Salisbury, Vol. II, pp. 107–111.

The general plan followed is that indicated on pages 149–159 in connection with tests of possible compensation in a stratum 10 miles thick, and of possible compensation decreasing uniformly from the surface to the limiting depth.

Illustration No. 16 is a copy of the figure published by Professor Chamberlin in the Journal of Geology. According to Professor Chamberlin the most probable mode of distribution of the compensation with respect to depth is indicated by the curve CCC. On this figure the ordinates represent depths below the surface and the abscissæ represent the compensating defects (or excesses) of density. The origin is at the upper left-hand corner of the figure. The rectangle AAAA represents the G compensation, uniformly distributed from the surface to the depth 113.7 kilometers (71 miles). The triangle included between the line BBB and the axes represents compensation which is a maximum at the surface and decreases uniformly downward to the limiting depth 109.0 miles. The figure serves, therefore, to compare three assumed modes of distribution of the compensation with respect to depth.



No. 16.

It is the writer's understanding of the curve CCC of this figure that Professor Chamberlin intends it to represent the most probable mode of distribution of the compensation with respect to depth, but does not intend it to indicate the particular depths at which the compensation has certain values. If this understanding is correct the curve CCC would represent Chamberlin's hypothesis equally well if it were either somewhat contracted or somewhat expanded in the vertical direction without other change. This interpretation of the curve has been used in the following investigation.

To make the problem concrete, and as a first approximation, it was assumed that the depth indicated at the two dots* near the bottom of the figure is 100 miles, that the curve CCC is a straight line below this point, and that this straight line intersects the vertical axis, and consequently the compensation becomes zero at the depth 141.3 miles.

Following the same line of reasoning as on pages 156, 157, and using the corresponding notation, let h_1 be the limiting depth of compensation (which is for the moment assumed to be 141.3 miles) and let $\delta_U = -\frac{h}{h_1}\delta$ be that average value of the compensating defect of density necessary for complete compensation within the depth h_1. By scaling off the abscissæ of curve CCC at short intervals, and examination of the values so found, it becomes evident that the Chamberlin compensation is approximated closely by the sum of the following six compensations, each uniformly distributed from the surface down to its own limiting depth.

No. 1	$\delta_d = 1.49\delta_U$ from the surface to depth	$.1h_1$
No. 2	$\delta_d = 1.27\delta_U$ from the surface to depth	$.3h_1$
No. 3	$\delta_d = .490\delta_U$ from the surface to depth	$.5h_1$
No. 4	$\delta_d = .143\delta_U$ from the surface to depth	$.7h_1$
No. 5	$\delta_d = .092\delta_U$ from the surface to depth	$.9h_1$
No. 6	$\delta_d = .028\delta_U$ from the surface to depth	$1.0h_1$

Also, by the same reasoning as on page 157 it appeared that

$$F_{C_6} = .151F_{(.1h_1)} + .386\,F_{(.3h_1)} + .249F_{(.5h_1)} + .101\,F_{(.7h_1)} + .084\,F_{(.9h_1)} + .028\,F_{(1.0h_1)}$$

In this formula F_{C_6} is a compensation factor by which the computed topographic deflection must be multiplied to secure the resultant deflection if the sum of the six compensations indicated above exists. $F_{(.1h_1)}$ is the F factor such as those shown in the table on page 70, or in the solid curve on illustration No. 15 at the end of the volume, for complete compensation extending from the surface to depth $.1h_1$, $F_{(.3h_1)}$, $F_{(.5h_1)}$ etc., each has the corresponding meaning with respect to depths $.3h_1$, $.5h_1$, etc., respectively.

* These dots have been added to the original figure.

Let F_c be the compensation factor corresponding to complete Chamberlin compensation, extending to the depth h_1. Then, since the sum of the six compensations indicated above is nearly equivalent to this Chamberlin compensation, F_{c_6} is a close approximation to F_c.

In the concrete case to be investigated, in which h_1 has been assumed to be 141.3 miles, F_{c_6} was then computed for each ring from the above formula. The necessary values of $F_{(.1h_1)}$, $F_{(.3h_1)}$, etc., in the formula, were those which had already been utilized (see p. 157).

In order to ascertain the closeness with which F_{c_6} approximates to F_c, eleven (instead of six) separate compensations were next assumed, extending to depths $\frac{1}{20}h_1$, $\frac{3}{20}h_1$, $\frac{5}{20}h_1$, ... $\frac{20}{20}h_1$, and representing as closely as possible the Chamberlin compensation (curve CCC). The formula for the compensation factors was then found to be—

$$F_{c_{11}} = .024F_{(.05h_1)} + .178F_{(.15h_1)} + .211F_{(.25h_1)} + .131F_{(.35h_1)} + .151F_{(.45h_1)} + .098F_{(.55h_1)} + .049F_{(.65h_1)} + .049F_{(.75h_1)} + .053F_{(.85h_1)} + .042F_{(.95h_1)} + .015F_{(1.0h_1)}$$

The values of $F_{c_{11}}$ were computed from this formula for rings 7, 14, and 20. The differences $F_{c_{11}} - F_{c_6}$ were found to be $-.001$, $-.009$, and $+.002$, respectively. It appeared from an examination in detail that if the number of separate assumed uniform compensations were increased indefinitely beyond 11 to secure a closer approximation of $F_{c_{11}}$ to F_c probably no computed factor would be changed by more than .003. Therefore it was assumed that, with sufficient accuracy, $F_{c_{11}}$ may be assumed equal to F_c. By inspection the necessary corrections, not exceeding .009, were applied to the values of F_{c_6} for all rings to make them agree with $F_{c_{11}}$ and the corrected values were taken as F_c for each ring, corresponding to the limiting depth 141.3 miles.

These values for F_c were then compared with the corresponding values of F for depth 70.67 miles (see p. 70), and it appeared that the two sets of factors would agree much more closely if the limiting depth for the Chamberlin compensation were made 201.5 instead of 141.3 miles. Accordingly, new factors F_c, corresponding to this limiting depth, were computed.

The resultant deflections, due to both the topography and assumed Chamberlin compensation, extending to the depth 201.5 miles, were then computed for the same 10 meridian stations and 10 prime vertical stations which were taken as representative in the investigation of possible compensation complete within a stratum 10 miles thick (see p. 151). These computed deflections were compared with the corresponding values for these stations, computed for complete compensation uniformly distributed from the surface to depth 70.67 miles. The maximum difference was found to be only 1″.30. But an examination in detail showed that the difference would be still smaller and probably a minimum if the limiting depth for the Chamberlin compensation were changed to 178.6 miles. Accordingly, the factors and the resultant deflections for the 20 representative stations were computed for the depth 178.6 miles, and are shown below in comparison with corresponding quantities for uniform compensation extending to depth 70.67 miles.

Ring.	Factor Fc for depth 287.4 kilometers or 178.6 miles.	Factor F for depth 113.7 kilometers or 70.67 miles.	Difference.	Ring.	Factor Fc for depth 287.4 kilometers or 178.6 miles.	Factor F for depth 113.7 kilometers or 70.67 miles.	Difference.
27	.996	.997	−.001	13	.555	.618	−.063
26	.995	.996	−.001	12	.453	.493	−.040
25	.993	.995	−.002	11	.344	.358	−.014
24	.989	.992	−.003	10	.245	.234	+.011
23	.983	.988	−.005	9	.160	.139	+.021
22	.974	.983	−.009	8	.098	.077	+.021
21	.965	.976	−.011	7	.055	.040	+.015
20	.952	.965	−.013	6	.029	.020	+.009
19	.930	.951	−.021	5	.015	.010	+.005
18	.902	.930	−.028	4	.008	.005	+.003
17	.861	.900	−.039	3	.005	.003	+.002
16	.805	.859	−.054	2	.003	.001	+.002
15	.737	.801	−.064	1	.000	.001	−.001
14	.651	.721	−.070				

The factors F_c are shown graphically on illustration No. 15 at the end of the volume, on the curve marked C and drawn with dots. If this curve is read with the first line of abscissæ below the illustration, it gives such factors for the assumed limiting depth 178.6 miles. If it is read with the third line of abscissæ above the illustration, it gives such factors for any depth.

Computed deflection.

MERIDIAN STATIONS.

No.	Name.	Chamberlin compensation, depth 178.6 miles.	Compensation uniform to depth 113.7 kilometers or 70.67 miles.	Difference.
		″	″	″
152	Mount Independence, Maine	− 1.90	− 1.80	−.10
169	Howlett, New York	+ 3.10	+ 3.33	−.23
100	Hill, Maryland	− 0.73	− 0.59	−.14
123	Sawnee, Georgia	− 2.72	− 2.72	.00
189	Thones Hill, Michigan	+ 1.96	+ 2.14	−.18
76	Weed Patch, Indiana	− 0.32	− 0.30	−.02
60	El Paso E. B., Colorado	− 3.26	− 3.49	+.23
35	Ibepah, Utah	+ 0.57	+ 0.68	−.11
17	Yolo NW. B., California	− 2.28	− 2.03	−.25
238	Santa Barbara, California	−12.12	−12.35	+.23

PRIME VERTICAL STATIONS.

163	Mount Independence, Maine	−3.36	−3.33	−.03
175	Howlett, New York	+0.51	+0.66	−.15
103	Hill, Maryland	−1.23	−1.02	−.21
128	Sawnee, Georgia	− .21	−0.15	−.06
202	Thones Hill, Michigan	−2.05	−2.14	+.09
81	Weed Patch, Indiana	+0.57	+0.62	−.05
62	El Paso E. B., Colorado	−6.61	−6.67	+.06
35	Ibepah, Utah	−1.87	−2.28	+.41
14	Yolo NW. B., California	+2.20	+1.40	+.80
229	Santa Barbara, California	+5.14	+4.76	+.38

Seven of the twenty differences are each less than 0″.10, and the maximum difference is only 0″.80. These differences are small in comparison with the residuals of solution G of which the average without regard to sign is 3″.04, and of which only 43 per cent are less than 2″.00 (see p. 136).

By the lines of reasoning indicated on pages 152, 153 the following conclusions have been reached. First, the most probable limiting depth is 178.6 miles for complete compensation with the mode of distribution with respect to depth postulated by Professor Chamberlin. Second, it is not possible to ascertain whether this compensation is more probable than the G compensation. uniformly distributed from the surface to the depth 70.67 miles, since the two sets of computed deflections agree so closely that their differences are much smaller than the accidental errors.

The first of the above conclusions may be made more definite. The details of the postulated mode of distribution of the Chamberlin compensation with the limiting depth 178.6 miles are shown in the curve CCC, on illustration No. 16, on which the depth indicated by the two dots must now be considered to be 127 miles, and from that point downward the curve must be considered to be a straight line intersecting the vertical axis at the depth 178.6 miles. Let δ_v be the defect of density necessary for complete compensation uniformly distributed from the surface to the depth 178.6 miles. Then, according to the above conclusion, the Chamberlin compensation at various depths is as follows:

Depth.	Chamberlin compensation.	Depth.	Chamberlin compensation.
Miles.		*Miles.*	
0	3.6 δ_U	80	.6 δ_U
5	3.7 δ_U	89	.4 δ_U
9	3.6 δ_U	98	.3 δ_U
18	3.2 δ_U	107	.3 δ_U
27	2.6 δ_U	116	.2 δ_U
36	2.0 δ_U	125	.2 δ_U
44	1.4 δ_U	142	.1 δ_U
53	1.1 δ_U	160	.1 δ_U
62	.9 δ_U	178.6	.0
71	.8 δ_U		

For a concrete case in which the mean elevation of the surface is .67 mile (3 540 feet) $\delta_U = -\frac{h}{h_1}\delta = -\frac{.67}{178.6}2.67 = -.01$. For this case the defect of density at depth 5 miles is .037 or about $\frac{1}{72}$ of the normal surface density $\left(\frac{.037}{2.67} = \frac{1}{72}\right)$. At depth 62 miles the defect of density is only .009, and is less than .002 for all depths between 100 and 178.6 miles.

It is the judgment of the writer that it is not advisable, at this time, to apply to the Chamberlin compensation the two additional tests, indicated on page 153. It is probable that the additional evidence obtained would be too weak to be convincing. When more data are available it may become advisable to apply these additional tests.

THE FLOATING CRUST HYPOTHESIS.

The hypothesis has been made at various times and by various persons that the outer portion of the earth is a solid crust a few miles thick, floating on a liquid substratum of greater density. In order to show the bearing of the present investigation on this hypothesis, it will be taken in the concrete form in which it has recently been used in a discussion of deflections of the plumb line in India.* It is there assumed that the solid crust is 25 miles thick below areas which are at sea level, that its density (uniform) is 2.68 (that of granite) and that the density of the liquid substratum on which it floats is 2.96 (that of basalt). It is assumed that this crust floats in the denser liquid substratum in the same manner that a large field of ice of variable thickness floats on water. To each elevated area on the surface of the crust there corresponds, on this assumption, a protuberance on the lower side of the crust such that the buoyancy due to the protuberance corresponds to the weight of the excess of mass above sea level. The buoyancy of the protuberance is represented by the excess of density ($0.28 = 2.96 - 2.68$) of the liquid substratum. This is $\frac{1}{9.6}$ part of the density of the crust $\left(\frac{0.28}{2.68} = \frac{1}{9.6}\right)$ and therefore the protuberance under each surface feature must be 9.6 times the elevation of that feature above sea level. In the article referred to the elevation of the Tibetan plateau is stated to be 3 miles, and the corresponding protuberance into the substratum to be 29 miles.

Apply this hypothesis to the United States. The average protuberance, under the area covered by group 8 of the residuals (Nevada, and parts of California and Utah, see p. 135 and illustration No. 13) should be about 67 000 feet (20 000 meters) or nearly 13 miles, since the average elevation of this area is about 7 000 feet. On the other hand, the area covered by group 3 (portions of New Jersey, Delaware, Pennsylvania, Maryland, and Virginia) is nearly at sea level, the average elevation being less than 100 feet, and should have a corresponding protuberance of less than 1 000 feet. So, also, the average elevations for groups 1 to 6, inclusive, is about 1 000 feet, and the corresponding protuberance should be about 9 600 feet (2 900

* On Deflections of the Plumb Line in India, by Rev. O. Fisher, Appendix No. 1 of Vol. XVIII of Account of the Operations of the Great Trigonometrical Survey of India. The title of this volume is Astronomical Observations for Latitude Made During the Period 1885–1905 and the Deduced Values of the Deflections of the Plumb Line.

meters) or nearly 2 miles. According to the hypothesis the thickness of the crust below sea level should be 38 (=25+13) miles under group 8, 25 miles only under group 3, and about 27 miles, on an average, under groups 1 to 6 combined. All of the isostatic compensation would, therefore, on this hypothesis occur between the depths 25 and 38 miles below sea level, under group 8, between 25 and 27 miles below sea level under groups 1 to 6, and the little compensation under group 3 would occur at depths slightly greater than 25 miles. It is obvious that, if this state of affairs actually existed, the mean depth of compensation would be much greater under group 8 than under groups 1 to 6, and the various tests applied in this investigation, see pages 142 and 143, to determine whether the depth of compensation is variable should show this relation of the depths of compensation. As a matter of fact the tests indicate that the depth of compensation is greater under groups 1 to 6, inclusive, than under group 8, instead of less. Though, as stated on page 143, the evidence of these tests is not strong enough to prove that there is a real difference of depth of compensation in the different regions, it is certainly sufficient to show that the depth of compensation can not be so much less under groups 1 to 6 than under group 8, as is required by the hypothesis of a floating crust. The evidence is, therefore, sufficient to show that the floating crust hypothesis in the form stated above is not true for the United States.

DEGREE OF COMPLETENESS OF COMPENSATION.

An indirect definition has already been given on page 67 for the expression "complete isostatic compensation," "Let the depth within which the isostatic compensation is complete be called the depth of compensation. At and below this depth the condition as to stress of any element of mass is isostatic; that is, any element of mass is subject to equal pressures from all directions as if it were a portion of a perfect fluid. Above this depth, on the other hand, each element of mass is subject, in general, to different pressures in different directions—to stresses which tend to distort it and to move it."

In other words, above the depth of compensation each element of mass is subject to a compressive stress which is appreciably greater in the direction of maximum compressive stress at that point than is the compressive stress in the direction of minimum stress at that point. This difference between the maximum compressive stress and the minimum compressive stress at a given point is conveniently called a stress-difference.

It is impossible, from any investigation based on deflections of the vertical alone, to determine accurately the degree of completeness of the isostatic compensation, unless the mode of distribution of the isostatic compensation with respect to depth and the limiting depth of compensation are both known. For the deflections of the vertical, the observed quantities, depend upon the distribution and depth of the isostatic compensation as well as upon its degree of completeness.

The residuals of solution G furnish a test of the departures of the facts from the assumed condition of complete isostatic compensation uniformly distributed to a limiting depth of 113.7 kilometers. In order to obtain definite ideas, let the whole of the residuals of this solution be credited to the incompleteness of the compensation. The conclusion as to the completeness of compensation will then be in error in that the actual approach to completeness will be considerably closer than that represented by the conclusion—that is, the conclusion will be an extreme limit of incompleteness rather than a direct measure. For by this process of reasoning every portion of a residual of solution G, due to the departure of the actual distribution of compensation with respect to depth from the assumed distribution, or due to the error in the assumed mean depth of compensation, or to regional variation from a fixed depth of compensation, or due to errors of observation in the astronomic determinations and the triangulation which affect the observed deflection of the vertical, or due to errors of computation, is credited to incompleteness of compensation.

Such a limit to the degree of incompleteness of compensation is furnished by the following table. The ten groups of stations represented in the table are the same that were used in the tables on pages 135–137. Their geographic limits are shown on illustration No. 13.

Group.	Number of stations.	Mean of topographic deflections without regard to sign.	Mean residual of solution G without regard to sign.	Value in fourth column divided by value in third column.
		″	″	
1	52	35.48	2.50	.07
2	54	20.86	2.53	.12
3	49	35.79	3.33	.09
4	50	23.85	2.72	.11
5	53	9.13	3.86	.42
6	51	18.42	2.37	.13
7	46	16.80	2.26	.13
8	43	31.84	3.48	.11
9	57	60.15	2.83	.05
10	52	66.35	4.57	.07
All combined	507	32.26	3.04	.094

The fourth column of the table furnishes a limit of the degree of incompleteness of the isostatic compensation. The approach to complete isostatic compensation is certainly closer than is represented by mean residuals without regard to sign. These means vary from 2″.26 in group 7 to 4″.57 in group 10, and the mean is 3″.04 for all observations.

This measure is indefinite and not enlightening considered by itself, for the reason that from it alone one obtains no idea as to whether these residuals would be much larger if no isostatic compensation existed.

The computed topographic deflections are the deflections which would necessarily be found if no isostatic compensation existed and if the densities beneath the surface were merely a function of the depth. Their mean values without regard to sign, as shown in the third column of the table, vary from 9″.13 in group 5 to 66″.35 in group 10, and is 32″.26 for all observations. The reduction of the residuals below these mean values is the effect of the isostatic compensation. In group 1 the isostatic compensation of the kind postulated in solution G has reduced the mean residual from 35″.48 to 2″.50 (the mean residual of solution G)—that is, to only 0.07 of 35″.48. The departure from complete compensation as measured by this group is less than 0.07.

The various groups separately indicate the departure from complete compensation to be from less than 0.42 for group 5 to less than 0.05 for group 9, and all observations combined indicate the departure to be less than 0.094. Group 5 shows a much wider departure from the mean value 0.094 than does any other group. The remaining groups show a range from 0.05 to 0.13 only.

The various values in the last column of the table do not indicate the departures from complete compensation within the areas covered by the separate groups—that is, one must not reason that complete isostatic compensation is approached within less than 0.05 by the arrangement of densities beneath the area covered by group 9, and that it is only approached within somewhat less than 0.42 beneath the area covered by group 5. It should be recalled, in this connection, that in computing the topographic deflections all areas within 4 126 kilometers of each station were considered (see p. 131) and that the computed topographic deflection is not small for the outer rings of topography (see p. 127). These outer rings of topography which lie far beyond the limits of the particular group of observations which may be under consideration, necessarily produce deflections at the station unless isostatic compensation exists beneath said outer rings. Hence each value in the last column of the table is a test furnished by one group of the completeness of the isostatic compensation in and around the United States, rather than a test of the completeness within the area covered by that group alone.

For the United States and adjacent areas it is safe to conclude from the evidence just summarized that the isostatic compensation is so nearly complete on an average that the deflections of the vertical are thereby reduced to less than one-tenth of the mean values which they would have if no isostatic compensation existed. One may properly characterize the isostatic compensation as departing on an average less than one-tenth from completeness or perfection.

This statement should not be interpreted as meaning that there is everywhere a slight deficiency in compensation. It is probable that under some areas there is overcompensation, as well as undercompensation in others.

If one wishes to form some idea of the extent to which the stress-differences in the earth have been reduced by the isostatic compensation below what they would have been if no isostatic compensation existed, it is necessary to supplement the evidence given by the above table by that given on illustrations Nos. 10, 11, and 12. A departure of one-tenth from completeness in the isostatic compensation under a given small land * area represents an excess or deficiency of mass above the 114-kilometer level (the assumed depth of compensation) of one-tenth of the mass which is above sea level within that area, being an excess of mass if there is undercompensation and a deficiency if there is overcompensation. In order to estimate stress-differences it is important to know whether the continuous land areas beneath which there is undercompensation, and therefore excess of load, are large or small, and similarly for the overcompensated areas. The average elevation of the surface of the United States above sea level is about 2 500 feet.† Therefore an average departure of one-tenth part from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet thick on an average. The greater the size of the continuous areas over which there is an excess corresponding to such a stratum the greater will be the existing stress differences. Illustrations Nos. 10, 11, and 12 show that the residuals of solution G place themselves in many groups with respect to sign—that is, the continuous groups in which the residuals of the same kind are all of one sign are small, there being from 40 to 60 such groups within the area covered by this investigation. From this it may be fairly inferred that the land areas of continuous undercompensation (excess of load at the 114-kilometer level), or, on the other hand, of overcompensation (deficiency of load at the 114-kilometer level), are about 50 in number in the area covered by this investigation, and are, therefore, of small average extent. If no isostatic compensation existed there would be an excess of load over the whole of the one continuous area covered by the continent.

Keeping this contrast in mind, the writer believes that the stress-differences in and about the United States have been so reduced by the isostatic compensation that they are less than one-twentieth as great as they would be if the continent were maintained in its elevated position and the ocean floor maintained in its depressed position by the rigidity of the earth.

In making this estimate, the writer has also kept in mind that, even if the isostatic compensation were everywhere complete, the stress-differences in the material would not be zero, for as stated in the definition of isostasy (see p. 66), the condition called isostasy is a condition of approximate equilibrium (that is, small stress-differences), not perfect equilibrium (no stress-differences).

EXTREME RIGIDITY VERSUS ISOSTASY.

As stated in connection with the definition of isostasy, page 66, if the heterogeneous material composing the earth were not at the outset so arranged that the density at any point was simply a function of the depth, the stresses produced by gravity would tend to bring about such an arrangement. As the material is not a perfect fluid, as it has considerable viscosity, at least near the surface, the rearrangement by gravity will be imperfect. The condition reached will tend to be that condition of approximate equilibrium known as isostasy.

In this investigation it has been found that the existing condition in and around the United States is a close approach to isostasy. It has been tacitly assumed throughout this publication, as shown by the wording at various points, as for example on page 114, that the proofs that

* The word "land" is introduced here merely to avoid confusion in the statement. The corresponding statements of this and later sentences may be made for ocean areas, but, in making them, certain obvious exchanges of position of the words "excess" and "deficiency" and of "overcompensation" and "undercompensation" must be made.

† "The Average Elevation of the United States," by Henry Gannett, Thirteenth Annual Report U. S. Geological Survey. Part 2, pp. 284-289.

the existing condition is a close approach to perfect isostasy are necessarily proofs that isostatic readjustment has taken place, and that, therefore, the material concerned in this readjustment is sufficiently lacking in rigidity to allow of the readjustment. It has thus been tacitly assumed that the closer is the approach to perfect isostasy at present the less must be the effective rigidity of the material.

It may be urged in objection to this tacit assumption that possibly the heterogeneous material composing the earth was arranged at the outset in the particular manner corresponding to isostasy.

Prof. T. C. Chamberlin has reached the conclusion that, according to the planetesimal theory of the formation of the earth, the effect of differential weathering and of vulcanism would tend to arrange the densities in the growing earth in this particular manner.* When the earth had attained sensibly its present size it would, according to this conclusion, be in an initial isostatic condition. On this basis it may appear, at first sight, that the discovery that isostasy now exists does not disprove the present existence of extreme rigidity.

To proceed no farther than this in one's thinking is to ignore the overwhelming geologic evidence that, within the interval covered by the geologic record, which interval must have begun after the earth attained approximately its present size, many thousands of feet of material have been eroded from some parts of the United States and adjacent regions and have been transferred to and deposited upon other parts; that changes of elevation of the solid surface brought about in these and other ways, amounting to thousands of feet in each area critically examined by geologists, are the rule rather than the exception; and that these changes in elevation have continued to take place in recent geologic time, and are probably still in progress. As soon as this mass of geologic evidence is considered it becomes evident that if an initial condition of complete isostasy existed, and if no isostatic readjustment had taken place since that time, the departure from perfect isostasy would now be measured by the algebraic sum for each region of the great changes in elevation of its solid surface which have certainly taken place. But the present investigation shows that a much closer approach to perfect isostasy than this now exists. As stated on page 166, the excesses and deficiencies of mass which constitute the present departure from perfect isostasy are measured by a stratum only 250 feet thick on an average. The algebraic sum of the changes in elevation of the solid surface have probably been as much as ten times this, on an average, for the United States and adjacent regions.

Moreover, it is not simply the variation in the elevation of the solid surface which must be considered. It is estimated by competent authority that a series of strata from 8 to 10 miles thick has been eroded and carried away from certain areas in the western part of the United States, which are now broad and lofty platforms carrying mountain ranges. The present elevation of these areas is less than 3 miles—the average elevation, not the elevation of the summits. The solid surface of this area was probably never as much as 11 to 13 miles above sea level. The region probably rose as erosion and transportation took place and the solid surface may never have been more than 5 miles above sea level. The total change of elevation of the solid surface may have been only 2 miles. But the disturbance of initial isostasy by such a change is measured by the difference of elevation between the present solid surface and the position which the restored surface would have if the removed material were returned to its original position in relation to the present surface. The disturbance of the initial isostasy is measured by the 8 to 10 miles of material which has been removed. The geologic evidence indicates many large changes of this character. It is, therefore, safe to say that the departures from perfect isostatic compensation which exist at present in the United States and adjacent areas are much less than one-tenth of the disturbances of a possible initial isostatic condition which have occurred since the earth attained approximately its present size and the geologic record began.

If isostatic readjustment had not been in progress during the interval in which the geologic record was being made, it would be impossible for the isostatic compensation to be so

* See reference in footnote on p. 148.

nearly complete as it is now. It is necessarily true that isostatic readjustment has occurred and, therefore, it is necessarily true that the effective rigidity of the portions of the earth involved in this readjustment has been low enough to allow the readjustment to take place.

These are the considerations that lead to the belief that the smallness of the present departures from complete isostatic compensation is a proof of small effective rigidity.

SOLUTION C, DEALING WITH LAND ONLY.

There are many things in the literature of geography that tend to lead one to the belief that the sea bottom under the great oceanic areas is nearly horizontal and unbroken, having moderate slopes only, in contrast with the surface of the land areas, which are broken up by many mountain chains and are characterized by much steeper and more irregular slopes than seem to occur in the deep portions of the ocean. So, too, the literature of geology seems to indicate that many geologists believe that there has been comparatively little folding and crushing of the strata beneath the ocean, that the strata lie comparatively unbroken in nearly horizontal position, whereas on every continent folded, crumpled, and broken strata abound. These observations and beliefs lead naturally to the supposition that possibly under the oceans the vertical forces control in fixing the elevation of the sea bottom, and that, on the other hand, the continents are built, or at least largely modified, by the influence of the horizontal forces which produce folding, crumpling, overthrusts, and other phenomena. Upon this supposition one must expect that isostatic compensation will be found to exist under the ocean where the vertical forces control, and that under the land, though there might be some tendency toward isostasy, it would be so greatly obscured by the effects of horizontal forces as not to be clearly recognizable.

It seemed desirable to make a solution of the observation equations, during the progress of this investigation, which would serve to test the validity of the above supposition. Accordingly, a solution, known as solution C, was made, in which the absolute term in each observation equation was the observed apparent deflection minus the topographic deflection as computed from the land areas only, all ocean areas being entirely ignored. This was readily done, since in the regular computations of the topographic deflections all values corresponding to ocean areas (negative elevations) were written in red, while for land areas (positive elevations) they were written in black (see p. 26). Solution C corresponds to the assumption that there is complete isostatic compensation of the defects of mass represented by the water of the oceans, by excesses of density in the material at the ocean bottom, and that under land areas there is no isostatic compensation. The assumption is the same as for solution A (see p. 93) for sea areas (complete isostatic compensation at depth zero), and is the same as for solution B for land areas (no isostatic compensation).

The following table shows the statistics of the residuals of this solution in comparison with those of the five principal solutions:

	Solution B	Solution E	Solution H	Solution G	Solution A	Solution C
Mean value of the squares of the residuals	129.06	16.21	15.82	15.81	27.46	43.75
Mean values of residuals without regard to sign	8.86	3.06	3.04	3.04	3.92	4.59
Percentage of residuals less than 2″.00	15	41	43	43	34	31
Percentage of residuals greater than 5″.00	66	18	18	18	29	32
Maximum residual	+43.84	−16.47	−15.74	+15.94	−22.35	+30.06

These five tests made with all the residuals in one group all agree in showing that solution C is intermediate between solution A and solution B, being somewhat farther from the truth than solution A and much nearer the truth than solution B. It is evidently much farther from the truth than any one of solutions E, H, and G.

Similar tests applied to the smaller groups of residuals which were used in the tables on pages 135–137 confirm the tests made above. The only important exception to this is that for

group 5 (Michigan, Minnesota, and Wisconsin), the four tests by the mean square of the residuals, by mean residuals, by percentage of residuals less than 2″.00, and by percentage of residuals greater than 5″.00, each indicates that solution C is as near or nearer to the truth than solutions E, H, and G.

The evidence as a whole indicates that the assumption used in solution C, that there is complete isostatic compensation close to the bottom of the sea and no isostatic compensation under the land, is somewhat farther from the truth than the assumption used in solution A, that there is complete isostatic compensation at depth zero everywhere under both sea and land. It also proves that the assumption of solution C is much farther from the truth than the assumption used in solution G, that everywhere under both sea and land there is complete isostatic compensation uniformly distributed throughout the depth 113.7 kilometers.

The evidence contains no indication that sea and land areas differ with respect to isostatic compensation.

The assumption upon which solution C is based differs from that on which solution A is based in one respect only, namely, that, whereas in solution C there is assumed to be no isostatic compensation beneath land areas, in solution A there is assumed to be complete isostatic compensation at depth zero beneath the land areas. The fact, then, that the residuals are considerably smaller in solution A than in solution C, the mean square of the residuals being less than two-thirds as great, is positive proof that isostatic compensation exists beneath the land areas.

It follows, moreover, that it is an isostatic compensation of the separate topographic features of the continent, not a compensation merely of the continent as a whole. In solution A it is a compensation of separate features which is assumed. An inspection of the numerical values of the computed topographic deflections, and of the deflections computed with isostatic compensation considered, shows that merely to have assumed the continent as a whole to be compensated, not its separate topographic features, would have given a solution resembling solution C much more closely than solution A.

In the above statement that the separate topographic features of the continent are compensated, it is not intended to assert that every minute topographic feature, such, for example, as a hill covering a single square mile, is separately compensated. It is believed that the larger topographic features are compensated. It is an interesting and important problem for future study to determine the maximum size, in the horizontal sense, which a topographic feature may have and still not have beneath it an approximation to complete isostatic compensation. It is certain, from the results of this investigation, that the continent as a whole is closely compensated, and that areas as large as States are also closely compensated. It is the writer's belief that each area as large as one degree square is generally largely compensated. The writer predicts that future investigations will show that the maximum horizontal extent which a topographic feature may have and still escape compensation is between 1 square mile and 1 square degree. This prediction is based, in part, upon a consideration of the mechanics of the problem.

THE AREA METHOD SUPERIOR TO THE ARC METHOD.

The arc method of deducing the figure of the earth may be illustrated by supposing that a skilled model maker is given several stiff wires, each representing a geodetic arc, either of a parallel or a meridian, each bent to the radius deduced from the astronomic observations on that arc, and is told in what latitude each is located on the geoid, and then requested to construct the ellipsoid of revolution which will conform most closely to the bent wires. Similarly, the area method is illustrated by supposing that the model maker is given a piece of sheet metal cut to the outline of the continuous triangulation which is supplied with the necessary astronomic observations, and accurately molded to fit the curvatures of the geoid as shown by the astronomic observations, and that he is then requested to construct the ellipsoid of revolution which will conform most accurately to the bent sheet. Such a bent sheet essentially

includes within itself the bent wires referred to in the first illustration, and, moreover, the wires are now held rigidly in their proper relative positions. The sheet is much more, however, than this rigid system of bent lines, for each arc usually treated as a line is really a belt of considerable width, which is now utilized fully. It is obvious that the model maker would succeed much better in constructing accurately the required ellipsoid of revolution from the one bent sheet than from the several bent wires.

The preceding paragraph is a general statement by analogy of the advantage of the area method over the arc method. The three paragraphs which follow give, in more accurate and definite form, the advantages of the area method as considered from three different points of view.

If one is to use the area method it is not necessary to carry the triangulation approximately along arcs of meridians or along arcs of parallels, and similarly to limit the location of astronomic stations. One is perfectly free to carry the triangulation forward wherever and in whatever direction the topography and other conditions are most favorable to rapid and economical progress. The solution by the area method is general, and will fully utilize all the observations of astronomic latitude, longitude, and azimuth in whatever way they are distributed with respect to each other. This is the important and essential advantage of the area method over the arc method, as seen from the point of view of one who is in general charge of field operations.

In the arc method some important rigid conditions, connecting arcs which intersect each other, are neglected, with a consequent loss of accuracy in the derived results. This neglect does not occur in the area method. For example, if an arc of a meridian and an arc of a parallel cross each other, the triangulation involved in the two arcs being in reality continuous, no account is taken in the arc method of the fact that for any triangulation station common to the two arcs the two latitudes derived from the two arcs must be identical, the two longitudes must be identical, and the two azimuths from that station to some other station must be identical. These are three rigid conditions which exist in nature, but do not occur in the arc method of solution. On the other hand, in the area method all such conditions are fully taken into account in the equations used, with a consequent increase in the accuracy of the derived results. The increase in accuracy will evidently be greater, the greater is the number of arcs involved and the greater is the number of connections which exist between them. For the United States the increase in accuracy is believed to be great. This is the advantage of the area method over the arc method from the point of view of one who is in general charge of computations having for their purpose the determination of the figure and size of the earth.

The determination of the figure and size of the earth is not the only valuable scientific result to be obtained from geodetic observations. Probably the slight increase in accuracy of this determination is not the most important result to be obtained from a further study of existing observations and from additional observations. Still more important than this are the determinations of the existence or nonexistence of isostasy, the determination of the completeness and distribution with respect to depth of the isostatic compensation, and, in general, the determination of the distribution of subsurface densities. These are probably more important because they have a broad bearing upon many of the fundamental problems of geology and geophysics. These determinations may be made by studying the residuals of computations of the figure and size of the earth, these residuals representing the unexplained deflections of the vertical. Evidently the greater the continuous area covered by any one computation all on one geodetic datum, and the greater the number of observed deflections within this area, the more reliable will be the conclusions from that investigation. If the area is small, the conclusions may be vitiated by some abnormal or unusual distribution of densities having a systematic effect over a considerable portion of the area. If a small number of deflections are used, the conclusions reached may be biased by the existence of one or a few very large

residuals. Hence, there is great advantage, as seen from the point of view of one who desires to study the distribution of subsurface densities, in the flexibility of the area method, which allows of the ready extension of both field and office work continuously and on one basis over large areas of any shape.

INCREASED ACCURACY RESULTING FROM THE RECOGNITION OF ISOSTASY.

In the computations of the figure and size of the earth heretofore made from geodetic observations, as, for example, the Bessel and the Clarke computations, the possible existence of isostasy was not recognized in the methods used. The computations were made on the same basis as solution A of this investigation. This basis is the assumption that there is complete isostatic compensation at depth zero; that there exists immediately below every elevation (either mountain or continent) the full compensating defect of density, and that at the very surface of the ocean floor there lies material of the excessive density necessary to compensate for the depression of this floor. But that this was really the basis of the computation has not apparently been recognized. The practice has been merely to treat deflections of the vertical as accidental errors. This is an implicit assumption that the deflections of the vertical are independent of the known topography, which can not be true unless complete isostatic compensation exists at depth zero.

If in the present investigation the existence of isostasy had not been definitely recognized, solution A would have been the only solution. But the existence of isostasy was recognized. It was realized that it must be dealt with in some more reasonable way than that represented by solution A with its extreme and improbable assumption of complete isostatic compensation at the very surface. There was introduced into the investigation the idea that the isostatic compensation must be distributed through some finite depth much less than the earth's radius, and the processes of computation were so arranged as to give a determination of the most probable value of the depth of compensation. The final outcome of the introduction of the assumption of isostasy in a definite and reasonable form into the problem of determining the figure and size of the earth has been solution G of this investigation. The increased accuracy of solution G over solution A is, therefore, the measure of the improvement in the determination of the figure and size of the earth due to a proper recognition of isostasy. The substitution of solution G for solution A reduced the sum of the squares of the residuals from 13922 to 8013. If the weight of the determination of the figure and size of the earth—that is, of the equatorial radius, flattening, and polar semidiameter—from solution A be called unity that of the determination from solution G is $1.7\left(\frac{13922}{8013}=1.7\right)$.

If the isostatic compensation is as nearly complete under other countries as it is under the United States, and the writer sees no reason to doubt that it is, a similar increase in accuracy may be made by a similar recognition of isostasy in connection with computations of the figure and size of the earth from the great mass of geodetic observations which have already been made in various parts of the world.

COMPARISON WITH OTHER DETERMINATIONS OF THE DIMENSIONS OF THE EARTH.

It is desirable to compare the dimensions of the earth, derived from such an investigation as this, in which wide departures have been made from past practice, with the dimensions previously derived by methods which have been generally accepted. It is desirable that one who is skeptical in regard to the validity of the new departures should see such a comparison in order that he may have a check upon the new methods. On the other hand, it is desirable that one who believes fully in the new methods should have a definite idea of the size and sign of the corrections which must, according to the new determination, be applied to previously accepted values.

The following tables show a comparison of the accepted final values from this investigation (from solution G) which will, for convenience, be designated as Coast and Geodetic Survey 1906 values, with the well-known Bessel and Clarke values:

	Equatorial radius.	Reciprocal of flattening.	Polar semi-diameter.
	Meters.		*Meters.*
Bessel 1841	6377397	299.2	6356079
Clarke 1866	6378206	295.0	6356584
Clarke 1880	6378249	293.5	6356515
Coast and Geodetic Survey 1906	6378283	297.8	6356868
	±34	±0.9	

The comparisons expressed in the form of corrections which must be applied to the older values to give the new values are as follows:

	Equatorial radius.	Reciprocal of flattening.	Polar semi-diameter.
	Meters.		*Meters.*
C. & G. S. 1906 – Bessel 1841	+886	−1.4	+789
C. & G. S. 1906 – Clarke 1866	+ 76	+2.8	+284
C. & G. S. 1906 – Clarke 1880	+ 34	+4.3	+353

The flattening as derived from this investigation falls between the Bessel and the Clarke values. The equatorial radius agrees closely with the Clarke values. The polar semidiameter agrees more closely with the Clarke values than with the Bessel values. Both the equatorial radius and polar semidiameter are greater than the values derived by either Bessel or Clarke. The comparison shows no wide departures of the new values from previously accepted values, but does contain a strong indication that, as judged by the new investigation, both the equatorial radius and polar semidiameter formerly accepted are too small.

It is desirable to supplement these comparisons with the old classical values by comparisons with the best recent values.

According to Dr. F. R. Helmert, the director of the Central Bureau of the International Geodetic Association, the best value now available for the flattening is $\frac{1}{298.3}$. This value was derived from pendulum observations alone, being the flattening corresponding to Helmert's formula of 1901, expressing the relation between gravity at sea level and the latitude, namely,

$$\gamma_0 = 978.046^{cm}\,(1 + 0.005302 \sin^2 \phi - 0.000007 \sin^2 2\phi).*$$

Helmert considers this value to be more exact than any value which can be obtained from measurements of geodetic arcs even though the arcs of recent decades be utilized.†

* Der normale Theil der Schwerkraft im Meeresniveau, von F. R. Helmert, S. 328–336. Sitzungsberichte / der Königlich Preussischen / Akademie der Wissenschaften / zu Berlin, / Jahrgang 1901, / Erster Halbband, Januar bis Juni. See also, Bericht über die relativen Messungen der Schwerkraft mit Pendelapparaten für den Zeitraum von 1900 bis 1903, unter Mitwirkung von F. R. Helmert erstattet von E. Borrass, S. 133–136, Verhandlungen / der vom 4 bis 13 August 1903 in Kopenhagen abgehaltenen / Vierzehnten Allgemeinen Conferenz der / Internationalen Erdmessung / Redigirt vom ständigen Secretär / H. G. van de Sande Bakhuyzen. / II. Theil : Spezialberichte.

† His statement on this point is so important that it is here reproduced in full. "In den Sitzungsberichten von 1901, März, habe ich die Abplattung der Erde aus den Messungsergebnissen für die Schwerkraft zu 1:298.3 abgeleitet. Der mittlere Fehler des Nenners dieser Zahl ist nur ± 1.1. Sie erscheint dadurch besonders gesichert, dass Kustenstationen und Festlandsstationen je für sich ganz zu demselben Werte führen. Seitdem ist durch Heckers Bestimmung der Schwerkraft auf dem Atlantischen Ozean die Grösse der Schwere daselbst als gut entsprechend der Festlandsformel für die Anderung der Schwerkraft mit der geographischen Breite erkannt worden (Sitzungsberichte von 1902, Februar). Ich glaube darin eine weitere Bestätigung des Abplattungswertes 1:298.3 erblicken zu können, insofern dadurch die Existenz der elliptischen Gestalt gesicherter geworden ist. Jedenfalls dürfte derselbe erheblich genauer sein als derjenige, den man aus den Gradmessungen ableiten kann, trotzdem dieselben in den letzten Jahrzehnten ausserordentlich an Umfang zugenommen haben." See Die Grösse der Erde, von F. R. Helmert, S. 525–537. Sitzungsberichte / der / Königlich Preussischen / Akademie der Wissenschaften. / Jahrgang 1906 / Erster Halbband, Januar bis Juni.

Again in 1907 in response to a request for best values to be used in concluding the triangulation of Egypt, Doctor Helmert furnished* this same value for the flattening, $\frac{1}{298.3}$. It is important to note that this value was furnished for use in connection with triangulation.

In view of the great confidence placed by Doctor Helmert in this value 298.3 ± 0.7† it is important to note the close agreement with it of the C. & G. S. 1906 value for the reciprocal of the flattening, namely, 297.8 ± 0.9. The two values agree within the probable error of either one, though the two values depend on different kinds of observations made in different parts of the earth. The close agreement of the value from this investigation with Helmert's value is significant as showing what accuracy can be secured with the geodetic observations of a single country when treated by the method pursued in this investigation.

The latest statement known to the writer from Doctor Helmert in regard to the most probable value of the equatorial radius is dated February, 1907.* Referring to his own article, entitled, "Die Grösse der Erde," from which the quotation in the footnote on the preceding page is made, he gave as the mean value derived from that investigation, $a = 6\ 378\ 150$ meters, and adds that this becomes about $a = 6\ 378\ 140$ meters, if the condition be introduced that the flattening is $\frac{1}{298.3}$. The C. & G. S. 1906 value is $a = 6\ 378\ 283 \pm 34$ meters, only 143 meters greater than the above value which is considered to be the most probable by Doctor Helmert when he excludes from consideration the measures made in the United States.‡ It is significant that here again the value from this investigation is slightly larger than that derived from arc measures elsewhere computed by a different method.

From the data given in the preceding paragraph, being those considered most probable by Doctor Helmert, namely, $a = 6\ 378\ 140$ and the flattening $\frac{1}{298.3}$, it follows that the polar semidiameter is $b = 6\ 356\ 758$ meters. The C. & G. S. 1906 value is $b = 6\ 356\ 868$ meters, only 110 meters greater than the value considered by Helmert as most probable when observations comprised in the present investigation are excluded from consideration. It is again significant that though the agreement is close the C. & G. S. 1906 value is the larger of the two.

Is it mere chance that for both the equatorial radius and the polar semidiameter the C. & G. S. 1906 values are larger than the Bessel, Clarke, and Helmert values? Or is there a fundamental reason for expecting such a relation? The Bessel, Clarke, and Helmert values have been derived from computations in which the deflections of the vertical are treated as being accidental errors, this being the practically universal method of treatment. Unusually large deflections observed in mountainous regions have frequently been rejected from such computations, but the remaining deflections which were retained have been treated as accidental errors. According to the present investigation they should not be so treated. According to this investigation, the deflections are systematic with respect to the topography, and the relation is approximately such an one for each continent as that fixed (in solution G) by the computations of topographic deflections with isostatic compensation considered as complete and extending to a finite depth. Considered broadly, such deflections are displacements of the zenith at each station away from the center of mass of the continent, or a displacement of the nadir toward it. Consult, for example, the computed deflections for depth 113.7 kilometers, as given on pages 48–56, and use illustration No. 13, at the end of this volume, to show the relation of each station to the continent. Such systematic deflections when allowed to

*Rapport sur les Travaux du Bureau Central de l'Association Géodésique Internationale en 1906 et Programme des Travaux pour l'Exercice de 1907, p. 5.

†A probable error of ± 0.7 corresponds to the mean error of ± 1.1, given by Helmert in the quotation shown in the last footnote on the preceding page.

‡In the same publication, after taking into account the recent geodetic measurements made in the United States, those concerned in this investigation, the most probable value as given by Helmert is $a = 6\ 378\ 200$ meters, differing by only 83 meters from the value derived in this investigation from observations in the United States alone.

enter as if they were accidental errors into a computation of the earth's dimensions, in accordance with the usual past practice, must evidently tend to make the computed values of the radii of curvature and of the earth's dimensions somewhat too small if the geodetic arcs used are confined to one continent. As this is true for each separate determination from a single continent, it is also true that even when results from various continents are combined the computed values must still be slightly too small. Of course, any particular one of the arcs used, if it spans a part of a continent only, and especially if it simply spans a single broad valley, may be affected by a systematic deflection of the zenith toward, instead of from, its center and so give too large a value for the earth's dimensions. But, broadly speaking, and for averages of many arcs, continental arcs span elevations, rather than depressions, and the topography within the area covered by the triangulation of each continent is properly represented by a dome rather than a basin. Moreover, the effect of the negative masses represented by the great oceanic depths which surround every continent produce deflections of the zenith which are systematically toward the oceans and so cooperate with the effect of the topography of the continent itself in tending to make the derived dimensions of the earth too small as ordinarily computed in the past. In considering the relations of deflections of the vertical to a continent, one should think of the continent as the whole of the great mass which stands above the level of the ocean bottoms rather than the comparatively small mass which stands above sea level.

In this connection an examination of illustration No. 17 at the end of the volume and the comments upon it, on pages 63–65, will be instructive. This illustration shows the shape of that portion of the geoid which lies beneath the United States. Note that the highest part of the geoid corresponds approximately in horizontal position to the highest great land masses in the United States; that the contour lines of the geoid near the coast by their approach to parallelism with the coast and by steep slopes indicate that the oceans have a strong influence on the deflections of the vertical; and that, as a whole, the geoid surface for the United States as referred to the Clarke spheroid, tends to be convex upward.

For the reasons given in the preceding paragraphs, one should expect the C. & G. S. 1906 values of the dimensions of the earth to be systematically larger than values previously derived, and that is the observed fact. Moreover, if an approach to complete isostatic compensation exists, extending to a moderate depth, the C. & G. S. 1906 equatorial radius and polar semidiameter have been brought nearer to the truth in being made larger than formerly accepted values by properly taking into account the systematic portion of the deflections of the vertical.

SUMMARY OF CONCLUSIONS.

The principal conclusions are here summarized for the convenience of anyone who wishes to obtain a general view of the scope and of the results of this investigation. Page references are given for the convenience of those who desire to look up the basis of particular conclusions.

The most probable values for the dimensions of the earth which can be derived from the observed deflections of the vertical in the United States used in this investigation are:

Equatorial radius of the earth, meters	6 378 283 ± 34
Reciprocal of flattening	297.8 ± 0.9
Polar semidiameter, meters	6 356 868

The probable errors given above were computed directly from the residuals on the assumption that all the errors involved are accidental in character. As there is some evidence of systematic errors, these probable errors should be slightly increased to represent the actual uncertainties in the values (pp. 105, 106, 115, and 116).

From general studies of observed deflections of the vertical in the United States, corroborated by similar observations in other countries, it appears that there must be some general law of distribution of subsurface densities which fixes a relation between subsurface densities and surface elevations such as to bring about an incomplete balancing of deflections produced by the topography, on the one hand, against deflections produced by variations in subsurface

densities, on the other hand. The theory of isostasy postulates such a relation between subsurface densities and surface elevations. Therefore it is important to investigate the condition called isostasy in connection with any extensive investigation based on observed deflections of the vertical (pp. 65–66). In the condition called isostasy the excess of matter above sea level at each part of the earth's land surface is compensated by a defect of density below that part, and the surface defect of matter at each part of the earth's surface which lies below sea level is compensated by an excess of density below that part. For full definitions of "Isostasy," "Isostatic compensation," and "Depth of compensation" see pages 66–68.

For the United States and adjacent areas it is certain that the assumption that the condition called isostasy exists is a much closer approximation to the truth than the assumption that it does not exist (pp. 114, 115, 139).

The best working hypothesis as to the relation of the isostatic compensation to depths below the surface is that the isostatic compensation is uniformly distributed with respect to depth from the surface to the limiting depth of compensation. This hypothesis lends itself very readily to computation. It seems to the writer to be the most probable simple assumption. Other plausible assumptions which are not mutually exclusive differ in different directions from it in such a manner that it may be considered to be a mean of several such assumptions (pp. 147–149).

For the United States and adjacent areas, if the isostatic compensation is uniformly distributed with respect to depth, the most probable value of the limiting depth is 70 miles (113 kilometers), and it is practically certain that the limiting depth is not less than 50 miles (80 kilometers) nor more than 100 miles (150 kilometers) (p. 146).

The observations of deflections of the vertical now available in the United States are not competent to determine the distribution of the isostatic compensation with respect to depth (pp. 153, 159, 162).

For the United States and adjacent areas, if the isostatic compensation is greatest at the surface and decreases uniformly with respect to depth until it becomes zero at the limiting depth, the most probable value of the limiting depth is 109 miles (175 kilometers) (p. 159).

For the United States and adjacent areas, if the isostatic compensation is uniformly distributed through a stratum 10 miles (16 kilometers) thick, the most probable depth for the bottom of this stratum is 37 miles (60 kilometers) (p. 153).

For the United States and adjacent areas, if the isostatic compensation is distributed with respect to depth according to the law postulated by Prof. T. C. Chamberlin, the most probable value of the limiting depth is 179 miles (287 kilometers) (p. 162). The Chamberlin distribution of compensation within this limiting depth is illustrated by numerical values on pages 162, 163.

The assumption, which, from certain geographic and geologic evidence appears reasonable, that isostatic compensation exists under sea areas, but not under land areas, is far from the truth (pp. 168, 169).

The isostatic compensation existing under land areas is not merely a compensation of a continent as a whole. It is a compensation of the separate large topographic features of the continent (p. 169).

There are some indications in the evidence that the depth of compensation is greater in the eastern and central portions of the United States than in the western portion. The indications are not sufficient, however, to prove that there is any real variation in depth of compensation (pp. 142, 143).

The hypothesis that the outer portion of the earth is a solid crust a few miles thick floating on a liquid substratum of greater density is not true for the United States (pp. 163, 164).

For the United States and adjacent areas it is a safe conclusion that the isostatic compensation is now so nearly complete, on an average, that the deflections of the vertical are thereby reduced to less than one-tenth of the mean values which they would have if no isostatic compensation existed. In this sense one may properly characterize the isostatic compensation as departing, on an average, less than one-tenth from completeness or perfection. It is probable

that under some areas there is overcompensation, as well as undercompensation in others (pp. 165, 166).

The present close approach to perfect isostatic compensation in the United States and adjacent areas is not due to an initial arrangement of densities giving such a close approximation at the time of the beginning of the geologic record. The present close approach to perfect isostatic compensation in the United States and adjacent areas is a proof that the outer portion of this part of the earth has a small effective rigidity. The departures from perfect isostatic compensation are a measure of the effective rigidity of the material involved in isostatic readjustment (pp. 166–168).

The two paragraphs which immediately precede this are combined in the statement in the following two sentences: For the United States and adjacent areas the assumption of extreme rigidity is far from the truth. The United States is not maintained in its position above sea level by the rigidity of the earth but is, in the main, buoyed up, floated, because it is composed of material of deficient density.

The writer estimates that the stress-differences in the material in and about the United States have been so reduced by isostatic compensation that they are less than one-twentieth as great as they would be if the continent were maintained in its elevated position and the ocean floor maintained in its depressed position by the rigidity of the earth (p. 166).

This investigation of the figure and size of the earth and of isostasy would be appreciably strengthened by the introduction into the triangulation of more Laplace points (points at which there are coincident determinations of astronomic longitude and astronomic azimuth) (pp. 101, 122).

The area method of dealing with observed deflections of the vertical is superior to the arc method, both in furnishing a higher degree of accuracy in the determinations of the figure and size of the earth, and in being more effective in developing the facts as to the distribution of subsurface densities (pp. 169–171).

The introduction into the computation of the assumption of isostasy in a definite and reasonable form has reduced the sum of the squares of the residuals from 13 922 to 8 013, and has increased the weight of the determinations of the equatorial radius, flattening and polar semidiameter from 1.0 to 1.7 (p. 171).

The writer believes that a similar increase in accuracy may be made by a similar recognition of isostasy in other parts of the world, in connection with computations of the figure and size of the earth from the great mass of geodetic observations which have already been made (p. 171).

The size of the earth as computed from observed deflections of the vertical without taking isostasy into account should, in general, be smaller than the truth (pp. 173, 174).

A comparison of the value of the flattening of the earth, as derived from this investigation based on observed deflections of the vertical in a single country, with the value derived from observations of a different kind (of the intensity of gravity), scattered over the whole earth, indicates that the method pursued in this investigation is very effective in confining both the accidental and the systematic errors of the final results within narrow limits. A similar comparison of the equatorial radius, flattening and polar semidiameter, as derived from this investigation, with values based upon the general consensus of all evidence available from geodetic observations, furnishes a confirmation of this indication. In other words, the external evidence cited confirms the internal evidence of this investigation that the methods used in it are very effective in eliminating the effects of systematic as well as accidental errors (pp. 171–173).

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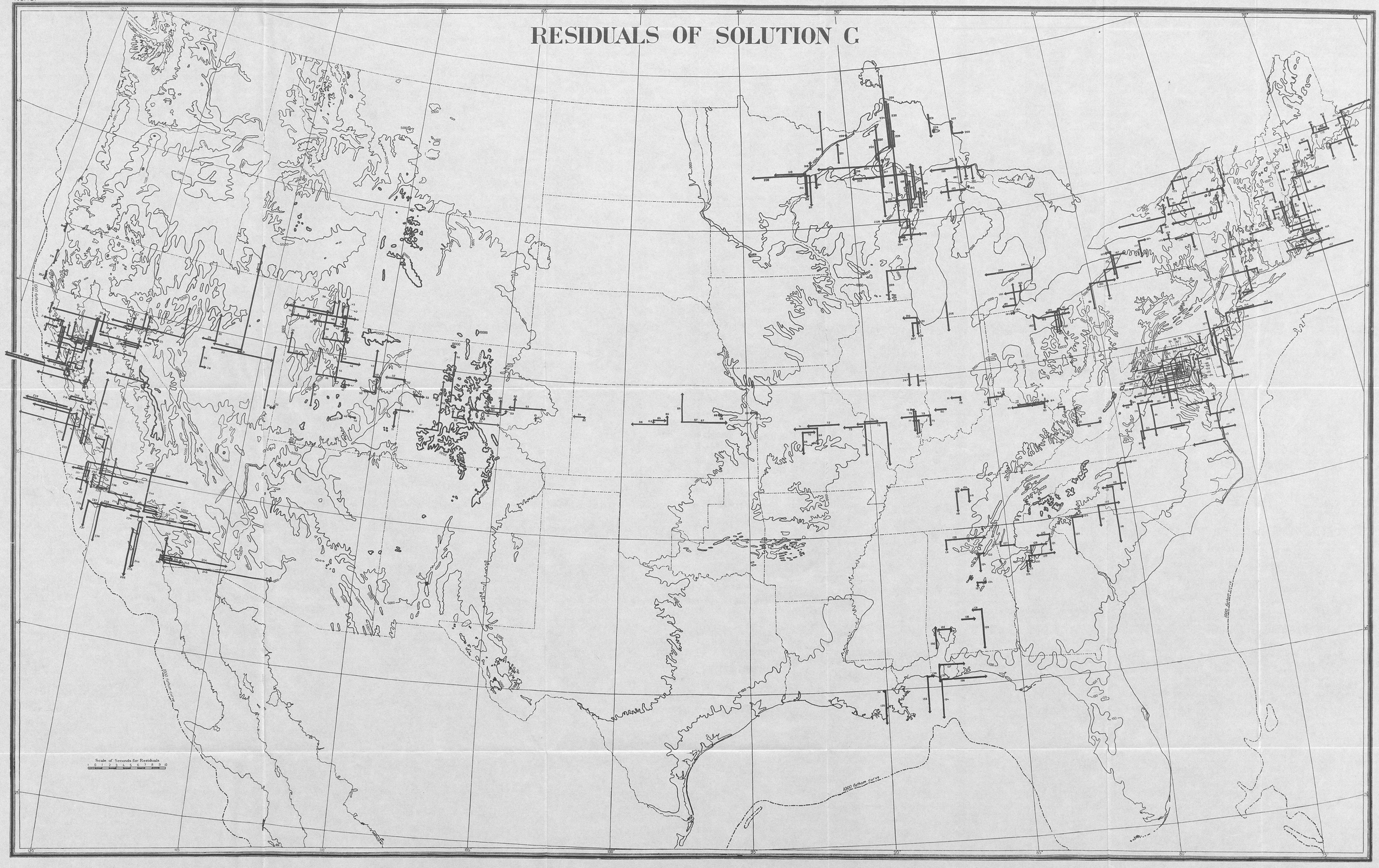
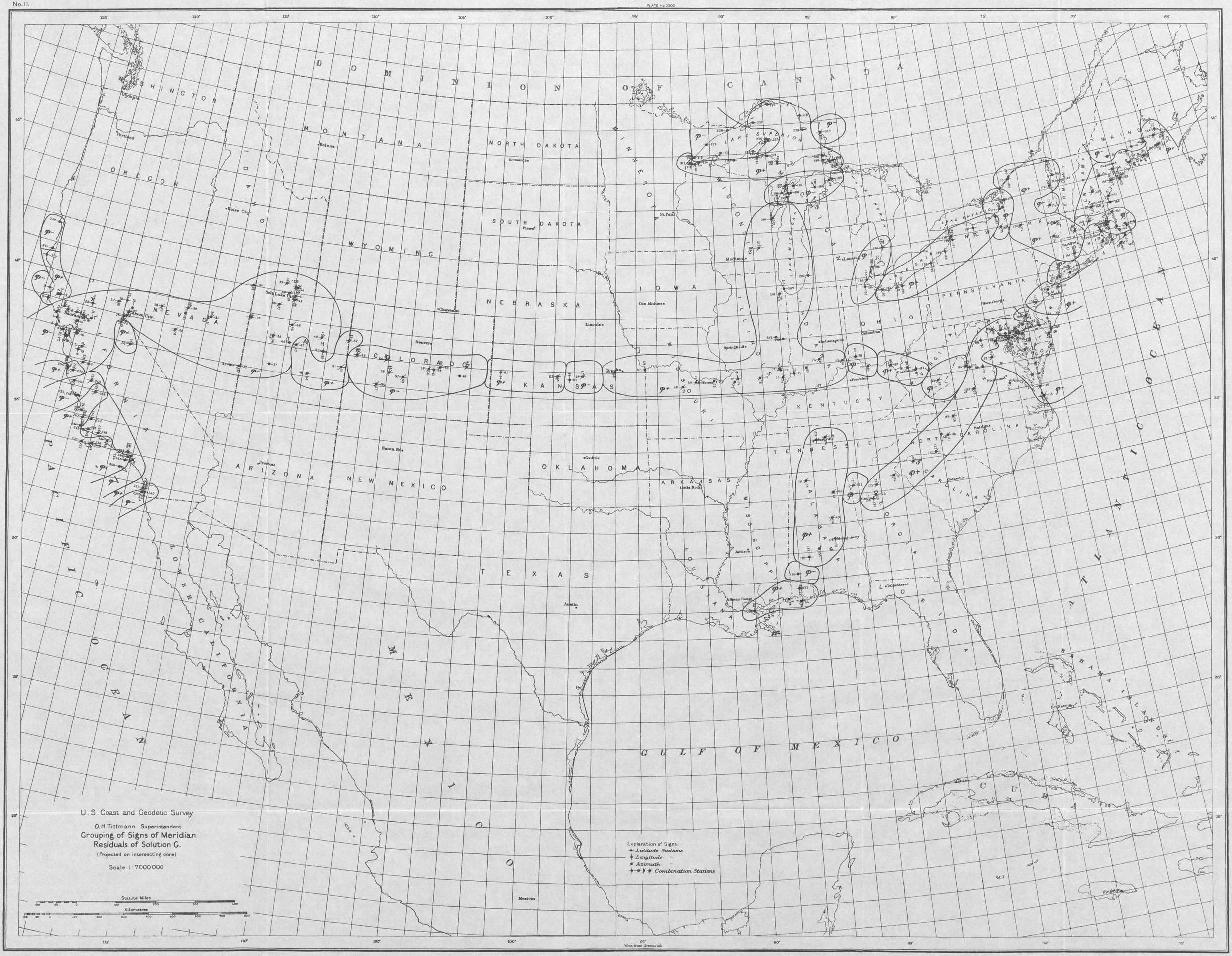
No. 10.

RESIDUALS OF SOLUTION G

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O. H. Tittmann Superintendent
Grouping of Signs of Meridian Residuals of Solution G.
(Projected on intersecting cone)
Scale 1:7000000
Statute Miles
Kilometres
Explanation of Signs:
Latitude Stations
Longitude "
Azimuth "
Combination Stations
DOMINION OF CANADA
WASHINGTON
OREGON
IDAHO
MONTANA
WYOMING
NORTH DAKOTA
SOUTH DAKOTA
NEBRASKA
NEVADA
UTAH
COLORADO
KANSAS
OKLAHOMA
TEXAS
ARIZONA
NEW MEXICO
CALIFORNIA
LOWER CALIFORNIA
MINNESOTA
IOWA
MISSOURI
ARKANSAS
LOUISIANA
MISSISSIPPI
ALABAMA
GEORGIA
FLORIDA
TENNESSEE
KENTUCKY
OHIO
INDIANA
ILLINOIS
WISCONSIN
MICHIGAN
PENNSYLVANIA
NEW YORK
VERMONT
NEW HAMPSHIRE
MAINE
VIRGINIA
NORTH CAROLINA
SOUTH CAROLINA
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO
GULF OF MEXICO
PACIFIC OCEAN
ATLANTIC OCEAN
CUBA
BAHAMA ISLANDS
Olympia
Portland
Helena
Boise City
Bismarck
Pierre
St. Paul
Madison
Lansing
Salt Lake City
Sacramento
Carson City
Cheyenne
Denver
Lincoln
Des Moines
Springfield
Topeka
Jefferson City
Indianapolis
Columbus
Frankfort
Harrisburg
Augusta
Montpelier
Concord
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Albany
Richmond
Raleigh
Columbia
Tallahassee
Montgomery
Atlanta
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Little Rock
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Mexico
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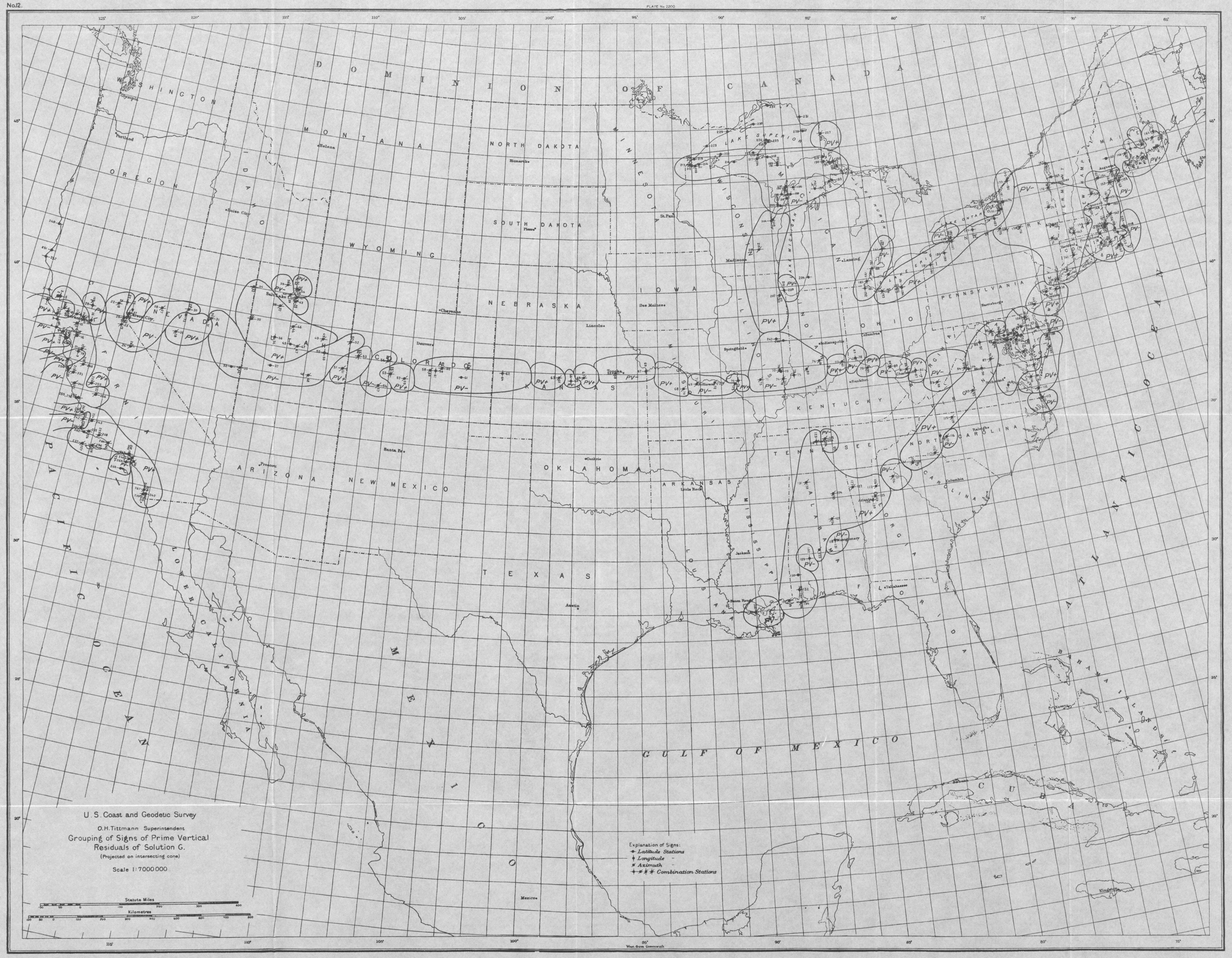
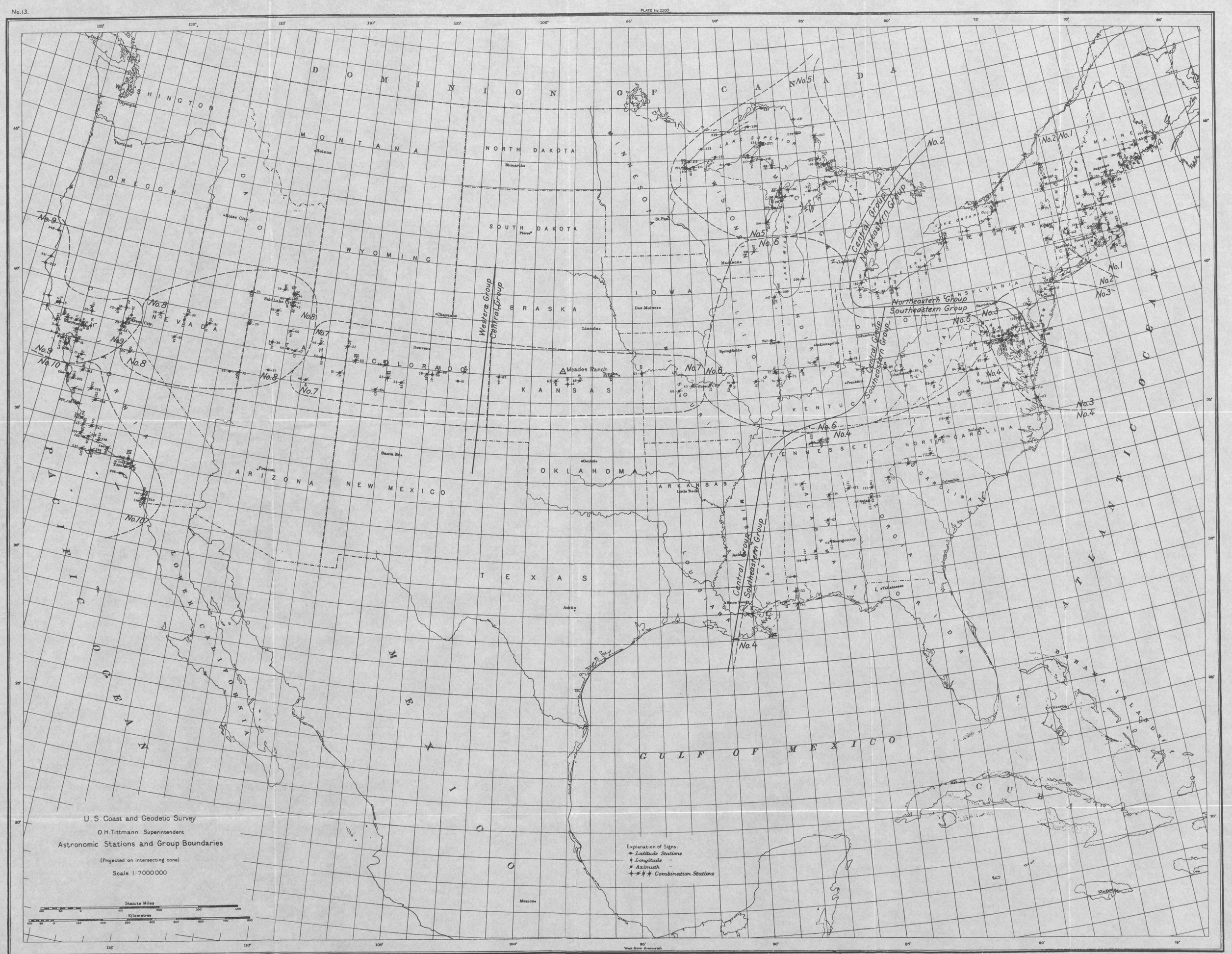
Kingston

No.12.

PLATE No 2200




U. S. Coast and Geodetic Survey
O. H. Tittmann Superintendent
Astronomic Stations and Group Boundaries
(Projected on intersecting cone)
Scale 1:7000000
Statute Miles
Kilometres
Explanation of Signs:
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Longitude "
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DOMINION OF CANADA
WASHINGTON
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NORTH DAKOTA
SOUTH DAKOTA
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KANSAS
NEVADA
UTAH
ARIZONA
NEW MEXICO
OKLAHOMA
TEXAS
IOWA
MINNESOTA
WISCONSIN
MICHIGAN
ILLINOIS
INDIANA
OHIO
KENTUCKY
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ARKANSAS
LOUISIANA
MISSISSIPPI
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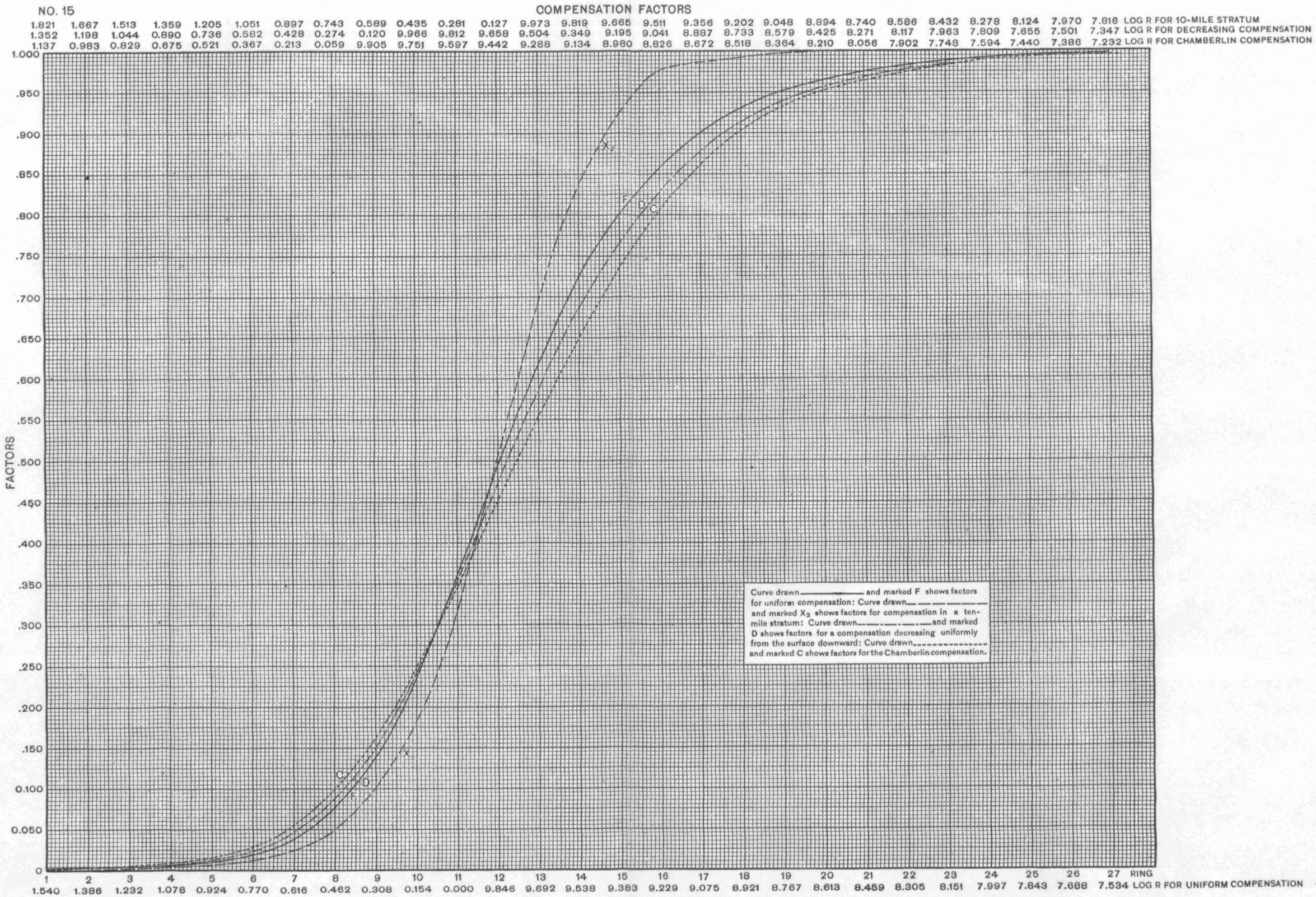
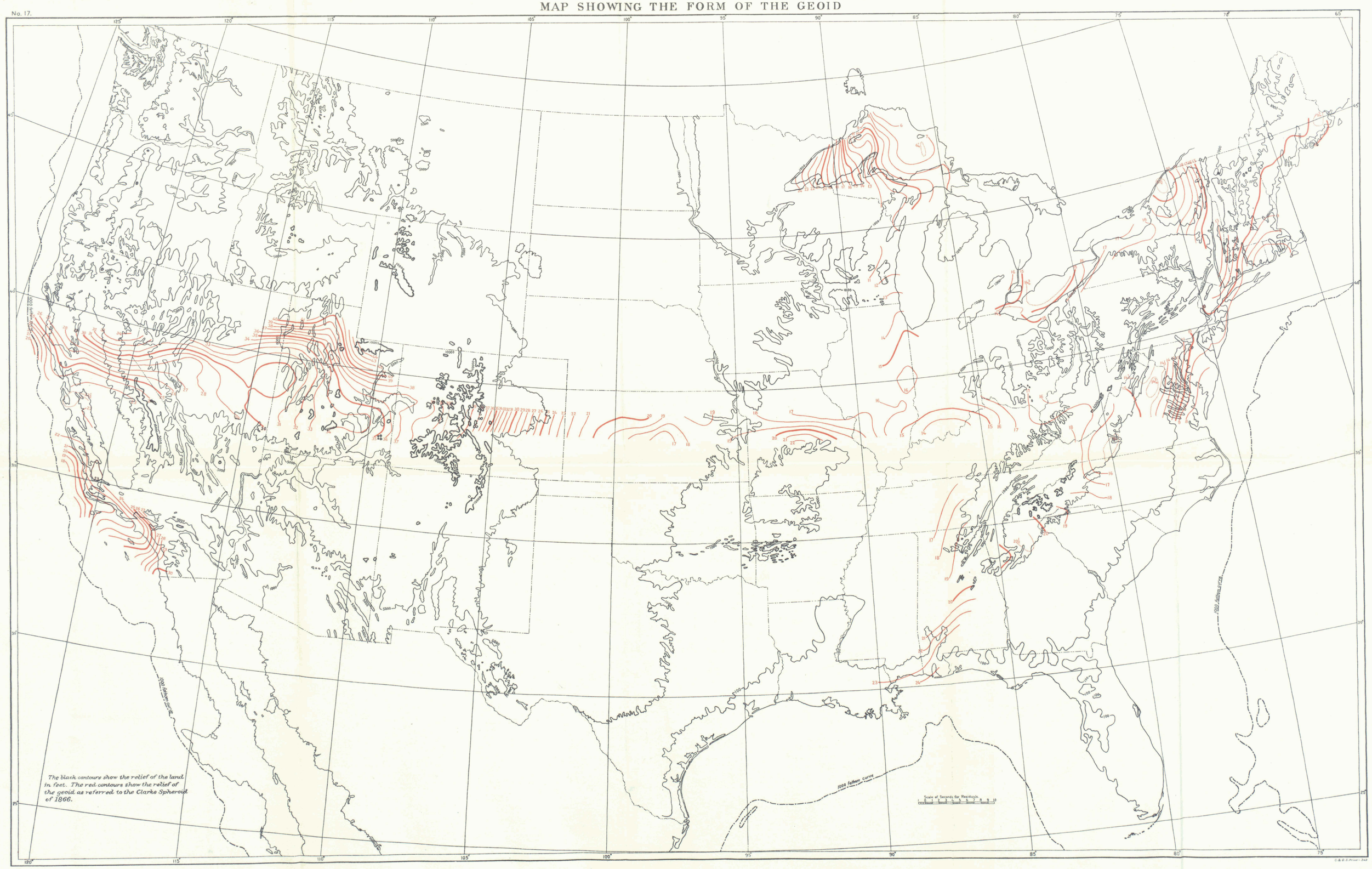
NORTH CAROLINA
VIRGINIA
PENNSYLVANIA
NEW YORK
VERMONT
NEW HAMPSHIRE
MAINE
CONNECTICUT
LOWER CALIFORNIA
MEXICO
CUBA
BAHAMA ISLANDS
GULF OF MEXICO
PACIFIC OCEAN
ATLANTIC OCEAN
LAKE SUPERIOR
LAKE MICHIGAN
LAKE ONTARIO
Olympia
Portland
Helena
Bismarck
Boise City
Pierre
St. Paul
Madison
Cheyenne
Salt Lake City
Denver
Lincoln
Des Moines
Springfield
Indianapolis
Columbus
Harrisburg
Frankfort
Richmond
Raleigh
Columbia
Atlanta
Montgomery
Tallahassee
Baton Rouge
Jackson
Little Rock
Oklahoma
Santa Fe
Prescott
Austin
Mexico
Meades Ranch
Kingston
Nassau
Western Group
Central Group
Northeastern Group
Southeastern Group
No. 1
No. 2
No. 3
No. 4
No. 5
No. 6
No. 7
No. 8
No. 9
No. 10


NO. 15
COMPENSATION FACTORS
1.821 1.667 1.513 1.359 1.205 1.051 0.897 0.743 0.589 0.435 0.281 0.127 9.973 9.819 9.665 9.511 9.356 9.202 9.048 8.894 8.740 8.586 8.432 8.278 8.124 7.970 7.816 LOG R FOR 10-MILE STRATUM
1.352 1.198 1.044 0.890 0.736 0.582 0.428 0.274 0.120 9.966 9.812 9.658 9.504 9.349 9.195 9.041 8.887 8.733 8.579 8.425 8.271 8.117 7.963 7.809 7.655 7.501 7.347 LOG R FOR DECREASING COMPENSATION
1.137 0.983 0.829 0.675 0.521 0.367 0.213 0.059 9.905 9.751 9.597 9.442 9.288 9.134 8.980 8.826 8.672 8.518 8.364 8.210 8.056 7.902 7.748 7.594 7.440 7.386 7.232 LOG R FOR CHAMBERLIN COMPENSATION
FACTORS
1.000 .950 .900 .850 .800 .750 .700 .650 .600 .550 .500 .450 .400 .350 .300 .250 .200 .150 0.100 0.050 0
X2
F
D
C
Curve drawn ———— and marked F shows factors for uniform compensation: Curve drawn — — — — and marked X2 shows factors for compensation in a ten-mile stratum: Curve drawn — · — · — and marked D shows factors for a compensation decreasing uniformly from the surface downward: Curve drawn - - - - - and marked C shows factors for the Chamberlin compensation.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 RING
1.540 1.386 1.232 1.078 0.924 0.770 0.616 0.462 0.308 0.154 0.000 9.846 9.692 9.538 9.383 9.229 9.075 8.921 8.767 8.613 8.459 8.305 8.151 7.997 7.843 7.688 7.534 LOG R FOR UNIFORM COMPENSATION

No. 17.

MAP SHOWING THE FORM OF THE GEOID



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