CONVERTING GPS HEIGHT INTO NAVD88 ELEVATION WITH THE GEOID96 GEOID HEIGHT MODEL

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ABSTRACT

The Global Positioning System (GPS) is commonly considered a three-dimensional system. But, the heights obtained from GPS are typically heights above an ellipsoidal model of the Earth. These GPS ellipsoidal heights are not consistent with leveled heights above mean sea level, often known as orthometric height. The conversion from ellipsoid to orthometric height requires a geoid height model. Geoid heights in the conterminous United States range from about -8 meters to -53 meters, and display considerable variation in the mountains. Through the use of careful GPS survey procedures coupled with high-resolution geoid models, surveyors have obtained orthometric heights with an accuracy commensurate with that of leveling.

In mid-October the National Geodetic Survey will release the GEOID96 geoid height model. It is computed with nearly 1.8 million terrestrial, ship, and altimetry gravity data, and is modeled on a 2' x 2' grid (about 3 km resolution). GEOID96 is fundamentally different from the earlier GEOID90 and GEOID93 models, in that over 2700 GPS heights on leveled benchmarks are used as a supplemental data set. These GPS benchmark data capture datum definition issues inherent in NAD 83 ellipsoid heights and NAVD 88 orthometric heights, and also provide control of long wavelength geoid model error. While the GEOID96 model is biased relative to a geocentric ellipsoid, this bias is deliberate; and, it enables one to directly convert between NAD 83 GPS and the NAVD 88 height systems. Tests of preliminary models show this conversion can be done to better than 3 cm (one-sigma), and that random error in GPS height is of greater concern.

INTRODUCTION

The impact of the Global Positioning System (GPS) is undeniable. In the span of just a few years GPS has become the leading positioning technology. This revolution has not been confined to the surveying community, but has extended into mapping, navigation, and Geographic Information System (GIS) arenas. We are witnessing widespread adoption of GPS with an equivalently widespread range of accuracy requirements. Many of these applications require accurate vertical positioning. And, a common requirement is the transformation of GPS heights into heights above mean sea level. In this paper we review height relationships and vertical datums, and show how the GEOID96 model was developed to convert between height systems.

HEIGHT RELATIONSHIPS

The procedure of geodetic leveling provides a height that is commonly known as a height above mean sea level. In this process, level differences, which express the alignment of a level bubble, are combined with surface gravity readings to obtain final values. Formally, heights from geodetic leveling are known as orthometric heights, and are denoted H (Figure 1). It should come as no surprise that orthometric heights will reflect local variations in gravity as well as changes in topography. The reference surface for orthometric heights is a level surface of the Earth that is closely associated with mean sea level on a global basis. This particular level surface is called the geoid. Orthometric heights are the vertical distance from the geoid to the surface of the Earth. And, by convention, the sign is considered positive as one moves radially outward.

GPS, on the other hand, produces a much different kind of height. Whether one uses point, differential, or differenced carrier phase positioning, one obtains a set of XYZ coordinates that depend upon locations of base stations and satellite positions. These three-dimensional, geometric relationships, in contrast to geodetic leveling, do not depend on local gravity variations. Since XYZ coordinates do not directly express the notion of height, it is necessary to transform them into a different coordinate system. Typically, XYZ are transformed into geodetic latitude, longitude and ellipsoidal height. This transformation is performed using a very simple, two-parameter, model of the Earth: the ellipsoid. The ellipsoid heights obtained from the transformation are denoted h.

These height relationships are portrayed in Figure 1. The difference between GPS ellipsoid heights, *h*, and leveled orthometric heights, *H*, is called geoid height, *N*. As seen in the figure, the geoid height is the vertical distance from the ellipsoid to the geoid level surface. These heights obey a simple equation

$$h = H + N \quad .$$

Figure 1 illustrates the relative alignment of the surfaces in the conterminous United States. Here, the geoid surface is beneath the ellipsoid. Thus, geoid heights are negative, and the ellipsoidal height is smaller in magnitude than the orthometric height at a given point. As an example, if one is near the seashore, orthometric height may only be +1 meter, while the ellipsoid height may be -30 meters. In this case, the ellipsoid surface would be overhead.

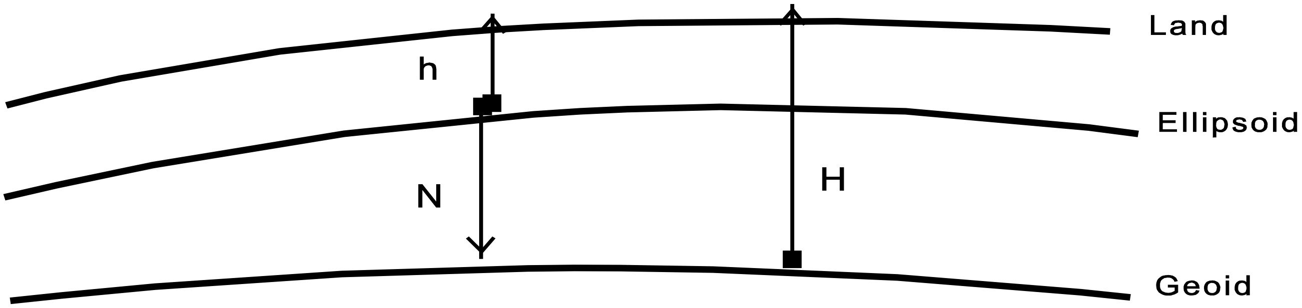

Figure 1. -- Types of Heights

As implied in the last paragraph, the differences between ellipsoidal height, *h*, and orthometric height, *H*, are sizable. These differences, *h-H*, are the geoid height. Globally, geoid heights can range from about +75 to -100 meters. In the conterminous United States, geoid heights vary from -8 to -53 meters. In addition to this considerable magnitude, the variation is extremely complex. This complexity relates to topographic relief as well as subsurface rock density variations. Because of this complexity, high resolution geoid models, and associated interpolation software, have been developed to support GPS height conversion.

It is a straightforward procedure to algebraically subtract an interpolated geoid height, *N*, from a GPS ellipsoidal height, *h*, to obtain an orthometric height, *H*:

$$H = h - N \quad .$$

However, when this is done on a point by point basis, the results often display a systematic offset in *H* for a given region (Milbert 1991). The cause of this offset is predominantly due to datum definition.

DATUMS AND REFERENCE SYSTEMS

The simple expressions shown above are built on an underlying assumption. That is, it is assumed that the quantities are referred to a common reference system. However, it is necessary to *realize* a reference system through measurements and definitions. Often, this realization of a system is referred to as a datum. Height datums can be inconsistent due to measurement error or definitional issues. These effects generally operate as a near-constant bias in a given area. At different levels of accuracy, height bias, or datum ambiguity, is evident in all three height components: ellipsoidal, orthometric, and geoid. The situation is best expressed by the equation:

$$h + \delta h = (H + \delta H) + (N + \delta N)$$

where the new terms indicate height bias in each component. Let us consider each of the three height systems.

Ellipsoidal Height Bias

The principal cause of systematic bias in ellipsoidal heights is the non-geocentric realization of a reference system. In particular, it is known that the NAD 83 (86) reference system is non-geocentric. While being primarily a horizontal, classical network, the NAD 83 (86) was controlled by VLBI and Doppler data sets, and this subset can be considered three-dimensional. Dr. Richard Snay, National Geodetic Survey, has computed a seven parameter Helmert transformation from NAD 83 (86) to ITRF94(1996.0) with 8 points common to both reference systems. The results are summarized in Table 1. The RMS of fit was 13 millimeters (mm).

Table 1 -- Transformation from NAD 83 (86) to ITRF94(1996.0)

ΔX	-0.9738	± 0.0261	m
ΔY	+1.9453	± 0.0215	m
ΔZ	+0.5486	± 0.0221	m
ω_X	-0.02755	± 0.00087	arc sec
ω_Y	-0.01005	± 0.00081	arc sec
ω_Z	-0.01136	± 0.00066	arc sec
scale	-0.00778	± 0.00264	ppm

Figure 2 portrays the datum differences between NAD 83 (86) and ITRF94(1996.0) ellipsoidal heights referred to the GRS80 ellipsoid. It is seen that the non-geocentricity of the NAD 83 (86) reference frame induces a smooth, systematic difference in ellipsoidal heights. The values in the U.S. range from -0.28 to -1.64 m, and have an average tilt of about 0.3 ppm.

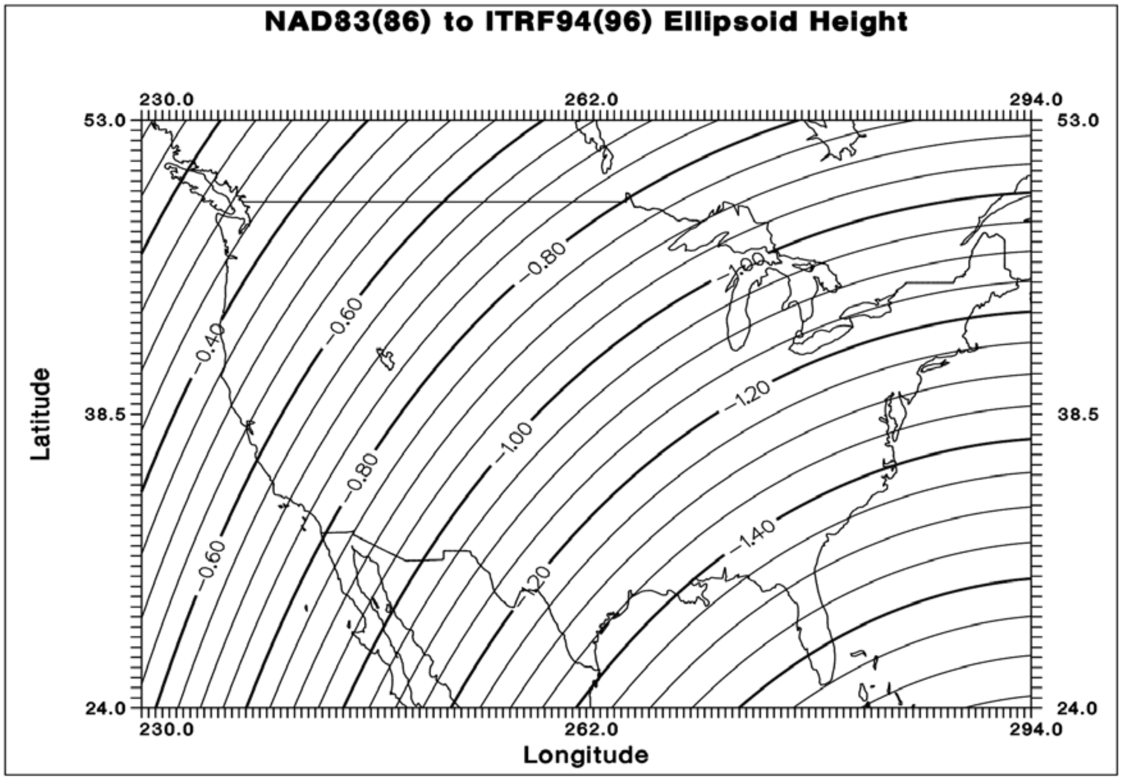

Figure 2. -- Ellipsoid height differences between NAD 83 (86) and ITRF94(1996)

Orthometric Height Bias

The NAVD 88 datum was realized by a single datum point, Father Point/Rimouski, in Quebec, Canada. A number of factors entered into selection of the datum and its adopted value. But, the primary requirement was to minimize the recompilation of national mapping products (Zilkoski et al. 1992). There was no guarantee that the NAVD 88 datum corresponds to the theoretical level surface defined by the GRS80 definitions. A recent study by Rapp (1996) compares ITRF93 GPS positions and a global geopotential model against the NAVD 88 vertical datum. Rapp found a mean offset for the NAVD88 datum of -27 cm when computed with a set of 397 GPS points. The sense of the offset is portrayed in Figure 3.

Figure 3. -- NAVD 88 offset

Geoid Height Bias

Modern high resolution geoid height models are invariably computed by a remove/restore technique (Schwarz et al., 1990). In this process, a global geopotential model is evaluated to obtain base geoid and gravity anomaly grids. Measured gravity data are reduced and the base gravity anomalies are subtracted in the removal stage. Some type of Fast Fourier Transform (FFT) technique is used to evaluate Stokes integral, and the residual geoid quantities are added back to the base geoid grid in the restore stage. However, due to its problematic nature, most practitioners do not perform trend removal prior to invocation of the FFT. While such trends are small, their spectrum will be folded, or aliased, across the recoverable frequency band (Hamming 1989, pp. 224-227). This effect is expected to be of long wavelength, and certainly deserves future study by the geodetic community.

THE GEOID96 AND G96SSS GEOID MODELS

Because of the increasing use of GPS in the United States for both horizontal and vertical positioning, the National Geodetic Survey (NGS) must support the surveying, mapping, navigational, and GIS communities in obtaining heights referred to the NAVD 88 datum using GPS technology. Studies by Milbert (1995) have found that it is possible to relate a given high-resolution gravimetric geoid model to the reference system of GPS ellipsoidal heights and to the vertical datum of an orthometric height system by means of an empirical conversion surface. Such conversion values, *s*, contain the datum definition and realization problems discussed above. Formally,

$$h = H + N + (\delta H + \delta N - \delta h) = H + N + s$$

It is possible to absorb the datum biases contained in the conversion surface, s, into a gravimetric geoid model, N. If this is done, then a new geoid model, denoted N_{96}, is obtained.

$$N_{96} = N + s$$

The advantage of such a geoid model is that it will support direct conversion between an ellipsoidal reference system and orthometric vertical datum, even if they are not defined on a common reference. This procedure was followed in GEOID96. If subscripts are used to denote the datums associated with the height systems, then we may write:

$$h_{83} = H_{88} + N_{96}$$

The equation shown above is the conversion we supply to our users. The major component of the datum definition error is the non-geocentricity of NAD 83, illustrated in Figure 2. In essence, GEOID96 has been defined relative to a non-geocentric ellipsoid. Alternatively, it is correct to state that GEOID96 is biased with respect to a geocentric ellipsoid. However, this bias is deliberate. GEOID96 supports the direct conversion between NAD 83 GPS heights and NAVD 88 orthometric heights.

The technique for obtaining the gravimetric geoid model, G96SSS, is to use gravity data to compute high frequency corrections to an existing model of the Earth's geoid heights. This base model was obtained by evaluating the new, EGM96, global geopotential model computed by the joint NASA Goddard Space Flight Center and DoD Defense Mapping Agency project (Rapp and Nerem 1994). By means of a one-dimensional, spherical FFT algorithm, we computed the high frequency geoid component from nearly 1.8 million point gravity data in the NGS data base. The high frequency geoid grid was combined with the EGM96 geoid heights, and a correction derived from digital terrain data, to yield the final gravimetric geoid heights. We are particularly grateful for the support of the Defense Mapping Agency in supplying certain gravity and digital terrain data.

The technique for obtaining the GEOID96 geoid model in the conterminous United States is to compute an empirical conversion surface that is added to the G96SSS model. We first compute the departures of the G96SSS geoid heights from the NAD 83 ellipsoid heights and NAVD 88 orthometric heights at nearly 3000 points. We remove a tilted plane trend, which captures the non-geocentricity of the NAD 83 (86) reference system, as well as vertical datum bias in NAVD 88. Then an empirical covariance function is fit to the covariance statistics of the detrended geoid residuals (Figure 4). The fit is made using a simple function of the form

$$C=C_0 e^{(-d^2/L^2)}$$

where d is the spherical distance between points; characteristic length, L = 400 km; and function variance $C_0 = (9.7)^2$ cm^2. Through collocation, predictions of the detrended residuals are made on a regular grid. In the prediction, GPS heights are assigned a random noise component of 5.8 cm. The trend is then restored to prediction grid, generating the final empirical conversion surface (See Figure 5 for an example). The similarity of the conversion surface and the non-geocentric effect in Figure 2 is evident.

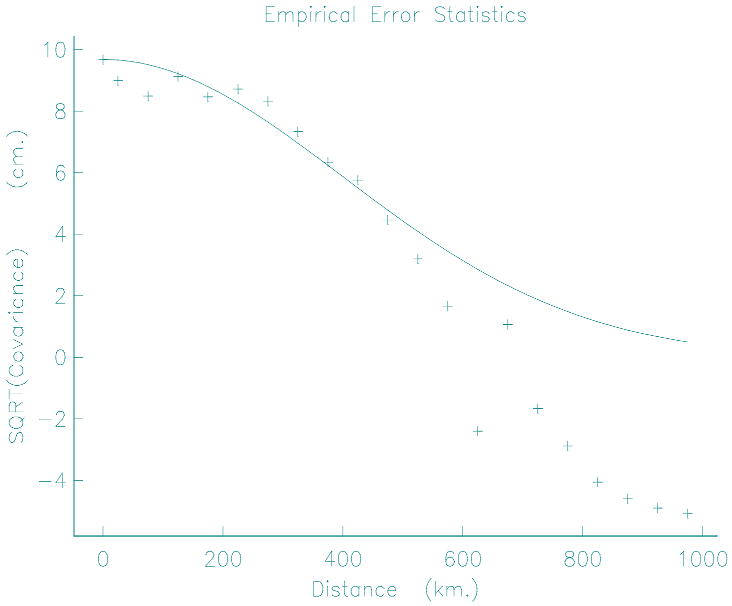

Figure 4. -- Sample empirical covariance function fit.

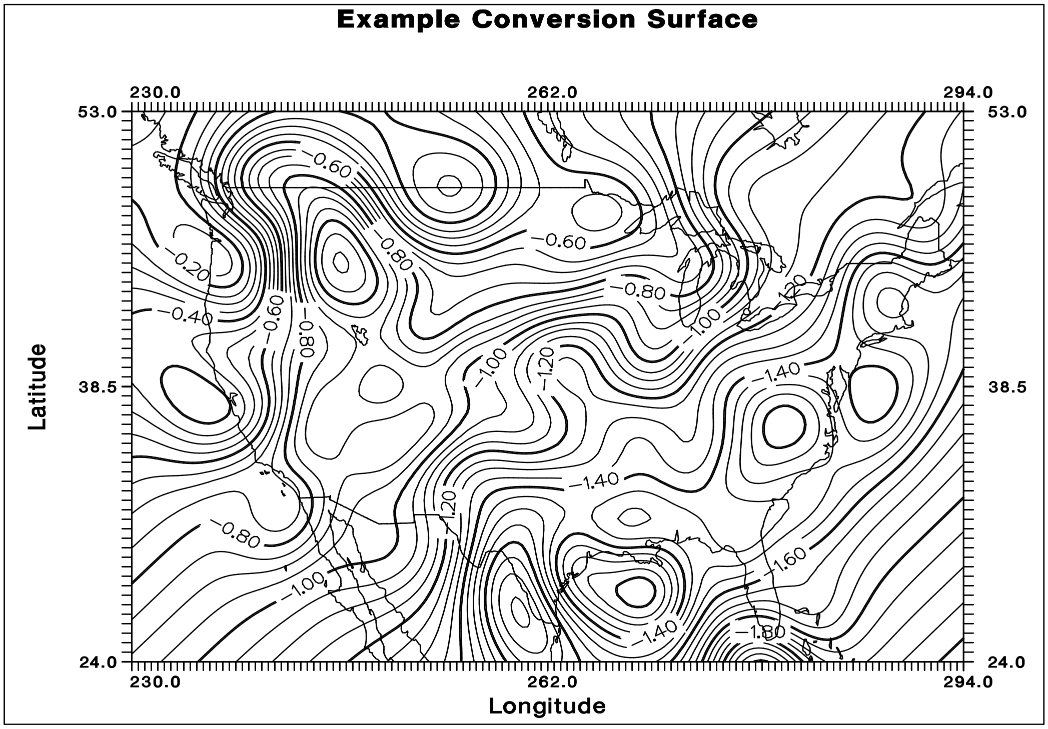

Figure 5. -- Example of a conversion surface between GEOID96 and G96SSS.

The smoothness of the conversion surface is remarkable. This is directly related to the long characteristic length, *L* = 400 km, that was obtained in the function fit. The conversion surface incorporates datum definition effects, as well as smoothly varying errors in ellipsoid, orthometric, and geoid height. A shaded relief image of a preliminary version of GEOID96 is shown in Figure 6.

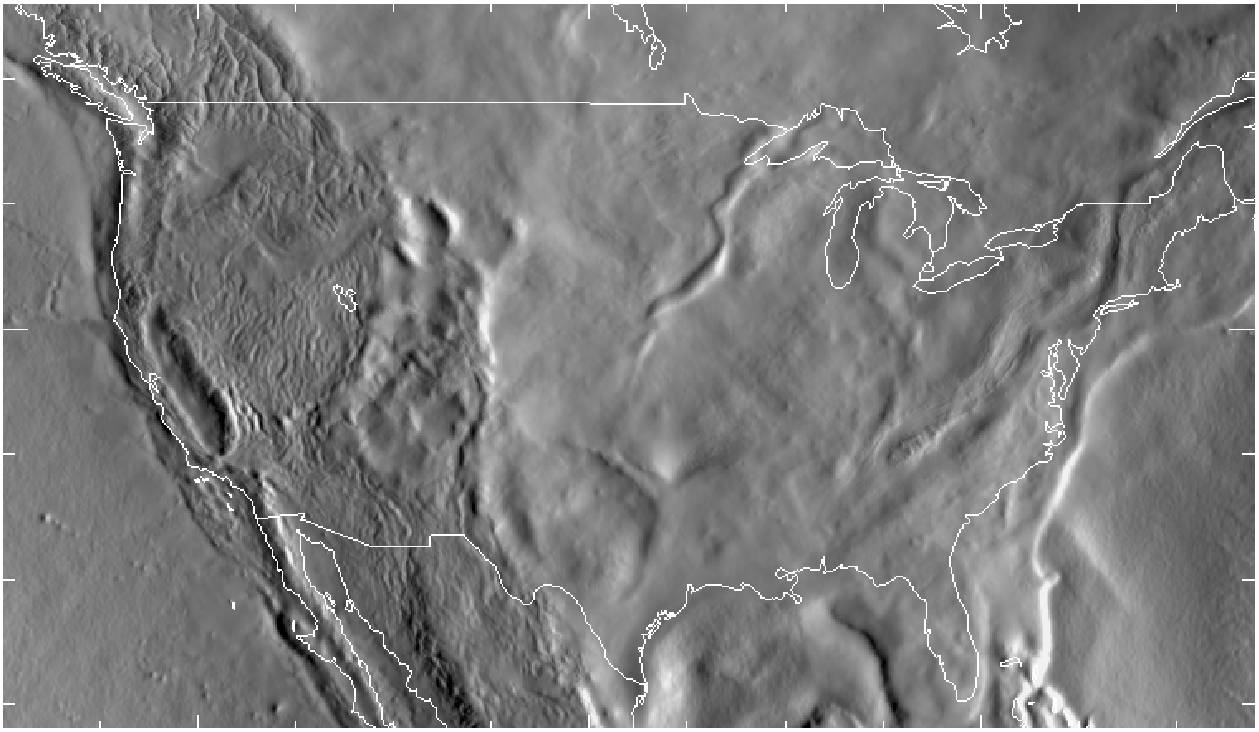

Figure 6. -- Preliminary version of GEOID96 geoid height model.

RESULTS WITH THE GEOID96 PROCEDURE

Our objective has been to develop a geoid model that directly converts between the inconsistent definitions of our height systems. To evaluate a preliminary GEOID96 model, the relationship

$$h_{83} = H_{88} + N_{96}$$

was tested at 2729 GPS points on leveled benchmarks. It was found that the RMS error was 5.8 cm, with no offsets or trends removed. Thus, the conversion process is seen to be quite successful.

Perhaps of even more interest, the covariance statistics for the residuals to the preliminary GEOID96 model were computed, and an empirical covariance function was then developed. These results are portrayed in Figure 7. Unlike Figure 4, which was plotted to a distance of 1000 km, Figure 7 is only plotted out to 100 km. At this much closer scale, a drop is seen in the statistics from 5.8 cm at d = 0, down to 2.6 cm at d = 5 km. This reduction is evidence of an uncorrelated (white-noise) process. The source of the 5.8 cm of uncorrelated error is random error in the GPS ellipsoidal heights. The sources of the 2.6 cm of correlated error are not obvious, but geoid error is certainly one component. Further discussion on the error sources can be found in Milbert (1995).

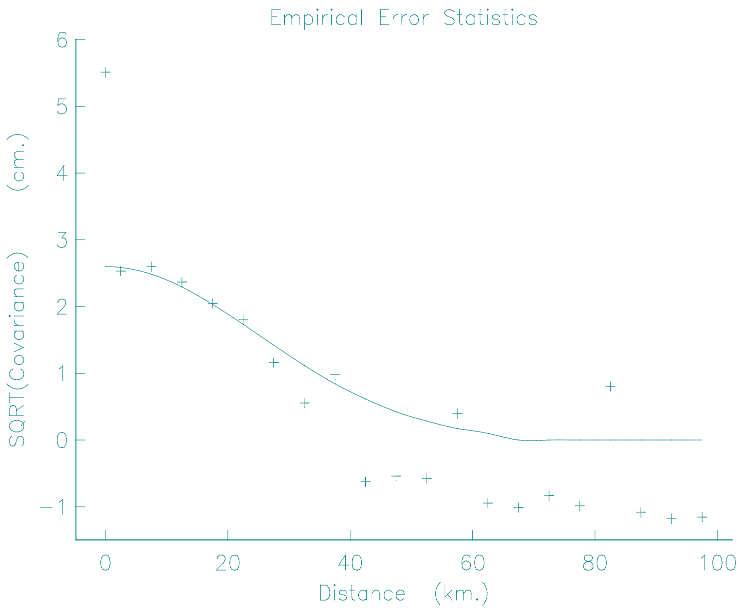

Figure 7. -- Empirical covariance function fit, preliminary GEOID96 model.

In closing this section, two points must be made. The GEOID96 model does not eliminate the need for leveled benchmark ties in geodetic surveys. However, one should obtain better results with adjustment procedures (as described in Milbert 1991) when benchmark distribution is not ideal. And, it has been found that particular care must be taken to get accurate GPS ellipsoidal heights. Zilkoski (1996), for example, gives guidelines that address numerous error sources; such as antenna phase center variation with elevation angle, tropospheric scaling parameters, and multipathing.

CONCLUSIONS

Gravimetric geoid models, such as GEOID93 or G96SSS, show systematic departures from NAD 83 GPS ellipsoidal heights at leveled benchmarks with NAVD 88 orthometric heights. These departures are dominated by datum definition and datum realization problems. It is possible to fit these departures into a very smooth conversion surface, and add this surface to a gravimetric geoid model. The GEOID96 geoid height model, which incorporates such a conversion surface, displays about

2.5 cm of accuracy (one sigma) between points spaced at 50 km or greater. GPS ellipsoidal height error of about 6 cm was observed after the computations.

AVAILABILITY

The geoid height and deflection of the vertical components have a target release date of mid-October 1996. They will be available on a single CD-ROM from the NGS Information Services Branch. GEOID96 is organized into 9 regions; 6 for the conterminous United States, and the remainder for Alaska, Hawaii, Puerto Rico, and the Virgin Islands. The GEOID96 model in the conterminous United States incorporates NAD 83 GPS heights on NAVD 88 benchmarks. For specialized studies, the G96SSS gravimetric geoid model is supplied in 6 regions covering the conterminous United States. The DEFLEC96 model, which is compatible with the GEOID96 model, is organized in 17 regions; spanning the conterminous United States, Alaska, Hawaii, Puerto Rico, and the Virgin Islands. In addition to the CD-ROM, users may obtain regional groupings individually on diskette. These models will also be available for anonymous FTP access at **ftp.ngs.noaa.gov**; on the World Wide Web at **http://www.ngs.noaa.gov**; and on the electronic bulletin board system -- telephone 301-713-4181 and 301-713-4182 (8-bit, no parity, 1 stop bit, 300 to 14400 baud). For more information, please contact:

Information Services Branch
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1315 East-West Highway, Station 9202
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Phone: 301-713-3242; Fax: 301-713-4172

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