Evaluation of the EGM96 Model of the Geopotential in the United States

(Addendum to: Evaluation of Preliminary Models of the Geopotential in the United States)

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Introduction

As a final report for the Special Working Group of the International Geoid Service, we have evaluated the geopotential model EGM96. The tests are all similar to those performed on models EGM-X01 through EGM-X05, and comparisons between the final model and the preliminary models are made. The EGM96 model served as the base model in the remove-compute-restore procedure in the calculation of G96SSS, the recently released high-resolution model of the geoid for the United States (and the foundation of the GEOID96 product).

It must be noted that some of the data sets used in this evaluation are not the final data sets used in the computation of G96SSS. These older data sets were used in this evaluation strictly to maintain compatibility with the previous beta test report (Smith and Milbert, 1996).

Use of EGM96 Coefficients in Computing Geoid Heights

Between the time of the beta report and the testing of EGM96, some clarifications were made regarding the computation of geoid undulations from a spherical harmonic model of the Earth's external gravitational potential. Specifically, the treatment of the degree zero component of the geoid undulations was investigated. It was pointed out that a correction for the difference between the normal potential of GRS-80 and the true potential of the geoid (as best we know it), should be applied to geoid undulation computations, when using GRS-80 as the chosen reference field. However, in an effort to maintain compatibility with results in the beta model report, we will continue to neglect this correction in this report. However, it has been implemented in the computation of GEOID96. The effect of this term amounts to a constant bias of 41 cm in the geoid undulation computations.

Low Resolution Geoid Computation and Evaluation

Using the methods described in the previous section, combined with the height anomaly to geoid undulation correction (the "up/down" procedure) discussed in the beta report, geoid undulations were computed from EGM96, incorporating a 3'x3' ellipsoid height DTED for the conterminous United States, and a 3'x3' grid of simple Bouguer anomalies. This grid of geoid undulations (N_{EGM96}) was compared to the geoid undulations implied by 2497 GPS ellipsoidal heights on leveled benchmarks (N_{BM}). A tilted plane was fit through the residuals (N_{EGM96} - N_{BM}) in the same way as the beta models, and the results of that fit are shown below.

Model	Offset (cm.)	Tilt (ppm)	RMS about plane (cm)	Azimuth (deg)
X01	-2.16	0.40	26.52	338
X02	1.02	0.32	29.77	336
X03	0.07	0.35	26.22	334
X04	0.43	0.35	25.99	335
X05	0.76	0.35	26.10	335
EGM96	**2.01**	**0.41**	**27.03**	**343**

Table 1. Tilted plane fits to 2497 low resolution geoid model residuals .

These statistics show EGM96 is something of an outlier, relative to the five beta models, without being unreasonably different. In order to help illustrate the continental scale of the tilt, the following figures have been provided. Figure 1 shows a scatter plot of geoid undulation residuals relative to orthometric height. It is encouraging to see that there is little dependence upon elevation, though a small correlation could possibly be inferred. Figure 2 shows the scatter plot of geoid undulation residuals relative to latitude, and Figure 3 shows them relative to longitude. The combined message of these two plots is that a northerly and westerly (i.e. azimuth of 343) upward tilt is clearly occurring across the entire United States, and is not just being driven by any cluster of localized residuals. Please keep in mind that these statistics do not reflect the final data sets and theory used in G96SSS, but rather are being kept internally consistent with the report on the beta models.

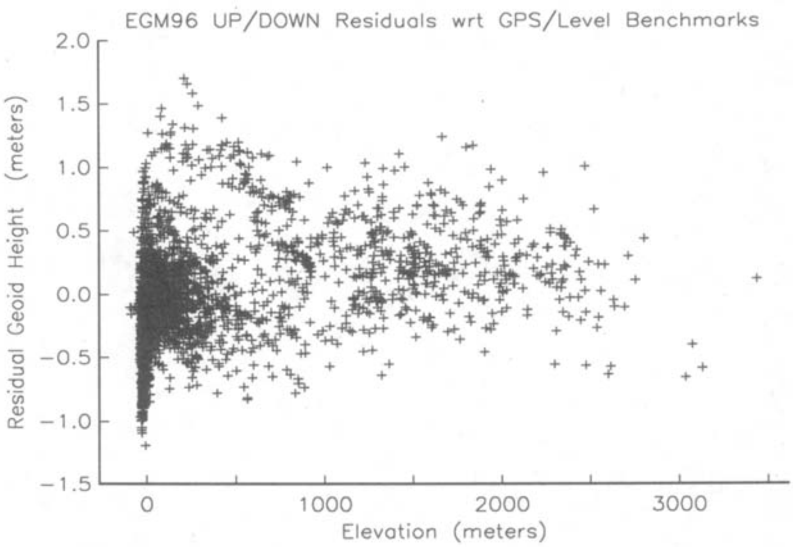

Figure 1: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to height

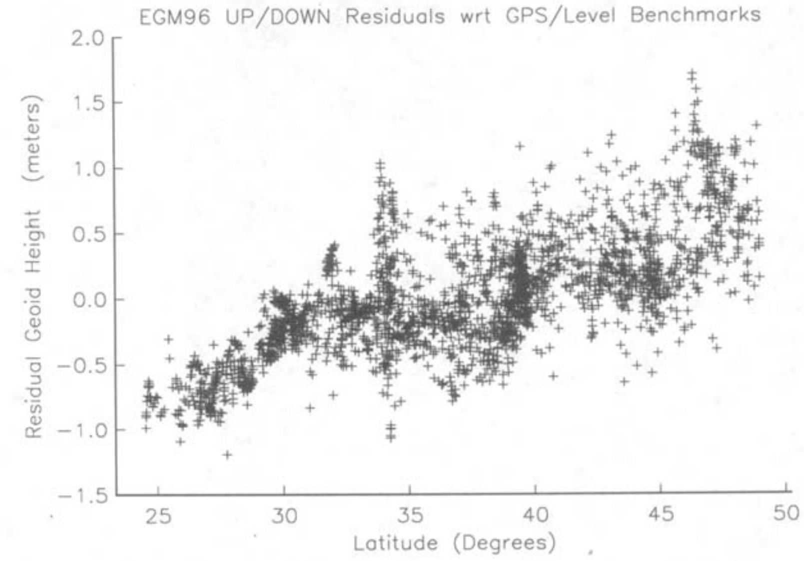

Figure 2: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to latitude

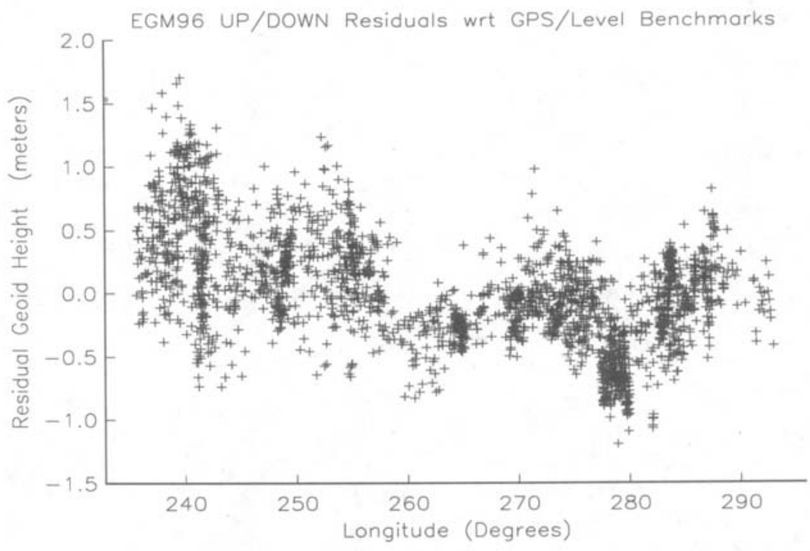

Figure 3: Residual geoid undulations, GPS/Benchmarks vs. EGM96, relative to longitude

Although it is important to maintain consistency with the Smith and Milbert, 1996 (for the rest of this paper, called the "beta report"), it is also quite useful to see how EGM96 compares to our final, best gravimetric geoid model, G96SSS (Milbert and Smith, 1996). As such, Figure 4 shows the difference between the geoid undulations of G96SSS and the undulations of EGM96, corrected for the height anomaly to geoid undulation (Rapp, 1996). The color scale in Figure 4 ranges from -150 cm (magenta) to +150 cm (red), though the data actually range from -186 cm to +334 cm. In this figure only, we use the more recent gravity data sets and theory to show our final version of potential "commission error" which is inherent in EGM96. This figure is not completely consistent with Figure 6 of the beta report, but is very useful in showing the areas where surface data and EGM96 differ from one another.

The behavior of the geoid in the Pacific Northwest is of special interest due to its rugged terrain and difficulty in matching GPS benchmarks with the geoid in the area. As per the beta model test, a study of the EGM96 model in the area of 38°-49° N and 230°-255° E was made. In that area, 505 GPS benchmarks are found. To avoid redundancy with the beta model report, only the EGM-X05 results are repeated below, along with the EGM96 results. The statistics are ordered by ascending elevation groups. Additionally, first, second and third differences are presented.

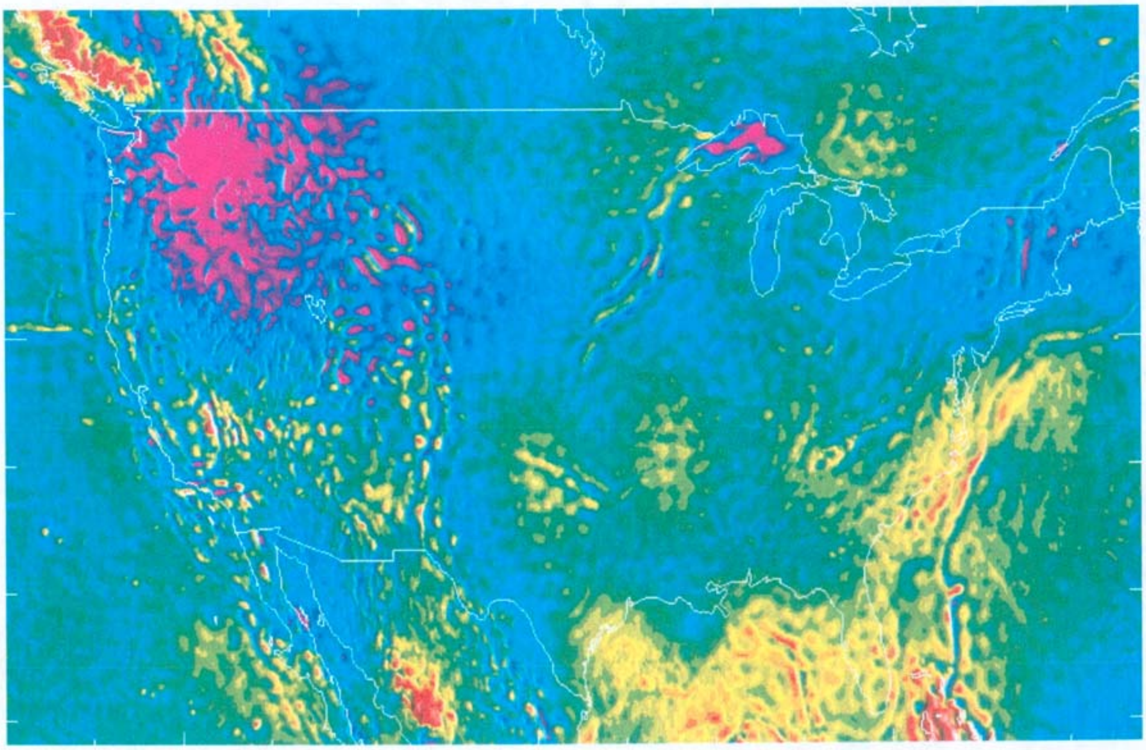

Figure 4: Geoid Undulation Residuals, G96SSS vs. EGM96

Table 2 shows that EGM96, as mentioned earlier, is something of an outlier relative to the five beta models (although, to be succinct we did not repeat the results of X01-X04). While its 2nd and 3rd differences have similarities to EGM-X05, the differences in offsets at various elevation groups show EGM96 to lie outside the three families (X01, X02 and X03-X05) identified in the beta model report. Table 2 corresponds with Table 5 in the beta model report. Results in Table 2, as well as the information contained in Figure 4, show that EGM96 also contains the elevation dependent commission error evident in the earlier beta models. Additional investigations with subsets of the GPS benchmark residuals (not reproduced here) show the elevation dependence is not due to GPS ellipsoid height scatter above 42 degrees latitude (discussed in the next section).

X05 Model	**Cohort**	**n**	**RMS** (cm)	**Offset** (cm.)	**Diff**	**2nd-Diff**	**3rd-Diff**
	0- 500	138	49.27	71.02			
	500-1000	97	31.76	69.20			
	1000-1500	93	28.49	42.93	-26.27		
	1500-2000	91	32.98	34.90	-8.03	18.24	
	2000-2500	71	29.93	18.54	-16.36	-8.33	-26.57
	Over 2500	15	35.05	-12.43			
	All	505	43.14	49.13			
EGM96 Model	**Cohort**	**n**	**RMS** (cm)	**Offset** (cm.)	**Diff**	**2nd-Diff**	**3rd-Diff**
	0- 500	138	45.68	68.07			
	500-1000	97	31.45	66.24			
	1000-1500	93	29.16	44.42	-21.83		
	1500-2000	91	33.49	39.00	-5.42	16.41	
	2000-2500	71	28.69	25.18	-13.82	-8.41	-24.81
	Over 2500	15	38.22	-3.46			
	All	505	40.21	49.97			

Table 2. Geoid residual statistics ordered by 500 m elevation cohorts.

High Resolution Geoid Computation and Evaluation

The details surrounding the computation of high resolution geoid models are found in the beta model report. Using the identical procedures and data sets as the beta models, a new high resolution geoid model for the United States was produced, which we call model 9696 (Please note that this is *not* the same as G96SSS, as the methods and data sets used in the beta model report are somewhat inconsistent with the final data and procedures of G96SSS). Model 9696 is consistent with models 9620 - 9624 (see beta report), which are computed using EGM-X01 - EGM-X05 respectively.

In a way similar to the low resolution model test, a tilted plane was fit to the high resolution models, to find how additional gravity data can be used to remove commission error in the low resolution models. The results were encouraging, showing that most of the tilt which appears in the low resolution models is removable by computing a high resolution model. The statistics of the fitted planes are found in Table 3 below, which corresponds to Table 2 of the beta report.

Model	**Offset** (cm.)	**Tilt** (ppm)	**RMS about plane** (cm)	**Azimuth** (deg)
9620	-4.58	0.04	35.07	293
9621	-5.90	0.06	36.14	317
9622	-2.07	0.05	36.14	319
9623	-2.05	0.06	36.56	321
9624	-2.05	0.05	36.52	321
9696	**-1.99**	**0.02**	**34.63**	**304**

Table 3. Tilted plane fits to 2497 high resolution geoid model residuals .

Table 3 shows an interesting, and somewhat surprising result. Considering the larger magnitude of the tilt of EGM96, compared with the beta models, it is surprising that the final impact on the high resolution geoid undulation grid would be to show a slight improvement. As will be mentioned in a later section, the ability to remove commission error through high-resolution geoid computation does not include the ability to remove any form of very long wavelength (i.e. tilt) errors in the models. As such, we are pleased for the reduction in the magnitude of tilt of a high resolution geoid, due to the use of EGM96.

Care should be taken, however, in interpreting the small magnitude of the tilt in the high resolution models. Scatter plots of the residual geoid undulations show distinct problems in certain areas, which happen to have the property of canceling one another in their contribution to a continental tilt. These scatter plots are presented in the next 3 figures. Figure 5 shows the geoid residuals relative to orthometric height. As we expect, there is significantly less scatter than in Figure 1, and a dependence of geoid undulation residual upon height does not appear to be present. Figure 6 shows the scatter relative to latitude, and Figure 7 shows it relative to longitude. In all three figures, certain characteristic problems are noted. First, a low-elevation, south and east cluster of negative residuals is part of a difficulty in modeling the Florida geoid (a problem that has subsequently been corrected for G96SSS). Also, a mid/high elevation, north and west spread of residuals represents the difficulties in modeling the Pacific northwest (again, some of this spread has tightened up with G96SSS). These features happen to have canceling effects upon the tilt of the high-resolution model (i.e. a least-squares adjustment fits a flat line through Figure 6, even though the general pattern is curved).

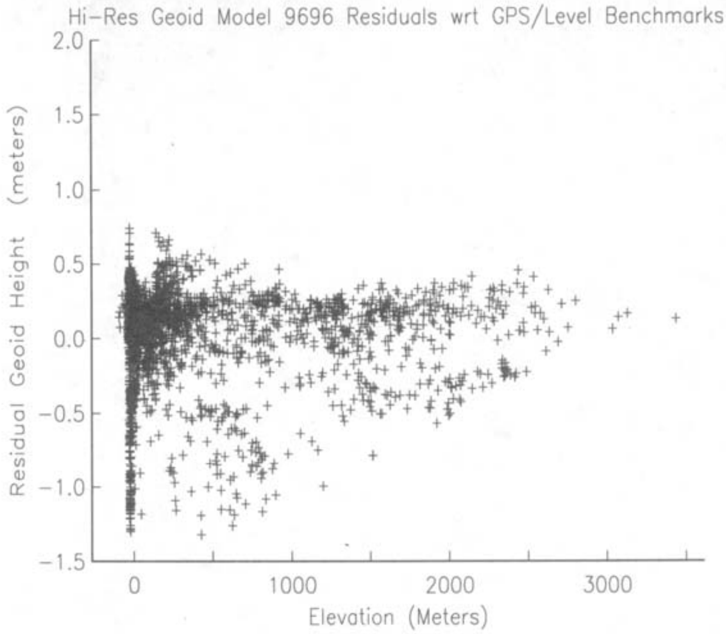

Figure 5: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to elevation

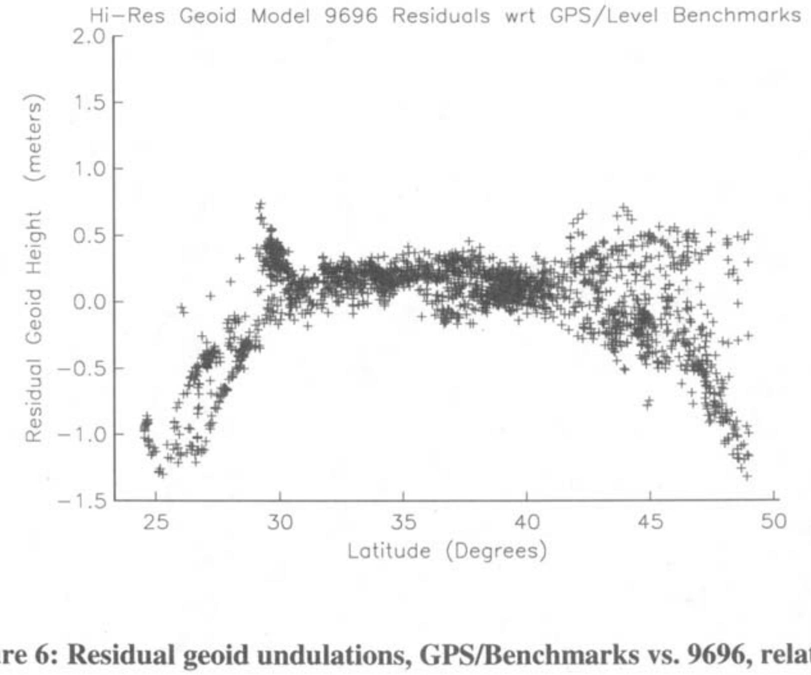

Figure 6: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to latitude

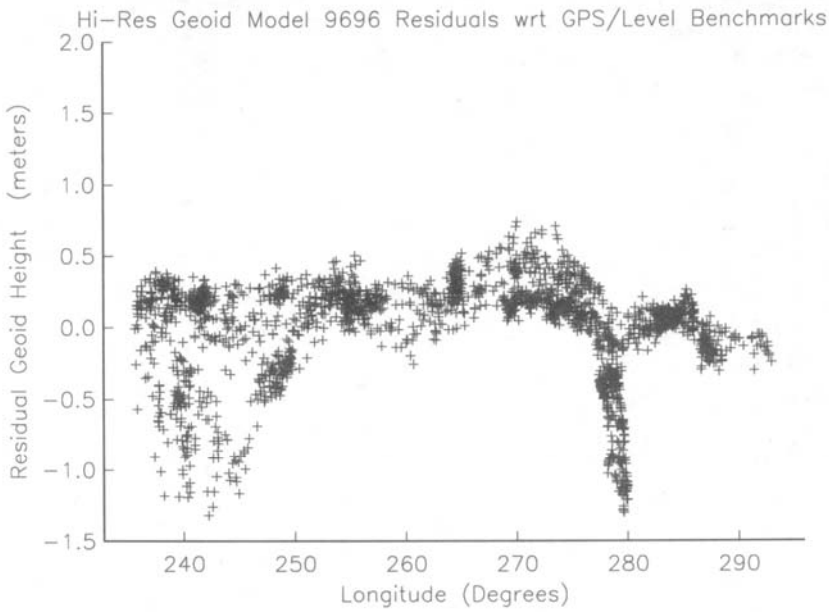

Figure 7: Residual geoid undulations, GPS/Benchmarks vs. 9696, relative to longitude

To better get an understanding of any potential elevation dependence that may not be immediately obvious in Figure 5, an "elevation cohort" test, identical with that of the beta model report, was conducted. In this test, geoid undulations residuals were grouped into elevation groups, where benchmarks near Florida (south of 30°) and Washington State (north of 48°) are withheld from the analysis. The results are presented in Table 4 below. Table 4 corresponds with Table 3 of the beta model report.

9624 Model	**Cutoff**(m)	**n**	**RMS**(cm)	**Offset**(cm.)	**Diff**	**2nd-Diff**
(X05)	0	2190	24.73	9.84		
	1000	448	23.83	6.97		
	1500	268	24.80	6.88	-0.09	
	2000	107	24.50	7.43	0.55	0.64
9696 Model	**Cutoff**(m)	**n**	**RMS**(cm)	**Offset**(cm.)	**Diff**	**2nd-Diff**
(EGM96)	0	2190	25.15	8.52		
	1000	448	24.52	3.22		
	1500	268	25.43	3.16	-0.06	
	2000	107	25.17	3.58	0.42	0.36

Table 4. Geoid residual statistics ordered by elevation cutoff tolerance.

Table 4 enables us to discern more about the potential for elevation dependence in the geoid undulation residuals for the high resolution models. Previously, model 9624 (based on

EGM-X05) showed the most uniform average offsets (as seen in the magnitudes of 1st and 2nd differences). However, model 9696 (based on EGM96) shows even greater stability in the average offsets. On the other hand, a larger RMS and greater discrepancies between average offset of all GPS benchmarks and those above 1000 m shows a greater dependence on elevation, and thus could be considered an artifact of a tilt in the model.

Comparison of EGM96 with EGM-X05

Prior to the release of EGM96, our preference amongst the beta models was EGM-X05. This was the model used in the final preparations and tests for G96SSS/GEOID96, in anticipation of the final EGM96 release. As such, it was interesting to compare these two models with one another, and see what was the final effect of using EGM96 rather than EGM-X05 in high resolution geoid calculations. Geoid undulations were evaluated directly on the geoid (rather than using an up/down procedure) for both EGM-X05 and EGM96, with the differences (EGM96 minus EGM-X05) shown in Figure 8, covering the range of 24-53 degrees North in latitude, and 230-294 degrees East in longitude. This image, done in color to aid in clarity, shows large bowl shaped discrepancies, with a peak-to-peak magnitude of 95 cm (magenta values are -49 cm, red values are +46 cm). Reports from the EGM96 team indicate that the most significant change between EGM-X05 and EGM96 occurred in the satellite (i.e. degree 2 to 70) solution. This agrees with the most pronounced undulation differences, which seem to have a characteristic wavelength of about 8-10 degrees (around spherical harmonic degree 40).

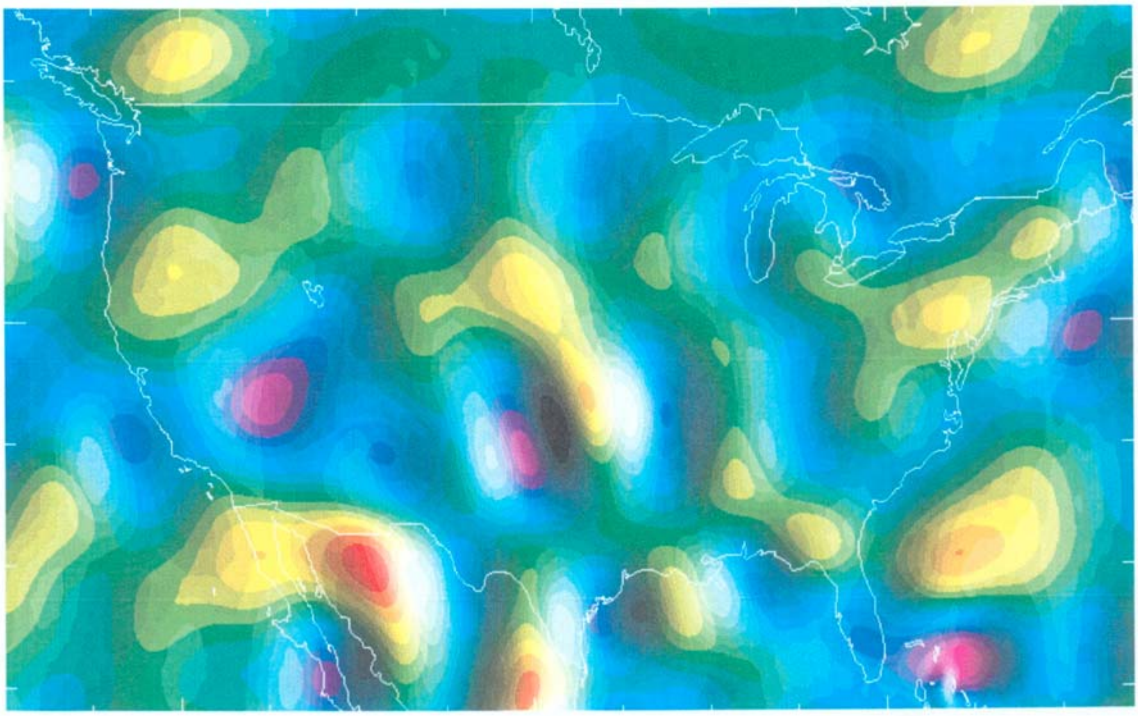

Figure 8: Geoid undulation differences, EGM96 minus EGM-X05

To confirm our visual findings, radially averaged power was computed for the region 25°-50° N, 258°-283° E on the differences seen in Figure 8. A plot of the power against the frequency is shown in Figure 9 on a log scale, and in Figure 10 on a log-log scale. The spike at the left of Figure 9 is indicative of additional power at a specific frequency band. To help isolate the frequency band, Figure 10 is plotted on a log-log scale. In this case, any deviation from a linear graph indicates a signal that is specific to one particular scale. In this case, we see the spike of Figure 9 translates into a large bump, deviating from the linear trend, around 0.1 - 0.2 cycles/degree, or a wavelength of about 5-10 degrees. This agrees nicely with the visual estimate obtained from Figure 8. A signal of this wavelength corresponds roughly to spherical harmonic degrees 30-50, which is within the spectral range of the satellite only, and combination solutions of EGM96 (to degree 70).

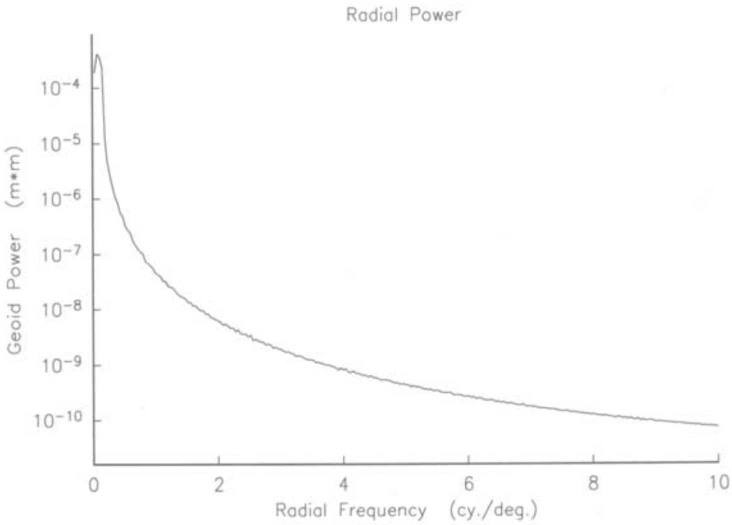

Figure 9: Radial power of EGM96 minus EGM-X05 undulation differences

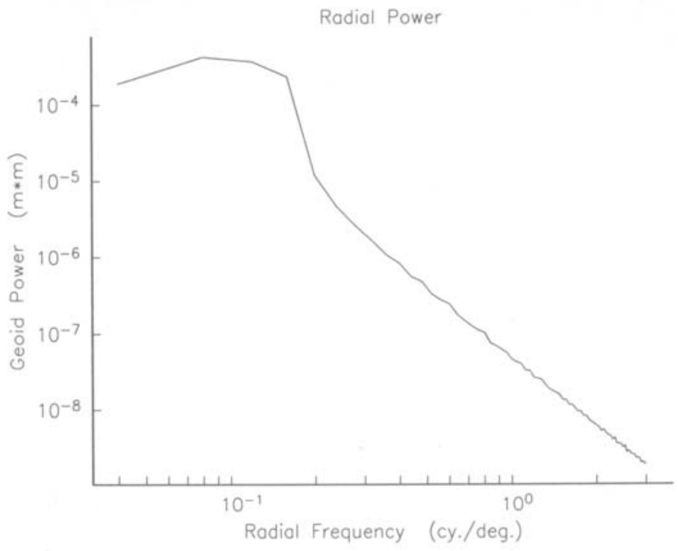

Figure 10: Radial power of EGM96 minus EGM-X05 undulation differences

In an effort to determine whether these large undulating differences were corrections, we investigated one of the largest differences between EGM-X05 and EGM96: the 1 ppm tilt across Oklahoma. As seen in Figure 8, this tilt connects a low (blue) in Texas to a high (red) in Kansas. Figure 11 shows a contour plot of the tilt, and the position of 16 GPS/level benchmarks in the area (indicated by "+"). Two tests against "ground truth" were made. In the first test, surface gravity data were gridded (see Smith and Milbert, 1996) and the anomalies implied by both EGM-X05 and EGM96 were removed. These two 2' x 2' residual gravity anomaly grids were run through the 1-D spherical FFT form of Stokes' integral to generate two 2' x 2' residual geoid undulation grids for the United States (with 8056 points in the study area). The size of the residual undulations is indicative of the fit between the surface gravity data and the two models (a comparison of the residual free-air anomalies could also be made, but we were interested in the fit of undulations at this point). Table 5 shows the results of this comparison.

Model name	Average residual undulation	RMS about the average
EGM-X05	13.1 cm	12.9 cm
EGM96	18.5 cm	16.6 cm

Table 5. Residual geoid undulations in Oklahoma

Table 5 shows a clear indication that for this area, the 1 ppm slope induced by changing from EGM-X05 to EGM96 is introducing a disagreement with the geoid implied by surface gravity. To further investigate this finding, a second test was made. In the second test, the geoid undulations implied by the 16 GPS/level benchmarks in Oklahoma were compared with the undulations computed solely from the n=360 models. The orthometric heights were originally

NAVD 88 values, but were corrected for a 43.4 cm bias in NAVD 88. As in the "low resolution geoid" section of this report , the undulations implied by the two models were corrected from surface height anomalies to geoid undulations. The undulations from the two models were subtracted from those implied by the GPS/level benchmarks, and the results of this comparison are shown in Table 6 below.

Model Name	Average difference	RMS about the average
EGM-X05	-9.1 cm	13.3 cm
EGM96	-17.5 cm	14.9 cm

Table 6. Residual geoid undulations on GPS/level benchmarks in Oklahoma

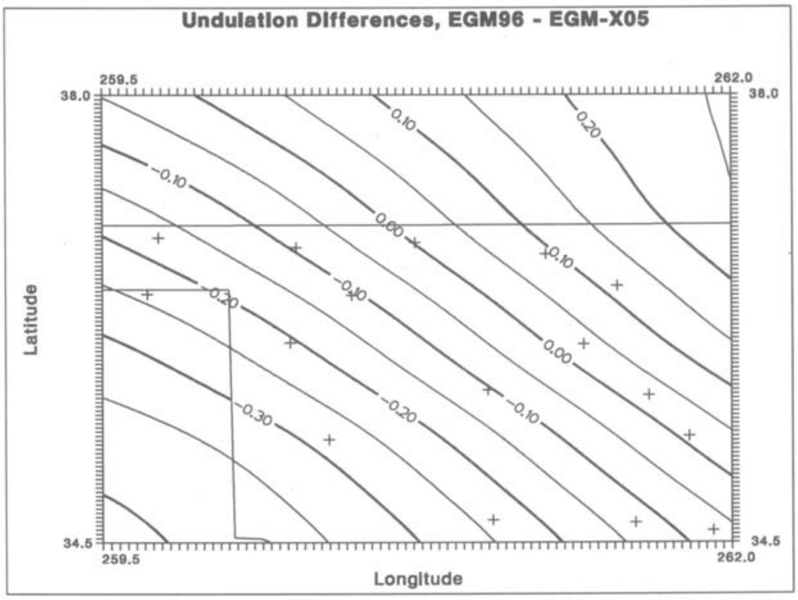

Figure 11: Undulation differences in Oklahoma, EGM96 minus EGM-X05, with GPS Benchmarks

Table 6 confirms the findings of Table 5, that the features introduced in Oklahoma by EGM96 are causing larger disagreements with surface data. It must be noted, however, that due to the semi-periodic nature of the differences between EGM96 and EGM-X05, a number of such slopes are induced across the United States, but time prevented us from studying them further. Oklahoma was chosen as an area with good gravity and GPS on benchmarks coverage, as well as coinciding with one of the larger differences between the models. Further studies should be done to determine the magnitude of the other differences in their respective regions.

As previously mentioned, much of the commission error in the spherical harmonic models can be removed by the use of the remove-compute-restore procedure in the creation of a high resolution model. However, any commission error with a wavelength longer then 29 degrees (i.e. spherical harmonic degrees 0 through 6 or 7) will not be repairable, due to the limited size (29 degrees north/south) of the grid which covers the United States. To exemplify this point, two high resolution geoids were produced using our most recent data sets, and procedures: one geoid based on EGM-X05, and one based on EGM96 (this second geoid is what we've termed G96SSS). The differences between the two geoids has been calculated, and a plot (minus a few degrees along the edges to reduce the visual impact of edge effects) of the differences is shown in Figure 12. Aside from the edge effects, the difference between the geoids is seen to be a very long wavelength structure across the United States. There is no indication of the large bowl-shaped differences seen in Figure 8. Thus, it seems clear that the structure of the geoid is not significantly dependent upon the shorter (29 degrees or smaller) wavelengths of the spherical harmonic model, but is rather controlled by the gravity data themselves. Any long wavelength (29 degrees or greater) errors, however, that may be in the model are not removable using this procedure.

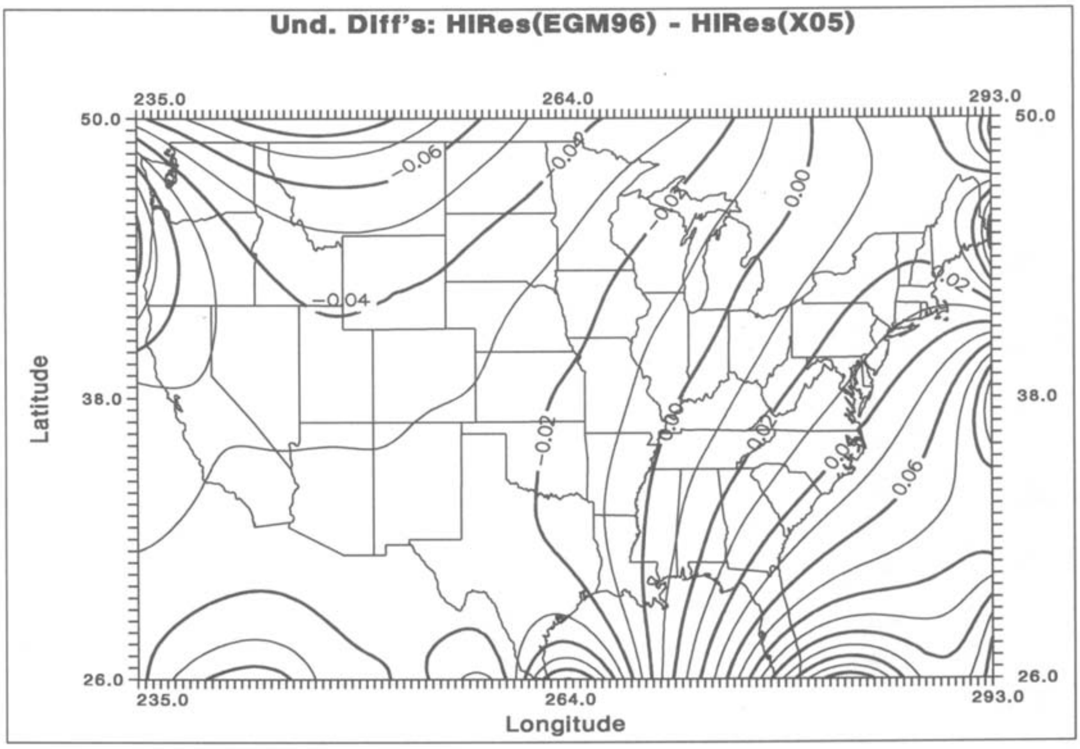

Figure 12: High-resolution undulation differences, CI = 1 cm

Conclusions

Upon first inspecting the differences between EGM96 and EGM-X05, it was surprising to see how large, and especially how semi-periodic, the differences were. These differences were primarily seen as bowl-shaped features approximately 1000 km wide, superimposed over a slight tilt. Overall, however, no significant change in the continental GPS/BM residual statistics occurred with EGM96, aside from the larger tilt. A regional study shows that one of the larger differences between EGM96 and EGM-X05 causes disagreement with surface data, but these regional features are removed in the remove-compute-restore procedures used in high-resolution geoid modeling. The importance of EGM96 became evident when it was used in the creation of model 9696, a high resolution model that was less tilted, and slightly more consistent at high elevations, relative to GPS benchmarks. This property, coupled with the known global improvements of EGM96, make us confident that EGM96 will serve as a firm foundation for the G96SSS and GEOID96 high-resolution geoid height models.

Acknowledgments

We would like to extend our sincere thanks and congratulations to all persons involved in the production and dissemination of EGM96. In addition, the coordinating efforts of Michael Sideris allowed an outstanding availability of global results from all members of the SWG. The efforts of numerous NGS employees in the creation and evaluation of the gravity, NAVD88 and GPS data were essential to this effort. The National Imagery and Mapping Agency (NIMA, formerly DMA) provided a major portion of the NGS land gravity data, and was instrumental in the creation of various 3" and 30" digital elevation data grids in use. Dr. Walter Smith, NOAA, provided the altimeter-derived gravity anomalies used in the high-resolution geoid computations.

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